Computer Algebra Independent Integration Tests

Summer 2024

4-Trig-functions/4.3-Tangent/215-4.3.1.2

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**709**]. This is test number [215].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (709)	0.00 (0)
Mathematica	100.00 (709)	0.00 (0)
Maple	83.22 (590)	16.78 (119)
Fricas	81.95 (581)	18.05 (128)
Maxima	58.11 (412)	41.89 (297)
Mupad	53.17 (377)	46.83 (332)
Giac	37.38 (265)	62.62 (444)
Reduce	26.94 (191)	73.06 (518)
Sympy	17.91 (127)	82.09 (582)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

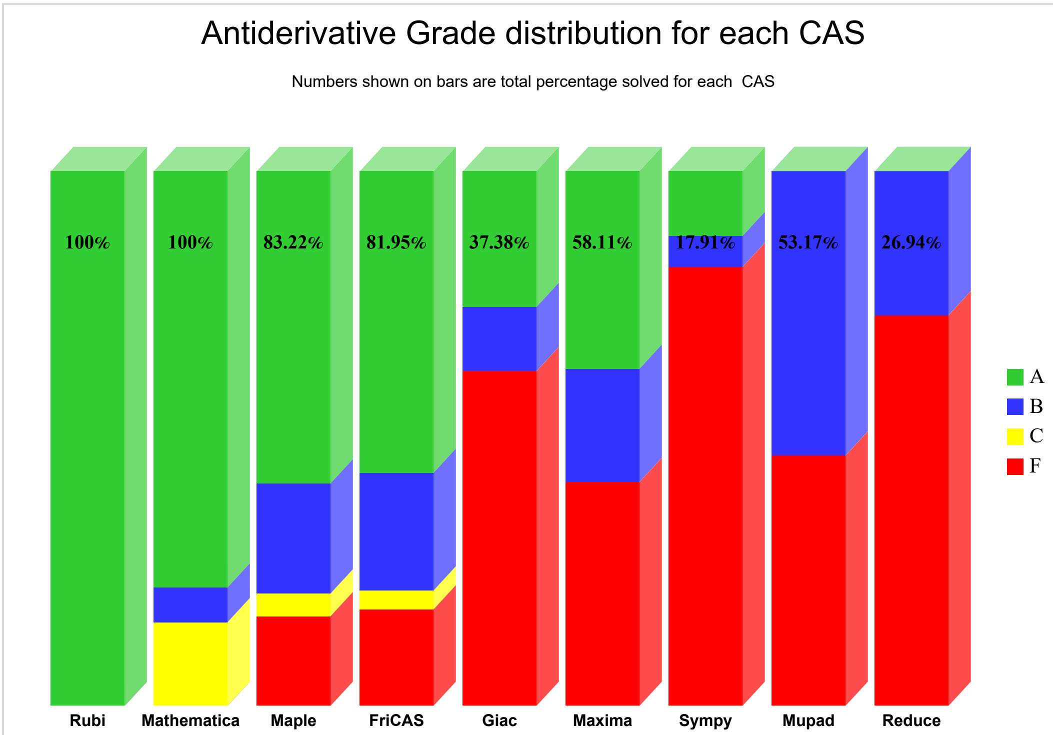
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

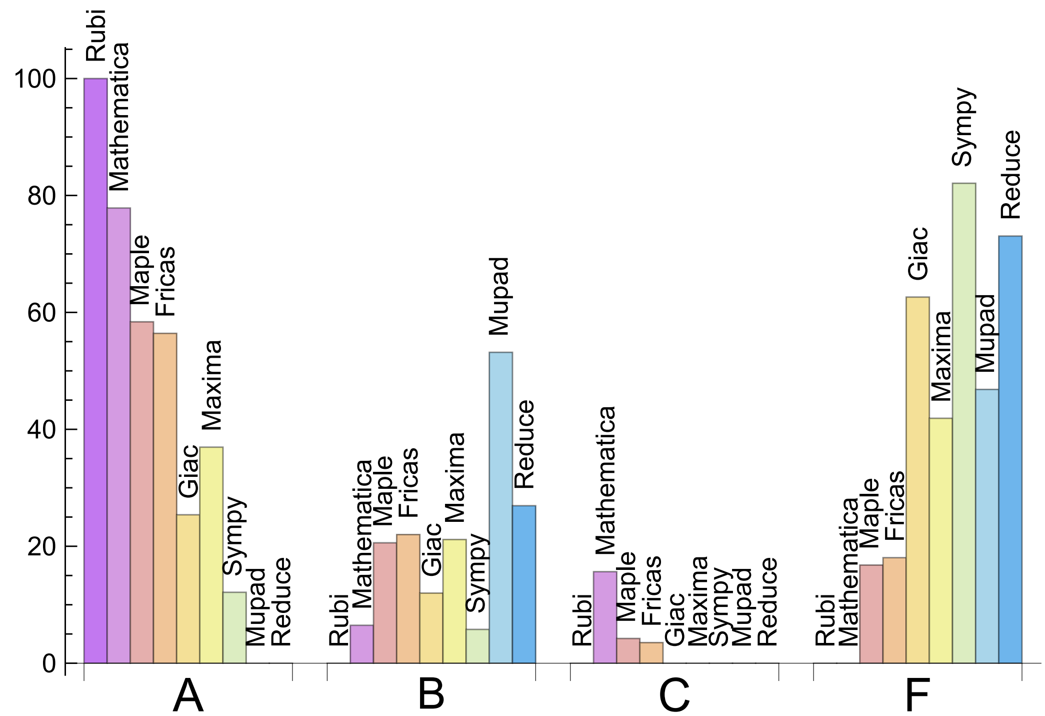
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	77.856	6.488	15.656	0.000
Maple	58.392	20.592	4.231	16.784
Fricas	56.417	22.003	3.526	18.054
Maxima	36.953	21.157	0.000	41.890
Giac	25.388	11.989	0.000	62.623
Sympy	12.130	5.783	0.000	82.087
Mupad	0.000	53.173	0.000	46.827
Reduce	0.000	26.939	0.000	73.061

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	119	100.00	0.00	0.00
Fricas	128	78.91	19.53	1.56
Maxima	297	61.28	6.06	32.66
Mupad	332	0.00	100.00	0.00
Giac	444	55.63	0.90	43.47
Reduce	518	100.00	0.00	0.00
Sympy	582	72.68	26.98	0.34

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.09
Reduce	0.18
Maxima	0.20
Rubi	0.50
Sympy	1.15
Giac	1.79
Mupad	2.06
Mathematica	2.97
Maple	25.36

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	138.83	1.01	114.00	1.00
Fricas	158.07	1.37	118.00	1.16
Mupad	168.21	1.71	111.00	1.36
Sympy	208.80	2.45	184.00	1.52
Mathematica	348.26	1.49	103.00	0.92
Maxima	364.70	2.43	130.00	1.22
Reduce	581.19	4.64	153.00	1.79
Maple	739.28	2.59	118.00	1.04
Giac	1743.81	18.38	118.00	1.23

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

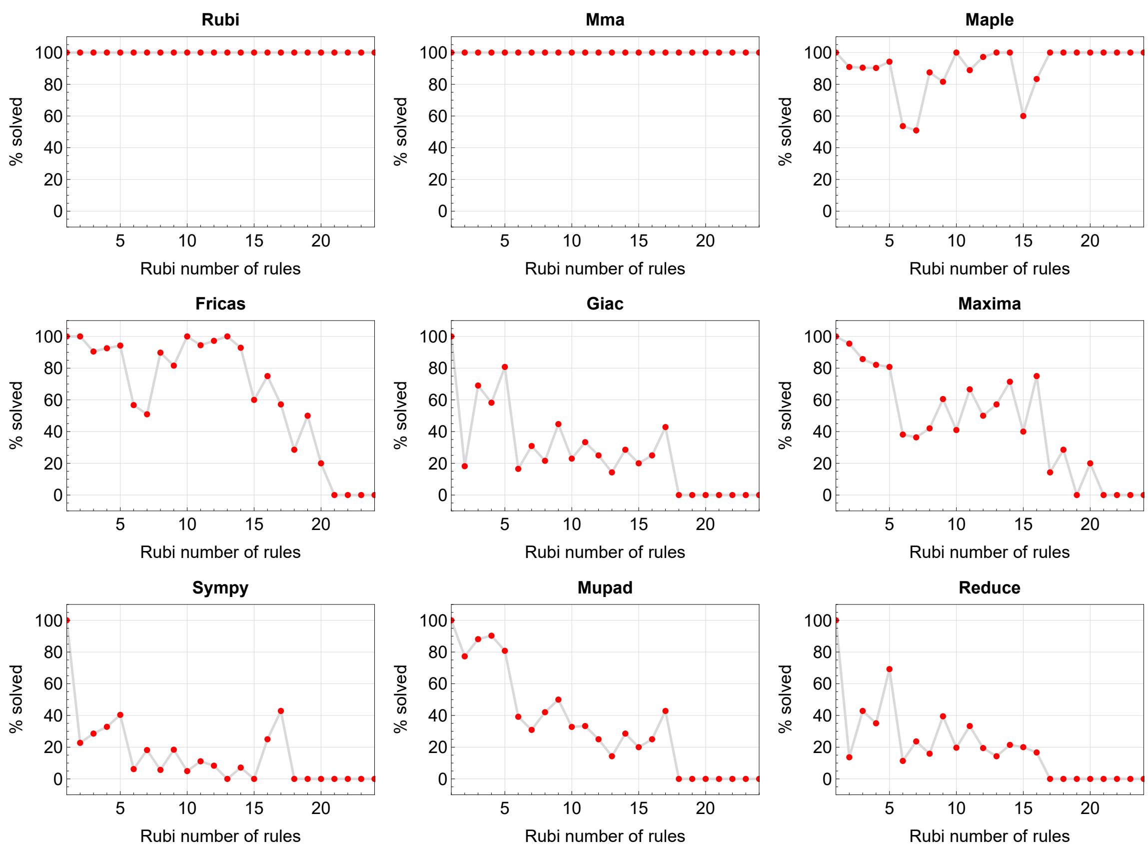


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

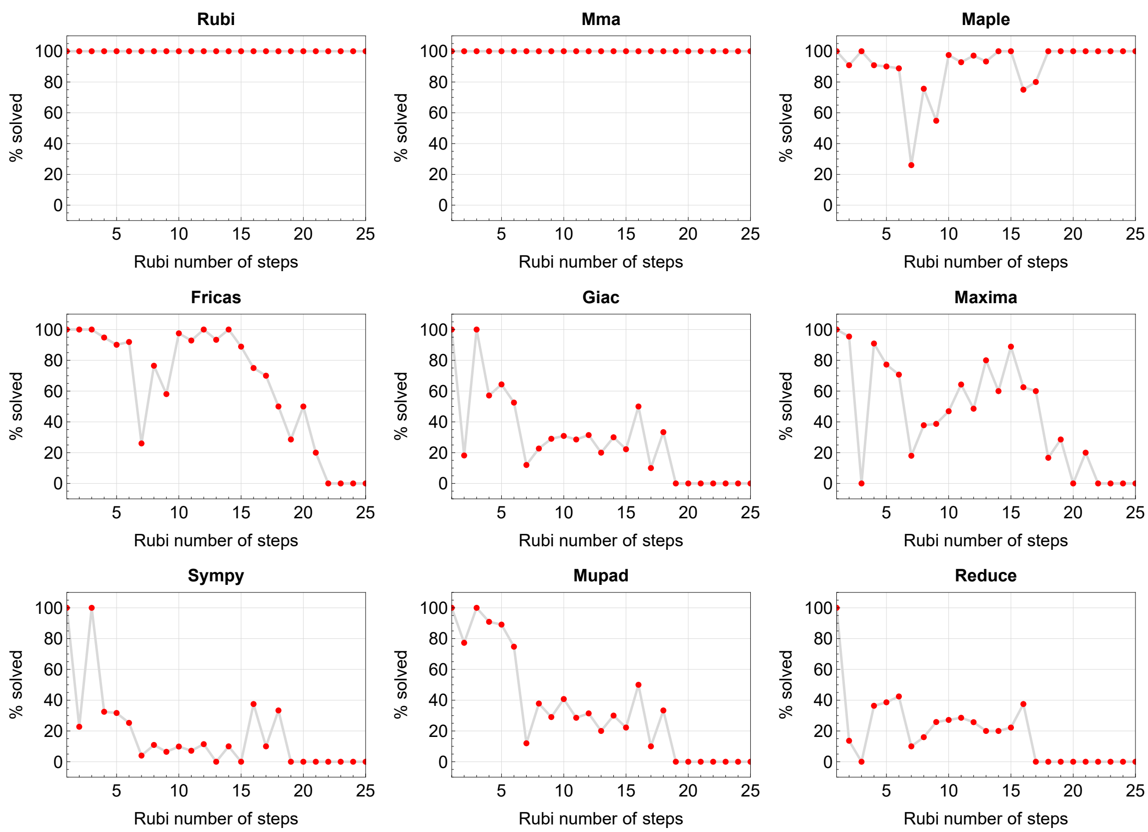


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

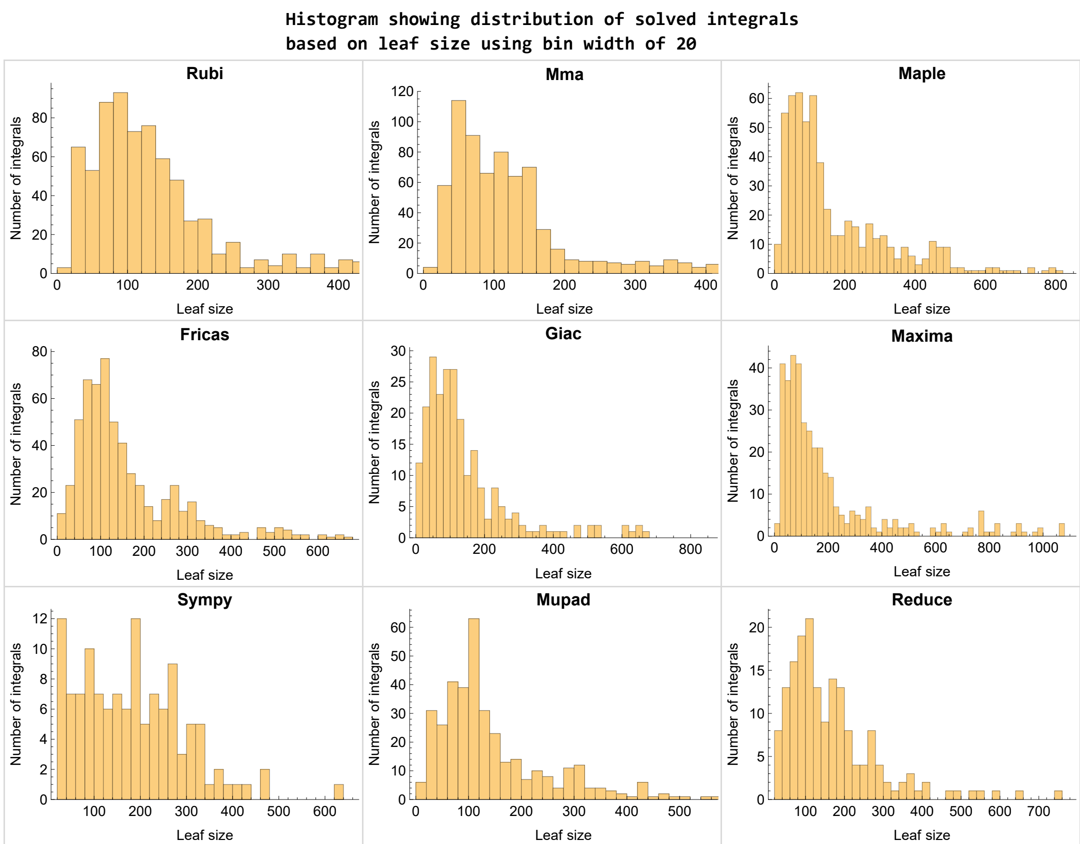


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

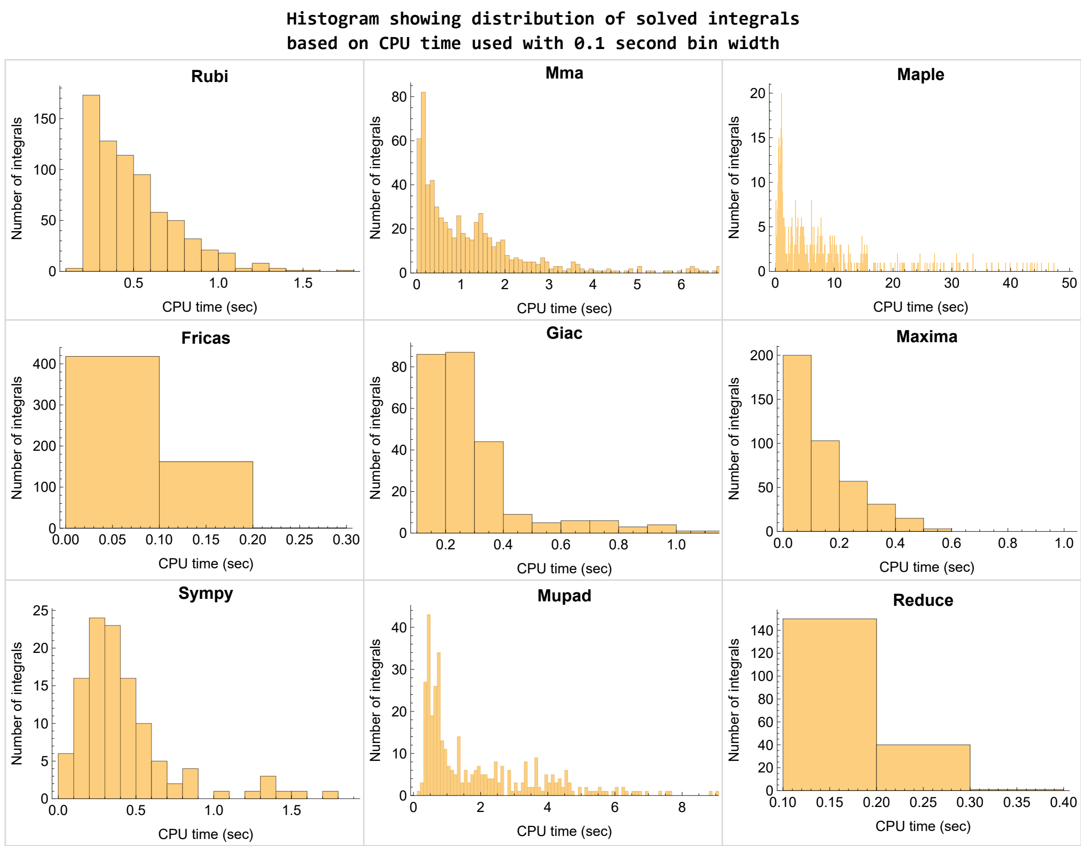


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

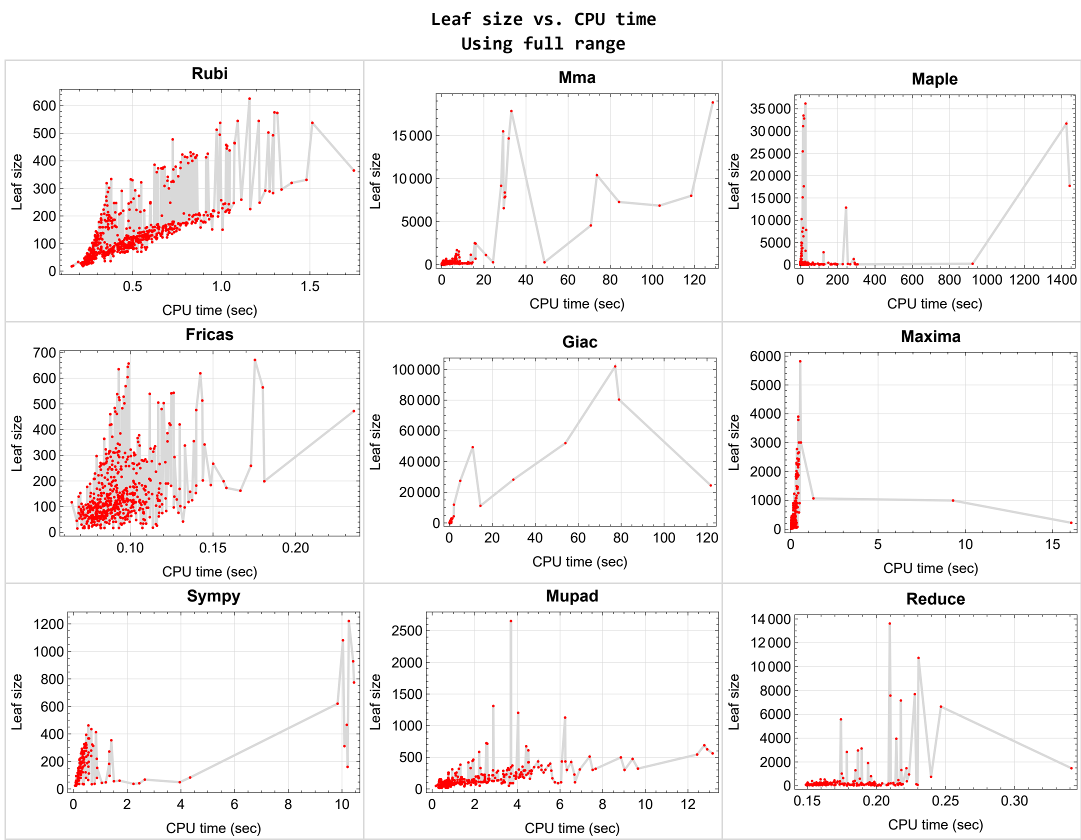


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {283, 284, 285, 297, 298, 299, 310, 311, 312, 323, 324, 325, 337, 338, 339, 351, 352, 353, 367, 368, 383, 384, 443, 444, 445, 568, 569, 570, 571, 572, 573, 580, 581, 582, 583, 584, 585, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 640, 641, 642, 643, 651, 707}

Mathematica {92, 93, 159, 176, 177, 395, 400, 401, 402, 403, 408, 409, 410, 411, 416, 423, 424, 430, 431, 462, 463, 471, 472, 475, 476, 499, 568, 611, 613, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 640, 641, 642, 643, 644, 645, 646, 647, 651, 652, 653, 660, 661, 662, 663, 665, 667, 668, 669, 671, 673, 675, 679, 681, 700, 703, 707, 708, 709}

Maple {23, 40, 63, 81, 285, 297, 298, 299, 304, 305, 310, 311, 312, 317, 318, 323, 324, 325, 330, 331, 332, 351, 359, 360, 368, 373, 375, 376, 377, 383, 384, 388, 389, 390, 391, 392, 393, 403, 410, 411, 417, 483, 484, 485, 486, 504, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 694}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
  if x is None:
     return 1
  else:
     return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

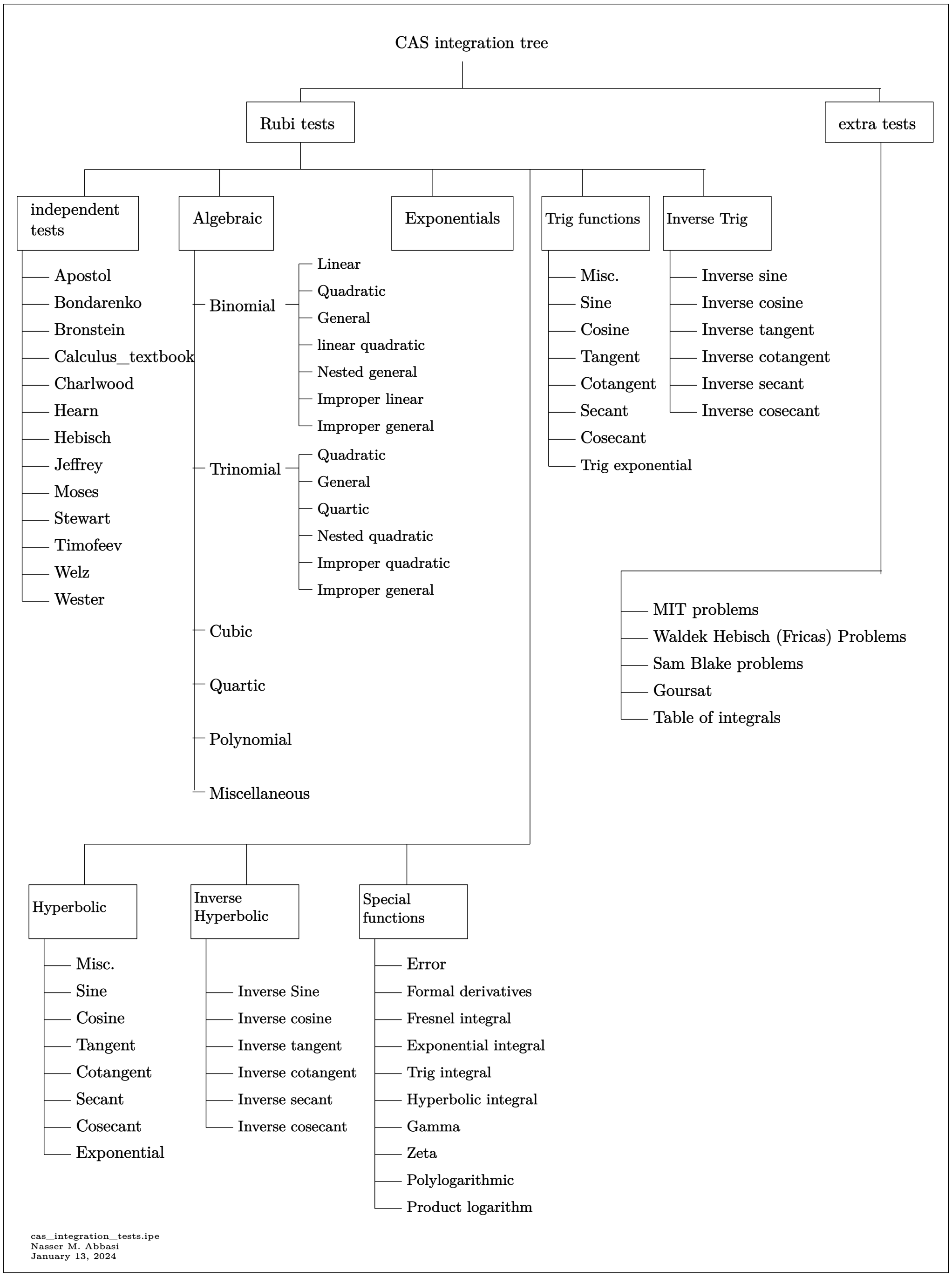
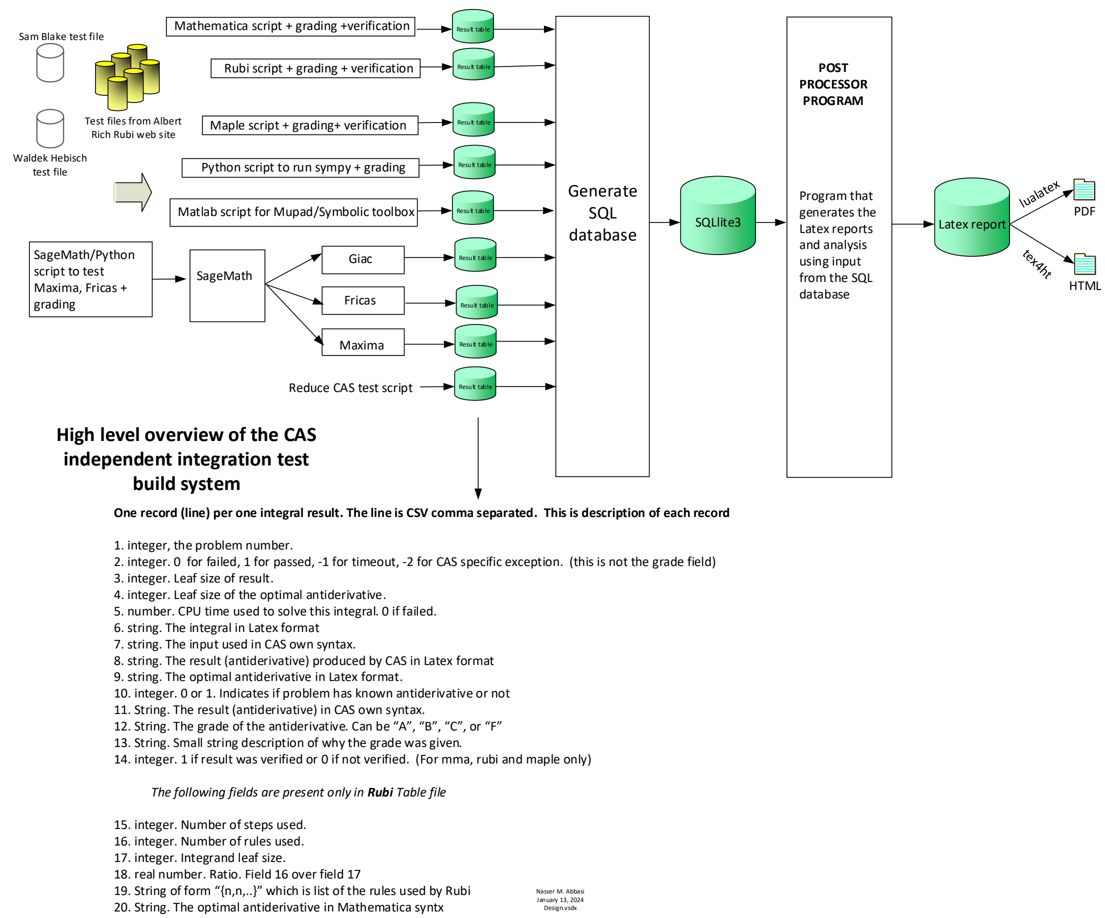


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 124, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 178, 179, 180, 181, 182, 183, 184, 186, 188, 190, 192, 194, 196, 198, 200, 202, 204, 206, 208, 210, 212, 214, 216, 218, 220, 222, 224, 226, 228, 230, 232, 234, 236, 238, 240, 242, 244, 246, 248, 250, 252, 254, 256, 258, 260, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 326, 327, 328, 330, 331, 332, 333, 334, 335, 336, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 498, 500, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 526, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 547, 548, 549, 550, 551, 552, 553, 554, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 570, 571, 572, 573, 574, 575, 576, 577, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 614, 632, 633, 634, 635, 636, 637, 638, 639, 648, 649, 650, 654, 655, 656, 658, 659, 664, 666, 670, 672, 674, 676, 678, 680, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 701, 702, 703, 704, 705, 706 }

B grade { 31, 47, 54, 55, 56, 62, 80, 88, 92, 93, 94, 95, 122, 123, 125, 150, 159, 160, 176, 177, 278, 329, 450, 497, 499, 501, 502, 503, 524, 527, 539, 541, 542, 543, 544, 545, 546, 557, 578, 579, 581, 611, 643, 645, 647, 700 }

C grade { 185, 187, 189, 191, 193, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225, 227, 229, 231, 233, 235, 237, 239, 241, 243, 245, 247, 249, 251, 253, 255, 257, 259, 261, 283, 284, 285, 298, 299, 311, 312, 323, 324, 325, 337, 338, 339, 351, 352, 353, 367, 368, 383, 384, 525, 540, 555, 568, 569, 580, 582, 612, 613, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 640, 641, 642, 644, 646, 651, 652, 653, 657, 660, 661, 662, 663, 665, 667, 668, 669, 671, 673, 675, 677, 679, 681, 707, 708, 709 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 63, 64, 65, 66, 69, 70, 71, 72, 73, 74, 81, 82, 83, 91, 92, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 188, 190, 192, 193, 194, 196, 198, 200, 201, 202, 203, 204, 206, 208, 210, 212, 213, 214, 216, 218, 220, 222, 223, 224, 226, 230, 232, 233, 234, 235, 236, 240, 242, 244, 245, 246, 247, 248, 250, 252, 254, 255, 256, 258, 260, 262, 279, 280, 281, 282, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 300, 301, 302, 303, 306, 307, 308, 309, 313, 314, 315, 316, 319, 320, 321, 322, 326, 327, 328, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 347, 348, 349, 350, 353, 354, 355, 356, 357, 362, 363, 364, 365, 366, 369, 370, 371, 378, 379, 380, 381, 382, 385, 386, 388, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 467, 487, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 583, 584, 585, 656, 667, 668, 679, 680, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696 }

B grade { 39, 59, 62, 67, 68, 75, 76, 77, 78, 79, 80, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 98, 109, 185, 187, 189, 191, 195, 197, 199, 205, 207, 209, 211, 215, 217, 219, 221, 225, 227, 228, 229, 231, 237, 238, 239, 241, 243, 249, 251, 253, 257, 259, 261, 283, 284, 290, 291, 297, 298, 299, 304, 305, 310, 311, 312, 317, 318, 323, 324, 325, 329, 330, 331, 332, 337, 344, 345, 346, 351, 352, 358, 359, 360, 361, 367, 368, 372, 373, 374, 375, 376, 377, 383, 384, 387, 389, 390, 391, 392, 393, 465, 466, 524, 539, 557, 582, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 654, 655, 664, 665, 666, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 681 }

C grade { 483, 484, 485, 486, 504, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610 }

F normal fail { 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 505, 506, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 657, 658, 659, 660, 661, 662, 663, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 6, 7, 8, 9, 10, 15, 16, 17, 18, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 40, 41, 42, 43, 44, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 63, 64, 65, 66, 68, 69, 71, 72, 73, 74, 75, 76, 81, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 110, 111, 112, 113, 117, 118, 119, 120, 121, 125, 126, 127, 128, 129, 135, 136, 137, 138, 139, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 279, 280, 281, 284, 285, 286, 287, 288, 289, 292, 293, 299, 300, 301, 302, 303, 305, 312, 313, 314, 315, 318, 325, 326, 327, 328, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 346, 347, 348, 349, 352, 353, 354, 355, 356, 361, 362, 363, 364, 365, 368, 369, 370, 376, 377, 378, 379, 380, 381, 383, 384, 392, 393, 394, 395, 397, 398, 399, 401, 403, 405, 406, 407, 408, 410, 413, 414, 415, 417, 419, 420, 421, 422, 423, 426, 427, 428, 429, 431, 433, 434, 435, 436, 443, 444, 445, 446, 447, 448, 449, 483, 484, 485, 486, 491, 493, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 518, 519, 520, 521, 522, 523, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 555, 556, 557, 561, 566, 567, 568, 572, 573, 655, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 694, 695, 696 }

B grade { 1, 2, 3, 4, 5, 11, 12, 13, 14, 19, 20, 21, 22, 28, 29, 30, 36, 37, 38, 39, 45, 46, 52, 53, 59, 60, 61, 62, 67, 70, 77, 78, 79, 80, 82, 88, 107, 108, 109, 114, 115, 116, 122, 123, 124, 130, 131, 132, 133, 134, 140, 141, 148, 149, 150, 166, 172, 282, 283, 290, 291, 294, 295, 296, 297, 298, 304, 306, 307, 308, 309, 310, 311, 316, 317, 319, 320, 321, 322, 323, 324, 329, 337, 344, 345, 350, 351, 357, 358, 359, 360, 366, 367, 371, 372, 373, 374, 375, 382, 385, 386, 387, 388, 389, 390, 391, 396, 400, 402, 404, 409, 411, 412, 416, 418, 424, 425, 430, 432, 465, 466, 467, 487, 495, 504, 516, 524, 539, 553, 554, 558, 559, 560, 562, 563, 564, 565, 569, 570, 571, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 654, 656, 670, 693 }

C grade { 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610 }

F normal fail { 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 437, 438, 439, 440, 441, 442, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 488, 489, 490, 492, 494, 496, 497, 498, 499, 500, 501, 502, 503, 612, 618, 632, 633, 634, 635, 636, 637, 638, 639, 648, 649, 650, 651, 652, 653, 657, 658, 659, 660, 661, 662, 663, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709 }

F(-1) timedout fail { 611, 614, 615, 616, 617, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 640, 641, 642, 643, 644, 645, 646, 647 }

F(-2) exception fail { 613, 619 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 62, 63, 64, 65, 68, 69, 70, 71, 72, 75, 76, 80, 81, 82, 83, 84, 89, 90, 99, 100, 101, 102, 103, 110, 114, 115, 116, 117, 118, 126, 130, 131, 132, 134, 135, 136, 143, 144, 148, 149, 151, 152, 154, 160, 161, 162, 166, 167, 168, 172, 178, 179, 180, 181, 182, 279, 280, 281, 282, 283, 284, 285, 293, 294, 295, 296, 297, 298, 299, 306, 307, 308, 309, 310, 311, 312, 319, 320, 321, 322, 323, 324, 325, 335, 336, 337, 338, 339, 347, 348, 349, 350, 351, 352, 353, 362, 363, 364, 365, 366, 367, 368, 378, 379, 380, 381, 382, 383, 384, 397, 398, 399, 405, 406, 407, 413, 414, 415, 419, 420, 421, 422, 426, 427, 428, 429, 433, 434, 435, 436, 467, 483, 484, 485, 486, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 559, 560, 562, 563, 564, 565, 566, 574, 575, 576, 577, 654, 655, 656, 682, 683, 684, 690, 691, 692 }

B grade { 42, 48, 56, 61, 66, 67, 73, 74, 77, 78, 79, 85, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 98, 107, 108, 109, 122, 123, 124, 125, 133, 140, 141, 142, 150, 153, 158, 159, 169, 170, 171, 176, 177, 288, 290, 291, 292, 300, 301, 303, 304, 315, 316, 317, 327, 328, 329, 330, 333, 334, 340, 341, 342, 343, 345, 346, 354, 355, 356, 357, 358, 360, 361, 369, 370, 371, 372, 373, 374, 376, 377, 385, 386, 387, 388, 390, 392, 393, 394, 395, 396, 400, 401, 402, 403, 404, 408, 409, 410, 411, 412, 416, 417, 418, 423, 424, 425, 430, 431, 432, 443, 444, 445, 446, 447, 448, 449, 487, 491, 493, 495, 504, 505, 557, 558, 561, 567, 568, 569, 570, 571, 572, 573, 578, 579, 580, 581, 582, 583, 584, 585, 685, 686, 687, 688, 689, 693, 694, 695, 696 }

C grade { }

F normal fail { 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 263, 264, 265, 266, 267, 268, 269, 270, 289, 302, 314, 326, 344, 359, 375, 391, 437, 438, 439, 440, 441, 442, 450, 451, 452, 453, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 488, 489, 490, 492, 494, 496, 497, 498, 499, 500, 503, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 621, 622, 623, 624, 629, 630, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 645, 646, 647, 648, 649, 650, 651, 652, 653, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 697, 698, 699, 702, 703, 704, 705, 706, 707, 708, 709 }

F(-1) timedout fail { 286, 287, 305, 313, 318, 331, 332, 389, 602, 603, 618, 619, 620, 625, 626, 627, 628, 644 }

F(-2) exception fail { 104, 105, 106, 111, 112, 113, 119, 120, 121, 127, 128, 129, 137, 138, 139, 145, 146, 147, 155, 156, 157, 163, 164, 165, 173, 174, 175, 183, 184, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 271, 272, 273, 274, 275, 276, 277, 278, 454, 455, 456, 501, 502, 506, 617, 631, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 700, 701 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 19, 20, 21, 22, 23, 24, 25, 26, 27, 30, 31, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 52, 53, 59, 60, 63, 64, 65, 66, 67, 68, 69, 70, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 467, 507, 508, 509, 510, 511, 512, 513, 521, 522, 523, 525, 526, 527, 536, 537, 538, 540, 541, 542, 543, 552, 553, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 584, 656 }

B grade { 11, 12, 13, 14, 15, 16, 17, 18, 28, 29, 32, 33, 34, 35, 39, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 61, 62, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 91, 92, 93, 94, 95, 96, 97, 98, 104, 112, 113, 129, 133, 150, 179, 465, 466, 514, 515, 516, 517, 518, 519, 520, 524, 528, 529, 530, 531, 532, 533, 534, 535, 539, 544, 545, 546, 547, 548, 550, 557, 568, 579, 580, 581, 582, 583, 585 }

C grade { }

F normal fail { 185, 186, 187, 189, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 464, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 657, 658, 659, 660, 661, 662, 663, 665, 666, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 697, 698, 699, 700, 701, 704, 705, 706, 707, 708, 709 }

F(-1) timedout fail { 549, 551, 664, 667 }

F(-2) exception fail { 188, 194, 204, 214, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 457, 458, 459, 460, 461, 462, 463, 668, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 702, 703 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 188, 279, 280, 281, 282, 286, 287, 288, 289, 293, 294, 295, 296, 300, 301, 302, 303, 306, 307, 308, 309, 313, 314, 315, 316, 319, 320, 321, 322, 326, 327, 328, 329, 333, 334, 335, 336, 340, 341, 342, 343, 347, 348, 349, 350, 354, 355, 356, 357, 362, 363, 364, 365, 366, 369, 370, 371, 372, 378, 379, 380, 382, 385, 386, 387, 397, 398, 399, 405, 406, 407, 412, 413, 414, 415, 418, 419, 420, 421, 422, 426, 427, 428, 429, 433, 434, 435, 436, 446, 447, 448, 449, 465, 466, 467, 483, 484, 485, 486, 491, 493, 495, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 589, 656, 668, 682, 683, 684, 690, 691, 692 }

C grade { }

F normal fail { }

F(-1) timedout fail { 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 283, 284, 285, 290, 291, 292, 297, 298, 299, 304, 305, 310, 311, 312, 317, 318, 323, 324, 325, 330, 331, 332, 337, 338, 339, 344, 345, 346, 351, 352, 353, 358, 359, 360, 361, 367, 368, 373, 374, 375, 376, 377, 381, 383, 384, 388, 389, 390, 391, 392, 393, 394, 395, 396, 400, 401, 402, 403, 404, 408, 409, 410, 411, 416, 417, 423, 424, 425, 430, 431, 432, 437, 438, 439, 440, 441, 442, 443, 444, 445, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 487, 488, 489, 490, 492, 494, 496, 497, 498, 499, 500, 501, 502, 503, 586, 587, 588, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 685, 686, 687, 688, 689, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 23, 24, 25, 26, 27, 31, 32, 40, 41, 43, 44, 47, 48, 49, 50, 54, 55, 56, 57, 63, 64, 65, 66, 68, 69, 71, 72, 73, 74, 75, 76, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 104, 105, 106, 110, 119, 120, 121, 137, 138, 139, 155, 156, 157, 163, 173, 174, 175, 183, 184, 487, 507, 508, 509, 510, 516, 525, 540 }

B grade { 16, 17, 18, 33, 34, 35, 42, 51, 58, 67, 87, 88, 111, 112, 113, 118, 126, 127, 128, 129, 136, 143, 144, 145, 146, 147, 153, 154, 161, 162, 164, 165, 169, 170, 171, 172, 179, 180, 181, 182, 486 }

C grade { }

F normal fail { 11, 12, 13, 19, 20, 21, 22, 28, 29, 30, 36, 37, 38, 39, 45, 46, 52, 53, 59, 60, 61, 62, 70, 77, 78, 79, 80, 99, 100, 101, 102, 103, 107, 108, 109, 114, 115, 116, 117, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 140, 141, 142, 148, 149, 150, 151, 152, 158, 159, 160, 166, 167, 168, 176, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 203, 204, 205, 206, 207, 208, 213, 214, 215, 216, 217, 226, 227, 228, 229, 230, 231, 232, 238, 239, 240, 241, 242, 243, 250, 251, 252, 253, 254, 260, 261, 262, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 295, 296, 297, 300, 301, 302, 303, 309, 314, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 353, 355, 356, 357, 358, 359, 360, 361, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 380, 381, 382, 388, 389, 390, 391, 394, 395, 396, 397, 398, 402, 403, 404, 417, 418, 419, 420, 421, 425, 426, 427, 428, 433, 434, 435, 439, 440, 441, 442, 444, 445, 446, 450, 451, 452, 453, 454, 455, 456, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 478, 479, 480, 481, 482, 483, 484, 485, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 511, 512, 513, 514, 515, 517, 518, 519, 520, 521, 522, 523, 524, 526, 527, 528, 529, 530, 531, 532, 533, 534, 536, 537, 538, 539, 541, 542, 543, 544, 545, 546, 547, 548, 549, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 568, 569, 570, 571, 572, 574, 575, 576, 577, 580, 581, 582, 583, 584, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 603, 604, 605, 606, 607, 608, 612, 613, 614, 615, 616, 617, 619, 620, 621, 622, 623, 624, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 655, 656, 657, 658, 660, 661, 662, 667, 668, 669, 675, 676, 685, 686, 687, 692, 693, 694, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709 }

F(-1) timedout fail { 185, 199, 200, 201, 202, 209, 210, 211, 212, 218, 219, 220, 221, 222, 223, 224, 225, 233, 234, 235, 236, 237, 244, 245, 246, 247, 248, 249, 255, 256, 257, 258, 259, 267, 291, 292, 293, 294, 298, 299, 304, 305, 306, 307, 308, 310, 311, 312, 313, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 346, 354, 362, 369, 370, 371, 378, 379, 383, 384, 385, 386, 387, 392, 393, 399, 400, 401, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 422, 423, 424, 429, 430, 431, 432, 436, 437, 438, 443, 447, 448, 449, 457, 458, 475, 476, 477, 535, 550, 551, 566, 567, 573, 585, 586, 601, 602, 609, 610, 611, 618, 625, 636, 654, 659, 663, 664, 665, 666, 670, 671, 672, 673, 674, 677, 678, 679, 680, 681, 682, 683, 684, 688, 689, 690, 691, 695, 696 }

F(-2) exception fail { 578, 579 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 130, 131, 132, 133, 134, 140, 141, 148, 149, 150, 151, 158, 336, 418, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585 }

C grade { }

F normal fail { 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 135, 136, 137, 138, 139, 142, 143, 144, 145, 146, 147, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	82	79	78	114	189	83	114	190	106
N.S.	1	0.87	0.84	0.83	1.21	2.01	0.88	1.21	2.02	1.13
time (sec)	N/A	0.305	0.224	162.023	0.040	0.076	4.343	0.174	0.161	0.667

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	68	63	67	92	153	68	92	153	149
N.S.	1	0.91	0.84	0.89	1.23	2.04	0.91	1.23	2.04	1.99
time (sec)	N/A	0.294	0.073	58.354	0.036	0.076	2.655	0.147	0.183	0.344

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	58	55	56	70	117	60	70	116	112
N.S.	1	0.94	0.89	0.90	1.13	1.89	0.97	1.13	1.87	1.81
time (sec)	N/A	0.297	0.075	14.390	0.033	0.065	1.715	0.140	0.163	0.322

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	45	48	81	48	48	79	48
N.S.	1	1.00	0.93	0.98	1.04	1.76	1.04	1.04	1.72	1.04
time (sec)	N/A	0.291	0.031	3.215	0.042	0.079	1.204	0.141	0.154	0.318

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	30	30	28	21	45	37	26	42	23
N.S.	1	1.11	1.11	1.04	0.78	1.67	1.37	0.96	1.56	0.85
time (sec)	N/A	0.270	0.020	0.612	0.028	0.068	0.784	0.128	0.154	0.296

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	23	17	18	24	18	24	17
N.S.	1	1.00	1.00	1.21	0.89	0.95	1.26	0.95	1.26	0.89
time (sec)	N/A	0.164	0.006	0.089	0.037	0.114	0.097	0.128	0.186	0.354

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	46	48	22	38	23	39	46	33	22
N.S.	1	1.02	1.07	0.49	0.84	0.51	0.87	1.02	0.73	0.49
time (sec)	N/A	0.265	0.068	0.593	0.115	0.072	0.095	0.136	0.154	0.347

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	72	46	53	61	56	136	77	63	64
N.S.	1	1.07	0.69	0.79	0.91	0.84	2.03	1.15	0.94	0.96
time (sec)	N/A	0.335	0.064	2.937	0.114	0.072	0.184	0.129	0.154	0.429

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	98	56	63	82	80	211	97	90	108
N.S.	1	1.10	0.63	0.71	0.92	0.90	2.37	1.09	1.01	1.21
time (sec)	N/A	0.420	0.076	12.924	0.117	0.076	0.219	0.128	0.153	0.685

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	124	68	73	103	104	279	117	117	152
N.S.	1	1.12	0.61	0.66	0.93	0.94	2.51	1.05	1.05	1.37
time (sec)	N/A	0.505	0.140	44.458	0.152	0.084	0.320	0.130	0.153	1.974

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	107	98	74	106	372	0	181	364	247
N.S.	1	1.09	1.00	0.76	1.08	3.80	0.00	1.85	3.71	2.52
time (sec)	N/A	0.534	0.018	32.582	0.040	0.089	0.000	0.173	0.172	4.507

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	81	76	64	86	276	0	139	265	178
N.S.	1	1.07	1.00	0.84	1.13	3.63	0.00	1.83	3.49	2.34
time (sec)	N/A	0.437	0.016	6.928	0.039	0.086	0.000	0.162	0.167	4.064

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	55	54	51	61	180	0	97	166	107
N.S.	1	1.02	1.00	0.94	1.13	3.33	0.00	1.80	3.07	1.98
time (sec)	N/A	0.344	0.012	1.628	0.051	0.109	0.000	0.156	0.184	2.321

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	34	32	82	41	52	63	39
N.S.	1	1.00	1.00	1.26	1.19	3.04	1.52	1.93	2.33	1.44
time (sec)	N/A	0.247	0.008	0.309	0.040	0.080	2.450	0.145	0.159	0.405

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	51	17	22	15	26	84	22	20
N.S.	1	1.00	1.96	0.65	0.85	0.58	1.00	3.23	0.85	0.77
time (sec)	N/A	0.250	0.047	0.372	0.041	0.068	0.079	0.142	0.179	0.346

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	36	42	105	196	51	54
N.S.	1	1.00	1.00	0.80	0.78	0.91	2.28	4.26	1.11	1.17
time (sec)	N/A	0.282	0.010	1.417	0.046	0.132	0.176	0.160	0.151	0.503

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	58	62	47	49	66	184	220	81	70
N.S.	1	0.94	1.00	0.76	0.79	1.06	2.97	3.55	1.31	1.13
time (sec)	N/A	0.283	0.014	6.537	0.035	0.079	0.266	0.193	0.156	2.041

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	66	76	57	58	90	253	244	108	93
N.S.	1	0.87	1.00	0.75	0.76	1.18	3.33	3.21	1.42	1.22
time (sec)	N/A	0.291	0.031	27.092	0.043	0.080	0.345	0.204	0.163	3.262

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	94	79	80	108	189	0	108	172	151
N.S.	1	0.86	0.72	0.73	0.99	1.73	0.00	0.99	1.58	1.39
time (sec)	N/A	0.297	0.501	107.834	0.036	0.087	0.000	0.165	0.160	0.375

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	72	63	69	95	151	0	95	135	132
N.S.	1	0.88	0.77	0.84	1.16	1.84	0.00	1.16	1.65	1.61
time (sec)	N/A	0.270	0.329	33.528	0.028	0.088	0.000	0.166	0.164	0.338

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	50	34	58	56	113	0	56	98	56
N.S.	1	0.91	0.62	1.05	1.02	2.05	0.00	1.02	1.78	1.02
time (sec)	N/A	0.254	0.157	7.170	0.034	0.081	0.000	0.152	0.156	0.332

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	50	47	21	75	0	42	61	35
N.S.	1	1.00	1.85	1.74	0.78	2.78	0.00	1.56	2.26	1.30
time (sec)	N/A	0.227	0.086	1.713	0.036	0.103	0.000	0.149	0.157	0.313

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	34	40	41	56	53	32	33	29
N.S.	1	1.00	0.89	1.05	1.08	1.47	1.39	0.84	0.87	0.76
time (sec)	N/A	0.255	0.120	0.116	0.118	0.075	0.141	0.135	0.150	0.329

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	31	19	32	17	36	17	28	18
N.S.	1	1.00	1.24	0.76	1.28	0.68	1.44	0.68	1.12	0.72
time (sec)	N/A	0.228	0.111	1.385	0.108	0.072	0.108	0.150	0.184	0.323

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	71	52	44	67	41	87	57	64	50
N.S.	1	1.06	0.78	0.66	1.00	0.61	1.30	0.85	0.96	0.75
time (sec)	N/A	0.283	0.195	5.940	0.118	0.077	0.162	0.186	0.149	0.373

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	115	99	79	92	78	185	89	92	88
N.S.	1	0.92	0.79	0.63	0.74	0.62	1.48	0.71	0.74	0.70
time (sec)	N/A	0.310	0.256	24.302	0.116	0.108	0.245	0.177	0.159	0.487

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	159	142	114	115	106	270	109	119	144
N.S.	1	0.89	0.79	0.64	0.64	0.59	1.51	0.61	0.66	0.80
time (sec)	N/A	0.347	0.323	75.840	0.116	0.087	0.303	0.184	0.153	1.191

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	121	108	133	181	364	0	237	274	290
N.S.	1	1.03	0.92	1.13	1.53	3.08	0.00	2.01	2.32	2.46
time (sec)	N/A	0.631	0.218	15.062	0.038	0.084	0.000	0.201	0.159	4.301

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	95	84	111	130	256	0	173	199	198
N.S.	1	1.01	0.89	1.18	1.38	2.72	0.00	1.84	2.12	2.11
time (sec)	N/A	0.511	0.113	3.524	0.044	0.094	0.000	0.200	0.186	3.921

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	65	70	86	83	148	0	107	122	104
N.S.	1	0.96	1.03	1.26	1.22	2.18	0.00	1.57	1.79	1.53
time (sec)	N/A	0.369	0.116	0.741	0.035	0.087	0.000	0.162	0.163	0.906

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	180	56	61	52	68	56	54	41
N.S.	1	1.00	3.91	1.22	1.33	1.13	1.48	1.22	1.17	0.89
time (sec)	N/A	0.294	0.471	0.664	0.042	0.082	0.171	0.188	0.163	0.445

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	52	38	52	34	75	531	56	78
N.S.	1	1.00	1.02	0.75	1.02	0.67	1.47	10.41	1.10	1.53
time (sec)	N/A	0.285	0.072	2.892	0.062	0.074	0.158	0.238	0.161	0.444

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	66	68	67	79	62	153	613	83	71
N.S.	1	0.96	0.99	0.97	1.14	0.90	2.22	8.88	1.20	1.03
time (sec)	N/A	0.311	0.121	12.795	0.036	0.070	0.234	0.297	0.182	1.324

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	78	85	102	98	90	238	641	110	256
N.S.	1	0.90	0.98	1.17	1.13	1.03	2.74	7.37	1.26	2.94
time (sec)	N/A	0.318	0.122	44.045	0.040	0.080	0.337	0.325	0.165	0.777

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	86	104	131	119	118	314	669	137	330
N.S.	1	0.82	0.99	1.25	1.13	1.12	2.99	6.37	1.30	3.14
time (sec)	N/A	0.323	0.282	124.663	0.036	0.096	0.425	0.367	0.155	2.267

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	94	79	91	108	215	0	108	192	151
N.S.	1	0.86	0.72	0.83	0.99	1.97	0.00	0.99	1.76	1.39
time (sec)	N/A	0.279	0.607	169.061	0.051	0.088	0.000	0.196	0.166	0.377

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	72	63	80	108	177	0	108	155	151
N.S.	1	0.88	0.77	0.98	1.32	2.16	0.00	1.32	1.89	1.84
time (sec)	N/A	0.264	0.345	59.881	0.034	0.074	0.000	0.203	0.162	0.359

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	50	34	69	82	139	0	82	118	114
N.S.	1	0.91	0.62	1.25	1.49	2.53	0.00	1.49	2.15	2.07
time (sec)	N/A	0.247	0.283	15.386	0.039	0.069	0.000	0.186	0.164	0.351

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	50	58	21	101	0	56	81	56
N.S.	1	1.00	1.85	2.15	0.78	3.74	0.00	2.07	3.00	2.07
time (sec)	N/A	0.219	0.128	3.667	0.036	0.086	0.000	0.168	0.170	0.341

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	63	67	46	51	76	97	94	50	46	41
N.S.	1	1.06	0.73	0.81	1.21	1.54	1.49	0.79	0.73	0.65
time (sec)	N/A	0.337	0.135	0.124	0.117	0.079	0.181	0.152	0.154	0.346

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	43	40	50	62	36	61	36	85	39
N.S.	1	0.90	0.83	1.04	1.29	0.75	1.27	0.75	1.77	0.81
time (sec)	N/A	0.249	0.095	2.723	0.119	0.096	0.199	0.184	0.149	0.363

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	38	57	34	80	18	59	36
N.S.	1	1.00	0.89	1.41	2.11	1.26	2.96	0.67	2.19	1.33
time (sec)	N/A	0.225	0.074	12.587	0.107	0.087	0.179	0.216	0.177	0.399

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	93	65	62	105	55	131	69	92	77
N.S.	1	0.95	0.66	0.63	1.07	0.56	1.34	0.70	0.94	0.79
time (sec)	N/A	0.288	0.229	43.668	0.116	0.085	0.229	0.224	0.191	0.444

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	137	137	97	128	92	226	99	119	125
N.S.	1	0.86	0.86	0.61	0.80	0.58	1.41	0.62	0.74	0.78
time (sec)	N/A	0.318	0.291	128.325	0.120	0.092	0.352	0.266	0.162	0.741

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	134	102	122	155	310	0	189	278	228
N.S.	1	1.06	0.80	0.96	1.22	2.44	0.00	1.49	2.19	1.80
time (sec)	N/A	0.665	1.664	7.762	0.038	0.091	0.000	0.250	0.162	4.119

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	102	93	100	109	202	0	125	179	136
N.S.	1	1.03	0.94	1.01	1.10	2.04	0.00	1.26	1.81	1.37
time (sec)	N/A	0.495	1.329	1.663	0.041	0.091	0.000	0.232	0.162	2.456

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	62	123	93	82	107	107	234	93	102
N.S.	1	1.02	2.02	1.52	1.34	1.75	1.75	3.84	1.52	1.67
time (sec)	N/A	0.386	1.440	1.421	0.040	0.085	0.219	0.310	0.164	0.775

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	19	75	17	36	901	54	66
N.S.	1	1.00	0.97	0.59	2.34	0.53	1.12	28.16	1.69	2.06
time (sec)	N/A	0.222	0.165	6.287	0.048	0.082	0.143	0.380	0.201	0.443

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	87	145	56	105	48	116	929	83	130
N.S.	1	0.99	1.65	0.64	1.19	0.55	1.32	10.56	0.94	1.48
time (sec)	N/A	0.429	0.664	24.548	0.038	0.077	0.254	0.454	0.202	0.659

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	99	180	85	123	76	190	465	110	134
N.S.	1	0.93	1.70	0.80	1.16	0.72	1.79	4.39	1.04	1.26
time (sec)	N/A	0.418	0.576	77.052	0.037	0.085	0.330	0.529	0.209	1.630

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	107	226	120	145	104	275	1039	137	330
N.S.	1	0.86	1.82	0.97	1.17	0.84	2.22	8.38	1.10	2.66
time (sec)	N/A	0.432	0.852	198.495	0.067	0.079	0.409	0.612	0.165	1.805

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	173	171	133	246	364	0	237	291	290
N.S.	1	1.06	1.05	0.82	1.51	2.23	0.00	1.45	1.79	1.78
time (sec)	N/A	0.833	1.843	15.612	0.047	0.084	0.000	0.322	0.157	4.313

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	139	237	111	180	256	0	173	216	198
N.S.	1	1.05	1.78	0.83	1.35	1.92	0.00	1.30	1.62	1.49
time (sec)	N/A	0.647	1.358	3.456	0.056	0.091	0.000	0.299	0.155	3.947

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	100	906	107	137	162	153	372	151	159
N.S.	1	1.03	9.34	1.10	1.41	1.67	1.58	3.84	1.56	1.64
time (sec)	N/A	0.503	6.710	2.958	0.040	0.092	0.269	0.401	0.174	2.478

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	85	246	79	121	68	109	1299	76	88
N.S.	1	1.09	3.15	1.01	1.55	0.87	1.40	16.65	0.97	1.13
time (sec)	N/A	0.417	0.864	12.561	0.037	0.093	0.266	0.588	0.186	0.659

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	145	38	118	34	80	915	83	130
N.S.	1	1.00	2.20	0.58	1.79	0.52	1.21	13.86	1.26	1.97
time (sec)	N/A	0.343	0.497	43.372	0.039	0.094	0.226	0.752	0.216	0.617

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	107	181	74	149	62	156	1327	110	186
N.S.	1	1.05	1.77	0.73	1.46	0.61	1.53	13.01	1.08	1.82
time (sec)	N/A	0.445	0.568	123.247	0.039	0.117	0.291	0.609	0.167	1.388

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	119	216	103	181	90	228	1409	137	145
N.S.	1	0.99	1.80	0.86	1.51	0.75	1.90	11.74	1.14	1.21
time (sec)	N/A	0.466	0.808	306.935	0.241	0.089	0.399	0.462	0.219	1.881

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	94	81	377	160	267	0	160	229	146
N.S.	1	0.86	0.74	3.46	1.47	2.45	0.00	1.47	2.10	1.34
time (sec)	N/A	0.284	0.785	0.414	0.044	0.083	0.000	0.275	0.163	0.803

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	72	63	102	108	229	0	108	192	151
N.S.	1	0.88	0.77	1.24	1.32	2.79	0.00	1.32	2.34	1.84
time (sec)	N/A	0.269	0.488	166.158	0.042	0.079	0.000	0.257	0.158	0.409

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	50	36	91	108	191	0	108	155	151
N.S.	1	0.91	0.65	1.65	1.96	3.47	0.00	1.96	2.82	2.75
time (sec)	N/A	0.248	0.348	59.603	0.039	0.083	0.000	0.268	0.164	0.414

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	72	80	21	153	0	82	118	114
N.S.	1	1.00	2.67	2.96	0.78	5.67	0.00	3.04	4.37	4.22
time (sec)	N/A	0.222	0.320	15.513	0.037	0.070	0.000	0.244	0.164	0.392

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	117	125	71	72	165	177	178	86	67	73
N.S.	1	1.07	0.61	0.62	1.41	1.51	1.52	0.74	0.57	0.62
time (sec)	N/A	0.554	0.161	0.153	0.148	0.100	0.275	0.192	0.191	0.400

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	78	62	85	86	125	131	68	221	70
N.S.	1	0.94	0.75	1.02	1.04	1.51	1.58	0.82	2.66	0.84
time (sec)	N/A	0.267	0.533	12.457	0.108	0.118	0.294	0.295	0.217	0.403

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	65	49	68	88	51	102	50	96	64
N.S.	1	0.92	0.69	0.96	1.24	0.72	1.44	0.70	1.35	0.90
time (sec)	N/A	0.267	0.246	42.712	0.131	0.078	0.275	0.356	0.194	0.445

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	50	36	38	93	34	80	30	86	53
N.S.	1	0.91	0.65	0.69	1.69	0.62	1.45	0.55	1.56	0.96
time (sec)	N/A	0.256	0.107	122.739	0.113	0.108	0.266	0.242	0.181	0.408

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	74	103	62	162	18	113	63
N.S.	1	1.00	0.89	2.74	3.81	2.30	6.00	0.67	4.19	2.33
time (sec)	N/A	0.225	0.161	298.986	0.120	0.085	0.358	0.245	0.168	0.438

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	137	123	331	164	83	209	89	146	122
N.S.	1	0.88	0.79	2.12	1.05	0.53	1.34	0.57	0.94	0.78
time (sec)	N/A	0.335	0.296	0.646	0.136	0.122	0.369	0.262	0.162	0.778

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	181	159	361	187	120	301	119	173	171
N.S.	1	0.85	0.74	1.69	0.87	0.56	1.41	0.56	0.81	0.80
time (sec)	N/A	0.357	0.590	0.604	0.110	0.108	0.483	0.286	0.164	2.177

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	176	115	122	215	310	0	189	289	228
N.S.	1	1.05	0.69	0.73	1.29	1.86	0.00	1.13	1.73	1.37
time (sec)	N/A	0.803	2.005	7.194	0.040	0.102	0.000	0.371	0.164	4.231

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	137	151	118	173	216	197	510	206	222
N.S.	1	1.05	1.16	0.91	1.33	1.66	1.52	3.92	1.58	1.71
time (sec)	N/A	0.665	1.643	6.112	0.057	0.095	0.305	0.500	0.197	4.420

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	103	130	111	154	122	148	1683	121	162
N.S.	1	1.05	1.33	1.13	1.57	1.24	1.51	17.17	1.23	1.65
time (sec)	N/A	0.535	1.816	23.620	0.040	0.092	0.326	0.779	0.219	2.636

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	19	152	17	36	1669	81	104
N.S.	1	1.00	0.97	0.59	4.75	0.53	1.12	52.16	2.53	3.25
time (sec)	N/A	0.222	0.380	74.598	0.042	0.090	0.211	0.710	0.181	0.581

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	104	122	56	187	48	121	1697	110	186
N.S.	1	1.03	1.21	0.55	1.85	0.48	1.20	16.80	1.09	1.84
time (sec)	N/A	0.485	0.556	194.967	0.047	0.104	0.345	0.569	0.190	1.363

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	140	146	287	217	76	192	1725	137	79
N.S.	1	0.99	1.04	2.04	1.54	0.54	1.36	12.23	0.97	0.56
time (sec)	N/A	0.588	0.672	1.477	0.050	0.129	0.402	0.642	0.176	0.995

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	148	170	317	246	104	265	1807	164	139
N.S.	1	0.93	1.07	1.99	1.55	0.65	1.67	11.36	1.03	0.87
time (sec)	N/A	0.595	0.958	0.690	0.042	0.101	0.560	0.669	0.175	1.716

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	94	56	611	186	345	0	186	283	153
N.S.	1	0.86	0.51	5.61	1.71	3.17	0.00	1.71	2.60	1.40
time (sec)	N/A	0.302	1.731	1.241	0.054	0.092	0.000	0.336	0.170	1.948

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	72	44	475	173	307	0	173	246	190
N.S.	1	0.88	0.54	5.79	2.11	3.74	0.00	2.11	3.00	2.32
time (sec)	N/A	0.274	0.931	1.233	0.033	0.087	0.000	0.320	0.189	2.474

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	50	34	124	134	269	0	134	209	107
N.S.	1	0.91	0.62	2.25	2.44	4.89	0.00	2.44	3.80	1.95
time (sec)	N/A	0.257	0.650	267.171	0.036	0.101	0.000	0.295	0.197	1.347

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	102	113	21	231	0	120	172	83
N.S.	1	1.00	3.78	4.19	0.78	8.56	0.00	4.44	6.37	3.07
time (sec)	N/A	0.220	0.588	103.982	0.037	0.086	0.000	0.294	0.167	0.927

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	200	212	100	103	121	297	301	134	98	113
N.S.	1	1.06	0.50	0.52	0.60	1.48	1.50	0.67	0.49	0.56
time (sec)	N/A	0.985	0.462	0.353	0.125	0.080	0.396	0.273	0.218	0.424

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	121	96	118	124	245	257	119	460	102
N.S.	1	0.91	0.72	0.89	0.93	1.84	1.93	0.89	3.46	0.77
time (sec)	N/A	0.301	0.814	75.777	0.119	0.089	0.391	0.234	0.170	0.415

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	113	86	114	135	179	216	101	309	111
N.S.	1	0.91	0.69	0.92	1.09	1.44	1.74	0.81	2.49	0.90
time (sec)	N/A	0.298	1.381	193.975	0.113	0.091	0.387	0.240	0.165	0.408

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	101	77	319	146	113	172	76	190	103
N.S.	1	0.89	0.68	2.80	1.28	0.99	1.51	0.67	1.67	0.90
time (sec)	N/A	0.292	0.821	0.779	0.111	0.094	0.417	0.271	0.163	0.466

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	42	31	451	136	17	36	39	110	66
N.S.	1	0.98	0.72	10.49	3.16	0.40	0.84	0.91	2.56	1.53
time (sec)	N/A	0.234	1.555	0.769	0.113	0.084	0.344	0.293	0.157	0.462

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	70	44	588	152	48	121	45	140	82
N.S.	1	0.88	0.55	7.35	1.90	0.60	1.51	0.56	1.75	1.02
time (sec)	N/A	0.266	0.221	1.280	0.134	0.109	0.449	0.294	0.197	0.452

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	50	34	639	162	76	197	32	167	82
N.S.	1	0.91	0.62	11.62	2.95	1.38	3.58	0.58	3.04	1.49
time (sec)	N/A	0.248	0.388	0.860	0.115	0.117	0.565	0.331	0.160	0.457

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	116	689	171	104	279	18	194	105
N.S.	1	1.00	4.30	25.52	6.33	3.85	10.33	0.67	7.19	3.89
time (sec)	N/A	0.228	2.871	0.871	0.116	0.116	0.658	0.330	0.166	0.453

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	203	152	739	246	125	323	119	227	195
N.S.	1	0.87	0.65	3.17	1.06	0.54	1.39	0.51	0.97	0.84
time (sec)	N/A	0.355	0.903	0.844	0.133	0.137	0.697	0.337	0.168	2.046

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	247	186	789	269	162	413	149	254	231
N.S.	1	0.86	0.65	2.75	0.94	0.56	1.44	0.52	0.89	0.80
time (sec)	N/A	0.407	1.074	1.474	0.137	0.167	0.836	0.356	0.159	2.315

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	248	205	151	396	378	320	924	335	399
N.S.	1	1.06	0.87	0.64	1.69	1.61	1.36	3.93	1.43	1.70
time (sec)	N/A	1.216	2.617	80.802	0.048	0.105	0.478	0.957	0.199	5.538

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	A	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	215	1540	147	352	284	277	2835	270	343
N.S.	1	1.05	7.51	0.72	1.72	1.39	1.35	13.83	1.32	1.67
time (sec)	N/A	1.024	8.147	226.998	0.058	0.098	0.464	0.882	0.162	4.988

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	A	A	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	182	1162	322	326	190	235	2849	205	281
N.S.	1	1.05	6.72	1.86	1.88	1.10	1.36	16.47	1.18	1.62
time (sec)	N/A	0.861	7.938	2.782	0.045	0.122	0.475	0.967	0.158	4.576

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	163	305	385	309	96	187	2863	130	207
N.S.	1	1.07	2.01	2.53	2.03	0.63	1.23	18.84	0.86	1.36
time (sec)	N/A	0.740	2.990	1.417	0.051	0.110	0.506	1.063	0.159	3.850

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	146	447	302	34	80	2451	137	37
N.S.	1	1.00	2.21	6.77	4.58	0.52	1.21	37.14	2.08	0.56
time (sec)	N/A	0.339	0.716	1.388	0.040	0.110	0.464	1.160	0.159	0.741

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	142	151	567	355	62	162	2863	164	65
N.S.	1	1.04	1.11	4.17	2.61	0.46	1.19	21.05	1.21	0.48
time (sec)	N/A	0.611	0.875	1.660	0.050	0.098	0.553	1.247	0.190	0.940

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	218	168	617	405	90	240	2891	191	93
N.S.	1	1.03	0.80	2.92	1.92	0.43	1.14	13.70	0.91	0.44
time (sec)	N/A	0.960	1.655	2.910	0.044	0.108	0.658	1.338	0.159	1.055

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	209	173	667	453	118	313	2919	218	222
N.S.	1	0.99	0.82	3.15	2.14	0.56	1.48	13.77	1.03	1.05
time (sec)	N/A	0.789	1.965	1.679	0.056	0.135	0.732	1.409	0.163	2.386

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	92	56	58	87	146	0	87	27	92
N.S.	1	0.86	0.52	0.54	0.81	1.36	0.00	0.81	0.25	0.86
time (sec)	N/A	0.278	0.253	0.494	0.034	0.087	0.000	0.168	0.152	0.598

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	70	46	47	67	109	0	67	27	114
N.S.	1	0.88	0.58	0.59	0.84	1.36	0.00	0.84	0.34	1.42
time (sec)	N/A	0.268	0.141	0.460	0.035	0.086	0.000	0.163	0.162	0.407

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	50	50	36	47	72	0	47	27	77
N.S.	1	0.91	0.91	0.65	0.85	1.31	0.00	0.85	0.49	1.40
time (sec)	N/A	0.259	0.091	0.457	0.054	0.079	0.000	0.158	0.184	0.418

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	34	23	27	33	0	27	27	25
N.S.	1	1.00	1.26	0.85	1.00	1.22	0.00	1.00	1.00	0.93
time (sec)	N/A	0.228	0.049	1.298	0.050	0.095	0.000	0.160	0.157	0.391

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	24	23	23	20	26	0	17	27	19
N.S.	1	1.04	1.00	1.00	0.87	1.13	0.00	0.74	1.17	0.83
time (sec)	N/A	0.223	0.024	0.672	0.056	0.116	0.000	0.147	0.152	0.443

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	35	26	0	32	60	53	18	29
N.S.	1	1.00	1.06	0.79	0.00	0.97	1.82	1.61	0.55	0.88
time (sec)	N/A	0.193	0.074	0.232	0.000	0.115	0.097	0.141	0.164	0.453

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	93	87	61	0	54	151	83	27	60
N.S.	1	1.06	0.99	0.69	0.00	0.61	1.72	0.94	0.31	0.68
time (sec)	N/A	0.295	0.173	0.984	0.000	0.082	0.173	0.154	0.152	0.519

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	137	131	96	0	76	219	103	27	123
N.S.	1	0.91	0.87	0.64	0.00	0.51	1.46	0.69	0.18	0.82
time (sec)	N/A	0.320	0.194	1.678	0.000	0.079	0.232	0.156	0.186	0.819

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	85	60	122	288	266	0	138	27	193
N.S.	1	1.01	0.71	1.45	3.43	3.17	0.00	1.64	0.32	2.30
time (sec)	N/A	0.459	0.724	0.749	0.044	0.118	0.000	0.225	0.151	4.141

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	59	50	100	186	174	0	99	27	116
N.S.	1	0.98	0.83	1.67	3.10	2.90	0.00	1.65	0.45	1.93
time (sec)	N/A	0.362	0.494	0.690	0.041	0.083	0.000	0.206	0.158	2.454

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	34	70	83	80	0	58	27	43
N.S.	1	1.00	1.10	2.26	2.68	2.58	0.00	1.87	0.87	1.39
time (sec)	N/A	0.277	0.420	1.117	0.046	0.082	0.000	0.199	0.158	0.513

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	19	29	17	34	21	25	25
N.S.	1	1.00	0.89	0.68	1.04	0.61	1.21	0.75	0.89	0.89
time (sec)	N/A	0.208	0.186	0.680	0.041	0.071	0.382	0.166	0.187	0.428

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	50	49	0	41	126	67	25	78
N.S.	1	1.00	1.06	1.04	0.00	0.87	2.68	1.43	0.53	1.66
time (sec)	N/A	0.274	0.271	0.806	0.000	0.090	0.190	0.183	0.167	0.605

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	64	72	84	0	63	196	119	27	134
N.S.	1	0.96	1.07	1.25	0.00	0.94	2.93	1.78	0.40	2.00
time (sec)	N/A	0.314	0.402	1.289	0.000	0.099	0.261	0.179	0.163	1.912

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	76	94	119	0	85	264	171	27	188
N.S.	1	0.89	1.11	1.40	0.00	1.00	3.11	2.01	0.32	2.21
time (sec)	N/A	0.320	0.549	1.099	0.000	0.086	0.344	0.192	0.188	4.267

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	72	44	47	77	138	0	77	37	93
N.S.	1	0.88	0.54	0.57	0.94	1.68	0.00	0.94	0.45	1.13
time (sec)	N/A	0.273	0.201	0.470	0.041	0.104	0.000	0.178	0.167	0.528

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	50	50	36	47	97	0	47	37	77
N.S.	1	0.91	0.91	0.65	0.85	1.76	0.00	0.85	0.67	1.40
time (sec)	N/A	0.253	0.123	0.460	0.036	0.091	0.000	0.186	0.170	0.398

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	50	20	35	54	0	35	37	33
N.S.	1	1.00	1.85	0.74	1.30	2.00	0.00	1.30	1.37	1.22
time (sec)	N/A	0.225	0.044	0.447	0.039	0.082	0.000	0.173	0.162	0.406

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	39	38	30	32	70	0	32	37	28
N.S.	1	1.03	1.00	0.79	0.84	1.84	0.00	0.84	0.97	0.74
time (sec)	N/A	0.247	0.041	0.780	0.030	0.085	0.000	0.169	0.153	0.419

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	25	19	19	21	17	65	17	37	22
N.S.	1	0.96	0.73	0.73	0.81	0.65	2.50	0.65	1.42	0.85
time (sec)	N/A	0.223	0.108	1.408	0.042	0.082	0.547	0.147	0.190	0.390

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	65	52	44	0	43	117	61	28	39
N.S.	1	1.07	0.85	0.72	0.00	0.70	1.92	1.00	0.46	0.64
time (sec)	N/A	0.271	0.121	0.272	0.000	0.074	0.152	0.145	0.196	0.451

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	115	97	79	0	65	189	93	37	71
N.S.	1	1.01	0.85	0.69	0.00	0.57	1.66	0.82	0.32	0.62
time (sec)	N/A	0.296	0.257	1.854	0.000	0.086	0.230	0.187	0.199	0.563

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	159	142	114	0	87	258	113	37	149
N.S.	1	0.96	0.86	0.69	0.00	0.53	1.56	0.68	0.22	0.90
time (sec)	N/A	0.332	0.278	1.069	0.000	0.105	0.280	0.161	0.204	1.313

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	130	294	133	421	326	0	203	37	191
N.S.	1	1.05	2.37	1.07	3.40	2.63	0.00	1.64	0.30	1.54
time (sec)	N/A	0.606	2.230	0.829	0.050	0.112	0.000	0.246	0.199	3.376

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	104	215	111	295	230	0	151	37	136
N.S.	1	1.04	2.15	1.11	2.95	2.30	0.00	1.51	0.37	1.36
time (sec)	N/A	0.484	1.291	0.793	0.061	0.104	0.000	0.236	0.193	3.104

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	146	89	167	134	0	95	37	104
N.S.	1	1.00	1.97	1.20	2.26	1.81	0.00	1.28	0.50	1.41
time (sec)	N/A	0.382	0.675	0.798	0.039	0.087	0.000	0.220	0.189	1.046

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	184	54	117	64	0	57	37	44
N.S.	1	1.00	3.83	1.12	2.44	1.33	0.00	1.19	0.77	0.92
time (sec)	N/A	0.291	0.395	0.820	0.154	0.082	0.000	0.211	0.152	0.535

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	64	38	38	45	30	112	47	35	79
N.S.	1	0.98	0.58	0.58	0.69	0.46	1.72	0.72	0.54	1.22
time (sec)	N/A	0.305	0.220	1.128	0.065	0.095	0.571	0.189	0.157	0.515

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	68	68	67	0	52	163	93	35	90
N.S.	1	0.96	0.96	0.94	0.00	0.73	2.30	1.31	0.49	1.27
time (sec)	N/A	0.310	0.454	1.191	0.000	0.071	0.231	0.220	0.166	0.876

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	80	95	102	0	74	231	145	37	161
N.S.	1	0.90	1.07	1.15	0.00	0.83	2.60	1.63	0.42	1.81
time (sec)	N/A	0.323	0.490	0.984	0.000	0.099	0.337	0.221	0.186	3.924

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	88	117	137	0	96	299	197	37	216
N.S.	1	0.82	1.09	1.28	0.00	0.90	2.79	1.84	0.35	2.02
time (sec)	N/A	0.323	0.712	1.298	0.000	0.075	0.413	0.240	0.201	2.862

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	94	56	58	87	194	0	87	197	119
N.S.	1	0.86	0.51	0.53	0.80	1.78	0.00	0.80	1.81	1.09
time (sec)	N/A	0.283	0.247	0.536	0.048	0.102	0.000	0.240	0.194	0.642

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	72	46	47	87	153	0	87	160	103
N.S.	1	0.88	0.56	0.57	1.06	1.87	0.00	1.06	1.95	1.26
time (sec)	N/A	0.273	0.189	0.510	0.037	0.082	0.000	0.243	0.216	0.571

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	50	34	36	67	112	0	67	123	114
N.S.	1	0.91	0.62	0.65	1.22	2.04	0.00	1.22	2.24	2.07
time (sec)	N/A	0.254	0.106	0.510	0.041	0.078	0.000	0.236	0.212	0.414

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	50	21	47	69	0	47	85	77
N.S.	1	1.00	1.85	0.78	1.74	2.56	0.00	1.74	3.15	2.85
time (sec)	N/A	0.220	0.089	0.491	0.045	0.070	0.000	0.219	0.202	0.420

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	48	68	45	113	0	53	299	41
N.S.	1	1.00	0.83	1.17	0.78	1.95	0.00	0.91	5.16	0.71
time (sec)	N/A	0.246	0.085	0.567	0.035	0.081	0.000	0.208	0.195	0.435

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	45	44	56	66	55	0	36	0	42
N.S.	1	0.94	0.92	1.17	1.38	1.15	0.00	0.75	0.00	0.88
time (sec)	N/A	0.245	0.077	0.561	0.031	0.096	0.000	0.203	0.290	0.428

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	24	21	30	153	18	386	20
N.S.	1	1.00	0.89	0.89	0.78	1.11	5.67	0.67	14.30	0.74
time (sec)	N/A	0.225	0.113	0.480	0.034	0.070	0.880	0.183	0.191	0.439

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	97	65	62	0	54	155	73	40	50
N.S.	1	1.10	0.74	0.70	0.00	0.61	1.76	0.83	0.45	0.57
time (sec)	N/A	0.356	0.147	0.333	0.000	0.069	0.198	0.165	0.169	0.531

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	137	137	97	0	76	224	103	49	124
N.S.	1	0.94	0.94	0.67	0.00	0.52	1.54	0.71	0.34	0.86
time (sec)	N/A	0.301	0.218	0.961	0.000	0.088	0.274	0.221	0.192	0.826

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	181	159	129	0	98	292	123	49	173
N.S.	1	0.87	0.76	0.62	0.00	0.47	1.40	0.59	0.23	0.83
time (sec)	N/A	0.349	0.333	1.133	0.000	0.076	0.312	0.222	0.201	2.195

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	128	113	122	341	278	0	164	278	150
N.S.	1	1.08	0.95	1.03	2.87	2.34	0.00	1.38	2.34	1.26
time (sec)	N/A	0.659	0.915	0.836	0.051	0.089	0.000	0.359	0.172	3.233

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	98	63	100	215	182	0	112	179	135
N.S.	1	1.05	0.68	1.08	2.31	1.96	0.00	1.20	1.92	1.45
time (sec)	N/A	0.507	0.710	0.884	0.046	0.089	0.000	0.325	0.176	2.663

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	70	108	86	319	112	0	110	0	105
N.S.	1	1.08	1.66	1.32	4.91	1.72	0.00	1.69	0.00	1.62
time (sec)	N/A	0.415	0.620	0.874	0.144	0.084	0.000	0.341	0.485	0.891

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	19	29	17	80	36	759	68
N.S.	1	1.00	1.00	0.59	0.91	0.53	2.50	1.12	23.72	2.12
time (sec)	N/A	0.219	0.316	1.519	0.039	0.093	0.873	0.309	0.214	0.527

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	102	54	56	69	41	219	73	821	133
N.S.	1	1.04	0.55	0.57	0.70	0.42	2.23	0.74	8.38	1.36
time (sec)	N/A	0.427	0.320	0.775	0.058	0.100	0.865	0.271	0.212	0.730

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	106	76	85	0	63	197	119	47	134
N.S.	1	1.05	0.75	0.84	0.00	0.62	1.95	1.18	0.47	1.33
time (sec)	N/A	0.436	0.363	0.832	0.000	0.090	0.285	0.329	0.185	3.049

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	120	98	120	0	85	265	171	49	188
N.S.	1	0.99	0.81	0.99	0.00	0.70	2.19	1.41	0.40	1.55
time (sec)	N/A	0.458	0.506	1.098	0.000	0.089	0.381	0.320	0.196	3.634

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	128	120	155	0	107	333	223	49	136
N.S.	1	0.92	0.86	1.12	0.00	0.77	2.40	1.60	0.35	0.98
time (sec)	N/A	0.469	0.930	1.367	0.000	0.107	0.464	0.344	0.201	2.677

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	72	44	47	97	168	0	97	184	120
N.S.	1	0.88	0.54	0.57	1.18	2.05	0.00	1.18	2.24	1.46
time (sec)	N/A	0.278	0.215	0.654	0.045	0.090	0.000	0.249	0.199	0.658

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	50	34	36	67	127	0	67	147	113
N.S.	1	0.91	0.62	0.65	1.22	2.31	0.00	1.22	2.67	2.05
time (sec)	N/A	0.250	0.149	0.629	0.040	0.098	0.000	0.240	0.179	0.443

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	58	20	55	84	0	55	110	93
N.S.	1	1.00	2.15	0.74	2.04	3.11	0.00	2.04	4.07	3.44
time (sec)	N/A	0.226	0.116	0.608	0.038	0.091	0.000	0.226	0.173	0.434

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	79	56	84	53	156	0	72	413	60
N.S.	1	0.88	0.62	0.93	0.59	1.73	0.00	0.80	4.59	0.67
time (sec)	N/A	0.262	0.101	0.624	0.061	0.111	0.000	0.225	0.184	0.460

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	59	52	41	95	102	0	49	0	55
N.S.	1	0.94	0.83	0.65	1.51	1.62	0.00	0.78	0.00	0.87
time (sec)	N/A	0.258	0.232	0.965	0.051	0.090	0.000	0.207	0.656	0.472

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	28	36	19	66	17	95	24	542	25
N.S.	1	0.97	1.24	0.66	2.28	0.59	3.28	0.83	18.69	0.86
time (sec)	N/A	0.219	0.046	1.256	0.052	0.075	1.342	0.208	0.205	0.460

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	21	41	272	18	542	19
N.S.	1	1.00	0.81	0.89	0.78	1.52	10.07	0.67	20.07	0.70
time (sec)	N/A	0.223	0.158	0.534	0.042	0.091	1.331	0.194	0.204	0.451

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	129	113	80	0	65	189	83	48	60
N.S.	1	1.11	0.97	0.69	0.00	0.56	1.63	0.72	0.41	0.52
time (sec)	N/A	0.465	0.182	0.379	0.000	0.076	0.240	0.189	0.172	0.618

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	159	142	115	0	87	258	113	57	90
N.S.	1	0.94	0.84	0.68	0.00	0.51	1.53	0.67	0.34	0.53
time (sec)	N/A	0.319	0.256	0.995	0.000	0.077	0.320	0.226	0.261	1.252

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	203	164	143	0	109	326	133	57	197
N.S.	1	0.89	0.72	0.63	0.00	0.48	1.43	0.58	0.25	0.86
time (sec)	N/A	0.367	0.449	1.178	0.000	0.084	0.400	0.226	0.210	2.124

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	143	237	111	295	230	0	151	216	197
N.S.	1	1.08	1.78	0.83	2.22	1.73	0.00	1.14	1.62	1.48
time (sec)	N/A	0.668	1.381	0.997	0.049	0.092	0.000	0.392	0.217	4.109

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	113	988	107	457	160	0	113	0	162
N.S.	1	1.06	9.23	1.00	4.27	1.50	0.00	1.06	0.00	1.51
time (sec)	N/A	0.549	6.504	1.013	0.160	0.089	0.000	0.374	1.016	2.679

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	89	247	71	141	76	0	71	0	88
N.S.	1	1.09	3.01	0.87	1.72	0.93	0.00	0.87	0.00	1.07
time (sec)	N/A	0.436	0.561	1.002	0.140	0.108	0.000	0.333	0.362	0.799

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	40	38	53	30	182	73	669	133
N.S.	1	1.00	0.59	0.56	0.78	0.44	2.68	1.07	9.84	1.96
time (sec)	N/A	0.336	0.319	0.962	0.049	0.082	1.325	0.302	0.256	0.754

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	140	73	74	91	52	354	99	746	64
N.S.	1	1.06	0.55	0.56	0.69	0.39	2.68	0.75	5.65	0.48
time (sec)	N/A	0.549	0.336	0.866	0.047	0.077	1.408	0.282	0.270	0.853

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	144	95	103	0	74	231	145	55	161
N.S.	1	1.07	0.71	0.77	0.00	0.55	1.72	1.08	0.41	1.20
time (sec)	N/A	0.582	0.391	0.961	0.000	0.108	0.353	0.374	0.198	4.586

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	160	117	138	0	96	299	197	57	216
N.S.	1	1.03	0.75	0.88	0.00	0.62	1.92	1.26	0.37	1.38
time (sec)	N/A	0.625	0.688	1.069	0.000	0.076	0.481	0.374	0.210	2.829

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	168	139	173	0	118	367	249	57	262
N.S.	1	0.97	0.80	0.99	0.00	0.68	2.11	1.43	0.33	1.51
time (sec)	N/A	0.638	1.040	1.357	0.000	0.082	0.551	0.380	0.228	4.180

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	121	98	124	229	273	0	122	0	105
N.S.	1	0.90	0.73	0.93	1.71	2.04	0.00	0.91	0.00	0.78
time (sec)	N/A	0.294	0.789	0.790	0.044	0.121	0.000	0.450	9.298	0.544

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	113	88	120	212	199	0	100	0	114
N.S.	1	0.90	0.70	0.95	1.68	1.58	0.00	0.79	0.00	0.90
time (sec)	N/A	0.285	0.770	0.780	0.063	0.156	0.000	0.459	4.578	0.515

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	101	79	68	191	124	0	69	0	104
N.S.	1	0.87	0.68	0.59	1.65	1.07	0.00	0.59	0.00	0.90
time (sec)	N/A	0.282	0.707	1.184	0.051	0.096	0.000	0.418	2.593	0.593

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	42	36	19	158	17	160	35	0	73
N.S.	1	0.98	0.84	0.44	3.67	0.40	3.72	0.81	0.00	1.70
time (sec)	N/A	0.232	0.041	1.570	0.054	0.107	10.208	0.396	0.586	0.591

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	69	44	49	141	41	466	38	0	85
N.S.	1	0.85	0.54	0.60	1.74	0.51	5.75	0.47	0.00	1.05
time (sec)	N/A	0.265	0.146	0.675	0.044	0.074	10.179	0.368	0.618	0.591

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	50	34	36	121	63	774	28	0	85
N.S.	1	0.91	0.62	0.65	2.20	1.15	14.07	0.51	0.00	1.55
time (sec)	N/A	0.246	0.112	0.663	0.059	0.081	10.448	0.331	0.634	0.576

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	21	85	1081	18	0	19
N.S.	1	1.00	0.81	0.89	0.78	3.15	40.04	0.67	0.00	0.70
time (sec)	N/A	0.214	0.180	0.585	0.036	0.099	10.036	0.304	0.584	0.637

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	257	152	152	0	109	325	123	90	198
N.S.	1	1.12	0.66	0.66	0.00	0.48	1.42	0.54	0.39	0.86
time (sec)	N/A	1.000	0.444	0.702	0.000	0.074	0.406	0.266	0.179	2.168

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	247	189	169	0	131	394	153	99	235
N.S.	1	0.89	0.68	0.61	0.00	0.47	1.42	0.55	0.36	0.85
time (sec)	N/A	0.395	0.742	1.141	0.000	0.085	0.471	0.370	0.331	3.337

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	291	208	197	0	153	462	173	99	294
N.S.	1	0.87	0.62	0.59	0.00	0.46	1.39	0.52	0.30	0.88
time (sec)	N/A	0.439	0.956	1.337	0.000	0.084	0.554	0.379	0.341	2.857

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	225	1704	147	786	267	0	195	0	344
N.S.	1	1.10	8.31	0.72	3.83	1.30	0.00	0.95	0.00	1.68
time (sec)	N/A	1.163	7.185	1.232	0.227	0.150	0.000	0.930	5.820	4.781

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	199	1244	143	531	182	0	165	0	284
N.S.	1	1.09	6.80	0.78	2.90	0.99	0.00	0.90	0.00	1.55
time (sec)	N/A	0.943	6.846	1.145	0.249	0.140	0.000	0.915	3.442	4.673

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	171	304	115	185	98	0	123	0	207
N.S.	1	1.10	1.95	0.74	1.19	0.63	0.00	0.79	0.00	1.33
time (sec)	N/A	0.772	1.355	1.088	0.170	0.102	0.000	0.834	1.478	4.124

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	40	38	53	30	311	125	0	37
N.S.	1	1.00	0.59	0.56	0.78	0.44	4.57	1.84	0.00	0.54
time (sec)	N/A	0.340	0.408	1.051	0.061	0.080	10.092	0.791	0.737	0.876

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	148	73	74	97	52	620	151	0	64
N.S.	1	1.07	0.53	0.54	0.70	0.38	4.49	1.09	0.00	0.46
time (sec)	N/A	0.624	0.681	1.048	0.056	0.079	9.835	0.706	0.747	1.031

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	228	95	110	141	74	928	177	0	159
N.S.	1	1.07	0.45	0.52	0.66	0.35	4.36	0.83	0.00	0.75
time (sec)	N/A	1.006	0.581	1.035	0.078	0.079	10.416	0.660	0.855	1.373

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	292	117	146	179	96	1221	203	0	224
N.S.	1	1.09	0.43	0.54	0.67	0.36	4.54	0.75	0.00	0.83
time (sec)	N/A	1.248	0.696	0.925	0.075	0.082	10.256	0.550	0.874	2.467

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	296	139	175	0	118	367	249	97	262
N.S.	1	1.09	0.51	0.65	0.00	0.44	1.35	0.92	0.36	0.97
time (sec)	N/A	1.340	1.126	1.059	0.000	0.076	0.580	0.775	0.336	3.931

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	320	161	210	0	140	435	301	99	308
N.S.	1	1.06	0.53	0.70	0.00	0.47	1.45	1.00	0.33	1.02
time (sec)	N/A	1.397	1.490	1.240	0.000	0.076	0.664	0.818	0.330	6.927

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	125	156	229	0	208	0	0	51	0
N.S.	1	1.02	1.27	1.86	0.00	1.69	0.00	0.00	0.41	0.00
time (sec)	N/A	0.577	1.965	33.579	0.000	0.078	0.000	0.000	0.198	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	95	57	102	0	154	0	0	51	0
N.S.	1	1.01	0.61	1.09	0.00	1.64	0.00	0.00	0.54	0.00
time (sec)	N/A	0.456	0.751	32.657	0.000	0.101	0.000	0.000	0.223	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	91	102	196	0	116	0	0	45	0
N.S.	1	1.01	1.13	2.18	0.00	1.29	0.00	0.00	0.50	0.00
time (sec)	N/A	0.450	0.918	7.308	0.000	0.078	0.000	0.000	0.227	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	44	82	0	59	0	0	29	40
N.S.	1	1.00	0.73	1.37	0.00	0.98	0.00	0.00	0.48	0.67
time (sec)	N/A	0.330	0.527	5.345	0.000	0.075	0.000	0.000	0.205	0.548

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	73	194	0	27	0	0	52	0
N.S.	1	1.00	1.22	3.23	0.00	0.45	0.00	0.00	0.87	0.00
time (sec)	N/A	0.335	0.655	4.981	0.000	0.079	0.000	0.000	0.220	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	97	62	102	0	76	0	0	52	0
N.S.	1	1.01	0.65	1.06	0.00	0.79	0.00	0.00	0.54	0.00
time (sec)	N/A	0.456	0.686	6.140	0.000	0.089	0.000	0.000	0.170	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	97	99	210	0	109	0	0	52	0
N.S.	1	1.01	1.03	2.19	0.00	1.14	0.00	0.00	0.54	0.00
time (sec)	N/A	0.437	1.054	10.004	0.000	0.091	0.000	0.000	0.168	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	133	121	116	0	118	0	0	52	0
N.S.	1	1.06	0.97	0.93	0.00	0.94	0.00	0.00	0.42	0.00
time (sec)	N/A	0.579	1.079	10.941	0.000	0.097	0.000	0.000	0.168	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	135	267	220	0	166	0	0	74	0
N.S.	1	0.98	1.93	1.59	0.00	1.20	0.00	0.00	0.54	0.00
time (sec)	N/A	0.656	2.986	13.818	0.000	0.088	0.000	0.000	0.191	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	104	67	92	0	108	0	0	52	0
N.S.	1	0.98	0.63	0.87	0.00	1.02	0.00	0.00	0.49	0.00
time (sec)	N/A	0.497	1.463	10.220	0.000	0.098	0.000	0.000	0.205	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	111	132	379	0	69	0	0	83	0
N.S.	1	1.04	1.23	3.54	0.00	0.64	0.00	0.00	0.78	0.00
time (sec)	N/A	0.500	2.070	9.870	0.000	0.091	0.000	0.000	0.211	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	114	102	0	82	0	0	83	0
N.S.	1	1.00	1.34	1.20	0.00	0.96	0.00	0.00	0.98	0.00
time (sec)	N/A	0.392	1.484	10.615	0.000	0.085	0.000	0.000	0.198	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	114	226	0	94	0	0	83	0
N.S.	1	1.00	1.34	2.66	0.00	1.11	0.00	0.00	0.98	0.00
time (sec)	N/A	0.389	1.893	13.750	0.000	0.080	0.000	0.000	0.231	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	121	133	116	0	96	0	0	83	0
N.S.	1	1.04	1.15	1.00	0.00	0.83	0.00	0.00	0.72	0.00
time (sec)	N/A	0.510	1.778	15.165	0.000	0.081	0.000	0.000	0.222	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	121	133	236	0	131	0	0	83	0
N.S.	1	1.04	1.15	2.03	0.00	1.13	0.00	0.00	0.72	0.00
time (sec)	N/A	0.524	2.445	24.535	0.000	0.111	0.000	0.000	0.204	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	157	155	128	0	142	0	0	83	0
N.S.	1	1.07	1.05	0.87	0.00	0.97	0.00	0.00	0.56	0.00
time (sec)	N/A	0.640	2.102	28.623	0.000	0.086	0.000	0.000	0.225	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	212	161	272	0	316	0	0	108	0
N.S.	1	1.05	0.80	1.35	0.00	1.56	0.00	0.00	0.53	0.00
time (sec)	N/A	1.044	4.565	5.877	0.000	0.093	0.000	0.000	0.201	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	182	89	134	0	258	0	0	108	0
N.S.	1	1.04	0.51	0.77	0.00	1.47	0.00	0.00	0.62	0.00
time (sec)	N/A	0.887	2.589	5.246	0.000	0.099	0.000	0.000	0.211	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	178	129	242	0	208	0	0	98	0
N.S.	1	1.02	0.74	1.38	0.00	1.19	0.00	0.00	0.56	0.00
time (sec)	N/A	0.838	3.192	8.317	0.000	0.078	0.000	0.000	0.229	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	147	79	103	0	148	0	0	72	0
N.S.	1	1.06	0.57	0.74	0.00	1.06	0.00	0.00	0.52	0.00
time (sec)	N/A	0.687	2.304	7.858	0.000	0.110	0.000	0.000	0.210	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	154	101	473	0	123	0	0	112	0
N.S.	1	1.24	0.81	3.81	0.00	0.99	0.00	0.00	0.90	0.00
time (sec)	N/A	0.722	2.532	7.257	0.000	0.086	0.000	0.000	0.230	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	108	123	115	0	82	0	0	112	0
N.S.	1	0.97	1.11	1.04	0.00	0.74	0.00	0.00	1.01	0.00
time (sec)	N/A	0.519	1.872	7.197	0.000	0.101	0.000	0.000	0.207	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	108	108	324	0	94	0	0	112	0
N.S.	1	0.97	0.97	2.92	0.00	0.85	0.00	0.00	1.01	0.00
time (sec)	N/A	0.527	2.256	8.454	0.000	0.077	0.000	0.000	0.238	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	130	133	129	0	96	0	0	112	0
N.S.	1	1.05	1.07	1.04	0.00	0.77	0.00	0.00	0.90	0.00
time (sec)	N/A	0.610	1.781	11.739	0.000	0.082	0.000	0.000	0.233	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	130	118	258	0	108	0	0	112	0
N.S.	1	1.05	0.95	2.08	0.00	0.87	0.00	0.00	0.90	0.00
time (sec)	N/A	0.635	2.869	18.480	0.000	0.083	0.000	0.000	0.221	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	166	148	139	0	110	0	0	112	0
N.S.	1	1.07	0.95	0.90	0.00	0.71	0.00	0.00	0.72	0.00
time (sec)	N/A	0.801	2.002	21.196	0.000	0.109	0.000	0.000	0.235	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	166	145	282	0	145	0	0	112	0
N.S.	1	1.07	0.94	1.82	0.00	0.94	0.00	0.00	0.72	0.00
time (sec)	N/A	0.770	4.171	30.819	0.000	0.093	0.000	0.000	0.174	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	202	170	151	0	156	0	0	112	0
N.S.	1	1.09	0.91	0.81	0.00	0.84	0.00	0.00	0.60	0.00
time (sec)	N/A	0.923	2.499	32.798	0.000	0.092	0.000	0.000	0.183	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	221	327	269	0	250	0	0	125	0
N.S.	1	1.03	1.52	1.25	0.00	1.16	0.00	0.00	0.58	0.00
time (sec)	N/A	1.048	6.914	17.098	0.000	0.106	0.000	0.000	0.232	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	190	101	119	0	188	0	0	93	0
N.S.	1	1.04	0.55	0.65	0.00	1.03	0.00	0.00	0.51	0.00
time (sec)	N/A	0.867	2.677	14.788	0.000	0.088	0.000	0.000	0.189	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	179	123	498	0	164	0	0	138	0
N.S.	1	1.01	0.69	2.80	0.00	0.92	0.00	0.00	0.78	0.00
time (sec)	N/A	0.838	4.792	13.465	0.000	0.109	0.000	0.000	0.238	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	150	130	129	0	129	0	0	138	0
N.S.	1	1.03	0.89	0.88	0.00	0.88	0.00	0.00	0.95	0.00
time (sec)	N/A	0.692	2.830	12.387	0.000	0.095	0.000	0.000	0.209	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	159	110	496	0	94	0	0	138	0
N.S.	1	1.02	0.71	3.18	0.00	0.60	0.00	0.00	0.88	0.00
time (sec)	N/A	0.712	3.766	14.069	0.000	0.106	0.000	0.000	0.236	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	133	133	129	0	96	0	0	138	0
N.S.	1	1.06	1.06	1.03	0.00	0.77	0.00	0.00	1.10	0.00
time (sec)	N/A	0.573	2.552	17.347	0.000	0.111	0.000	0.000	0.235	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	133	108	270	0	108	0	0	138	0
N.S.	1	1.06	0.86	2.16	0.00	0.86	0.00	0.00	1.10	0.00
time (sec)	N/A	0.577	3.978	26.880	0.000	0.081	0.000	0.000	0.202	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	169	148	139	0	110	0	0	138	0
N.S.	1	1.08	0.95	0.89	0.00	0.71	0.00	0.00	0.88	0.00
time (sec)	N/A	0.722	2.403	31.253	0.000	0.092	0.000	0.000	0.221	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	169	121	278	0	122	0	0	138	0
N.S.	1	1.08	0.78	1.78	0.00	0.78	0.00	0.00	0.88	0.00
time (sec)	N/A	0.711	7.971	42.074	0.000	0.094	0.000	0.000	0.181	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	205	155	151	0	124	0	0	138	0
N.S.	1	1.10	0.83	0.81	0.00	0.66	0.00	0.00	0.74	0.00
time (sec)	N/A	0.870	3.002	46.421	0.000	0.099	0.000	0.000	0.185	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	135	128	235	0	205	0	0	39	0
N.S.	1	0.99	0.94	1.73	0.00	1.51	0.00	0.00	0.29	0.00
time (sec)	N/A	0.636	2.148	4.261	0.000	0.086	0.000	0.000	0.162	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	62	105	0	152	0	0	39	0
N.S.	1	1.00	0.59	1.00	0.00	1.45	0.00	0.00	0.37	0.00
time (sec)	N/A	0.500	1.531	3.804	0.000	0.103	0.000	0.000	0.164	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	102	200	0	123	0	0	39	0
N.S.	1	1.00	1.01	1.98	0.00	1.22	0.00	0.00	0.39	0.00
time (sec)	N/A	0.505	1.694	2.863	0.000	0.087	0.000	0.000	0.191	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	49	87	0	63	0	0	39	0
N.S.	1	1.00	0.70	1.24	0.00	0.90	0.00	0.00	0.56	0.00
time (sec)	N/A	0.386	1.208	2.237	0.000	0.083	0.000	0.000	0.188	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	74	477	0	96	0	0	35	0
N.S.	1	1.00	1.06	6.81	0.00	1.37	0.00	0.00	0.50	0.00
time (sec)	N/A	0.391	1.238	1.967	0.000	0.078	0.000	0.000	0.219	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	83	161	0	90	0	0	28	0
N.S.	1	1.00	1.04	2.01	0.00	1.12	0.00	0.00	0.35	0.00
time (sec)	N/A	0.386	0.964	2.804	0.000	0.093	0.000	0.000	0.213	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	109	407	0	107	0	0	42	0
N.S.	1	1.00	1.36	5.09	0.00	1.34	0.00	0.00	0.52	0.00
time (sec)	N/A	0.391	1.443	3.683	0.000	0.090	0.000	0.000	0.213	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	116	125	118	0	115	0	0	46	0
N.S.	1	1.02	1.10	1.04	0.00	1.01	0.00	0.00	0.40	0.00
time (sec)	N/A	0.520	1.424	3.772	0.000	0.091	0.000	0.000	0.191	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	116	134	444	0	129	0	0	46	0
N.S.	1	1.02	1.18	3.89	0.00	1.13	0.00	0.00	0.40	0.00
time (sec)	N/A	0.519	1.903	4.197	0.000	0.095	0.000	0.000	0.211	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	152	142	128	0	137	0	0	46	0
N.S.	1	1.05	0.98	0.88	0.00	0.94	0.00	0.00	0.32	0.00
time (sec)	N/A	0.648	1.739	10.991	0.000	0.090	0.000	0.000	0.164	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	188	302	258	0	256	0	0	49	0
N.S.	1	1.03	1.65	1.41	0.00	1.40	0.00	0.00	0.27	0.00
time (sec)	N/A	0.819	2.755	5.671	0.000	0.082	0.000	0.000	0.150	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	158	85	121	0	201	0	0	49	0
N.S.	1	1.04	0.56	0.80	0.00	1.32	0.00	0.00	0.32	0.00
time (sec)	N/A	0.670	1.456	5.110	0.000	0.090	0.000	0.000	0.154	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	154	123	222	0	170	0	0	49	0
N.S.	1	1.01	0.81	1.46	0.00	1.12	0.00	0.00	0.32	0.00
time (sec)	N/A	0.671	1.823	3.956	0.000	0.080	0.000	0.000	0.155	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	120	67	95	0	115	0	0	49	0
N.S.	1	1.01	0.56	0.80	0.00	0.97	0.00	0.00	0.41	0.00
time (sec)	N/A	0.502	1.322	3.188	0.000	0.092	0.000	0.000	0.188	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	116	80	219	0	100	0	0	49	0
N.S.	1	1.01	0.70	1.90	0.00	0.87	0.00	0.00	0.43	0.00
time (sec)	N/A	0.530	1.436	4.126	0.000	0.082	0.000	0.000	0.216	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	101	164	0	97	0	0	49	0
N.S.	1	1.00	1.12	1.82	0.00	1.08	0.00	0.00	0.54	0.00
time (sec)	N/A	0.396	1.267	3.485	0.000	0.133	0.000	0.000	0.180	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	102	428	0	108	0	0	45	0
N.S.	1	1.00	1.13	4.76	0.00	1.20	0.00	0.00	0.50	0.00
time (sec)	N/A	0.418	1.217	3.408	0.000	0.079	0.000	0.000	0.207	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	126	112	177	0	101	0	0	38	0
N.S.	1	1.09	0.97	1.53	0.00	0.87	0.00	0.00	0.33	0.00
time (sec)	N/A	0.525	1.047	3.358	0.000	0.079	0.000	0.000	0.203	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	126	123	480	0	118	0	0	55	0
N.S.	1	1.09	1.06	4.14	0.00	1.02	0.00	0.00	0.47	0.00
time (sec)	N/A	0.549	1.969	4.465	0.000	0.103	0.000	0.000	0.189	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	162	134	128	0	126	0	0	76	0
N.S.	1	1.08	0.89	0.85	0.00	0.84	0.00	0.00	0.51	0.00
time (sec)	N/A	0.688	1.461	4.245	0.000	0.106	0.000	0.000	0.171	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	162	149	520	0	140	0	0	82	0
N.S.	1	1.08	0.99	3.47	0.00	0.93	0.00	0.00	0.55	0.00
time (sec)	N/A	0.678	2.655	4.533	0.000	0.094	0.000	0.000	0.177	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	198	151	138	0	148	0	0	82	0
N.S.	1	1.09	0.83	0.76	0.00	0.82	0.00	0.00	0.45	0.00
time (sec)	N/A	0.838	1.829	17.704	0.000	0.098	0.000	0.000	0.159	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	188	128	246	0	213	0	0	61	0
N.S.	1	1.06	0.72	1.38	0.00	1.20	0.00	0.00	0.34	0.00
time (sec)	N/A	0.931	2.173	4.781	0.000	0.085	0.000	0.000	0.162	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	154	74	107	0	158	0	0	61	0
N.S.	1	1.09	0.52	0.76	0.00	1.12	0.00	0.00	0.43	0.00
time (sec)	N/A	0.735	1.557	3.793	0.000	0.095	0.000	0.000	0.176	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	150	93	229	0	150	0	0	61	0
N.S.	1	1.06	0.66	1.62	0.00	1.06	0.00	0.00	0.43	0.00
time (sec)	N/A	0.701	1.796	4.223	0.000	0.078	0.000	0.000	0.178	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	123	125	117	0	97	0	0	61	0
N.S.	1	1.06	1.08	1.01	0.00	0.84	0.00	0.00	0.53	0.00
time (sec)	N/A	0.580	1.403	4.008	0.000	0.081	0.000	0.000	0.219	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	123	117	446	0	114	0	0	61	0
N.S.	1	1.06	1.01	3.84	0.00	0.98	0.00	0.00	0.53	0.00
time (sec)	N/A	0.583	1.546	3.633	0.000	0.079	0.000	0.000	0.204	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	133	104	191	0	111	0	0	61	0
N.S.	1	1.01	0.79	1.45	0.00	0.84	0.00	0.00	0.46	0.00
time (sec)	N/A	0.568	1.469	3.197	0.000	0.086	0.000	0.000	0.230	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	133	140	497	0	120	0	0	57	0
N.S.	1	1.01	1.06	3.77	0.00	0.91	0.00	0.00	0.43	0.00
time (sec)	N/A	0.583	1.591	3.572	0.000	0.077	0.000	0.000	0.226	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	170	129	143	0	112	0	0	50	0
N.S.	1	1.12	0.85	0.94	0.00	0.74	0.00	0.00	0.33	0.00
time (sec)	N/A	0.703	1.225	3.405	0.000	0.088	0.000	0.000	0.219	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	170	145	279	0	129	0	0	78	0
N.S.	1	1.12	0.95	1.84	0.00	0.85	0.00	0.00	0.51	0.00
time (sec)	N/A	0.724	1.998	4.492	0.000	0.098	0.000	0.000	0.203	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	206	151	151	0	137	0	0	86	0
N.S.	1	1.11	0.81	0.81	0.00	0.74	0.00	0.00	0.46	0.00
time (sec)	N/A	0.942	1.587	4.589	0.000	0.093	0.000	0.000	0.230	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	203	124	257	0	193	0	0	69	0
N.S.	1	1.06	0.65	1.34	0.00	1.01	0.00	0.00	0.36	0.00
time (sec)	N/A	0.902	2.044	5.438	0.000	0.081	0.000	0.000	0.180	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	171	134	122	0	147	0	0	69	0
N.S.	1	1.09	0.85	0.78	0.00	0.94	0.00	0.00	0.44	0.00
time (sec)	N/A	0.718	1.489	4.748	0.000	0.110	0.000	0.000	0.173	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	169	106	444	0	114	0	0	69	0
N.S.	1	1.04	0.65	2.72	0.00	0.70	0.00	0.00	0.42	0.00
time (sec)	N/A	0.726	1.538	4.493	0.000	0.078	0.000	0.000	0.179	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	143	137	191	0	111	0	0	69	0
N.S.	1	1.08	1.04	1.45	0.00	0.84	0.00	0.00	0.52	0.00
time (sec)	N/A	0.589	1.446	3.801	0.000	0.088	0.000	0.000	0.200	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	143	149	501	0	128	0	0	69	0
N.S.	1	1.08	1.13	3.80	0.00	0.97	0.00	0.00	0.52	0.00
time (sec)	N/A	0.590	1.661	3.726	0.000	0.080	0.000	0.000	0.234	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	179	144	144	0	125	0	0	69	0
N.S.	1	1.10	0.88	0.88	0.00	0.77	0.00	0.00	0.42	0.00
time (sec)	N/A	0.726	1.411	3.616	0.000	0.100	0.000	0.000	0.201	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	179	142	291	0	132	0	0	65	0
N.S.	1	1.10	0.87	1.79	0.00	0.81	0.00	0.00	0.40	0.00
time (sec)	N/A	0.732	1.898	3.863	0.000	0.088	0.000	0.000	0.225	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	214	137	153	0	123	0	0	58	0
N.S.	1	1.12	0.72	0.80	0.00	0.64	0.00	0.00	0.30	0.00
time (sec)	N/A	0.914	1.148	3.807	0.000	0.077	0.000	0.000	0.235	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	71	0	0	0	0	0	34	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.455	0.242	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	61	0	0	0	0	0	32	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.430	0.156	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	61	0	0	0	0	0	35	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.456	0.124	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	63	0	0	0	0	0	35	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.443	0.207	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	111	0	0	0	0	0	58	0
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.463	0.397	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	102	0	0	0	0	0	56	0
N.S.	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.446	0.340	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	80	103	0	0	0	0	0	59	0
N.S.	1	0.96	1.24	0.00	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.449	0.321	0.000	0.000	0.000	0.000	0.000	0.246	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	82	105	0	0	0	0	0	59	0
N.S.	1	0.96	1.24	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.453	0.472	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	84	0	0	0	0	0	30	0
N.S.	1	0.99	1.00	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.469	1.432	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	81	103	0	0	0	0	0	30	0
N.S.	1	0.99	1.26	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.448	1.179	0.000	0.000	0.000	0.000	0.000	0.239	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	112	0	0	0	0	0	36	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.439	1.587	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	119	0	0	0	0	0	36	0
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.471	1.791	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	86	128	0	0	0	0	0	40	0
N.S.	1	1.02	1.52	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.470	1.495	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	86	121	0	0	0	0	0	40	0
N.S.	1	1.02	1.44	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.462	1.265	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	141	0	0	0	0	0	57	0
N.S.	1	1.00	1.99	0.00	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.468	1.998	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	143	0	0	0	0	0	57	0
N.S.	1	1.00	2.01	0.00	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.472	1.736	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	102	70	82	76	154	0	0	60	474
N.S.	1	0.87	0.60	0.70	0.65	1.32	0.00	0.00	0.51	4.05
time (sec)	N/A	0.281	0.304	0.913	0.030	0.094	0.000	0.000	0.235	9.403

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	78	58	63	58	119	0	0	60	352
N.S.	1	0.89	0.66	0.72	0.66	1.35	0.00	0.00	0.68	4.00
time (sec)	N/A	0.276	0.190	0.503	0.035	0.085	0.000	0.000	0.216	4.240

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	54	48	44	40	84	0	0	60	230
N.S.	1	0.92	0.81	0.75	0.68	1.42	0.00	0.00	1.02	3.90
time (sec)	N/A	0.267	0.143	0.504	0.055	0.102	0.000	0.000	0.195	3.482

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	21	46	0	0	60	82
N.S.	1	1.00	1.00	0.83	0.72	1.59	0.00	0.00	2.07	2.83
time (sec)	N/A	0.254	0.063	0.366	0.028	0.087	0.000	0.000	0.191	0.652

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	112	51	328	122	253	0	0	25	0
N.S.	1	0.90	0.41	2.65	0.98	2.04	0.00	0.00	0.20	0.00
time (sec)	N/A	0.290	0.110	4.437	0.116	0.088	0.000	0.000	0.162	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	191	53	351	176	275	0	0	25	0
N.S.	1	0.97	0.27	1.78	0.89	1.40	0.00	0.00	0.13	0.00
time (sec)	N/A	0.321	0.113	4.237	0.131	0.110	0.000	0.000	0.177	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	274	270	53	371	230	297	0	0	25	0
N.S.	1	0.99	0.19	1.35	0.84	1.08	0.00	0.00	0.09	0.00
time (sec)	N/A	0.351	0.223	4.659	0.137	0.100	0.000	0.000	0.222	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	155	95	115	0	132	0	0	60	289
N.S.	1	1.05	0.65	0.78	0.00	0.90	0.00	0.00	0.41	1.97
time (sec)	N/A	0.687	0.875	3.222	0.000	0.116	0.000	0.000	0.216	5.593

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	114	77	89	0	97	0	0	60	102
N.S.	1	1.04	0.70	0.81	0.00	0.88	0.00	0.00	0.55	0.93
time (sec)	N/A	0.518	0.597	2.811	0.000	0.126	0.000	0.000	0.245	3.318

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	63	222	62	0	0	60	88
N.S.	1	1.00	0.86	0.86	3.04	0.85	0.00	0.00	0.82	1.21
time (sec)	N/A	0.364	0.480	2.664	16.067	0.089	0.000	0.000	0.242	3.264

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	39	52	0	25	0	0	56	61
N.S.	1	1.00	1.26	1.68	0.00	0.81	0.00	0.00	1.81	1.97
time (sec)	N/A	0.215	0.262	2.600	0.000	0.075	0.000	0.000	0.219	0.375

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	87	320	774	184	0	0	23	0
N.S.	1	1.00	1.05	3.86	9.33	2.22	0.00	0.00	0.28	0.00
time (sec)	N/A	0.352	0.598	4.364	0.250	0.092	0.000	0.000	0.218	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	163	126	340	935	245	0	0	25	0
N.S.	1	1.06	0.82	2.21	6.07	1.59	0.00	0.00	0.16	0.00
time (sec)	N/A	0.645	0.649	4.665	0.270	0.095	0.000	0.000	0.161	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	245	152	361	2220	267	0	0	25	0
N.S.	1	1.10	0.68	1.62	9.96	1.20	0.00	0.00	0.11	0.00
time (sec)	N/A	1.021	0.844	4.475	0.404	0.105	0.000	0.000	0.156	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	102	73	82	76	170	0	0	62	544
N.S.	1	0.87	0.62	0.70	0.65	1.45	0.00	0.00	0.53	4.65
time (sec)	N/A	0.274	0.380	0.549	0.034	0.117	0.000	0.000	0.217	12.431

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	78	61	63	58	134	0	0	62	420
N.S.	1	0.89	0.69	0.72	0.66	1.52	0.00	0.00	0.70	4.77
time (sec)	N/A	0.265	0.236	0.553	0.042	0.096	0.000	0.000	0.207	4.512

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	54	51	44	40	98	0	0	62	296
N.S.	1	0.92	0.86	0.75	0.68	1.66	0.00	0.00	1.05	5.02
time (sec)	N/A	0.265	0.149	0.555	0.043	0.093	0.000	0.000	0.225	3.514

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	21	59	0	0	62	153
N.S.	1	1.00	1.00	0.83	0.72	2.03	0.00	0.00	2.14	5.28
time (sec)	N/A	0.221	0.092	0.483	0.044	0.078	0.000	0.000	0.201	1.513

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	97	84	92	396	98	244	0	0	57	0
N.S.	1	0.87	0.95	4.08	1.01	2.52	0.00	0.00	0.59	0.00
time (sec)	N/A	0.259	0.154	4.713	0.144	0.082	0.000	0.000	0.187	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	170	159	53	772	158	287	0	0	57	0
N.S.	1	0.94	0.31	4.54	0.93	1.69	0.00	0.00	0.34	0.00
time (sec)	N/A	0.314	0.103	4.344	0.138	0.086	0.000	0.000	0.194	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	247	238	53	815	212	311	0	0	57	0
N.S.	1	0.96	0.21	3.30	0.86	1.26	0.00	0.00	0.23	0.00
time (sec)	N/A	0.357	0.135	5.057	0.117	0.090	0.000	0.000	0.212	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	155	109	100	994	125	0	0	62	293
N.S.	1	1.05	0.74	0.68	6.76	0.85	0.00	0.00	0.42	1.99
time (sec)	N/A	0.717	1.160	2.872	9.297	0.102	0.000	0.000	0.245	4.656

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	114	91	74	580	89	0	0	62	103
N.S.	1	1.04	0.83	0.67	5.27	0.81	0.00	0.00	0.56	0.94
time (sec)	N/A	0.535	0.674	2.624	0.472	0.087	0.000	0.000	0.193	3.319

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	57	48	0	53	0	0	58	98
N.S.	1	1.00	0.83	0.70	0.00	0.77	0.00	0.00	0.84	1.42
time (sec)	N/A	0.324	0.363	2.446	0.000	0.080	0.000	0.000	0.190	1.727

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	201	40	0	0	53	60
N.S.	1	1.00	1.00	0.94	6.48	1.29	0.00	0.00	1.71	1.94
time (sec)	N/A	0.238	0.308	3.855	0.192	0.085	0.000	0.000	0.184	0.254

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	122	123	101	735	884	222	0	0	57	0
N.S.	1	1.01	0.83	6.02	7.25	1.82	0.00	0.00	0.47	0.00
time (sec)	N/A	0.514	0.913	4.890	0.279	0.083	0.000	0.000	0.207	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	192	203	160	795	0	274	0	0	57	0
N.S.	1	1.06	0.83	4.14	0.00	1.43	0.00	0.00	0.30	0.00
time (sec)	N/A	0.845	1.387	4.830	0.000	0.097	0.000	0.000	0.250	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	102	73	82	76	190	0	0	97	626
N.S.	1	0.87	0.62	0.70	0.65	1.62	0.00	0.00	0.83	5.35
time (sec)	N/A	0.289	0.487	0.192	0.037	0.123	0.000	0.000	0.231	12.903

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	78	61	63	58	152	0	0	97	498
N.S.	1	0.89	0.69	0.72	0.66	1.73	0.00	0.00	1.10	5.66
time (sec)	N/A	0.275	0.292	0.162	0.038	0.096	0.000	0.000	0.242	8.853

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	54	51	44	40	114	0	0	97	370
N.S.	1	0.92	0.86	0.75	0.68	1.93	0.00	0.00	1.64	6.27
time (sec)	N/A	0.270	0.237	100.755	0.038	0.106	0.000	0.000	0.226	3.765

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	21	73	0	0	97	242
N.S.	1	1.00	1.00	0.83	0.72	2.52	0.00	0.00	3.34	8.34
time (sec)	N/A	0.242	0.136	2.279	0.028	0.093	0.000	0.000	0.221	3.699

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	89	76	83	425	98	236	0	0	92	0
N.S.	1	0.85	0.93	4.78	1.10	2.65	0.00	0.00	1.03	0.00
time (sec)	N/A	0.274	0.440	6.140	0.118	0.116	0.000	0.000	0.207	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	141	131	53	472	140	263	0	0	92	0
N.S.	1	0.93	0.38	3.35	0.99	1.87	0.00	0.00	0.65	0.00
time (sec)	N/A	0.303	0.080	67.116	0.115	0.089	0.000	0.000	0.213	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	218	206	53	1315	194	309	0	0	92	0
N.S.	1	0.94	0.24	6.03	0.89	1.42	0.00	0.00	0.42	0.00
time (sec)	N/A	0.323	0.108	2.571	0.114	0.097	0.000	0.000	0.285	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	155	103	92	0	121	0	0	97	301
N.S.	1	1.05	0.70	0.63	0.00	0.82	0.00	0.00	0.66	2.05
time (sec)	N/A	0.716	1.102	21.631	0.000	0.097	0.000	0.000	0.239	5.129

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	108	93	66	0	83	0	0	91	105
N.S.	1	1.04	0.89	0.63	0.00	0.80	0.00	0.00	0.88	1.01
time (sec)	N/A	0.445	0.528	2.594	0.000	0.086	0.000	0.000	0.207	3.410

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	46	41	331	45	0	0	86	64
N.S.	1	1.00	0.71	0.63	5.09	0.69	0.00	0.00	1.32	0.98
time (sec)	N/A	0.358	0.460	6.200	0.219	0.080	0.000	0.000	0.201	0.451

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	69	51	328	59	0	0	92	89
N.S.	1	1.00	1.97	1.46	9.37	1.69	0.00	0.00	2.63	2.54
time (sec)	N/A	0.246	0.689	14.982	0.216	0.094	0.000	0.000	0.211	0.977

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	159	163	118	1279	1076	244	0	0	92	0
N.S.	1	1.03	0.74	8.04	6.77	1.53	0.00	0.00	0.58	0.00
time (sec)	N/A	0.674	1.262	285.869	0.288	0.091	0.000	0.000	0.258	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	231	243	155	1343	0	300	0	0	92	0
N.S.	1	1.05	0.67	5.81	0.00	1.30	0.00	0.00	0.40	0.00
time (sec)	N/A	1.043	1.688	3.422	0.000	0.102	0.000	0.000	0.226	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	102	73	82	76	202	0	0	131	690
N.S.	1	0.87	0.62	0.70	0.65	1.73	0.00	0.00	1.12	5.90
time (sec)	N/A	0.280	0.424	0.176	0.050	0.144	0.000	0.000	0.299	12.769

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	78	63	63	58	164	0	0	131	562
N.S.	1	0.89	0.72	0.72	0.66	1.86	0.00	0.00	1.49	6.39
time (sec)	N/A	0.266	0.383	0.154	0.041	0.104	0.000	0.000	0.275	13.158

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	54	51	44	40	126	0	0	131	434
N.S.	1	0.92	0.86	0.75	0.68	2.14	0.00	0.00	2.22	7.36
time (sec)	N/A	0.265	0.229	100.940	0.037	0.102	0.000	0.000	0.265	4.970

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	21	85	0	0	131	306
N.S.	1	1.00	1.00	0.83	0.72	2.93	0.00	0.00	4.52	10.55
time (sec)	N/A	0.248	0.170	2.286	0.032	0.084	0.000	0.000	0.240	3.610

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	116	100	51	462	117	235	0	0	125	0
N.S.	1	0.86	0.44	3.98	1.01	2.03	0.00	0.00	1.08	0.00
time (sec)	N/A	0.287	0.123	10.759	0.124	0.093	0.000	0.000	0.243	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	141	125	53	485	138	261	0	0	125	0
N.S.	1	0.89	0.38	3.44	0.98	1.85	0.00	0.00	0.89	0.00
time (sec)	N/A	0.300	0.105	66.063	0.139	0.082	0.000	0.000	0.239	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	189	178	53	532	176	277	0	0	125	0
N.S.	1	0.94	0.28	2.81	0.93	1.47	0.00	0.00	0.66	0.00
time (sec)	N/A	0.310	0.106	2.202	0.126	0.113	0.000	0.000	0.246	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	147	109	76	0	109	0	0	123	286
N.S.	1	1.06	0.78	0.55	0.00	0.78	0.00	0.00	0.88	2.06
time (sec)	N/A	0.608	0.965	2.756	0.000	0.085	0.000	0.000	0.278	3.337

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	59	50	418	71	0	0	117	102
N.S.	1	1.00	0.57	0.48	4.02	0.68	0.00	0.00	1.12	0.98
time (sec)	N/A	0.478	0.723	7.050	0.226	0.085	0.000	0.000	0.263	1.923

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	86	61	504	59	0	0	125	85
N.S.	1	1.00	1.21	0.86	7.10	0.83	0.00	0.00	1.76	1.20
time (sec)	N/A	0.380	0.951	15.208	0.231	0.080	0.000	0.000	0.257	0.919

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	73	62	454	73	0	0	125	112
N.S.	1	1.00	2.09	1.77	12.97	2.09	0.00	0.00	3.57	3.20
time (sec)	N/A	0.250	1.292	3.248	0.276	0.086	0.000	0.000	0.258	2.497

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	196	203	131	1926	1253	258	0	0	125	0
N.S.	1	1.04	0.67	9.83	6.39	1.32	0.00	0.00	0.64	0.00
time (sec)	N/A	0.879	2.320	3.976	0.432	0.097	0.000	0.000	0.234	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	268	283	188	2005	0	314	0	0	125	0
N.S.	1	1.06	0.70	7.48	0.00	1.17	0.00	0.00	0.47	0.00
time (sec)	N/A	1.292	3.548	6.883	0.000	0.114	0.000	0.000	0.232	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	342	365	194	2085	0	342	0	0	125	0
N.S.	1	1.07	0.57	6.10	0.00	1.00	0.00	0.00	0.37	0.00
time (sec)	N/A	1.748	5.750	7.722	0.000	0.145	0.000	0.000	0.255	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	102	73	82	297	150	0	0	139	434
N.S.	1	0.87	0.62	0.70	2.54	1.28	0.00	0.00	1.19	3.71
time (sec)	N/A	0.272	0.271	1.130	0.045	0.098	0.000	0.000	0.229	6.251

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	78	63	63	169	113	0	0	139	306
N.S.	1	0.89	0.72	0.72	1.92	1.28	0.00	0.00	1.58	3.48
time (sec)	N/A	0.262	0.167	1.135	0.049	0.085	0.000	0.000	0.240	3.618

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	54	49	44	79	76	0	0	139	155
N.S.	1	0.92	0.83	0.75	1.34	1.29	0.00	0.00	2.36	2.63
time (sec)	N/A	0.259	0.118	1.120	0.042	0.083	0.000	0.000	0.233	1.446

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	21	37	0	0	42	47
N.S.	1	1.00	1.00	0.89	0.78	1.37	0.00	0.00	1.56	1.74
time (sec)	N/A	0.233	0.065	0.826	0.028	0.076	0.000	0.000	0.214	0.177

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	144	51	304	138	271	0	0	84	0
N.S.	1	0.96	0.34	2.03	0.92	1.81	0.00	0.00	0.56	0.00
time (sec)	N/A	0.293	0.116	9.303	0.117	0.093	0.000	0.000	0.185	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	53	312	192	293	0	0	84	0
N.S.	1	1.00	0.24	1.40	0.86	1.31	0.00	0.00	0.38	0.00
time (sec)	N/A	0.336	0.193	7.745	0.127	0.093	0.000	0.000	0.202	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	302	53	350	246	315	0	0	84	0
N.S.	1	1.01	0.18	1.17	0.82	1.05	0.00	0.00	0.28	0.00
time (sec)	N/A	0.367	0.325	7.716	0.129	0.093	0.000	0.000	0.162	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	155	95	111	608	153	0	0	139	301
N.S.	1	1.05	0.65	0.76	4.14	1.04	0.00	0.00	0.95	2.05
time (sec)	N/A	0.749	1.084	6.714	0.259	0.140	0.000	0.000	0.175	6.321

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	114	77	91	474	116	0	0	139	105
N.S.	1	1.04	0.70	0.83	4.31	1.05	0.00	0.00	1.26	0.95
time (sec)	N/A	0.540	0.755	4.909	0.216	0.103	0.000	0.000	0.185	3.360

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	65	72	340	79	0	0	139	91
N.S.	1	1.00	0.89	0.99	4.66	1.08	0.00	0.00	1.90	1.25
time (sec)	N/A	0.392	0.535	4.631	0.182	0.094	0.000	0.000	0.216	5.908

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	40	44	206	40	0	0	137	98
N.S.	1	1.00	1.14	1.26	5.89	1.14	0.00	0.00	3.91	2.80
time (sec)	N/A	0.235	0.359	4.516	0.133	0.078	0.000	0.000	0.221	1.104

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	70	108	0	149	0	0	130	0
N.S.	1	1.00	1.35	2.08	0.00	2.87	0.00	0.00	2.50	0.00
time (sec)	N/A	0.235	0.399	6.551	0.000	0.113	0.000	0.000	0.176	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	123	96	322	837	245	0	0	80	0
N.S.	1	1.01	0.79	2.64	6.86	2.01	0.00	0.00	0.66	0.00
time (sec)	N/A	0.491	0.567	9.281	0.251	0.087	0.000	0.000	0.167	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	205	117	361	1938	267	0	0	84	0
N.S.	1	1.06	0.61	1.87	10.04	1.38	0.00	0.00	0.44	0.00
time (sec)	N/A	0.857	0.743	9.387	0.382	0.102	0.000	0.000	0.175	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	102	73	82	76	149	0	0	310	370
N.S.	1	0.87	0.62	0.70	0.65	1.27	0.00	0.00	2.65	3.16
time (sec)	N/A	0.277	0.350	1.021	0.039	0.086	0.000	0.000	0.390	5.126

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	78	59	63	58	108	0	0	310	242
N.S.	1	0.89	0.67	0.72	0.66	1.23	0.00	0.00	3.52	2.75
time (sec)	N/A	0.282	0.189	1.004	0.044	0.088	0.000	0.000	0.420	3.941

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	52	37	44	38	67	0	0	310	85
N.S.	1	0.91	0.65	0.77	0.67	1.18	0.00	0.00	5.44	1.49
time (sec)	N/A	0.274	0.100	0.942	0.048	0.077	0.000	0.000	0.427	0.783

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	21	49	0	0	52	67
N.S.	1	1.00	1.00	0.89	0.78	1.81	0.00	0.00	1.93	2.48
time (sec)	N/A	0.245	0.067	0.990	0.039	0.084	0.000	0.000	0.166	0.267

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	179	176	51	378	153	294	0	0	52	0
N.S.	1	0.98	0.28	2.11	0.85	1.64	0.00	0.00	0.29	0.00
time (sec)	N/A	0.323	0.183	9.458	0.126	0.088	0.000	0.000	0.168	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	255	53	421	207	316	0	0	52	0
N.S.	1	1.01	0.21	1.67	0.82	1.25	0.00	0.00	0.21	0.00
time (sec)	N/A	0.353	0.313	8.864	0.131	0.088	0.000	0.000	0.170	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	334	53	462	261	338	0	0	52	0
N.S.	1	1.02	0.16	1.40	0.79	1.03	0.00	0.00	0.16	0.00
time (sec)	N/A	0.379	0.474	9.948	0.142	0.133	0.000	0.000	0.191	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	155	108	134	764	184	0	0	310	301
N.S.	1	1.05	0.73	0.91	5.20	1.25	0.00	0.00	2.11	2.05
time (sec)	N/A	0.751	1.839	9.916	0.300	0.149	0.000	0.000	0.415	7.530

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	114	92	114	626	143	0	0	310	105
N.S.	1	1.04	0.84	1.04	5.69	1.30	0.00	0.00	2.82	0.95
time (sec)	N/A	0.530	1.402	5.385	0.252	0.116	0.000	0.000	0.399	5.757

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	80	86	488	102	0	0	310	91
N.S.	1	1.00	1.10	1.18	6.68	1.40	0.00	0.00	4.25	1.25
time (sec)	N/A	0.398	0.955	4.786	0.217	0.130	0.000	0.000	0.382	3.692

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	59	51	350	59	0	0	310	139
N.S.	1	1.00	1.69	1.46	10.00	1.69	0.00	0.00	8.86	3.97
time (sec)	N/A	0.250	0.679	4.635	0.180	0.112	0.000	0.000	0.364	1.872

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	101	169	813	196	0	0	308	0
N.S.	1	1.00	1.17	1.97	9.45	2.28	0.00	0.00	3.58	0.00
time (sec)	N/A	0.389	0.900	6.523	0.259	0.089	0.000	0.000	0.394	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	95	343	0	246	0	0	50	0
N.S.	1	1.00	1.09	3.94	0.00	2.83	0.00	0.00	0.57	0.00
time (sec)	N/A	0.358	0.672	7.804	0.000	0.094	0.000	0.000	0.166	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	157	163	120	374	1821	270	0	0	50	0
N.S.	1	1.04	0.76	2.38	11.60	1.72	0.00	0.00	0.32	0.00
time (sec)	N/A	0.684	1.002	9.346	0.304	0.085	0.000	0.000	0.174	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	247	145	413	2632	292	0	0	52	0
N.S.	1	1.06	0.62	1.77	11.30	1.25	0.00	0.00	0.22	0.00
time (sec)	N/A	1.070	1.651	9.938	0.414	0.096	0.000	0.000	0.218	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	126	85	101	94	175	0	0	76	434
N.S.	1	0.86	0.58	0.69	0.64	1.20	0.00	0.00	0.52	2.97
time (sec)	N/A	0.305	0.338	1.074	0.039	0.102	0.000	0.000	0.202	6.085

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	102	73	82	76	134	0	0	76	306
N.S.	1	0.87	0.62	0.70	0.65	1.15	0.00	0.00	0.65	2.62
time (sec)	N/A	0.291	0.279	1.054	0.034	0.093	0.000	0.000	0.162	4.068

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	76	51	63	58	93	0	0	76	155
N.S.	1	0.88	0.59	0.73	0.67	1.08	0.00	0.00	0.88	1.80
time (sec)	N/A	0.280	0.160	0.959	0.043	0.090	0.000	0.000	0.164	1.369

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	50	36	42	44	50	0	0	76	72
N.S.	1	0.91	0.65	0.76	0.80	0.91	0.00	0.00	1.38	1.31
time (sec)	N/A	0.265	0.114	0.953	0.031	0.097	0.000	0.000	0.164	0.306

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	21	61	0	0	76	23
N.S.	1	1.00	1.00	0.83	0.72	2.10	0.00	0.00	2.62	0.79
time (sec)	N/A	0.257	0.102	0.965	0.037	0.079	0.000	0.000	0.184	0.747

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	51	449	175	305	0	0	76	0
N.S.	1	1.00	0.25	2.16	0.84	1.47	0.00	0.00	0.37	0.00
time (sec)	N/A	0.340	0.171	10.184	0.121	0.085	0.000	0.000	0.173	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	281	287	53	489	229	327	0	0	76	0
N.S.	1	1.02	0.19	1.74	0.81	1.16	0.00	0.00	0.27	0.00
time (sec)	N/A	0.360	0.454	9.479	0.118	0.091	0.000	0.000	0.165	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	155	112	145	902	199	0	0	76	301
N.S.	1	1.05	0.76	0.99	6.14	1.35	0.00	0.00	0.52	2.05
time (sec)	N/A	0.718	1.897	25.754	0.366	0.181	0.000	0.000	0.160	9.025

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	114	94	125	764	158	0	0	76	105
N.S.	1	1.04	0.85	1.14	6.95	1.44	0.00	0.00	0.69	0.95
time (sec)	N/A	0.554	1.678	6.645	0.392	0.136	0.000	0.000	0.159	6.050

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	80	102	626	117	0	0	76	91
N.S.	1	1.00	1.10	1.40	8.58	1.60	0.00	0.00	1.04	1.25
time (sec)	N/A	0.373	1.544	5.165	0.273	0.108	0.000	0.000	0.215	3.655

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	57	75	488	74	0	0	76	50
N.S.	1	1.00	1.63	2.14	13.94	2.11	0.00	0.00	2.17	1.43
time (sec)	N/A	0.240	1.357	4.891	0.211	0.096	0.000	0.000	0.216	2.228

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	123	129	82	232	1074	270	0	0	76	0
N.S.	1	1.05	0.67	1.89	8.73	2.20	0.00	0.00	0.62	0.00
time (sec)	N/A	0.550	1.264	6.995	0.323	0.082	0.000	0.000	0.198	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	126	149	260	827	245	0	0	76	0
N.S.	1	1.47	1.73	3.02	9.62	2.85	0.00	0.00	0.88	0.00
time (sec)	N/A	0.505	1.330	7.180	0.318	0.089	0.000	0.000	0.208	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	122	127	121	413	0	267	0	0	74	0
N.S.	1	1.04	0.99	3.39	0.00	2.19	0.00	0.00	0.61	0.00
time (sec)	N/A	0.475	0.936	7.970	0.000	0.084	0.000	0.000	0.203	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	192	203	143	433	2297	289	0	0	74	0
N.S.	1	1.06	0.74	2.26	11.96	1.51	0.00	0.00	0.39	0.00
time (sec)	N/A	0.820	1.219	9.645	0.326	0.086	0.000	0.000	0.197	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	270	289	165	480	3789	311	0	0	76	0
N.S.	1	1.07	0.61	1.78	14.03	1.15	0.00	0.00	0.28	0.00
time (sec)	N/A	1.272	1.687	9.088	0.432	0.092	0.000	0.000	0.206	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	126	81	101	94	160	0	0	0	370
N.S.	1	0.86	0.55	0.69	0.64	1.10	0.00	0.00	0.00	2.53
time (sec)	N/A	0.288	0.406	1.077	0.043	0.098	0.000	0.000	1.338	5.454

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	98	61	82	76	119	0	0	0	242
N.S.	1	0.87	0.54	0.73	0.67	1.05	0.00	0.00	0.00	2.14
time (sec)	N/A	0.286	0.251	1.017	0.045	0.081	0.000	0.000	1.353	4.650

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	74	49	63	62	77	0	0	939	110
N.S.	1	0.88	0.58	0.75	0.74	0.92	0.00	0.00	11.18	1.31
time (sec)	N/A	0.274	0.127	0.984	0.049	0.081	0.000	0.000	1.379	0.810

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	52	52	44	32	61	0	0	759	0
N.S.	1	0.91	0.91	0.77	0.56	1.07	0.00	0.00	13.32	0.00
time (sec)	N/A	0.271	0.148	1.069	0.051	0.083	0.000	0.000	1.048	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	21	72	0	0	1123	23
N.S.	1	1.00	1.00	0.83	0.72	2.48	0.00	0.00	38.72	0.79
time (sec)	N/A	0.243	0.139	1.010	0.037	0.080	0.000	0.000	1.049	0.748

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	237	240	51	455	195	316	0	0	169	0
N.S.	1	1.01	0.22	1.92	0.82	1.33	0.00	0.00	0.71	0.00
time (sec)	N/A	0.342	0.420	9.445	0.125	0.087	0.000	0.000	0.480	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	310	319	53	501	249	338	0	0	169	0
N.S.	1	1.03	0.17	1.62	0.80	1.09	0.00	0.00	0.55	0.00
time (sec)	N/A	0.353	0.672	10.430	0.120	0.106	0.000	0.000	0.500	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	114	92	143	902	173	0	0	0	105
N.S.	1	1.04	0.84	1.30	8.20	1.57	0.00	0.00	0.00	0.95
time (sec)	N/A	0.550	1.900	10.074	0.504	0.158	0.000	0.000	1.264	6.700

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	82	113	764	132	0	0	0	91
N.S.	1	1.00	1.12	1.55	10.47	1.81	0.00	0.00	0.00	1.25
time (sec)	N/A	0.384	1.702	5.570	0.380	0.121	0.000	0.000	1.286	3.792

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	59	86	626	89	0	0	0	50
N.S.	1	1.00	1.69	2.46	17.89	2.54	0.00	0.00	0.00	1.43
time (sec)	N/A	0.241	1.509	5.033	0.258	0.097	0.000	0.000	1.292	3.600

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	160	172	130	235	1164	326	0	0	939	0
N.S.	1	1.08	0.81	1.47	7.28	2.04	0.00	0.00	5.87	0.00
time (sec)	N/A	0.729	1.987	7.227	0.333	0.114	0.000	0.000	1.015	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	121	167	126	267	0	245	0	0	939	0
N.S.	1	1.38	1.04	2.21	0.00	2.02	0.00	0.00	7.76	0.00
time (sec)	N/A	0.686	1.806	7.261	0.000	0.089	0.000	0.000	1.208	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	125	170	120	469	977	267	0	0	98	0
N.S.	1	1.36	0.96	3.75	7.82	2.14	0.00	0.00	0.78	0.00
time (sec)	N/A	0.643	1.706	8.435	0.301	0.084	0.000	0.000	0.238	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	157	167	119	471	0	278	0	0	1457	0
N.S.	1	1.06	0.76	3.00	0.00	1.77	0.00	0.00	9.28	0.00
time (sec)	N/A	0.604	1.479	8.057	0.000	0.108	0.000	0.000	1.021	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	227	243	141	492	2779	300	0	0	165	0
N.S.	1	1.07	0.62	2.17	12.24	1.32	0.00	0.00	0.73	0.00
time (sec)	N/A	1.013	2.018	10.003	0.357	0.093	0.000	0.000	128.992	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	307	331	175	552	5821	322	0	0	169	0
N.S.	1	1.08	0.57	1.80	18.96	1.05	0.00	0.00	0.55	0.00
time (sec)	N/A	1.481	2.964	9.763	0.540	0.100	0.000	0.000	0.498	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	413	373	265	1870	418	0	0	74	0
N.S.	1	1.04	0.94	0.67	4.70	1.05	0.00	0.00	0.19	0.00
time (sec)	N/A	0.914	1.848	7.923	0.440	0.088	0.000	0.000	0.214	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	333	277	204	1400	323	0	0	60	0
N.S.	1	1.38	1.15	0.85	5.81	1.34	0.00	0.00	0.25	0.00
time (sec)	N/A	0.489	1.704	7.778	0.360	0.085	0.000	0.000	0.212	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	31	76	64	0	0	37	0
N.S.	1	1.00	1.00	0.86	2.11	1.78	0.00	0.00	1.03	0.00
time (sec)	N/A	0.234	0.596	8.365	0.268	0.074	0.000	0.000	0.157	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	48	51	54	75	0	0	37	86
N.S.	1	1.00	0.59	0.63	0.67	0.93	0.00	0.00	0.46	1.06
time (sec)	N/A	0.379	0.620	6.457	0.338	0.071	0.000	0.000	0.158	1.666

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	127	63	62	130	86	0	0	37	101
N.S.	1	1.04	0.52	0.51	1.07	0.70	0.00	0.00	0.30	0.83
time (sec)	N/A	0.537	0.824	6.589	0.262	0.070	0.000	0.000	0.160	1.950

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	174	80	79	178	97	0	0	37	109
N.S.	1	1.06	0.49	0.48	1.09	0.59	0.00	0.00	0.23	0.66
time (sec)	N/A	0.784	1.095	6.569	0.246	0.074	0.000	0.000	0.159	2.370

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	465	376	316	3005	644	0	0	80	0
N.S.	1	1.26	1.02	0.86	8.14	1.75	0.00	0.00	0.22	0.00
time (sec)	N/A	1.074	3.734	7.582	0.578	0.098	0.000	0.000	0.290	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	458	375	288	2367	538	0	0	74	0
N.S.	1	1.02	0.84	0.64	5.30	1.20	0.00	0.00	0.17	0.00
time (sec)	N/A	1.028	3.218	7.570	0.441	0.092	0.000	0.000	0.220	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	373	338	266	1871	420	0	0	61	0
N.S.	1	1.34	1.22	0.96	6.73	1.51	0.00	0.00	0.22	0.00
time (sec)	N/A	0.644	2.816	7.559	0.421	0.089	0.000	0.000	0.212	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	398	411	401	286	1462	460	0	0	76	0
N.S.	1	1.03	1.01	0.72	3.67	1.16	0.00	0.00	0.19	0.00
time (sec)	N/A	0.855	2.570	7.573	0.366	0.088	0.000	0.000	0.159	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	76	76	0	0	76	0
N.S.	1	1.00	1.00	0.82	2.00	2.00	0.00	0.00	2.00	0.00
time (sec)	N/A	0.250	0.732	7.229	0.169	0.074	0.000	0.000	0.176	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	84	63	59	79	0	0	76	102
N.S.	1	1.00	1.04	0.78	0.73	0.98	0.00	0.00	0.94	1.26
time (sec)	N/A	0.387	1.099	6.063	0.282	0.075	0.000	0.000	0.203	1.833

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	128	98	80	84	91	0	0	76	110
N.S.	1	1.02	0.78	0.64	0.67	0.73	0.00	0.00	0.61	0.88
time (sec)	N/A	0.593	1.385	6.148	0.245	0.070	0.000	0.000	0.165	2.274

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	174	113	90	160	103	0	0	76	125
N.S.	1	1.04	0.68	0.54	0.96	0.62	0.00	0.00	0.46	0.75
time (sec)	N/A	0.796	1.490	6.156	0.291	0.076	0.000	0.000	0.194	2.892

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	488	503	386	316	3005	635	0	0	115	0
N.S.	1	1.03	0.79	0.65	6.16	1.30	0.00	0.00	0.24	0.00
time (sec)	N/A	1.266	2.815	7.692	0.469	0.093	0.000	0.000	0.277	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	418	387	289	2431	525	0	0	96	0
N.S.	1	1.28	1.18	0.88	7.43	1.61	0.00	0.00	0.29	0.00
time (sec)	N/A	0.829	3.698	7.799	0.454	0.093	0.000	0.000	0.236	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	437	454	359	305	2013	484	0	0	118	0
N.S.	1	1.04	0.82	0.70	4.61	1.11	0.00	0.00	0.27	0.00
time (sec)	N/A	1.033	3.122	7.940	0.329	0.091	0.000	0.000	0.182	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	280	372	343	296	1492	505	0	0	118	0
N.S.	1	1.33	1.22	1.06	5.33	1.80	0.00	0.00	0.42	0.00
time (sec)	N/A	0.670	3.553	7.652	0.332	0.117	0.000	0.000	0.185	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	76	80	0	0	118	104
N.S.	1	1.00	1.00	0.82	2.00	2.11	0.00	0.00	3.11	2.74
time (sec)	N/A	0.243	1.124	8.380	0.181	0.071	0.000	0.000	0.180	1.747

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	92	82	94	94	0	0	118	112
N.S.	1	1.00	1.14	1.01	1.16	1.16	0.00	0.00	1.46	1.38
time (sec)	N/A	0.431	1.354	6.214	0.265	0.074	0.000	0.000	0.183	2.174

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	128	104	89	96	99	0	0	118	127
N.S.	1	1.02	0.83	0.71	0.77	0.79	0.00	0.00	0.94	1.02
time (sec)	N/A	0.555	1.421	6.247	0.242	0.069	0.000	0.000	0.183	2.817

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	175	121	97	124	99	0	0	118	133
N.S.	1	1.04	0.72	0.57	0.73	0.59	0.00	0.00	0.70	0.79
time (sec)	N/A	0.806	1.574	6.467	0.251	0.072	0.000	0.000	0.182	3.359

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	375	350	307	2258	461	0	0	162	0
N.S.	1	1.33	1.24	1.08	7.98	1.63	0.00	0.00	0.57	0.00
time (sec)	N/A	0.671	3.419	10.243	0.430	0.092	0.000	0.000	0.202	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	361	373	302	232	726	385	0	0	191	0
N.S.	1	1.03	0.84	0.64	2.01	1.07	0.00	0.00	0.53	0.00
time (sec)	N/A	0.664	1.643	10.137	0.307	0.094	0.000	0.000	0.203	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	31	76	64	0	0	50	40
N.S.	1	1.00	1.00	0.86	2.11	1.78	0.00	0.00	1.39	1.11
time (sec)	N/A	0.233	0.475	11.037	0.186	0.084	0.000	0.000	0.170	1.510

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	48	42	80	78	0	0	111	78
N.S.	1	1.00	0.60	0.52	1.00	0.98	0.00	0.00	1.39	0.98
time (sec)	N/A	0.384	0.582	6.161	0.384	0.083	0.000	0.000	0.180	1.391

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	127	68	62	130	89	0	0	119	86
N.S.	1	1.05	0.56	0.51	1.07	0.74	0.00	0.00	0.98	0.71
time (sec)	N/A	0.600	0.795	6.121	0.246	0.076	0.000	0.000	0.178	1.135

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	173	79	70	178	100	0	0	119	101
N.S.	1	1.05	0.48	0.42	1.08	0.61	0.00	0.00	0.72	0.61
time (sec)	N/A	0.758	0.998	6.152	0.311	0.075	0.000	0.000	0.179	1.357

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	220	87	88	226	111	0	0	119	109
N.S.	1	1.07	0.42	0.43	1.10	0.54	0.00	0.00	0.58	0.53
time (sec)	N/A	0.956	1.279	6.282	0.336	0.071	0.000	0.000	0.178	1.510

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	464	337	289	1817	483	0	0	349	0
N.S.	1	1.15	0.84	0.72	4.51	1.20	0.00	0.00	0.87	0.00
time (sec)	N/A	1.076	3.532	10.368	0.387	0.095	0.000	0.000	0.436	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	378	338	275	778	539	0	0	202	0
N.S.	1	1.34	1.19	0.97	2.75	1.90	0.00	0.00	0.71	0.00
time (sec)	N/A	0.680	3.681	10.567	0.356	0.112	0.000	0.000	0.404	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	76	67	0	0	178	0
N.S.	1	1.00	1.00	0.82	2.00	1.76	0.00	0.00	4.68	0.00
time (sec)	N/A	0.251	0.930	10.669	0.153	0.072	0.000	0.000	0.219	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	63	62	80	75	0	0	201	84
N.S.	1	1.00	0.79	0.78	1.00	0.94	0.00	0.00	2.51	1.05
time (sec)	N/A	0.388	0.732	8.696	0.238	0.069	0.000	0.000	0.236	1.099

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	126	83	70	130	89	0	0	171	104
N.S.	1	1.04	0.69	0.58	1.07	0.74	0.00	0.00	1.41	0.86
time (sec)	N/A	0.538	0.903	6.306	0.244	0.072	0.000	0.000	0.243	1.278

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	173	100	91	178	100	0	0	187	112
N.S.	1	1.05	0.61	0.55	1.08	0.61	0.00	0.00	1.13	0.68
time (sec)	N/A	0.809	1.195	6.319	0.271	0.072	0.000	0.000	0.220	1.421

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	219	100	103	226	111	0	0	187	127
N.S.	1	1.05	0.48	0.49	1.08	0.53	0.00	0.00	0.89	0.61
time (sec)	N/A	0.953	1.328	6.354	0.247	0.081	0.000	0.000	0.232	1.736

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	426	370	314	2449	541	0	0	88	0
N.S.	1	1.31	1.14	0.97	7.54	1.66	0.00	0.00	0.27	0.00
time (sec)	N/A	0.924	5.053	11.130	0.373	0.125	0.000	0.000	0.203	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	420	357	313	1466	543	0	0	88	0
N.S.	1	1.04	0.88	0.77	3.62	1.34	0.00	0.00	0.22	0.00
time (sec)	N/A	0.862	3.878	10.958	0.396	0.126	0.000	0.000	0.205	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	76	71	0	0	88	0
N.S.	1	1.00	1.00	0.82	2.00	1.87	0.00	0.00	2.32	0.00
time (sec)	N/A	0.256	1.151	11.606	0.178	0.071	0.000	0.000	0.197	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	63	79	86	79	0	0	84	102
N.S.	1	1.00	0.79	0.99	1.08	0.99	0.00	0.00	1.05	1.28
time (sec)	N/A	0.416	1.199	9.253	0.238	0.070	0.000	0.000	0.187	1.328

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	126	85	87	130	86	0	0	77	109
N.S.	1	1.04	0.70	0.72	1.07	0.71	0.00	0.00	0.64	0.90
time (sec)	N/A	0.561	1.090	9.154	0.242	0.076	0.000	0.000	0.213	1.428

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	172	102	97	178	89	0	0	92	118
N.S.	1	1.06	0.63	0.60	1.10	0.55	0.00	0.00	0.57	0.73
time (sec)	N/A	0.757	1.186	6.627	0.269	0.076	0.000	0.000	0.153	1.688

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	219	107	115	226	111	0	0	113	135
N.S.	1	1.06	0.52	0.56	1.10	0.54	0.00	0.00	0.55	0.66
time (sec)	N/A	1.029	1.488	6.612	0.252	0.086	0.000	0.000	0.154	1.953

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	118	0	0	0	0	0	140	0
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	1.63	0.00
time (sec)	N/A	0.503	1.447	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	116	0	0	0	0	0	142	0
N.S.	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	1.65	0.00
time (sec)	N/A	0.476	1.168	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	85	116	0	0	0	0	0	142	0
N.S.	1	0.99	1.35	0.00	0.00	0.00	0.00	0.00	1.65	0.00
time (sec)	N/A	0.491	0.976	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	95	0	0	0	0	0	142	0
N.S.	1	0.99	1.13	0.00	0.00	0.00	0.00	0.00	1.69	0.00
time (sec)	N/A	0.477	0.922	0.000	0.000	0.000	0.000	0.000	0.251	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	95	0	0	0	0	0	103	0
N.S.	1	0.99	1.13	0.00	0.00	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.479	1.189	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	88	112	0	0	0	0	0	103	0
N.S.	1	1.02	1.30	0.00	0.00	0.00	0.00	0.00	1.20	0.00
time (sec)	N/A	0.499	1.189	0.000	0.000	0.000	0.000	0.000	0.262	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	437	248	240	0	3902	528	0	0	78	0
N.S.	1	0.57	0.55	0.00	8.93	1.21	0.00	0.00	0.18	0.00
time (sec)	N/A	0.712	2.339	0.000	0.427	0.096	0.000	0.000	0.225	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	378	201	220	0	1906	515	0	0	53	0
N.S.	1	0.53	0.58	0.00	5.04	1.36	0.00	0.00	0.14	0.00
time (sec)	N/A	0.665	1.798	0.000	0.306	0.093	0.000	0.000	0.218	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	340	154	161	0	1753	367	0	0	30	0
N.S.	1	0.45	0.47	0.00	5.16	1.08	0.00	0.00	0.09	0.00
time (sec)	N/A	0.466	1.064	0.000	0.320	0.104	0.000	0.000	0.195	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	47	0	107	55	0	0	30	81
N.S.	1	1.00	1.27	0.00	2.89	1.49	0.00	0.00	0.81	2.19
time (sec)	N/A	0.269	0.873	0.000	0.219	0.091	0.000	0.000	0.176	2.066

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	316	58	0	0	62	90
N.S.	1	1.00	0.86	0.00	3.90	0.72	0.00	0.00	0.77	1.11
time (sec)	N/A	0.425	0.991	0.000	0.212	0.077	0.000	0.000	0.196	1.294

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	126	100	0	402	107	0	0	96	122
N.S.	1	1.03	0.82	0.00	3.30	0.88	0.00	0.00	0.79	1.00
time (sec)	N/A	0.572	1.339	0.000	0.252	0.073	0.000	0.000	0.208	2.494

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	171	116	0	983	133	0	0	130	303
N.S.	1	1.05	0.71	0.00	6.03	0.82	0.00	0.00	0.80	1.86
time (sec)	N/A	0.762	1.961	0.000	0.240	0.078	0.000	0.000	0.238	5.313

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	88	210	0	0	0	0	0	335	0
N.S.	1	1.02	2.44	0.00	0.00	0.00	0.00	0.00	3.90	0.00
time (sec)	N/A	0.463	4.894	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	88	147	0	0	0	0	0	139	0
N.S.	1	1.02	1.71	0.00	0.00	0.00	0.00	0.00	1.62	0.00
time (sec)	N/A	0.455	1.078	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	114	114	0	0	0	0	0	61	0
N.S.	1	1.33	1.33	0.00	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.469	0.320	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	114	67	0	0	0	0	0	35	0
N.S.	1	1.39	0.82	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.424	0.143	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	88	152	0	0	0	0	0	30	0
N.S.	1	1.02	1.77	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.460	1.318	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	88	156	0	0	0	0	0	40	0
N.S.	1	1.02	1.81	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.473	1.378	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	88	153	0	0	0	0	0	52	0
N.S.	1	1.02	1.78	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.467	1.584	0.000	0.000	0.000	0.000	0.000	0.225	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	121	186	0	0	0	0	0	167	0
N.S.	1	1.13	1.74	0.00	0.00	0.00	0.00	0.00	1.56	0.00
time (sec)	N/A	0.516	2.998	0.000	0.000	0.000	0.000	0.000	0.280	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	121	186	0	0	0	0	0	133	0
N.S.	1	1.13	1.74	0.00	0.00	0.00	0.00	0.00	1.24	0.00
time (sec)	N/A	0.478	3.239	0.000	0.000	0.000	0.000	0.000	0.282	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	114	186	0	0	0	0	0	95	0
N.S.	1	1.07	1.74	0.00	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.500	1.622	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	114	174	0	0	0	0	0	94	0
N.S.	1	1.07	1.63	0.00	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.453	1.044	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	118	174	0	0	0	0	0	236	0
N.S.	1	1.10	1.63	0.00	0.00	0.00	0.00	0.00	2.21	0.00
time (sec)	N/A	0.482	1.463	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	121	178	0	0	0	0	0	27	0
N.S.	1	1.13	1.66	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.496	1.955	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	121	178	0	0	0	0	0	79	0
N.S.	1	1.13	1.66	0.00	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.497	3.362	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	105	165	0	0	0	0	0	28	0
N.S.	1	0.96	1.51	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.443	6.237	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	86	80	272	0	247	0	623	24	168
N.S.	1	0.89	0.82	2.80	0.00	2.55	0.00	6.42	0.25	1.73
time (sec)	N/A	0.277	0.555	1.835	0.000	0.102	0.000	0.622	0.176	5.641

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	60	60	150	0	142	0	206	24	216
N.S.	1	0.92	0.92	2.31	0.00	2.18	0.00	3.17	0.37	3.32
time (sec)	N/A	0.257	0.169	1.166	0.000	0.083	0.000	0.488	0.187	2.663

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	28	60	0	28	24	104
N.S.	1	1.00	1.00	0.97	0.88	1.88	0.00	0.88	0.75	3.25
time (sec)	N/A	0.227	0.115	0.556	0.034	0.085	0.000	0.358	0.150	0.461

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	60	54	0	0	0	0	0	24	0
N.S.	1	1.07	0.96	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.249	0.137	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	64	58	0	0	0	0	0	24	0
N.S.	1	1.07	0.97	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.243	0.183	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	64	58	0	0	0	0	0	24	0
N.S.	1	1.07	0.97	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.245	0.326	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	94	149	0	0	0	0	0	24	0
N.S.	1	1.02	1.62	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.497	12.486	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	149	0	0	0	0	0	24	0
N.S.	1	1.00	1.62	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.457	11.820	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	146	0	0	0	0	0	22	0
N.S.	1	1.00	1.66	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.415	6.347	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	85	146	0	0	0	0	0	22	0
N.S.	1	0.97	1.66	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.458	12.163	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	94	149	0	0	0	0	0	24	0
N.S.	1	1.02	1.62	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.473	13.253	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	94	149	0	0	0	0	0	24	0
N.S.	1	1.02	1.62	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.472	14.246	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	181	0	0	0	0	0	126	0
N.S.	1	1.00	1.89	0.00	0.00	0.00	0.00	0.00	1.31	0.00
time (sec)	N/A	0.482	6.950	0.000	0.000	0.000	0.000	0.000	0.279	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	170	0	0	0	0	0	118	0
N.S.	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.474	6.852	0.000	0.000	0.000	0.000	0.000	0.298	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	170	0	0	0	0	0	98	0
N.S.	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	1.04	0.00
time (sec)	N/A	0.456	6.255	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	91	143	0	0	0	0	0	37	0
N.S.	1	0.97	1.52	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.479	7.108	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	93	143	0	0	0	0	0	37	0
N.S.	1	0.97	1.49	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.485	7.765	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	98	157	0	0	0	0	0	37	0
N.S.	1	1.02	1.64	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.488	8.031	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	269	259	165	4331	435	335	0	0	43	511
N.S.	1	0.96	0.61	16.10	1.62	1.25	0.00	0.00	0.16	1.90
time (sec)	N/A	1.113	1.527	6.088	0.275	0.115	0.000	0.000	0.202	7.387

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	205	209	119	4982	346	265	0	0	43	425
N.S.	1	1.02	0.58	24.30	1.69	1.29	0.00	0.00	0.21	2.07
time (sec)	N/A	0.875	1.529	5.841	0.260	0.084	0.000	0.000	0.181	6.523

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	148	147	82	2581	175	178	0	0	43	227
N.S.	1	0.99	0.55	17.44	1.18	1.20	0.00	0.00	0.29	1.53
time (sec)	N/A	0.620	1.212	5.471	0.231	0.109	0.000	0.000	0.168	6.673

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	94	97	58	2484	113	129	246	0	43	121
N.S.	1	1.03	0.62	26.43	1.20	1.37	2.62	0.00	0.46	1.29
time (sec)	N/A	0.424	0.954	5.898	0.227	0.082	0.536	0.000	0.168	2.055

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	36	86	84	49	0	32	0
N.S.	1	1.00	1.00	0.97	2.32	2.27	1.32	0.00	0.86	0.00
time (sec)	N/A	0.236	0.233	5.298	0.204	0.086	3.957	0.000	0.162	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	156	87	0	0	0	0	0	39	0
N.S.	1	1.29	0.72	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.538	3.667	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	146	112	0	0	0	0	0	43	0
N.S.	1	1.29	0.99	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.506	8.815	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	156	116	0	0	0	0	0	43	0
N.S.	1	1.29	0.96	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.574	12.358	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	156	159	122	0	1067	166	0	0	47	318
N.S.	1	1.02	0.78	0.00	6.84	1.06	0.00	0.00	0.30	2.04
time (sec)	N/A	0.650	2.610	0.000	1.298	0.082	0.000	0.000	0.191	7.667

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	142	166	0	0	0	0	0	47	0
N.S.	1	1.38	1.61	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.559	10.658	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	102	91	0	595	134	0	0	47	174
N.S.	1	1.04	0.93	0.00	6.07	1.37	0.00	0.00	0.48	1.78
time (sec)	N/A	0.434	1.989	0.000	0.368	0.087	0.000	0.000	0.189	2.951

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	140	166	0	0	0	0	0	47	0
N.S.	1	1.36	1.61	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.540	9.486	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	59	0	217	109	0	0	47	106
N.S.	1	1.00	1.28	0.00	4.72	2.37	0.00	0.00	1.02	2.30
time (sec)	N/A	0.246	1.360	0.000	0.203	0.085	0.000	0.000	0.158	1.507

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	138	154	0	0	0	0	0	43	0
N.S.	1	1.37	1.52	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.532	6.334	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	69	146	0	0	0	0	0	36	0
N.S.	1	1.06	2.25	0.00	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.341	1.291	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	137	157	0	0	0	0	0	47	0
N.S.	1	1.36	1.55	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.540	9.880	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	78	151	0	0	0	0	0	47	0
N.S.	1	1.05	2.04	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.575	9.896	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	142	166	0	0	0	0	0	47	0
N.S.	1	1.38	1.61	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.565	11.465	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	70	165	0	0	0	0	0	85	0
N.S.	1	1.06	2.50	0.00	0.00	0.00	0.00	0.00	1.29	0.00
time (sec)	N/A	0.549	8.560	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	70	165	0	0	0	0	0	59	0
N.S.	1	1.06	2.50	0.00	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.527	7.886	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	67	150	0	0	0	0	0	34	0
N.S.	1	1.06	2.38	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.352	1.491	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	1261	137	115	0	0	72	62
N.S.	1	1.00	1.00	31.52	3.42	2.88	0.00	0.00	1.80	1.55
time (sec)	N/A	0.256	1.398	7.037	0.151	0.088	0.000	0.000	0.210	1.815

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	61	0	304	139	0	0	113	260
N.S.	1	1.00	0.66	0.00	3.30	1.51	0.00	0.00	1.23	2.83
time (sec)	N/A	0.416	1.891	0.000	0.300	0.086	0.000	0.000	0.161	5.229

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	145	129	0	0	172	0	0	152	321
N.S.	1	0.98	0.87	0.00	0.00	1.16	0.00	0.00	1.03	2.17
time (sec)	N/A	0.606	2.430	0.000	0.000	0.098	0.000	0.000	0.175	9.653

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	56	53	69	70	57	56	70	118	68
N.S.	1	0.93	0.88	1.15	1.17	0.95	0.93	1.17	1.97	1.13
time (sec)	N/A	0.291	0.128	14.594	0.030	0.078	1.503	0.202	0.160	0.657

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	47	48	45	44	48	80	46
N.S.	1	1.00	0.93	1.07	1.09	1.02	1.00	1.09	1.82	1.05
time (sec)	N/A	0.284	0.067	3.662	0.037	0.077	1.054	0.178	0.181	0.651

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	20	30	34	25	42	23
N.S.	1	1.00	1.00	0.89	0.71	1.07	1.21	0.89	1.50	0.82
time (sec)	N/A	0.268	0.011	0.869	0.031	0.078	0.679	0.156	0.202	0.639

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	16	27	24	18	24	21
N.S.	1	1.00	1.00	1.29	0.94	1.59	1.41	1.06	1.41	1.24
time (sec)	N/A	0.157	0.006	0.197	0.044	0.082	0.058	0.136	0.201	0.662

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	44	46	36	38	35	0	41	34	31
N.S.	1	1.02	1.07	0.84	0.88	0.81	0.00	0.95	0.79	0.72
time (sec)	N/A	0.262	0.038	1.158	0.131	0.106	0.000	0.168	0.203	0.693

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	70	62	52	61	51	0	53	65	41
N.S.	1	1.08	0.95	0.80	0.94	0.78	0.00	0.82	1.00	0.63
time (sec)	N/A	0.325	0.068	4.884	0.112	0.098	0.000	0.153	0.197	0.708

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	96	57	62	82	62	0	64	93	76
N.S.	1	1.10	0.66	0.71	0.94	0.71	0.00	0.74	1.07	0.87
time (sec)	N/A	0.408	0.073	20.726	0.107	0.082	0.000	0.175	0.159	1.004

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	79	74	63	86	88	0	141	272	175
N.S.	1	1.07	1.00	0.85	1.16	1.19	0.00	1.91	3.68	2.36
time (sec)	N/A	0.419	0.009	8.039	0.042	0.096	0.000	0.217	0.150	4.310

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	53	52	50	61	74	0	99	170	105
N.S.	1	1.02	1.00	0.96	1.17	1.42	0.00	1.90	3.27	2.02
time (sec)	N/A	0.331	0.009	2.394	0.048	0.093	0.000	0.212	0.151	2.607

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	31	54	37	54	64	38
N.S.	1	1.00	1.00	1.33	1.29	2.25	1.54	2.25	2.67	1.58
time (sec)	N/A	0.243	0.007	0.626	0.029	0.089	2.233	0.161	0.158	0.721

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	46	23	23	23	0	129	23	38
N.S.	1	1.00	1.92	0.96	0.96	0.96	0.00	5.38	0.96	1.58
time (sec)	N/A	0.239	0.029	0.753	0.038	0.077	0.000	0.163	0.199	0.697

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	35	38	0	11886	52	47
N.S.	1	1.00	1.00	0.82	0.80	0.86	0.00	270.14	1.18	1.07
time (sec)	N/A	0.274	0.005	2.666	0.031	0.086	0.000	2.126	0.187	0.759

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	56	60	46	49	51	0	27424	83	67
N.S.	1	0.93	1.00	0.77	0.82	0.85	0.00	457.07	1.38	1.12
time (sec)	N/A	0.276	0.007	9.598	0.034	0.108	0.000	5.078	0.189	0.752

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	64	74	56	58	62	0	49370	111	87
N.S.	1	0.86	1.00	0.76	0.78	0.84	0.00	667.16	1.50	1.18
time (sec)	N/A	0.281	0.024	36.894	0.029	0.083	0.000	10.875	0.209	0.795

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	129	133	138	133	122	0	156	241	132
N.S.	1	1.08	1.12	1.16	1.12	1.03	0.00	1.31	2.03	1.11
time (sec)	N/A	0.321	0.384	115.438	0.045	0.088	0.000	0.230	0.212	0.729

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	107	104	110	104	100	0	118	187	102
N.S.	1	1.10	1.07	1.13	1.07	1.03	0.00	1.22	1.93	1.05
time (sec)	N/A	0.295	0.481	30.104	0.039	0.079	0.000	0.214	0.209	0.688

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	64	54	82	71	79	0	80	133	71
N.S.	1	0.85	0.72	1.09	0.95	1.05	0.00	1.07	1.77	0.95
time (sec)	N/A	0.274	0.139	8.134	0.031	0.108	0.000	0.198	0.184	0.663

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	46	48	20	55	0	41	79	39
N.S.	1	1.00	2.09	2.18	0.91	2.50	0.00	1.86	3.59	1.77
time (sec)	N/A	0.221	0.029	2.544	0.033	0.081	0.000	0.184	0.208	0.691

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	69	43	41	44	48	49	42	136
N.S.	1	1.00	1.77	1.10	1.05	1.13	1.23	1.26	1.08	3.49
time (sec)	N/A	0.258	0.095	0.251	0.115	0.081	0.082	0.141	0.179	0.786

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	78	52	64	55	52	0	61	63	50
N.S.	1	1.59	1.06	1.31	1.12	1.06	0.00	1.24	1.29	1.02
time (sec)	N/A	0.254	0.272	2.112	0.113	0.084	0.000	0.189	0.153	0.723

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	139	197	97	85	75	0	89	115	83
N.S.	1	1.46	2.07	1.02	0.89	0.79	0.00	0.94	1.21	0.87
time (sec)	N/A	0.301	2.141	9.248	0.145	0.119	0.000	0.210	0.154	0.798

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	130	168	149	180	142	0	343	530	328
N.S.	1	1.01	1.30	1.16	1.40	1.10	0.00	2.66	4.11	2.54
time (sec)	N/A	0.415	0.052	15.413	0.038	0.096	0.000	0.292	0.163	3.560

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	101	120	118	129	120	0	249	382	216
N.S.	1	1.04	1.24	1.22	1.33	1.24	0.00	2.57	3.94	2.23
time (sec)	N/A	0.400	0.036	4.398	0.043	0.098	0.000	0.252	0.174	3.385

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	68	67	83	82	96	0	122	221	106
N.S.	1	1.11	1.10	1.36	1.34	1.57	0.00	2.00	3.62	1.74
time (sec)	N/A	0.357	0.027	1.112	0.060	0.100	0.000	0.238	0.196	1.177

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	46	84	53	60	62	0	1100	73	66
N.S.	1	0.98	1.79	1.13	1.28	1.32	0.00	23.40	1.55	1.40
time (sec)	N/A	0.341	0.351	1.182	0.037	0.090	0.000	0.424	0.156	0.771

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	54	67	52	52	53	0	11162	74	77
N.S.	1	0.98	1.22	0.95	0.95	0.96	0.00	202.95	1.35	1.40
time (sec)	N/A	0.392	0.036	4.784	0.034	0.080	0.000	14.483	0.198	0.797

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	77	69	88	77	74	0	28204	119	115
N.S.	1	0.95	0.85	1.09	0.95	0.91	0.00	348.20	1.47	1.42
time (sec)	N/A	0.422	0.119	20.719	0.054	0.088	0.000	29.803	0.198	0.863

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	100	92	108	98	94	0	52002	163	176
N.S.	1	0.93	0.86	1.01	0.92	0.88	0.00	486.00	1.52	1.64
time (sec)	N/A	0.451	0.195	67.698	0.048	0.089	0.000	54.047	0.217	0.956

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	123	113	128	117	113	0	80368	207	192
N.S.	1	0.92	0.85	0.96	0.88	0.85	0.00	604.27	1.56	1.44
time (sec)	N/A	0.469	0.375	186.213	0.041	0.093	0.000	79.009	0.210	1.205

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	192	177	219	176	150	0	220	351	175
N.S.	1	0.99	0.91	1.13	0.91	0.77	0.00	1.13	1.81	0.90
time (sec)	N/A	0.363	1.350	188.985	0.037	0.100	0.000	0.272	0.173	0.755

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	115	115	173	142	128	0	166	275	139
N.S.	1	0.83	0.83	1.25	1.03	0.93	0.00	1.20	1.99	1.01
time (sec)	N/A	0.329	0.403	68.531	0.047	0.093	0.000	0.237	0.170	0.705

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	64	54	127	98	105	0	112	199	97
N.S.	1	0.85	0.72	1.69	1.31	1.40	0.00	1.49	2.65	1.29
time (sec)	N/A	0.276	0.261	15.345	0.031	0.083	0.000	0.234	0.179	0.652

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	57	72	20	78	0	57	122	55
N.S.	1	1.00	2.59	3.27	0.91	3.55	0.00	2.59	5.55	2.50
time (sec)	N/A	0.215	0.113	4.440	0.036	0.082	0.000	0.198	0.193	0.655

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	79	73	78	71	94	80	78	106
N.S.	1	1.00	1.10	1.01	1.08	0.99	1.31	1.11	1.08	1.47
time (sec)	N/A	0.378	0.171	0.286	0.161	0.083	0.101	0.173	0.230	0.679

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	119	401	98	81	79	0	87	146	141
N.S.	1	1.23	4.13	1.01	0.84	0.81	0.00	0.90	1.51	1.45
time (sec)	N/A	0.319	0.507	4.334	0.122	0.100	0.000	0.231	0.189	0.728

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	130	488	114	110	100	0	112	136	109
N.S.	1	1.55	5.81	1.36	1.31	1.19	0.00	1.33	1.62	1.30
time (sec)	N/A	0.288	0.785	17.803	0.121	0.118	0.000	0.295	0.229	0.816

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	214	390	155	147	128	0	142	202	141
N.S.	1	1.46	2.65	1.05	1.00	0.87	0.00	0.97	1.37	0.96
time (sec)	N/A	0.361	5.633	66.299	0.172	0.128	0.000	0.307	0.204	1.332

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	159	637	248	208	170	0	465	809	423
N.S.	1	0.99	3.96	1.54	1.29	1.06	0.00	2.89	5.02	2.63
time (sec)	N/A	0.581	2.065	30.783	0.041	0.108	0.000	0.348	0.196	4.527

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	130	464	198	157	147	0	333	595	293
N.S.	1	1.02	3.62	1.55	1.23	1.15	0.00	2.60	4.65	2.29
time (sec)	N/A	0.558	1.518	8.771	0.052	0.095	0.000	0.318	0.188	4.359

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	97	293	146	111	123	0	171	364	160
N.S.	1	1.08	3.26	1.62	1.23	1.37	0.00	1.90	4.04	1.78
time (sec)	N/A	0.483	1.513	2.413	0.040	0.117	0.000	0.275	0.183	2.640

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	70	131	96	84	109	0	4309	156	116
N.S.	1	0.96	1.79	1.32	1.15	1.49	0.00	59.03	2.14	1.59
time (sec)	N/A	0.455	1.126	2.679	0.057	0.119	0.000	2.045	0.150	1.312

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	82	81	75	77	77	0	24430	117	104
N.S.	1	0.93	0.92	0.85	0.88	0.88	0.00	277.61	1.33	1.18
time (sec)	N/A	0.521	0.483	9.711	0.033	0.086	0.000	121.745	0.157	0.851

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	113	121	125	107	102	0	0	183	147
N.S.	1	1.02	1.09	1.13	0.96	0.92	0.00	0.00	1.65	1.32
time (sec)	N/A	0.582	0.817	35.876	0.039	0.092	0.000	0.000	0.150	0.971

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	130	141	145	126	123	0	101962	249	214
N.S.	1	0.98	1.07	1.10	0.95	0.93	0.00	772.44	1.89	1.62
time (sec)	N/A	0.581	1.905	113.409	0.039	0.091	0.000	77.226	0.160	1.110

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	159	167	165	147	147	0	0	315	259
N.S.	1	0.96	1.01	1.00	0.89	0.89	0.00	0.00	1.91	1.57
time (sec)	N/A	0.614	4.373	292.115	0.048	0.102	0.000	0.000	0.169	1.143

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	103	99	106	108	183	0	139	954	119
N.S.	1	0.89	0.85	0.91	0.93	1.58	0.00	1.20	8.22	1.03
time (sec)	N/A	0.306	0.818	29.983	0.029	0.107	0.000	0.147	0.224	0.918

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	52	52	53	53	117	0	57	378	57
N.S.	1	0.88	0.88	0.90	0.90	1.98	0.00	0.97	6.41	0.97
time (sec)	N/A	0.267	0.096	7.482	0.050	0.103	0.000	0.142	0.222	0.713

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	59	0	19	66	18
N.S.	1	1.00	1.00	1.06	1.00	3.28	0.00	1.06	3.67	1.00
time (sec)	N/A	0.220	0.012	1.561	0.032	0.097	0.000	0.148	0.207	0.723

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	158	143	120	141	119	0	168	174	156
N.S.	1	1.70	1.54	1.29	1.52	1.28	0.00	1.81	1.87	1.68
time (sec)	N/A	0.384	0.384	2.063	0.116	0.101	0.000	0.144	0.180	0.948

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	245	225	197	271	208	0	277	353	318
N.S.	1	1.61	1.48	1.30	1.78	1.37	0.00	1.82	2.32	2.09
time (sec)	N/A	0.477	0.819	8.100	0.132	0.111	0.000	0.153	0.164	1.254

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	150	321	269	361	259	0	278	549	724
N.S.	1	1.07	2.29	1.92	2.58	1.85	0.00	1.99	3.92	5.17
time (sec)	N/A	1.007	1.583	15.611	0.130	0.173	0.000	0.270	0.160	2.543

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	80	109	129	163	191	0	136	113	310
N.S.	1	1.01	1.38	1.63	2.06	2.42	0.00	1.72	1.43	3.92
time (sec)	N/A	0.503	0.332	3.850	0.124	0.118	0.000	0.215	0.165	0.943

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	43	80	131	0	74	52	39
N.S.	1	1.00	0.98	0.93	1.74	2.85	0.00	1.61	1.13	0.85
time (sec)	N/A	0.227	0.087	1.074	0.124	0.096	0.000	0.199	0.196	0.771

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	83	79	90	142	187	0	118	118	110
N.S.	1	0.92	0.88	1.00	1.58	2.08	0.00	1.31	1.31	1.22
time (sec)	N/A	0.504	0.352	1.349	0.109	0.103	0.000	0.204	0.159	0.863

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	151	137	221	379	262	0	286	265	342
N.S.	1	0.92	0.83	1.34	2.30	1.59	0.00	1.73	1.61	2.07
time (sec)	N/A	0.949	1.071	4.009	0.140	0.104	0.000	0.227	0.163	3.696

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	156	229	201	186	386	0	242	7571	258
N.S.	1	0.88	1.29	1.13	1.04	2.17	0.00	1.36	42.53	1.45
time (sec)	N/A	0.365	1.642	254.471	0.039	0.122	0.000	0.203	0.210	0.762

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	100	122	114	115	281	0	137	3952	130
N.S.	1	0.86	1.05	0.98	0.99	2.42	0.00	1.18	34.07	1.12
time (sec)	N/A	0.311	1.992	58.227	0.047	0.107	0.000	0.188	0.214	0.717

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	51	51	57	60	178	0	62	1477	67
N.S.	1	0.84	0.84	0.93	0.98	2.92	0.00	1.02	24.21	1.10
time (sec)	N/A	0.269	0.078	15.566	0.039	0.102	0.000	0.178	0.222	0.727

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	32	21	20	57	0	20	169	20
N.S.	1	1.00	1.60	1.05	1.00	2.85	0.00	1.00	8.45	1.00
time (sec)	N/A	0.223	0.100	3.419	0.036	0.094	0.000	0.169	0.215	0.697

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	210	304	154	282	279	0	269	480	246
N.S.	1	1.38	2.00	1.01	1.86	1.84	0.00	1.77	3.16	1.62
time (sec)	N/A	0.432	2.738	4.225	0.115	0.108	0.000	0.201	0.216	1.083

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	322	416	248	502	424	0	400	752	463
N.S.	1	1.37	1.77	1.06	2.14	1.80	0.00	1.70	3.20	1.97
time (sec)	N/A	0.548	2.205	16.877	0.128	0.124	0.000	0.193	0.240	1.933

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	B	B	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	222	747	454	827	472	0	530	6643	2654
N.S.	1	0.87	2.93	1.78	3.24	1.85	0.00	2.08	26.05	10.41
time (sec)	N/A	0.521	6.210	150.115	0.139	0.235	0.000	0.302	0.247	3.697

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	161	709	259	471	355	0	280	2966	585
N.S.	1	0.88	3.85	1.41	2.56	1.93	0.00	1.52	16.12	3.18
time (sec)	N/A	0.406	6.410	30.712	0.143	0.138	0.000	0.278	0.187	2.205

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	104	120	135	212	293	0	166	1912	383
N.S.	1	1.14	1.32	1.48	2.33	3.22	0.00	1.82	21.01	4.21
time (sec)	N/A	0.342	0.807	7.899	0.116	0.127	0.000	0.274	0.194	1.326

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	103	78	118	182	215	0	138	186	136
N.S.	1	1.26	0.95	1.44	2.22	2.62	0.00	1.68	2.27	1.66
time (sec)	N/A	0.310	0.372	1.922	0.123	0.090	0.000	0.245	0.177	1.053

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	173	153	172	348	302	0	286	403	286
N.S.	1	1.10	0.97	1.10	2.22	1.92	0.00	1.82	2.57	1.82
time (sec)	N/A	0.383	0.574	2.324	0.130	0.103	0.000	0.268	0.153	3.090

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	273	249	320	772	418	0	438	654	674
N.S.	1	1.13	1.03	1.33	3.20	1.73	0.00	1.82	2.71	2.80
time (sec)	N/A	0.536	1.096	8.433	0.166	0.124	0.000	0.286	0.176	4.413

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	160	272	195	200	476	0	220	13621	234
N.S.	1	0.86	1.47	1.05	1.08	2.57	0.00	1.19	73.63	1.26
time (sec)	N/A	0.409	0.935	0.820	0.033	0.140	0.000	0.229	0.210	0.762

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	102	140	115	128	354	0	123	7160	143
N.S.	1	0.84	1.16	0.95	1.06	2.93	0.00	1.02	59.17	1.18
time (sec)	N/A	0.318	2.422	164.796	0.060	0.122	0.000	0.219	0.218	0.719

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	57	57	63	78	284	0	65	3133	80
N.S.	1	0.83	0.83	0.91	1.13	4.12	0.00	0.94	45.41	1.16
time (sec)	N/A	0.280	0.176	36.720	0.045	0.109	0.000	0.209	0.189	0.720

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	142	0	20	65	39
N.S.	1	1.00	1.00	0.95	0.91	6.45	0.00	0.91	2.95	1.77
time (sec)	N/A	0.225	0.116	7.442	0.037	0.095	0.000	0.194	0.187	0.678

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	263	458	219	458	503	0	369	1303	419
N.S.	1	1.30	2.27	1.08	2.27	2.49	0.00	1.83	6.45	2.07
time (sec)	N/A	0.505	6.196	10.330	0.159	0.120	0.000	0.228	0.185	1.701

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	386	596	312	738	671	0	613	1475	715
N.S.	1	1.31	2.02	1.06	2.50	2.27	0.00	2.08	5.00	2.42
time (sec)	N/A	0.623	6.165	41.042	0.150	0.175	0.000	0.244	0.341	2.587

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	223	688	444	902	564	0	510	10737	1203
N.S.	1	0.87	2.68	1.73	3.51	2.19	0.00	1.98	41.78	4.68
time (sec)	N/A	0.482	2.083	291.074	0.144	0.180	0.000	0.397	0.230	4.034

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	164	396	269	518	513	0	314	7694	1311
N.S.	1	1.11	2.68	1.82	3.50	3.47	0.00	2.12	51.99	8.86
time (sec)	N/A	0.399	2.219	93.885	0.140	0.144	0.000	0.383	0.228	2.870

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	117	132	191	326	294	0	221	2842	260
N.S.	1	1.23	1.39	2.01	3.43	3.09	0.00	2.33	29.92	2.74
time (sec)	N/A	0.325	0.280	16.752	0.132	0.099	0.000	0.347	0.179	3.030

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	170	110	280	412	352	0	293	5579	443
N.S.	1	1.10	0.71	1.81	2.66	2.27	0.00	1.89	35.99	2.86
time (sec)	N/A	0.362	0.959	3.502	0.129	0.105	0.000	0.320	0.174	1.895

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	250	183	283	658	480	0	399	1025	610
N.S.	1	1.13	0.83	1.28	2.98	2.17	0.00	1.81	4.64	2.76
time (sec)	N/A	0.491	1.550	4.958	0.141	0.119	0.000	0.380	0.175	4.513

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	359	371	457	1229	619	0	640	1324	1128
N.S.	1	1.16	1.20	1.47	3.96	2.00	0.00	2.06	4.27	3.64
time (sec)	N/A	0.636	1.736	24.188	0.199	0.142	0.000	0.407	0.218	6.238

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	123	69	225	0	145	0	0	51	0
N.S.	1	1.02	0.57	1.86	0.00	1.20	0.00	0.00	0.42	0.00
time (sec)	N/A	0.625	0.725	45.288	0.000	0.102	0.000	0.000	0.200	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	93	58	103	0	123	0	0	51	0
N.S.	1	1.01	0.63	1.12	0.00	1.34	0.00	0.00	0.55	0.00
time (sec)	N/A	0.473	0.494	37.912	0.000	0.118	0.000	0.000	0.181	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	89	58	199	0	120	0	0	45	0
N.S.	1	1.01	0.66	2.26	0.00	1.36	0.00	0.00	0.51	0.00
time (sec)	N/A	0.464	0.407	12.400	0.000	0.105	0.000	0.000	0.167	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	42	83	0	74	0	0	29	39
N.S.	1	1.00	0.72	1.43	0.00	1.28	0.00	0.00	0.50	0.67
time (sec)	N/A	0.345	0.334	9.277	0.000	0.092	0.000	0.000	0.168	0.387

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	54	193	0	89	0	0	53	0
N.S.	1	1.00	0.93	3.33	0.00	1.53	0.00	0.00	0.91	0.00
time (sec)	N/A	0.351	0.363	8.906	0.000	0.088	0.000	0.000	0.153	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	95	69	103	0	103	0	0	53	0
N.S.	1	1.01	0.73	1.10	0.00	1.10	0.00	0.00	0.56	0.00
time (sec)	N/A	0.463	0.336	10.365	0.000	0.095	0.000	0.000	0.178	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	95	74	209	0	111	0	0	53	0
N.S.	1	1.01	0.79	2.22	0.00	1.18	0.00	0.00	0.56	0.00
time (sec)	N/A	0.450	0.666	19.632	0.000	0.102	0.000	0.000	0.193	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	131	94	117	0	118	0	0	53	0
N.S.	1	1.07	0.76	0.95	0.00	0.96	0.00	0.00	0.43	0.00
time (sec)	N/A	0.574	0.531	21.794	0.000	0.110	0.000	0.000	0.164	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	139	127	204	0	172	0	0	86	0
N.S.	1	0.97	0.89	1.43	0.00	1.20	0.00	0.00	0.60	0.00
time (sec)	N/A	0.708	1.186	165.453	0.000	0.092	0.000	0.000	0.223	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	135	126	392	0	171	0	0	78	0
N.S.	1	0.94	0.88	2.74	0.00	1.20	0.00	0.00	0.55	0.00
time (sec)	N/A	0.723	0.946	23.286	0.000	0.097	0.000	0.000	0.214	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	104	87	163	0	133	0	0	55	0
N.S.	1	1.01	0.84	1.58	0.00	1.29	0.00	0.00	0.53	0.00
time (sec)	N/A	0.545	0.888	15.451	0.000	0.095	0.000	0.000	0.221	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	64	377	0	122	0	0	87	0
N.S.	1	1.00	0.67	3.97	0.00	1.28	0.00	0.00	0.92	0.00
time (sec)	N/A	0.554	1.480	14.792	0.000	0.087	0.000	0.000	0.233	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	132	101	187	0	131	0	0	87	0
N.S.	1	0.95	0.73	1.35	0.00	0.94	0.00	0.00	0.63	0.00
time (sec)	N/A	0.698	1.390	15.512	0.000	0.098	0.000	0.000	0.217	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	141	92	398	0	139	0	0	87	0
N.S.	1	0.97	0.63	2.74	0.00	0.96	0.00	0.00	0.60	0.00
time (sec)	N/A	0.734	1.692	25.415	0.000	0.127	0.000	0.000	0.235	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	177	127	212	0	154	0	0	87	0
N.S.	1	0.96	0.69	1.15	0.00	0.84	0.00	0.00	0.47	0.00
time (sec)	N/A	0.899	2.846	26.705	0.000	0.117	0.000	0.000	0.218	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	177	126	442	0	164	0	0	87	0
N.S.	1	0.96	0.68	2.40	0.00	0.89	0.00	0.00	0.47	0.00
time (sec)	N/A	0.910	3.523	39.139	0.000	0.109	0.000	0.000	0.215	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-1)	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	193	157	232	0	202	0	0	137	0
N.S.	1	0.90	0.73	1.08	0.00	0.94	0.00	0.00	0.64	0.00
time (sec)	N/A	0.379	2.356	922.671	0.000	0.112	0.000	0.000	0.213	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-1)	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	186	155	437	0	200	0	0	125	0
N.S.	1	0.98	0.82	2.30	0.00	1.05	0.00	0.00	0.66	0.00
time (sec)	N/A	0.390	2.221	26.263	0.000	0.102	0.000	0.000	0.186	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	152	132	183	0	164	0	0	93	0
N.S.	1	1.09	0.95	1.32	0.00	1.18	0.00	0.00	0.67	0.00
time (sec)	N/A	0.363	2.279	23.489	0.000	0.096	0.000	0.000	0.165	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	184	130	627	0	169	0	0	121	0
N.S.	1	1.03	0.73	3.52	0.00	0.95	0.00	0.00	0.68	0.00
time (sec)	N/A	0.362	2.851	21.906	0.000	0.130	0.000	0.000	0.176	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	161	117	210	0	148	0	0	121	0
N.S.	1	1.08	0.79	1.41	0.00	0.99	0.00	0.00	0.81	0.00
time (sec)	N/A	0.359	2.339	21.582	0.000	0.119	0.000	0.000	0.215	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	199	150	652	0	164	0	0	121	0
N.S.	1	1.19	0.90	3.90	0.00	0.98	0.00	0.00	0.72	0.00
time (sec)	N/A	0.369	2.681	27.076	0.000	0.098	0.000	0.000	0.222	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	183	150	241	0	181	0	0	121	0
N.S.	1	0.87	0.71	1.14	0.00	0.86	0.00	0.00	0.57	0.00
time (sec)	N/A	0.374	3.662	28.533	0.000	0.125	0.000	0.000	0.214	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	181	146	493	0	191	0	0	121	0
N.S.	1	0.80	0.64	2.17	0.00	0.84	0.00	0.00	0.53	0.00
time (sec)	N/A	0.384	5.274	39.855	0.000	0.105	0.000	0.000	0.219	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	213	246	263	0	206	0	0	121	0
N.S.	1	0.80	0.92	0.99	0.00	0.77	0.00	0.00	0.45	0.00
time (sec)	N/A	0.402	7.571	47.362	0.000	0.112	0.000	0.000	0.221	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	456	324	7284	7821	0	0	0	0	36	0
N.S.	1	0.71	15.97	17.15	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.731	84.173	31.313	0.000	0.000	0.000	0.000	0.222	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	396	300	290	3131	0	0	0	0	36	0
N.S.	1	0.76	0.73	7.91	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.705	24.341	28.029	0.000	0.000	0.000	0.000	0.204	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	334	246	276	3618	0	0	0	0	32	0
N.S.	1	0.74	0.83	10.83	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.600	2.035	7.692	0.000	0.000	0.000	0.000	0.191	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	324	250	218	3037	0	0	0	0	25	0
N.S.	1	0.77	0.67	9.37	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.601	1.673	7.276	0.000	0.000	0.000	0.000	0.176	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	451	344	285	4862	0	0	0	0	41	0
N.S.	1	0.76	0.63	10.78	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.757	2.538	9.163	0.000	0.000	0.000	0.000	0.165	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	422	327	418	3759	0	0	0	0	45	0
N.S.	1	0.77	0.99	8.91	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.728	4.274	11.084	0.000	0.000	0.000	0.000	0.172	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	593	425	17838	6387	0	0	0	0	45	0
N.S.	1	0.72	30.08	10.77	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.842	33.063	16.633	0.000	0.000	0.000	0.000	0.175	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	480	328	1129	12832	0	0	0	0	408	0
N.S.	1	0.68	2.35	26.73	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.696	20.985	244.870	0.000	0.000	0.000	0.000	0.238	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	F(-2)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	440	309	314	2820	0	0	0	0	408	0
N.S.	1	0.70	0.71	6.41	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	0.680	8.202	124.405	0.000	0.000	0.000	0.000	0.199	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	477	342	6560	10259	0	0	0	0	392	0
N.S.	1	0.72	13.75	21.51	0.00	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.698	29.433	7.001	0.000	0.000	0.000	0.000	0.199	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	430	324	422	7438	0	0	0	0	349	0
N.S.	1	0.75	0.98	17.30	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.719	8.618	10.563	0.000	0.000	0.000	0.000	0.198	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	555	431	8379	15133	0	0	0	0	63	0
N.S.	1	0.78	15.10	27.27	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.827	29.956	12.448	0.000	0.000	0.000	0.000	0.217	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	520	413	351	8277	0	0	0	0	69	0
N.S.	1	0.79	0.68	15.92	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.854	6.864	14.766	0.000	0.000	0.000	0.000	0.237	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	691	538	9161	17616	0	0	0	0	69	0
N.S.	1	0.78	13.26	25.49	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.994	28.178	18.587	0.000	0.000	0.000	0.000	0.202	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	583	395	7817	31692	0	0	0	0	68	0
N.S.	1	0.68	13.41	54.36	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.771	29.850	1424.856	0.000	0.000	0.000	0.000	0.277	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	532	374	352	17718	0	0	0	0	68	0
N.S.	1	0.70	0.66	33.30	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.770	5.913	1442.465	0.000	0.000	0.000	0.000	0.267	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	566	413	7905	25457	0	0	0	0	64	0
N.S.	1	0.73	13.97	44.98	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.808	30.147	12.890	0.000	0.000	0.000	0.000	0.231	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	515	392	369	31118	0	0	0	0	57	0
N.S.	1	0.76	0.72	60.42	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.814	5.316	14.838	0.000	0.000	0.000	0.000	0.190	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	664	513	14652	33496	0	0	0	0	85	0
N.S.	1	0.77	22.07	50.45	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.974	31.794	16.923	0.000	0.000	0.000	0.000	0.175	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	620	495	409	32827	0	0	0	0	93	0
N.S.	1	0.80	0.66	52.95	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.992	7.440	20.388	0.000	0.000	0.000	0.000	0.173	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	820	626	15481	36194	0	0	0	0	93	0
N.S.	1	0.76	18.88	44.14	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.160	29.141	27.495	0.000	0.000	0.000	0.000	0.173	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	69	0	0	0	0	0	34	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.375	0.278	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	59	0	0	0	0	0	32	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.366	0.242	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	59	0	0	0	0	0	36	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.367	0.155	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	61	0	0	0	0	0	36	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.368	0.180	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	124	109	0	0	0	0	0	62	0
N.S.	1	1.04	0.92	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.586	0.424	0.000	0.000	0.000	0.000	0.000	0.290	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	122	106	0	0	0	0	0	59	0
N.S.	1	1.03	0.89	0.00	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.562	0.505	0.000	0.000	0.000	0.000	0.000	0.239	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	122	107	0	0	0	0	0	63	0
N.S.	1	1.03	0.90	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.576	0.551	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	124	118	0	0	0	0	0	63	0
N.S.	1	1.04	0.99	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.593	0.601	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	552	379	276	0	0	0	0	0	27	0
N.S.	1	0.69	0.50	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.743	14.601	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	552	369	280	0	0	0	0	0	27	0
N.S.	1	0.67	0.51	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.730	12.664	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	579	423	285	0	0	0	0	0	35	0
N.S.	1	0.73	0.49	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.780	48.792	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	581	415	6862	0	0	0	0	0	35	0
N.S.	1	0.71	11.81	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.795	103.415	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-1)	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	687	545	8003	0	0	0	0	0	357	0
N.S.	1	0.79	11.65	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	1.211	118.428	0.000	0.000	0.000	0.000	0.000	0.294	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	687	545	4560	0	0	0	0	0	357	0
N.S.	1	0.79	6.64	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	1.093	70.798	0.000	0.000	0.000	0.000	0.000	0.275	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	715	574	18832	0	0	0	0	0	59	0
N.S.	1	0.80	26.34	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.316	128.701	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	717	576	10396	0	0	0	0	0	59	0
N.S.	1	0.80	14.50	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.300	73.687	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	206	159	0	0	0	0	0	170	0
N.S.	1	1.22	0.94	0.00	0.00	0.00	0.00	0.00	1.01	0.00
time (sec)	N/A	0.425	1.217	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	119	0	0	0	0	0	64	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.670	0.455	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	65	0	0	0	0	0	35	0
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.373	0.148	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	160	1158	0	0	0	0	0	27	0
N.S.	1	1.13	8.21	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.384	13.800	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	227	219	2453	0	0	0	0	0	362	0
N.S.	1	0.96	10.81	0.00	0.00	0.00	0.00	0.00	1.59	0.00
time (sec)	N/A	0.447	16.080	0.000	0.000	0.000	0.000	0.000	0.263	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	182	186	699	0	0	0	0	0	27	0
N.S.	1	1.02	3.84	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.399	5.059	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	138	161	462	286	420	0	0	23	0
N.S.	1	0.86	1.00	2.87	1.78	2.61	0.00	0.00	0.14	0.00
time (sec)	N/A	0.331	2.187	0.329	0.064	0.130	0.000	0.000	0.183	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	77	71	204	116	176	0	0	23	0
N.S.	1	0.88	0.81	2.32	1.32	2.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.281	0.170	181.339	0.043	0.104	0.000	0.000	0.206	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	26	64	0	26	23	51
N.S.	1	1.00	1.00	1.04	1.00	2.46	0.00	1.00	0.88	1.96
time (sec)	N/A	0.229	0.055	15.161	0.035	0.101	0.000	0.357	0.176	1.522

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	164	118	0	0	0	0	0	14	0
N.S.	1	0.98	0.71	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.400	0.210	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	272	293	225	0	0	0	0	0	23	0
N.S.	1	1.08	0.83	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.514	0.870	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	432	478	360	0	0	0	0	0	23	0
N.S.	1	1.11	0.83	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.726	2.920	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	185	306	0	0	0	0	0	23	0
N.S.	1	1.16	1.92	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.367	12.547	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	185	340	0	0	0	0	0	21	0
N.S.	1	1.16	2.14	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.322	3.185	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	185	341	0	0	0	0	0	21	0
N.S.	1	1.15	2.12	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.324	4.075	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	185	341	0	0	0	0	0	23	0
N.S.	1	1.15	2.12	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.344	5.071	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	158	133	229	0	110	0	0	52	0
N.S.	1	1.27	1.07	1.85	0.00	0.89	0.00	0.00	0.42	0.00
time (sec)	N/A	0.883	0.713	11.121	0.000	0.093	0.000	0.000	0.223	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	122	250	205	0	80	0	0	52	0
N.S.	1	1.36	2.78	2.28	0.00	0.89	0.00	0.00	0.58	0.00
time (sec)	N/A	0.694	2.763	9.010	0.000	0.080	0.000	0.000	0.200	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	122	100	168	0	58	0	0	46	0
N.S.	1	1.36	1.11	1.87	0.00	0.64	0.00	0.00	0.51	0.00
time (sec)	N/A	0.668	0.399	4.543	0.000	0.085	0.000	0.000	0.179	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	85	192	108	0	27	0	0	32	0
N.S.	1	1.42	3.20	1.80	0.00	0.45	0.00	0.00	0.53	0.00
time (sec)	N/A	0.495	2.257	3.771	0.000	0.077	0.000	0.000	0.168	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	85	143	55	0	86	0	0	52	74
N.S.	1	1.42	2.38	0.92	0.00	1.43	0.00	0.00	0.87	1.23
time (sec)	N/A	0.488	1.384	2.226	0.000	0.084	0.000	0.000	0.175	1.367

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	116	190	214	0	144	0	0	52	0
N.S.	1	1.30	2.13	2.40	0.00	1.62	0.00	0.00	0.58	0.00
time (sec)	N/A	0.635	3.015	2.885	0.000	0.081	0.000	0.000	0.210	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	120	57	260	0	180	0	0	52	0
N.S.	1	1.25	0.59	2.71	0.00	1.88	0.00	0.00	0.54	0.00
time (sec)	N/A	0.638	0.993	2.940	0.000	0.085	0.000	0.000	0.223	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	150	666	385	0	222	0	0	52	0
N.S.	1	1.15	5.12	2.96	0.00	1.71	0.00	0.00	0.40	0.00
time (sec)	N/A	0.765	6.968	3.496	0.000	0.081	0.000	0.000	0.201	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	223	156	387	0	151	0	0	49	0
N.S.	1	1.17	0.82	2.04	0.00	0.79	0.00	0.00	0.26	0.00
time (sec)	N/A	1.038	1.990	14.992	0.000	0.101	0.000	0.000	0.214	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	187	464	351	0	140	0	0	49	0
N.S.	1	1.21	3.01	2.28	0.00	0.91	0.00	0.00	0.32	0.00
time (sec)	N/A	0.858	3.991	12.761	0.000	0.093	0.000	0.000	0.214	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	187	131	315	0	115	0	0	45	0
N.S.	1	1.21	0.85	2.05	0.00	0.75	0.00	0.00	0.29	0.00
time (sec)	N/A	0.855	1.409	10.136	0.000	0.102	0.000	0.000	0.193	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	151	268	277	0	102	0	0	38	0
N.S.	1	1.26	2.23	2.31	0.00	0.85	0.00	0.00	0.32	0.00
time (sec)	N/A	0.725	4.340	7.917	0.000	0.084	0.000	0.000	0.213	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	151	158	239	0	91	0	0	55	0
N.S.	1	1.26	1.32	1.99	0.00	0.76	0.00	0.00	0.46	0.00
time (sec)	N/A	0.702	1.097	6.012	0.000	0.090	0.000	0.000	0.172	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	115	114	206	0	94	0	0	76	0
N.S.	1	1.25	1.24	2.24	0.00	1.02	0.00	0.00	0.83	0.00
time (sec)	N/A	0.561	2.102	4.657	0.000	0.091	0.000	0.000	0.230	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	115	116	171	0	79	0	0	82	0
N.S.	1	1.25	1.26	1.86	0.00	0.86	0.00	0.00	0.89	0.00
time (sec)	N/A	0.574	1.011	3.459	0.000	0.085	0.000	0.000	0.164	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	141	1106	135	0	108	0	0	82	0
N.S.	1	1.16	9.07	1.11	0.00	0.89	0.00	0.00	0.67	0.00
time (sec)	N/A	0.704	6.965	4.124	0.000	0.080	0.000	0.000	0.154	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	145	67	208	0	144	0	0	82	0
N.S.	1	1.15	0.53	1.65	0.00	1.14	0.00	0.00	0.65	0.00
time (sec)	N/A	0.747	1.317	4.155	0.000	0.083	0.000	0.000	0.166	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	179	414	321	0	187	0	0	82	0
N.S.	1	1.09	2.52	1.96	0.00	1.14	0.00	0.00	0.50	0.00
time (sec)	N/A	0.904	3.593	5.310	0.000	0.083	0.000	0.000	0.160	0.000

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	199	80	79	202	100	0	0	37	96
N.S.	1	1.11	0.45	0.44	1.13	0.56	0.00	0.00	0.21	0.54
time (sec)	N/A	0.952	1.660	10.007	0.227	0.081	0.000	0.000	0.206	2.560

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	152	63	62	148	86	0	0	37	84
N.S.	1	1.15	0.48	0.47	1.12	0.65	0.00	0.00	0.28	0.64
time (sec)	N/A	0.706	1.092	9.753	0.252	0.087	0.000	0.000	0.176	0.864

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	106	56	50	59	68	0	0	33	88
N.S.	1	1.25	0.66	0.59	0.69	0.80	0.00	0.00	0.39	1.04
time (sec)	N/A	0.539	0.719	9.601	0.227	0.073	0.000	0.000	0.168	0.606

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	31	76	52	0	0	97	0
N.S.	1	1.00	1.00	0.86	2.11	1.44	0.00	0.00	2.69	0.00
time (sec)	N/A	0.343	0.671	11.109	0.177	0.075	0.000	0.000	0.191	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	330	125	203	1400	313	0	0	37	0
N.S.	1	1.43	0.54	0.88	6.09	1.36	0.00	0.00	0.16	0.00
time (sec)	N/A	0.499	1.100	11.552	0.372	0.087	0.000	0.000	0.160	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	438	274	269	1836	470	0	0	37	0
N.S.	1	0.93	0.58	0.57	3.91	1.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.047	3.202	11.567	0.299	0.091	0.000	0.000	0.162	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	445	227	291	2249	569	0	0	37	0
N.S.	1	1.10	0.56	0.72	5.57	1.41	0.00	0.00	0.09	0.00
time (sec)	N/A	1.031	2.331	11.602	0.470	0.097	0.000	0.000	0.168	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	571	538	305	315	2661	657	0	0	37	0
N.S.	1	0.94	0.53	0.55	4.66	1.15	0.00	0.00	0.06	0.00
time (sec)	N/A	1.514	2.750	11.808	0.462	0.099	0.000	0.000	0.177	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	198	80	70	202	103	0	0	103	110
N.S.	1	1.13	0.46	0.40	1.15	0.59	0.00	0.00	0.59	0.63
time (sec)	N/A	0.982	1.405	9.386	0.238	0.075	0.000	0.000	0.204	2.420

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	152	63	61	136	83	0	0	97	100
N.S.	1	1.21	0.50	0.48	1.08	0.66	0.00	0.00	0.77	0.79
time (sec)	N/A	0.717	0.970	8.994	0.234	0.075	0.000	0.000	0.206	2.015

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	105	48	42	80	68	0	0	253	82
N.S.	1	1.31	0.60	0.52	1.00	0.85	0.00	0.00	3.16	1.02
time (sec)	N/A	0.531	0.604	9.752	0.220	0.098	0.000	0.000	0.227	1.553

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	31	76	58	0	0	111	0
N.S.	1	1.00	1.00	0.86	2.11	1.61	0.00	0.00	3.08	0.00
time (sec)	N/A	0.376	0.651	14.652	0.141	0.073	0.000	0.000	0.222	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	350	373	209	231	714	341	0	0	119	0
N.S.	1	1.07	0.60	0.66	2.04	0.97	0.00	0.00	0.34	0.00
time (sec)	N/A	0.662	1.386	14.542	0.250	0.094	0.000	0.000	0.195	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	400	250	299	2147	502	0	0	119	0
N.S.	1	1.11	0.69	0.83	5.96	1.39	0.00	0.00	0.33	0.00
time (sec)	N/A	0.837	1.865	14.757	0.419	0.096	0.000	0.000	0.191	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	493	245	322	2254	604	0	0	119	0
N.S.	1	0.92	0.46	0.60	4.22	1.13	0.00	0.00	0.22	0.00
time (sec)	N/A	1.292	1.906	15.120	0.340	0.098	0.000	0.000	0.177	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	105	202	0	0	0	0	0	28	0
N.S.	1	0.96	1.85	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.610	13.503	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	0	0	0	0	0	59	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.551	8.334	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	78	0	0	0	0	0	35	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.507	0.226	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	433	0	0	0	0	0	30	0
N.S.	1	1.00	5.03	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.568	4.820	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	100	0	0	0	0	0	40	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.581	1.815	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	117	122	0	0	0	0	0	102	0
N.S.	1	1.11	1.16	0.00	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.590	1.391	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	116	143	0	0	0	0	0	262	0
N.S.	1	1.10	1.36	0.00	0.00	0.00	0.00	0.00	2.50	0.00
time (sec)	N/A	0.607	1.666	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	220	212	0	0	0	0	0	90	0
N.S.	1	1.24	1.19	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.566	4.487	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	175	140	0	0	0	0	0	63	0
N.S.	1	1.13	0.90	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.854	3.476	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	113	75	0	0	0	0	0	36	0
N.S.	1	1.26	0.83	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.494	0.126	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	157	869	0	0	0	0	0	27	0
N.S.	1	1.12	6.21	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.484	8.398	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	227	215	2502	0	0	0	0	0	43	0
N.S.	1	0.95	11.02	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.544	15.681	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	698	0	0	0	0	0	27	0
N.S.	1	1.00	3.84	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.477	16.040	0.000	0.000	0.000	0.000	0.000	0.190	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [173] had the largest ratio of [1.1333299999999995]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	0.87	22	0.227
2	A	6	5	0.91	22	0.227
3	A	6	5	0.94	22	0.227
4	A	6	5	1.00	22	0.227
5	A	6	5	1.11	22	0.227
6	A	1	1	1.00	13	0.077
7	A	5	5	1.02	22	0.227
8	A	7	7	1.07	22	0.318
9	A	9	9	1.10	22	0.409
10	A	11	11	1.12	22	0.500
11	A	10	10	1.09	22	0.455
12	A	8	8	1.07	22	0.364
13	A	6	6	1.02	22	0.273
14	A	4	4	1.00	20	0.200
15	A	4	4	1.00	20	0.200
16	A	6	5	1.00	22	0.227
17	A	6	5	0.94	22	0.227
18	A	6	5	0.87	22	0.227
19	A	5	4	0.86	24	0.167
20	A	5	4	0.88	24	0.167
21	A	5	4	0.91	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	4	3	1.00	24	0.125
23	A	4	4	1.00	15	0.267
24	A	4	3	1.00	24	0.125
25	A	5	4	1.06	24	0.167
26	A	5	4	0.92	24	0.167
27	A	5	4	0.89	24	0.167
28	A	10	10	1.03	24	0.417
29	A	8	8	1.01	24	0.333
30	A	6	6	0.96	22	0.273
31	A	4	4	1.00	22	0.182
32	A	4	4	1.00	24	0.167
33	A	6	5	0.96	24	0.208
34	A	6	5	0.90	24	0.208
35	A	6	5	0.82	24	0.208
36	A	5	4	0.86	24	0.167
37	A	5	4	0.88	24	0.167
38	A	5	4	0.91	24	0.167
39	A	4	3	1.00	24	0.125
40	A	6	6	1.06	15	0.400
41	A	5	4	0.90	24	0.167
42	A	4	3	1.00	24	0.125
43	A	5	4	0.95	24	0.167
44	A	5	4	0.86	24	0.167
45	A	10	10	1.06	24	0.417
46	A	8	8	1.03	22	0.364
47	A	6	6	1.02	22	0.273
48	A	2	2	1.00	24	0.083
49	A	8	7	0.99	24	0.292
50	A	8	7	0.93	24	0.292
51	A	8	7	0.86	24	0.292
52	A	12	12	1.06	24	0.500
53	A	10	10	1.05	22	0.455

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	8	8	1.03	22	0.364
55	A	6	6	1.09	24	0.250
56	A	4	4	1.00	24	0.167
57	A	8	7	1.05	24	0.292
58	A	8	7	0.99	24	0.292
59	A	5	4	0.86	24	0.167
60	A	5	4	0.88	24	0.167
61	A	5	4	0.91	24	0.167
62	A	4	3	1.00	24	0.125
63	A	10	10	1.07	15	0.667
64	A	5	4	0.94	24	0.167
65	A	5	4	0.92	24	0.167
66	A	5	4	0.91	24	0.167
67	A	4	3	1.00	24	0.125
68	A	5	4	0.88	24	0.167
69	A	5	4	0.85	24	0.167
70	A	12	12	1.05	22	0.545
71	A	10	10	1.05	22	0.455
72	A	8	8	1.05	24	0.333
73	A	2	2	1.00	24	0.083
74	A	6	6	1.03	24	0.250
75	A	10	9	0.99	24	0.375
76	A	10	9	0.93	24	0.375
77	A	5	4	0.86	24	0.167
78	A	5	4	0.88	24	0.167
79	A	5	4	0.91	24	0.167
80	A	4	3	1.00	24	0.125
81	A	16	16	1.06	15	1.067
82	A	5	4	0.91	24	0.167
83	A	5	4	0.91	24	0.167
84	A	5	4	0.89	24	0.167
85	A	4	3	0.98	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	5	4	0.88	24	0.167
87	A	5	4	0.91	24	0.167
88	A	4	3	1.00	24	0.125
89	A	5	4	0.87	24	0.167
90	A	5	4	0.86	24	0.167
91	A	16	16	1.06	22	0.727
92	A	14	14	1.05	24	0.583
93	A	12	12	1.05	24	0.500
94	A	10	10	1.07	24	0.417
95	A	4	4	1.00	24	0.167
96	A	8	8	1.04	24	0.333
97	A	12	12	1.03	24	0.500
98	A	12	11	0.99	24	0.458
99	A	5	4	0.86	24	0.167
100	A	5	4	0.88	24	0.167
101	A	5	4	0.91	24	0.167
102	A	4	3	1.00	24	0.125
103	A	4	3	1.04	24	0.125
104	A	3	3	1.00	15	0.200
105	A	5	4	1.06	24	0.167
106	A	5	4	0.91	24	0.167
107	A	8	8	1.01	24	0.333
108	A	6	6	0.98	24	0.250
109	A	4	4	1.00	24	0.167
110	A	2	2	1.00	22	0.091
111	A	4	4	1.00	22	0.182
112	A	6	5	0.96	24	0.208
113	A	6	5	0.89	24	0.208
114	A	5	4	0.88	24	0.167
115	A	5	4	0.91	24	0.167
116	A	4	3	1.00	24	0.125
117	A	5	4	1.03	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	4	3	0.96	24	0.125
119	A	5	5	1.07	15	0.333
120	A	5	4	1.01	24	0.167
121	A	5	4	0.96	24	0.167
122	A	10	10	1.05	24	0.417
123	A	8	8	1.04	24	0.333
124	A	6	6	1.00	24	0.250
125	A	4	4	1.00	24	0.167
126	A	4	4	0.98	22	0.182
127	A	6	5	0.96	22	0.227
128	A	6	5	0.90	24	0.208
129	A	6	5	0.82	24	0.208
130	A	5	4	0.86	24	0.167
131	A	5	4	0.88	24	0.167
132	A	5	4	0.91	24	0.167
133	A	4	3	1.00	24	0.125
134	A	5	4	1.00	24	0.167
135	A	5	4	0.94	24	0.167
136	A	4	3	1.00	24	0.125
137	A	7	7	1.10	15	0.467
138	A	5	4	0.94	24	0.167
139	A	5	4	0.87	24	0.167
140	A	10	10	1.08	24	0.417
141	A	8	8	1.05	24	0.333
142	A	6	6	1.08	24	0.250
143	A	2	2	1.00	24	0.083
144	A	6	6	1.04	22	0.273
145	A	8	7	1.05	22	0.318
146	A	8	7	0.99	24	0.292
147	A	8	7	0.92	24	0.292
148	A	5	4	0.88	24	0.167
149	A	5	4	0.91	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	4	3	1.00	24	0.125
151	A	5	4	0.88	24	0.167
152	A	5	4	0.94	24	0.167
153	A	4	3	0.97	24	0.125
154	A	4	3	1.00	24	0.125
155	A	9	9	1.11	15	0.600
156	A	5	4	0.94	24	0.167
157	A	5	4	0.89	24	0.167
158	A	10	10	1.08	24	0.417
159	A	8	8	1.06	24	0.333
160	A	6	6	1.09	24	0.250
161	A	4	4	1.00	24	0.167
162	A	8	8	1.06	22	0.364
163	A	10	9	1.07	22	0.409
164	A	10	9	1.03	24	0.375
165	A	10	9	0.97	24	0.375
166	A	5	4	0.90	24	0.167
167	A	5	4	0.90	24	0.167
168	A	5	4	0.87	24	0.167
169	A	4	3	0.98	24	0.125
170	A	5	4	0.85	24	0.167
171	A	5	4	0.91	24	0.167
172	A	4	3	1.00	24	0.125
173	A	17	17	1.12	15	1.133
174	A	5	4	0.89	24	0.167
175	A	5	4	0.87	24	0.167
176	A	14	14	1.10	24	0.583
177	A	12	12	1.09	24	0.500
178	A	10	10	1.10	24	0.417
179	A	4	4	1.00	24	0.167
180	A	8	8	1.07	24	0.333
181	A	12	12	1.07	24	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	16	16	1.09	22	0.727
183	A	18	17	1.09	22	0.773
184	A	18	17	1.06	24	0.708
185	A	10	10	1.02	26	0.385
186	A	8	8	1.01	26	0.308
187	A	8	8	1.01	26	0.308
188	A	6	6	1.00	26	0.231
189	A	6	6	1.00	26	0.231
190	A	8	8	1.01	26	0.308
191	A	8	8	1.01	26	0.308
192	A	10	10	1.06	26	0.385
193	A	10	10	0.98	28	0.357
194	A	8	8	0.98	28	0.286
195	A	8	8	1.04	28	0.286
196	A	6	6	1.00	28	0.214
197	A	6	6	1.00	28	0.214
198	A	8	8	1.04	28	0.286
199	A	8	8	1.04	28	0.286
200	A	10	10	1.07	28	0.357
201	A	14	14	1.05	28	0.500
202	A	12	12	1.04	28	0.429
203	A	12	12	1.02	28	0.429
204	A	10	10	1.06	28	0.357
205	A	10	10	1.24	28	0.357
206	A	8	8	0.97	28	0.286
207	A	8	8	0.97	28	0.286
208	A	8	8	1.05	28	0.286
209	A	8	8	1.05	28	0.286
210	A	10	10	1.07	28	0.357
211	A	10	10	1.07	28	0.357
212	A	12	12	1.09	28	0.429
213	A	14	14	1.03	28	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	12	12	1.04	28	0.429
215	A	12	12	1.01	28	0.429
216	A	10	10	1.03	28	0.357
217	A	10	10	1.02	28	0.357
218	A	8	8	1.06	28	0.286
219	A	8	8	1.06	28	0.286
220	A	10	10	1.08	28	0.357
221	A	10	10	1.08	28	0.357
222	A	12	12	1.10	28	0.429
223	A	10	10	0.99	28	0.357
224	A	8	8	1.00	28	0.286
225	A	8	8	1.00	28	0.286
226	A	6	6	1.00	28	0.214
227	A	6	6	1.00	28	0.214
228	A	6	6	1.00	28	0.214
229	A	6	6	1.00	28	0.214
230	A	8	8	1.02	28	0.286
231	A	8	8	1.02	28	0.286
232	A	10	10	1.05	28	0.357
233	A	12	12	1.03	28	0.429
234	A	10	10	1.04	28	0.357
235	A	10	10	1.01	28	0.357
236	A	8	8	1.01	28	0.286
237	A	8	8	1.01	28	0.286
238	A	6	6	1.00	28	0.214
239	A	6	6	1.00	28	0.214
240	A	8	8	1.09	28	0.286
241	A	8	8	1.09	28	0.286
242	A	10	10	1.08	28	0.357
243	A	10	10	1.08	28	0.357
244	A	12	12	1.09	28	0.429
245	A	12	12	1.06	28	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	10	10	1.09	28	0.357
247	A	10	10	1.06	28	0.357
248	A	8	8	1.06	28	0.286
249	A	8	8	1.06	28	0.286
250	A	8	8	1.01	28	0.286
251	A	8	8	1.01	28	0.286
252	A	10	10	1.12	28	0.357
253	A	10	10	1.12	28	0.357
254	A	12	12	1.11	28	0.429
255	A	12	12	1.06	28	0.429
256	A	10	10	1.09	28	0.357
257	A	10	10	1.04	28	0.357
258	A	8	8	1.08	28	0.286
259	A	8	8	1.08	28	0.286
260	A	10	10	1.10	28	0.357
261	A	10	10	1.10	28	0.357
262	A	12	12	1.12	28	0.429
263	A	8	7	1.00	26	0.269
264	A	8	7	1.00	26	0.269
265	A	8	7	1.00	26	0.269
266	A	8	7	1.00	26	0.269
267	A	8	7	1.00	28	0.250
268	A	8	7	1.00	28	0.250
269	A	8	7	0.96	28	0.250
270	A	8	7	0.96	28	0.250
271	A	8	7	0.99	28	0.250
272	A	8	7	0.99	28	0.250
273	A	8	7	1.00	28	0.250
274	A	8	7	1.00	28	0.250
275	A	8	7	1.02	28	0.250
276	A	8	7	1.02	28	0.250
277	A	8	7	1.00	28	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	8	7	1.00	28	0.250
279	A	5	4	0.87	26	0.154
280	A	5	4	0.89	26	0.154
281	A	5	4	0.92	26	0.154
282	A	4	3	1.00	26	0.115
283	A	7	6	0.90	26	0.231
284	A	9	8	0.97	26	0.308
285	A	11	10	0.99	26	0.385
286	A	8	8	1.05	26	0.308
287	A	6	6	1.04	26	0.231
288	A	4	4	1.00	26	0.154
289	A	2	2	1.00	24	0.083
290	A	6	5	1.00	24	0.208
291	A	10	9	1.06	26	0.346
292	A	14	13	1.10	26	0.500
293	A	5	4	0.87	26	0.154
294	A	5	4	0.89	26	0.154
295	A	5	4	0.92	26	0.154
296	A	4	3	1.00	26	0.115
297	A	6	5	0.87	26	0.192
298	A	8	7	0.94	26	0.269
299	A	10	9	0.96	26	0.346
300	A	8	8	1.05	26	0.308
301	A	6	6	1.04	26	0.231
302	A	4	4	1.00	24	0.167
303	A	2	2	1.00	24	0.083
304	A	8	7	1.01	26	0.269
305	A	12	11	1.06	26	0.423
306	A	5	4	0.87	26	0.154
307	A	5	4	0.89	26	0.154
308	A	5	4	0.92	26	0.154
309	A	4	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	6	5	0.85	26	0.192
311	A	7	6	0.93	26	0.231
312	A	9	8	0.94	26	0.308
313	A	8	8	1.05	26	0.308
314	A	6	6	1.04	24	0.250
315	A	4	4	1.00	24	0.167
316	A	2	2	1.00	26	0.077
317	A	10	9	1.03	26	0.346
318	A	14	13	1.05	26	0.500
319	A	5	4	0.87	26	0.154
320	A	5	4	0.89	26	0.154
321	A	5	4	0.92	26	0.154
322	A	4	3	1.00	26	0.115
323	A	7	6	0.86	26	0.231
324	A	7	6	0.89	26	0.231
325	A	8	7	0.94	26	0.269
326	A	8	8	1.06	24	0.333
327	A	6	6	1.00	24	0.250
328	A	4	4	1.00	26	0.154
329	A	2	2	1.00	26	0.077
330	A	12	11	1.04	26	0.423
331	A	16	15	1.06	26	0.577
332	A	20	19	1.07	26	0.731
333	A	5	4	0.87	26	0.154
334	A	5	4	0.89	26	0.154
335	A	5	4	0.92	26	0.154
336	A	4	3	1.00	26	0.115
337	A	8	7	0.96	26	0.269
338	A	10	9	1.00	26	0.346
339	A	12	11	1.01	26	0.423
340	A	8	8	1.05	26	0.308
341	A	6	6	1.04	26	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	4	4	1.00	26	0.154
343	A	2	2	1.00	26	0.077
344	A	4	3	1.00	24	0.125
345	A	8	7	1.01	24	0.292
346	A	12	11	1.06	26	0.423
347	A	5	4	0.87	26	0.154
348	A	5	4	0.89	26	0.154
349	A	5	4	0.91	26	0.154
350	A	4	3	1.00	26	0.115
351	A	9	8	0.98	26	0.308
352	A	11	10	1.01	26	0.385
353	A	13	12	1.02	26	0.462
354	A	8	8	1.05	26	0.308
355	A	6	6	1.04	26	0.231
356	A	4	4	1.00	26	0.154
357	A	2	2	1.00	26	0.077
358	A	6	5	1.00	26	0.192
359	A	6	5	1.00	24	0.208
360	A	10	9	1.04	24	0.375
361	A	14	13	1.06	26	0.500
362	A	5	4	0.86	26	0.154
363	A	5	4	0.87	26	0.154
364	A	5	4	0.88	26	0.154
365	A	5	4	0.91	26	0.154
366	A	4	3	1.00	26	0.115
367	A	10	9	1.00	26	0.346
368	A	12	11	1.02	26	0.423
369	A	8	8	1.05	26	0.308
370	A	6	6	1.04	26	0.231
371	A	4	4	1.00	26	0.154
372	A	2	2	1.00	26	0.077
373	A	8	7	1.05	26	0.269

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	8	7	1.47	26	0.269
375	A	8	7	1.04	24	0.292
376	A	12	11	1.06	24	0.458
377	A	16	15	1.07	26	0.577
378	A	5	4	0.86	26	0.154
379	A	5	4	0.87	26	0.154
380	A	5	4	0.88	26	0.154
381	A	5	4	0.91	26	0.154
382	A	4	3	1.00	26	0.115
383	A	11	10	1.01	26	0.385
384	A	13	12	1.03	26	0.462
385	A	6	6	1.04	26	0.231
386	A	4	4	1.00	26	0.154
387	A	2	2	1.00	26	0.077
388	A	10	9	1.08	26	0.346
389	A	10	9	1.38	26	0.346
390	A	10	9	1.36	26	0.346
391	A	10	9	1.06	24	0.375
392	A	14	13	1.07	24	0.542
393	A	18	17	1.08	26	0.654
394	A	15	14	1.04	30	0.467
395	A	11	10	1.38	30	0.333
396	A	2	2	1.00	30	0.067
397	A	4	4	1.00	30	0.133
398	A	6	6	1.04	30	0.200
399	A	8	8	1.06	30	0.267
400	A	17	16	1.26	30	0.533
401	A	17	16	1.02	30	0.533
402	A	13	12	1.34	30	0.400
403	A	15	14	1.03	30	0.467
404	A	2	2	1.00	30	0.067
405	A	4	4	1.00	30	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	6	6	1.02	30	0.200
407	A	8	8	1.04	30	0.267
408	A	19	18	1.03	30	0.600
409	A	15	14	1.28	30	0.467
410	A	17	16	1.04	30	0.533
411	A	13	12	1.33	30	0.400
412	A	2	2	1.00	30	0.067
413	A	4	4	1.00	30	0.133
414	A	6	6	1.02	30	0.200
415	A	8	8	1.04	30	0.267
416	A	13	12	1.33	30	0.400
417	A	13	12	1.03	30	0.400
418	A	2	2	1.00	30	0.067
419	A	4	4	1.00	30	0.133
420	A	6	6	1.05	30	0.200
421	A	8	8	1.05	30	0.267
422	A	10	10	1.07	30	0.333
423	A	17	16	1.15	30	0.533
424	A	13	12	1.34	30	0.400
425	A	2	2	1.00	30	0.067
426	A	4	4	1.00	30	0.133
427	A	6	6	1.04	30	0.200
428	A	8	8	1.05	30	0.267
429	A	10	10	1.05	30	0.333
430	A	15	14	1.31	30	0.467
431	A	15	14	1.04	30	0.467
432	A	2	2	1.00	30	0.067
433	A	4	4	1.00	30	0.133
434	A	6	6	1.04	30	0.200
435	A	8	8	1.06	30	0.267
436	A	10	10	1.06	30	0.333
437	A	8	7	1.00	30	0.233

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
438	A	8	7	1.00	30	0.233
439	A	8	7	0.99	30	0.233
440	A	8	7	0.99	30	0.233
441	A	8	7	0.99	30	0.233
442	A	8	7	1.02	30	0.233
443	A	13	12	0.57	30	0.400
444	A	12	11	0.53	30	0.367
445	A	9	8	0.45	30	0.267
446	A	2	2	1.00	30	0.067
447	A	4	4	1.00	30	0.133
448	A	6	6	1.03	30	0.200
449	A	8	8	1.05	30	0.267
450	A	7	6	1.02	26	0.231
451	A	7	6	1.02	26	0.231
452	A	7	6	1.33	26	0.231
453	A	7	6	1.39	24	0.250
454	A	7	6	1.02	26	0.231
455	A	7	6	1.02	26	0.231
456	A	7	6	1.02	26	0.231
457	A	7	6	1.13	28	0.214
458	A	7	6	1.13	28	0.214
459	A	7	6	1.07	28	0.214
460	A	7	6	1.07	28	0.214
461	A	7	6	1.10	28	0.214
462	A	7	6	1.13	28	0.214
463	A	7	6	1.13	28	0.214
464	A	7	6	0.96	26	0.231
465	A	5	4	0.89	24	0.167
466	A	5	4	0.92	24	0.167
467	A	4	3	1.00	24	0.125
468	A	4	3	1.07	24	0.125
469	A	4	3	1.07	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
470	A	4	3	1.07	24	0.125
471	A	7	6	1.02	24	0.250
472	A	7	6	1.00	24	0.250
473	A	7	6	1.00	22	0.273
474	A	7	6	0.97	22	0.273
475	A	7	6	1.02	24	0.250
476	A	7	6	1.02	24	0.250
477	A	7	6	1.00	28	0.214
478	A	7	6	1.00	28	0.214
479	A	7	6	1.00	28	0.214
480	A	7	6	0.97	28	0.214
481	A	7	6	0.97	28	0.214
482	A	7	6	1.02	28	0.214
483	A	10	10	0.96	30	0.333
484	A	8	8	1.02	30	0.267
485	A	6	6	0.99	30	0.200
486	A	4	4	1.03	30	0.133
487	A	2	2	1.00	28	0.071
488	A	7	6	1.29	30	0.200
489	A	7	6	1.29	30	0.200
490	A	7	6	1.29	30	0.200
491	A	6	6	1.02	30	0.200
492	A	7	6	1.38	30	0.200
493	A	4	4	1.04	30	0.133
494	A	7	6	1.36	30	0.200
495	A	2	2	1.00	30	0.067
496	A	7	6	1.37	30	0.200
497	A	6	5	1.06	28	0.179
498	A	7	6	1.36	30	0.200
499	A	8	7	1.05	30	0.233
500	A	7	6	1.38	30	0.200
501	A	8	7	1.06	32	0.219

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
502	A	8	7	1.06	32	0.219
503	A	6	5	1.06	30	0.167
504	A	2	2	1.00	32	0.062
505	A	4	4	1.00	32	0.125
506	A	6	6	0.98	32	0.188
507	A	6	5	0.93	19	0.263
508	A	6	5	1.00	19	0.263
509	A	6	5	1.00	19	0.263
510	A	1	1	1.00	10	0.100
511	A	5	5	1.02	19	0.263
512	A	7	7	1.08	19	0.368
513	A	9	9	1.10	19	0.474
514	A	8	8	1.07	19	0.421
515	A	6	6	1.02	19	0.316
516	A	4	4	1.00	17	0.235
517	A	4	4	1.00	17	0.235
518	A	6	5	1.00	19	0.263
519	A	6	5	0.93	19	0.263
520	A	6	5	0.86	19	0.263
521	A	6	5	1.08	21	0.238
522	A	6	5	1.10	21	0.238
523	A	6	5	0.85	21	0.238
524	A	4	3	1.00	21	0.143
525	A	4	4	1.00	12	0.333
526	A	6	5	1.59	21	0.238
527	A	7	6	1.46	21	0.286
528	A	12	11	1.01	21	0.524
529	A	11	10	1.04	21	0.476
530	A	10	9	1.11	19	0.474
531	A	9	8	0.98	19	0.421
532	A	9	8	0.98	21	0.381
533	A	10	9	0.95	21	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
534	A	10	9	0.93	21	0.429
535	A	10	9	0.92	21	0.429
536	A	6	5	0.99	21	0.238
537	A	6	5	0.83	21	0.238
538	A	6	5	0.85	21	0.238
539	A	4	3	1.00	21	0.143
540	A	6	6	1.00	12	0.500
541	A	7	6	1.23	21	0.286
542	A	7	6	1.55	21	0.286
543	A	9	8	1.46	21	0.381
544	A	14	13	0.99	21	0.619
545	A	13	12	1.02	21	0.571
546	A	12	11	1.08	19	0.579
547	A	12	11	0.96	19	0.579
548	A	8	7	0.93	21	0.333
549	A	10	9	1.02	21	0.429
550	A	10	9	0.98	21	0.429
551	A	10	9	0.96	21	0.429
552	A	6	5	0.89	21	0.238
553	A	6	5	0.88	21	0.238
554	A	4	3	1.00	21	0.143
555	A	8	7	1.70	21	0.333
556	A	10	9	1.61	21	0.429
557	A	15	14	1.07	21	0.667
558	A	9	8	1.01	21	0.381
559	A	4	3	1.00	19	0.158
560	A	9	8	0.92	19	0.421
561	A	15	14	0.92	21	0.667
562	A	6	5	0.88	21	0.238
563	A	6	5	0.86	21	0.238
564	A	6	5	0.84	21	0.238
565	A	4	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
566	A	8	7	1.38	21	0.333
567	A	10	9	1.37	21	0.429
568	A	12	11	0.87	21	0.524
569	A	10	9	0.88	21	0.429
570	A	8	7	1.14	21	0.333
571	A	6	5	1.26	19	0.263
572	A	9	8	1.10	19	0.421
573	A	13	12	1.13	21	0.571
574	A	6	5	0.86	21	0.238
575	A	6	5	0.84	21	0.238
576	A	6	5	0.83	21	0.238
577	A	4	3	1.00	21	0.143
578	A	8	7	1.30	21	0.333
579	A	10	9	1.31	21	0.429
580	A	13	12	0.87	21	0.571
581	A	11	10	1.11	21	0.476
582	A	6	5	1.23	21	0.238
583	A	8	7	1.10	19	0.368
584	A	11	10	1.13	19	0.526
585	A	16	15	1.16	21	0.714
586	A	10	10	1.02	23	0.435
587	A	8	8	1.01	23	0.348
588	A	8	8	1.01	23	0.348
589	A	6	6	1.00	23	0.261
590	A	6	6	1.00	23	0.261
591	A	8	8	1.01	23	0.348
592	A	8	8	1.01	23	0.348
593	A	10	10	1.07	23	0.435
594	A	11	11	0.97	25	0.440
595	A	11	11	0.94	25	0.440
596	A	9	9	1.01	25	0.360
597	A	9	9	1.00	25	0.360

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
598	A	11	11	0.95	25	0.440
599	A	11	11	0.97	25	0.440
600	A	13	13	0.96	25	0.520
601	A	13	13	0.96	25	0.520
602	A	9	8	0.90	25	0.320
603	A	9	8	0.98	25	0.320
604	A	8	7	1.09	25	0.280
605	A	9	8	1.03	25	0.320
606	A	7	6	1.08	25	0.240
607	A	8	7	1.19	25	0.280
608	A	7	6	0.87	25	0.240
609	A	7	6	0.80	25	0.240
610	A	8	7	0.80	25	0.280
611	A	18	17	0.71	25	0.680
612	A	19	18	0.76	25	0.720
613	A	13	12	0.74	25	0.480
614	A	15	14	0.77	25	0.560
615	A	19	18	0.76	25	0.720
616	A	19	18	0.77	25	0.720
617	A	21	20	0.72	25	0.800
618	A	17	16	0.68	25	0.640
619	A	18	17	0.70	25	0.680
620	A	18	17	0.72	25	0.680
621	A	19	18	0.75	25	0.720
622	A	21	20	0.78	25	0.800
623	A	21	20	0.79	25	0.800
624	A	22	21	0.78	25	0.840
625	A	19	18	0.68	25	0.720
626	A	20	19	0.70	25	0.760
627	A	21	20	0.73	25	0.800
628	A	22	21	0.76	25	0.840
629	A	23	22	0.77	25	0.880

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
630	A	24	23	0.80	25	0.920
631	A	25	24	0.76	25	0.960
632	A	6	6	1.00	23	0.261
633	A	6	6	1.00	23	0.261
634	A	6	6	1.00	23	0.261
635	A	6	6	1.00	23	0.261
636	A	9	9	1.04	25	0.360
637	A	9	9	1.03	25	0.360
638	A	9	9	1.03	25	0.360
639	A	9	9	1.04	25	0.360
640	A	16	15	0.69	25	0.600
641	A	16	15	0.67	25	0.600
642	A	17	16	0.73	25	0.640
643	A	17	16	0.71	25	0.640
644	A	5	4	0.79	25	0.160
645	A	5	4	0.79	25	0.160
646	A	5	4	0.80	25	0.160
647	A	5	4	0.80	25	0.160
648	A	8	7	1.22	23	0.304
649	A	9	9	1.00	23	0.391
650	A	6	6	1.00	21	0.286
651	A	7	6	1.13	23	0.261
652	A	5	4	0.96	23	0.174
653	A	4	3	1.02	23	0.130
654	A	6	5	0.86	21	0.238
655	A	6	5	0.88	21	0.238
656	A	4	3	1.00	21	0.143
657	A	5	4	0.98	12	0.333
658	A	8	7	1.08	21	0.333
659	A	10	9	1.11	21	0.429
660	A	5	4	1.16	21	0.190
661	A	5	4	1.16	19	0.211

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
662	A	5	4	1.15	19	0.211
663	A	5	4	1.15	21	0.190
664	A	12	12	1.27	26	0.462
665	A	10	10	1.36	26	0.385
666	A	10	10	1.36	26	0.385
667	A	8	8	1.42	26	0.308
668	A	8	8	1.42	26	0.308
669	A	10	10	1.30	26	0.385
670	A	10	10	1.25	26	0.385
671	A	12	12	1.15	26	0.462
672	A	14	14	1.17	28	0.500
673	A	12	12	1.21	28	0.429
674	A	12	12	1.21	28	0.429
675	A	10	10	1.26	28	0.357
676	A	10	10	1.26	28	0.357
677	A	8	8	1.25	28	0.286
678	A	8	8	1.25	28	0.286
679	A	10	10	1.16	28	0.357
680	A	10	10	1.15	28	0.357
681	A	12	12	1.09	28	0.429
682	A	10	10	1.11	30	0.333
683	A	8	8	1.15	30	0.267
684	A	6	6	1.25	30	0.200
685	A	4	4	1.00	30	0.133
686	A	11	10	1.43	30	0.333
687	A	17	16	0.93	30	0.533
688	A	17	16	1.10	30	0.533
689	A	21	20	0.94	30	0.667
690	A	10	10	1.13	30	0.333
691	A	8	8	1.21	30	0.267
692	A	6	6	1.31	30	0.200
693	A	4	4	1.00	30	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
694	A	13	12	1.07	30	0.400
695	A	15	14	1.11	30	0.467
696	A	19	18	0.92	30	0.600
697	A	9	8	0.96	26	0.308
698	A	9	8	1.00	26	0.308
699	A	9	8	1.00	24	0.333
700	A	9	8	1.00	26	0.308
701	A	9	8	1.00	26	0.308
702	A	9	8	1.11	28	0.286
703	A	9	8	1.10	28	0.286
704	A	10	9	1.24	23	0.391
705	A	11	11	1.13	23	0.478
706	A	8	8	1.26	21	0.381
707	A	9	8	1.12	23	0.348
708	A	7	6	0.95	23	0.261
709	A	6	5	1.00	23	0.217

CHAPTER 3

Listing of integrals

3.1 $\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^{10}(c+dx)}{10d}+\frac{a\tan(c+dx)}{d}+\frac{4a\tan^3(c+dx)}{3d}+\frac{6a\tan^5(c+dx)}{5d}+\frac{4a\tan^7(c+dx)}{7d}+\frac{a\tan^9(c+dx)}{9d}$$

output
```
1/10*I*a*sec(d*x+c)^10/d+a*tan(d*x+c)/d+4/3*a*tan(d*x+c)^3/d+6/5*a*tan(d*x
+c)^5/d+4/7*a*tan(d*x+c)^7/d+1/9*a*tan(d*x+c)^9/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.84

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^{10}(c+dx)}{10d}+\frac{a\left(\tan(c+dx)+\frac{4}{3}\tan^3(c+dx)+\frac{6}{5}\tan^5(c+dx)+\frac{4}{7}\tan^7(c+dx)+\frac{1}{9}\tan^9(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^10*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/10)*a*Sec[c + d*x]^10)/d + (a*(Tan[c + d*x] + (4*Tan[c + d*x]^3)/3 + (
6*Tan[c + d*x]^5)/5 + (4*Tan[c + d*x]^7)/7 + Tan[c + d*x]^9/9))/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 3967, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^{10}(a+ia\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^{10}(c+dx)dx+\frac{ia\sec^{10}(c+dx)}{10d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{10}dx+\frac{ia\sec^{10}(c+dx)}{10d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{a\int \left(\tan^8(c+dx)+4\tan^6(c+dx)+6\tan^4(c+dx)+4\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}+\frac{ia\sec^{10}(c+dx)}{10d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(-\frac{1}{9}\tan^9(c+dx)-\frac{4}{7}\tan^7(c+dx)-\frac{6}{5}\tan^5(c+dx)-\frac{4}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{ia\sec^{10}(c+dx)}{10d}$$

input
```
Int[Sec[c + d*x]^10*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/10)*a*Sec[c + d*x]^10)/d - (a*(-Tan[c + d*x] - (4*Tan[c + d*x]^3)/3 -
(6*Tan[c + d*x]^5)/5 - (4*Tan[c + d*x]^7)/7 - Tan[c + d*x]^9/9))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 162.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

method	result
risch	$\frac{256 i a \left(252 \,{\mathrm e}^{10 i \left(d x +c \right)}+210 \,{\mathrm e}^{8 i \left(d x +c \right)}+120 \,{\mathrm e}^{6 i \left(d x +c \right)}+45 \,{\mathrm e}^{4 i \left(d x +c \right)}+10 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{315 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{10}}$
derivativedivides	$\frac{a \left(\tan \left(d x +c \right)+\frac{i \tan \left(d x +c \right)^{10}}{10}+\frac{\tan \left(d x +c \right)^{9}}{9}+\frac{i \tan \left(d x +c \right)^{8}}{2}+\frac{4 \tan \left(d x +c \right)^{7}}{7}+i \tan \left(d x +c \right)^{6}+\frac{6 \tan \left(d x +c \right)^{5}}{5}+i \tan \left(d x +c \right)^{4}+4\right.}{d}$
default	$\frac{a \left(\tan \left(d x +c \right)+\frac{i \tan \left(d x +c \right)^{10}}{10}+\frac{\tan \left(d x +c \right)^{9}}{9}+\frac{i \tan \left(d x +c \right)^{8}}{2}+\frac{4 \tan \left(d x +c \right)^{7}}{7}+i \tan \left(d x +c \right)^{6}+\frac{6 \tan \left(d x +c \right)^{5}}{5}+i \tan \left(d x +c \right)^{4}+4\right.}{d}$

input
```
int(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
256/315*I*a*(252*exp(10*I*(d*x+c))+210*exp(8*I*(d*x+c))+120*exp(6*I*(d*x+c
))+45*exp(4*I*(d*x+c))+10*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^10
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(82) = 164$.

Time = 0.08 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.01

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{256\left(-252i\,ae^{(10i\,dx+10i\,c)} - 210i\,ae^{(8i\,dx+8i\,c)} - 120i\,ae^{(6i\,dx+\cdots}\right.}{315\left(de^{(20i\,dx+20i\,c)} + 10\,de^{(18i\,dx+18i\,c)} + 45\,de^{(16i\,dx+16i\,c)} + 120\,de^{(14i\,dx+14i\,c)} + 210\,de^{(12i\,dx+12i\,c)} + 252\,\cdots\right.}$$

input
```
integrate(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-256/315*(-252*I*a*e^(10*I*d*x + 10*I*c) - 210*I*a*e^(8*I*d*x + 8*I*c) - 1
20*I*a*e^(6*I*d*x + 6*I*c) - 45*I*a*e^(4*I*d*x + 4*I*c) - 10*I*a*e^(2*I*d*
x + 2*I*c) - I*a)/(d*e^(20*I*d*x + 20*I*c) + 10*d*e^(18*I*d*x + 18*I*c) +
45*d*e^(16*I*d*x + 16*I*c) + 120*d*e^(14*I*d*x + 14*I*c) + 210*d*e^(12*I*d
*x + 12*I*c) + 252*d*e^(10*I*d*x + 10*I*c) + 210*d*e^(8*I*d*x + 8*I*c) + 1
20*d*e^(6*I*d*x + 6*I*c) + 45*d*e^(4*I*d*x + 4*I*c) + 10*d*e^(2*I*d*x + 2*
I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx$$

$$= \begin{cases} \frac{a\left(\frac{\tan^{9}(c+dx)}{9} + \frac{4\tan^{7}(c+dx)}{7} + \frac{6\tan^{5}(c+dx)}{5} + \frac{4\tan^{3}(c+dx)}{3} + \tan(c+dx)\right) + \frac{ia\sec^{10}(c+dx)}{10}}{d} & \text{for } d \neq 0 \\ x(ia\tan(c)+a)\sec^{10}(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**10*(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((a*(tan(c + d*x)**9/9 + 4*tan(c + d*x)**7/7 + 6*tan(c + d*x)**5
/5 + 4*tan(c + d*x)**3/3 + tan(c + d*x)) + I*a*sec(c + d*x)**10/10)/d, Ne(
d, 0)), (x*(I*a*tan(c) + a)*sec(c)**10, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.21

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{63i\,a\tan{(dx+c)}^{10} + 70\,a\tan{(dx+c)}^{9} + 315i\,a\tan{(dx+c)}^{8} + 360\,a\tan{(dx+c)}^{7} + 630i\,a\tan{(dx+c}$$

input
```
integrate(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/630*(63*I*a*tan(d*x + c)^10 + 70*a*tan(d*x + c)^9 + 315*I*a*tan(d*x + c)
^8 + 360*a*tan(d*x + c)^7 + 630*I*a*tan(d*x + c)^6 + 756*a*tan(d*x + c)^5
+ 630*I*a*tan(d*x + c)^4 + 840*a*tan(d*x + c)^3 + 315*I*a*tan(d*x + c)^2 +
 630*a*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.21

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx =$$
$$-\frac{-63i\,a\tan{(dx+c)}^{10} - 70\,a\tan{(dx+c)}^{9} - 315i\,a\tan{(dx+c)}^{8} - 360\,a\tan{(dx+c)}^{7} - 630i\,a\tan{(d}$$

input
```
integrate(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/630*(-63*I*a*tan(d*x + c)^10 - 70*a*tan(d*x + c)^9 - 315*I*a*tan(d*x +
c)^8 - 360*a*tan(d*x + c)^7 - 630*I*a*tan(d*x + c)^6 - 756*a*tan(d*x + c)^
5 - 630*I*a*tan(d*x + c)^4 - 840*a*tan(d*x + c)^3 - 315*I*a*tan(d*x + c)^2
 - 630*a*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\left(-\cos\left(c+d\,x\right)^{10}\,63\mathrm{i}+256\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^{9}+128\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^{7}+96\,\sin\left(c+d\right.\right.}{630\,d\cos\left(c+d\,x\right)}$$

input
```
int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^10,x)
```

output
```
(a*(70*cos(c + d*x)*sin(c + d*x) + 80*cos(c + d*x)^3*sin(c + d*x) + 96*cos
(c + d*x)^5*sin(c + d*x) + 128*cos(c + d*x)^7*sin(c + d*x) + 256*cos(c + d
*x)^9*sin(c + d*x) - cos(c + d*x)^10*63i + 63i))/(630*d*cos(c + d*x)^10)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.02

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\sin\left(dx+c\right)a\left(-256\cos\left(dx+c\right)\sin\left(dx+c\right)^{8}+1152\cos\left(dx+c\right)\sin\left(dx+c\right)^{6}-2016\cos\left(dx+c\right)\sin\left(d\right.\right.}{630d\left(\sin\left(dx+c\right)^{1}\right.}$$

input
```
int(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x)
```

output
```
(sin(c + d*x)*a*( - 256*cos(c + d*x)*sin(c + d*x)**8 + 1152*cos(c + d*x)*s
in(c + d*x)**6 - 2016*cos(c + d*x)*sin(c + d*x)**4 + 1680*cos(c + d*x)*sin
(c + d*x)**2 - 630*cos(c + d*x) - 63*sin(c + d*x)**9*i + 315*sin(c + d*x)*
*7*i - 630*sin(c + d*x)**5*i + 630*sin(c + d*x)**3*i - 315*sin(c + d*x)*i)
)/(630*d*(sin(c + d*x)**10 - 5*sin(c + d*x)**8 + 10*sin(c + d*x)**6 - 10*s
in(c + d*x)**4 + 5*sin(c + d*x)**2 - 1))
```

3.2 $\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^8(c+dx)}{8d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{d}+\frac{3a\tan^5(c+dx)}{5d}+\frac{a\tan^7(c+dx)}{7d}$$

output
```
1/8*I*a*sec(d*x+c)^8/d+a*tan(d*x+c)/d+a*tan(d*x+c)^3/d+3/5*a*tan(d*x+c)^5/
d+1/7*a*tan(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^8(c+dx)}{8d}+\frac{a\left(\tan(c+dx)+\tan^3(c+dx)+\frac{3}{5}\tan^5(c+dx)+\frac{1}{7}\tan^7(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/8)*a*Sec[c + d*x]^8)/d + (a*(Tan[c + d*x] + Tan[c + d*x]^3 + (3*Tan[c
+ d*x]^5)/5 + Tan[c + d*x]^7/7))/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 3967, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^8(a+ia\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^8(c+dx)dx+\frac{ia\sec^8(c+dx)}{8d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^8dx+\frac{ia\sec^8(c+dx)}{8d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{a\int \left(\tan^6(c+dx)+3\tan^4(c+dx)+3\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}+\frac{ia\sec^8(c+dx)}{8d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(-\frac{1}{7}\tan^7(c+dx)-\frac{3}{5}\tan^5(c+dx)-\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{ia\sec^8(c+dx)}{8d}$$

input
```
Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/8)*a*Sec[c + d*x]^8)/d - (a*(-Tan[c + d*x] - Tan[c + d*x]^3 - (3*Tan[c
 + d*x]^5)/5 - Tan[c + d*x]^7/7))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 58.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

method	result
risch	$\frac{32ia\left(70\,\mathrm{e}^{8i(dx+c)}+56\,\mathrm{e}^{6i(dx+c)}+28\,\mathrm{e}^{4i(dx+c)}+8\,\mathrm{e}^{2i(dx+c)}+1\right)}{35d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{8}}$
derivativedivides	$\frac{a\left(\tan(dx+c)+\frac{i\tan(dx+c)^{8}}{8}+\frac{\tan(dx+c)^{7}}{7}+\frac{i\tan(dx+c)^{6}}{2}+\frac{3\tan(dx+c)^{5}}{5}+\frac{3i\tan(dx+c)^{4}}{4}+\tan(dx+c)^{3}+\frac{i\tan(dx+c)^{2}}{2}\right)}{d}$
default	$\frac{a\left(\tan(dx+c)+\frac{i\tan(dx+c)^{8}}{8}+\frac{\tan(dx+c)^{7}}{7}+\frac{i\tan(dx+c)^{6}}{2}+\frac{3\tan(dx+c)^{5}}{5}+\frac{3i\tan(dx+c)^{4}}{4}+\tan(dx+c)^{3}+\frac{i\tan(dx+c)^{2}}{2}\right)}{d}$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
32/35*I*a*(70*exp(8*I*(d*x+c))+56*exp(6*I*(d*x+c))+28*exp(4*I*(d*x+c))+8*e
xp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^8
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(67) = 134$.

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.04

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{32\left(-70i\,ae^{(8i\,dx+8i\,c)}-56i\,ae^{(6i\,dx+6i\,c)}-28i\,ae^{(4i\,dx+4i\,c)}-8i\,ae^{(2i\,dx+2}\right.}{35\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+}\right.}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-32/35*(-70*I*a*e^(8*I*d*x + 8*I*c) - 56*I*a*e^(6*I*d*x + 6*I*c) - 28*I*a*
e^(4*I*d*x + 4*I*c) - 8*I*a*e^(2*I*d*x + 2*I*c) - I*a)/(d*e^(16*I*d*x + 16
*I*c) + 8*d*e^(14*I*d*x + 14*I*c) + 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(1
0*I*d*x + 10*I*c) + 70*d*e^(8*I*d*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) +
28*d*e^(4*I*d*x + 4*I*c) + 8*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{a\left(\frac{\tan^7(c+dx)}{7}+\frac{3\tan^5(c+dx)}{5}+\tan^3(c+dx)+\tan(c+dx)\right)+\frac{ia\sec^8(c+dx)}{8}}{d} & \text{for } d\neq 0 \\ x(ia\tan(c)+a)\sec^8(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((a*(tan(c + d*x)**7/7 + 3*tan(c + d*x)**5/5 + tan(c + d*x)**3 +
 tan(c + d*x)) + I*a*sec(c + d*x)**8/8)/d, Ne(d, 0)), (x*(I*a*tan(c) + a)*
sec(c)**8, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{35i\,a\tan\left(dx+c\right)^8+40\,a\tan\left(dx+c\right)^7+140i\,a\tan\left(dx+c\right)^6+168\,a\tan\left(dx+c\right)^5+210i\,a\tan\left(dx+c\right.}{280\,d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/280*(35*I*a*tan(d*x + c)^8 + 40*a*tan(d*x + c)^7 + 140*I*a*tan(d*x + c)^
6 + 168*a*tan(d*x + c)^5 + 210*I*a*tan(d*x + c)^4 + 280*a*tan(d*x + c)^3 +
 140*I*a*tan(d*x + c)^2 + 280*a*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx =$$
$$-\frac{-35i\,a\tan\left(dx+c\right)^8-40\,a\tan\left(dx+c\right)^7-140i\,a\tan\left(dx+c\right)^6-168\,a\tan\left(dx+c\right)^5-210i\,a\tan\left(dx\right.}{280\,d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/280*(-35*I*a*tan(d*x + c)^8 - 40*a*tan(d*x + c)^7 - 140*I*a*tan(d*x + c
)^6 - 168*a*tan(d*x + c)^5 - 210*I*a*tan(d*x + c)^4 - 280*a*tan(d*x + c)^3
 - 140*I*a*tan(d*x + c)^2 - 280*a*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.99

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{a\,\sin\left(c+d\,x\right)\,\left(280\cos\left(c+d\,x\right)^7+\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)\,140\mathrm{i}+280\cos\left(c+d\,x\right)^5\sin\left(c+d\,x\right)^2+\right.$$

input
```
int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^8,x)
```

output
```
(a*sin(c + d*x)*(40*cos(c + d*x)*sin(c + d*x)^6 + cos(c + d*x)^6*sin(c + d
*x)*140i + 280*cos(c + d*x)^7 + sin(c + d*x)^7*35i + cos(c + d*x)^2*sin(c
+ d*x)^5*140i + 168*cos(c + d*x)^3*sin(c + d*x)^4 + cos(c + d*x)^4*sin(c +
 d*x)^3*210i + 280*cos(c + d*x)^5*sin(c + d*x)^2))/(280*d*cos(c + d*x)^8)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.04

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{\sin\left(dx+c\right)a\left(-128\cos\left(dx+c\right)\sin\left(dx+c\right)^6+448\cos\left(dx+c\right)\sin\left(dx+c\right)^4-560\cos\left(dx+c\right)\sin\left(dx\right.\right.}{280d\left(\sin\left(dx+c\right)^8-4\sin\left(dx+c\right)^6\right.}$$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x)
```

output
```
(sin(c + d*x)*a*( - 128*cos(c + d*x)*sin(c + d*x)**6 + 448*cos(c + d*x)*si
n(c + d*x)**4 - 560*cos(c + d*x)*sin(c + d*x)**2 + 280*cos(c + d*x) - 35*s
in(c + d*x)**7*i + 140*sin(c + d*x)**5*i - 210*sin(c + d*x)**3*i + 140*sin
(c + d*x)*i))/(280*d*(sin(c + d*x)**8 - 4*sin(c + d*x)**6 + 6*sin(c + d*x)
**4 - 4*sin(c + d*x)**2 + 1))
```

3.3 $\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx=\frac{ia\sec^6(c+dx)}{6d}+\frac{a\tan(c+dx)}{d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}$$

output
```
1/6*I*a*sec(d*x+c)^6/d+a*tan(d*x+c)/d+2/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c
)^5/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx=\frac{ia\sec^6(c+dx)}{6d}+\frac{a\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/6)*a*Sec[c + d*x]^6)/d + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan
[c + d*x]^5/5))/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 3967, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+ia\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^6(c+dx)dx+\frac{ia\sec^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^6dx+\frac{ia\sec^6(c+dx)}{6d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{a\int\left(\tan^4(c+dx)+2\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}+\frac{ia\sec^6(c+dx)}{6d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{ia\sec^6(c+dx)}{6d}$$

input
```
Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/6)*a*Sec[c + d*x]^6)/d - (a*(-Tan[c + d*x] - (2*Tan[c + d*x]^3)/3 - Ta
n[c + d*x]^5/5))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 14.39 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{16 i a \left(20 \,{\mathrm e}^{6 i (d x +c)}+15 \,{\mathrm e}^{4 i (d x +c)}+6 \,{\mathrm e}^{2 i (d x +c)}+1\right)}{15 d \left({\mathrm e}^{2 i (d x +c)}+1\right)^{6}}$	56
derivativedivides	$\frac{a \left(\tan \left(d x +c \right)+\frac{i \tan \left(d x +c \right)^{6}}{6}+\frac{\tan \left(d x +c \right)^{5}}{5}+\frac{i \tan \left(d x +c \right)^{4}}{2}+\frac{2 \tan \left(d x +c \right)^{3}}{3}+\frac{i \tan \left(d x +c \right)^{2}}{2}\right)}{d}$	66
default	$\frac{a \left(\tan \left(d x +c \right)+\frac{i \tan \left(d x +c \right)^{6}}{6}+\frac{\tan \left(d x +c \right)^{5}}{5}+\frac{i \tan \left(d x +c \right)^{4}}{2}+\frac{2 \tan \left(d x +c \right)^{3}}{3}+\frac{i \tan \left(d x +c \right)^{2}}{2}\right)}{d}$	66

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
16/15*I*a*(20*exp(6*I*(d*x+c))+15*exp(4*I*(d*x+c))+6*exp(2*I*(d*x+c))+1)/d
/(exp(2*I*(d*x+c))+1)^6
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(54) = 108$.

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.89

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{16\left(-20i\,ae^{(6i\,dx+6i\,c)}-15i\,ae^{(4i\,dx+4i\,c)}-6i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)}{15\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+\right.}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-16/15*(-20*I*a*e^(6*I*d*x + 6*I*c) - 15*I*a*e^(4*I*d*x + 4*I*c) - 6*I*a*e
^(2*I*d*x + 2*I*c) - I*a)/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*
I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d
*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{a\left(\frac{\tan^5(c+dx)}{5}+\frac{2\tan^3(c+dx)}{3}+\tan(c+dx)\right)+\frac{ia\sec^6(c+dx)}{6}}{d} & \text{for } d\neq 0 \\ x(ia\tan(c)+a)\sec^6(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((a*(tan(c + d*x)**5/5 + 2*tan(c + d*x)**3/3 + tan(c + d*x)) + I
*a*sec(c + d*x)**6/6)/d, Ne(d, 0)), (x*(I*a*tan(c) + a)*sec(c)**6, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{5i\,a\tan\left(dx+c\right)^6+6\,a\tan\left(dx+c\right)^5+15i\,a\tan\left(dx+c\right)^4+20\,a\tan\left(dx+c\right)^3+15i\,a\tan\left(dx+c\right)^2+3}{30\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/30*(5*I*a*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*I*a*tan(d*x + c)^4 +
20*a*tan(d*x + c)^3 + 15*I*a*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx =$$
$$-\frac{-5i\,a\tan\left(dx+c\right)^6-6\,a\tan\left(dx+c\right)^5-15i\,a\tan\left(dx+c\right)^4-20\,a\tan\left(dx+c\right)^3-15i\,a\tan\left(dx+c\right)^2}{30\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/30*(-5*I*a*tan(d*x + c)^6 - 6*a*tan(d*x + c)^5 - 15*I*a*tan(d*x + c)^4
- 20*a*tan(d*x + c)^3 - 15*I*a*tan(d*x + c)^2 - 30*a*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.81

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{a\,\sin\left(c+d\,x\right)\,\left(30\cos\left(c+d\,x\right)^5+\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)\,15\mathrm{i}+20\cos\left(c+d\,x\right)^3\sin\left(c+d\,x\right)^2+\cos\right.}{30\,d\cos\left(c+d\,x\right)^6}$$

input
```
int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^6,x)
```

output
```
(a*sin(c + d*x)*(6*cos(c + d*x)*sin(c + d*x)^4 + cos(c + d*x)^4*sin(c + d*
x)*15i + 30*cos(c + d*x)^5 + sin(c + d*x)^5*5i + cos(c + d*x)^2*sin(c + d*
x)^3*15i + 20*cos(c + d*x)^3*sin(c + d*x)^2))/(30*d*cos(c + d*x)^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.87

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{\sin\left(dx+c\right)a\left(-16\cos\left(dx+c\right)\sin\left(dx+c\right)^4+40\cos\left(dx+c\right)\sin\left(dx+c\right)^2-30\cos\left(dx+c\right)-5\sin\left(dx\right.\right.}{30d\left(\sin\left(dx+c\right)^6-3\sin\left(dx+c\right)^4+3\sin\left(dx+c\right)^2-1\right)}$$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x)
```

output
```
(sin(c + d*x)*a*( - 16*cos(c + d*x)*sin(c + d*x)**4 + 40*cos(c + d*x)*sin(
c + d*x)**2 - 30*cos(c + d*x) - 5*sin(c + d*x)**5*i + 15*sin(c + d*x)**3*i
 - 15*sin(c + d*x)*i))/(30*d*(sin(c + d*x)**6 - 3*sin(c + d*x)**4 + 3*sin(
c + d*x)**2 - 1))
```

3.4 $\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx=\frac{ia\sec^4(c+dx)}{4d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

output
```
1/4*I*a*sec(d*x+c)^4/d+a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx=\frac{ia\sec^4(c+dx)}{4d}+\frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/4)*a*Sec[c + d*x]^4)/d + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 3967, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4(a+ia\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^4(c+dx)dx+\frac{ia\sec^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{ia\sec^4(c+dx)}{4d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{a\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}+\frac{ia\sec^4(c+dx)}{4d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{ia\sec^4(c+dx)}{4d}$$

input
```
Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/4)*a*Sec[c + d*x]^4)/d - (a*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{a\left(\tan(dx+c)+\frac{i\tan(dx+c)^4}{4}+\frac{\tan(dx+c)^3}{3}+\frac{i\tan(dx+c)^2}{2}\right)}{d}$	45
default	$\frac{a\left(\tan(dx+c)+\frac{i\tan(dx+c)^4}{4}+\frac{\tan(dx+c)^3}{3}+\frac{i\tan(dx+c)^2}{2}\right)}{d}$	45
risch	$\frac{4ia\left(6\,e^{4i(dx+c)}+4\,e^{2i(dx+c)}+1\right)}{3d\left(e^{2i(dx+c)}+1\right)^4}$	45

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a/d*(tan(d*x+c)+1/4*I*tan(d*x+c)^4+1/3*tan(d*x+c)^3+1/2*I*tan(d*x+c)^2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(40) = 80.

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{4\left(-6i\,ae^{(4i\,dx+4i\,c)}-4i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)}{3\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-4/3*(-6*I*a*e^(4*I*d*x + 4*I*c) - 4*I*a*e^(2*I*d*x + 2*I*c) - I*a)/(d*e^(
8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d
*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{a\left(\frac{\tan^3(c+dx)}{3}+\tan(c+dx)\right)+\frac{ia\sec^4(c+dx)}{4}}{d} & \text{for } d\neq 0 \\ x(ia\tan(c)+a)\sec^4(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((a*(tan(c + d*x)**3/3 + tan(c + d*x)) + I*a*sec(c + d*x)**4/4)/
d, Ne(d, 0)), (x*(I*a*tan(c) + a)*sec(c)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{3i\,a\tan\left(dx+c\right)^4+4\,a\tan\left(dx+c\right)^3+6i\,a\tan\left(dx+c\right)^2+12\,a\tan\left(dx+c\right)}{12\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(3*I*a*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*I*a*tan(d*x + c)^2 + 1
2*a*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=-\frac{-3i\,a\tan\left(dx+c\right)^4-4\,a\tan\left(dx+c\right)^3-6i\,a\tan\left(dx+c\right)^2-12\,a\tan\left(dx+c\right)}{12\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/12*(-3*I*a*tan(d*x + c)^4 - 4*a*tan(d*x + c)^3 - 6*I*a*tan(d*x + c)^2 -
 12*a*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\frac{1i\,a\tan(c+d\,x)^4}{4}+\frac{a\tan(c+d\,x)^3}{3}+\frac{1i\,a\tan(c+d\,x)^2}{2}+a\tan(c+d\,x)}{d}$$

input
```
int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^4,x)
```

output
```
(a*tan(c + d*x) + (a*tan(c + d*x)^2*1i)/2 + (a*tan(c + d*x)^3)/3 + (a*tan(
c + d*x)^4*1i)/4)/d
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.72

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\sin(dx+c)\,a\left(-8\cos(dx+c)\sin(dx+c)^2+12\cos(dx+c)-3\sin(dx+c)^3 i+6\sin(dx+c)\,i\right)}{12d\left(\sin(dx+c)^4-2\sin(dx+c)^2+1\right)}$$

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x)
```

output
```
(sin(c + d*x)*a*( - 8*cos(c + d*x)*sin(c + d*x)**2 + 12*cos(c + d*x) - 3*s
in(c + d*x)**3*i + 6*sin(c + d*x)*i))/(12*d*(sin(c + d*x)**4 - 2*sin(c + d
*x)**2 + 1))
```

3.5 $\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 27

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{i(a+ia\tan(c+dx))^2}{2ad}$$

output
```
-1/2*I*(a+I*a*tan(d*x+c))^2/a/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^2(c+dx)}{2d} + \frac{a\tan(c+dx)}{d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/2)*a*Sec[c + d*x]^2)/d + (a*Tan[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 3967, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+ia\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^2(c+dx)dx+\frac{ia\sec^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{ia\sec^2(c+dx)}{2d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{a\int 1d(-\tan(c+dx))}{d}+\frac{ia\sec^2(c+dx)}{2d}$$

$$\Big\downarrow \text{24}$$

$$\frac{a\tan(c+dx)}{d}+\frac{ia\sec^2(c+dx)}{2d}$$

input
```
Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/2)*a*Sec[c + d*x]^2)/d + (a*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$-\frac{ia\left(-\frac{\tan(dx+c)^2}{2}+i\tan(dx+c)\right)}{d}$	28
default	$-\frac{ia\left(-\frac{\tan(dx+c)^2}{2}+i\tan(dx+c)\right)}{d}$	28
risch	$\frac{2ia\left(2\,\mathrm{e}^{2i(dx+c)}+1\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}$	34

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-I*a/d*(-1/2*tan(d*x+c)^2+I*tan(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(21) = 42$.

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{2\left(-2i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2*(-2*I*a*e^(2*I*d*x + 2*I*c) - I*a)/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*
d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{\frac{ia\tan^2{(c+dx)}}{2}+a\tan{(c+dx)}}{d} & \text{for } d\neq 0 \\ x(ia\tan{(c)}+a)\sec^2{(c)} & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((I*a*tan(c + d*x)**2/2 + a*tan(c + d*x))/d, Ne(d, 0)), (x*(I*a*
tan(c) + a)*sec(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{i\left(i\,a\tan\left(dx+c\right)+a\right)^2}{2\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*I*(I*a*tan(d*x + c) + a)^2/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{-i\,a\tan\left(dx+c\right)^2-2\,a\tan\left(dx+c\right)}{2\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(-I*a*tan(d*x + c)^2 - 2*a*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\tan(c+d\,x)\,\left(2+\tan(c+d\,x)\,1\mathrm{i}\right)}{2\,d}$$

input
```
int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^2,x)
```

output
```
(a*tan(c + d*x)*(tan(c + d*x)*1i + 2))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\sin(dx+c)\,a(-2\cos(dx+c)-\sin(dx+c)\,i)}{2d\left(\sin(dx+c)^2-1\right)}$$

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x)
```

output
```
(sin(c + d*x)*a*( - 2*cos(c + d*x) - sin(c + d*x)*i))/(2*d*(sin(c + d*x)**
2 - 1))
```

3.6 $\int (a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int (a+ia\tan(c+dx))\,dx=ax-\frac{ia\log(\cos(c+dx))}{d}$$

output
```
a*x-I*a*ln(cos(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (a+ia\tan(c+dx))\,dx=ax-\frac{ia\log(\cos(c+dx))}{d}$$

input
```
Integrate[a + I*a*Tan[c + d*x],x]
```

output
```
a*x - (I*a*Log[Cos[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + ia \tan(c + dx)) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax - \frac{ia \log(\cos(c + dx))}{d}$$

input
```
Int[a + I*a*Tan[c + d*x],x]
```

output
```
a*x - (I*a*Log[Cos[c + d*x]])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
default	$ax+\frac{ia\ln\left(1+\tan(dx+c)^{2}\right)}{2d}$	23
norman	$ax+\frac{ia\ln\left(1+\tan(dx+c)^{2}\right)}{2d}$	23
parallelrisch	$ax+\frac{ia\ln\left(1+\tan(dx+c)^{2}\right)}{2d}$	23
parts	$ax+\frac{ia\ln\left(1+\tan(dx+c)^{2}\right)}{2d}$	23
derivativedivides	$\frac{a\left(\frac{i\ln\left(1+\tan(dx+c)^{2}\right)}{2}+\arctan(\tan(dx+c))\right)}{d}$	28
risch	$-\frac{ia\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{2ac}{d}$	28

input
```
int(a+I*a*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+1/2*I*a/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int (a+ia\tan(c+dx))\,dx=-\frac{i\,a\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{d}$$

input
```
integrate(a+I*a*tan(d*x+c),x, algorithm="fricas")
```

output
```
-I*a*log(e^(2*I*d*x + 2*I*c) + 1)/d
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int (a + ia\tan(c + dx))\,dx = -\frac{ia\log\left(e^{2idx} + e^{-2ic}\right)}{d}$$

input
```
integrate(a+I*a*tan(d*x+c),x)
```

output
```
-I*a*log(exp(2*I*d*x) + exp(-2*I*c))/d
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int (a + ia\tan(c + dx))\,dx = ax + \frac{i\,a\log\left(\sec\left(dx + c\right)\right)}{d}$$

input
```
integrate(a+I*a*tan(d*x+c),x, algorithm="maxima")
```

output
```
a*x + I*a*log(sec(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int (a + ia\tan(c + dx))\,dx = ax - \frac{i\,a\log\left(\left|\cos\left(dx + c\right)\right|\right)}{d}$$

input
```
integrate(a+I*a*tan(d*x+c),x, algorithm="giac")
```

output
```
a*x - I*a*log(abs(cos(d*x + c)))/d
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int (a + ia\tan(c + dx))\,dx = \frac{a\,\ln\left(\tan(c + d\,x) + 1\mathrm{i}\right)\,1\mathrm{i}}{d}$$

input
```
int(a + a*tan(c + d*x)*1i,x)
```

output
```
(a*log(tan(c + d*x) + 1i)*1i)/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int (a + ia\tan(c + dx))\,dx = \frac{a\left(\log\left(\tan\left(dx + c\right)^2 + 1\right)i + 2dx\right)}{2d}$$

input
```
int(a+I*a*tan(d*x+c),x)
```

output
```
(a*(log(tan(c + d*x)**2 + 1)*i + 2*d*x))/(2*d)
```

3.7 $\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx=\frac{ax}{2}-\frac{ia\cos^2(c+dx)}{2d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*a*x-1/2*I*a*cos(d*x+c)^2/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx=\frac{a(c+dx)}{2d}-\frac{ia\cos^2(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*(c + d*x))/(2*d) - ((I/2)*a*Cos[c + d*x]^2)/d + (a*Sin[2*(c + d*x)])/(4
*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 3967, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a+ia\tan(c+dx)}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \ 3967$$

$$a\int \cos^2(c+dx)dx-\frac{ia\cos^2(c+dx)}{2d}$$

$$\Big\downarrow \ 3042$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{ia\cos^2(c+dx)}{2d}$$

$$\Big\downarrow \ 3115$$

$$a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{ia\cos^2(c+dx)}{2d}$$

$$\Big\downarrow \ 24$$

$$a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{ia\cos^2(c+dx)}{2d}$$

input
```
Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/2*I)*a*Cos[c + d*x]^2)/d + a*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)
)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

method	result	size
risch	$\frac{ax}{2}-\frac{ia\,e^{2i(dx+c)}}{4d}$	22
derivativedivides	$\frac{-\frac{ia\cos(dx+c)^2}{2}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	42
default	$\frac{-\frac{ia\cos(dx+c)^2}{2}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	42

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x-1/4*I/d*a*exp(2*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.51

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{2\,adx - i\,ae^{(2i\,dx+2i\,c)}}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*a*d*x - I*a*e^(2*I*d*x + 2*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ax}{2} + \begin{cases} -\frac{iae^{2ic}e^{2idx}}{4d} & \text{for } d \neq 0 \\ \frac{axe^{2ic}}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c)),x)
```

output
```
a*x/2 + Piecewise((-I*a*exp(2*I*c)*exp(2*I*d*x)/(4*d), Ne(d, 0)), (a*x*exp
(2*I*c)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{(dx+c)a + \frac{a\tan(dx+c)-i\,a}{\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*((d*x + c)*a + (a*tan(d*x + c) - I*a)/(tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{1}{4}i\,a\left(\frac{\log\left(\tan\left(dx+c\right)+i\right)}{d}-\frac{\log\left(\tan\left(dx+c\right)-i\right)}{d}-\frac{2i}{d(\tan\left(dx+c\right)+i)}\right)$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/4*I*a*(log(tan(d*x + c) + I)/d - log(tan(d*x + c) - I)/d - 2*I/(d*(tan(d
*x + c) + I)))
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx=\frac{a\,x}{2}+\frac{a}{2\,d\,\left(\tan\left(c+d\,x\right)+1{}\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i),x)
```

output
```
(a*x)/2 + a/(2*d*(tan(c + d*x) + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx=\frac{a\left(\cos\left(dx+c\right)\sin\left(dx+c\right)+\sin\left(dx+c\right)^2 i+dx\right)}{2d}$$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x)
```

output
```
(a*(cos(c + d*x)*sin(c + d*x) + sin(c + d*x)**2*i + d*x))/(2*d)
```

3.8 $\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3ax}{8}-\frac{ia\cos^4(c+dx)}{4d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}$$

output
```
3/8*a*x-1/4*I*a*cos(d*x+c)^4/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x
+c)^3*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a(12c+12dx-8i\cos^4(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*(12*c + 12*d*x - (8*I)*Cos[c + d*x]^4 + 8*Sin[2*(c + d*x)] + Sin[4*(c +
 d*x)]))/(32*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3042, 3967, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos^4(c+dx)dx-\frac{ia\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{ia\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{ia\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{ia\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{ia\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{24}$$

$$a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{ia\cos^4(c+dx)}{4d}$$

input
```
Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/4*I)*a*Cos[c + d*x]^4)/d + a*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (
3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{-\frac{ia\cos(dx+c)^4}{4}+a\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$	53
default	$\frac{-\frac{ia\cos(dx+c)^4}{4}+a\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$	53
risch	$\frac{3ax}{8}-\frac{ia\,e^{4i(dx+c)}}{32d}-\frac{ia\cos(2dx+2c)}{8d}+\frac{a\sin(2dx+2c)}{4d}$	53

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*I*a*cos(d*x+c)^4+a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)
+3/8*d*x+3/8*c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\left(12\,adxe^{(2i\,dx+2i\,c)}-i\,ae^{(6i\,dx+6i\,c)}-6i\,ae^{(4i\,dx+4i\,c)}+2i\,a\right)e^{(-2i\,dx-2i\,c)}}{32\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/32*(12*a*d*x*e^(2*I*d*x + 2*I*c) - I*a*e^(6*I*d*x + 6*I*c) - 6*I*a*e^(4*
I*d*x + 4*I*c) + 2*I*a)*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.03

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3ax}{8} + \begin{cases} \frac{(-256iad^2e^{6ic}e^{4idx}-1536iad^2e^{4ic}e^{2idx}+512iad^2e^{-2idx})e^{-2ic}}{8192d^3} & \text{for } d^3e^{2ic} \neq 0 \\ x\left(-\frac{3a}{8}+\frac{(ae^{6ic}+3ae^{4ic}+3ae^{2ic}+a)e^{-2ic}}{8}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c)),x)
```

output
```
3*a*x/8 + Piecewise(((-256*I*a*d**2*exp(6*I*c)*exp(4*I*d*x) - 1536*I*a*d**
2*exp(4*I*c)*exp(2*I*d*x) + 512*I*a*d**2*exp(-2*I*d*x))*exp(-2*I*c)/(8192*
d**3), Ne(d**3*exp(2*I*c), 0)), (x*(-3*a/8 + (a*exp(6*I*c) + 3*a*exp(4*I*c
) + 3*a*exp(2*I*c) + a)*exp(-2*I*c)/8), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3\,(dx+c)a + \frac{3\,a\tan(dx+c)^3+5\,a\tan(dx+c)-2i\,a}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/8*(3*(d*x + c)*a + (3*a*tan(d*x + c)^3 + 5*a*tan(d*x + c) - 2*I*a)/(tan(
d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{1}{16}i\,a\left(\frac{3\,\log\left(\tan\left(dx+c\right)+i\right)}{d}-\frac{3\,\log\left(\tan\left(dx+c\right)-i\right)}{d}-\frac{2\,\left(3i\,\tan\left(dx+c\right)^{2}-3\,\tan\left(dx+c\right)+2i\right)}{d(\tan\left(dx+c\right)+i)^{2}(\tan\left(dx+c\right)-i)}\right)$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/16*I*a*(3*log(tan(d*x + c) + I)/d - 3*log(tan(d*x + c) - I)/d - 2*(3*I*t
an(d*x + c)^2 - 3*tan(d*x + c) + 2*I)/(d*(tan(d*x + c) + I)^2*(tan(d*x + c
) - I)))
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3\,a\,x}{8}+\frac{\frac{3\,a\,\mathrm{tan}\left(c+d\,x\right)^2}{8}+\frac{3{}\mathrm{i}\,a\,\mathrm{tan}\left(c+d\,x\right)}{8}+\frac{a}{4}}{d\,\left({\mathrm{tan}\left(c+d\,x\right)}^3+{\mathrm{tan}\left(c+d\,x\right)}^2\,1{}\mathrm{i}+\mathrm{tan}\left(c+d\,x\right)+1{}\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i),x)
```

output
```
(3*a*x)/8 + (a/4 + (a*tan(c + d*x)*3i)/8 + (3*a*tan(c + d*x)^2)/8)/(d*(tan
(c + d*x) + tan(c + d*x)^2*1i + tan(c + d*x)^3 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{a\left(-2\cos\left(dx+c\right)\sin\left(dx+c\right)^3+5\cos\left(dx+c\right)\sin\left(dx+c\right)-2\sin\left(dx+c\right)^4i+4\sin\left(dx+c\right)^2i+3dx\right)}{8d}$$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x)
```

output
```
(a*( - 2*cos(c + d*x)*sin(c + d*x)**3 + 5*cos(c + d*x)*sin(c + d*x) - 2*si
n(c + d*x)**4*i + 4*sin(c + d*x)**2*i + 3*d*x))/(8*d)
```

3.9 $\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5ax}{16} - \frac{ia\cos^6(c+dx)}{6d} + \frac{5a\cos(c+dx)\sin(c+dx)}{16d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{a\cos^5(c+dx)\sin(c+dx)}{6d}$$

output
```
5/16*a*x-1/6*I*a*cos(d*x+c)^6/d+5/16*a*cos(d*x+c)*sin(d*x+c)/d+5/24*a*cos(
d*x+c)^3*sin(d*x+c)/d+1/6*a*cos(d*x+c)^5*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.63

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a(60c+60dx-32i\cos^6(c+dx)+45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx)))}{192d}$$

input
```
Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*(60*c + 60*d*x - (32*I)*Cos[c + d*x]^6 + 45*Sin[2*(c + d*x)] + 9*Sin[4*
(c + d*x)] + Sin[6*(c + d*x)]))/(192*d)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {3042, 3967, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos^6(c+dx)dx-\frac{ia\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^6dx-\frac{ia\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{6}\int \cos^4(c+dx)dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{ia\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{ia\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{ia\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{ia\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{ia\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{24}$$

$$a\left(\frac{\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5}{6}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)\right)-\frac{ia\cos^6(c+dx)}{6d}$$

input
```
Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/6*I)*a*Cos[c + d*x]^6)/d + a*((Cos[c + d*x]^5*Sin[c + d*x])/(6*d) + (
5*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d
*x])/(2*d)))/4))/6)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

Maple [A] (verified)

Time = 12.92 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{-\frac{i a \cos(dx+c)^{6}}{6}+a\left(\frac{\left(\cos(dx+c)^{5}+\frac{5\cos(dx+c)^{3}}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}$	63
default	$\frac{-\frac{i a \cos(dx+c)^{6}}{6}+a\left(\frac{\left(\cos(dx+c)^{5}+\frac{5\cos(dx+c)^{3}}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}$	63
risch	$\frac{5ax}{16}-\frac{i a\,{\mathrm e}^{6i(dx+c)}}{192d}-\frac{i a\cos(4dx+4c)}{32d}+\frac{3a\sin(4dx+4c)}{64d}-\frac{5i a\cos(2dx+2c)}{64d}+\frac{15a\sin(2dx+2c)}{64d}$	84

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/6*I*a*cos(d*x+c)^6+a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(
d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\left(120\,adxe^{(4i\,dx+4i\,c)} - 2i\,ae^{(10i\,dx+10i\,c)} - 15i\,ae^{(8i\,dx+8i\,c)} - 60i\,ae^{(6i\,dx+6i\,c)} + 30i\,ae^{(2i\,dx+2i\,c)} + 3i\,a\right)e^{(-4i\,dx}}{384\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/384*(120*a*d*x*e^(4*I*d*x + 4*I*c) - 2*I*a*e^(10*I*d*x + 10*I*c) - 15*I*
a*e^(8*I*d*x + 8*I*c) - 60*I*a*e^(6*I*d*x + 6*I*c) + 30*I*a*e^(2*I*d*x + 2
*I*c) + 3*I*a)*e^(-4*I*d*x - 4*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.37

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5ax}{16} + \begin{cases} \frac{(-33554432iad^4e^{12ic}e^{6idx} - 251658240iad^4e^{10ic}e^{4idx} - 1006632960iad^4e^{8ic}e^{2idx} + 503316480iad^4e^{4ic}e^{-2idx} + 50331648iad^4e^{2ic}e^{-4idx})e}{6442450944d^5} \\ x\left(-\frac{5a}{16} + \frac{(ae^{10ic}+5ae^{8ic}+10ae^{6ic}+10ae^{4ic}+5ae^{2ic}+a)e^{-4ic}}{32}\right) \end{cases}$$

input
```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c)),x)
```

output
```
5*a*x/16 + Piecewise(((-33554432*I*a*d**4*exp(12*I*c)*exp(6*I*d*x) - 25165
8240*I*a*d**4*exp(10*I*c)*exp(4*I*d*x) - 1006632960*I*a*d**4*exp(8*I*c)*ex
p(2*I*d*x) + 503316480*I*a*d**4*exp(4*I*c)*exp(-2*I*d*x) + 50331648*I*a*d*
*4*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*c)/(6442450944*d**5), Ne(d**5*exp(6*
I*c), 0)), (x*(-5*a/16 + (a*exp(10*I*c) + 5*a*exp(8*I*c) + 10*a*exp(6*I*c)
 + 10*a*exp(4*I*c) + 5*a*exp(2*I*c) + a)*exp(-4*I*c)/32), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{15\,(dx+c)a + \frac{15\,a\tan(dx+c)^5+40\,a\tan(dx+c)^3+33\,a\tan(dx+c)-8i\,a}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{48\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/48*(15*(d*x + c)*a + (15*a*tan(d*x + c)^5 + 40*a*tan(d*x + c)^3 + 33*a*t
an(d*x + c) - 8*I*a)/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2
 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{1}{96}i\,a\left(\frac{15\,\log\left(\tan\left(dx+c\right)+i\right)}{d} - \frac{15\,\log\left(\tan\left(dx+c\right)-i\right)}{d} - \frac{2\,\left(15i\,\tan\left(dx+c\right)^4 - 15\,\tan\left(dx+c\right)^3 +\right.}{d(\tan\left(dx+c\right)+i)}\right.$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/96*I*a*(15*log(tan(d*x + c) + I)/d - 15*log(tan(d*x + c) - I)/d - 2*(15*
I*tan(d*x + c)^4 - 15*tan(d*x + c)^3 + 25*I*tan(d*x + c)^2 - 25*tan(d*x +
c) + 8*I)/(d*(tan(d*x + c) + I)^3*(tan(d*x + c) - I)^2))
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.21

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5\,a\,x}{16} + \frac{\frac{5\,a\tan(c+d\,x)^4}{16}+\frac{5\mathrm{i}\,a\tan(c+d\,x)^3}{16}+\frac{25\,a\tan(c+d\,x)^2}{48}+\frac{25\mathrm{i}\,a\tan(c+d\,x)}{48}+\frac{a}{6}}{d\,\left(\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4\,1\mathrm{i}+2\tan\left(c+d\,x\right)^3+\tan\left(c+d\,x\right)^2\,2\mathrm{i}+\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i),x)
```

output
```
(5*a*x)/16 + (a/6 + (a*tan(c + d*x)*25i)/48 + (25*a*tan(c + d*x)^2)/48 + (
a*tan(c + d*x)^3*5i)/16 + (5*a*tan(c + d*x)^4)/16)/(d*(tan(c + d*x) + tan(
c + d*x)^2*2i + 2*tan(c + d*x)^3 + tan(c + d*x)^4*1i + tan(c + d*x)^5 + 1i
))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\left(8\cos\left(dx+c\right)\sin\left(dx+c\right)^5-26\cos\left(dx+c\right)\sin\left(dx+c\right)^3+33\cos\left(dx+c\right)\sin\left(dx+c\right)+8\sin\left(dx+c\right)\right.}{48d}$$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x)
```

output
```
(a*(8*cos(c + d*x)*sin(c + d*x)**5 - 26*cos(c + d*x)*sin(c + d*x)**3 + 33*
cos(c + d*x)*sin(c + d*x) + 8*sin(c + d*x)**6*i - 24*sin(c + d*x)**4*i + 2
4*sin(c + d*x)**2*i + 15*d*x))/(48*d)
```

3.10 $\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx = \frac{35ax}{128} - \frac{ia\cos^8(c+dx)}{8d} + \frac{35a\cos(c+dx)\sin(c+dx)}{128d} + \frac{35a\cos^3(c+dx)\sin(c+dx)}{192d} + \frac{7a\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{a\cos^7(c+dx)\sin(c+dx)}{8d}$$

output
```
35/128*a*x-1/8*I*a*cos(d*x+c)^8/d+35/128*a*cos(d*x+c)*sin(d*x+c)/d+35/192*
a*cos(d*x+c)^3*sin(d*x+c)/d+7/48*a*cos(d*x+c)^5*sin(d*x+c)/d+1/8*a*cos(d*x
+c)^7*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.61

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{a(840c+840dx-384i\cos^8(c+dx)+672\sin(2(c+dx))+168\sin(4(c+dx))+32\sin(6(c+dx))+3\,\mathrm{s}}{3072d}$$

input
```
Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*(840*c + 840*d*x - (384*I)*Cos[c + d*x]^8 + 672*Sin[2*(c + d*x)] + 168*
Sin[4*(c + d*x)] + 32*Sin[6*(c + d*x)] + 3*Sin[8*(c + d*x)]))/(3072*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3967, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{a+ia\tan(c+dx)}{\sec(c+dx)^8}dx$$
$$\Big\downarrow \; 3967$$
$$a\int \cos^8(c+dx)dx-\frac{ia\cos^8(c+dx)}{8d}$$
$$\Big\downarrow \; 3042$$
$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^8dx-\frac{ia\cos^8(c+dx)}{8d}$$
$$\Big\downarrow \; 3115$$

$$a\left(\frac{7}{8}\int \cos^6(c+dx)dx+\frac{\sin(c+dx)\cos^7(c+dx)}{8d}\right)-\frac{ia\cos^8(c+dx)}{8d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{7}{8}\int \sin\left(c+dx+\frac{\pi}{2}\right)^6dx+\frac{\sin(c+dx)\cos^7(c+dx)}{8d}\right)-\frac{ia\cos^8(c+dx)}{8d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{7}{8}\left(\frac{5}{6}\int \cos^4(c+dx)dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\frac{\sin(c+dx)\cos^7(c+dx)}{8d}\right)-\frac{ia\cos^8(c+dx)}{8d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{7}{8}\left(\frac{5}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\frac{\sin(c+dx)\cos^7(c+dx)}{8d}\right)-\frac{ia\cos^8(c+dx)}{8d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\frac{\sin(c+dx)\cos^7(c+}{8d}\right.$$
$$\frac{ia\cos^8(c+dx)}{8d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\frac{\sin(c+dx)\cos}{8d}\right.$$
$$\frac{ia\cos^8(c+dx)}{8d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\text{sin}\right.$$
$$\frac{ia\cos^8(c+dx)}{8d}$$

$$\Big\downarrow \text{24}$$

$$a\left(\frac{\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7}{8}\left(\frac{\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5}{6}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c}{2d}\right.\right.\right.\right.$$
$$\frac{ia\cos^8(c+dx)}{8d}$$

input
```
Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/8*I)*a*Cos[c + d*x]^8)/d + a*((Cos[c + d*x]^7*Sin[c + d*x])/(8*d) + (
7*((Cos[c + d*x]^5*Sin[c + d*x])/(6*d) + (5*((Cos[c + d*x]^3*Sin[c + d*x])
/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/6))/8)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

Maple [A] (verified)

Time = 44.46 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{-\frac{ia\cos(dx+c)^8}{8}+a\left(\frac{\left(\cos(dx+c)^7+\frac{7\cos(dx+c)^5}{6}+\frac{35\cos(dx+c)^3}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)}{d}$
default	$\frac{-\frac{ia\cos(dx+c)^8}{8}+a\left(\frac{\left(\cos(dx+c)^7+\frac{7\cos(dx+c)^5}{6}+\frac{35\cos(dx+c)^3}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)}{d}$
risch	$\frac{35ax}{128}-\frac{ia\,e^{8i(dx+c)}}{1024d}-\frac{ia\cos(6dx+6c)}{128d}+\frac{a\sin(6dx+6c)}{96d}-\frac{7ia\cos(4dx+4c)}{256d}+\frac{7a\sin(4dx+4c)}{128d}-\frac{7ia\cos(2dx}{128d}$

input
```
int(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/8*I*a*cos(d*x+c)^8+a*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos
(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{\left(840\,adxe^{(6i\,dx+6i\,c)}-3i\,ae^{(14i\,dx+14i\,c)}-28i\,ae^{(12i\,dx+12i\,c)}-126i\,ae^{(10i\,dx+10i\,c)}-420i\,ae^{(8i\,dx+8i\,c)}+252i\,a}{3072\,d}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/3072*(840*a*d*x*e^(6*I*d*x + 6*I*c) - 3*I*a*e^(14*I*d*x + 14*I*c) - 28*I
*a*e^(12*I*d*x + 12*I*c) - 126*I*a*e^(10*I*d*x + 10*I*c) - 420*I*a*e^(8*I*
d*x + 8*I*c) + 252*I*a*e^(4*I*d*x + 4*I*c) + 42*I*a*e^(2*I*d*x + 2*I*c) +
4*I*a)*e^(-6*I*d*x - 6*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.51

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx = \frac{35ax}{128} + \begin{cases} \frac{(-10133099161583616iad^6e^{20ic}e^{8idx} - 94575592174780416iad^6e^{18ic}e^{6idx} - 425590164786511872iad^6e^{16ic}e^{4idx} - 1418633882621706240i}{10376293541} \\ x\left(-\frac{35a}{128} + \frac{(ae^{14ic}+7ae^{12ic}+21ae^{10ic}+35ae^{8ic}+35ae^{6ic}+21ae^{4ic}+7ae^{2ic}+a)e^{-6ic}}{128}\right) \end{cases}$$

input
```
integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c)),x)
```

output
```
35*a*x/128 + Piecewise(((-10133099161583616*I*a*d**6*exp(20*I*c)*exp(8*I*d
*x) - 94575592174780416*I*a*d**6*exp(18*I*c)*exp(6*I*d*x) - 42559016478651
1872*I*a*d**6*exp(16*I*c)*exp(4*I*d*x) - 1418633882621706240*I*a*d**6*exp(
14*I*c)*exp(2*I*d*x) + 851180329573023744*I*a*d**6*exp(10*I*c)*exp(-2*I*d*
x) + 141863388262170624*I*a*d**6*exp(8*I*c)*exp(-4*I*d*x) + 13510798882111
488*I*a*d**6*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(10376293541461622784*
d**7), Ne(d**7*exp(12*I*c), 0)), (x*(-35*a/128 + (a*exp(14*I*c) + 7*a*exp(
12*I*c) + 21*a*exp(10*I*c) + 35*a*exp(8*I*c) + 35*a*exp(6*I*c) + 21*a*exp(
4*I*c) + 7*a*exp(2*I*c) + a)*exp(-6*I*c)/128), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.93

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx = \frac{105\,(dx+c)a + \frac{105\,a\tan(dx+c)^7+385\,a\tan(dx+c)^5+511\,a\tan(dx+c)^3+279\,a\tan(dx+c)-48i\,a}{\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1}}{384\,d}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/384*(105*(d*x + c)*a + (105*a*tan(d*x + c)^7 + 385*a*tan(d*x + c)^5 + 51
1*a*tan(d*x + c)^3 + 279*a*tan(d*x + c) - 48*I*a)/(tan(d*x + c)^8 + 4*tan(
d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.05

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{1}{768}i\,a\left(\frac{105\,\log\left(\tan\left(dx+c\right)+i\right)}{d}-\frac{105\,\log\left(\tan\left(dx+c\right)-i\right)}{d}-\frac{2\left(105i\,\tan\left(dx+c\right)^6-105\,\tan\left(dx+\right.\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/768*I*a*(105*log(tan(d*x + c) + I)/d - 105*log(tan(d*x + c) - I)/d - 2*(
105*I*tan(d*x + c)^6 - 105*tan(d*x + c)^5 + 280*I*tan(d*x + c)^4 - 280*tan
(d*x + c)^3 + 231*I*tan(d*x + c)^2 - 231*tan(d*x + c) + 48*I)/(d*(tan(d*x
+ c) + I)^4*(tan(d*x + c) - I)^3))
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.37

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx=\frac{35\,a\,x}{128}$$
$$+\frac{\frac{35\,a\tan(c+d\,x)^6}{128}+\frac{35\mathrm{i}\,a\tan(c+d\,x)^5}{128}+\frac{35\,a\tan(c+d\,x)^4}{48}+\frac{35\mathrm{i}\,a\tan(c+d\,x)^3}{48}+\frac{77\,a\tan(c+d\,x)^2}{128}+\frac{77\mathrm{i}\,a\tan}{1}}{d\,\left(\tan\left(c+d\,x\right)^7+\tan\left(c+d\,x\right)^6\,1\mathrm{i}+3\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4\,3\mathrm{i}+3\tan\left(c+d\,x\right)^3+\tan\left(c+\right.\right.}$$

input
```
int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i),x)
```

output
```
(35*a*x)/128 + (a/8 + (a*tan(c + d*x)*77i)/128 + (77*a*tan(c + d*x)^2)/128
 + (a*tan(c + d*x)^3*35i)/48 + (35*a*tan(c + d*x)^4)/48 + (a*tan(c + d*x)^
5*35i)/128 + (35*a*tan(c + d*x)^6)/128)/(d*(tan(c + d*x) + tan(c + d*x)^2*
3i + 3*tan(c + d*x)^3 + tan(c + d*x)^4*3i + 3*tan(c + d*x)^5 + tan(c + d*x
)^6*1i + tan(c + d*x)^7 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.05

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{a\big(-48\cos{(dx+c)}\sin{(dx+c)}^7+200\cos{(dx+c)}\sin{(dx+c)}^5-326\cos{(dx+c)}\sin{(dx+c)}^3+279\,\mathrm{co}}{}$$

input
```
int(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x)
```

output
```
(a*( - 48*cos(c + d*x)*sin(c + d*x)**7 + 200*cos(c + d*x)*sin(c + d*x)**5
- 326*cos(c + d*x)*sin(c + d*x)**3 + 279*cos(c + d*x)*sin(c + d*x) - 48*si
n(c + d*x)**8*i + 192*sin(c + d*x)**6*i - 288*sin(c + d*x)**4*i + 192*sin(
c + d*x)**2*i + 105*d*x))/(384*d)
```

3.11 $\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5a\text{arctanh}(\sin(c+dx))}{16d} + \frac{ia\sec^7(c+dx)}{7d} + \frac{5a\sec(c+dx)\tan(c+dx)}{16d} + \frac{5a\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{a\sec^5(c+dx)\tan(c+dx)}{6d}$$

output
```
5/16*a*arctanh(sin(d*x+c))/d+1/7*I*a*sec(d*x+c)^7/d+5/16*a*sec(d*x+c)*tan(
d*x+c)/d+5/24*a*sec(d*x+c)^3*tan(d*x+c)/d+1/6*a*sec(d*x+c)^5*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5a\text{arctanh}(\sin(c+dx))}{16d} + \frac{ia\sec^7(c+dx)}{7d} + \frac{5a\sec(c+dx)\tan(c+dx)}{16d} + \frac{5a\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{a\sec^5(c+dx)\tan(c+dx)}{6d}$$

input
```
Integrate[Sec[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

output
```
(5*a*ArcTanh[Sin[c + d*x]])/(16*d) + ((I/7)*a*Sec[c + d*x]^7)/d + (5*a*Sec
[c + d*x]*Tan[c + d*x])/(16*d) + (5*a*Sec[c + d*x]^3*Tan[c + d*x])/(24*d)
+ (a*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3967, 3042, 4255, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^7(a+ia\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^7(c+dx)dx + \frac{ia\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^7dx+\frac{ia\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{5}{6}\int \sec^5(c+dx)dx+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)+\frac{ia\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{6}\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)+\frac{ia\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)+\frac{ia\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)+\frac{ia\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(c+d}{6d}\right.$$
$$\frac{ia\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5}{6d}\right.$$
$$\frac{ia\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(c+\right.$$
$$\frac{ia\sec^7(c+dx)}{7d}$$

input
```
Int[Sec[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/7)*a*Sec[c + d*x]^7)/d + a*((Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (5*(
(Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTanh[Sin[c + d*x]]/(2*d) + (S
ec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/6)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 32.58 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{\frac{ia}{7\cos(dx+c)^7}+a\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}$
default	$\frac{\frac{ia}{7\cos(dx+c)^7}+a\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}$
risch	$-\frac{ia\left(105\,\mathrm{e}^{13i(dx+c)}+700\,\mathrm{e}^{11i(dx+c)}+1981\,\mathrm{e}^{9i(dx+c)}-3072\,\mathrm{e}^{7i(dx+c)}-1981\,\mathrm{e}^{5i(dx+c)}-700\,\mathrm{e}^{3i(dx+c)}-105\,\mathrm{e}^{i(dx+c)}\right)}{168d\left(\mathrm{e}^{2i(dx+c)}+1\right)^7}-$

input
```
int(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/7*I*a/cos(d*x+c)^7+a*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*se
c(d*x+c))*tan(d*x+c)+5/16*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(86) = 172.

Time = 0.09 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.80

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{-210i\,ae^{(13i\,dx+13i\,c)}-1400i\,ae^{(11i\,dx+11i\,c)}-3962i\,ae^{(9i\,dx+9i\,c)}+6144i\,ae^{(7i\,dx+7i\,c)}+3962i\,ae^{(5i\,dx+5i\,c)}+}{}$$

input
```
integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/336*(-210*I*a*e^(13*I*d*x + 13*I*c) - 1400*I*a*e^(11*I*d*x + 11*I*c) - 3
962*I*a*e^(9*I*d*x + 9*I*c) + 6144*I*a*e^(7*I*d*x + 7*I*c) + 3962*I*a*e^(5
*I*d*x + 5*I*c) + 1400*I*a*e^(3*I*d*x + 3*I*c) + 210*I*a*e^(I*d*x + I*c) +
 105*(a*e^(14*I*d*x + 14*I*c) + 7*a*e^(12*I*d*x + 12*I*c) + 21*a*e^(10*I*d
*x + 10*I*c) + 35*a*e^(8*I*d*x + 8*I*c) + 35*a*e^(6*I*d*x + 6*I*c) + 21*a*
e^(4*I*d*x + 4*I*c) + 7*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) + I
) - 105*(a*e^(14*I*d*x + 14*I*c) + 7*a*e^(12*I*d*x + 12*I*c) + 21*a*e^(10*
I*d*x + 10*I*c) + 35*a*e^(8*I*d*x + 8*I*c) + 35*a*e^(6*I*d*x + 6*I*c) + 21
*a*e^(4*I*d*x + 4*I*c) + 7*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c)
- I))/(d*e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*
d*x + 10*I*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d
*e^(4*I*d*x + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx = ia\left(\int\left(-i\sec^7(c+dx)\right)dx + \int \tan(c+dx)\sec^7(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**7*(a+I*a*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*sec(c + d*x)**7, x) + Integral(tan(c + d*x)*sec(c + d*x)*
*7, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{7a\left(\frac{2\left(15\sin(dx+c)^5-40\sin(dx+c)^3+33\sin(dx+c)\right)}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1} - 15\log(\sin(dx+c)+1) + 15\log(\sin(dx+c)-1)\right) -}{672\,d}$$

input
```
integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/672*(7*a*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(
sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x
 + c) + 1) + 15*log(sin(d*x + c) - 1)) - 96*I*a/cos(d*x + c)^7)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(86) = 172$.

Time = 0.17 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.85

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{105\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-105\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(231\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-336i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/336*(105*a*log(tan(1/2*d*x + 1/2*c) + 1) - 105*a*log(tan(1/2*d*x + 1/2*c
) - 1) + 2*(231*a*tan(1/2*d*x + 1/2*c)^13 - 336*I*a*tan(1/2*d*x + 1/2*c)^1
2 - 196*a*tan(1/2*d*x + 1/2*c)^11 + 595*a*tan(1/2*d*x + 1/2*c)^9 - 1680*I*
a*tan(1/2*d*x + 1/2*c)^8 - 595*a*tan(1/2*d*x + 1/2*c)^5 - 1008*I*a*tan(1/2
*d*x + 1/2*c)^4 + 196*a*tan(1/2*d*x + 1/2*c)^3 - 231*a*tan(1/2*d*x + 1/2*c
) - 48*I*a)/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 4.51 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.52

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$
$$-\frac{-\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8}+2\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{6}-\frac{85\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+10\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\frac{85\,a\tan}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+35\tan\left(\right.\right.}$$

input
```
int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^7,x)
```

output
```
(5*a*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((a*2i)/7 + (11*a*tan(c/2 + (d*x)/
2))/8 - (7*a*tan(c/2 + (d*x)/2)^3)/6 + a*tan(c/2 + (d*x)/2)^4*6i + (85*a*t
an(c/2 + (d*x)/2)^5)/24 + a*tan(c/2 + (d*x)/2)^8*10i - (85*a*tan(c/2 + (d*
x)/2)^9)/24 + (7*a*tan(c/2 + (d*x)/2)^11)/6 + a*tan(c/2 + (d*x)/2)^12*2i -
 (11*a*tan(c/2 + (d*x)/2)^13)/8)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 +
 (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c
/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.71

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{a\left(-105\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^6+315\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx\right.\right.}{}$$

input
```
int(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x)
```

output
```
(a*( - 105*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**6 + 315*co
s(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 - 315*cos(c + d*x)*lo
g(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 + 105*cos(c + d*x)*log(tan((c + d*
x)/2) - 1) + 105*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**6 -
315*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4 + 315*cos(c + d
*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 - 105*cos(c + d*x)*log(tan((
c + d*x)/2) + 1) - 48*cos(c + d*x)*sin(c + d*x)**6*i - 105*cos(c + d*x)*si
n(c + d*x)**5 + 144*cos(c + d*x)*sin(c + d*x)**4*i + 280*cos(c + d*x)*sin(
c + d*x)**3 - 144*cos(c + d*x)*sin(c + d*x)**2*i - 231*cos(c + d*x)*sin(c
+ d*x) + 48*cos(c + d*x)*i - 48*i))/(336*cos(c + d*x)*d*(sin(c + d*x)**6 -
 3*sin(c + d*x)**4 + 3*sin(c + d*x)**2 - 1))
```

3.12 $\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3a\text{arctanh}(\sin(c+dx))}{8d} + \frac{ia\sec^5(c+dx)}{5d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$

output
```
3/8*a*arctanh(sin(d*x+c))/d+1/5*I*a*sec(d*x+c)^5/d+3/8*a*sec(d*x+c)*tan(d*
x+c)/d+1/4*a*sec(d*x+c)^3*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3a\text{arctanh}(\sin(c+dx))}{8d} + \frac{ia\sec^5(c+dx)}{5d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$

input
```
Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

output
```
(3*a*ArcTanh[Sin[c + d*x]])/(8*d) + ((I/5)*a*Sec[c + d*x]^5)/d + (3*a*Sec[
c + d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3967, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^5(a+ia\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^5(c+dx)dx+\frac{ia\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx+\frac{ia\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{3}{4}\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}$$

input
```
Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/5)*a*Sec[c + d*x]^5)/d + a*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(
ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 6.93 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{ia}{5\cos(dx+c)^5}+a\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
default	$\frac{\frac{ia}{5\cos(dx+c)^5}+a\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
risch	$-\frac{ia(15\,e^{9i(dx+c)}+70\,e^{7i(dx+c)}-128\,e^{5i(dx+c)}-70\,e^{3i(dx+c)}-15\,e^{i(dx+c)})}{20d(e^{2i(dx+c)}+1)^5}+\frac{3a\ln(e^{i(dx+c)}+i)}{8d}-\frac{3a\ln(e^{i(dx+c)}-}{8d}$

input
```
int(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*I*a/cos(d*x+c)^5+a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c
)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(66) = 132.

Time = 0.09 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.63

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{-30i\,ae^{(9i\,dx+9i\,c)}-140i\,ae^{(7i\,dx+7i\,c)}+256i\,ae^{(5i\,dx+5i\,c)}+140i\,ae^{(3i\,dx+3i\,c)}+30i\,ae^{(i\,dx+i\,c)}+15\,(ae^{(10i\,dx-}}{}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/40*(-30*I*a*e^(9*I*d*x + 9*I*c) - 140*I*a*e^(7*I*d*x + 7*I*c) + 256*I*a*
e^(5*I*d*x + 5*I*c) + 140*I*a*e^(3*I*d*x + 3*I*c) + 30*I*a*e^(I*d*x + I*c)
 + 15*(a*e^(10*I*d*x + 10*I*c) + 5*a*e^(8*I*d*x + 8*I*c) + 10*a*e^(6*I*d*x
 + 6*I*c) + 10*a*e^(4*I*d*x + 4*I*c) + 5*a*e^(2*I*d*x + 2*I*c) + a)*log(e^
(I*d*x + I*c) + I) - 15*(a*e^(10*I*d*x + 10*I*c) + 5*a*e^(8*I*d*x + 8*I*c)
 + 10*a*e^(6*I*d*x + 6*I*c) + 10*a*e^(4*I*d*x + 4*I*c) + 5*a*e^(2*I*d*x +
2*I*c) + a)*log(e^(I*d*x + I*c) - I))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*
I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d
*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx = ia\left(\int \left(-i\sec^5\left(c+dx\right)\right)\,dx + \int \tan\left(c+dx\right)\sec^5\left(c+dx\right)dx\right)$$

input
```
integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*sec(c + d*x)**5, x) + Integral(tan(c + d*x)*sec(c + d*x)*
*5, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{5\,a\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - 3\,\log\left(\sin\left(dx+c\right)+1\right) + 3\,\log\left(\sin\left(dx+c\right)-1\right)\right) - \frac{16i\,a}{\cos(dx+c)^5}}{80\,d}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/80*(5*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(
d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 16*
I*a/cos(d*x + c)^5)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(66) = 132$.

Time $= 0.16$ (sec) , antiderivative size $= 139$, normalized size of antiderivative $= 1.83$

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{15\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-15\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-40i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-10\right.}{}}{40\,d}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/40*(15*a*log(tan(1/2*d*x + 1/2*c) + 1) - 15*a*log(tan(1/2*d*x + 1/2*c) -
 1) + 2*(25*a*tan(1/2*d*x + 1/2*c)^9 - 40*I*a*tan(1/2*d*x + 1/2*c)^8 - 10*
a*tan(1/2*d*x + 1/2*c)^7 - 80*I*a*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(1/2*d*
x + 1/2*c)^3 - 25*a*tan(1/2*d*x + 1/2*c) - 8*I*a)/(tan(1/2*d*x + 1/2*c)^2
- 1)^5)/d
```

Mupad [B] (verification not implemented)

Time $= 4.06$ (sec) , antiderivative size $= 178$, normalized size of antiderivative $= 2.34$

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$
$$-\frac{-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+2\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+4\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^5,x)
```

output
```
(3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a*2i)/5 + (5*a*tan(c/2 + (d*x)/2
))/4 - (a*tan(c/2 + (d*x)/2)^3)/2 + a*tan(c/2 + (d*x)/2)^4*4i + (a*tan(c/2
 + (d*x)/2)^7)/2 + a*tan(c/2 + (d*x)/2)^8*2i - (5*a*tan(c/2 + (d*x)/2)^9)/
4)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*
x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.49

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{a\left(-15\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^4+30\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+\right.\right.}{}$$

input
```
int(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x)
```

output
```
(a*( - 15*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 + 30*cos(
c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 - 15*cos(c + d*x)*log(t
an((c + d*x)/2) - 1) + 15*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d
*x)**4 - 30*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 + 15*co
s(c + d*x)*log(tan((c + d*x)/2) + 1) - 8*cos(c + d*x)*sin(c + d*x)**4*i -
15*cos(c + d*x)*sin(c + d*x)**3 + 16*cos(c + d*x)*sin(c + d*x)**2*i + 25*c
os(c + d*x)*sin(c + d*x) - 8*cos(c + d*x)*i + 8*i))/(40*cos(c + d*x)*d*(si
n(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.13 $\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{ia\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*a*arctanh(sin(d*x+c))/d+1/3*I*a*sec(d*x+c)^3/d+1/2*a*sec(d*x+c)*tan(d*
x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{ia\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/(2*d) + ((I/3)*a*Sec[c + d*x]^3)/d + (a*Sec[c +
d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3967, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3(a+ia\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^3(c+dx)dx+\frac{ia\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{ia\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}$$

input
```
Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/3)*a*Sec[c + d*x]^3)/d + a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x
]*Tan[c + d*x])/(2*d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\frac{ia}{3\cos(dx+c)^3}+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	51
default	$\frac{\frac{ia}{3\cos(dx+c)^3}+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	51
risch	$-\frac{ia\left(3\,\mathrm{e}^{5i(dx+c)}-8\,\mathrm{e}^{3i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d}$	94

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*I*a/cos(d*x+c)^3+a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+t
an(d*x+c))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(46) = 92$.

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.33

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{-6i\,ae^{(5i\,dx+5i\,c)} + 16i\,ae^{(3i\,dx+3i\,c)} + 6i\,ae^{(i\,dx+i\,c)} + 3\left(ae^{(6i\,dx+6i\,c)} + 3\,ae^{(4i\,dx+4i\,c)} + 3\,ae^{(2i\,dx+2i\,c)} + a\right)\log}{6\left(de^{(6i\,dx+6i\,c)} + 3\,de^{(4i\,dx+4i\,c)} + 3\right.}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(-6*I*a*e^(5*I*d*x + 5*I*c) + 16*I*a*e^(3*I*d*x + 3*I*c) + 6*I*a*e^(I*
d*x + I*c) + 3*(a*e^(6*I*d*x + 6*I*c) + 3*a*e^(4*I*d*x + 4*I*c) + 3*a*e^(2
*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) + I) - 3*(a*e^(6*I*d*x + 6*I*c) +
 3*a*e^(4*I*d*x + 4*I*c) + 3*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c
) - I))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x
+ 2*I*c) + d)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx = ia\left(\int \left(-i\sec^3\left(c+dx\right)\right)\,dx + \int \tan\left(c+dx\right)\sec^3\left(c+dx\right)dx\right)$$

input
```
integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*sec(c + d*x)**3, x) + Integral(tan(c + d*x)*sec(c + d*x)*
*3, x))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= -\frac{3\,a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) - \frac{4i\,a}{\cos(dx+c)^3}}{12\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/12*(3*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
log(sin(d*x + c) - 1)) - 4*I*a/cos(d*x + c)^3)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(46) = 92$.

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.80

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{3\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right) - 3\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right) + \frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 6i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 3\,a\tan(\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3*a*log(tan(1/2*d*x + 1/2*c) + 1) - 3*a*log(tan(1/2*d*x + 1/2*c) - 1)
 + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*I*a*tan(1/2*d*x + 1/2*c)^4 - 3*a*tan(
1/2*d*x + 1/2*c) - 2*I*a)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.98

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+2\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a\,2\mathrm{i}}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^3,x)
```

output
```
(a*atanh(tan(c/2 + (d*x)/2)))/d - ((a*2i)/3 + a*tan(c/2 + (d*x)/2) + a*tan
(c/2 + (d*x)/2)^4*2i - a*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2
- 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.07

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{a\left(-3\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2+3\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+3\cos\left(dx\right.\right.}{}$$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x)
```

output
```
(a*( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 + 3*cos(c
+ d*x)*log(tan((c + d*x)/2) - 1) + 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1
)*sin(c + d*x)**2 - 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1) - 2*cos(c + d
*x)*sin(c + d*x)**2*i - 3*cos(c + d*x)*sin(c + d*x) + 2*cos(c + d*x)*i - 2
*i))/(6*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.14 $\int \sec(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 27

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx=\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}$$

output
```
a*arctanh(sin(d*x+c))/d+I*a*sec(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx=\frac{a\coth^{-1}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}$$

input
```
Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*ArcCoth[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+ia\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

method	result	size
derivativedivides	$\frac{\frac{ia}{\cos(dx+c)}+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	34
default	$\frac{\frac{ia}{\cos(dx+c)}+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	34
risch	$\frac{2i\mathrm{e}^{i(dx+c)}a}{d(\mathrm{e}^{2i(dx+c)}+1)}+\frac{a\ln(\mathrm{e}^{i(dx+c)}+i)}{d}-\frac{a\ln(\mathrm{e}^{i(dx+c)}-i)}{d}$	68

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(I*a/cos(d*x+c)+a*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(25) = 50$.

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.04

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx = \frac{2i\,ae^{(i\,dx+i\,c)} + \left(ae^{(2i\,dx+2i\,c)} + a\right)\log\left(e^{(i\,dx+i\,c)} + i\right) - \left(ae^{(2i\,dx+2i\,c)} + a\right)\log\left(e^{(i\,dx+i\,c)} - i\right)}{de^{(2i\,dx+2i\,c)} + d}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
(2*I*a*e^(I*d*x + I*c) + (a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) +
 I) - (a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) - I))/(d*e^(2*I*d*x
+ 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{a\log(\tan(c+dx)+\sec(c+dx))+ia\sec(c+dx)}{d} & \text{for } d \neq 0 \\ x(ia\tan(c)+a)\sec(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + I*a*sec(c + d*x))/d, Ne(d
, 0)), (x*(I*a*tan(c) + a)*sec(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+\frac{i\,a}{\cos(dx+c)}}{d}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
(a*log(sec(d*x + c) + tan(d*x + c)) + I*a/cos(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(25) = 50$.

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.93

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2i\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
(a*log(tan(1/2*d*x + 1/2*c) + 1) - a*log(tan(1/2*d*x + 1/2*c) - 1) - 2*I*a
/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\,2\mathrm{i}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

input
```
int((a + a*tan(c + d*x)*1i)/cos(c + d*x),x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)))/d - (a*2i)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{a\left(-\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\cos\left(dx+c\right)i+i\right)}{\cos\left(dx+c\right)d}$$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c)),x)
```

output
```
(a*( - cos(c + d*x)*log(tan((c + d*x)/2) - 1) + cos(c + d*x)*log(tan((c +
d*x)/2) + 1) - cos(c + d*x)*i + i))/(cos(c + d*x)*d)
```

3.15 $\int \cos(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 26

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx=-\frac{ia\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}$$

output
```
-I*a*cos(d*x+c)/d+a*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx=-\frac{ia\cos(c)\cos(dx)}{d}+\frac{a\cos(dx)\sin(c)}{d}+\frac{a\cos(c)\sin(dx)}{d}+\frac{ia\sin(c)\sin(dx)}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*a*Cos[c]*Cos[d*x])/d + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
 + (I*a*Sin[c]*Sin[d*x])/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3967, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos(c+dx)dx-\frac{ia\cos(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx-\frac{ia\cos(c+dx)}{d}$$

$$\Big\downarrow \text{3117}$$

$$\frac{a\sin(c+dx)}{d}-\frac{ia\cos(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*a*Cos[c + d*x])/d + (a*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
risch	$-\frac{ia\,e^{i(dx+c)}}{d}$	17
derivativedivides	$\frac{-ia\cos(dx+c)+a\sin(dx+c)}{d}$	24
default	$\frac{-ia\cos(dx+c)+a\sin(dx+c)}{d}$	24
orering	$-\frac{i\cos(dx+c)(a+ia\tan(dx+c))}{d}$	25

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-I/d*a*exp(I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{i\,ae^{(i\,dx+i\,c)}}{d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-I*a*e^(I*d*x + I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} -\frac{iae^{ic}e^{idx}}{d} & \text{for } d \neq 0 \\ axe^{ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise((-I*a*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (a*x*exp(I*c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx = \frac{-i\,a\cos{(dx+c)}+a\sin{(dx+c)}}{d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
(-I*a*cos(d*x + c) + a*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(24) = 48.

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.23

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{4i\,ae^{(i\,dx+i\,c)} + a\log\left(i\,e^{(i\,dx+i\,c)}+1\right) + a\log\left(i\,e^{(i\,dx+i\,c)}-1\right) - a\log\left(-i\,e^{(i\,dx+i\,c)}+1\right) - a\log\left(-i\,e^{(i\,dx}\right.}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(4*I*a*e^(I*d*x + I*c) + a*log(I*e^(I*d*x + I*c) + 1) + a*log(I*e^(I*
d*x + I*c) - 1) - a*log(-I*e^(I*d*x + I*c) + 1) - a*log(-I*e^(I*d*x + I*c)
 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx = \frac{2\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i),x)
```

output
```
(2*a)/(d*(tan(c/2 + (d*x)/2) + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a(-\cos(dx+c)\,i+\sin(dx+c)+i)}{d}$$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c)),x)
```

output
```
(a*( - cos(c + d*x)*i + sin(c + d*x) + i))/d
```

3.16 $\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

output
```
-1/3*I*a*cos(d*x+c)^3/d+a*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/3*I)*a*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3
*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos^3(c+dx)dx-\frac{ia\cos^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{ia\cos^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{a\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}-\frac{ia\cos^3(c+dx)}{3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{ia\cos^3(c+dx)}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/3*I)*a*Cos[c + d*x]^3)/d - (a*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{-\frac{ia\cos(dx+c)^3}{3}+\frac{a\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$	37
default	$\frac{-\frac{ia\cos(dx+c)^3}{3}+\frac{a\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$	37
risch	$-\frac{ia\,e^{3i(dx+c)}}{12d}-\frac{ia\cos(dx+c)}{4d}+\frac{3a\sin(dx+c)}{4d}$	43

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*I*a*cos(d*x+c)^3+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\left(-i\,ae^{(4i\,dx+4i\,c)} - 6i\,ae^{(2i\,dx+2i\,c)} + 3i\,a\right)e^{(-i\,dx-i\,c)}}{12\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(-I*a*e^(4*I*d*x + 4*I*c) - 6*I*a*e^(2*I*d*x + 2*I*c) + 3*I*a)*e^(-I*
d*x - I*c)/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(37) = 74$.

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.28

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{(-8iad^2e^{4ic}e^{3idx}-48iad^2e^{2ic}e^{idx}+24iad^2e^{-idx})e^{-ic}}{96d^3} & \text{for } d^3e^{ic} \neq 0 \\ \frac{x(ae^{4ic}+2ae^{2ic}+a)e^{-ic}}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((-8*I*a*d**2*exp(4*I*c)*exp(3*I*d*x) - 48*I*a*d**2*exp(2*I*c)*e
xp(I*d*x) + 24*I*a*d**2*exp(-I*d*x))*exp(-I*c)/(96*d**3), Ne(d**3*exp(I*c)
, 0)), (x*(a*exp(4*I*c) + 2*a*exp(2*I*c) + a)*exp(-I*c)/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= -\frac{i\,a\cos\left(dx+c\right)^3+\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/3*(I*a*cos(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(40) = 80.

Time = 0.16 (sec) , antiderivative size = 196, normalized size of antiderivative = 4.26

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx =$$
$$-\frac{\left(9\,ae^{(i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+6\,ae^{(i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-9\,ae^{(i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/48*(9*a*e^(I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 6*a*e^(I*d*x + I*c
)*log(I*e^(I*d*x + I*c) - 1) - 9*a*e^(I*d*x + I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 6*a*e^(I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3*a*e^(I*d*x + I*
c)*log(I*e^(I*d*x) + e^(-I*c)) + 3*a*e^(I*d*x + I*c)*log(-I*e^(I*d*x) + e^
(-I*c)) + 4*I*a*e^(4*I*d*x + 4*I*c) + 24*I*a*e^(2*I*d*x + 2*I*c) - 12*I*a)
*e^(-I*d*x - I*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{2\,a\,\left(-\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3{}\mathrm{i}}{4}-\frac{\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2\,1{}\mathrm{i}}{4}+\frac{9\,\sin(c+d\,x)}{8}+\frac{\sin(3\,c+3\,d\,x)}{8}\right)}{3\,d}$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i),x)
```

output
```
(2*a*((9*sin(c + d*x))/8 + sin(3*c + 3*d*x)/8 - (cos(c/2 + (d*x)/2)^2*3i)/
4 - (cos((3*c)/2 + (3*d*x)/2)^2*1i)/4))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{a\left(\cos(dx+c)\sin(dx+c)^2 i-\cos(dx+c) i-\sin(dx+c)^3+3\sin(dx+c)+i\right)}{3d}$$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x)
```

output
```
(a*(cos(c + d*x)*sin(c + d*x)**2*i - cos(c + d*x)*i - sin(c + d*x)**3 + 3*
sin(c + d*x) + i))/(3*d)
```

3.17 $\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx=-\frac{ia\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}$$

output
```
-1/5*I*a*cos(d*x+c)^5/d+a*sin(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin(d*x+
c)^5/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx=-\frac{ia\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/5*I)*a*Cos[c + d*x]^5)/d + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/
(3*d) + (a*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos^5(c+dx)dx-\frac{ia\cos^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{ia\cos^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{a\int \left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}-\frac{ia\cos^5(c+dx)}{5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{ia\cos^5(c+dx)}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/5*I)*a*Cos[c + d*x]^5)/d - (a*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 -
 Sin[c + d*x]^5/5))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

Maple [A] (verified)

Time = 6.54 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{-\frac{ia\cos(dx+c)^5}{5}+\frac{a\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}}{d}$	47
default	$\frac{-\frac{ia\cos(dx+c)^5}{5}+\frac{a\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}}{d}$	47
risch	$-\frac{ia\,e^{5i(dx+c)}}{80d}-\frac{ia\cos(dx+c)}{8d}+\frac{5a\sin(dx+c)}{8d}-\frac{ia\cos(3dx+3c)}{16d}+\frac{5a\sin(3dx+3c)}{48d}$	74

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/5*I*a*cos(d*x+c)^5+1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d
*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\left(-3i\,ae^{(8i\,dx+8i\,c)}-20i\,ae^{(6i\,dx+6i\,c)}-90i\,ae^{(4i\,dx+4i\,c)}+60i\,ae^{(2i\,dx+2i\,c)}+5i\,a\right)e^{(-3i\,dx-3i\,c)}}{240\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/240*(-3*I*a*e^(8*I*d*x + 8*I*c) - 20*I*a*e^(6*I*d*x + 6*I*c) - 90*I*a*e^
(4*I*d*x + 4*I*c) + 60*I*a*e^(2*I*d*x + 2*I*c) + 5*I*a)*e^(-3*I*d*x - 3*I*
c)/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(53) = 106.

Time = 0.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.97

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{\left(-18432iad^4e^{9ic}e^{5idx}-122880iad^4e^{7ic}e^{3idx}-552960iad^4e^{5ic}e^{idx}+368640iad^4e^{3ic}e^{-idx}+30720iad^4e^{ic}e^{-3idx}\right)e^{-4ic}}{1474560d^5} & \text{for}\ d^5e^{4ic}\neq 0 \\ \frac{x\left(ae^{8ic}+4ae^{6ic}+6ae^{4ic}+4ae^{2ic}+a\right)e^{-3ic}}{16} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((-18432*I*a*d**4*exp(9*I*c)*exp(5*I*d*x) - 122880*I*a*d**4*exp(
7*I*c)*exp(3*I*d*x) - 552960*I*a*d**4*exp(5*I*c)*exp(I*d*x) + 368640*I*a*d
**4*exp(3*I*c)*exp(-I*d*x) + 30720*I*a*d**4*exp(I*c)*exp(-3*I*d*x))*exp(-4
*I*c)/(1474560*d**5), Ne(d**5*exp(4*I*c), 0)), (x*(a*exp(8*I*c) + 4*a*exp(
6*I*c) + 6*a*exp(4*I*c) + 4*a*exp(2*I*c) + a)*exp(-3*I*c)/16, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= -\frac{3i\,a\cos\left(dx+c\right)^5 - \left(3\,\sin\left(dx+c\right)^5 - 10\,\sin\left(dx+c\right)^3 + 15\,\sin\left(dx+c\right)\right)a}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/15*(3*I*a*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*s
in(d*x + c))*a)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(54) = 108$.

Time = 0.19 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.55

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx =$$
$$-\frac{\left(135\,ae^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right) + 90\,ae^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right) - 135\,ae^{(3i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx+\cdots}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/960*(135*a*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 90*a*e^(3*I*d
*x + I*c)*log(I*e^(I*d*x + I*c) - 1) - 135*a*e^(3*I*d*x + I*c)*log(-I*e^(I
*d*x + I*c) + 1) - 90*a*e^(3*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 45
*a*e^(3*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 45*a*e^(3*I*d*x + I*c)*
log(-I*e^(I*d*x) + e^(-I*c)) + 12*I*a*e^(8*I*d*x + 6*I*c) + 80*I*a*e^(6*I*
d*x + 4*I*c) + 360*I*a*e^(4*I*d*x + 2*I*c) - 240*I*a*e^(2*I*d*x) - 20*I*a*
e^(-2*I*c))*e^(-3*I*d*x - I*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{2\,a\,\left(-\frac{75\,\sin(c+d\,x)}{16}-\frac{25\,\sin(3\,c+3\,d\,x)}{32}-\frac{3\,\sin(5\,c+5\,d\,x)}{32}+\frac{\cos(c+d\,x)\,15{}\mathrm{i}}{16}+\frac{\cos(3\,c+3\,d\,x)\,15{}\mathrm{i}}{32}+\frac{\cos(5\,c+5\,d\,x)\,3{}\mathrm{i}}{32}\right)}{15\,d}$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i),x)
```

output
```
-(2*a*((cos(c + d*x)*15i)/16 - (75*sin(c + d*x))/16 + (cos(3*c + 3*d*x)*15
i)/32 + (cos(5*c + 5*d*x)*3i)/32 - (25*sin(3*c + 3*d*x))/32 - (3*sin(5*c +
 5*d*x))/32))/(15*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.31

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\left(-3\cos\left(dx+c\right)\sin\left(dx+c\right)^4 i+6\cos\left(dx+c\right)\sin\left(dx+c\right)^2 i-3\cos\left(dx+c\right)i+3\sin\left(dx+c\right)^5-10\,\mathrm{s}\right.}{15d}$$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x)
```

output
```
(a*( - 3*cos(c + d*x)*sin(c + d*x)**4*i + 6*cos(c + d*x)*sin(c + d*x)**2*i
 - 3*cos(c + d*x)*i + 3*sin(c + d*x)**5 - 10*sin(c + d*x)**3 + 15*sin(c +
d*x) + 3*i))/(15*d)
```

3.18 $\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^7(c+dx)}{7d} + \frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{d} + \frac{3a\sin^5(c+dx)}{5d} - \frac{a\sin^7(c+dx)}{7d}$$

output
```
-1/7*I*a*cos(d*x+c)^7/d+a*sin(d*x+c)/d-a*sin(d*x+c)^3/d+3/5*a*sin(d*x+c)^5
/d-1/7*a*sin(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^7(c+dx)}{7d} + \frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{d} + \frac{3a\sin^5(c+dx)}{5d} - \frac{a\sin^7(c+dx)}{7d}$$

input
```
Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/7*I)*a*Cos[c + d*x]^7)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/d
+ (3*a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos^7(c+dx)dx-\frac{ia\cos^7(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^7dx-\frac{ia\cos^7(c+dx)}{7d}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{a\int \left(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}-\frac{ia\cos^7(c+dx)}{7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{ia\cos^7(c+dx)}{7d}$$

input
```
Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/7*I)*a*Cos[c + d*x]^7)/d - (a*(-Sin[c + d*x] + Sin[c + d*x]^3 - (3*Si
n[c + d*x]^5)/5 + Sin[c + d*x]^7/7))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

Maple [A] (verified)

Time = 27.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{-\frac{ia\cos(dx+c)^7}{7}+\frac{a\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}}{d}$
default	$\frac{-\frac{ia\cos(dx+c)^7}{7}+\frac{a\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}}{d}$
risch	$-\frac{ia\,e^{7i(dx+c)}}{448d}-\frac{5ia\cos(dx+c)}{64d}+\frac{35a\sin(dx+c)}{64d}-\frac{ia\cos(5dx+5c)}{64d}+\frac{7a\sin(5dx+5c)}{320d}-\frac{3ia\cos(3dx+3c)}{64d}+$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/7*I*a*cos(d*x+c)^7+1/7*a*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*c
os(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.18

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{\left(-5i\,ae^{(12i\,dx+12i\,c)} - 42i\,ae^{(10i\,dx+10i\,c)} - 175i\,ae^{(8i\,dx+8i\,c)} - 700i\,ae^{(6i\,dx+6i\,c)} + 525i\,ae^{(4i\,dx+4i\,c)} + 70i\,ae^{(}\right.}{2240\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2240*(-5*I*a*e^(12*I*d*x + 12*I*c) - 42*I*a*e^(10*I*d*x + 10*I*c) - 175*
I*a*e^(8*I*d*x + 8*I*c) - 700*I*a*e^(6*I*d*x + 6*I*c) + 525*I*a*e^(4*I*d*x
 + 4*I*c) + 70*I*a*e^(2*I*d*x + 2*I*c) + 7*I*a)*e^(-5*I*d*x - 5*I*c)/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(65) = 130.

Time = 0.34 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.33

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \begin{cases} \frac{\left(-107374182400iad^6e^{16ic}e^{7idx} - 901943132160iad^6e^{14ic}e^{5idx} - 3758096384000iad^6e^{12ic}e^{3idx} - 15032385536000iad^6e^{10ic}e^{idx} + 11274289152\right.}{48103633715200d^7} \\ \frac{x\left(ae^{12ic} + 6ae^{10ic} + 15ae^{8ic} + 20ae^{6ic} + 15ae^{4ic} + 6ae^{2ic} + a\right)e^{-5ic}}{64} \end{cases}$$

input
```
integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((-107374182400*I*a*d**6*exp(16*I*c)*exp(7*I*d*x) - 901943132160
*I*a*d**6*exp(14*I*c)*exp(5*I*d*x) - 3758096384000*I*a*d**6*exp(12*I*c)*ex
p(3*I*d*x) - 15032385536000*I*a*d**6*exp(10*I*c)*exp(I*d*x) + 112742891520
00*I*a*d**6*exp(8*I*c)*exp(-I*d*x) + 1503238553600*I*a*d**6*exp(6*I*c)*exp
(-3*I*d*x) + 150323855360*I*a*d**6*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(
48103633715200*d**7), Ne(d**7*exp(9*I*c), 0)), (x*(a*exp(12*I*c) + 6*a*exp
(10*I*c) + 15*a*exp(8*I*c) + 20*a*exp(6*I*c) + 15*a*exp(4*I*c) + 6*a*exp(2
*I*c) + a)*exp(-5*I*c)/64, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{5i\,a\cos\left(dx+c\right)^7+\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a}{35\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/35*(5*I*a*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*s
in(d*x + c)^3 - 35*sin(d*x + c))*a)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(68) = 136.

Time = 0.20 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.21

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{\left(1015\,ae^{(5i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+700\,ae^{(5i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-1015\,ae^{(5i\,dx+i\,c)}\log\left(-i\,e^{(i\right.\right.}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/8960*(1015*a*e^(5*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 700*a*e^(5*
I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) - 1015*a*e^(5*I*d*x + I*c)*log(-I*
e^(I*d*x + I*c) + 1) - 700*a*e^(5*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1)
 - 315*a*e^(5*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 315*a*e^(5*I*d*x
+ I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 20*I*a*e^(12*I*d*x + 8*I*c) + 168*I*
a*e^(10*I*d*x + 6*I*c) + 700*I*a*e^(8*I*d*x + 4*I*c) + 2800*I*a*e^(6*I*d*x
 + 2*I*c) - 280*I*a*e^(2*I*d*x - 2*I*c) - 2100*I*a*e^(4*I*d*x) - 28*I*a*e^
(-4*I*c))*e^(-5*I*d*x - I*c)/d
```

Mupad [B] (verification not implemented)

Time = 3.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.22

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{2\,a\left(-\frac{1225\,\sin(c+d\,x)}{128}-\frac{245\,\sin(3\,c+3\,d\,x)}{128}-\frac{49\,\sin(5\,c+5\,d\,x)}{128}-\frac{5\,\sin(7\,c+7\,d\,x)}{128}+\frac{\cos(c+d\,x)\,175\mathrm{i}}{128}+\frac{\cos(3\,c+3\,d\,x)\,105\mathrm{i}}{128}+\mathrm{co}\right.}{35\,d}$$

input
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i),x)
```

output
```
-(2*a*((cos(c + d*x)*175i)/128 - (1225*sin(c + d*x))/128 + (cos(3*c + 3*d*
x)*105i)/128 + (cos(5*c + 5*d*x)*35i)/128 + (cos(7*c + 7*d*x)*5i)/128 - (2
45*sin(3*c + 3*d*x))/128 - (49*sin(5*c + 5*d*x))/128 - (5*sin(7*c + 7*d*x)
)/128))/(35*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.42

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\left(5\cos\left(dx+c\right)\sin\left(dx+c\right)^6 i-15\cos\left(dx+c\right)\sin\left(dx+c\right)^4 i+15\cos\left(dx+c\right)\sin\left(dx+c\right)^2 i-5\cos\left(d\right.\right.}{35d}$$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x)
```

output
```
(a*(5*cos(c + d*x)*sin(c + d*x)**6*i - 15*cos(c + d*x)*sin(c + d*x)**4*i +
 15*cos(c + d*x)*sin(c + d*x)**2*i - 5*cos(c + d*x)*i - 5*sin(c + d*x)**7
+ 21*sin(c + d*x)**5 - 35*sin(c + d*x)**3 + 35*sin(c + d*x) + 5*i))/(35*d)
```

3.19 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{4i(a+ia\tan(c+dx))^6}{3a^4d} + \frac{12i(a+ia\tan(c+dx))^7}{7a^5d} - \frac{3i(a+ia\tan(c+dx))^8}{4a^6d} + \frac{i(a+ia\tan(c+dx))^9}{9a^7d}$$

output
```
-4/3*I*(a+I*a*tan(d*x+c))^6/a^4/d+12/7*I*(a+I*a*tan(d*x+c))^7/a^5/d-3/4*I*
(a+I*a*tan(d*x+c))^8/a^6/d+1/9*I*(a+I*a*tan(d*x+c))^9/a^7/d
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2\sec^8(c+dx)(\cos(6(c+dx))+i\sin(6(c+dx)))(-40i+170i\cos(2(c+dx))+83\sec(c+dx)\sin(3(}{504d}$$

input
```
Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/504*(a^2*Sec[c + d*x]^8*(Cos[6*(c + d*x)] + I*Sin[6*(c + d*x)])*(-40*I
+ (170*I)*Cos[2*(c + d*x)] + 83*Sec[c + d*x]*Sin[3*(c + d*x)] + 27*Tan[c +
 d*x]))/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \sec(c+dx)^8(a+ia\tan(c+dx))^2dx$$

$$\Big\downarrow \textbf{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^5d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \textbf{49}$$

$$-\frac{i\int \left(-(i\tan(c+dx)a+a)^8+6a(i\tan(c+dx)a+a)^7-12a^2(i\tan(c+dx)a+a)^6+8a^3(i\tan(c+dx)a+a)^5\right)}{a^7d}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{i\left(\frac{4}{3}a^3(a+ia\tan(c+dx))^6-\frac{12}{7}a^2(a+ia\tan(c+dx))^7-\frac{1}{9}(a+ia\tan(c+dx))^9+\frac{3}{4}a(a+ia\tan(c+dx))^8\right)}{a^7d}$$

input
```
Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*((4*a^3*(a + I*a*Tan[c + d*x])^6)/3 - (12*a^2*(a + I*a*Tan[c + d*x])
^7)/7 + (3*a*(a + I*a*Tan[c + d*x])^8)/4 - (a + I*a*Tan[c + d*x])^9/9))/(a
^7*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 107.83 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

method	result
risch	$\frac{64 i a^2 \left(126 \,\mathrm{e}^{10 i (dx+c)}+126 \,\mathrm{e}^{8 i (dx+c)}+84 \,\mathrm{e}^{6 i (dx+c)}+36 \,\mathrm{e}^{4 i (dx+c)}+9 \,\mathrm{e}^{2 i (dx+c)}+1\right)}{63 d \left(\mathrm{e}^{2 i (dx+c)}+1\right)^9}$
derivativedivides	$\frac{-a^2 \left(\frac{\sin(dx+c)^3}{9 \cos(dx+c)^9}+\frac{2 \sin(dx+c)^3}{21 \cos(dx+c)^7}+\frac{8 \sin(dx+c)^3}{105 \cos(dx+c)^5}+\frac{16 \sin(dx+c)^3}{315 \cos(dx+c)^3}\right)+\frac{i a^2}{4 \cos(dx+c)^8}-a^2 \left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6 \sec(dx+c)}{35}\right.}{d}$
default	$\frac{-a^2 \left(\frac{\sin(dx+c)^3}{9 \cos(dx+c)^9}+\frac{2 \sin(dx+c)^3}{21 \cos(dx+c)^7}+\frac{8 \sin(dx+c)^3}{105 \cos(dx+c)^5}+\frac{16 \sin(dx+c)^3}{315 \cos(dx+c)^3}\right)+\frac{i a^2}{4 \cos(dx+c)^8}-a^2 \left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6 \sec(dx+c)}{35}\right.}{d}$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
64/63*I*a^2*(126*exp(10*I*(d*x+c))+126*exp(8*I*(d*x+c))+84*exp(6*I*(d*x+c)
)+36*exp(4*I*(d*x+c))+9*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^9
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(85) = 170.

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.73

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{64\left(-126i\,a^2e^{(10i\,dx+10i\,c)}-126i\,a^2e^{(8i\,dx+8i\,c)}-84i\,a^2e^{(6i\,dx+6i\,c)}-36i\,a^2e^{(}\right.}{63\left(de^{(18i\,dx+18i\,c)}+9\,de^{(16i\,dx+16i\,c)}+36\,de^{(14i\,dx+14i\,c)}+84\,de^{(12i\,dx+12i\,c)}+126\,de^{(10i\,dx+10i\,c)}+126\,de^{(8}\right.}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-64/63*(-126*I*a^2*e^(10*I*d*x + 10*I*c) - 126*I*a^2*e^(8*I*d*x + 8*I*c) -
 84*I*a^2*e^(6*I*d*x + 6*I*c) - 36*I*a^2*e^(4*I*d*x + 4*I*c) - 9*I*a^2*e^(
2*I*d*x + 2*I*c) - I*a^2)/(d*e^(18*I*d*x + 18*I*c) + 9*d*e^(16*I*d*x + 16*
I*c) + 36*d*e^(14*I*d*x + 14*I*c) + 84*d*e^(12*I*d*x + 12*I*c) + 126*d*e^(
10*I*d*x + 10*I*c) + 126*d*e^(8*I*d*x + 8*I*c) + 84*d*e^(6*I*d*x + 6*I*c)
+ 36*d*e^(4*I*d*x + 4*I*c) + 9*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2(c+dx)\sec^8(c+dx)\,dx + \int \left(-2i\tan(c+dx)\sec^8(c+dx)\right)dx + \int \left(-\sec^8(c+dx)\right)dx\right)$$

input
```
integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**8, x) + Integral(-2*I*tan(c
+ d*x)*sec(c + d*x)**8, x) + Integral(-sec(c + d*x)**8, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx =$$
$$-\frac{28\,a^2\tan{(dx+c)}^9 - 63i\,a^2\tan{(dx+c)}^8 + 72\,a^2\tan{(dx+c)}^7 - 252i\,a^2\tan{(dx+c)}^6 - 378i\,a^2\tan{(}}{252\,d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/252*(28*a^2*tan(d*x + c)^9 - 63*I*a^2*tan(d*x + c)^8 + 72*a^2*tan(d*x +
 c)^7 - 252*I*a^2*tan(d*x + c)^6 - 378*I*a^2*tan(d*x + c)^4 - 168*a^2*tan(
d*x + c)^3 - 252*I*a^2*tan(d*x + c)^2 - 252*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx =$$
$$-\frac{28\,a^2\tan{(dx+c)}^9 - 63i\,a^2\tan{(dx+c)}^8 + 72\,a^2\tan{(dx+c)}^7 - 252i\,a^2\tan{(dx+c)}^6 - 378i\,a^2\tan{(}}{252\,d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/252*(28*a^2*tan(d*x + c)^9 - 63*I*a^2*tan(d*x + c)^8 + 72*a^2*tan(d*x +
 c)^7 - 252*I*a^2*tan(d*x + c)^6 - 378*I*a^2*tan(d*x + c)^4 - 168*a^2*tan(
d*x + c)^3 - 252*I*a^2*tan(d*x + c)^2 - 252*a^2*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.39

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{a^2\,\sin\left(c+d\,x\right)\,\left(252\cos\left(c+d\,x\right)^8+\cos\left(c+d\,x\right)^7\,\sin\left(c+d\,x\right)\,252\mathrm{i}+168\cos\left(c+d\,x\right)^6\sin\left(c+d\,x\right)^2+\right.}{}$$

input
```
int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^8,x)
```

output
```
(a^2*sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^7*63i + cos(c + d*x)^7*sin(c
+ d*x)*252i + 252*cos(c + d*x)^8 - 28*sin(c + d*x)^8 - 72*cos(c + d*x)^2*s
in(c + d*x)^6 + cos(c + d*x)^3*sin(c + d*x)^5*252i + cos(c + d*x)^5*sin(c
+ d*x)^3*378i + 168*cos(c + d*x)^6*sin(c + d*x)^2))/(252*d*cos(c + d*x)^9)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.58

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{\sin\left(dx+c\right)a^2\left(-63\cos\left(dx+c\right)\sin\left(dx+c\right)^7i+252\cos\left(dx+c\right)\sin\left(dx+c\right)^5i-378\cos\left(dx+c\right)\sin\left(d\right.\right.}{252\cos\left(dx+c\right)d\left(\sin\left(dx+c\right)^8\right.}$$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(sin(c + d*x)*a**2*( - 63*cos(c + d*x)*sin(c + d*x)**7*i + 252*cos(c + d*x
)*sin(c + d*x)**5*i - 378*cos(c + d*x)*sin(c + d*x)**3*i + 252*cos(c + d*x
)*sin(c + d*x)*i + 128*sin(c + d*x)**8 - 576*sin(c + d*x)**6 + 1008*sin(c
+ d*x)**4 - 840*sin(c + d*x)**2 + 252))/(252*cos(c + d*x)*d*(sin(c + d*x)*
*8 - 4*sin(c + d*x)**6 + 6*sin(c + d*x)**4 - 4*sin(c + d*x)**2 + 1))
```

3.20 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{4i(a+ia\tan(c+dx))^5}{5a^3d} + \frac{2i(a+ia\tan(c+dx))^6}{3a^4d} - \frac{i(a+ia\tan(c+dx))^7}{7a^5d}$$

output
```
-4/5*I*(a+I*a*tan(d*x+c))^5/a^3/d+2/3*I*(a+I*a*tan(d*x+c))^6/a^4/d-1/7*I*(
a+I*a*tan(d*x+c))^7/a^5/d
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\sec^7(c+dx)(7+22\cos(2(c+dx))-20i\sin(2(c+dx)))(-i\cos(5(c+dx))+\sin(5(c+dx)))}{105d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(a^2*Sec[c + d*x]^7*(7 + 22*Cos[2*(c + d*x)] - (20*I)*Sin[2*(c + d*x)])*((
-I)*Cos[5*(c + d*x)] + Sin[5*(c + d*x)]))/(105*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \sec(c+dx)^6(a+ia\tan(c+dx))^2dx$$

$$\Big\downarrow \textbf{3968}$$

$$-\frac{i\int(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^4d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \textbf{49}$$

$$-\frac{i\int\left((i\tan(c+dx)a+a)^6-4a(i\tan(c+dx)a+a)^5+4a^2(i\tan(c+dx)a+a)^4\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{i\left(\frac{4}{5}a^2(a+ia\tan(c+dx))^5+\frac{1}{7}(a+ia\tan(c+dx))^7-\frac{2}{3}a(a+ia\tan(c+dx))^6\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*((4*a^2*(a + I*a*Tan[c + d*x])^5)/5 - (2*a*(a + I*a*Tan[c + d*x])^6)
/3 + (a + I*a*Tan[c + d*x])^7/7))/(a^5*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 33.53 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result
risch	$\frac{128ia^2\left(35\,\mathrm{e}^{8i(dx+c)}+35\,\mathrm{e}^{6i(dx+c)}+21\,\mathrm{e}^{4i(dx+c)}+7\,\mathrm{e}^{2i(dx+c)}+1\right)}{105d\left(\mathrm{e}^{2i(dx+c)}+1\right)^7}$
derivativedivides	$\frac{-a^2\left(\frac{\sin(dx+c)^3}{7\cos(dx+c)^7}+\frac{4\sin(dx+c)^3}{35\cos(dx+c)^5}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^3}\right)+\frac{ia^2}{3\cos(dx+c)^6}-a^2\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$
default	$\frac{-a^2\left(\frac{\sin(dx+c)^3}{7\cos(dx+c)^7}+\frac{4\sin(dx+c)^3}{35\cos(dx+c)^5}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^3}\right)+\frac{ia^2}{3\cos(dx+c)^6}-a^2\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
128/105*I*a^2*(35*exp(8*I*(d*x+c))+35*exp(6*I*(d*x+c))+21*exp(4*I*(d*x+c))
+7*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^7
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(64) = 128$.

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{128\left(-35i\,a^2e^{(8i\,dx+8i\,c)} - 35i\,a^2e^{(6i\,dx+6i\,c)} - 21i\,a^2e^{(4i\,dx+4i\,c)} - 7i\,a^2e^{(2i\,dx+2i\,c)} - i\right.}{105\left(de^{(14i\,dx+14i\,c)} + 7\,de^{(12i\,dx+12i\,c)} + 21\,de^{(10i\,dx+10i\,c)} + 35\,de^{(8i\,dx+8i\,c)} + 35\,de^{(6i\,dx+6i\,c)} + 21\,de^{(4i\,dx+4\right.}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-128/105*(-35*I*a^2*e^(8*I*d*x + 8*I*c) - 35*I*a^2*e^(6*I*d*x + 6*I*c) - 2
1*I*a^2*e^(4*I*d*x + 4*I*c) - 7*I*a^2*e^(2*I*d*x + 2*I*c) - I*a^2)/(d*e^(1
4*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x + 10*I*c)
 + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I*d*x +
 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2(c+dx)\sec^6(c+dx)\,dx + \int \left(-2i\tan(c+dx)\sec^6(c+dx)\right)dx + \int \left(-\sec^6(c+dx)\right)dx\right)$$

input
```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**6, x) + Integral(-2*I*tan(c
+ d*x)*sec(c + d*x)**6, x) + Integral(-sec(c + d*x)**6, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{15\,a^2\tan\left(dx+c\right)^7 - 35i\,a^2\tan\left(dx+c\right)^6 + 21\,a^2\tan\left(dx+c\right)^5 - 105i\,a^2\tan\left(dx+c\right)^4 - 35\,a^2\tan\left(dx\right.}{105\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/105*(15*a^2*tan(d*x + c)^7 - 35*I*a^2*tan(d*x + c)^6 + 21*a^2*tan(d*x +
 c)^5 - 105*I*a^2*tan(d*x + c)^4 - 35*a^2*tan(d*x + c)^3 - 105*I*a^2*tan(d
*x + c)^2 - 105*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{15\,a^2\tan\left(dx+c\right)^7 - 35i\,a^2\tan\left(dx+c\right)^6 + 21\,a^2\tan\left(dx+c\right)^5 - 105i\,a^2\tan\left(dx+c\right)^4 - 35\,a^2\tan\left(dx\right.}{105\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/105*(15*a^2*tan(d*x + c)^7 - 35*I*a^2*tan(d*x + c)^6 + 21*a^2*tan(d*x +
 c)^5 - 105*I*a^2*tan(d*x + c)^4 - 35*a^2*tan(d*x + c)^3 - 105*I*a^2*tan(d
*x + c)^2 - 105*a^2*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.61

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{a^2\sin(c+d\,x)\,\left(105\cos(c+d\,x)^6+\cos(c+d\,x)^5\,\sin(c+d\,x)\,105\mathrm{i}+35\cos(c+d\,x)^4\sin(c+d\,x)^2+\cdots\right.}{105\cdots}$$

input
```
int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^6,x)
```

output
```
(a^2*sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^5*35i + cos(c + d*x)^5*sin(c
+ d*x)*105i + 105*cos(c + d*x)^6 - 15*sin(c + d*x)^6 - 21*cos(c + d*x)^2*s
in(c + d*x)^4 + cos(c + d*x)^3*sin(c + d*x)^3*105i + 35*cos(c + d*x)^4*sin
(c + d*x)^2))/(105*d*cos(c + d*x)^7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.65

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{\sin(dx+c)\,a^2\left(-35\cos(dx+c)\sin(dx+c)^5 i+105\cos(dx+c)\sin(dx+c)^3 i-105\cos(dx+c)\sin(d\cdots\right.}{105\cos(dx+c)\,d\left(\sin(dx+c)^6-3\sin(dx+c)^4+\cdots\right.}$$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(sin(c + d*x)*a**2*( - 35*cos(c + d*x)*sin(c + d*x)**5*i + 105*cos(c + d*x
)*sin(c + d*x)**3*i - 105*cos(c + d*x)*sin(c + d*x)*i + 64*sin(c + d*x)**6
 - 224*sin(c + d*x)**4 + 280*sin(c + d*x)**2 - 105))/(105*cos(c + d*x)*d*(
sin(c + d*x)**6 - 3*sin(c + d*x)**4 + 3*sin(c + d*x)**2 - 1))
```

3.21 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx=-\frac{i(a+ia\tan(c+dx))^4}{2a^2d}+\frac{i(a+ia\tan(c+dx))^5}{5a^3d}$$

output
```
-1/2*I*(a+I*a*tan(d*x+c))^4/a^2/d+1/5*I*(a+I*a*tan(d*x+c))^5/a^3/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx=-\frac{a^2(-i+\tan(c+dx))^4(3i+2\tan(c+dx))}{10d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/10*(a^2*(-I + Tan[c + d*x])^4*(3*I + 2*Tan[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4(a+ia\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))(i\tan(c+dx)a+a)^3d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(2a(i\tan(c+dx)a+a)^3-(i\tan(c+dx)a+a)^4\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{1}{2}a(a+ia\tan(c+dx))^4-\frac{1}{5}(a+ia\tan(c+dx))^5\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*((a*(a + I*a*Tan[c + d*x])^4)/2 - (a + I*a*Tan[c + d*x])^5/5))/(a^3*
d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 7.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

method	result	size
risch	$\frac{8 i a^{2} \left(10 \,{\mathrm e}^{6 i \left(d x +c \right)}+10 \,{\mathrm e}^{4 i \left(d x +c \right)}+5 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{5 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{5}}$	58
derivativedivides	$\frac{-a^{2} \left(\frac{\sin \left(d x +c \right)^{3}}{5 \cos \left(d x +c \right)^{5}}+\frac{2 \sin \left(d x +c \right)^{3}}{15 \cos \left(d x +c \right)^{3}}\right)+\frac{i a^{2}}{2 \cos \left(d x +c \right)^{4}}-a^{2} \left(-\frac{2}{3}-\frac{\sec \left(d x +c \right)^{2}}{3}\right) \tan \left(d x +c \right)}{d}$	85
default	$\frac{-a^{2} \left(\frac{\sin \left(d x +c \right)^{3}}{5 \cos \left(d x +c \right)^{5}}+\frac{2 \sin \left(d x +c \right)^{3}}{15 \cos \left(d x +c \right)^{3}}\right)+\frac{i a^{2}}{2 \cos \left(d x +c \right)^{4}}-a^{2} \left(-\frac{2}{3}-\frac{\sec \left(d x +c \right)^{2}}{3}\right) \tan \left(d x +c \right)}{d}$	85

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
8/5*I*a^2*(10*exp(6*I*(d*x+c))+10*exp(4*I*(d*x+c))+5*exp(2*I*(d*x+c))+1)/d
/(exp(2*I*(d*x+c))+1)^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(43) = 86$.

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{8\left(-10i\,a^2e^{(6i\,dx+6i\,c)} - 10i\,a^2e^{(4i\,dx+4i\,c)} - 5i\,a^2e^{(2i\,dx+2i\,c)} - i\,a^2\right)}{5\left(de^{(10i\,dx+10i\,c)} + 5\,de^{(8i\,dx+8i\,c)} + 10\,de^{(6i\,dx+6i\,c)} + 10\,de^{(4i\,dx+4i\,c)} + 5\,de^{(2i\,dx+2i\,c)} + d\right)}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-8/5*(-10*I*a^2*e^(6*I*d*x + 6*I*c) - 10*I*a^2*e^(4*I*d*x + 4*I*c) - 5*I*a
^2*e^(2*I*d*x + 2*I*c) - I*a^2)/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x
+ 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*
I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2(c+dx)\sec^4(c+dx)\,dx + \int \left(-2i\tan(c+dx)\sec^4(c+dx)\right)dx + \int \left(-\sec^4(c+dx)\right)dx\right)$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**4, x) + Integral(-2*I*tan(c
+ d*x)*sec(c + d*x)**4, x) + Integral(-sec(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2\,a^2\tan\left(dx+c\right)^5 - 5i\,a^2\tan\left(dx+c\right)^4 - 10i\,a^2\tan\left(dx+c\right)^2 - 10\,a^2\tan\left(dx+c\right)}{10\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/10*(2*a^2*tan(d*x + c)^5 - 5*I*a^2*tan(d*x + c)^4 - 10*I*a^2*tan(d*x +
c)^2 - 10*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2\,a^2\tan\left(dx+c\right)^5 - 5i\,a^2\tan\left(dx+c\right)^4 - 10i\,a^2\tan\left(dx+c\right)^2 - 10\,a^2\tan\left(dx+c\right)}{10\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/10*(2*a^2*tan(d*x + c)^5 - 5*I*a^2*tan(d*x + c)^4 - 10*I*a^2*tan(d*x +
c)^2 - 10*a^2*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{-\frac{a^2\tan(c+d\,x)^5}{5}+\frac{a^2\tan(c+d\,x)^4\,1\mathrm{i}}{2}+a^2\tan(c+d\,x)^2\,1\mathrm{i}+a^2\tan(c+d\,x)}{d}$$

input
```
int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^4,x)
```

output
```
(a^2*tan(c + d*x) + a^2*tan(c + d*x)^2*1i + (a^2*tan(c + d*x)^4*1i)/2 - (a
^2*tan(c + d*x)^5)/5)/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.78

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{\sin(dx+c)\,a^2\left(-5\cos(dx+c)\sin(dx+c)^3\,i+10\cos(dx+c)\sin(dx+c)\,i+8\sin(dx+c)^4-20\sin(d\right.}{10\cos(dx+c)\,d\left(\sin(dx+c)^4-2\sin(dx+c)^2+1\right)}$$

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(sin(c + d*x)*a**2*( - 5*cos(c + d*x)*sin(c + d*x)**3*i + 10*cos(c + d*x)*
sin(c + d*x)*i + 8*sin(c + d*x)**4 - 20*sin(c + d*x)**2 + 10))/(10*cos(c +
 d*x)*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.22 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{i(a+ia\tan(c+dx))^3}{3ad}$$

output
```
-1/3*I*(a+I*a*tan(d*x+c))^3/a/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\tan(c+dx)}{d} + \frac{ia^2\tan^2(c+dx)}{d} - \frac{a^2\tan^3(c+dx)}{3d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(a^2*Tan[c + d*x])/d + (I*a^2*Tan[c + d*x]^2)/d - (a^2*Tan[c + d*x]^3)/(3*
d)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+ia\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int(i\tan(c+dx)a+a)^2d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$-\frac{i(a+ia\tan(c+dx))^3}{3ad}$$

input
```
Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-1/3*I)*(a + I*a*Tan[c + d*x])^3)/(a*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

method	result	size
risch	$\frac{8ia^2\left(3\,\mathrm{e}^{4i(dx+c)}+3\,\mathrm{e}^{2i(dx+c)}+1\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	47
derivativedivides	$\frac{-\frac{a^2\sin(dx+c)^3}{3\cos(dx+c)^3}+\frac{ia^2}{\cos(dx+c)^2}+a^2\tan(dx+c)}{d}$	51
default	$\frac{-\frac{a^2\sin(dx+c)^3}{3\cos(dx+c)^3}+\frac{ia^2}{\cos(dx+c)^2}+a^2\tan(dx+c)}{d}$	51

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
8/3*I*a^2*(3*exp(4*I*(d*x+c))+3*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)
^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(21) = 42.

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{8\left(-3i\,a^2e^{(4i\,dx+4i\,c)}-3i\,a^2e^{(2i\,dx+2i\,c)}-i\,a^2\right)}{3\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-8/3*(-3*I*a^2*e^(4*I*d*x + 4*I*c) - 3*I*a^2*e^(2*I*d*x + 2*I*c) - I*a^2)/
(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c)
 + d)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2(c+dx)\sec^2(c+dx)\,dx + \int \left(-2i\tan(c+dx)\sec^2(c+dx)\right)\,dx + \int \left(-\sec^2(c+dx)\right)\,dx\right)$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**2, x) + Integral(-2*I*tan(c
+ d*x)*sec(c + d*x)**2, x) + Integral(-sec(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{i\,(i\,a\tan(dx+c)+a)^3}{3\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/3*I*(I*a*tan(d*x + c) + a)^3/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2\tan\left(dx+c\right)^3-3i\,a^2\tan\left(dx+c\right)^2-3\,a^2\tan\left(dx+c\right)}{3\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/3*(a^2*tan(d*x + c)^3 - 3*I*a^2*tan(d*x + c)^2 - 3*a^2*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\tan(c+d\,x)\,\left(-\tan(c+d\,x)^2+\tan(c+d\,x)\,3\mathrm{i}+3\right)}{3\,d}$$

input
```
int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^2,x)
```

output
```
(a^2*tan(c + d*x)*(tan(c + d*x)*3i - tan(c + d*x)^2 + 3))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.26

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{\sin\left(dx+c\right)a^2\left(-3\cos\left(dx+c\right)\sin\left(dx+c\right)i+4\sin\left(dx+c\right)^2-3\right)}{3\cos\left(dx+c\right)d\left(\sin\left(dx+c\right)^2-1\right)}$$

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(sin(c + d*x)*a**2*( - 3*cos(c + d*x)*sin(c + d*x)*i + 4*sin(c + d*x)**2 -
 3))/(3*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.23 $\int (a + ia\tan(c + dx))^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int (a + ia\tan(c + dx))^2 \, dx = 2a^2x - \frac{2ia^2\log(\cos(c + dx))}{d} - \frac{a^2\tan(c + dx)}{d}$$

output
```
2*a^2*x-2*I*a^2*ln(cos(d*x+c))/d-a^2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int (a + ia\tan(c + dx))^2 \, dx = -\frac{ia(-2a\log(i + \tan(c + dx)) - ia\tan(c + dx))}{d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*a*(-2*a*Log[I + Tan[c + d*x]] - I*a*Tan[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3958}$$

$$2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x$$

$$\Big\downarrow \text{3042}$$

$$2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x$$

$$\Big\downarrow \text{3956}$$

$$-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x$$

input
```
Int[(a + I*a*Tan[c + d*x])^2,x]
```

output
```
2*a^2*x - ((2*I)*a^2*Log[Cos[c + d*x]])/d - (a^2*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (warning: unable to verify)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{a^2\left(-\tan(dx+c)+i\ln\left(1+\tan(dx+c)^2\right)+2\arctan(\tan(dx+c))\right)}{d}$	40
default	$\frac{a^2\left(-\tan(dx+c)+i\ln\left(1+\tan(dx+c)^2\right)+2\arctan(\tan(dx+c))\right)}{d}$	40
parallelrisch	$\frac{ia^2\ln\left(1+\tan(dx+c)^2\right)+2a^2xd-a^2\tan(dx+c)}{d}$	41
norman	$2a^2x-\frac{a^2\tan(dx+c)}{d}+\frac{ia^2\ln\left(1+\tan(dx+c)^2\right)}{d}$	42
parts	$a^2x+\frac{ia^2\ln\left(1+\tan(dx+c)^2\right)}{d}-\frac{a^2(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}$	51
risch	$-\frac{4a^2c}{d}-\frac{2ia^2}{d\left(e^{2i(dx+c)}+1\right)}-\frac{2ia^2\ln\left(e^{2i(dx+c)}+1\right)}{d}$	54

input
```
int((a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*a^2*(-tan(d*x+c)+I*ln(1+tan(d*x+c)^2)+2*arctan(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int (a+ia\tan(c+dx))^2\,dx = -\frac{2\left(i\,a^2+\left(i\,a^2e^{(2i\,dx+2i\,c)}+i\,a^2\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{de^{(2i\,dx+2i\,c)}+d}$$

input
```
integrate((a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2*(I*a^2 + (I*a^2*e^(2*I*d*x + 2*I*c) + I*a^2)*log(e^(2*I*d*x + 2*I*c) +
1))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int (a+ia\tan(c+dx))^2\,dx = -\frac{2ia^2}{de^{2ic}e^{2idx}+d}-\frac{2ia^2\log\left(e^{2idx}+e^{-2ic}\right)}{d}$$

input
```
integrate((a+I*a*tan(d*x+c))**2,x)
```

output
```
-2*I*a**2/(d*exp(2*I*c)*exp(2*I*d*x) + d) - 2*I*a**2*log(exp(2*I*d*x) + ex
p(-2*I*c))/d
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int (a+ia\tan(c+dx))^2\,dx = a^2x+\frac{(dx+c-\tan{(dx+c)})a^2}{d}+\frac{2i\,a^2\log{(\sec{(dx+c)})}}{d}$$

input
```
integrate((a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
a^2*x + (d*x + c - tan(d*x + c))*a^2/d + 2*I*a^2*log(sec(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int (a+ia\tan(c+dx))^2\,dx=\frac{2i\,a^2\log(\tan(dx+c)+i)}{d}-\frac{a^2\tan(dx+c)}{d}$$

input
```
integrate((a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
2*I*a^2*log(tan(d*x + c) + I)/d - a^2*tan(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int (a+ia\tan(c+dx))^2\,dx=\frac{a^2\,(-\tan(c+d\,x)+\ln(\tan(c+d\,x)+1\mathrm{i})\,2\mathrm{i})}{d}$$

input
```
int((a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(a^2*(log(tan(c + d*x) + 1i)*2i - tan(c + d*x)))/d
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int (a+ia\tan(c+dx))^2\,dx=\frac{a^2\left(\log\left(\tan(dx+c)^2+1\right)i-\tan(dx+c)+2dx\right)}{d}$$

input
```
int((a+I*a*tan(d*x+c))^2,x)
```

output
```
(a**2*(log(tan(c + d*x)**2 + 1)*i - tan(c + d*x) + 2*d*x))/d
```

3.24 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 25

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{ia^3}{d(a-ia\tan(c+dx))}$$

output
```
-I*a^3/d/(a-I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{ia^2(\cos(c+dx)+i\sin(c+dx))^2}{2d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-1/2*I)*a^2*(Cos[c + d*x] + I*Sin[c + d*x])^2)/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \ 3968$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \ 17$$

$$-\frac{ia^3}{d(a-ia\tan(c+dx))}$$

input
```
Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*a^3)/(d*(a - I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
risch	$-\frac{i a^{2} {\mathrm e}^{2 i \left(d x +c \right)}}{2 d}$	19
orering	$-\frac{i \cos \left(d x +c \right)^{2} \left(a +i a \tan \left(d x +c \right)\right)^{2}}{2 d}$	29
derivativedivides	$\frac{-a^{2} \left(-\frac{\cos \left(d x +c \right) \sin \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)-i a^{2} \cos \left(d x +c \right)^{2}+a^{2} \left(\frac{\cos \left(d x +c \right) \sin \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}$	73
default	$\frac{-a^{2} \left(-\frac{\cos \left(d x +c \right) \sin \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)-i a^{2} \cos \left(d x +c \right)^{2}+a^{2} \left(\frac{\cos \left(d x +c \right) \sin \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}$	73

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/d*a^2*exp(2*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx=-\frac{i\,a^2e^{(2i\,dx+2i\,c)}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*I*a^2*e^(2*I*d*x + 2*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = \begin{cases} -\frac{ia^2e^{2ic}e^{2idx}}{2d} & \text{for } d \neq 0 \\ a^2xe^{2ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise((-I*a**2*exp(2*I*c)*exp(2*I*d*x)/(2*d), Ne(d, 0)), (a**2*x*exp(2
*I*c), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\tan(dx+c) - i\,a^2}{\left(\tan(dx+c)^2+1\right)d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
(a^2*tan(d*x + c) - I*a^2)/((tan(d*x + c)^2 + 1)*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2}{d(\tan(dx+c)+i)}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
a^2/(d*(tan(d*x + c) + I))
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2}{d\,(\tan(c+d\,x)+1\mathrm{i})}$$

input
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
a^2/(d*(tan(c + d*x) + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{\sin(dx+c)\,a^2(\cos(dx+c)+\sin(dx+c)\,i)}{d}$$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(sin(c + d*x)*a**2*(cos(c + d*x) + sin(c + d*x)*i))/d
```

3.25 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2x}{4} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2} - \frac{ia^5}{4d\left(a^3-ia^3\tan(c+dx)\right)}$$

output
```
1/4*a^2*x-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2-1/4*I*a^5/d/(a^3-I*a^3*tan(d*x+
c))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2(2i+\tan(c+dx)+\arctan(\tan(c+dx))(i+\tan(c+dx))^2)}{4d(i+\tan(c+dx))^2}$$

input
```
Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(a^2*(2*I + Tan[c + d*x] + ArcTan[Tan[c + d*x]]*(I + Tan[c + d*x])^2))/(4*
d*(I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \; 54$$

$$-\frac{ia^5\int \left(\frac{1}{2(a-ia\tan(c+dx))^3a}+\frac{1}{4(\tan^2(c+dx)a^2+a^2)a^2}+\frac{1}{4(a-ia\tan(c+dx))^2a^2}\right)d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{ia^5\left(\frac{i\arctan(\tan(c+dx))}{4a^3}+\frac{1}{4a^2(a-ia\tan(c+dx))}+\frac{1}{4a(a-ia\tan(c+dx))^2}\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*a^5*(((I/4)*ArcTan[Tan[c + d*x]])/a^3 + 1/(4*a*(a - I*a*Tan[c + d*x]
)^2) + 1/(4*a^2*(a - I*a*Tan[c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 5.94 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

method	result
risch	$\frac{a^{2} x}{4}-\frac{i a^{2} {\mathrm e}^{4 i \left(d x +c \right)}}{16 d}-\frac{i a^{2} {\mathrm e}^{2 i \left(d x +c \right)}}{4 d}$
derivativedivides	$\frac{-a^{2} \left(-\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)^{3}}{4}+\frac{\cos \left(d x +c \right) \sin \left(d x +c \right)}{8}+\frac{d x}{8}+\frac{c}{8}\right)-\frac{i a^{2} \cos \left(d x +c \right)^{4}}{2}+a^{2} \left(\frac{\left(\cos \left(d x +c \right)^{3}+\frac{3 \cos \left(d x +c \right)}{2}\right) \sin \left(d x +c \right)}{4}\right.}{d}$
default	$\frac{-a^{2} \left(-\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)^{3}}{4}+\frac{\cos \left(d x +c \right) \sin \left(d x +c \right)}{8}+\frac{d x}{8}+\frac{c}{8}\right)-\frac{i a^{2} \cos \left(d x +c \right)^{4}}{2}+a^{2} \left(\frac{\left(\cos \left(d x +c \right)^{3}+\frac{3 \cos \left(d x +c \right)}{2}\right) \sin \left(d x +c \right)}{4}\right.}{d}$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x-1/16*I/d*a^2*exp(4*I*(d*x+c))-1/4*I/d*a^2*exp(2*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{4\,a^2dx - i\,a^2e^{(4i\,dx+4i\,c)} - 4i\,a^2e^{(2i\,dx+2i\,c)}}{16\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/16*(4*a^2*d*x - I*a^2*e^(4*I*d*x + 4*I*c) - 4*I*a^2*e^(2*I*d*x + 2*I*c))
/d
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.30

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2x}{4} + \begin{cases} \frac{-4ia^2de^{4ic}e^{4idx}-16ia^2de^{2ic}e^{2idx}}{64d^2} & \text{for } d^2 \neq 0 \\ x\left(\frac{a^2e^{4ic}}{4} + \frac{a^2e^{2ic}}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**2,x)
```

output
```
a**2*x/4 + Piecewise(((-4*I*a**2*d*exp(4*I*c)*exp(4*I*d*x) - 16*I*a**2*d*e
xp(2*I*c)*exp(2*I*d*x))/(64*d**2), Ne(d**2, 0)), (x*(a**2*exp(4*I*c)/4 + a
**2*exp(2*I*c)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{(dx+c)a^2 + \frac{a^2\tan(dx+c)^3+3\,a^2\tan(dx+c)-2i\,a^2}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{4\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*((d*x + c)*a^2 + (a^2*tan(d*x + c)^3 + 3*a^2*tan(d*x + c) - 2*I*a^2)/(
tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{1}{8}a^2\left(\frac{i\log(\tan(dx+c)+i)}{d}-\frac{i\log(\tan(dx+c)-i)}{d}+\frac{2(\tan(dx+c)+2i)}{d(\tan(dx+c)+i)^2}\right)$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/8*a^2*(I*log(tan(d*x + c) + I)/d - I*log(tan(d*x + c) - I)/d + 2*(tan(d*
x + c) + 2*I)/(d*(tan(d*x + c) + I)^2))
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx=\frac{a^2\,x}{4}+\frac{\frac{a^2\tan(c+d\,x)}{4}+\frac{a^2\,1\mathrm{i}}{2}}{d\,\left(\tan(c+d\,x)^2+\tan(c+d\,x)\,2\mathrm{i}-1\right)}$$

input
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(a^2*x)/4 + ((a^2*tan(c + d*x))/4 + (a^2*1i)/2)/(d*(tan(c + d*x)*2i + tan(
c + d*x)^2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{a^2\left(-2\cos\left(dx+c\right)\sin\left(dx+c\right)^3+3\cos\left(dx+c\right)\sin\left(dx+c\right)-2\sin\left(dx+c\right)^4 i+4\sin\left(dx+c\right)^2 i+dx\right)}{4d}$$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(a**2*( - 2*cos(c + d*x)*sin(c + d*x)**3 + 3*cos(c + d*x)*sin(c + d*x) - 2
*sin(c + d*x)**4*i + 4*sin(c + d*x)**2*i + d*x))/(4*d)
```

3.26 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 125

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2x}{4} - \frac{ia^5}{12d(a-ia\tan(c+dx))^3} - \frac{ia^4}{8d(a-ia\tan(c+dx))^2} - \frac{3ia^7}{16d\,(a^5-ia^5\tan(c+dx))} + \frac{ia^7}{16d\,(a^5+ia^5\tan(c+dx))}$$

output
```
1/4*a^2*x-1/12*I*a^5/d/(a-I*a*tan(d*x+c))^3-1/8*I*a^4/d/(a-I*a*tan(d*x+c))
^2-3/16*I*a^7/d/(a^5-I*a^5*tan(d*x+c))+1/16*I*a^7/d/(a^5+I*a^5*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2(4i-\tan(c+dx)+6i\tan^2(c+dx)+3\tan^3(c+dx)+3\arctan(\tan(c+dx))(-i+\tan(c+dx))(i+}{12d(-i+\tan(c+dx))(i+\tan(c+dx))^3}$$

input
```
Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(a^2*(4*I - Tan[c + d*x] + (6*I)*Tan[c + d*x]^2 + 3*Tan[c + d*x]^3 + 3*Arc
Tan[Tan[c + d*x]]*(-I + Tan[c + d*x])*(I + Tan[c + d*x])^3))/(12*d*(-I + T
an[c + d*x])*(I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^7\int \frac{1}{(a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^7\int\left(\frac{3}{16a^4(a-ia\tan(c+dx))^2}+\frac{1}{16a^4(i\tan(c+dx)a+a)^2}+\frac{1}{4a^3(a-ia\tan(c+dx))^3}+\frac{1}{4a^2(a-ia\tan(c+dx))^4}+\frac{1}{4a^4(\tan^2(c+dx)a^2+a}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^7\left(\frac{i\arctan(\tan(c+dx))}{4a^5}+\frac{3}{16a^4(a-ia\tan(c+dx))}-\frac{1}{16a^4(a+ia\tan(c+dx))}+\frac{1}{8a^3(a-ia\tan(c+dx))^2}+\frac{1}{12a^2(a-ia\tan(c+dx))^3}\right)}{d}$$

input
```
Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*a^7*(((I/4)*ArcTan[Tan[c + d*x]])/a^5 + 1/(12*a^2*(a - I*a*Tan[c + d
*x])^3) + 1/(8*a^3*(a - I*a*Tan[c + d*x])^2) + 3/(16*a^4*(a - I*a*Tan[c +
d*x])) - 1/(16*a^4*(a + I*a*Tan[c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 24.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.63

method	result
risch	$\frac{a^2x}{4}-\frac{ia^2 e^{6i(dx+c)}}{96d}-\frac{ia^2 e^{4i(dx+c)}}{16d}-\frac{5ia^2\cos(2dx+2c)}{32d}+\frac{7a^2\sin(2dx+2c)}{32d}$
derivativedivides	$\frac{-a^2\left(-\frac{\sin(dx+c)\cos(dx+c)^5}{6}+\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{ia^2\cos(dx+c)^6}{3}+a^2\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos}{}\right.}{}\right.}{d}$
default	$\frac{-a^2\left(-\frac{\sin(dx+c)\cos(dx+c)^5}{6}+\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{ia^2\cos(dx+c)^6}{3}+a^2\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos}{}\right.}{}\right.}{d}$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x-1/96*I/d*a^2*exp(6*I*(d*x+c))-1/16*I/d*a^2*exp(4*I*(d*x+c))-5/32
*I/d*a^2*cos(2*d*x+2*c)+7/32/d*a^2*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.62

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{\left(24\,a^2dxe^{(2i\,dx+2i\,c)}-i\,a^2e^{(8i\,dx+8i\,c)}-6i\,a^2e^{(6i\,dx+6i\,c)}-18i\,a^2e^{(4i\,dx+4i\,c)}+3i\,a^2\right)e^{(-2i\,dx-2i\,c)}}{96\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/96*(24*a^2*d*x*e^(2*I*d*x + 2*I*c) - I*a^2*e^(8*I*d*x + 8*I*c) - 6*I*a^2
*e^(6*I*d*x + 6*I*c) - 18*I*a^2*e^(4*I*d*x + 4*I*c) + 3*I*a^2)*e^(-2*I*d*x
 - 2*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.48

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2x}{4} + \begin{cases} \frac{(-8192ia^2d^3e^{8ic}e^{6idx}-49152ia^2d^3e^{6ic}e^{4idx}-147456ia^2d^3e^{4ic}e^{2idx}+24576ia^2d^3e^{-2idx})e^{-2ic}}{786432d^4} & \text{for } d^4e^{2ic} \neq 0 \\ x\left(-\frac{a^2}{4}+\frac{(a^2e^{8ic}+4a^2e^{6ic}+6a^2e^{4ic}+4a^2e^{2ic}+a^2)e^{-2ic}}{16}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**2,x)
```

output
```
a**2*x/4 + Piecewise(((-8192*I*a**2*d**3*exp(8*I*c)*exp(6*I*d*x) - 49152*I
*a**2*d**3*exp(6*I*c)*exp(4*I*d*x) - 147456*I*a**2*d**3*exp(4*I*c)*exp(2*I
*d*x) + 24576*I*a**2*d**3*exp(-2*I*d*x))*exp(-2*I*c)/(786432*d**4), Ne(d**
4*exp(2*I*c), 0)), (x*(-a**2/4 + (a**2*exp(8*I*c) + 4*a**2*exp(6*I*c) + 6*
a**2*exp(4*I*c) + 4*a**2*exp(2*I*c) + a**2)*exp(-2*I*c)/16), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.74

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{3\,(dx+c)a^2 + \frac{3\,a^2\tan(dx+c)^5+8\,a^2\tan(dx+c)^3+9\,a^2\tan(dx+c)-4i\,a^2}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{12\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/12*(3*(d*x + c)*a^2 + (3*a^2*tan(d*x + c)^5 + 8*a^2*tan(d*x + c)^3 + 9*a
^2*tan(d*x + c) - 4*I*a^2)/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x
+ c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.71

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{1}{24}\,a^2\left(\frac{3i\,\log\left(\tan\left(dx+c\right)+i\right)}{d}-\frac{3i\,\log\left(\tan\left(dx+c\right)-i\right)}{d}+\frac{2\left(3\,\tan\left(dx+c\right)^3+6i\,\tan\left(dx+c\right)^2-\text{ta}\right.}{d(\tan\left(dx+c\right)+i)^3(\tan\left(dx+c\right.}\right.$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*a^2*(3*I*log(tan(d*x + c) + I)/d - 3*I*log(tan(d*x + c) - I)/d + 2*(3
*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - tan(d*x + c) + 4*I)/(d*(tan(d*x + c
) + I)^3*(tan(d*x + c) - I)))
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{a^2\,x}{4}+\frac{\frac{a^2\,\tan(c+d\,x)^3}{4}+\frac{a^2\,\tan(c+d\,x)^2\,1\mathrm{i}}{2}-\frac{a^2\,\tan(c+d\,x)}{12}+\frac{a^2\,1\mathrm{i}}{3}}{d\,\left(\tan\left(c+d\,x\right)^4+\tan\left(c+d\,x\right)^3\,2\mathrm{i}+\tan\left(c+d\,x\right)\,2\mathrm{i}-1\right)}$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(a^2*x)/4 + ((a^2*1i)/3 - (a^2*tan(c + d*x))/12 + (a^2*tan(c + d*x)^2*1i)/
2 + (a^2*tan(c + d*x)^3)/4)/(d*(tan(c + d*x)*2i + tan(c + d*x)^3*2i + tan(
c + d*x)^4 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.74

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{a^2\left(4\cos\left(dx+c\right)\sin\left(dx+c\right)^5-10\cos\left(dx+c\right)\sin\left(dx+c\right)^3+9\cos\left(dx+c\right)\sin\left(dx+c\right)+4\sin\left(dx+c\right.\right.}{12d}$$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(a**2*(4*cos(c + d*x)*sin(c + d*x)**5 - 10*cos(c + d*x)*sin(c + d*x)**3 +
9*cos(c + d*x)*sin(c + d*x) + 4*sin(c + d*x)**6*i - 12*sin(c + d*x)**4*i +
 12*sin(c + d*x)**2*i + 3*d*x))/(12*d)
```

3.27 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 179

$$\begin{aligned}\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx =& \frac{15a^2x}{64} - \frac{ia^6}{32d(a-ia\tan(c+dx))^4} \\ &- \frac{ia^5}{16d(a-ia\tan(c+dx))^3} \\ &- \frac{3ia^4}{32d(a-ia\tan(c+dx))^2} \\ &+ \frac{ia^4}{64d(a+ia\tan(c+dx))^2} \\ &- \frac{5ia^9}{32d\,(a^7-ia^7\tan(c+dx))} \\ &+ \frac{5ia^9}{64d\,(a^7+ia^7\tan(c+dx))}\end{aligned}$$

output
```
15/64*a^2*x-1/32*I*a^6/d/(a-I*a*tan(d*x+c))^4-1/16*I*a^5/d/(a-I*a*tan(d*x+
c))^3-3/32*I*a^4/d/(a-I*a*tan(d*x+c))^2+1/64*I*a^4/d/(a+I*a*tan(d*x+c))^2-
5/32*I*a^9/d/(a^7-I*a^7*tan(d*x+c))+5/64*I*a^9/d/(a^7+I*a^7*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.79

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx =$$
$$-\frac{a^2\sec^6(c+dx)(-80i-65i\cos(2(c+dx))+16i\cos(4(c+dx))+i\cos(6(c+dx))+120\arctan(\tan(c}{512d(-i+\tan(c+dx}$$

input
```
Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/512*(a^2*Sec[c + d*x]^6*(-80*I - (65*I)*Cos[2*(c + d*x)] + (16*I)*Cos[4
*(c + d*x)] + I*Cos[6*(c + d*x)] + 120*ArcTan[Tan[c + d*x]]*(Cos[2*(c + d*
x)] - I*Sin[2*(c + d*x)]) - 5*Sin[2*(c + d*x)] + 32*Sin[4*(c + d*x)] + 3*S
in[6*(c + d*x)]))/(d*(-I + Tan[c + d*x])^2*(I + Tan[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sec(c+dx)^8}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^9\int \frac{1}{(a-ia\tan(c+dx))^5(i\tan(c+dx)a+a)^3}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^9 \int \left(\frac{5}{32a^6(a-ia\tan(c+dx))^2} + \frac{5}{64a^6(i\tan(c+dx)a+a)^2} + \frac{3}{16a^5(a-ia\tan(c+dx))^3} + \frac{1}{32a^5(i\tan(c+dx)a+a)^3} + \frac{3}{16a^4(a-ia\tan(c+d}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^9 \left(\frac{15i\arctan(\tan(c+dx))}{64a^7} + \frac{5}{32a^6(a-ia\tan(c+dx))} - \frac{5}{64a^6(a+ia\tan(c+dx))} + \frac{3}{32a^5(a-ia\tan(c+dx))^2} - \frac{1}{64a^5(a+ia\tan(c+dx))^2} +\right.}{d}$$

input
```
Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*a^9*((((15*I)/64)*ArcTan[Tan[c + d*x]])/a^7 + 1/(32*a^3*(a - I*a*Tan
[c + d*x])^4) + 1/(16*a^4*(a - I*a*Tan[c + d*x])^3) + 3/(32*a^5*(a - I*a*T
an[c + d*x])^2) + 5/(32*a^6*(a - I*a*Tan[c + d*x])) - 1/(64*a^5*(a + I*a*T
an[c + d*x])^2) - 5/(64*a^6*(a + I*a*Tan[c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 75.84 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.64

method	result
risch	$\frac{15a^2x}{64} - \frac{ia^2 e^{8i(dx+c)}}{512d} - \frac{ia^2 e^{6i(dx+c)}}{64d} - \frac{7ia^2\cos(4dx+4c)}{128d} + \frac{a^2\sin(4dx+4c)}{16d} - \frac{7ia^2\cos(2dx+2c)}{64d} + \frac{13a^2\sin}{6}$
derivativedivides	$\frac{-a^2\left(-\frac{\sin(dx+c)\cos(dx+c)^7}{8}+\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)-\frac{ia^2\cos(dx+c)^8}{4}+a^2\left(\right.}{d}$
default	$\frac{-a^2\left(-\frac{\sin(dx+c)\cos(dx+c)^7}{8}+\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)-\frac{ia^2\cos(dx+c)^8}{4}+a^2\left(\right.}{d}$

input
```
int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
15/64*a^2*x-1/512*I/d*a^2*exp(8*I*(d*x+c))-1/64*I/d*a^2*exp(6*I*(d*x+c))-7
/128*I/d*a^2*cos(4*d*x+4*c)+1/16/d*a^2*sin(4*d*x+4*c)-7/64*I/d*a^2*cos(2*d
*x+2*c)+13/64/d*a^2*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.59

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{\left(120\,a^2dxe^{(4i\,dx+4i\,c)} - i\,a^2e^{(12i\,dx+12i\,c)} - 8i\,a^2e^{(10i\,dx+10i\,c)} - 30i\,a^2e^{(8i\,dx+8i\,c)} - 80i\,a^2e^{(6i\,dx+6i\,c)} + 24i\,a^2e\right.}{512\,d}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/512*(120*a^2*d*x*e^(4*I*d*x + 4*I*c) - I*a^2*e^(12*I*d*x + 12*I*c) - 8*I
*a^2*e^(10*I*d*x + 10*I*c) - 30*I*a^2*e^(8*I*d*x + 8*I*c) - 80*I*a^2*e^(6*
I*d*x + 6*I*c) + 24*I*a^2*e^(2*I*d*x + 2*I*c) + 2*I*a^2)*e^(-4*I*d*x - 4*I
*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.51

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{15a^2x}{64} + \begin{cases} \frac{(-8589934592ia^2d^5e^{14ic}e^{8idx} - 68719476736ia^2d^5e^{12ic}e^{6idx} - 257698037760ia^2d^5e^{10ic}e^{4idx} - 687194767360ia^2d^5e^{8ic}e^{2idx} + 2061584302}{4398046511104d^6} \\ x\left(-\frac{15a^2}{64} + \frac{(a^2e^{12ic}+6a^2e^{10ic}+15a^2e^{8ic}+20a^2e^{6ic}+15a^2e^{4ic}+6a^2e^{2ic}+a^2)e^{-4ic}}{64}\right) \end{cases}$$

input
```
integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**2,x)
```

output
```
15*a**2*x/64 + Piecewise(((-8589934592*I*a**2*d**5*exp(14*I*c)*exp(8*I*d*x
) - 68719476736*I*a**2*d**5*exp(12*I*c)*exp(6*I*d*x) - 257698037760*I*a**2
*d**5*exp(10*I*c)*exp(4*I*d*x) - 687194767360*I*a**2*d**5*exp(8*I*c)*exp(2
*I*d*x) + 206158430208*I*a**2*d**5*exp(4*I*c)*exp(-2*I*d*x) + 17179869184*
I*a**2*d**5*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*c)/(4398046511104*d**6), Ne
(d**6*exp(6*I*c), 0)), (x*(-15*a**2/64 + (a**2*exp(12*I*c) + 6*a**2*exp(10
*I*c) + 15*a**2*exp(8*I*c) + 20*a**2*exp(6*I*c) + 15*a**2*exp(4*I*c) + 6*a
**2*exp(2*I*c) + a**2)*exp(-4*I*c)/64), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.64

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{15\,(dx+c)a^2 + \frac{15\,a^2\tan(dx+c)^7+55\,a^2\tan(dx+c)^5+73\,a^2\tan(dx+c)^3+49\,a^2\tan(dx+c)-16i\,a^2}{\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1}}{64\,d}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/64*(15*(d*x + c)*a^2 + (15*a^2*tan(d*x + c)^7 + 55*a^2*tan(d*x + c)^5 +
73*a^2*tan(d*x + c)^3 + 49*a^2*tan(d*x + c) - 16*I*a^2)/(tan(d*x + c)^8 +
4*tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.61

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{1}{128}a^2\left(\frac{15i\log\left(\tan\left(dx+c\right)+i\right)}{d}-\frac{15i\log\left(\tan\left(dx+c\right)-i\right)}{d}+\frac{2\left(15\tan\left(dx+c\right)^5+30i\tan\left(dx+c\right)\right.}{d(\tan}\right.$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/128*a^2*(15*I*log(tan(d*x + c) + I)/d - 15*I*log(tan(d*x + c) - I)/d + 2
*(15*tan(d*x + c)^5 + 30*I*tan(d*x + c)^4 + 10*tan(d*x + c)^3 + 50*I*tan(d
*x + c)^2 - 17*tan(d*x + c) + 16*I)/(d*(tan(d*x + c) + I)^4*(tan(d*x + c)
- I)^2))
```

Mupad [B] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.80

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx=\frac{15\,a^2\,x}{64}$$
$$+\frac{\frac{15\,a^2\,\tan(c+d\,x)^5}{64}+\frac{a^2\,\tan(c+d\,x)^4\,15\mathrm{i}}{32}+\frac{5\,a^2\,\tan(c+d\,x)^3}{32}+\frac{a^2\,\tan(c+d\,x)^2\,25\mathrm{i}}{32}-\frac{17\,a^2\,\tan(c+d\,x)}{64}+\frac{a^2\,1\mathrm{i}}{4}}{d\,\left(\tan\left(c+d\,x\right)^6+\tan\left(c+d\,x\right)^5\,2\mathrm{i}+\tan\left(c+d\,x\right)^4+\tan\left(c+d\,x\right)^3\,4\mathrm{i}-\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right.\right.}$$

input
```
int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(15*a^2*x)/64 + ((a^2*1i)/4 - (17*a^2*tan(c + d*x))/64 + (a^2*tan(c + d*x)
^2*25i)/32 + (5*a^2*tan(c + d*x)^3)/32 + (a^2*tan(c + d*x)^4*15i)/32 + (15
*a^2*tan(c + d*x)^5)/64)/(d*(tan(c + d*x)*2i - tan(c + d*x)^2 + tan(c + d*
x)^3*4i + tan(c + d*x)^4 + tan(c + d*x)^5*2i + tan(c + d*x)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.66

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{a^2\left(-16\cos\left(dx+c\right)\sin\left(dx+c\right)^7+56\cos\left(dx+c\right)\sin\left(dx+c\right)^5-74\cos\left(dx+c\right)\sin\left(dx+c\right)^3+49\cos\left(\right.\right.}{}$$

input
```
int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(a**2*( - 16*cos(c + d*x)*sin(c + d*x)**7 + 56*cos(c + d*x)*sin(c + d*x)**
5 - 74*cos(c + d*x)*sin(c + d*x)**3 + 49*cos(c + d*x)*sin(c + d*x) - 16*si
n(c + d*x)**8*i + 64*sin(c + d*x)**6*i - 96*sin(c + d*x)**4*i + 64*sin(c +
 d*x)**2*i + 15*d*x))/(64*d)
```

3.28 $\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 118

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{7a^2\text{arctanh}(\sin(c+dx))}{16d} + \frac{7ia^2\sec^5(c+dx)}{30d} + \frac{7a^2\sec(c+dx)\tan(c+dx)}{16d} + \frac{7a^2\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}$$

output
```
7/16*a^2*arctanh(sin(d*x+c))/d+7/30*I*a^2*sec(d*x+c)^5/d+7/16*a^2*sec(d*x+
c)*tan(d*x+c)/d+7/24*a^2*sec(d*x+c)^3*tan(d*x+c)/d+1/6*I*sec(d*x+c)^5*(a^2
+I*a^2*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{7a^2\text{arctanh}(\sin(c+dx))}{16d}+\frac{2ia^2\sec^5(c+dx)}{5d}+\frac{7a^2\sec(c+dx)\tan(c+dx)}{16d}+\frac{7a^2\sec^3(c+dx)\tan(c+dx)}{24d}-\frac{a^2\sec^5(c+dx)\tan(c+dx)}{6d}$$

input
```
Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(7*a^2*ArcTanh[Sin[c + d*x]])/(16*d) + (((2*I)/5)*a^2*Sec[c + d*x]^5)/d +
(7*a^2*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (7*a^2*Sec[c + d*x]^3*Tan[c + d
*x])/(24*d) - (a^2*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 3979, 3042, 3967, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(c+dx)^5(a+ia\tan(c+dx))^2dx$$

$$\Big\downarrow \text{ 3979}$$

$$\frac{7}{6}a\int \sec^5(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{7}{6}a\int \sec(c+dx)^5(i\tan(c+dx)a+a)dx+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}$$

$$\Big\downarrow \textcolor{blue}{3967}$$

$$\frac{7}{6}a\left(a\int \sec^5(c+dx)dx+\frac{ia\sec^5(c+dx)}{5d}\right)+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{7}{6}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx+\frac{ia\sec^5(c+dx)}{5d}\right)+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}$$

$$\Big\downarrow \textcolor{blue}{4255}$$

$$\frac{7}{6}a\left(a\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}\right)+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{7}{6}a\left(a\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}\right)+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}$$

$$\Big\downarrow \textcolor{blue}{4255}$$

$$\frac{7}{6}a\left(a\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}\right)+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{7}{6}a\left(a\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}\right)+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}$$

$$\Big\downarrow \textcolor{blue}{4257}$$

$$\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}+\frac{7}{6}a\left(a\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}\right)$$

input
```
Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((I/6)*Sec[c + d*x]^5*(a^2 + I*a^2*Tan[c + d*x]))/d + (7*a*(((I/5)*a*Sec[c
 + d*x]^5)/d + a*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTanh[Sin[c
+ d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4)))/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 15.06 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{i a^2 \left(105\,\mathrm{e}^{11 i (d x+c)}+595\,\mathrm{e}^{9 i (d x+c)}-1686\,\mathrm{e}^{7 i (d x+c)}-1386\,\mathrm{e}^{5 i (d x+c)}-595\,\mathrm{e}^{3 i (d x+c)}-105\,\mathrm{e}^{i (d x+c)}\right)}{120 d \left(\mathrm{e}^{2 i (d x+c)}+1\right)^6}-\frac{7 a^2 \ln\left(\mathrm{e}^{i (d x+\right.}}{16 d}$
derivativedivides	$\frac{-a^2 \left(\frac{\sin(d x+c)^3}{6 \cos(d x+c)^6}+\frac{\sin(d x+c)^3}{8 \cos(d x+c)^4}+\frac{\sin(d x+c)^3}{16 \cos(d x+c)^2}+\frac{\sin(d x+c)}{16}-\frac{\ln(\sec(d x+c)+\tan(d x+c))}{16}\right)+\frac{2 i a^2}{5 \cos(d x+c)^5}+a^2 \left(-\left(-\frac{\sec(d}{4}\right.\right.}{d}$
default	$\frac{-a^2 \left(\frac{\sin(d x+c)^3}{6 \cos(d x+c)^6}+\frac{\sin(d x+c)^3}{8 \cos(d x+c)^4}+\frac{\sin(d x+c)^3}{16 \cos(d x+c)^2}+\frac{\sin(d x+c)}{16}-\frac{\ln(\sec(d x+c)+\tan(d x+c))}{16}\right)+\frac{2 i a^2}{5 \cos(d x+c)^5}+a^2 \left(-\left(-\frac{\sec(d}{4}\right.\right.}{d}$

input
```
int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/120*I*a^2/d/(exp(2*I*(d*x+c))+1)^6*(105*exp(11*I*(d*x+c))+595*exp(9*I*(
d*x+c))-1686*exp(7*I*(d*x+c))-1386*exp(5*I*(d*x+c))-595*exp(3*I*(d*x+c))-1
05*exp(I*(d*x+c)))-7/16/d*a^2*ln(exp(I*(d*x+c))-I)+7/16/d*a^2*ln(exp(I*(d*
x+c))+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(102) = 204.

Time = 0.08 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.08

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{-210 i\,a^2 e^{(11 i\,dx+11 i\,c)}-1190 i\,a^2 e^{(9 i\,dx+9 i\,c)}+3372 i\,a^2 e^{(7 i\,dx+7 i\,c)}+2772 i\,a^2 e^{(5 i\,dx+5 i\,c)}+1190 i\,a^2 e^{(3 i\,dx+3 i\,c)}}{}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/240*(-210*I*a^2*e^(11*I*d*x + 11*I*c) - 1190*I*a^2*e^(9*I*d*x + 9*I*c) +
 3372*I*a^2*e^(7*I*d*x + 7*I*c) + 2772*I*a^2*e^(5*I*d*x + 5*I*c) + 1190*I*
a^2*e^(3*I*d*x + 3*I*c) + 210*I*a^2*e^(I*d*x + I*c) + 105*(a^2*e^(12*I*d*x
 + 12*I*c) + 6*a^2*e^(10*I*d*x + 10*I*c) + 15*a^2*e^(8*I*d*x + 8*I*c) + 20
*a^2*e^(6*I*d*x + 6*I*c) + 15*a^2*e^(4*I*d*x + 4*I*c) + 6*a^2*e^(2*I*d*x +
 2*I*c) + a^2)*log(e^(I*d*x + I*c) + I) - 105*(a^2*e^(12*I*d*x + 12*I*c) +
 6*a^2*e^(10*I*d*x + 10*I*c) + 15*a^2*e^(8*I*d*x + 8*I*c) + 20*a^2*e^(6*I*
d*x + 6*I*c) + 15*a^2*e^(4*I*d*x + 4*I*c) + 6*a^2*e^(2*I*d*x + 2*I*c) + a^
2)*log(e^(I*d*x + I*c) - I))/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x +
10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*
I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2{(c+dx)}\sec^5{(c+dx)}\,dx + \int \left(-2i\tan{(c+dx)}\sec^5{(c+dx)}\right)\,dx + \int \left(-\sec^5{(c+dx)}\right)\,dx\right)$$

input
```
integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**5, x) + Integral(-2*I*tan(c
+ d*x)*sec(c + d*x)**5, x) + Integral(-sec(c + d*x)**5, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.53

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{5\,a^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1} - 3\,\log\left(\sin\left(dx+c\right)+1\right) + 3\,\log\left(\sin\left(dx+c\right)-1\right)\right) + 30\,a^2}{480\,d}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/480*(5*a^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(s
in(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x +
 c) + 1) + 3*log(sin(d*x + c) - 1)) + 30*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(
d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) - 192*I*a^2/cos(d*x + c)^5)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(102) = 204$.

Time = 0.20 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.01

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$= \frac{105\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-105\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(135\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-480i\,a^2\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/240*(105*a^2*log(tan(1/2*d*x + 1/2*c) + 1) - 105*a^2*log(tan(1/2*d*x + 1
/2*c) - 1) + 2*(135*a^2*tan(1/2*d*x + 1/2*c)^11 - 480*I*a^2*tan(1/2*d*x +
1/2*c)^10 - 445*a^2*tan(1/2*d*x + 1/2*c)^9 + 480*I*a^2*tan(1/2*d*x + 1/2*c
)^8 - 330*a^2*tan(1/2*d*x + 1/2*c)^7 - 960*I*a^2*tan(1/2*d*x + 1/2*c)^6 -
330*a^2*tan(1/2*d*x + 1/2*c)^5 + 960*I*a^2*tan(1/2*d*x + 1/2*c)^4 - 445*a^
2*tan(1/2*d*x + 1/2*c)^3 - 96*I*a^2*tan(1/2*d*x + 1/2*c)^2 + 135*a^2*tan(1
/2*d*x + 1/2*c) + 96*I*a^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.46

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{7\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d} - \frac{-\frac{9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}4\mathrm{i}+\frac{89\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}-a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}4\mathrm{i}+\frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+a^2\,\mathrm{t}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

input
```
int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^5,x)
```

output
```
(7*a^2*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((a^2*tan(c/2 + (d*x)/2)^2*4i)/5
 + (89*a^2*tan(c/2 + (d*x)/2)^3)/24 - a^2*tan(c/2 + (d*x)/2)^4*8i + (11*a^
2*tan(c/2 + (d*x)/2)^5)/4 + a^2*tan(c/2 + (d*x)/2)^6*8i + (11*a^2*tan(c/2
+ (d*x)/2)^7)/4 - a^2*tan(c/2 + (d*x)/2)^8*4i + (89*a^2*tan(c/2 + (d*x)/2)
^9)/24 + a^2*tan(c/2 + (d*x)/2)^10*4i - (9*a^2*tan(c/2 + (d*x)/2)^11)/8 -
(a^2*4i)/5 - (9*a^2*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 + (d*x)/2)^4 - 6
*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8
- 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.32

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\left(-96\cos\left(dx+c\right)i-105\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^6+315\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)\right.}{}$$

input
```
int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(a**2*( - 96*cos(c + d*x)*i - 105*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**
6 + 315*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 - 315*log(tan((c + d*x)/
2) - 1)*sin(c + d*x)**2 + 105*log(tan((c + d*x)/2) - 1) + 105*log(tan((c +
 d*x)/2) + 1)*sin(c + d*x)**6 - 315*log(tan((c + d*x)/2) + 1)*sin(c + d*x)
**4 + 315*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 - 105*log(tan((c + d*x
)/2) + 1) - 96*sin(c + d*x)**6*i - 105*sin(c + d*x)**5 + 288*sin(c + d*x)*
*4*i + 280*sin(c + d*x)**3 - 288*sin(c + d*x)**2*i - 135*sin(c + d*x) + 96
*i))/(240*d*(sin(c + d*x)**6 - 3*sin(c + d*x)**4 + 3*sin(c + d*x)**2 - 1))
```

3.29 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{5a^2\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{5ia^2\sec^3(c+dx)}{12d}+\frac{5a^2\sec(c+dx)\tan(c+dx)}{8d}+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

output
```
5/8*a^2*arctanh(sin(d*x+c))/d+5/12*I*a^2*sec(d*x+c)^3/d+5/8*a^2*sec(d*x+c)
*tan(d*x+c)/d+1/4*I*sec(d*x+c)^3*(a^2+I*a^2*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{5a^2\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{2ia^2\sec^3(c+dx)}{3d}+\frac{5a^2\sec(c+dx)\tan(c+dx)}{8d}-\frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(5*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (((2*I)/3)*a^2*Sec[c + d*x]^3)/d + (
5*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (a^2*Sec[c + d*x]^3*Tan[c + d*x])
/(4*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3979, 3042, 3967, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3(a+ia\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{5}{4}a\int \sec^3(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{4}a\int \sec(c+dx)^3(i\tan(c+dx)a+a)dx+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{5}{4}a\left(a\int \sec^3(c+dx)dx+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{4}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{5}{4}a\left(a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{4}a\left(a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}+\frac{5}{4}a\left(a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}\right)$$

input
```
Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((I/4)*Sec[c + d*x]^3*(a^2 + I*a^2*Tan[c + d*x]))/d + (5*a*(((I/3)*a*Sec[c
 + d*x]^3)/d + a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x]
)/(2*d))))/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{i a^{2}\left(15\,\mathrm{e}^{7 i(d x+c)}-73\,\mathrm{e}^{5 i(d x+c)}-55\,\mathrm{e}^{3 i(d x+c)}-15\,\mathrm{e}^{i(d x+c)}\right)}{12 d\left(\mathrm{e}^{2 i(d x+c)}+1\right)^{4}}+\frac{5 a^{2} \ln\left(\mathrm{e}^{i(d x+c)}+i\right)}{8 d}-\frac{5 a^{2} \ln\left(\mathrm{e}^{i(d x+c)}-i\right)}{8 d}$
derivativedivides	$\frac{-a^{2}\left(\frac{\sin(d x+c)^{3}}{4\cos(d x+c)^{4}}+\frac{\sin(d x+c)^{3}}{8\cos(d x+c)^{2}}+\frac{\sin(d x+c)}{8}-\frac{\ln(\sec(d x+c)+\tan(d x+c))}{8}\right)+\frac{2 i a^{2}}{3\cos(d x+c)^{3}}+a^{2}\left(\frac{\sec(d x+c)\tan(d x+c)}{2}+\frac{\ln(\sec}{}\right.}{d}$
default	$\frac{-a^{2}\left(\frac{\sin(d x+c)^{3}}{4\cos(d x+c)^{4}}+\frac{\sin(d x+c)^{3}}{8\cos(d x+c)^{2}}+\frac{\sin(d x+c)}{8}-\frac{\ln(\sec(d x+c)+\tan(d x+c))}{8}\right)+\frac{2 i a^{2}}{3\cos(d x+c)^{3}}+a^{2}\left(\frac{\sec(d x+c)\tan(d x+c)}{2}+\frac{\ln(\sec}{}\right.}{d}$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/12*I*a^2/d/(exp(2*I*(d*x+c))+1)^4*(15*exp(7*I*(d*x+c))-73*exp(5*I*(d*x+
c))-55*exp(3*I*(d*x+c))-15*exp(I*(d*x+c)))+5/8/d*a^2*ln(exp(I*(d*x+c))+I)-
5/8/d*a^2*ln(exp(I*(d*x+c))-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(80) = 160$.

Time = 0.09 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.72

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{-30i\,a^2e^{(7i\,dx+7i\,c)} + 146i\,a^2e^{(5i\,dx+5i\,c)} + 110i\,a^2e^{(3i\,dx+3i\,c)} + 30i\,a^2e^{(i\,dx+i\,c)} + 15\left(a^2e^{(8i\,dx+8i\,c)} + 4\,a^2e^{(6i\,d}\right.}{24\,(de^{(8}}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/24*(-30*I*a^2*e^(7*I*d*x + 7*I*c) + 146*I*a^2*e^(5*I*d*x + 5*I*c) + 110*
I*a^2*e^(3*I*d*x + 3*I*c) + 30*I*a^2*e^(I*d*x + I*c) + 15*(a^2*e^(8*I*d*x
+ 8*I*c) + 4*a^2*e^(6*I*d*x + 6*I*c) + 6*a^2*e^(4*I*d*x + 4*I*c) + 4*a^2*e
^(2*I*d*x + 2*I*c) + a^2)*log(e^(I*d*x + I*c) + I) - 15*(a^2*e^(8*I*d*x +
8*I*c) + 4*a^2*e^(6*I*d*x + 6*I*c) + 6*a^2*e^(4*I*d*x + 4*I*c) + 4*a^2*e^(
2*I*d*x + 2*I*c) + a^2)*log(e^(I*d*x + I*c) - I))/(d*e^(8*I*d*x + 8*I*c) +
 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*
c) + d)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2(c+dx)\sec^3(c+dx)\,dx + \int \left(-2i\tan(c+dx)\sec^3(c+dx)\right)dx + \int \left(-\sec^3(c+dx)\right)dx\right)$$

input
```
integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**3, x) + Integral(-2*I*tan(c
+ d*x)*sec(c + d*x)**3, x) + Integral(-sec(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.38

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx =$$

$$-\frac{3\,a^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+12\,a^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\mathrm{lo}\right.}{48\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/48*(3*a^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*
x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^2*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
 c) - 1)) - 32*I*a^2/cos(d*x + c)^3)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(80) = 160.

Time = 0.20 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.84

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$=\frac{15\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-15\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6\cdots\right.}{}}{24\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(15*a^2*log(tan(1/2*d*x + 1/2*c) + 1) - 15*a^2*log(tan(1/2*d*x + 1/2*
c) - 1) + 2*(9*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*I*a^2*tan(1/2*d*x + 1/2*c)^
6 - 33*a^2*tan(1/2*d*x + 1/2*c)^5 + 48*I*a^2*tan(1/2*d*x + 1/2*c)^4 - 33*a
^2*tan(1/2*d*x + 1/2*c)^3 - 16*I*a^2*tan(1/2*d*x + 1/2*c)^2 + 9*a^2*tan(1/
2*d*x + 1/2*c) + 16*I*a^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.11

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{5\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$
$$-\frac{-\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 4\mathrm{i}+\frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 4\mathrm{i}+\frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{a^2\tan}{}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right.}$$

input
```
int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^3,x)
```

output
```
(5*a^2*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a^2*tan(c/2 + (d*x)/2)^2*4i)/3
 + (11*a^2*tan(c/2 + (d*x)/2)^3)/4 - a^2*tan(c/2 + (d*x)/2)^4*4i + (11*a^2
*tan(c/2 + (d*x)/2)^5)/4 + a^2*tan(c/2 + (d*x)/2)^6*4i - (3*a^2*tan(c/2 +
(d*x)/2)^7)/4 - (a^2*4i)/3 - (3*a^2*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 +
 (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 +
(d*x)/2)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.12

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{a^2\left(16\cos\left(dx+c\right)i-15\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^4+30\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2-\right.}{}$$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(a**2*(16*cos(c + d*x)*i - 15*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 +
30*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 - 15*log(tan((c + d*x)/2) - 1
) + 15*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4 - 30*log(tan((c + d*x)/2)
 + 1)*sin(c + d*x)**2 + 15*log(tan((c + d*x)/2) + 1) - 16*sin(c + d*x)**4*
i - 15*sin(c + d*x)**3 + 32*sin(c + d*x)**2*i + 9*sin(c + d*x) - 16*i))/(2
4*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.30 $\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 68

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{3a^2\text{arctanh}(\sin(c+dx))}{2d} + \frac{3ia^2\sec(c+dx)}{2d} + \frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

output
```
3/2*a^2*arctanh(sin(d*x+c))/d+3/2*I*a^2*sec(d*x+c)/d+1/2*I*sec(d*x+c)*(a^2
+I*a^2*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\coth^{-1}(\sin(c+dx))}{d} + \frac{a^2\text{arctanh}(\sin(c+dx))}{2d} + \frac{2ia^2\sec(c+dx)}{d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(a^2*ArcCoth[Sin[c + d*x]])/d + (a^2*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*I)
*a^2*Sec[c + d*x])/d - (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3979, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(c+dx)(a+ia\tan(c+dx))^2dx$$
$$\Big\downarrow \text{3979}$$
$$\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$
$$\Big\downarrow \text{3042}$$
$$\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$
$$\Big\downarrow \text{3967}$$
$$\frac{3}{2}a\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$
$$\Big\downarrow \text{3042}$$
$$\frac{3}{2}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$
$$\Big\downarrow \text{4257}$$
$$\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{3}{2}a\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)$$

input
```
Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(3*a*((a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d))/2 + ((I/2)*Sec[
c + d*x]*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.26

method	result	size
derivativedivides	$\frac{-a^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ia^2}{\cos(dx+c)}+a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$	86
default	$\frac{-a^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ia^2}{\cos(dx+c)}+a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$	86
risch	$\frac{ia^2\left(5\,\mathrm{e}^{3i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{3a^2\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d}+\frac{3a^2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}$	89

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+
tan(d*x+c)))+2*I*a^2/cos(d*x+c)+a^2*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(56) = 112$.

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.18

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{10i\,a^2e^{(3i\,dx+3i\,c)}+6i\,a^2e^{(i\,dx+i\,c)}+3\left(a^2e^{(4i\,dx+4i\,c)}+2\,a^2e^{(2i\,dx+2i\,c)}+a^2\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(a^2e^{(4i\,dx+\cdots}\right.}{2\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(10*I*a^2*e^(3*I*d*x + 3*I*c) + 6*I*a^2*e^(I*d*x + I*c) + 3*(a^2*e^(4*
I*d*x + 4*I*c) + 2*a^2*e^(2*I*d*x + 2*I*c) + a^2)*log(e^(I*d*x + I*c) + I)
 - 3*(a^2*e^(4*I*d*x + 4*I*c) + 2*a^2*e^(2*I*d*x + 2*I*c) + a^2)*log(e^(I*
d*x + I*c) - I))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2{(c+dx)}\sec{(c+dx)}\,dx + \int (-2i\tan{(c+dx)}\sec{(c+dx)})\,dx + \int (-\sec{(c+dx)})\,dx\right)$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(tan(c + d*x)**2*sec(c + d*x), x) + Integral(-2*I*tan(c + d
*x)*sec(c + d*x), x) + Integral(-sec(c + d*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log{(\sin{(dx+c)}+1)}-\log{(\sin{(dx+c)}-1)}\right)+4\,a^2\log{(\sec{(dx+c)}+\tan{(dx+c)})}}{4\,d}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*(a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) + 4*a^2*log(sec(d*x + c) + tan(d*x + c)) + 8*I*a^2/co
s(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.57

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{3\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-3\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^2\,\mathrm{ta}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(3*a^2*log(tan(1/2*d*x + 1/2*c) + 1) - 3*a^2*log(tan(1/2*d*x + 1/2*c)
- 1) - 2*(a^2*tan(1/2*d*x + 1/2*c)^3 + 4*I*a^2*tan(1/2*d*x + 1/2*c)^2 + a^
2*tan(1/2*d*x + 1/2*c) - 4*I*a^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.53

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{3\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 4\mathrm{i}+a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a^2\,4\mathrm{i}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x),x)
```

output
```
(3*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (a^2*tan(c/2 + (d*x)/2)^2*4i + a^2*t
an(c/2 + (d*x)/2)^3 - a^2*4i + a^2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)
/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec), antiderivative size = 122, normalized size of antiderivative = 1.79

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{a^2\left(-4\cos(dx+c)\,i-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2+3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+3\log\left(\tan\left(\frac{dx}{2}+\right.\right.\right.}{2d\left(\sin(dx+c)^2-\right.}$$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(a**2*( - 4*cos(c + d*x)*i - 3*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 +
 3*log(tan((c + d*x)/2) - 1) + 3*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2
 - 3*log(tan((c + d*x)/2) + 1) - 4*sin(c + d*x)**2*i + sin(c + d*x) + 4*i)
)/(2*d*(sin(c + d*x)**2 - 1))
```

3.31 $\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2\text{arctanh}(\sin(c+dx))}{d} - \frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

output
```
-a^2*arctanh(sin(d*x+c))/d-2*I*cos(d*x+c)*(a^2+I*a^2*tan(d*x+c))/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 180 vs. 2(46) = 92.

Time = 0.47 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.91

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{a^2\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(-2i+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+a\right.\right.\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(a^2*(Cos[(c + d*x)/2]*(-2*I + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (2 - I*Log[Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]] + I*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[(c + d
*x)/2])*(Cos[(c + 5*d*x)/2] + I*Sin[(c + 5*d*x)/2]))/(d*(Cos[d*x] + I*Sin[
d*x])^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3977, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3977}$$

$$a^2(-\int \sec(c+dx)dx) - \frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a^2\left(-\int \csc\left(c+dx+\frac{\pi}{2}\right)dx\right) - \frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{a^2\text{arctanh}(\sin(c+dx))}{d} - \frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-((a^2*ArcTanh[Sin[c + d*x]])/d) - ((2*I)*Cos[c + d*x]*(a^2 + I*a^2*Tan[c
+ d*x]))/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{-a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-2ia^2\cos(dx+c)+a^2\sin(dx+c)}{d}$	56
default	$\frac{-a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-2ia^2\cos(dx+c)+a^2\sin(dx+c)}{d}$	56
risch	$-\frac{2ia^2 e^{i(dx+c)}}{d}+\frac{a^2\ln(e^{i(dx+c)}-i)}{d}-\frac{a^2\ln(e^{i(dx+c)}+i)}{d}$	61

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-2*I*a^2*cos(d*x+c)+a^2*s
in(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{-2i\,a^2e^{(i\,dx+i\,c)}-a^2\log\left(e^{(i\,dx+i\,c)}+i\right)+a^2\log\left(e^{(i\,dx+i\,c)}-i\right)}{d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
(-2*I*a^2*e^(I*d*x + I*c) - a^2*log(e^(I*d*x + I*c) + I) + a^2*log(e^(I*d*
x + I*c) - I))/d
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx=\frac{a^2\left(\log\left(e^{idx}-ie^{-ic}\right)-\log\left(e^{idx}+ie^{-ic}\right)\right)}{d}+\begin{cases}-\frac{2ia^2e^{ic}e^{idx}}{d} & \text{for } d\neq 0\\ 2a^2xe^{ic} & \text{otherwise}\end{cases}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**2,x)
```

output
```
a**2*(log(exp(I*d*x) - I*exp(-I*c)) - log(exp(I*d*x) + I*exp(-I*c)))/d + P
iecewise((-2*I*a**2*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (2*a**2*x*exp(I*c),
True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\sin(dx+c))+4i\,a^2\cos(dx+c)-2\,a^2\sin(dx+}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*(a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c))
 + 4*I*a^2*cos(d*x + c) - 2*a^2*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{-2i\,a^2e^{(i\,dx+i\,c)}-a^2\log\left(i\,e^{(i\,dx+i\,c)}-1\right)+a^2\log\left(-i\,e^{(i\,dx+i\,c)}-1\right)}{d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
(-2*I*a^2*e^(I*d*x + I*c) - a^2*log(I*e^(I*d*x + I*c) - 1) + a^2*log(-I*e^
(I*d*x + I*c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{4\,a^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(4*a^2)/(d*(tan(c/2 + (d*x)/2) + 1i)) - (2*a^2*atanh(tan(c/2 + (d*x)/2)))/
d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\left(-2\cos\left(dx+c\right)i+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2\sin\left(dx+c\right)+2i\right)}{d}$$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(a**2*( - 2*cos(c + d*x)*i + log(tan((c + d*x)/2) - 1) - log(tan((c + d*x)
/2) + 1) + 2*sin(c + d*x) + 2*i))/d
```

3.32 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\sin(c+dx)}{3d} - \frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

output
```
1/3*a^2*sin(d*x+c)/d-2/3*I*cos(d*x+c)^3*(a^2+I*a^2*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2ia^2\cos^3(c+dx)}{3d} + \frac{a^2\sin(c+dx)}{d} - \frac{2a^2\sin^3(c+dx)}{3d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(((-2*I)/3)*a^2*Cos[c + d*x]^3)/d + (a^2*Sin[c + d*x])/d - (2*a^2*Sin[c +
d*x]^3)/(3*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3977, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{ 3977}$$

$$\frac{1}{3}a^2\int \cos(c+dx)dx-\frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)dx-\frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{a^2\sin(c+dx)}{3d}-\frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(a^2*Sin[c + d*x])/(3*d) - (((2*I)/3)*Cos[c + d*x]^3*(a^2 + I*a^2*Tan[c +
d*x]))/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{i a^{2} {\mathrm e}^{3 i \left(d x +c \right)}}{6 d}-\frac{i a^{2} {\mathrm e}^{i \left(d x +c \right)}}{2 d}$	38
derivativedivides	$\frac{-\frac{a^{2} \sin \left(d x +c \right)^{3}}{3}-\frac{2 i a^{2} \cos \left(d x +c \right)^{3}}{3}+\frac{a^{2} \left(2+\cos \left(d x +c \right)^{2}\right) \sin \left(d x +c \right)}{3}}{d}$	54
default	$\frac{-\frac{a^{2} \sin \left(d x +c \right)^{3}}{3}-\frac{2 i a^{2} \cos \left(d x +c \right)^{3}}{3}+\frac{a^{2} \left(2+\cos \left(d x +c \right)^{2}\right) \sin \left(d x +c \right)}{3}}{d}$	54

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*I/d*a^2*exp(3*I*(d*x+c))-1/2*I/d*a^2*exp(I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{-i\,a^2e^{(3i\,dx+3i\,c)} - 3i\,a^2e^{(i\,dx+i\,c)}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(-I*a^2*e^(3*I*d*x + 3*I*c) - 3*I*a^2*e^(I*d*x + I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.47

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \begin{cases} \frac{-2ia^2de^{3ic}e^{3idx}-6ia^2de^{ic}e^{idx}}{12d^2} & \text{for } d^2 \neq 0 \\ x\left(\frac{a^2e^{3ic}}{2}+\frac{a^2e^{ic}}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-2*I*a**2*d*exp(3*I*c)*exp(3*I*d*x) - 6*I*a**2*d*exp(I*c)*exp(
I*d*x))/(12*d**2), Ne(d**2, 0)), (x*(a**2*exp(3*I*c)/2 + a**2*exp(I*c)/2),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2i\,a^2\cos(dx+c)^3 + a^2\sin(dx+c)^3 + \left(\sin(dx+c)^3 - 3\,\sin(dx+c)\right)a^2}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/3*(2*I*a^2*cos(d*x + c)^3 + a^2*sin(d*x + c)^3 + (sin(d*x + c)^3 - 3*si
n(d*x + c))*a^2)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(43) = 86$.

Time = 0.24 (sec) , antiderivative size = 531, normalized size of antiderivative = 10.41

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/96*(24*a^2*e^(4*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 48*a^2*e^(2
*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 24*a^2*e^(-2*I*c)*log(I*e^(I*d*x + I*
c) + 1) + 27*a^2*e^(4*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 54*a^2*e
^(2*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 27*a^2*e^(-2*I*c)*log(I*e^(I*d*x +
 I*c) - 1) - 24*a^2*e^(4*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 48*a
^2*e^(2*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 24*a^2*e^(-2*I*c)*log(-I*e^(I
*d*x + I*c) + 1) - 27*a^2*e^(4*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1)
- 54*a^2*e^(2*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 27*a^2*e^(-2*I*c)*log(-
I*e^(I*d*x + I*c) - 1) + 3*a^2*e^(4*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I
*c)) + 6*a^2*e^(2*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) + 3*a^2*e^(-2*I*c)*lo
g(I*e^(I*d*x) + e^(-I*c)) - 3*a^2*e^(4*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e
^(-I*c)) - 6*a^2*e^(2*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) - 3*a^2*e^(-2*I*
c)*log(-I*e^(I*d*x) + e^(-I*c)) + 16*I*a^2*e^(7*I*d*x + 5*I*c) + 80*I*a^2*
e^(5*I*d*x + 3*I*c) + 112*I*a^2*e^(3*I*d*x + I*c) + 48*I*a^2*e^(I*d*x - I*
c))/(d*e^(4*I*d*x + 2*I*c) + 2*d*e^(2*I*d*x) + d*e^(-2*I*c))
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2\,a^2\,\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}-2\right)}{3\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
-(2*a^2*(tan(c/2 + (d*x)/2)*3i + 3*tan(c/2 + (d*x)/2)^2 - 2))/(3*d*(3*tan(
c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*3i - tan(c/2 + (d*x)/2)^3 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\left(2\cos\left(dx+c\right)\sin\left(dx+c\right)^2 i-2\cos\left(dx+c\right)i-2\sin\left(dx+c\right)^3+3\sin\left(dx+c\right)+2i\right)}{3d}$$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(a**2*(2*cos(c + d*x)*sin(c + d*x)**2*i - 2*cos(c + d*x)*i - 2*sin(c + d*x
)**3 + 3*sin(c + d*x) + 2*i))/(3*d)
```

3.33 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 69

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{3a^2\sin(c+dx)}{5d} - \frac{a^2\sin^3(c+dx)}{5d} - \frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

output
```
3/5*a^2*sin(d*x+c)/d-1/5*a^2*sin(d*x+c)^3/d-2/5*I*cos(d*x+c)^5*(a^2+I*a^2*
tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2ia^2\cos^5(c+dx)}{5d} + \frac{a^2\sin(c+dx)}{d} - \frac{a^2\sin^3(c+dx)}{d} + \frac{2a^2\sin^5(c+dx)}{5d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(((-2*I)/5)*a^2*Cos[c + d*x]^5)/d + (a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*
x]^3)/d + (2*a^2*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3042, 3977, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{3}{5}a^2\int \cos^3(c+dx)dx-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{3a^2\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{5d}-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{5d}-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(-3*a^2*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/(5*d) - (((2*I)/5)*Cos[c + d*x
]^5*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

Maple [A] (verified)

Time = 12.80 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{i a^{2} {\mathrm e}^{5 i (d x +c)}}{40 d}-\frac{i a^{2} {\mathrm e}^{3 i (d x +c)}}{8 d}-\frac{i a^{2} \cos \left(d x +c \right)}{4 d}+\frac{a^{2} \sin \left(d x +c \right)}{2 d}$
derivativedivides	$\frac{-a^{2} \left(-\frac{\cos \left(d x +c \right)^{4} \sin \left(d x +c \right)}{5}+\frac{\left(2+\cos \left(d x +c \right)^{2}\right) \sin \left(d x +c \right)}{15}\right)-\frac{2 i a^{2} \cos \left(d x +c \right)^{5}}{5}+\frac{a^{2} \left(\frac{8}{3}+\cos \left(d x +c \right)^{4}+\frac{4 \cos \left(d x +c \right)^{2}}{3}\right) \sin (d x +c}{5}}{d}$
default	$\frac{-a^{2} \left(-\frac{\cos \left(d x +c \right)^{4} \sin \left(d x +c \right)}{5}+\frac{\left(2+\cos \left(d x +c \right)^{2}\right) \sin \left(d x +c \right)}{15}\right)-\frac{2 i a^{2} \cos \left(d x +c \right)^{5}}{5}+\frac{a^{2} \left(\frac{8}{3}+\cos \left(d x +c \right)^{4}+\frac{4 \cos \left(d x +c \right)^{2}}{3}\right) \sin (d x +c}{5}}{d}$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/40*I/d*a^2*exp(5*I*(d*x+c))-1/8*I/d*a^2*exp(3*I*(d*x+c))-1/4*I/d*a^2*co
s(d*x+c)+1/2*a^2*sin(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{\left(-i\,a^2e^{(6i\,dx+6i\,c)}-5i\,a^2e^{(4i\,dx+4i\,c)}-15i\,a^2e^{(2i\,dx+2i\,c)}+5i\,a^2\right)e^{(-i\,dx-i\,c)}}{40\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/40*(-I*a^2*e^(6*I*d*x + 6*I*c) - 5*I*a^2*e^(4*I*d*x + 4*I*c) - 15*I*a^2*
e^(2*I*d*x + 2*I*c) + 5*I*a^2)*e^(-I*d*x - I*c)/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(60) = 120$.

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.22

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \begin{cases} \frac{\left(-512ia^2d^3e^{6ic}e^{5idx}-2560ia^2d^3e^{4ic}e^{3idx}-7680ia^2d^3e^{2ic}e^{idx}+2560ia^2d^3e^{-idx}\right)e^{-ic}}{20480d^4} & \text{for } d^4e^{ic}\neq 0 \\ \frac{x\left(a^2e^{6ic}+3a^2e^{4ic}+3a^2e^{2ic}+a^2\right)e^{-ic}}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-512*I*a**2*d**3*exp(6*I*c)*exp(5*I*d*x) - 2560*I*a**2*d**3*ex
p(4*I*c)*exp(3*I*d*x) - 7680*I*a**2*d**3*exp(2*I*c)*exp(I*d*x) + 2560*I*a*
*2*d**3*exp(-I*d*x))*exp(-I*c)/(20480*d**4), Ne(d**4*exp(I*c), 0)), (x*(a*
*2*exp(6*I*c) + 3*a**2*exp(4*I*c) + 3*a**2*exp(2*I*c) + a**2)*exp(-I*c)/8,
 True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{6i\,a^2\cos{(dx+c)}^5 - \left(3\,\sin{(dx+c)}^5 - 5\,\sin{(dx+c)}^3\right)a^2 - \left(3\,\sin{(dx+c)}^5 - 10\,\sin{(dx+c)}^3 + 15\,\mathrm{s}\right.}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/15*(6*I*a^2*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^2
- (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(59) = 118.

Time = 0.30 (sec) , antiderivative size = 613, normalized size of antiderivative = 8.88

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/160*(45*a^2*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 90*a^2*e^(
3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 45*a^2*e^(I*d*x - I*c)*log(I*e
^(I*d*x + I*c) + 1) + 40*a^2*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) - 1
) + 80*a^2*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 40*a^2*e^(I*d*x
- I*c)*log(I*e^(I*d*x + I*c) - 1) - 45*a^2*e^(5*I*d*x + 3*I*c)*log(-I*e^(I
*d*x + I*c) + 1) - 90*a^2*e^(3*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) + 1) -
45*a^2*e^(I*d*x - I*c)*log(-I*e^(I*d*x + I*c) + 1) - 40*a^2*e^(5*I*d*x + 3
*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 80*a^2*e^(3*I*d*x + I*c)*log(-I*e^(I*d
*x + I*c) - 1) - 40*a^2*e^(I*d*x - I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5*a^
2*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 10*a^2*e^(3*I*d*x + I*
c)*log(I*e^(I*d*x) + e^(-I*c)) - 5*a^2*e^(I*d*x - I*c)*log(I*e^(I*d*x) + e
^(-I*c)) + 5*a^2*e^(5*I*d*x + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 10*a^2
*e^(3*I*d*x + I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 5*a^2*e^(I*d*x - I*c)*lo
g(-I*e^(I*d*x) + e^(-I*c)) + 4*I*a^2*e^(10*I*d*x + 8*I*c) + 28*I*a^2*e^(8*
I*d*x + 6*I*c) + 104*I*a^2*e^(6*I*d*x + 4*I*c) + 120*I*a^2*e^(4*I*d*x + 2*
I*c) + 20*I*a^2*e^(2*I*d*x) - 20*I*a^2*e^(-2*I*c))/(d*e^(5*I*d*x + 3*I*c)
+ 2*d*e^(3*I*d*x + I*c) + d*e^(I*d*x - I*c))
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{2\,a^2\,\left(\frac{5\,\sin(3\,c+3\,d\,x)}{16}-\frac{\cos(5\,c+5\,d\,x)\,1\mathrm{i}}{16}-\frac{\cos(3\,c+3\,d\,x)\,5\mathrm{i}}{16}+\frac{\sin(5\,c+5\,d\,x)}{16}+\frac{5\,\sqrt{3}\,\sin\left(c+d\,x-\frac{\ln(3)\,1\mathrm{i}}{2}\right)}{8}\right)}{5\,d}$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(2*a^2*((5*sin(3*c + 3*d*x))/16 - (cos(5*c + 5*d*x)*1i)/16 - (cos(3*c + 3*
d*x)*5i)/16 + sin(5*c + 5*d*x)/16 + (5*3^(1/2)*sin(c - (log(3)*1i)/2 + d*x
))/8))/(5*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.20

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{a^2\left(-2\cos\left(dx+c\right)\sin\left(dx+c\right)^4 i+4\cos\left(dx+c\right)\sin\left(dx+c\right)^2 i-2\cos\left(dx+c\right) i+2\sin\left(dx+c\right)^5-5\,\mathrm{s}\right.}{5d}$$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(a**2*( - 2*cos(c + d*x)*sin(c + d*x)**4*i + 4*cos(c + d*x)*sin(c + d*x)**
2*i - 2*cos(c + d*x)*i + 2*sin(c + d*x)**5 - 5*sin(c + d*x)**3 + 5*sin(c +
 d*x) + 2*i))/(5*d)
```

3.34 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{5a^2\sin(c+dx)}{7d}-\frac{10a^2\sin^3(c+dx)}{21d}+\frac{a^2\sin^5(c+dx)}{7d}-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

output
```
5/7*a^2*sin(d*x+c)/d-10/21*a^2*sin(d*x+c)^3/d+1/7*a^2*sin(d*x+c)^5/d-2/7*I
*cos(d*x+c)^7*(a^2+I*a^2*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2ia^2\cos^7(c+dx)}{7d}+\frac{a^2\sin(c+dx)}{d}-\frac{4a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^5(c+dx)}{d}-\frac{2a^2\sin^7(c+dx)}{7d}$$

input
```
Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(((-2*I)/7)*a^2*Cos[c + d*x]^7)/d + (a^2*Sin[c + d*x])/d - (4*a^2*Sin[c +
d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^5)/d - (2*a^2*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3042, 3977, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{5}{7}a^2\int \cos^5(c+dx)dx-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{5a^2\int \left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{7d}-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5a^2\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{7d}-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

input
```
Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(-5*a^2*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]^5/5))/(7*d) -
 (((2*I)/7)*Cos[c + d*x]^7*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

Maple [A] (verified)

Time = 44.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{i a^2 e^{7 i (d x+c)}}{224 d}-\frac{i a^2 e^{5 i (d x+c)}}{32 d}-\frac{5 i a^2 \cos(d x+c)}{32 d}+\frac{15 a^2 \sin(d x+c)}{32 d}-\frac{3 i a^2 \cos(3 d x+3 c)}{32 d}+\frac{11 a^2 \sin(3 d x+3 c)}{96 d}$
derivativedivides	$\frac{-a^2\left(-\frac{\sin(dx+c)\cos(dx+c)^6}{7}+\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{35}\right)-\frac{2ia^2\cos(dx+c)^7}{7}+\frac{a^2\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(}{}\right.}{}}{d}$
default	$\frac{-a^2\left(-\frac{\sin(dx+c)\cos(dx+c)^6}{7}+\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{35}\right)-\frac{2ia^2\cos(dx+c)^7}{7}+\frac{a^2\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(}{}\right.}{}}{d}$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/224*I/d*a^2*exp(7*I*(d*x+c))-1/32*I/d*a^2*exp(5*I*(d*x+c))-5/32*I/d*a^2
*cos(d*x+c)+15/32*a^2*sin(d*x+c)/d-3/32*I/d*a^2*cos(3*d*x+3*c)+11/96/d*a^2
*sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{\left(-3i\,a^2e^{(10i\,dx+10i\,c)}-21i\,a^2e^{(8i\,dx+8i\,c)}-70i\,a^2e^{(6i\,dx+6i\,c)}-210i\,a^2e^{(4i\,dx+4i\,c)}+105i\,a^2e^{(2i\,dx+2i\,c)}+7i\,a^2\right.}{672\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/672*(-3*I*a^2*e^(10*I*d*x + 10*I*c) - 21*I*a^2*e^(8*I*d*x + 8*I*c) - 70*
I*a^2*e^(6*I*d*x + 6*I*c) - 210*I*a^2*e^(4*I*d*x + 4*I*c) + 105*I*a^2*e^(2
*I*d*x + 2*I*c) + 7*I*a^2)*e^(-3*I*d*x - 3*I*c)/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(76) = 152$.

Time = 0.34 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.74

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \begin{cases} \frac{(-75497472ia^2d^5e^{11ic}e^{7idx}-528482304ia^2d^5e^{9ic}e^{5idx}-1761607680ia^2d^5e^{7ic}e^{3idx}-5284823040ia^2d^5e^{5ic}e^{idx}+2642411520ia^2d^5e^{3ic}e^{-idx}}{16911433728d^6} \\ \frac{x(a^2e^{10ic}+5a^2e^{8ic}+10a^2e^{6ic}+10a^2e^{4ic}+5a^2e^{2ic}+a^2)e^{-3ic}}{32} \end{cases}$$

input
```
integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-75497472*I*a**2*d**5*exp(11*I*c)*exp(7*I*d*x) - 528482304*I*a
**2*d**5*exp(9*I*c)*exp(5*I*d*x) - 1761607680*I*a**2*d**5*exp(7*I*c)*exp(3
*I*d*x) - 5284823040*I*a**2*d**5*exp(5*I*c)*exp(I*d*x) + 2642411520*I*a**2
*d**5*exp(3*I*c)*exp(-I*d*x) + 176160768*I*a**2*d**5*exp(I*c)*exp(-3*I*d*x
))*exp(-4*I*c)/(16911433728*d**6), Ne(d**6*exp(4*I*c), 0)), (x*(a**2*exp(1
0*I*c) + 5*a**2*exp(8*I*c) + 10*a**2*exp(6*I*c) + 10*a**2*exp(4*I*c) + 5*a
**2*exp(2*I*c) + a**2)*exp(-3*I*c)/32, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx =$$
$$-\frac{30i\,a^2\cos(dx+c)^7 + \left(15\sin(dx+c)^7 - 42\sin(dx+c)^5 + 35\sin(dx+c)^3\right)a^2 + 3\left(5\sin(dx+c)^7 -\right.}{105\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/105*(30*I*a^2*cos(d*x + c)^7 + (15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 +
 35*sin(d*x + c)^3)*a^2 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin
(d*x + c)^3 - 35*sin(d*x + c))*a^2)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(75) = 150$.

Time = 0.32 (sec) , antiderivative size = 641, normalized size of antiderivative = 7.37

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/10752*(2583*a^2*e^(7*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5166*a
^2*e^(5*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 2583*a^2*e^(3*I*d*x - I*
c)*log(I*e^(I*d*x + I*c) + 1) + 2121*a^2*e^(7*I*d*x + 3*I*c)*log(I*e^(I*d*
x + I*c) - 1) + 4242*a^2*e^(5*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 21
21*a^2*e^(3*I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) - 2583*a^2*e^(7*I*d*x
+ 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5166*a^2*e^(5*I*d*x + I*c)*log(-I*e
^(I*d*x + I*c) + 1) - 2583*a^2*e^(3*I*d*x - I*c)*log(-I*e^(I*d*x + I*c) +
1) - 2121*a^2*e^(7*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 4242*a^2*e
^(5*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 2121*a^2*e^(3*I*d*x - I*c)*
log(-I*e^(I*d*x + I*c) - 1) - 462*a^2*e^(7*I*d*x + 3*I*c)*log(I*e^(I*d*x)
+ e^(-I*c)) - 924*a^2*e^(5*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 462*
a^2*e^(3*I*d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 462*a^2*e^(7*I*d*x + 3
*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 924*a^2*e^(5*I*d*x + I*c)*log(-I*e^(I
*d*x) + e^(-I*c)) + 462*a^2*e^(3*I*d*x - I*c)*log(-I*e^(I*d*x) + e^(-I*c))
 + 48*I*a^2*e^(14*I*d*x + 10*I*c) + 432*I*a^2*e^(12*I*d*x + 8*I*c) + 1840*
I*a^2*e^(10*I*d*x + 6*I*c) + 5936*I*a^2*e^(8*I*d*x + 4*I*c) + 6160*I*a^2*e
^(6*I*d*x + 2*I*c) - 1904*I*a^2*e^(2*I*d*x - 2*I*c) - 112*I*a^2*e^(4*I*d*x
) - 112*I*a^2*e^(-4*I*c))/(d*e^(7*I*d*x + 3*I*c) + 2*d*e^(5*I*d*x + I*c) +
 d*e^(3*I*d*x - I*c))
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.94

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{2\,a^2\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2{}\mathrm{i}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)} + \frac{256\,a^2\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)}{7\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^7} - \frac{8\,a^2\,\left(4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9{}\mathrm{i}\right)}{3\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2} - \frac{128\,a^2\,\left(6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-7{}\mathrm{i}\right)}{7\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^6} + \frac{16\,a^2\,\left(8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-15{}\mathrm{i}\right)}{3\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^3} - \frac{32\,a^2\,\left(22\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-35{}\mathrm{i}\right)}{7\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4} + \frac{32\,a^2\,\left(31\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-42{}\mathrm{i}\right)}{7\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

input
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(2*a^2*(tan(c/2 + (d*x)/2) - 2i))/(d*(tan(c/2 + (d*x)/2)^2 + 1)) + (256*a^
2*(tan(c/2 + (d*x)/2) - 1i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^7) - (8*a^2*(
4*tan(c/2 + (d*x)/2) - 9i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^2) - (128*a^2*
(6*tan(c/2 + (d*x)/2) - 7i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^6) + (16*a^2*
(8*tan(c/2 + (d*x)/2) - 15i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^3) - (32*a^2
*(22*tan(c/2 + (d*x)/2) - 35i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^4) + (32*a
^2*(31*tan(c/2 + (d*x)/2) - 42i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.26

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{a^2\left(6\cos(dx+c)\sin(dx+c)^6 i-18\cos(dx+c)\sin(dx+c)^4 i+18\cos(dx+c)\sin(dx+c)^2 i-6\cos(\right.}{21d}$$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(a**2*(6*cos(c + d*x)*sin(c + d*x)**6*i - 18*cos(c + d*x)*sin(c + d*x)**4*
i + 18*cos(c + d*x)*sin(c + d*x)**2*i - 6*cos(c + d*x)*i - 6*sin(c + d*x)*
*7 + 21*sin(c + d*x)**5 - 28*sin(c + d*x)**3 + 21*sin(c + d*x) + 6*i))/(21
*d)
```

3.35 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{7a^2\sin(c+dx)}{9d} - \frac{7a^2\sin^3(c+dx)}{9d} + \frac{7a^2\sin^5(c+dx)}{15d} - \frac{a^2\sin^7(c+dx)}{9d} - \frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

output
```
7/9*a^2*sin(d*x+c)/d-7/9*a^2*sin(d*x+c)^3/d+7/15*a^2*sin(d*x+c)^5/d-1/9*a^
2*sin(d*x+c)^7/d-2/9*I*cos(d*x+c)^9*(a^2+I*a^2*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2ia^2\cos^9(c+dx)}{9d} + \frac{a^2\sin(c+dx)}{d} - \frac{5a^2\sin^3(c+dx)}{3d} + \frac{9a^2\sin^5(c+dx)}{5d} - \frac{a^2\sin^7(c+dx)}{d} + \frac{2a^2\sin^9(c+dx)}{9d}$$

input
```
Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(((-2*I)/9)*a^2*Cos[c + d*x]^9)/d + (a^2*Sin[c + d*x])/d - (5*a^2*Sin[c +
d*x]^3)/(3*d) + (9*a^2*Sin[c + d*x]^5)/(5*d) - (a^2*Sin[c + d*x]^7)/d + (2
*a^2*Sin[c + d*x]^9)/(9*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 3977, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sec(c+dx)^9}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{7}{9}a^2\int \cos^7(c+dx)dx-\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{9}a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^7dx-\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{7a^2\int \left(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{9d}-\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{7a^2\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{9d}-\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

input
```
Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(-7*a^2*(-Sin[c + d*x] + Sin[c + d*x]^3 - (3*Sin[c + d*x]^5)/5 + Sin[c + d
*x]^7/7))/(9*d) - (((2*I)/9)*Cos[c + d*x]^9*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

Maple [A] (verified)

Time = 124.66 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{-a^2\left(-\frac{\cos(dx+c)^8\sin(dx+c)}{9}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{63}\right)-\frac{2ia^2\cos(dx+c)^9}{9}+\frac{a^2\left(\frac{128}{35}+\mathrm{co}\right.}{}}{d}$
default	$\frac{-a^2\left(-\frac{\cos(dx+c)^8\sin(dx+c)}{9}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{63}\right)-\frac{2ia^2\cos(dx+c)^9}{9}+\frac{a^2\left(\frac{128}{35}+\mathrm{co}\right.}{}}{d}$
risch	$-\frac{ia^2e^{9i(dx+c)}}{1152d}-\frac{ia^2e^{7i(dx+c)}}{128d}-\frac{7ia^2\cos(dx+c)}{64d}+\frac{7a^2\sin(dx+c)}{16d}-\frac{ia^2\cos(5dx+5c)}{32d}+\frac{11a^2\sin(5dx+5c)}{320d}-$

input
```
int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*(-1/9*cos(d*x+c)^8*sin(d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*
x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-2/9*I*a^2*cos(d*x+c)^9+1/9*a^2*(128/3
5+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin
(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{\left(-5i\,a^2e^{(14i\,dx+14i\,c)}-45i\,a^2e^{(12i\,dx+12i\,c)}-189i\,a^2e^{(10i\,dx+10i\,c)}-525i\,a^2e^{(8i\,dx+8i\,c)}-1575i\,a^2e^{(6i\,dx+6i\,c)}+\right.}{5760\,d}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/5760*(-5*I*a^2*e^(14*I*d*x + 14*I*c) - 45*I*a^2*e^(12*I*d*x + 12*I*c) -
189*I*a^2*e^(10*I*d*x + 10*I*c) - 525*I*a^2*e^(8*I*d*x + 8*I*c) - 1575*I*a
^2*e^(6*I*d*x + 6*I*c) + 945*I*a^2*e^(4*I*d*x + 4*I*c) + 105*I*a^2*e^(2*I*
d*x + 2*I*c) + 9*I*a^2)*e^(-5*I*d*x - 5*I*c)/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(94) = 188$.

Time = 0.43 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.99

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \begin{cases} \frac{(-126663739519795200ia^2d^7e^{18ic}e^{9idx} - 1139973655678156800ia^2d^7e^{16ic}e^{7idx} - 4787889353848258560ia^2d^7e^{14ic}e^{5idx} - 13299692649578496\cdots}{} \\ \frac{x(a^2e^{14ic}+7a^2e^{12ic}+21a^2e^{10ic}+35a^2e^{8ic}+35a^2e^{6ic}+21a^2e^{4ic}+7a^2e^{2ic}+a^2)e^{-5ic}}{128} \end{cases}$$

input
```
integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-126663739519795200*I*a**2*d**7*exp(18*I*c)*exp(9*I*d*x) - 113
9973655678156800*I*a**2*d**7*exp(16*I*c)*exp(7*I*d*x) - 478788935384825856
0*I*a**2*d**7*exp(14*I*c)*exp(5*I*d*x) - 13299692649578496000*I*a**2*d**7*
exp(12*I*c)*exp(3*I*d*x) - 39899077948735488000*I*a**2*d**7*exp(10*I*c)*ex
p(I*d*x) + 23939446769241292800*I*a**2*d**7*exp(8*I*c)*exp(-I*d*x) + 26599
38529915699200*I*a**2*d**7*exp(6*I*c)*exp(-3*I*d*x) + 227994731135631360*I
*a**2*d**7*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(14591662792680407040 0*d*
*8), Ne(d**8*exp(9*I*c), 0)), (x*(a**2*exp(14*I*c) + 7*a**2*exp(12*I*c) +
21*a**2*exp(10*I*c) + 35*a**2*exp(8*I*c) + 35*a**2*exp(6*I*c) + 21*a**2*ex
p(4*I*c) + 7*a**2*exp(2*I*c) + a**2)*exp(-5*I*c)/128, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.13

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx =$$
$$-\frac{70i\,a^2\cos(dx+c)^9 - \left(35\,\sin(dx+c)^9 - 135\,\sin(dx+c)^7 + 189\,\sin(dx+c)^5 - 105\,\sin(dx+c)^3\right)a^2\cdots}{31\cdots}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/315*(70*I*a^2*cos(d*x + c)^9 - (35*sin(d*x + c)^9 - 135*sin(d*x + c)^7
+ 189*sin(d*x + c)^5 - 105*sin(d*x + c)^3)*a^2 - (35*sin(d*x + c)^9 - 180*
sin(d*x + c)^7 + 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c
))*a^2)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(91) = 182$.

Time = 0.37 (sec) , antiderivative size = 669, normalized size of antiderivative = 6.37

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/92160*(18585*a^2*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 37170
*a^2*e^(7*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 18585*a^2*e^(5*I*d*x -
 I*c)*log(I*e^(I*d*x + I*c) + 1) + 14625*a^2*e^(9*I*d*x + 3*I*c)*log(I*e^(
I*d*x + I*c) - 1) + 29250*a^2*e^(7*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1)
 + 14625*a^2*e^(5*I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) - 18585*a^2*e^(9
*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 37170*a^2*e^(7*I*d*x + I*c)*
log(-I*e^(I*d*x + I*c) + 1) - 18585*a^2*e^(5*I*d*x - I*c)*log(-I*e^(I*d*x
+ I*c) + 1) - 14625*a^2*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) -
29250*a^2*e^(7*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 14625*a^2*e^(5*I
*d*x - I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3960*a^2*e^(9*I*d*x + 3*I*c)*log
(I*e^(I*d*x) + e^(-I*c)) - 7920*a^2*e^(7*I*d*x + I*c)*log(I*e^(I*d*x) + e^
(-I*c)) - 3960*a^2*e^(5*I*d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 3960*a^
2*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 7920*a^2*e^(7*I*d*x +
 I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 3960*a^2*e^(5*I*d*x - I*c)*log(-I*e^(
I*d*x) + e^(-I*c)) + 80*I*a^2*e^(18*I*d*x + 12*I*c) + 880*I*a^2*e^(16*I*d*
x + 10*I*c) + 4544*I*a^2*e^(14*I*d*x + 8*I*c) + 15168*I*a^2*e^(12*I*d*x +
6*I*c) + 45024*I*a^2*e^(10*I*d*x + 4*I*c) + 43680*I*a^2*e^(8*I*d*x + 2*I*c
) - 18624*I*a^2*e^(4*I*d*x - 2*I*c) - 1968*I*a^2*e^(2*I*d*x - 4*I*c) - 672
0*I*a^2*e^(6*I*d*x) - 144*I*a^2*e^(-6*I*c))/(d*e^(9*I*d*x + 3*I*c) + 2*d*e
^(7*I*d*x + I*c) + d*e^(5*I*d*x - I*c))
```

Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 330, normalized size of antiderivative = 3.14

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{2a^2\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2\mathrm{i}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)} + \frac{1024a^2\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{i}\right)}{9d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^9} - \frac{8a^2\left(5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-12\mathrm{i}\right)}{3d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^2} - \frac{512a^2\left(8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-9\mathrm{i}\right)}{9d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^8} + \frac{128a^2\left(19\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-24\mathrm{i}\right)}{3d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^7} - \frac{64a^2\left(19\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-35\mathrm{i}\right)}{5d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^4} + \frac{56a^2\left(19\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-40\mathrm{i}\right)}{15d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^3} - \frac{128a^2\left(59\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-84\mathrm{i}\right)}{9d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^6} + \frac{32a^2\left(781\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1260\mathrm{i}\right)}{45d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^5}$$

input
```
int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(2*a^2*(tan(c/2 + (d*x)/2) - 2i))/(d*(tan(c/2 + (d*x)/2)^2 + 1)) + (1024*a
^2*(tan(c/2 + (d*x)/2) - 1i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^9) - (8*a^2*
(5*tan(c/2 + (d*x)/2) - 12i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^2) - (512*a^
2*(8*tan(c/2 + (d*x)/2) - 9i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^8) + (128*a
^2*(19*tan(c/2 + (d*x)/2) - 24i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^7) - (64
*a^2*(19*tan(c/2 + (d*x)/2) - 35i))/(5*d*(tan(c/2 + (d*x)/2)^2 + 1)^4) + (
56*a^2*(19*tan(c/2 + (d*x)/2) - 40i))/(15*d*(tan(c/2 + (d*x)/2)^2 + 1)^3)
- (128*a^2*(59*tan(c/2 + (d*x)/2) - 84i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^
6) + (32*a^2*(781*tan(c/2 + (d*x)/2) - 1260i))/(45*d*(tan(c/2 + (d*x)/2)^2
 + 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.30

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\left(-10\cos(dx+c)\sin(dx+c)^8 i + 40\cos(dx+c)\sin(dx+c)^6 i - 60\cos(dx+c)\sin(dx+c)^4 i + 40 c\right.}{}$$

input
```
int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(a**2*( - 10*cos(c + d*x)*sin(c + d*x)**8*i + 40*cos(c + d*x)*sin(c + d*x)
**6*i - 60*cos(c + d*x)*sin(c + d*x)**4*i + 40*cos(c + d*x)*sin(c + d*x)**
2*i - 10*cos(c + d*x)*i + 10*sin(c + d*x)**9 - 45*sin(c + d*x)**7 + 81*sin
(c + d*x)**5 - 75*sin(c + d*x)**3 + 45*sin(c + d*x) + 10*i))/(45*d)
```

3.36 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{8i(a+ia\tan(c+dx))^7}{7a^4d} + \frac{3i(a+ia\tan(c+dx))^8}{2a^5d} - \frac{2i(a+ia\tan(c+dx))^9}{3a^6d} + \frac{i(a+ia\tan(c+dx))^{10}}{10a^7d}$$

output
```
-8/7*I*(a+I*a*tan(d*x+c))^7/a^4/d+3/2*I*(a+I*a*tan(d*x+c))^8/a^5/d-2/3*I*(
a+I*a*tan(d*x+c))^9/a^6/d+1/10*I*(a+I*a*tan(d*x+c))^10/a^7/d
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{a^3\sec^9(c+dx)(\cos(7(c+dx))+i\sin(7(c+dx)))(-66i+242i\cos(2(c+dx))+119\sec(c+dx)\sin(3}{840d}$$

input
```
Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
-1/840*(a^3*Sec[c + d*x]^9*(Cos[7*(c + d*x)] + I*Sin[7*(c + d*x)])*(-66*I
+ (242*I)*Cos[2*(c + d*x)] + 119*Sec[c + d*x]*Sin[3*(c + d*x)] + 35*Tan[c
+ d*x]))/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^8(a+ia\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^6d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int\left(-(i\tan(c+dx)a+a)^9+6a(i\tan(c+dx)a+a)^8-12a^2(i\tan(c+dx)a+a)^7+8a^3(i\tan(c+dx)a+a)^6\right)}{a^7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{8}{7}a^3(a+ia\tan(c+dx))^7-\frac{3}{2}a^2(a+ia\tan(c+dx))^8-\frac{1}{10}(a+ia\tan(c+dx))^{10}+\frac{2}{3}a(a+ia\tan(c+dx))^9\right)}{a^7d}$$

input
```
Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*((8*a^3*(a + I*a*Tan[c + d*x])^7)/7 - (3*a^2*(a + I*a*Tan[c + d*x])^
8)/2 + (2*a*(a + I*a*Tan[c + d*x])^9)/3 - (a + I*a*Tan[c + d*x])^10/10))/(
a^7*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 169.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

method	result
risch	$\frac{128 i a^{3} \left(210 \,{\mathrm e}^{12 i \left(d x +c \right)}+252 \,{\mathrm e}^{10 i \left(d x +c \right)}+210 \,{\mathrm e}^{8 i \left(d x +c \right)}+120 \,{\mathrm e}^{6 i \left(d x +c \right)}+45 \,{\mathrm e}^{4 i \left(d x +c \right)}+10 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{105 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{10}}$
derivativedivides	$\frac{-i a^{3} \left(\frac{\sin \left(d x +c \right)^{4}}{10 \cos \left(d x +c \right)^{10}}+\frac{3 \sin \left(d x +c \right)^{4}}{40 \cos \left(d x +c \right)^{8}}+\frac{\sin \left(d x +c \right)^{4}}{20 \cos \left(d x +c \right)^{6}}+\frac{\sin \left(d x +c \right)^{4}}{40 \cos \left(d x +c \right)^{4}}\right)-3 a^{3} \left(\frac{\sin \left(d x +c \right)^{3}}{9 \cos \left(d x +c \right)^{9}}+\frac{2 \sin \left(d x +c \right)^{3}}{21 \cos \left(d x +c \right)^{7}}+\frac{8 \sin \left(d x +c \right)^{3}}{105 \cos \left(d x +c \right)}\right.}{d}$
default	$\frac{-i a^{3} \left(\frac{\sin \left(d x +c \right)^{4}}{10 \cos \left(d x +c \right)^{10}}+\frac{3 \sin \left(d x +c \right)^{4}}{40 \cos \left(d x +c \right)^{8}}+\frac{\sin \left(d x +c \right)^{4}}{20 \cos \left(d x +c \right)^{6}}+\frac{\sin \left(d x +c \right)^{4}}{40 \cos \left(d x +c \right)^{4}}\right)-3 a^{3} \left(\frac{\sin \left(d x +c \right)^{3}}{9 \cos \left(d x +c \right)^{9}}+\frac{2 \sin \left(d x +c \right)^{3}}{21 \cos \left(d x +c \right)^{7}}+\frac{8 \sin \left(d x +c \right)^{3}}{105 \cos \left(d x +c \right)}\right.}{d}$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
128/105*I*a^3*(210*exp(12*I*(d*x+c))+252*exp(10*I*(d*x+c))+210*exp(8*I*(d*
x+c))+120*exp(6*I*(d*x+c))+45*exp(4*I*(d*x+c))+10*exp(2*I*(d*x+c))+1)/d/(e
xp(2*I*(d*x+c))+1)^10
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(85) = 170$.

Time = 0.09 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.97

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{128\left(-210i\,a^3e^{(12i\,dx+12i\,c)} - 252i\,a^3e^{(10i\,dx+10i\,c)} - 210i\,a^3e^{(8i\,dx+8i\,c)} - 120\right.}{105\left(de^{(20i\,dx+20i\,c)} + 10\,de^{(18i\,dx+18i\,c)} + 45\,de^{(16i\,dx+16i\,c)} + 120\,de^{(14i\,dx+14i\,c)} + 210\,de^{(12i\,dx+12i\,c)} + 252\right.}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-128/105*(-210*I*a^3*e^(12*I*d*x + 12*I*c) - 252*I*a^3*e^(10*I*d*x + 10*I*
c) - 210*I*a^3*e^(8*I*d*x + 8*I*c) - 120*I*a^3*e^(6*I*d*x + 6*I*c) - 45*I*
a^3*e^(4*I*d*x + 4*I*c) - 10*I*a^3*e^(2*I*d*x + 2*I*c) - I*a^3)/(d*e^(20*I
*d*x + 20*I*c) + 10*d*e^(18*I*d*x + 18*I*c) + 45*d*e^(16*I*d*x + 16*I*c) +
 120*d*e^(14*I*d*x + 14*I*c) + 210*d*e^(12*I*d*x + 12*I*c) + 252*d*e^(10*I
*d*x + 10*I*c) + 210*d*e^(8*I*d*x + 8*I*c) + 120*d*e^(6*I*d*x + 6*I*c) + 4
5*d*e^(4*I*d*x + 4*I*c) + 10*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\begin{aligned}\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -ia^3\Bigg(&\int i\sec^8(c+dx)\,dx \\ &+ \int \left(-3\tan(c+dx)\sec^8(c+dx)\right)\,dx \\ &+ \int \tan^3(c+dx)\sec^8(c+dx)\,dx \\ &+ \int \left(-3i\tan^2(c+dx)\sec^8(c+dx)\right)\,dx\Bigg)\end{aligned}$$

input
```
integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-I*a**3*(Integral(I*sec(c + d*x)**8, x) + Integral(-3*tan(c + d*x)*sec(c +
 d*x)**8, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**8, x) + Integral(-3*
I*tan(c + d*x)**2*sec(c + d*x)**8, x))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{21i\,a^3\tan{(dx+c)}^{10}+70\,a^3\tan{(dx+c)}^9+240\,a^3\tan{(dx+c)}^7-210i\,a^3\tan{(dx+c)}^6+252\,a^3\tan{(}}{210\,d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/210*(21*I*a^3*tan(d*x + c)^10 + 70*a^3*tan(d*x + c)^9 + 240*a^3*tan(d*x
 + c)^7 - 210*I*a^3*tan(d*x + c)^6 + 252*a^3*tan(d*x + c)^5 - 420*I*a^3*ta
n(d*x + c)^4 - 315*I*a^3*tan(d*x + c)^2 - 210*a^3*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{21i\,a^3\tan{(dx+c)}^{10}+70\,a^3\tan{(dx+c)}^9+240\,a^3\tan{(dx+c)}^7-210i\,a^3\tan{(dx+c)}^6+252\,a^3\tan{(}}{210\,d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/210*(21*I*a^3*tan(d*x + c)^10 + 70*a^3*tan(d*x + c)^9 + 240*a^3*tan(d*x
 + c)^7 - 210*I*a^3*tan(d*x + c)^6 + 252*a^3*tan(d*x + c)^5 - 420*I*a^3*ta
n(d*x + c)^4 - 315*I*a^3*tan(d*x + c)^2 - 210*a^3*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.39

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{a^3\,\sin\left(c+d\,x\right)\,\left(-210\cos\left(c+d\,x\right)^9-\cos\left(c+d\,x\right)^8\,\sin\left(c+d\,x\right)\,315\mathrm{i}-\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)^3\,4$$

input
```
int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^8,x)
```

output
```
-(a^3*sin(c + d*x)*(70*cos(c + d*x)*sin(c + d*x)^8 - cos(c + d*x)^8*sin(c
+ d*x)*315i - 210*cos(c + d*x)^9 + sin(c + d*x)^9*21i + 240*cos(c + d*x)^3
*sin(c + d*x)^6 - cos(c + d*x)^4*sin(c + d*x)^5*210i + 252*cos(c + d*x)^5*
sin(c + d*x)^4 - cos(c + d*x)^6*sin(c + d*x)^3*420i))/(210*d*cos(c + d*x)^
10)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.76

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{\sin(dx+c)\,a^3\left(-128\cos(dx+c)\sin(dx+c)^8+576\cos(dx+c)\sin(dx+c)^6-1008\cos(dx+c)\sin(d}{210d\left(\sin(dx+c)^{10}\right.}$$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(sin(c + d*x)*a**3*( - 128*cos(c + d*x)*sin(c + d*x)**8 + 576*cos(c + d*x)
*sin(c + d*x)**6 - 1008*cos(c + d*x)*sin(c + d*x)**4 + 840*cos(c + d*x)*si
n(c + d*x)**2 - 210*cos(c + d*x) - 84*sin(c + d*x)**9*i + 420*sin(c + d*x)
**7*i - 840*sin(c + d*x)**5*i + 840*sin(c + d*x)**3*i - 315*sin(c + d*x)*i
))/(210*d*(sin(c + d*x)**10 - 5*sin(c + d*x)**8 + 10*sin(c + d*x)**6 - 10*
sin(c + d*x)**4 + 5*sin(c + d*x)**2 - 1))
```

3.37 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2i(a+ia\tan(c+dx))^6}{3a^3d} + \frac{4i(a+ia\tan(c+dx))^7}{7a^4d} - \frac{i(a+ia\tan(c+dx))^8}{8a^5d}$$

output
```
-2/3*I*(a+I*a*tan(d*x+c))^6/a^3/d+4/7*I*(a+I*a*tan(d*x+c))^7/a^4/d-1/8*I*(
a+I*a*tan(d*x+c))^8/a^5/d
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{a^3\sec^8(c+dx)(8+29\cos(2(c+dx))-27i\sin(2(c+dx)))(-i\cos(6(c+dx))+\sin(6(c+dx)))}{168d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*Sec[c + d*x]^8*(8 + 29*Cos[2*(c + d*x)] - (27*I)*Sin[2*(c + d*x)])*((
-I)*Cos[6*(c + d*x)] + Sin[6*(c + d*x)]))/(168*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(c+dx)^6(a+ia\tan(c+dx))^3dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int (a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^5d(ia\tan(c+dx))}{a^5d}$$
$$\Big\downarrow \text{49}$$
$$-\frac{i\int \left((i\tan(c+dx)a+a)^7-4a(i\tan(c+dx)a+a)^6+4a^2(i\tan(c+dx)a+a)^5\right)d(ia\tan(c+dx))}{a^5d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{i\left(\frac{2}{3}a^2(a+ia\tan(c+dx))^6+\frac{1}{8}(a+ia\tan(c+dx))^8-\frac{4}{7}a(a+ia\tan(c+dx))^7\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*((2*a^2*(a + I*a*Tan[c + d*x])^6)/3 - (4*a*(a + I*a*Tan[c + d*x])^7)
/7 + (a + I*a*Tan[c + d*x])^8/8))/(a^5*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 59.88 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

method	result
risch	$\frac{32ia^3\left(56\,\mathrm{e}^{10i(dx+c)}+70\,\mathrm{e}^{8i(dx+c)}+56\,\mathrm{e}^{6i(dx+c)}+28\,\mathrm{e}^{4i(dx+c)}+8\,\mathrm{e}^{2i(dx+c)}+1\right)}{21d\left(\mathrm{e}^{2i(dx+c)}+1\right)^8}$
derivativedivides	$\frac{-ia^3\left(\frac{\sin(dx+c)^4}{8\cos(dx+c)^8}+\frac{\sin(dx+c)^4}{12\cos(dx+c)^6}+\frac{\sin(dx+c)^4}{24\cos(dx+c)^4}\right)-3a^3\left(\frac{\sin(dx+c)^3}{7\cos(dx+c)^7}+\frac{4\sin(dx+c)^3}{35\cos(dx+c)^5}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^3}\right)+\frac{ia^3}{2\cos(dx+c)^6}}{d}$
default	$\frac{-ia^3\left(\frac{\sin(dx+c)^4}{8\cos(dx+c)^8}+\frac{\sin(dx+c)^4}{12\cos(dx+c)^6}+\frac{\sin(dx+c)^4}{24\cos(dx+c)^4}\right)-3a^3\left(\frac{\sin(dx+c)^3}{7\cos(dx+c)^7}+\frac{4\sin(dx+c)^3}{35\cos(dx+c)^5}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^3}\right)+\frac{ia^3}{2\cos(dx+c)^6}}{d}$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
32/21*I*a^3*(56*exp(10*I*(d*x+c))+70*exp(8*I*(d*x+c))+56*exp(6*I*(d*x+c))+
28*exp(4*I*(d*x+c))+8*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^8
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(64) = 128$.

Time = 0.07 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.16

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{32\left(-56i\,a^3e^{(10i\,dx+10i\,c)}-70i\,a^3e^{(8i\,dx+8i\,c)}-56i\,a^3e^{(6i\,dx+6i\,c)}-28i\,a^3e^{(4i\,dx+4i\,c)}-\cdots\right.}{21\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+\cdots}\right.}$$

input

```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output

```
-32/21*(-56*I*a^3*e^(10*I*d*x + 10*I*c) - 70*I*a^3*e^(8*I*d*x + 8*I*c) - 5
6*I*a^3*e^(6*I*d*x + 6*I*c) - 28*I*a^3*e^(4*I*d*x + 4*I*c) - 8*I*a^3*e^(2*
I*d*x + 2*I*c) - I*a^3)/(d*e^(16*I*d*x + 16*I*c) + 8*d*e^(14*I*d*x + 14*I*
c) + 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(10*I*d*x + 10*I*c) + 70*d*e^(8*I
*d*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) + 28*d*e^(4*I*d*x + 4*I*c) + 8*d*
e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx = -ia^3\left(\int i\sec^6(c+dx)\,dx + \int\left(-3\tan(c+dx)\sec^6(c+dx)\right)dx + \int\tan^3(c+dx)\sec^6(c+dx)\,dx + \int\left(-3i\tan^2(c+dx)\sec^6(c+dx)\right)dx\right)$$

input

```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**3,x)
```

output

```
-I*a**3*(Integral(I*sec(c + d*x)**6, x) + Integral(-3*tan(c + d*x)*sec(c +
 d*x)**6, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**6, x) + Integral(-3*
I*tan(c + d*x)**2*sec(c + d*x)**6, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.32

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{21i\,a^3\tan\left(dx+c\right)^8+72\,a^3\tan\left(dx+c\right)^7-28i\,a^3\tan\left(dx+c\right)^6+168\,a^3\tan\left(dx+c\right)^5-210i\,a^3\tan\left(c\right.}{168\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/168*(21*I*a^3*tan(d*x + c)^8 + 72*a^3*tan(d*x + c)^7 - 28*I*a^3*tan(d*x
 + c)^6 + 168*a^3*tan(d*x + c)^5 - 210*I*a^3*tan(d*x + c)^4 + 56*a^3*tan(d
*x + c)^3 - 252*I*a^3*tan(d*x + c)^2 - 168*a^3*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.32

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{21i\,a^3\tan\left(dx+c\right)^8+72\,a^3\tan\left(dx+c\right)^7-28i\,a^3\tan\left(dx+c\right)^6+168\,a^3\tan\left(dx+c\right)^5-210i\,a^3\tan\left(c\right.}{168\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/168*(21*I*a^3*tan(d*x + c)^8 + 72*a^3*tan(d*x + c)^7 - 28*I*a^3*tan(d*x
 + c)^6 + 168*a^3*tan(d*x + c)^5 - 210*I*a^3*tan(d*x + c)^4 + 56*a^3*tan(d
*x + c)^3 - 252*I*a^3*tan(d*x + c)^2 - 168*a^3*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx =$$
$$-\frac{a^3\,\sin\left(c+d\,x\right)\,\left(-168\cos\left(c+d\,x\right)^7-\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)\,252\mathrm{i}+56\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)\right.}{}$$

input
```
int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^6,x)
```

output
```
-(a^3*sin(c + d*x)*(72*cos(c + d*x)*sin(c + d*x)^6 - cos(c + d*x)^6*sin(c
+ d*x)*252i - 168*cos(c + d*x)^7 + sin(c + d*x)^7*21i - cos(c + d*x)^2*sin
(c + d*x)^5*28i + 168*cos(c + d*x)^3*sin(c + d*x)^4 - cos(c + d*x)^4*sin(c
 + d*x)^3*210i + 56*cos(c + d*x)^5*sin(c + d*x)^2))/(168*d*cos(c + d*x)^8)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.89

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{\sin\left(dx+c\right)a^3\left(-128\cos\left(dx+c\right)\sin\left(dx+c\right)^6+448\cos\left(dx+c\right)\sin\left(dx+c\right)^4-560\cos\left(dx+c\right)\sin\left(dx\right.\right.}{168d\left(\sin\left(dx+c\right)^8-4\sin\left(dx+c\right)^6\right.}$$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(sin(c + d*x)*a**3*( - 128*cos(c + d*x)*sin(c + d*x)**6 + 448*cos(c + d*x)
*sin(c + d*x)**4 - 560*cos(c + d*x)*sin(c + d*x)**2 + 168*cos(c + d*x) - 9
1*sin(c + d*x)**7*i + 364*sin(c + d*x)**5*i - 546*sin(c + d*x)**3*i + 252*
sin(c + d*x)*i))/(168*d*(sin(c + d*x)**8 - 4*sin(c + d*x)**6 + 6*sin(c + d
*x)**4 - 4*sin(c + d*x)**2 + 1))
```

3.38 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2i(a+ia\tan(c+dx))^5}{5a^2d} + \frac{i(a+ia\tan(c+dx))^6}{6a^3d}$$

output
```
-2/5*I*(a+I*a*tan(d*x+c))^5/a^2/d+1/6*I*(a+I*a*tan(d*x+c))^6/a^3/d
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{a^3(7-5i\tan(c+dx))(-i+\tan(c+dx))^5}{30d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*(7 - (5*I)*Tan[c + d*x])*(-I + Tan[c + d*x])^5)/(30*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(c+dx)^4(a+ia\tan(c+dx))^3dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))(i\tan(c+dx)a+a)^4d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{i\int \left(2a(i\tan(c+dx)a+a)^4-(i\tan(c+dx)a+a)^5\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{i\left(\frac{2}{5}a(a+ia\tan(c+dx))^5-\frac{1}{6}(a+ia\tan(c+dx))^6\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*((2*a*(a + I*a*Tan[c + d*x])^5)/5 - (a + I*a*Tan[c + d*x])^6/6))/(a^
3*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 15.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

method	result
risch	$\frac{32 i a^{3} \left(15 \,{\mathrm e}^{8 i \left(d x +c \right)}+20 \,{\mathrm e}^{6 i \left(d x +c \right)}+15 \,{\mathrm e}^{4 i \left(d x +c \right)}+6 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{15 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{6}}$
derivativedivides	$\frac{-i a^{3} \left(\frac{\sin \left(d x +c \right)^{4}}{6 \cos \left(d x +c \right)^{6}}+\frac{\sin \left(d x +c \right)^{4}}{12 \cos \left(d x +c \right)^{4}}\right)-3 a^{3} \left(\frac{\sin \left(d x +c \right)^{3}}{5 \cos \left(d x +c \right)^{5}}+\frac{2 \sin \left(d x +c \right)^{3}}{15 \cos \left(d x +c \right)^{3}}\right)+\frac{3 i a^{3}}{4 \cos \left(d x +c \right)^{4}}-a^{3} \left(-\frac{2}{3}-\frac{\sec \left(d x +c \right)^{2}}{3}\right) \tan(dx}{d}$
default	$\frac{-i a^{3} \left(\frac{\sin \left(d x +c \right)^{4}}{6 \cos \left(d x +c \right)^{6}}+\frac{\sin \left(d x +c \right)^{4}}{12 \cos \left(d x +c \right)^{4}}\right)-3 a^{3} \left(\frac{\sin \left(d x +c \right)^{3}}{5 \cos \left(d x +c \right)^{5}}+\frac{2 \sin \left(d x +c \right)^{3}}{15 \cos \left(d x +c \right)^{3}}\right)+\frac{3 i a^{3}}{4 \cos \left(d x +c \right)^{4}}-a^{3} \left(-\frac{2}{3}-\frac{\sec \left(d x +c \right)^{2}}{3}\right) \tan(dx}{d}$

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
32/15*I*a^3*(15*exp(8*I*(d*x+c))+20*exp(6*I*(d*x+c))+15*exp(4*I*(d*x+c))+6
*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^6
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(43) = 86$.

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.53

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{32\left(-15i\,a^3e^{(8i\,dx+8i\,c)}-20i\,a^3e^{(6i\,dx+6i\,c)}-15i\,a^3e^{(4i\,dx+4i\,c)}-6i\,a^3e^{(2i\,dx+2i\,c)}-i\,a^3\right)}{15\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+\right.}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-32/15*(-15*I*a^3*e^(8*I*d*x + 8*I*c) - 20*I*a^3*e^(6*I*d*x + 6*I*c) - 15*
I*a^3*e^(4*I*d*x + 4*I*c) - 6*I*a^3*e^(2*I*d*x + 2*I*c) - I*a^3)/(d*e^(12*
I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 2
0*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*
c) + d)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx = -ia^3\left(\int i\sec^4{(c+dx)}\,dx + \int \left(-3\tan{(c+dx)}\sec^4{(c+dx)}\right)dx + \int \tan^3{(c+dx)}\sec^4{(c+dx)}\,dx + \int \left(-3i\tan^2{(c+dx)}\sec^4{(c+dx)}\right)dx\right)$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-I*a**3*(Integral(I*sec(c + d*x)**4, x) + Integral(-3*tan(c + d*x)*sec(c +
 d*x)**4, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**4, x) + Integral(-3*
I*tan(c + d*x)**2*sec(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{5i\,a^3\tan\left(dx+c\right)^6+18\,a^3\tan\left(dx+c\right)^5-15i\,a^3\tan\left(dx+c\right)^4+20\,a^3\tan\left(dx+c\right)^3-45i\,a^3\tan\left(dx+\right.}{30\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/30*(5*I*a^3*tan(d*x + c)^6 + 18*a^3*tan(d*x + c)^5 - 15*I*a^3*tan(d*x +
 c)^4 + 20*a^3*tan(d*x + c)^3 - 45*I*a^3*tan(d*x + c)^2 - 30*a^3*tan(d*x +
 c))/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{5i\,a^3\tan\left(dx+c\right)^6+18\,a^3\tan\left(dx+c\right)^5-15i\,a^3\tan\left(dx+c\right)^4+20\,a^3\tan\left(dx+c\right)^3-45i\,a^3\tan\left(dx+\right.}{30\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/30*(5*I*a^3*tan(d*x + c)^6 + 18*a^3*tan(d*x + c)^5 - 15*I*a^3*tan(d*x +
 c)^4 + 20*a^3*tan(d*x + c)^3 - 45*I*a^3*tan(d*x + c)^2 - 30*a^3*tan(d*x +
 c))/d
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.07

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx =$$
$$-\frac{a^3\,\sin\left(c+d\,x\right)\,\left(-30\cos\left(c+d\,x\right)^5-\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)\,45\mathrm{i}+20\cos\left(c+d\,x\right)^3\sin\left(c+d\,x\right)^2\right.}{30\,d\cos\left(c+d\,x\right)^6}$$

input
```
int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^4,x)
```

output
```
-(a^3*sin(c + d*x)*(18*cos(c + d*x)*sin(c + d*x)^4 - cos(c + d*x)^4*sin(c
+ d*x)*45i - 30*cos(c + d*x)^5 + sin(c + d*x)^5*5i - cos(c + d*x)^2*sin(c
+ d*x)^3*15i + 20*cos(c + d*x)^3*sin(c + d*x)^2))/(30*d*cos(c + d*x)^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.15

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{\sin\left(dx+c\right)a^3\left(-32\cos\left(dx+c\right)\sin\left(dx+c\right)^4+80\cos\left(dx+c\right)\sin\left(dx+c\right)^2-30\cos\left(dx+c\right)-25\sin\left(\right.\right.}{30d\left(\sin\left(dx+c\right)^6-3\sin\left(dx+c\right)^4+3\sin\left(dx+c\right)^2-1\right.}$$

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(sin(c + d*x)*a**3*( - 32*cos(c + d*x)*sin(c + d*x)**4 + 80*cos(c + d*x)*s
in(c + d*x)**2 - 30*cos(c + d*x) - 25*sin(c + d*x)**5*i + 75*sin(c + d*x)*
*3*i - 45*sin(c + d*x)*i))/(30*d*(sin(c + d*x)**6 - 3*sin(c + d*x)**4 + 3*
sin(c + d*x)**2 - 1))
```

3.39 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{i(a+ia\tan(c+dx))^4}{4ad}$$

output
```
-1/4*I*(a+I*a*tan(d*x+c))^4/a/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{a^3\tan(c+dx)\left(4+6i\tan(c+dx)-4\tan^2(c+dx)-i\tan^3(c+dx)\right)}{4d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*Tan[c + d*x]*(4 + (6*I)*Tan[c + d*x] - 4*Tan[c + d*x]^2 - I*Tan[c + d
*x]^3))/(4*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+ia\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int(i\tan(c+dx)a+a)^3d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$-\frac{i(a+ia\tan(c+dx))^4}{4ad}$$

input
```
Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-1/4*I)*(a + I*a*Tan[c + d*x])^4)/(a*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(23) = 46$.

Time = 3.67 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

method	result	size
risch	$\frac{4ia^3\left(4\,\mathrm{e}^{6i(dx+c)}+6\,\mathrm{e}^{4i(dx+c)}+4\,\mathrm{e}^{2i(dx+c)}+1\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}$	58
derivativedivides	$\frac{-\frac{ia^3\sin(dx+c)^4}{4\cos(dx+c)^4}-\frac{a^3\sin(dx+c)^3}{\cos(dx+c)^3}+\frac{3ia^3}{2\cos(dx+c)^2}+a^3\tan(dx+c)}{d}$	73
default	$\frac{-\frac{ia^3\sin(dx+c)^4}{4\cos(dx+c)^4}-\frac{a^3\sin(dx+c)^3}{\cos(dx+c)^3}+\frac{3ia^3}{2\cos(dx+c)^2}+a^3\tan(dx+c)}{d}$	73

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
4*I*a^3*(4*exp(6*I*(d*x+c))+6*exp(4*I*(d*x+c))+4*exp(2*I*(d*x+c))+1)/d/(ex
p(2*I*(d*x+c))+1)^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(21) = 42$.

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.74

$$\begin{aligned}&\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx\\&=-\frac{4\left(-4i\,a^3e^{(6i\,dx+6i\,c)}-6i\,a^3e^{(4i\,dx+4i\,c)}-4i\,a^3e^{(2i\,dx+2i\,c)}-i\,a^3\right)}{de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d}\end{aligned}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-4*(-4*I*a^3*e^(6*I*d*x + 6*I*c) - 6*I*a^3*e^(4*I*d*x + 4*I*c) - 4*I*a^3*e
^(2*I*d*x + 2*I*c) - I*a^3)/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*
c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx = -ia^3\left(\int i\sec^2(c+dx)\,dx + \int \left(-3\tan(c+dx)\sec^2(c+dx)\right)\,dx + \int \tan^3(c+dx)\sec^2(c+dx)\,dx + \int \left(-3i\tan^2(c+dx)\sec^2(c+dx)\right)\,dx\right)$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-I*a**3*(Integral(I*sec(c + d*x)**2, x) + Integral(-3*tan(c + d*x)*sec(c +
 d*x)**2, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**2, x) + Integral(-3*
I*tan(c + d*x)**2*sec(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{i\,(i\,a\tan(dx+c)+a)^4}{4\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/4*I*(I*a*tan(d*x + c) + a)^4/(a*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(21) = 42$.

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= -\frac{i\,a^3\tan\left(dx+c\right)^4+4\,a^3\tan\left(dx+c\right)^3-6i\,a^3\tan\left(dx+c\right)^2-4\,a^3\tan\left(dx+c\right)}{4\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/4*(I*a^3*tan(d*x + c)^4 + 4*a^3*tan(d*x + c)^3 - 6*I*a^3*tan(d*x + c)^2
 - 4*a^3*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{-\frac{a^3\tan(c+d\,x)^4\,1\mathrm{i}}{4}-a^3\tan(c+d\,x)^3+\frac{a^3\tan(c+d\,x)^2\,3\mathrm{i}}{2}+a^3\tan(c+d\,x)}{d}$$

input
```
int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^2,x)
```

output
```
(a^3*tan(c + d*x) + (a^3*tan(c + d*x)^2*3i)/2 - a^3*tan(c + d*x)^3 - (a^3*
tan(c + d*x)^4*1i)/4)/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.00

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{\sin(dx+c)\,a^3\left(-8\cos(dx+c)\sin(dx+c)^2+4\cos(dx+c)-7\sin(dx+c)^3 i+6\sin(dx+c)\,i\right)}{4d\left(\sin(dx+c)^4-2\sin(dx+c)^2+1\right)}$$

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(sin(c + d*x)*a**3*( - 8*cos(c + d*x)*sin(c + d*x)**2 + 4*cos(c + d*x) - 7
*sin(c + d*x)**3*i + 6*sin(c + d*x)*i))/(4*d*(sin(c + d*x)**4 - 2*sin(c +
d*x)**2 + 1))
```

3.40 $\int (a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int (a+ia\tan(c+dx))^3\,dx = 4a^3x - \frac{4ia^3\log(\cos(c+dx))}{d} - \frac{2a^3\tan(c+dx)}{d} + \frac{ia(a+ia\tan(c+dx))^2}{2d}$$

output
```
4*a^3*x-4*I*a^3*ln(cos(d*x+c))/d-2*a^3*tan(d*x+c)/d+1/2*I*a*(a+I*a*tan(d*x
+c))^2/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int (a+ia\tan(c+dx))^3\,dx = \frac{ia^3(8\log(i+\tan(c+dx))+6i\tan(c+dx)-\tan^2(c+dx))}{2d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/2)*a^3*(8*Log[I + Tan[c + d*x]] + (6*I)*Tan[c + d*x] - Tan[c + d*x]^2)
)/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3959, 3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3959}$$

$$2a\int (i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$2a\int (i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3958}$$

$$2a\left(2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$2a\left(2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3956}$$

$$2a\left(-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}$$

input
```
Int[(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/2)*a*(a + I*a*Tan[c + d*x])^2)/d + 2*a*(2*a^2*x - ((2*I)*a^2*Log[Cos[c
 + d*x]])/d - (a^2*Tan[c + d*x])/d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

Maple [A] (warning: unable to verify)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{a^3\left(-3\tan(dx+c)-\frac{i\tan(dx+c)^2}{2}+2i\ln\left(1+\tan(dx+c)^2\right)+4\arctan(\tan(dx+c))\right)}{d}$
default	$\frac{a^3\left(-3\tan(dx+c)-\frac{i\tan(dx+c)^2}{2}+2i\ln\left(1+\tan(dx+c)^2\right)+4\arctan(\tan(dx+c))\right)}{d}$
parallelrisch	$\frac{-ia^3\tan(dx+c)^2+4ia^3\ln\left(1+\tan(dx+c)^2\right)+8a^3xd-6a^3\tan(dx+c)}{2d}$
norman	$4a^3x-\frac{3a^3\tan(dx+c)}{d}-\frac{ia^3\tan(dx+c)^2}{2d}+\frac{2ia^3\ln\left(1+\tan(dx+c)^2\right)}{d}$
risch	$-\frac{8a^3c}{d}-\frac{2ia^3\left(4\,\mathrm{e}^{2i(dx+c)}+3\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{4ia^3\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$
parts	$a^3x-\frac{ia^3\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{3ia^3\ln\left(1+\tan(dx+c)^2\right)}{2d}-\frac{3a^3(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}$

input
```
int((a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*a^3*(-3*tan(d*x+c)-1/2*I*tan(d*x+c)^2+2*I*ln(1+tan(d*x+c)^2)+4*arctan(
tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.54

$$\int (a+ia\tan(c+dx))^3\,dx = -\frac{2\left(4i\,a^3e^{(2i\,dx+2i\,c)}+3i\,a^3+2\left(i\,a^3e^{(4i\,dx+4i\,c)}+2i\,a^3e^{(2i\,dx+2i\,c)}+i\,a^3\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

input
```
integrate((a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2*(4*I*a^3*e^(2*I*d*x + 2*I*c) + 3*I*a^3 + 2*(I*a^3*e^(4*I*d*x + 4*I*c) +
 2*I*a^3*e^(2*I*d*x + 2*I*c) + I*a^3)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(
4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.49

$$\int (a+ia\tan(c+dx))^3\,dx=-\frac{4ia^3\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{-8ia^3e^{2ic}e^{2idx}-6ia^3}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d}$$

input
```
integrate((a+I*a*tan(d*x+c))**3,x)
```

output
```
-4*I*a**3*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-8*I*a**3*exp(2*I*c)*exp(2*
I*d*x) - 6*I*a**3)/(d*exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*x
) + d)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\begin{aligned}\int (a+ia\tan(c+dx))^3\,dx=a^3x+\frac{3\,(dx+c-\tan(dx+c))a^3}{d}\\ &+\frac{i\,a^3\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin(dx+c)^2-1\right)\right)}{2\,d}\\ &+\frac{3i\,a^3\log(\sec(dx+c))}{d}\end{aligned}$$

input
```
integrate((a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
a^3*x + 3*(d*x + c - tan(d*x + c))*a^3/d + 1/2*I*a^3*(1/(sin(d*x + c)^2 -
1) - log(sin(d*x + c)^2 - 1))/d + 3*I*a^3*log(sec(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int (a+ia\tan(c+dx))^3\,dx = \frac{4i\,a^3\log\left(\tan\left(dx+c\right)+i\right)}{d} - \frac{i\,a^3d\tan\left(dx+c\right)^2+6\,a^3d\tan\left(dx+c\right)}{2\,d^2}$$

input
```
integrate((a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
4*I*a^3*log(tan(d*x + c) + I)/d - 1/2*(I*a^3*d*tan(d*x + c)^2 + 6*a^3*d*ta
n(d*x + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.65

$$\int (a+ia\tan(c+dx))^3\,dx = -\frac{a^3\left(6\tan(c+d\,x)-\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,8\mathrm{i}+\tan(c+d\,x)^2\,1\mathrm{i}\right)}{2\,d}$$

input
```
int((a + a*tan(c + d*x)*1i)^3,x)
```

output
```
-(a^3*(6*tan(c + d*x) - log(tan(c + d*x) + 1i)*8i + tan(c + d*x)^2*1i))/(2
*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int (a+ia\tan(c+dx))^3\,dx$$
$$=\frac{a^3\left(4\log\left(\tan\left(dx+c\right)^2+1\right)i-\tan\left(dx+c\right)^2i-6\tan\left(dx+c\right)+8dx\right)}{2d}$$

input
```
int((a+I*a*tan(d*x+c))^3,x)
```

output
```
(a**3*(4*log(tan(c + d*x)**2 + 1)*i - tan(c + d*x)**2*i - 6*tan(c + d*x) +
 8*d*x))/(2*d)
```

3.41 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 48

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx = -a^3x+\frac{ia^3\log(\cos(c+dx))}{d} - \frac{2ia^3}{d(1-i\tan(c+dx))}$$

output
```
-a^3*x+I*a^3*ln(cos(d*x+c))/d-2*I*a^3/d/(1-I*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{ia^3\left(\log(i+\tan(c+dx))+\frac{2a}{a-ia\tan(c+dx)}\right)}{d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*a^3*(Log[I + Tan[c + d*x]] + (2*a)/(a - I*a*Tan[c + d*x])))/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{i\tan(c+dx)a+a}{(a-ia\tan(c+dx))^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{ia^3\int \left(\frac{2a}{(a-ia\tan(c+dx))^2}+\frac{1}{ia\tan(c+dx)-a}\right)d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^3\left(\frac{2a}{a-ia\tan(c+dx)}+\log(a-ia\tan(c+dx))\right)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*a^3*(Log[a - I*a*Tan[c + d*x]] + (2*a)/(a - I*a*Tan[c + d*x])))/d
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{i a^{3} {\mathrm e}^{2 i \left(d x +c \right)}}{d}+\frac{2 a^{3} c}{d}+\frac{i a^{3} \ln \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{d}$
derivativedivides	$\frac{-i a^{3} \left(-\frac{\sin \left(d x +c \right)^{2}}{2}-\ln \left(\cos \left(d x +c \right)\right)\right)-3 a^{3} \left(-\frac{\cos \left(d x +c \right) \sin \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)-\frac{3 i a^{3} \cos \left(d x +c \right)^{2}}{2}+a^{3} \left(\frac{\cos \left(d x +c \right) \sin \left(d x +c \right)}{2}+\cdots\right)}{d}$
default	$\frac{-i a^{3} \left(-\frac{\sin \left(d x +c \right)^{2}}{2}-\ln \left(\cos \left(d x +c \right)\right)\right)-3 a^{3} \left(-\frac{\cos \left(d x +c \right) \sin \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)-\frac{3 i a^{3} \cos \left(d x +c \right)^{2}}{2}+a^{3} \left(\frac{\cos \left(d x +c \right) \sin \left(d x +c \right)}{2}+\cdots\right)}{d}$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-I/d*a^3*exp(2*I*(d*x+c))+2/d*a^3*c+I/d*a^3*ln(exp(2*I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{-i\,a^3e^{(2i\,dx+2i\,c)}+i\,a^3\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
(-I*a^3*e^(2*I*d*x + 2*I*c) + I*a^3*log(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{ia^3\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\begin{cases} -\frac{ia^3e^{2ic}e^{2idx}}{d} & \text{for } d\neq 0 \\ 2a^3xe^{2ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*a**3*log(exp(2*I*d*x) + exp(-2*I*c))/d + Piecewise((-I*a**3*exp(2*I*c)*e
xp(2*I*d*x)/d, Ne(d, 0)), (2*a**3*x*exp(2*I*c), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2\,(dx+c)a^3+i\,a^3\log\left(\tan\left(dx+c\right)^2+1\right)-\frac{4\,(a^3\tan(dx+c)-i\,a^3)}{\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(2*(d*x + c)*a^3 + I*a^3*log(tan(d*x + c)^2 + 1) - 4*(a^3*tan(d*x + c
) - I*a^3)/(tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{i\,a^3\log\left(\tan\left(dx+c\right)+i\right)}{d}+\frac{2\,a^3}{d(\tan\left(dx+c\right)+i)}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-I*a^3*log(tan(d*x + c) + I)/d + 2*a^3/(d*(tan(d*x + c) + I))
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx=\frac{2\,a^3}{d\,\left(\tan\left(c+d\,x\right)+1\mathrm{i}\right)}-\frac{a^3\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{d}$$

input
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(2*a^3)/(d*(tan(c + d*x) + 1i)) - (a^3*log(tan(c + d*x) + 1i)*1i)/d
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.77

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{a^3\left(2\cos\left(dx+c\right)\sin\left(dx+c\right)-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)i+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)i+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{d}$$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(a**3*(2*cos(c + d*x)*sin(c + d*x) - log(tan((c + d*x)/2)**2 + 1)*i + log(
tan((c + d*x)/2) - 1)*i + log(tan((c + d*x)/2) + 1)*i + 2*sin(c + d*x)**2*
i - c - d*x))/d
```

3.42 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{ia^5}{2d(a-ia\tan(c+dx))^2}$$

output
```
-1/2*I*a^5/d/(a-I*a*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx=\frac{ia^3}{2d(i+\tan(c+dx))^2}$$

input
```
Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/2)*a^3)/(d*(I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{17}$$

$$-\frac{ia^5}{2d(a-ia\tan(c+dx))^2}$$

input
```
Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-1/2*I)*a^5)/(d*(a - I*a*Tan[c + d*x])^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 12.59 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{i a^3 \mathrm{e}^{4i(dx+c)}}{8d}-\frac{i a^3 \mathrm{e}^{2i(dx+c)}}{4d}$
derivativedivides	$\frac{-\frac{i a^3 \sin(dx+c)^4}{4}-3a^3\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{3i a^3\cos(dx+c)^4}{4}+a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos}{}\right.}{4}\right.}{d}$
default	$\frac{-\frac{i a^3 \sin(dx+c)^4}{4}-3a^3\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{3i a^3\cos(dx+c)^4}{4}+a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos}{}\right.}{4}\right.}{d}$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*I/d*a^3*exp(4*I*(d*x+c))-1/4*I/d*a^3*exp(2*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx=\frac{-i\,a^3e^{(4i\,dx+4i\,c)}-2i\,a^3e^{(2i\,dx+2i\,c)}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/8*(-I*a^3*e^(4*I*d*x + 4*I*c) - 2*I*a^3*e^(2*I*d*x + 2*I*c))/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(22) = 44.

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.96

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx = \begin{cases} \frac{-4ia^3de^{4ic}e^{4idx}-8ia^3de^{2ic}e^{2idx}}{32d^2} & \text{for } d^2\neq 0 \\ x\left(\frac{a^3e^{4ic}}{2}+\frac{a^3e^{2ic}}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise(((-4*I*a**3*d*exp(4*I*c)*exp(4*I*d*x) - 8*I*a**3*d*exp(2*I*c)*ex
p(2*I*d*x))/(32*d**2), Ne(d**2, 0)), (x*(a**3*exp(4*I*c)/2 + a**3*exp(2*I*
c)/2), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(21) = 42.

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{-i\,a^3\tan\left(dx+c\right)^2-2\,a^3\tan\left(dx+c\right)+i\,a^3}{2\left(\tan\left(dx+c\right)^4+2\,\tan\left(dx+c\right)^2+1\right)d}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(-I*a^3*tan(d*x + c)^2 - 2*a^3*tan(d*x + c) + I*a^3)/((tan(d*x + c)^4
 + 2*tan(d*x + c)^2 + 1)*d)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{i\,a^3}{2\,d(\tan(dx+c)+i)^2}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*I*a^3/(d*(tan(d*x + c) + I)^2)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{a^3\left(\frac{e^{c\,2i+d\,x\,2i}}{2}+\frac{e^{c\,4i+d\,x\,4i}}{4}\right)\,1i}{2\,d}$$

input
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
-(a^3*(exp(c*2i + d*x*2i)/2 + exp(c*4i + d*x*4i)/4)*1i)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{\sin(dx+c)\,a^3\left(-2\cos(dx+c)\sin(dx+c)^2+2\cos(dx+c)-2\sin(dx+c)^3 i+3\sin(dx+c)\,i\right)}{2d}$$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(sin(c + d*x)*a**3*( - 2*cos(c + d*x)*sin(c + d*x)**2 + 2*cos(c + d*x) - 2
*sin(c + d*x)**3*i + 3*sin(c + d*x)*i))/(2*d)
```

3.43 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 98

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{a^3x}{8}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3}-\frac{ia^7}{8d\left(a^2-ia^2\tan(c+dx)\right)^2}-\frac{ia^7}{8d\left(a^4-ia^4\tan(c+dx)\right)}$$

output
```
1/8*a^3*x-1/6*I*a^6/d/(a-I*a*tan(d*x+c))^3-1/8*I*a^7/d/(a^2-I*a^2*tan(d*x+
c))^2-1/8*I*a^7/d/(a^4-I*a^4*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.66

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{a^3(-10+9i\tan(c+dx)+3\tan^2(c+dx)+3\arctan(\tan(c+dx))(i+\tan(c+dx))^3)}{24d(i+\tan(c+dx))^3}$$

input
```
Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*(-10 + (9*I)*Tan[c + d*x] + 3*Tan[c + d*x]^2 + 3*ArcTan[Tan[c + d*x]]
*(I + Tan[c + d*x])^3))/(24*d*(I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^7\int \frac{1}{(a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^7\int \left(\frac{1}{2(a-ia\tan(c+dx))^4a}+\frac{1}{4(a-ia\tan(c+dx))^3a^2}+\frac{1}{8(\tan^2(c+dx)a^2+a^2)a^3}+\frac{1}{8(a-ia\tan(c+dx))^2a^3}\right)d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^7\left(\frac{i\arctan(\tan(c+dx))}{8a^4}+\frac{1}{8a^3(a-ia\tan(c+dx))}+\frac{1}{8a^2(a-ia\tan(c+dx))^2}+\frac{1}{6a(a-ia\tan(c+dx))^3}\right)}{d}$$

input
```
Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*a^7*(((I/8)*ArcTan[Tan[c + d*x]])/a^4 + 1/(6*a*(a - I*a*Tan[c + d*x]
)^3) + 1/(8*a^2*(a - I*a*Tan[c + d*x])^2) + 1/(8*a^3*(a - I*a*Tan[c + d*x]
))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 43.67 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.63

method	result
risch	$\frac{a^3 x}{8}-\frac{i a^3 e^{6 i (dx+c)}}{48 d}-\frac{3 i a^3 e^{4 i (dx+c)}}{32 d}-\frac{3 i a^3 e^{2 i (dx+c)}}{16 d}$
derivativedivides	$\frac{-i a^3\left(-\frac{\cos(dx+c)^4 \sin(dx+c)^2}{6}-\frac{\cos(dx+c)^4}{12}\right)-3 a^3\left(-\frac{\sin(dx+c)\cos(dx+c)^5}{6}+\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\right.}{d}$
default	$\frac{-i a^3\left(-\frac{\cos(dx+c)^4 \sin(dx+c)^2}{6}-\frac{\cos(dx+c)^4}{12}\right)-3 a^3\left(-\frac{\sin(dx+c)\cos(dx+c)^5}{6}+\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\right.}{d}$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*a^3*x-1/48*I/d*a^3*exp(6*I*(d*x+c))-3/32*I/d*a^3*exp(4*I*(d*x+c))-3/16
*I/d*a^3*exp(2*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{12\,a^3dx - 2i\,a^3e^{(6i\,dx+6i\,c)} - 9i\,a^3e^{(4i\,dx+4i\,c)} - 18i\,a^3e^{(2i\,dx+2i\,c)}}{96\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/96*(12*a^3*d*x - 2*I*a^3*e^(6*I*d*x + 6*I*c) - 9*I*a^3*e^(4*I*d*x + 4*I*
c) - 18*I*a^3*e^(2*I*d*x + 2*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.34

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{a^3x}{8} + \begin{cases} \frac{-512ia^3d^2e^{6ic}e^{6idx} - 2304ia^3d^2e^{4ic}e^{4idx} - 4608ia^3d^2e^{2ic}e^{2idx}}{24576d^3} & \text{for } d^3 \neq 0 \\ x\left(\frac{a^3e^{6ic}}{8} + \frac{3a^3e^{4ic}}{8} + \frac{3a^3e^{2ic}}{8}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**3,x)
```

output
```
a**3*x/8 + Piecewise(((-512*I*a**3*d**2*exp(6*I*c)*exp(6*I*d*x) - 2304*I*a
**3*d**2*exp(4*I*c)*exp(4*I*d*x) - 4608*I*a**3*d**2*exp(2*I*c)*exp(2*I*d*x
))/(24576*d**3), Ne(d**3, 0)), (x*(a**3*exp(6*I*c)/8 + 3*a**3*exp(4*I*c)/8
 + 3*a**3*exp(2*I*c)/8), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.07

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{3\,(dx+c)a^3 + \frac{3\,a^3\tan(dx+c)^5+8\,a^3\tan(dx+c)^3+6i\,a^3\tan(dx+c)^2+21\,a^3\tan(dx+c)-10i\,a^3}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{24\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/24*(3*(d*x + c)*a^3 + (3*a^3*tan(d*x + c)^5 + 8*a^3*tan(d*x + c)^3 + 6*I
*a^3*tan(d*x + c)^2 + 21*a^3*tan(d*x + c) - 10*I*a^3)/(tan(d*x + c)^6 + 3*
tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{1}{48}i\,a^3\left(\frac{3\,\log\left(\tan\left(dx+c\right)+i\right)}{d} - \frac{3\,\log\left(\tan\left(dx+c\right)-i\right)}{d} + \frac{2\,\left(-3i\,\tan\left(dx+c\right)^2+9\,\tan\left(dx+c\right)+10\right.}{d(\tan\left(dx+c\right)+i)^3}\right.$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/48*I*a^3*(3*log(tan(d*x + c) + I)/d - 3*log(tan(d*x + c) - I)/d + 2*(-3*
I*tan(d*x + c)^2 + 9*tan(d*x + c) + 10*I)/(d*(tan(d*x + c) + I)^3))
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{a^3\,x}{8}-\frac{\frac{a^3\tan(c+d\,x)^2}{8}+\frac{a^3\tan(c+d\,x)\,3\mathrm{i}}{8}-\frac{5\,a^3}{12}}{d\,\left(-\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2\,3\mathrm{i}+3\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(a^3*x)/8 - ((a^3*tan(c + d*x)*3i)/8 - (5*a^3)/12 + (a^3*tan(c + d*x)^2)/8
)/(d*(3*tan(c + d*x) - tan(c + d*x)^2*3i - tan(c + d*x)^3 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{a^3\left(16\cos\left(dx+c\right)\sin\left(dx+c\right)^5-34\cos\left(dx+c\right)\sin\left(dx+c\right)^3+21\cos\left(dx+c\right)\sin\left(dx+c\right)+16\sin\left(dx\right.\right.}{24d}$$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(a**3*(16*cos(c + d*x)*sin(c + d*x)**5 - 34*cos(c + d*x)*sin(c + d*x)**3 +
 21*cos(c + d*x)*sin(c + d*x) + 16*sin(c + d*x)**6*i - 42*sin(c + d*x)**4*
i + 36*sin(c + d*x)**2*i + 3*d*x))/(24*d)
```

3.44 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 160

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{5a^3x}{32} - \frac{ia^7}{16d(a-ia\tan(c+dx))^4} - \frac{ia^9}{12d\left(a^2-ia^2\tan(c+dx)\right)^3} - \frac{3ia^9}{32d\left(a^3-ia^3\tan(c+dx)\right)^2} - \frac{ia^9}{8d\left(a^6-ia^6\tan(c+dx)\right)} + \frac{ia^9}{32d\left(a^6+ia^6\tan(c+dx)\right)}$$

output
```
5/32*a^3*x-1/16*I*a^7/d/(a-I*a*tan(d*x+c))^4-1/12*I*a^9/d/(a^2-I*a^2*tan(d
*x+c))^3-3/32*I*a^9/d/(a^3-I*a^3*tan(d*x+c))^2-1/8*I*a^9/d/(a^6-I*a^6*tan(
d*x+c))+1/32*I*a^9/d/(a^6+I*a^6*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.86

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{ia^9\left(\frac{5i\arctan(\tan(c+dx))}{32a^6}+\frac{1}{16a^2(a-ia\tan(c+dx))^4}+\frac{1}{12a^3(a-ia\tan(c+dx))^3}+\frac{3}{32a^4(a-ia\tan(c+dx))^2}+\frac{1}{8a^5(a-ia\tan(c+dx))}\right.}{d}$$

input
```
Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*a^9*((((5*I)/32)*ArcTan[Tan[c + d*x]])/a^6 + 1/(16*a^2*(a - I*a*Tan[
c + d*x])^4) + 1/(12*a^3*(a - I*a*Tan[c + d*x])^3) + 3/(32*a^4*(a - I*a*Ta
n[c + d*x])^2) + 1/(8*a^5*(a - I*a*Tan[c + d*x])) - 1/(32*a^5*(a + I*a*Tan
[c + d*x]))))/d
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{\sec(c+dx)^8}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^9\int \frac{1}{(a-ia\tan(c+dx))^5(i\tan(c+dx)a+a)^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^9\int\left(\frac{1}{8a^5(a-ia\tan(c+dx))^2}+\frac{1}{32a^5(i\tan(c+dx)a+a)^2}+\frac{3}{16a^4(a-ia\tan(c+dx))^3}+\frac{1}{4a^3(a-ia\tan(c+dx))^4}+\frac{1}{4a^2(a-ia\tan(c+dx))^5}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^9\left(\frac{5i\arctan(\tan(c+dx))}{32a^6}+\frac{1}{8a^5(a-ia\tan(c+dx))}-\frac{1}{32a^5(a+ia\tan(c+dx))}+\frac{3}{32a^4(a-ia\tan(c+dx))^2}+\frac{1}{12a^3(a-ia\tan(c+dx))^3}+\right.}{d}$$

input
```
Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*a^9*((((5*I)/32)*ArcTan[Tan[c + d*x]])/a^6 + 1/(16*a^2*(a - I*a*Tan[
c + d*x])^4) + 1/(12*a^3*(a - I*a*Tan[c + d*x])^3) + 3/(32*a^4*(a - I*a*Ta
n[c + d*x])^2) + 1/(8*a^5*(a - I*a*Tan[c + d*x])) - 1/(32*a^5*(a + I*a*Tan
[c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 128.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.61

method	result
risch	$\frac{5a^3x}{32} - \frac{ia^3 e^{8i(dx+c)}}{256d} - \frac{5ia^3 e^{6i(dx+c)}}{192d} - \frac{5ia^3 e^{4i(dx+c)}}{64d} - \frac{9ia^3 \cos(2dx+2c)}{64d} + \frac{11a^3 \sin(2dx+2c)}{64d}$
derivativedivides	$\frac{-ia^3\left(-\frac{\cos(dx+c)^6 \sin(dx+c)^2}{8} - \frac{\cos(dx+c)^6}{24}\right) - 3a^3\left(-\frac{\sin(dx+c)\cos(dx+c)^7}{8} + \frac{\left(\cos(dx+c)^5 + \frac{5\cos(dx+c)^3}{4} + \frac{15\cos(dx+c)}{8}\right)}{48}\right.}{}$
default	$\frac{-ia^3\left(-\frac{\cos(dx+c)^6 \sin(dx+c)^2}{8} - \frac{\cos(dx+c)^6}{24}\right) - 3a^3\left(-\frac{\sin(dx+c)\cos(dx+c)^7}{8} + \frac{\left(\cos(dx+c)^5 + \frac{5\cos(dx+c)^3}{4} + \frac{15\cos(dx+c)}{8}\right)}{48}\right.}{}$

input
```
int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
5/32*a^3*x-1/256*I/d*a^3*exp(8*I*(d*x+c))-5/192*I/d*a^3*exp(6*I*(d*x+c))-5
/64*I/d*a^3*exp(4*I*(d*x+c))-9/64*I/d*a^3*cos(2*d*x+2*c)+11/64/d*a^3*sin(2
*d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.58

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{\left(120\,a^3dxe^{(2i\,dx+2i\,c)} - 3i\,a^3e^{(10i\,dx+10i\,c)} - 20i\,a^3e^{(8i\,dx+8i\,c)} - 60i\,a^3e^{(6i\,dx+6i\,c)} - 120i\,a^3e^{(4i\,dx+4i\,c)} + 12i\,a\right.}{768\,d}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/768*(120*a^3*d*x*e^(2*I*d*x + 2*I*c) - 3*I*a^3*e^(10*I*d*x + 10*I*c) - 2
0*I*a^3*e^(8*I*d*x + 8*I*c) - 60*I*a^3*e^(6*I*d*x + 6*I*c) - 120*I*a^3*e^(
4*I*d*x + 4*I*c) + 12*I*a^3)*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.41

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{5a^3x}{32} + \begin{cases} \frac{(-25165824ia^3d^4e^{10ic}e^{8idx}-167772160ia^3d^4e^{8ic}e^{6idx}-503316480ia^3d^4e^{6ic}e^{4idx}-1006632960ia^3d^4e^{4ic}e^{2idx}+100663296ia^3d^4e^{-2idx})}{6442450944d^5} \\ x\left(-\frac{5a^3}{32}+\frac{(a^3e^{10ic}+5a^3e^{8ic}+10a^3e^{6ic}+10a^3e^{4ic}+5a^3e^{2ic}+a^3)e^{-2ic}}{32}\right) \end{cases}$$

input
```
integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**3,x)
```

output
```
5*a**3*x/32 + Piecewise(((-25165824*I*a**3*d**4*exp(10*I*c)*exp(8*I*d*x) -
 167772160*I*a**3*d**4*exp(8*I*c)*exp(6*I*d*x) - 503316480*I*a**3*d**4*exp
(6*I*c)*exp(4*I*d*x) - 1006632960*I*a**3*d**4*exp(4*I*c)*exp(2*I*d*x) + 10
0663296*I*a**3*d**4*exp(-2*I*d*x))*exp(-2*I*c)/(6442450944*d**5), Ne(d**5*
exp(2*I*c), 0)), (x*(-5*a**3/32 + (a**3*exp(10*I*c) + 5*a**3*exp(8*I*c) +
10*a**3*exp(6*I*c) + 10*a**3*exp(4*I*c) + 5*a**3*exp(2*I*c) + a**3)*exp(-2
*I*c)/32), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.80

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{15\,(dx+c)a^3 + \frac{15\,a^3\tan(dx+c)^7+55\,a^3\tan(dx+c)^5+73\,a^3\tan(dx+c)^3+16i\,a^3\tan(dx+c)^2+81\,a^3\tan(dx+c)-32i\,a^3}{\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1}}{96\,d}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/96*(15*(d*x + c)*a^3 + (15*a^3*tan(d*x + c)^7 + 55*a^3*tan(d*x + c)^5 +
73*a^3*tan(d*x + c)^3 + 16*I*a^3*tan(d*x + c)^2 + 81*a^3*tan(d*x + c) - 32
*I*a^3)/(tan(d*x + c)^8 + 4*tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*x
+ c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.62

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{1}{192}i\,a^3\left(\frac{15\,\log\left(\tan\left(dx+c\right)+i\right)}{d} - \frac{15\,\log\left(\tan\left(dx+c\right)-i\right)}{d} + \frac{2\left(-15i\,\tan\left(dx+c\right)^4+45\,\tan\left(dx+c\right.\right.}{d(\tan\left(dx+c\right)}\right.$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/192*I*a^3*(15*log(tan(d*x + c) + I)/d - 15*log(tan(d*x + c) - I)/d + 2*(
-15*I*tan(d*x + c)^4 + 45*tan(d*x + c)^3 + 35*I*tan(d*x + c)^2 + 15*tan(d*
x + c) + 32*I)/(d*(tan(d*x + c) + I)^4*(tan(d*x + c) - I)))
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.78

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{5\,a^3\,x}{32} - \frac{\frac{5\,a^3\,\tan(c+d\,x)^4}{32}+\frac{a^3\,\tan(c+d\,x)^3\,15\mathrm{i}}{32}-\frac{35\,a^3\,\tan(c+d\,x)^2}{96}+\frac{a^3\,\tan(c+d\,x)\,5\mathrm{i}}{32}-\frac{a^3}{3}}{d\,\left(-{\tan\left(c+d\,x\right)}^5-{\tan\left(c+d\,x\right)}^4\,3\mathrm{i}+2\,{\tan\left(c+d\,x\right)}^3-{\tan\left(c+d\,x\right)}^2\,2\mathrm{i}+3\,\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(5*a^3*x)/32 - ((a^3*tan(c + d*x)*5i)/32 - a^3/3 - (35*a^3*tan(c + d*x)^2)
/96 + (a^3*tan(c + d*x)^3*15i)/32 + (5*a^3*tan(c + d*x)^4)/32)/(d*(3*tan(c
 + d*x) - tan(c + d*x)^2*2i + 2*tan(c + d*x)^3 - tan(c + d*x)^4*3i - tan(c
 + d*x)^5 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.74

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{a^3\left(-48\cos\left(dx+c\right)\sin\left(dx+c\right)^7+152\cos\left(dx+c\right)\sin\left(dx+c\right)^5-170\cos\left(dx+c\right)\sin\left(dx+c\right)^3+81\,\mathrm{co}\right.}{}$$

input
```
int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(a**3*( - 48*cos(c + d*x)*sin(c + d*x)**7 + 152*cos(c + d*x)*sin(c + d*x)*
*5 - 170*cos(c + d*x)*sin(c + d*x)**3 + 81*cos(c + d*x)*sin(c + d*x) - 48*
sin(c + d*x)**8*i + 176*sin(c + d*x)**6*i - 240*sin(c + d*x)**4*i + 144*si
n(c + d*x)**2*i + 15*d*x))/(96*d)
```

3.45 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 127

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{7a^3\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{7ia^3\sec^3(c+dx)}{12d}+\frac{7a^3\sec(c+dx)\tan(c+dx)}{8d}+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}+\frac{7i\sec^3(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{20d}$$

output
```
7/8*a^3*arctanh(sin(d*x+c))/d+7/12*I*a^3*sec(d*x+c)^3/d+7/8*a^3*sec(d*x+c)
*tan(d*x+c)/d+1/5*I*a*sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2/d+7/20*I*sec(d*x+c
)^3*(a^3+I*a^3*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{a^3(\cos(3dx)+i\sin(3dx))\left(1680\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec^5(c+dx)(448i+640i\cos(2(c+\right.}{960d(\cos(dx)+i\sin(dx))^3}$$

input
```
Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*(Cos[3*d*x] + I*Sin[3*d*x])*(1680*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2
]] + Sec[c + d*x]^5*(448*I + (640*I)*Cos[2*(c + d*x)] - 150*Sin[2*(c + d*x
)] + 105*Sin[4*(c + d*x)])))/(960*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3979, 3042, 3979, 3042, 3967, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(c+dx)^3(a+ia\tan(c+dx))^3dx$$
$$\Big\downarrow \text{3979}$$
$$\frac{7}{5}a\int \sec^3(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$
$$\Big\downarrow \text{3042}$$
$$\frac{7}{5}a\int \sec(c+dx)^3(i\tan(c+dx)a+a)^2dx+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{7}{5}a\left(\frac{5}{4}a\int \sec^3(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\right)+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{5}a\left(\frac{5}{4}a\int \sec(c+dx)^3(i\tan(c+dx)a+a)dx+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\right)+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{7}{5}a\left(\frac{5}{4}a\left(a\int \sec^3(c+dx)dx+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\right)+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{5}a\left(\frac{5}{4}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\right)+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7}{5}a\left(\frac{5}{4}a\left(a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c\right.}{4d}\right.$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{5}a\left(\frac{5}{4}a\left(a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^{2}\right.}{4d}\right.$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{7}{5}a\left(\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}+\frac{5}{4}a\left(a\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}\right)\right)+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

input
```
Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/5)*a*Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^2)/d + (7*a*(((I/4)*Sec[c +
 d*x]^3*(a^2 + I*a^2*Tan[c + d*x]))/d + (5*a*(((I/3)*a*Sec[c + d*x]^3)/d +
 a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d))))/4))
/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 7.76 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{i a^3 \left(105\, e^{9i(dx+c)}-790\, e^{7i(dx+c)}-896\, e^{5i(dx+c)}-490\, e^{3i(dx+c)}-105\, e^{i(dx+c)}\right)}{60d\left(e^{2i(dx+c)}+1\right)^5}-\frac{7a^3\ln\left(e^{i(dx+c)}-i\right)}{8d}+\frac{7a^3\ln(e^{i}}{}$
derivativedivides	$\frac{-i a^3\left(\frac{\sin(dx+c)^4}{5\cos(dx+c)^5}+\frac{\sin(dx+c)^4}{15\cos(dx+c)^3}-\frac{\sin(dx+c)^4}{15\cos(dx+c)}-\frac{\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{15}\right)-3a^3\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(d}{8}\right.}{d}$
default	$\frac{-i a^3\left(\frac{\sin(dx+c)^4}{5\cos(dx+c)^5}+\frac{\sin(dx+c)^4}{15\cos(dx+c)^3}-\frac{\sin(dx+c)^4}{15\cos(dx+c)}-\frac{\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{15}\right)-3a^3\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(d}{8}\right.}{d}$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*I*a^3/d/(exp(2*I*(d*x+c))+1)^5*(105*exp(9*I*(d*x+c))-790*exp(7*I*(d*
x+c))-896*exp(5*I*(d*x+c))-490*exp(3*I*(d*x+c))-105*exp(I*(d*x+c)))-7/8/d*
a^3*ln(exp(I*(d*x+c))-I)+7/8/d*a^3*ln(exp(I*(d*x+c))+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(107) = 214.

Time = 0.09 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.44

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{-210i\,a^3e^{(9i\,dx+9i\,c)}+1580i\,a^3e^{(7i\,dx+7i\,c)}+1792i\,a^3e^{(5i\,dx+5i\,c)}+980i\,a^3e^{(3i\,dx+3i\,c)}+210i\,a^3e^{(i\,dx+i\,c)}+10}{}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/120*(-210*I*a^3*e^(9*I*d*x + 9*I*c) + 1580*I*a^3*e^(7*I*d*x + 7*I*c) + 1
792*I*a^3*e^(5*I*d*x + 5*I*c) + 980*I*a^3*e^(3*I*d*x + 3*I*c) + 210*I*a^3*
e^(I*d*x + I*c) + 105*(a^3*e^(10*I*d*x + 10*I*c) + 5*a^3*e^(8*I*d*x + 8*I*
c) + 10*a^3*e^(6*I*d*x + 6*I*c) + 10*a^3*e^(4*I*d*x + 4*I*c) + 5*a^3*e^(2*
I*d*x + 2*I*c) + a^3)*log(e^(I*d*x + I*c) + I) - 105*(a^3*e^(10*I*d*x + 10
*I*c) + 5*a^3*e^(8*I*d*x + 8*I*c) + 10*a^3*e^(6*I*d*x + 6*I*c) + 10*a^3*e^
(4*I*d*x + 4*I*c) + 5*a^3*e^(2*I*d*x + 2*I*c) + a^3)*log(e^(I*d*x + I*c) -
 I))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x
+ 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx = -ia^3\left(\int i\sec^3(c+dx)\,dx + \int \left(-3\tan(c+dx)\sec^3(c+dx)\right)\,dx + \int \tan^3(c+dx)\sec^3(c+dx)\,dx + \int \left(-3i\tan^2(c+dx)\sec^3(c+dx)\right)\,dx\right)$$

input
```
integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-I*a**3*(Integral(I*sec(c + d*x)**3, x) + Integral(-3*tan(c + d*x)*sec(c +
 d*x)**3, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**3, x) + Integral(-3*
I*tan(c + d*x)**2*sec(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.22

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{45\,a^3\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) + 60\,a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \right.}{240\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/240*(45*a^3*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(
d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^3*
(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
 + c) - 1)) - 240*I*a^3/cos(d*x + c)^3 - 16*I*(5*cos(d*x + c)^2 - 3)*a^3/c
os(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.49

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{105\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-105\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-360i\,a^3\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/120*(105*a^3*log(tan(1/2*d*x + 1/2*c) + 1) - 105*a^3*log(tan(1/2*d*x + 1
/2*c) - 1) + 2*(15*a^3*tan(1/2*d*x + 1/2*c)^9 - 360*I*a^3*tan(1/2*d*x + 1/
2*c)^8 - 390*a^3*tan(1/2*d*x + 1/2*c)^7 + 960*I*a^3*tan(1/2*d*x + 1/2*c)^6
 - 400*I*a^3*tan(1/2*d*x + 1/2*c)^4 + 390*a^3*tan(1/2*d*x + 1/2*c)^3 + 320
*I*a^3*tan(1/2*d*x + 1/2*c)^2 - 15*a^3*tan(1/2*d*x + 1/2*c) - 136*I*a^3)/(
tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 4.12 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.80

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{7\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$
$$-\frac{-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 6\mathrm{i}+\frac{13\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}-a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 16\mathrm{i}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 20\mathrm{i}}{3}-\frac{13\,a^3}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{}\right.}$$

input
```
int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^3,x)
```

output
```
(7*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a^3*tan(c/2 + (d*x)/2)^4*20i)/
3 - (13*a^3*tan(c/2 + (d*x)/2)^3)/2 - (a^3*tan(c/2 + (d*x)/2)^2*16i)/3 - a
^3*tan(c/2 + (d*x)/2)^6*16i + (13*a^3*tan(c/2 + (d*x)/2)^7)/2 + a^3*tan(c/
2 + (d*x)/2)^8*6i - (a^3*tan(c/2 + (d*x)/2)^9)/4 + (a^3*34i)/15 + (a^3*tan
(c/2 + (d*x)/2))/4)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 +
 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10
- 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.19

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{a^3\left(-105\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^4+210\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(d\right.\right.}{}$$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(a**3*( - 105*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 + 210
*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 - 105*cos(c + d*x)
*log(tan((c + d*x)/2) - 1) + 105*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*si
n(c + d*x)**4 - 210*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2
 + 105*cos(c + d*x)*log(tan((c + d*x)/2) + 1) - 136*cos(c + d*x)*sin(c + d
*x)**4*i - 105*cos(c + d*x)*sin(c + d*x)**3 + 272*cos(c + d*x)*sin(c + d*x
)**2*i + 15*cos(c + d*x)*sin(c + d*x) - 136*cos(c + d*x)*i - 160*sin(c + d
*x)**2*i + 136*i))/(120*cos(c + d*x)*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**
2 + 1))
```

3.46 $\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 99

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{5a^3\text{arctanh}(\sin(c+dx))}{2d} + \frac{5ia^3\sec(c+dx)}{2d} + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} + \frac{5i\sec(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{6d}$$

output
```
5/2*a^3*arctanh(sin(d*x+c))/d+5/2*I*a^3*sec(d*x+c)/d+1/3*I*a*sec(d*x+c)*(a
+I*a*tan(d*x+c))^2/d+5/6*I*sec(d*x+c)*(a^3+I*a^3*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{a^3(\cos(3dx)+i\sin(3dx))\left(60\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+i\sec^3(c+dx)(20+24\cos(2(c+dx))\right.}{12d(\cos(dx)+i\sin(dx))^3}$$

input
```
Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*(Cos[3*d*x] + I*Sin[3*d*x])*(60*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]
 + I*Sec[c + d*x]^3*(20 + 24*Cos[2*(c + d*x)] + (9*I)*Sin[2*(c + d*x)])))/
(12*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3979, 3042, 3979, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{5}{3}a\left(\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{3}{2}a\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)\right)+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

input
```
Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/3)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d + (5*a*((3*a*((a*ArcTanh
[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d))/2 + ((I/2)*Sec[c + d*x]*(a^2 +
I*a^2*Tan[c + d*x]))/d))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

method	result
risch	$\frac{i a^{3}\left(33\,{\mathrm e}^{5 i (d x+c)}+40\,{\mathrm e}^{3 i (d x+c)}+15\,{\mathrm e}^{i (d x+c)}\right)}{3 d \left({\mathrm e}^{2 i (d x+c)}+1\right)^{3}}-\frac{5 a^{3} \ln \left({\mathrm e}^{i (d x+c)}-i\right)}{2 d}+\frac{5 a^{3} \ln \left({\mathrm e}^{i (d x+c)}+i\right)}{2 d}$
derivativedivides	$\frac{-i a^{3}\left(\frac{\sin(d x+c)^{4}}{3 \cos(d x+c)^{3}}-\frac{\sin(d x+c)^{4}}{3 \cos(d x+c)}-\frac{\left(2+\sin(d x+c)^{2}\right) \cos(d x+c)}{3}\right)-3 a^{3}\left(\frac{\sin(d x+c)^{3}}{2 \cos(d x+c)^{2}}+\frac{\sin(d x+c)}{2}-\frac{\ln(\sec(d x+c)+\tan(d x+c))}{2}\right.}{d}$
default	$\frac{-i a^{3}\left(\frac{\sin(d x+c)^{4}}{3 \cos(d x+c)^{3}}-\frac{\sin(d x+c)^{4}}{3 \cos(d x+c)}-\frac{\left(2+\sin(d x+c)^{2}\right) \cos(d x+c)}{3}\right)-3 a^{3}\left(\frac{\sin(d x+c)^{3}}{2 \cos(d x+c)^{2}}+\frac{\sin(d x+c)}{2}-\frac{\ln(\sec(d x+c)+\tan(d x+c))}{2}\right.}{d}$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*I*a^3/d/(exp(2*I*(d*x+c))+1)^3*(33*exp(5*I*(d*x+c))+40*exp(3*I*(d*x+c)
)+15*exp(I*(d*x+c)))-5/2/d*a^3*ln(exp(I*(d*x+c))-I)+5/2/d*a^3*ln(exp(I*(d*
x+c))+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(81) = 162$.

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.04

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{66i\,a^3e^{(5i\,dx+5i\,c)} + 80i\,a^3e^{(3i\,dx+3i\,c)} + 30i\,a^3e^{(i\,dx+i\,c)} + 15\left(a^3e^{(6i\,dx+6i\,c)} + 3\,a^3e^{(4i\,dx+4i\,c)} + 3\,a^3e^{(2i\,dx+2i\,c)}\right.}{6\left(de^{(6i\,dx+6i\,c)} + 3\,de^{(4i\,dx+4\right.}}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/6*(66*I*a^3*e^(5*I*d*x + 5*I*c) + 80*I*a^3*e^(3*I*d*x + 3*I*c) + 30*I*a^
3*e^(I*d*x + I*c) + 15*(a^3*e^(6*I*d*x + 6*I*c) + 3*a^3*e^(4*I*d*x + 4*I*c
) + 3*a^3*e^(2*I*d*x + 2*I*c) + a^3)*log(e^(I*d*x + I*c) + I) - 15*(a^3*e^
(6*I*d*x + 6*I*c) + 3*a^3*e^(4*I*d*x + 4*I*c) + 3*a^3*e^(2*I*d*x + 2*I*c)
+ a^3)*log(e^(I*d*x + I*c) - I))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x +
 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx = -ia^3\left(\int i\sec(c+dx)\,dx + \int(-3\tan(c+dx)\sec(c+dx))\,dx + \int\tan^3(c+dx)\sec(c+dx)\,dx + \int\left(-3i\tan^2(c+dx)\sec(c+dx)\right)dx\right)$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-I*a**3*(Integral(I*sec(c + d*x), x) + Integral(-3*tan(c + d*x)*sec(c + d*
x), x) + Integral(tan(c + d*x)**3*sec(c + d*x), x) + Integral(-3*I*tan(c +
 d*x)**2*sec(c + d*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.10

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{9\,a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} + \log\left(\sin\left(dx+c\right)+1\right) - \log\left(\sin\left(dx+c\right)-1\right)\right) + 12\,a^3\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right.\right.}{12\,d}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/12*(9*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 12*a^3*log(sec(d*x + c) + tan(d*x + c)) + 36*I*a
^3/cos(d*x + c) + 4*I*(3*cos(d*x + c)^2 - 1)*a^3/cos(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.26

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{15\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right) - 15\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right) - \frac{2\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4\right.}{\left(\tan\left(\frac{1}{2}\right.\right.}}{6\,d}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(15*a^3*log(tan(1/2*d*x + 1/2*c) + 1) - 15*a^3*log(tan(1/2*d*x + 1/2*c
) - 1) - 2*(9*a^3*tan(1/2*d*x + 1/2*c)^5 + 18*I*a^3*tan(1/2*d*x + 1/2*c)^4
 - 48*I*a^3*tan(1/2*d*x + 1/2*c)^2 - 9*a^3*tan(1/2*d*x + 1/2*c) + 22*I*a^3
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.37

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{5\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 6\mathrm{i}-a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 16\mathrm{i}-3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^3\,22\mathrm{i}}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x),x)
```

output
```
(5*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (a^3*tan(c/2 + (d*x)/2)^4*6i - a^3*t
an(c/2 + (d*x)/2)^2*16i + 3*a^3*tan(c/2 + (d*x)/2)^5 + (a^3*22i)/3 - 3*a^3
*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 +
 tan(c/2 + (d*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.81

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$= \frac{a^3\left(-15\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2+15\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+15\cos\right.}{}$$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(a**3*( - 15*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 + 15*c
os(c + d*x)*log(tan((c + d*x)/2) - 1) + 15*cos(c + d*x)*log(tan((c + d*x)/
2) + 1)*sin(c + d*x)**2 - 15*cos(c + d*x)*log(tan((c + d*x)/2) + 1) - 22*c
os(c + d*x)*sin(c + d*x)**2*i + 9*cos(c + d*x)*sin(c + d*x) + 22*cos(c + d
*x)*i + 24*sin(c + d*x)**2*i - 22*i))/(6*cos(c + d*x)*d*(sin(c + d*x)**2 -
 1))
```

3.47 $\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{3a^3\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{3ia^3\sec(c+dx)}{d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}$$

output
```
-3*a^3*arctanh(sin(d*x+c))/d-3*I*a^3*sec(d*x+c)/d-2*I*a*cos(d*x+c)*(a+I*a*
tan(d*x+c))^2/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 123 vs. 2(61) = 122.

Time = 1.44 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.02

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{a^3\cos^2(c+dx)\left(6\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)\cos(c+dx)(i\cos(3c)+\sin(3c))+(-\cos(2c-dx)+\right.}{d(\cos(dx)+i\sin(dx))^3}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*Cos[c + d*x]^2*(6*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]*
(I*Cos[3*c] + Sin[3*c]) + (-Cos[2*c - d*x] + I*Sin[2*c - d*x])*(5*Cos[c +
d*x] - I*Sin[c + d*x]))*(-I + Tan[c + d*x])^3)/(d*(Cos[d*x] + I*Sin[d*x])^
3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3977, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3977}$$

$$-3a^2\int \sec(c+dx)(i\tan(c+dx)a+a)dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$-3a^2\int \sec(c+dx)(i\tan(c+dx)a+a)dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{3967}$$

$$-3a^2\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$-3a^2\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{4257}$$

$$-3a^2\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
-3*a^2*((a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d) - ((2*I)*a*Cos
[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{4ia^3 e^{i(dx+c)}}{d}-\frac{2i e^{i(dx+c)} a^3}{d\left(e^{2i(dx+c)}+1\right)}+\frac{3a^3 \ln\left(e^{i(dx+c)}-i\right)}{d}-\frac{3a^3 \ln\left(e^{i(dx+c)}+i\right)}{d}$
derivativedivides	$\frac{-ia^3\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)-3a^3(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-3ia^3\cos(dx+c)+a}{d}$
default	$\frac{-ia^3\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)-3a^3(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-3ia^3\cos(dx+c)+a}{d}$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-4*I/d*a^3*exp(I*(d*x+c))-2*I*exp(I*(d*x+c))*a^3/d/(exp(2*I*(d*x+c))+1)+3/
d*a^3*ln(exp(I*(d*x+c))-I)-3/d*a^3*ln(exp(I*(d*x+c))+I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.75

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{-4i\,a^3e^{(3i\,dx+3i\,c)}-6i\,a^3e^{(i\,dx+i\,c)}-3\left(a^3e^{(2i\,dx+2i\,c)}+a^3\right)\log\left(e^{(i\,dx+i\,c)}+i\right)+3\left(a^3e^{(2i\,dx+2i\,c)}+a^3\right)\log\left(e\right.}{de^{(2i\,dx+2i\,c)}+d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
(-4*I*a^3*e^(3*I*d*x + 3*I*c) - 6*I*a^3*e^(I*d*x + I*c) - 3*(a^3*e^(2*I*d*
x + 2*I*c) + a^3)*log(e^(I*d*x + I*c) + I) + 3*(a^3*e^(2*I*d*x + 2*I*c) +
a^3)*log(e^(I*d*x + I*c) - I))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.75

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2ia^3e^{ic}e^{idx}}{de^{2ic}e^{2idx}+d} + \frac{3a^3\left(\log\left(e^{idx}-ie^{-ic}\right)-\log\left(e^{idx}+ie^{-ic}\right)\right)}{d} + \begin{cases} -\frac{4ia^3e^{ic}e^{idx}}{d} & \text{for } d\neq 0 \\ 4a^3xe^{ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-2*I*a**3*exp(I*c)*exp(I*d*x)/(d*exp(2*I*c)*exp(2*I*d*x) + d) + 3*a**3*(lo
g(exp(I*d*x) - I*exp(-I*c)) - log(exp(I*d*x) + I*exp(-I*c)))/d + Piecewise
((-4*I*a**3*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (4*a**3*x*exp(I*c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2i\,a^3\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)+3\,a^3(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right))}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(2*I*a^3*(1/cos(d*x + c) + cos(d*x + c)) + 3*a^3*(log(sin(d*x + c) +
1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) + 6*I*a^3*cos(d*x + c) - 2*a^
3*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(55) = 110$.

Time = 0.31 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.84

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{63\,a^3e^{(2i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)-33\,a^3e^{(2i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-63\,a^3e^{(2i\,dx+2i\,c)}\log\left(-i\,e^{(i\,dx+i}\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/32*(63*a^3*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) - 33*a^3*e^(2*
I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) - 63*a^3*e^(2*I*d*x + 2*I*c)*log
(-I*e^(I*d*x + I*c) + 1) + 33*a^3*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*
c) - 1) - 128*I*a^3*e^(3*I*d*x + 3*I*c) - 192*I*a^3*e^(I*d*x + I*c) + 63*a
^3*log(I*e^(I*d*x + I*c) + 1) - 33*a^3*log(I*e^(I*d*x + I*c) - 1) - 63*a^3
*log(-I*e^(I*d*x + I*c) + 1) + 33*a^3*log(-I*e^(I*d*x + I*c) - 1))/(d*e^(2
*I*d*x + 2*I*c) + d)
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.67

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= -\frac{6\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,2\mathrm{i}-10\,a^3}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
- (6*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (8*a^3*tan(c/2 + (d*x)/2)^2 - 10*a
^3 + a^3*tan(c/2 + (d*x)/2)*2i)/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2
)^2*1i - tan(c/2 + (d*x)/2)^3 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.52

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{a^3\left(3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+4\cos(dx+c)\sin(dx+\right.}{\cos(dx+c)\,d}$$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(a**3*(3*cos(c + d*x)*log(tan((c + d*x)/2) - 1) - 3*cos(c + d*x)*log(tan((
c + d*x)/2) + 1) + 4*cos(c + d*x)*sin(c + d*x) + 5*cos(c + d*x)*i + 4*sin(
c + d*x)**2*i - 5*i))/(cos(c + d*x)*d)
```

3.48 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 32

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

output
```
-1/3*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{ia^3(\cos(c+dx)+i\sin(c+dx))^3}{3d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-1/3*I)*a^3*(Cos[c + d*x] + I*Sin[c + d*x])^3)/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^3}{\sec(c+dx)^3}dx$$
$$\Big\downarrow \text{3969}$$
$$-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-1/3*I)*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Maple [A] (verified)

Time = 6.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

method	result	size
risch	$-\frac{i a^{3} {\mathrm e}^{3 i \left(d x +c \right)}}{3 d}$	19
orering	$-\frac{i \cos\left(d x +c \right)^{3} \left(a +i a \tan\left(d x +c \right)\right)^{3}}{3 d}$	29
derivativedivides	$\frac{\frac{i a^{3} \left(2+\sin\left(d x +c \right)^{2}\right) \cos\left(d x +c \right)}{3}-a^{3} \sin\left(d x +c \right)^{3}-i a^{3} \cos\left(d x +c \right)^{3}+\frac{a^{3} \left(2+\cos\left(d x +c \right)^{2}\right) \sin\left(d x +c \right)}{3}}{d}$	76
default	$\frac{\frac{i a^{3} \left(2+\sin\left(d x +c \right)^{2}\right) \cos\left(d x +c \right)}{3}-a^{3} \sin\left(d x +c \right)^{3}-i a^{3} \cos\left(d x +c \right)^{3}+\frac{a^{3} \left(2+\cos\left(d x +c \right)^{2}\right) \sin\left(d x +c \right)}{3}}{d}$	76

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/d*a^3*exp(3*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{i\,a^3e^{(3i\,dx+3i\,c)}}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/3*I*a^3*e^(3*I*d*x + 3*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx = \begin{cases} -\frac{ia^3e^{3ic}e^{3idx}}{3d} & \text{for } d\neq 0 \\ a^3xe^{3ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise((-I*a**3*exp(3*I*c)*exp(3*I*d*x)/(3*d), Ne(d, 0)), (a**3*x*exp(3
*I*c), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(26) = 52.

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.34

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx =$$
$$-\frac{3i\,a^3\cos\left(dx+c\right)^3+3\,a^3\sin\left(dx+c\right)^3+i\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)a^3+\left(\sin\left(dx+c\right)^3-3\,\sin\left(\right.\right.}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/3*(3*I*a^3*cos(d*x + c)^3 + 3*a^3*sin(d*x + c)^3 + I*(cos(d*x + c)^3 -
3*cos(d*x + c))*a^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a^3)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(26) = 52$.

Time = 0.38 (sec) , antiderivative size = 901, normalized size of antiderivative = 28.16

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/384*(108*a^3*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 432*a^3*e
^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 432*a^3*e^(2*I*d*x - 2*I*c
)*log(I*e^(I*d*x + I*c) + 1) + 648*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) +
 1) + 108*a^3*e^(-4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 111*a^3*e^(8*I*d*x +
 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 444*a^3*e^(6*I*d*x + 2*I*c)*log(I*e^(
I*d*x + I*c) - 1) + 444*a^3*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1)
 + 666*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 111*a^3*e^(-4*I*c)*log
(I*e^(I*d*x + I*c) - 1) - 108*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*
c) + 1) - 432*a^3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 432*a^
3*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 648*a^3*e^(4*I*d*x)*lo
g(-I*e^(I*d*x + I*c) + 1) - 108*a^3*e^(-4*I*c)*log(-I*e^(I*d*x + I*c) + 1)
 - 111*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 444*a^3*e^(6*
I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 444*a^3*e^(2*I*d*x - 2*I*c)*l
og(-I*e^(I*d*x + I*c) - 1) - 666*a^3*e^(4*I*d*x)*log(-I*e^(I*d*x + I*c) -
1) - 111*a^3*e^(-4*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 3*a^3*e^(8*I*d*x + 4
*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 12*a^3*e^(6*I*d*x + 2*I*c)*log(I*e^(I*
d*x) + e^(-I*c)) + 12*a^3*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c))
+ 18*a^3*e^(4*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) + 3*a^3*e^(-4*I*c)*log(I*
e^(I*d*x) + e^(-I*c)) - 3*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I
*c)) - 12*a^3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 12*a^3...
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.06

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3 \, dx$$
$$= -\frac{2\,a^3\,\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}{3\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 3\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
-(2*a^3*(3*tan(c/2 + (d*x)/2)^2 - 1))/(3*d*(3*tan(c/2 + (d*x)/2) - tan(c/2
 + (d*x)/2)^2*3i - tan(c/2 + (d*x)/2)^3 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3 \, dx$$
$$= \frac{a^3(4\cos(dx+c)\sin(dx+c)^2 i-\cos(dx+c) i-4\sin(dx+c)^3+3\sin(dx+c)+i)}{3d}$$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(a**3*(4*cos(c + d*x)*sin(c + d*x)**2*i - cos(c + d*x)*i - 4*sin(c + d*x)*
*3 + 3*sin(c + d*x) + i))/(3*d)
```

3.49 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{ia^3\cos^3(c+dx)}{15d} + \frac{a^3\sin(c+dx)}{5d} - \frac{a^3\sin^3(c+dx)}{15d} - \frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

output
```
-1/15*I*a^3*cos(d*x+c)^3/d+1/5*a^3*sin(d*x+c)/d-1/15*a^3*sin(d*x+c)^3/d-2/
5*I*a*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2/d
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.65

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{a^3(-i\cos(2(c+dx))+\sin(2(c+dx)))\left(\cos(c+dx)\left(4+25\sqrt{\cos^2(c+dx)}\right)+\left(4+3\sqrt{\cos^2(c+dx)}\right)\mathrm{c}\right.}{60d\sqrt{\cos^2(c}}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])*(Cos[c + d*x]*(4 + 25*Sqrt
[Cos[c + d*x]^2]) + (4 + 3*Sqrt[Cos[c + d*x]^2])*Cos[3*(c + d*x)] - I*((4
+ 5*Sqrt[Cos[c + d*x]^2])*Sin[c + d*x] + (4 - 3*Sqrt[Cos[c + d*x]^2])*Sin[
3*(c + d*x)])))/(60*d*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = $\{3042, 3977, 3042, 3967, 3042, 3113, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{1}{5}a^2\int \cos^3(c+dx)(i\tan(c+dx)a+a)dx-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}a^2\int \frac{i\tan(c+dx)a+a}{\sec(c+dx)^3}dx-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{5}a^2\left(a\int \cos^3(c+dx)dx-\frac{ia\cos^3(c+dx)}{3d}\right)-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}a^2\left(a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{ia\cos^3(c+dx)}{3d}\right)-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{1}{5}a^2\left(-\frac{a\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}-\frac{ia\cos^3(c+dx)}{3d}\right)-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}a^2\left(-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{ia\cos^3(c+dx)}{3d}\right)-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^2*(((-1/3*I)*a*Cos[c + d*x]^3)/d - (a*(-Sin[c + d*x] + Sin[c + d*x]^3/3
))/d))/5 - (((2*I)/5)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^2)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

Maple [A] (verified)

Time = 24.55 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{i a^3 e^{5 i (dx+c)}}{20 d}-\frac{i a^3 e^{3 i (dx+c)}}{6 d}-\frac{i a^3 e^{i (dx+c)}}{4 d}$
derivativedivides	$\frac{-i a^3\left(-\frac{\cos(dx+c)^3\sin(dx+c)^2}{5}-\frac{2\cos(dx+c)^3}{15}\right)-3a^3\left(-\frac{\cos(dx+c)^4\sin(dx+c)}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right)-\frac{3 i a^3\cos(dx+}{5}}{d}$
default	$\frac{-i a^3\left(-\frac{\cos(dx+c)^3\sin(dx+c)^2}{5}-\frac{2\cos(dx+c)^3}{15}\right)-3a^3\left(-\frac{\cos(dx+c)^4\sin(dx+c)}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right)-\frac{3 i a^3\cos(dx+}{5}}{d}$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/20*I/d*a^3*exp(5*I*(d*x+c))-1/6*I/d*a^3*exp(3*I*(d*x+c))-1/4*I/d*a^3*ex
p(I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.55

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{-3i\,a^3e^{(5i\,dx+5i\,c)}-10i\,a^3e^{(3i\,dx+3i\,c)}-15i\,a^3e^{(i\,dx+i\,c)}}{60\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/60*(-3*I*a^3*e^(5*I*d*x + 5*I*c) - 10*I*a^3*e^(3*I*d*x + 3*I*c) - 15*I*a
^3*e^(I*d*x + I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx = \begin{cases} \frac{-24ia^3d^2e^{5ic}e^{5idx}-80ia^3d^2e^{3ic}e^{3idx}-120ia^3d^2e^{ic}e^{idx}}{480d^3} & \text{for } d^3 \neq 0 \\ x\left(\frac{a^3e^{5ic}}{4}+\frac{a^3e^{3ic}}{2}+\frac{a^3e^{ic}}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise(((-24*I*a**3*d**2*exp(5*I*c)*exp(5*I*d*x) - 80*I*a**3*d**2*exp(3
*I*c)*exp(3*I*d*x) - 120*I*a**3*d**2*exp(I*c)*exp(I*d*x))/(480*d**3), Ne(d
**3, 0)), (x*(a**3*exp(5*I*c)/4 + a**3*exp(3*I*c)/2 + a**3*exp(I*c)/4), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{9i\,a^3\cos\left(dx+c\right)^5+i\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)a^3-3\left(3\,\sin\left(dx+c\right)^5-5\,\sin\left(dx+c\right)^3\right)a^3 -}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/15*(9*I*a^3*cos(d*x + c)^5 + I*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^
3 - 3*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^3 - (3*sin(d*x + c)^5 - 10*s
in(d*x + c)^3 + 15*sin(d*x + c))*a^3)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(74) = 148$.

Time = 0.45 (sec) , antiderivative size = 929, normalized size of antiderivative = 10.56

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/7680*(1785*a^3*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 7140*a^3
*e^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 7140*a^3*e^(2*I*d*x - 2*
I*c)*log(I*e^(I*d*x + I*c) + 1) + 10710*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I
*c) + 1) + 1785*a^3*e^(-4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1530*a^3*e^(8*
I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 6120*a^3*e^(6*I*d*x + 2*I*c)*l
og(I*e^(I*d*x + I*c) - 1) + 6120*a^3*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x +
I*c) - 1) + 9180*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 1530*a^3*e^(
-4*I*c)*log(I*e^(I*d*x + I*c) - 1) - 1785*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e
^(I*d*x + I*c) + 1) - 7140*a^3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 7140*a^3*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 10710*a^
3*e^(4*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 1785*a^3*e^(-4*I*c)*log(-I*e^(
I*d*x + I*c) + 1) - 1530*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) -
1) - 6120*a^3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 6120*a^3*e
^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 9180*a^3*e^(4*I*d*x)*log(
-I*e^(I*d*x + I*c) - 1) - 1530*a^3*e^(-4*I*c)*log(-I*e^(I*d*x + I*c) - 1)
- 255*a^3*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1020*a^3*e^(6*
I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1020*a^3*e^(2*I*d*x - 2*I*c)*
log(I*e^(I*d*x) + e^(-I*c)) - 1530*a^3*e^(4*I*d*x)*log(I*e^(I*d*x) + e^(-I
*c)) - 255*a^3*e^(-4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 255*a^3*e^(8*I*d*x
 + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 1020*a^3*e^(6*I*d*x + 2*I*c)*l...
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.48

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{2\,a^3\,\left(15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,30\mathrm{i}-40\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,20\mathrm{i}+7\right)}{15\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,5\mathrm{i}-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,10\mathrm{i}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(2*a^3*(tan(c/2 + (d*x)/2)^3*30i - 40*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*
x)/2)*20i + 15*tan(c/2 + (d*x)/2)^4 + 7))/(15*d*(5*tan(c/2 + (d*x)/2) - ta
n(c/2 + (d*x)/2)^2*10i - 10*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*5i
 + tan(c/2 + (d*x)/2)^5 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{a^3\left(-12\cos\left(dx+c\right)\sin\left(dx+c\right)^4i+19\cos\left(dx+c\right)\sin\left(dx+c\right)^2i-7\cos\left(dx+c\right)i+12\sin\left(dx+c\right)^5-\right.}{15d}$$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(a**3*( - 12*cos(c + d*x)*sin(c + d*x)**4*i + 19*cos(c + d*x)*sin(c + d*x)
**2*i - 7*cos(c + d*x)*i + 12*sin(c + d*x)**5 - 25*sin(c + d*x)**3 + 15*si
n(c + d*x) + 7*i))/(15*d)
```

3.50 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 106

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{3ia^3\cos^5(c+dx)}{35d}+\frac{3a^3\sin(c+dx)}{7d}-\frac{2a^3\sin^3(c+dx)}{7d}+\frac{3a^3\sin^5(c+dx)}{35d}-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

output
```
-3/35*I*a^3*cos(d*x+c)^5/d+3/7*a^3*sin(d*x+c)/d-2/7*a^3*sin(d*x+c)^3/d+3/3
5*a^3*sin(d*x+c)^5/d-2/7*I*a*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2/d
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.70

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{a^3(-i\cos(3(c+dx))+\sin(3(c+dx)))\left(35\sqrt{\cos^2(c+dx)}+\left(8+84\sqrt{\cos^2(c+dx)}\right)\cos(2(c+dx))+\left(\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(35*Sqrt[Cos[c + d*x]^2] +
 (8 + 84*Sqrt[Cos[c + d*x]^2])*Cos[2*(c + d*x)] + (8 - 15*Sqrt[Cos[c + d*x
]^2])*Cos[4*(c + d*x)] - (8*I)*Sin[2*(c + d*x)] - (56*I)*Sqrt[Cos[c + d*x]
^2]*Sin[2*(c + d*x)] - (8*I)*Sin[4*(c + d*x)] + (20*I)*Sqrt[Cos[c + d*x]^2
]*Sin[4*(c + d*x)]))/(280*d*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 3977, 3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{3}{7}a^2\int \cos^5(c+dx)(i\tan(c+dx)a+a)dx-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{7}a^2\int \frac{i\tan(c+dx)a+a}{\sec(c+dx)^5}dx-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{3}{7}a^2\left(a\int \cos^5(c+dx)dx-\frac{ia\cos^5(c+dx)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{7}a^2\left(a\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{ia\cos^5(c+dx)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{3}{7}a^2\left(-\frac{a\int\left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}-\frac{ia\cos^5(c+dx)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{7}a^2\left(-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{ia\cos^5(c+dx)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

input
```
Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(3*a^2*(((-1/5*I)*a*Cos[c + d*x]^5)/d - (a*(-Sin[c + d*x] + (2*Sin[c + d*x
]^3)/3 - Sin[c + d*x]^5/5))/d))/7 - (((2*I)/7)*a*Cos[c + d*x]^7*(a + I*a*T
an[c + d*x])^2)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

Maple [A] (verified)

Time = 77.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{i a^{3} {\mathrm e}^{7 i \left(d x +c \right)}}{112 d}-\frac{i a^{3} {\mathrm e}^{5 i \left(d x +c \right)}}{20 d}-\frac{i a^{3} {\mathrm e}^{3 i \left(d x +c \right)}}{8 d}-\frac{3 i a^{3} \cos \left(d x +c \right)}{16 d}+\frac{5 a^{3} \sin \left(d x +c \right)}{16 d}$
derivativedivides	$\frac{-i a^{3} \left(-\frac{\cos \left(d x +c \right)^{5} \sin \left(d x +c \right)^{2}}{7}-\frac{2 \cos \left(d x +c \right)^{5}}{35}\right)-3 a^{3} \left(-\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)^{6}}{7}+\frac{\left(\frac{8}{3}+\cos \left(d x +c \right)^{4}+\frac{4 \cos \left(d x +c \right)^{2}}{3}\right) \sin \left(d x +c \right)}{35}\right.}{d}$
default	$\frac{-i a^{3} \left(-\frac{\cos \left(d x +c \right)^{5} \sin \left(d x +c \right)^{2}}{7}-\frac{2 \cos \left(d x +c \right)^{5}}{35}\right)-3 a^{3} \left(-\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)^{6}}{7}+\frac{\left(\frac{8}{3}+\cos \left(d x +c \right)^{4}+\frac{4 \cos \left(d x +c \right)^{2}}{3}\right) \sin \left(d x +c \right)}{35}\right.}{d}$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/112*I/d*a^3*exp(7*I*(d*x+c))-1/20*I/d*a^3*exp(5*I*(d*x+c))-1/8*I/d*a^3*
exp(3*I*(d*x+c))-3/16*I/d*a^3*cos(d*x+c)+5/16*a^3*sin(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{\left(-5i\,a^3e^{(8i\,dx+8i\,c)}-28i\,a^3e^{(6i\,dx+6i\,c)}-70i\,a^3e^{(4i\,dx+4i\,c)}-140i\,a^3e^{(2i\,dx+2i\,c)}+35i\,a^3\right)e^{(-i\,dx-i\,c)}}{560\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/560*(-5*I*a^3*e^(8*I*d*x + 8*I*c) - 28*I*a^3*e^(6*I*d*x + 6*I*c) - 70*I*
a^3*e^(4*I*d*x + 4*I*c) - 140*I*a^3*e^(2*I*d*x + 2*I*c) + 35*I*a^3)*e^(-I*
d*x - I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.79

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \begin{cases} \frac{(-10240ia^3d^4e^{8ic}e^{7idx}-57344ia^3d^4e^{6ic}e^{5idx}-143360ia^3d^4e^{4ic}e^{3idx}-286720ia^3d^4e^{2ic}e^{idx}+71680ia^3d^4e^{-idx})e^{-ic}}{1146880d^5} & \text{for } d^5e^{ic}\neq 0 \\ \frac{x(a^3e^{8ic}+4a^3e^{6ic}+6a^3e^{4ic}+4a^3e^{2ic}+a^3)e^{-ic}}{16} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise(((-10240*I*a**3*d**4*exp(8*I*c)*exp(7*I*d*x) - 57344*I*a**3*d**4
*exp(6*I*c)*exp(5*I*d*x) - 143360*I*a**3*d**4*exp(4*I*c)*exp(3*I*d*x) - 28
6720*I*a**3*d**4*exp(2*I*c)*exp(I*d*x) + 71680*I*a**3*d**4*exp(-I*d*x))*ex
p(-I*c)/(1146880*d**5), Ne(d**5*exp(I*c), 0)), (x*(a**3*exp(8*I*c) + 4*a**
3*exp(6*I*c) + 6*a**3*exp(4*I*c) + 4*a**3*exp(2*I*c) + a**3)*exp(-I*c)/16,
 True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.16

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx =$$
$$-\frac{15i\,a^3\cos(dx+c)^7+i\left(5\cos(dx+c)^7-7\cos(dx+c)^5\right)a^3+\left(15\sin(dx+c)^7-42\sin(dx+c)^5+\right.}{35\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/35*(15*I*a^3*cos(d*x + c)^7 + I*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a
^3 + (15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^3 + (5*
sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*
a^3)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(90) = 180$.

Time = 0.53 (sec) , antiderivative size = 465, normalized size of antiderivative = 4.39

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{19635\,a^3e^{(5i\,dx+3i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+39270\,a^3e^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+19635\,a^3e^{(i\,dx-i\,c)}\log\left(i\right.}{}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/71680*(19635*a^3*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 39270*
a^3*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 19635*a^3*e^(I*d*x - I*
c)*log(I*e^(I*d*x + I*c) + 1) + 19635*a^3*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d
*x + I*c) - 1) + 39270*a^3*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) +
19635*a^3*e^(I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) - 19635*a^3*e^(5*I*d*
x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 39270*a^3*e^(3*I*d*x + I*c)*log(-
I*e^(I*d*x + I*c) + 1) - 19635*a^3*e^(I*d*x - I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 19635*a^3*e^(5*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 39270*a
^3*e^(3*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 19635*a^3*e^(I*d*x - I*
c)*log(-I*e^(I*d*x + I*c) - 1) - 640*I*a^3*e^(12*I*d*x + 10*I*c) - 4864*I*
a^3*e^(10*I*d*x + 8*I*c) - 16768*I*a^3*e^(8*I*d*x + 6*I*c) - 39424*I*a^3*e
^(6*I*d*x + 4*I*c) - 40320*I*a^3*e^(4*I*d*x + 2*I*c) - 8960*I*a^3*e^(2*I*d
*x) + 4480*I*a^3*e^(-2*I*c))/(d*e^(5*I*d*x + 3*I*c) + 2*d*e^(3*I*d*x + I*c
) + d*e^(I*d*x - I*c))
```

Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.26

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx =$$
$$-\frac{2\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{17\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}-\frac{17\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{2}+\frac{31\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{2}-\frac{5\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{2}+\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,35\mathrm{i}}{8}-\frac{\cos}{}\right.}{35\,d\,\left(\cos\left(3\,c+3\,d\,x\right)-\sin\left(3\,c+3\,d\,x\right)\,1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
-(2*a^3*cos(c/2 + (d*x)/2)*((cos(c/2 + (d*x)/2)*35i)/8 - (cos((3*c)/2 + (3
*d*x)/2)*35i)/8 + (cos((5*c)/2 + (5*d*x)/2)*119i)/8 - (cos((7*c)/2 + (7*d*
x)/2)*15i)/8 + (17*sin(c/2 + (d*x)/2))/2 - (17*sin((3*c)/2 + (3*d*x)/2))/2
 + (31*sin((5*c)/2 + (5*d*x)/2))/2 - (5*sin((7*c)/2 + (7*d*x)/2))/2))/(35*
d*(cos(3*c + 3*d*x) - sin(3*c + 3*d*x)*1i))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{a^3\left(20\cos\left(dx+c\right)\sin\left(dx+c\right)^6 i-53\cos\left(dx+c\right)\sin\left(dx+c\right)^4 i+46\cos\left(dx+c\right)\sin\left(dx+c\right)^2 i-13\,\mathrm{co}\right.}{35d}$$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(a**3*(20*cos(c + d*x)*sin(c + d*x)**6*i - 53*cos(c + d*x)*sin(c + d*x)**4
*i + 46*cos(c + d*x)*sin(c + d*x)**2*i - 13*cos(c + d*x)*i - 20*sin(c + d*
x)**7 + 63*sin(c + d*x)**5 - 70*sin(c + d*x)**3 + 35*sin(c + d*x) + 13*i))
/(35*d)
```

3.51 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{5ia^3\cos^7(c+dx)}{63d} + \frac{5a^3\sin(c+dx)}{9d} - \frac{5a^3\sin^3(c+dx)}{9d} + \frac{a^3\sin^5(c+dx)}{3d} - \frac{5a^3\sin^7(c+dx)}{63d} - \frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

output
```
-5/63*I*a^3*cos(d*x+c)^7/d+5/9*a^3*sin(d*x+c)/d-5/9*a^3*sin(d*x+c)^3/d+1/3
*a^3*sin(d*x+c)^5/d-5/63*a^3*sin(d*x+c)^7/d-2/9*I*a*cos(d*x+c)^9*(a+I*a*ta
n(d*x+c))^2/d
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.82

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{a^3(-i\cos(3(c+dx))+\sin(3(c+dx)))\left(210\sqrt{\cos^2(c+dx)}+\left(32+567\sqrt{\cos^2(c+dx)}\right)\cos(2(c+dx))\right.}{}$$

input
```
Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(210*Sqrt[Cos[c + d*x]^2]
+ (32 + 567*Sqrt[Cos[c + d*x]^2])*Cos[2*(c + d*x)] + (32 - 162*Sqrt[Cos[c
+ d*x]^2])*Cos[4*(c + d*x)] - 7*Sqrt[Cos[c + d*x]^2]*Cos[6*(c + d*x)] - (3
2*I)*Sin[2*(c + d*x)] - (378*I)*Sqrt[Cos[c + d*x]^2]*Sin[2*(c + d*x)] - (3
2*I)*Sin[4*(c + d*x)] + (216*I)*Sqrt[Cos[c + d*x]^2]*Sin[4*(c + d*x)] + (1
4*I)*Sqrt[Cos[c + d*x]^2]*Sin[6*(c + d*x)]))/(2016*d*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 3977, 3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^3}{\sec(c+dx)^9}dx$$
$$\Big\downarrow \text{3977}$$
$$\frac{5}{9}a^2\int \cos^7(c+dx)(i\tan(c+dx)a+a)dx-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{9}a^2\int \frac{i\tan(c+dx)a+a}{\sec(c+dx)^7}dx-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{5}{9}a^2\left(a\int \cos^7(c+dx)dx-\frac{ia\cos^7(c+dx)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{9}a^2\left(a\int \sin\left(c+dx+\frac{\pi}{2}\right)^7dx-\frac{ia\cos^7(c+dx)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{5}{9}a^2\left(-\frac{a\int \left(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}-\frac{ia\cos^7(c+dx)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{9}a^2\left(-\frac{a\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{ia\cos^7(c+dx)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

input
```
Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(5*a^2*(((-1/7*I)*a*Cos[c + d*x]^7)/d - (a*(-Sin[c + d*x] + Sin[c + d*x]^3
 - (3*Sin[c + d*x]^5)/5 + Sin[c + d*x]^7/7))/d))/9 - (((2*I)/9)*a*Cos[c +
d*x]^9*(a + I*a*Tan[c + d*x])^2)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

Maple [A] (verified)

Time = 198.50 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{i a^3 e^{9 i (dx+c)}}{576 d}-\frac{3 i a^3 e^{7 i (dx+c)}}{224 d}-\frac{3 i a^3 e^{5 i (dx+c)}}{64 d}-\frac{9 i a^3 \cos (dx+c)}{64 d}+\frac{21 a^3 \sin (dx+c)}{64 d}-\frac{19 i a^3 \cos (3dx+3c)}{192 d}$ -
derivativedivides	$\frac{-i a^3\left(-\frac{\cos(dx+c)^7 \sin(dx+c)^2}{9}-\frac{2\cos(dx+c)^7}{63}\right)-3a^3\left(-\frac{\cos(dx+c)^8 \sin(dx+c)}{9}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+}{5}\right.}{63}\right.}{d}$
default	$\frac{-i a^3\left(-\frac{\cos(dx+c)^7 \sin(dx+c)^2}{9}-\frac{2\cos(dx+c)^7}{63}\right)-3a^3\left(-\frac{\cos(dx+c)^8 \sin(dx+c)}{9}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+}{5}\right.}{63}\right.}{d}$

input
```
int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/576*I/d*a^3*exp(9*I*(d*x+c))-3/224*I/d*a^3*exp(7*I*(d*x+c))-3/64*I/d*a^
3*exp(5*I*(d*x+c))-9/64*I/d*a^3*cos(d*x+c)+21/64*a^3*sin(d*x+c)/d-19/192*I
/d*a^3*cos(3*d*x+3*c)+7/64/d*a^3*sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.84

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{\left(-7i\,a^3e^{(12i\,dx+12i\,c)}-54i\,a^3e^{(10i\,dx+10i\,c)}-189i\,a^3e^{(8i\,dx+8i\,c)}-420i\,a^3e^{(6i\,dx+6i\,c)}-945i\,a^3e^{(4i\,dx+4i\,c)}+37\right.}{4032\,d}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4032*(-7*I*a^3*e^(12*I*d*x + 12*I*c) - 54*I*a^3*e^(10*I*d*x + 10*I*c) -
189*I*a^3*e^(8*I*d*x + 8*I*c) - 420*I*a^3*e^(6*I*d*x + 6*I*c) - 945*I*a^3*
e^(4*I*d*x + 4*I*c) + 378*I*a^3*e^(2*I*d*x + 2*I*c) + 21*I*a^3)*e^(-3*I*d*
x - 3*I*c)/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(112) = 224.

Time = 0.41 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.22

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \begin{cases} \frac{(-270582939648ia^3d^6e^{13ic}e^{9idx}-2087354105856ia^3d^6e^{11ic}e^{7idx}-7305739370496ia^3d^6e^{9ic}e^{5idx}-16234976378880ia^3d^6e^{7ic}e^{3idx}-3652869}{155855773237248d^7} \\ \frac{x(a^3e^{12ic}+6a^3e^{10ic}+15a^3e^{8ic}+20a^3e^{6ic}+15a^3e^{4ic}+6a^3e^{2ic}+a^3)e^{-3ic}}{64} \end{cases}$$

input
```
integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise(((-270582939648*I*a**3*d**6*exp(13*I*c)*exp(9*I*d*x) - 208735410
5856*I*a**3*d**6*exp(11*I*c)*exp(7*I*d*x) - 7305739370496*I*a**3*d**6*exp(
9*I*c)*exp(5*I*d*x) - 16234976378880*I*a**3*d**6*exp(7*I*c)*exp(3*I*d*x) -
 36528696852480*I*a**3*d**6*exp(5*I*c)*exp(I*d*x) + 14611478740992*I*a**3*
d**6*exp(3*I*c)*exp(-I*d*x) + 811748818944*I*a**3*d**6*exp(I*c)*exp(-3*I*d
*x))*exp(-4*I*c)/(155855773237248*d**7), Ne(d**7*exp(4*I*c), 0)), (x*(a**3
*exp(12*I*c) + 6*a**3*exp(10*I*c) + 15*a**3*exp(8*I*c) + 20*a**3*exp(6*I*c
) + 15*a**3*exp(4*I*c) + 6*a**3*exp(2*I*c) + a**3)*exp(-3*I*c)/64, True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx =$$
$$-\frac{105i\,a^3\cos(dx+c)^9+5i\left(7\cos(dx+c)^9-9\cos(dx+c)^7\right)a^3-3\left(35\sin(dx+c)^9-135\sin(dx+}{}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/315*(105*I*a^3*cos(d*x + c)^9 + 5*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^
7)*a^3 - 3*(35*sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 -
105*sin(d*x + c)^3)*a^3 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7 + 378*si
n(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^3)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1039 vs. $2(106) = 212$.

Time = 0.61 (sec) , antiderivative size = 1039, normalized size of antiderivative = 8.38

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/516096*(119511*a^3*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 478
044*a^3*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 717066*a^3*e^(7*I
*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 478044*a^3*e^(5*I*d*x - I*c)*log(
I*e^(I*d*x + I*c) + 1) + 119511*a^3*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x + I
*c) + 1) + 128898*a^3*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) - 1) + 51
5592*a^3*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) - 1) + 773388*a^3*e^(7*
I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 515592*a^3*e^(5*I*d*x - I*c)*log
(I*e^(I*d*x + I*c) - 1) + 128898*a^3*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x +
I*c) - 1) - 119511*a^3*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) + 1) -
478044*a^3*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 717066*a^3*e^
(7*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) + 1) - 478044*a^3*e^(5*I*d*x - I*c)
*log(-I*e^(I*d*x + I*c) + 1) - 119511*a^3*e^(3*I*d*x - 3*I*c)*log(-I*e^(I*
d*x + I*c) + 1) - 128898*a^3*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) -
 1) - 515592*a^3*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 773388*
a^3*e^(7*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 515592*a^3*e^(5*I*d*x
- I*c)*log(-I*e^(I*d*x + I*c) - 1) - 128898*a^3*e^(3*I*d*x - 3*I*c)*log(-I
*e^(I*d*x + I*c) - 1) + 9387*a^3*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x) + e^
(-I*c)) + 37548*a^3*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 5632
2*a^3*e^(7*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 37548*a^3*e^(5*I*d*x
 - I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 9387*a^3*e^(3*I*d*x - 3*I*c)*log(...
```

Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.66

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{2\,a^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-3\mathrm{i}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)} + \frac{2048\,a^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)}{9\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^9} - \frac{1024\,a^3\,\left(8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9\mathrm{i}\right)}{9\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^8} - \frac{4\,a^3\,\left(14\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-39\mathrm{i}\right)}{3\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2} + \frac{8\,a^3\,\left(43\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-97\mathrm{i}\right)}{3\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^3} - \frac{16\,a^3\,\left(188\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-357\mathrm{i}\right)}{7\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4} + \frac{128\,a^3\,\left(263\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-333\mathrm{i}\right)}{21\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^7} - \frac{64\,a^3\,\left(1598\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2289\mathrm{i}\right)}{63\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^6} + \frac{32\,a^3\,\left(2041\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-3339\mathrm{i}\right)}{63\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

input
```
int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(2*a^3*(tan(c/2 + (d*x)/2) - 3i))/(d*(tan(c/2 + (d*x)/2)^2 + 1)) + (2048*a
^3*(tan(c/2 + (d*x)/2) - 1i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^9) - (1024*a
^3*(8*tan(c/2 + (d*x)/2) - 9i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^8) - (4*a^
3*(14*tan(c/2 + (d*x)/2) - 39i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^2) + (8*a
^3*(43*tan(c/2 + (d*x)/2) - 97i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^3) - (16
*a^3*(188*tan(c/2 + (d*x)/2) - 357i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^4) +
 (128*a^3*(263*tan(c/2 + (d*x)/2) - 333i))/(21*d*(tan(c/2 + (d*x)/2)^2 + 1
)^7) - (64*a^3*(1598*tan(c/2 + (d*x)/2) - 2289i))/(63*d*(tan(c/2 + (d*x)/2
)^2 + 1)^6) + (32*a^3*(2041*tan(c/2 + (d*x)/2) - 3339i))/(63*d*(tan(c/2 +
(d*x)/2)^2 + 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{a^3\left(-28\cos(dx+c)\sin(dx+c)^8 i+103\cos(dx+c)\sin(dx+c)^6 i-141\cos(dx+c)\sin(dx+c)^4 i+8\right.}{}$$

input
```
int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(a**3*( - 28*cos(c + d*x)*sin(c + d*x)**8*i + 103*cos(c + d*x)*sin(c + d*x
)**6*i - 141*cos(c + d*x)*sin(c + d*x)**4*i + 85*cos(c + d*x)*sin(c + d*x)
**2*i - 19*cos(c + d*x)*i + 28*sin(c + d*x)**9 - 117*sin(c + d*x)**7 + 189
*sin(c + d*x)**5 - 147*sin(c + d*x)**3 + 63*sin(c + d*x) + 19*i))/(63*d)
```

3.52 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{21a^4\text{arctanh}(\sin(c+dx))}{16d}+\frac{7ia^4\sec^3(c+dx)}{8d}+\frac{21a^4\sec(c+dx)\tan(c+dx)}{16d}+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}+\frac{3i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{10d}+\frac{21i\sec^3(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{40d}$$

output
```
21/16*a^4*arctanh(sin(d*x+c))/d+7/8*I*a^4*sec(d*x+c)^3/d+21/16*a^4*sec(d*x
+c)*tan(d*x+c)/d+1/6*I*a*sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d+3/10*I*sec(d*
x+c)^3*(a^2+I*a^2*tan(d*x+c))^2/d+21/40*I*sec(d*x+c)^3*(a^4+I*a^4*tan(d*x+
c))/d
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.05

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx =$$
$$-\frac{a^4\sec^2(c+dx)(\cos(4c)-i\sin(4c))\left(-4608i\cos(c+dx)+5040\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\mathrm{s}\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
-1/3840*(a^4*Sec[c + d*x]^2*(Cos[4*c] - I*Sin[4*c])*((-4608*I)*Cos[c + d*x
] + 5040*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 5*((-512*I)*Cos[3*(c + d*x)] + 90*Si
n[c + d*x] + 155*Sin[3*(c + d*x)] - 63*Sin[5*(c + d*x)]))*(-I + Tan[c + d*
x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3979, 3042, 3979, 3042, 3979, 3042, 3967, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec(c+dx)^3(a+ia\tan(c+dx))^4dx$$

$$\Big\downarrow \ 3979$$

$$\frac{3}{2}a\int \sec^3(c+dx)(i\tan(c+dx)a+a)^3dx+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \ 3042$$

$$\frac{3}{2}a\int \sec(c+dx)^3(i\tan(c+dx)a+a)^3dx+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{3}{2}a\left(\frac{7}{5}a\int \sec^3(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}\right)+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\left(\frac{7}{5}a\int \sec(c+dx)^3(i\tan(c+dx)a+a)^2dx+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}\right)+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{3}{2}a\left(\frac{7}{5}a\left(\frac{5}{4}a\int \sec^3(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\right)+\frac{ia\sec^3(c+dx)(a\right.$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\left(\frac{7}{5}a\left(\frac{5}{4}a\int \sec(c+dx)^3(i\tan(c+dx)a+a)dx+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\right)+\frac{ia\sec^3(c+dx)(a\right.$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{3}{2}a\left(\frac{7}{5}a\left(\frac{5}{4}a\left(a\int \sec^3(c+dx)dx+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\right)+\frac{ia\sec^3(c+dx\right.$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\left(\frac{7}{5}a\left(\frac{5}{4}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\right)+\frac{ia\sec^3(\right.$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{2}a\left(\frac{7}{5}a\left(\frac{5}{4}a\left(a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\right.\right.$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\left(\frac{7}{5}a\left(\frac{5}{4}a\left(a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2\right.}{4}\right.\right.$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{3}{2}a\left(\frac{7}{5}a\left(\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}+\frac{5}{4}a\left(a\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia}{}\right.\right.\right.$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

input
```
Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/6)*a*Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d + (3*a*(((I/5)*a*Sec[c
 + d*x]^3*(a + I*a*Tan[c + d*x])^2)/d + (7*a*(((I/4)*Sec[c + d*x]^3*(a^2 +
 I*a^2*Tan[c + d*x]))/d + (5*a*(((I/3)*a*Sec[c + d*x]^3)/d + a*(ArcTanh[Si
n[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d))))/4))/5))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 15.61 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{i a^4 \left(315\,\mathrm{e}^{11 i (dx+c)}-3335\,\mathrm{e}^{9 i (dx+c)}-5058\,\mathrm{e}^{7 i (dx+c)}-4158\,\mathrm{e}^{5 i (dx+c)}-1785\,\mathrm{e}^{3 i (dx+c)}-315\,\mathrm{e}^{i (dx+c)}\right)}{120 d \left(\mathrm{e}^{2 i (dx+c)}+1\right)^6}+\frac{21 a^4 \ln\left(\mathrm{e}^{i(c}\right.}{16 c}$
derivativedivides	$\frac{a^4\left(\frac{\sin(dx+c)^5}{6\cos(dx+c)^6}+\frac{\sin(dx+c)^5}{24\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{48\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{48}-\frac{\sin(dx+c)}{16}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)-4ia^4\left(\frac{\sin(dx+c)^4}{5\cos(dx+c)^5}\right.}{}$
default	$\frac{a^4\left(\frac{\sin(dx+c)^5}{6\cos(dx+c)^6}+\frac{\sin(dx+c)^5}{24\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{48\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{48}-\frac{\sin(dx+c)}{16}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)-4ia^4\left(\frac{\sin(dx+c)^4}{5\cos(dx+c)^5}\right.}{}$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/120*I*a^4/d/(exp(2*I*(d*x+c))+1)^6*(315*exp(11*I*(d*x+c))-3335*exp(9*I*
(d*x+c))-5058*exp(7*I*(d*x+c))-4158*exp(5*I*(d*x+c))-1785*exp(3*I*(d*x+c))
-315*exp(I*(d*x+c)))+21/16*a^4/d*ln(exp(I*(d*x+c))+I)-21/16*a^4/d*ln(exp(I
*(d*x+c))-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(137) = 274.

Time = 0.08 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.23

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$=\frac{-630i\,a^4e^{(11i\,dx+11i\,c)}+6670i\,a^4e^{(9i\,dx+9i\,c)}+10116i\,a^4e^{(7i\,dx+7i\,c)}+8316i\,a^4e^{(5i\,dx+5i\,c)}+3570i\,a^4e^{(3i\,dx+3i}}{}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/240*(-630*I*a^4*e^(11*I*d*x + 11*I*c) + 6670*I*a^4*e^(9*I*d*x + 9*I*c) +
 10116*I*a^4*e^(7*I*d*x + 7*I*c) + 8316*I*a^4*e^(5*I*d*x + 5*I*c) + 3570*I
*a^4*e^(3*I*d*x + 3*I*c) + 630*I*a^4*e^(I*d*x + I*c) + 315*(a^4*e^(12*I*d*
x + 12*I*c) + 6*a^4*e^(10*I*d*x + 10*I*c) + 15*a^4*e^(8*I*d*x + 8*I*c) + 2
0*a^4*e^(6*I*d*x + 6*I*c) + 15*a^4*e^(4*I*d*x + 4*I*c) + 6*a^4*e^(2*I*d*x
+ 2*I*c) + a^4)*log(e^(I*d*x + I*c) + I) - 315*(a^4*e^(12*I*d*x + 12*I*c)
+ 6*a^4*e^(10*I*d*x + 10*I*c) + 15*a^4*e^(8*I*d*x + 8*I*c) + 20*a^4*e^(6*I
*d*x + 6*I*c) + 15*a^4*e^(4*I*d*x + 4*I*c) + 6*a^4*e^(2*I*d*x + 2*I*c) + a
^4)*log(e^(I*d*x + I*c) - I))/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x +
 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4
*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx = a^4\left(\int\left(-6\tan^2(c+dx)\sec^3(c+dx)\right)dx + \int\tan^4(c+dx)\sec^3(c+dx)\,dx + \int 4i\tan(c+dx)\sec^3(c+dx)\,dx + \int\left(-4i\tan^3(c+dx)\sec^3(c+dx)\right)dx + \int\sec^3(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**4,x)
```

output
```
a**4*(Integral(-6*tan(c + d*x)**2*sec(c + d*x)**3, x) + Integral(tan(c + d
*x)**4*sec(c + d*x)**3, x) + Integral(4*I*tan(c + d*x)*sec(c + d*x)**3, x)
 + Integral(-4*I*tan(c + d*x)**3*sec(c + d*x)**3, x) + Integral(sec(c + d*
x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.51

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx =$$

$$-\frac{5\,a^4\left(\frac{2\left(3\,\sin(dx+c)^5+8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+180\,a}{}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/480*(5*a^4*(2*(3*sin(d*x + c)^5 + 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(s
in(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x +
 c) + 1) + 3*log(sin(d*x + c) - 1)) + 180*a^4*(2*(sin(d*x + c)^3 + sin(d*x
 + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) + 120*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log
(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 640*I*a^4/cos(d*x + c)^3 - 1
28*I*(5*cos(d*x + c)^2 - 3)*a^4/cos(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.45

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$=\frac{315\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-315\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2\left(75\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+960i\,a^4\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/240*(315*a^4*log(tan(1/2*d*x + 1/2*c) + 1) - 315*a^4*log(tan(1/2*d*x + 1
/2*c) - 1) - 2*(75*a^4*tan(1/2*d*x + 1/2*c)^11 + 960*I*a^4*tan(1/2*d*x + 1
/2*c)^10 + 1175*a^4*tan(1/2*d*x + 1/2*c)^9 - 4800*I*a^4*tan(1/2*d*x + 1/2*
c)^8 - 1890*a^4*tan(1/2*d*x + 1/2*c)^7 + 4480*I*a^4*tan(1/2*d*x + 1/2*c)^6
 - 1890*a^4*tan(1/2*d*x + 1/2*c)^5 - 1920*I*a^4*tan(1/2*d*x + 1/2*c)^4 + 1
175*a^4*tan(1/2*d*x + 1/2*c)^3 + 1728*I*a^4*tan(1/2*d*x + 1/2*c)^2 + 75*a^
4*tan(1/2*d*x + 1/2*c) - 448*I*a^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.78

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{21\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d} - \frac{\frac{5\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8} + a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,8\mathrm{i} + \frac{235\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24} - a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\,40\mathrm{i} - \frac{63\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4} + \cdots}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - 6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + 15\tan\left(\frac{c}{2}+\cdots\right.\right.}$$

input
```
int((a + a*tan(c + d*x)*1i)^4/cos(c + d*x)^3,x)
```

output
```
(21*a^4*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((a^4*tan(c/2 + (d*x)/2)^2*72i)
/5 + (235*a^4*tan(c/2 + (d*x)/2)^3)/24 - a^4*tan(c/2 + (d*x)/2)^4*16i - (6
3*a^4*tan(c/2 + (d*x)/2)^5)/4 + (a^4*tan(c/2 + (d*x)/2)^6*112i)/3 - (63*a^
4*tan(c/2 + (d*x)/2)^7)/4 - a^4*tan(c/2 + (d*x)/2)^8*40i + (235*a^4*tan(c/
2 + (d*x)/2)^9)/24 + a^4*tan(c/2 + (d*x)/2)^10*8i + (5*a^4*tan(c/2 + (d*x)
/2)^11)/8 - (a^4*56i)/15 + (5*a^4*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 +
(d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2
 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.79

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{a^4\left(640\cos\left(dx+c\right)\sin\left(dx+c\right)^2 i - 448\cos\left(dx+c\right) i - 315\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^6 + 945\,\mathrm{lo}\right.}{}$$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x)
```

output
```
(a**4*(640*cos(c + d*x)*sin(c + d*x)**2*i - 448*cos(c + d*x)*i - 315*log(t
an((c + d*x)/2) - 1)*sin(c + d*x)**6 + 945*log(tan((c + d*x)/2) - 1)*sin(c
 + d*x)**4 - 945*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 + 315*log(tan((
c + d*x)/2) - 1) + 315*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**6 - 945*log
(tan((c + d*x)/2) + 1)*sin(c + d*x)**4 + 945*log(tan((c + d*x)/2) + 1)*sin
(c + d*x)**2 - 315*log(tan((c + d*x)/2) + 1) - 448*sin(c + d*x)**6*i - 315
*sin(c + d*x)**5 + 1344*sin(c + d*x)**4*i + 200*sin(c + d*x)**3 - 1344*sin
(c + d*x)**2*i + 75*sin(c + d*x) + 448*i))/(240*d*(sin(c + d*x)**6 - 3*sin
(c + d*x)**4 + 3*sin(c + d*x)**2 - 1))
```

3.53 $\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 133

$$\begin{aligned}\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx &= \frac{35a^4\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{35ia^4\sec(c+dx)}{8d}\\ &+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}\\ &+\frac{7i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{12d}\\ &+\frac{35i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{24d}\end{aligned}$$

output
```
35/8*a^4*arctanh(sin(d*x+c))/d+35/8*I*a^4*sec(d*x+c)/d+1/4*I*a*sec(d*x+c)*
(a+I*a*tan(d*x+c))^3/d+7/12*I*sec(d*x+c)*(a^2+I*a^2*tan(d*x+c))^2/d+35/24*
I*sec(d*x+c)*(a^4+I*a^4*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.78

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx =$$
$$-\frac{a^4\sec^4(c+dx)\left(-896i\cos(c+dx)+3\left(-128i\cos(3(c+dx))+105\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
-1/192*(a^4*Sec[c + d*x]^4*((-896*I)*Cos[c + d*x] + 3*((-128*I)*Cos[3*(c +
 d*x)] + 105*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 35*Cos[4*(c + d*x)
]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 140*Cos[2*(c + d*x)]*(Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]]) - 105*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 35*Cos[4*(c + d*x)]*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 42*Sin[c + d*x] + 58*Sin[3*(c +
d*x)])))/d
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3979, 3042, 3979, 3042, 3979, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(c+dx)(a+ia\tan(c+dx))^4dx$$
$$\Big\downarrow \text{3979}$$
$$\frac{7}{4}a\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{7}{4}a\left(\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\left(\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a+}{3d}\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a+}{3d}\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a}{3}\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+\right.$$

$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{3}{2}a\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)\right)+\frac{ia\sec(c+dx\right.$$

$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

input
```
Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/4)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3)/d + (7*a*(((I/3)*a*Sec[c +
 d*x]*(a + I*a*Tan[c + d*x])^2)/d + (5*a*((3*a*((a*ArcTanh[Sin[c + d*x]])/
d + (I*a*Sec[c + d*x])/d))/2 + ((I/2)*Sec[c + d*x]*(a^2 + I*a^2*Tan[c + d*
x]))/d))/3))/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

method	result
risch	$\frac{i a^{4} \left(279 \,{\mathrm e}^{7 i \left(d x +c \right)}+511 \,{\mathrm e}^{5 i \left(d x +c \right)}+385 \,{\mathrm e}^{3 i \left(d x +c \right)}+105 \,{\mathrm e}^{i \left(d x +c \right)}\right)}{12 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{4}}+\frac{35 a^{4} \ln \left({\mathrm e}^{i \left(d x +c \right)}+i\right)}{8 d}-\frac{35 a^{4} \ln \left({\mathrm e}^{i \left(d x +c \right)}-i\right)}{8 d}$
derivativedivides	$a^{4} \left(\frac{\sin \left(d x +c \right)^{5}}{4 \cos \left(d x +c \right)^{4}}-\frac{\sin \left(d x +c \right)^{5}}{8 \cos \left(d x +c \right)^{2}}-\frac{\sin \left(d x +c \right)^{3}}{8}-\frac{3 \sin \left(d x +c \right)}{8}+\frac{3 \ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)}{8}\right)-4 i a^{4} \left(\frac{\sin \left(d x +c \right)^{4}}{3 \cos \left(d x +c \right)^{3}}-\frac{\sin \left(d x +c \right)^{4}}{3 \cos \left(d x +c \right)}\right.$
default	$a^{4} \left(\frac{\sin \left(d x +c \right)^{5}}{4 \cos \left(d x +c \right)^{4}}-\frac{\sin \left(d x +c \right)^{5}}{8 \cos \left(d x +c \right)^{2}}-\frac{\sin \left(d x +c \right)^{3}}{8}-\frac{3 \sin \left(d x +c \right)}{8}+\frac{3 \ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)}{8}\right)-4 i a^{4} \left(\frac{\sin \left(d x +c \right)^{4}}{3 \cos \left(d x +c \right)^{3}}-\frac{\sin \left(d x +c \right)^{4}}{3 \cos \left(d x +c \right)}\right.$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/12*I*a^4/d/(exp(2*I*(d*x+c))+1)^4*(279*exp(7*I*(d*x+c))+511*exp(5*I*(d*x
+c))+385*exp(3*I*(d*x+c))+105*exp(I*(d*x+c)))+35/8*a^4/d*ln(exp(I*(d*x+c))
+I)-35/8*a^4/d*ln(exp(I*(d*x+c))-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(109) = 218.

Time = 0.09 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.92

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{558i\,a^4e^{(7i\,dx+7i\,c)} + 1022i\,a^4e^{(5i\,dx+5i\,c)} + 770i\,a^4e^{(3i\,dx+3i\,c)} + 210i\,a^4e^{(i\,dx+i\,c)} + 105\left(a^4e^{(8i\,dx+8i\,c)} + 4\,a^4e^{\cdots}\right.}{24\,(d\cdots}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/24*(558*I*a^4*e^(7*I*d*x + 7*I*c) + 1022*I*a^4*e^(5*I*d*x + 5*I*c) + 770
*I*a^4*e^(3*I*d*x + 3*I*c) + 210*I*a^4*e^(I*d*x + I*c) + 105*(a^4*e^(8*I*d
*x + 8*I*c) + 4*a^4*e^(6*I*d*x + 6*I*c) + 6*a^4*e^(4*I*d*x + 4*I*c) + 4*a^
4*e^(2*I*d*x + 2*I*c) + a^4)*log(e^(I*d*x + I*c) + I) - 105*(a^4*e^(8*I*d*
x + 8*I*c) + 4*a^4*e^(6*I*d*x + 6*I*c) + 6*a^4*e^(4*I*d*x + 4*I*c) + 4*a^4
*e^(2*I*d*x + 2*I*c) + a^4)*log(e^(I*d*x + I*c) - I))/(d*e^(8*I*d*x + 8*I*
c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x +
2*I*c) + d)
```

Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx = a^4\left(\int\left(-6\tan^2(c+dx)\sec(c+dx)\right)\,dx + \int\tan^4(c+dx)\sec(c+dx)\,dx + \int 4i\tan(c+dx)\sec(c+dx)\,dx + \int\left(-4i\tan^3(c+dx)\sec(c+dx)\right)\,dx + \int\sec(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**4,x)
```

output
```
a**4*(Integral(-6*tan(c + d*x)**2*sec(c + d*x), x) + Integral(tan(c + d*x)
**4*sec(c + d*x), x) + Integral(4*I*tan(c + d*x)*sec(c + d*x), x) + Integr
al(-4*I*tan(c + d*x)**3*sec(c + d*x), x) + Integral(sec(c + d*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.35

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{3a^4\left(\frac{2\left(5\sin(dx+c)^3-3\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}+3\log\left(\sin\left(dx+c\right)+1\right)-3\log\left(\sin\left(dx+c\right)-1\right)\right)+72a^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}\right.}{}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/48*(3*a^4*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin
(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)) + 72
*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(si
n(d*x + c) - 1)) + 48*a^4*log(sec(d*x + c) + tan(d*x + c)) + 192*I*a^4/cos
(d*x + c) + 64*I*(3*cos(d*x + c)^2 - 1)*a^4/cos(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.30

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{105a^4\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-105a^4\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)-\frac{2\left(81a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+96ia^4\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{}}{24\,d}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/24*(105*a^4*log(tan(1/2*d*x + 1/2*c) + 1) - 105*a^4*log(tan(1/2*d*x + 1/
2*c) - 1) - 2*(81*a^4*tan(1/2*d*x + 1/2*c)^7 + 96*I*a^4*tan(1/2*d*x + 1/2*
c)^6 - 105*a^4*tan(1/2*d*x + 1/2*c)^5 - 480*I*a^4*tan(1/2*d*x + 1/2*c)^4 -
 105*a^4*tan(1/2*d*x + 1/2*c)^3 + 544*I*a^4*tan(1/2*d*x + 1/2*c)^2 + 81*a^
4*tan(1/2*d*x + 1/2*c) - 160*I*a^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.49

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{35\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$
$$-\frac{\frac{27\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 8\mathrm{i}-\frac{35\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 40\mathrm{i}-\frac{35\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{a^4\tan}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right.}$$

input
```
int((a + a*tan(c + d*x)*1i)^4/cos(c + d*x),x)
```

output
```
(35*a^4*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a^4*tan(c/2 + (d*x)/2)^2*136i
)/3 - (35*a^4*tan(c/2 + (d*x)/2)^3)/4 - a^4*tan(c/2 + (d*x)/2)^4*40i - (35
*a^4*tan(c/2 + (d*x)/2)^5)/4 + a^4*tan(c/2 + (d*x)/2)^6*8i + (27*a^4*tan(c
/2 + (d*x)/2)^7)/4 - (a^4*40i)/3 + (27*a^4*tan(c/2 + (d*x)/2))/4)/(d*(6*ta
n(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan
(c/2 + (d*x)/2)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.62

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$=\frac{a^4\left(-192\cos\left(dx+c\right)\sin\left(dx+c\right)^2 i+160\cos\left(dx+c\right)i-105\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^4+210\right.}{}$$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x)
```

output
```
(a**4*( - 192*cos(c + d*x)*sin(c + d*x)**2*i + 160*cos(c + d*x)*i - 105*lo
g(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 + 210*log(tan((c + d*x)/2) - 1)*si
n(c + d*x)**2 - 105*log(tan((c + d*x)/2) - 1) + 105*log(tan((c + d*x)/2) +
 1)*sin(c + d*x)**4 - 210*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 + 105*
log(tan((c + d*x)/2) + 1) - 160*sin(c + d*x)**4*i + 87*sin(c + d*x)**3 + 3
20*sin(c + d*x)**2*i - 81*sin(c + d*x) - 160*i))/(24*d*(sin(c + d*x)**4 -
2*sin(c + d*x)**2 + 1))
```

3.54 $\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{15a^4\text{arctanh}(\sin(c+dx))}{2d} - \frac{15ia^4\sec(c+dx)}{2d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d} - \frac{5i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{2d}$$

output
```
-15/2*a^4*arctanh(sin(d*x+c))/d-15/2*I*a^4*sec(d*x+c)/d-2*I*a*cos(d*x+c)*(
a+I*a*tan(d*x+c))^3/d-5/2*I*sec(d*x+c)*(a^4+I*a^4*tan(d*x+c))/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 906 vs. $2(97) = 194$.

Time = 6.71 (sec) , antiderivative size = 906, normalized size of antiderivative = 9.34

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(15*Cos[4*c]*Cos[c + d*x]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*(
a + I*a*Tan[c + d*x])^4)/(2*d*(Cos[d*x] + I*Sin[d*x])^4) - (15*Cos[4*c]*Co
s[c + d*x]^4*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(a + I*a*Tan[c +
 d*x])^4)/(2*d*(Cos[d*x] + I*Sin[d*x])^4) + (Cos[d*x]*Cos[c + d*x]^4*((-8*
I)*Cos[3*c] - 8*Sin[3*c])*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x] + I*Sin[d
*x])^4) + (Cos[c + d*x]^4*Sec[c]*((-4*I)*Cos[4*c] - 4*Sin[4*c])*(a + I*a*T
an[c + d*x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4) - (((15*I)/2)*Cos[c + d*x]^4
*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sin[4*c]*(a + I*a*Tan[c + d*
x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4) + (((15*I)/2)*Cos[c + d*x]^4*Log[Cos[
c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sin[4*c]*(a + I*a*Tan[c + d*x])^4)/(d
*(Cos[d*x] + I*Sin[d*x])^4) + (Cos[c + d*x]^4*(8*Cos[3*c] - (8*I)*Sin[3*c]
)*Sin[d*x]*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4) + (Cos[
c + d*x]^4*(Cos[4*c]/4 - (I/4)*Sin[4*c])*(a + I*a*Tan[c + d*x])^4)/(d*(Cos
[d*x] + I*Sin[d*x])^4*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) - (I*Co
s[c + d*x]^4*(4*Cos[4*c] - (4*I)*Sin[4*c])*Sin[(d*x)/2]*(a + I*a*Tan[c + d
*x])^4)/(d*(Cos[c/2] - Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^4*(Cos[c/2 + (d*x
)/2] - Sin[c/2 + (d*x)/2])) + (Cos[c + d*x]^4*(-1/4*Cos[4*c] + (I/4)*Sin[4
*c])*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4*(Cos[c/2 + (d*
x)/2] + Sin[c/2 + (d*x)/2])^2) + (I*Cos[c + d*x]^4*(4*Cos[4*c] - (4*I)*Sin
[4*c])*Sin[(d*x)/2]*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[c/2] + Sin[c/2])*...
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3977, 3042, 3979, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{\sec(c+dx)}dx$$

$$\Big\downarrow \; 3977$$

$$-5a^2 \int \sec(c+dx)(i\tan(c+dx)a+a)^2dx - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \; 3042$$

$$-5a^2 \int \sec(c+dx)(i\tan(c+dx)a+a)^2dx - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \; 3979$$

$$-5a^2\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx + \frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right) - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \; 3042$$

$$-5a^2\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx + \frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right) - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \; 3967$$

$$-5a^2\left(\frac{3}{2}a\left(a\int \sec(c+dx)dx + \frac{ia\sec(c+dx)}{d}\right) + \frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right) - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \; 3042$$

$$-5a^2\left(\frac{3}{2}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx + \frac{ia\sec(c+dx)}{d}\right) + \frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right) - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \; 4257$$

$$-5a^2\left(\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d} + \frac{3}{2}a\left(\frac{a\text{arctanh}(\sin(c+dx))}{d} + \frac{ia\sec(c+dx)}{d}\right)\right) - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-2*I)*a*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^3)/d - 5*a^2*((3*a*((a*ArcTa
nh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d))/2 + ((I/2)*Sec[c + d*x]*(a^2
+ I*a^2*Tan[c + d*x]))/d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

method	result
risch	$-\frac{8ia^4 e^{i(dx+c)}}{d}-\frac{ia^4\left(9\,e^{3i(dx+c)}+7\,e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{15a^4\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{15a^4\ln\left(e^{i(dx+c)}-i\right)}{2d}$
derivativedivides	$\frac{a^4\left(\frac{\sin(dx+c)^5}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^3}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4ia^4\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c\right.}{d}$
default	$\frac{a^4\left(\frac{\sin(dx+c)^5}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^3}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4ia^4\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c\right.}{d}$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-8*I*a^4/d*exp(I*(d*x+c))-I*a^4/d/(exp(2*I*(d*x+c))+1)^2*(9*exp(3*I*(d*x+c
))+7*exp(I*(d*x+c)))-15/2*a^4/d*ln(exp(I*(d*x+c))+I)+15/2*a^4/d*ln(exp(I*(
d*x+c))-I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.67

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$=\frac{-16i\,a^4e^{(5i\,dx+5i\,c)}-50i\,a^4e^{(3i\,dx+3i\,c)}-30i\,a^4e^{(i\,dx+i\,c)}-15\left(a^4e^{(4i\,dx+4i\,c)}+2\,a^4e^{(2i\,dx+2i\,c)}+a^4\right)\log\left(e^{(i\,dx\right.}}{2\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/2*(-16*I*a^4*e^(5*I*d*x + 5*I*c) - 50*I*a^4*e^(3*I*d*x + 3*I*c) - 30*I*a
^4*e^(I*d*x + I*c) - 15*(a^4*e^(4*I*d*x + 4*I*c) + 2*a^4*e^(2*I*d*x + 2*I*
c) + a^4)*log(e^(I*d*x + I*c) + I) + 15*(a^4*e^(4*I*d*x + 4*I*c) + 2*a^4*e
^(2*I*d*x + 2*I*c) + a^4)*log(e^(I*d*x + I*c) - I))/(d*e^(4*I*d*x + 4*I*c)
 + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.58

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{15a^4\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{2}-\frac{\log\left(e^{idx}+ie^{-ic}\right)}{2}\right)}{d} + \frac{-9ia^4e^{3ic}e^{3idx}-7ia^4e^{ic}e^{idx}}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d} + \begin{cases} -\frac{8ia^4e^{ic}e^{idx}}{d} & \text{for } d \neq 0 \\ 8a^4xe^{ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**4,x)
```

output
```
15*a**4*(log(exp(I*d*x) - I*exp(-I*c))/2 - log(exp(I*d*x) + I*exp(-I*c))/2
)/d + (-9*I*a**4*exp(3*I*c)*exp(3*I*d*x) - 7*I*a**4*exp(I*c)*exp(I*d*x))/(
d*exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*x) + d) + Piecewise((
-8*I*a**4*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (8*a**4*x*exp(I*c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.41

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{a^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+3\log\left(\sin\left(dx+c\right)+1\right)-3\log\left(\sin\left(dx+c\right)-1\right)-4\sin\left(dx+c\right)\right)+16i\,a^4\left(\frac{1}{\cos(dx+}\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/4*(a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) -
 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) + 16*I*a^4*(1/cos(d*x + c) + co
s(d*x + c)) + 12*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*si
n(d*x + c)) + 16*I*a^4*cos(d*x + c) - 4*a^4*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(81) = 162$.

Time = 0.40 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.84

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{235\,a^4e^{(4i\,dx+4i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+470\,a^4e^{(2i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)-5\,a^4e^{(4i\,dx+4i\,c)}\log\left(i\,e^{(i\,dx+i}\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/32*(235*a^4*e^(4*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 470*a^4*e^(
2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) - 5*a^4*e^(4*I*d*x + 4*I*c)*lo
g(I*e^(I*d*x + I*c) - 1) - 10*a^4*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c
) - 1) - 235*a^4*e^(4*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 470*a^4
*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) + 5*a^4*e^(4*I*d*x + 4*I*
c)*log(-I*e^(I*d*x + I*c) - 1) + 10*a^4*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*
x + I*c) - 1) - 256*I*a^4*e^(5*I*d*x + 5*I*c) - 800*I*a^4*e^(3*I*d*x + 3*I
*c) - 480*I*a^4*e^(I*d*x + I*c) + 235*a^4*log(I*e^(I*d*x + I*c) + 1) - 5*a
^4*log(I*e^(I*d*x + I*c) - 1) - 235*a^4*log(-I*e^(I*d*x + I*c) + 1) + 5*a^
4*log(-I*e^(I*d*x + I*c) - 1))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2
*I*c) + d)
```

Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.64

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{15\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{17\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,9\mathrm{i}-39\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,7\mathrm{i}+24\,a^4}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,1\mathrm{i}-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,2\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(a^4*tan(c/2 + (d*x)/2)^3*9i - 39*a^4*tan(c/2 + (d*x)/2)^2 + 17*a^4*tan(c/
2 + (d*x)/2)^4 + 24*a^4 - a^4*tan(c/2 + (d*x)/2)*7i)/(d*(tan(c/2 + (d*x)/2
) - tan(c/2 + (d*x)/2)^2*2i - 2*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^
4*1i + tan(c/2 + (d*x)/2)^5 + 1i)) - (15*a^4*atanh(tan(c/2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.56

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{a^4\left(-16\cos\left(dx+c\right)\sin\left(dx+c\right)^2 i+24\cos\left(dx+c\right) i+15\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2-15\log\left(\right.}{}$$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x)
```

output
```
(a**4*( - 16*cos(c + d*x)*sin(c + d*x)**2*i + 24*cos(c + d*x)*i + 15*log(t
an((c + d*x)/2) - 1)*sin(c + d*x)**2 - 15*log(tan((c + d*x)/2) - 1) - 15*l
og(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 + 15*log(tan((c + d*x)/2) + 1) +
16*sin(c + d*x)**3 + 24*sin(c + d*x)**2*i - 17*sin(c + d*x) - 24*i))/(2*d*
(sin(c + d*x)**2 - 1))
```

3.55 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 24, antiderivative size = 78

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx=\frac{a^4\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}+\frac{2i\cos(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}$$

output
```
a^4*arctanh(sin(d*x+c))/d-2/3*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d+2*I*
cos(d*x+c)*(a^4+I*a^4*tan(d*x+c))/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 246 vs. 2(78) = 156.

Time = 0.86 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.15

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$=\frac{a^4\left(-3\cos(4c)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(4c)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.}{}$$

input
```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(a^4*(-3*Cos[4*c]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Cos[4*c]*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*Cos[3*d*x]*Sin[c] + 6*Cos[d*x]*
Sin[3*c] + (3*I)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[4*c] - (3*I)
*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[4*c] + Cos[3*c]*((6*I)*Cos[d
*x] - 6*Sin[d*x]) + (6*I)*Sin[3*c]*Sin[d*x] - (2*I)*Sin[c]*Sin[3*d*x] + 2*
Cos[c]*((-I)*Cos[3*d*x] + Sin[3*d*x]))*(Cos[c + d*x] + I*Sin[c + d*x])^4)/
(3*d*(Cos[d*x] + I*Sin[d*x])^4)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3977, 3042, 3977, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^4}{\sec(c+dx)^3}dx$$
$$\Big\downarrow \text{3977}$$
$$-a^2\int \cos(c+dx)(i\tan(c+dx)a+a)^2dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$
$$\Big\downarrow \text{3042}$$
$$-a^2\int \frac{(i\tan(c+dx)a+a)^2}{\sec(c+dx)}dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$
$$\Big\downarrow \text{3977}$$
$$-a^2\left(a^2(-\int \sec(c+dx)dx)-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \textbf{3042}$$

$$-a^2\left(a^2\left(-\int \csc\left(c+dx+\frac{\pi}{2}\right)dx\right)-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \textbf{4257}$$

$$-a^2\left(-\frac{a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((-2*I)/3)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d - a^2*(-((a^2*Arc
Tanh[Sin[c + d*x]])/d) - ((2*I)*Cos[c + d*x]*(a^2 + I*a^2*Tan[c + d*x]))/d
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 12.56 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{2ia^4 e^{3i(dx+c)}}{3d}+\frac{2ia^4 e^{i(dx+c)}}{d}+\frac{a^4 \ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a^4 \ln\left(e^{i(dx+c)}-i\right)}{d}$
derivativedivides	$\frac{a^4\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+\frac{4ia^4\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}-2a^4\sin(dx+c)^3-\frac{4ia^4\cos(}{3}}{d}$
default	$\frac{a^4\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+\frac{4ia^4\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}-2a^4\sin(dx+c)^3-\frac{4ia^4\cos(}{3}}{d}$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*a^4/d*exp(3*I*(d*x+c))+2*I*a^4/d*exp(I*(d*x+c))+a^4/d*ln(exp(I*(d*x
+c))+I)-a^4/d*ln(exp(I*(d*x+c))-I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{-2i\,a^4e^{(3i\,dx+3i\,c)}+6i\,a^4e^{(i\,dx+i\,c)}+3\,a^4\log\left(e^{(i\,dx+i\,c)}+i\right)-3\,a^4\log\left(e^{(i\,dx+i\,c)}-i\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/3*(-2*I*a^4*e^(3*I*d*x + 3*I*c) + 6*I*a^4*e^(I*d*x + I*c) + 3*a^4*log(e^
(I*d*x + I*c) + I) - 3*a^4*log(e^(I*d*x + I*c) - I))/d
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.40

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{a^4\left(-\log\left(e^{idx}-ie^{-ic}\right)+\log\left(e^{idx}+ie^{-ic}\right)\right)}{d} + \begin{cases} \frac{-2ia^4de^{3ic}e^{3idx}+6ia^4de^{ic}e^{idx}}{3d^2} & \text{for } d^2 \neq 0 \\ x\left(2a^4e^{3ic}-2a^4e^{ic}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**4,x)
```

output
```
a**4*(-log(exp(I*d*x) - I*exp(-I*c)) + log(exp(I*d*x) + I*exp(-I*c)))/d +
Piecewise(((-2*I*a**4*d*exp(3*I*c)*exp(3*I*d*x) + 6*I*a**4*d*exp(I*c)*exp(
I*d*x))/(3*d**2), Ne(d**2, 0)), (x*(2*a**4*exp(3*I*c) - 2*a**4*exp(I*c)),
True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.55

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx =$$

$$-\frac{8i\,a^4\cos\left(dx+c\right)^3+12\,a^4\sin\left(dx+c\right)^3+8i\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)a^4+\left(2\,\sin\left(dx+c\right)^3-3\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/6*(8*I*a^4*cos(d*x + c)^3 + 12*a^4*sin(d*x + c)^3 + 8*I*(cos(d*x + c)^3
 - 3*cos(d*x + c))*a^4 + (2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*l
og(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^4 + 2*(sin(d*x + c)^3 - 3*sin(d*x
 + c))*a^4)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1299 vs. $2(68) = 136$.

Time = 0.59 (sec) , antiderivative size = 1299, normalized size of antiderivative = 16.65

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/768*(1110*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 6660*a^4
*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 16650*a^4*e^(8*I*d*x +
2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 16650*a^4*e^(4*I*d*x - 2*I*c)*log(I*e^
(I*d*x + I*c) + 1) + 6660*a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) +
1) + 22200*a^4*e^(6*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 1110*a^4*e^(-6*I*c
)*log(I*e^(I*d*x + I*c) + 1) + 1875*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*
x + I*c) - 1) + 11250*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1)
+ 28125*a^4*e^(8*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 28125*a^4*e^(
4*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 11250*a^4*e^(2*I*d*x - 4*I*c
)*log(I*e^(I*d*x + I*c) - 1) + 37500*a^4*e^(6*I*d*x)*log(I*e^(I*d*x + I*c)
 - 1) + 1875*a^4*e^(-6*I*c)*log(I*e^(I*d*x + I*c) - 1) - 1110*a^4*e^(12*I*
d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 6660*a^4*e^(10*I*d*x + 4*I*c)*l
og(-I*e^(I*d*x + I*c) + 1) - 16650*a^4*e^(8*I*d*x + 2*I*c)*log(-I*e^(I*d*x
 + I*c) + 1) - 16650*a^4*e^(4*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) -
 6660*a^4*e^(2*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 22200*a^4*e^(6
*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 1110*a^4*e^(-6*I*c)*log(-I*e^(I*d*x
+ I*c) + 1) - 1875*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) -
11250*a^4*e^(10*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 28125*a^4*e^(
8*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 28125*a^4*e^(4*I*d*x - 2*I*
c)*log(-I*e^(I*d*x + I*c) - 1) - 11250*a^4*e^(2*I*d*x - 4*I*c)*log(-I*e...
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{2\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{\frac{8\,a^4}{3}-a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,8\mathrm{i}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(2*a^4*atanh(tan(c/2 + (d*x)/2)))/d - ((8*a^4)/3 - a^4*tan(c/2 + (d*x)/2)*
8i)/(d*(3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*3i - tan(c/2 + (d*x)/2
)^3 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{a^4\left(8\cos(dx+c)\sin(dx+c)^2\,i+4\cos(dx+c)\,i-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\right.}{3d}$$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x)
```

output
```
(a**4*(8*cos(c + d*x)*sin(c + d*x)**2*i + 4*cos(c + d*x)*i - 3*log(tan((c
+ d*x)/2) - 1) + 3*log(tan((c + d*x)/2) + 1) - 8*sin(c + d*x)**3 - 4*i))/(
3*d)
```

3.56 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 24, antiderivative size = 66

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{15d} - \frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

output
```
-1/15*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d-1/5*I*cos(d*x+c)^5*(a+I*a*ta
n(d*x+c))^4/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 145 vs. 2(66) = 132.

Time = 0.50 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.20

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{a^4(-i\cos(2(c+dx))+\sin(2(c+dx)))\left(\cos(c+dx)\left(8+5\sqrt{\cos^2(c+dx)}\right)+\left(8+3\sqrt{\cos^2(c+dx)}\right)\text{co}\right.}{30d\sqrt{\cos^2(}}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(a^4*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])*(Cos[c + d*x]*(8 + 5*Sqrt[
Cos[c + d*x]^2]) + (8 + 3*Sqrt[Cos[c + d*x]^2])*Cos[3*(c + d*x)] + I*((-8
+ 5*Sqrt[Cos[c + d*x]^2])*Sin[c + d*x] + (-8 + 3*Sqrt[Cos[c + d*x]^2])*Sin
[3*(c + d*x)])))/(30*d*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{1}{5}a\int \cos^3(c+dx)(i\tan(c+dx)a+a)^3dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}a\int \frac{(i\tan(c+dx)a+a)^3}{\sec(c+dx)^3}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$$\Big\downarrow \text{3969}$$

$$-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}-\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{15d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-1/15*I)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d - ((I/5)*Cos[c + d
*x]^5*(a + I*a*Tan[c + d*x])^4)/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 43.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{i a^4 e^{5i(dx+c)}}{10d}-\frac{i a^4 e^{3i(dx+c)}}{6d}$
derivativedivides	$\frac{\frac{a^4\sin(dx+c)^5}{5}-4ia^4\left(-\frac{\cos(dx+c)^3\sin(dx+c)^2}{5}-\frac{2\cos(dx+c)^3}{15}\right)-6a^4\left(-\frac{\cos(dx+c)^4\sin(dx+c)}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right.}{d}$
default	$\frac{\frac{a^4\sin(dx+c)^5}{5}-4ia^4\left(-\frac{\cos(dx+c)^3\sin(dx+c)^2}{5}-\frac{2\cos(dx+c)^3}{15}\right)-6a^4\left(-\frac{\cos(dx+c)^4\sin(dx+c)}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right.}{d}$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/10*I*a^4/d*exp(5*I*(d*x+c))-1/6*I*a^4/d*exp(3*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{-3i\,a^4e^{(5i\,dx+5i\,c)}-5i\,a^4e^{(3i\,dx+3i\,c)}}{30\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/30*(-3*I*a^4*e^(5*I*d*x + 5*I*c) - 5*I*a^4*e^(3*I*d*x + 3*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx = \begin{cases} \frac{-6ia^4de^{5ic}e^{5idx}-10ia^4de^{3ic}e^{3idx}}{60d^2} & \text{for } d^2\neq 0 \\ x\left(\frac{a^4e^{5ic}}{2}+\frac{a^4e^{3ic}}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((-6*I*a**4*d*exp(5*I*c)*exp(5*I*d*x) - 10*I*a**4*d*exp(3*I*c)*e
xp(3*I*d*x))/(60*d**2), Ne(d**2, 0)), (x*(a**4*exp(5*I*c)/2 + a**4*exp(3*I
*c)/2), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(54) = 108$.

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.79

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{12i\,a^4\cos\left(dx+c\right)^5 - 3\,a^4\sin\left(dx+c\right)^5 + 4i\left(3\,\cos\left(dx+c\right)^5 - 5\,\cos\left(dx+c\right)^3\right)a^4 - 6\left(3\,\sin\left(dx+c\right)^5\right.}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/15*(12*I*a^4*cos(d*x + c)^5 - 3*a^4*sin(d*x + c)^5 + 4*I*(3*cos(d*x + c
)^5 - 5*cos(d*x + c)^3)*a^4 - 6*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^4
- (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(54) = 108$.

Time = 0.75 (sec) , antiderivative size = 915, normalized size of antiderivative = 13.86

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/7680*(9075*a^4*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 36300*a^
4*e^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 36300*a^4*e^(2*I*d*x -
2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 54450*a^4*e^(4*I*d*x)*log(I*e^(I*d*x +
 I*c) + 1) + 9075*a^4*e^(-4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 9000*a^4*e^(
8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 36000*a^4*e^(6*I*d*x + 2*I*c
)*log(I*e^(I*d*x + I*c) - 1) + 36000*a^4*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*
x + I*c) - 1) + 54000*a^4*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 9000*a^
4*e^(-4*I*c)*log(I*e^(I*d*x + I*c) - 1) - 9075*a^4*e^(8*I*d*x + 4*I*c)*log
(-I*e^(I*d*x + I*c) + 1) - 36300*a^4*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x +
 I*c) + 1) - 36300*a^4*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5
4450*a^4*e^(4*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 9075*a^4*e^(-4*I*c)*log
(-I*e^(I*d*x + I*c) + 1) - 9000*a^4*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x +
I*c) - 1) - 36000*a^4*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 36
000*a^4*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 54000*a^4*e^(4*I
*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 9000*a^4*e^(-4*I*c)*log(-I*e^(I*d*x +
I*c) - 1) - 75*a^4*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 300*a
^4*e^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 300*a^4*e^(2*I*d*x -
2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 450*a^4*e^(4*I*d*x)*log(I*e^(I*d*x) +
 e^(-I*c)) - 75*a^4*e^(-4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 75*a^4*e^(8*I
*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 300*a^4*e^(6*I*d*x + 2*I*c...
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.97

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{2\,a^4\,\left(15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,15\mathrm{i}-25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,5\mathrm{i}+4\right)}{15\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,5\mathrm{i}-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,10\mathrm{i}+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(2*a^4*(tan(c/2 + (d*x)/2)^3*15i - 25*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*
x)/2)*5i + 15*tan(c/2 + (d*x)/2)^4 + 4))/(15*d*(5*tan(c/2 + (d*x)/2) - tan
(c/2 + (d*x)/2)^2*10i - 10*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*5i
+ tan(c/2 + (d*x)/2)^5 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{a^4\left(-24\cos\left(dx+c\right)\sin\left(dx+c\right)^4 i+28\cos\left(dx+c\right)\sin\left(dx+c\right)^2 i-4\cos\left(dx+c\right)i+24\sin\left(dx+c\right)^5 -\right.}{15d}$$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x)
```

output
```
(a**4*( - 24*cos(c + d*x)*sin(c + d*x)**4*i + 28*cos(c + d*x)*sin(c + d*x)
**2*i - 4*cos(c + d*x)*i + 24*sin(c + d*x)**5 - 40*sin(c + d*x)**3 + 15*si
n(c + d*x) + 4*i))/(15*d)
```

3.57 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 24, antiderivative size = 102

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{3a^4\sin(c+dx)}{35d} - \frac{a^4\sin^3(c+dx)}{35d} - \frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d} - \frac{2i\cos^5(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{35d}$$

output
```
3/35*a^4*sin(d*x+c)/d-1/35*a^4*sin(d*x+c)^3/d-2/7*I*a*cos(d*x+c)^7*(a+I*a*
tan(d*x+c))^3/d-2/35*I*cos(d*x+c)^5*(a^4+I*a^4*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.77

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{a^4(-i\cos(3(c+dx))+\sin(3(c+dx)))\left(35\sqrt{\cos^2(c+dx)}+8\left(4+7\sqrt{\cos^2(c+dx)}\right)\cos(2(c+dx))+\left(\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(a^4*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(35*Sqrt[Cos[c + d*x]^2] +
 8*(4 + 7*Sqrt[Cos[c + d*x]^2])*Cos[2*(c + d*x)] + (32 + 5*Sqrt[Cos[c + d*
x]^2])*Cos[4*(c + d*x)] - (32*I)*Sin[2*(c + d*x)] - (14*I)*Sqrt[Cos[c + d*
x]^2]*Sin[2*(c + d*x)] - (32*I)*Sin[4*(c + d*x)] + (5*I)*Sqrt[Cos[c + d*x]
^2]*Sin[4*(c + d*x)]))/(280*d*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 3977, 3042, 3977, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{1}{7}a^2\int \cos^5(c+dx)(i\tan(c+dx)a+a)^2dx-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}a^2\int \frac{(i\tan(c+dx)a+a)^2}{\sec(c+dx)^5}dx-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}$$

$$\Big\downarrow \text{3977}$$

$$\frac{1}{7}a^2\left(\frac{3}{5}a^2\int \cos^3(c+dx)dx-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}a^2\left(\frac{3}{5}a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{1}{7}a^2\left(-\frac{3a^2\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{5d}-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}a^2\left(-\frac{3a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{5d}-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}$$

input
```
Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((-2*I)/7)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^3)/d + (a^2*((-3*a^2*(
-Sin[c + d*x] + Sin[c + d*x]^3/3))/(5*d) - (((2*I)/5)*Cos[c + d*x]^5*(a^2
+ I*a^2*Tan[c + d*x]))/d))/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

Maple [A] (verified)

Time = 123.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{i a^4 e^{7 i (dx+c)}}{56 d}-\frac{3 i a^4 e^{5 i (dx+c)}}{40 d}-\frac{i a^4 e^{3 i (dx+c)}}{8 d}-\frac{i a^4 e^{i (dx+c)}}{8 d}$
derivativedivides	$\frac{a^4\left(-\frac{\sin(dx+c)^3\cos(dx+c)^4}{7}-\frac{3\cos(dx+c)^4\sin(dx+c)}{35}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{35}\right)-4ia^4\left(-\frac{\cos(dx+c)^5\sin(dx+c)^2}{7}-\frac{2\cos(}{}\right.}{}$
default	$\frac{a^4\left(-\frac{\sin(dx+c)^3\cos(dx+c)^4}{7}-\frac{3\cos(dx+c)^4\sin(dx+c)}{35}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{35}\right)-4ia^4\left(-\frac{\cos(dx+c)^5\sin(dx+c)^2}{7}-\frac{2\cos(}{}\right.}{}$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/56*I*a^4/d*exp(7*I*(d*x+c))-3/40*I*a^4/d*exp(5*I*(d*x+c))-1/8*I*a^4/d*e
xp(3*I*(d*x+c))-1/8*I*a^4/d*exp(I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{-5i\,a^4e^{(7i\,dx+7i\,c)}-21i\,a^4e^{(5i\,dx+5i\,c)}-35i\,a^4e^{(3i\,dx+3i\,c)}-35i\,a^4e^{(i\,dx+i\,c)}}{280\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/280*(-5*I*a^4*e^(7*I*d*x + 7*I*c) - 21*I*a^4*e^(5*I*d*x + 5*I*c) - 35*I*
a^4*e^(3*I*d*x + 3*I*c) - 35*I*a^4*e^(I*d*x + I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.53

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx = \begin{cases} \frac{-2560ia^4d^3e^{7ic}e^{7idx}-10752ia^4d^3e^{5ic}e^{5idx}-17920ia^4d^3e^{3ic}e^{3idx}-17920ia^4d^3e^{ic}e^{idx}}{143360d^4} & \text{for } d^4 \neq 0 \\ x\left(\frac{a^4e^{7ic}}{8}+\frac{3a^4e^{5ic}}{8}+\frac{3a^4e^{3ic}}{8}+\frac{a^4e^{ic}}{8}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((-2560*I*a**4*d**3*exp(7*I*c)*exp(7*I*d*x) - 10752*I*a**4*d**3*
exp(5*I*c)*exp(5*I*d*x) - 17920*I*a**4*d**3*exp(3*I*c)*exp(3*I*d*x) - 1792
0*I*a**4*d**3*exp(I*c)*exp(I*d*x))/(143360*d**4), Ne(d**4, 0)), (x*(a**4*e
xp(7*I*c)/8 + 3*a**4*exp(5*I*c)/8 + 3*a**4*exp(3*I*c)/8 + a**4*exp(I*c)/8)
, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.46

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{20i\,a^4\cos(dx+c)^7+4i\left(5\cos(dx+c)^7-7\cos(dx+c)^5\right)a^4+2\left(15\sin(dx+c)^7-42\sin(dx+c)^5\right.}{}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/35*(20*I*a^4*cos(d*x + c)^7 + 4*I*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)
*a^4 + 2*(15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^4 +
 (5*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*a^4 + (5*sin(d*x + c)^7 - 21*sin(d*
x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^4)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1327 vs. $2(86) = 172$.

Time = 0.61 (sec) , antiderivative size = 1327, normalized size of antiderivative = 13.01

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/143360*(89950*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5397
00*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1349250*a^4*e^(8*
I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1349250*a^4*e^(4*I*d*x - 2*I*c
)*log(I*e^(I*d*x + I*c) + 1) + 539700*a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d
*x + I*c) + 1) + 1799000*a^4*e^(6*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 8995
0*a^4*e^(-6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 86065*a^4*e^(12*I*d*x + 6*I*
c)*log(I*e^(I*d*x + I*c) - 1) + 516390*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I
*d*x + I*c) - 1) + 1290975*a^4*e^(8*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) -
 1) + 1290975*a^4*e^(4*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 516390*
a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1721300*a^4*e^(6*I*d*
x)*log(I*e^(I*d*x + I*c) - 1) + 86065*a^4*e^(-6*I*c)*log(I*e^(I*d*x + I*c)
 - 1) - 89950*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 53970
0*a^4*e^(10*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1349250*a^4*e^(8*
I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1349250*a^4*e^(4*I*d*x - 2*I*
c)*log(-I*e^(I*d*x + I*c) + 1) - 539700*a^4*e^(2*I*d*x - 4*I*c)*log(-I*e^(
I*d*x + I*c) + 1) - 1799000*a^4*e^(6*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) -
89950*a^4*e^(-6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 86065*a^4*e^(12*I*d*x +
 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 516390*a^4*e^(10*I*d*x + 4*I*c)*log(
-I*e^(I*d*x + I*c) - 1) - 1290975*a^4*e^(8*I*d*x + 2*I*c)*log(-I*e^(I*d*x
+ I*c) - 1) - 1290975*a^4*e^(4*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - ...
```

Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.82

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx =$$

$$-\frac{2\,a^4\left(35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 105\mathrm{i}-210\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 210\mathrm{i}+147\tan\right.}{35\,d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 7\mathrm{i}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 35\mathrm{i}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right.}$$

input
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
-(2*a^4*(tan(c/2 + (d*x)/2)*49i + 147*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*
x)/2)^3*210i - 210*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*105i + 35*t
an(c/2 + (d*x)/2)^6 - 12))/(35*d*(7*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2
)^2*21i - 35*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*35i + 21*tan(c/2
+ (d*x)/2)^5 - tan(c/2 + (d*x)/2)^6*7i - tan(c/2 + (d*x)/2)^7 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$= \frac{a^4\left(40\cos(dx+c)\sin(dx+c)^6 i-92\cos(dx+c)\sin(dx+c)^4 i+64\cos(dx+c)\sin(dx+c)^2 i-12\cos\right.}{35d}$$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x)
```

output
```
(a**4*(40*cos(c + d*x)*sin(c + d*x)**6*i - 92*cos(c + d*x)*sin(c + d*x)**4
*i + 64*cos(c + d*x)*sin(c + d*x)**2*i - 12*cos(c + d*x)*i - 40*sin(c + d*
x)**7 + 112*sin(c + d*x)**5 - 105*sin(c + d*x)**3 + 35*sin(c + d*x) + 12*i
))/(35*d)
```

3.58 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{5a^4\sin(c+dx)}{21d}-\frac{10a^4\sin^3(c+dx)}{63d}+\frac{a^4\sin^5(c+dx)}{21d}-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}-\frac{2i\cos^7(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{21d}$$

output
```
5/21*a^4*sin(d*x+c)/d-10/63*a^4*sin(d*x+c)^3/d+1/21*a^4*sin(d*x+c)^5/d-2/9
*I*a*cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3/d-2/21*I*cos(d*x+c)^7*(a^4+I*a^4*ta
n(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.80

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{a^4(-i\cos(4(c+dx))+\sin(4(c+dx)))\left(168\cos(c+dx)\sqrt{\cos^2(c+dx)}+4\left(16+45\sqrt{\cos^2(c+dx)}\right)\cos\right.}{}$$

input
```
Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(a^4*((-I)*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])*(168*Cos[c + d*x]*Sqrt[Cos
[c + d*x]^2] + 4*(16 + 45*Sqrt[Cos[c + d*x]^2])*Cos[3*(c + d*x)] + 64*Cos[
5*(c + d*x)] - 28*Sqrt[Cos[c + d*x]^2]*Cos[5*(c + d*x)] - (42*I)*Sqrt[Cos[
c + d*x]^2]*Sin[c + d*x] - (64*I)*Sin[3*(c + d*x)] - (135*I)*Sqrt[Cos[c +
d*x]^2]*Sin[3*(c + d*x)] - (64*I)*Sin[5*(c + d*x)] + (35*I)*Sqrt[Cos[c + d
*x]^2]*Sin[5*(c + d*x)]))/(1008*d*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 3977, 3042, 3977, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^4}{\sec(c+dx)^9}dx$$
$$\Big\downarrow \text{3977}$$
$$\frac{1}{3}a^2\int \cos^7(c+dx)(i\tan(c+dx)a+a)^2dx-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}a^2\int \frac{(i\tan(c+dx)a+a)^2}{\sec(c+dx)^7}dx-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}$$

$$\Big\downarrow \text{3977}$$

$$\frac{1}{3}a^2\left(\frac{5}{7}a^2\int \cos^5(c+dx)dx-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}a^2\left(\frac{5}{7}a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{1}{3}a^2\left(-\frac{5a^2\int \left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{7d}-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^2\left(-\frac{5a^2\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{7d}-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}$$

input
```
Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((-2*I)/9)*a*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^3)/d + (a^2*((-5*a^2*(
-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]^5/5))/(7*d) - (((2*I)/
7)*Cos[c + d*x]^7*(a^2 + I*a^2*Tan[c + d*x]))/d))/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

Maple [A] (verified)

Time = 306.94 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{i a^{4} e^{9 i (d x +c)}}{288 d}-\frac{5 i a^{4} e^{7 i (d x +c)}}{224 d}-\frac{i a^{4} e^{5 i (d x +c)}}{16 d}-\frac{5 i a^{4} e^{3 i (d x +c)}}{48 d}-\frac{i a^{4} \cos (d x +c)}{8 d}+\frac{3 a^{4} \sin (d x +c)}{16 d}$
derivativedivides	$a^{4} \left(-\frac{\sin (d x +c)^{3} \cos (d x +c)^{6}}{9}-\frac{\sin (d x +c) \cos (d x +c)^{6}}{21}+\frac{\left(\frac{8}{3}+\cos (d x +c)^{4}+\frac{4 \cos (d x +c)^{2}}{3}\right) \sin (d x +c)}{105}\right)-4 i a^{4} \left(-\frac{\cos (d x +c)^{7} \sin}{9}\right.$
default	$a^{4} \left(-\frac{\sin (d x +c)^{3} \cos (d x +c)^{6}}{9}-\frac{\sin (d x +c) \cos (d x +c)^{6}}{21}+\frac{\left(\frac{8}{3}+\cos (d x +c)^{4}+\frac{4 \cos (d x +c)^{2}}{3}\right) \sin (d x +c)}{105}\right)-4 i a^{4} \left(-\frac{\cos (d x +c)^{7} \sin}{9}\right.$

input
```
int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/288*I*a^4/d*exp(9*I*(d*x+c))-5/224*I*a^4/d*exp(7*I*(d*x+c))-1/16*I*a^4/
d*exp(5*I*(d*x+c))-5/48*I*a^4/d*exp(3*I*(d*x+c))-1/8*I*a^4/d*cos(d*x+c)+3/
16*a^4*sin(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.75

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{\left(-7i\,a^4e^{(10i\,dx+10i\,c)}-45i\,a^4e^{(8i\,dx+8i\,c)}-126i\,a^4e^{(6i\,dx+6i\,c)}-210i\,a^4e^{(4i\,dx+4i\,c)}-315i\,a^4e^{(2i\,dx+2i\,c)}+63i\right.}{2016\,d}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/2016*(-7*I*a^4*e^(10*I*d*x + 10*I*c) - 45*I*a^4*e^(8*I*d*x + 8*I*c) - 12
6*I*a^4*e^(6*I*d*x + 6*I*c) - 210*I*a^4*e^(4*I*d*x + 4*I*c) - 315*I*a^4*e^
(2*I*d*x + 2*I*c) + 63*I*a^4)*e^(-I*d*x - I*c)/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(107) = 214.

Time = 0.40 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.90

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx = \begin{cases} \frac{\left(-176160768ia^4d^5e^{10ic}e^{9idx}-1132462080ia^4d^5e^{8ic}e^{7idx}-3170893824ia^4d^5e^{6ic}e^{5idx}-5284823040ia^4d^5e^{4ic}e^{3idx}-7927234560ia^4d^5e^{2ic}e^{ic}\right.}{50734301184d^6} \\ \frac{x\left(a^4e^{10ic}+5a^4e^{8ic}+10a^4e^{6ic}+10a^4e^{4ic}+5a^4e^{2ic}+a^4\right)e^{-ic}}{32} \end{cases}$$

input
```
integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((-176160768*I*a**4*d**5*exp(10*I*c)*exp(9*I*d*x) - 1132462080*I
*a**4*d**5*exp(8*I*c)*exp(7*I*d*x) - 3170893824*I*a**4*d**5*exp(6*I*c)*exp
(5*I*d*x) - 5284823040*I*a**4*d**5*exp(4*I*c)*exp(3*I*d*x) - 7927234560*I*
a**4*d**5*exp(2*I*c)*exp(I*d*x) + 1585446912*I*a**4*d**5*exp(-I*d*x))*exp(
-I*c)/(50734301184*d**6), Ne(d**6*exp(I*c), 0)), (x*(a**4*exp(10*I*c) + 5*
a**4*exp(8*I*c) + 10*a**4*exp(6*I*c) + 10*a**4*exp(4*I*c) + 5*a**4*exp(2*I
*c) + a**4)*exp(-I*c)/32, True))
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.51

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{140i\,a^4\cos\left(dx+c\right)^9+20i\,\left(7\,\cos\left(dx+c\right)^9-9\,\cos\left(dx+c\right)^7\right)a^4-\left(35\,\sin\left(dx+c\right)^9-90\,\sin\left(dx+c\right)\right.}{}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/315*(140*I*a^4*cos(d*x + c)^9 + 20*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)
^7)*a^4 - (35*sin(d*x + c)^9 - 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a^4
- 6*(35*sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin
(d*x + c)^3)*a^4 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7 + 378*sin(d*x +
 c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^4)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1409 vs. 2(102) = 204.

Time = 0.46 (sec) , antiderivative size = 1409, normalized size of antiderivative = 11.74

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/516096*(435267*a^4*e^(13*I*d*x + 7*I*c)*log(I*e^(I*d*x + I*c) + 1) + 261
1602*a^4*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 6529005*a^4*e^(
9*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 8705340*a^4*e^(7*I*d*x + I*c
)*log(I*e^(I*d*x + I*c) + 1) + 6529005*a^4*e^(5*I*d*x - I*c)*log(I*e^(I*d*
x + I*c) + 1) + 2611602*a^4*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 435267*a^4*e^(I*d*x - 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 427896*a^4*e^
(13*I*d*x + 7*I*c)*log(I*e^(I*d*x + I*c) - 1) + 2567376*a^4*e^(11*I*d*x +
5*I*c)*log(I*e^(I*d*x + I*c) - 1) + 6418440*a^4*e^(9*I*d*x + 3*I*c)*log(I*
e^(I*d*x + I*c) - 1) + 8557920*a^4*e^(7*I*d*x + I*c)*log(I*e^(I*d*x + I*c)
 - 1) + 6418440*a^4*e^(5*I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) + 2567376
*a^4*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c) - 1) + 427896*a^4*e^(I*d*x
- 5*I*c)*log(I*e^(I*d*x + I*c) - 1) - 435267*a^4*e^(13*I*d*x + 7*I*c)*log(
-I*e^(I*d*x + I*c) + 1) - 2611602*a^4*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x
 + I*c) + 1) - 6529005*a^4*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1)
 - 8705340*a^4*e^(7*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) + 1) - 6529005*a^4
*e^(5*I*d*x - I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2611602*a^4*e^(3*I*d*x -
3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 435267*a^4*e^(I*d*x - 5*I*c)*log(-I*e
^(I*d*x + I*c) + 1) - 427896*a^4*e^(13*I*d*x + 7*I*c)*log(-I*e^(I*d*x + I*
c) - 1) - 2567376*a^4*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 6
418440*a^4*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 8557920*a^...
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.21

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{2\,a^4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{89\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}-\frac{55\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{4}+\frac{55\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{4}-\frac{355\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{16}+\frac{35\,\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{16}-\frac{\mathrm{co}}{}\right.}{63\,d\,\left(\cos\left(4\,c+4\,d\,x\right)-\sin\left(4\,c+4\,d\,x\right)\,1\right.}$$

input
```
int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(2*a^4*cos(c/2 + (d*x)/2)*((cos((5*c)/2 + (5*d*x)/2)*21i)/2 - (cos((3*c)/2
 + (3*d*x)/2)*21i)/2 - (cos((7*c)/2 + (7*d*x)/2)*87i)/4 + (cos((9*c)/2 + (
9*d*x)/2)*7i)/4 + (89*sin(c/2 + (d*x)/2))/8 - (55*sin((3*c)/2 + (3*d*x)/2)
)/4 + (55*sin((5*c)/2 + (5*d*x)/2))/4 - (355*sin((7*c)/2 + (7*d*x)/2))/16
+ (35*sin((9*c)/2 + (9*d*x)/2))/16))/(63*d*(cos(4*c + 4*d*x) - sin(4*c + 4
*d*x)*1i))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.14

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{a^4\left(-56\cos\left(dx+c\right)\sin\left(dx+c\right)^8 i+188\cos\left(dx+c\right)\sin\left(dx+c\right)^6 i-228\cos\left(dx+c\right)\sin\left(dx+c\right)^4 i+1\right.}{}$$

input
```
int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x)
```

output
```
(a**4*( - 56*cos(c + d*x)*sin(c + d*x)**8*i + 188*cos(c + d*x)*sin(c + d*x
)**6*i - 228*cos(c + d*x)*sin(c + d*x)**4*i + 116*cos(c + d*x)*sin(c + d*x
)**2*i - 20*cos(c + d*x)*i + 56*sin(c + d*x)**9 - 216*sin(c + d*x)**7 + 31
5*sin(c + d*x)**5 - 210*sin(c + d*x)**3 + 63*sin(c + d*x) + 20*i))/(63*d)
```

3.59 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx=-\frac{8i(a+ia\tan(c+dx))^9}{9a^4d}+\frac{6i(a+ia\tan(c+dx))^{10}}{5a^5d}-\frac{6i(a+ia\tan(c+dx))^{11}}{11a^6d}+\frac{i(a+ia\tan(c+dx))^{12}}{12a^7d}$$

output
```
-8/9*I*(a+I*a*tan(d*x+c))^9/a^4/d+6/5*I*(a+I*a*tan(d*x+c))^10/a^5/d-6/11*I
*(a+I*a*tan(d*x+c))^11/a^6/d+1/12*I*(a+I*a*tan(d*x+c))^12/a^7/d
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx=\frac{a^5\sec^{12}(c+dx)(78\cos(c+dx)+221\cos(3(c+dx))-3i(18\sin(c+dx)+73\sin(3(c+dx))))(-i\cos(9}{1980d}$$

input
```
Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*Sec[c + d*x]^12*(78*Cos[c + d*x] + 221*Cos[3*(c + d*x)] - (3*I)*(18*S
in[c + d*x] + 73*Sin[3*(c + d*x)]))*((-I)*Cos[9*(c + d*x)] + Sin[9*(c + d*
x)]))/(1980*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^8(a+ia\tan(c+dx))^5dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^8d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(-(i\tan(c+dx)a+a)^{11}+6a(i\tan(c+dx)a+a)^{10}-12a^2(i\tan(c+dx)a+a)^9+8a^3(i\tan(c+dx)a+a\right.}{a^7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{8}{9}a^3(a+ia\tan(c+dx))^9-\frac{6}{5}a^2(a+ia\tan(c+dx))^{10}-\frac{1}{12}(a+ia\tan(c+dx))^{12}+\frac{6}{11}a(a+ia\tan(c+dx))^{1}\right.}{a^7d}$$

input
```
Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-I)*((8*a^3*(a + I*a*Tan[c + d*x])^9)/9 - (6*a^2*(a + I*a*Tan[c + d*x])^
10)/5 + (6*a*(a + I*a*Tan[c + d*x])^11)/11 - (a + I*a*Tan[c + d*x])^12/12)
)/(a^7*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(93) = 186.

Time = 0.41 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.46

$$\frac{ia^5\left(\frac{\sin(dx+c)^6}{12\cos(dx+c)^{12}}+\frac{\sin(dx+c)^6}{20\cos(dx+c)^{10}}+\frac{\sin(dx+c)^6}{40\cos(dx+c)^8}+\frac{\sin(dx+c)^6}{120\cos(dx+c)^6}\right)+5a^5\left(\frac{\sin(dx+c)^5}{11\cos(dx+c)^{11}}+\frac{2\sin(dx+c)^5}{33\cos(dx+c)^9}+\frac{8\sin(dx+c)}{231\cos(dx+c}\right.}{}$$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x)
```

output
```
1/d*(I*a^5*(1/12*sin(d*x+c)^6/cos(d*x+c)^12+1/20*sin(d*x+c)^6/cos(d*x+c)^1
0+1/40*sin(d*x+c)^6/cos(d*x+c)^8+1/120*sin(d*x+c)^6/cos(d*x+c)^6)+5*a^5*(1
/11*sin(d*x+c)^5/cos(d*x+c)^11+2/33*sin(d*x+c)^5/cos(d*x+c)^9+8/231*sin(d*
x+c)^5/cos(d*x+c)^7+16/1155*sin(d*x+c)^5/cos(d*x+c)^5)-10*I*a^5*(1/10*sin(
d*x+c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)^4/cos(d*x+c)^8+1/20*sin(d*x+c)^4/co
s(d*x+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)^4)-10*a^5*(1/9*sin(d*x+c)^3/cos(d*
x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/3
15*sin(d*x+c)^3/cos(d*x+c)^3)+5/8*I*a^5/cos(d*x+c)^8-a^5*(-16/35-1/7*sec(d
*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(85) = 170$.

Time = 0.08 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.45

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{1024\left(-495i\,a^5e^{(16i\,dx+16i\,c)} - 792i\,a^5e^{(14i\,dx+14i\,c)} - 924i\,a^5e^{(12i\,dx+12i\,c)} - 79\right.}{495\left(de^{(24i\,dx+24i\,c)} + 12\,de^{(22i\,dx+22i\,c)} + 66\,de^{(20i\,dx+20i\,c)} + 220\,de^{(18i\,dx+18i\,c)} + 495\,de^{(16i\,dx+16i\,c)} + 792\right.}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
-1024/495*(-495*I*a^5*e^(16*I*d*x + 16*I*c) - 792*I*a^5*e^(14*I*d*x + 14*I
*c) - 924*I*a^5*e^(12*I*d*x + 12*I*c) - 792*I*a^5*e^(10*I*d*x + 10*I*c) -
495*I*a^5*e^(8*I*d*x + 8*I*c) - 220*I*a^5*e^(6*I*d*x + 6*I*c) - 66*I*a^5*e
^(4*I*d*x + 4*I*c) - 12*I*a^5*e^(2*I*d*x + 2*I*c) - I*a^5)/(d*e^(24*I*d*x
+ 24*I*c) + 12*d*e^(22*I*d*x + 22*I*c) + 66*d*e^(20*I*d*x + 20*I*c) + 220*
d*e^(18*I*d*x + 18*I*c) + 495*d*e^(16*I*d*x + 16*I*c) + 792*d*e^(14*I*d*x
+ 14*I*c) + 924*d*e^(12*I*d*x + 12*I*c) + 792*d*e^(10*I*d*x + 10*I*c) + 49
5*d*e^(8*I*d*x + 8*I*c) + 220*d*e^(6*I*d*x + 6*I*c) + 66*d*e^(4*I*d*x + 4*
I*c) + 12*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx = ia^5\left(\int \left(-i\sec^8{(c+dx)}\right)\,dx + \int 5\tan{(c+dx)}\sec^8{(c+dx)}\,dx + \int \left(-10\tan^3{(c+dx)}\sec^8{(c+dx)}\right)\,dx + \int \tan^5{(c+dx)}\sec^8{(c+dx)}\,dx + \int 10i\tan^2{(c+dx)}\sec^8{(c+dx)}\,dx + \int \left(-5i\tan^4{(c+dx)}\sec^8{(c+dx)}\right)\,dx\right)$$

input
```
integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**5,x)
```

output
```
I*a**5*(Integral(-I*sec(c + d*x)**8, x) + Integral(5*tan(c + d*x)*sec(c +
d*x)**8, x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x)**8, x) + Integral(
tan(c + d*x)**5*sec(c + d*x)**8, x) + Integral(10*I*tan(c + d*x)**2*sec(c
+ d*x)**8, x) + Integral(-5*I*tan(c + d*x)**4*sec(c + d*x)**8, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.47

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{-165i\,a^5\tan{(dx+c)}^{12} - 900\,a^5\tan{(dx+c)}^{11} + 1386i\,a^5\tan{(dx+c)}^{10} - 1100\,a^5\tan{(dx+c)}^{9} + 54$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/1980*(-165*I*a^5*tan(d*x + c)^12 - 900*a^5*tan(d*x + c)^11 + 1386*I*a^5
*tan(d*x + c)^10 - 1100*a^5*tan(d*x + c)^9 + 5445*I*a^5*tan(d*x + c)^8 + 3
960*a^5*tan(d*x + c)^7 + 4620*I*a^5*tan(d*x + c)^6 + 8712*a^5*tan(d*x + c)
^5 - 2475*I*a^5*tan(d*x + c)^4 + 4620*a^5*tan(d*x + c)^3 - 4950*I*a^5*tan(
d*x + c)^2 - 1980*a^5*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.47

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{-165i\,a^5\tan{(dx+c)}^{12} - 900\,a^5\tan{(dx+c)}^{11} + 1386i\,a^5\tan{(dx+c)}^{10} - 1100\,a^5\tan{(dx+c)}^9 + 54}{}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/1980*(-165*I*a^5*tan(d*x + c)^12 - 900*a^5*tan(d*x + c)^11 + 1386*I*a^5
*tan(d*x + c)^10 - 1100*a^5*tan(d*x + c)^9 + 5445*I*a^5*tan(d*x + c)^8 + 3
960*a^5*tan(d*x + c)^7 + 4620*I*a^5*tan(d*x + c)^6 + 8712*a^5*tan(d*x + c)
^5 - 2475*I*a^5*tan(d*x + c)^4 + 4620*a^5*tan(d*x + c)^3 - 4950*I*a^5*tan(
d*x + c)^2 - 1980*a^5*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.34

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\left(-\cos{(c+d\,x)}^{12}\,1749\mathrm{i} + 2048\,\sin{(c+d\,x)}\,\cos{(c+d\,x)}^{11} + 1024\,\sin{(c+d\,x)}\,\cos{(c+d\,x)}^9 + 768\,\mathrm{s}\right.}{}$$

input
```
int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x)^8,x)
```

output
```
(a^5*(900*cos(c + d*x)*sin(c + d*x) - 3400*cos(c + d*x)^3*sin(c + d*x) + 6
40*cos(c + d*x)^5*sin(c + d*x) + 768*cos(c + d*x)^7*sin(c + d*x) + 1024*co
s(c + d*x)^9*sin(c + d*x) + 2048*cos(c + d*x)^11*sin(c + d*x) - cos(c + d*
x)^2*2376i + cos(c + d*x)^4*3960i - cos(c + d*x)^12*1749i + 165i))/(1980*d
*cos(c + d*x)^12)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.10

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$= \frac{\sin(dx+c)\,a^5\left(-2048\cos(dx+c)\sin(dx+c)^{10}+11264\cos(dx+c)\sin(dx+c)^8-25344\cos(dx+c)\right.}{}$$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x)
```

output
```
(sin(c + d*x)*a**5*( - 2048*cos(c + d*x)*sin(c + d*x)**10 + 11264*cos(c +
d*x)*sin(c + d*x)**8 - 25344*cos(c + d*x)*sin(c + d*x)**6 + 29568*cos(c +
d*x)*sin(c + d*x)**4 - 14520*cos(c + d*x)*sin(c + d*x)**2 + 1980*cos(c + d
*x) - 1749*sin(c + d*x)**11*i + 10494*sin(c + d*x)**9*i - 26235*sin(c + d*
x)**7*i + 34980*sin(c + d*x)**5*i - 22275*sin(c + d*x)**3*i + 4950*sin(c +
 d*x)*i))/(1980*d*(sin(c + d*x)**12 - 6*sin(c + d*x)**10 + 15*sin(c + d*x)
**8 - 20*sin(c + d*x)**6 + 15*sin(c + d*x)**4 - 6*sin(c + d*x)**2 + 1))
```

3.60 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i(a+ia\tan(c+dx))^8}{2a^3d} + \frac{4i(a+ia\tan(c+dx))^9}{9a^4d} - \frac{i(a+ia\tan(c+dx))^{10}}{10a^5d}$$

output
```
-1/2*I*(a+I*a*tan(d*x+c))^8/a^3/d+4/9*I*(a+I*a*tan(d*x+c))^9/a^4/d-1/10*I*
(a+I*a*tan(d*x+c))^10/a^5/d
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\sec^{10}(c+dx)(5+23\cos(2(c+dx))-22i\sin(2(c+dx)))(-i\cos(8(c+dx))+\sin(8(c+dx)))}{180d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*Sec[c + d*x]^10*(5 + 23*Cos[2*(c + d*x)] - (22*I)*Sin[2*(c + d*x)])*(
(-I)*Cos[8*(c + d*x)] + Sin[8*(c + d*x)]))/(180*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+ia\tan(c+dx))^5dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^7d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left((i\tan(c+dx)a+a)^9-4a(i\tan(c+dx)a+a)^8+4a^2(i\tan(c+dx)a+a)^7\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{1}{2}a^2(a+ia\tan(c+dx))^8+\frac{1}{10}(a+ia\tan(c+dx))^{10}-\frac{4}{9}a(a+ia\tan(c+dx))^9\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-I)*((a^2*(a + I*a*Tan[c + d*x])^8)/2 - (4*a*(a + I*a*Tan[c + d*x])^9)/9
 + (a + I*a*Tan[c + d*x])^10/10))/(a^5*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 166.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

method	result
risch	$\frac{128 i a^{5} \left(120 \,{\mathrm e}^{14 i (d x +c)}+210 \,{\mathrm e}^{12 i (d x +c)}+252 \,{\mathrm e}^{10 i (d x +c)}+210 \,{\mathrm e}^{8 i (d x +c)}+120 \,{\mathrm e}^{6 i (d x +c)}+45 \,{\mathrm e}^{4 i (d x +c)}+10 \,{\mathrm e}^{2 i (d x +c)}+1\right)}{45 d \left({\mathrm e}^{2 i (d x +c)}+1\right)^{10}}$
derivativedivides	$i a^{5} \left(\frac{\sin\left(d x +c \right)^{6}}{10 \cos\left(d x +c \right)^{10}}+\frac{\sin\left(d x +c \right)^{6}}{20 \cos\left(d x +c \right)^{8}}+\frac{\sin\left(d x +c \right)^{6}}{60 \cos\left(d x +c \right)^{6}}\right)+5 a^{5} \left(\frac{\sin\left(d x +c \right)^{5}}{9 \cos\left(d x +c \right)^{9}}+\frac{4 \sin\left(d x +c \right)^{5}}{63 \cos\left(d x +c \right)^{7}}+\frac{8 \sin\left(d x +c \right)^{5}}{315 \cos\left(d x +c \right)^{5}}\right)-10 i a^{5} \left(\frac{\sin(}{8 \cos}\right.$
default	$i a^{5} \left(\frac{\sin\left(d x +c \right)^{6}}{10 \cos\left(d x +c \right)^{10}}+\frac{\sin\left(d x +c \right)^{6}}{20 \cos\left(d x +c \right)^{8}}+\frac{\sin\left(d x +c \right)^{6}}{60 \cos\left(d x +c \right)^{6}}\right)+5 a^{5} \left(\frac{\sin\left(d x +c \right)^{5}}{9 \cos\left(d x +c \right)^{9}}+\frac{4 \sin\left(d x +c \right)^{5}}{63 \cos\left(d x +c \right)^{7}}+\frac{8 \sin\left(d x +c \right)^{5}}{315 \cos\left(d x +c \right)^{5}}\right)-10 i a^{5} \left(\frac{\sin(}{8 \cos}\right.$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
128/45*I*a^5*(120*exp(14*I*(d*x+c))+210*exp(12*I*(d*x+c))+252*exp(10*I*(d*
x+c))+210*exp(8*I*(d*x+c))+120*exp(6*I*(d*x+c))+45*exp(4*I*(d*x+c))+10*exp
(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^10
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(64) = 128$.

Time = 0.08 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.79

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{128\left(-120i\,a^5e^{(14i\,dx+14i\,c)} - 210i\,a^5e^{(12i\,dx+12i\,c)} - 252i\,a^5e^{(10i\,dx+10i\,c)} - 210i\,a^5e^{(8i\,dx+}\right.}{45\left(de^{(20i\,dx+20i\,c)} + 10\,de^{(18i\,dx+18i\,c)} + 45\,de^{(16i\,dx+16i\,c)} + 120\,de^{(14i\,dx+14i\,c)} + 210\,de^{(12i\,dx+12i\,c)} + 252\,d\right.}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
-128/45*(-120*I*a^5*e^(14*I*d*x + 14*I*c) - 210*I*a^5*e^(12*I*d*x + 12*I*c
) - 252*I*a^5*e^(10*I*d*x + 10*I*c) - 210*I*a^5*e^(8*I*d*x + 8*I*c) - 120*
I*a^5*e^(6*I*d*x + 6*I*c) - 45*I*a^5*e^(4*I*d*x + 4*I*c) - 10*I*a^5*e^(2*I
*d*x + 2*I*c) - I*a^5)/(d*e^(20*I*d*x + 20*I*c) + 10*d*e^(18*I*d*x + 18*I*
c) + 45*d*e^(16*I*d*x + 16*I*c) + 120*d*e^(14*I*d*x + 14*I*c) + 210*d*e^(1
2*I*d*x + 12*I*c) + 252*d*e^(10*I*d*x + 10*I*c) + 210*d*e^(8*I*d*x + 8*I*c
) + 120*d*e^(6*I*d*x + 6*I*c) + 45*d*e^(4*I*d*x + 4*I*c) + 10*d*e^(2*I*d*x
 + 2*I*c) + d)
```

Sympy [F]

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx = ia^5\Bigg(&\int \left(-i\sec^6(c+dx)\right)\,dx \\ &+ \int 5\tan(c+dx)\sec^6(c+dx)\,dx \\ &+ \int \left(-10\tan^3(c+dx)\sec^6(c+dx)\right)\,dx \\ &+ \int \tan^5(c+dx)\sec^6(c+dx)\,dx \\ &+ \int 10i\tan^2(c+dx)\sec^6(c+dx)\,dx \\ &+ \int \left(-5i\tan^4(c+dx)\sec^6(c+dx)\right)\,dx\Bigg)\end{aligned}$$

input
```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**5,x)
```

output
```
I*a**5*(Integral(-I*sec(c + d*x)**6, x) + Integral(5*tan(c + d*x)*sec(c +
d*x)**6, x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x)**6, x) + Integral(
tan(c + d*x)**5*sec(c + d*x)**6, x) + Integral(10*I*tan(c + d*x)**2*sec(c
+ d*x)**6, x) + Integral(-5*I*tan(c + d*x)**4*sec(c + d*x)**6, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.32

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{-9i\,a^5\tan\left(dx+c\right)^{10} - 50\,a^5\tan\left(dx+c\right)^9 + 90i\,a^5\tan\left(dx+c\right)^8 + 210i\,a^5\tan\left(dx+c\right)^6 + 252\,a^5\tan}{90\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/90*(-9*I*a^5*tan(d*x + c)^10 - 50*a^5*tan(d*x + c)^9 + 90*I*a^5*tan(d*x
 + c)^8 + 210*I*a^5*tan(d*x + c)^6 + 252*a^5*tan(d*x + c)^5 + 240*a^5*tan(
d*x + c)^3 - 225*I*a^5*tan(d*x + c)^2 - 90*a^5*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.32

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{-9i\,a^5\tan\left(dx+c\right)^{10} - 50\,a^5\tan\left(dx+c\right)^9 + 90i\,a^5\tan\left(dx+c\right)^8 + 210i\,a^5\tan\left(dx+c\right)^6 + 252\,a^5\tan}{90\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/90*(-9*I*a^5*tan(d*x + c)^10 - 50*a^5*tan(d*x + c)^9 + 90*I*a^5*tan(d*x
 + c)^8 + 210*I*a^5*tan(d*x + c)^6 + 252*a^5*tan(d*x + c)^5 + 240*a^5*tan(
d*x + c)^3 - 225*I*a^5*tan(d*x + c)^2 - 90*a^5*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{a^5\,\sin\left(c+d\,x\right)\,\left(90\cos\left(c+d\,x\right)^9+\cos\left(c+d\,x\right)^8\,\sin\left(c+d\,x\right)\,225\mathrm{i}-240\cos\left(c+d\,x\right)^7\sin\left(c+d\,x\right)^2-\right.}{}$$

input
```
int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x)^6,x)
```

output
```
(a^5*sin(c + d*x)*(50*cos(c + d*x)*sin(c + d*x)^8 + cos(c + d*x)^8*sin(c +
 d*x)*225i + 90*cos(c + d*x)^9 + sin(c + d*x)^9*9i - cos(c + d*x)^2*sin(c
+ d*x)^7*90i - cos(c + d*x)^4*sin(c + d*x)^5*210i - 252*cos(c + d*x)^5*sin
(c + d*x)^4 - 240*cos(c + d*x)^7*sin(c + d*x)^2))/(90*d*cos(c + d*x)^10)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.34

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{\sin\left(dx+c\right)a^5\left(-128\cos\left(dx+c\right)\sin\left(dx+c\right)^8+576\cos\left(dx+c\right)\sin\left(dx+c\right)^6-1008\cos\left(dx+c\right)\sin\left(d\right.\right.}{90d\left(\sin\left(dx+c\right)^{10}\right.}$$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x)
```

output
```
(sin(c + d*x)*a**5*( - 128*cos(c + d*x)*sin(c + d*x)**8 + 576*cos(c + d*x)
*sin(c + d*x)**6 - 1008*cos(c + d*x)*sin(c + d*x)**4 + 600*cos(c + d*x)*si
n(c + d*x)**2 - 90*cos(c + d*x) - 114*sin(c + d*x)**9*i + 570*sin(c + d*x)
**7*i - 1140*sin(c + d*x)**5*i + 900*sin(c + d*x)**3*i - 225*sin(c + d*x)*
i))/(90*d*(sin(c + d*x)**10 - 5*sin(c + d*x)**8 + 10*sin(c + d*x)**6 - 10*
sin(c + d*x)**4 + 5*sin(c + d*x)**2 - 1))
```

3.61 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx=-\frac{2i(a+ia\tan(c+dx))^7}{7a^2d}+\frac{i(a+ia\tan(c+dx))^8}{8a^3d}$$

output
```
-2/7*I*(a+I*a*tan(d*x+c))^7/a^2/d+1/8*I*(a+I*a*tan(d*x+c))^8/a^3/d
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx=\frac{ia^5(-i+\tan(c+dx))^7(9i+7\tan(c+dx))}{56d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((I/56)*a^5*(-I + Tan[c + d*x])^7*(9*I + 7*Tan[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4(a+ia\tan(c+dx))^5dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))(i\tan(c+dx)a+a)^6d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(2a(i\tan(c+dx)a+a)^6-(i\tan(c+dx)a+a)^7\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{2}{7}a(a+ia\tan(c+dx))^7-\frac{1}{8}(a+ia\tan(c+dx))^8\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-I)*((2*a*(a + I*a*Tan[c + d*x])^7)/7 - (a + I*a*Tan[c + d*x])^8/8))/(a^
3*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 59.60 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.65

method	result
risch	$\frac{32 i a^{5} \left(28 \,{\mathrm e}^{12 i \left(d x +c \right)}+56 \,{\mathrm e}^{10 i \left(d x +c \right)}+70 \,{\mathrm e}^{8 i \left(d x +c \right)}+56 \,{\mathrm e}^{6 i \left(d x +c \right)}+28 \,{\mathrm e}^{4 i \left(d x +c \right)}+8 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{7 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{8}}$
derivativedivides	$\frac{i a^{5} \left(\frac{\sin \left(d x +c \right)^{6}}{8 \cos \left(d x +c \right)^{8}}+\frac{\sin \left(d x +c \right)^{6}}{24 \cos \left(d x +c \right)^{6}}\right)+5 a^{5} \left(\frac{\sin \left(d x +c \right)^{5}}{7 \cos \left(d x +c \right)^{7}}+\frac{2 \sin \left(d x +c \right)^{5}}{35 \cos \left(d x +c \right)^{5}}\right)-10 i a^{5} \left(\frac{\sin \left(d x +c \right)^{4}}{6 \cos \left(d x +c \right)^{6}}+\frac{\sin \left(d x +c \right)^{4}}{12 \cos \left(d x +c \right)^{4}}\right)-10 a^{5} \left(\right.}{d}$
default	$\frac{i a^{5} \left(\frac{\sin \left(d x +c \right)^{6}}{8 \cos \left(d x +c \right)^{8}}+\frac{\sin \left(d x +c \right)^{6}}{24 \cos \left(d x +c \right)^{6}}\right)+5 a^{5} \left(\frac{\sin \left(d x +c \right)^{5}}{7 \cos \left(d x +c \right)^{7}}+\frac{2 \sin \left(d x +c \right)^{5}}{35 \cos \left(d x +c \right)^{5}}\right)-10 i a^{5} \left(\frac{\sin \left(d x +c \right)^{4}}{6 \cos \left(d x +c \right)^{6}}+\frac{\sin \left(d x +c \right)^{4}}{12 \cos \left(d x +c \right)^{4}}\right)-10 a^{5} \left(\right.}{d}$

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
32/7*I*a^5*(28*exp(12*I*(d*x+c))+56*exp(10*I*(d*x+c))+70*exp(8*I*(d*x+c))+
56*exp(6*I*(d*x+c))+28*exp(4*I*(d*x+c))+8*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(
d*x+c))+1)^8
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(43) = 86$.

Time = 0.08 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.47

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{32\left(-28i\,a^5e^{(12i\,dx+12i\,c)} - 56i\,a^5e^{(10i\,dx+10i\,c)} - 70i\,a^5e^{(8i\,dx+8i\,c)} - 56i\,a^5e^{(6i\,dx+6i\,c)} - 28i\,a^5e^{(4i\,dx+4i\,c)}\right.}{7\left(de^{(16i\,dx+16i\,c)} + 8\,de^{(14i\,dx+14i\,c)} + 28\,de^{(12i\,dx+12i\,c)} + 56\,de^{(10i\,dx+10i\,c)} + 70\,de^{(8i\,dx+8i\,c)} + 56\,de^{(6i\,dx+6i\,c)}\right.}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
-32/7*(-28*I*a^5*e^(12*I*d*x + 12*I*c) - 56*I*a^5*e^(10*I*d*x + 10*I*c) -
70*I*a^5*e^(8*I*d*x + 8*I*c) - 56*I*a^5*e^(6*I*d*x + 6*I*c) - 28*I*a^5*e^(
4*I*d*x + 4*I*c) - 8*I*a^5*e^(2*I*d*x + 2*I*c) - I*a^5)/(d*e^(16*I*d*x + 1
6*I*c) + 8*d*e^(14*I*d*x + 14*I*c) + 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(
10*I*d*x + 10*I*c) + 70*d*e^(8*I*d*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) +
 28*d*e^(4*I*d*x + 4*I*c) + 8*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\begin{aligned}\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx = ia^5\Bigg(\int\left(-i\sec^4(c+dx)\right)dx \\ +\int 5\tan(c+dx)\sec^4(c+dx)\,dx \\ +\int\left(-10\tan^3(c+dx)\sec^4(c+dx)\right)dx \\ +\int\tan^5(c+dx)\sec^4(c+dx)\,dx \\ +\int 10i\tan^2(c+dx)\sec^4(c+dx)\,dx \\ +\int\left(-5i\tan^4(c+dx)\sec^4(c+dx)\right)dx\Bigg)\end{aligned}$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**5,x)
```

output
```
I*a**5*(Integral(-I*sec(c + d*x)**4, x) + Integral(5*tan(c + d*x)*sec(c +
d*x)**4, x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x)**4, x) + Integral(
tan(c + d*x)**5*sec(c + d*x)**4, x) + Integral(10*I*tan(c + d*x)**2*sec(c
+ d*x)**4, x) + Integral(-5*I*tan(c + d*x)**4*sec(c + d*x)**4, x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(43) = 86.

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.96

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{-7i\,a^5\tan(dx+c)^8 - 40\,a^5\tan(dx+c)^7 + 84i\,a^5\tan(dx+c)^6 + 56\,a^5\tan(dx+c)^5 + 70i\,a^5\tan(dx}{56\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/56*(-7*I*a^5*tan(d*x + c)^8 - 40*a^5*tan(d*x + c)^7 + 84*I*a^5*tan(d*x
+ c)^6 + 56*a^5*tan(d*x + c)^5 + 70*I*a^5*tan(d*x + c)^4 + 168*a^5*tan(d*x
 + c)^3 - 140*I*a^5*tan(d*x + c)^2 - 56*a^5*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(43) = 86.

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.96

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{-7i\,a^5\tan(dx+c)^8 - 40\,a^5\tan(dx+c)^7 + 84i\,a^5\tan(dx+c)^6 + 56\,a^5\tan(dx+c)^5 + 70i\,a^5\tan(dx}{56\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/56*(-7*I*a^5*tan(d*x + c)^8 - 40*a^5*tan(d*x + c)^7 + 84*I*a^5*tan(d*x
+ c)^6 + 56*a^5*tan(d*x + c)^5 + 70*I*a^5*tan(d*x + c)^4 + 168*a^5*tan(d*x
 + c)^3 - 140*I*a^5*tan(d*x + c)^2 - 56*a^5*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.75

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\,\sin\left(c+d\,x\right)\,\left(56\cos\left(c+d\,x\right)^7+\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)\,140\mathrm{i}-168\cos\left(c+d\,x\right)^5\sin\left(c+d\,x\right)^2-\right.}{}$$

input
```
int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x)^4,x)
```

output
```
(a^5*sin(c + d*x)*(40*cos(c + d*x)*sin(c + d*x)^6 + cos(c + d*x)^6*sin(c +
 d*x)*140i + 56*cos(c + d*x)^7 + sin(c + d*x)^7*7i - cos(c + d*x)^2*sin(c
+ d*x)^5*84i - 56*cos(c + d*x)^3*sin(c + d*x)^4 - cos(c + d*x)^4*sin(c + d
*x)^3*70i - 168*cos(c + d*x)^5*sin(c + d*x)^2))/(56*d*cos(c + d*x)^8)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.82

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{\sin\left(dx+c\right)a^5\left(-128\cos\left(dx+c\right)\sin\left(dx+c\right)^6+448\cos\left(dx+c\right)\sin\left(dx+c\right)^4-336\cos\left(dx+c\right)\sin\left(dx\right.\right.}{56d\left(\sin\left(dx+c\right)^8-4\sin\left(dx+c\right)^6\right.}$$

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x)
```

output
```
(sin(c + d*x)*a**5*( - 128*cos(c + d*x)*sin(c + d*x)**6 + 448*cos(c + d*x)
*sin(c + d*x)**4 - 336*cos(c + d*x)*sin(c + d*x)**2 + 56*cos(c + d*x) - 11
9*sin(c + d*x)**7*i + 476*sin(c + d*x)**5*i - 490*sin(c + d*x)**3*i + 140*
sin(c + d*x)*i))/(56*d*(sin(c + d*x)**8 - 4*sin(c + d*x)**6 + 6*sin(c + d*
x)**4 - 4*sin(c + d*x)**2 + 1))
```

3.62 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i(a+ia\tan(c+dx))^6}{6ad}$$

output
```
-1/6*I*(a+I*a*tan(d*x+c))^6/a/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 72 vs. 2(27) = 54.

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.67

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\tan(c+dx)\left(6+15i\tan(c+dx)-20\tan^2(c+dx)-15i\tan^3(c+dx)+6\tan^4(c+dx)+i\tan^5(c+\right.}{6d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*Tan[c + d*x]*(6 + (15*I)*Tan[c + d*x] - 20*Tan[c + d*x]^2 - (15*I)*Ta
n[c + d*x]^3 + 6*Tan[c + d*x]^4 + I*Tan[c + d*x]^5))/(6*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(c+dx)^2(a+ia\tan(c+dx))^5dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{i\int(i\tan(c+dx)a+a)^5d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \; 17$$

$$-\frac{i(a+ia\tan(c+dx))^6}{6ad}$$

input
```
Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-1/6*I)*(a + I*a*Tan[c + d*x])^6)/(a*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(23) = 46$.

Time = 15.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.96

method	result	size
risch	$\frac{32 i a^{5} \left(6 \,{\mathrm e}^{10 i (d x +c)}+15 \,{\mathrm e}^{8 i (d x +c)}+20 \,{\mathrm e}^{6 i (d x +c)}+15 \,{\mathrm e}^{4 i (d x +c)}+6 \,{\mathrm e}^{2 i (d x +c)}+1\right)}{3 d \left({\mathrm e}^{2 i (d x +c)}+1\right)^{6}}$	80
derivativedivides	$\frac{\frac{i a^{5} \sin \left(d x +c \right)^{6}}{6 \cos \left(d x +c \right)^{6}}+\frac{a^{5} \sin \left(d x +c \right)^{5}}{\cos \left(d x +c \right)^{5}}-\frac{5 i a^{5} \sin \left(d x +c \right)^{4}}{2 \cos \left(d x +c \right)^{4}}-\frac{10 a^{5} \sin \left(d x +c \right)^{3}}{3 \cos \left(d x +c \right)^{3}}+\frac{5 i a^{5}}{2 \cos \left(d x +c \right)^{2}}+a^{5} \tan \left(d x +c \right)}{d}$	115
default	$\frac{\frac{i a^{5} \sin \left(d x +c \right)^{6}}{6 \cos \left(d x +c \right)^{6}}+\frac{a^{5} \sin \left(d x +c \right)^{5}}{\cos \left(d x +c \right)^{5}}-\frac{5 i a^{5} \sin \left(d x +c \right)^{4}}{2 \cos \left(d x +c \right)^{4}}-\frac{10 a^{5} \sin \left(d x +c \right)^{3}}{3 \cos \left(d x +c \right)^{3}}+\frac{5 i a^{5}}{2 \cos \left(d x +c \right)^{2}}+a^{5} \tan \left(d x +c \right)}{d}$	115

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
32/3*I*a^5*(6*exp(10*I*(d*x+c))+15*exp(8*I*(d*x+c))+20*exp(6*I*(d*x+c))+15
*exp(4*I*(d*x+c))+6*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^6
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(21) = 42$.

Time $= 0.07$ (sec) , antiderivative size $= 153$, normalized size of antiderivative $= 5.67$

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{32\left(-6i\,a^5e^{(10i\,dx+10i\,c)} - 15i\,a^5e^{(8i\,dx+8i\,c)} - 20i\,a^5e^{(6i\,dx+6i\,c)} - 15i\,a^5e^{(4i\,dx+4i\,c)} - 6i\,a^5e^{(2i\,dx+2i\,c)} - i\,a}{3\left(de^{(12i\,dx+12i\,c)} + 6\,de^{(10i\,dx+10i\,c)} + 15\,de^{(8i\,dx+8i\,c)} + 20\,de^{(6i\,dx+6i\,c)} + 15\,de^{(4i\,dx+4i\,c)} + 6\,de^{(2i\,dx+2i\,c)} + \right.}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
-32/3*(-6*I*a^5*e^(10*I*d*x + 10*I*c) - 15*I*a^5*e^(8*I*d*x + 8*I*c) - 20*
I*a^5*e^(6*I*d*x + 6*I*c) - 15*I*a^5*e^(4*I*d*x + 4*I*c) - 6*I*a^5*e^(2*I*
d*x + 2*I*c) - I*a^5)/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c)
 + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x +
 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx = ia^5\left(\int\left(-i\sec^2(c+dx)\right)dx + \int 5\tan(c+dx)\sec^2(c+dx)\,dx + \int\left(-10\tan^3(c+dx)\sec^2(c+dx)\right)dx + \int \tan^5(c+dx)\sec^2(c+dx)\,dx + \int 10i\tan^2(c+dx)\sec^2(c+dx)\,dx + \int\left(-5i\tan^4(c+dx)\sec^2(c+dx)\right)dx\right)$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**5,x)
```

output
```
I*a**5*(Integral(-I*sec(c + d*x)**2, x) + Integral(5*tan(c + d*x)*sec(c +
d*x)**2, x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x)**2, x) + Integral(
tan(c + d*x)**5*sec(c + d*x)**2, x) + Integral(10*I*tan(c + d*x)**2*sec(c
+ d*x)**2, x) + Integral(-5*I*tan(c + d*x)**4*sec(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i\,(i\,a\tan\left(dx+c\right)+a)^6}{6\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/6*I*(I*a*tan(d*x + c) + a)^6/(a*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(21) = 42.

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.04

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{-i\,a^5\tan\left(dx+c\right)^6-6\,a^5\tan\left(dx+c\right)^5+15i\,a^5\tan\left(dx+c\right)^4+20\,a^5\tan\left(dx+c\right)^3-15i\,a^5\tan\left(dx+\right.}{6\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/6*(-I*a^5*tan(d*x + c)^6 - 6*a^5*tan(d*x + c)^5 + 15*I*a^5*tan(d*x + c)
^4 + 20*a^5*tan(d*x + c)^3 - 15*I*a^5*tan(d*x + c)^2 - 6*a^5*tan(d*x + c))
/d
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.22

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\,\sin\left(c+d\,x\right)\,\left(6\cos\left(c+d\,x\right)^5+\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)\,15\mathrm{i}-20\cos\left(c+d\,x\right)^3\sin\left(c+d\,x\right)^2-\cos\right.}{6\,d\cos\left(c+d\,x\right)^6}$$

input
```
int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x)^2,x)
```

output
```
(a^5*sin(c + d*x)*(6*cos(c + d*x)*sin(c + d*x)^4 + cos(c + d*x)^4*sin(c +
d*x)*15i + 6*cos(c + d*x)^5 + sin(c + d*x)^5*1i - cos(c + d*x)^2*sin(c + d
*x)^3*15i - 20*cos(c + d*x)^3*sin(c + d*x)^2))/(6*d*cos(c + d*x)^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 4.37

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{\sin\left(dx+c\right)a^5\left(-32\cos\left(dx+c\right)\sin\left(dx+c\right)^4+32\cos\left(dx+c\right)\sin\left(dx+c\right)^2-6\cos\left(dx+c\right)-31\sin\left(dx\right.\right.}{6d\left(\sin\left(dx+c\right)^6-3\sin\left(dx+c\right)^4+3\sin\left(dx+c\right)^2-1\right)}$$

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x)
```

output
```
(sin(c + d*x)*a**5*( - 32*cos(c + d*x)*sin(c + d*x)**4 + 32*cos(c + d*x)*s
in(c + d*x)**2 - 6*cos(c + d*x) - 31*sin(c + d*x)**5*i + 45*sin(c + d*x)**
3*i - 15*sin(c + d*x)*i))/(6*d*(sin(c + d*x)**6 - 3*sin(c + d*x)**4 + 3*si
n(c + d*x)**2 - 1))
```

3.63 $\int (a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 15, antiderivative size = 117

$$\int (a+ia\tan(c+dx))^5\,dx = 16a^5x - \frac{16ia^5\log(\cos(c+dx))}{d} - \frac{8a^5\tan(c+dx)}{d} + \frac{2ia^2(a+ia\tan(c+dx))^3}{3d} + \frac{ia(a+ia\tan(c+dx))^4}{4d} + \frac{2ia(a^2+ia^2\tan(c+dx))^2}{d}$$

output
```
16*a^5*x-16*I*a^5*ln(cos(d*x+c))/d-8*a^5*tan(d*x+c)/d+2/3*I*a^2*(a+I*a*tan
(d*x+c))^3/d+1/4*I*a*(a+I*a*tan(d*x+c))^4/d+2*I*a*(a^2+I*a^2*tan(d*x+c))^2
/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.61

$$\int (a+ia\tan(c+dx))^5\,dx = \frac{a^5(35i+192i\log(i+\tan(c+dx))-180\tan(c+dx)-66i\tan^2(c+dx)+20\tan^3(c+dx)+3i\tan^4(c+$$
$$12d$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*(35*I + (192*I)*Log[I + Tan[c + d*x]] - 180*Tan[c + d*x] - (66*I)*Tan
[c + d*x]^2 + 20*Tan[c + d*x]^3 + (3*I)*Tan[c + d*x]^4))/(12*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3959, 3042, 3959, 3042, 3959, 3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^5dx$$

$$\Big\downarrow \text{3959}$$

$$2a\int (i\tan(c+dx)a+a)^4dx+\frac{ia(a+ia\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3042}$$

$$2a\int (i\tan(c+dx)a+a)^4dx+\frac{ia(a+ia\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3959}$$

$$2a\left(2a\int (i\tan(c+dx)a+a)^3dx+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{ia(a+ia\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3042}$$

$$2a\left(2a\int (i\tan(c+dx)a+a)^3dx+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{ia(a+ia\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3959}$$

$$2a\left(2a\left(2a\int (i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{ia(a+ia\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3042}$$

$$2a\left(2a\left(2a\int (i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{ia(a+ia\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3958}$$

$$2a\left(2a\left(2a\left(2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+d}{3d}\right.\right.$$
$$\frac{ia(a+ia\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3042}$$

$$2a\left(2a\left(2a\left(2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+d}{3d}\right.\right.$$
$$\frac{ia(a+ia\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3956}$$

$$2a\left(2a\left(2a\left(-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+}{3d}\right.\right.$$
$$\frac{ia(a+ia\tan(c+dx))^4}{4d}$$

input
```
Int[(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((I/4)*a*(a + I*a*Tan[c + d*x])^4)/d + 2*a*(((I/3)*a*(a + I*a*Tan[c + d*x]
)^3)/d + 2*a*(((I/2)*a*(a + I*a*Tan[c + d*x])^2)/d + 2*a*(2*a^2*x - ((2*I)
*a^2*Log[Cos[c + d*x]])/d - (a^2*Tan[c + d*x])/d)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

Maple [A] (warning: unable to verify)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{a^{5}\left(-15\tan(dx+c)+\frac{i\tan(dx+c)^{4}}{4}+\frac{5\tan(dx+c)^{3}}{3}-\frac{11i\tan(dx+c)^{2}}{2}+8i\ln\left(1+\tan(dx+c)^{2}\right)+16\arctan(\tan(dx+c))\right)}{d}$
default	$\frac{a^{5}\left(-15\tan(dx+c)+\frac{i\tan(dx+c)^{4}}{4}+\frac{5\tan(dx+c)^{3}}{3}-\frac{11i\tan(dx+c)^{2}}{2}+8i\ln\left(1+\tan(dx+c)^{2}\right)+16\arctan(\tan(dx+c))\right)}{d}$
parallelrisch	$\frac{3ia^{5}\tan(dx+c)^{4}-66ia^{5}\tan(dx+c)^{2}+20\tan(dx+c)^{3}a^{5}+96ia^{5}\ln\left(1+\tan(dx+c)^{2}\right)+192a^{5}xd-180a^{5}\tan(dx+c)}{12d}$
risch	$-\frac{32a^{5}c}{d}-\frac{4ia^{5}\left(48\,e^{6i(dx+c)}+108\,e^{4i(dx+c)}+88\,e^{2i(dx+c)}+25\right)}{3d\left(e^{2i(dx+c)}+1\right)^{4}}-\frac{16ia^{5}\ln\left(e^{2i(dx+c)}+1\right)}{d}$
norman	$16a^{5}x-\frac{15a^{5}\tan(dx+c)}{d}+\frac{5a^{5}\tan(dx+c)^{3}}{3d}-\frac{11ia^{5}\tan(dx+c)^{2}}{2d}+\frac{ia^{5}\tan(dx+c)^{4}}{4d}+\frac{8ia^{5}\ln\left(1+\tan(dx+c\right.}{d}$
parts	$a^{5}x-\frac{10ia^{5}\left(\frac{\tan(dx+c)^{2}}{2}-\frac{\ln\left(1+\tan(dx+c)^{2}\right)}{2}\right)}{d}+\frac{ia^{5}\left(\frac{\tan(dx+c)^{4}}{4}-\frac{\tan(dx+c)^{2}}{2}+\frac{\ln\left(1+\tan(dx+c)^{2}\right)}{2}\right)}{d}+\frac{5ia^{5}\ln\left(\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*a^5*(-15*tan(d*x+c)+1/4*I*tan(d*x+c)^4+5/3*tan(d*x+c)^3-11/2*I*tan(d*x
+c)^2+8*I*ln(1+tan(d*x+c)^2)+16*arctan(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.51

$$\int (a+ia\tan(c+dx))^5\,dx = -\frac{4\left(48i\,a^5e^{(6i\,dx+6i\,c)}+108i\,a^5e^{(4i\,dx+4i\,c)}+88i\,a^5e^{(2i\,dx+2i\,c)}+25i\,a^5+12\left(i\,a^5e^{(8i\,dx+8i\,c)}+4i\,a^5e^{(6i\,dx+6i}\right.\right.}{3\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,d\right.}$$

input
```
integrate((a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
-4/3*(48*I*a^5*e^(6*I*d*x + 6*I*c) + 108*I*a^5*e^(4*I*d*x + 4*I*c) + 88*I*
a^5*e^(2*I*d*x + 2*I*c) + 25*I*a^5 + 12*(I*a^5*e^(8*I*d*x + 8*I*c) + 4*I*a
^5*e^(6*I*d*x + 6*I*c) + 6*I*a^5*e^(4*I*d*x + 4*I*c) + 4*I*a^5*e^(2*I*d*x
+ 2*I*c) + I*a^5)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(8*I*d*x + 8*I*c) + 4
*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c)
 + d)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.52

$$\int (a+ia\tan(c+dx))^5\,dx = -\frac{16ia^5\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{-192ia^5e^{6ic}e^{6idx}-432ia^5e^{4ic}e^{4idx}-352ia^5e^{2ic}e^{2idx}-100ia^5}{3de^{8ic}e^{8idx}+12de^{6ic}e^{6idx}+18de^{4ic}e^{4idx}+12de^{2ic}e^{2idx}+3d}$$

input
```
integrate((a+I*a*tan(d*x+c))**5,x)
```

output
```
-16*I*a**5*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-192*I*a**5*exp(6*I*c)*exp
(6*I*d*x) - 432*I*a**5*exp(4*I*c)*exp(4*I*d*x) - 352*I*a**5*exp(2*I*c)*exp
(2*I*d*x) - 100*I*a**5)/(3*d*exp(8*I*c)*exp(8*I*d*x) + 12*d*exp(6*I*c)*exp
(6*I*d*x) + 18*d*exp(4*I*c)*exp(4*I*d*x) + 12*d*exp(2*I*c)*exp(2*I*d*x) +
3*d)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.41

$$\begin{aligned}\int (a+ia\tan(c+dx))^5\,dx &= a^5x+\frac{5\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^5}{3\,d}\\&\quad+\frac{10\left(dx+c-\tan\left(dx+c\right)\right)a^5}{d}\\&\quad+\frac{i\,a^5\left(\frac{4\,\sin(dx+c)^2-3}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-2\,\log\left(\sin\left(dx+c\right)^2-1\right)\right)}{4\,d}\\&\quad+\frac{5i\,a^5\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)^2-1\right)\right)}{d}\\&\quad+\frac{5i\,a^5\log\left(\sec\left(dx+c\right)\right)}{d}\end{aligned}$$

input
```
integrate((a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
a^5*x + 5/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^5/d + 10*(d*
x + c - tan(d*x + c))*a^5/d + 1/4*I*a^5*((4*sin(d*x + c)^2 - 3)/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 2*log(sin(d*x + c)^2 - 1))/d + 5*I*a^5*(1/
(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d + 5*I*a^5*log(sec(d*x +
c))/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int (a+ia\tan(c+dx))^5\,dx = \frac{16i\,a^5\log\left(\tan\left(dx+c\right)+i\right)}{d} - \frac{-3i\,a^5d^3\tan\left(dx+c\right)^4 - 20\,a^5d^3\tan\left(dx+c\right)^3 + 66i\,a^5d^3\tan\left(dx+c\right)^2 + 180\,a^5d^3\tan\left(dx+c\right)}{12\,d^4}$$

input
```
integrate((a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
16*I*a^5*log(tan(d*x + c) + I)/d - 1/12*(-3*I*a^5*d^3*tan(d*x + c)^4 - 20*
a^5*d^3*tan(d*x + c)^3 + 66*I*a^5*d^3*tan(d*x + c)^2 + 180*a^5*d^3*tan(d*x
 + c))/d^4
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int (a+ia\tan(c+dx))^5\,dx = \frac{a^5\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,16\mathrm{i} - 15\,a^5\tan(c+d\,x) - \frac{a^5\tan(c+d\,x)^2\,11\mathrm{i}}{2} + \frac{5\,a^5\tan(c+d\,x)^3}{3} + \frac{a^5\tan(c+d\,x)^4\,1\mathrm{i}}{4}}{d}$$

input
```
int((a + a*tan(c + d*x)*1i)^5,x)
```

output
```
(a^5*log(tan(c + d*x) + 1i)*16i - 15*a^5*tan(c + d*x) - (a^5*tan(c + d*x)^
2*11i)/2 + (5*a^5*tan(c + d*x)^3)/3 + (a^5*tan(c + d*x)^4*1i)/4)/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.57

$$\int (a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\left(96\log\left(\tan(dx+c)^2+1\right)i+3\tan(dx+c)^4 i+20\tan(dx+c)^3-66\tan(dx+c)^2 i-180\tan(dx+$$
$$12d$$

input
```
int((a+I*a*tan(d*x+c))^5,x)
```

output
```
(a**5*(96*log(tan(c + d*x)**2 + 1)*i + 3*tan(c + d*x)**4*i + 20*tan(c + d*
x)**3 - 66*tan(c + d*x)**2*i - 180*tan(c + d*x) + 192*d*x))/(12*d)
```

3.64 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -12a^5x+\frac{12ia^5\log(\cos(c+dx))}{d}+\frac{5a^5\tan(c+dx)}{d}+\frac{ia^5\tan^2(c+dx)}{2d}-\frac{8ia^6}{d(a-ia\tan(c+dx))}$$

output
```
-12*a^5*x+12*I*a^5*ln(cos(d*x+c))/d+5*a^5*tan(d*x+c)/d+1/2*I*a^5*tan(d*x+c
)^2/d-8*I*a^6/d/(a-I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^5\left(24\log(i+\tan(c+dx))+10i\tan(c+dx)-\tan^2(c+dx)+\frac{16i}{i+\tan(c+dx)}\right)}{2d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-1/2*I)*a^5*(24*Log[I + Tan[c + d*x]] + (10*I)*Tan[c + d*x] - Tan[c + d*
x]^2 + (16*I)/(I + Tan[c + d*x])))/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^2}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{ia^3\int \frac{(i\tan(c+dx)a+a)^3}{(a-ia\tan(c+dx))^2}d(ia\tan(c+dx))}{d}$$
$$\Big\downarrow \text{49}$$
$$-\frac{ia^3\int \left(\frac{8a^3}{(a-ia\tan(c+dx))^2}-\frac{12a^2}{a-ia\tan(c+dx)}+i\tan(c+dx)a+5a\right)d(ia\tan(c+dx))}{d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{ia^3\left(\frac{8a^3}{a-ia\tan(c+dx)}-\frac{1}{2}a^2\tan^2(c+dx)+5ia^2\tan(c+dx)+12a^2\log(a-ia\tan(c+dx))\right)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-I)*a^3*(12*a^2*Log[a - I*a*Tan[c + d*x]] + (5*I)*a^2*Tan[c + d*x] - (a^
2*Tan[c + d*x]^2)/2 + (8*a^3)/(a - I*a*Tan[c + d*x])))/d
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 12.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{4 i a^{5} {\mathrm e}^{2 i (d x+c)}}{d}+\frac{24 a^{5} c}{d}+\frac{2 i a^{5} \left(6 \,{\mathrm e}^{2 i (d x+c)}+5\right)}{d \left({\mathrm e}^{2 i (d x+c)}+1\right)^{2}}+\frac{12 i a^{5} \ln \left({\mathrm e}^{2 i (d x+c)}+1\right)}{d}$
derivativedivides	$i a^{5} \left(\frac{\sin (d x+c)^{6}}{2 \cos (d x+c)^{2}}+\frac{\sin (d x+c)^{4}}{2}+\sin (d x+c)^{2}+2 \ln (\cos (d x+c))\right)+5 a^{5} \Bigl(\frac{\sin (d x+c)^{5}}{\cos (d x+c)}+\left(\sin (d x+c)^{3}+\frac{3 \sin (d x+c)}{2}\right) \cos (d x+$
default	$i a^{5} \left(\frac{\sin (d x+c)^{6}}{2 \cos (d x+c)^{2}}+\frac{\sin (d x+c)^{4}}{2}+\sin (d x+c)^{2}+2 \ln (\cos (d x+c))\right)+5 a^{5} \Bigl(\frac{\sin (d x+c)^{5}}{\cos (d x+c)}+\left(\sin (d x+c)^{3}+\frac{3 \sin (d x+c)}{2}\right) \cos (d x+$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-4*I/d*a^5*exp(2*I*(d*x+c))+24/d*a^5*c+2*I*a^5*(6*exp(2*I*(d*x+c))+5)/d/(e
xp(2*I*(d*x+c))+1)^2+12*I/d*a^5*ln(exp(2*I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.51

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2\left(2i\,a^5e^{(6i\,dx+6i\,c)}+4i\,a^5e^{(4i\,dx+4i\,c)}-4i\,a^5e^{(2i\,dx+2i\,c)}-5i\,a^5+6\left(-i\,a^5e^{(4i\,dx+4i\,c)}-2i\,a^5e^{(2i\,dx+2i\,c)}-i\right.\right.}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
-2*(2*I*a^5*e^(6*I*d*x + 6*I*c) + 4*I*a^5*e^(4*I*d*x + 4*I*c) - 4*I*a^5*e^
(2*I*d*x + 2*I*c) - 5*I*a^5 + 6*(-I*a^5*e^(4*I*d*x + 4*I*c) - 2*I*a^5*e^(2
*I*d*x + 2*I*c) - I*a^5)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(4*I*d*x + 4*I
*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.58

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{12ia^5\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{12ia^5e^{2ic}e^{2idx}+10ia^5}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d} + \begin{cases} -\frac{4ia^5e^{2ic}e^{2idx}}{d} & \text{for } d\neq 0 \\ 8a^5xe^{2ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**5,x)
```

output
```
12*I*a**5*log(exp(2*I*d*x) + exp(-2*I*c))/d + (12*I*a**5*exp(2*I*c)*exp(2*
I*d*x) + 10*I*a**5)/(d*exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*
x) + d) + Piecewise((-4*I*a**5*exp(2*I*c)*exp(2*I*d*x)/d, Ne(d, 0)), (8*a*
*5*x*exp(2*I*c), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{-i\,a^5\tan\left(dx+c\right)^2+24\left(dx+c\right)a^5+12i\,a^5\log\left(\tan\left(dx+c\right)^2+1\right)-10\,a^5\tan\left(dx+c\right)-\frac{16\left(a^5\tan(dx+c)-i\,a^5\right)}{\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/2*(-I*a^5*tan(d*x + c)^2 + 24*(d*x + c)*a^5 + 12*I*a^5*log(tan(d*x + c)
^2 + 1) - 10*a^5*tan(d*x + c) - 16*(a^5*tan(d*x + c) - I*a^5)/(tan(d*x + c
)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{12i\,a^5\log\left(\tan\left(dx+c\right)+i\right)}{d} + \frac{8\,a^5}{d(\tan\left(dx+c\right)+i)} - \frac{-i\,a^5d\tan\left(dx+c\right)^2-10\,a^5d\tan\left(dx+c\right)}{2\,d^2}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-12*I*a^5*log(tan(d*x + c) + I)/d + 8*a^5/(d*(tan(d*x + c) + I)) - 1/2*(-I
*a^5*d*tan(d*x + c)^2 - 10*a^5*d*tan(d*x + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{8\,a^5}{d\,\left(\tan\left(c+d\,x\right)+1{}\mathrm{i}\right)} - \frac{a^5\,\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,12{}\mathrm{i}}{d} + \frac{5\,a^5\tan(c+d\,x)}{d} + \frac{a^5\tan(c+d\,x)^2\,1{}\mathrm{i}}{2\,d}$$

input
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
(8*a^5)/(d*(tan(c + d*x) + 1i)) - (a^5*log(tan(c + d*x) + 1i)*12i)/d + (5*
a^5*tan(c + d*x))/d + (a^5*tan(c + d*x)^2*1i)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.66

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\left(16\cos\left(dx+c\right)\sin\left(dx+c\right)^3-26\cos\left(dx+c\right)\sin\left(dx+c\right)-24\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(dx+c\right)^2\right.}{}$$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x)
```

output
```
(a**5*(16*cos(c + d*x)*sin(c + d*x)**3 - 26*cos(c + d*x)*sin(c + d*x) - 24
*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*i + 24*log(tan((c + d*x)/2)*
*2 + 1)*i + 24*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*i - 24*log(tan((c
 + d*x)/2) - 1)*i + 24*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*i - 24*lo
g(tan((c + d*x)/2) + 1)*i + 16*sin(c + d*x)**4*i - 24*sin(c + d*x)**2*c -
24*sin(c + d*x)**2*d*x - 17*sin(c + d*x)**2*i + 24*c + 24*d*x))/(2*d*(sin(
c + d*x)**2 - 1))
```

3.65 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 71

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = a^5x-\frac{ia^5\log(\cos(c+dx))}{d}-\frac{2ia^5}{d(1-i\tan(c+dx))^2}+\frac{4ia^5}{d(1-i\tan(c+dx))}$$

output
```
a^5*x-I*a^5*ln(cos(d*x+c))/d-2*I*a^5/d/(1-I*tan(d*x+c))^2+4*I*a^5/d/(1-I*t
an(d*x+c))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.69

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^5\left(-\log(i+\tan(c+dx))+\frac{2-4i\tan(c+dx)}{(i+\tan(c+dx))^2}\right)}{d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-I)*a^5*(-Log[I + Tan[c + d*x]] + (2 - (4*I)*Tan[c + d*x])/(I + Tan[c +
d*x])^2))/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{(i\tan(c+dx)a+a)^2}{(a-ia\tan(c+dx))^3}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{ia^5\int \left(\frac{4a^2}{(a-ia\tan(c+dx))^3}-\frac{4a}{(a-ia\tan(c+dx))^2}+\frac{1}{a-ia\tan(c+dx)}\right)d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^5\left(\frac{2a^2}{(a-ia\tan(c+dx))^2}-\frac{4a}{a-ia\tan(c+dx)}-\log(a-ia\tan(c+dx))\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-I)*a^5*(-Log[a - I*a*Tan[c + d*x]] + (2*a^2)/(a - I*a*Tan[c + d*x])^2 -
 (4*a)/(a - I*a*Tan[c + d*x])))/d
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 42.71 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{i a^{5} {\mathrm e}^{4 i (d x +c)}}{2 d}+\frac{i a^{5} {\mathrm e}^{2 i (d x +c)}}{d}-\frac{2 a^{5} c}{d}-\frac{i a^{5} \ln \left({\mathrm e}^{2 i (d x +c)}+1\right)}{d}$
derivativedivides	$\frac{i a^{5} \left(-\frac{\sin \left(d x +c \right)^{4}}{4}-\frac{\sin \left(d x +c \right)^{2}}{2}-\ln \left(\cos \left(d x +c \right)\right)\right)+5 a^{5} \left(-\frac{\left(\sin \left(d x +c \right)^{3}+\frac{3 \sin \left(d x +c \right)}{2}\right) \cos \left(d x +c \right)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)-\frac{5 i a^{5} \sin (d x}{2}}{}$
default	$\frac{i a^{5} \left(-\frac{\sin \left(d x +c \right)^{4}}{4}-\frac{\sin \left(d x +c \right)^{2}}{2}-\ln \left(\cos \left(d x +c \right)\right)\right)+5 a^{5} \left(-\frac{\left(\sin \left(d x +c \right)^{3}+\frac{3 \sin \left(d x +c \right)}{2}\right) \cos \left(d x +c \right)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)-\frac{5 i a^{5} \sin (d x}{2}}{}$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/d*a^5*exp(4*I*(d*x+c))+I/d*a^5*exp(2*I*(d*x+c))-2/d*a^5*c-I/d*a^5*l
n(exp(2*I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{-i\,a^5e^{(4i\,dx+4i\,c)}+2i\,a^5e^{(2i\,dx+2i\,c)}-2i\,a^5\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/2*(-I*a^5*e^(4*I*d*x + 4*I*c) + 2*I*a^5*e^(2*I*d*x + 2*I*c) - 2*I*a^5*lo
g(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.44

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^5\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \begin{cases} \frac{-ia^5de^{4ic}e^{4idx}+2ia^5de^{2ic}e^{2idx}}{2d^2} & \text{for } d^2\neq 0 \\ x\left(2a^5e^{4ic}-2a^5e^{2ic}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**5,x)
```

output
```
-I*a**5*log(exp(2*I*d*x) + exp(-2*I*c))/d + Piecewise(((-I*a**5*d*exp(4*I*
c)*exp(4*I*d*x) + 2*I*a**5*d*exp(2*I*c)*exp(2*I*d*x))/(2*d**2), Ne(d**2, 0
)), (x*(2*a**5*exp(4*I*c) - 2*a**5*exp(2*I*c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.24

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{2\,(dx+c)a^5 + i\,a^5\log\left(\tan\left(dx+c\right)^2+1\right) - \frac{4\,\left(2\,a^5\tan(dx+c)^3 - 3i\,a^5\tan(dx+c)^2 - i\,a^5\right)}{\tan(dx+c)^4 + 2\,\tan(dx+c)^2 + 1}}{2\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/2*(2*(d*x + c)*a^5 + I*a^5*log(tan(d*x + c)^2 + 1) - 4*(2*a^5*tan(d*x +
c)^3 - 3*I*a^5*tan(d*x + c)^2 - I*a^5)/(tan(d*x + c)^4 + 2*tan(d*x + c)^2
+ 1))/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.70

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{i\,a^5\log\left(\tan\left(dx+c\right)+i\right)}{d} - \frac{2\,\left(2\,a^5\tan\left(dx+c\right) + i\,a^5\right)}{d\left(\tan\left(dx+c\right)+i\right)^2}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
I*a^5*log(tan(d*x + c) + I)/d - 2*(2*a^5*tan(d*x + c) + I*a^5)/(d*(tan(d*x
 + c) + I)^2)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\,\ln(\tan(c+d\,x)+1\mathrm{i})\,1\mathrm{i}}{d} - \frac{4\,a^5\tan(c+d\,x)+a^5\,2\mathrm{i}}{d\,\left(\tan(c+d\,x)^2+\tan(c+d\,x)\,2\mathrm{i}-1\right)}$$

input
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
(a^5*log(tan(c + d*x) + 1i)*1i)/d - (4*a^5*tan(c + d*x) + a^5*2i)/(d*(tan(
c + d*x)*2i + tan(c + d*x)^2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.35

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\left(-4\cos(dx+c)\sin(dx+c)^3+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)i-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)i-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{d}$$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x)
```

output
```
(a**5*( - 4*cos(c + d*x)*sin(c + d*x)**3 + log(tan((c + d*x)/2)**2 + 1)*i
- log(tan((c + d*x)/2) - 1)*i - log(tan((c + d*x)/2) + 1)*i - 4*sin(c + d*
x)**4*i + 2*sin(c + d*x)**2*i + c + d*x))/d
```

3.66 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2ia^8}{3d(a-ia\tan(c+dx))^3} + \frac{ia^7}{2d(a-ia\tan(c+dx))^2}$$

output
```
-2/3*I*a^8/d/(a-I*a*tan(d*x+c))^3+1/2*I*a^7/d/(a-I*a*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^5(-i+3\tan(c+dx))}{6d(i+\tan(c+dx))^3}$$

input
```
Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-1/6*I)*a^5*(-I + 3*Tan[c + d*x]))/(d*(I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\frac{ia^7\int \frac{i\tan(c+dx)a+a}{(a-ia\tan(c+dx))^4}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 53}$$

$$-\frac{ia^7\int \left(\frac{2a}{(a-ia\tan(c+dx))^4}-\frac{1}{(a-ia\tan(c+dx))^3}\right)d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{ia^7\left(\frac{2a}{3(a-ia\tan(c+dx))^3}-\frac{1}{2(a-ia\tan(c+dx))^2}\right)}{d}$$

input
```
Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-I)*a^7*((2*a)/(3*(a - I*a*Tan[c + d*x])^3) - 1/(2*(a - I*a*Tan[c + d*x]
)^2)))/d
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 122.74 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{i a^{5} {\mathrm e}^{6 i \left(d x +c \right)}}{12 d}-\frac{i a^{5} {\mathrm e}^{4 i \left(d x +c \right)}}{8 d}$
derivativedivides	$\frac{\frac{i a^{5} \sin\left(d x +c \right)^{6}}{6}+5 a^{5} \left(-\frac{\sin\left(d x +c \right)^{3} \cos\left(d x +c \right)^{3}}{6}-\frac{\sin\left(d x +c \right) \cos\left(d x +c \right)^{3}}{8}+\frac{\cos\left(d x +c \right) \sin\left(d x +c \right)}{16}+\frac{d x}{16}+\frac{c}{16}\right)-10 i a^{5} \left(-\frac{\cos(d x +c}{}\right.}{}$
default	$\frac{\frac{i a^{5} \sin\left(d x +c \right)^{6}}{6}+5 a^{5} \left(-\frac{\sin\left(d x +c \right)^{3} \cos\left(d x +c \right)^{3}}{6}-\frac{\sin\left(d x +c \right) \cos\left(d x +c \right)^{3}}{8}+\frac{\cos\left(d x +c \right) \sin\left(d x +c \right)}{16}+\frac{d x}{16}+\frac{c}{16}\right)-10 i a^{5} \left(-\frac{\cos(d x +c}{}\right.}{}$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12*I/d*a^5*exp(6*I*(d*x+c))-1/8*I/d*a^5*exp(4*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{-2i\,a^5e^{(6i\,dx+6i\,c)}-3i\,a^5e^{(4i\,dx+4i\,c)}}{24\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/24*(-2*I*a^5*e^(6*I*d*x + 6*I*c) - 3*I*a^5*e^(4*I*d*x + 4*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx = \begin{cases} \frac{-8ia^5de^{6ic}e^{6idx}-12ia^5de^{4ic}e^{4idx}}{96d^2} & \text{for } d^2\neq 0 \\ x\left(\frac{a^5e^{6ic}}{2}+\frac{a^5e^{4ic}}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**5,x)
```

output
```
Piecewise(((-8*I*a**5*d*exp(6*I*c)*exp(6*I*d*x) - 12*I*a**5*d*exp(4*I*c)*e
xp(4*I*d*x))/(96*d**2), Ne(d**2, 0)), (x*(a**5*exp(6*I*c)/2 + a**5*exp(4*I
*c)/2), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(43) = 86$.

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.69

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{3i\,a^5\tan\left(dx+c\right)^4+10\,a^5\tan\left(dx+c\right)^3-12i\,a^5\tan\left(dx+c\right)^2-6\,a^5\tan\left(dx+c\right)+i\,a^5}{6\left(\tan\left(dx+c\right)^6+3\,\tan\left(dx+c\right)^4+3\,\tan\left(dx+c\right)^2+1\right)d}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/6*(3*I*a^5*tan(d*x + c)^4 + 10*a^5*tan(d*x + c)^3 - 12*I*a^5*tan(d*x +
c)^2 - 6*a^5*tan(d*x + c) + I*a^5)/((tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3
*tan(d*x + c)^2 + 1)*d)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.55

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{3i\,a^5\tan(dx+c)+a^5}{6\,d(\tan(dx+c)+i)^3}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/6*(3*I*a^5*tan(d*x + c) + a^5)/(d*(tan(d*x + c) + I)^3)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\,(1+\tan(c+d\,x)\ 3\mathrm{i})}{6\,d\,\left(-\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2\,3\mathrm{i}+3\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
(a^5*(tan(c + d*x)*3i + 1))/(6*d*(3*tan(c + d*x) - tan(c + d*x)^2*3i - tan
(c + d*x)^3 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.56

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{\sin(dx+c)\,a^5\left(16\cos(dx+c)\sin(dx+c)^4-22\cos(dx+c)\sin(dx+c)^2+6\cos(dx+c)+16\sin(dx+c)\right.}{6d}$$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x)
```

output
```
(sin(c + d*x)*a**5*(16*cos(c + d*x)*sin(c + d*x)**4 - 22*cos(c + d*x)*sin(
c + d*x)**2 + 6*cos(c + d*x) + 16*sin(c + d*x)**5*i - 30*sin(c + d*x)**3*i
 + 15*sin(c + d*x)*i))/(6*d)
```

3.67 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^9}{4d(a-ia\tan(c+dx))^4}$$

output
```
-1/4*I*a^9/d/(a-I*a*tan(d*x+c))^4
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^5}{4d(i+\tan(c+dx))^4}$$

input
```
Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-1/4*I)*a^5)/(d*(I + Tan[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^8}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^9\int \frac{1}{(a-ia\tan(c+dx))^5}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{17}$$

$$-\frac{ia^9}{4d(a-ia\tan(c+dx))^4}$$

input
```
Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-1/4*I)*a^9)/(d*(a - I*a*Tan[c + d*x])^4)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(23) = 46$.

Time = 298.99 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.74

method	result
risch	$-\frac{i a^5 e^{8i(dx+c)}}{64d} - \frac{i a^5 e^{6i(dx+c)}}{16d} - \frac{3i a^5 e^{4i(dx+c)}}{32d} - \frac{i a^5 e^{2i(dx+c)}}{16d}$
derivativedivides	$\frac{i a^5\left(-\frac{\cos(dx+c)^4 \sin(dx+c)^4}{8} - \frac{\cos(dx+c)^4 \sin(dx+c)^2}{12} - \frac{\cos(dx+c)^4}{24}\right) + 5a^5\left(-\frac{\sin(dx+c)^3 \cos(dx+c)^5}{8} - \frac{\sin(dx+c)\cos(dx+c)^5}{16}\right.}{}$
default	$\frac{i a^5\left(-\frac{\cos(dx+c)^4 \sin(dx+c)^4}{8} - \frac{\cos(dx+c)^4 \sin(dx+c)^2}{12} - \frac{\cos(dx+c)^4}{24}\right) + 5a^5\left(-\frac{\sin(dx+c)^3 \cos(dx+c)^5}{8} - \frac{\sin(dx+c)\cos(dx+c)^5}{16}\right.}{}$

input
```
int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/64*I/d*a^5*exp(8*I*(d*x+c))-1/16*I/d*a^5*exp(6*I*(d*x+c))-3/32*I/d*a^5*
exp(4*I*(d*x+c))-1/16*I/d*a^5*exp(2*I*(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(21) = 42$.

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.30

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{-i\,a^5e^{(8i\,dx+8i\,c)} - 4i\,a^5e^{(6i\,dx+6i\,c)} - 6i\,a^5e^{(4i\,dx+4i\,c)} - 4i\,a^5e^{(2i\,dx+2i\,c)}}{64\,d}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/64*(-I*a^5*e^(8*I*d*x + 8*I*c) - 4*I*a^5*e^(6*I*d*x + 6*I*c) - 6*I*a^5*e
^(4*I*d*x + 4*I*c) - 4*I*a^5*e^(2*I*d*x + 2*I*c))/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(22) = 44$.

Time = 0.36 (sec) , antiderivative size = 162, normalized size of antiderivative = 6.00

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \begin{cases} \frac{-8192ia^5d^3e^{8ic}e^{8idx}-32768ia^5d^3e^{6ic}e^{6idx}-49152ia^5d^3e^{4ic}e^{4idx}-32768ia^5d^3e^{2ic}e^{2idx}}{524288d^4} & \text{for } d^4 \neq 0 \\ x\left(\frac{a^5e^{8ic}}{8}+\frac{3a^5e^{6ic}}{8}+\frac{3a^5e^{4ic}}{8}+\frac{a^5e^{2ic}}{8}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**5,x)
```

output
```
Piecewise(((-8192*I*a**5*d**3*exp(8*I*c)*exp(8*I*d*x) - 32768*I*a**5*d**3*
exp(6*I*c)*exp(6*I*d*x) - 49152*I*a**5*d**3*exp(4*I*c)*exp(4*I*d*x) - 3276
8*I*a**5*d**3*exp(2*I*c)*exp(2*I*d*x))/(524288*d**4), Ne(d**4, 0)), (x*(a*
*5*exp(8*I*c)/8 + 3*a**5*exp(6*I*c)/8 + 3*a**5*exp(4*I*c)/8 + a**5*exp(2*I
*c)/8), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(21) = 42$.

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.81

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= -\frac{i\,a^5\tan(dx+c)^4+4\,a^5\tan(dx+c)^3-6i\,a^5\tan(dx+c)^2-4\,a^5\tan(dx+c)+i\,a^5}{4\left(\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1\right)d}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/4*(I*a^5*tan(d*x + c)^4 + 4*a^5*tan(d*x + c)^3 - 6*I*a^5*tan(d*x + c)^2
 - 4*a^5*tan(d*x + c) + I*a^5)/((tan(d*x + c)^8 + 4*tan(d*x + c)^6 + 6*tan
(d*x + c)^4 + 4*tan(d*x + c)^2 + 1)*d)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i\,a^5}{4\,d(\tan{(dx+c)}+i)^4}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/4*I*a^5/(d*(tan(d*x + c) + I)^4)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{\frac{a^5\cos(c+d\,x)^4\,1\mathrm{i}}{4}+a^5\cos{(c+d\,x)}^6\,(\tan(c+d\,x)-2\mathrm{i})-2\,a^5\cos{(c+d\,x)}^8\,(\tan(c+d\,x)-\mathrm{i})}{d}$$

input
```
int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
-((a^5*cos(c + d*x)^4*1i)/4 + a^5*cos(c + d*x)^6*(tan(c + d*x) - 2i) - 2*a
^5*cos(c + d*x)^8*(tan(c + d*x) - 1i))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.19

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{\sin(dx+c)\,a^5\left(-8\cos(dx+c)\sin(dx+c)^6+20\cos(dx+c)\sin(dx+c)^4-16\cos(dx+c)\sin(dx+c)\right.}{4d}$$

input
```
int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x)
```

output
```
(sin(c + d*x)*a**5*( - 8*cos(c + d*x)*sin(c + d*x)**6 + 20*cos(c + d*x)*si
n(c + d*x)**4 - 16*cos(c + d*x)*sin(c + d*x)**2 + 4*cos(c + d*x) - 8*sin(c
 + d*x)**7*i + 24*sin(c + d*x)**5*i - 25*sin(c + d*x)**3*i + 10*sin(c + d*
x)*i))/(4*d)
```

3.68 $\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 156

$$\begin{aligned}\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx = {} & \frac{a^5x}{32}-\frac{ia^{10}}{10d(a-ia\tan(c+dx))^5} \\ & -\frac{ia^9}{16d(a-ia\tan(c+dx))^4} \\ & -\frac{ia^{11}}{24d\left(a^2-ia^2\tan(c+dx)\right)^3} \\ & -\frac{ia^{11}}{32d\left(a^3-ia^3\tan(c+dx)\right)^2} \\ & -\frac{ia^{11}}{32d\left(a^6-ia^6\tan(c+dx)\right)}\end{aligned}$$

output
```
1/32*a^5*x-1/10*I*a^10/d/(a-I*a*tan(d*x+c))^5-1/16*I*a^9/d/(a-I*a*tan(d*x+
c))^4-1/24*I*a^11/d/(a^2-I*a^2*tan(d*x+c))^3-1/32*I*a^11/d/(a^3-I*a^3*tan(
d*x+c))^2-1/32*I*a^11/d/(a^6-I*a^6*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.79

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\sec^5(c+dx)(500\cos(c+dx)+375\cos(3(c+dx))+149\cos(5(c+dx))-100i\sin(c+dx)-225i\sin}{3840d(i+\tan(c}$$

input
```
Integrate[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*Sec[c + d*x]^5*(500*Cos[c + d*x] + 375*Cos[3*(c + d*x)] + 149*Cos[5*(
c + d*x)] - (100*I)*Sin[c + d*x] - (225*I)*Sin[3*(c + d*x)] - (125*I)*Sin[
5*(c + d*x)] + 120*ArcTan[Tan[c + d*x]]*(I*Cos[5*(c + d*x)] + Sin[5*(c + d
*x)])))/(3840*d*(I + Tan[c + d*x])^5)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^{10}}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{ia^{11}\int \frac{1}{(a-ia\tan(c+dx))^6(i\tan(c+dx)a+a)}d(ia\tan(c+dx))}{d}$$
$$\Big\downarrow \text{54}$$

$$-\frac{ia^{11}\int\left(\frac{1}{2(a-ia\tan(c+dx))^6a}+\frac{1}{4(a-ia\tan(c+dx))^5a^2}+\frac{1}{8(a-ia\tan(c+dx))^4a^3}+\frac{1}{16(a-ia\tan(c+dx))^3a^4}+\frac{1}{32(\tan^2(c+dx)a^2+a^2)a}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^{11}\left(\frac{i\arctan(\tan(c+dx))}{32a^6}+\frac{1}{32a^5(a-ia\tan(c+dx))}+\frac{1}{32a^4(a-ia\tan(c+dx))^2}+\frac{1}{24a^3(a-ia\tan(c+dx))^3}+\frac{1}{16a^2(a-ia\tan(c+dx))^4}+\right.}{d}$$

input
```
Int[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-I)*a^11*(((I/32)*ArcTan[Tan[c + d*x]])/a^6 + 1/(10*a*(a - I*a*Tan[c + d
*x])^5) + 1/(16*a^2*(a - I*a*Tan[c + d*x])^4) + 1/(24*a^3*(a - I*a*Tan[c +
 d*x])^3) + 1/(32*a^4*(a - I*a*Tan[c + d*x])^2) + 1/(32*a^5*(a - I*a*Tan[c
 + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(134) = 268$.

Time = 0.65 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.12

$$ia^5\left(-\frac{\cos(dx+c)^6\sin(dx+c)^4}{10}-\frac{\cos(dx+c)^6\sin(dx+c)^2}{20}-\frac{\cos(dx+c)^6}{60}\right)+5a^5\left(-\frac{\sin(dx+c)^3\cos(dx+c)^7}{10}-\frac{3\sin(dx+c)\cos(dx+c)^7}{80}\right.$$

input
```
int(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x)
```

output
```
1/d*(I*a^5*(-1/10*cos(d*x+c)^6*sin(d*x+c)^4-1/20*cos(d*x+c)^6*sin(d*x+c)^2
-1/60*cos(d*x+c)^6)+5*a^5*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)
*cos(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*
x+c)+3/256*d*x+3/256*c)-10*I*a^5*(-1/10*cos(d*x+c)^8*sin(d*x+c)^2-1/40*cos
(d*x+c)^8)-10*a^5*(-1/10*sin(d*x+c)*cos(d*x+c)^9+1/80*(cos(d*x+c)^7+7/6*co
s(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+7/256*d*x+7/256
*c)-1/2*I*a^5*cos(d*x+c)^10+a^5*(1/10*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16
*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)+63/256*d*
x+63/256*c))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.53

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{120\,a^5dx-12i\,a^5e^{(10i\,dx+10i\,c)}-75i\,a^5e^{(8i\,dx+8i\,c)}-200i\,a^5e^{(6i\,dx+6i\,c)}-300i\,a^5e^{(4i\,dx+4i\,c)}-300i\,a^5e^{(2i\,dx+\cdots}}{3840\,d}$$

input
```
integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/3840*(120*a^5*d*x - 12*I*a^5*e^(10*I*d*x + 10*I*c) - 75*I*a^5*e^(8*I*d*x
 + 8*I*c) - 200*I*a^5*e^(6*I*d*x + 6*I*c) - 300*I*a^5*e^(4*I*d*x + 4*I*c)
- 300*I*a^5*e^(2*I*d*x + 2*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.34

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5x}{32} + \begin{cases} \frac{-100663296ia^5d^4e^{10ic}e^{10idx}-629145600ia^5d^4e^{8ic}e^{8idx}-1677721600ia^5d^4e^{6ic}e^{6idx}-2516582400ia^5d^4e^{4ic}e^{4idx}-2516582400ia^5d^4e^{2ic}e^{2idx}}{32212254720d^5} \\ x\left(\frac{a^5e^{10ic}}{32}+\frac{5a^5e^{8ic}}{32}+\frac{5a^5e^{6ic}}{16}+\frac{5a^5e^{4ic}}{16}+\frac{5a^5e^{2ic}}{32}\right) \end{cases}$$

input
```
integrate(cos(d*x+c)**10*(a+I*a*tan(d*x+c))**5,x)
```

output
```
a**5*x/32 + Piecewise(((-100663296*I*a**5*d**4*exp(10*I*c)*exp(10*I*d*x) -
 629145600*I*a**5*d**4*exp(8*I*c)*exp(8*I*d*x) - 1677721600*I*a**5*d**4*ex
p(6*I*c)*exp(6*I*d*x) - 2516582400*I*a**5*d**4*exp(4*I*c)*exp(4*I*d*x) - 2
516582400*I*a**5*d**4*exp(2*I*c)*exp(2*I*d*x))/(32212254720*d**5), Ne(d**5
, 0)), (x*(a**5*exp(10*I*c)/32 + 5*a**5*exp(8*I*c)/32 + 5*a**5*exp(6*I*c)/
16 + 5*a**5*exp(4*I*c)/16 + 5*a**5*exp(2*I*c)/32), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.05

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{15\,(dx+c)a^5 + \frac{15\,a^5\tan(dx+c)^9+70\,a^5\tan(dx+c)^7+128\,a^5\tan(dx+c)^5-80i\,a^5\tan(dx+c)^4-230\,a^5\tan(dx+c)^3+560i\,a^5\tan(dx+c)^2+\cdots}{\tan(dx+c)^{10}+5\,\tan(dx+c)^8+10\,\tan(dx+c)^6+10\,\tan(dx+c)^4+5\,\tan(dx+c)^2+1}}{480\,d}$$

input
```
integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/480*(15*(d*x + c)*a^5 + (15*a^5*tan(d*x + c)^9 + 70*a^5*tan(d*x + c)^7 +
 128*a^5*tan(d*x + c)^5 - 80*I*a^5*tan(d*x + c)^4 - 230*a^5*tan(d*x + c)^3
 + 560*I*a^5*tan(d*x + c)^2 + 465*a^5*tan(d*x + c) - 128*I*a^5)/(tan(d*x +
 c)^10 + 5*tan(d*x + c)^8 + 10*tan(d*x + c)^6 + 10*tan(d*x + c)^4 + 5*tan(
d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.57

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{1}{960}i\,a^5\left(\frac{15\,\log\left(\tan\left(dx+c\right)+i\right)}{d}-\frac{15\,\log\left(\tan\left(dx+c\right)-i\right)}{d}-\frac{2\left(15i\,\tan\left(dx+c\right)^4-75\,\tan\left(dx+c\right)^3\right.}{d(\tan}\right.$$

input
```
integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
1/960*I*a^5*(15*log(tan(d*x + c) + I)/d - 15*log(tan(d*x + c) - I)/d - 2*(
15*I*tan(d*x + c)^4 - 75*tan(d*x + c)^3 - 155*I*tan(d*x + c)^2 + 175*tan(d
*x + c) + 128*I)/(d*(tan(d*x + c) + I)^5))
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx=\frac{a^5\,x}{32}$$
$$+\frac{\frac{a^5\tan(c+d\,x)^4}{32}+\frac{a^5\tan(c+d\,x)^3\,5\mathrm{i}}{32}-\frac{31\,a^5\tan(c+d\,x)^2}{96}-\frac{a^5\tan(c+d\,x)\,35\mathrm{i}}{96}+\frac{4\,a^5}{15}}{d\,\left(\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4\,5\mathrm{i}-10\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2\,10\mathrm{i}+5\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
(a^5*x)/32 + ((4*a^5)/15 - (a^5*tan(c + d*x)*35i)/96 - (31*a^5*tan(c + d*x
)^2)/96 + (a^5*tan(c + d*x)^3*5i)/32 + (a^5*tan(c + d*x)^4)/32)/(d*(5*tan(
c + d*x) - tan(c + d*x)^2*10i - 10*tan(c + d*x)^3 + tan(c + d*x)^4*5i + ta
n(c + d*x)^5 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\left(768\cos\left(dx+c\right)\sin\left(dx+c\right)^9-2736\cos\left(dx+c\right)\sin\left(dx+c\right)^7+3608\cos\left(dx+c\right)\sin\left(dx+c\right)^5-209\right.}{}$$

input
```
int(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x)
```

output
```
(a**5*(768*cos(c + d*x)*sin(c + d*x)**9 - 2736*cos(c + d*x)*sin(c + d*x)**
7 + 3608*cos(c + d*x)*sin(c + d*x)**5 - 2090*cos(c + d*x)*sin(c + d*x)**3
+ 465*cos(c + d*x)*sin(c + d*x) + 768*sin(c + d*x)**10*i - 3120*sin(c + d*
x)**8*i + 4880*sin(c + d*x)**6*i - 3600*sin(c + d*x)**4*i + 1200*sin(c + d
*x)**2*i + 15*d*x))/(480*d)
```

3.69 $\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 214

$$\begin{aligned}\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx &= \frac{7a^5x}{128}-\frac{ia^{11}}{24d(a-ia\tan(c+dx))^6}\\ &\quad-\frac{ia^{10}}{20d(a-ia\tan(c+dx))^5}\\ &\quad-\frac{ia^8}{24d(a-ia\tan(c+dx))^3}\\ &\quad-\frac{3ia^{13}}{64d\left(a^2-ia^2\tan(c+dx)\right)^4}\\ &\quad-\frac{5ia^{13}}{128d\left(a^4-ia^4\tan(c+dx)\right)^2}\\ &\quad-\frac{3ia^{13}}{64d\left(a^8-ia^8\tan(c+dx)\right)}\\ &\quad+\frac{ia^{13}}{128d\left(a^8+ia^8\tan(c+dx)\right)}\end{aligned}$$

output
```
7/128*a^5*x-1/24*I*a^11/d/(a-I*a*tan(d*x+c))^6-1/20*I*a^10/d/(a-I*a*tan(d*
x+c))^5-1/24*I*a^8/d/(a-I*a*tan(d*x+c))^3-3/64*I*a^13/d/(a^2-I*a^2*tan(d*x
+c))^4-5/128*I*a^13/d/(a^4-I*a^4*tan(d*x+c))^2-3/64*I*a^13/d/(a^8-I*a^8*ta
n(d*x+c))+1/128*I*a^13/d/(a^8+I*a^8*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.74

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{a^5\sec^7(c+dx)(-1750\cos(c+dx)-1575\cos(3(c+dx))-693\cos(5(c+dx))+50\cos(7(c+dx))+\ldots}{1\ldots}$$

input
```
Integrate[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
-1/15360*(a^5*Sec[c + d*x]^7*(-1750*Cos[c + d*x] - 1575*Cos[3*(c + d*x)] -
 693*Cos[5*(c + d*x)] + 50*Cos[7*(c + d*x)] + (350*I)*Sin[c + d*x] + (945*
I)*Sin[3*(c + d*x)] - (840*I)*ArcTan[Tan[c + d*x]]*(Cos[5*(c + d*x)] - I*S
in[5*(c + d*x)]) + (525*I)*Sin[5*(c + d*x)] - (70*I)*Sin[7*(c + d*x)]))/(d
*(-I + Tan[c + d*x])*(I + Tan[c + d*x])^6)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^{12}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^{13}\int \frac{1}{(a-ia\tan(c+dx))^7(i\tan(c+dx)a+a)^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^{13}\int\left(\frac{3}{64a^7(a-ia\tan(c+dx))^2}+\frac{1}{128a^7(i\tan(c+dx)a+a)^2}+\frac{5}{64a^6(a-ia\tan(c+dx))^3}+\frac{1}{8a^5(a-ia\tan(c+dx))^4}+\frac{3}{16a^4(a-ia\tan(c+\right.}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{ia^{13}\left(\frac{7i\arctan(\tan(c+dx))}{128a^8}+\frac{3}{64a^7(a-ia\tan(c+dx))}-\frac{1}{128a^7(a+ia\tan(c+dx))}+\frac{5}{128a^6(a-ia\tan(c+dx))^2}+\frac{1}{24a^5(a-ia\tan(c+dx))^3}\right.}{d}$$

input
```
Int[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-I)*a^13*(((((7*I)/128)*ArcTan[Tan[c + d*x]])/a^8 + 1/(24*a^2*(a - I*a*Ta
n[c + d*x])^6) + 1/(20*a^3*(a - I*a*Tan[c + d*x])^5) + 3/(64*a^4*(a - I*a*
Tan[c + d*x])^4) + 1/(24*a^5*(a - I*a*Tan[c + d*x])^3) + 5/(128*a^6*(a - I
*a*Tan[c + d*x])^2) + 3/(64*a^7*(a - I*a*Tan[c + d*x])) - 1/(128*a^7*(a +
I*a*Tan[c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.69

$$\frac{ia^5\left(-\frac{\cos(dx+c)^8\sin(dx+c)^4}{12}-\frac{\cos(dx+c)^8\sin(dx+c)^2}{30}-\frac{\cos(dx+c)^8}{120}\right)+5a^5\left(-\frac{\sin(dx+c)^3\cos(dx+c)^9}{12}-\frac{\sin(dx+c)\cos(dx+c)^9}{40}+\right.}{}$$

input
```
int(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x)
```

output
```
1/d*(I*a^5*(-1/12*cos(d*x+c)^8*sin(d*x+c)^4-1/30*cos(d*x+c)^8*sin(d*x+c)^2
-1/120*cos(d*x+c)^8)+5*a^5*(-1/12*sin(d*x+c)^3*cos(d*x+c)^9-1/40*sin(d*x+c
)*cos(d*x+c)^9+1/320*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/
16*cos(d*x+c))*sin(d*x+c)+7/1024*d*x+7/1024*c)-10*I*a^5*(-1/12*cos(d*x+c)^
10*sin(d*x+c)^2-1/60*cos(d*x+c)^10)-10*a^5*(-1/12*sin(d*x+c)*cos(d*x+c)^11
+1/120*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)
^3+315/128*cos(d*x+c))*sin(d*x+c)+21/1024*d*x+21/1024*c)-5/12*I*a^5*cos(d*
x+c)^12+a^5*(1/12*(cos(d*x+c)^11+11/10*cos(d*x+c)^9+99/80*cos(d*x+c)^7+231
/160*cos(d*x+c)^5+231/128*cos(d*x+c)^3+693/256*cos(d*x+c))*sin(d*x+c)+231/
1024*d*x+231/1024*c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.56

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{\left(840\,a^5dxe^{(2i\,dx+2i\,c)}-10i\,a^5e^{(14i\,dx+14i\,c)}-84i\,a^5e^{(12i\,dx+12i\,c)}-315i\,a^5e^{(10i\,dx+10i\,c)}-700i\,a^5e^{(8i\,dx+8i\,c)}-\right.}{15360\,d}$$

input
```
integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/15360*(840*a^5*d*x*e^(2*I*d*x + 2*I*c) - 10*I*a^5*e^(14*I*d*x + 14*I*c)
- 84*I*a^5*e^(12*I*d*x + 12*I*c) - 315*I*a^5*e^(10*I*d*x + 10*I*c) - 700*I
*a^5*e^(8*I*d*x + 8*I*c) - 1050*I*a^5*e^(6*I*d*x + 6*I*c) - 1260*I*a^5*e^(
4*I*d*x + 4*I*c) + 60*I*a^5)*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.41

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{7a^5x}{128} + \begin{cases} \frac{(-33776997205278720ia^5d^6e^{14ic}e^{12idx}-283726776524341248ia^5d^6e^{12ic}e^{10idx}-1063975411966279680ia^5d^6e^{10ic}e^{8idx}-2364389804369\ldots}{518814677\ldots} \\ x\left(-\frac{7a^5}{128}+\frac{(a^5e^{14ic}+7a^5e^{12ic}+21a^5e^{10ic}+35a^5e^{8ic}+35a^5e^{6ic}+21a^5e^{4ic}+7a^5e^{2ic}+a^5)e^{-2ic}}{128}\right) \end{cases}$$

input
```
integrate(cos(d*x+c)**12*(a+I*a*tan(d*x+c))**5,x)
```

output
```
7*a**5*x/128 + Piecewise(((-33776997205278720*I*a**5*d**6*exp(14*I*c)*exp(
12*I*d*x) - 283726776524341248*I*a**5*d**6*exp(12*I*c)*exp(10*I*d*x) - 106
3975411966279680*I*a**5*d**6*exp(10*I*c)*exp(8*I*d*x) - 236438980436951040
0*I*a**5*d**6*exp(8*I*c)*exp(6*I*d*x) - 3546584706554265600*I*a**5*d**6*ex
p(6*I*c)*exp(4*I*d*x) - 4255901647865118720*I*a**5*d**6*exp(4*I*c)*exp(2*I
*d*x) + 202661983231672320*I*a**5*d**6*exp(-2*I*d*x))*exp(-2*I*c)/(5188146
7707308113920*d**7), Ne(d**7*exp(2*I*c), 0)), (x*(-7*a**5/128 + (a**5*exp(
14*I*c) + 7*a**5*exp(12*I*c) + 21*a**5*exp(10*I*c) + 35*a**5*exp(8*I*c) +
35*a**5*exp(6*I*c) + 21*a**5*exp(4*I*c) + 7*a**5*exp(2*I*c) + a**5)*exp(-2
*I*c)/128), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.87

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{105\,(dx+c)a^5 + \frac{105\,a^5\tan(dx+c)^{11}+595\,a^5\tan(dx+c)^9+1386\,a^5\tan(dx+c)^7+1686\,a^5\tan(dx+c)^5-240i\,a^5\tan(dx+c)^4+45\,a^5\tan(dx\ldots}{\tan(dx+c)^{12}+6\,\tan(dx+c)^{10}+15\,\tan(dx+c)^8+20\,\tan(dx+c)^6+15\,\tan(dx+c)^4\ldots}}{1920\,d}$$

input
```
integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/1920*(105*(d*x + c)*a^5 + (105*a^5*tan(d*x + c)^11 + 595*a^5*tan(d*x + c
)^9 + 1386*a^5*tan(d*x + c)^7 + 1686*a^5*tan(d*x + c)^5 - 240*I*a^5*tan(d*
x + c)^4 + 45*a^5*tan(d*x + c)^3 + 1824*I*a^5*tan(d*x + c)^2 + 1815*a^5*ta
n(d*x + c) - 496*I*a^5)/(tan(d*x + c)^12 + 6*tan(d*x + c)^10 + 15*tan(d*x
+ c)^8 + 20*tan(d*x + c)^6 + 15*tan(d*x + c)^4 + 6*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.56

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{1}{3840}i\,a^5\left(\frac{105\,\log\left(\tan\left(dx+c\right)+i\right)}{d}-\frac{105\,\log\left(\tan\left(dx+c\right)-i\right)}{d}-\frac{2\,\left(105i\,\tan\left(dx+c\right)^6-525\,\tan\left(dx\right.\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
1/3840*I*a^5*(105*log(tan(d*x + c) + I)/d - 105*log(tan(d*x + c) - I)/d -
2*(105*I*tan(d*x + c)^6 - 525*tan(d*x + c)^5 - 980*I*tan(d*x + c)^4 + 700*
tan(d*x + c)^3 - 189*I*tan(d*x + c)^2 + 665*tan(d*x + c) + 496*I)/(d*(tan(
d*x + c) + I)^6*(tan(d*x + c) - I)))
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.80

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx=\frac{7\,a^5\,x}{128}$$
$$-\frac{-\frac{7\,a^5\,\tan(c+d\,x)^6}{128}-\frac{a^5\,\tan(c+d\,x)^5\,35\mathrm{i}}{128}+\frac{49\,a^5\,\tan(c+d\,x)^4}{96}+\frac{a^5\,\tan(c+d\,x)^3\,35\mathrm{i}}{96}+\frac{63\,a^5\,\tan(c+d\,x)^2}{640}+\frac{a^5\,\mathrm{tan}}{}}{d\,\left(\tan\left(c+d\,x\right)^7+\tan\left(c+d\,x\right)^6\,5\mathrm{i}-9\,\tan\left(c+d\,x\right)^5-\tan\left(c+d\,x\right)^4\,5\mathrm{i}-5\,\tan\left(c+d\,x\right)^3-\tan\left(c+\right.\right.}$$

input
```
int(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
(7*a^5*x)/128 - ((a^5*tan(c + d*x)*133i)/384 - (31*a^5)/120 + (63*a^5*tan(
c + d*x)^2)/640 + (a^5*tan(c + d*x)^3*35i)/96 + (49*a^5*tan(c + d*x)^4)/96
 - (a^5*tan(c + d*x)^5*35i)/128 - (7*a^5*tan(c + d*x)^6)/128)/(d*(5*tan(c
+ d*x) - tan(c + d*x)^2*9i - 5*tan(c + d*x)^3 - tan(c + d*x)^4*5i - 9*tan(
c + d*x)^5 + tan(c + d*x)^6*5i + tan(c + d*x)^7 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.81

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\left(-2560\cos(dx+c)\sin(dx+c)^{11}+11776\cos(dx+c)\sin(dx+c)^9-21552\cos(dx+c)\sin(dx+c)^7\right.}{}$$

input
```
int(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x)
```

output
```
(a**5*( - 2560*cos(c + d*x)*sin(c + d*x)**11 + 11776*cos(c + d*x)*sin(c +
d*x)**9 - 21552*cos(c + d*x)*sin(c + d*x)**7 + 19656*cos(c + d*x)*sin(c +
d*x)**5 - 9030*cos(c + d*x)*sin(c + d*x)**3 + 1815*cos(c + d*x)*sin(c + d*
x) - 2560*sin(c + d*x)**12*i + 13056*sin(c + d*x)**10*i - 27120*sin(c + d*
x)**8*i + 29120*sin(c + d*x)**6*i - 16800*sin(c + d*x)**4*i + 4800*sin(c +
 d*x)**2*i + 105*d*x))/(1920*d)
```

3.70 $\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 22, antiderivative size = 167

$$\begin{aligned}\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx &= \frac{63a^5\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{63ia^5\sec(c+dx)}{8d}\\&\quad+\frac{9ia^2\sec(c+dx)(a+ia\tan(c+dx))^3}{20d}\\&\quad+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}\\&\quad+\frac{21ia\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{20d}\\&\quad+\frac{21i\sec(c+dx)\left(a^5+ia^5\tan(c+dx)\right)}{8d}\end{aligned}$$

output
```
63/8*a^5*arctanh(sin(d*x+c))/d+63/8*I*a^5*sec(d*x+c)/d+9/20*I*a^2*sec(d*x+
c)*(a+I*a*tan(d*x+c))^3/d+1/5*I*a*sec(d*x+c)*(a+I*a*tan(d*x+c))^4/d+21/20*
I*a*sec(d*x+c)*(a^2+I*a^2*tan(d*x+c))^2/d+21/8*I*sec(d*x+c)*(a^5+I*a^5*tan
(d*x+c))/d
```

Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.69

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5(\cos(5dx)+i\sin(5dx))\left(5040\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+i\sec^5(c+dx)(1344+1920\cos(2(c-}{320d(\cos(dx)+i\sin(dx))^5}$$

input
```
Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*(Cos[5*d*x] + I*Sin[5*d*x])*(5040*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2
]] + I*Sec[c + d*x]^5*(1344 + 1920*Cos[2*(c + d*x)] + 640*Cos[4*(c + d*x)]
 + (450*I)*Sin[2*(c + d*x)] + (325*I)*Sin[4*(c + d*x)])))/(320*d*(Cos[d*x]
 + I*Sin[d*x])^5)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 3979, 3042, 3979, 3042, 3979, 3042, 3979, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(c+dx)(a+ia\tan(c+dx))^5dx$$
$$\Big\downarrow \text{3979}$$
$$\frac{9}{5}a\int \sec(c+dx)(i\tan(c+dx)a+a)^4dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{9}{5}a\int \sec(c+dx)(i\tan(c+dx)a+a)^4dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$\big\downarrow$ 3979

$$\frac{9}{5}a\left(\frac{7}{4}a\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}\right)+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$\big\downarrow$ 3042

$$\frac{9}{5}a\left(\frac{7}{4}a\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}\right)+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$\big\downarrow$ 3979

$$\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)+\frac{ia\sec(c+dx)(a+}{4d}\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$\big\downarrow$ 3042

$$\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)+\frac{ia\sec(c+dx)(a+}{4d}\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$\big\downarrow$ 3979

$$\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)}{}\right.\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$\big\downarrow$ 3042

$$\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)}{}\right.\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$\big\downarrow$ 3967

$$\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int\sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{3}{2}a\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)\right)+\frac{ia\sec(c\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

input
```
Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((I/5)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^4)/d + (9*a*(((I/4)*a*Sec[c +
 d*x]*(a + I*a*Tan[c + d*x])^3)/d + (7*a*(((I/3)*a*Sec[c + d*x]*(a + I*a*T
an[c + d*x])^2)/d + (5*a*((3*a*((a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c +
 d*x])/d))/2 + ((I/2)*Sec[c + d*x]*(a^2 + I*a^2*Tan[c + d*x]))/d))/3))/4))
/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 7.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

method	result
risch	$\frac{ia^5\left(965\,\mathrm{e}^{9i(dx+c)}+2370\,\mathrm{e}^{7i(dx+c)}+2688\,\mathrm{e}^{5i(dx+c)}+1470\,\mathrm{e}^{3i(dx+c)}+315\,\mathrm{e}^{i(dx+c)}\right)}{20d\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}+\frac{63a^5\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8d}-\frac{63a^5\ln}{}$
derivativedivides	$ia^5\left(\frac{\sin(dx+c)^6}{5\cos(dx+c)^5}-\frac{\sin(dx+c)^6}{15\cos(dx+c)^3}+\frac{\sin(dx+c)^6}{5\cos(dx+c)}+\frac{\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}\right)+5a^5\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx}{8\cos(d}\right.$
default	$ia^5\left(\frac{\sin(dx+c)^6}{5\cos(dx+c)^5}-\frac{\sin(dx+c)^6}{15\cos(dx+c)^3}+\frac{\sin(dx+c)^6}{5\cos(dx+c)}+\frac{\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}\right)+5a^5\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx}{8\cos(d}\right.$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/20*I*a^5/d/(exp(2*I*(d*x+c))+1)^5*(965*exp(9*I*(d*x+c))+2370*exp(7*I*(d*
x+c))+2688*exp(5*I*(d*x+c))+1470*exp(3*I*(d*x+c))+315*exp(I*(d*x+c)))+63/8
/d*a^5*ln(exp(I*(d*x+c))+I)-63/8/d*a^5*ln(exp(I*(d*x+c))-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(137) = 274$.

Time = 0.10 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.86

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{1930i\,a^5e^{(9i\,dx+9i\,c)} + 4740i\,a^5e^{(7i\,dx+7i\,c)} + 5376i\,a^5e^{(5i\,dx+5i\,c)} + 2940i\,a^5e^{(3i\,dx+3i\,c)} + 630i\,a^5e^{(i\,dx+i\,c)} + 31}{}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/40*(1930*I*a^5*e^(9*I*d*x + 9*I*c) + 4740*I*a^5*e^(7*I*d*x + 7*I*c) + 53
76*I*a^5*e^(5*I*d*x + 5*I*c) + 2940*I*a^5*e^(3*I*d*x + 3*I*c) + 630*I*a^5*
e^(I*d*x + I*c) + 315*(a^5*e^(10*I*d*x + 10*I*c) + 5*a^5*e^(8*I*d*x + 8*I*
c) + 10*a^5*e^(6*I*d*x + 6*I*c) + 10*a^5*e^(4*I*d*x + 4*I*c) + 5*a^5*e^(2*
I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) + I) - 315*(a^5*e^(10*I*d*x + 10
*I*c) + 5*a^5*e^(8*I*d*x + 8*I*c) + 10*a^5*e^(6*I*d*x + 6*I*c) + 10*a^5*e^
(4*I*d*x + 4*I*c) + 5*a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) -
 I))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x
+ 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx = ia^5\left(\int\left(-i\sec\left(c+dx\right)\right)\,dx + \int 5\tan\left(c+dx\right)\sec\left(c+dx\right)dx + \int\left(-10\tan^3\left(c+dx\right)\sec\left(c+dx\right)\right)\,dx + \int\tan^5\left(c+dx\right)\sec\left(c+dx\right)dx + \int 10i\tan^2\left(c+dx\right)\sec\left(c+dx\right)dx + \int\left(-5i\tan^4\left(c+dx\right)\sec\left(c+dx\right)\right)\,dx\right)$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**5,x)
```

output
```
I*a**5*(Integral(-I*sec(c + d*x), x) + Integral(5*tan(c + d*x)*sec(c + d*x
), x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x), x) + Integral(tan(c + d
*x)**5*sec(c + d*x), x) + Integral(10*I*tan(c + d*x)**2*sec(c + d*x), x) +
 Integral(-5*I*tan(c + d*x)**4*sec(c + d*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.29

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{75\,a^5\left(\frac{2\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+600\,a^5\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-\cdots}\right.}{}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/240*(75*a^5*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*s
in(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)) +
600*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1)) + 240*a^5*log(sec(d*x + c) + tan(d*x + c)) + 1200*I*a^
5/cos(d*x + c) + 800*I*(3*cos(d*x + c)^2 - 1)*a^5/cos(d*x + c)^3 + 16*I*(1
5*cos(d*x + c)^4 - 10*cos(d*x + c)^2 + 3)*a^5/cos(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.13

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{315\,a^5\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-315\,a^5\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2\left(275\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+200i\,a^5\tan\left(\frac{1}{2}\,dx\cdots\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
1/40*(315*a^5*log(tan(1/2*d*x + 1/2*c) + 1) - 315*a^5*log(tan(1/2*d*x + 1/
2*c) - 1) - 2*(275*a^5*tan(1/2*d*x + 1/2*c)^9 + 200*I*a^5*tan(1/2*d*x + 1/
2*c)^8 - 750*a^5*tan(1/2*d*x + 1/2*c)^7 - 1600*I*a^5*tan(1/2*d*x + 1/2*c)^
6 + 3280*I*a^5*tan(1/2*d*x + 1/2*c)^4 + 750*a^5*tan(1/2*d*x + 1/2*c)^3 - 2
240*I*a^5*tan(1/2*d*x + 1/2*c)^2 - 275*a^5*tan(1/2*d*x + 1/2*c) + 488*I*a^
5)/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.37

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{63\,a^5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$
$$-\frac{\frac{55\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 10\mathrm{i}-\frac{75\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}-a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 80\mathrm{i}+a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 164\mathrm{i}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\right.\right.}$$

input
```
int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x),x)
```

output
```
(63*a^5*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((75*a^5*tan(c/2 + (d*x)/2)^3)/
2 - a^5*tan(c/2 + (d*x)/2)^2*112i + a^5*tan(c/2 + (d*x)/2)^4*164i - a^5*ta
n(c/2 + (d*x)/2)^6*80i - (75*a^5*tan(c/2 + (d*x)/2)^7)/2 + a^5*tan(c/2 + (
d*x)/2)^8*10i + (55*a^5*tan(c/2 + (d*x)/2)^9)/4 + (a^5*122i)/5 - (55*a^5*t
an(c/2 + (d*x)/2))/4)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4
 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^1
0 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.73

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\left(-315\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^4+630\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(d\right.\right.}{}$$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x)
```

output
```
(a**5*( - 315*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 + 630
*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 - 315*cos(c + d*x)
*log(tan((c + d*x)/2) - 1) + 315*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*si
n(c + d*x)**4 - 630*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2
 + 315*cos(c + d*x)*log(tan((c + d*x)/2) + 1) - 488*cos(c + d*x)*sin(c + d
*x)**4*i + 325*cos(c + d*x)*sin(c + d*x)**3 + 976*cos(c + d*x)*sin(c + d*x
)**2*i - 275*cos(c + d*x)*sin(c + d*x) - 488*cos(c + d*x)*i + 640*sin(c +
d*x)**4*i - 1120*sin(c + d*x)**2*i + 488*i))/(40*cos(c + d*x)*d*(sin(c + d
*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.71 $\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 22, antiderivative size = 130

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{35a^5\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{35ia^5\sec(c+dx)}{2d} - \frac{7ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d} - \frac{35i\sec(c+dx)\left(a^5+ia^5\tan(c+dx)\right)}{6d}$$

output
```
-35/2*a^5*arctanh(sin(d*x+c))/d-35/2*I*a^5*sec(d*x+c)/d-7/3*I*a^3*sec(d*x+
c)*(a+I*a*tan(d*x+c))^2/d-2*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^4/d-35/6*I*s
ec(d*x+c)*(a^5+I*a^5*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.16

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\cos^2(c+dx)\left(-840i\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)\cos^3(c+dx)(\cos(5c)-i\sin(5c))+(\cos(4c-d}{24d(}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*Cos[c + d*x]^2*((-840*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c
+ d*x]^3*(Cos[5*c] - I*Sin[5*c]) + (Cos[4*c - d*x] - I*Sin[4*c - d*x])*(51
1*Cos[c + d*x] + 153*Cos[3*(c + d*x)] - I*(49*Sin[c + d*x] + 57*Sin[3*(c +
 d*x)])))*(-I + Tan[c + d*x])^5)/(24*d*(Cos[d*x] + I*Sin[d*x])^5)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3977, 3042, 3979, 3042, 3979, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)}dx$$

$$\Big\downarrow \; 3977$$

$$-7a^2\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

$$\Big\downarrow \; 3042$$

$$-7a^2\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

$$\Big\downarrow \text{3979}$$

$$-7a^2\left(\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

$$\Big\downarrow \text{3042}$$

$$-7a^2\left(\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

$$\Big\downarrow \text{3979}$$

$$-7a^2\left(\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a}{}\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

$$\Big\downarrow \text{3042}$$

$$-7a^2\left(\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a}{}\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

$$\Big\downarrow \text{3967}$$

$$-7a^2\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(}{}\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

$$\Big\downarrow \text{3042}$$

$$-7a^2\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c}{}\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

$$\Big\downarrow \text{4257}$$

$$-7a^2\left(\frac{5}{3}a\left(\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{3}{2}a\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)\right)+\frac{ia\sec(c+\right.$$

$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-2*I)*a*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^4)/d - 7*a^2*(((I/3)*a*Sec[c
 + d*x]*(a + I*a*Tan[c + d*x])^2)/d + (5*a*((3*a*((a*ArcTanh[Sin[c + d*x]]
)/d + (I*a*Sec[c + d*x])/d))/2 + ((I/2)*Sec[c + d*x]*(a^2 + I*a^2*Tan[c +
d*x]))/d))/3)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 6.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{16 i a^5 e^{i(dx+c)}}{d}-\frac{i a^5\left(87\, e^{5i(dx+c)}+136\, e^{3i(dx+c)}+57\, e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{35a^5 \ln\left(e^{i(dx+c)}-i\right)}{2d}-\frac{35a^5 \ln\left(e^{i(dx+c)}+i\right)}{2d}$
derivativedivides	$\frac{i a^5\left(\frac{\sin(dx+c)^6}{3\cos(dx+c)^3}-\frac{\sin(dx+c)^6}{\cos(dx+c)}-\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)\right)+5a^5\left(\frac{\sin(dx+c)^5}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^3}{2}+\frac{3\sin(dx}{2}\right.}{}$
default	$\frac{i a^5\left(\frac{\sin(dx+c)^6}{3\cos(dx+c)^3}-\frac{\sin(dx+c)^6}{\cos(dx+c)}-\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)\right)+5a^5\left(\frac{\sin(dx+c)^5}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^3}{2}+\frac{3\sin(dx}{2}\right.}{}$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-16*I/d*a^5*exp(I*(d*x+c))-1/3*I*a^5/d/(exp(2*I*(d*x+c))+1)^3*(87*exp(5*I*
(d*x+c))+136*exp(3*I*(d*x+c))+57*exp(I*(d*x+c)))+35/2/d*a^5*ln(exp(I*(d*x+
c))-I)-35/2/d*a^5*ln(exp(I*(d*x+c))+I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.66

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{-96i\,a^5e^{(7i\,dx+7i\,c)}-462i\,a^5e^{(5i\,dx+5i\,c)}-560i\,a^5e^{(3i\,dx+3i\,c)}-210i\,a^5e^{(i\,dx+i\,c)}-105\left(a^5e^{(6i\,dx+6i\,c)}+3\,a^5e^{(}\right.}{6\left(de^{(6i\,dx+6i\,c)}\right.}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/6*(-96*I*a^5*e^(7*I*d*x + 7*I*c) - 462*I*a^5*e^(5*I*d*x + 5*I*c) - 560*I
*a^5*e^(3*I*d*x + 3*I*c) - 210*I*a^5*e^(I*d*x + I*c) - 105*(a^5*e^(6*I*d*x
 + 6*I*c) + 3*a^5*e^(4*I*d*x + 4*I*c) + 3*a^5*e^(2*I*d*x + 2*I*c) + a^5)*l
og(e^(I*d*x + I*c) + I) + 105*(a^5*e^(6*I*d*x + 6*I*c) + 3*a^5*e^(4*I*d*x
+ 4*I*c) + 3*a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) - I))/(d*e
^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d
)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.52

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{35a^5\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{2}-\frac{\log\left(e^{idx}+ie^{-ic}\right)}{2}\right)}{d}$$
$$+\frac{-87ia^5e^{5ic}e^{5idx}-136ia^5e^{3ic}e^{3idx}-57ia^5e^{ic}e^{idx}}{3de^{6ic}e^{6idx}+9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}+3d}+\begin{cases}-\frac{16ia^5e^{ic}e^{idx}}{d} & \text{for } d\neq 0\\ 16a^5xe^{ic} & \text{otherwise}\end{cases}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**5,x)
```

output
```
35*a**5*(log(exp(I*d*x) - I*exp(-I*c))/2 - log(exp(I*d*x) + I*exp(-I*c))/2
)/d + (-87*I*a**5*exp(5*I*c)*exp(5*I*d*x) - 136*I*a**5*exp(3*I*c)*exp(3*I*
d*x) - 57*I*a**5*exp(I*c)*exp(I*d*x))/(3*d*exp(6*I*c)*exp(6*I*d*x) + 9*d*e
xp(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) + 3*d) + Piecewise((-
16*I*a**5*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (16*a**5*x*exp(I*c), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.33

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{15\,a^5\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)-4\,\sin\left(dx+c\right)\right)+120i\,a^5\left(\frac{}{\cos}\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/12*(15*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) +
1) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) + 120*I*a^5*(1/cos(d*x + c)
 + cos(d*x + c)) + 4*I*a^5*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(
d*x + c)) + 60*a^5*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(
d*x + c)) + 60*I*a^5*cos(d*x + c) - 12*a^5*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 510 vs. 2(108) = 216.

Time = 0.50 (sec) , antiderivative size = 510, normalized size of antiderivative = 3.92

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
1/1536*(8295*a^5*e^(6*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 24885*a^
5*e^(4*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 24885*a^5*e^(2*I*d*x +
2*I*c)*log(I*e^(I*d*x + I*c) + 1) - 18585*a^5*e^(6*I*d*x + 6*I*c)*log(I*e^
(I*d*x + I*c) - 1) - 55755*a^5*e^(4*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) -
 1) - 55755*a^5*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) - 8295*a^5*
e^(6*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 24885*a^5*e^(4*I*d*x + 4
*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 24885*a^5*e^(2*I*d*x + 2*I*c)*log(-I*e
^(I*d*x + I*c) + 1) + 18585*a^5*e^(6*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c)
 - 1) + 55755*a^5*e^(4*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 55755*
a^5*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 24576*I*a^5*e^(7*I*d
*x + 7*I*c) - 118272*I*a^5*e^(5*I*d*x + 5*I*c) - 143360*I*a^5*e^(3*I*d*x +
 3*I*c) - 53760*I*a^5*e^(I*d*x + I*c) + 8295*a^5*log(I*e^(I*d*x + I*c) + 1
) - 18585*a^5*log(I*e^(I*d*x + I*c) - 1) - 8295*a^5*log(-I*e^(I*d*x + I*c)
 + 1) + 18585*a^5*log(-I*e^(I*d*x + I*c) - 1))/(d*e^(6*I*d*x + 6*I*c) + 3*
d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B] (verification not implemented)

Time = 4.42 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.71

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{35\,a^5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{37\,a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 27\mathrm{i} - 118\,a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 48\mathrm{i} + 139\,a^5\tan\left(\frac{c}{2}\cdots\right.}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7 - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 1\mathrm{i} + 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 3\mathrm{i} - 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 - \tan\cdots\right.}$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
- (35*a^5*atanh(tan(c/2 + (d*x)/2)))/d - (139*a^5*tan(c/2 + (d*x)/2)^2 - a
^5*tan(c/2 + (d*x)/2)^3*48i - 118*a^5*tan(c/2 + (d*x)/2)^4 + a^5*tan(c/2 +
 (d*x)/2)^5*27i + 37*a^5*tan(c/2 + (d*x)/2)^6 - (166*a^5)/3 + (a^5*tan(c/2
 + (d*x)/2)*55i)/3)/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*3i - 3*t
an(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*3i + 3*tan(c/2 + (d*x)/2)^5 - t
an(c/2 + (d*x)/2)^6*1i - tan(c/2 + (d*x)/2)^7 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.58

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\left(105\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2-105\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-105\,\mathrm{c}\right.}{}$$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x)
```

output
```
(a**5*(105*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 - 105*co
s(c + d*x)*log(tan((c + d*x)/2) - 1) - 105*cos(c + d*x)*log(tan((c + d*x)/
2) + 1)*sin(c + d*x)**2 + 105*cos(c + d*x)*log(tan((c + d*x)/2) + 1) + 96*
cos(c + d*x)*sin(c + d*x)**3 + 166*cos(c + d*x)*sin(c + d*x)**2*i - 111*co
s(c + d*x)*sin(c + d*x) - 166*cos(c + d*x)*i + 96*sin(c + d*x)**4*i - 264*
sin(c + d*x)**2*i + 166*i))/(6*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.72 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 98

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{5a^5\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{5ia^5\sec(c+dx)}{d}+\frac{10ia^3\cos(c+dx)(a+ia\tan(c+dx))^2}{3d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}$$

output
```
5*a^5*arctanh(sin(d*x+c))/d+5*I*a^5*sec(d*x+c)/d+10/3*I*a^3*cos(d*x+c)*(a+
I*a*tan(d*x+c))^2/d-2/3*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4/d
```

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.33

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\cos^4(c+dx)\left(30\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)\cos(c+dx)(i\cos(5c)+\sin(5c))-(\cos(3c-2dx)\right.\cdots}{3d(\cos(dx)+i\sin(dx)\cdots}$$

input
```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*Cos[c + d*x]^4*(30*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]
*(I*Cos[5*c] + Sin[5*c]) - (Cos[3*c - 2*d*x] - I*Sin[3*c - 2*d*x])*(10 + 1
3*Cos[2*(c + d*x)] - (17*I)*Sin[2*(c + d*x)]))*(-I + Tan[c + d*x])^5)/(3*d
*(Cos[d*x] + I*Sin[d*x])^5)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3977, 3042, 3977, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{5}{3}a^2\int \cos(c+dx)(i\tan(c+dx)a+a)^3dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5}{3}a^2\int \frac{(i\tan(c+dx)a+a)^3}{\sec(c+dx)}dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}$$

$$\Big\downarrow \text{3977}$$

$$-\frac{5}{3}a^2\left(-3a^2\int \sec(c+dx)(i\tan(c+dx)a+a)dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5}{3}a^2\left(-3a^2\int \sec(c+dx)(i\tan(c+dx)a+a)dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}$$

$$\Big\downarrow \text{3967}$$

$$-\frac{5}{3}a^2\left(-3a^2\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5}{3}a^2\left(-3a^2\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{5}{3}a^2\left(-3a^2\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(((-2*I)/3)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^4)/d - (5*a^2*(-3*a^2*
((a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d) - ((2*I)*a*Cos[c + d*
x]*(a + I*a*Tan[c + d*x])^2)/d))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 23.62 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{4 i a^{5} {\mathrm e}^{3 i (d x+c)}}{3 d}+\frac{8 i a^{5} {\mathrm e}^{i (d x+c)}}{d}+\frac{2 i a^{5} {\mathrm e}^{i (d x+c)}}{d \left({\mathrm e}^{2 i (d x+c)}+1\right)}-\frac{5 a^{5} \ln \left({\mathrm e}^{i (d x+c)}-i\right)}{d}+\frac{5 a^{5} \ln \left({\mathrm e}^{i (d x+c)}+i\right)}{d}$
derivativedivides	$\frac{i a^{5} \left(\frac{\sin(d x+c)^{6}}{\cos(d x+c)}+\left(\frac{8}{3}+\sin(d x+c)^{4}+\frac{4 \sin(d x+c)^{2}}{3}\right) \cos(d x+c)\right)+5 a^{5} \left(-\frac{\sin(d x+c)^{3}}{3}-\sin(d x+c)+\ln(\sec(d x+c)+\tan(dx}{d}$
default	$\frac{i a^{5} \left(\frac{\sin(d x+c)^{6}}{\cos(d x+c)}+\left(\frac{8}{3}+\sin(d x+c)^{4}+\frac{4 \sin(d x+c)^{2}}{3}\right) \cos(d x+c)\right)+5 a^{5} \left(-\frac{\sin(d x+c)^{3}}{3}-\sin(d x+c)+\ln(\sec(d x+c)+\tan(dx}{d}$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-4/3*I/d*a^5*exp(3*I*(d*x+c))+8*I/d*a^5*exp(I*(d*x+c))+2*I*a^5*exp(I*(d*x+
c))/d/(exp(2*I*(d*x+c))+1)-5/d*a^5*ln(exp(I*(d*x+c))-I)+5/d*a^5*ln(exp(I*(
d*x+c))+I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.24

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{-4i\,a^5e^{(5i\,dx+5i\,c)}+20i\,a^5e^{(3i\,dx+3i\,c)}+30i\,a^5e^{(i\,dx+i\,c)}+15\left(a^5e^{(2i\,dx+2i\,c)}+a^5\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(a^5\right.}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/3*(-4*I*a^5*e^(5*I*d*x + 5*I*c) + 20*I*a^5*e^(3*I*d*x + 3*I*c) + 30*I*a^
5*e^(I*d*x + I*c) + 15*(a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c)
 + I) - 15*(a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) - I))/(d*e^
(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.51

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{2ia^5e^{ic}e^{idx}}{de^{2ic}e^{2idx}+d}+\frac{5a^5\left(-\log\left(e^{idx}-ie^{-ic}\right)+\log\left(e^{idx}+ie^{-ic}\right)\right)}{d}$$
$$+\begin{cases}\frac{-4ia^5de^{3ic}e^{3idx}+24ia^5de^{ic}e^{idx}}{3d^2} & \text{for } d^2\neq 0\\ x(4a^5e^{3ic}-8a^5e^{ic}) & \text{otherwise}\end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**5,x)
```

output
```
2*I*a**5*exp(I*c)*exp(I*d*x)/(d*exp(2*I*c)*exp(2*I*d*x) + d) + 5*a**5*(-lo
g(exp(I*d*x) - I*exp(-I*c)) + log(exp(I*d*x) + I*exp(-I*c)))/d + Piecewise
(((-4*I*a**5*d*exp(3*I*c)*exp(3*I*d*x) + 24*I*a**5*d*exp(I*c)*exp(I*d*x))/
(3*d**2), Ne(d**2, 0)), (x*(4*a**5*exp(3*I*c) - 8*a**5*exp(I*c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.57

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{10i\,a^5\cos\left(dx+c\right)^3+20\,a^5\sin\left(dx+c\right)^3+2i\left(\cos\left(dx+c\right)^3-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)a^5+20i\,(\cos$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/6*(10*I*a^5*cos(d*x + c)^3 + 20*a^5*sin(d*x + c)^3 + 2*I*(cos(d*x + c)^
3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^5 + 20*I*(cos(d*x + c)^3 - 3*cos(d*
x + c))*a^5 + 5*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*
x + c) - 1) + 6*sin(d*x + c))*a^5 + 2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^
5)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1683 vs. 2(84) = 168.

Time = 0.78 (sec) , antiderivative size = 1683, normalized size of antiderivative = 17.17

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/6144*(39225*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 31380
0*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1098300*a^5*e^(12*
I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2196600*a^5*e^(10*I*d*x + 2*I*
c)*log(I*e^(I*d*x + I*c) + 1) + 2196600*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I
*d*x + I*c) + 1) + 1098300*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) +
 1) + 313800*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2745750*
a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 39225*a^5*e^(-8*I*c)*log(I*e^
(I*d*x + I*c) + 1) + 8520*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) -
 1) + 68160*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 238560*a
^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 477120*a^5*e^(10*I*d*
x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 477120*a^5*e^(6*I*d*x - 2*I*c)*log
(I*e^(I*d*x + I*c) - 1) + 238560*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x +
I*c) - 1) + 68160*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 596
400*a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 8520*a^5*e^(-8*I*c)*log(I
*e^(I*d*x + I*c) - 1) - 39225*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I
*c) + 1) - 313800*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1
098300*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2196600*a^5*
e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2196600*a^5*e^(6*I*d*x
- 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1098300*a^5*e^(4*I*d*x - 4*I*c)*log
(-I*e^(I*d*x + I*c) + 1) - 313800*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d...
```

Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.65

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{10\,a^5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{8\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,34\mathrm{i}-\frac{82\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,38\mathrm{i}+\frac{46\,a^5}{3}}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,3\mathrm{i}-4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,4\mathrm{i}+3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
(10*a^5*atanh(tan(c/2 + (d*x)/2)))/d - (a^5*tan(c/2 + (d*x)/2)^3*34i - (82
*a^5*tan(c/2 + (d*x)/2)^2)/3 + 8*a^5*tan(c/2 + (d*x)/2)^4 + (46*a^5)/3 - a
^5*tan(c/2 + (d*x)/2)*38i)/(d*(3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2
*4i - 4*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*3i + tan(c/2 + (d*x)/2
)^5 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.23

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\left(-15\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+15\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-16\cos\left(dx+c\right)\sin\right.}{3\cos\left(dx\right.}$$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x)
```

output
```
(a**5*( - 15*cos(c + d*x)*log(tan((c + d*x)/2) - 1) + 15*cos(c + d*x)*log(
tan((c + d*x)/2) + 1) - 16*cos(c + d*x)*sin(c + d*x)**3 - 12*cos(c + d*x)*
sin(c + d*x) - 23*cos(c + d*x)*i - 16*sin(c + d*x)**4*i - 4*sin(c + d*x)**
2*i + 23*i))/(3*cos(c + d*x)*d)
```

3.73 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 32

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}$$

output
```
-1/5*I*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5/d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^5(\cos(c+dx)+i\sin(c+dx))^5}{5d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-1/5*I)*a^5*(Cos[c + d*x] + I*Sin[c + d*x])^5)/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \; 3969$$

$$-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-1/5*I)*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5)/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Maple [A] (verified)

Time = 74.60 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

method	result
risch	$-\frac{i a^{5} {\mathrm e}^{5 i \left(d x +c \right)}}{5 d}$
orering	$-\frac{i \cos \left(d x +c \right)^{5} \left(a +i a \tan \left(d x +c \right)\right)^{5}}{5 d}$
derivativedivides	$\frac{-\frac{i a^{5} \left(\frac{8}{3}+\sin \left(d x +c \right)^{4}+\frac{4 \sin \left(d x +c \right)^{2}}{3}\right) \cos \left(d x +c \right)}{5}+a^{5} \sin \left(d x +c \right)^{5}-10 i a^{5} \left(-\frac{\cos \left(d x +c \right)^{3} \sin \left(d x +c \right)^{2}}{5}-\frac{2 \cos \left(d x +c \right)^{3}}{15}\right)-10 a^{5} \left(\right.}{d}$
default	$\frac{-\frac{i a^{5} \left(\frac{8}{3}+\sin \left(d x +c \right)^{4}+\frac{4 \sin \left(d x +c \right)^{2}}{3}\right) \cos \left(d x +c \right)}{5}+a^{5} \sin \left(d x +c \right)^{5}-10 i a^{5} \left(-\frac{\cos \left(d x +c \right)^{3} \sin \left(d x +c \right)^{2}}{5}-\frac{2 \cos \left(d x +c \right)^{3}}{15}\right)-10 a^{5} \left(\right.}{d}$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/5*I/d*a^5*exp(5*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i\,a^{5}e^{\left(5i\,dx+5i\,c\right)}}{5\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
-1/5*I*a^5*e^(5*I*d*x + 5*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx = \begin{cases} -\frac{ia^5e^{5ic}e^{5idx}}{5d} & \text{for } d\neq 0 \\ a^5xe^{5ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**5,x)
```

output
```
Piecewise((-I*a**5*exp(5*I*c)*exp(5*I*d*x)/(5*d), Ne(d, 0)), (a**5*x*exp(5
*I*c), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(26) = 52.

Time = 0.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 4.75

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx =$$
$$-\frac{15i\,a^5\cos(dx+c)^5 - 15\,a^5\sin(dx+c)^5 + 10i\left(3\,\cos(dx+c)^5 - 5\,\cos(dx+c)^3\right)a^5 + i\left(3\,\cos(dx+c\right.}{}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/15*(15*I*a^5*cos(d*x + c)^5 - 15*a^5*sin(d*x + c)^5 + 10*I*(3*cos(d*x +
 c)^5 - 5*cos(d*x + c)^3)*a^5 + I*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 +
15*cos(d*x + c))*a^5 - 10*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^5 - (3*s
in(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^5)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1669 vs. $2(26) = 52$.

Time = 0.71 (sec) , antiderivative size = 1669, normalized size of antiderivative = 52.16

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/40960*(11375*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 9100
0*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 318500*a^5*e^(12*I
*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 637000*a^5*e^(10*I*d*x + 2*I*c)
*log(I*e^(I*d*x + I*c) + 1) + 637000*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*
x + I*c) + 1) + 318500*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1)
+ 91000*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 796250*a^5*e^
(8*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 11375*a^5*e^(-8*I*c)*log(I*e^(I*d*x
 + I*c) + 1) + 11590*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) +
 92720*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 324520*a^5*e^
(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 649040*a^5*e^(10*I*d*x + 2
*I*c)*log(I*e^(I*d*x + I*c) - 1) + 649040*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^
(I*d*x + I*c) - 1) + 324520*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c)
- 1) + 92720*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 811300*a
^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 11590*a^5*e^(-8*I*c)*log(I*e^(
I*d*x + I*c) - 1) - 11375*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 91000*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 318500
*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 637000*a^5*e^(10*I
*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 637000*a^5*e^(6*I*d*x - 2*I*c)
*log(-I*e^(I*d*x + I*c) + 1) - 318500*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*
d*x + I*c) + 1) - 91000*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) ...
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.25

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{2\,a^5\,\left(5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{5\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,5\mathrm{i}-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,10\mathrm{i}+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
(2*a^5*(5*tan(c/2 + (d*x)/2)^4 - 10*tan(c/2 + (d*x)/2)^2 + 1))/(5*d*(5*tan
(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*10i - 10*tan(c/2 + (d*x)/2)^3 + tan
(c/2 + (d*x)/2)^4*5i + tan(c/2 + (d*x)/2)^5 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.53

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{a^5\left(-16\cos(dx+c)\sin(dx+c)^4 i+12\cos(dx+c)\sin(dx+c)^2 i-\cos(dx+c)\,i+16\sin(dx+c)^5-2\right.}{5d}$$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x)
```

output
```
(a**5*( - 16*cos(c + d*x)*sin(c + d*x)**4*i + 12*cos(c + d*x)*sin(c + d*x)
**2*i - cos(c + d*x)*i + 16*sin(c + d*x)**5 - 20*sin(c + d*x)**3 + 5*sin(c
 + d*x) + i))/(5*d)
```

3.74 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 101

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^3}{105d} - \frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^4}{35d} - \frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}$$

output
```
-2/105*I*a^2*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d-2/35*I*a*cos(d*x+c)^5*(a+
I*a*tan(d*x+c))^4/d-1/7*I*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5/d
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.21

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\sec(c+dx)(-i\cos(4(c+dx))+\sin(4(c+dx)))\left(77+92\cos(2(c+dx))+\left(15+416\sqrt{\cos^2(c+dx)}\right.\right.}{840d}$$

input
```
Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*Sec[c + d*x]*((-I)*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])*(77 + 92*Cos[
2*(c + d*x)] + (15 + 416*Sqrt[Cos[c + d*x]^2])*Cos[4*(c + d*x)] + (22*I)*S
in[2*(c + d*x)] + (15*I)*Sin[4*(c + d*x)] - (416*I)*Sqrt[Cos[c + d*x]^2]*S
in[4*(c + d*x)]))/(840*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3978, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{2}{7}a\int \cos^5(c+dx)(i\tan(c+dx)a+a)^4dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}a\int \frac{(i\tan(c+dx)a+a)^4}{\sec(c+dx)^5}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{2}{7}a\left(\frac{1}{5}a\int \cos^3(c+dx)(i\tan(c+dx)a+a)^3dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}\right)-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}a\left(\frac{1}{5}a\int \frac{(i\tan(c+dx)a+a)^3}{\sec(c+dx)^3}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}\right)-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}$$

$$\Big\downarrow \text{3969}$$

$$\frac{2}{7}a\left(-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}-\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{15d}\right)-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}$$

input
```
Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-1/7*I)*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^5)/d + (2*a*(((-1/15*I)*a*
Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d - ((I/5)*Cos[c + d*x]^5*(a + I*
a*Tan[c + d*x])^4)/d))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 194.97 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55

method	result
risch	$-\frac{i a^5 \mathrm{e}^{7 i (d x+c)}}{28 d}-\frac{i a^5 \mathrm{e}^{5 i (d x+c)}}{10 d}-\frac{i a^5 \mathrm{e}^{3 i (d x+c)}}{12 d}$
derivativedivides	$\frac{i a^5\left(-\frac{\cos(dx+c)^3\sin(dx+c)^4}{7}-\frac{4\cos(dx+c)^3\sin(dx+c)^2}{35}-\frac{8\cos(dx+c)^3}{105}\right)+5a^5\left(-\frac{\sin(dx+c)^3\cos(dx+c)^4}{7}-\frac{3\cos(dx+c)^4\sin(d}{35}\right.}{}$
default	$\frac{i a^5\left(-\frac{\cos(dx+c)^3\sin(dx+c)^4}{7}-\frac{4\cos(dx+c)^3\sin(dx+c)^2}{35}-\frac{8\cos(dx+c)^3}{105}\right)+5a^5\left(-\frac{\sin(dx+c)^3\cos(dx+c)^4}{7}-\frac{3\cos(dx+c)^4\sin(d}{35}\right.}{}$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/28*I/d*a^5*exp(7*I*(d*x+c))-1/10*I/d*a^5*exp(5*I*(d*x+c))-1/12*I/d*a^5*
exp(3*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.48

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{-15i\,a^5e^{(7i\,dx+7i\,c)}-42i\,a^5e^{(5i\,dx+5i\,c)}-35i\,a^5e^{(3i\,dx+3i\,c)}}{420\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/420*(-15*I*a^5*e^(7*I*d*x + 7*I*c) - 42*I*a^5*e^(5*I*d*x + 5*I*c) - 35*I
*a^5*e^(3*I*d*x + 3*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.20

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \begin{cases} \frac{-120ia^5d^2e^{7ic}e^{7idx}-336ia^5d^2e^{5ic}e^{5idx}-280ia^5d^2e^{3ic}e^{3idx}}{3360d^3} & \text{for } d^3 \neq 0 \\ x\left(\frac{a^5e^{7ic}}{4}+\frac{a^5e^{5ic}}{2}+\frac{a^5e^{3ic}}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**5,x)
```

output
```
Piecewise(((-120*I*a**5*d**2*exp(7*I*c)*exp(7*I*d*x) - 336*I*a**5*d**2*exp
(5*I*c)*exp(5*I*d*x) - 280*I*a**5*d**2*exp(3*I*c)*exp(3*I*d*x))/(3360*d**3
), Ne(d**3, 0)), (x*(a**5*exp(7*I*c)/4 + a**5*exp(5*I*c)/2 + a**5*exp(3*I*
c)/4), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(83) = 166.

Time = 0.05 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.85

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx =$$
$$-\frac{75i\,a^5\cos(dx+c)^7+i\left(15\cos(dx+c)^7-42\cos(dx+c)^5+35\cos(dx+c)^3\right)a^5+30i\left(5\cos(dx+\right.}{}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/105*(75*I*a^5*cos(d*x + c)^7 + I*(15*cos(d*x + c)^7 - 42*cos(d*x + c)^5
 + 35*cos(d*x + c)^3)*a^5 + 30*I*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^5
 + 10*(15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^5 + 15
*(5*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*a^5 + 3*(5*sin(d*x + c)^7 - 21*sin(
d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^5)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1697 vs. $2(83) = 166$.

Time = 0.57 (sec) , antiderivative size = 1697, normalized size of antiderivative = 16.80

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/3440640*(7357770*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) +
58862160*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 206017560*a
^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 412035120*a^5*e^(10*I
*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 412035120*a^5*e^(6*I*d*x - 2*I*
c)*log(I*e^(I*d*x + I*c) + 1) + 206017560*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^
(I*d*x + I*c) + 1) + 58862160*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c
) + 1) + 515043900*a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 7357770*a^
5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 7390425*a^5*e^(16*I*d*x + 8*I*c)
*log(I*e^(I*d*x + I*c) - 1) + 59123400*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I
*d*x + I*c) - 1) + 206931900*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c
) - 1) + 413863800*a^5*e^(10*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 4
13863800*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 206931900*a^
5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 59123400*a^5*e^(2*I*d*x
 - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 517329750*a^5*e^(8*I*d*x)*log(I*e^(
I*d*x + I*c) - 1) + 7390425*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) - 1) - 73
57770*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 58862160*a^5*
e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 206017560*a^5*e^(12*I*d
*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 412035120*a^5*e^(10*I*d*x + 2*I*
c)*log(-I*e^(I*d*x + I*c) + 1) - 412035120*a^5*e^(6*I*d*x - 2*I*c)*log(-I*
e^(I*d*x + I*c) + 1) - 206017560*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*...
```

Mupad [B] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.84

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx =$$
$$-\frac{2\,a^5\,\left(105\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 210\mathrm{i}-455\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 350\mathrm{i}+273\,\mathrm{ta}\right.}{105\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 7\mathrm{i}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 35\mathrm{i}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right.}$$

input
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
-(2*a^5*(tan(c/2 + (d*x)/2)*56i + 273*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*
x)/2)^3*350i - 455*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*210i + 105*
tan(c/2 + (d*x)/2)^6 - 23))/(105*d*(7*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)
/2)^2*21i - 35*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*35i + 21*tan(c/
2 + (d*x)/2)^5 - tan(c/2 + (d*x)/2)^6*7i - tan(c/2 + (d*x)/2)^7 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{a^5\left(240\cos(dx+c)\sin(dx+c)^6 i-468\cos(dx+c)\sin(dx+c)^4 i+251\cos(dx+c)\sin(dx+c)^2 i-2\right.}{105d}$$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x)
```

output
```
(a**5*(240*cos(c + d*x)*sin(c + d*x)**6*i - 468*cos(c + d*x)*sin(c + d*x)*
*4*i + 251*cos(c + d*x)*sin(c + d*x)**2*i - 23*cos(c + d*x)*i - 240*sin(c
+ d*x)**7 + 588*sin(c + d*x)**5 - 455*sin(c + d*x)**3 + 105*sin(c + d*x) +
 23*i))/(105*d)
```

3.75 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 141

$$\begin{aligned}\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{ia^5\cos^5(c+dx)}{105d}+\frac{a^5\sin(c+dx)}{21d}\\ &\quad-\frac{2a^5\sin^3(c+dx)}{63d}+\frac{a^5\sin^5(c+dx)}{105d}\\ &\quad-\frac{2ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^2}{63d}\\ &\quad-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}\end{aligned}$$

output
```
-1/105*I*a^5*cos(d*x+c)^5/d+1/21*a^5*sin(d*x+c)/d-2/63*a^5*sin(d*x+c)^3/d+
1/105*a^5*sin(d*x+c)^5/d-2/63*I*a^3*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2/d-2/
9*I*a*cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4/d
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\sec(c+dx)(-i\cos(5(c+dx))+\sin(5(c+dx)))\left(678\cos(c+dx)+475\cos(3(c+dx))+175\cos(5(c\right.}{}$$

input
```
Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*Sec[c + d*x]*((-I)*Cos[5*(c + d*x)] + Sin[5*(c + d*x)])*(678*Cos[c +
d*x] + 475*Cos[3*(c + d*x)] + 175*Cos[5*(c + d*x)] + 1472*Sqrt[Cos[c + d*x
]^2]*Cos[5*(c + d*x)] - (120*I)*Sin[c + d*x] - (260*I)*Sin[3*(c + d*x)] -
(140*I)*Sin[5*(c + d*x)] - (1472*I)*Sqrt[Cos[c + d*x]^2]*Sin[5*(c + d*x)])
)/(5040*d)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3977, 3042, 3977, 3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^9}dx$$
$$\Big\downarrow \text{3977}$$
$$\frac{1}{9}a^2\int \cos^7(c+dx)(i\tan(c+dx)a+a)^3dx-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}a^2\int\frac{(i\tan(c+dx)a+a)^3}{\sec(c+dx)^7}dx-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

$$\Big\downarrow \text{3977}$$

$$\frac{1}{9}a^2\left(\frac{3}{7}a^2\int\cos^5(c+dx)(i\tan(c+dx)a+a)dx-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}a^2\left(\frac{3}{7}a^2\int\frac{i\tan(c+dx)a+a}{\sec(c+dx)^5}dx-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{9}a^2\left(\frac{3}{7}a^2\left(a\int\cos^5(c+dx)dx-\frac{ia\cos^5(c+dx)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}a^2\left(\frac{3}{7}a^2\left(a\int\sin\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{ia\cos^5(c+dx)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{1}{9}a^2\left(\frac{3}{7}a^2\left(-\frac{a\int\left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}-\frac{ia\cos^5(c+dx)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a-}{7c}\right.$$
$$\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}a^2\left(\frac{3}{7}a^2\left(-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{ia\cos^5(c+dx)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\,t}{7d}\right.$$
$$\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

input
```
Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(((-2*I)/9)*a*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^4)/d + (a^2*((3*a^2*((
(-1/5*I)*a*Cos[c + d*x]^5)/d - (a*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 -
Sin[c + d*x]^5/5))/d))/7 - (((2*I)/7)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*
x])^2)/d))/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(124) = 248.

Time = 1.48 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.04

$$ia^5\left(-\frac{\cos(dx+c)^5\sin(dx+c)^4}{9}-\frac{4\cos(dx+c)^5\sin(dx+c)^2}{63}-\frac{8\cos(dx+c)^5}{315}\right)+5a^5\left(-\frac{\sin(dx+c)^3\cos(dx+c)^6}{9}-\frac{\sin(dx+c)\cos(dx+c)^6}{21}\right.$$

input
```
int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x)
```

output
```
1/d*(I*a^5*(-1/9*cos(d*x+c)^5*sin(d*x+c)^4-4/63*cos(d*x+c)^5*sin(d*x+c)^2-
8/315*cos(d*x+c)^5)+5*a^5*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*
cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-10*I*a^
5*(-1/9*cos(d*x+c)^7*sin(d*x+c)^2-2/63*cos(d*x+c)^7)-10*a^5*(-1/9*cos(d*x+
c)^8*sin(d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)
*sin(d*x+c))-5/9*I*a^5*cos(d*x+c)^9+1/9*a^5*(128/35+cos(d*x+c)^8+8/7*cos(d
*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.54

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{-35i\,a^5e^{(9i\,dx+9i\,c)}-180i\,a^5e^{(7i\,dx+7i\,c)}-378i\,a^5e^{(5i\,dx+5i\,c)}-420i\,a^5e^{(3i\,dx+3i\,c)}-315i\,a^5e^{(i\,dx+i\,c)}}{5040\,d}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/5040*(-35*I*a^5*e^(9*I*d*x + 9*I*c) - 180*I*a^5*e^(7*I*d*x + 7*I*c) - 37
8*I*a^5*e^(5*I*d*x + 5*I*c) - 420*I*a^5*e^(3*I*d*x + 3*I*c) - 315*I*a^5*e^
(I*d*x + I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.36

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \begin{cases} \frac{-215040ia^5d^4e^{9ic}e^{9idx}-1105920ia^5d^4e^{7ic}e^{7idx}-2322432ia^5d^4e^{5ic}e^{5idx}-2580480ia^5d^4e^{3ic}e^{3idx}-1935360ia^5d^4e^{ic}e^{idx}}{30965760d^5} & \text{for } d^5 \neq 0 \\ x\left(\frac{a^5e^{9ic}}{16}+\frac{a^5e^{7ic}}{4}+\frac{3a^5e^{5ic}}{8}+\frac{a^5e^{3ic}}{4}+\frac{a^5e^{ic}}{16}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**5,x)
```

output
```
Piecewise(((-215040*I*a**5*d**4*exp(9*I*c)*exp(9*I*d*x) - 1105920*I*a**5*d
**4*exp(7*I*c)*exp(7*I*d*x) - 2322432*I*a**5*d**4*exp(5*I*c)*exp(5*I*d*x)
- 2580480*I*a**5*d**4*exp(3*I*c)*exp(3*I*d*x) - 1935360*I*a**5*d**4*exp(I*
c)*exp(I*d*x))/(30965760*d**5), Ne(d**5, 0)), (x*(a**5*exp(9*I*c)/16 + a**
5*exp(7*I*c)/4 + 3*a**5*exp(5*I*c)/8 + a**5*exp(3*I*c)/4 + a**5*exp(I*c)/1
6), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.54

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx =$$
$$-\frac{175i\,a^5\cos{(dx+c)}^9+i\left(35\cos{(dx+c)}^9-90\cos{(dx+c)}^7+63\cos{(dx+c)}^5\right)a^5+50i\left(7\cos{(dx+\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/315*(175*I*a^5*cos(d*x + c)^9 + I*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^
7 + 63*cos(d*x + c)^5)*a^5 + 50*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^
5 - 5*(35*sin(d*x + c)^9 - 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a^5 - 10
*(35*sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*
x + c)^3)*a^5 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7 + 378*sin(d*x + c)
^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^5)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1725 vs. $2(119) = 238$.

Time = 0.64 (sec) , antiderivative size = 1725, normalized size of antiderivative = 12.23

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/41287680*(69853455*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1)
+ 558827640*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 19558967
40*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3911793480*a^5*e^
(10*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3911793480*a^5*e^(6*I*d*x
- 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1955896740*a^5*e^(4*I*d*x - 4*I*c)*l
og(I*e^(I*d*x + I*c) + 1) + 558827640*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d
*x + I*c) + 1) + 4889741850*a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 6
9853455*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 70703325*a^5*e^(16*I*d
*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 565626600*a^5*e^(14*I*d*x + 6*I*c
)*log(I*e^(I*d*x + I*c) - 1) + 1979693100*a^5*e^(12*I*d*x + 4*I*c)*log(I*e
^(I*d*x + I*c) - 1) + 3959386200*a^5*e^(10*I*d*x + 2*I*c)*log(I*e^(I*d*x +
 I*c) - 1) + 3959386200*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1)
 + 1979693100*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5656266
00*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 4949232750*a^5*e^(
8*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 70703325*a^5*e^(-8*I*c)*log(I*e^(I*d
*x + I*c) - 1) - 69853455*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 558827640*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 19
55896740*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3911793480
*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3911793480*a^5*e^(
6*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1955896740*a^5*e^(4*I*d*...
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.56

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= -\frac{a^5\left(\frac{e^{c\,1i+d\,x\,1i}\,1i}{16}+\frac{e^{c\,3i+d\,x\,3i}\,1i}{12}+\frac{e^{c\,5i+d\,x\,5i}\,3i}{40}+\frac{e^{c\,7i+d\,x\,7i}\,1i}{28}+\frac{e^{c\,9i+d\,x\,9i}\,1i}{144}\right)}{d}$$

input
```
int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
-(a^5*((exp(c*1i + d*x*1i)*1i)/16 + (exp(c*3i + d*x*3i)*1i)/12 + (exp(c*5i
 + d*x*5i)*3i)/40 + (exp(c*7i + d*x*7i)*1i)/28 + (exp(c*9i + d*x*9i)*1i)/1
44))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.97

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\left(-560\cos(dx+c)\sin(dx+c)^8 i+1700\cos(dx+c)\sin(dx+c)^6 i-1803\cos(dx+c)\sin(dx+c)^4 i\right.}{}$$

input
```
int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x)
```

output
```
(a**5*( - 560*cos(c + d*x)*sin(c + d*x)**8*i + 1700*cos(c + d*x)*sin(c + d
*x)**6*i - 1803*cos(c + d*x)*sin(c + d*x)**4*i + 746*cos(c + d*x)*sin(c +
d*x)**2*i - 83*cos(c + d*x)*i + 560*sin(c + d*x)**9 - 1980*sin(c + d*x)**7
 + 2583*sin(c + d*x)**5 - 1470*sin(c + d*x)**3 + 315*sin(c + d*x) + 83*i))
/(315*d)
```

3.76 $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 159

$$\begin{aligned}\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx = &-\frac{5ia^5\cos^7(c+dx)}{231d}\\ &+\frac{5a^5\sin(c+dx)}{33d}-\frac{5a^5\sin^3(c+dx)}{33d}\\ &+\frac{a^5\sin^5(c+dx)}{11d}-\frac{5a^5\sin^7(c+dx)}{231d}\\ &-\frac{2ia^3\cos^9(c+dx)(a+ia\tan(c+dx))^2}{33d}\\ &-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}\end{aligned}$$

output
```
-5/231*I*a^5*cos(d*x+c)^7/d+5/33*a^5*sin(d*x+c)/d-5/33*a^5*sin(d*x+c)^3/d+
1/11*a^5*sin(d*x+c)^5/d-5/231*a^5*sin(d*x+c)^7/d-2/33*I*a^3*cos(d*x+c)^9*(
a+I*a*tan(d*x+c))^2/d-2/11*I*a*cos(d*x+c)^11*(a+I*a*tan(d*x+c))^4/d
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.07

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{ia^5\sec(c+dx)(\cos(5(c+dx))+i\sin(5(c+dx)))\left(-1749\cos(c+dx)-1595\cos(3(c+dx))-665\cos(\right.}{}$$

input
```
Integrate[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((I/14784)*a^5*Sec[c + d*x]*(Cos[5*(c + d*x)] + I*Sin[5*(c + d*x)])*(-1749
*Cos[c + d*x] - 1595*Cos[3*(c + d*x)] - 665*Cos[5*(c + d*x)] - 2816*Sqrt[C
os[c + d*x]^2]*Cos[5*(c + d*x)] + 105*Cos[7*(c + d*x)] + (330*I)*Sin[c + d
*x] + (946*I)*Sin[3*(c + d*x)] + (490*I)*Sin[5*(c + d*x)] + (2816*I)*Sqrt[
Cos[c + d*x]^2]*Sin[5*(c + d*x)] - (126*I)*Sin[7*(c + d*x)]))/d
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3977, 3042, 3977, 3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^{11}}dx$$
$$\Big\downarrow \text{3977}$$
$$\frac{3}{11}a^2\int \cos^9(c+dx)(i\tan(c+dx)a+a)^3dx-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{3}{11}a^2\int\frac{(i\tan(c+dx)a+a)^3}{\sec(c+dx)^9}dx-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$

$$\Big\downarrow \text{3977}$$

$$\frac{3}{11}a^2\left(\frac{5}{9}a^2\int\cos^7(c+dx)(i\tan(c+dx)a+a)dx-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}\right)-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{11}a^2\left(\frac{5}{9}a^2\int\frac{i\tan(c+dx)a+a}{\sec(c+dx)^7}dx-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}\right)-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{3}{11}a^2\left(\frac{5}{9}a^2\left(a\int\cos^7(c+dx)dx-\frac{ia\cos^7(c+dx)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}\right)-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{11}a^2\left(\frac{5}{9}a^2\left(a\int\sin\left(c+dx+\frac{\pi}{2}\right)^7dx-\frac{ia\cos^7(c+dx)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}\right)-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{3}{11}a^2\left(\frac{5}{9}a^2\left(-\frac{a\int\left(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}-\frac{ia\cos^7(c+dx)}{7d}\right)-\right.$$
$$\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{11}a^2\left(\frac{5}{9}a^2\left(-\frac{a\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{ia\cos^7(c+dx)}{7d}\right)-\frac{2ia\cos^9(c\right.$$
$$\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$

input
```
Int[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(((-2*I)/11)*a*Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^4)/d + (3*a^2*((5*a^
2*(((-1/7*I)*a*Cos[c + d*x]^7)/d - (a*(-Sin[c + d*x] + Sin[c + d*x]^3 - (3
*Sin[c + d*x]^5)/5 + Sin[c + d*x]^7/7))/d))/9 - (((2*I)/9)*a*Cos[c + d*x]^
9*(a + I*a*Tan[c + d*x])^2)/d))/11
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(140) = 280$.

Time = 0.69 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.99

$$ia^5\left(-\frac{\cos(dx+c)^7\sin(dx+c)^4}{11}-\frac{4\cos(dx+c)^7\sin(dx+c)^2}{99}-\frac{8\cos(dx+c)^7}{693}\right)+5a^5\left(-\frac{\sin(dx+c)^3\cos(dx+c)^8}{11}-\frac{\cos(dx+c)^8\sin(dx+c)}{33}\right.$$

input
```
int(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x)
```

output
```
1/d*(I*a^5*(-1/11*cos(d*x+c)^7*sin(d*x+c)^4-4/99*cos(d*x+c)^7*sin(d*x+c)^2
-8/693*cos(d*x+c)^7)+5*a^5*(-1/11*sin(d*x+c)^3*cos(d*x+c)^8-1/33*cos(d*x+c
)^8*sin(d*x+c)+1/231*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)
*sin(d*x+c))-10*I*a^5*(-1/11*cos(d*x+c)^9*sin(d*x+c)^2-2/99*cos(d*x+c)^9)-
10*a^5*(-1/11*sin(d*x+c)*cos(d*x+c)^10+1/99*(128/35+cos(d*x+c)^8+8/7*cos(d
*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))-5/11*I*a^5*cos(
d*x+c)^11+1/11*a^5*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)^8+80/63*cos(d*x+c
)^6+32/21*cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.65

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{\left(-21i\,a^5e^{(12i\,dx+12i\,c)}-154i\,a^5e^{(10i\,dx+10i\,c)}-495i\,a^5e^{(8i\,dx+8i\,c)}-924i\,a^5e^{(6i\,dx+6i\,c)}-1155i\,a^5e^{(4i\,dx+4i\,c)}-\right.}{14784\,d}$$

input
```
integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/14784*(-21*I*a^5*e^(12*I*d*x + 12*I*c) - 154*I*a^5*e^(10*I*d*x + 10*I*c)
 - 495*I*a^5*e^(8*I*d*x + 8*I*c) - 924*I*a^5*e^(6*I*d*x + 6*I*c) - 1155*I*
a^5*e^(4*I*d*x + 4*I*c) - 1386*I*a^5*e^(2*I*d*x + 2*I*c) + 231*I*a^5)*e^(-
I*d*x - I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.67

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \begin{cases} \frac{(-90194313216ia^5d^6e^{12ic}e^{11idx}-661424963584ia^5d^6e^{10ic}e^{9idx}-2126008811520ia^5d^6e^{8ic}e^{7idx}-3968549781504ia^5d^6e^{6ic}e^{5idx}-496068722}{63496796504064d^7} \\ \frac{x(a^5e^{12ic}+6a^5e^{10ic}+15a^5e^{8ic}+20a^5e^{6ic}+15a^5e^{4ic}+6a^5e^{2ic}+a^5)e^{-ic}}{64} \end{cases}$$

input
```
integrate(cos(d*x+c)**11*(a+I*a*tan(d*x+c))**5,x)
```

output
```
Piecewise(((-90194313216*I*a**5*d**6*exp(12*I*c)*exp(11*I*d*x) - 661424963
584*I*a**5*d**6*exp(10*I*c)*exp(9*I*d*x) - 2126008811520*I*a**5*d**6*exp(8
*I*c)*exp(7*I*d*x) - 3968549781504*I*a**5*d**6*exp(6*I*c)*exp(5*I*d*x) - 4
960687226880*I*a**5*d**6*exp(4*I*c)*exp(3*I*d*x) - 5952824672256*I*a**5*d*
*6*exp(2*I*c)*exp(I*d*x) + 992137445376*I*a**5*d**6*exp(-I*d*x))*exp(-I*c)
/(63496796504064*d**7), Ne(d**7*exp(I*c), 0)), (x*(a**5*exp(12*I*c) + 6*a*
*5*exp(10*I*c) + 15*a**5*exp(8*I*c) + 20*a**5*exp(6*I*c) + 15*a**5*exp(4*I
*c) + 6*a**5*exp(2*I*c) + a**5)*exp(-I*c)/64, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.55

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx =$$
$$-\frac{315i\,a^5\cos(dx+c)^{11}+i\left(63\cos(dx+c)^{11}-154\cos(dx+c)^9+99\cos(dx+c)^7\right)a^5+70i\left(9\cos(d\right.}{}$$

input
```
integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/693*(315*I*a^5*cos(d*x + c)^11 + I*(63*cos(d*x + c)^11 - 154*cos(d*x +
c)^9 + 99*cos(d*x + c)^7)*a^5 + 70*I*(9*cos(d*x + c)^11 - 11*cos(d*x + c)^
9)*a^5 + 2*(315*sin(d*x + c)^11 - 1540*sin(d*x + c)^9 + 2970*sin(d*x + c)^
7 - 2772*sin(d*x + c)^5 + 1155*sin(d*x + c)^3)*a^5 + 3*(105*sin(d*x + c)^1
1 - 385*sin(d*x + c)^9 + 495*sin(d*x + c)^7 - 231*sin(d*x + c)^5)*a^5 + (6
3*sin(d*x + c)^11 - 385*sin(d*x + c)^9 + 990*sin(d*x + c)^7 - 1386*sin(d*x
 + c)^5 + 1155*sin(d*x + c)^3 - 693*sin(d*x + c))*a^5)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1807 vs. $2(135) = 270$.

Time = 0.67 (sec) , antiderivative size = 1807, normalized size of antiderivative = 11.36

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/121110528*(168111405*a^5*e^(17*I*d*x + 9*I*c)*log(I*e^(I*d*x + I*c) + 1
) + 1344891240*a^5*e^(15*I*d*x + 7*I*c)*log(I*e^(I*d*x + I*c) + 1) + 47071
19340*a^5*e^(13*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 9414238680*a^5
*e^(11*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 11767798350*a^5*e^(9*I*
d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 9414238680*a^5*e^(7*I*d*x - I*c)*l
og(I*e^(I*d*x + I*c) + 1) + 4707119340*a^5*e^(5*I*d*x - 3*I*c)*log(I*e^(I*
d*x + I*c) + 1) + 1344891240*a^5*e^(3*I*d*x - 5*I*c)*log(I*e^(I*d*x + I*c)
 + 1) + 168111405*a^5*e^(I*d*x - 7*I*c)*log(I*e^(I*d*x + I*c) + 1) + 17025
1620*a^5*e^(17*I*d*x + 9*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1362012960*a^5*
e^(15*I*d*x + 7*I*c)*log(I*e^(I*d*x + I*c) - 1) + 4767045360*a^5*e^(13*I*d
*x + 5*I*c)*log(I*e^(I*d*x + I*c) - 1) + 9534090720*a^5*e^(11*I*d*x + 3*I*
c)*log(I*e^(I*d*x + I*c) - 1) + 11917613400*a^5*e^(9*I*d*x + I*c)*log(I*e^
(I*d*x + I*c) - 1) + 9534090720*a^5*e^(7*I*d*x - I*c)*log(I*e^(I*d*x + I*c
) - 1) + 4767045360*a^5*e^(5*I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1
362012960*a^5*e^(3*I*d*x - 5*I*c)*log(I*e^(I*d*x + I*c) - 1) + 170251620*a
^5*e^(I*d*x - 7*I*c)*log(I*e^(I*d*x + I*c) - 1) - 168111405*a^5*e^(17*I*d*
x + 9*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1344891240*a^5*e^(15*I*d*x + 7*I*
c)*log(-I*e^(I*d*x + I*c) + 1) - 4707119340*a^5*e^(13*I*d*x + 5*I*c)*log(-
I*e^(I*d*x + I*c) + 1) - 9414238680*a^5*e^(11*I*d*x + 3*I*c)*log(-I*e^(I*d
*x + I*c) + 1) - 11767798350*a^5*e^(9*I*d*x + I*c)*log(-I*e^(I*d*x + I*...
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\left(\frac{5\,\sin(3\,c+3\,d\,x)}{64}-\frac{\cos(5\,c+5\,d\,x)\,1\mathrm{i}}{16}-\frac{\cos(7\,c+7\,d\,x)\,15\mathrm{i}}{448}-\frac{\cos(9\,c+9\,d\,x)\,1\mathrm{i}}{96}-\frac{\cos(11\,c+11\,d\,x)\,1\mathrm{i}}{704}-\frac{\cos(3\,c+3\,d\,x)\,5\mathrm{i}}{64}+\frac{\sin(5\,c+5}{16}\right.}{d}$$

input
```
int(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
(a^5*((5*sin(3*c + 3*d*x))/64 - (cos(5*c + 5*d*x)*1i)/16 - (cos(7*c + 7*d*
x)*15i)/448 - (cos(9*c + 9*d*x)*1i)/96 - (cos(11*c + 11*d*x)*1i)/704 - (co
s(3*c + 3*d*x)*5i)/64 + sin(5*c + 5*d*x)/16 + (15*sin(7*c + 7*d*x))/448 +
sin(9*c + 9*d*x)/96 + sin(11*c + 11*d*x)/704 + (24^(1/2)*cos(c - atanh(7/5
)*1i + d*x))/64))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.03

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\big(336\cos(dx+c)\sin(dx+c)^{10}i - 1372\cos(dx+c)\sin(dx+c)^8 i + 2161\cos(dx+c)\sin(dx+c)^6 i -$$

input
```
int(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x)
```

output
```
(a**5*(336*cos(c + d*x)*sin(c + d*x)**10*i - 1372*cos(c + d*x)*sin(c + d*x
)**8*i + 2161*cos(c + d*x)*sin(c + d*x)**6*i - 1611*cos(c + d*x)*sin(c + d
*x)**4*i + 547*cos(c + d*x)*sin(c + d*x)**2*i - 61*cos(c + d*x)*i - 336*si
n(c + d*x)**11 + 1540*sin(c + d*x)**9 - 2805*sin(c + d*x)**7 + 2541*sin(c
+ d*x)**5 - 1155*sin(c + d*x)**3 + 231*sin(c + d*x) + 61*i))/(231*d)
```

3.77 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{2i(a+ia\tan(c+dx))^{12}}{3a^4d}+\frac{12i(a+ia\tan(c+dx))^{13}}{13a^5d}-\frac{3i(a+ia\tan(c+dx))^{14}}{7a^6d}+\frac{i(a+ia\tan(c+dx))^{15}}{15a^7d}$$

output
```
-2/3*I*(a+I*a*tan(d*x+c))^12/a^4/d+12/13*I*(a+I*a*tan(d*x+c))^13/a^5/d-3/7
*I*(a+I*a*tan(d*x+c))^14/a^6/d+1/15*I*(a+I*a*tan(d*x+c))^15/a^7/d
```

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.51

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8(-i+\tan(c+dx))^{12}\left(-144i-363\tan(c+dx)+312i\tan^2(c+dx)+91\tan^3(c+dx)\right)}{1365d}$$

input
```
Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*(-I + Tan[c + d*x])^12*(-144*I - 363*Tan[c + d*x] + (312*I)*Tan[c + d
*x]^2 + 91*Tan[c + d*x]^3))/(1365*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^8(a+ia\tan(c+dx))^8dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{11}d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(-(i\tan(c+dx)a+a)^{14}+6a(i\tan(c+dx)a+a)^{13}-12a^2(i\tan(c+dx)a+a)^{12}+8a^3(i\tan(c+dx)a+a\right.}{a^7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{2}{3}a^3(a+ia\tan(c+dx))^{12}-\frac{12}{13}a^2(a+ia\tan(c+dx))^{13}-\frac{1}{15}(a+ia\tan(c+dx))^{15}+\frac{3}{7}a(a+ia\tan(c+dx))\right.}{a^7d}$$

input
```
Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*((2*a^3*(a + I*a*Tan[c + d*x])^12)/3 - (12*a^2*(a + I*a*Tan[c + d*x]
)^13)/13 + (3*a*(a + I*a*Tan[c + d*x])^14)/7 - (a + I*a*Tan[c + d*x])^15/1
5))/(a^7*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(93) = 186$.

Time = 1.24 (sec) , antiderivative size = 611, normalized size of antiderivative = 5.61

$$a^8\left(\frac{\sin(dx+c)^9}{15\cos(dx+c)^{15}}+\frac{2\sin(dx+c)^9}{65\cos(dx+c)^{13}}+\frac{8\sin(dx+c)^9}{715\cos(dx+c)^{11}}+\frac{16\sin(dx+c)^9}{6435\cos(dx+c)^9}\right)-8ia^8\left(\frac{\sin(dx+c)^8}{14\cos(dx+c)^{14}}+\frac{\sin(dx+c)^8}{28\cos(dx+c)^{12}}+\frac{\sin(dx}{70\cos(dx}\right.$$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(1/15*sin(d*x+c)^9/cos(d*x+c)^15+2/65*sin(d*x+c)^9/cos(d*x+c)^13+
8/715*sin(d*x+c)^9/cos(d*x+c)^11+16/6435*sin(d*x+c)^9/cos(d*x+c)^9)-8*I*a^
8*(1/14*sin(d*x+c)^8/cos(d*x+c)^14+1/28*sin(d*x+c)^8/cos(d*x+c)^12+1/70*si
n(d*x+c)^8/cos(d*x+c)^10+1/280*sin(d*x+c)^8/cos(d*x+c)^8)-28*a^8*(1/13*sin
(d*x+c)^7/cos(d*x+c)^13+6/143*sin(d*x+c)^7/cos(d*x+c)^11+8/429*sin(d*x+c)^
7/cos(d*x+c)^9+16/3003*sin(d*x+c)^7/cos(d*x+c)^7)+56*I*a^8*(1/12*sin(d*x+c
)^6/cos(d*x+c)^12+1/20*sin(d*x+c)^6/cos(d*x+c)^10+1/40*sin(d*x+c)^6/cos(d*
x+c)^8+1/120*sin(d*x+c)^6/cos(d*x+c)^6)+70*a^8*(1/11*sin(d*x+c)^5/cos(d*x+
c)^11+2/33*sin(d*x+c)^5/cos(d*x+c)^9+8/231*sin(d*x+c)^5/cos(d*x+c)^7+16/11
55*sin(d*x+c)^5/cos(d*x+c)^5)+I*a^8/cos(d*x+c)^8-28*a^8*(1/9*sin(d*x+c)^3/
cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^
5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)-56*I*a^8*(1/10*sin(d*x+c)^4/cos(d*x+c)
^10+3/40*sin(d*x+c)^4/cos(d*x+c)^8+1/20*sin(d*x+c)^4/cos(d*x+c)^6+1/40*sin
(d*x+c)^4/cos(d*x+c)^4)-a^8*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/3
5*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(85) = 170$.

Time = 0.09 (sec) , antiderivative size = 345, normalized size of antiderivative = 3.17

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{8192\left(-1365i\,a^8e^{(22i\,dx+22i\,c)} - 3003i\,a^8e^{(20i\,dx+20i\,c)} - 5005i\,a^8e^{(18i\,dx+18i\,c)} - 6\right.}{1365\left(de^{(30i\,dx+30i\,c)} + 15\,de^{(28i\,dx+28i\,c)} + 105\,de^{(26i\,dx+26i\,c)} + 455\,de^{(24i\,dx+24i\,c)} + 1365\,de^{(22i\,dx+22i\,c)} + 3\right.}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-8192/1365*(-1365*I*a^8*e^(22*I*d*x + 22*I*c) - 3003*I*a^8*e^(20*I*d*x + 2
0*I*c) - 5005*I*a^8*e^(18*I*d*x + 18*I*c) - 6435*I*a^8*e^(16*I*d*x + 16*I*
c) - 6435*I*a^8*e^(14*I*d*x + 14*I*c) - 5005*I*a^8*e^(12*I*d*x + 12*I*c) -
 3003*I*a^8*e^(10*I*d*x + 10*I*c) - 1365*I*a^8*e^(8*I*d*x + 8*I*c) - 455*I
*a^8*e^(6*I*d*x + 6*I*c) - 105*I*a^8*e^(4*I*d*x + 4*I*c) - 15*I*a^8*e^(2*I
*d*x + 2*I*c) - I*a^8)/(d*e^(30*I*d*x + 30*I*c) + 15*d*e^(28*I*d*x + 28*I*
c) + 105*d*e^(26*I*d*x + 26*I*c) + 455*d*e^(24*I*d*x + 24*I*c) + 1365*d*e^
(22*I*d*x + 22*I*c) + 3003*d*e^(20*I*d*x + 20*I*c) + 5005*d*e^(18*I*d*x +
18*I*c) + 6435*d*e^(16*I*d*x + 16*I*c) + 6435*d*e^(14*I*d*x + 14*I*c) + 50
05*d*e^(12*I*d*x + 12*I*c) + 3003*d*e^(10*I*d*x + 10*I*c) + 1365*d*e^(8*I*
d*x + 8*I*c) + 455*d*e^(6*I*d*x + 6*I*c) + 105*d*e^(4*I*d*x + 4*I*c) + 15*
d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx = a^8\left(\int \left(-28\tan^2(c+dx)\sec^8(c+dx)\right)dx + \int 70\tan^4(c+dx)\sec^8(c+dx)\,dx + \int \left(-28\tan^6(c+dx)\sec^8(c+dx)\right)dx + \int \tan^8(c+dx)\sec^8(c+dx)\,dx + \int 8i\tan(c+dx)\sec^8(c+dx)\,dx + \int \left(-56i\tan^3(c+dx)\sec^8(c+dx)\right)dx + \int 56i\tan^5(c+dx)\sec^8(c+dx)\,dx + \int \left(-8i\tan^7(c+dx)\sec^8(c+dx)\right)dx + \int \sec^8(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**8,x)
```

output
```
a**8*(Integral(-28*tan(c + d*x)**2*sec(c + d*x)**8, x) + Integral(70*tan(c
 + d*x)**4*sec(c + d*x)**8, x) + Integral(-28*tan(c + d*x)**6*sec(c + d*x)
**8, x) + Integral(tan(c + d*x)**8*sec(c + d*x)**8, x) + Integral(8*I*tan(
c + d*x)*sec(c + d*x)**8, x) + Integral(-56*I*tan(c + d*x)**3*sec(c + d*x)
**8, x) + Integral(56*I*tan(c + d*x)**5*sec(c + d*x)**8, x) + Integral(-8*
I*tan(c + d*x)**7*sec(c + d*x)**8, x) + Integral(sec(c + d*x)**8, x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(85) = 170$.

Time = 0.05 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.71

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{91\,a^8\tan\left(dx+c\right)^{15} - 780i\,a^8\tan\left(dx+c\right)^{14} - 2625\,a^8\tan\left(dx+c\right)^{13} + 3640i\,a^8\tan\left(dx+c\right)^{12} - 1365\,a^8}{}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/1365*(91*a^8*tan(d*x + c)^15 - 780*I*a^8*tan(d*x + c)^14 - 2625*a^8*tan(
d*x + c)^13 + 3640*I*a^8*tan(d*x + c)^12 - 1365*a^8*tan(d*x + c)^11 + 1201
2*I*a^8*tan(d*x + c)^10 + 15015*a^8*tan(d*x + c)^9 + 19305*a^8*tan(d*x + c
)^7 - 20020*I*a^8*tan(d*x + c)^6 - 3003*a^8*tan(d*x + c)^5 - 10920*I*a^8*t
an(d*x + c)^4 - 11375*a^8*tan(d*x + c)^3 + 5460*I*a^8*tan(d*x + c)^2 + 136
5*a^8*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(85) = 170$.

Time = 0.34 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.71

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{91\,a^8\tan\left(dx+c\right)^{15} - 780i\,a^8\tan\left(dx+c\right)^{14} - 2625\,a^8\tan\left(dx+c\right)^{13} + 3640i\,a^8\tan\left(dx+c\right)^{12} - 1365\,a^8}{}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/1365*(91*a^8*tan(d*x + c)^15 - 780*I*a^8*tan(d*x + c)^14 - 2625*a^8*tan(
d*x + c)^13 + 3640*I*a^8*tan(d*x + c)^12 - 1365*a^8*tan(d*x + c)^11 + 1201
2*I*a^8*tan(d*x + c)^10 + 15015*a^8*tan(d*x + c)^9 + 19305*a^8*tan(d*x + c
)^7 - 20020*I*a^8*tan(d*x + c)^6 - 3003*a^8*tan(d*x + c)^5 - 10920*I*a^8*t
an(d*x + c)^4 - 11375*a^8*tan(d*x + c)^3 + 5460*I*a^8*tan(d*x + c)^2 + 136
5*a^8*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.40

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(\frac{\sin(9\,c+9\,d\,x)}{12}+\frac{\sin(11\,c+11\,d\,x)}{52}+\frac{\sin(13\,c+13\,d\,x)}{364}+\frac{\sin(15\,c+15\,d\,x)}{5460}+\frac{\cos(c+d\,x)\,297\mathrm{i}}{7168}+\frac{\cos(3\,c+3\,d\,x)\,33\mathrm{i}}{1024}+\frac{\cos(5\,c+5\,d\,x)\,9}{5120}\right.}{d\cos\left(c+d\,x\right)^{15}}$$

input
```
int((a + a*tan(c + d*x)*1i)^8/cos(c + d*x)^8,x)
```

output
```
(a^8*((cos(c + d*x)*297i)/7168 + (cos(3*c + 3*d*x)*33i)/1024 + (cos(5*c +
5*d*x)*99i)/5120 + (cos(7*c + 7*d*x)*9i)/1024 - (cos(9*c + 9*d*x)*247i)/30
72 - (cos(11*c + 11*d*x)*19i)/1024 - (cos(13*c + 13*d*x)*19i)/7168 - (cos(
15*c + 15*d*x)*19i)/107520 + sin(9*c + 9*d*x)/12 + sin(11*c + 11*d*x)/52 +
 sin(13*c + 13*d*x)/364 + sin(15*c + 15*d*x)/5460))/(d*cos(c + d*x)^15)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.60

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{\sin\left(dx+c\right)a^8\left(-3952\cos\left(dx+c\right)\sin\left(dx+c\right)^{13}i+27664\cos\left(dx+c\right)\sin\left(dx+c\right)^{11}i-82992\cos\left(dx+\right.\right.}{}$$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(sin(c + d*x)*a**8*( - 3952*cos(c + d*x)*sin(c + d*x)**13*i + 27664*cos(c
+ d*x)*sin(c + d*x)**11*i - 82992*cos(c + d*x)*sin(c + d*x)**9*i + 138320*
cos(c + d*x)*sin(c + d*x)**7*i - 116480*cos(c + d*x)*sin(c + d*x)**5*i + 4
3680*cos(c + d*x)*sin(c + d*x)**3*i - 5460*cos(c + d*x)*sin(c + d*x)*i + 4
096*sin(c + d*x)**14 - 30720*sin(c + d*x)**12 + 99840*sin(c + d*x)**10 - 1
83040*sin(c + d*x)**8 + 184080*sin(c + d*x)**6 - 93912*sin(c + d*x)**4 + 2
0930*sin(c + d*x)**2 - 1365))/(1365*cos(c + d*x)*d*(sin(c + d*x)**14 - 7*s
in(c + d*x)**12 + 21*sin(c + d*x)**10 - 35*sin(c + d*x)**8 + 35*sin(c + d*
x)**6 - 21*sin(c + d*x)**4 + 7*sin(c + d*x)**2 - 1))
```

3.78 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{4i(a+ia\tan(c+dx))^{11}}{11a^3d} + \frac{i(a+ia\tan(c+dx))^{12}}{3a^4d} - \frac{i(a+ia\tan(c+dx))^{13}}{13a^5d}$$

output
```
-4/11*I*(a+I*a*tan(d*x+c))^11/a^3/d+1/3*I*(a+I*a*tan(d*x+c))^12/a^4/d-1/13
*I*(a+I*a*tan(d*x+c))^13/a^5/d
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.54

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8(-i+\tan(c+dx))^{11}\left(-46+77i\tan(c+dx)+33\tan^2(c+dx)\right)}{429d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*(-I + Tan[c + d*x])^11*(-46 + (77*I)*Tan[c + d*x] + 33*Tan[c + d*x]^2
))/(429*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(c+dx)^6(a+ia\tan(c+dx))^8dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{10}d(ia\tan(c+dx))}{a^5d}$$
$$\Big\downarrow \text{49}$$
$$-\frac{i\int\left((i\tan(c+dx)a+a)^{12}-4a(i\tan(c+dx)a+a)^{11}+4a^2(i\tan(c+dx)a+a)^{10}\right)d(ia\tan(c+dx))}{a^5d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{i\left(\frac{4}{11}a^2(a+ia\tan(c+dx))^{11}+\frac{1}{13}(a+ia\tan(c+dx))^{13}-\frac{1}{3}a(a+ia\tan(c+dx))^{12}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*((4*a^2*(a + I*a*Tan[c + d*x])^11)/11 - (a*(a + I*a*Tan[c + d*x])^12
)/3 + (a + I*a*Tan[c + d*x])^13/13))/(a^5*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(70) = 140$.

Time = 1.23 (sec) , antiderivative size = 475, normalized size of antiderivative = 5.79

$$\frac{a^8\left(\frac{\sin(dx+c)^9}{13\cos(dx+c)^{13}}+\frac{4\sin(dx+c)^9}{143\cos(dx+c)^{11}}+\frac{8\sin(dx+c)^9}{1287\cos(dx+c)^9}\right)+56ia^8\left(\frac{\sin(dx+c)^6}{10\cos(dx+c)^{10}}+\frac{\sin(dx+c)^6}{20\cos(dx+c)^8}+\frac{\sin(dx+c)^6}{60\cos(dx+c)^6}\right)-28a^8(}{}$$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(1/13*sin(d*x+c)^9/cos(d*x+c)^13+4/143*sin(d*x+c)^9/cos(d*x+c)^11
+8/1287*sin(d*x+c)^9/cos(d*x+c)^9)+56*I*a^8*(1/10*sin(d*x+c)^6/cos(d*x+c)^
10+1/20*sin(d*x+c)^6/cos(d*x+c)^8+1/60*sin(d*x+c)^6/cos(d*x+c)^6)-28*a^8*(
1/11*sin(d*x+c)^7/cos(d*x+c)^11+4/99*sin(d*x+c)^7/cos(d*x+c)^9+8/693*sin(d
*x+c)^7/cos(d*x+c)^7)-8*I*a^8*(1/12*sin(d*x+c)^8/cos(d*x+c)^12+1/30*sin(d*
x+c)^8/cos(d*x+c)^10+1/120*sin(d*x+c)^8/cos(d*x+c)^8)+70*a^8*(1/9*sin(d*x+
c)^5/cos(d*x+c)^9+4/63*sin(d*x+c)^5/cos(d*x+c)^7+8/315*sin(d*x+c)^5/cos(d*
x+c)^5)-56*I*a^8*(1/8*sin(d*x+c)^4/cos(d*x+c)^8+1/12*sin(d*x+c)^4/cos(d*x+
c)^6+1/24*sin(d*x+c)^4/cos(d*x+c)^4)-28*a^8*(1/7*sin(d*x+c)^3/cos(d*x+c)^7
+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+4/3*I*a^8
/cos(d*x+c)^6-a^8*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(64) = 128$.

Time = 0.09 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.74

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{4096\left(-286i\,a^8e^{(20i\,dx+20i\,c)} - 715i\,a^8e^{(18i\,dx+18i\,c)} - 1287i\,a^8e^{(16i\,dx+16i\,c)} - 1716i\,a^8e^{(14i\,c}\right.}{429\left(de^{(26i\,dx+26i\,c)} + 13\,de^{(24i\,dx+24i\,c)} + 78\,de^{(22i\,dx+22i\,c)} + 286\,de^{(20i\,dx+20i\,c)} + 715\,de^{(18i\,dx+18i\,c)} + 1287\right.}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-4096/429*(-286*I*a^8*e^(20*I*d*x + 20*I*c) - 715*I*a^8*e^(18*I*d*x + 18*I
*c) - 1287*I*a^8*e^(16*I*d*x + 16*I*c) - 1716*I*a^8*e^(14*I*d*x + 14*I*c)
- 1716*I*a^8*e^(12*I*d*x + 12*I*c) - 1287*I*a^8*e^(10*I*d*x + 10*I*c) - 71
5*I*a^8*e^(8*I*d*x + 8*I*c) - 286*I*a^8*e^(6*I*d*x + 6*I*c) - 78*I*a^8*e^(
4*I*d*x + 4*I*c) - 13*I*a^8*e^(2*I*d*x + 2*I*c) - I*a^8)/(d*e^(26*I*d*x +
26*I*c) + 13*d*e^(24*I*d*x + 24*I*c) + 78*d*e^(22*I*d*x + 22*I*c) + 286*d*
e^(20*I*d*x + 20*I*c) + 715*d*e^(18*I*d*x + 18*I*c) + 1287*d*e^(16*I*d*x +
 16*I*c) + 1716*d*e^(14*I*d*x + 14*I*c) + 1716*d*e^(12*I*d*x + 12*I*c) + 1
287*d*e^(10*I*d*x + 10*I*c) + 715*d*e^(8*I*d*x + 8*I*c) + 286*d*e^(6*I*d*x
 + 6*I*c) + 78*d*e^(4*I*d*x + 4*I*c) + 13*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx = a^8\left(\int \left(-28\tan^2(c+dx)\sec^6(c+dx)\right)\,dx + \int 70\tan^4(c+dx)\sec^6(c+dx)\,dx + \int \left(-28\tan^6(c+dx)\sec^6(c+dx)\right)\,dx + \int \tan^8(c+dx)\sec^6(c+dx)\,dx + \int 8i\tan(c+dx)\sec^6(c+dx)\,dx + \int \left(-56i\tan^3(c+dx)\sec^6(c+dx)\right)\,dx + \int 56i\tan^5(c+dx)\sec^6(c+dx)\,dx + \int \left(-8i\tan^7(c+dx)\sec^6(c+dx)\right)\,dx + \int \sec^6(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**8,x)
```

output
```
a**8*(Integral(-28*tan(c + d*x)**2*sec(c + d*x)**6, x) + Integral(70*tan(c
 + d*x)**4*sec(c + d*x)**6, x) + Integral(-28*tan(c + d*x)**6*sec(c + d*x)
**6, x) + Integral(tan(c + d*x)**8*sec(c + d*x)**6, x) + Integral(8*I*tan(
c + d*x)*sec(c + d*x)**6, x) + Integral(-56*I*tan(c + d*x)**3*sec(c + d*x)
**6, x) + Integral(56*I*tan(c + d*x)**5*sec(c + d*x)**6, x) + Integral(-8*
I*tan(c + d*x)**7*sec(c + d*x)**6, x) + Integral(sec(c + d*x)**6, x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(64) = 128$.

Time = 0.03 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.11

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{33\,a^8\tan\left(dx+c\right)^{13} - 286i\,a^8\tan\left(dx+c\right)^{12} - 1014\,a^8\tan\left(dx+c\right)^{11} + 1716i\,a^8\tan\left(dx+c\right)^{10} + 715\,a^8\,\mathrm{t}}{}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/429*(33*a^8*tan(d*x + c)^13 - 286*I*a^8*tan(d*x + c)^12 - 1014*a^8*tan(d
*x + c)^11 + 1716*I*a^8*tan(d*x + c)^10 + 715*a^8*tan(d*x + c)^9 + 2574*I*
a^8*tan(d*x + c)^8 + 5148*a^8*tan(d*x + c)^7 - 3432*I*a^8*tan(d*x + c)^6 +
 1287*a^8*tan(d*x + c)^5 - 4290*I*a^8*tan(d*x + c)^4 - 3718*a^8*tan(d*x +
c)^3 + 1716*I*a^8*tan(d*x + c)^2 + 429*a^8*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(64) = 128$.

Time = 0.32 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.11

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{33\,a^8\tan\left(dx+c\right)^{13} - 286i\,a^8\tan\left(dx+c\right)^{12} - 1014\,a^8\tan\left(dx+c\right)^{11} + 1716i\,a^8\tan\left(dx+c\right)^{10} + 715\,a^8\,\mathrm{t}}{}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/429*(33*a^8*tan(d*x + c)^13 - 286*I*a^8*tan(d*x + c)^12 - 1014*a^8*tan(d
*x + c)^11 + 1716*I*a^8*tan(d*x + c)^10 + 715*a^8*tan(d*x + c)^9 + 2574*I*
a^8*tan(d*x + c)^8 + 5148*a^8*tan(d*x + c)^7 - 3432*I*a^8*tan(d*x + c)^6 +
 1287*a^8*tan(d*x + c)^5 - 4290*I*a^8*tan(d*x + c)^4 - 3718*a^8*tan(d*x +
c)^3 + 1716*I*a^8*tan(d*x + c)^2 + 429*a^8*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.32

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\,\sin\left(c+d\,x\right)\,\left(2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,\left(-184\sin\left(c+d\,x\right)^2-184\sin\left(2\,c+2\,d\,x\right)^2+\frac{\sin(2\,c+2\,d\,x)\,9867\mathrm{i}}{256}-1\right.}{}$$

input
```
int((a + a*tan(c + d*x)*1i)^8/cos(c + d*x)^6,x)
```

output
```
(a^8*sin(c + d*x)*(2*sin(c/2 + (d*x)/2)^2 - 1)*((sin(2*c + 2*d*x)*9867i)/2
56 + (sin(4*c + 4*d*x)*69069i)/1024 + (sin(6*c + 6*d*x)*42757i)/512 + (sin
(8*c + 8*d*x)*23023i)/256 + (sin(10*c + 10*d*x)*7007i)/512 + (sin(12*c + 1
2*d*x)*1001i)/1024 - 184*sin(2*c + 2*d*x)^2 - 184*sin(3*c + 3*d*x)^2 - 184
*sin(4*c + 4*d*x)^2 - 28*sin(5*c + 5*d*x)^2 - 2*sin(6*c + 6*d*x)^2 - 184*s
in(c + d*x)^2 + 429))/(429*d*(sin(c + d*x)^2 - 1)^7)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.00

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{\sin\left(dx+c\right)a^8\left(-2002\cos\left(dx+c\right)\sin\left(dx+c\right)^{11}i+12012\cos\left(dx+c\right)\sin\left(dx+c\right)^9i-30030\cos\left(dx+\right.\right.}{}$$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(sin(c + d*x)*a**8*( - 2002*cos(c + d*x)*sin(c + d*x)**11*i + 12012*cos(c
+ d*x)*sin(c + d*x)**9*i - 30030*cos(c + d*x)*sin(c + d*x)**7*i + 30888*co
s(c + d*x)*sin(c + d*x)**5*i - 12870*cos(c + d*x)*sin(c + d*x)**3*i + 1716
*cos(c + d*x)*sin(c + d*x)*i + 2048*sin(c + d*x)**12 - 13312*sin(c + d*x)*
*10 + 36608*sin(c + d*x)**8 - 45760*sin(c + d*x)**6 + 26312*sin(c + d*x)**
4 - 6292*sin(c + d*x)**2 + 429))/(429*cos(c + d*x)*d*(sin(c + d*x)**12 - 6
*sin(c + d*x)**10 + 15*sin(c + d*x)**8 - 20*sin(c + d*x)**6 + 15*sin(c + d
*x)**4 - 6*sin(c + d*x)**2 + 1))
```

3.79 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i(a+ia\tan(c+dx))^{10}}{5a^2d} + \frac{i(a+ia\tan(c+dx))^{11}}{11a^3d}$$

output
```
-1/5*I*(a+I*a*tan(d*x+c))^10/a^2/d+1/11*I*(a+I*a*tan(d*x+c))^11/a^3/d
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8(-i+\tan(c+dx))^{10}(6i+5\tan(c+dx))}{55d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*(-I + Tan[c + d*x])^10*(6*I + 5*Tan[c + d*x]))/(55*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4(a+ia\tan(c+dx))^8dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))(i\tan(c+dx)a+a)^9d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(2a(i\tan(c+dx)a+a)^9-(i\tan(c+dx)a+a)^{10}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{1}{5}a(a+ia\tan(c+dx))^{10}-\frac{1}{11}(a+ia\tan(c+dx))^{11}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*((a*(a + I*a*Tan[c + d*x])^10)/5 - (a + I*a*Tan[c + d*x])^11/11))/(a
^3*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(47) = 94$.

Time = 267.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.25

method	result
risch	$\frac{1024 i a^{8} \left(55 \,{\mathrm e}^{18 i \left(d x +c \right)}+165 \,{\mathrm e}^{16 i \left(d x +c \right)}+330 \,{\mathrm e}^{14 i \left(d x +c \right)}+462 \,{\mathrm e}^{12 i \left(d x +c \right)}+462 \,{\mathrm e}^{10 i \left(d x +c \right)}+330 \,{\mathrm e}^{8 i \left(d x +c \right)}+165 \,{\mathrm e}^{6 i \left(d x +c \right)}\right.}{55 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{11}}$
derivativedivides	$\frac{a^{8} \left(\frac{\sin \left(d x +c \right)^{9}}{11 \cos \left(d x +c \right)^{11}}+\frac{2 \sin \left(d x +c \right)^{9}}{99 \cos \left(d x +c \right)^{9}}\right)-8 i a^{8} \left(\frac{\sin \left(d x +c \right)^{8}}{10 \cos \left(d x +c \right)^{10}}+\frac{\sin \left(d x +c \right)^{8}}{40 \cos \left(d x +c \right)^{8}}\right)-28 a^{8} \left(\frac{\sin \left(d x +c \right)^{7}}{9 \cos \left(d x +c \right)^{9}}+\frac{2 \sin \left(d x +c \right)^{7}}{63 \cos \left(d x +c \right)^{7}}\right)-56 i}{}$
default	$\frac{a^{8} \left(\frac{\sin \left(d x +c \right)^{9}}{11 \cos \left(d x +c \right)^{11}}+\frac{2 \sin \left(d x +c \right)^{9}}{99 \cos \left(d x +c \right)^{9}}\right)-8 i a^{8} \left(\frac{\sin \left(d x +c \right)^{8}}{10 \cos \left(d x +c \right)^{10}}+\frac{\sin \left(d x +c \right)^{8}}{40 \cos \left(d x +c \right)^{8}}\right)-28 a^{8} \left(\frac{\sin \left(d x +c \right)^{7}}{9 \cos \left(d x +c \right)^{9}}+\frac{2 \sin \left(d x +c \right)^{7}}{63 \cos \left(d x +c \right)^{7}}\right)-56 i}{}$

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1024/55*I*a^8*(55*exp(18*I*(d*x+c))+165*exp(16*I*(d*x+c))+330*exp(14*I*(d*
x+c))+462*exp(12*I*(d*x+c))+462*exp(10*I*(d*x+c))+330*exp(8*I*(d*x+c))+165
*exp(6*I*(d*x+c))+55*exp(4*I*(d*x+c))+11*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d
*x+c))+1)^11
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(43) = 86$.

Time = 0.10 (sec) , antiderivative size = 269, normalized size of antiderivative = 4.89

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{1024\left(-55i\,a^8e^{(18i\,dx+18i\,c)} - 165i\,a^8e^{(16i\,dx+16i\,c)} - 330i\,a^8e^{(14i\,dx+14i\,c)} - 462i\,a^8e^{(12i\,dx+12i\,c)} - 462\right.}{55\left(de^{(22i\,dx+22i\,c)} + 11\,de^{(20i\,dx+20i\,c)} + 55\,de^{(18i\,dx+18i\,c)} + 165\,de^{(16i\,dx+16i\,c)} + 330\,de^{(14i\,dx+14i\,c)} + 462\,d\right.}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-1024/55*(-55*I*a^8*e^(18*I*d*x + 18*I*c) - 165*I*a^8*e^(16*I*d*x + 16*I*c
) - 330*I*a^8*e^(14*I*d*x + 14*I*c) - 462*I*a^8*e^(12*I*d*x + 12*I*c) - 46
2*I*a^8*e^(10*I*d*x + 10*I*c) - 330*I*a^8*e^(8*I*d*x + 8*I*c) - 165*I*a^8*
e^(6*I*d*x + 6*I*c) - 55*I*a^8*e^(4*I*d*x + 4*I*c) - 11*I*a^8*e^(2*I*d*x +
 2*I*c) - I*a^8)/(d*e^(22*I*d*x + 22*I*c) + 11*d*e^(20*I*d*x + 20*I*c) + 5
5*d*e^(18*I*d*x + 18*I*c) + 165*d*e^(16*I*d*x + 16*I*c) + 330*d*e^(14*I*d*
x + 14*I*c) + 462*d*e^(12*I*d*x + 12*I*c) + 462*d*e^(10*I*d*x + 10*I*c) +
330*d*e^(8*I*d*x + 8*I*c) + 165*d*e^(6*I*d*x + 6*I*c) + 55*d*e^(4*I*d*x +
4*I*c) + 11*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx = a^8\left(\int\left(-28\tan^2(c+dx)\sec^4(c+dx)\right)\,dx + \int 70\tan^4(c+dx)\sec^4(c+dx)\,dx + \int\left(-28\tan^6(c+dx)\sec^4(c+dx)\right)\,dx + \int \tan^8(c+dx)\sec^4(c+dx)\,dx + \int 8i\tan(c+dx)\sec^4(c+dx)\,dx + \int\left(-56i\tan^3(c+dx)\sec^4(c+dx)\right)\,dx + \int 56i\tan^5(c+dx)\sec^4(c+dx)\,dx + \int\left(-8i\tan^7(c+dx)\sec^4(c+dx)\right)\,dx + \int \sec^4(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**8,x)
```

output
```
a**8*(Integral(-28*tan(c + d*x)**2*sec(c + d*x)**4, x) + Integral(70*tan(c
 + d*x)**4*sec(c + d*x)**4, x) + Integral(-28*tan(c + d*x)**6*sec(c + d*x)
**4, x) + Integral(tan(c + d*x)**8*sec(c + d*x)**4, x) + Integral(8*I*tan(
c + d*x)*sec(c + d*x)**4, x) + Integral(-56*I*tan(c + d*x)**3*sec(c + d*x)
**4, x) + Integral(56*I*tan(c + d*x)**5*sec(c + d*x)**4, x) + Integral(-8*
I*tan(c + d*x)**7*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4, x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(43) = 86$.

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.44

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{5\,a^8\tan{(dx+c)}^{11} - 44i\,a^8\tan{(dx+c)}^{10} - 165\,a^8\tan{(dx+c)}^9 + 330i\,a^8\tan{(dx+c)}^8 + 330\,a^8\tan{(dx}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/55*(5*a^8*tan(d*x + c)^11 - 44*I*a^8*tan(d*x + c)^10 - 165*a^8*tan(d*x +
 c)^9 + 330*I*a^8*tan(d*x + c)^8 + 330*a^8*tan(d*x + c)^7 + 462*a^8*tan(d*
x + c)^5 - 660*I*a^8*tan(d*x + c)^4 - 495*a^8*tan(d*x + c)^3 + 220*I*a^8*t
an(d*x + c)^2 + 55*a^8*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(43) = 86$.

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.44

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{5\,a^8\tan{(dx+c)}^{11} - 44i\,a^8\tan{(dx+c)}^{10} - 165\,a^8\tan{(dx+c)}^9 + 330i\,a^8\tan{(dx+c)}^8 + 330\,a^8\tan{(dx}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/55*(5*a^8*tan(d*x + c)^11 - 44*I*a^8*tan(d*x + c)^10 - 165*a^8*tan(d*x +
 c)^9 + 330*I*a^8*tan(d*x + c)^8 + 330*a^8*tan(d*x + c)^7 + 462*a^8*tan(d*
x + c)^5 - 660*I*a^8*tan(d*x + c)^4 - 495*a^8*tan(d*x + c)^3 + 220*I*a^8*t
an(d*x + c)^2 + 55*a^8*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.95

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8\left(\frac{\sin(9\,c+9\,d\,x)}{10}+\frac{\sin(11\,c+11\,d\,x)}{110}+\frac{\cos(c+d\,x)\,63\mathrm{i}}{1280}+\frac{\cos(3\,c+3\,d\,x)\,9\mathrm{i}}{256}+\frac{\cos(5\,c+5\,d\,x)\,9\mathrm{i}}{512}+\frac{\cos(7\,c+7\,d\,x)\,3\mathrm{i}}{512}-\frac{\cos(9\,c+9\,d\,x)\,25}{2560}\right.}{d\cos\left(c+d\,x\right)^{11}}$$

input
```
int((a + a*tan(c + d*x)*1i)^8/cos(c + d*x)^4,x)
```

output
```
(a^8*((cos(c + d*x)*63i)/1280 + (cos(3*c + 3*d*x)*9i)/256 + (cos(5*c + 5*d
*x)*9i)/512 + (cos(7*c + 7*d*x)*3i)/512 - (cos(9*c + 9*d*x)*253i)/2560 - (
cos(11*c + 11*d*x)*23i)/2560 + sin(9*c + 9*d*x)/10 + sin(11*c + 11*d*x)/11
0))/(d*cos(c + d*x)^11)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.80

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{\sin\left(dx+c\right)a^8\left(-506\cos\left(dx+c\right)\sin\left(dx+c\right)^9 i+2530\cos\left(dx+c\right)\sin\left(dx+c\right)^7 i-3300\cos\left(dx+c\right)\sin\right.}{55\cos\left(dx+\right.}$$

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(sin(c + d*x)*a**8*( - 506*cos(c + d*x)*sin(c + d*x)**9*i + 2530*cos(c + d
*x)*sin(c + d*x)**7*i - 3300*cos(c + d*x)*sin(c + d*x)**5*i + 1540*cos(c +
 d*x)*sin(c + d*x)**3*i - 220*cos(c + d*x)*sin(c + d*x)*i + 512*sin(c + d*
x)**10 - 2816*sin(c + d*x)**8 + 4576*sin(c + d*x)**6 - 2992*sin(c + d*x)**
4 + 770*sin(c + d*x)**2 - 55))/(55*cos(c + d*x)*d*(sin(c + d*x)**10 - 5*si
n(c + d*x)**8 + 10*sin(c + d*x)**6 - 10*sin(c + d*x)**4 + 5*sin(c + d*x)**
2 - 1))
```

3.80 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i(a+ia\tan(c+dx))^9}{9ad}$$

output
```
-1/9*I*(a+I*a*tan(d*x+c))^9/a/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 102 vs. 2(27) = 54.

Time = 0.59 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\tan(c+dx)\left(9+36i\tan(c+dx)-84\tan^2(c+dx)-126i\tan^3(c+dx)+126\tan^4(c+dx)+84i\tan\right.}{9d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*Tan[c + d*x]*(9 + (36*I)*Tan[c + d*x] - 84*Tan[c + d*x]^2 - (126*I)*T
an[c + d*x]^3 + 126*Tan[c + d*x]^4 + (84*I)*Tan[c + d*x]^5 - 36*Tan[c + d*
x]^6 - (9*I)*Tan[c + d*x]^7 + Tan[c + d*x]^8))/(9*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(c+dx)^2(a+ia\tan(c+dx))^8dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int(i\tan(c+dx)a+a)^8d(ia\tan(c+dx))}{ad}$$
$$\Big\downarrow \text{17}$$
$$-\frac{i(a+ia\tan(c+dx))^9}{9ad}$$

input
```
Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/9*I)*(a + I*a*Tan[c + d*x])^9)/(a*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(23) = 46$.

Time = 103.98 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.19

method	result
risch	$\frac{512ia^8\left(9\,\mathrm{e}^{16i(dx+c)}+36\,\mathrm{e}^{14i(dx+c)}+84\,\mathrm{e}^{12i(dx+c)}+126\,\mathrm{e}^{10i(dx+c)}+126\,\mathrm{e}^{8i(dx+c)}+84\,\mathrm{e}^{6i(dx+c)}+36\,\mathrm{e}^{4i(dx+c)}}{9d\left(\mathrm{e}^{2i(dx+c)}+1\right)^9}$
derivativedivides	$\frac{\frac{a^8\sin(dx+c)^9}{9\cos(dx+c)^9}+\frac{28ia^8\sin(dx+c)^6}{3\cos(dx+c)^6}-\frac{4a^8\sin(dx+c)^7}{\cos(dx+c)^7}-\frac{ia^8\sin(dx+c)^8}{\cos(dx+c)^8}+\frac{14a^8\sin(dx+c)^5}{\cos(dx+c)^5}+\frac{4ia^8}{\cos(dx+c)^2}-\frac{28a^8\sin(dx+c)^3}{3\cos(dx+c)^3}}{d}$
default	$\frac{\frac{a^8\sin(dx+c)^9}{9\cos(dx+c)^9}+\frac{28ia^8\sin(dx+c)^6}{3\cos(dx+c)^6}-\frac{4a^8\sin(dx+c)^7}{\cos(dx+c)^7}-\frac{ia^8\sin(dx+c)^8}{\cos(dx+c)^8}+\frac{14a^8\sin(dx+c)^5}{\cos(dx+c)^5}+\frac{4ia^8}{\cos(dx+c)^2}-\frac{28a^8\sin(dx+c)^3}{3\cos(dx+c)^3}}{d}$

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
512/9*I*a^8*(9*exp(16*I*(d*x+c))+36*exp(14*I*(d*x+c))+84*exp(12*I*(d*x+c))
+126*exp(10*I*(d*x+c))+126*exp(8*I*(d*x+c))+84*exp(6*I*(d*x+c))+36*exp(4*I
*(d*x+c))+9*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^9
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(21) = 42$.

Time = 0.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 8.56

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{512\left(-9i\,a^8e^{(16i\,dx+16i\,c)} - 36i\,a^8e^{(14i\,dx+14i\,c)} - 84i\,a^8e^{(12i\,dx+12i\,c)} - 126i\,a^8e^{(10i\,dx+10i\,c)} - 126i\,a^8e^{(8i\,dx+}\right.}{9\left(de^{(18i\,dx+18i\,c)} + 9\,de^{(16i\,dx+16i\,c)} + 36\,de^{(14i\,dx+14i\,c)} + 84\,de^{(12i\,dx+12i\,c)} + 126\,de^{(10i\,dx+10i\,c)} + 126\,de\right.}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-512/9*(-9*I*a^8*e^(16*I*d*x + 16*I*c) - 36*I*a^8*e^(14*I*d*x + 14*I*c) -
84*I*a^8*e^(12*I*d*x + 12*I*c) - 126*I*a^8*e^(10*I*d*x + 10*I*c) - 126*I*a
^8*e^(8*I*d*x + 8*I*c) - 84*I*a^8*e^(6*I*d*x + 6*I*c) - 36*I*a^8*e^(4*I*d*
x + 4*I*c) - 9*I*a^8*e^(2*I*d*x + 2*I*c) - I*a^8)/(d*e^(18*I*d*x + 18*I*c)
 + 9*d*e^(16*I*d*x + 16*I*c) + 36*d*e^(14*I*d*x + 14*I*c) + 84*d*e^(12*I*d
*x + 12*I*c) + 126*d*e^(10*I*d*x + 10*I*c) + 126*d*e^(8*I*d*x + 8*I*c) + 8
4*d*e^(6*I*d*x + 6*I*c) + 36*d*e^(4*I*d*x + 4*I*c) + 9*d*e^(2*I*d*x + 2*I*
c) + d)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx = a^8\bigg(\int \left(-28\tan^2{(c+dx)}\sec^2{(c+dx)}\right)\,dx + \int 70\tan^4{(c+dx)}\sec^2{(c+dx)}\,dx + \int \left(-28\tan^6{(c+dx)}\sec^2{(c+dx)}\right)\,dx + \int \tan^8{(c+dx)}\sec^2{(c+dx)}\,dx + \int 8i\tan{(c+dx)}\sec^2{(c+dx)}\,dx + \int \left(-56i\tan^3{(c+dx)}\sec^2{(c+dx)}\right)\,dx + \int 56i\tan^5{(c+dx)}\sec^2{(c+dx)}\,dx + \int \left(-8i\tan^7{(c+dx)}\sec^2{(c+dx)}\right)\,dx + \int \sec^2{(c+dx)}\,dx\bigg)$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**8,x)
```

output
```
a**8*(Integral(-28*tan(c + d*x)**2*sec(c + d*x)**2, x) + Integral(70*tan(c
 + d*x)**4*sec(c + d*x)**2, x) + Integral(-28*tan(c + d*x)**6*sec(c + d*x)
**2, x) + Integral(tan(c + d*x)**8*sec(c + d*x)**2, x) + Integral(8*I*tan(
c + d*x)*sec(c + d*x)**2, x) + Integral(-56*I*tan(c + d*x)**3*sec(c + d*x)
**2, x) + Integral(56*I*tan(c + d*x)**5*sec(c + d*x)**2, x) + Integral(-8*
I*tan(c + d*x)**7*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i\,(i\,a\tan\left(dx+c\right)+a)^9}{9\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/9*I*(I*a*tan(d*x + c) + a)^9/(a*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(21) = 42.

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 4.44

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\tan\left(dx+c\right)^9 - 9i\,a^8\tan\left(dx+c\right)^8 - 36\,a^8\tan\left(dx+c\right)^7 + 84i\,a^8\tan\left(dx+c\right)^6 + 126\,a^8\tan\left(dx+c\right)^5}{9\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/9*(a^8*tan(d*x + c)^9 - 9*I*a^8*tan(d*x + c)^8 - 36*a^8*tan(d*x + c)^7 +
 84*I*a^8*tan(d*x + c)^6 + 126*a^8*tan(d*x + c)^5 - 126*I*a^8*tan(d*x + c)
^4 - 84*a^8*tan(d*x + c)^3 + 36*I*a^8*tan(d*x + c)^2 + 9*a^8*tan(d*x + c))
/d
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.07

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8\left(\sin(9c+9dx)+\frac{\cos(c+dx)\,63i}{128}+\frac{\cos(3c+3dx)\,21i}{64}+\frac{\cos(5c+5dx)\,9i}{64}+\frac{\cos(7c+7dx)\,9i}{256}-\frac{\cos(9c+9dx)\,255i}{256}\right)}{9d\cos(c+dx)^9}$$

input
```
int((a + a*tan(c + d*x)*1i)^8/cos(c + d*x)^2,x)
```

output
```
(a^8*((cos(c + d*x)*63i)/128 + (cos(3*c + 3*d*x)*21i)/64 + (cos(5*c + 5*d*
x)*9i)/64 + (cos(7*c + 7*d*x)*9i)/256 - (cos(9*c + 9*d*x)*255i)/256 + sin(
9*c + 9*d*x)))/(9*d*cos(c + d*x)^9)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 172, normalized size of antiderivative = 6.37

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{\sin(dx+c)\,a^8\left(-255\cos(dx+c)\sin(dx+c)^7 i+444\cos(dx+c)\sin(dx+c)^5 i-234\cos(dx+c)\sin(\right.}{9\cos(dx+c)\,d\left(\sin(dx+c)^8-4\right.}$$

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(sin(c + d*x)*a**8*( - 255*cos(c + d*x)*sin(c + d*x)**7*i + 444*cos(c + d*
x)*sin(c + d*x)**5*i - 234*cos(c + d*x)*sin(c + d*x)**3*i + 36*cos(c + d*x
)*sin(c + d*x)*i + 256*sin(c + d*x)**8 - 576*sin(c + d*x)**6 + 432*sin(c +
 d*x)**4 - 120*sin(c + d*x)**2 + 9))/(9*cos(c + d*x)*d*(sin(c + d*x)**8 -
4*sin(c + d*x)**6 + 6*sin(c + d*x)**4 - 4*sin(c + d*x)**2 + 1))
```

3.81 $\int (a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 15, antiderivative size = 200

$$\begin{aligned}\int (a+ia\tan(c+dx))^8\,dx = {} & 128a^8x-\frac{128ia^8\log(\cos(c+dx))}{d}-\frac{64a^8\tan(c+dx)}{d} \\ & +\frac{4ia^3(a+ia\tan(c+dx))^5}{5d}+\frac{ia^2(a+ia\tan(c+dx))^6}{3d} \\ & +\frac{ia(a+ia\tan(c+dx))^7}{7d}+\frac{16ia^2\left(a^2+ia^2\tan(c+dx)\right)^3}{3d} \\ & +\frac{2i\left(a^2+ia^2\tan(c+dx)\right)^4}{d}+\frac{16i\left(a^4+ia^4\tan(c+dx)\right)^2}{d}\end{aligned}$$

output
```
128*a^8*x-128*I*a^8*ln(cos(d*x+c))/d-64*a^8*tan(d*x+c)/d+4/5*I*a^3*(a+I*a*
tan(d*x+c))^5/d+1/3*I*a^2*(a+I*a*tan(d*x+c))^6/d+1/7*I*a*(a+I*a*tan(d*x+c)
)^7/d+16/3*I*a^2*(a^2+I*a^2*tan(d*x+c))^3/d+2*I*(a^2+I*a^2*tan(d*x+c))^4/d
+16*I*(a^4+I*a^4*tan(d*x+c))^2/d
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.50

$$\int (a+ia\tan(c+dx))^8\,dx$$
$$=\frac{a^8(13440i\log(i+\tan(c+dx))-13335\tan(c+dx)-6300i\tan^2(c+dx)+3465\tan^3(c+dx)+1680i\,\mathrm{t}}{105d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*((13440*I)*Log[I + Tan[c + d*x]] - 13335*Tan[c + d*x] - (6300*I)*Tan[
c + d*x]^2 + 3465*Tan[c + d*x]^3 + (1680*I)*Tan[c + d*x]^4 - 609*Tan[c + d
*x]^5 - (140*I)*Tan[c + d*x]^6 + 15*Tan[c + d*x]^7))/(105*d)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.067, Rules used = {3042, 3959, 3042, 3959, 3042, 3959, 3042, 3959, 3042, 3959, 3042, 3959, 3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^8\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a+ia\tan(c+dx))^8dx$$
$$\Big\downarrow \text{3959}$$
$$2a\int (i\tan(c+dx)a+a)^7dx+\frac{ia(a+ia\tan(c+dx))^7}{7d}$$
$$\Big\downarrow \text{3042}$$
$$2a\int (i\tan(c+dx)a+a)^7dx+\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \ 3959$$

$$2a\left(2a\int (i\tan(c+dx)a+a)^6dx+\frac{ia(a+ia\tan(c+dx))^6}{6d}\right)+\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \ 3042$$

$$2a\left(2a\int (i\tan(c+dx)a+a)^6dx+\frac{ia(a+ia\tan(c+dx))^6}{6d}\right)+\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \ 3959$$

$$2a\left(2a\left(2a\int (i\tan(c+dx)a+a)^5dx+\frac{ia(a+ia\tan(c+dx))^5}{5d}\right)+\frac{ia(a+ia\tan(c+dx))^6}{6d}\right)+\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \ 3042$$

$$2a\left(2a\left(2a\int (i\tan(c+dx)a+a)^5dx+\frac{ia(a+ia\tan(c+dx))^5}{5d}\right)+\frac{ia(a+ia\tan(c+dx))^6}{6d}\right)+\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \ 3959$$

$$2a\left(2a\left(2a\left(2a\int (i\tan(c+dx)a+a)^4dx+\frac{ia(a+ia\tan(c+dx))^4}{4d}\right)+\frac{ia(a+ia\tan(c+dx))^5}{5d}\right)+\frac{ia(a+ia\,\mathrm{ta}}{6}\right.$$
$$\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \ 3042$$

$$2a\left(2a\left(2a\left(2a\int (i\tan(c+dx)a+a)^4dx+\frac{ia(a+ia\tan(c+dx))^4}{4d}\right)+\frac{ia(a+ia\tan(c+dx))^5}{5d}\right)+\frac{ia(a+ia\,\mathrm{ta}}{6}\right.$$
$$\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \ 3959$$

$$2a\left(2a\left(2a\left(2a\left(2a\int (i\tan(c+dx)a+a)^3dx+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{ia(a+ia\tan(c+dx))^4}{4d}\right)+\frac{ia(a+}{}\right.\right.$$
$$\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \ 3042$$

$$2a\left(2a\left(2a\left(2a\left(2a\int(i\tan(c+dx)a+a)^3dx+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{ia(a+ia\tan(c+dx))^4}{4d}\right)+\frac{ia(a+\right.\right.\right.$$
$$\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$\downarrow$ 3959

$$2a\left(2a\left(2a\left(2a\left(2a\left(2a\int(i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{ia(\right.\right.\right.\right.$$
$$\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$\downarrow$ 3042

$$2a\left(2a\left(2a\left(2a\left(2a\left(2a\int(i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{ia(\right.\right.\right.\right.$$
$$\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$\downarrow$ 3958

$$2a\left(2a\left(2a\left(2a\left(2a\left(2a\left(2ia^2\int\tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+\right.\right.\right.\right.\right.$$
$$\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$\downarrow$ 3042

$$2a\left(2a\left(2a\left(2a\left(2a\left(2a\left(2ia^2\int\tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+\right.\right.\right.\right.\right.$$
$$\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$\downarrow$ 3956

$$2a\left(2a\left(2a\left(2a\left(2a\left(2a\left(-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a-\right.\right.\right.\right.\right.$$
$$\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

input
```
Int[(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/7)*a*(a + I*a*Tan[c + d*x])^7)/d + 2*a*(((I/6)*a*(a + I*a*Tan[c + d*x]
)^6)/d + 2*a*(((I/5)*a*(a + I*a*Tan[c + d*x])^5)/d + 2*a*(((I/4)*a*(a + I*
a*Tan[c + d*x])^4)/d + 2*a*(((I/3)*a*(a + I*a*Tan[c + d*x])^3)/d + 2*a*(((
I/2)*a*(a + I*a*Tan[c + d*x])^2)/d + 2*a*(2*a^2*x - ((2*I)*a^2*Log[Cos[c +
 d*x]])/d - (a^2*Tan[c + d*x])/d))))))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

Maple [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.52

method	result
derivativedivides	$\frac{a^{8}\left(-127\tan(dx+c)+\frac{\tan(dx+c)^{7}}{7}-\frac{4i\tan(dx+c)^{6}}{3}-\frac{29\tan(dx+c)^{5}}{5}+16i\tan(dx+c)^{4}+33\tan(dx+c)^{3}-60i\tan(dx+c)^{2}+6\right.}{d}$
default	$\frac{a^{8}\left(-127\tan(dx+c)+\frac{\tan(dx+c)^{7}}{7}-\frac{4i\tan(dx+c)^{6}}{3}-\frac{29\tan(dx+c)^{5}}{5}+16i\tan(dx+c)^{4}+33\tan(dx+c)^{3}-60i\tan(dx+c)^{2}+6\right.}{d}$
risch	$-\frac{256a^{8}c}{d}-\frac{32ia^{8}\left(2940\,\mathrm{e}^{12i(dx+c)}+13230\,\mathrm{e}^{10i(dx+c)}+26950\,\mathrm{e}^{8i(dx+c)}+30625\,\mathrm{e}^{6i(dx+c)}+20139\,\mathrm{e}^{4i(dx+c)}+7203\,\mathrm{e}^{2i(\right.}{105d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{7}}$
parallelrisch	$\frac{-140ia^{8}\tan(dx+c)^{6}+15\tan(dx+c)^{7}a^{8}+1680ia^{8}\tan(dx+c)^{4}-609\tan(dx+c)^{5}a^{8}-6300ia^{8}\tan(dx+c)^{2}+3465\tan(dx}{105d}$
norman	$128a^{8}x-\frac{127a^{8}\tan(dx+c)}{d}+\frac{33a^{8}\tan(dx+c)^{3}}{d}-\frac{29a^{8}\tan(dx+c)^{5}}{5d}+\frac{a^{8}\tan(dx+c)^{7}}{7d}-\frac{60ia^{8}\tan(dx+c)^{2}}{d}$
parts	$a^{8}x+\frac{a^{8}\left(\frac{\tan(dx+c)^{7}}{7}-\frac{\tan(dx+c)^{5}}{5}+\frac{\tan(dx+c)^{3}}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}-\frac{56ia^{8}\left(\frac{\tan(dx+c)^{2}}{2}-\frac{\ln\left(1+\mathrm{ta}\right.}{}\right.}{d}$

input
```
int((a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/d*a^8*(-127*tan(d*x+c)+1/7*tan(d*x+c)^7-4/3*I*tan(d*x+c)^6-29/5*tan(d*x+
c)^5+16*I*tan(d*x+c)^4+33*tan(d*x+c)^3-60*I*tan(d*x+c)^2+64*I*ln(1+tan(d*x
+c)^2)+128*arctan(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.48

$$\int (a+ia\tan(c+dx))^{8}\,dx =$$
$$-\frac{32\left(2940i\,a^{8}e^{(12i\,dx+12i\,c)}+13230i\,a^{8}e^{(10i\,dx+10i\,c)}+26950i\,a^{8}e^{(8i\,dx+8i\,c)}+30625i\,a^{8}e^{(6i\,dx+6i\,c)}+20139i\right.}{105\left(de^{(1}\right.}$$

input
```
integrate((a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-32/105*(2940*I*a^8*e^(12*I*d*x + 12*I*c) + 13230*I*a^8*e^(10*I*d*x + 10*I
*c) + 26950*I*a^8*e^(8*I*d*x + 8*I*c) + 30625*I*a^8*e^(6*I*d*x + 6*I*c) +
20139*I*a^8*e^(4*I*d*x + 4*I*c) + 7203*I*a^8*e^(2*I*d*x + 2*I*c) + 1089*I*
a^8 + 420*(I*a^8*e^(14*I*d*x + 14*I*c) + 7*I*a^8*e^(12*I*d*x + 12*I*c) + 2
1*I*a^8*e^(10*I*d*x + 10*I*c) + 35*I*a^8*e^(8*I*d*x + 8*I*c) + 35*I*a^8*e^
(6*I*d*x + 6*I*c) + 21*I*a^8*e^(4*I*d*x + 4*I*c) + 7*I*a^8*e^(2*I*d*x + 2*
I*c) + I*a^8)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(14*I*d*x + 14*I*c) + 7*d
*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x + 10*I*c) + 35*d*e^(8*I*d*x + 8*
I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I*d*x + 4*I*c) + 7*d*e^(2*I*d*
x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.50

$$\int (a+ia\tan(c+dx))^8\,dx = -\frac{128ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{-94080ia^8e^{12ic}e^{12idx}-423360ia^8e^{10ic}e^{10idx}-862400ia^8e^{8ic}e^{8idx}-980000ia^8e^{6ic}e^{6idx}-644448ia^8e^{4ic}\cdots}{105de^{14ic}e^{14idx}+735de^{12ic}e^{12idx}+2205de^{10ic}e^{10idx}+3675de^{8ic}e^{8idx}+3675de^{6ic}e^{6idx}+2205de^{4}\cdots}$$

input
```
integrate((a+I*a*tan(d*x+c))**8,x)
```

output
```
-128*I*a**8*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-94080*I*a**8*exp(12*I*c)
*exp(12*I*d*x) - 423360*I*a**8*exp(10*I*c)*exp(10*I*d*x) - 862400*I*a**8*e
xp(8*I*c)*exp(8*I*d*x) - 980000*I*a**8*exp(6*I*c)*exp(6*I*d*x) - 644448*I*
a**8*exp(4*I*c)*exp(4*I*d*x) - 230496*I*a**8*exp(2*I*c)*exp(2*I*d*x) - 348
48*I*a**8)/(105*d*exp(14*I*c)*exp(14*I*d*x) + 735*d*exp(12*I*c)*exp(12*I*d
*x) + 2205*d*exp(10*I*c)*exp(10*I*d*x) + 3675*d*exp(8*I*c)*exp(8*I*d*x) +
3675*d*exp(6*I*c)*exp(6*I*d*x) + 2205*d*exp(4*I*c)*exp(4*I*d*x) + 735*d*ex
p(2*I*c)*exp(2*I*d*x) + 105*d)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.60

$$\int (a+ia\tan(c+dx))^8\,dx$$
$$= \frac{15\,a^8\tan\left(dx+c\right)^7 - 140i\,a^8\tan\left(dx+c\right)^6 - 609\,a^8\tan\left(dx+c\right)^5 + 1680i\,a^8\tan\left(dx+c\right)^4 + 3465\,a^8\tan}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/105*(15*a^8*tan(d*x + c)^7 - 140*I*a^8*tan(d*x + c)^6 - 609*a^8*tan(d*x
+ c)^5 + 1680*I*a^8*tan(d*x + c)^4 + 3465*a^8*tan(d*x + c)^3 - 6300*I*a^8*
tan(d*x + c)^2 + 13440*(d*x + c)*a^8 + 6720*I*a^8*log(tan(d*x + c)^2 + 1)
- 13335*a^8*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.67

$$\int (a+ia\tan(c+dx))^8\,dx = \frac{128i\,a^8\log\left(\tan\left(dx+c\right)+i\right)}{d}$$
$$+ \frac{15\,a^8d^6\tan\left(dx+c\right)^7 - 140i\,a^8d^6\tan\left(dx+c\right)^6 - 609\,a^8d^6\tan\left(dx+c\right)^5 + 1680i\,a^8d^6\tan\left(dx+c\right)^4 + 3}{105\,d^7}$$

input
```
integrate((a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
128*I*a^8*log(tan(d*x + c) + I)/d + 1/105*(15*a^8*d^6*tan(d*x + c)^7 - 140
*I*a^8*d^6*tan(d*x + c)^6 - 609*a^8*d^6*tan(d*x + c)^5 + 1680*I*a^8*d^6*ta
n(d*x + c)^4 + 3465*a^8*d^6*tan(d*x + c)^3 - 6300*I*a^8*d^6*tan(d*x + c)^2
 - 13335*a^8*d^6*tan(d*x + c))/d^7
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.56

$$\int (a+ia\tan(c+dx))^8\,dx$$
$$= \frac{33\,a^8\tan(c+d\,x)^3-127\,a^8\tan(c+d\,x)-\frac{29\,a^8\tan(c+d\,x)^5}{5}+\frac{a^8\tan(c+d\,x)^7}{7}+a^8\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,128\mathrm{i}}{d}$$

input
```
int((a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(a^8*log(tan(c + d*x) + 1i)*128i - 127*a^8*tan(c + d*x) - a^8*tan(c + d*x)
^2*60i + 33*a^8*tan(c + d*x)^3 + a^8*tan(c + d*x)^4*16i - (29*a^8*tan(c +
d*x)^5)/5 - (a^8*tan(c + d*x)^6*4i)/3 + (a^8*tan(c + d*x)^7)/7)/d
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.49

$$\int (a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(6720\log\left(\tan(dx+c)^2+1\right)i+15\tan(dx+c)^7-140\tan(dx+c)^6i-609\tan(dx+c)^5+1680\tan}{105d}$$

input
```
int((a+I*a*tan(d*x+c))^8,x)
```

output
```
(a**8*(6720*log(tan(c + d*x)**2 + 1)*i + 15*tan(c + d*x)**7 - 140*tan(c +
d*x)**6*i - 609*tan(c + d*x)**5 + 1680*tan(c + d*x)**4*i + 3465*tan(c + d*
x)**3 - 6300*tan(c + d*x)**2*i - 13335*tan(c + d*x) + 13440*d*x))/(105*d)
```

3.82 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 133

$$\begin{aligned}\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx &= -192a^8x+\frac{192ia^8\log(\cos(c+dx))}{d}\\&\quad+\frac{129a^8\tan(c+dx)}{d}+\frac{36ia^8\tan^2(c+dx)}{d}\\&\quad-\frac{10a^8\tan^3(c+dx)}{d}-\frac{2ia^8\tan^4(c+dx)}{d}\\&\quad+\frac{a^8\tan^5(c+dx)}{5d}-\frac{64ia^9}{d(a-ia\tan(c+dx))}\end{aligned}$$

output
```
-192*a^8*x+192*I*a^8*ln(cos(d*x+c))/d+129*a^8*tan(d*x+c)/d+36*I*a^8*tan(d*
x+c)^2/d-10*a^8*tan(d*x+c)^3/d-2*I*a^8*tan(d*x+c)^4/d+1/5*a^8*tan(d*x+c)^5
/d-64*I*a^9/d/(a-I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.72

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{ia^8\Big(960\log(i+\tan(c+dx))+645i\tan(c+dx)-180\tan^2(c+dx)-50i\tan^3(c+dx)+10\tan^4(c+}{5d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/5*I)*a^8*(960*Log[I + Tan[c + d*x]] + (645*I)*Tan[c + d*x] - 180*Tan[
c + d*x]^2 - (50*I)*Tan[c + d*x]^3 + 10*Tan[c + d*x]^4 + I*Tan[c + d*x]^5
+ (320*I)/(I + Tan[c + d*x])))/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{(i\tan(c+dx)a+a)^6}{(a-ia\tan(c+dx))^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{ia^3\int\left(\frac{64a^6}{(a-ia\tan(c+dx))^2}-\frac{192a^5}{a-ia\tan(c+dx)}+\tan^4(c+dx)a^4-8i\tan^3(c+dx)a^4-30\tan^2(c+dx)a^4+72i\tan(c-\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^3\left(\frac{64a^6}{a-ia\tan(c+dx)}+\frac{1}{5}ia^5\tan^5(c+dx)+2a^5\tan^4(c+dx)-10ia^5\tan^3(c+dx)-36a^5\tan^2(c+dx)+129ia^5\text{t}\right.}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*a^3*(192*a^5*Log[a - I*a*Tan[c + d*x]] + (129*I)*a^5*Tan[c + d*x] -
36*a^5*Tan[c + d*x]^2 - (10*I)*a^5*Tan[c + d*x]^3 + 2*a^5*Tan[c + d*x]^4 +
 (I/5)*a^5*Tan[c + d*x]^5 + (64*a^6)/(a - I*a*Tan[c + d*x])))/d
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 75.78 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{32 i a^{8} \mathrm{e}^{2 i (d x+c)}}{d}+\frac{384 a^{8} c}{d}+\frac{16 i a^{8} \left(150 \,\mathrm{e}^{8 i (d x+c)}+500 \,\mathrm{e}^{6 i (d x+c)}+650 \,\mathrm{e}^{4 i (d x+c)}+385 \,\mathrm{e}^{2 i (d x+c)}+87\right)}{5 d \left(\mathrm{e}^{2 i (d x+c)}+1\right)^{5}}+\frac{192 i a^{8} \ln}{}$
derivativedivides	$\frac{a^{8} \left(\frac{\sin(dx+c)^{9}}{5 \cos(dx+c)^{5}}-\frac{4 \sin(dx+c)^{9}}{15 \cos(dx+c)^{3}}+\frac{8 \sin(dx+c)^{9}}{5 \cos(dx+c)}+\frac{8 \left(\sin(dx+c)^{7}+\frac{7 \sin(dx+c)^{5}}{6}+\frac{35 \sin(dx+c)^{3}}{24}+\frac{35 \sin(dx+c)}{16}\right) \cos(dx+c)}{5}-\frac{7c}{2}\right.}{}$
default	$\frac{a^{8} \left(\frac{\sin(dx+c)^{9}}{5 \cos(dx+c)^{5}}-\frac{4 \sin(dx+c)^{9}}{15 \cos(dx+c)^{3}}+\frac{8 \sin(dx+c)^{9}}{5 \cos(dx+c)}+\frac{8 \left(\sin(dx+c)^{7}+\frac{7 \sin(dx+c)^{5}}{6}+\frac{35 \sin(dx+c)^{3}}{24}+\frac{35 \sin(dx+c)}{16}\right) \cos(dx+c)}{5}-\frac{7c}{2}\right.}{}$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
-32*I/d*a^8*exp(2*I*(d*x+c))+384/d*a^8*c+16/5*I*a^8*(150*exp(8*I*(d*x+c))+
500*exp(6*I*(d*x+c))+650*exp(4*I*(d*x+c))+385*exp(2*I*(d*x+c))+87)/d/(exp(
2*I*(d*x+c))+1)^5+192*I/d*a^8*ln(exp(2*I*(d*x+c))+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(121) = 242.

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.84

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{16\left(10i\,a^8e^{(12i\,dx+12i\,c)}+50i\,a^8e^{(10i\,dx+10i\,c)}-50i\,a^8e^{(8i\,dx+8i\,c)}-400i\,a^8e^{(6i\,dx+6i\,c)}-600i\,a^8e^{(4i\,dx+4i\,c)}-\right.}{5\left(de^{(10i\,dx+10i\,c)}+\right.}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-16/5*(10*I*a^8*e^(12*I*d*x + 12*I*c) + 50*I*a^8*e^(10*I*d*x + 10*I*c) - 5
0*I*a^8*e^(8*I*d*x + 8*I*c) - 400*I*a^8*e^(6*I*d*x + 6*I*c) - 600*I*a^8*e^
(4*I*d*x + 4*I*c) - 375*I*a^8*e^(2*I*d*x + 2*I*c) - 87*I*a^8 + 60*(-I*a^8*
e^(10*I*d*x + 10*I*c) - 5*I*a^8*e^(8*I*d*x + 8*I*c) - 10*I*a^8*e^(6*I*d*x
+ 6*I*c) - 10*I*a^8*e^(4*I*d*x + 4*I*c) - 5*I*a^8*e^(2*I*d*x + 2*I*c) - I*
a^8)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d
*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^
(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.93

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{192ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{2400ia^8e^{8ic}e^{8idx}+8000ia^8e^{6ic}e^{6idx}+10400ia^8e^{4ic}e^{4idx}+6160ia^8e^{2ic}e^{2idx}+1392ia^8}{5de^{10ic}e^{10idx}+25de^{8ic}e^{8idx}+50de^{6ic}e^{6idx}+50de^{4ic}e^{4idx}+25de^{2ic}e^{2idx}+5d} + \begin{cases} -\frac{32ia^8e^{2ic}e^{2idx}}{d} & \text{for } d\neq 0 \\ 64a^8xe^{2ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**8,x)
```

output
```
192*I*a**8*log(exp(2*I*d*x) + exp(-2*I*c))/d + (2400*I*a**8*exp(8*I*c)*exp
(8*I*d*x) + 8000*I*a**8*exp(6*I*c)*exp(6*I*d*x) + 10400*I*a**8*exp(4*I*c)*
exp(4*I*d*x) + 6160*I*a**8*exp(2*I*c)*exp(2*I*d*x) + 1392*I*a**8)/(5*d*exp
(10*I*c)*exp(10*I*d*x) + 25*d*exp(8*I*c)*exp(8*I*d*x) + 50*d*exp(6*I*c)*ex
p(6*I*d*x) + 50*d*exp(4*I*c)*exp(4*I*d*x) + 25*d*exp(2*I*c)*exp(2*I*d*x) +
 5*d) + Piecewise((-32*I*a**8*exp(2*I*c)*exp(2*I*d*x)/d, Ne(d, 0)), (64*a*
*8*x*exp(2*I*c), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\tan{(dx+c)}^5 - 10i\,a^8\tan{(dx+c)}^4 - 50\,a^8\tan{(dx+c)}^3 + 180i\,a^8\tan{(dx+c)}^2 - 960\,(dx+c)a^8 - 4}{5\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/5*(a^8*tan(d*x + c)^5 - 10*I*a^8*tan(d*x + c)^4 - 50*a^8*tan(d*x + c)^3
+ 180*I*a^8*tan(d*x + c)^2 - 960*(d*x + c)*a^8 - 480*I*a^8*log(tan(d*x + c
)^2 + 1) + 645*a^8*tan(d*x + c) + 320*(a^8*tan(d*x + c) - I*a^8)/(tan(d*x
+ c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= -\frac{192i\,a^8\log{(\tan{(dx+c)}+i)}}{d} + \frac{64\,a^8}{d(\tan{(dx+c)}+i)}$$
$$+ \frac{a^8d^4\tan{(dx+c)}^5 - 10i\,a^8d^4\tan{(dx+c)}^4 - 50\,a^8d^4\tan{(dx+c)}^3 + 180i\,a^8d^4\tan{(dx+c)}^2 + 645\,a^8d}{5\,d^5}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-192*I*a^8*log(tan(d*x + c) + I)/d + 64*a^8/(d*(tan(d*x + c) + I)) + 1/5*(
a^8*d^4*tan(d*x + c)^5 - 10*I*a^8*d^4*tan(d*x + c)^4 - 50*a^8*d^4*tan(d*x
+ c)^3 + 180*I*a^8*d^4*tan(d*x + c)^2 + 645*a^8*d^4*tan(d*x + c))/d^5
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.77

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{\frac{64\,a^8}{\tan(c+d\,x)+1\mathrm{i}} + 129\,a^8\tan(c+d\,x) - 10\,a^8\tan(c+d\,x)^3 + \frac{a^8\tan(c+d\,x)^5}{5} - a^8\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,192\mathrm{i} + \cdots}{d}$$

input
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
((64*a^8)/(tan(c + d*x) + 1i) - a^8*log(tan(c + d*x) + 1i)*192i + 129*a^8*
tan(c + d*x) + a^8*tan(c + d*x)^2*36i - 10*a^8*tan(c + d*x)^3 - a^8*tan(c
+ d*x)^4*2i + (a^8*tan(c + d*x)^5)/5)/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 460, normalized size of antiderivative = 3.46

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(-960\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)^4 i + 1920\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\cdots\right.}{}$$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x)
```

output

```
(a**8*( - 960*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**4*i
+ 1920*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*i - 960*c
os(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*i + 960*cos(c + d*x)*log(tan((c +
 d*x)/2) - 1)*sin(c + d*x)**4*i - 1920*cos(c + d*x)*log(tan((c + d*x)/2) -
 1)*sin(c + d*x)**2*i + 960*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*i + 960
*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*i - 1920*cos(c + d
*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*i + 960*cos(c + d*x)*log(tan
((c + d*x)/2) + 1)*i + 320*cos(c + d*x)*sin(c + d*x)**6*i - 960*cos(c + d*
x)*sin(c + d*x)**4*c - 960*cos(c + d*x)*sin(c + d*x)**4*d*x - 830*cos(c +
d*x)*sin(c + d*x)**4*i + 1920*cos(c + d*x)*sin(c + d*x)**2*c + 1920*cos(c
+ d*x)*sin(c + d*x)**2*d*x + 500*cos(c + d*x)*sin(c + d*x)**2*i - 960*cos(
c + d*x)*c - 960*cos(c + d*x)*d*x - 320*sin(c + d*x)**7 + 1656*sin(c + d*x
)**5 - 2300*sin(c + d*x)**3 + 965*sin(c + d*x)))/(5*cos(c + d*x)*d*(sin(c
+ d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.83 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = 80a^8x - \frac{80ia^8\log(\cos(c+dx))}{d} - \frac{31a^8\tan(c+dx)}{d} - \frac{4ia^8\tan^2(c+dx)}{d} + \frac{a^8\tan^3(c+dx)}{3d} - \frac{16ia^{10}}{d(a-ia\tan(c+dx))^2} + \frac{80ia^9}{d(a-ia\tan(c+dx))}$$

output
```
80*a^8*x-80*I*a^8*ln(cos(d*x+c))/d-31*a^8*tan(d*x+c)/d-4*I*a^8*tan(d*x+c)^
2/d+1/3*a^8*tan(d*x+c)^3/d-16*I*a^10/d/(a-I*a*tan(d*x+c))^2+80*I*a^9/d/(a-
I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.69

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx =$$
$$-\frac{ia^8\left(-93i\tan(c+dx)+12\tan^2(c+dx)+i\tan^3(c+dx)+48\left(-5\log(i+\tan(c+dx))+\frac{4-5i\tan(c+dx}{(i+\tan(c+dx))}\right.\right.}{3d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/3*I)*a^8*((-93*I)*Tan[c + d*x] + 12*Tan[c + d*x]^2 + I*Tan[c + d*x]^3
 + 48*(-5*Log[I + Tan[c + d*x]] + (4 - (5*I)*Tan[c + d*x])/(I + Tan[c + d*
x])^2)))/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{(i\tan(c+dx)a+a)^5}{(a-ia\tan(c+dx))^3}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{ia^5 \int \left(\frac{32a^5}{(a-ia\tan(c+dx))^3} - \frac{80a^4}{(a-ia\tan(c+dx))^2} + \frac{80a^3}{a-ia\tan(c+dx)} + \tan^2(c+dx)a^2 - 8i\tan(c+dx)a^2 - 31a^2\right) d(ia\tan}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^5 \left(\frac{16a^5}{(a-ia\tan(c+dx))^2} - \frac{80a^4}{a-ia\tan(c+dx)} + \frac{1}{3}ia^3\tan^3(c+dx) + 4a^3\tan^2(c+dx) - 31ia^3\tan(c+dx) - 80a^3\log(a}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*a^5*(-80*a^3*Log[a - I*a*Tan[c + d*x]] - (31*I)*a^3*Tan[c + d*x] + 4
*a^3*Tan[c + d*x]^2 + (I/3)*a^3*Tan[c + d*x]^3 + (16*a^5)/(a - I*a*Tan[c +
 d*x])^2 - (80*a^4)/(a - I*a*Tan[c + d*x])))/d
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 193.98 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{4ia^8 e^{4i(dx+c)}}{d}+\frac{32ia^8 e^{2i(dx+c)}}{d}-\frac{160a^8c}{d}-\frac{4ia^8(60\,e^{4i(dx+c)}+105\,e^{2i(dx+c)}+47)}{3d(e^{2i(dx+c)}+1)^3}-\frac{80ia^8\ln(e^{2i(dx+c)}+1)}{d}$
derivativedivides	$\frac{a^8\left(\frac{\sin(dx+c)^9}{3\cos(dx+c)^3}-\frac{2\sin(dx+c)^9}{\cos(dx+c)}-2\left(\sin(dx+c)^7+\frac{7\sin(dx+c)^5}{6}+\frac{35\sin(dx+c)^3}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)+\frac{35dx}{8}+\frac{35c}{8}\right)-}{}$
default	$\frac{a^8\left(\frac{\sin(dx+c)^9}{3\cos(dx+c)^3}-\frac{2\sin(dx+c)^9}{\cos(dx+c)}-2\left(\sin(dx+c)^7+\frac{7\sin(dx+c)^5}{6}+\frac{35\sin(dx+c)^3}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)+\frac{35dx}{8}+\frac{35c}{8}\right)-}{}$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
-4*I/d*a^8*exp(4*I*(d*x+c))+32*I/d*a^8*exp(2*I*(d*x+c))-160/d*a^8*c-4/3*I*
a^8*(60*exp(4*I*(d*x+c))+105*exp(2*I*(d*x+c))+47)/d/(exp(2*I*(d*x+c))+1)^3
-80*I/d*a^8*ln(exp(2*I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.44

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{4\left(3i\,a^8e^{(10i\,dx+10i\,c)}-15i\,a^8e^{(8i\,dx+8i\,c)}-63i\,a^8e^{(6i\,dx+6i\,c)}-9i\,a^8e^{(4i\,dx+4i\,c)}+81i\,a^8e^{(2i\,dx+2i\,c)}+47i\,a^8\right.}{3\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,\right.}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-4/3*(3*I*a^8*e^(10*I*d*x + 10*I*c) - 15*I*a^8*e^(8*I*d*x + 8*I*c) - 63*I*
a^8*e^(6*I*d*x + 6*I*c) - 9*I*a^8*e^(4*I*d*x + 4*I*c) + 81*I*a^8*e^(2*I*d*
x + 2*I*c) + 47*I*a^8 + 60*(I*a^8*e^(6*I*d*x + 6*I*c) + 3*I*a^8*e^(4*I*d*x
 + 4*I*c) + 3*I*a^8*e^(2*I*d*x + 2*I*c) + I*a^8)*log(e^(2*I*d*x + 2*I*c) +
 1))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2
*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.74

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{80ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{-240ia^8e^{4ic}e^{4idx}-420ia^8e^{2ic}e^{2idx}-188ia^8}{3de^{6ic}e^{6idx}+9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}+3d} + \begin{cases} \frac{-4ia^8de^{4ic}e^{4idx}+32ia^8de^{2ic}e^{2idx}}{d^2} & \text{for } d^2\neq 0 \\ x\left(16a^8e^{4ic}-64a^8e^{2ic}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**8,x)
```

output
```
-80*I*a**8*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-240*I*a**8*exp(4*I*c)*exp
(4*I*d*x) - 420*I*a**8*exp(2*I*c)*exp(2*I*d*x) - 188*I*a**8)/(3*d*exp(6*I*
c)*exp(6*I*d*x) + 9*d*exp(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x
) + 3*d) + Piecewise(((-4*I*a**8*d*exp(4*I*c)*exp(4*I*d*x) + 32*I*a**8*d*e
xp(2*I*c)*exp(2*I*d*x))/d**2, Ne(d**2, 0)), (x*(16*a**8*exp(4*I*c) - 64*a*
*8*exp(2*I*c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.09

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\tan(dx+c)^3 - 12i\,a^8\tan(dx+c)^2 + 240\,(dx+c)a^8 + 120i\,a^8\log\left(\tan(dx+c)^2+1\right) - 93\,a^8\tan(dx}{3\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/3*(a^8*tan(d*x + c)^3 - 12*I*a^8*tan(d*x + c)^2 + 240*(d*x + c)*a^8 + 12
0*I*a^8*log(tan(d*x + c)^2 + 1) - 93*a^8*tan(d*x + c) - 48*(5*a^8*tan(d*x
+ c)^3 - 6*I*a^8*tan(d*x + c)^2 + 3*a^8*tan(d*x + c) - 4*I*a^8)/(tan(d*x +
 c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.81

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{80i\,a^8\log\left(\tan\left(dx+c\right)+i\right)}{d} + \frac{a^8d^2\tan\left(dx+c\right)^3 - 12i\,a^8d^2\tan\left(dx+c\right)^2 - 93\,a^8d^2\tan\left(dx+c\right)}{3\,d^3} - \frac{16\left(5\,a^8\tan\left(dx+c\right)+4i\,a^8\right)}{d(\tan\left(dx+c\right)+i)^2}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
80*I*a^8*log(tan(d*x + c) + I)/d + 1/3*(a^8*d^2*tan(d*x + c)^3 - 12*I*a^8*
d^2*tan(d*x + c)^2 - 93*a^8*d^2*tan(d*x + c))/d^3 - 16*(5*a^8*tan(d*x + c)
 + 4*I*a^8)/(d*(tan(d*x + c) + I)^2)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8\tan(c+d\,x)^3}{3\,d} - \frac{80\,a^8\tan(c+d\,x)+a^8\,64\mathrm{i}}{d\,\left(\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,2\mathrm{i}-1\right)} - \frac{31\,a^8\tan(c+d\,x)}{d} + \frac{a^8\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,80\mathrm{i}}{d} - \frac{a^8\tan(c+d\,x)^2\,4\mathrm{i}}{d}$$

input
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(a^8*log(tan(c + d*x) + 1i)*80i)/d - (31*a^8*tan(c + d*x))/d - (80*a^8*tan
(c + d*x) + a^8*64i)/(d*(tan(c + d*x)*2i + tan(c + d*x)^2 - 1)) - (a^8*tan
(c + d*x)^2*4i)/d + (a^8*tan(c + d*x)^3)/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.49

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(240\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(dx+c\right)^2 i-240\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)i-\right.}{}$$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(a**8*(240*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*i - 2
40*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*i - 240*cos(c + d*x)*log(tan(
(c + d*x)/2) - 1)*sin(c + d*x)**2*i + 240*cos(c + d*x)*log(tan((c + d*x)/2
) - 1)*i - 240*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*i +
240*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*i - 96*cos(c + d*x)*sin(c + d*x
)**6*i + 240*cos(c + d*x)*sin(c + d*x)**2*c + 240*cos(c + d*x)*sin(c + d*x
)**2*d*x + 108*cos(c + d*x)*sin(c + d*x)**2*i - 240*cos(c + d*x)*c - 240*c
os(c + d*x)*d*x + 96*sin(c + d*x)**7 - 48*sin(c + d*x)**5 - 286*sin(c + d*
x)**3 + 237*sin(c + d*x)))/(3*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.84 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx = -8a^8x + \frac{8ia^8\log(\cos(c+dx))}{d} + \frac{a^8\tan(c+dx)}{d} - \frac{16ia^{11}}{3d(a-ia\tan(c+dx))^3} + \frac{16ia^{10}}{d(a-ia\tan(c+dx))^2} - \frac{24ia^9}{d(a-ia\tan(c+dx))}$$

output
```
-8*a^8*x+8*I*a^8*ln(cos(d*x+c))/d+a^8*tan(d*x+c)/d-16/3*I*a^11/d/(a-I*a*ta
n(d*x+c))^3+16*I*a^10/d/(a-I*a*tan(d*x+c))^2-24*I*a^9/d/(a-I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=-\frac{ia^7\left(8a\log(i+\tan(c+dx))+ia\tan(c+dx)+\frac{8ia(-5+12i\tan(c+dx)+9\tan^2(c+dx))}{3(i+\tan(c+dx))^3}\right)}{d}$$

input
```
Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*a^7*(8*a*Log[I + Tan[c + d*x]] + I*a*Tan[c + d*x] + (((8*I)/3)*a*(-5
 + (12*I)*Tan[c + d*x] + 9*Tan[c + d*x]^2))/(I + Tan[c + d*x])^3))/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^6}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{ia^7\int \frac{(i\tan(c+dx)a+a)^4}{(a-ia\tan(c+dx))^4}d(ia\tan(c+dx))}{d}$$
$$\Big\downarrow \text{49}$$
$$-\frac{ia^7\int \left(\frac{16a^4}{(a-ia\tan(c+dx))^4}-\frac{32a^3}{(a-ia\tan(c+dx))^3}+\frac{24a^2}{(a-ia\tan(c+dx))^2}-\frac{8a}{a-ia\tan(c+dx)}+1\right)d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$-\frac{ia^7\left(\frac{16a^4}{3(a-ia\tan(c+dx))^3}-\frac{16a^3}{(a-ia\tan(c+dx))^2}+\frac{24a^2}{a-ia\tan(c+dx)}+ia\tan(c+dx)+8a\log(a-ia\tan(c+dx))\right)}{d}$$

input
```
Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*a^7*(8*a*Log[a - I*a*Tan[c + d*x]] + I*a*Tan[c + d*x] + (16*a^4)/(3*
(a - I*a*Tan[c + d*x])^3) - (16*a^3)/(a - I*a*Tan[c + d*x])^2 + (24*a^2)/(
a - I*a*Tan[c + d*x])))/d
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(105) = 210.

Time = 0.78 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.80

$$\frac{29a^8 \sin(dx+c)\cos(dx+c)^5}{6d} + \frac{32ia^8 \sin(dx+c)^6}{3d} + \frac{a^8 \sin(dx+c)^7 \cos(dx+c)}{d} - \frac{35a^8 \sin(dx+c)^3 \cos(\ldots}{3d}$$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x)
```

output
```
29/6/d*a^8*sin(d*x+c)*cos(d*x+c)^5+32/3*I/d*a^8*sin(d*x+c)^6+1/d*a^8*sin(d
*x+c)^7*cos(d*x+c)-35/3/d*a^8*sin(d*x+c)^3*cos(d*x+c)^3-233/24/d*a^8*sin(d
*x+c)*cos(d*x+c)^3+14/3*I/d*a^8*cos(d*x+c)^4+4*I/d*a^8*sin(d*x+c)^2-4/3*I/
d*a^8*cos(d*x+c)^6+2*I/d*a^8*sin(d*x+c)^4-8*a^8*x+1/d*a^8*sin(d*x+c)^9/cos
(d*x+c)+35/6/d*a^8*cos(d*x+c)*sin(d*x+c)^5+175/24/d*a^8*cos(d*x+c)*sin(d*x
+c)^3+111/8/d*a^8*cos(d*x+c)*sin(d*x+c)+28/3*I/d*a^8*sin(d*x+c)^2*cos(d*x+
c)^4-8/d*a^8*c+8*I*a^8*ln(cos(d*x+c))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{2\left(i\,a^8 e^{(8i\,dx+8i\,c)} - 2i\,a^8 e^{(6i\,dx+6i\,c)} + 6i\,a^8 e^{(4i\,dx+4i\,c)} + 9i\,a^8 e^{(2i\,dx+2i\,c)} - 3i\,a^8 + 12\left(-i\,a^8 e^{(2i\,dx+2i\,c)} - i\ldots\right.\right.}{3\left(de^{(2i\,dx+2i\,c)} + d\right)}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-2/3*(I*a^8*e^(8*I*d*x + 8*I*c) - 2*I*a^8*e^(6*I*d*x + 6*I*c) + 6*I*a^8*e^
(4*I*d*x + 4*I*c) + 9*I*a^8*e^(2*I*d*x + 2*I*c) - 3*I*a^8 + 12*(-I*a^8*e^(
2*I*d*x + 2*I*c) - I*a^8)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(2*I*d*x + 2*
I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.51

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{2ia^8}{de^{2ic}e^{2idx}+d} + \frac{8ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \begin{cases} \frac{-2ia^8d^2e^{6ic}e^{6idx}+6ia^8d^2e^{4ic}e^{4idx}-18ia^8d^2e^{2ic}e^{2idx}}{3d^3} & \text{for } d^3 \neq 0 \\ x(4a^8e^{6ic}-8a^8e^{4ic}+12a^8e^{2ic}) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**8,x)
```

output
```
2*I*a**8/(d*exp(2*I*c)*exp(2*I*d*x) + d) + 8*I*a**8*log(exp(2*I*d*x) + exp
(-2*I*c))/d + Piecewise(((-2*I*a**8*d**2*exp(6*I*c)*exp(6*I*d*x) + 6*I*a**
8*d**2*exp(4*I*c)*exp(4*I*d*x) - 18*I*a**8*d**2*exp(2*I*c)*exp(2*I*d*x))/(
3*d**3), Ne(d**3, 0)), (x*(4*a**8*exp(6*I*c) - 8*a**8*exp(4*I*c) + 12*a**8
*exp(2*I*c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.28

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx =$$
$$-\frac{24\,(dx+c)a^8 + 12i\,a^8\log\left(\tan\left(dx+c\right)^2+1\right) - 3\,a^8\tan\left(dx+c\right) - \frac{8\left(9\,a^8\tan(dx+c)^5 - 15i\,a^8\tan(dx+c)^4 + 4\,a^8\,\mathrm{t}\right.}{\tan(dx+c)^6 + 3\,\mathrm{tar}}}{3\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/3*(24*(d*x + c)*a^8 + 12*I*a^8*log(tan(d*x + c)^2 + 1) - 3*a^8*tan(d*x
+ c) - 8*(9*a^8*tan(d*x + c)^5 - 15*I*a^8*tan(d*x + c)^4 + 4*a^8*tan(d*x +
 c)^3 - 12*I*a^8*tan(d*x + c)^2 + 3*a^8*tan(d*x + c) - 5*I*a^8)/(tan(d*x +
 c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= -\frac{8i\,a^8\log\left(\tan\left(dx+c\right)+i\right)}{d} + \frac{a^8\tan\left(dx+c\right)}{d} + \frac{8\left(9\,a^8\tan\left(dx+c\right)^2+12i\,a^8\tan\left(dx+c\right)-5\,a^8\right)}{3\,d(\tan\left(dx+c\right)+i)^3}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-8*I*a^8*log(tan(d*x + c) + I)/d + a^8*tan(d*x + c)/d + 8/3*(9*a^8*tan(d*x
 + c)^2 + 12*I*a^8*tan(d*x + c) - 5*a^8)/(d*(tan(d*x + c) + I)^3)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\tan(c+d\,x)}{d} - \frac{24\,a^8\tan(c+d\,x)^2+a^8\tan(c+d\,x)\;32\mathrm{i}-\frac{40\,a^8}{3}}{d\,\left(-\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2 3\mathrm{i}+3\tan\left(c+d\,x\right)+1\mathrm{i}\right)} - \frac{a^8\;\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\;8\mathrm{i}}{d}$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(a^8*tan(c + d*x))/d - (a^8*log(tan(c + d*x) + 1i)*8i)/d - (a^8*tan(c + d*
x)*32i - (40*a^8)/3 + 24*a^8*tan(c + d*x)^2)/(d*(3*tan(c + d*x) - tan(c +
d*x)^2*3i - tan(c + d*x)^3 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.67

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(-24\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)i+24\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)i+24\cos(dx+c\right.}{}$$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(a**8*( - 24*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*i + 24*cos(c + d*x)
*log(tan((c + d*x)/2) - 1)*i + 24*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*i
 + 64*cos(c + d*x)*sin(c + d*x)**6*i - 48*cos(c + d*x)*sin(c + d*x)**4*i +
 24*cos(c + d*x)*sin(c + d*x)**2*i - 24*cos(c + d*x)*c - 24*cos(c + d*x)*d
*x - 64*sin(c + d*x)**7 + 80*sin(c + d*x)**5 - 40*sin(c + d*x)**3 + 27*sin
(c + d*x)))/(3*cos(c + d*x)*d)
```

3.85 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 43

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx=-\frac{i(a^3+ia^3\tan(c+dx))^4}{8d(a-ia\tan(c+dx))^4}$$

output
```
-1/8*I*(a^3+I*a^3*tan(d*x+c))^4/d/(a-I*a*tan(d*x+c))^4
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx=-\frac{ia^8(\cos(c+dx)+i\sin(c+dx))^8}{8d}$$

input
```
Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/8*I)*a^8*(Cos[c + d*x] + I*Sin[c + d*x])^8)/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^8}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^9\int \frac{(i\tan(c+dx)a+a)^3}{(a-ia\tan(c+dx))^5}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{48}$$

$$-\frac{ia^8(a+ia\tan(c+dx))^4}{8d(a-ia\tan(c+dx))^4}$$

input
```
Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/8*I)*a^8*(a + I*a*Tan[c + d*x])^4)/(d*(a - I*a*Tan[c + d*x])^4)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(38) = 76.

Time = 0.77 (sec) , antiderivative size = 451, normalized size of antiderivative = 10.49

$$a^{8}\left(-\frac{\left(\sin(dx+c)^{7}+\frac{7\sin(dx+c)^{5}}{6}+\frac{35\sin(dx+c)^{3}}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)-ia^{8}\sin\left(dx+c\right)^{8}-28a^{8}\left(-\frac{\sin(dx+\cdots}{\cdots}\right.$$

input
```
int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(-1/8*(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin
(d*x+c))*cos(d*x+c)+35/128*d*x+35/128*c)-I*a^8*sin(d*x+c)^8-28*a^8*(-1/8*s
in(d*x+c)^5*cos(d*x+c)^3-5/48*sin(d*x+c)^3*cos(d*x+c)^3-5/64*sin(d*x+c)*co
s(d*x+c)^3+5/128*cos(d*x+c)*sin(d*x+c)+5/128*d*x+5/128*c)-I*a^8*cos(d*x+c)
^8+70*a^8*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/6
4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)-56*I*a^8*(-1
/8*cos(d*x+c)^6*sin(d*x+c)^2-1/24*cos(d*x+c)^6)-28*a^8*(-1/8*sin(d*x+c)*co
s(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)
+5/128*d*x+5/128*c)+56*I*a^8*(-1/8*cos(d*x+c)^4*sin(d*x+c)^4-1/12*cos(d*x+
c)^4*sin(d*x+c)^2-1/24*cos(d*x+c)^4)+a^8*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)
^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.40

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i\,a^8e^{(8i\,dx+8i\,c)}}{8\,d}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-1/8*I*a^8*e^(8*I*d*x + 8*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx = \begin{cases} -\frac{ia^8e^{8ic}e^{8idx}}{8d} & \text{for } d\neq 0 \\ a^8xe^{8ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise((-I*a**8*exp(8*I*c)*exp(8*I*d*x)/(8*d), Ne(d, 0)), (a**8*x*exp(8
*I*c), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(35) = 70.

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.16

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{a^8\tan(dx+c)^7 - 4i\,a^8\tan(dx+c)^6 - 7\,a^8\tan(dx+c)^5 + 8i\,a^8\tan(dx+c)^4 + 7\,a^8\tan(dx+c)^3 -}{(\tan(dx+c)^8 + 4\,\tan(dx+c)^6 + 6\,\tan(dx+c)^4 + 4\,\tan(dx+c)^2 +}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-(a^8*tan(d*x + c)^7 - 4*I*a^8*tan(d*x + c)^6 - 7*a^8*tan(d*x + c)^5 + 8*I
*a^8*tan(d*x + c)^4 + 7*a^8*tan(d*x + c)^3 - 4*I*a^8*tan(d*x + c)^2 - a^8*
tan(d*x + c))/((tan(d*x + c)^8 + 4*tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*t
an(d*x + c)^2 + 1)*d)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{a^8\tan\left(dx+c\right)^3 - a^8\tan\left(dx+c\right)}{d(\tan\left(dx+c\right)+i)^4}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-(a^8*tan(d*x + c)^3 - a^8*tan(d*x + c))/(d*(tan(d*x + c) + I)^4)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= -\frac{a^8\tan(c+d\,x)\,\left(\tan(c+d\,x)^2-1\right)}{d\,\left(\tan\left(c+d\,x\right)^4+\tan\left(c+d\,x\right)^3\,4\mathrm{i}-6\tan\left(c+d\,x\right)^2-\tan\left(c+d\,x\right)\,4\mathrm{i}+1\right)}$$

input
```
int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
-(a^8*tan(c + d*x)*(tan(c + d*x)^2 - 1))/(d*(tan(c + d*x)^3*4i - 6*tan(c +
 d*x)^2 - tan(c + d*x)*4i + tan(c + d*x)^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.56

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{\sin(dx+c)\,a^8\left(-16\cos(dx+c)\sin(dx+c)^6+24\cos(dx+c)\sin(dx+c)^4-10\cos(dx+c)\sin(dx+c\right.}{d}$$

input
```
int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(sin(c + d*x)*a**8*( - 16*cos(c + d*x)*sin(c + d*x)**6 + 24*cos(c + d*x)*s
in(c + d*x)**4 - 10*cos(c + d*x)*sin(c + d*x)**2 + cos(c + d*x) - 16*sin(c
 + d*x)**7*i + 32*sin(c + d*x)**5*i - 20*sin(c + d*x)**3*i + 4*sin(c + d*x
)*i))/d
```

3.86 $\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{4ia^{13}}{5d(a-ia\tan(c+dx))^5} + \frac{ia^{12}}{d(a-ia\tan(c+dx))^4} - \frac{ia^{11}}{3d(a-ia\tan(c+dx))^3}$$

output
```
-4/5*I*a^13/d/(a-I*a*tan(d*x+c))^5+I*a^12/d/(a-I*a*tan(d*x+c))^4-1/3*I*a^1
1/d/(a-I*a*tan(d*x+c))^3
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.55

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{a^8(-2-5i\tan(c+dx)+5\tan^2(c+dx))}{15d(i+\tan(c+dx))^5}$$

input
```
Integrate[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
-1/15*(a^8*(-2 - (5*I)*Tan[c + d*x] + 5*Tan[c + d*x]^2))/(d*(I + Tan[c + d
*x])^5)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^{10}}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{ia^{11}\int \frac{(i\tan(c+dx)a+a)^2}{(a-ia\tan(c+dx))^6}d(ia\tan(c+dx))}{d}$$
$$\Big\downarrow \text{53}$$
$$-\frac{ia^{11}\int \left(\frac{4a^2}{(a-ia\tan(c+dx))^6}-\frac{4a}{(a-ia\tan(c+dx))^5}+\frac{1}{(a-ia\tan(c+dx))^4}\right)d(ia\tan(c+dx))}{d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{ia^{11}\left(\frac{4a^2}{5(a-ia\tan(c+dx))^5}-\frac{a}{(a-ia\tan(c+dx))^4}+\frac{1}{3(a-ia\tan(c+dx))^3}\right)}{d}$$

input
```
Int[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*a^11*((4*a^2)/(5*(a - I*a*Tan[c + d*x])^5) - a/(a - I*a*Tan[c + d*x]
)^4 + 1/(3*(a - I*a*Tan[c + d*x])^3)))/d
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(70) = 140$.

Time = 1.28 (sec) , antiderivative size = 588, normalized size of antiderivative = 7.35

$$a^8\left(-\frac{\sin(dx+c)^7\cos(dx+c)^3}{10}-\frac{7\sin(dx+c)^5\cos(dx+c)^3}{80}-\frac{7\sin(dx+c)^3\cos(dx+c)^3}{96}-\frac{7\sin(dx+c)\cos(dx+c)^3}{128}+\frac{7\cos(dx+c)\sin(dx+c)}{256}\right.$$

input
```
int(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(-1/10*sin(d*x+c)^7*cos(d*x+c)^3-7/80*sin(d*x+c)^5*cos(d*x+c)^3-7
/96*sin(d*x+c)^3*cos(d*x+c)^3-7/128*sin(d*x+c)*cos(d*x+c)^3+7/256*cos(d*x+
c)*sin(d*x+c)+7/256*d*x+7/256*c)-8*I*a^8*(-1/10*cos(d*x+c)^4*sin(d*x+c)^6-
3/40*cos(d*x+c)^4*sin(d*x+c)^4-1/20*cos(d*x+c)^4*sin(d*x+c)^2-1/40*cos(d*x
+c)^4)-28*a^8*(-1/10*sin(d*x+c)^5*cos(d*x+c)^5-1/16*sin(d*x+c)^3*cos(d*x+c
)^5-1/32*sin(d*x+c)*cos(d*x+c)^5+1/128*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d
*x+c)+3/256*d*x+3/256*c)+56*I*a^8*(-1/10*cos(d*x+c)^6*sin(d*x+c)^4-1/20*co
s(d*x+c)^6*sin(d*x+c)^2-1/60*cos(d*x+c)^6)+70*a^8*(-1/10*sin(d*x+c)^3*cos(
d*x+c)^7-3/80*sin(d*x+c)*cos(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3
+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c)-56*I*a^8*(-1/10*cos(d*x+c)
^8*sin(d*x+c)^2-1/40*cos(d*x+c)^8)-28*a^8*(-1/10*sin(d*x+c)*cos(d*x+c)^9+1
/80*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*si
n(d*x+c)+7/256*d*x+7/256*c)-4/5*I*a^8*cos(d*x+c)^10+a^8*(1/10*(cos(d*x+c)^
9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+
c))*sin(d*x+c)+63/256*d*x+63/256*c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{-6i\,a^8e^{(10i\,dx+10i\,c)} - 15i\,a^8e^{(8i\,dx+8i\,c)} - 10i\,a^8e^{(6i\,dx+6i\,c)}}{240\,d}$$

input
```
integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/240*(-6*I*a^8*e^(10*I*d*x + 10*I*c) - 15*I*a^8*e^(8*I*d*x + 8*I*c) - 10*
I*a^8*e^(6*I*d*x + 6*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.51

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx = \begin{cases} \frac{-384ia^8d^2e^{10ic}e^{10idx}-960ia^8d^2e^{8ic}e^{8idx}-640ia^8d^2e^{6ic}e^{6idx}}{15360d^3} & \text{for } d^3 \neq 0 \\ x\left(\frac{a^8e^{10ic}}{4}+\frac{a^8e^{8ic}}{2}+\frac{a^8e^{6ic}}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**10*(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-384*I*a**8*d**2*exp(10*I*c)*exp(10*I*d*x) - 960*I*a**8*d**2*e
xp(8*I*c)*exp(8*I*d*x) - 640*I*a**8*d**2*exp(6*I*c)*exp(6*I*d*x))/(15360*d
**3), Ne(d**3, 0)), (x*(a**8*exp(10*I*c)/4 + a**8*exp(8*I*c)/2 + a**8*exp(
6*I*c)/4), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(64) = 128.

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.90

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{5\,a^8\tan(dx+c)^7-30i\,a^8\tan(dx+c)^6-77\,a^8\tan(dx+c)^5+110i\,a^8\tan(dx+c)^4+95\,a^8\tan(dx-}{15\left(\tan(dx+c)^{10}+5\,\tan(dx+c)^8+10\,\tan(dx+c)^6+10\,\tan(dx+c\right.}$$

input
```
integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/15*(5*a^8*tan(d*x + c)^7 - 30*I*a^8*tan(d*x + c)^6 - 77*a^8*tan(d*x + c
)^5 + 110*I*a^8*tan(d*x + c)^4 + 95*a^8*tan(d*x + c)^3 - 50*I*a^8*tan(d*x
+ c)^2 - 15*a^8*tan(d*x + c) + 2*I*a^8)/((tan(d*x + c)^10 + 5*tan(d*x + c)
^8 + 10*tan(d*x + c)^6 + 10*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)*d)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.56

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{5\,a^8\tan\left(dx+c\right)^2-5i\,a^8\tan\left(dx+c\right)-2\,a^8}{15\,d(\tan\left(dx+c\right)+i)^5}$$

input
```
integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-1/15*(5*a^8*tan(d*x + c)^2 - 5*I*a^8*tan(d*x + c) - 2*a^8)/(d*(tan(d*x +
c) + I)^5)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(-5\tan(c+d\,x)^2+\tan(c+d\,x)\;5\mathrm{i}+2\right)}{15\,d\;\left(\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4 5\mathrm{i}-10\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2 10\mathrm{i}+5\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(a^8*(tan(c + d*x)*5i - 5*tan(c + d*x)^2 + 2))/(15*d*(5*tan(c + d*x) - tan
(c + d*x)^2*10i - 10*tan(c + d*x)^3 + tan(c + d*x)^4*5i + tan(c + d*x)^5 +
 1i))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.75

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{\sin(dx+c)\,a^8\left(192\cos(dx+c)\sin(dx+c)^8-504\cos(dx+c)\sin(dx+c)^6+452\cos(dx+c)\sin(dx+\right.}{}$$

input
```
int(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(sin(c + d*x)*a**8*(192*cos(c + d*x)*sin(c + d*x)**8 - 504*cos(c + d*x)*si
n(c + d*x)**6 + 452*cos(c + d*x)*sin(c + d*x)**4 - 155*cos(c + d*x)*sin(c
+ d*x)**2 + 15*cos(c + d*x) + 192*sin(c + d*x)**9*i - 600*sin(c + d*x)**7*
i + 680*sin(c + d*x)**5*i - 330*sin(c + d*x)**3*i + 60*sin(c + d*x)*i))/(1
5*d)
```

3.87 $\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{ia^{14}}{3d(a-ia\tan(c+dx))^6} + \frac{ia^{13}}{5d(a-ia\tan(c+dx))^5}$$

output
```
-1/3*I*a^14/d/(a-I*a*tan(d*x+c))^6+1/5*I*a^13/d/(a-I*a*tan(d*x+c))^5
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{a^8(-2i+3\tan(c+dx))}{15d(i+\tan(c+dx))^6}$$

input
```
Integrate[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
-1/15*(a^8*(-2*I + 3*Tan[c + d*x]))/(d*(I + Tan[c + d*x])^6)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^{12}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^{13}\int \frac{i\tan(c+dx)a+a}{(a-ia\tan(c+dx))^7}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{ia^{13}\int \left(\frac{2a}{(a-ia\tan(c+dx))^7}-\frac{1}{(a-ia\tan(c+dx))^6}\right)d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^{13}\left(\frac{a}{3(a-ia\tan(c+dx))^6}-\frac{1}{5(a-ia\tan(c+dx))^5}\right)}{d}$$

input
```
Int[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*a^13*(a/(3*(a - I*a*Tan[c + d*x])^6) - 1/(5*(a - I*a*Tan[c + d*x])^5
)))/d
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(47) = 94$.

Time = 0.86 (sec) , antiderivative size = 639, normalized size of antiderivative = 11.62

$$a^{8}\left(-\frac{\sin(dx+c)^{7}\cos(dx+c)^{5}}{12}-\frac{7\sin(dx+c)^{5}\cos(dx+c)^{5}}{120}-\frac{7\sin(dx+c)^{3}\cos(dx+c)^{5}}{192}-\frac{7\sin(dx+c)\cos(dx+c)^{5}}{384}+\frac{7\left(\cos(dx+c)^{3}+\frac{3\cos(d}{2}\right.}{1536}\right.$$

input
```
int(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(-1/12*sin(d*x+c)^7*cos(d*x+c)^5-7/120*sin(d*x+c)^5*cos(d*x+c)^5-
7/192*sin(d*x+c)^3*cos(d*x+c)^5-7/384*sin(d*x+c)*cos(d*x+c)^5+7/1536*(cos(
d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+7/1024*d*x+7/1024*c)+56*I*a^8*(-1/12*c
os(d*x+c)^8*sin(d*x+c)^4-1/30*cos(d*x+c)^8*sin(d*x+c)^2-1/120*cos(d*x+c)^8
)-28*a^8*(-1/12*sin(d*x+c)^5*cos(d*x+c)^7-1/24*sin(d*x+c)^3*cos(d*x+c)^7-1
/64*sin(d*x+c)*cos(d*x+c)^7+1/384*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(
d*x+c))*sin(d*x+c)+5/1024*d*x+5/1024*c)-56*I*a^8*(-1/12*cos(d*x+c)^10*sin(
d*x+c)^2-1/60*cos(d*x+c)^10)+70*a^8*(-1/12*sin(d*x+c)^3*cos(d*x+c)^9-1/40*
sin(d*x+c)*cos(d*x+c)^9+1/320*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x
+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+7/1024*d*x+7/1024*c)-8*I*a^8*(-1/12*cos
(d*x+c)^6*sin(d*x+c)^6-1/20*cos(d*x+c)^6*sin(d*x+c)^4-1/40*cos(d*x+c)^6*si
n(d*x+c)^2-1/120*cos(d*x+c)^6)-28*a^8*(-1/12*sin(d*x+c)*cos(d*x+c)^11+1/12
0*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+31
5/128*cos(d*x+c))*sin(d*x+c)+21/1024*d*x+21/1024*c)-2/3*I*a^8*cos(d*x+c)^1
2+a^8*(1/12*(cos(d*x+c)^11+11/10*cos(d*x+c)^9+99/80*cos(d*x+c)^7+231/160*c
os(d*x+c)^5+231/128*cos(d*x+c)^3+693/256*cos(d*x+c))*sin(d*x+c)+231/1024*d
*x+231/1024*c))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.38

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{-5i\,a^8e^{(12i\,dx+12i\,c)}-24i\,a^8e^{(10i\,dx+10i\,c)}-45i\,a^8e^{(8i\,dx+8i\,c)}-40i\,a^8e^{(6i\,dx+6i\,c)}-15i\,a^8e^{(4i\,dx+4i\,c)}}{960\,d}$$

input
```
integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/960*(-5*I*a^8*e^(12*I*d*x + 12*I*c) - 24*I*a^8*e^(10*I*d*x + 10*I*c) - 4
5*I*a^8*e^(8*I*d*x + 8*I*c) - 40*I*a^8*e^(6*I*d*x + 6*I*c) - 15*I*a^8*e^(4
*I*d*x + 4*I*c))/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(42) = 84$.

Time = 0.56 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.58

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \begin{cases} \frac{-3932160ia^8d^4e^{12ic}e^{12idx}-18874368ia^8d^4e^{10ic}e^{10idx}-35389440ia^8d^4e^{8ic}e^{8idx}-31457280ia^8d^4e^{6ic}e^{6idx}-11796480ia^8d^4e^{4ic}e^{4idx}}{754974720d^5} & \text{for} \\ x\left(\frac{a^8e^{12ic}}{16}+\frac{a^8e^{10ic}}{4}+\frac{3a^8e^{8ic}}{8}+\frac{a^8e^{6ic}}{4}+\frac{a^8e^{4ic}}{16}\right) & \text{otl} \end{cases}$$

input
```
integrate(cos(d*x+c)**12*(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-3932160*I*a**8*d**4*exp(12*I*c)*exp(12*I*d*x) - 18874368*I*a*
*8*d**4*exp(10*I*c)*exp(10*I*d*x) - 35389440*I*a**8*d**4*exp(8*I*c)*exp(8*
I*d*x) - 31457280*I*a**8*d**4*exp(6*I*c)*exp(6*I*d*x) - 11796480*I*a**8*d*
*4*exp(4*I*c)*exp(4*I*d*x))/(754974720*d**5), Ne(d**5, 0)), (x*(a**8*exp(1
2*I*c)/16 + a**8*exp(10*I*c)/4 + 3*a**8*exp(8*I*c)/8 + a**8*exp(6*I*c)/4 +
 a**8*exp(4*I*c)/16), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(43) = 86$.

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.95

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$
$$-\frac{3\,a^8\tan(dx+c)^7-20i\,a^8\tan(dx+c)^6-57\,a^8\tan(dx+c)^5+90i\,a^8\tan(dx+c)^4+85\,a^8\tan(dx+}{15\left(\tan(dx+c)^{12}+6\,\tan(dx+c)^{10}+15\,\tan(dx+c)^8+20\,\tan(dx+c)^6+15\,\text{ta}\right.}$$

input
```
integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/15*(3*a^8*tan(d*x + c)^7 - 20*I*a^8*tan(d*x + c)^6 - 57*a^8*tan(d*x + c
)^5 + 90*I*a^8*tan(d*x + c)^4 + 85*a^8*tan(d*x + c)^3 - 48*I*a^8*tan(d*x +
 c)^2 - 15*a^8*tan(d*x + c) + 2*I*a^8)/((tan(d*x + c)^12 + 6*tan(d*x + c)^
10 + 15*tan(d*x + c)^8 + 20*tan(d*x + c)^6 + 15*tan(d*x + c)^4 + 6*tan(d*x
 + c)^2 + 1)*d)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.58

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{3\,a^8\tan\left(dx+c\right)-2i\,a^8}{15\,d(\tan\left(dx+c\right)+i)^6}$$

input
```
integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-1/15*(3*a^8*tan(d*x + c) - 2*I*a^8)/(d*(tan(d*x + c) + I)^6)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$
$$-\frac{a^8\left(3\tan(c+d\,x)-2\mathrm{i}\right)}{15\,d\,\left(\tan\left(c+d\,x\right)^6+\tan\left(c+d\,x\right)^5\,6\mathrm{i}-15\tan\left(c+d\,x\right)^4-\tan\left(c+d\,x\right)^3\,20\mathrm{i}+15\tan\left(c+d\,x\right)^2+\mathrm{ta}\right.}$$

input
```
int(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
-(a^8*(3*tan(c + d*x) - 2i))/(15*d*(tan(c + d*x)*6i + 15*tan(c + d*x)^2 -
tan(c + d*x)^3*20i - 15*tan(c + d*x)^4 + tan(c + d*x)^5*6i + tan(c + d*x)^
6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.04

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{\sin(dx+c)\,a^8\left(-160\cos(dx+c)\sin(dx+c)^{10}+592\cos(dx+c)\sin(dx+c)^8-834\cos(dx+c)\sin(dx+c)\right.}{}$$

input
```
int(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(sin(c + d*x)*a**8*( - 160*cos(c + d*x)*sin(c + d*x)**10 + 592*cos(c + d*x
)*sin(c + d*x)**8 - 834*cos(c + d*x)*sin(c + d*x)**6 + 547*cos(c + d*x)*si
n(c + d*x)**4 - 160*cos(c + d*x)*sin(c + d*x)**2 + 15*cos(c + d*x) - 160*s
in(c + d*x)**11*i + 672*sin(c + d*x)**9*i - 1110*sin(c + d*x)**7*i + 900*s
in(c + d*x)**5*i - 360*sin(c + d*x)**3*i + 60*sin(c + d*x)*i))/(15*d)
```

3.88 $\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{ia^{15}}{7d(a-ia\tan(c+dx))^7}$$

output
```
-1/7*I*a^15/d/(a-I*a*tan(d*x+c))^7
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 116 vs. 2(27) = 54.

Time = 2.87 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.30

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8(35+56\cos(2(c+dx))+28\cos(4(c+dx))+8\cos(6(c+dx))-14i\sin(2(c+dx))-14i\sin(4(c+d}{896d(\cos(dx)+i\sin(dx))^8}$$

input
```
Integrate[Cos[c + d*x]^14*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*(35 + 56*Cos[2*(c + d*x)] + 28*Cos[4*(c + d*x)] + 8*Cos[6*(c + d*x)]
- (14*I)*Sin[2*(c + d*x)] - (14*I)*Sin[4*(c + d*x)] - (6*I)*Sin[6*(c + d*x
)])*((-I)*Cos[8*(c + 2*d*x)] + Sin[8*(c + 2*d*x)]))/(896*d*(Cos[d*x] + I*S
in[d*x])^8)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^{14}}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{ia^{15}\int \frac{1}{(a-ia\tan(c+dx))^8}d(ia\tan(c+dx))}{d}$$
$$\Big\downarrow \text{17}$$
$$-\frac{ia^{15}}{7d(a-ia\tan(c+dx))^7}$$

input
```
Int[Cos[c + d*x]^14*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/7*I)*a^15)/(d*(a - I*a*Tan[c + d*x])^7)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(23) = 46$.

Time = 0.87 (sec) , antiderivative size = 689, normalized size of antiderivative = 25.52

$$a^{8}\left(-\frac{\sin(dx+c)^{7}\cos(dx+c)^{7}}{14}-\frac{\sin(dx+c)^{5}\cos(dx+c)^{7}}{24}-\frac{\sin(dx+c)^{3}\cos(dx+c)^{7}}{48}-\frac{\sin(dx+c)\cos(dx+c)^{7}}{128}+\frac{\left(\cos(dx+c)^{5}+\frac{5\cos(dx+c)^{3}}{4}\right.}{76}\right.$$

input
```
int(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(-1/14*sin(d*x+c)^7*cos(d*x+c)^7-1/24*sin(d*x+c)^5*cos(d*x+c)^7-1
/48*sin(d*x+c)^3*cos(d*x+c)^7-1/128*sin(d*x+c)*cos(d*x+c)^7+1/768*(cos(d*x
+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2048*d*x+5/2048*c)-8*
I*a^8*(-1/14*cos(d*x+c)^8*sin(d*x+c)^6-1/28*cos(d*x+c)^8*sin(d*x+c)^4-1/70
*cos(d*x+c)^8*sin(d*x+c)^2-1/280*cos(d*x+c)^8)-28*a^8*(-1/14*sin(d*x+c)^5*
cos(d*x+c)^9-5/168*sin(d*x+c)^3*cos(d*x+c)^9-1/112*sin(d*x+c)*cos(d*x+c)^9
+1/896*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))
*sin(d*x+c)+5/2048*d*x+5/2048*c)-56*I*a^8*(-1/14*cos(d*x+c)^12*sin(d*x+c)^
2-1/84*cos(d*x+c)^12)+70*a^8*(-1/14*sin(d*x+c)^3*cos(d*x+c)^11-1/56*sin(d*
x+c)*cos(d*x+c)^11+1/560*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5
+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)+9/2048*d*x+9/2048*c)-4
/7*I*a^8*cos(d*x+c)^14-28*a^8*(-1/14*sin(d*x+c)*cos(d*x+c)^13+1/168*(cos(d
*x+c)^11+11/10*cos(d*x+c)^9+99/80*cos(d*x+c)^7+231/160*cos(d*x+c)^5+231/12
8*cos(d*x+c)^3+693/256*cos(d*x+c))*sin(d*x+c)+33/2048*d*x+33/2048*c)+56*I*
a^8*(-1/14*cos(d*x+c)^10*sin(d*x+c)^4-1/42*cos(d*x+c)^10*sin(d*x+c)^2-1/21
0*cos(d*x+c)^10)+a^8*(1/14*(cos(d*x+c)^13+13/12*cos(d*x+c)^11+143/120*cos(
d*x+c)^9+429/320*cos(d*x+c)^7+1001/640*cos(d*x+c)^5+1001/512*cos(d*x+c)^3+
3003/1024*cos(d*x+c))*sin(d*x+c)+429/2048*d*x+429/2048*c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(21) = 42$.

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.85

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{-i\,a^8e^{(14i\,dx+14i\,c)} - 7i\,a^8e^{(12i\,dx+12i\,c)} - 21i\,a^8e^{(10i\,dx+10i\,c)} - 35i\,a^8e^{(8i\,dx+8i\,c)} - 35i\,a^8e^{(6i\,dx+6i\,c)} - 21i\,a^8e^{(}}{896\,d}$$

input
```
integrate(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/896*(-I*a^8*e^(14*I*d*x + 14*I*c) - 7*I*a^8*e^(12*I*d*x + 12*I*c) - 21*I
*a^8*e^(10*I*d*x + 10*I*c) - 35*I*a^8*e^(8*I*d*x + 8*I*c) - 35*I*a^8*e^(6*
I*d*x + 6*I*c) - 21*I*a^8*e^(4*I*d*x + 4*I*c) - 7*I*a^8*e^(2*I*d*x + 2*I*c
))/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(22) = 44$.

Time = 0.66 (sec) , antiderivative size = 279, normalized size of antiderivative = 10.33

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \begin{cases} \frac{-4398046511104ia^8d^6e^{14ic}e^{14idx} - 30786325577728ia^8d^6e^{12ic}e^{12idx} - 92358976733184ia^8d^6e^{10ic}e^{10idx} - 153931627888640ia^8d^6e^{8ic}e^{8idx} - \ldots}{3940649673949184d^7} \\ x\left(\frac{a^8e^{14ic}}{64} + \frac{3a^8e^{12ic}}{32} + \frac{15a^8e^{10ic}}{64} + \frac{5a^8e^{8ic}}{16} + \frac{15a^8e^{6ic}}{64} + \frac{3a^8e^{4ic}}{32} + \frac{a^8e^{2ic}}{64}\right) \end{cases}$$

input
```
integrate(cos(d*x+c)**14*(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-4398046511104*I*a**8*d**6*exp(14*I*c)*exp(14*I*d*x) - 3078632
5577728*I*a**8*d**6*exp(12*I*c)*exp(12*I*d*x) - 92358976733184*I*a**8*d**6
*exp(10*I*c)*exp(10*I*d*x) - 153931627888640*I*a**8*d**6*exp(8*I*c)*exp(8*
I*d*x) - 153931627888640*I*a**8*d**6*exp(6*I*c)*exp(6*I*d*x) - 92358976733
184*I*a**8*d**6*exp(4*I*c)*exp(4*I*d*x) - 30786325577728*I*a**8*d**6*exp(2
*I*c)*exp(2*I*d*x))/(3940649673949184*d**7), Ne(d**7, 0)), (x*(a**8*exp(14
*I*c)/64 + 3*a**8*exp(12*I*c)/32 + 15*a**8*exp(10*I*c)/64 + 5*a**8*exp(8*I
*c)/16 + 15*a**8*exp(6*I*c)/64 + 3*a**8*exp(4*I*c)/32 + a**8*exp(2*I*c)/64
), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(21) = 42$.

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 6.33

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$
$$-\frac{a^8\tan(dx+c)^7 - 7i\,a^8\tan(dx+c)^6 - 21\,a^8\tan(dx+c)^5 + 35i\,a^8\tan(dx+c)^4 + 35\,a^8\tan(dx+c)^3 \ldots}{7\left(\tan(dx+c)^{14} + 7\,\tan(dx+c)^{12} + 21\,\tan(dx+c)^{10} + 35\,\tan(dx+c)^8 + 35\,\tan(dx+c)^6 \ldots\right.}$$

input
```
integrate(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/7*(a^8*tan(d*x + c)^7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5
+ 35*I*a^8*tan(d*x + c)^4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^
2 - 7*a^8*tan(d*x + c) + I*a^8)/((tan(d*x + c)^14 + 7*tan(d*x + c)^12 + 21
*tan(d*x + c)^10 + 35*tan(d*x + c)^8 + 35*tan(d*x + c)^6 + 21*tan(d*x + c)
^4 + 7*tan(d*x + c)^2 + 1)*d)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{a^8}{7\,d(\tan{(dx+c)}+i)^7}$$

input
```
integrate(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-1/7*a^8/(d*(tan(d*x + c) + I)^7)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.89

$$\begin{aligned}\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{a^8\cos{(c+d\,x)}^8\,(\tan(c+d\,x)-7\mathrm{i})}{7\,d}\\ &\quad+\frac{64\,a^8\cos{(c+d\,x)}^{14}\,(\tan(c+d\,x)-\mathrm{i})}{7\,d}\\ &\quad+\frac{8\,a^8\cos{(c+d\,x)}^{10}\,(3\tan(c+d\,x)-7\mathrm{i})}{7\,d}\\ &\quad-\frac{16\,a^8\cos{(c+d\,x)}^{12}\,(5\tan(c+d\,x)-7\mathrm{i})}{7\,d}\end{aligned}$$

input
```
int(cos(c + d*x)^14*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(64*a^8*cos(c + d*x)^14*(tan(c + d*x) - 1i))/(7*d) - (a^8*cos(c + d*x)^8*(
tan(c + d*x) - 7i))/(7*d) + (8*a^8*cos(c + d*x)^10*(3*tan(c + d*x) - 7i))/
(7*d) - (16*a^8*cos(c + d*x)^12*(5*tan(c + d*x) - 7i))/(7*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 194, normalized size of antiderivative = 7.19

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{\sin(dx+c)\,a^8\left(64\cos(dx+c)\sin(dx+c)^{12}-304\cos(dx+c)\sin(dx+c)^{10}+584\cos(dx+c)\sin(dx-\right.}{}$$

input
```
int(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(sin(c + d*x)*a**8*(64*cos(c + d*x)*sin(c + d*x)**12 - 304*cos(c + d*x)*si
n(c + d*x)**10 + 584*cos(c + d*x)*sin(c + d*x)**8 - 575*cos(c + d*x)*sin(c
 + d*x)**6 + 301*cos(c + d*x)*sin(c + d*x)**4 - 77*cos(c + d*x)*sin(c + d*
x)**2 + 7*cos(c + d*x) + 64*sin(c + d*x)**13*i - 336*sin(c + d*x)**11*i +
728*sin(c + d*x)**9*i - 833*sin(c + d*x)**7*i + 532*sin(c + d*x)**5*i - 18
2*sin(c + d*x)**3*i + 28*sin(c + d*x)*i))/(7*d)
```

3.89 $\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 233

$$\begin{aligned}\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx =& \frac{a^8x}{256} - \frac{ia^{16}}{16d(a-ia\tan(c+dx))^8} \\ &- \frac{ia^{15}}{28d(a-ia\tan(c+dx))^7} \\ &- \frac{ia^{14}}{48d(a-ia\tan(c+dx))^6} \\ &- \frac{ia^{13}}{80d(a-ia\tan(c+dx))^5} \\ &- \frac{ia^{12}}{128d(a-ia\tan(c+dx))^4} \\ &- \frac{ia^{10}}{256d(a-ia\tan(c+dx))^2} \\ &- \frac{ia^{17}}{192d\,(a^3-ia^3\tan(c+dx))^3} \\ &- \frac{ia^{17}}{256d\,(a^9-ia^9\tan(c+dx))}\end{aligned}$$

output
```
1/256*a^8*x-1/16*I*a^16/d/(a-I*a*tan(d*x+c))^8-1/28*I*a^15/d/(a-I*a*tan(d*
x+c))^7-1/48*I*a^14/d/(a-I*a*tan(d*x+c))^6-1/80*I*a^13/d/(a-I*a*tan(d*x+c)
)^5-1/128*I*a^12/d/(a-I*a*tan(d*x+c))^4-1/256*I*a^10/d/(a-I*a*tan(d*x+c))^
2-1/192*I*a^17/d/(a^3-I*a^3*tan(d*x+c))^3-1/256*I*a^17/d/(a^9-I*a^9*tan(d*
x+c))
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.65

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$
$$-\frac{ia^8\sec^8(c+dx)(7350+12544\cos(2(c+dx))+7840\cos(4(c+dx))+3840\cos(6(c+dx))+1194\cos}{}$$

input
```
Integrate[Cos[c + d*x]^16*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/215040*I)*a^8*Sec[c + d*x]^8*(7350 + 12544*Cos[2*(c + d*x)] + 7840*Co
s[4*(c + d*x)] + 3840*Cos[6*(c + d*x)] + 1194*Cos[8*(c + d*x)] - (3136*I)*
Sin[2*(c + d*x)] - (3920*I)*Sin[4*(c + d*x)] - (2880*I)*Sin[6*(c + d*x)] -
 (1089*I)*Sin[8*(c + d*x)] + 840*ArcTan[Tan[c + d*x]]*(I*Cos[8*(c + d*x)]
+ Sin[8*(c + d*x)])))/(d*(I + Tan[c + d*x])^8)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^{16}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^{17}\int \frac{1}{(a-ia\tan(c+dx))^9(i\tan(c+dx)a+a)}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^{17}\int \left(\frac{1}{2(a-ia\tan(c+dx))^9a}+\frac{1}{4(a-ia\tan(c+dx))^8a^2}+\frac{1}{8(a-ia\tan(c+dx))^7a^3}+\frac{1}{16(a-ia\tan(c+dx))^6a^4}+\frac{1}{32(a-ia\tan(c+dx))^5a^5}\right.}{}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^{17}\left(\frac{i\arctan(\tan(c+dx))}{256a^9}+\frac{1}{256a^8(a-ia\tan(c+dx))}+\frac{1}{256a^7(a-ia\tan(c+dx))^2}+\frac{1}{192a^6(a-ia\tan(c+dx))^3}+\frac{1}{128a^5(a-ia\tan(c+dx))}\right.}{d}$$

input
```
Int[Cos[c + d*x]^16*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*a^17*(((I/256)*ArcTan[Tan[c + d*x]])/a^9 + 1/(16*a*(a - I*a*Tan[c +
d*x])^8) + 1/(28*a^2*(a - I*a*Tan[c + d*x])^7) + 1/(48*a^3*(a - I*a*Tan[c
+ d*x])^6) + 1/(80*a^4*(a - I*a*Tan[c + d*x])^5) + 1/(128*a^5*(a - I*a*Tan
[c + d*x])^4) + 1/(192*a^6*(a - I*a*Tan[c + d*x])^3) + 1/(256*a^7*(a - I*a
*Tan[c + d*x])^2) + 1/(256*a^8*(a - I*a*Tan[c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(199) = 398$.

Time = 0.84 (sec) , antiderivative size = 739, normalized size of antiderivative = 3.17

Expression too large to display

input
```
int(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(-1/16*sin(d*x+c)^7*cos(d*x+c)^9-1/32*sin(d*x+c)^5*cos(d*x+c)^9-5
/384*sin(d*x+c)^3*cos(d*x+c)^9-1/256*sin(d*x+c)*cos(d*x+c)^9+1/2048*(cos(d
*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+3
5/32768*d*x+35/32768*c)-8*I*a^8*(-1/16*cos(d*x+c)^10*sin(d*x+c)^6-3/112*co
s(d*x+c)^10*sin(d*x+c)^4-1/112*cos(d*x+c)^10*sin(d*x+c)^2-1/560*cos(d*x+c)
^10)-28*a^8*(-1/16*sin(d*x+c)^5*cos(d*x+c)^11-5/224*sin(d*x+c)^3*cos(d*x+c
)^11-5/896*sin(d*x+c)*cos(d*x+c)^11+1/1792*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+
21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)+45/3
2768*d*x+45/32768*c)-1/2*I*a^8*cos(d*x+c)^16+70*a^8*(-1/16*sin(d*x+c)^3*co
s(d*x+c)^13-3/224*sin(d*x+c)*cos(d*x+c)^13+1/896*(cos(d*x+c)^11+11/10*cos(
d*x+c)^9+99/80*cos(d*x+c)^7+231/160*cos(d*x+c)^5+231/128*cos(d*x+c)^3+693/
256*cos(d*x+c))*sin(d*x+c)+99/32768*d*x+99/32768*c)-56*I*a^8*(-1/16*cos(d*
x+c)^14*sin(d*x+c)^2-1/112*cos(d*x+c)^14)-28*a^8*(-1/16*sin(d*x+c)*cos(d*x
+c)^15+1/224*(cos(d*x+c)^13+13/12*cos(d*x+c)^11+143/120*cos(d*x+c)^9+429/3
20*cos(d*x+c)^7+1001/640*cos(d*x+c)^5+1001/512*cos(d*x+c)^3+3003/1024*cos(
d*x+c))*sin(d*x+c)+429/32768*d*x+429/32768*c)+56*I*a^8*(-1/16*cos(d*x+c)^1
2*sin(d*x+c)^4-1/56*cos(d*x+c)^12*sin(d*x+c)^2-1/336*cos(d*x+c)^12)+a^8*(1
/16*(cos(d*x+c)^15+15/14*cos(d*x+c)^13+65/56*cos(d*x+c)^11+143/112*cos(d*x
+c)^9+1287/896*cos(d*x+c)^7+429/256*cos(d*x+c)^5+2145/1024*cos(d*x+c)^3+64
35/2048*cos(d*x+c))*sin(d*x+c)+6435/32768*d*x+6435/32768*c))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.54

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{1680\,a^8dx - 105i\,a^8e^{(16i\,dx+16i\,c)} - 960i\,a^8e^{(14i\,dx+14i\,c)} - 3920i\,a^8e^{(12i\,dx+12i\,c)} - 9408i\,a^8e^{(10i\,dx+10i\,c)} - 14}{430080\,d}$$

input
```
integrate(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/430080*(1680*a^8*d*x - 105*I*a^8*e^(16*I*d*x + 16*I*c) - 960*I*a^8*e^(14
*I*d*x + 14*I*c) - 3920*I*a^8*e^(12*I*d*x + 12*I*c) - 9408*I*a^8*e^(10*I*d
*x + 10*I*c) - 14700*I*a^8*e^(8*I*d*x + 8*I*c) - 15680*I*a^8*e^(6*I*d*x +
6*I*c) - 11760*I*a^8*e^(4*I*d*x + 4*I*c) - 6720*I*a^8*e^(2*I*d*x + 2*I*c))
/d
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.39

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8x}{256} + \begin{cases} \frac{-354658470655426560ia^8d^7e^{16ic}e^{16idx} - 324259173170675712 0ia^8d^7e^{14ic}e^{14idx} - 13240582904469258240ia^8d^7e^{12ic}e^{12idx} - 31777398970}{} \\ x\left(\frac{a^8e^{16ic}}{256} + \frac{a^8e^{14ic}}{32} + \frac{7a^8e^{12ic}}{64} + \frac{7a^8e^{10ic}}{32} + \frac{35a^8e^{8ic}}{128} + \frac{7a^8e^{6ic}}{32} + \frac{7a^8e^{4ic}}{64} + \frac{a^8e^{2ic}}{32}\right) \end{cases}$$

input
```
integrate(cos(d*x+c)**16*(a+I*a*tan(d*x+c))**8,x)
```

output
```
a**8*x/256 + Piecewise(((-3546584706554265 60*I*a**8*d**7*exp(16*I*c)*exp(1
6*I*d*x) - 3242591731706757120*I*a**8*d**7*exp(14*I*c)*exp(14*I*d*x) - 132
40582904469258240*I*a**8*d**7*exp(12*I*c)*exp(12*I*d*x) - 3177739897072621
9776*I*a**8*d**7*exp(10*I*c)*exp(10*I*d*x) - 49652185891759718400*I*a**8*d
**7*exp(8*I*c)*exp(8*I*d*x) - 52962331617877032960*I*a**8*d**7*exp(6*I*c)*
exp(6*I*d*x) - 39721748713407774720*I*a**8*d**7*exp(4*I*c)*exp(4*I*d*x) -
22698142121947299840*I*a**8*d**7*exp(2*I*c)*exp(2*I*d*x))/(145268109580462
7189760*d**8), Ne(d**8, 0)), (x*(a**8*exp(16*I*c)/256 + a**8*exp(14*I*c)/3
2 + 7*a**8*exp(12*I*c)/64 + 7*a**8*exp(10*I*c)/32 + 35*a**8*exp(8*I*c)/128
 + 7*a**8*exp(6*I*c)/32 + 7*a**8*exp(4*I*c)/64 + a**8*exp(2*I*c)/32), True
))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.06

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{105\,(dx+c)a^8 + \frac{105\,a^8\tan(dx+c)^{15}+805\,a^8\tan(dx+c)^{13}+2681\,a^8\tan(dx+c)^{11}+5053\,a^8\tan(dx+c)^{9}+2883\,a^8\tan(dx+c)^{7}+21504i\,a^8\,\mathrm{t}}{\tan(dx+c)^{16}+8\,\tan(dx+c)^{14}+28\,\tan(dx+c)^{12}+56\,\tan(d}}{26880}$$

input
```
integrate(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/26880*(105*(d*x + c)*a^8 + (105*a^8*tan(d*x + c)^15 + 805*a^8*tan(d*x +
c)^13 + 2681*a^8*tan(d*x + c)^11 + 5053*a^8*tan(d*x + c)^9 + 2883*a^8*tan(
d*x + c)^7 + 21504*I*a^8*tan(d*x + c)^6 + 70791*a^8*tan(d*x + c)^5 - 11468
8*I*a^8*tan(d*x + c)^4 - 117285*a^8*tan(d*x + c)^3 + 74752*I*a^8*tan(d*x +
 c)^2 + 26775*a^8*tan(d*x + c) - 4096*I*a^8)/(tan(d*x + c)^16 + 8*tan(d*x
+ c)^14 + 28*tan(d*x + c)^12 + 56*tan(d*x + c)^10 + 70*tan(d*x + c)^8 + 56
*tan(d*x + c)^6 + 28*tan(d*x + c)^4 + 8*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.51

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{1}{53760}a^8\left(-\frac{105i\log\left(\tan\left(dx+c\right)+i\right)}{d}+\frac{105i\log\left(\tan\left(dx+c\right)-i\right)}{d}-\frac{2\left(105\tan\left(dx+c\right)^7+840i\,\mathrm{t}\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-1/53760*a^8*(-105*I*log(tan(d*x + c) + I)/d + 105*I*log(tan(d*x + c) - I)
/d - 2*(105*tan(d*x + c)^7 + 840*I*tan(d*x + c)^6 - 2975*tan(d*x + c)^5 -
6160*I*tan(d*x + c)^4 + 8351*tan(d*x + c)^3 + 8008*I*tan(d*x + c)^2 - 5993
*tan(d*x + c) - 4096*I)/(d*(tan(d*x + c) + I)^8))
```

Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.84

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8\,x}{256} - \frac{-\frac{a^8\tan(c+d\,x)^7}{256}-\frac{a^8\tan(c+d\,x)^6\,1\mathrm{i}}{32}+\frac{85\,a^8\tan(c+d\,x)^5}{768}+\frac{a^8\tan(c+d\,x)^4\,11\mathrm{i}}{48}-\frac{1193\,a^8\tan(c+d\,x)^3}{3840}-\frac{a^8}{}}{d\,\left(\tan\left(c+d\,x\right)^8+\tan\left(c+d\,x\right)^7\,8\mathrm{i}-28\tan\left(c+d\,x\right)^6-\tan\left(c+d\,x\right)^5\,56\mathrm{i}+70\tan\left(c+d\,x\right)^4+\tan\left(\right.\right.}$$

input
```
int(cos(c + d*x)^16*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(a^8*x)/256 - ((5993*a^8*tan(c + d*x))/26880 + (a^8*16i)/105 - (a^8*tan(c
+ d*x)^2*143i)/480 - (1193*a^8*tan(c + d*x)^3)/3840 + (a^8*tan(c + d*x)^4*
11i)/48 + (85*a^8*tan(c + d*x)^5)/768 - (a^8*tan(c + d*x)^6*1i)/32 - (a^8*
tan(c + d*x)^7)/256)/(d*(tan(c + d*x)^3*56i - 28*tan(c + d*x)^2 - tan(c +
d*x)*8i + 70*tan(c + d*x)^4 - tan(c + d*x)^5*56i - 28*tan(c + d*x)^6 + tan
(c + d*x)^7*8i + tan(c + d*x)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.97

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(-215040\cos(dx+c)\sin(dx+c)^{15}+1244160\cos(dx+c)\sin(dx+c)^{13}-3024640\cos(dx+c)\sin(\right.}{}$$

input
```
int(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(a**8*( - 215040*cos(c + d*x)*sin(c + d*x)**15 + 1244160*cos(c + d*x)*sin(
c + d*x)**13 - 3024640*cos(c + d*x)*sin(c + d*x)**11 + 3984256*cos(c + d*x
)*sin(c + d*x)**9 - 3047472*cos(c + d*x)*sin(c + d*x)**7 + 1336776*cos(c +
 d*x)*sin(c + d*x)**5 - 304710*cos(c + d*x)*sin(c + d*x)**3 + 26775*cos(c
+ d*x)*sin(c + d*x) - 215040*sin(c + d*x)**16*i + 1351680*sin(c + d*x)**14
*i - 3619840*sin(c + d*x)**12*i + 5354496*sin(c + d*x)**10*i - 4730880*sin
(c + d*x)**8*i + 2508800*sin(c + d*x)**6*i - 752640*sin(c + d*x)**4*i + 10
7520*sin(c + d*x)**2*i + 105*d*x))/(26880*d)
```

3.90 $\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 287

$$\begin{aligned}
&\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx\\
&=\frac{5a^8x}{512}-\frac{ia^{17}}{36d(a-ia\tan(c+dx))^9}-\frac{ia^{16}}{32d(a-ia\tan(c+dx))^8}\\
&\quad-\frac{3ia^{15}}{112d(a-ia\tan(c+dx))^7}-\frac{ia^{14}}{48d(a-ia\tan(c+dx))^6}\\
&\quad-\frac{ia^{13}}{64d(a-ia\tan(c+dx))^5}-\frac{3ia^{12}}{256d(a-ia\tan(c+dx))^4}\\
&\quad-\frac{7ia^{11}}{768d(a-ia\tan(c+dx))^3}-\frac{ia^{10}}{128d(a-ia\tan(c+dx))^2}\\
&\quad-\frac{9ia^{19}}{1024d\,(a^{11}-ia^{11}\tan(c+dx))}+\frac{ia^{19}}{1024d\,(a^{11}+ia^{11}\tan(c+dx))}
\end{aligned}$$

output
```
5/512*a^8*x-1/36*I*a^17/d/(a-I*a*tan(d*x+c))^9-1/32*I*a^16/d/(a-I*a*tan(d*
x+c))^8-3/112*I*a^15/d/(a-I*a*tan(d*x+c))^7-1/48*I*a^14/d/(a-I*a*tan(d*x+c
))^6-1/64*I*a^13/d/(a-I*a*tan(d*x+c))^5-3/256*I*a^12/d/(a-I*a*tan(d*x+c))^
4-7/768*I*a^11/d/(a-I*a*tan(d*x+c))^3-1/128*I*a^10/d/(a-I*a*tan(d*x+c))^2-
9/1024*I*a^19/d/(a^11-I*a^11*tan(d*x+c))+1/1024*I*a^19/d/(a^11+I*a^11*tan(
d*x+c))
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.65

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$
$$-\frac{ia^8\sec^{10}(c+dx)(7938+14112\cos(2(c+dx))+10080\cos(4(c+dx))+6480\cos(6(c+dx))+2462\cos$$

input
```
Integrate[Cos[c + d*x]^18*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/258048*I)*a^8*Sec[c + d*x]^10*(7938 + 14112*Cos[2*(c + d*x)] + 10080*
Cos[4*(c + d*x)] + 6480*Cos[6*(c + d*x)] + 2462*Cos[8*(c + d*x)] - 112*Cos
[10*(c + d*x)] - (3528*I)*Sin[2*(c + d*x)] - (5040*I)*Sin[4*(c + d*x)] - (
4860*I)*Sin[6*(c + d*x)] - (2147*I)*Sin[8*(c + d*x)] + 2520*ArcTan[Tan[c +
 d*x]]*(I*Cos[8*(c + d*x)] + Sin[8*(c + d*x)]) + (140*I)*Sin[10*(c + d*x)]
))/(d*(-I + Tan[c + d*x])*(I + Tan[c + d*x])^9)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^{18}}dx$$
$$\Big\downarrow \ 3968$$
$$-\frac{ia^{19}\int \frac{1}{(a-ia\tan(c+dx))^{10}(i\tan(c+dx)a+a)^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^{19}\int\left(\frac{9}{1024a^{10}(a-ia\tan(c+dx))^2}+\frac{1}{1024a^{10}(i\tan(c+dx)a+a)^2}+\frac{1}{64a^9(a-ia\tan(c+dx))^3}+\frac{7}{256a^8(a-ia\tan(c+dx))^4}+\frac{}{64a^7(a-ia}\right.}{}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^{19}\left(\frac{5i\arctan(\tan(c+dx))}{512a^{11}}+\frac{9}{1024a^{10}(a-ia\tan(c+dx))}-\frac{1}{1024a^{10}(a+ia\tan(c+dx))}+\frac{1}{128a^9(a-ia\tan(c+dx))^2}+\frac{7}{768a^8(a-ia\tan(c-}\right.}{}$$

input
```
Int[Cos[c + d*x]^18*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*a^19*((((5*I)/512)*ArcTan[Tan[c + d*x]])/a^11 + 1/(36*a^2*(a - I*a*T
an[c + d*x])^9) + 1/(32*a^3*(a - I*a*Tan[c + d*x])^8) + 3/(112*a^4*(a - I*
a*Tan[c + d*x])^7) + 1/(48*a^5*(a - I*a*Tan[c + d*x])^6) + 1/(64*a^6*(a -
I*a*Tan[c + d*x])^5) + 3/(256*a^7*(a - I*a*Tan[c + d*x])^4) + 7/(768*a^8*(
a - I*a*Tan[c + d*x])^3) + 1/(128*a^9*(a - I*a*Tan[c + d*x])^2) + 9/(1024*
a^10*(a - I*a*Tan[c + d*x])) - 1/(1024*a^10*(a + I*a*Tan[c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 788 vs. $2(245) = 490$.

Time = 1.47 (sec) , antiderivative size = 789, normalized size of antiderivative = 2.75

Expression too large to display

input
```
int(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(-1/18*sin(d*x+c)^7*cos(d*x+c)^11-7/288*sin(d*x+c)^5*cos(d*x+c)^1
1-5/576*sin(d*x+c)^3*cos(d*x+c)^11-5/2304*sin(d*x+c)*cos(d*x+c)^11+1/4608*
(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/
128*cos(d*x+c))*sin(d*x+c)+35/65536*d*x+35/65536*c)-8*I*a^8*(-1/18*sin(d*x
+c)^6*cos(d*x+c)^12-1/48*cos(d*x+c)^12*sin(d*x+c)^4-1/168*cos(d*x+c)^12*si
n(d*x+c)^2-1/1008*cos(d*x+c)^12)-28*a^8*(-1/18*sin(d*x+c)^5*cos(d*x+c)^13-
5/288*sin(d*x+c)^3*cos(d*x+c)^13-5/1344*sin(d*x+c)*cos(d*x+c)^13+5/16128*(
cos(d*x+c)^11+11/10*cos(d*x+c)^9+99/80*cos(d*x+c)^7+231/160*cos(d*x+c)^5+2
31/128*cos(d*x+c)^3+693/256*cos(d*x+c))*sin(d*x+c)+55/65536*d*x+55/65536*c
)+56*I*a^8*(-1/18*sin(d*x+c)^4*cos(d*x+c)^14-1/72*cos(d*x+c)^14*sin(d*x+c)
^2-1/504*cos(d*x+c)^14)+70*a^8*(-1/18*sin(d*x+c)^3*cos(d*x+c)^15-1/96*sin(
d*x+c)*cos(d*x+c)^15+1/1344*(cos(d*x+c)^13+13/12*cos(d*x+c)^11+143/120*cos
(d*x+c)^9+429/320*cos(d*x+c)^7+1001/640*cos(d*x+c)^5+1001/512*cos(d*x+c)^3
+3003/1024*cos(d*x+c))*sin(d*x+c)+143/65536*d*x+143/65536*c)-56*I*a^8*(-1/
18*sin(d*x+c)^2*cos(d*x+c)^16-1/144*cos(d*x+c)^16)-28*a^8*(-1/18*sin(d*x+c
)*cos(d*x+c)^17+1/288*(cos(d*x+c)^15+15/14*cos(d*x+c)^13+65/56*cos(d*x+c)^
11+143/112*cos(d*x+c)^9+1287/896*cos(d*x+c)^7+429/256*cos(d*x+c)^5+2145/10
24*cos(d*x+c)^3+6435/2048*cos(d*x+c))*sin(d*x+c)+715/65536*d*x+715/65536*c
)-4/9*I*a^8*cos(d*x+c)^18+a^8*(1/18*(cos(d*x+c)^17+17/16*cos(d*x+c)^15+255
/224*cos(d*x+c)^13+1105/896*cos(d*x+c)^11+2431/1792*cos(d*x+c)^9+21879/...
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.56

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{\left(5040\,a^8dxe^{(2i\,dx+2i\,c)} - 28i\,a^8e^{(20i\,dx+20i\,c)} - 315i\,a^8e^{(18i\,dx+18i\,c)} - 1620i\,a^8e^{(16i\,dx+16i\,c)} - 5040i\,a^8e^{(14i\,dx+\cdots}\right.}{}$$

input
```
integrate(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/516096*(5040*a^8*d*x*e^(2*I*d*x + 2*I*c) - 28*I*a^8*e^(20*I*d*x + 20*I*c
) - 315*I*a^8*e^(18*I*d*x + 18*I*c) - 1620*I*a^8*e^(16*I*d*x + 16*I*c) - 5
040*I*a^8*e^(14*I*d*x + 14*I*c) - 10584*I*a^8*e^(12*I*d*x + 12*I*c) - 1587
6*I*a^8*e^(10*I*d*x + 10*I*c) - 17640*I*a^8*e^(8*I*d*x + 8*I*c) - 15120*I*
a^8*e^(6*I*d*x + 6*I*c) - 11340*I*a^8*e^(4*I*d*x + 4*I*c) + 252*I*a^8)*e^(
-2*I*d*x - 2*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.44

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{5a^8x}{512} + \begin{cases} \frac{\left(-277298568799925181577403826176ia^8d^9e^{20ic}e^{18idx} - 3119608898999158292745793044480ia^8d^9e^{18ic}e^{16idx} - 1604370290913852836 26\cdots\right.}{} \\ x\left(-\frac{5a^8}{512} + \frac{\left(a^8e^{20ic}+10a^8e^{18ic}+45a^8e^{16ic}+120a^8e^{14ic}+210a^8e^{12ic}+252a^8e^{10ic}+210a^8e^{8ic}+120a^8e^{6ic}+45a^8e^{4ic}+10a^8e^{2ic}+a^8\right)e^{-\cdots}}{1024}\right. \end{cases}$$

input
```
integrate(cos(d*x+c)**18*(a+I*a*tan(d*x+c))**8,x)
```

output
```
5*a**8*x/512 + Piecewise(((-27729856879992518157740382617 6*I*a**8*d**9*exp
(20*I*c)*exp(18*I*d*x) - 311960889899915829274579304448 0*I*a**8*d**9*exp(1
8*I*c)*exp(16*I*d*x) - 1604370290913852836269264994304 0*I*a**8*d**9*exp(16
*I*c)*exp(14*I*d*x) - 4991374238398653268393268871168 0*I*a**8*d**9*exp(14*
I*c)*exp(12*I*d*x) - 10481885900637171863625864629452 8*I*a**8*d**9*exp(12*
I*c)*exp(10*I*d*x) - 15722828850955757795438796944179 2*I*a**8*d**9*exp(10*
I*c)*exp(8*I*d*x) - 17469809834395286439376441049088 0*I*a**8*d**9*exp(8*I*
c)*exp(6*I*d*x) - 14974122715195959805179806613504 0*I*a**8*d**9*exp(6*I*c)
*exp(4*I*d*x) - 11230592036396969853884854960128 0*I*a**8*d**9*exp(4*I*c)*e
xp(2*I*d*x) + 24956871191993266341966344355 84*I*a**8*d**9*exp(-2*I*d*x))*e
xp(-2*I*c)/(51111672201202209468347073240760 32*d**10), Ne(d**10*exp(2*I*c)
, 0)), (x*(-5*a**8/512 + (a**8*exp(20*I*c) + 10*a**8*exp(18*I*c) + 45*a**8
*exp(16*I*c) + 120*a**8*exp(14*I*c) + 210*a**8*exp(12*I*c) + 252*a**8*exp(
10*I*c) + 210*a**8*exp(8*I*c) + 120*a**8*exp(6*I*c) + 45*a**8*exp(4*I*c) +
 10*a**8*exp(2*I*c) + a**8)*exp(-2*I*c)/1024), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.94

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{315\,(dx+c)a^8 + \frac{315\,a^8\tan(dx+c)^{17}+2730\,a^8\tan(dx+c)^{15}+10458\,a^8\tan(dx+c)^{13}+23202\,a^8\tan(dx+c)^{11}+32768\,a^8\tan(dx+c)^9+2748}{\tan(dx+c)^{18}+9\,\tan(dx+c)^{16}+36\,\tan(dx+c)^{14}+84\,\mathrm{t}}}{}$$

input
```
integrate(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/32256*(315*(d*x + c)*a^8 + (315*a^8*tan(d*x + c)^17 + 2730*a^8*tan(d*x +
 c)^15 + 10458*a^8*tan(d*x + c)^13 + 23202*a^8*tan(d*x + c)^11 + 32768*a^8
*tan(d*x + c)^9 + 27486*a^8*tan(d*x + c)^7 + 21504*I*a^8*tan(d*x + c)^6 +
86310*a^8*tan(d*x + c)^5 - 119808*I*a^8*tan(d*x + c)^4 - 121002*a^8*tan(d*
x + c)^3 + 82944*I*a^8*tan(d*x + c)^2 + 31941*a^8*tan(d*x + c) - 5120*I*a^
8)/(tan(d*x + c)^18 + 9*tan(d*x + c)^16 + 36*tan(d*x + c)^14 + 84*tan(d*x
+ c)^12 + 126*tan(d*x + c)^10 + 126*tan(d*x + c)^8 + 84*tan(d*x + c)^6 + 3
6*tan(d*x + c)^4 + 9*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.52

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{1}{64512}a^8\left(-\frac{315i\log(\tan(dx+c)+i)}{d}+\frac{315i\log(\tan(dx+c)-i)}{d}-\frac{2\left(315\tan(dx+c)^9+2520i\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-1/64512*a^8*(-315*I*log(tan(d*x + c) + I)/d + 315*I*log(tan(d*x + c) - I)
/d - 2*(315*tan(d*x + c)^9 + 2520*I*tan(d*x + c)^8 - 8610*tan(d*x + c)^7 -
 15960*I*tan(d*x + c)^6 + 16128*tan(d*x + c)^5 + 5544*I*tan(d*x + c)^4 + 7
074*tan(d*x + c)^3 + 11736*I*tan(d*x + c)^2 - 9019*tan(d*x + c) - 5120*I)/
(d*(tan(d*x + c) + I)^9*(tan(d*x + c) - I)))
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.80

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{5a^8x}{512} + \frac{\frac{5a^8\tan(c+dx)^9}{512}+\frac{a^8\tan(c+dx)^8\,5\mathrm{i}}{64}-\frac{205a^8\tan(c+dx)^7}{768}-\frac{a^8\tan(c+dx)^6\,95\mathrm{i}}{192}+\frac{a^8\tan(c+dx)^5}{2}+\frac{a^8\tan(c+dx)}{64}}{d\left(\tan(c+dx)^{10}+\tan(c+dx)^9\,8\mathrm{i}-27\tan(c+dx)^8-\tan(c+dx)^7\,48\mathrm{i}+42\tan(c+dx)^6+42\,\mathrm{t}\right.}$$

input
```
int(cos(c + d*x)^18*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(5*a^8*x)/512 + ((a^8*tan(c + d*x)^2*163i)/448 - (a^8*10i)/63 - (9019*a^8*
tan(c + d*x))/32256 + (393*a^8*tan(c + d*x)^3)/1792 + (a^8*tan(c + d*x)^4*
11i)/64 + (a^8*tan(c + d*x)^5)/2 - (a^8*tan(c + d*x)^6*95i)/192 - (205*a^8
*tan(c + d*x)^7)/768 + (a^8*tan(c + d*x)^8*5i)/64 + (5*a^8*tan(c + d*x)^9)
/512)/(d*(tan(c + d*x)^3*48i - 27*tan(c + d*x)^2 - tan(c + d*x)*8i + 42*ta
n(c + d*x)^4 + 42*tan(c + d*x)^6 - tan(c + d*x)^7*48i - 27*tan(c + d*x)^8
+ tan(c + d*x)^9*8i + tan(c + d*x)^10 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.89

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(229376\cos\left(dx+c\right)\sin\left(dx+c\right)^{17}-1562624\cos\left(dx+c\right)\sin\left(dx+c\right)^{15}+4592640\cos\left(dx+c\right)\sin\left(dx\right.\right.}{}$$

input
```
int(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(a**8*(229376*cos(c + d*x)*sin(c + d*x)**17 - 1562624*cos(c + d*x)*sin(c +
 d*x)**15 + 4592640*cos(c + d*x)*sin(c + d*x)**13 - 7582976*cos(c + d*x)*s
in(c + d*x)**11 + 7660928*cos(c + d*x)*sin(c + d*x)**9 - 4820112*cos(c + d
*x)*sin(c + d*x)**7 + 1827672*cos(c + d*x)*sin(c + d*x)**5 - 376530*cos(c
+ d*x)*sin(c + d*x)**3 + 31941*cos(c + d*x)*sin(c + d*x) + 229376*sin(c +
d*x)**18*i - 1677312*sin(c + d*x)**16*i + 5345280*sin(c + d*x)**14*i - 969
8304*sin(c + d*x)**12*i + 10967040*sin(c + d*x)**10*i - 7934976*sin(c + d*
x)**8*i + 3612672*sin(c + d*x)**6*i - 967680*sin(c + d*x)**4*i + 129024*si
n(c + d*x)**2*i + 315*d*x))/(32256*d)
```

3.91 $\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 22, antiderivative size = 235

$$\begin{aligned}\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx = &-\frac{3003a^8\operatorname{arctanh}(\sin(c+dx))}{16d}\\ &-\frac{3003ia^8\sec(c+dx)}{16d}\\ &-\frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d}\\ &-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}\\ &-\frac{429ia^2\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{40d}\\ &-\frac{143i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^4}{30d}\\ &-\frac{1001i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)^2}{40d}\\ &-\frac{1001i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{16d}\end{aligned}$$

output
```
-3003/16*a^8*arctanh(sin(d*x+c))/d-3003/16*I*a^8*sec(d*x+c)/d-13/6*I*a^3*s
ec(d*x+c)*(a+I*a*tan(d*x+c))^5/d-2*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^7/d-4
29/40*I*a^2*sec(d*x+c)*(a^2+I*a^2*tan(d*x+c))^3/d-143/30*I*sec(d*x+c)*(a^2
+I*a^2*tan(d*x+c))^4/d-1001/40*I*sec(d*x+c)*(a^4+I*a^4*tan(d*x+c))^2/d-100
1/16*I*sec(d*x+c)*(a^8+I*a^8*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 2.62 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.87

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\cos^2(c+dx)(\cos(8c)-i\sin(8c))\left(-658944i\cos(c+dx)+720720\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*Cos[c + d*x]^2*(Cos[8*c] - I*Sin[8*c])*((-658944*I)*Cos[c + d*x] + 72
0720*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]]) + 5*((-73216*I)*Cos[3*(c + d*x)] - (19968*
I)*Cos[5*(c + d*x)] - (1536*I)*Cos[7*(c + d*x)] + 12870*Sin[c + d*x] + 221
65*Sin[3*(c + d*x)] + 10959*Sin[5*(c + d*x)] + 1536*Sin[7*(c + d*x)]))*(-I
 + Tan[c + d*x])^8)/(3840*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {3042, 3977, 3042, 3979, 3042, 3979, 3042, 3979, 3042, 3979, 3042, 3979, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)}dx$$
$$\Big\downarrow \text{3977}$$
$$-13a^2\int \sec(c+dx)(i\tan(c+dx)a+a)^6dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\downarrow \text{3042}$$

$$-13a^2 \int \sec(c+dx)(i\tan(c+dx)a+a)^6dx - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\downarrow \text{3979}$$

$$-13a^2\left(\frac{11}{6}a\int \sec(c+dx)(i\tan(c+dx)a+a)^5dx + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^5}{6d}\right) - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\downarrow \text{3042}$$

$$-13a^2\left(\frac{11}{6}a\int \sec(c+dx)(i\tan(c+dx)a+a)^5dx + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^5}{6d}\right) - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\downarrow \text{3979}$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\int \sec(c+dx)(i\tan(c+dx)a+a)^4dx + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}\right) + \frac{ia\sec(c+dx)}{}\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\downarrow \text{3042}$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\int \sec(c+dx)(i\tan(c+dx)a+a)^4dx + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}\right) + \frac{ia\sec(c+dx)}{}\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\downarrow \text{3979}$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\left(\frac{7}{4}a\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}\right) + \frac{ia\sec(c+}{}\right.\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\downarrow \text{3042}$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\left(\frac{7}{4}a\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}\right) + \frac{ia\sec(c+}{}\right.\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \text{3979}$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)+\frac{ia\,\mathrm{se}}{}\right.\right.\right.\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \text{3042}$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)+\frac{ia\,\mathrm{se}}{}\right.\right.\right.\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \text{3979}$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\right.\right.\right.\right.\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \text{3042}$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\right.\right.\right.\right.\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \text{3967}$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)\right.\right.\right.\right.\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \text{3042}$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+d\right.}{2d}\right.\right.\right.\right.\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \text{4257}$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{3}{2}a\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right.\right.\right.\right.\right.\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-2*I)*a*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^7)/d - 13*a^2*(((I/6)*a*Sec[
c + d*x]*(a + I*a*Tan[c + d*x])^5)/d + (11*a*(((I/5)*a*Sec[c + d*x]*(a + I
*a*Tan[c + d*x])^4)/d + (9*a*(((I/4)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])
^3)/d + (7*a*(((I/3)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d + (5*a*((3
*a*((a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d))/2 + ((I/2)*Sec[c
+ d*x]*(a^2 + I*a^2*Tan[c + d*x]))/d))/3))/4))/5))/6)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 80.80 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{128 i a^{8} e^{i(dx+c)}}{d}-\frac{i a^{8}\left(62475\,e^{11 i(dx+c)}+246505\,e^{9 i(dx+c)}+416094\,e^{7 i(dx+c)}+364194\,e^{5 i(dx+c)}+163095\,e^{3 i(dx+c)}\right.}{120 d\left(e^{2 i(dx+c)}+1\right)^{6}}$
derivativedivides	$\frac{a^{8}\left(\frac{\sin(dx+c)^{9}}{6\cos(dx+c)^{6}}-\frac{\sin(dx+c)^{9}}{8\cos(dx+c)^{4}}+\frac{5\sin(dx+c)^{9}}{16\cos(dx+c)^{2}}+\frac{5\sin(dx+c)^{7}}{16}+\frac{7\sin(dx+c)^{5}}{16}+\frac{35\sin(dx+c)^{3}}{48}+\frac{35\sin(dx+c)}{16}-\frac{35\ln(\sec(dx+c}{1}\right.}{}$
default	$\frac{a^{8}\left(\frac{\sin(dx+c)^{9}}{6\cos(dx+c)^{6}}-\frac{\sin(dx+c)^{9}}{8\cos(dx+c)^{4}}+\frac{5\sin(dx+c)^{9}}{16\cos(dx+c)^{2}}+\frac{5\sin(dx+c)^{7}}{16}+\frac{7\sin(dx+c)^{5}}{16}+\frac{35\sin(dx+c)^{3}}{48}+\frac{35\sin(dx+c)}{16}-\frac{35\ln(\sec(dx+c}{1}\right.}{}$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
-128*I/d*a^8*exp(I*(d*x+c))-1/120*I*a^8/d/(exp(2*I*(d*x+c))+1)^6*(62475*ex
p(11*I*(d*x+c))+246505*exp(9*I*(d*x+c))+416094*exp(7*I*(d*x+c))+364194*exp
(5*I*(d*x+c))+163095*exp(3*I*(d*x+c))+29685*exp(I*(d*x+c)))-3003/16/d*a^8*
ln(exp(I*(d*x+c))+I)+3003/16/d*a^8*ln(exp(I*(d*x+c))-I)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.61

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{-30720i\,a^8e^{(13i\,dx+13i\,c)} - 309270i\,a^8e^{(11i\,dx+11i\,c)} - 953810i\,a^8e^{(9i\,dx+9i\,c)} - 1446588i\,a^8e^{(7i\,dx+7i\,c)} - 1189}{}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/240*(-30720*I*a^8*e^(13*I*d*x + 13*I*c) - 309270*I*a^8*e^(11*I*d*x + 11*
I*c) - 953810*I*a^8*e^(9*I*d*x + 9*I*c) - 1446588*I*a^8*e^(7*I*d*x + 7*I*c
) - 1189188*I*a^8*e^(5*I*d*x + 5*I*c) - 510510*I*a^8*e^(3*I*d*x + 3*I*c) -
 90090*I*a^8*e^(I*d*x + I*c) - 45045*(a^8*e^(12*I*d*x + 12*I*c) + 6*a^8*e^
(10*I*d*x + 10*I*c) + 15*a^8*e^(8*I*d*x + 8*I*c) + 20*a^8*e^(6*I*d*x + 6*I
*c) + 15*a^8*e^(4*I*d*x + 4*I*c) + 6*a^8*e^(2*I*d*x + 2*I*c) + a^8)*log(e^
(I*d*x + I*c) + I) + 45045*(a^8*e^(12*I*d*x + 12*I*c) + 6*a^8*e^(10*I*d*x
+ 10*I*c) + 15*a^8*e^(8*I*d*x + 8*I*c) + 20*a^8*e^(6*I*d*x + 6*I*c) + 15*a
^8*e^(4*I*d*x + 4*I*c) + 6*a^8*e^(2*I*d*x + 2*I*c) + a^8)*log(e^(I*d*x + I
*c) - I))/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8
*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*
d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.36

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{3003a^8\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{16} - \frac{\log\left(e^{idx}+ie^{-ic}\right)}{16}\right)}{d}$$
$$+ \frac{-62475ia^8e^{11ic}e^{11idx} - 246505ia^8e^{9ic}e^{9idx} - 416094ia^8e^{7ic}e^{7idx} - 364194ia^8e^{5ic}e^{5idx} - 163095ia^8e^{3ic}e^{3}}{120de^{12ic}e^{12idx} + 720de^{10ic}e^{10idx} + 1800de^{8ic}e^{8idx} + 2400de^{6ic}e^{6idx} + 1800de^{4ic}e^{4idx} + 720de^{2ic}}$$
$$+ \begin{cases} -\frac{128ia^8e^{ic}e^{idx}}{d} & \text{for } d \neq 0 \\ 128a^8xe^{ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**8,x)
```

output
```
3003*a**8*(log(exp(I*d*x) - I*exp(-I*c))/16 - log(exp(I*d*x) + I*exp(-I*c)
)/16)/d + (-62475*I*a**8*exp(11*I*c)*exp(11*I*d*x) - 246505*I*a**8*exp(9*I
*c)*exp(9*I*d*x) - 416094*I*a**8*exp(7*I*c)*exp(7*I*d*x) - 364194*I*a**8*e
xp(5*I*c)*exp(5*I*d*x) - 163095*I*a**8*exp(3*I*c)*exp(3*I*d*x) - 29685*I*a
**8*exp(I*c)*exp(I*d*x))/(120*d*exp(12*I*c)*exp(12*I*d*x) + 720*d*exp(10*I
*c)*exp(10*I*d*x) + 1800*d*exp(8*I*c)*exp(8*I*d*x) + 2400*d*exp(6*I*c)*exp
(6*I*d*x) + 1800*d*exp(4*I*c)*exp(4*I*d*x) + 720*d*exp(2*I*c)*exp(2*I*d*x)
 + 120*d) + Piecewise((-128*I*a**8*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (128*
a**8*x*exp(I*c), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(195) = 390.

Time = 0.05 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.69

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{5\,a^8\left(\frac{2\left(87\,\sin(dx+c)^5-136\,\sin(dx+c)^3+57\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}+105\,\log\left(\sin\left(dx+c\right)+1\right)-105\,\log\left(\sin\left(dx+c\right)-1\right)\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/480*(5*a^8*(2*(87*sin(d*x + c)^5 - 136*sin(d*x + c)^3 + 57*sin(d*x + c)
)/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 105*log(sin
(d*x + c) + 1) - 105*log(sin(d*x + c) - 1) - 96*sin(d*x + c)) + 840*a^8*(2
*(9*sin(d*x + c)^3 - 7*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 +
1) + 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1) - 16*sin(d*x + c)
) + 8400*a^8*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1
) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) + 26880*I*a^8*(1/cos(d*x + c
) + cos(d*x + c)) + 8960*I*a^8*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*
cos(d*x + c)) + 768*I*a^8*((15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(
d*x + c)^5 + 5*cos(d*x + c)) + 6720*a^8*(log(sin(d*x + c) + 1) - log(sin(d
*x + c) - 1) - 2*sin(d*x + c)) + 3840*I*a^8*cos(d*x + c) - 480*a^8*sin(d*x
 + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(195) = 390$.

Time = 0.96 (sec) , antiderivative size = 924, normalized size of antiderivative = 3.93

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/61440*(11512215*a^8*e^(12*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 6
9073290*a^8*e^(10*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 172683225*a
^8*e^(8*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 230244300*a^8*e^(6*I*d
*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 172683225*a^8*e^(4*I*d*x + 4*I*c)
*log(I*e^(I*d*x + I*c) + 1) + 69073290*a^8*e^(2*I*d*x + 2*I*c)*log(I*e^(I*
d*x + I*c) + 1) - 19305*a^8*e^(12*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) -
1) - 115830*a^8*e^(10*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) - 289575*
a^8*e^(8*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) - 386100*a^8*e^(6*I*d*x
 + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) - 289575*a^8*e^(4*I*d*x + 4*I*c)*log(
I*e^(I*d*x + I*c) - 1) - 115830*a^8*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I
*c) - 1) - 11512215*a^8*e^(12*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) + 1)
- 69073290*a^8*e^(10*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1726832
25*a^8*e^(8*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 230244300*a^8*e^(
6*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 172683225*a^8*e^(4*I*d*x +
4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 69073290*a^8*e^(2*I*d*x + 2*I*c)*log(
-I*e^(I*d*x + I*c) + 1) + 19305*a^8*e^(12*I*d*x + 12*I*c)*log(-I*e^(I*d*x
+ I*c) - 1) + 115830*a^8*e^(10*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1)
 + 289575*a^8*e^(8*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 386100*a^8
*e^(6*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 289575*a^8*e^(4*I*d*x +
 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 115830*a^8*e^(2*I*d*x + 2*I*c)*lo...
```

Mupad [B] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.70

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
((a^8*tan(c/2 + (d*x)/2)^3*160729i)/120 - (127113*a^8*tan(c/2 + (d*x)/2)^2
)/40 + (167237*a^8*tan(c/2 + (d*x)/2)^4)/24 - (a^8*tan(c/2 + (d*x)/2)^5*12
977i)/4 - (97811*a^8*tan(c/2 + (d*x)/2)^6)/12 + (a^8*tan(c/2 + (d*x)/2)^7*
43757i)/12 + (22415*a^8*tan(c/2 + (d*x)/2)^8)/4 - (a^8*tan(c/2 + (d*x)/2)^
9*45115i)/24 - (52795*a^8*tan(c/2 + (d*x)/2)^10)/24 + (a^8*tan(c/2 + (d*x)
/2)^11*2891i)/8 + (3019*a^8*tan(c/2 + (d*x)/2)^12)/8 + (8848*a^8)/15 - (a^
8*tan(c/2 + (d*x)/2)*25499i)/120)/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)
/2)^2*6i - 6*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*15i + 15*tan(c/2
+ (d*x)/2)^5 - tan(c/2 + (d*x)/2)^6*20i - 20*tan(c/2 + (d*x)/2)^7 + tan(c/
2 + (d*x)/2)^8*15i + 15*tan(c/2 + (d*x)/2)^9 - tan(c/2 + (d*x)/2)^10*6i -
6*tan(c/2 + (d*x)/2)^11 + tan(c/2 + (d*x)/2)^12*1i + tan(c/2 + (d*x)/2)^13
 + 1i)) - (3003*a^8*atanh(tan(c/2 + (d*x)/2)))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.43

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(-30720\cos\left(dx+c\right)\sin\left(dx+c\right)^6 i+138240\cos\left(dx+c\right)\sin\left(dx+c\right)^4 i-177920\cos\left(dx+c\right)\sin\left(dx\right.\right.}{}$$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(a**8*( - 30720*cos(c + d*x)*sin(c + d*x)**6*i + 138240*cos(c + d*x)*sin(c
 + d*x)**4*i - 177920*cos(c + d*x)*sin(c + d*x)**2*i + 70784*cos(c + d*x)*
i + 45045*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**6 - 135135*log(tan((c +
d*x)/2) - 1)*sin(c + d*x)**4 + 135135*log(tan((c + d*x)/2) - 1)*sin(c + d*
x)**2 - 45045*log(tan((c + d*x)/2) - 1) - 45045*log(tan((c + d*x)/2) + 1)*
sin(c + d*x)**6 + 135135*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4 - 13513
5*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 + 45045*log(tan((c + d*x)/2) +
 1) + 30720*sin(c + d*x)**7 + 70784*sin(c + d*x)**6*i - 108555*sin(c + d*x
)**5 - 212352*sin(c + d*x)**4*i + 123080*sin(c + d*x)**3 + 212352*sin(c +
d*x)**2*i - 45285*sin(c + d*x) - 70784*i))/(240*d*(sin(c + d*x)**6 - 3*sin
(c + d*x)**4 + 3*sin(c + d*x)**2 - 1))
```

3.92 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 205

$$\begin{aligned}\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx &= \frac{1155a^8\operatorname{arctanh}(\sin(c+dx))}{8d} \\ &+ \frac{1155ia^8\sec(c+dx)}{8d} \\ &+ \frac{22ia^3\cos(c+dx)(a+ia\tan(c+dx))^5}{3d} \\ &- \frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d} \\ &+ \frac{33ia^2\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{4d} \\ &+ \frac{77i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)^2}{4d} \\ &+ \frac{385i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{8d}\end{aligned}$$

output
```
1155/8*a^8*arctanh(sin(d*x+c))/d+1155/8*I*a^8*sec(d*x+c)/d+22/3*I*a^3*cos(
d*x+c)*(a+I*a*tan(d*x+c))^5/d-2/3*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^7/d+
33/4*I*a^2*sec(d*x+c)*(a^2+I*a^2*tan(d*x+c))^3/d+77/4*I*sec(d*x+c)*(a^4+I*
a^4*tan(d*x+c))^2/d+385/8*I*sec(d*x+c)*(a^8+I*a^8*tan(d*x+c))/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1540 vs. $2(205) = 410$.

Time = 8.15 (sec) , antiderivative size = 1540, normalized size of antiderivative = 7.51

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(-1155*Cos[8*c]*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]
]*(a + I*a*Tan[c + d*x])^8)/(8*d*(Cos[d*x] + I*Sin[d*x])^8) + (1155*Cos[8*
c]*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(a + I*a*Ta
n[c + d*x])^8)/(8*d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[3*d*x]*Cos[c + d*x]^
8*(((-32*I)/3)*Cos[5*c] - (32*Sin[5*c])/3)*(a + I*a*Tan[c + d*x])^8)/(d*(C
os[d*x] + I*Sin[d*x])^8) + (Cos[d*x]*Cos[c + d*x]^8*((160*I)*Cos[7*c] + 16
0*Sin[7*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (((1
155*I)/8)*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sin[
8*c]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) - (((1155*I)/
8)*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sin[8*c]*(a
 + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*Se
c[c]*(((236*I)/3)*Cos[8*c] + (236*Sin[8*c])/3)*(a + I*a*Tan[c + d*x])^8)/(
d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*(-160*Cos[7*c] + (160*I)*Si
n[7*c])*Sin[d*x]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) +
 (Cos[c + d*x]^8*((32*Cos[5*c])/3 - ((32*I)/3)*Sin[5*c])*Sin[3*d*x]*(a + I
*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*(Cos[8
*c]/16 - (I/16)*Sin[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d
*x])^8*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^4) - (I*Cos[c + d*x]^8*((
4*Cos[8*c])/3 - ((4*I)/3)*Sin[8*c])*Sin[(d*x)/2]*(a + I*a*Tan[c + d*x])^8)
/(d*(Cos[c/2] - Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2]...
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 3977, 3042, 3977, 3042, 3979, 3042, 3979, 3042, 3979, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{11}{3}a^2\int \cos(c+dx)(i\tan(c+dx)a+a)^6dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{11}{3}a^2\int \frac{(i\tan(c+dx)a+a)^6}{\sec(c+dx)}dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{3977}$$

$$-\frac{11}{3}a^2\left(-9a^2\int \sec(c+dx)(i\tan(c+dx)a+a)^4dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^5}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{11}{3}a^2\left(-9a^2\int \sec(c+dx)(i\tan(c+dx)a+a)^4dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^5}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{3979}$$

$$-\frac{11}{3}a^2\left(-9a^2\left(\frac{7}{4}a\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}\right)-\frac{2ia\cos(c+
$$

$$\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{11}{3}a^2\left(-9a^2\left(\frac{7}{4}a\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}\right)-\frac{2ia\cos(c+
$$

$$\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{3979}$$

$$-\frac{11}{3}a^2\left(-9a^2\left(\frac{7}{4}a\left(\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)+\frac{ia\sec(c
$$

$$\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{11}{3}a^2\left(-9a^2\left(\frac{7}{4}a\left(\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)+\frac{ia\sec(c
$$

$$\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{3979}$$

$$-\frac{11}{3}a^2\left(-9a^2\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia
$$

$$\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{11}{3}a^2\left(-9a^2\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia
$$

$$\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{3967}$$

$$-\frac{11}{3}a^2\left(-9a^2\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{i\ldots}{}\right.\right.\right.\right.$$
$$\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{11}{3}a^2\left(-9a^2\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right.\right.\right.\right.\right.$$
$$\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{11}{3}a^2\left(-9a^2\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{3}{2}a\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)\right)\right.\right.\right.\right.$$
$$\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(((-2*I)/3)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^7)/d - (11*a^2*(((-2*I
)*a*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^5)/d - 9*a^2*(((I/4)*a*Sec[c + d*x
]*(a + I*a*Tan[c + d*x])^3)/d + (7*a*(((I/3)*a*Sec[c + d*x]*(a + I*a*Tan[c
 + d*x])^2)/d + (5*a*((3*a*((a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x
])/d))/2 + ((I/2)*Sec[c + d*x]*(a^2 + I*a^2*Tan[c + d*x]))/d))/3))/4)))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 227.00 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{32 i a^8 e^{3i(dx+c)}}{3d}+\frac{160 i a^8 e^{i(dx+c)}}{d}+\frac{i a^8\left(2295\, e^{7i(dx+c)}+5855\, e^{5i(dx+c)}+5153\, e^{3i(dx+c)}+1545\, e^{i(dx+c)}\right)}{12 d\left(e^{2i(dx+c)}+1\right)^4}-115$
derivativedivides	$a^8\left(\frac{\sin(dx+c)^9}{4\cos(dx+c)^4}-\frac{5\sin(dx+c)^9}{8\cos(dx+c)^2}-\frac{5\sin(dx+c)^7}{8}-\frac{7\sin(dx+c)^5}{8}-\frac{35\sin(dx+c)^3}{24}-\frac{35\sin(dx+c)}{8}+\frac{35\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)$
default	$a^8\left(\frac{\sin(dx+c)^9}{4\cos(dx+c)^4}-\frac{5\sin(dx+c)^9}{8\cos(dx+c)^2}-\frac{5\sin(dx+c)^7}{8}-\frac{7\sin(dx+c)^5}{8}-\frac{35\sin(dx+c)^3}{24}-\frac{35\sin(dx+c)}{8}+\frac{35\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
-32/3*I/d*a^8*exp(3*I*(d*x+c))+160*I/d*a^8*exp(I*(d*x+c))+1/12*I*a^8/d/(ex
p(2*I*(d*x+c))+1)^4*(2295*exp(7*I*(d*x+c))+5855*exp(5*I*(d*x+c))+5153*exp(
3*I*(d*x+c))+1545*exp(I*(d*x+c)))-1155/8/d*a^8*ln(exp(I*(d*x+c))-I)+1155/8
/d*a^8*ln(exp(I*(d*x+c))+I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.39

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{-256i\,a^8e^{(11i\,dx+11i\,c)}+2816i\,a^8e^{(9i\,dx+9i\,c)}+18414i\,a^8e^{(7i\,dx+7i\,c)}+33726i\,a^8e^{(5i\,dx+5i\,c)}+25410i\,a^8e^{(3i\,dx-}}{}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/24*(-256*I*a^8*e^(11*I*d*x + 11*I*c) + 2816*I*a^8*e^(9*I*d*x + 9*I*c) +
18414*I*a^8*e^(7*I*d*x + 7*I*c) + 33726*I*a^8*e^(5*I*d*x + 5*I*c) + 25410*
I*a^8*e^(3*I*d*x + 3*I*c) + 6930*I*a^8*e^(I*d*x + I*c) + 3465*(a^8*e^(8*I*
d*x + 8*I*c) + 4*a^8*e^(6*I*d*x + 6*I*c) + 6*a^8*e^(4*I*d*x + 4*I*c) + 4*a
^8*e^(2*I*d*x + 2*I*c) + a^8)*log(e^(I*d*x + I*c) + I) - 3465*(a^8*e^(8*I*
d*x + 8*I*c) + 4*a^8*e^(6*I*d*x + 6*I*c) + 6*a^8*e^(4*I*d*x + 4*I*c) + 4*a
^8*e^(2*I*d*x + 2*I*c) + a^8)*log(e^(I*d*x + I*c) - I))/(d*e^(8*I*d*x + 8*
I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x
+ 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.35

$$
\begin{aligned}
&\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx \\
&= \frac{1155a^8\left(-\frac{\log\left(e^{idx}-ie^{-ic}\right)}{8}+\frac{\log\left(e^{idx}+ie^{-ic}\right)}{8}\right)}{d} \\
&\quad + \frac{2295ia^8e^{7ic}e^{7idx}+5855ia^8e^{5ic}e^{5idx}+5153ia^8e^{3ic}e^{3idx}+1545ia^8e^{ic}e^{idx}}{12de^{8ic}e^{8idx}+48de^{6ic}e^{6idx}+72de^{4ic}e^{4idx}+48de^{2ic}e^{2idx}+12d} \\
&\quad + \begin{cases} \frac{-32ia^8de^{3ic}e^{3idx}+480ia^8de^{ic}e^{idx}}{3d^2} & \text{for } d^2\neq 0 \\ x(32a^8e^{3ic}-160a^8e^{ic}) & \text{otherwise} \end{cases}
\end{aligned}
$$

input

```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**8,x)
```

output

```
1155*a**8*(-log(exp(I*d*x) - I*exp(-I*c))/8 + log(exp(I*d*x) + I*exp(-I*c)
)/8)/d + (2295*I*a**8*exp(7*I*c)*exp(7*I*d*x) + 5855*I*a**8*exp(5*I*c)*exp
(5*I*d*x) + 5153*I*a**8*exp(3*I*c)*exp(3*I*d*x) + 1545*I*a**8*exp(I*c)*exp
(I*d*x))/(12*d*exp(8*I*c)*exp(8*I*d*x) + 48*d*exp(6*I*c)*exp(6*I*d*x) + 72
*d*exp(4*I*c)*exp(4*I*d*x) + 48*d*exp(2*I*c)*exp(2*I*d*x) + 12*d) + Piecew
ise(((-32*I*a**8*d*exp(3*I*c)*exp(3*I*d*x) + 480*I*a**8*d*exp(I*c)*exp(I*d
*x))/(3*d**2), Ne(d**2, 0)), (x*(32*a**8*exp(3*I*c) - 160*a**8*exp(I*c)),
True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(169) = 338.

Time = 0.06 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.72

$$
\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx =
$$

$$
-\frac{128i\,a^8\cos(dx+c)^3+448\,a^8\sin(dx+c)^3+896i\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\,\cos(dx+c)\right)a^8+128}{}
$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/48*(128*I*a^8*cos(d*x + c)^3 + 448*a^8*sin(d*x + c)^3 + 896*I*(cos(d*x
+ c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^8 + 128*I*(cos(d*x + c)^3 - (9
*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x + c))*a^8 + 896*I*(cos(d*x
 + c)^3 - 3*cos(d*x + c))*a^8 + (16*sin(d*x + c)^3 - 6*(13*sin(d*x + c)^3
- 11*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 105*log(sin(d
*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 144*sin(d*x + c))*a^8 + 112*(4*
sin(d*x + c)^3 - 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c)
 + 1) + 15*log(sin(d*x + c) - 1) + 24*sin(d*x + c))*a^8 + 560*(2*sin(d*x +
 c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c)
)*a^8 + 16*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^8)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2835 vs. $2(169) = 338$.

Time = 0.88 (sec) , antiderivative size = 2835, normalized size of antiderivative = 13.83

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/98304*(763587*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 106
90218*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 69486417*a^8*
e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 277945668*a^8*e^(22*I*d
*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 764350587*a^8*e^(20*I*d*x + 6*I*c
)*log(I*e^(I*d*x + I*c) + 1) + 1528701174*a^8*e^(18*I*d*x + 4*I*c)*log(I*e
^(I*d*x + I*c) + 1) + 2293051761*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x +
 I*c) + 1) + 2293051761*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1
) + 1528701174*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 76435
0587*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 277945668*a^8*e^
(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 69486417*a^8*e^(4*I*d*x - 1
0*I*c)*log(I*e^(I*d*x + I*c) + 1) + 10690218*a^8*e^(2*I*d*x - 12*I*c)*log(
I*e^(I*d*x + I*c) + 1) + 2620630584*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c)
 + 1) + 763587*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 14956128*a^8*e
^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) - 1) + 209385792*a^8*e^(26*I*d*
x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1361007648*a^8*e^(24*I*d*x + 10*I
*c)*log(I*e^(I*d*x + I*c) - 1) + 5444030592*a^8*e^(22*I*d*x + 8*I*c)*log(I
*e^(I*d*x + I*c) - 1) + 14971084128*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*
x + I*c) - 1) + 29942168256*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c)
 - 1) + 44913252384*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) +
44913252384*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 29942...
```

Mupad [B] (verification not implemented)

Time = 4.99 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.67

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{\frac{1147\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{4}+\frac{a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,3505\mathrm{i}}{4}-\frac{5639\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}-a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,3585\mathrm{i}+\frac{25993\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{6}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,3\mathrm{i}+7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,13\mathrm{i}-18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\tan\right.}$$
$$+\frac{1155\,a^8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
((27565*a^8*tan(c/2 + (d*x)/2)^2)/12 - (a^8*tan(c/2 + (d*x)/2)^3*12041i)/3
 - 4575*a^8*tan(c/2 + (d*x)/2)^4 + (a^8*tan(c/2 + (d*x)/2)^5*33847i)/6 + (
25993*a^8*tan(c/2 + (d*x)/2)^6)/6 - a^8*tan(c/2 + (d*x)/2)^7*3585i - (5639
*a^8*tan(c/2 + (d*x)/2)^8)/3 + (a^8*tan(c/2 + (d*x)/2)^9*3505i)/4 + (1147*
a^8*tan(c/2 + (d*x)/2)^10)/4 - (1360*a^8)/3 + (a^8*tan(c/2 + (d*x)/2)*4293
i)/4)/(d*(3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*7i - 13*tan(c/2 + (d
*x)/2)^3 + tan(c/2 + (d*x)/2)^4*18i + 22*tan(c/2 + (d*x)/2)^5 - tan(c/2 +
(d*x)/2)^6*22i - 18*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8*13i + 7*ta
n(c/2 + (d*x)/2)^9 - tan(c/2 + (d*x)/2)^10*3i - tan(c/2 + (d*x)/2)^11 + 1i
)) + (1155*a^8*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.32

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(1024\cos\left(dx+c\right)\sin\left(dx+c\right)^6 i+1536\cos\left(dx+c\right)\sin\left(dx+c\right)^4 i-8064\cos\left(dx+c\right)\sin\left(dx+c\right)^2 i\right.}{}$$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(a**8*(1024*cos(c + d*x)*sin(c + d*x)**6*i + 1536*cos(c + d*x)*sin(c + d*x
)**4*i - 8064*cos(c + d*x)*sin(c + d*x)**2*i + 5440*cos(c + d*x)*i - 3465*
log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 + 6930*log(tan((c + d*x)/2) - 1)
*sin(c + d*x)**2 - 3465*log(tan((c + d*x)/2) - 1) + 3465*log(tan((c + d*x)
/2) + 1)*sin(c + d*x)**4 - 6930*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2
+ 3465*log(tan((c + d*x)/2) + 1) - 1024*sin(c + d*x)**7 - 1024*sin(c + d*x
)**5 - 5440*sin(c + d*x)**4*i + 5495*sin(c + d*x)**3 + 10880*sin(c + d*x)*
*2*i - 3441*sin(c + d*x) - 5440*i))/(24*d*(sin(c + d*x)**4 - 2*sin(c + d*x
)**2 + 1))
```

3.93 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 173

$$
\begin{aligned}
\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx = & -\frac{63a^8\operatorname{arctanh}(\sin(c+dx))}{2d} \\
& -\frac{63ia^8\sec(c+dx)}{2d} \\
& +\frac{6ia^3\cos^3(c+dx)(a+ia\tan(c+dx))^5}{5d} \\
& -\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d} \\
& -\frac{42ia^2\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{5d} \\
& -\frac{21i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{2d}
\end{aligned}
$$

output
```
-63/2*a^8*arctanh(sin(d*x+c))/d-63/2*I*a^8*sec(d*x+c)/d+6/5*I*a^3*cos(d*x+
c)^3*(a+I*a*tan(d*x+c))^5/d-2/5*I*a*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^7/d-42
/5*I*a^2*cos(d*x+c)*(a^2+I*a^2*tan(d*x+c))^3/d-21/2*I*sec(d*x+c)*(a^8+I*a^
8*tan(d*x+c))/d
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1162 vs. $2(173) = 346$.

Time = 7.94 (sec) , antiderivative size = 1162, normalized size of antiderivative = 6.72

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(63*Cos[8*c]*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*(
a + I*a*Tan[c + d*x])^8)/(2*d*(Cos[d*x] + I*Sin[d*x])^8) - (63*Cos[8*c]*Co
s[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(a + I*a*Tan[c +
 d*x])^8)/(2*d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[5*d*x]*Cos[c + d*x]^8*(((
-8*I)/5)*Cos[3*c] - (8*Sin[3*c])/5)*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x]
 + I*Sin[d*x])^8) + (Cos[3*d*x]*Cos[c + d*x]^8*((8*I)*Cos[5*c] + 8*Sin[5*c
])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[d*x]*Cos
[c + d*x]^8*((-48*I)*Cos[7*c] - 48*Sin[7*c])*(a + I*a*Tan[c + d*x])^8)/(d*
(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*Sec[c]*((-8*I)*Cos[8*c] - 8*S
in[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) - (((63*I
)/2)*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sin[8*c]*
(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (((63*I)/2)*Cos[
c + d*x]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sin[8*c]*(a + I*a*
Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*(48*Cos[7
*c] - (48*I)*Sin[7*c])*Sin[d*x]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I
*Sin[d*x])^8) + (Cos[c + d*x]^8*(-8*Cos[5*c] + (8*I)*Sin[5*c])*Sin[3*d*x]*
(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*
((8*Cos[3*c])/5 - ((8*I)/5)*Sin[3*c])*Sin[5*d*x]*(a + I*a*Tan[c + d*x])^8)
/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*(Cos[8*c]/4 - (I/4)*Sin[8
*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + ...
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3977, 3042, 3977, 3042, 3977, 3042, 3979, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{9}{5}a^2\int \cos^3(c+dx)(i\tan(c+dx)a+a)^6dx-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{9}{5}a^2\int \frac{(i\tan(c+dx)a+a)^6}{\sec(c+dx)^3}dx-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \text{3977}$$

$$-\frac{9}{5}a^2\left(-\frac{7}{3}a^2\int \cos(c+dx)(i\tan(c+dx)a+a)^4dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^5}{3d}\right)-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{9}{5}a^2\left(-\frac{7}{3}a^2\int \frac{(i\tan(c+dx)a+a)^4}{\sec(c+dx)}dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^5}{3d}\right)-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \text{3977}$$

$$-\frac{9}{5}a^2\left(-\frac{7}{3}a^2\left(-5a^2\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}\right)-\frac{2ia\cos^3(\right.$$
$$\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{9}{5}a^2\left(-\frac{7}{3}a^2\left(-5a^2\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}\right)-\frac{2ia\cos^3(c\right.$$
$$\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \text{3979}$$

$$-\frac{9}{5}a^2\left(-\frac{7}{3}a^2\left(-5a^2\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)-\frac{2ia\cos\right.\right.$$
$$\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{9}{5}a^2\left(-\frac{7}{3}a^2\left(-5a^2\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)-\frac{2ia\cos\right.\right.$$
$$\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \text{3967}$$

$$-\frac{9}{5}a^2\left(-\frac{7}{3}a^2\left(-5a^2\left(\frac{3}{2}a\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)-\frac{2ia\right.\right.$$
$$\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{9}{5}a^2\left(-\frac{7}{3}a^2\left(-5a^2\left(\frac{3}{2}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)\right.\right.$$
$$\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{9}{5}a^2\left(-\frac{7}{3}a^2\left(-5a^2\left(\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{3}{2}a\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)\right)-\right.\right.$$
$$\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(((-2*I)/5)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^7)/d - (9*a^2*((((-2*I
)/3)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^5)/d - (7*a^2*(((-2*I)*a*Cos[
c + d*x]*(a + I*a*Tan[c + d*x])^3)/d - 5*a^2*((3*a*((a*ArcTanh[Sin[c + d*x
]])/d + (I*a*Sec[c + d*x])/d))/2 + ((I/2)*Sec[c + d*x]*(a^2 + I*a^2*Tan[c
+ d*x]))/d)))/3))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(152) = 304$.

Time = 2.78 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.86

$$\frac{a^8 \sin\left(dx+c\right)^9}{2d\cos\left(dx+c\right)^2}+\frac{a^8 \sin\left(dx+c\right)^7}{2d}+\frac{203a^8 \sin\left(dx+c\right)^5}{10d}+\frac{21a^8 \sin\left(dx+c\right)^3}{2d}+\frac{283a^8 \sin\left(dx+c\right)}{10d}-\frac{63a^8 \ln\left(s\right.}{}$$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/2/d*a^8*sin(d*x+c)^9/cos(d*x+c)^2+1/2*a^8*sin(d*x+c)^7/d+203/10*a^8*sin(
d*x+c)^5/d+21/2*a^8*sin(d*x+c)^3/d+283/10*a^8*sin(d*x+c)/d-63/2/d*a^8*ln(s
ec(d*x+c)+tan(d*x+c))+112/15*I/d*a^8*cos(d*x+c)^3-832/15*I/d*a^8*cos(d*x+c
)-104/5*I/d*a^8*cos(d*x+c)*sin(d*x+c)^4-8/5*I/d*a^8*cos(d*x+c)^5+29/5/d*a^
8*sin(d*x+c)*cos(d*x+c)^4-8/5/d*a^8*cos(d*x+c)^2*sin(d*x+c)-8*I/d*a^8*cos(
d*x+c)*sin(d*x+c)^6-8*I/d*a^8*sin(d*x+c)^8/cos(d*x+c)-416/15*I/d*a^8*cos(d
*x+c)*sin(d*x+c)^2+56/5*I/d*a^8*cos(d*x+c)^3*sin(d*x+c)^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.10

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{-16i\,a^8e^{(9i\,dx+9i\,c)}+48i\,a^8e^{(7i\,dx+7i\,c)}-336i\,a^8e^{(5i\,dx+5i\,c)}-1050i\,a^8e^{(3i\,dx+3i\,c)}-630i\,a^8e^{(i\,dx+i\,c)}-315\,(a}{10\,(de^{(4i\,dx+4i}}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/10*(-16*I*a^8*e^(9*I*d*x + 9*I*c) + 48*I*a^8*e^(7*I*d*x + 7*I*c) - 336*I
*a^8*e^(5*I*d*x + 5*I*c) - 1050*I*a^8*e^(3*I*d*x + 3*I*c) - 630*I*a^8*e^(I
*d*x + I*c) - 315*(a^8*e^(4*I*d*x + 4*I*c) + 2*a^8*e^(2*I*d*x + 2*I*c) + a
^8)*log(e^(I*d*x + I*c) + I) + 315*(a^8*e^(4*I*d*x + 4*I*c) + 2*a^8*e^(2*I
*d*x + 2*I*c) + a^8)*log(e^(I*d*x + I*c) - I))/(d*e^(4*I*d*x + 4*I*c) + 2*
d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.36

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{63a^8\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{2}-\frac{\log\left(e^{idx}+ie^{-ic}\right)}{2}\right)}{d}+\frac{-17ia^8e^{3ic}e^{3idx}-15ia^8e^{ic}e^{idx}}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d} + \begin{cases}\frac{-8ia^8d^2e^{5ic}e^{5idx}+40ia^8d^2e^{3ic}e^{3idx}-240ia^8d^2e^{ic}e^{idx}}{5d^3} & \text{for } d^3\neq 0\\ x\left(8a^8e^{5ic}-24a^8e^{3ic}+48a^8e^{ic}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**8,x)
```

output
```
63*a**8*(log(exp(I*d*x) - I*exp(-I*c))/2 - log(exp(I*d*x) + I*exp(-I*c))/2
)/d + (-17*I*a**8*exp(3*I*c)*exp(3*I*d*x) - 15*I*a**8*exp(I*c)*exp(I*d*x))
/(d*exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*x) + d) + Piecewise
(((-8*I*a**8*d**2*exp(5*I*c)*exp(5*I*d*x) + 40*I*a**8*d**2*exp(3*I*c)*exp(
3*I*d*x) - 240*I*a**8*d**2*exp(I*c)*exp(I*d*x))/(5*d**3), Ne(d**3, 0)), (x
*(8*a**8*exp(5*I*c) - 24*a**8*exp(3*I*c) + 48*a**8*exp(I*c)), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(143) = 286.

Time = 0.05 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.88

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{96i\,a^8\cos\left(dx+c\right)^5-840\,a^8\sin\left(dx+c\right)^5+224i\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)a^8+224i\left(3\,\cos\left(\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/60*(96*I*a^8*cos(d*x + c)^5 - 840*a^8*sin(d*x + c)^5 + 224*I*(3*cos(d*x
 + c)^5 - 5*cos(d*x + c)^3)*a^8 + 224*I*(3*cos(d*x + c)^5 - 10*cos(d*x + c
)^3 + 15*cos(d*x + c))*a^8 + 96*I*(cos(d*x + c)^5 - 5*cos(d*x + c)^3 + 5/c
os(d*x + c) + 15*cos(d*x + c))*a^8 - (12*sin(d*x + c)^5 + 40*sin(d*x + c)^
3 - 30*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 105*log(sin(d*x + c) + 1) + 105
*log(sin(d*x + c) - 1) + 180*sin(d*x + c))*a^8 - 56*(6*sin(d*x + c)^5 + 10
*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30
*sin(d*x + c))*a^8 - 112*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^8 - 4*(3*
sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^8)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2849 vs. $2(143) = 286$.

Time = 0.97 (sec) , antiderivative size = 2849, normalized size of antiderivative = 16.47

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/655360*(42021645*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) + 1) +
588303030*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 382396969
5*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 15295878780*a^8*e
^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 42063666645*a^8*e^(20*I*d
*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 84127333290*a^8*e^(18*I*d*x + 4*I
*c)*log(I*e^(I*d*x + I*c) + 1) + 126190999935*a^8*e^(16*I*d*x + 2*I*c)*log
(I*e^(I*d*x + I*c) + 1) + 126190999935*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I
*d*x + I*c) + 1) + 84127333290*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I
*c) + 1) + 42063666645*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1)
+ 15295878780*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3823969
695*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 588303030*a^8*e^
(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 144218285640*a^8*e^(14*I*d
*x)*log(I*e^(I*d*x + I*c) + 1) + 42021645*a^8*e^(-14*I*c)*log(I*e^(I*d*x +
 I*c) + 1) + 21376575*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) - 1)
 + 299272050*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 194526
8325*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 7781073300*a^8
*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 21397951575*a^8*e^(20*I
*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 42795903150*a^8*e^(18*I*d*x + 4
*I*c)*log(I*e^(I*d*x + I*c) - 1) + 64193854725*a^8*e^(16*I*d*x + 2*I*c)*lo
g(I*e^(I*d*x + I*c) - 1) + 64193854725*a^8*e^(12*I*d*x - 2*I*c)*log(I*e...
```

Mupad [B] (verification not implemented)

Time = 4.58 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.62

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{63\,a^8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{65\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,309\mathrm{i}-761\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,1109\mathrm{i}+\frac{7351\,a^8\tan}{5}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,5\mathrm{i}-12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,20\mathrm{i}+26\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\right.}$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(a^8*tan(c/2 + (d*x)/2)^3*1223i - (4407*a^8*tan(c/2 + (d*x)/2)^2)/5 + (735
1*a^8*tan(c/2 + (d*x)/2)^4)/5 - a^8*tan(c/2 + (d*x)/2)^5*1109i - 761*a^8*t
an(c/2 + (d*x)/2)^6 + a^8*tan(c/2 + (d*x)/2)^7*309i + 65*a^8*tan(c/2 + (d*
x)/2)^8 + (496*a^8)/5 - a^8*tan(c/2 + (d*x)/2)*431i)/(d*(5*tan(c/2 + (d*x)
/2) - tan(c/2 + (d*x)/2)^2*12i - 20*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)
/2)^4*26i + 26*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x)/2)^6*20i - 12*tan(c/
2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8*5i + tan(c/2 + (d*x)/2)^9 + 1i)) - (
63*a^8*atanh(tan(c/2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8\left(-256\cos\left(dx+c\right)\sin\left(dx+c\right)^6 i+128\cos\left(dx+c\right)\sin\left(dx+c\right)^4 i-288\cos\left(dx+c\right)\sin\left(dx+c\right)^2 i+\right.}{}$$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(a**8*( - 256*cos(c + d*x)*sin(c + d*x)**6*i + 128*cos(c + d*x)*sin(c + d*
x)**4*i - 288*cos(c + d*x)*sin(c + d*x)**2*i + 496*cos(c + d*x)*i + 315*lo
g(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 - 315*log(tan((c + d*x)/2) - 1) -
315*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 + 315*log(tan((c + d*x)/2) +
 1) + 256*sin(c + d*x)**7 - 256*sin(c + d*x)**5 + 320*sin(c + d*x)**3 + 49
6*sin(c + d*x)**2*i - 325*sin(c + d*x) - 496*i))/(10*d*(sin(c + d*x)**2 -
1))
```

3.94 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 152

$$\begin{aligned}\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx &= \frac{a^8\text{arctanh}(\sin(c+dx))}{d}\\ &+\frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}\\ &-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}\\ &-\frac{2ia^2\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{3d}\\ &+\frac{2i\cos(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{d}\end{aligned}$$

output
```
a^8*arctanh(sin(d*x+c))/d+2/5*I*a^3*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5/d-2/
7*I*a*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^7/d-2/3*I*a^2*cos(d*x+c)^3*(a^2+I*a^
2*tan(d*x+c))^3/d+2*I*cos(d*x+c)*(a^8+I*a^8*tan(d*x+c))/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 305 vs. 2(152) = 304.

Time = 2.99 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.01

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(-70i\cos\left(\frac{1}{2}(c+dx)\right)+42i\cos\left(\frac{3}{2}(c+dx)\right)+210i\cos\left(\frac{5}{2}(c+dx)\right)-30i\cos\left(\frac{7}{2}(c+dx)\right)-105\cos\left(\frac{7}{2}\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*((-70*I)*Cos[(c + d*x)/2] + (42*I)*Cos[(3*(c + d*x))/2] + (210*I)*Cos
[(5*(c + d*x))/2] - (30*I)*Cos[(7*(c + d*x))/2] - 105*Cos[(7*(c + d*x))/2]
*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*Cos[(7*(c + d*x))/2]*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] - 70*Sin[(c + d*x)/2] - 42*Sin[(3*(c +
 d*x))/2] + 210*Sin[(5*(c + d*x))/2] + 30*Sin[(7*(c + d*x))/2] + (105*I)*L
og[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2] - (105*I)*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2])*(Cos[(7*c + 23
*d*x)/2] + I*Sin[(7*c + 23*d*x)/2]))/(105*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3977, 3042, 3977, 3042, 3977, 3042, 3977, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \text{3977}$$

$$-a^2\int \cos^5(c+dx)(i\tan(c+dx)a+a)^6dx-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \text{3042}$$

$$-a^2\int \frac{(i\tan(c+dx)a+a)^6}{\sec(c+dx)^5}dx-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \text{3977}$$

$$-a^2\left(-a^2\int \cos^3(c+dx)(i\tan(c+dx)a+a)^4dx-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \text{3042}$$

$$-a^2\left(-a^2\int \frac{(i\tan(c+dx)a+a)^4}{\sec(c+dx)^3}dx-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \text{3977}$$

$$-a^2\left(-a^2\left(-a^2\int \cos(c+dx)(i\tan(c+dx)a+a)^2dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}\right)-\frac{2ia\cos^5(c+\right.$$
$$\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \text{3042}$$

$$-a^2\left(-a^2\left(-a^2\int \frac{(i\tan(c+dx)a+a)^2}{\sec(c+dx)}dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}\right)-\frac{2ia\cos^5(c+dx)(a+ia}{5d}\right.$$
$$\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \text{3977}$$

$$-a^2\left(-a^2\left(-a^2\left(a^2(-\int \sec(c+dx)dx)-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan}{3d}\right.\right.$$
$$\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \text{3042}$$

$$-a^2\left(-a^2\left(-a^2\left(a^2\left(-\int \csc\left(c+dx+\frac{\pi}{2}\right)dx\right)-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}\right)-\frac{2ia\cos^3(c+dx)(a\right.\right.\right.$$
$$\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \text{4257}$$

$$-a^2\left(-a^2\left(-a^2\left(-\frac{a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia}{3d}\right.\right.$$
$$\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

input
```
Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(((-2*I)/7)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^7)/d - a^2*(((((-2*I)/5
)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5)/d - a^2*(((((-2*I)/3)*a*Cos[c
+ d*x]^3*(a + I*a*Tan[c + d*x])^3)/d - a^2*(-((a^2*ArcTanh[Sin[c + d*x]])/
d) - ((2*I)*Cos[c + d*x]*(a^2 + I*a^2*Tan[c + d*x]))/d)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(138) = 276.

Time = 1.42 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.53

$$-\frac{29a^8\sin(dx+c)^7}{7d}-\frac{a^8\sin(dx+c)^5}{5d}-\frac{a^8\sin(dx+c)^3}{3d}+\frac{139a^8\sin(dx+c)}{105d}+\frac{a^8\ln(\sec(dx+c)+\tan(dx+$$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x)
```

output
```
-29/7*a^8*sin(d*x+c)^7/d-1/5*a^8*sin(d*x+c)^5/d-1/3*a^8*sin(d*x+c)^3/d+139
/105*a^8*sin(d*x+c)/d+1/d*a^8*ln(sec(d*x+c)+tan(d*x+c))-8*I/d*a^8*sin(d*x+
c)^4*cos(d*x+c)^3-32/5*I/d*a^8*cos(d*x+c)^3*sin(d*x+c)^2+128/35*I/d*a^8*co
s(d*x+c)-8/7*I/d*a^8*cos(d*x+c)^7-64/15*I/d*a^8*cos(d*x+c)^3+48/35*I/d*a^8
*cos(d*x+c)*sin(d*x+c)^4+8*I/d*a^8*sin(d*x+c)^2*cos(d*x+c)^5+16/5*I/d*a^8*
cos(d*x+c)^5-10/d*a^8*sin(d*x+c)^3*cos(d*x+c)^4-232/35/d*a^8*sin(d*x+c)*co
s(d*x+c)^4+122/105/d*a^8*cos(d*x+c)^2*sin(d*x+c)+64/35*I/d*a^8*cos(d*x+c)*
sin(d*x+c)^2+29/7/d*a^8*sin(d*x+c)*cos(d*x+c)^6+8/7*I/d*a^8*cos(d*x+c)*sin
(d*x+c)^6
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.63

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{-30i\,a^8e^{(7i\,dx+7i\,c)}+42i\,a^8e^{(5i\,dx+5i\,c)}-70i\,a^8e^{(3i\,dx+3i\,c)}+210i\,a^8e^{(i\,dx+i\,c)}+105\,a^8\log\left(e^{(i\,dx+i\,c)}+i\right)-1}{105\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/105*(-30*I*a^8*e^(7*I*d*x + 7*I*c) + 42*I*a^8*e^(5*I*d*x + 5*I*c) - 70*I
*a^8*e^(3*I*d*x + 3*I*c) + 210*I*a^8*e^(I*d*x + I*c) + 105*a^8*log(e^(I*d*
x + I*c) + I) - 105*a^8*log(e^(I*d*x + I*c) - I))/d
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.23

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8\left(-\log\left(e^{idx}-ie^{-ic}\right)+\log\left(e^{idx}+ie^{-ic}\right)\right)}{d} + \begin{cases} \frac{-30ia^8d^3e^{7ic}e^{7idx}+42ia^8d^3e^{5ic}e^{5idx}-70ia^8d^3e^{3ic}e^{3idx}+210ia^8d^3e^{ic}e^{idx}}{105d^4} & \text{for } d^4\neq 0 \\ x(2a^8e^{7ic}-2a^8e^{5ic}+2a^8e^{3ic}-2a^8e^{ic}) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**8,x)
```

output
```
a**8*(-log(exp(I*d*x) - I*exp(-I*c)) + log(exp(I*d*x) + I*exp(-I*c)))/d +
Piecewise(((-30*I*a**8*d**3*exp(7*I*c)*exp(7*I*d*x) + 42*I*a**8*d**3*exp(5
*I*c)*exp(5*I*d*x) - 70*I*a**8*d**3*exp(3*I*c)*exp(3*I*d*x) + 210*I*a**8*d
**3*exp(I*c)*exp(I*d*x))/(105*d**4), Ne(d**4, 0)), (x*(2*a**8*exp(7*I*c) -
 2*a**8*exp(5*I*c) + 2*a**8*exp(3*I*c) - 2*a**8*exp(I*c)), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(130) = 260.

Time = 0.05 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.03

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{240i\,a^8\cos(dx+c)^7+840\,a^8\sin(dx+c)^7+112i\left(15\cos(dx+c)^7-42\cos(dx+c)^5+35\cos(dx+\right.}{}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/210*(240*I*a^8*cos(d*x + c)^7 + 840*a^8*sin(d*x + c)^7 + 112*I*(15*cos(
d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*a^8 + 336*I*(5*cos(d*x
 + c)^7 - 7*cos(d*x + c)^5)*a^8 + 48*I*(5*cos(d*x + c)^7 - 21*cos(d*x + c)
^5 + 35*cos(d*x + c)^3 - 35*cos(d*x + c))*a^8 + (30*sin(d*x + c)^7 + 42*si
n(d*x + c)^5 + 70*sin(d*x + c)^3 - 105*log(sin(d*x + c) + 1) + 105*log(sin
(d*x + c) - 1) + 210*sin(d*x + c))*a^8 + 56*(15*sin(d*x + c)^7 - 42*sin(d*
x + c)^5 + 35*sin(d*x + c)^3)*a^8 + 420*(5*sin(d*x + c)^7 - 7*sin(d*x + c)
^5)*a^8 + 6*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35
*sin(d*x + c))*a^8)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2863 vs. $2(130) = 260$.

Time = 1.06 (sec) , antiderivative size = 2863, normalized size of antiderivative = 18.84

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/55050240*(1635552135*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) + 1
) + 22897729890*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 148
835244285*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 595340977
140*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1637187687135*a^
8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3274375374270*a^8*e^(1
8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 4911563061405*a^8*e^(16*I*d*
x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 4911563061405*a^8*e^(12*I*d*x - 2*
I*c)*log(I*e^(I*d*x + I*c) + 1) + 3274375374270*a^8*e^(10*I*d*x - 4*I*c)*l
og(I*e^(I*d*x + I*c) + 1) + 1637187687135*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^
(I*d*x + I*c) + 1) + 595340977140*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x +
 I*c) + 1) + 148835244285*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) +
 1) + 22897729890*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 56
13214927320*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 1635552135*a^8*e
^(-14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1690450650*a^8*e^(28*I*d*x + 14*I*
c)*log(I*e^(I*d*x + I*c) - 1) + 23666309100*a^8*e^(26*I*d*x + 12*I*c)*log(
I*e^(I*d*x + I*c) - 1) + 153831009150*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I
*d*x + I*c) - 1) + 615324036600*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x +
I*c) - 1) + 1692141100650*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) -
 1) + 3384282201300*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) +
5076423301950*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 507...
```

Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.36

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{2\,a^8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,16\mathrm{i}-\frac{80\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,224\mathrm{i}}{3}+\frac{224\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5}+\frac{a^8\,\mathrm{ta}}{}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,7\mathrm{i}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,35\mathrm{i}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\mathrm{t}\right.}$$

input
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(2*a^8*atanh(tan(c/2 + (d*x)/2)))/d + ((224*a^8*tan(c/2 + (d*x)/2)^2)/5 -
(a^8*tan(c/2 + (d*x)/2)^3*224i)/3 - (80*a^8*tan(c/2 + (d*x)/2)^4)/3 + a^8*
tan(c/2 + (d*x)/2)^5*16i - (304*a^8)/105 + (a^8*tan(c/2 + (d*x)/2)*304i)/1
5)/(d*(7*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*21i - 35*tan(c/2 + (d*x
)/2)^3 + tan(c/2 + (d*x)/2)^4*35i + 21*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d
*x)/2)^6*7i - tan(c/2 + (d*x)/2)^7 + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.86

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(1920\cos\left(dx+c\right)\sin\left(dx+c\right)^6 i-1728\cos\left(dx+c\right)\sin\left(dx+c\right)^4 i+496\cos\left(dx+c\right)\sin\left(dx+c\right)^2 i+\right.}{}$$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(a**8*(1920*cos(c + d*x)*sin(c + d*x)**6*i - 1728*cos(c + d*x)*sin(c + d*x
)**4*i + 496*cos(c + d*x)*sin(c + d*x)**2*i + 152*cos(c + d*x)*i - 105*log
(tan((c + d*x)/2) - 1) + 105*log(tan((c + d*x)/2) + 1) - 1920*sin(c + d*x)
**7 + 2688*sin(c + d*x)**5 - 1120*sin(c + d*x)**3 - 152*i))/(105*d)
```

3.95 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 66

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{63d} - \frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d}$$

output
```
-1/63*I*a*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^7/d-1/9*I*cos(d*x+c)^9*(a+I*a*ta
n(d*x+c))^8/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 146 vs. 2(66) = 132.

Time = 0.72 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.21

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\sec(c+dx)(-i\cos(5(c+dx))+\sin(5(c+dx)))\left(9\cos(c+dx)+16\cos(3(c+dx))+7\cos(5(c+dx)\right.}{}$$

input
```
Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*Sec[c + d*x]*((-I)*Cos[5*(c + d*x)] + Sin[5*(c + d*x)])*(9*Cos[c + d*
x] + 16*Cos[3*(c + d*x)] + 7*Cos[5*(c + d*x)] + 192*Sqrt[Cos[c + d*x]^2]*C
os[5*(c + d*x)] + (9*I)*Sin[c + d*x] + (16*I)*Sin[3*(c + d*x)] + (7*I)*Sin
[5*(c + d*x)] - (192*I)*Sqrt[Cos[c + d*x]^2]*Sin[5*(c + d*x)]))/(252*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^9}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{1}{9}a\int \cos^7(c+dx)(i\tan(c+dx)a+a)^7dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}a\int \frac{(i\tan(c+dx)a+a)^7}{\sec(c+dx)^7}dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d}$$

$$\Big\downarrow \text{3969}$$

$$-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d}-\frac{ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{63d}$$

input
```
Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/63*I)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^7)/d - ((I/9)*Cos[c + d
*x]^9*(a + I*a*Tan[c + d*x])^8)/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(58) = 116$.

Time = 1.39 (sec) , antiderivative size = 447, normalized size of antiderivative = 6.77

$$\frac{a^8\sin(dx+c)^9}{9} - 56ia^8\left(-\frac{\cos(dx+c)^7\sin(dx+c)^2}{9} - \frac{2\cos(dx+c)^7}{63}\right) - 28a^8\left(-\frac{\sin(dx+c)^5\cos(dx+c)^4}{9} - \frac{5\sin(dx+c)^3\cos(dx+c)^4}{63} - \right.$$

input
```
int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(1/9*a^8*sin(d*x+c)^9-56*I*a^8*(-1/9*cos(d*x+c)^7*sin(d*x+c)^2-2/63*co
s(d*x+c)^7)-28*a^8*(-1/9*sin(d*x+c)^5*cos(d*x+c)^4-5/63*sin(d*x+c)^3*cos(d
*x+c)^4-1/21*cos(d*x+c)^4*sin(d*x+c)+1/63*(2+cos(d*x+c)^2)*sin(d*x+c))-8/9
*I*a^8*cos(d*x+c)^9+70*a^8*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)
*cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-8*I*a^
8*(-1/9*cos(d*x+c)^3*sin(d*x+c)^6-2/21*cos(d*x+c)^3*sin(d*x+c)^4-8/105*cos
(d*x+c)^3*sin(d*x+c)^2-16/315*cos(d*x+c)^3)-28*a^8*(-1/9*cos(d*x+c)^8*sin(
d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+
c))+56*I*a^8*(-1/9*cos(d*x+c)^5*sin(d*x+c)^4-4/63*cos(d*x+c)^5*sin(d*x+c)^
2-8/315*cos(d*x+c)^5)+1/9*a^8*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*
cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{-7i\,a^8e^{(9i\,dx+9i\,c)}-9i\,a^8e^{(7i\,dx+7i\,c)}}{126\,d}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/126*(-7*I*a^8*e^(9*I*d*x + 9*I*c) - 9*I*a^8*e^(7*I*d*x + 7*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx = \begin{cases} \frac{-14ia^8de^{9ic}e^{9idx}-18ia^8de^{7ic}e^{7idx}}{252d^2} & \text{for } d^2\neq 0 \\ x\left(\frac{a^8e^{9ic}}{2}+\frac{a^8e^{7ic}}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-14*I*a**8*d*exp(9*I*c)*exp(9*I*d*x) - 18*I*a**8*d*exp(7*I*c)*
exp(7*I*d*x))/(252*d**2), Ne(d**2, 0)), (x*(a**8*exp(9*I*c)/2 + a**8*exp(7
*I*c)/2), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(54) = 108$.

Time = 0.04 (sec) , antiderivative size = 302, normalized size of antiderivative = 4.58

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx =$$
$$-\frac{280i\,a^8\cos{(dx+c)}^9 - 35\,a^8\sin{(dx+c)}^9 + 56i\,\left(35\,\cos{(dx+c)}^9 - 90\,\cos{(dx+c)}^7 + 63\,\cos{(dx+c)}\right.}{}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/315*(280*I*a^8*cos(d*x + c)^9 - 35*a^8*sin(d*x + c)^9 + 56*I*(35*cos(d*
x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*a^8 + 8*I*(35*cos(d*x +
c)^9 - 135*cos(d*x + c)^7 + 189*cos(d*x + c)^5 - 105*cos(d*x + c)^3)*a^8 +
 280*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^8 - 70*(35*sin(d*x + c)^9 -
 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a^8 - 28*(35*sin(d*x + c)^9 - 135*
sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*x + c)^3)*a^8 - (35*sin(d*
x + c)^9 - 180*sin(d*x + c)^7 + 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 +
315*sin(d*x + c))*a^8 - 140*(7*sin(d*x + c)^9 - 9*sin(d*x + c)^7)*a^8)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2451 vs. $2(54) = 108$.

Time = 1.16 (sec) , antiderivative size = 2451, normalized size of antiderivative = 37.14

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/66060288*(1419343317*a^8*e^(24*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1
) + 17032119804*a^8*e^(22*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 936
76658922*a^8*e^(20*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 31225552974
0*a^8*e^(18*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 702574941915*a^8*e
^(16*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1124119907064*a^8*e^(14*I
*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1124119907064*a^8*e^(10*I*d*x -
 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 702574941915*a^8*e^(8*I*d*x - 4*I*c)*
log(I*e^(I*d*x + I*c) + 1) + 312255529740*a^8*e^(6*I*d*x - 6*I*c)*log(I*e^
(I*d*x + I*c) + 1) + 93676658922*a^8*e^(4*I*d*x - 8*I*c)*log(I*e^(I*d*x +
I*c) + 1) + 17032119804*a^8*e^(2*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1
) + 1311473224908*a^8*e^(12*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 1419343317
*a^8*e^(-12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1419097050*a^8*e^(24*I*d*x +
 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 17029164600*a^8*e^(22*I*d*x + 10*I*c
)*log(I*e^(I*d*x + I*c) - 1) + 93660405300*a^8*e^(20*I*d*x + 8*I*c)*log(I*
e^(I*d*x + I*c) - 1) + 312201351000*a^8*e^(18*I*d*x + 6*I*c)*log(I*e^(I*d*
x + I*c) - 1) + 702453039750*a^8*e^(16*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c
) - 1) + 1123924863600*a^8*e^(14*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1)
 + 1123924863600*a^8*e^(10*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 702
453039750*a^8*e^(8*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 31220135100
0*a^8*e^(6*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 93660405300*a^8*...
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.56

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{2\,a^8\,\left(\frac{\mathrm{e}^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}\,9\mathrm{i}}{4}+\frac{\mathrm{e}^{c\,9\mathrm{i}+d\,x\,9\mathrm{i}}\,7\mathrm{i}}{4}\right)}{63\,d}$$

input
```
int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
-(2*a^8*((exp(c*7i + d*x*7i)*9i)/4 + (exp(c*9i + d*x*9i)*7i)/4))/(63*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.08

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(-896\cos\left(dx+c\right)\sin\left(dx+c\right)^8 i+1856\cos\left(dx+c\right)\sin\left(dx+c\right)^6 i-1200\cos\left(dx+c\right)\sin\left(dx+c\right)^4 i\right.}{}$$

input
```
int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(a**8*( - 896*cos(c + d*x)*sin(c + d*x)**8*i + 1856*cos(c + d*x)*sin(c + d
*x)**6*i - 1200*cos(c + d*x)*sin(c + d*x)**4*i + 248*cos(c + d*x)*sin(c +
d*x)**2*i - 8*cos(c + d*x)*i + 896*sin(c + d*x)**9 - 2304*sin(c + d*x)**7
+ 2016*sin(c + d*x)**5 - 672*sin(c + d*x)**3 + 63*sin(c + d*x) + 8*i))/(63
*d)
```

3.96 $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 136

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{1155d} - \frac{2ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^6}{231d} - \frac{ia\cos^9(c+dx)(a+ia\tan(c+dx))^7}{33d} - \frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}$$

output
```
-2/1155*I*a^3*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5/d-2/231*I*a^2*cos(d*x+c)^7
*(a+I*a*tan(d*x+c))^6/d-1/33*I*a*cos(d*x+c)^9*(a+I*a*tan(d*x+c))^7/d-1/11*
I*cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8/d
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\sec(c+dx)(-i\cos(6(c+dx))+\sin(6(c+dx)))\left(726+1111\cos(2(c+dx))+490\cos(4(c+dx))+1\right.}{}$$

input
```
Integrate[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*Sec[c + d*x]*((-I)*Cos[6*(c + d*x)] + Sin[6*(c + d*x)])*(726 + 1111*C
os[2*(c + d*x)] + 490*Cos[4*(c + d*x)] + 105*Cos[6*(c + d*x)] + 11008*Sqrt
[Cos[c + d*x]^2]*Cos[6*(c + d*x)] + (649*I)*Sin[2*(c + d*x)] + (490*I)*Sin
[4*(c + d*x)] + (105*I)*Sin[6*(c + d*x)] - (11008*I)*Sqrt[Cos[c + d*x]^2]*
Sin[6*(c + d*x)]))/(18480*d)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3978, 3042, 3978, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^{11}}dx$$
$$\Big\downarrow \text{3978}$$
$$\frac{3}{11}a\int \cos^9(c+dx)(i\tan(c+dx)a+a)^7dx-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{3}{11}a\int \frac{(i\tan(c+dx)a+a)^7}{\sec(c+dx)^9}dx-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{3}{11}a\left(\frac{2}{9}a\int \cos^7(c+dx)(i\tan(c+dx)a+a)^6dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^7}{9d}\right)-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{11}a\left(\frac{2}{9}a\int \frac{(i\tan(c+dx)a+a)^6}{\sec(c+dx)^7}dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^7}{9d}\right)-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{3}{11}a\left(\frac{2}{9}a\left(\frac{1}{7}a\int \cos^5(c+dx)(i\tan(c+dx)a+a)^5dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^6}{7d}\right)-\frac{i\cos^9(c+dx)(a$$
$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{11}a\left(\frac{2}{9}a\left(\frac{1}{7}a\int \frac{(i\tan(c+dx)a+a)^5}{\sec(c+dx)^5}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^6}{7d}\right)-\frac{i\cos^9(c+dx)(a+ia\tan(c+}{9d}$$
$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}$$

$$\Big\downarrow \text{3969}$$

$$\frac{3}{11}a\left(\frac{2}{9}a\left(-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^6}{7d}-\frac{ia\cos^5(c+dx)(a+ia\tan(c+dx))^5}{35d}\right)-\frac{i\cos^9(c+dx)(a+ia}{9d}$$
$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}$$

input
```
Int[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/11*I)*Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^8)/d + (3*a*(((-1/9*I)*C
os[c + d*x]^9*(a + I*a*Tan[c + d*x])^7)/d + (2*a*(((-1/35*I)*a*Cos[c + d*x
]^5*(a + I*a*Tan[c + d*x])^5)/d - ((I/7)*Cos[c + d*x]^7*(a + I*a*Tan[c + d
*x])^6)/d))/9))/11
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(120) = 240$.

Time = 1.66 (sec) , antiderivative size = 567, normalized size of antiderivative = 4.17

$$\frac{a^8\left(-\frac{\sin(dx+c)^7\cos(dx+c)^4}{11}-\frac{7\sin(dx+c)^5\cos(dx+c)^4}{99}-\frac{5\sin(dx+c)^3\cos(dx+c)^4}{99}-\frac{\cos(dx+c)^4\sin(dx+c)}{33}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx}{99}\right.}{}$$

input
```
int(cos(d*x+c)^11*(a+I*a*Tan[d*x+c))^8,x)
```

output
```
1/d*(a^8*(-1/11*sin(d*x+c)^7*cos(d*x+c)^4-7/99*sin(d*x+c)^5*cos(d*x+c)^4-5
/99*sin(d*x+c)^3*cos(d*x+c)^4-1/33*cos(d*x+c)^4*sin(d*x+c)+1/99*(2+cos(d*x
+c)^2)*sin(d*x+c))-8/11*I*a^8*cos(d*x+c)^11-28*a^8*(-1/11*sin(d*x+c)^5*cos
(d*x+c)^6-5/99*sin(d*x+c)^3*cos(d*x+c)^6-5/231*sin(d*x+c)*cos(d*x+c)^6+1/2
31*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+56*I*a^8*(-1/11*cos(d*x
+c)^7*sin(d*x+c)^4-4/99*cos(d*x+c)^7*sin(d*x+c)^2-8/693*cos(d*x+c)^7)+70*a
^8*(-1/11*sin(d*x+c)^3*cos(d*x+c)^8-1/33*cos(d*x+c)^8*sin(d*x+c)+1/231*(16
/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-56*I*a^8*(-
1/11*cos(d*x+c)^9*sin(d*x+c)^2-2/99*cos(d*x+c)^9)-28*a^8*(-1/11*sin(d*x+c)
*cos(d*x+c)^10+1/99*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)
^4+64/35*cos(d*x+c)^2)*sin(d*x+c))-8*I*a^8*(-1/11*cos(d*x+c)^5*sin(d*x+c)^
6-2/33*cos(d*x+c)^5*sin(d*x+c)^4-8/231*cos(d*x+c)^5*sin(d*x+c)^2-16/1155*c
os(d*x+c)^5)+1/11*a^8*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)^8+80/63*cos(d*
x+c)^6+32/21*cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.46

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{-105i\,a^8e^{(11i\,dx+11i\,c)}-385i\,a^8e^{(9i\,dx+9i\,c)}-495i\,a^8e^{(7i\,dx+7i\,c)}-231i\,a^8e^{(5i\,dx+5i\,c)}}{9240\,d}$$

input
```
integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/9240*(-105*I*a^8*e^(11*I*d*x + 11*I*c) - 385*I*a^8*e^(9*I*d*x + 9*I*c) -
 495*I*a^8*e^(7*I*d*x + 7*I*c) - 231*I*a^8*e^(5*I*d*x + 5*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.19

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx = \begin{cases} \frac{-53760ia^8d^3e^{11ic}e^{11idx}-197120ia^8d^3e^{9ic}e^{9idx}-253440ia^8d^3e^{7ic}e^{7idx}-118272ia^8d^3e^{5ic}e^{5idx}}{4730880d^4} & \text{for } d^4 \neq 0 \\ x\left(\frac{a^8e^{11ic}}{8}+\frac{3a^8e^{9ic}}{8}+\frac{3a^8e^{7ic}}{8}+\frac{a^8e^{5ic}}{8}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**11*(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-53760*I*a**8*d**3*exp(11*I*c)*exp(11*I*d*x) - 197120*I*a**8*d
**3*exp(9*I*c)*exp(9*I*d*x) - 253440*I*a**8*d**3*exp(7*I*c)*exp(7*I*d*x) -
 118272*I*a**8*d**3*exp(5*I*c)*exp(5*I*d*x))/(4730880*d**4), Ne(d**4, 0)),
 (x*(a**8*exp(11*I*c)/8 + 3*a**8*exp(9*I*c)/8 + 3*a**8*exp(7*I*c)/8 + a**8
*exp(5*I*c)/8), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(112) = 224.

Time = 0.05 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.61

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{2520i\,a^8\cos(dx+c)^{11}+24i\left(105\cos(dx+c)^{11}-385\cos(dx+c)^9+495\cos(dx+c)^7-231\cos(d\right.}{}$$

input
```
integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/3465*(2520*I*a^8*cos(d*x + c)^11 + 24*I*(105*cos(d*x + c)^11 - 385*cos(
d*x + c)^9 + 495*cos(d*x + c)^7 - 231*cos(d*x + c)^5)*a^8 + 280*I*(63*cos(
d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x + c)^7)*a^8 + 1960*I*(9*cos(
d*x + c)^11 - 11*cos(d*x + c)^9)*a^8 + 28*(315*sin(d*x + c)^11 - 1540*sin(
d*x + c)^9 + 2970*sin(d*x + c)^7 - 2772*sin(d*x + c)^5 + 1155*sin(d*x + c)
^3)*a^8 + 210*(105*sin(d*x + c)^11 - 385*sin(d*x + c)^9 + 495*sin(d*x + c)
^7 - 231*sin(d*x + c)^5)*a^8 + 140*(63*sin(d*x + c)^11 - 154*sin(d*x + c)^
9 + 99*sin(d*x + c)^7)*a^8 + 5*(63*sin(d*x + c)^11 - 385*sin(d*x + c)^9 +
990*sin(d*x + c)^7 - 1386*sin(d*x + c)^5 + 1155*sin(d*x + c)^3 - 693*sin(d
*x + c))*a^8 + 35*(9*sin(d*x + c)^11 - 11*sin(d*x + c)^9)*a^8)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2863 vs. $2(112) = 224$.

Time = 1.25 (sec) , antiderivative size = 2863, normalized size of antiderivative = 21.05

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/4844421120*(82027951005*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c)
+ 1) + 1148391314070*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1)
+ 7464543541455*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 298
58174165820*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 82109978
956005*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1642199579120
10*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 246329936868015*a
^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 246329936868015*a^8*e
^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 164219957912010*a^8*e^(10
*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 82109978956005*a^8*e^(8*I*d*x
 - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 29858174165820*a^8*e^(6*I*d*x - 8*I
*c)*log(I*e^(I*d*x + I*c) + 1) + 7464543541455*a^8*e^(4*I*d*x - 10*I*c)*lo
g(I*e^(I*d*x + I*c) + 1) + 1148391314070*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^
(I*d*x + I*c) + 1) + 281519927849160*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c
) + 1) + 82027951005*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 82004266
575*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1148059732050*a
^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 7462388258325*a^8*e^
(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 29849553033300*a^8*e^(22*
I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 82086270841575*a^8*e^(20*I*d*x
 + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 164172541683150*a^8*e^(18*I*d*x + 4
*I*c)*log(I*e^(I*d*x + I*c) - 1) + 246258812524725*a^8*e^(16*I*d*x + 2*...
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.48

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= -\frac{a^8\left(\frac{e^{c\,5i+d\,x\,5i}\,1i}{40}+\frac{e^{c\,7i+d\,x\,7i}\,3i}{56}+\frac{e^{c\,9i+d\,x\,9i}\,1i}{24}+\frac{e^{c\,11i+d\,x\,11i}\,1i}{88}\right)}{d}$$

input
```
int(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
-(a^8*((exp(c*5i + d*x*5i)*1i)/40 + (exp(c*7i + d*x*7i)*3i)/56 + (exp(c*9i
 + d*x*9i)*1i)/24 + (exp(c*11i + d*x*11i)*1i)/88))/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.21

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(13440\cos(dx+c)\sin(dx+c)^{10}i - 42560\cos(dx+c)\sin(dx+c)^8i + 49040\cos(dx+c)\sin(dx+c\right.}{}$$

input
```
int(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(a**8*(13440*cos(c + d*x)*sin(c + d*x)**10*i - 42560*cos(c + d*x)*sin(c +
d*x)**8*i + 49040*cos(c + d*x)*sin(c + d*x)**6*i - 24312*cos(c + d*x)*sin(
c + d*x)**4*i + 4544*cos(c + d*x)*sin(c + d*x)**2*i - 152*cos(c + d*x)*i -
 13440*sin(c + d*x)**11 + 49280*sin(c + d*x)**9 - 68640*sin(c + d*x)**7 +
44352*sin(c + d*x)**5 - 12705*sin(c + d*x)**3 + 1155*sin(c + d*x) + 152*i)
)/(1155*d)
```

3.97 $\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 211

$$\begin{aligned}\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx = &-\frac{20ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^5}{3003d}\\&-\frac{20ia^2\cos^9(c+dx)(a+ia\tan(c+dx))^6}{1287d}\\&-\frac{5ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^7}{143d}\\&-\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}\\&-\frac{8ia^2\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{9009d}\\&-\frac{8i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^4}{3003d}\end{aligned}$$

output
```
-20/3003*I*a^3*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5/d-20/1287*I*a^2*cos(d*x+c
)^9*(a+I*a*tan(d*x+c))^6/d-5/143*I*a*cos(d*x+c)^11*(a+I*a*tan(d*x+c))^7/d-
1/13*I*cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8/d-8/9009*I*a^2*cos(d*x+c)^3*(a^2
+I*a^2*tan(d*x+c))^3/d-8/3003*I*cos(d*x+c)^5*(a^2+I*a^2*tan(d*x+c))^4/d
```

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.80

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\sec(c+dx)(-i\cos(7(c+dx))+\sin(7(c+dx)))\left(44759\cos(c+dx)+26117\cos(3(c+dx))+7791\,\mathrm{co}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^13*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*Sec[c + d*x]*((-I)*Cos[7*(c + d*x)] + Sin[7*(c + d*x)])*(44759*Cos[c
+ d*x] + 26117*Cos[3*(c + d*x)] + 7791*Cos[5*(c + d*x)] + 693*Cos[7*(c + d
*x)] + 275456*Sqrt[Cos[c + d*x]^2]*Cos[7*(c + d*x)] + (1001*I)*Sin[c + d*x
] + (2093*I)*Sin[3*(c + d*x)] + (1785*I)*Sin[5*(c + d*x)] + (693*I)*Sin[7*
(c + d*x)] - (275456*I)*Sqrt[Cos[c + d*x]^2]*Sin[7*(c + d*x)]))/(576576*d)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3978, 3042, 3978, 3042, 3978, 3042, 3978, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^{13}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{5}{13}a\int \cos^{11}(c+dx)(i\tan(c+dx)a+a)^7dx-\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{13}a\int \frac{(i\tan(c+dx)a+a)^7}{\sec(c+dx)^{11}}dx-\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$\Big\downarrow$ 3978

$$\frac{5}{13}a\left(\frac{4}{11}a\int \cos^9(c+dx)(i\tan(c+dx)a+a)^6dx-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^7}{11d}\right)-\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$\Big\downarrow$ 3042

$$\frac{5}{13}a\left(\frac{4}{11}a\int \frac{(i\tan(c+dx)a+a)^6}{\sec(c+dx)^9}dx-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^7}{11d}\right)-\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$\Big\downarrow$ 3978

$$\frac{5}{13}a\left(\frac{4}{11}a\left(\frac{1}{3}a\int \cos^7(c+dx)(i\tan(c+dx)a+a)^5dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^6}{9d}\right)-\frac{i\cos^{11}(c+dx)(\right. \frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$\Big\downarrow$ 3042

$$\frac{5}{13}a\left(\frac{4}{11}a\left(\frac{1}{3}a\int \frac{(i\tan(c+dx)a+a)^5}{\sec(c+dx)^7}dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^6}{9d}\right)-\frac{i\cos^{11}(c+dx)(a+ia\tan(c}{11d}\right. \frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$\Big\downarrow$ 3978

$$\frac{5}{13}a\left(\frac{4}{11}a\left(\frac{1}{3}a\left(\frac{2}{7}a\int \cos^5(c+dx)(i\tan(c+dx)a+a)^4dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}\right)-\frac{i\cos^9(c+}{}\right.\right. \frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$\Big\downarrow$ 3042

$$\frac{5}{13}a\left(\frac{4}{11}a\left(\frac{1}{3}a\left(\frac{2}{7}a\int \frac{(i\tan(c+dx)a+a)^4}{\sec(c+dx)^5}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}\right)-\frac{i\cos^9(c+dx)(a+ia\,\mathrm{ta}}{9d}\right.\right. \frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$\Big\downarrow$ 3978

$$\frac{5}{13}a\left(\frac{4}{11}a\left(\frac{1}{3}a\left(\frac{2}{7}a\left(\frac{1}{5}a\int\cos^3(c+dx)(i\tan(c+dx)a+a)^3dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}\right)-\frac{i\cos^7(\right.\right.\right.\right.$$
$$\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{13}a\left(\frac{4}{11}a\left(\frac{1}{3}a\left(\frac{2}{7}a\left(\frac{1}{5}a\int\frac{(i\tan(c+dx)a+a)^3}{\sec(c+dx)^3}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}\right)-\frac{i\cos^7(c+dx)(a+}{7\right.\right.\right.\right.$$
$$\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$$\Big\downarrow \text{3969}$$

$$\frac{5}{13}a\left(\frac{4}{11}a\left(\frac{1}{3}a\left(\frac{2}{7}a\left(-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}-\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{15d}\right)-\frac{i\cos^7(c+}{\right.\right.\right.\right.$$
$$\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

input
```
Int[Cos[c + d*x]^13*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/13*I)*Cos[c + d*x]^13*(a + I*a*Tan[c + d*x])^8)/d + (5*a*(((-1/11*I)*
Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^7)/d + (4*a*(((-1/9*I)*Cos[c + d*x]
^9*(a + I*a*Tan[c + d*x])^6)/d + (a*(((-1/7*I)*Cos[c + d*x]^7*(a + I*a*Tan
[c + d*x])^5)/d + (2*a*(((-1/15*I)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])
^3)/d - ((I/5)*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^4)/d))/7))/3))/11))/1
3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 616 vs. 2(187) = 374.

Time = 2.91 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.92

$$\frac{a^8\left(-\frac{\sin(dx+c)^7\cos(dx+c)^6}{13}-\frac{7\sin(dx+c)^5\cos(dx+c)^6}{143}-\frac{35\sin(dx+c)^3\cos(dx+c)^6}{1287}-\frac{5\sin(dx+c)\cos(dx+c)^6}{429}+\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\,c}{}\right.}{4}\right.}{}$$

input
```
int(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(-1/13*sin(d*x+c)^7*cos(d*x+c)^6-7/143*sin(d*x+c)^5*cos(d*x+c)^6-
35/1287*sin(d*x+c)^3*cos(d*x+c)^6-5/429*sin(d*x+c)*cos(d*x+c)^6+1/429*(8/3
+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+56*I*a^8*(-1/13*cos(d*x+c)^9*s
in(d*x+c)^4-4/143*cos(d*x+c)^9*sin(d*x+c)^2-8/1287*cos(d*x+c)^9)-28*a^8*(-
1/13*sin(d*x+c)^5*cos(d*x+c)^8-5/143*sin(d*x+c)^3*cos(d*x+c)^8-5/429*cos(d
*x+c)^8*sin(d*x+c)+5/3003*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+
c)^2)*sin(d*x+c))-8/13*I*a^8*cos(d*x+c)^13+70*a^8*(-1/13*sin(d*x+c)^3*cos(
d*x+c)^10-3/143*sin(d*x+c)*cos(d*x+c)^10+1/429*(128/35+cos(d*x+c)^8+8/7*co
s(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))-8*I*a^8*(-1/
13*cos(d*x+c)^7*sin(d*x+c)^6-6/143*cos(d*x+c)^7*sin(d*x+c)^4-8/429*cos(d*x
+c)^7*sin(d*x+c)^2-16/3003*cos(d*x+c)^7)-28*a^8*(-1/13*sin(d*x+c)*cos(d*x+
c)^12+1/143*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)^8+80/63*cos(d*x+c)^6+32/
21*cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))-56*I*a^8*(-1/13*cos(d*x+c
)^11*sin(d*x+c)^2-2/143*cos(d*x+c)^11)+1/13*a^8*(1024/231+cos(d*x+c)^12+12
/11*cos(d*x+c)^10+40/33*cos(d*x+c)^8+320/231*cos(d*x+c)^6+128/77*cos(d*x+c
)^4+512/231*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.43

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\frac{-693i\,a^8e^{(13i\,dx+13i\,c)}-4095i\,a^8e^{(11i\,dx+11i\,c)}-10010i\,a^8e^{(9i\,dx+9i\,c)}-12870i\,a^8e^{(7i\,dx+7i\,c)}-9009i\,a^8e^{(5i\,dx}}{288288\,d}$$

input
```
integrate(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/288288*(-693*I*a^8*e^(13*I*d*x + 13*I*c) - 4095*I*a^8*e^(11*I*d*x + 11*I
*c) - 10010*I*a^8*e^(9*I*d*x + 9*I*c) - 12870*I*a^8*e^(7*I*d*x + 7*I*c) -
9009*I*a^8*e^(5*I*d*x + 5*I*c) - 3003*I*a^8*e^(3*I*d*x + 3*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.14

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\begin{cases}\frac{-17439916032ia^8d^5e^{13ic}e^{13idx}-103054049280ia^8d^5e^{11ic}e^{11idx}-251909898240ia^8d^5e^{9ic}e^{9idx}-323884154880ia^8d^5e^{7ic}e^{7idx}-22671890841}{7255005069312d^6} \\ x\left(\frac{a^8e^{13ic}}{32}+\frac{5a^8e^{11ic}}{32}+\frac{5a^8e^{9ic}}{16}+\frac{5a^8e^{7ic}}{16}+\frac{5a^8e^{5ic}}{32}+\frac{a^8e^{3ic}}{32}\right)\end{cases}$$

input
```
integrate(cos(d*x+c)**13*(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-17439916032*I*a**8*d**5*exp(13*I*c)*exp(13*I*d*x) - 103054049
280*I*a**8*d**5*exp(11*I*c)*exp(11*I*d*x) - 251909898240*I*a**8*d**5*exp(9
*I*c)*exp(9*I*d*x) - 323884154880*I*a**8*d**5*exp(7*I*c)*exp(7*I*d*x) - 22
6718908416*I*a**8*d**5*exp(5*I*c)*exp(5*I*d*x) - 75572969472*I*a**8*d**5*e
xp(3*I*c)*exp(3*I*d*x))/(7255005069312*d**6), Ne(d**6, 0)), (x*(a**8*exp(1
3*I*c)/32 + 5*a**8*exp(11*I*c)/32 + 5*a**8*exp(9*I*c)/16 + 5*a**8*exp(7*I*
c)/16 + 5*a**8*exp(5*I*c)/32 + a**8*exp(3*I*c)/32), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(175) = 350.

Time = 0.04 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.92

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$
$$-\frac{5544i\,a^8\cos(dx+c)^{13}+24i\left(231\cos(dx+c)^{13}-819\cos(dx+c)^{11}+1001\cos(dx+c)^9-429\cos\right.}{}$$

input
```
integrate(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/9009*(5544*I*a^8*cos(d*x + c)^13 + 24*I*(231*cos(d*x + c)^13 - 819*cos(
d*x + c)^11 + 1001*cos(d*x + c)^9 - 429*cos(d*x + c)^7)*a^8 + 392*I*(99*co
s(d*x + c)^13 - 234*cos(d*x + c)^11 + 143*cos(d*x + c)^9)*a^8 + 3528*I*(11
*cos(d*x + c)^13 - 13*cos(d*x + c)^11)*a^8 - 42*(1155*sin(d*x + c)^13 - 54
60*sin(d*x + c)^11 + 10010*sin(d*x + c)^9 - 8580*sin(d*x + c)^7 + 3003*sin
(d*x + c)^5)*a^8 - 28*(693*sin(d*x + c)^13 - 4095*sin(d*x + c)^11 + 10010*
sin(d*x + c)^9 - 12870*sin(d*x + c)^7 + 9009*sin(d*x + c)^5 - 3003*sin(d*x
 + c)^3)*a^8 - 84*(231*sin(d*x + c)^13 - 819*sin(d*x + c)^11 + 1001*sin(d*
x + c)^9 - 429*sin(d*x + c)^7)*a^8 - 3*(231*sin(d*x + c)^13 - 1638*sin(d*x
 + c)^11 + 5005*sin(d*x + c)^9 - 8580*sin(d*x + c)^7 + 9009*sin(d*x + c)^5
 - 6006*sin(d*x + c)^3 + 3003*sin(d*x + c))*a^8 - 7*(99*sin(d*x + c)^13 -
234*sin(d*x + c)^11 + 143*sin(d*x + c)^9)*a^8)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2891 vs. 2(175) = 350.

Time = 1.34 (sec) , antiderivative size = 2891, normalized size of antiderivative = 13.70

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/151145938944*(1945052766657*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I
*c) + 1) + 27230738733198*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c)
+ 1) + 176999801765787*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1
) + 707999207063148*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) +
1946997819423657*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 389
3995638847314*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 584099
3458270971*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 584099345
8270971*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 389399563884
7314*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 194699781942365
7*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 707999207063148*a^8
*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 176999801765787*a^8*e^(4
*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 27230738733198*a^8*e^(2*I*d*
x - 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 6675421095166824*a^8*e^(14*I*d*x)
*log(I*e^(I*d*x + I*c) + 1) + 1945052766657*a^8*e^(-14*I*c)*log(I*e^(I*d*x
 + I*c) + 1) + 1944080407269*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 27217125701766*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) -
 1) + 176911317061479*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1)
 + 707645268245916*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1
946024487676269*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 3892
048975352538*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5838...
```

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.44

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= -\frac{a^8\left(\frac{e^{c\,3i+d\,x\,3i}\,1i}{96}+\frac{e^{c\,5i+d\,x\,5i}\,1i}{32}+\frac{e^{c\,7i+d\,x\,7i}\,5i}{112}+\frac{e^{c\,9i+d\,x\,9i}\,5i}{144}+\frac{e^{c\,11i+d\,x\,11i}\,5i}{352}+\frac{e^{c\,13i+d\,x\,13i}\,1i}{416}\right)}{d}$$

input
```
int(cos(c + d*x)^13*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
-(a^8*((exp(c*3i + d*x*3i)*1i)/96 + (exp(c*5i + d*x*5i)*1i)/32 + (exp(c*7i
 + d*x*7i)*5i)/112 + (exp(c*9i + d*x*9i)*5i)/144 + (exp(c*11i + d*x*11i)*5
i)/352 + (exp(c*13i + d*x*13i)*1i)/416))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.91

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(-88704\cos\left(dx+c\right)\sin\left(dx+c\right)^{12}i+374976\cos\left(dx+c\right)\sin\left(dx+c\right)^{10}i-624400\cos\left(dx+c\right)\sin\left(d\right.\right.}{}$$

input
```
int(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(a**8*( - 88704*cos(c + d*x)*sin(c + d*x)**12*i + 374976*cos(c + d*x)*sin(
c + d*x)**10*i - 624400*cos(c + d*x)*sin(c + d*x)**8*i + 511624*cos(c + d*
x)*sin(c + d*x)**6*i - 207672*cos(c + d*x)*sin(c + d*x)**4*i + 35416*cos(c
 + d*x)*sin(c + d*x)**2*i - 1240*cos(c + d*x)*i + 88704*sin(c + d*x)**13 -
 419328*sin(c + d*x)**11 + 800800*sin(c + d*x)**9 - 782496*sin(c + d*x)**7
 + 405405*sin(c + d*x)**5 - 102102*sin(c + d*x)**3 + 9009*sin(c + d*x) + 1
240*i))/(9009*d)
```

3.98 $\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 212

$$
\begin{aligned}
\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx &= \frac{7a^8\sin(c+dx)}{1287d} - \frac{7a^8\sin^3(c+dx)}{1287d} \\
&\quad + \frac{7a^8\sin^5(c+dx)}{2145d} - \frac{a^8\sin^7(c+dx)}{1287d} \\
&\quad - \frac{2ia^3\cos^{13}(c+dx)(a+ia\tan(c+dx))^5}{195d} \\
&\quad - \frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d} \\
&\quad - \frac{2ia^2\cos^{11}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{715d} \\
&\quad - \frac{2i\cos^9(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{1287d}
\end{aligned}
$$

output

```
7/1287*a^8*sin(d*x+c)/d-7/1287*a^8*sin(d*x+c)^3/d+7/2145*a^8*sin(d*x+c)^5/
d-1/1287*a^8*sin(d*x+c)^7/d-2/195*I*a^3*cos(d*x+c)^13*(a+I*a*tan(d*x+c))^5
/d-2/15*I*a*cos(d*x+c)^15*(a+I*a*tan(d*x+c))^7/d-2/715*I*a^2*cos(d*x+c)^11
*(a^2+I*a^2*tan(d*x+c))^3/d-2/1287*I*cos(d*x+c)^9*(a^8+I*a^8*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.82

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\frac{a^8\sec(c+dx)(-i\cos(8(c+dx))+\sin(8(c+dx)))\left(28600+48256\cos(2(c+dx))+28896\cos(4(c+dx\right.}{}$$

input
```
Integrate[Cos[c + d*x]^15*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*Sec[c + d*x]*((-I)*Cos[8*(c + d*x)] + Sin[8*(c + d*x)])*(28600 + 4825
6*Cos[2*(c + d*x)] + 28896*Cos[4*(c + d*x)] + 12672*Cos[6*(c + d*x)] + 343
2*Cos[8*(c + d*x)] + 317440*Sqrt[Cos[c + d*x]^2]*Cos[8*(c + d*x)] - (10946
*I)*Sin[2*(c + d*x)] - (13146*I)*Sin[4*(c + d*x)] - (8778*I)*Sin[6*(c + d*
x)] - (3003*I)*Sin[8*(c + d*x)] - (317440*I)*Sqrt[Cos[c + d*x]^2]*Sin[8*(c
 + d*x)]))/(823680*d)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {3042, 3977, 3042, 3977, 3042, 3977, 3042, 3977, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^{15}}dx$$
$$\Big\downarrow \text{3977}$$
$$\frac{1}{15}a^2\int \cos^{13}(c+dx)(i\tan(c+dx)a+a)^6dx-\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{15}a^2\int \frac{(i\tan(c+dx)a+a)^6}{\sec(c+dx)^{13}}dx-\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

$$\Big\downarrow \text{3977}$$

$$\frac{1}{15}a^2\left(\frac{3}{13}a^2\int \cos^{11}(c+dx)(i\tan(c+dx)a+a)^4dx-\frac{2ia\cos^{13}(c+dx)(a+ia\tan(c+dx))^5}{13d}\right)-\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{15}a^2\left(\frac{3}{13}a^2\int \frac{(i\tan(c+dx)a+a)^4}{\sec(c+dx)^{11}}dx-\frac{2ia\cos^{13}(c+dx)(a+ia\tan(c+dx))^5}{13d}\right)-\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

$$\Big\downarrow \text{3977}$$

$$\frac{1}{15}a^2\left(\frac{3}{13}a^2\left(\frac{5}{11}a^2\int \cos^9(c+dx)(i\tan(c+dx)a+a)^2dx-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^3}{11d}\right)-\frac{2ia\cos^1}{}\right.$$
$$\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{15}a^2\left(\frac{3}{13}a^2\left(\frac{5}{11}a^2\int \frac{(i\tan(c+dx)a+a)^2}{\sec(c+dx)^9}dx-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^3}{11d}\right)-\frac{2ia\cos^{13}(c+dx)(a}{13}\right.$$
$$\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

$$\Big\downarrow \text{3977}$$

$$\frac{1}{15}a^2\left(\frac{3}{13}a^2\left(\frac{5}{11}a^2\left(\frac{7}{9}a^2\int \cos^7(c+dx)dx-\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}\right)-\frac{2ia\cos^{11}(c+dx)(a+i}{11d}\right.\right.$$
$$\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{15}a^2\left(\frac{3}{13}a^2\left(\frac{5}{11}a^2\left(\frac{7}{9}a^2\int\sin\left(c+dx+\frac{\pi}{2}\right)^7dx-\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}\right)-\frac{2ia\cos^{11}(c+dx)}{}\right.\right.\right.$$
$$\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{1}{15}a^2\left(\frac{3}{13}a^2\left(\frac{5}{11}a^2\left(-\frac{7a^2\int\left(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{9d}-\frac{2i\cos^9(c}{}\right.\right.\right.$$
$$\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{15}a^2\left(\frac{3}{13}a^2\left(\frac{5}{11}a^2\left(-\frac{7a^2\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{9d}-\frac{2i\cos^9(c+dx)\left(a^2\right.}{9d}\right.\right.\right.$$
$$\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

input
```
Int[Cos[c + d*x]^15*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(((-2*I)/15)*a*Cos[c + d*x]^15*(a + I*a*Tan[c + d*x])^7)/d + (a^2*((((-2*I
)/13)*a*Cos[c + d*x]^13*(a + I*a*Tan[c + d*x])^5)/d + (3*a^2*((((-2*I)/11)
*a*Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^3)/d + (5*a^2*((-7*a^2*(-Sin[c +
 d*x] + Sin[c + d*x]^3 - (3*Sin[c + d*x]^5)/5 + Sin[c + d*x]^7/7))/(9*d) -
 (((2*I)/9)*Cos[c + d*x]^9*(a^2 + I*a^2*Tan[c + d*x]))/d))/11))/13))/15
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(188) = 376$.

Time = 1.68 (sec) , antiderivative size = 667, normalized size of antiderivative = 3.15

$$a^8\left(-\frac{\sin(dx+c)^7\cos(dx+c)^8}{15}-\frac{7\sin(dx+c)^5\cos(dx+c)^8}{195}-\frac{7\sin(dx+c)^3\cos(dx+c)^8}{429}-\frac{7\cos(dx+c)^8\sin(dx+c)}{1287}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos}{}\right.}{}\right.$$

input
```
int(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(-1/15*sin(d*x+c)^7*cos(d*x+c)^8-7/195*sin(d*x+c)^5*cos(d*x+c)^8-
7/429*sin(d*x+c)^3*cos(d*x+c)^8-7/1287*cos(d*x+c)^8*sin(d*x+c)+1/1287*(16/
5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-56*I*a^8*(-1
/15*sin(d*x+c)^2*cos(d*x+c)^13-2/195*cos(d*x+c)^13)-28*a^8*(-1/15*sin(d*x+
c)^5*cos(d*x+c)^10-1/39*sin(d*x+c)^3*cos(d*x+c)^10-1/143*sin(d*x+c)*cos(d*
x+c)^10+1/1287*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64
/35*cos(d*x+c)^2)*sin(d*x+c))+56*I*a^8*(-1/15*sin(d*x+c)^4*cos(d*x+c)^11-4
/195*cos(d*x+c)^11*sin(d*x+c)^2-8/2145*cos(d*x+c)^11)+70*a^8*(-1/15*sin(d*
x+c)^3*cos(d*x+c)^12-1/65*sin(d*x+c)*cos(d*x+c)^12+1/715*(256/63+cos(d*x+c
)^10+10/9*cos(d*x+c)^8+80/63*cos(d*x+c)^6+32/21*cos(d*x+c)^4+128/63*cos(d*
x+c)^2)*sin(d*x+c))-8*I*a^8*(-1/15*sin(d*x+c)^6*cos(d*x+c)^9-2/65*cos(d*x+
c)^9*sin(d*x+c)^4-8/715*cos(d*x+c)^9*sin(d*x+c)^2-16/6435*cos(d*x+c)^9)-28
*a^8*(-1/15*sin(d*x+c)*cos(d*x+c)^14+1/195*(1024/231+cos(d*x+c)^12+12/11*c
os(d*x+c)^10+40/33*cos(d*x+c)^8+320/231*cos(d*x+c)^6+128/77*cos(d*x+c)^4+5
12/231*cos(d*x+c)^2)*sin(d*x+c))-8/15*I*a^8*cos(d*x+c)^15+1/15*a^8*(2048/4
29+cos(d*x+c)^14+14/13*cos(d*x+c)^12+168/143*cos(d*x+c)^10+560/429*cos(d*x
+c)^8+640/429*cos(d*x+c)^6+256/143*cos(d*x+c)^4+1024/429*cos(d*x+c)^2)*sin
(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.56

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{-429i\,a^8e^{(15i\,dx+15i\,c)} - 3465i\,a^8e^{(13i\,dx+13i\,c)} - 12285i\,a^8e^{(11i\,dx+11i\,c)} - 25025i\,a^8e^{(9i\,dx+9i\,c)} - 32175i\,a^8e^{(7\cdots}}{823680\,d}$$

input
```
integrate(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/823680*(-429*I*a^8*e^(15*I*d*x + 15*I*c) - 3465*I*a^8*e^(13*I*d*x + 13*I
*c) - 12285*I*a^8*e^(11*I*d*x + 11*I*c) - 25025*I*a^8*e^(9*I*d*x + 9*I*c)
- 32175*I*a^8*e^(7*I*d*x + 7*I*c) - 27027*I*a^8*e^(5*I*d*x + 5*I*c) - 1501
5*I*a^8*e^(3*I*d*x + 3*I*c) - 6435*I*a^8*e^(I*d*x + I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.48

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \begin{cases} \frac{-10867748850798428160ia^8d^7e^{15ic}e^{15idx}-8777797148721807360 0ia^8d^7e^{13ic}e^{13idx}-311212808000136806400ia^8d^7e^{11ic}e^{11idx}-633952016\cdots}{\cdots} \\ x\left(\frac{a^8e^{15ic}}{128}+\frac{7a^8e^{13ic}}{128}+\frac{21a^8e^{11ic}}{128}+\frac{35a^8e^{9ic}}{128}+\frac{35a^8e^{7ic}}{128}+\frac{21a^8e^{5ic}}{128}+\frac{7a^8e^{3ic}}{128}+\frac{a^8e^{ic}}{128}\right) \end{cases}$$

input
```
integrate(cos(d*x+c)**15*(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-10867748850798428160*I*a**8*d**7*exp(15*I*c)*exp(15*I*d*x) -
87777971487218073600*I*a**8*d**7*exp(13*I*c)*exp(13*I*d*x) - 3112128080001
36806400*I*a**8*d**7*exp(11*I*c)*exp(11*I*d*x) - 633952016296574976000*I*a
**8*d**7*exp(9*I*c)*exp(9*I*d*x) - 815081163809882112000*I*a**8*d**7*exp(7
*I*c)*exp(7*I*d*x) - 684668177600300974080*I*a**8*d**7*exp(5*I*c)*exp(5*I*
d*x) - 380371209777944985600*I*a**8*d**7*exp(3*I*c)*exp(3*I*d*x) - 1630162
32761976422400*I*a**8*d**7*exp(I*c)*exp(I*d*x))/(20866077793532982067200*d
**8), Ne(d**8, 0)), (x*(a**8*exp(15*I*c)/128 + 7*a**8*exp(13*I*c)/128 + 21
*a**8*exp(11*I*c)/128 + 35*a**8*exp(9*I*c)/128 + 35*a**8*exp(7*I*c)/128 +
21*a**8*exp(5*I*c)/128 + 7*a**8*exp(3*I*c)/128 + a**8*exp(I*c)/128), True)
)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 453 vs. 2(180) = 360.

Time = 0.06 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.14

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$
$$-\frac{3432i\,a^8\cos(dx+c)^{15}+8i\left(429\cos(dx+c)^{15}-1485\cos(dx+c)^{13}+1755\cos(dx+c)^{11}-715\cos\cdots\right.}{\cdots}$$

input
```
integrate(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/6435*(3432*I*a^8*cos(d*x + c)^15 + 8*I*(429*cos(d*x + c)^15 - 1485*cos(
d*x + c)^13 + 1755*cos(d*x + c)^11 - 715*cos(d*x + c)^9)*a^8 + 168*I*(143*
cos(d*x + c)^15 - 330*cos(d*x + c)^13 + 195*cos(d*x + c)^11)*a^8 + 1848*I*
(13*cos(d*x + c)^15 - 15*cos(d*x + c)^13)*a^8 + 4*(3003*sin(d*x + c)^15 -
13860*sin(d*x + c)^13 + 24570*sin(d*x + c)^11 - 20020*sin(d*x + c)^9 + 643
5*sin(d*x + c)^7)*a^8 + 10*(3003*sin(d*x + c)^15 - 17325*sin(d*x + c)^13 +
 40950*sin(d*x + c)^11 - 50050*sin(d*x + c)^9 + 32175*sin(d*x + c)^7 - 900
9*sin(d*x + c)^5)*a^8 + 4*(3003*sin(d*x + c)^15 - 20790*sin(d*x + c)^13 +
61425*sin(d*x + c)^11 - 100100*sin(d*x + c)^9 + 96525*sin(d*x + c)^7 - 540
54*sin(d*x + c)^5 + 15015*sin(d*x + c)^3)*a^8 + (429*sin(d*x + c)^15 - 148
5*sin(d*x + c)^13 + 1755*sin(d*x + c)^11 - 715*sin(d*x + c)^9)*a^8 + (429*
sin(d*x + c)^15 - 3465*sin(d*x + c)^13 + 12285*sin(d*x + c)^11 - 25025*sin
(d*x + c)^9 + 32175*sin(d*x + c)^7 - 27027*sin(d*x + c)^5 + 15015*sin(d*x
+ c)^3 - 6435*sin(d*x + c))*a^8)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2919 vs. $2(180) = 360$.

Time = 1.41 (sec) , antiderivative size = 2919, normalized size of antiderivative = 13.77

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/863691079680*(5682101344920*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I
*c) + 1) + 79549418828880*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c)
+ 1) + 517071222387720*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1
) + 2068284889550880*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 5687783446264920*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 11
375566892529840*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1706
3350338794760*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 170633
50338794760*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 11375566
892529840*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5687783446
264920*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 20682848895508
80*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 517071222387720*a^
8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 79549418828880*a^8*e^(
2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 19500971815765440*a^8*e^(14
*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 5682101344920*a^8*e^(-14*I*c)*log(I*e
^(I*d*x + I*c) + 1) + 5674116082635*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d
*x + I*c) - 1) + 79437625156890*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x +
 I*c) - 1) + 516344563519785*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 2065378254079140*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c)
- 1) + 5679790198717635*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1
) + 11359580397435270*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - ...
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.05

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{2\,a^8\,\left(2\sin\left(\frac{c}{4}+\frac{d\,x}{4}\right)^2-1\right)\,\left(-\frac{44779\sin(c+d\,x)^2}{32}+\frac{\sin(c+d\,x)\,32175\mathrm{i}}{128}-\frac{26075\sin(2\,c+2\,d\,x)^2}{16}-\frac{\sin(2\,c+2\,d\,x)\,3575\mathrm{i}}{8}+\frac{11458}{}\right.}{}$$

input
```
int(cos(c + d*x)^15*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(2*a^8*(2*sin(c/4 + (d*x)/4)^2 - 1)*((sin(c + d*x)*32175i)/128 - (sin(2*c
+ 2*d*x)*3575i)/8 + (sin(3*c + 3*d*x)*84227i)/128 - sin(4*c + 4*d*x)*754i
+ (sin(5*c + 5*d*x)*111527i)/128 - (sin(6*c + 6*d*x)*7187i)/8 + (sin(7*c +
 7*d*x)*121427i)/128 - (26075*sin(2*c + 2*d*x)^2)/16 + (114583*sin(c/2 + (
d*x)/2)^2)/64 - (57925*sin(3*c + 3*d*x)^2)/32 + (116585*sin((3*c)/2 + (3*d
*x)/2)^2)/64 + (119315*sin((5*c)/2 + (5*d*x)/2)^2)/64 + (122285*sin((7*c)/
2 + (7*d*x)/2)^2)/64 - (44779*sin(c + d*x)^2)/32 - 952))/(6435*d*(sin((15*
c)/2 + (15*d*x)/2) - sin((15*c)/4 + (15*d*x)/4)^2*2i + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.03

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(54912\cos\left(dx+c\right)\sin\left(dx+c\right)^{14}i-289344\cos\left(dx+c\right)\sin\left(dx+c\right)^{12}i+629712\cos\left(dx+c\right)\sin\left(dx\right.\right.}{}$$

input
```
int(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x)
```

output
```
(a**8*(54912*cos(c + d*x)*sin(c + d*x)**14*i - 289344*cos(c + d*x)*sin(c +
 d*x)**12*i + 629712*cos(c + d*x)*sin(c + d*x)**10*i - 724600*cos(c + d*x)
*sin(c + d*x)**8*i + 466240*cos(c + d*x)*sin(c + d*x)**6*i - 161232*cos(c
+ d*x)*sin(c + d*x)**4*i + 25264*cos(c + d*x)*sin(c + d*x)**2*i - 952*cos(
c + d*x)*i - 54912*sin(c + d*x)**15 + 316800*sin(c + d*x)**13 - 767520*sin
(c + d*x)**11 + 1006720*sin(c + d*x)**9 - 765765*sin(c + d*x)**7 + 333333*
sin(c + d*x)**5 - 75075*sin(c + d*x)**3 + 6435*sin(c + d*x) + 952*i))/(643
5*d)
```

3.99 $\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{8i(a-ia\tan(c+dx))^5}{5a^6d} - \frac{2i(a-ia\tan(c+dx))^6}{a^7d} + \frac{6i(a-ia\tan(c+dx))^7}{7a^8d} - \frac{i(a-ia\tan(c+dx))^8}{8a^9d}$$

output
```
8/5*I*(a-I*a*tan(d*x+c))^5/a^6/d-2*I*(a-I*a*tan(d*x+c))^6/a^7/d+6/7*I*(a-I
*a*tan(d*x+c))^7/a^8/d-1/8*I*(a-I*a*tan(d*x+c))^8/a^9/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.52

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{(i+\tan(c+dx))^5\left(93+185i\tan(c+dx)-135\tan^2(c+dx)-35i\tan^3(c+dx)\right)}{280ad}$$

input
```
Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x]),x]
```

output
```
((I + Tan[c + d*x])^5*(93 + (185*I)*Tan[c + d*x] - 135*Tan[c + d*x]^2 - (3
5*I)*Tan[c + d*x]^3))/(280*a*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{10}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)^3d(ia\tan(c+dx))}{a^9d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(-(a-ia\tan(c+dx))^7+6a(a-ia\tan(c+dx))^6-12a^2(a-ia\tan(c+dx))^5+8a^3(a-ia\tan(c+dx))^4\right)}{a^9d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{8}{5}a^3(a-ia\tan(c+dx))^5+2a^2(a-ia\tan(c+dx))^6+\frac{1}{8}(a-ia\tan(c+dx))^8-\frac{6}{7}a(a-ia\tan(c+dx))^7\right)}{a^9d}$$

input
```
Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*((-8*a^3*(a - I*a*Tan[c + d*x])^5)/5 + 2*a^2*(a - I*a*Tan[c + d*x])^
6 - (6*a*(a - I*a*Tan[c + d*x])^7)/7 + (a - I*a*Tan[c + d*x])^8/8))/(a^9*d
)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

method	result
risch	$\frac{32 i \left(56 \,{\mathrm e}^{6 i \left(d x +c \right)}+28 \,{\mathrm e}^{4 i \left(d x +c \right)}+8 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{35 d a \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{8}}$
derivativedivides	$-\frac{-\tan \left(d x +c \right)+\frac{i \tan \left(d x +c \right)^{8}}{8}-\frac{\tan \left(d x +c \right)^{7}}{7}+\frac{i \tan \left(d x +c \right)^{6}}{2}-\frac{3 \tan \left(d x +c \right)^{5}}{5}+\frac{3 i \tan \left(d x +c \right)^{4}}{4}-\tan \left(d x +c \right)^{3}+\frac{i \tan \left(d x +c \right)^{2}}{2}}{a d}$
default	$-\frac{-\tan \left(d x +c \right)+\frac{i \tan \left(d x +c \right)^{8}}{8}-\frac{\tan \left(d x +c \right)^{7}}{7}+\frac{i \tan \left(d x +c \right)^{6}}{2}-\frac{3 \tan \left(d x +c \right)^{5}}{5}+\frac{3 i \tan \left(d x +c \right)^{4}}{4}-\tan \left(d x +c \right)^{3}+\frac{i \tan \left(d x +c \right)^{2}}{2}}{a d}$

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
32/35*I*(56*exp(6*I*(d*x+c))+28*exp(4*I*(d*x+c))+8*exp(2*I*(d*x+c))+1)/d/a
/(exp(2*I*(d*x+c))+1)^8
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.36

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{32\left(-56i\,e^{(6i\,dx+6i\,c)}-28i\,e^{(4i\,dx+4i\,c)}-8i\,e^{(2i\,dx+2i\,c)}-\right.}{35\left(ade^{(16i\,dx+16i\,c)}+8\,ade^{(14i\,dx+14i\,c)}+28\,ade^{(12i\,dx+12i\,c)}+56\,ade^{(10i\,dx+10i\,c)}+70\,ade^{(8i\,dx+8i\,c)}+56\,ad\right.}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-32/35*(-56*I*e^(6*I*d*x + 6*I*c) - 28*I*e^(4*I*d*x + 4*I*c) - 8*I*e^(2*I*
d*x + 2*I*c) - I)/(a*d*e^(16*I*d*x + 16*I*c) + 8*a*d*e^(14*I*d*x + 14*I*c)
 + 28*a*d*e^(12*I*d*x + 12*I*c) + 56*a*d*e^(10*I*d*x + 10*I*c) + 70*a*d*e^
(8*I*d*x + 8*I*c) + 56*a*d*e^(6*I*d*x + 6*I*c) + 28*a*d*e^(4*I*d*x + 4*I*c
) + 8*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^{10}(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(sec(c + d*x)**10/(tan(c + d*x) - I), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{35i\,\tan\left(dx+c\right)^{8}-40\,\tan\left(dx+c\right)^{7}+140i\,\tan\left(dx+c\right)^{6}-168\,\tan\left(dx+c\right)^{5}+210i\,\tan\left(dx+c\right)^{4}-}{280\,ad}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/280*(35*I*tan(d*x + c)^8 - 40*tan(d*x + c)^7 + 140*I*tan(d*x + c)^6 - 1
68*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 - 280*tan(d*x + c)^3 + 140*I*tan(
d*x + c)^2 - 280*tan(d*x + c))/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{35i\,\tan\left(dx+c\right)^{8}-40\,\tan\left(dx+c\right)^{7}+140i\,\tan\left(dx+c\right)^{6}-168\,\tan\left(dx+c\right)^{5}+210i\,\tan\left(dx+c\right)^{4}-}{280\,ad}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/280*(35*I*tan(d*x + c)^8 - 40*tan(d*x + c)^7 + 140*I*tan(d*x + c)^6 - 1
68*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 - 280*tan(d*x + c)^3 + 140*I*tan(
d*x + c)^2 - 280*tan(d*x + c))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.86

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{\cos\left(c+d\,x\right)^8\,35\mathrm{i}+128\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^7+64\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^5+48\,\sin\left(c+d\,x\right)\,\cos}{280\,a\,d\cos\left(c+d\,x\right)^8}$$

input
```
int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)),x)
```

output
```
(40*cos(c + d*x)*sin(c + d*x) + 48*cos(c + d*x)^3*sin(c + d*x) + 64*cos(c
+ d*x)^5*sin(c + d*x) + 128*cos(c + d*x)^7*sin(c + d*x) + cos(c + d*x)^8*3
5i - 35i)/(280*a*d*cos(c + d*x)^8)
```

Reduce [F]

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\int \frac{\sec(dx+c)^{10}}{\tan(dx+c)i+1}dx}{a}$$

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x)
```

output
```
int(sec(c + d*x)**10/(tan(c + d*x)*i + 1),x)/a
```

3.100 $\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{i(a-ia\tan(c+dx))^4}{a^5d} - \frac{4i(a-ia\tan(c+dx))^5}{5a^6d} + \frac{i(a-ia\tan(c+dx))^6}{6a^7d}$$

output
```
I*(a-I*a*tan(d*x+c))^4/a^5/d-4/5*I*(a-I*a*tan(d*x+c))^5/a^6/d+1/6*I*(a-I*a
*tan(d*x+c))^6/a^7/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i(i+\tan(c+dx))^4\left(-11-14i\tan(c+dx)+5\tan^2(c+dx)\right)}{30ad}$$

input
```
Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/30*I)*(I + Tan[c + d*x])^4*(-11 - (14*I)*Tan[c + d*x] + 5*Tan[c + d*x
]^2))/(a*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\sec(c+dx)^8}{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \ 3968$$
$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^2d(ia\tan(c+dx))}{a^7d}$$
$$\Big\downarrow \ 49$$
$$-\frac{i\int \left((a-ia\tan(c+dx))^5-4a(a-ia\tan(c+dx))^4+4a^2(a-ia\tan(c+dx))^3\right)d(ia\tan(c+dx))}{a^7d}$$
$$\Big\downarrow \ 2009$$
$$-\frac{i\left(-a^2(a-ia\tan(c+dx))^4-\frac{1}{6}(a-ia\tan(c+dx))^6+\frac{4}{5}a(a-ia\tan(c+dx))^5\right)}{a^7d}$$

input
```
Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*(-(a^2*(a - I*a*Tan[c + d*x])^4) + (4*a*(a - I*a*Tan[c + d*x])^5)/5
- (a - I*a*Tan[c + d*x])^6/6))/(a^7*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{16i\left(15\,\mathrm{e}^{4i(dx+c)}+6\,\mathrm{e}^{2i(dx+c)}+1\right)}{15da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{6}}$	47
derivativedivides	$-\frac{-\tan(dx+c)+\frac{i\tan(dx+c)^{6}}{6}-\frac{\tan(dx+c)^{5}}{5}+\frac{i\tan(dx+c)^{4}}{2}-\frac{2\tan(dx+c)^{3}}{3}+\frac{i\tan(dx+c)^{2}}{2}}{ad}$	71
default	$-\frac{-\tan(dx+c)+\frac{i\tan(dx+c)^{6}}{6}-\frac{\tan(dx+c)^{5}}{5}+\frac{i\tan(dx+c)^{4}}{2}-\frac{2\tan(dx+c)^{3}}{3}+\frac{i\tan(dx+c)^{2}}{2}}{ad}$	71

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
16/15*I*(15*exp(4*I*(d*x+c))+6*exp(2*I*(d*x+c))+1)/d/a/(exp(2*I*(d*x+c))+1
)^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{16\left(-15i\,e^{(4i\,dx+4i\,c)}-6i\,e^{(2i\,dx+2i\,c)}-i\right)}{15\left(ade^{(12i\,dx+12i\,c)}+6\,ade^{(10i\,dx+10i\,c)}+15\,ade^{(8i\,dx+8i\,c)}+20\,ade^{(6i\,dx+6i\,c)}+15\,ade^{(4i\,dx+4i\,c)}+6\,ade^{(2i\,c}\right.}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-16/15*(-15*I*e^(4*I*d*x + 4*I*c) - 6*I*e^(2*I*d*x + 2*I*c) - I)/(a*d*e^(1
2*I*d*x + 12*I*c) + 6*a*d*e^(10*I*d*x + 10*I*c) + 15*a*d*e^(8*I*d*x + 8*I*
c) + 20*a*d*e^(6*I*d*x + 6*I*c) + 15*a*d*e^(4*I*d*x + 4*I*c) + 6*a*d*e^(2*
I*d*x + 2*I*c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^8(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(sec(c + d*x)**8/(tan(c + d*x) - I), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{-5i\,\tan(dx+c)^6+6\,\tan(dx+c)^5-15i\,\tan(dx+c)^4+20\,\tan(dx+c)^3-15i\,\tan(dx+c)^2+30\,\mathrm{tan}}{30\,ad}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/30*(-5*I*tan(d*x + c)^6 + 6*tan(d*x + c)^5 - 15*I*tan(d*x + c)^4 + 20*ta
n(d*x + c)^3 - 15*I*tan(d*x + c)^2 + 30*tan(d*x + c))/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{5i\,\tan{(dx+c)}^6 - 6\,\tan{(dx+c)}^5 + 15i\,\tan{(dx+c)}^4 - 20\,\tan{(dx+c)}^3 + 15i\,\tan{(dx+c)}^2 - 30\,\mathrm{ta}}{30\,ad}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/30*(5*I*tan(d*x + c)^6 - 6*tan(d*x + c)^5 + 15*I*tan(d*x + c)^4 - 20*ta
n(d*x + c)^3 + 15*I*tan(d*x + c)^2 - 30*tan(d*x + c))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\sin{(c+d\,x)}\,\left(30\cos{(c+d\,x)}^5 - \cos{(c+d\,x)}^4\,\sin{(c+d\,x)}\,15\mathrm{i} + 20\cos{(c+d\,x)}^3\sin{(c+d\,x)}^2 - \cos{(c}\right.}{30\,a\,d\cos{(c+d\,x)}^6}$$

input
```
int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)),x)
```

output
```
(sin(c + d*x)*(6*cos(c + d*x)*sin(c + d*x)^4 - cos(c + d*x)^4*sin(c + d*x)
*15i + 30*cos(c + d*x)^5 - sin(c + d*x)^5*5i - cos(c + d*x)^2*sin(c + d*x)
^3*15i + 20*cos(c + d*x)^3*sin(c + d*x)^2))/(30*a*d*cos(c + d*x)^6)
```

Reduce [F]

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\int \frac{\sec(dx+c)^8}{\tan(dx+c)i+1}dx}{a}$$

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x)
```

output
```
int(sec(c + d*x)**8/(tan(c + d*x)*i + 1),x)/a
```

3.101 $\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2i(a-ia\tan(c+dx))^3}{3a^4d} - \frac{i(a-ia\tan(c+dx))^4}{4a^5d}$$

output
```
2/3*I*(a-I*a*tan(d*x+c))^3/a^4/d-1/4*I*(a-I*a*tan(d*x+c))^4/a^5/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\tan(c+dx)\left(12-6i\tan(c+dx)+4\tan^2(c+dx)-3i\tan^3(c+dx)\right)}{12ad}$$

input
```
Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x]),x]
```

output
```
(Tan[c + d*x]*(12 - (6*I)*Tan[c + d*x] + 4*Tan[c + d*x]^2 - (3*I)*Tan[c + 
d*x]^3))/(12*a*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(2a(a-ia\tan(c+dx))^2-(a-ia\tan(c+dx))^3\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{1}{4}(a-ia\tan(c+dx))^4-\frac{2}{3}a(a-ia\tan(c+dx))^3\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*((-2*a*(a - I*a*Tan[c + d*x])^3)/3 + (a - I*a*Tan[c + d*x])^4/4))/(a
^5*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{4i\left(4\,{\mathrm e}^{2i(dx+c)}+1\right)}{3da\left({\mathrm e}^{2i(dx+c)}+1\right)^{4}}$	36
derivativedivides	$-\frac{-\tan(dx+c)+\frac{i\tan(dx+c)^{4}}{4}-\frac{\tan(dx+c)^{3}}{3}+\frac{i\tan(dx+c)^{2}}{2}}{ad}$	50
default	$-\frac{-\tan(dx+c)+\frac{i\tan(dx+c)^{4}}{4}-\frac{\tan(dx+c)^{3}}{3}+\frac{i\tan(dx+c)^{2}}{2}}{ad}$	50

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
4/3*I*(4*exp(2*I*(d*x+c))+1)/d/a/(exp(2*I*(d*x+c))+1)^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{4\left(-4i\,e^{(2i\,dx+2i\,c)}-i\right)}{3\left(ade^{(8i\,dx+8i\,c)}+4\,ade^{(6i\,dx+6i\,c)}+6\,ade^{(4i\,dx+4i\,c)}+4\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-4/3*(-4*I*e^(2*I*d*x + 2*I*c) - I)/(a*d*e^(8*I*d*x + 8*I*c) + 4*a*d*e^(6*
I*d*x + 6*I*c) + 6*a*d*e^(4*I*d*x + 4*I*c) + 4*a*d*e^(2*I*d*x + 2*I*c) + a
*d)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^6(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(sec(c + d*x)**6/(tan(c + d*x) - I), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{-3i\,\tan(dx+c)^4+4\,\tan(dx+c)^3-6i\,\tan(dx+c)^2+12\,\tan(dx+c)}{12\,ad}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(-3*I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 - 6*I*tan(d*x + c)^2 + 12*tan
(d*x + c))/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{3i\tan(dx+c)^4-4\tan(dx+c)^3+6i\tan(dx+c)^2-12\tan(dx+c)}{12\,ad}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/12*(3*I*tan(d*x + c)^4 - 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 12*tan
(d*x + c))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\sin(c+d\,x)\,\left(12\cos(c+d\,x)^3-\cos(c+d\,x)^2\,\sin(c+d\,x)\,6\mathrm{i}+4\,\cos(c+d\,x)\,\sin(c+d\,x)^2-\sin(c+\right.}{12\,a\,d\cos(c+d\,x)^4}$$

input
```
int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)),x)
```

output
```
(sin(c + d*x)*(4*cos(c + d*x)*sin(c + d*x)^2 - cos(c + d*x)^2*sin(c + d*x)
*6i + 12*cos(c + d*x)^3 - sin(c + d*x)^3*3i))/(12*a*d*cos(c + d*x)^4)
```

Reduce [F]

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\int \frac{\sec(dx+c)^6}{\tan(dx+c)i+1}dx}{a}$$

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x)
```

output
```
int(sec(c + d*x)**6/(tan(c + d*x)*i + 1),x)/a
```

3.102 $\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{i(a-ia\tan(c+dx))^2}{2a^3d}$$

output
```
1/2*I*(a-I*a*tan(d*x+c))^2/a^3/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{\tan(c+dx)}{ad}-\frac{i\tan^2(c+dx)}{2ad}$$

input
```
Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

output
```
Tan[c + d*x]/(a*d) - ((I/2)*Tan[c + d*x]^2)/(a*d)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sec(c+dx)^4}{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \text{ 3968}$$
$$-\frac{i\int (a-ia\tan(c+dx))d(ia\tan(c+dx))}{a^3d}$$
$$\Big\downarrow \text{ 17}$$
$$\frac{i(a-ia\tan(c+dx))^2}{2a^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/2)*(a - I*a*Tan[c + d*x])^2)/(a^3*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{2i}{da\left(e^{2i(dx+c)}+1\right)^{2}}$	23
derivativedivides	$-\frac{i\left(\frac{\tan(dx+c)^{2}}{2}+i\tan(dx+c)\right)}{ad}$	30
default	$-\frac{i\left(\frac{\tan(dx+c)^{2}}{2}+i\tan(dx+c)\right)}{ad}$	30

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a/(exp(2*I*(d*x+c))+1)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2i}{ade^{(4i\,dx+4i\,c)}+2\,ade^{(2i\,dx+2i\,c)}+ad}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
2*I/(a*d*e^(4*I*d*x + 4*I*c) + 2*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^4(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(sec(c + d*x)**4/(tan(c + d*x) - I), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\tan(dx+c)^2 - 2\tan(dx+c)}{2\,ad}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(I*tan(d*x + c)^2 - 2*tan(d*x + c))/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\tan(dx+c)^2 - 2\tan(dx+c)}{2\,ad}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(I*tan(d*x + c)^2 - 2*tan(d*x + c))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\tan(c+d\,x)\,\left(-2+\tan(c+d\,x)\,1\mathrm{i}\right)}{2\,a\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)),x)
```

output
```
-(tan(c + d*x)*(tan(c + d*x)*1i - 2))/(2*a*d)
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\int \frac{\sec(dx+c)^4}{\tan(dx+c)i+1}dx}{a}$$

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x)
```

output
```
int(sec(c + d*x)**4/(tan(c + d*x)*i + 1),x)/a
```

3.103 $\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 23

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{x}{a} + \frac{i\log(\cos(c+dx))}{ad}$$

output
```
x/a+I*ln(cos(d*x+c))/a/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\log(i-\tan(c+dx))}{ad}$$

input
```
Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*Log[I - Tan[c + d*x]])/(a*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^2}{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int \frac{1}{i\tan(c+dx)a+a}d(ia\tan(c+dx))}{ad}$$
$$\Big\downarrow \text{16}$$
$$-\frac{i\log(a+ia\tan(c+dx))}{ad}$$

input
```
Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*Log[a + I*a*Tan[c + d*x]])/(a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\frac{i \ln(a+ia\tan(dx+c))}{ad}$	23
default	$-\frac{i \ln(a+ia\tan(dx+c))}{ad}$	23
risch	$\frac{2x}{a}+\frac{2c}{ad}+\frac{i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{ad}$	38

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-I/a/d*ln(a+I*a*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2\,dx+i\,\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
(2*d*x + I*log(e^(2*I*d*x + 2*I*c) + 1))/(a*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^2(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(sec(c + d*x)**2/(tan(c + d*x) - I), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\log(i\,a\tan(dx+c)+a)}{ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-I*log(I*a*tan(d*x + c) + a)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\log(\tan(dx+c)-i)}{ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-I*log(tan(d*x + c) - I)/(a*d)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{a\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)),x)
```

output
```
-(log(tan(c + d*x) - 1i)*1i)/(a*d)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\int \frac{\sec(dx+c)^2}{\tan(dx+c)i+1}dx}{a}$$

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x)
```

output
```
int(sec(c + d*x)**2/(tan(c + d*x)*i + 1),x)/a
```

3.104 $\int \frac{1}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 33

$$\int \frac{1}{a+ia\tan(c+dx)}\,dx=\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}$$

output
```
1/2*x/a+1/2*I/d/(a+I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{1}{a+ia\tan(c+dx)}\,dx=\frac{\frac{\arctan(\tan(c+dx))}{a}+\frac{1}{-ia+a\tan(c+dx)}}{2d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(-1),x]
```

output
```
(ArcTan[Tan[c + d*x]]/a + ((-I)*a + a*Tan[c + d*x])^(-1))/(2*d)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+ia\tan(c+dx)} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \text{3960}$$
$$\frac{\int 1dx}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}$$
$$\Big\downarrow \text{24}$$
$$\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(-1),x]
```

output
```
x/(2*a) + (I/2)/(d*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{x}{2a}+\frac{i\mathrm{e}^{-2i(dx+c)}}{4ad}$	26
derivativedivides	$\frac{\arctan(\tan(dx+c))}{2da}+\frac{1}{2da(-i+\tan(dx+c))}$	36
default	$\frac{\arctan(\tan(dx+c))}{2da}+\frac{1}{2da(-i+\tan(dx+c))}$	36
norman	$\frac{\frac{x}{2a}+\frac{i}{2ad}+\frac{\tan(dx+c)}{2ad}+\frac{x\tan(dx+c)^2}{2a}}{1+\tan(dx+c)^2}$	58

input
```
int(1/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a+1/4*I/a/d*exp(-2*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{1}{a+ia\tan(c+dx)}\,dx=\frac{\left(2\,dxe^{(2i\,dx+2i\,c)}+i\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

input
```
integrate(1/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*d*x*e^(2*I*d*x + 2*I*c) + I)*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.82

$$\int \frac{1}{a + ia\tan(c+dx)}\,dx = \begin{cases} \frac{ie^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic} \neq 0 \\ x\left(\frac{(e^{2ic}+1)e^{-2ic}}{2a} - \frac{1}{2a}\right) & \text{otherwise} \end{cases} + \frac{x}{2a}$$

input
```
integrate(1/(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((I*exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x
*((exp(2*I*c) + 1)*exp(-2*I*c)/(2*a) - 1/(2*a)), True)) + x/(2*a)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a + ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(25) = 50.

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.61

$$\int \frac{1}{a + ia\tan(c+dx)}\,dx = \frac{i\log(\tan(dx+c)+i)}{4\,ad} - \frac{i\log(\tan(dx+c)-i)}{4\,ad} + \frac{1}{2\,ad(\tan(dx+c)-i)}$$

input
```
integrate(1/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/4*I*log(tan(d*x + c) + I)/(a*d) - 1/4*I*log(tan(d*x + c) - I)/(a*d) + 1/
2/(a*d*(tan(d*x + c) - I))
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{a+ia\tan(c+dx)}\,dx = \frac{x}{2\,a}+\frac{1\mathrm{i}}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

input
```
int(1/(a + a*tan(c + d*x)*1i),x)
```

output
```
x/(2*a) + 1i/(2*a*d*(tan(c + d*x)*1i + 1))
```

Reduce [F]

$$\int \frac{1}{a+ia\tan(c+dx)}\,dx = \frac{\int \frac{1}{\tan(dx+c)i+1}dx}{a}$$

input
```
int(1/(a+I*a*tan(d*x+c)),x)
```

output
```
int(1/(tan(c + d*x)*i + 1),x)/a
```

3.105 $\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{3x}{8a} - \frac{i}{8d(a-ia\tan(c+dx))} + \frac{i}{4d(a+ia\tan(c+dx))} + \frac{ia^3}{8d\left(a^2+ia^2\tan(c+dx)\right)^2}$$

output
```
3/8*x/a-1/8*I/d/(a-I*a*tan(d*x+c))+1/4*I/d/(a+I*a*tan(d*x+c))+1/8*I*a^3/d/
(a^2+I*a^2*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2-3i\tan(c+dx)+3\tan^2(c+dx)+3\arctan(\tan(c+dx))(-i+\tan(c+dx))^2(i+\tan(c+dx))}{8ad(-i+\tan(c+dx))^2(i+\tan(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

output
```
(2 - (3*I)*Tan[c + d*x] + 3*Tan[c + d*x]^2 + 3*ArcTan[Tan[c + d*x]]*(-I +
Tan[c + d*x])^2*(I + Tan[c + d*x]))/(8*a*d*(-I + Tan[c + d*x])^2*(I + Tan[
c + d*x]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(c+dx)^2(a+ia\tan(c+dx))}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^3}d(ia\tan(c+dx))}{d}$$
$$\Big\downarrow \text{54}$$
$$-\frac{ia^3\int \left(\frac{1}{8a^3(a-ia\tan(c+dx))^2}+\frac{1}{4a^3(i\tan(c+dx)a+a)^2}+\frac{1}{4a^2(i\tan(c+dx)a+a)^3}+\frac{3}{8a^3(\tan^2(c+dx)a^2+a^2)}\right)d(ia\tan(c+dx))}{d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{ia^3\left(\frac{3i\arctan(\tan(c+dx))}{8a^4}+\frac{1}{8a^3(a-ia\tan(c+dx))}-\frac{1}{4a^3(a+ia\tan(c+dx))}-\frac{1}{8a^2(a+ia\tan(c+dx))^2}\right)}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*a^3*((((3*I)/8)*ArcTan[Tan[c + d*x]])/a^4 + 1/(8*a^3*(a - I*a*Tan[c
+ d*x])) - 1/(8*a^2*(a + I*a*Tan[c + d*x])^2) - 1/(4*a^3*(a + I*a*Tan[c +
d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{3x}{8a}+\frac{i\mathrm{e}^{-4i(dx+c)}}{32ad}+\frac{i\cos(2dx+2c)}{8ad}+\frac{\sin(2dx+2c)}{4ad}$	61
derivativedivides	$\frac{\frac{3i\ln(\tan(dx+c)+i)}{16}+\frac{1}{8\tan(dx+c)+8i}-\frac{3i\ln(-i+\tan(dx+c))}{16}-\frac{i}{8(-i+\tan(dx+c))^2}+\frac{1}{-4i+4\tan(dx+c)}}{da}$	75
default	$\frac{\frac{3i\ln(\tan(dx+c)+i)}{16}+\frac{1}{8\tan(dx+c)+8i}-\frac{3i\ln(-i+\tan(dx+c))}{16}-\frac{i}{8(-i+\tan(dx+c))^2}+\frac{1}{-4i+4\tan(dx+c)}}{da}$	75

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3/8*x/a+1/32*I/a/d*exp(-4*I*(d*x+c))+1/8*I/a/d*cos(2*d*x+2*c)+1/4/a/d*sin(
2*d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(12\,dxe^{(4i\,dx+4i\,c)}-2i\,e^{(6i\,dx+6i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{32\,ad}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/32*(12*d*x*e^(4*I*d*x + 4*I*c) - 2*I*e^(6*I*d*x + 6*I*c) + 6*I*e^(2*I*d*
x + 2*I*c) + I)*e^(-4*I*d*x - 4*I*c)/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.72

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \begin{cases} \frac{(-512ia^2d^2e^{8ic}e^{2idx}+1536ia^2d^2e^{4ic}e^{-2idx}+256ia^2d^2e^{2ic}e^{-4idx})e^{-6ic}}{8192a^3d^3} & \text{for } a^3d^3e^{6ic}\neq 0 \\ x\left(\frac{(e^{6ic}+3e^{4ic}+3e^{2ic}+1)e^{-4ic}}{8a}-\frac{3}{8a}\right) & \text{otherwise} \end{cases} + \frac{3x}{8a}$$

input
```
integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((-512*I*a**2*d**2*exp(8*I*c)*exp(2*I*d*x) + 1536*I*a**2*d**2*ex
p(4*I*c)*exp(-2*I*d*x) + 256*I*a**2*d**2*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*
I*c)/(8192*a**3*d**3), Ne(a**3*d**3*exp(6*I*c), 0)), (x*((exp(6*I*c) + 3*e
xp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-4*I*c)/(8*a) - 3/(8*a)), True)) + 3*x/(
8*a)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{3i\log(\tan(dx+c)+i)}{16\,ad} - \frac{3i\log(\tan(dx+c)-i)}{16\,ad} + \frac{i\left(-3i\tan(dx+c)^2 - 3\tan(dx+c) - 2i\right)}{8\,ad(\tan(dx+c)+i)(\tan(dx+c)-i)^2}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
3/16*I*log(tan(d*x + c) + I)/(a*d) - 3/16*I*log(tan(d*x + c) - I)/(a*d) +
1/8*I*(-3*I*tan(d*x + c)^2 - 3*tan(d*x + c) - 2*I)/(a*d*(tan(d*x + c) + I)
*(tan(d*x + c) - I)^2)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.68

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{3\,x}{8\,a} - \frac{\frac{3\tan(c+d\,x)^2}{8} - \frac{\tan(c+d\,x)\,3\mathrm{i}}{8} + \frac{1}{4}}{a\,d\,\left(1+\tan(c+d\,x)\,1\mathrm{i}\right)^2\,\left(\tan(c+d\,x)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i),x)
```

output
```
(3*x)/(8*a) - ((3*tan(c + d*x)^2)/8 - (tan(c + d*x)*3i)/8 + 1/4)/(a*d*(tan
(c + d*x)*1i + 1)^2*(tan(c + d*x) + 1i))
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\int \frac{\cos(dx+c)^2}{\tan(dx+c)i+1}dx}{a}$$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x)
```

output
```
int(cos(c + d*x)**2/(tan(c + d*x)*i + 1),x)/a
```

3.106 $\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 150

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx &= \frac{5x}{16a}-\frac{i}{8d(a-ia\tan(c+dx))}\\ &\quad+\frac{3i}{16d(a+ia\tan(c+dx))}+\frac{ia^5}{24d\left(a^2+ia^2\tan(c+dx)\right)^3}\\ &\quad-\frac{ia^5}{32d\left(a^3-ia^3\tan(c+dx)\right)^2}+\frac{3ia^5}{32d\left(a^3+ia^3\tan(c+dx)\right)^2}\end{aligned}$$

output
```
5/16*x/a-1/8*I/d/(a-I*a*tan(d*x+c))+3/16*I/d/(a+I*a*tan(d*x+c))+1/24*I*a^5
/d/(a^2+I*a^2*tan(d*x+c))^3-1/32*I*a^5/d/(a^3-I*a^3*tan(d*x+c))^2+3/32*I*a
^5/d/(a^3+I*a^3*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\sec^5(c+dx)(-80\cos(c+dx)+15\cos(3(c+dx))+\cos(5(c+dx))+120i\arctan(\tan(c+dx))(\cos(c}{384ad(-i+\tan(c+dx))^3(i+\text{ta}}$$

input
```
Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

output
```
-1/384*(Sec[c + d*x]^5*(-80*Cos[c + d*x] + 15*Cos[3*(c + d*x)] + Cos[5*(c
+ d*x)] + (120*I)*ArcTan[Tan[c + d*x]]*(Cos[c + d*x] + I*Sin[c + d*x]) + (
40*I)*Sin[c + d*x] + (45*I)*Sin[3*(c + d*x)] + (5*I)*Sin[5*(c + d*x)]))/(a
*d*(-I + Tan[c + d*x])^3*(I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^4(a+ia\tan(c+dx))}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^4}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^5\int \left(\frac{1}{8a^5(a-ia\tan(c+dx))^2}+\frac{3}{16a^5(i\tan(c+dx)a+a)^2}+\frac{1}{16a^4(a-ia\tan(c+dx))^3}+\frac{3}{16a^4(i\tan(c+dx)a+a)^3}+\frac{1}{8a^3(i\tan(c+dx)a+a}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^5\left(\frac{5i\arctan(\tan(c+dx))}{16a^6}+\frac{1}{8a^5(a-ia\tan(c+dx))}-\frac{3}{16a^5(a+ia\tan(c+dx))}+\frac{1}{32a^4(a-ia\tan(c+dx))^2}-\frac{3}{32a^4(a+ia\tan(c+dx))^2}-\right.}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*a^5*((((5*I)/16)*ArcTan[Tan[c + d*x]])/a^6 + 1/(32*a^4*(a - I*a*Tan[
c + d*x])^2) + 1/(8*a^5*(a - I*a*Tan[c + d*x])) - 1/(24*a^3*(a + I*a*Tan[c
 + d*x])^3) - 3/(32*a^4*(a + I*a*Tan[c + d*x])^2) - 3/(16*a^5*(a + I*a*Tan
[c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.64

method	result
risch	$\frac{5x}{16a}+\frac{i\mathrm{e}^{-6i(dx+c)}}{192ad}+\frac{i\cos(4dx+4c)}{32ad}+\frac{3\sin(4dx+4c)}{64ad}+\frac{5i\cos(2dx+2c)}{64ad}+\frac{15\sin(2dx+2c)}{64ad}$
derivativedivides	$\frac{\frac{i}{32(\tan(dx+c)+i)^2}+\frac{5i\ln(\tan(dx+c)+i)}{32}+\frac{1}{8\tan(dx+c)+8i}-\frac{5i\ln(-i+\tan(dx+c))}{32}-\frac{3i}{32(-i+\tan(dx+c))^2}-\frac{1}{24(-i+\tan(dx+c))^3}+}{da}$
default	$\frac{\frac{i}{32(\tan(dx+c)+i)^2}+\frac{5i\ln(\tan(dx+c)+i)}{32}+\frac{1}{8\tan(dx+c)+8i}-\frac{5i\ln(-i+\tan(dx+c))}{32}-\frac{3i}{32(-i+\tan(dx+c))^2}-\frac{1}{24(-i+\tan(dx+c))^3}+}{da}$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
5/16*x/a+1/192*I/a/d*exp(-6*I*(d*x+c))+1/32*I/a/d*cos(4*d*x+4*c)+3/64/a/d*
sin(4*d*x+4*c)+5/64*I/a/d*cos(2*d*x+2*c)+15/64/a/d*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.51

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(120\,dxe^{(6i\,dx+6i\,c)} - 3i\,e^{(10i\,dx+10i\,c)} - 30i\,e^{(8i\,dx+8i\,c)} + 60i\,e^{(4i\,dx+4i\,c)} + 15i\,e^{(2i\,dx+2i\,c)} + 2i\right)e^{(-6i\,dx-6i\,c)}}{384\,ad}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/384*(120*d*x*e^(6*I*d*x + 6*I*c) - 3*I*e^(10*I*d*x + 10*I*c) - 30*I*e^(8
*I*d*x + 8*I*c) + 60*I*e^(4*I*d*x + 4*I*c) + 15*I*e^(2*I*d*x + 2*I*c) + 2*
I)*e^(-6*I*d*x - 6*I*c)/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.46

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \begin{cases} \frac{(-50331648ia^4d^4e^{16ic}e^{4idx} - 503316480ia^4d^4e^{14ic}e^{2idx} + 1006632960ia^4d^4e^{10ic}e^{-2idx} + 251658240ia^4d^4e^{8ic}e^{-4idx} + 33554432ia^4d^4e^{6ic}e^{-6}}{6442450944a^5d^5} \\ x\left(\frac{(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1)e^{-6ic}}{32a} - \frac{5}{16a}\right) \end{cases} + \frac{5x}{16a}$$

input
```
integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((-50331648*I*a**4*d**4*exp(16*I*c)*exp(4*I*d*x) - 503316480*I*a
**4*d**4*exp(14*I*c)*exp(2*I*d*x) + 1006632960*I*a**4*d**4*exp(10*I*c)*exp
(-2*I*d*x) + 251658240*I*a**4*d**4*exp(8*I*c)*exp(-4*I*d*x) + 33554432*I*a
**4*d**4*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(6442450944*a**5*d**5), Ne
(a**5*d**5*exp(12*I*c), 0)), (x*((exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*
c) + 10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-6*I*c)/(32*a) - 5/(16*a)), Tru
e)) + 5*x/(16*a)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.69

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{5i\log\left(\tan\left(dx+c\right)+i\right)}{32\,ad} - \frac{5i\log\left(\tan\left(dx+c\right)-i\right)}{32\,ad} + \frac{i\left(-15i\tan\left(dx+c\right)^4 - 15\tan\left(dx+c\right)^3 - 25i\tan\left(dx+c\right)^2 - 25\tan\left(dx+c\right) - 8i\right)}{48\,ad(\tan\left(dx+c\right)+i)^2(\tan\left(dx+c\right)-i)^3}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
5/32*I*log(tan(d*x + c) + I)/(a*d) - 5/32*I*log(tan(d*x + c) - I)/(a*d) +
1/48*I*(-15*I*tan(d*x + c)^4 - 15*tan(d*x + c)^3 - 25*I*tan(d*x + c)^2 - 2
5*tan(d*x + c) - 8*I)/(a*d*(tan(d*x + c) + I)^2*(tan(d*x + c) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.82

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{5\,x}{16\,a} + \frac{\frac{25\tan(c+d\,x)}{48\,a}+\frac{1\mathrm{i}}{6\,a}+\frac{\tan(c+d\,x)^2\,25\mathrm{i}}{48\,a}+\frac{5\tan(c+d\,x)^3}{16\,a}+\frac{\tan(c+d\,x)^4\,5\mathrm{i}}{16\,a}}{d\,\left(\tan\left(c+d\,x\right)^5\,1\mathrm{i}+\tan\left(c+d\,x\right)^4+\tan\left(c+d\,x\right)^3\,2\mathrm{i}+2\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,1\mathrm{i}+1\right)}$$

input
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i),x)
```

output
```
(5*x)/(16*a) + ((25*tan(c + d*x))/(48*a) + 1i/(6*a) + (tan(c + d*x)^2*25i)
/(48*a) + (5*tan(c + d*x)^3)/(16*a) + (tan(c + d*x)^4*5i)/(16*a))/(d*(tan(
c + d*x)*1i + 2*tan(c + d*x)^2 + tan(c + d*x)^3*2i + tan(c + d*x)^4 + tan(
c + d*x)^5*1i + 1))
```

Reduce [F]

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\int \frac{\cos(dx+c)^4}{\tan(dx+c)i+1}dx}{a}$$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x)
```

output
```
int(cos(c + d*x)**4/(tan(c + d*x)*i + 1),x)/a
```

3.107 $\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 84

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{3\text{arctanh}(\sin(c+dx))}{8ad} - \frac{i\sec^5(c+dx)}{5ad} + \frac{3\sec(c+dx)\tan(c+dx)}{8ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{4ad}$$

output
```
3/8*arctanh(sin(d*x+c))/a/d-1/5*I*sec(d*x+c)^5/a/d+3/8*sec(d*x+c)*tan(d*x+
c)/a/d+1/4*sec(d*x+c)^3*tan(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{240\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec^5(c+dx)(-64i+70\sin(2(c+dx))+15\sin(4(c+dx)))}{320ad}$$

input
```
Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x]),x]
```

output
```
(240*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^5*(-64*I + 70*Si
n[2*(c + d*x)] + 15*Sin[4*(c + d*x)]))/(320*a*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3982, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^7}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3982}$$

$$\frac{\int \sec^5(c+dx)dx}{a}-\frac{i\sec^5(c+dx)}{5ad}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{a}-\frac{i\sec^5(c+dx)}{5ad}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}$$

input
```
Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/5*I)*Sec[c + d*x]^5)/(a*d) + ((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (
3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4)/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.45

method	result
risch	$-\frac{i\left(15\,\mathrm{e}^{9i(dx+c)}+70\,\mathrm{e}^{7i(dx+c)}+128\,\mathrm{e}^{5i(dx+c)}-70\,\mathrm{e}^{3i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}\right)}{20da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8ad}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8ad}$
derivativedivides	$\frac{\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}+\frac{2\left(\frac{7}{16}+\frac{5i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{5}{16}+\frac{3i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{4}+\frac{3i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(\frac{1}{8}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}$
default	$\frac{\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}+\frac{2\left(\frac{7}{16}+\frac{5i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{5}{16}+\frac{3i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{4}+\frac{3i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(\frac{1}{8}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/20*I/d/a/(exp(2*I*(d*x+c))+1)^5*(15*exp(9*I*(d*x+c))+70*exp(7*I*(d*x+c)
)+128*exp(5*I*(d*x+c))-70*exp(3*I*(d*x+c))-15*exp(I*(d*x+c)))-3/8/a/d*ln(e
xp(I*(d*x+c))-I)+3/8/a/d*ln(exp(I*(d*x+c))+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(74) = 148.

Time = 0.12 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.17

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx$$
$$=\frac{15\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15}{40\left(ade^{(1}\right.}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/40*(15*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x +
6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x
+ I*c) + I) - 15*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*
I*d*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e
^(I*d*x + I*c) - I) - 30*I*e^(9*I*d*x + 9*I*c) - 140*I*e^(7*I*d*x + 7*I*c)
 - 256*I*e^(5*I*d*x + 5*I*c) + 140*I*e^(3*I*d*x + 3*I*c) + 30*I*e^(I*d*x +
 I*c))/(a*d*e^(10*I*d*x + 10*I*c) + 5*a*d*e^(8*I*d*x + 8*I*c) + 10*a*d*e^(
6*I*d*x + 6*I*c) + 10*a*d*e^(4*I*d*x + 4*I*c) + 5*a*d*e^(2*I*d*x + 2*I*c)
+ a*d)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^7(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(sec(c + d*x)**7/(tan(c + d*x) - I), x)/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(74) = 148$.

Time = 0.04 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.43

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{3\left(\frac{16\left(\frac{25i\sin(dx+c)}{\cos(dx+c)+1}-\frac{10i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{80\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10i\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{40\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{25i\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+8\right)}{-120ia+\frac{600ia\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1200ia\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1200ia\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{600ia\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{120ia\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} -$$

$$8d$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-3/8*(16*(25*I*sin(d*x + c)/(cos(d*x + c) + 1) - 10*I*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 80*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*I*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7 + 40*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 25*I*
sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 8)/(-120*I*a + 600*I*a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 - 1200*I*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 12
00*I*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 600*I*a*sin(d*x + c)^8/(cos(d
*x + c) + 1)^8 + 120*I*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - log(sin(
d*x + c)/(cos(d*x + c) + 1) + 1)/a + log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a)/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.64

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{15\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a} - \frac{15\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a} + \frac{2\left(25\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+40i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+80i\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5 a}}{40\,d}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/40*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a - 15*log(tan(1/2*d*x + 1/2*c) - 1
)/a + 2*(25*tan(1/2*d*x + 1/2*c)^9 + 40*I*tan(1/2*d*x + 1/2*c)^8 - 10*tan(
1/2*d*x + 1/2*c)^7 + 80*I*tan(1/2*d*x + 1/2*c)^4 + 10*tan(1/2*d*x + 1/2*c)
^3 - 25*tan(1/2*d*x + 1/2*c) + 8*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a))/d
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.30

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a\,d} + \frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{2\,a}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{2\,a}+\frac{5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{4\,a}-\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,4{}\mathrm{i}}{a}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,2{}\mathrm{i}}{a}+\frac{2{}\mathrm{i}}{5\,a}}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-5\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-10\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

input
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)),x)
```

output
```
(3*atanh(tan(c/2 + (d*x)/2)))/(4*a*d) + (tan(c/2 + (d*x)/2)^3/(2*a) + (tan
(c/2 + (d*x)/2)^4*4i)/a - tan(c/2 + (d*x)/2)^7/(2*a) + (tan(c/2 + (d*x)/2)
^8*2i)/a + (5*tan(c/2 + (d*x)/2)^9)/(4*a) + 2i/(5*a) - (5*tan(c/2 + (d*x)/
2))/(4*a))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c
/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

Reduce [F]

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\int \frac{\sec(dx+c)^7}{\tan(dx+c)i+1}dx}{a}$$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x)
```

output
```
int(sec(c + d*x)**7/(tan(c + d*x)*i + 1),x)/a
```

3.108 $\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{2ad}-\frac{i\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)\tan(c+dx)}{2ad}$$

output
```
1/2*arctanh(sin(d*x+c))/a/d-1/3*I*sec(d*x+c)^3/a/d+1/2*sec(d*x+c)*tan(d*x+
c)/a/d
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{12\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec^3(c+dx)(-4i+3\sin(2(c+dx)))}{12ad}$$

input
```
Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

output
```
(12*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^3*(-4*I + 3*Sin[2
*(c + d*x)]))/(12*a*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3982, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^5}{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \text{3982}$$
$$\frac{\int \sec^3(c+dx)dx}{a}-\frac{i\sec^3(c+dx)}{3ad}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a}-\frac{i\sec^3(c+dx)}{3ad}$$
$$\Big\downarrow \text{4255}$$
$$\frac{\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{i\sec^3(c+dx)}{3ad}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{i\sec^3(c+dx)}{3ad}$$
$$\Big\downarrow \text{4257}$$
$$\frac{\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{i\sec^3(c+dx)}{3ad}$$

input
```
Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/3*I)*Sec[c + d*x]^3)/(a*d) + (ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c +
d*x]*Tan[c + d*x])/(2*d))/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{i\left(3\,\mathrm{e}^{5i(dx+c)}+8\,\mathrm{e}^{3i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}\right)}{3da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{3}}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2ad}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2ad}$
derivativedivides	$\frac{\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(\frac{1}{4}-\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+}{ad}$
default	$\frac{\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(\frac{1}{4}-\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+}{ad}$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/d/a/(exp(2*I*(d*x+c))+1)^3*(3*exp(5*I*(d*x+c))+8*exp(3*I*(d*x+c))-3
*exp(I*(d*x+c)))-1/2/a/d*ln(exp(I*(d*x+c))-I)+1/2/a/d*ln(exp(I*(d*x+c))+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(52) = 104.

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.90

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{3\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx}\right.}{6\left(ade^{(6i\,dx+6i\,c)}+3\,ade^{(4i\,dx+4i\,c)}+3\,ade^{(2i\,c}\right.}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c
) + 1)*log(e^(I*d*x + I*c) + I) - 3*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x +
4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 6*I*e^(5*I*
d*x + 5*I*c) - 16*I*e^(3*I*d*x + 3*I*c) + 6*I*e^(I*d*x + I*c))/(a*d*e^(6*I
*d*x + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2*I*d*x + 2*I*c) + a*
d)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^5(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(sec(c + d*x)**5/(tan(c + d*x) - I), x)/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(52) = 104$.

Time = 0.04 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.10

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{4\left(\frac{3i\sin(dx+c)}{\cos(dx+c)+1}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3i\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+2\right)}{6ia-\frac{18ia\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18ia\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6ia\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{2d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(4*(3*I*sin(d*x + c)/(cos(d*x + c) + 1) + 6*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 - 3*I*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2)/(6*I*a - 18*I*a*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 18*I*a*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4 - 6*I*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + log(sin(d*x + c)/(cos
(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.65

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a}-\frac{3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a}+\frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3a}}{6\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3*log(tan(1/2*d*x + 1/2*c) + 1)/a - 3*log(tan(1/2*d*x + 1/2*c) - 1)/a
 + 2*(3*tan(1/2*d*x + 1/2*c)^5 + 6*I*tan(1/2*d*x + 1/2*c)^4 - 3*tan(1/2*d*
x + 1/2*c) + 2*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))/d
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.93

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{a}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,2{}\mathrm{i}}{a}+\frac{2{}\mathrm{i}}{3\,a}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

input
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)),x)
```

output
```
atanh(tan(c/2 + (d*x)/2))/(a*d) + ((tan(c/2 + (d*x)/2)^4*2i)/a + tan(c/2 +
 (d*x)/2)^5/a + 2i/(3*a) - tan(c/2 + (d*x)/2)/a)/(d*(3*tan(c/2 + (d*x)/2)^
2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

Reduce [F]

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\int \frac{\sec(dx+c)^5}{\tan(dx+c)i+1}dx}{a}$$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x)
```

output
```
int(sec(c + d*x)**5/(tan(c + d*x)*i + 1),x)/a
```

3.109 $\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 31

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{\operatorname{arctanh}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}$$

output
```
arctanh(sin(d*x+c))/a/d-I*sec(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{2\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)-i\sec(c+dx)}{ad}$$

input
```
Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

output
```
(2*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] - I*Sec[c + d*x])/(a*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3982, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^3}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3982}$$

$$\frac{\int \sec(c+dx)dx}{a}-\frac{i\sec(c+dx)}{ad}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{i\sec(c+dx)}{ad}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\operatorname{arctanh}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}$$

input
```
Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

output
```
ArcTanh[Sin[c + d*x]]/(a*d) - (I*Sec[c + d*x])/(a*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(30) = 60$.

Time = 1.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.26

method	result	size
derivativedivides	$\frac{\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	70
default	$\frac{\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	70
risch	$-\frac{2i\mathrm{e}^{i(dx+c)}}{da\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{ad}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{ad}$	74

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d/a*(1/2*I/(tan(1/2*d*x+1/2*c)-1)-1/2*ln(tan(1/2*d*x+1/2*c)-1)-1/2*I/(ta
n(1/2*d*x+1/2*c)+1)+1/2*ln(tan(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(29) = 58$.

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.58

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-\left(e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-2i\,e^{(i\,dx+i\,c)}}{ade^{(2i\,dx+2i\,c)}+ad}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
((e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - (e^(2*I*d*x + 2*I*c)
 + 1)*log(e^(I*d*x + I*c) - I) - 2*I*e^(I*d*x + I*c))/(a*d*e^(2*I*d*x + 2*
I*c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^3(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(sec(c + d*x)**3/(tan(c + d*x) - I), x)/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(29) = 58.

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.68

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a} - \frac{2}{-i\,a+\frac{i\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x +
c) + 1) - 1)/a - 2/(-I*a + I*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a} - \frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a} + \frac{2i}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
(log(tan(1/2*d*x + 1/2*c) + 1)/a - log(tan(1/2*d*x + 1/2*c) - 1)/a + 2*I/(
(tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d} + \frac{2\mathrm{i}}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)),x)
```

output
```
(2*atanh(tan(c/2 + (d*x)/2)))/(a*d) + 2i/(a*d*(tan(c/2 + (d*x)/2)^2 - 1))
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\int \frac{\sec(dx+c)^3}{\tan(dx+c)i+1}dx}{a}$$

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x)
```

output
```
int(sec(c + d*x)**3/(tan(c + d*x)*i + 1),x)/a
```

3.110 $\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{i\sec(c+dx)}{d(a+ia\tan(c+dx))}$$

output
```
I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\sec(c+dx)}{ad(-i+\tan(c+dx))}$$

input
```
Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x]),x]
```

output
```
Sec[c + d*x]/(a*d*(-I + Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \text{3969}$$
$$\frac{i\sec(c+dx)}{d(a+ia\tan(c+dx))}$$

input
```
Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x]),x]
```

output
```
(I*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{i e^{-i(dx+c)}}{ad}$	19
derivativedivides	$\frac{2}{da\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	23
default	$\frac{2}{da\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	23
orering	$\frac{i\sec(dx+c)}{d(a+ia\tan(dx+c))}$	27

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
I/a/d*exp(-I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{i\,e^{(-i\,dx-i\,c)}}{ad}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
I*e^(-I*d*x - I*c)/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \begin{cases} \frac{\sec(c+dx)}{ad\tan(c+dx)-iad} & \text{for } d \neq 0 \\ \frac{x\sec(c)}{ia\tan(c)+a} & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise((sec(c + d*x)/(a*d*tan(c + d*x) - I*a*d), Ne(d, 0)), (x*sec(c)/(
I*a*tan(c) + a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2}{\left(-i\,a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)d}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
2/((-I*a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2}{ad\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
2/(a*d*(tan(1/2*d*x + 1/2*c) - I))
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2\mathrm{i}}{a\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}$$

input
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)),x)
```

output
```
2i/(a*d*(tan(c/2 + (d*x)/2)*1i + 1))
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\int \frac{\sec(dx+c)}{\tan(dx+c)i+1}dx}{a}$$

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c)),x)
```

output
```
int(sec(c + d*x)/(tan(c + d*x)*i + 1),x)/a
```

3.111 $\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{2\sin(c+dx)}{3ad}+\frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}$$

output
```
2/3*sin(d*x+c)/a/d+1/3*I*cos(d*x+c)/d/(a+I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx=-\frac{\sec(c+dx)(-3+\cos(2(c+dx))+2i\sin(2(c+dx)))}{6ad(-i+\tan(c+dx))}$$

input
```
Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x]),x]
```

output
```
-1/6*(Sec[c + d*x]*(-3 + Cos[2*(c + d*x)] + (2*I)*Sin[2*(c + d*x)]))/(a*d*
(-I + Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3983, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)(a+ia\tan(c+dx))}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{2\int \cos(c+dx)dx}{3a}+\frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{3a}+\frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3117}$$

$$\frac{2\sin(c+dx)}{3ad}+\frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}$$

input
```
Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x]),x]
```

output
```
(2*Sin[c + d*x])/(3*a*d) + ((I/3)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

method	result
risch	$\frac{i e^{-3i(dx+c)}}{12ad}+\frac{i\cos(dx+c)}{4ad}+\frac{3\sin(dx+c)}{4ad}$
derivativedivides	$\frac{-\frac{2}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{3}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4i}}{ad}$
default	$\frac{-\frac{2}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{3}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4i}}{ad}$
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}+\frac{2\tan(dx+c)}{3ad}-\frac{2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{3ad}-\frac{2i\tan(dx+c)^{2}}{3ad}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\tan(dx+c)^{2}}{3ad}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\tan(dx+c)}{3ad}+\frac{2i\tan\cdots}{\cdots}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\left(1+\tan(dx+c)^{2}\right)}$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/12*I/a/d*exp(-3*I*(d*x+c))+1/4*I/a/d*cos(d*x+c)+3/4*sin(d*x+c)/a/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(-3i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{12\,ad}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(-3*I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x
- 3*I*c)/(a*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(36) = 72.

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.68

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx = \begin{cases} \frac{(-24ia^2d^2e^{5ic}e^{idx}+48ia^2d^2e^{3ic}e^{-idx}+8ia^2d^2e^{ic}e^{-3idx})e^{-4ic}}{96a^3d^3} & \text{for } a^3d^3e^{4ic}\neq 0 \\ \frac{x(e^{4ic}+2e^{2ic}+1)e^{-3ic}}{4a} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((-24*I*a**2*d**2*exp(5*I*c)*exp(I*d*x) + 48*I*a**2*d**2*exp(3*I
*c)*exp(-I*d*x) + 8*I*a**2*d**2*exp(I*c)*exp(-3*I*d*x))*exp(-4*I*c)/(96*a*
*3*d**3), Ne(a**3*d**3*exp(4*I*c), 0)), (x*(exp(4*I*c) + 2*exp(2*I*c) + 1)
*exp(-3*I*c)/(4*a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}+\frac{9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{3}}}{6\,d}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3/(a*(tan(1/2*d*x + 1/2*c) + I)) + (9*tan(1/2*d*x + 1/2*c)^2 - 12*I*t
an(1/2*d*x + 1/2*c) - 7)/(a*(tan(1/2*d*x + 1/2*c) - I)^3))/d
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.66

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}3\mathrm{i}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)\,2\mathrm{i}}{3\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)\,\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,1\mathrm{i}\right)^{3}}$$

input
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i),x)
```

output
```
((tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*3i - 3*tan(c/2 + (d*x)/2)^3 +
1i)*2i)/(3*a*d*(tan(c/2 + (d*x)/2) + 1i)*(tan(c/2 + (d*x)/2)*1i + 1)^3)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\int \frac{\cos(dx+c)}{\tan(dx+c)i+1}dx}{a}$$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c)),x)
```

output
```
int(cos(c + d*x)/(tan(c + d*x)*i + 1),x)/a
```

3.112 $\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{4\sin(c+dx)}{5ad}-\frac{4\sin^3(c+dx)}{15ad}+\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}$$

output
```
4/5*sin(d*x+c)/a/d-4/15*sin(d*x+c)^3/a/d+1/5*I*cos(d*x+c)^3/d/(a+I*a*tan(d
*x+c))
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\sec(c+dx)(-45+20\cos(2(c+dx))+\cos(4(c+dx))+40i\sin(2(c+dx))+4i\sin(4(c+dx)))}{120ad(-i+\tan(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

output
```
-1/120*(Sec[c + d*x]*(-45 + 20*Cos[2*(c + d*x)] + Cos[4*(c + d*x)] + (40*I
)*Sin[2*(c + d*x)] + (4*I)*Sin[4*(c + d*x)]))/(a*d*(-I + Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 3983, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+ia\tan(c+dx))}dx$$

$$\Big\downarrow \text{ 3983}$$

$$\frac{4\int \cos^3(c+dx)dx}{5a}+\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{5a}+\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{ 3113}$$

$$-\frac{4\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{5ad}+\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{4\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{5ad}+\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}$$

input
```
Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

output
```
(-4*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/(5*a*d) + ((I/5)*Cos[c + d*x]^3)/(
d*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.25

method	result
risch	$\frac{i\,\mathrm{e}^{-5i(dx+c)}}{80ad}+\frac{i\cos(dx+c)}{8ad}+\frac{5\sin(dx+c)}{8ad}+\frac{i\cos(3dx+3c)}{16ad}+\frac{5\sin(3dx+3c)}{48ad}$
derivativedivides	$\frac{-\frac{i}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{2}}-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{3}}+\frac{5}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{3i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\cdots}{ad}$
default	$\frac{-\frac{i}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{2}}-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{3}}+\frac{5}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{3i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\cdots}{ad}$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/80*I/a/d*exp(-5*I*(d*x+c))+1/8*I/a/d*cos(d*x+c)+5/8*sin(d*x+c)/a/d+1/16*
I/a/d*cos(3*d*x+3*c)+5/48/a/d*sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(-5i\,e^{(8i\,dx+8i\,c)}-60i\,e^{(6i\,dx+6i\,c)}+90i\,e^{(4i\,dx+4i\,c)}+20i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{240\,ad}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/240*(-5*I*e^(8*I*d*x + 8*I*c) - 60*I*e^(6*I*d*x + 6*I*c) + 90*I*e^(4*I*d
*x + 4*I*c) + 20*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*x - 5*I*c)/(a*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(53) = 106.

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.93

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \begin{cases} \frac{\left(-30720ia^4d^4e^{12ic}e^{3idx}-368640ia^4d^4e^{10ic}e^{idx}+552960ia^4d^4e^{8ic}e^{-idx}+122880ia^4d^4e^{6ic}e^{-3idx}+18432ia^4d^4e^{4ic}e^{-5idx}\right)e^{-9ic}}{1474560a^5d^5} & \text{for } a^5 \\ \frac{x\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-5ic}}{16a} & \text{otherw} \end{cases}$$

input
```
integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((-30720*I*a**4*d**4*exp(12*I*c)*exp(3*I*d*x) - 368640*I*a**4*d*
*4*exp(10*I*c)*exp(I*d*x) + 552960*I*a**4*d**4*exp(8*I*c)*exp(-I*d*x) + 12
2880*I*a**4*d**4*exp(6*I*c)*exp(-3*I*d*x) + 18432*I*a**4*d**4*exp(4*I*c)*e
xp(-5*I*d*x))*exp(-9*I*c)/(1474560*a**5*d**5), Ne(a**5*d**5*exp(9*I*c), 0)
), (x*(exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*c) + 4*exp(2*I*c) + 1)*exp(-5
*I*c)/(16*a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(57) = 114$.

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.78

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{\frac{5\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+24i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-13\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+\frac{165\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-480i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-650\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+400i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^5}}{120\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/120*(5*(15*tan(1/2*d*x + 1/2*c)^2 + 24*I*tan(1/2*d*x + 1/2*c) - 13)/(a*(
tan(1/2*d*x + 1/2*c) + I)^3) + (165*tan(1/2*d*x + 1/2*c)^4 - 480*I*tan(1/2
*d*x + 1/2*c)^3 - 650*tan(1/2*d*x + 1/2*c)^2 + 400*I*tan(1/2*d*x + 1/2*c)
+ 113)/(a*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.00

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx =$$
$$-\frac{\left(-15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 15\mathrm{i}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 25\mathrm{i}-13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\mathrm{ta}\right.}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)^3\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,1\mathrm{i}\right)^5}$$

input
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i),x)
```

output
```
-((9*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*21i - 13*tan(c/2 + (d*x)/2)
^3 + tan(c/2 + (d*x)/2)^4*25i - 5*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2
)^6*15i - 15*tan(c/2 + (d*x)/2)^7 + 3i)*2i)/(15*a*d*(tan(c/2 + (d*x)/2) +
1i)^3*(tan(c/2 + (d*x)/2)*1i + 1)^5)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\int \frac{\cos(dx+c)^3}{\tan(dx+c)i+1}dx}{a}$$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x)
```

output
```
int(cos(c + d*x)**3/(tan(c + d*x)*i + 1),x)/a
```

3.113 $\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{6\sin(c+dx)}{7ad} - \frac{4\sin^3(c+dx)}{7ad} + \frac{6\sin^5(c+dx)}{35ad} + \frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}$$

output
```
6/7*sin(d*x+c)/a/d-4/7*sin(d*x+c)^3/a/d+6/35*sin(d*x+c)^5/a/d+1/7*I*cos(d*
x+c)^5/d/(a+I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\sec(c+dx)(-350+175\cos(2(c+dx))+14\cos(4(c+dx))+\cos(6(c+dx))+350i\sin(2(c+dx))+}{1120ad(-i+\tan(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

output
```
-1/1120*(Sec[c + d*x]*(-350 + 175*Cos[2*(c + d*x)] + 14*Cos[4*(c + d*x)] +
 Cos[6*(c + d*x)] + (350*I)*Sin[2*(c + d*x)] + (56*I)*Sin[4*(c + d*x)] + (
6*I)*Sin[6*(c + d*x)]))/(a*d*(-I + Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 3983, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(c+dx)^5(a+ia\tan(c+dx))}dx$$
$$\Big\downarrow \text{3983}$$
$$\frac{6\int \cos^5(c+dx)dx}{7a}+\frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}$$
$$\Big\downarrow \text{3042}$$
$$\frac{6\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx}{7a}+\frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}$$
$$\Big\downarrow \text{3113}$$
$$-\frac{6\int \left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{7ad}+\frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{6\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{7ad}+\frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}$$

input
```
Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

output
```
(-6*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]^5/5))/(7*a*d) + (
(I/7)*Cos[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.40

method	result
risch	$\frac{i e^{-7i(dx+c)}}{448ad}+\frac{5i\cos(dx+c)}{64ad}+\frac{35\sin(dx+c)}{64ad}+\frac{i\cos(5dx+5c)}{64ad}+\frac{7\sin(5dx+5c)}{320ad}+\frac{3i\cos(3dx+3c)}{64ad}+$
derivativedivides	$\frac{i}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{4}}-\frac{i}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{2}}+\frac{1}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{5}}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{3}}+\frac{11}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-$
default	$\frac{i}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{4}}-\frac{i}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{2}}+\frac{1}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{5}}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{3}}+\frac{11}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-$

input
```
int(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/448*I/a/d*exp(-7*I*(d*x+c))+5/64*I/a/d*cos(d*x+c)+35/64*sin(d*x+c)/a/d+1
/64*I/a/d*cos(5*d*x+5*c)+7/320/a/d*sin(5*d*x+5*c)+3/64*I/a/d*cos(3*d*x+3*c
)+7/64/a/d*sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(-7i\,e^{(12i\,dx+12i\,c)} - 70i\,e^{(10i\,dx+10i\,c)} - 525i\,e^{(8i\,dx+8i\,c)} + 700i\,e^{(6i\,dx+6i\,c)} + 175i\,e^{(4i\,dx+4i\,c)} + 42i\,e^{(2i\,dx+2i\,c)}}{2240\,ad}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2240*(-7*I*e^(12*I*d*x + 12*I*c) - 70*I*e^(10*I*d*x + 10*I*c) - 525*I*e^
(8*I*d*x + 8*I*c) + 700*I*e^(6*I*d*x + 6*I*c) + 175*I*e^(4*I*d*x + 4*I*c)
+ 42*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(68) = 136.

Time = 0.34 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.11

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \begin{cases} \frac{\left(-150323855360ia^6d^6e^{21ic}e^{5idx} - 1503238553600ia^6d^6e^{19ic}e^{3idx} - 11274289152000ia^6d^6e^{17ic}e^{idx} + 15032385536000ia^6d^6e^{15ic}e^{-idx} + 3758\right.}{48103633715200a^7d^7} \\ \frac{x\left(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1\right)e^{-7ic}}{64a} \end{cases}$$

input
```
integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((-150323855360*I*a**6*d**6*exp(21*I*c)*exp(5*I*d*x) - 150323855
3600*I*a**6*d**6*exp(19*I*c)*exp(3*I*d*x) - 11274289152000*I*a**6*d**6*exp
(17*I*c)*exp(I*d*x) + 15032385536000*I*a**6*d**6*exp(15*I*c)*exp(-I*d*x) +
 3758096384000*I*a**6*d**6*exp(13*I*c)*exp(-3*I*d*x) + 901943132160*I*a**6
*d**6*exp(11*I*c)*exp(-5*I*d*x) + 107374182400*I*a**6*d**6*exp(9*I*c)*exp(
-7*I*d*x))*exp(-16*I*c)/(48103633715200*a**7*d**7), Ne(a**7*d**7*exp(16*I*
c), 0)), (x*(exp(12*I*c) + 6*exp(10*I*c) + 15*exp(8*I*c) + 20*exp(6*I*c) +
 15*exp(4*I*c) + 6*exp(2*I*c) + 1)*exp(-7*I*c)/(64*a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(73) = 146.

Time = 0.19 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.01

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{7\left(55\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+180i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-250\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-160i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+43\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^5}+\frac{735\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-3360i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{}}{560\,d}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/560*(7*(55*tan(1/2*d*x + 1/2*c)^4 + 180*I*tan(1/2*d*x + 1/2*c)^3 - 250*t
an(1/2*d*x + 1/2*c)^2 - 160*I*tan(1/2*d*x + 1/2*c) + 43)/(a*(tan(1/2*d*x +
 1/2*c) + I)^5) + (735*tan(1/2*d*x + 1/2*c)^6 - 3360*I*tan(1/2*d*x + 1/2*c
)^5 - 7315*tan(1/2*d*x + 1/2*c)^4 + 8820*I*tan(1/2*d*x + 1/2*c)^3 + 6321*t
an(1/2*d*x + 1/2*c)^2 - 2492*I*tan(1/2*d*x + 1/2*c) - 461)/(a*(tan(1/2*d*x
 + 1/2*c) - I)^7))/d
```

Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.21

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(-35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}35\mathrm{i}-35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}105\mathrm{i}-126\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+\right.}{35\,a\,d\,\left(\tan\left(\frac{c}{2}\right.\right.}$$

input
```
int(cos(c + d*x)^5/(a + a*tan(c + d*x)*1i),x)
```

output
```
((25*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*55i - 15*tan(c/2 + (d*x)/2)
^3 + tan(c/2 + (d*x)/2)^4*130i + 26*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)
/2)^6*182i - 126*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8*105i - 35*tan
(c/2 + (d*x)/2)^9 + tan(c/2 + (d*x)/2)^10*35i - 35*tan(c/2 + (d*x)/2)^11 +
 5i)*2i)/(35*a*d*(tan(c/2 + (d*x)/2) + 1i)^5*(tan(c/2 + (d*x)/2)*1i + 1)^7
)
```

Reduce [F]

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\int \frac{\cos(dx+c)^5}{\tan(dx+c)i+1}dx}{a}$$

input
```
int(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x)
```

output
```
int(cos(c + d*x)**5/(tan(c + d*x)*i + 1),x)/a
```

3.114 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{4i(a-ia\tan(c+dx))^5}{5a^7d} - \frac{2i(a-ia\tan(c+dx))^6}{3a^8d} + \frac{i(a-ia\tan(c+dx))^7}{7a^9d}$$

output
```
4/5*I*(a-I*a*tan(d*x+c))^5/a^7/d-2/3*I*(a-I*a*tan(d*x+c))^6/a^8/d+1/7*I*(a
-I*a*tan(d*x+c))^7/a^9/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.54

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{(i+\tan(c+dx))^5\,(-29-40i\tan(c+dx)+15\tan^2(c+dx))}{105a^2d}$$

input
```
Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/105*((I + Tan[c + d*x])^5*(-29 - (40*I)*Tan[c + d*x] + 15*Tan[c + d*x]^
2))/(a^2*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sec(c+dx)^{10}}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{ 3968}$$
$$-\frac{i\int (a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)^2d(ia\tan(c+dx))}{a^9d}$$
$$\Big\downarrow \text{ 49}$$
$$-\frac{i\int \left((a-ia\tan(c+dx))^6-4a(a-ia\tan(c+dx))^5+4a^2(a-ia\tan(c+dx))^4\right)d(ia\tan(c+dx))}{a^9d}$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{i\left(-\frac{4}{5}a^2(a-ia\tan(c+dx))^5-\frac{1}{7}(a-ia\tan(c+dx))^7+\frac{2}{3}a(a-ia\tan(c+dx))^6\right)}{a^9d}$$

input
```
Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*((-4*a^2*(a - I*a*Tan[c + d*x])^5)/5 + (2*a*(a - I*a*Tan[c + d*x])^6
)/3 - (a - I*a*Tan[c + d*x])^7/7))/(a^9*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{128 i \left(21 \,{\mathrm e}^{4 i \left(d x +c \right)}+7 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{105 d \,a^{2} \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{7}}$	47
derivativedivides	$\frac{\tan \left(d x +c \right)-\frac{\tan \left(d x +c \right)^{7}}{7}-\frac{i \tan \left(d x +c \right)^{6}}{3}-\frac{\tan \left(d x +c \right)^{5}}{5}-i \tan \left(d x +c \right)^{4}+\frac{\tan \left(d x +c \right)^{3}}{3}-i \tan \left(d x +c \right)^{2}}{a^{2} d}$	78
default	$\frac{\tan \left(d x +c \right)-\frac{\tan \left(d x +c \right)^{7}}{7}-\frac{i \tan \left(d x +c \right)^{6}}{3}-\frac{\tan \left(d x +c \right)^{5}}{5}-i \tan \left(d x +c \right)^{4}+\frac{\tan \left(d x +c \right)^{3}}{3}-i \tan \left(d x +c \right)^{2}}{a^{2} d}$	78

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
128/105*I*(21*exp(4*I*(d*x+c))+7*exp(2*I*(d*x+c))+1)/d/a^2/(exp(2*I*(d*x+c
))+1)^7
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(64) = 128$.

Time = 0.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.68

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx =$$
$$-\frac{128\left(-21i\,e^{(4i\,dx+4i\,c)}-7i\,e^{(2i\,dx+2i\,c)}-i\right)}{105\left(a^2de^{(14i\,dx+14i\,c)}+7\,a^2de^{(12i\,dx+12i\,c)}+21\,a^2de^{(10i\,dx+10i\,c)}+35\,a^2de^{(8i\,dx+8i\,c)}+35\,a^2de^{(6i\,dx+6i\,c)}+\right.}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-128/105*(-21*I*e^(4*I*d*x + 4*I*c) - 7*I*e^(2*I*d*x + 2*I*c) - I)/(a^2*d*
e^(14*I*d*x + 14*I*c) + 7*a^2*d*e^(12*I*d*x + 12*I*c) + 21*a^2*d*e^(10*I*d
*x + 10*I*c) + 35*a^2*d*e^(8*I*d*x + 8*I*c) + 35*a^2*d*e^(6*I*d*x + 6*I*c)
 + 21*a^2*d*e^(4*I*d*x + 4*I*c) + 7*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^{10}(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sec(c + d*x)**10/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a*
*2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{15\,\tan\left(dx+c\right)^7+35i\,\tan\left(dx+c\right)^6+21\,\tan\left(dx+c\right)^5+105i\,\tan\left(dx+c\right)^4-35\,\tan\left(dx+c\right)^3+10}{105\,a^2d}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/105*(15*tan(d*x + c)^7 + 35*I*tan(d*x + c)^6 + 21*tan(d*x + c)^5 + 105*
I*tan(d*x + c)^4 - 35*tan(d*x + c)^3 + 105*I*tan(d*x + c)^2 - 105*tan(d*x
+ c))/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{15\,\tan\left(dx+c\right)^7+35i\,\tan\left(dx+c\right)^6+21\,\tan\left(dx+c\right)^5+105i\,\tan\left(dx+c\right)^4-35\,\tan\left(dx+c\right)^3+10}{105\,a^2d}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/105*(15*tan(d*x + c)^7 + 35*I*tan(d*x + c)^6 + 21*tan(d*x + c)^5 + 105*
I*tan(d*x + c)^4 - 35*tan(d*x + c)^3 + 105*I*tan(d*x + c)^2 - 105*tan(d*x
+ c))/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.13

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\cos\left(c+d\,x\right)^7\,35\mathrm{i}+64\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^6+32\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^4+24\,\sin\left(c+d\,x\right)\,\cos}{105\,a^2\,d\cos\left(c+d\,x\right)^7}$$

input
```
int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
(24*cos(c + d*x)^2*sin(c + d*x) - 15*sin(c + d*x) - cos(c + d*x)*35i + 32*
cos(c + d*x)^4*sin(c + d*x) + 64*cos(c + d*x)^6*sin(c + d*x) + cos(c + d*x
)^7*35i)/(105*a^2*d*cos(c + d*x)^7)
```

Reduce [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec(dx+c)^{10}}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx}{a^2}$$

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(sec(c + d*x)**10/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x))/a**2
```

3.115 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i(a-ia\tan(c+dx))^4}{2a^6d} - \frac{i(a-ia\tan(c+dx))^5}{5a^7d}$$

output
```
1/2*I*(a-I*a*tan(d*x+c))^4/a^6/d-1/5*I*(a-I*a*tan(d*x+c))^5/a^7/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\tan(c+dx)\left(-10+10i\tan(c+dx)+5i\tan^3(c+dx)+2\tan^4(c+dx)\right)}{10a^2d}$$

input
```
Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/10*(Tan[c + d*x]*(-10 + (10*I)*Tan[c + d*x] + (5*I)*Tan[c + d*x]^3 + 2*
Tan[c + d*x]^4))/(a^2*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^8}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)d(ia\tan(c+dx))}{a^7d}$$
$$\Big\downarrow \text{49}$$
$$-\frac{i\int \left(2a(a-ia\tan(c+dx))^3-(a-ia\tan(c+dx))^4\right)d(ia\tan(c+dx))}{a^7d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{i\left(\frac{1}{5}(a-ia\tan(c+dx))^5-\frac{1}{2}a(a-ia\tan(c+dx))^4\right)}{a^7d}$$

input
```
Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*(-1/2*(a*(a - I*a*Tan[c + d*x])^4) + (a - I*a*Tan[c + d*x])^5/5))/(a
^7*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{8i\left(5\,\mathrm{e}^{2i(dx+c)}+1\right)}{5d\,a^{2}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}$	36
derivativedivides	$\frac{\tan(dx+c)-\frac{\tan(dx+c)^{5}}{5}-\frac{i\tan(dx+c)^{4}}{2}-i\tan(dx+c)^{2}}{a^{2}d}$	47
default	$\frac{\tan(dx+c)-\frac{\tan(dx+c)^{5}}{5}-\frac{i\tan(dx+c)^{4}}{2}-i\tan(dx+c)^{2}}{a^{2}d}$	47

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
8/5*I*(5*exp(2*I*(d*x+c))+1)/d/a^2/(exp(2*I*(d*x+c))+1)^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(43) = 86$.

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.76

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{8\left(-5i\,e^{(2i\,dx+2i\,c)}-i\right)}{5\left(a^2de^{(10i\,dx+10i\,c)}+5\,a^2de^{(8i\,dx+8i\,c)}+10\,a^2de^{(6i\,dx+6i\,c)}+10\,a^2de^{(4i\,dx+4i\,c)}+5\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-8/5*(-5*I*e^(2*I*d*x + 2*I*c) - I)/(a^2*d*e^(10*I*d*x + 10*I*c) + 5*a^2*d
*e^(8*I*d*x + 8*I*c) + 10*a^2*d*e^(6*I*d*x + 6*I*c) + 10*a^2*d*e^(4*I*d*x
+ 4*I*c) + 5*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^8(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sec(c + d*x)**8/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**
2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\,\tan\left(dx+c\right)^5+5i\,\tan\left(dx+c\right)^4+10i\,\tan\left(dx+c\right)^2-10\,\tan\left(dx+c\right)}{10\,a^2d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/10*(2*tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 + 10*I*tan(d*x + c)^2 - 10*ta
n(d*x + c))/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\,\tan\left(dx+c\right)^5+5i\,\tan\left(dx+c\right)^4+10i\,\tan\left(dx+c\right)^2-10\,\tan\left(dx+c\right)}{10\,a^2d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/10*(2*tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 + 10*I*tan(d*x + c)^2 - 10*ta
n(d*x + c))/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sin\left(c+d\,x\right)\,\left(-10\cos\left(c+d\,x\right)^4+\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)\,10\mathrm{i}+\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)^3\,5\mathrm{i}+2\,\mathrm{s}\right.}{10\,a^2\,d\cos\left(c+d\,x\right)^5}$$

input
```
int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
-(sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^3*5i + cos(c + d*x)^3*sin(c + d*
x)*10i - 10*cos(c + d*x)^4 + 2*sin(c + d*x)^4))/(10*a^2*d*cos(c + d*x)^5)
```

Reduce [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec(dx+c)^8}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx}{a^2}$$

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(sec(c + d*x)**8/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x))/a**2
```

3.116 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2} dx = \frac{i(a-ia\tan(c+dx))^3}{3a^5d}$$

output
```
1/3*I*(a-I*a*tan(d*x+c))^3/a^5/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2} dx = \frac{\tan(c+dx)}{a^2d} - \frac{i\tan^2(c+dx)}{a^2d} - \frac{\tan^3(c+dx)}{3a^2d}$$

input
```
Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
Tan[c + d*x]/(a^2*d) - (I*Tan[c + d*x]^2)/(a^2*d) - Tan[c + d*x]^3/(3*a^2*
d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^6}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int (a-ia\tan(c+dx))^2d(ia\tan(c+dx))}{a^5d}$$
$$\Big\downarrow \text{17}$$
$$\frac{i(a-ia\tan(c+dx))^3}{3a^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((I/3)*(a - I*a*Tan[c + d*x])^3)/(a^5*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{(\tan(dx+c)+i)^3}{3a^2d}$	20
default	$-\frac{(\tan(dx+c)+i)^3}{3a^2d}$	20
risch	$\frac{8i}{3d\,a^2\left(e^{2i(dx+c)}+1\right)^3}$	23

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2/d*(tan(d*x+c)+I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(21) = 42.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{8i}{3\left(a^2de^{(6i\,dx+6i\,c)}+3\,a^2de^{(4i\,dx+4i\,c)}+3\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
8/3*I/(a^2*d*e^(6*I*d*x + 6*I*c) + 3*a^2*d*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e
^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^6(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sec(c + d*x)**6/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**
2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\tan(dx+c)^3 + 3i\tan(dx+c)^2 - 3\tan(dx+c)}{3a^2d}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/3*(tan(d*x + c)^3 + 3*I*tan(d*x + c)^2 - 3*tan(d*x + c))/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\tan(dx+c)^3 + 3i\tan(dx+c)^2 - 3\tan(dx+c)}{3a^2d}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/3*(tan(d*x + c)^3 + 3*I*tan(d*x + c)^2 - 3*tan(d*x + c))/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\tan(c+d\,x)\,\left(\tan(c+d\,x)^2+\tan(c+d\,x)\,3\mathrm{i}-3\right)}{3\,a^2\,d}$$

input
```
int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
-(tan(c + d*x)*(tan(c + d*x)*3i + tan(c + d*x)^2 - 3))/(3*a^2*d)
```

Reduce [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec(dx+c)^6}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx}{a^2}$$

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(sec(c + d*x)**6/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x))/a**2
```

3.117 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{2x}{a^2} + \frac{2i\log(\cos(c+dx))}{a^2d} - \frac{\tan(c+dx)}{a^2d}$$

output
```
2*x/a^2+2*I*ln(cos(d*x+c))/a^2/d-tan(d*x+c)/a^2/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2i\log(i-\tan(c+dx))}{a^2d} - \frac{\tan(c+dx)}{a^2d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-2*I)*Log[I - Tan[c + d*x]])/(a^2*d) - Tan[c + d*x]/(a^2*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sec(c+dx)^4}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{ 3968}$$
$$-\frac{i\int \frac{a-ia\tan(c+dx)}{i\tan(c+dx)a+a}d(ia\tan(c+dx))}{a^3d}$$
$$\Big\downarrow \text{ 49}$$
$$-\frac{i\int \left(\frac{2a}{i\tan(c+dx)a+a}-1\right)d(ia\tan(c+dx))}{a^3d}$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{i(2a\log(a+ia\tan(c+dx))-ia\tan(c+dx))}{a^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*(2*a*Log[a + I*a*Tan[c + d*x]] - I*a*Tan[c + d*x]))/(a^3*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{-\tan(dx+c)-2i\ln(-i+\tan(dx+c))}{d\,a^2}$	30
default	$\frac{-\tan(dx+c)-2i\ln(-i+\tan(dx+c))}{d\,a^2}$	30
risch	$\frac{4x}{a^2}+\frac{4c}{a^2d}-\frac{2i}{d\,a^2\left(e^{2i(dx+c)}+1\right)}+\frac{2i\ln\left(e^{2i(dx+c)}+1\right)}{a^2d}$	60

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(-tan(d*x+c)-2*I*ln(-I+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{2\left(2\,dxe^{(2i\,dx+2i\,c)}+2\,dx-\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\right)}{a^2de^{(2i\,dx+2i\,c)}+a^2d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
2*(2*d*x*e^(2*I*d*x + 2*I*c) + 2*d*x - (-I*e^(2*I*d*x + 2*I*c) - I)*log(e^
(2*I*d*x + 2*I*c) + 1) - I)/(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^4(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sec(c + d*x)**4/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**
2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{-\frac{2i\log(i\tan(dx+c)+1)}{a^2}-\frac{\tan(dx+c)}{a^2}}{d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
(-2*I*log(I*tan(d*x + c) + 1)/a^2 - tan(d*x + c)/a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2i\log(\tan(dx+c)-i)}{a^2d} - \frac{\tan(dx+c)}{a^2d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-2*I*log(tan(d*x + c) - I)/(a^2*d) - tan(d*x + c)/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\tan(c+d\,x)+\ln(\tan(c+d\,x)-\mathrm{i})\,2\mathrm{i}}{a^2\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
-(log(tan(c + d*x) - 1i)*2i + tan(c + d*x))/(a^2*d)
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec(dx+c)^4}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx}{a^2}$$

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(sec(c + d*x)**4/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x))/a**2
```

3.118 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 26

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i}{d\,(a^2+ia^2\tan(c+dx))}$$

output
```
I/d/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{1}{a^2d(-i+\tan(c+dx))}$$

input
```
Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
1/(a^2*d*(-I + Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^2}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int \frac{1}{(i\tan(c+dx)a+a)^2}d(ia\tan(c+dx))}{ad}$$
$$\Big\downarrow \text{17}$$
$$\frac{i}{ad(a+ia\tan(c+dx))}$$

input
```
Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
I/(a*d*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{1}{a^2 d(-i+\tan(dx+c))}$	19
default	$\frac{1}{a^2 d(-i+\tan(dx+c))}$	19
risch	$\frac{i e^{-2i(dx+c)}}{2a^2 d}$	19
orering	$\frac{i\sec(dx+c)^2}{2d(a+ia\tan(dx+c))^2}$	29

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2/d/(-I+tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i\,e^{(-2i\,dx-2i\,c)}}{2\,a^2d}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*I*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(17) = 34$.

Time = 0.55 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.50

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} -\frac{i\sec^2(c+dx)}{2a^2d\tan^2(c+dx)-4ia^2d\tan(c+dx)-2a^2d} & \text{for } d \neq 0 \\ \frac{x\sec^2(c)}{(ia\tan(c)+a)^2} & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise((-I*sec(c + d*x)**2/(2*a**2*d*tan(c + d*x)**2 - 4*I*a**2*d*tan(c
 + d*x) - 2*a**2*d), Ne(d, 0)), (x*sec(c)**2/(I*a*tan(c) + a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i}{(i\,a\tan(dx+c)+a)ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
I/((I*a*tan(d*x + c) + a)*a*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{1}{a^2d(\tan(dx+c)-i)}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/(a^2*d*(tan(d*x + c) - I))
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{1\mathrm{i}}{a^2\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
1i/(a^2*d*(tan(c + d*x)*1i + 1))
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec(dx+c)^2}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx}{a^2}$$

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(sec(c + d*x)**2/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x))/a**2
```

3.119 $\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx=\frac{x}{4a^2}+\frac{i}{4d(a+ia\tan(c+dx))^2}+\frac{i}{4d\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
1/4*x/a^2+1/4*I/d/(a+I*a*tan(d*x+c))^2+1/4*I/d/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx=\frac{-2i+\tan(c+dx)+\arctan(\tan(c+dx))(-i+\tan(c+dx))^2}{4a^2d(-i+\tan(c+dx))^2}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(-2),x]
```

output
```
(-2*I + Tan[c + d*x] + ArcTan[Tan[c + d*x]]*(-I + Tan[c + d*x])^2)/(4*a^2*
d*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3960, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3960}$$
$$\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}$$
$$\Big\downarrow \text{3960}$$
$$\frac{\frac{\int 1dx}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}$$
$$\Big\downarrow \text{24}$$
$$\frac{\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(-2),x]
```

output
```
(I/4)/(d*(a + I*a*Tan[c + d*x])^2) + (x/(2*a) + (I/2)/(d*(a + I*a*Tan[c +
d*x])))/(2*a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{x}{4a^2}+\frac{i\mathrm{e}^{-2i(dx+c)}}{4a^2d}+\frac{i\mathrm{e}^{-4i(dx+c)}}{16a^2d}$	44
derivativedivides	$\frac{\arctan(\tan(dx+c))}{4d\,a^2}-\frac{i}{4d\,a^2(-i+\tan(dx+c))^2}+\frac{1}{4a^2d(-i+\tan(dx+c))}$	56
default	$\frac{\arctan(\tan(dx+c))}{4d\,a^2}-\frac{i}{4d\,a^2(-i+\tan(dx+c))^2}+\frac{1}{4a^2d(-i+\tan(dx+c))}$	56
norman	$\frac{\frac{x}{4a}+\frac{3\tan(dx+c)}{4ad}+\frac{\tan(dx+c)^3}{4ad}+\frac{x\tan(dx+c)^2}{2a}+\frac{x\tan(dx+c)^4}{4a}+\frac{i}{2ad}}{a\left(1+\tan(dx+c)^2\right)^2}$	91

input
```
int(1/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x/a^2+1/4*I/a^2/d*exp(-2*I*(d*x+c))+1/16*I/a^2/d*exp(-4*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(4\,dxe^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{16\,a^2d}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/16*(4*d*x*e^(4*I*d*x + 4*I*c) + 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x
 - 4*I*c)/(a^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.92

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} \frac{(16ia^2de^{4ic}e^{-2idx}+4ia^2de^{2ic}e^{-4idx})e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic}\neq 0 \\ x\left(\frac{(e^{4ic}+2e^{2ic}+1)e^{-4ic}}{4a^2}-\frac{1}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x}{4a^2}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((16*I*a**2*d*exp(4*I*c)*exp(-2*I*d*x) + 4*I*a**2*d*exp(2*I*c)*e
xp(-4*I*d*x))*exp(-6*I*c)/(64*a**4*d**2), Ne(a**4*d**2*exp(6*I*c), 0)), (x
*((exp(4*I*c) + 2*exp(2*I*c) + 1)*exp(-4*I*c)/(4*a**2) - 1/(4*a**2)), True
)) + x/(4*a**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx = \frac{i\log(\tan(dx+c)+i)}{8a^2d} - \frac{i\log(\tan(dx+c)-i)}{8a^2d} + \frac{\tan(dx+c)-2i}{4a^2d(\tan(dx+c)-i)^2}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/8*I*log(tan(d*x + c) + I)/(a^2*d) - 1/8*I*log(tan(d*x + c) - I)/(a^2*d)
+ 1/4*(tan(d*x + c) - 2*I)/(a^2*d*(tan(d*x + c) - I)^2)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx = \frac{x}{4a^2} - \frac{\frac{\tan(c+dx)}{4} - \frac{1}{2}\mathrm{i}}{a^2 d\,(1+\tan(c+dx)\ 1\mathrm{i})^2}$$

input
```
int(1/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
x/(4*a^2) - (tan(c + d*x)/4 - 1i/2)/(a^2*d*(tan(c + d*x)*1i + 1)^2)
```

Reduce [F]

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx}{a^2}$$

input
```
int(1/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(1/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x))/a**2
```

3.120 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{x}{4a^2}+\frac{ia}{12d(a+ia\tan(c+dx))^3}+\frac{i}{8d(a+ia\tan(c+dx))^2} - \frac{i}{16d\,(a^2-ia^2\tan(c+dx))}+\frac{3i}{16d\,(a^2+ia^2\tan(c+dx))}$$

output
```
1/4*x/a^2+1/12*I*a/d/(a+I*a*tan(d*x+c))^3+1/8*I/d/(a+I*a*tan(d*x+c))^2-1/1
6*I/d/(a^2-I*a^2*tan(d*x+c))+3/16*I/d/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{4i+\tan(c+dx)+6i\tan^2(c+dx)-3\tan^3(c+dx)-3\arctan(\tan(c+dx))(-i+\tan(c+dx))^3(i+}{12a^2d(-i+\tan(c+dx))^3(i+\tan(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/12*(4*I + Tan[c + d*x] + (6*I)*Tan[c + d*x]^2 - 3*Tan[c + d*x]^3 - 3*Ar
cTan[Tan[c + d*x]]*(-I + Tan[c + d*x])^3*(I + Tan[c + d*x]))/(a^2*d*(-I +
Tan[c + d*x])^3*(I + Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^4}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^3\int \left(\frac{1}{16a^4(a-ia\tan(c+dx))^2}+\frac{3}{16a^4(i\tan(c+dx)a+a)^2}+\frac{1}{4a^3(i\tan(c+dx)a+a)^3}+\frac{1}{4a^2(i\tan(c+dx)a+a)^4}+\frac{1}{4a^4(\tan^2(c+dx)a^2+a [illegible]}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^3\left(\frac{i\arctan(\tan(c+dx))}{4a^5}+\frac{1}{16a^4(a-ia\tan(c+dx))}-\frac{3}{16a^4(a+ia\tan(c+dx))}-\frac{1}{8a^3(a+ia\tan(c+dx))^2}-\frac{1}{12a^2(a+ia\tan(c+dx))^3}\right)}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*a^3*(((I/4)*ArcTan[Tan[c + d*x]])/a^5 + 1/(16*a^4*(a - I*a*Tan[c + d
*x])) - 1/(12*a^2*(a + I*a*Tan[c + d*x])^3) - 1/(8*a^3*(a + I*a*Tan[c + d*
x])^2) - 3/(16*a^4*(a + I*a*Tan[c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

method	result
risch	$\frac{x}{4a^2}+\frac{i\mathrm{e}^{-4i(dx+c)}}{16a^2d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{96a^2d}+\frac{5i\cos(2dx+2c)}{32a^2d}+\frac{7\sin(2dx+2c)}{32a^2d}$
derivativedivides	$\frac{\frac{i\ln(\tan(dx+c)+i)}{8}+\frac{1}{16\tan(dx+c)+16i}-\frac{i\ln(-i+\tan(dx+c))}{8}-\frac{i}{8(-i+\tan(dx+c))^2}-\frac{1}{12(-i+\tan(dx+c))^3}+\frac{3}{16(-i+\tan(dx+c))}}{d\,a^2}$
default	$\frac{\frac{i\ln(\tan(dx+c)+i)}{8}+\frac{1}{16\tan(dx+c)+16i}-\frac{i\ln(-i+\tan(dx+c))}{8}-\frac{i}{8(-i+\tan(dx+c))^2}-\frac{1}{12(-i+\tan(dx+c))^3}+\frac{3}{16(-i+\tan(dx+c))}}{d\,a^2}$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x/a^2+1/16*I/a^2/d*exp(-4*I*(d*x+c))+1/96*I/a^2/d*exp(-6*I*(d*x+c))+5/
32*I/a^2/d*cos(2*d*x+2*c)+7/32/a^2/d*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.57

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(24\,dxe^{(6i\,dx+6i\,c)} - 3i\,e^{(8i\,dx+8i\,c)} + 18i\,e^{(4i\,dx+4i\,c)} + 6i\,e^{(2i\,dx+2i\,c)} + i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^2d}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/96*(24*d*x*e^(6*I*d*x + 6*I*c) - 3*I*e^(8*I*d*x + 8*I*c) + 18*I*e^(4*I*d
*x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-6*I*d*x - 6*I*c)/(a^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.66

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} \frac{(-24576ia^6d^3e^{14ic}e^{2idx}+147456ia^6d^3e^{10ic}e^{-2idx}+49152ia^6d^3e^{8ic}e^{-4idx}+8192ia^6d^3e^{6ic}e^{-6idx})e^{-12ic}}{786432a^8d^4} & \text{for } a^8d^4e^{12ic}\neq 0 \\ x\left(\frac{(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1)e^{-6ic}}{16a^2}-\frac{1}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x}{4a^2}$$

input
```
integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-24576*I*a**6*d**3*exp(14*I*c)*exp(2*I*d*x) + 147456*I*a**6*d*
*3*exp(10*I*c)*exp(-2*I*d*x) + 49152*I*a**6*d**3*exp(8*I*c)*exp(-4*I*d*x)
+ 8192*I*a**6*d**3*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(786432*a**8*d**
4), Ne(a**8*d**4*exp(12*I*c), 0)), (x*((exp(8*I*c) + 4*exp(6*I*c) + 6*exp(
4*I*c) + 4*exp(2*I*c) + 1)*exp(-6*I*c)/(16*a**2) - 1/(4*a**2)), True)) + x
/(4*a**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i\log(\tan(dx+c)+i)}{8a^2d} - \frac{i\log(\tan(dx+c)-i)}{8a^2d} + \frac{3\tan(dx+c)^3-6i\tan(dx+c)^2-\tan(dx+c)-4i}{12a^2d(\tan(dx+c)+i)(\tan(dx+c)-i)^3}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/8*I*log(tan(d*x + c) + I)/(a^2*d) - 1/8*I*log(tan(d*x + c) - I)/(a^2*d)
+ 1/12*(3*tan(d*x + c)^3 - 6*I*tan(d*x + c)^2 - tan(d*x + c) - 4*I)/(a^2*d
*(tan(d*x + c) + I)*(tan(d*x + c) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.62

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{x}{4\,a^2} - \frac{\frac{\tan(c+d\,x)^3\,1\mathrm{i}}{4} + \frac{\tan(c+d\,x)^2}{2} - \frac{\tan(c+d\,x)\,1\mathrm{i}}{12} + \frac{1}{3}}{a^2\,d\,(1+\tan\left(c+d\,x\right)\,1\mathrm{i})^3\,\left(\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
x/(4*a^2) - (tan(c + d*x)^2/2 - (tan(c + d*x)*1i)/12 + (tan(c + d*x)^3*1i)
/4 + 1/3)/(a^2*d*(tan(c + d*x)*1i + 1)^3*(tan(c + d*x) + 1i))
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\cos(dx+c)^2}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx}{a^2}$$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(cos(c + d*x)**2/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x))/a**2
```

3.121 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 165

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= \frac{15x}{64a^2}-\frac{i}{64d(a-ia\tan(c+dx))^2}\\&\quad+\frac{ia^2}{32d(a+ia\tan(c+dx))^4}+\frac{ia}{16d(a+ia\tan(c+dx))^3}\\&\quad+\frac{3i}{32d(a+ia\tan(c+dx))^2}-\frac{5i}{64d\,(a^2-ia^2\tan(c+dx))}\\&\quad+\frac{5i}{32d\,(a^2+ia^2\tan(c+dx))}\end{aligned}$$

output

```
15/64*x/a^2-1/64*I/d/(a-I*a*tan(d*x+c))^2+1/32*I*a^2/d/(a+I*a*tan(d*x+c))^
4+1/16*I*a/d/(a+I*a*tan(d*x+c))^3+3/32*I/d/(a+I*a*tan(d*x+c))^2-5/64*I/d/(
a^2-I*a^2*tan(d*x+c))+5/32*I/d/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.86

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{i\sec^6(c+dx)(-80-65\cos(2(c+dx))+16\cos(4(c+dx))+\cos(6(c+dx))+120i\arctan(\tan(c+dx)}{512a^2d(-i+\tan(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((I/512)*Sec[c + d*x]^6*(-80 - 65*Cos[2*(c + d*x)] + 16*Cos[4*(c + d*x)] +
 Cos[6*(c + d*x)] + (120*I)*ArcTan[Tan[c + d*x]]*(Cos[2*(c + d*x)] + I*Sin
[2*(c + d*x)]) - (5*I)*Sin[2*(c + d*x)] + (32*I)*Sin[4*(c + d*x)] + (3*I)*
Sin[6*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^4*(I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sec(c+dx)^4(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^5}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \; 54$$

$$-\frac{ia^5 \int \left(\frac{5}{64a^6(a-ia\tan(c+dx))^2}+\frac{5}{32a^6(i\tan(c+dx)a+a)^2}+\frac{1}{32a^5(a-ia\tan(c+dx))^3}+\frac{3}{16a^5(i\tan(c+dx)a+a)^3}+\frac{3}{16a^4(i\tan(c+dx)a-}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^5\left(\frac{15i\arctan(\tan(c+dx))}{64a^7}+\frac{5}{64a^6(a-ia\tan(c+dx))}-\frac{5}{32a^6(a+ia\tan(c+dx))}+\frac{1}{64a^5(a-ia\tan(c+dx))^2}-\frac{3}{32a^5(a+ia\tan(c+dx))^2}-\right.}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*a^5*((((15*I)/64)*ArcTan[Tan[c + d*x]])/a^7 + 1/(64*a^5*(a - I*a*Tan
[c + d*x])^2) + 5/(64*a^6*(a - I*a*Tan[c + d*x])) - 1/(32*a^3*(a + I*a*Tan
[c + d*x])^4) - 1/(16*a^4*(a + I*a*Tan[c + d*x])^3) - 3/(32*a^5*(a + I*a*T
an[c + d*x])^2) - 5/(32*a^6*(a + I*a*Tan[c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.69

method	result
risch	$\frac{15x}{64a^2}+\frac{i\mathrm{e}^{-6i(dx+c)}}{64a^2d}+\frac{i\mathrm{e}^{-8i(dx+c)}}{512a^2d}+\frac{7i\cos(4dx+4c)}{128a^2d}+\frac{\sin(4dx+4c)}{16a^2d}+\frac{7i\cos(2dx+2c)}{64a^2d}+\frac{13\sin(2dx+2c)}{64a^2d}$
derivativedivides	$\frac{-\frac{15i\ln(-i+\tan(dx+c))}{128}+\frac{i}{32(-i+\tan(dx+c))^4}-\frac{3i}{32(-i+\tan(dx+c))^2}-\frac{1}{16(-i+\tan(dx+c))^3}+\frac{5}{32(-i+\tan(dx+c))}+\frac{i}{64(\tan(dx+c)}}{d\,a^2}$
default	$\frac{-\frac{15i\ln(-i+\tan(dx+c))}{128}+\frac{i}{32(-i+\tan(dx+c))^4}-\frac{3i}{32(-i+\tan(dx+c))^2}-\frac{1}{16(-i+\tan(dx+c))^3}+\frac{5}{32(-i+\tan(dx+c))}+\frac{i}{64(\tan(dx+c)}}{d\,a^2}$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
15/64*x/a^2+1/64*I/a^2/d*exp(-6*I*(d*x+c))+1/512*I/a^2/d*exp(-8*I*(d*x+c))
+7/128*I/a^2/d*cos(4*d*x+4*c)+1/16/a^2/d*sin(4*d*x+4*c)+7/64*I/a^2/d*cos(2
*d*x+2*c)+13/64/a^2/d*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.53

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(120\,dxe^{(8i\,dx+8i\,c)}-2i\,e^{(12i\,dx+12i\,c)}-24i\,e^{(10i\,dx+10i\,c)}+80i\,e^{(6i\,dx+6i\,c)}+30i\,e^{(4i\,dx+4i\,c)}+8i\,e^{(2i\,dx+2i\,c)}+\right.}{512\,a^2d}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/512*(120*d*x*e^(8*I*d*x + 8*I*c) - 2*I*e^(12*I*d*x + 12*I*c) - 24*I*e^(1
0*I*d*x + 10*I*c) + 80*I*e^(6*I*d*x + 6*I*c) + 30*I*e^(4*I*d*x + 4*I*c) +
8*I*e^(2*I*d*x + 2*I*c) + I)*e^(-8*I*d*x - 8*I*c)/(a^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.56

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \begin{cases} \frac{(-17179869184ia^{10}d^5e^{24ic}e^{4idx} - 206158430208ia^{10}d^5e^{22ic}e^{2idx} + 687194767360ia^{10}d^5e^{18ic}e^{-2idx} + 257698037760ia^{10}d^5e^{16ic}e^{-4idx} + 6871\cdots}{4398046511104a^{12}d^6} \\ x\left(\frac{(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1)e^{-8ic}}{64a^2} - \frac{15}{64a^2}\right) \end{cases}$$
$$+ \frac{15x}{64a^2}$$

input
```
integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-17179869184*I*a**10*d**5*exp(24*I*c)*exp(4*I*d*x) - 206158430
208*I*a**10*d**5*exp(22*I*c)*exp(2*I*d*x) + 687194767360*I*a**10*d**5*exp(
18*I*c)*exp(-2*I*d*x) + 257698037760*I*a**10*d**5*exp(16*I*c)*exp(-4*I*d*x
) + 68719476736*I*a**10*d**5*exp(14*I*c)*exp(-6*I*d*x) + 8589934592*I*a**1
0*d**5*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(4398046511104*a**12*d**6),
 Ne(a**12*d**6*exp(20*I*c), 0)), (x*((exp(12*I*c) + 6*exp(10*I*c) + 15*exp
(8*I*c) + 20*exp(6*I*c) + 15*exp(4*I*c) + 6*exp(2*I*c) + 1)*exp(-8*I*c)/(6
4*a**2) - 15/(64*a**2)), True)) + 15*x/(64*a**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{15i\log(\tan(dx+c)+i)}{128a^2d} - \frac{15i\log(\tan(dx+c)-i)}{128a^2d} + \frac{15\tan(dx+c)^5 - 30i\tan(dx+c)^4 + 10\tan(dx+c)^3 - 50i\tan(dx+c)^2 - 17\tan(dx+c) - 16i}{64a^2d(\tan(dx+c)+i)^2(\tan(dx+c)-i)^4}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
15/128*I*log(tan(d*x + c) + I)/(a^2*d) - 15/128*I*log(tan(d*x + c) - I)/(a
^2*d) + 1/64*(15*tan(d*x + c)^5 - 30*I*tan(d*x + c)^4 + 10*tan(d*x + c)^3
- 50*I*tan(d*x + c)^2 - 17*tan(d*x + c) - 16*I)/(a^2*d*(tan(d*x + c) + I)^
2*(tan(d*x + c) - I)^4)
```

Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.90

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{15x}{64a^2} + \frac{\frac{1}{4a^2} - \frac{\tan(c+dx)\,17\mathrm{i}}{64a^2} + \frac{25\tan(c+dx)^2}{32a^2} + \frac{\tan(c+dx)^3\,5\mathrm{i}}{32a^2} + \frac{15\tan(c+dx)^4}{32a^2} + \frac{\tan(c+dx)^5\,15\mathrm{i}}{64a^2}}{d\left(\tan(c+dx)^6\,1\mathrm{i} + 2\tan(c+dx)^5 + \tan(c+dx)^4\,1\mathrm{i} + 4\tan(c+dx)^3 - \tan(c+dx)^2\,1\mathrm{i} + 2\tan(\right.}$$

input
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(15*x)/(64*a^2) + (1/(4*a^2) - (tan(c + d*x)*17i)/(64*a^2) + (25*tan(c + d
*x)^2)/(32*a^2) + (tan(c + d*x)^3*5i)/(32*a^2) + (15*tan(c + d*x)^4)/(32*a
^2) + (tan(c + d*x)^5*15i)/(64*a^2))/(d*(2*tan(c + d*x) - tan(c + d*x)^2*1
i + 4*tan(c + d*x)^3 + tan(c + d*x)^4*1i + 2*tan(c + d*x)^5 + tan(c + d*x)
^6*1i - 1i))
```

Reduce [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\cos(dx+c)^4}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx}{a^2}$$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(cos(c + d*x)**4/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x))/a**2
```

3.122 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{7\text{arctanh}(\sin(c+dx))}{16a^2d} + \frac{7\sec(c+dx)\tan(c+dx)}{16a^2d} + \frac{7\sec^3(c+dx)\tan(c+dx)}{24a^2d} + \frac{7\sec^5(c+dx)\tan(c+dx)}{30a^2d} - \frac{2i\sec^7(c+dx)}{5d\,(a^2+ia^2\tan(c+dx))}$$

output
```
7/16*arctanh(sin(d*x+c))/a^2/d+7/16*sec(d*x+c)*tan(d*x+c)/a^2/d+7/24*sec(d
*x+c)^3*tan(d*x+c)/a^2/d+7/30*sec(d*x+c)^5*tan(d*x+c)/a^2/d-2/5*I*sec(d*x+
c)^7/d/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 294 vs. 2(124) = 248.

Time = 2.23 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.37

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sec^6(c+dx)\left(3072i\cos(c+dx)+5\left(210\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+21\cos(6(c+dx))\,\mathrm{l}\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/7680*(Sec[c + d*x]^6*((3072*I)*Cos[c + d*x] + 5*(210*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]] + 21*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] + 315*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 126*Cos[4*(c + d*x)]*(Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]) - 210*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 21*Cos[6*(c + d*x
)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 60*Sin[c + d*x] - 238*Sin[3*
(c + d*x)] - 42*Sin[5*(c + d*x)])))/(a^2*d)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3981, 3042, 4255, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^9}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{7\int \sec^7(c+dx)dx}{5a^2}-\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int \csc\left(c+dx+\frac{\pi}{2}\right)^7dx}{5a^2}-\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7\left(\frac{5}{6}\int \sec^5(c+dx)dx+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)}{5a^2}-\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5}{6}\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)}{5a^2}-\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7\left(\frac{5}{6}\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)}{5a^2}-\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5}{6}\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)}{5a^2}-\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)}{5a^2}-\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)}{5a^2}-\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4257}$$

$$\frac{7\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)}{5a^2}-\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(((-2*I)/5)*Sec[c + d*x]^7)/(d*(a^2 + I*a^2*Tan[c + d*x])) + (7*((Sec[c +
d*x]^5*Tan[c + d*x])/(6*d) + (5*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*
(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/6))
/(5*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{i\left(105\,\mathrm{e}^{11 i(dx+c)}+595\,\mathrm{e}^{9 i(dx+c)}+1386\,\mathrm{e}^{7 i(dx+c)}+1686\,\mathrm{e}^{5 i(dx+c)}-595\,\mathrm{e}^{3 i(dx+c)}-105\,\mathrm{e}^{i(dx+c)}\right)}{120 d\,a^{2}\left(\mathrm{e}^{2 i(dx+c)}+1\right)^{6}}-\frac{7\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{16a^{2}d}$
derivativedivides	$\frac{\frac{2\left(\frac{1}{4}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{2\left(\frac{9}{32}-\frac{3i}{8}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{2\left(-\frac{1}{12}-\frac{3i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(-\frac{1}{4}-\frac{i}{5}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{2\left(-\frac{9}{32}+\frac{5i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}+}{}$
default	$\frac{\frac{2\left(\frac{1}{4}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{2\left(\frac{9}{32}-\frac{3i}{8}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{2\left(-\frac{1}{12}-\frac{3i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(-\frac{1}{4}-\frac{i}{5}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{2\left(-\frac{9}{32}+\frac{5i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}+}{}$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/120*I/d/a^2/(exp(2*I*(d*x+c))+1)^6*(105*exp(11*I*(d*x+c))+595*exp(9*I*(
d*x+c))+1386*exp(7*I*(d*x+c))+1686*exp(5*I*(d*x+c))-595*exp(3*I*(d*x+c))-1
05*exp(I*(d*x+c)))-7/16/a^2/d*ln(exp(I*(d*x+c))-I)+7/16/a^2/d*ln(exp(I*(d*
x+c))+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(110) = 220.

Time = 0.11 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.63

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{105\left(e^{(12i\,dx+12i\,c)}+6\,e^{(10i\,dx+10i\,c)}+15\,e^{(8i\,dx+8i\,c)}+20\,e^{(6i\,dx+6i\,c)}+15\,e^{(4i\,dx+4i\,c)}+6\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(\right.}{}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/240*(105*(e^(12*I*d*x + 12*I*c) + 6*e^(10*I*d*x + 10*I*c) + 15*e^(8*I*d*
x + 8*I*c) + 20*e^(6*I*d*x + 6*I*c) + 15*e^(4*I*d*x + 4*I*c) + 6*e^(2*I*d*
x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 105*(e^(12*I*d*x + 12*I*c) + 6*
e^(10*I*d*x + 10*I*c) + 15*e^(8*I*d*x + 8*I*c) + 20*e^(6*I*d*x + 6*I*c) +
15*e^(4*I*d*x + 4*I*c) + 6*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) -
I) - 210*I*e^(11*I*d*x + 11*I*c) - 1190*I*e^(9*I*d*x + 9*I*c) - 2772*I*e^(
7*I*d*x + 7*I*c) - 3372*I*e^(5*I*d*x + 5*I*c) + 1190*I*e^(3*I*d*x + 3*I*c)
 + 210*I*e^(I*d*x + I*c))/(a^2*d*e^(12*I*d*x + 12*I*c) + 6*a^2*d*e^(10*I*d
*x + 10*I*c) + 15*a^2*d*e^(8*I*d*x + 8*I*c) + 20*a^2*d*e^(6*I*d*x + 6*I*c)
 + 15*a^2*d*e^(4*I*d*x + 4*I*c) + 6*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^9(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sec(c + d*x)**9/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**
2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 421 vs. 2(110) = 220.

Time = 0.05 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.40

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{2\left(\frac{135\sin(dx+c)}{\cos(dx+c)+1}+\frac{96i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{445\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{960i\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{330\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{960i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{330\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{480i\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\cdots\right.}{a^2-\frac{6a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{20a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{6a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\cdots}}{240\,d}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/240*(2*(135*sin(d*x + c)/(cos(d*x + c) + 1) + 96*I*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 - 445*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 960*I*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 330*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 960
*I*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 330*sin(d*x + c)^7/(cos(d*x + c)
+ 1)^7 - 480*I*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 445*sin(d*x + c)^9/(c
os(d*x + c) + 1)^9 + 480*I*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 135*sin
(d*x + c)^11/(cos(d*x + c) + 1)^11 - 96*I)/(a^2 - 6*a^2*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 15*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*a^2*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a^2*sin(d*x + c)^8/(cos(d*x + c) +
1)^8 - 6*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^2*sin(d*x + c)^12/(
cos(d*x + c) + 1)^12) + 105*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 -
 105*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.64

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{105\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2} - \frac{105\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2} + \frac{2\left(135\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+480i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-445\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-480i\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/240*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 105*log(tan(1/2*d*x + 1/2*c
) - 1)/a^2 + 2*(135*tan(1/2*d*x + 1/2*c)^11 + 480*I*tan(1/2*d*x + 1/2*c)^1
0 - 445*tan(1/2*d*x + 1/2*c)^9 - 480*I*tan(1/2*d*x + 1/2*c)^8 - 330*tan(1/
2*d*x + 1/2*c)^7 + 960*I*tan(1/2*d*x + 1/2*c)^6 - 330*tan(1/2*d*x + 1/2*c)
^5 - 960*I*tan(1/2*d*x + 1/2*c)^4 - 445*tan(1/2*d*x + 1/2*c)^3 + 96*I*tan(
1/2*d*x + 1/2*c)^2 + 135*tan(1/2*d*x + 1/2*c) - 96*I)/((tan(1/2*d*x + 1/2*
c)^2 - 1)^6*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.54

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a^2\,d}$$
$$-\frac{-\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}4\mathrm{i}+\frac{89\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}4\mathrm{i}+\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}8}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-\right.}$$

input
```
int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
(7*atanh(tan(c/2 + (d*x)/2)))/(8*a^2*d) - ((89*tan(c/2 + (d*x)/2)^3)/24 -
(tan(c/2 + (d*x)/2)^2*4i)/5 - (9*tan(c/2 + (d*x)/2))/8 + tan(c/2 + (d*x)/2
)^4*8i + (11*tan(c/2 + (d*x)/2)^5)/4 - tan(c/2 + (d*x)/2)^6*8i + (11*tan(c
/2 + (d*x)/2)^7)/4 + tan(c/2 + (d*x)/2)^8*4i + (89*tan(c/2 + (d*x)/2)^9)/2
4 - tan(c/2 + (d*x)/2)^10*4i - (9*tan(c/2 + (d*x)/2)^11)/8 + 4i/5)/(a^2*d*
(tan(c/2 + (d*x)/2)^2 - 1)^6)
```

Reduce [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec(dx+c)^9}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx}{a^2}$$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(sec(c + d*x)**9/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x))/a**2
```

3.123 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 100

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2} dx = \frac{5\text{arctanh}(\sin(c+dx))}{8a^2d} + \frac{5\sec(c+dx)\tan(c+dx)}{8a^2d} + \frac{5\sec^3(c+dx)\tan(c+dx)}{12a^2d} - \frac{2i\sec^5(c+dx)}{3d(a^2+ia^2\tan(c+dx))}$$

output
```
5/8*arctanh(sin(d*x+c))/a^2/d+5/8*sec(d*x+c)*tan(d*x+c)/a^2/d+5/12*sec(d*x
+c)^3*tan(d*x+c)/a^2/d-2/3*I*sec(d*x+c)^5/d/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 215 vs. 2(100) = 200.

Time = 1.29 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.15

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2} dx = -\frac{\sec^4(c+dx)\left(128i\cos(c+dx) + 45\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + 60\cos(2(c+dx))\left(\log\left(\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/192*(Sec[c + d*x]^4*((128*I)*Cos[c + d*x] + 45*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 60*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 15*Cos[4*(c + d*x)]*(
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]]) - 45*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 18*Sin[c + d*x]
 - 30*Sin[3*(c + d*x)]))/(a^2*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3981, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^7}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3981}$$
$$\frac{5\int \sec^5(c+dx)dx}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$
$$\Big\downarrow \text{3042}$$
$$\frac{5\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$
$$\Big\downarrow \text{4255}$$
$$\frac{5\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$
$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{5\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4257}$$

$$\frac{5\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(((-2*I)/3)*Sec[c + d*x]^5)/(d*(a^2 + I*a^2*Tan[c + d*x])) + (5*((Sec[c +
d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*
x]*Tan[c + d*x])/(2*d)))/4))/(3*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{i\left(15\,\mathrm{e}^{7i(dx+c)}+55\,\mathrm{e}^{5i(dx+c)}+73\,\mathrm{e}^{3i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}\right)}{12d\,a^{2}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{4}}+\frac{5\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8a^{2}d}-\frac{5\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8a^{2}d}$
derivativedivides	$\frac{\frac{2\left(\frac{3}{16}+\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{16}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{1}{4}+\frac{i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2\left(\frac{3}{16}-\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+}{a^{2}d}$
default	$\frac{\frac{2\left(\frac{3}{16}+\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{16}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{1}{4}+\frac{i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2\left(\frac{3}{16}-\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+}{a^{2}d}$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/12*I/d/a^2/(exp(2*I*(d*x+c))+1)^4*(15*exp(7*I*(d*x+c))+55*exp(5*I*(d*x+
c))+73*exp(3*I*(d*x+c))-15*exp(I*(d*x+c)))+5/8/a^2/d*ln(exp(I*(d*x+c))+I)-
5/8/a^2/d*ln(exp(I*(d*x+c))-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(88) = 176.

Time = 0.10 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.30

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{15\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}+4\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6}\right.}{24\left(a^2de^{(8i\,dx+8i\,c)}+4\,a^2de^{(6i}\right.}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/24*(15*(e^(8*I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I
*c) + 4*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 15*(e^(8*I*d*x
 + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x +
 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 30*I*e^(7*I*d*x + 7*I*c) - 110*I*e
^(5*I*d*x + 5*I*c) - 146*I*e^(3*I*d*x + 3*I*c) + 30*I*e^(I*d*x + I*c))/(a^
2*d*e^(8*I*d*x + 8*I*c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6*a^2*d*e^(4*I*d*x
 + 4*I*c) + 4*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^7(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sec(c + d*x)**7/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**
2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(88) = 176$.

Time = 0.06 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.95

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{33\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{48i\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{33\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{48i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{9\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-16i\right)}{a^2-\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{15\log\left(\frac{\sin(dx}{\cos(dx-}\right.}{a^2}}{24d}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/24*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*I*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 33*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 48*I*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 33*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 48*I*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 + 9*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
16*I)/(a^2 - 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2
*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 15*log(sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)/a^2 - 15*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.51

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{15\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2}-\frac{15\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^2}+\frac{2\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+48i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-33\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-48i\tan\left(\frac{1}{2}dx+\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}}{24d}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 15*log(tan(1/2*d*x + 1/2*c) -
 1)/a^2 + 2*(9*tan(1/2*d*x + 1/2*c)^7 + 48*I*tan(1/2*d*x + 1/2*c)^6 - 33*t
an(1/2*d*x + 1/2*c)^5 - 48*I*tan(1/2*d*x + 1/2*c)^4 - 33*tan(1/2*d*x + 1/2
*c)^3 + 16*I*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) - 16*I)/((tan
(1/2*d*x + 1/2*c)^2 - 1)^4*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 3.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.36

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^2\,d} + \frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 4\mathrm{i} - \frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4} - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 4\mathrm{i} - \frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 4\mathrm{i}}{3} + \frac{3\,\mathrm{ta}}{}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^4}$$

input
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
(5*atanh(tan(c/2 + (d*x)/2)))/(4*a^2*d) + ((3*tan(c/2 + (d*x)/2))/4 + (tan
(c/2 + (d*x)/2)^2*4i)/3 - (11*tan(c/2 + (d*x)/2)^3)/4 - tan(c/2 + (d*x)/2)
^4*4i - (11*tan(c/2 + (d*x)/2)^5)/4 + tan(c/2 + (d*x)/2)^6*4i + (3*tan(c/2
 + (d*x)/2)^7)/4 - 4i/3)/(a^2*d*(tan(c/2 + (d*x)/2)^2 - 1)^4)
```

Reduce [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec(dx+c)^7}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx}{a^2}$$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(sec(c + d*x)**7/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x))/a**2
```

3.124 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{3\text{arctanh}(\sin(c+dx))}{2a^2d} + \frac{3\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{2i\sec^3(c+dx)}{d\,(a^2+ia^2\tan(c+dx))}$$

output
```
3/2*arctanh(sin(d*x+c))/a^2/d+3/2*sec(d*x+c)*tan(d*x+c)/a^2/d-2*I*sec(d*x+
c)^3/d/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.97

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sec^2(c+dx)\left(8i\cos(c+dx)+3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(2(c+dx))\left(\log\left(\cos\left(\right.\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/4*(Sec[c + d*x]^2*((8*I)*Cos[c + d*x] + 3*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] + 3*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + 2*Sin[c + d*x]))/(a^2*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3981, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{3\int \sec^3(c+dx)dx}{a^2}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{3\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4257}$$

$$\frac{3\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-2*I)*Sec[c + d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x])) + (3*(ArcTanh[Sin[c
 + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/a^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{i\left(3\,\mathrm{e}^{3i(dx+c)}+5\,\mathrm{e}^{i(dx+c)}\right)}{d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2a^2d}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2a^2d}$
derivativedivides	$\frac{\frac{2\left(-\frac{1}{4}-i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\frac{2\left(-\frac{1}{4}+i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}}{a^2d}$
default	$\frac{\frac{2\left(-\frac{1}{4}-i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\frac{2\left(-\frac{1}{4}+i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}}{a^2d}$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-I/d/a^2/(exp(2*I*(d*x+c))+1)^2*(3*exp(3*I*(d*x+c))+5*exp(I*(d*x+c)))+3/2/
a^2/d*ln(exp(I*(d*x+c))+I)-3/2/a^2/d*ln(exp(I*(d*x+c))-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(66) = 132.

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.81

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{3\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)}{2\left(a^2de^{(4i\,dx+4i\,c)}+2\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(3*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*
c) + I) - 3*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x
 + I*c) - I) - 6*I*e^(3*I*d*x + 3*I*c) - 10*I*e^(I*d*x + I*c))/(a^2*d*e^(4
*I*d*x + 4*I*c) + 2*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^5(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sec(c + d*x)**5/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**
2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(66) = 132.

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.26

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{4i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+4i\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{2d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 4*I*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4*I)/(a^2 - 2*a^2*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) -
3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 3*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2} - \frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^2} - \frac{2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+4i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2}}{2d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(3*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 3*log(tan(1/2*d*x + 1/2*c) - 1)
/a^2 - 2*(tan(1/2*d*x + 1/2*c)^3 - 4*I*tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*
x + 1/2*c) + 4*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.41

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2\,d} - \frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{a^2}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 4\mathrm{i}}{a^2}+\frac{4\mathrm{i}}{a^2}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$

input
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
(3*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (tan(c/2 + (d*x)/2)^3/a^2 - (tan(c
/2 + (d*x)/2)^2*4i)/a^2 + 4i/a^2 + tan(c/2 + (d*x)/2)/a^2)/(d*(tan(c/2 + (
d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

Reduce [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec(dx+c)^5}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx}{a^2}$$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(sec(c + d*x)**5/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x))/a**2
```

3.125 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 48

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\operatorname{arctanh}(\sin(c+dx))}{a^2d} + \frac{2i\sec(c+dx)}{d\,(a^2+ia^2\tan(c+dx))}$$

output
```
-arctanh(sin(d*x+c))/a^2/d+2*I*sec(d*x+c)/d/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 184 vs. 2(48) = 96.

Time = 0.40 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.83

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sec^2(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(2i+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+s\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-((Sec[c + d*x]^2*(Cos[(c + d*x)/2]*(2*I + Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (2 + I*Log[Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2]] - I*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
)*Sin[(c + d*x)/2])*(Cos[(3*(c + d*x))/2] + I*Sin[(3*(c + d*x))/2]))/(a^2*
d*(-I + Tan[c + d*x])^2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3981, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^3}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3981}$$
$$-\frac{\int \sec(c+dx)dx}{a^2}+\frac{2i\sec(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}+\frac{2i\sec(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}$$
$$\Big\downarrow \text{4257}$$
$$-\frac{\operatorname{arctanh}(\sin(c+dx))}{a^2d}+\frac{2i\sec(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-(ArcTanh[Sin[c + d*x]]/(a^2*d)) + ((2*I)*Sec[c + d*x])/(d*(a^2 + I*a^2*Ta
n[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\frac{\frac{4}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2d}$	54
default	$\frac{\frac{4}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2d}$	54
risch	$\frac{2i\mathrm{e}^{-i(dx+c)}}{a^2d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^2d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^2d}$	61

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/d/a^2*(2/(-I+tan(1/2*d*x+1/2*c))-1/2*ln(tan(1/2*d*x+1/2*c)+1)+1/2*ln(tan
(1/2*d*x+1/2*c)-1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.33

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\left(e^{(i\,dx+i\,c)}\log\left(e^{(i\,dx+i\,c)}+i\right)-e^{(i\,dx+i\,c)}\log\left(e^{(i\,dx+i\,c)}-i\right)-2i\right)e^{(-i\,dx-i\,c)}}{a^2d}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(e^(I*d*x + I*c)*log(e^(I*d*x + I*c) + I) - e^(I*d*x + I*c)*log(e^(I*d*x
+ I*c) - I) - 2*I)*e^(-I*d*x - I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^3(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sec(c + d*x)**3/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**
2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(44) = 88$.

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.44

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx =$$
$$-\frac{-2i\arctan(\cos(dx+c),\sin(dx+c)+1) - 2i\arctan(\cos(dx+c),-\sin(dx+c)+1) - 4i\cos(dx}{}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*(-2*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 2*I*arctan2(cos(d*x +
 c), -sin(d*x + c) + 1) - 4*I*cos(d*x + c) + log(cos(d*x + c)^2 + sin(d*x
+ c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin
(d*x + c) + 1) - 4*sin(d*x + c))/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2} - \frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2} - \frac{4}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-(log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - log(tan(1/2*d*x + 1/2*c) - 1)/a^2 -
4/(a^2*(tan(1/2*d*x + 1/2*c) - I)))/d
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d} + \frac{4\mathrm{i}}{a^2\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}$$

input
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
4i/(a^2*d*(tan(c/2 + (d*x)/2)*1i + 1)) - (2*atanh(tan(c/2 + (d*x)/2)))/(a^
2*d)
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec(dx+c)^3}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx}{a^2}$$

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(sec(c + d*x)**3/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x))/a**2
```

3.126 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 65

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2} + \frac{i\sec(c+dx)}{3d\,(a^2+ia^2\tan(c+dx))}$$

output
```
1/3*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^2+1/3*I*sec(d*x+c)/d/(a^2+I*a^2*tan(
d*x+c))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\sec(c+dx)(-2i+\tan(c+dx))}{3a^2d(-i+\tan(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*(-2*I + Tan[c + d*x]))/(3*a^2*d*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \ 3983$$

$$\frac{\int \frac{\sec(c+dx)}{i\tan(c+dx)a+a}dx}{3a}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\sec(c+dx)}{i\tan(c+dx)a+a}dx}{3a}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \ 3969$$

$$\frac{i\sec(c+dx)}{3ad(a+ia\tan(c+dx))}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}$$

input
```
Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((I/3)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^2) + ((I/3)*Sec[c + d*x])/(
a*d*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

method	result	size
risch	$\frac{i e^{-i(dx+c)}}{2a^2d}+\frac{i e^{-3i(dx+c)}}{6a^2d}$	38
derivativedivides	$\frac{-\frac{4}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{a^2d}$	57
default	$\frac{-\frac{4}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{a^2d}$	57

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*I/a^2/d*exp(-I*(d*x+c))+1/6*I/a^2/d*exp(-3*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.46

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{6\,a^2d}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(51) = 102.

Time = 0.57 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.72

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} \frac{\tan(c+dx)\sec(c+dx)}{3a^2d\tan^2(c+dx)-6ia^2d\tan(c+dx)-3a^2d} - \frac{2i\sec(c+dx)}{3a^2d\tan^2(c+dx)-6ia^2d\tan(c+dx)-3a^2d} & \text{for } d\neq 0 \\ \frac{x\sec(c)}{(ia\tan(c)+a)^2} & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise((tan(c + d*x)*sec(c + d*x)/(3*a**2*d*tan(c + d*x)**2 - 6*I*a**2*
d*tan(c + d*x) - 3*a**2*d) - 2*I*sec(c + d*x)/(3*a**2*d*tan(c + d*x)**2 -
6*I*a**2*d*tan(c + d*x) - 3*a**2*d), Ne(d, 0)), (x*sec(c)/(I*a*tan(c) + a)
**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i\cos(3\,dx+3\,c)+3i\cos(dx+c)+\sin(3\,dx+3\,c)+3\sin(dx+c)}{6\,a^2d}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(I*cos(3*d*x + 3*c) + 3*I*cos(d*x + c) + sin(3*d*x + 3*c) + 3*sin(d*x
+ c))/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right)}{3\,a^2d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 3*I*tan(1/2*d*x + 1/2*c) - 2)/(a^2*d*(tan(
1/2*d*x + 1/2*c) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 3\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\mathrm{i}\right)}{3\,a^2\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 1\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+1\right)}$$

input
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
-(2*(3*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*3i - 2i))/(3*a^2*d*(tan(c
/2 + (d*x)/2)*3i - 3*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec(dx+c)}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx}{a^2}$$

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(sec(c + d*x)/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x))/a**2
```

3.127 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx=\frac{3\sin(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
3/5*sin(d*x+c)/a^2/d-1/5*sin(d*x+c)^3/a^2/d+2/5*I*cos(d*x+c)^3/d/(a^2+I*a^
2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\sec(c+dx)(-12i+4i\cos(2(c+dx))-3\sec(c+dx)\sin(3(c+dx))+5\tan(c+dx))}{20a^2d(-i+\tan(c+dx))^2}$$

input
```
Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*(-12*I + (4*I)*Cos[2*(c + d*x)] - 3*Sec[c + d*x]*Sin[3*(c +
d*x)] + 5*Tan[c + d*x]))/(20*a^2*d*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(c+dx)(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3981}$$
$$\frac{3\int \cos^3(c+dx)dx}{5a^2}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$
$$\Big\downarrow \text{3042}$$
$$\frac{3\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{5a^2}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$
$$\Big\downarrow \text{3113}$$
$$-\frac{3\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{3\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(-3*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/(5*a^2*d) + (((2*I)/5)*Cos[c + d*x
]^3)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

method	result
risch	$\frac{i e^{-3i(dx+c)}}{8a^2 d}+\frac{i e^{-5i(dx+c)}}{40a^2 d}+\frac{i\cos(dx+c)}{4a^2 d}+\frac{\sin(dx+c)}{2a^2 d}$
derivativedivides	$\frac{-\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{5i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{4}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{3}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{7}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\cdots}{a^2 d}$
default	$\frac{-\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{5i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{4}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{3}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{7}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\cdots}{a^2 d}$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*I/a^2/d*exp(-3*I*(d*x+c))+1/40*I/a^2/d*exp(-5*I*(d*x+c))+1/4*I/a^2/d*c
os(d*x+c)+1/2*sin(d*x+c)/a^2/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(-5i\,e^{(6i\,dx+6i\,c)}+15i\,e^{(4i\,dx+4i\,c)}+5i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-5i\,dx-5i\,c)}}{40\,a^2d}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/40*(-5*I*e^(6*I*d*x + 6*I*c) + 15*I*e^(4*I*d*x + 4*I*c) + 5*I*e^(2*I*d*x
 + 2*I*c) + I)*e^(-5*I*d*x - 5*I*c)/(a^2*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(60) = 120.

Time = 0.23 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.30

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} \frac{(-2560ia^6d^3e^{10ic}e^{idx}+7680ia^6d^3e^{8ic}e^{-idx}+2560ia^6d^3e^{6ic}e^{-3idx}+512ia^6d^3e^{4ic}e^{-5idx})e^{-9ic}}{20480a^8d^4} & \text{for } a^8d^4e^{9ic}\neq 0 \\ \frac{x(e^{6ic}+3e^{4ic}+3e^{2ic}+1)e^{-5ic}}{8a^2} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-2560*I*a**6*d**3*exp(10*I*c)*exp(I*d*x) + 7680*I*a**6*d**3*ex
p(8*I*c)*exp(-I*d*x) + 2560*I*a**6*d**3*exp(6*I*c)*exp(-3*I*d*x) + 512*I*a
**6*d**3*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(20480*a**8*d**4), Ne(a**8*
d**4*exp(9*I*c), 0)), (x*(exp(6*I*c) + 3*exp(4*I*c) + 3*exp(2*I*c) + 1)*ex
p(-5*I*c)/(8*a**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.31

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{5}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}+\frac{35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-90i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+70i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+21}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^5}}{20\,d}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/20*(5/(a^2*(tan(1/2*d*x + 1/2*c) + I)) + (35*tan(1/2*d*x + 1/2*c)^4 - 90
*I*tan(1/2*d*x + 1/2*c)^3 - 120*tan(1/2*d*x + 1/2*c)^2 + 70*I*tan(1/2*d*x
+ 1/2*c) + 21)/(a^2*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.27

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= -\frac{2\left(-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 10\mathrm{i}+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2\mathrm{i}\right)}{5\,a^2\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{i}\right)^5\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
-(2*(3*tan(c/2 + (d*x)/2) + 10*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4
*10i - 5*tan(c/2 + (d*x)/2)^5 - 2i))/(5*a^2*d*(tan(c/2 + (d*x)/2) - 1i)^5*
(tan(c/2 + (d*x)/2) + 1i))
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\cos(dx+c)}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx}{a^2}$$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(cos(c + d*x)/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x))/a**2
```

3.128 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{5\sin(c+dx)}{7a^2d} - \frac{10\sin^3(c+dx)}{21a^2d} + \frac{\sin^5(c+dx)}{7a^2d} + \frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
5/7*sin(d*x+c)/a^2/d-10/21*sin(d*x+c)^3/a^2/d+1/7*sin(d*x+c)^5/a^2/d+2/7*I
*cos(d*x+c)^5/d/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i\sec^2(c+dx)(-140\cos(c+dx)+42\cos(3(c+dx))+2\cos(5(c+dx))-70i\sin(c+dx)+63i\sin(3(c+}{336a^2d(-i+\tan(c+dx))^2}$$

input
```
Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((I/336)*Sec[c + d*x]^2*(-140*Cos[c + d*x] + 42*Cos[3*(c + d*x)] + 2*Cos[5
*(c + d*x)] - (70*I)*Sin[c + d*x] + (63*I)*Sin[3*(c + d*x)] + (5*I)*Sin[5*
(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{5\int \cos^5(c+dx)dx}{7a^2}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx}{7a^2}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{5\int \left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(-5*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]^5/5))/(7*a^2*d) +
 (((2*I)/7)*Cos[c + d*x]^5)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.15

method	result
risch	$\frac{i \mathrm{e}^{-5 i (d x+c)}}{32 a^{2} d}+\frac{i \mathrm{e}^{-7 i (d x+c)}}{224 a^{2} d}+\frac{5 i \cos (d x+c)}{32 a^{2} d}+\frac{15 \sin (d x+c)}{32 a^{2} d}+\frac{3 i \cos (3 d x+3 c)}{32 a^{2} d}+\frac{11 \sin (3 d x+3 c)}{96 a^{2} d}$
derivativedivides	$\frac{\frac{2 i}{\left(-i+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{6}}-\frac{5 i}{\left(-i+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{23 i}{8 \left(-i+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{4}{7 \left(-i+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{7}}+\frac{4}{\left(-i+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{}{12\left(-i+\mathrm{ta}\right.}}{a^{2} d}$
default	$\frac{\frac{2 i}{\left(-i+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{6}}-\frac{5 i}{\left(-i+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{23 i}{8 \left(-i+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{4}{7 \left(-i+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{7}}+\frac{4}{\left(-i+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{}{12\left(-i+\mathrm{ta}\right.}}{a^{2} d}$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*I/a^2/d*exp(-5*I*(d*x+c))+1/224*I/a^2/d*exp(-7*I*(d*x+c))+5/32*I/a^2/
d*cos(d*x+c)+15/32*sin(d*x+c)/a^2/d+3/32*I/a^2/d*cos(3*d*x+3*c)+11/96/a^2/
d*sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(-7i\,e^{(10i\,dx+10i\,c)}-105i\,e^{(8i\,dx+8i\,c)}+210i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+21i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-7i\,dx-7i\,c)}}{672\,a^2d}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/672*(-7*I*e^(10*I*d*x + 10*I*c) - 105*I*e^(8*I*d*x + 8*I*c) + 210*I*e^(6
*I*d*x + 6*I*c) + 70*I*e^(4*I*d*x + 4*I*c) + 21*I*e^(2*I*d*x + 2*I*c) + 3*
I)*e^(-7*I*d*x - 7*I*c)/(a^2*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(76) = 152.

Time = 0.34 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.60

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} \frac{(-176160768ia^{10}d^5e^{19ic}e^{3idx}-2642411520ia^{10}d^5e^{17ic}e^{idx}+5284823040ia^{10}d^5e^{15ic}e^{-idx}+1761607680ia^{10}d^5e^{13ic}e^{-3idx}+528482304ia^{10}d}{16911433728a^{12}d^6} \\ \frac{x\left(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1\right)e^{-7ic}}{32a^2} \end{cases}$$

input
```
integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-176160768*I*a**10*d**5*exp(19*I*c)*exp(3*I*d*x) - 2642411520*
I*a**10*d**5*exp(17*I*c)*exp(I*d*x) + 5284823040*I*a**10*d**5*exp(15*I*c)*
exp(-I*d*x) + 1761607680*I*a**10*d**5*exp(13*I*c)*exp(-3*I*d*x) + 52848230
4*I*a**10*d**5*exp(11*I*c)*exp(-5*I*d*x) + 75497472*I*a**10*d**5*exp(9*I*c
)*exp(-7*I*d*x))*exp(-16*I*c)/(16911433728*a**12*d**6), Ne(a**12*d**6*exp(
16*I*c), 0)), (x*(exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*c) + 10*exp(4*I*
c) + 5*exp(2*I*c) + 1)*exp(-7*I*c)/(32*a**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.63

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{7\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-8\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+\frac{273\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-1155i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-2450\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2870i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^7}}{168\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/168*(7*(9*tan(1/2*d*x + 1/2*c)^2 + 15*I*tan(1/2*d*x + 1/2*c) - 8)/(a^2*(
tan(1/2*d*x + 1/2*c) + I)^3) + (273*tan(1/2*d*x + 1/2*c)^6 - 1155*I*tan(1/
2*d*x + 1/2*c)^5 - 2450*tan(1/2*d*x + 1/2*c)^4 + 2870*I*tan(1/2*d*x + 1/2*
c)^3 + 2037*tan(1/2*d*x + 1/2*c)^2 - 791*I*tan(1/2*d*x + 1/2*c) - 152)/(a^
2*(tan(1/2*d*x + 1/2*c) - I)^7))/d
```

Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.81

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(-21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 42\mathrm{i}+28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 56\mathrm{i}+42\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\right.}{21\,a^2\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)^3\left(1+\tan\left(\right.\right.}$$

input
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
((3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*24i + 76*tan(c/2 + (d*x)/2)^
3 + tan(c/2 + (d*x)/2)^4*28i + 42*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2
)^6*56i + 28*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8*42i - 21*tan(c/2
+ (d*x)/2)^9 - 6i)*2i)/(21*a^2*d*(tan(c/2 + (d*x)/2) + 1i)^3*(tan(c/2 + (d
*x)/2)*1i + 1)^7)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\cos(dx+c)^3}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx}{a^2}$$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(cos(c + d*x)**3/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x))/a**2
```

3.129 $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{7\sin(c+dx)}{9a^2d} - \frac{7\sin^3(c+dx)}{9a^2d} + \frac{7\sin^5(c+dx)}{15a^2d} - \frac{\sin^7(c+dx)}{9a^2d} + \frac{2i\cos^7(c+dx)}{9d\,(a^2+ia^2\tan(c+dx))}$$

output
```
7/9*sin(d*x+c)/a^2/d-7/9*sin(d*x+c)^3/a^2/d+7/15*sin(d*x+c)^5/a^2/d-1/9*si
n(d*x+c)^7/a^2/d+2/9*I*cos(d*x+c)^7/d/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i\sec^2(c+dx)(-1050\cos(c+dx)+378\cos(3(c+dx))+30\cos(5(c+dx))+2\cos(7(c+dx))-525i\sin}{2880a^2d(-i+\tan(c+dx))^2}$$

input
```
Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((I/2880)*Sec[c + d*x]^2*(-1050*Cos[c + d*x] + 378*Cos[3*(c + d*x)] + 30*C
os[5*(c + d*x)] + 2*Cos[7*(c + d*x)] - (525*I)*Sin[c + d*x] + (567*I)*Sin[
3*(c + d*x)] + (75*I)*Sin[5*(c + d*x)] + (7*I)*Sin[7*(c + d*x)]))/(a^2*d*(
-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = $\{3042, 3981, 3042, 3113, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(c+dx)^5(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3981}$$
$$\frac{7\int \cos^7(c+dx)dx}{9a^2}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}$$
$$\Big\downarrow \text{3042}$$
$$\frac{7\int \sin\left(c+dx+\frac{\pi}{2}\right)^7dx}{9a^2}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}$$
$$\Big\downarrow \text{3113}$$
$$-\frac{7\int \left(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{7\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(-7*(-Sin[c + d*x] + Sin[c + d*x]^3 - (3*Sin[c + d*x]^5)/5 + Sin[c + d*x]^
7/7))/(9*a^2*d) + (((2*I)/9)*Cos[c + d*x]^7)/(d*(a^2 + I*a^2*Tan[c + d*x])
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.28

method	result
risch	$\frac{i e^{-7i(dx+c)}}{128a^2d}+\frac{i e^{-9i(dx+c)}}{1152a^2d}+\frac{7i\cos(dx+c)}{64a^2d}+\frac{7\sin(dx+c)}{16a^2d}+\frac{i\cos(5dx+5c)}{32a^2d}+\frac{11\sin(5dx+5c)}{320a^2d}+\frac{7i\cos(3dx+$
derivativedivides	$-\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{51i}{16\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{49i}{6\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{35i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{4}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}-\frac{}{(-i}$
default	$-\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{51i}{16\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{49i}{6\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{35i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{4}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}-\frac{}{(-i}$

input
```
int(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/128*I/a^2/d*exp(-7*I*(d*x+c))+1/1152*I/a^2/d*exp(-9*I*(d*x+c))+7/64*I/a^
2/d*cos(d*x+c)+7/16*sin(d*x+c)/a^2/d+1/32*I/a^2/d*cos(5*d*x+5*c)+11/320/a^
2/d*sin(5*d*x+5*c)+7/96*I/a^2/d*cos(3*d*x+3*c)+7/64/a^2/d*sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(-9i\,e^{(14i\,dx+14i\,c)}-105i\,e^{(12i\,dx+12i\,c)}-945i\,e^{(10i\,dx+10i\,c)}+1575i\,e^{(8i\,dx+8i\,c)}+525i\,e^{(6i\,dx+6i\,c)}+189i\,e^{(4i\,d}\right.}{5760\,a^2d}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/5760*(-9*I*e^(14*I*d*x + 14*I*c) - 105*I*e^(12*I*d*x + 12*I*c) - 945*I*e
^(10*I*d*x + 10*I*c) + 1575*I*e^(8*I*d*x + 8*I*c) + 525*I*e^(6*I*d*x + 6*I
*c) + 189*I*e^(4*I*d*x + 4*I*c) + 45*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-9*I*
d*x - 9*I*c)/(a^2*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(94) = 188$.

Time = 0.41 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.79

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \begin{cases} \frac{(-227994731135631360ia^{14}d^7e^{30ic}e^{5idx} - 2659938529915699200ia^{14}d^7e^{28ic}e^{3idx} - 23939446769241292800ia^{14}d^7e^{26ic}e^{idx} + 3989907794873}{} \\ \frac{x(e^{14ic}+7e^{12ic}+21e^{10ic}+35e^{8ic}+35e^{6ic}+21e^{4ic}+7e^{2ic}+1)e^{-9ic}}{128a^2} \end{cases}$$

input
```
integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-227994731135631360*I*a**14*d**7*exp(30*I*c)*exp(5*I*d*x) - 26
59938529915699200*I*a**14*d**7*exp(28*I*c)*exp(3*I*d*x) - 2393944676924129
2800*I*a**14*d**7*exp(26*I*c)*exp(I*d*x) + 3989907794873548800 0*I*a**14*d*
*7*exp(24*I*c)*exp(-I*d*x) + 13299692649578496000*I*a**14*d**7*exp(22*I*c)
*exp(-3*I*d*x) + 4787889353848258560*I*a**14*d**7*exp(20*I*c)*exp(-5*I*d*x
) + 1139973655678156800*I*a**14*d**7*exp(18*I*c)*exp(-7*I*d*x) + 126663739
519795200*I*a**14*d**7*exp(16*I*c)*exp(-9*I*d*x))*exp(-25*I*c)/(1459166279
26804070400*a**16*d**8), Ne(a**16*d**8*exp(25*I*c), 0)), (x*(exp(14*I*c) +
 7*exp(12*I*c) + 21*exp(10*I*c) + 35*exp(8*I*c) + 35*exp(6*I*c) + 21*exp(4
*I*c) + 7*exp(2*I*c) + 1)*exp(-9*I*c)/(128*a**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(93) = 186$.

Time = 0.24 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.84

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{3\left(435\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1470i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2060\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1330i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+353\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^5}+\frac{4455\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-26460i\tan\left(\frac{1}{2}\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2880*(3*(435*tan(1/2*d*x + 1/2*c)^4 + 1470*I*tan(1/2*d*x + 1/2*c)^3 - 20
60*tan(1/2*d*x + 1/2*c)^2 - 1330*I*tan(1/2*d*x + 1/2*c) + 353)/(a^2*(tan(1
/2*d*x + 1/2*c) + I)^5) + (4455*tan(1/2*d*x + 1/2*c)^8 - 26460*I*tan(1/2*d
*x + 1/2*c)^7 - 78120*tan(1/2*d*x + 1/2*c)^6 + 137340*I*tan(1/2*d*x + 1/2*
c)^5 + 157374*tan(1/2*d*x + 1/2*c)^4 - 118356*I*tan(1/2*d*x + 1/2*c)^3 - 5
7744*tan(1/2*d*x + 1/2*c)^2 + 16596*I*tan(1/2*d*x + 1/2*c) + 2339)/(a^2*(t
an(1/2*d*x + 1/2*c) - I)^9))/d
```

Mupad [B] (verification not implemented)

Time = 2.86 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.02

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{191\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{16}-\frac{1289\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{64}+\frac{649\sin\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{64}-\frac{41\sin\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{32}+\frac{41\sin\left(\frac{9c}{2}+\frac{9dx}{2}\right)}{32}-\frac{7\sin}{}\right.}{45a^2d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right.}$$

input
```
int(cos(c + d*x)^5/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(cos(c/2 + (d*x)/2)*((cos((3*c)/2 + (3*d*x)/2)*525i)/32 - (cos((5*c)/2 + (
5*d*x)/2)*205i)/32 + (cos((7*c)/2 + (7*d*x)/2)*1i)/2 - (cos((9*c)/2 + (9*d
*x)/2)*1i)/2 + (cos((11*c)/2 + (11*d*x)/2)*1i)/32 - (cos((13*c)/2 + (13*d*
x)/2)*1i)/32 + (191*sin(c/2 + (d*x)/2))/16 - (1289*sin((3*c)/2 + (3*d*x)/2
))/64 + (649*sin((5*c)/2 + (5*d*x)/2))/64 - (41*sin((7*c)/2 + (7*d*x)/2))/
32 + (41*sin((9*c)/2 + (9*d*x)/2))/32 - (7*sin((11*c)/2 + (11*d*x)/2))/64
+ (7*sin((13*c)/2 + (13*d*x)/2))/64)*2i)/(45*a^2*d*(cos(c/2 + (d*x)/2) + s
in(c/2 + (d*x)/2)*1i)^9*(cos(c/2 + (d*x)/2)*1i + sin(c/2 + (d*x)/2))^5)
```

Reduce [F]

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\cos(dx+c)^5}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx}{a^2}$$

input
```
int(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(cos(c + d*x)**5/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x))/a**2
```

3.130 $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{8i(a-ia\tan(c+dx))^7}{7a^{10}d} - \frac{3i(a-ia\tan(c+dx))^8}{2a^{11}d} + \frac{2i(a-ia\tan(c+dx))^9}{3a^{12}d} - \frac{i(a-ia\tan(c+dx))^{10}}{10a^{13}d}$$

output
```
8/7*I*(a-I*a*tan(d*x+c))^7/a^10/d-3/2*I*(a-I*a*tan(d*x+c))^8/a^11/d+2/3*I*
(a-I*a*tan(d*x+c))^9/a^12/d-1/10*I*(a-I*a*tan(d*x+c))^10/a^13/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.51

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{(i+\tan(c+dx))^7\left(-44-98i\tan(c+dx)+77\tan^2(c+dx)+21i\tan^3(c+dx)\right)}{210a^3d}$$

input
```
Integrate[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I + Tan[c + d*x])^7*(-44 - (98*I)*Tan[c + d*x] + 77*Tan[c + d*x]^2 + (21
*I)*Tan[c + d*x]^3))/(210*a^3*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{3042, 3968, 49, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(c+dx)^{14}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{i\int(a-ia\tan(c+dx))^6(i\tan(c+dx)a+a)^3d(ia\tan(c+dx))}{a^{13}d}$$

$$\Big\downarrow \; 49$$

$$-\frac{i\int\left(-(a-ia\tan(c+dx))^9+6a(a-ia\tan(c+dx))^8-12a^2(a-ia\tan(c+dx))^7+8a^3(a-ia\tan(c+dx))^6\right.}{a^{13}d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{i\left(-\frac{8}{7}a^3(a-ia\tan(c+dx))^7+\frac{3}{2}a^2(a-ia\tan(c+dx))^8+\frac{1}{10}(a-ia\tan(c+dx))^{10}-\frac{2}{3}a(a-ia\tan(c+dx))^9\right.}{a^{13}d}$$

input
```
Int[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*((-8*a^3*(a - I*a*Tan[c + d*x])^7)/7 + (3*a^2*(a - I*a*Tan[c + d*x])
^8)/2 - (2*a*(a - I*a*Tan[c + d*x])^9)/3 + (a - I*a*Tan[c + d*x])^10/10))/
(a^13*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.53

method	result
risch	$\frac{128i\left(120\,{\mathrm e}^{6i(dx+c)}+45\,{\mathrm e}^{4i(dx+c)}+10\,{\mathrm e}^{2i(dx+c)}+1\right)}{105d\,a^{3}\left({\mathrm e}^{2i(dx+c)}+1\right)^{10}}$
derivativedivides	$-\frac{i\left(i\tan(dx+c)-\frac{\tan(dx+c)^{10}}{10}-\frac{i\tan(dx+c)^{9}}{3}-\frac{8i\tan(dx+c)^{7}}{7}+\tan(dx+c)^{6}-\frac{6i\tan(dx+c)^{5}}{5}+2\tan(dx+c)^{4}+\frac{3\tan(dx+c)^{2}}{2}\right.}{a^{3}d}$
default	$-\frac{i\left(i\tan(dx+c)-\frac{\tan(dx+c)^{10}}{10}-\frac{i\tan(dx+c)^{9}}{3}-\frac{8i\tan(dx+c)^{7}}{7}+\tan(dx+c)^{6}-\frac{6i\tan(dx+c)^{5}}{5}+2\tan(dx+c)^{4}+\frac{3\tan(dx+c)^{2}}{2}\right.}{a^{3}d}$

input
```
int(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
128/105*I*(120*exp(6*I*(d*x+c))+45*exp(4*I*(d*x+c))+10*exp(2*I*(d*x+c))+1)
/d/a^3/(exp(2*I*(d*x+c))+1)^10
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(85) = 170$.

Time = 0.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.78

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{128\left(-120i\,e^{(6i\,dx+6i\,c)} - \ldots\right.}{105\left(a^3de^{(20i\,dx+20i\,c)} + 10\,a^3de^{(18i\,dx+18i\,c)} + 45\,a^3de^{(16i\,dx+16i\,c)} + 120\,a^3de^{(14i\,dx+14i\,c)} + 210\,a^3de^{(12i\,dx+\ldots}\right.}$$

input
```
integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-128/105*(-120*I*e^(6*I*d*x + 6*I*c) - 45*I*e^(4*I*d*x + 4*I*c) - 10*I*e^(
2*I*d*x + 2*I*c) - I)/(a^3*d*e^(20*I*d*x + 20*I*c) + 10*a^3*d*e^(18*I*d*x
+ 18*I*c) + 45*a^3*d*e^(16*I*d*x + 16*I*c) + 120*a^3*d*e^(14*I*d*x + 14*I*
c) + 210*a^3*d*e^(12*I*d*x + 12*I*c) + 252*a^3*d*e^(10*I*d*x + 10*I*c) + 2
10*a^3*d*e^(8*I*d*x + 8*I*c) + 120*a^3*d*e^(6*I*d*x + 6*I*c) + 45*a^3*d*e^
(4*I*d*x + 4*I*c) + 10*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^{14}(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**14/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sec(c + d*x)**14/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan
(c + d*x) + I), x)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.80

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-21i\,\tan(dx+c)^{10}+70\,\tan(dx+c)^{9}+240\,\tan(dx+c)^{7}+210i\,\tan(dx+c)^{6}+252\,\tan(dx+c)^{5}}{210\,a^3d}$$

input
```
integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/210*(-21*I*tan(d*x + c)^10 + 70*tan(d*x + c)^9 + 240*tan(d*x + c)^7 + 2
10*I*tan(d*x + c)^6 + 252*tan(d*x + c)^5 + 420*I*tan(d*x + c)^4 + 315*I*ta
n(d*x + c)^2 - 210*tan(d*x + c))/(a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.80

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-21i\,\tan(dx+c)^{10}+70\,\tan(dx+c)^{9}+240\,\tan(dx+c)^{7}+210i\,\tan(dx+c)^{6}+252\,\tan(dx+c)^{5}}{210\,a^3d}$$

input
```
integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/210*(-21*I*tan(d*x + c)^10 + 70*tan(d*x + c)^9 + 240*tan(d*x + c)^7 + 2
10*I*tan(d*x + c)^6 + 252*tan(d*x + c)^5 + 420*I*tan(d*x + c)^4 + 315*I*ta
n(d*x + c)^2 - 210*tan(d*x + c))/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.09

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\cos\left(c+d\,x\right)^{10}\,84\mathrm{i}+128\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^9+64\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^7+48\,\sin\left(c+d\,x\right)\,\mathrm{co}}{210\,a^3\,d\cos\left(\right.}$$

input
```
int(1/(cos(c + d*x)^14*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
(40*cos(c + d*x)^3*sin(c + d*x) - 70*cos(c + d*x)*sin(c + d*x) + 48*cos(c
+ d*x)^5*sin(c + d*x) + 64*cos(c + d*x)^7*sin(c + d*x) + 128*cos(c + d*x)^
9*sin(c + d*x) - cos(c + d*x)^2*105i + cos(c + d*x)^10*84i + 21i)/(210*a^3
*d*cos(c + d*x)^10)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.81

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{-128\cos\left(dx+c\right)\sin\left(dx+c\right)^9+576\cos\left(dx+c\right)\sin\left(dx+c\right)^7-1008\cos\left(dx+c\right)\sin\left(dx+c\right)^5+840\,\mathrm{co}}{210a^3d\left(\sin\left(dx+\right.\right.}$$

input
```
int(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - 128*cos(c + d*x)*sin(c + d*x)**9 + 576*cos(c + d*x)*sin(c + d*x)**7 -
1008*cos(c + d*x)*sin(c + d*x)**5 + 840*cos(c + d*x)*sin(c + d*x)**3 - 210
*cos(c + d*x)*sin(c + d*x) - 42*sin(c + d*x)**10*i + 210*sin(c + d*x)**8*i
 - 420*sin(c + d*x)**6*i + 420*sin(c + d*x)**4*i - 315*sin(c + d*x)**2*i +
 126*i)/(210*a**3*d*(sin(c + d*x)**10 - 5*sin(c + d*x)**8 + 10*sin(c + d*x
)**6 - 10*sin(c + d*x)**4 + 5*sin(c + d*x)**2 - 1))
```

3.131 $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{2i(a-ia\tan(c+dx))^6}{3a^9d} - \frac{4i(a-ia\tan(c+dx))^7}{7a^{10}d} + \frac{i(a-ia\tan(c+dx))^8}{8a^{11}d}$$

output
```
2/3*I*(a-I*a*tan(d*x+c))^6/a^9/d-4/7*I*(a-I*a*tan(d*x+c))^7/a^10/d+1/8*I*(
a-I*a*tan(d*x+c))^8/a^11/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.56

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{(i+\tan(c+dx))^6\,(-37i+54\tan(c+dx)+21i\tan^2(c+dx))}{168a^3d}$$

input
```
Integrate[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I + Tan[c + d*x])^6*(-37*I + 54*Tan[c + d*x] + (21*I)*Tan[c + d*x]^2))/(
168*a^3*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^{12}}{(a+ia\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int (a-ia\tan(c+dx))^5(i\tan(c+dx)a+a)^2d(ia\tan(c+dx))}{a^{11}d}$$
$$\Big\downarrow \text{49}$$
$$-\frac{i\int \left((a-ia\tan(c+dx))^7-4a(a-ia\tan(c+dx))^6+4a^2(a-ia\tan(c+dx))^5\right)d(ia\tan(c+dx))}{a^{11}d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{i\left(-\frac{2}{3}a^2(a-ia\tan(c+dx))^6-\frac{1}{8}(a-ia\tan(c+dx))^8+\frac{4}{7}a(a-ia\tan(c+dx))^7\right)}{a^{11}d}$$

input
```
Int[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*((-2*a^2*(a - I*a*Tan[c + d*x])^6)/3 + (4*a*(a - I*a*Tan[c + d*x])^7
)/7 - (a - I*a*Tan[c + d*x])^8/8))/(a^11*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

method	result
risch	$\frac{32 i \left(28 \,{\mathrm e}^{4 i \left(d x +c \right)}+8 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{21 d \,a^{3} \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{8}}$
derivativedivides	$\frac{i \left(\frac{\tan \left(d x +c \right)^{8}}{8}-\frac{\tan \left(d x +c \right)^{6}}{6}+\frac{3 i \tan \left(d x +c \right)^{7}}{7}-\frac{5 \tan \left(d x +c \right)^{4}}{4}+i \tan \left(d x +c \right)^{5}-\frac{3 \tan \left(d x +c \right)^{2}}{2}+\frac{i \tan \left(d x +c \right)^{3}}{3}-i \tan \left(d x +c \right)\right)}{a^{3} d}$
default	$\frac{i \left(\frac{\tan \left(d x +c \right)^{8}}{8}-\frac{\tan \left(d x +c \right)^{6}}{6}+\frac{3 i \tan \left(d x +c \right)^{7}}{7}-\frac{5 \tan \left(d x +c \right)^{4}}{4}+i \tan \left(d x +c \right)^{5}-\frac{3 \tan \left(d x +c \right)^{2}}{2}+\frac{i \tan \left(d x +c \right)^{3}}{3}-i \tan \left(d x +c \right)\right)}{a^{3} d}$

input
```
int(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
32/21*I*(28*exp(4*I*(d*x+c))+8*exp(2*I*(d*x+c))+1)/d/a^3/(exp(2*I*(d*x+c))
+1)^8
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(64) = 128$.

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.87

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{32\left(-28i\,e^{(4i\,dx+4i\,c)} - 8i\,e^{(2i\,dx+2i\,c)} - i\right)}{21\left(a^3de^{(16i\,dx+16i\,c)} + 8\,a^3de^{(14i\,dx+14i\,c)} + 28\,a^3de^{(12i\,dx+12i\,c)} + 56\,a^3de^{(10i\,dx+10i\,c)} + 70\,a^3de^{(8i\,dx+8i\,c)} + \right.}$$

input
```
integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-32/21*(-28*I*e^(4*I*d*x + 4*I*c) - 8*I*e^(2*I*d*x + 2*I*c) - I)/(a^3*d*e^
(16*I*d*x + 16*I*c) + 8*a^3*d*e^(14*I*d*x + 14*I*c) + 28*a^3*d*e^(12*I*d*x
 + 12*I*c) + 56*a^3*d*e^(10*I*d*x + 10*I*c) + 70*a^3*d*e^(8*I*d*x + 8*I*c)
 + 56*a^3*d*e^(6*I*d*x + 6*I*c) + 28*a^3*d*e^(4*I*d*x + 4*I*c) + 8*a^3*d*e
^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^{12}(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**12/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sec(c + d*x)**12/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan
(c + d*x) + I), x)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-21i\tan(dx+c)^8 + 72\tan(dx+c)^7 + 28i\tan(dx+c)^6 + 168\tan(dx+c)^5 + 210i\tan(dx+c)^4 +}{168\,a^3d}$$

input
```
integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/168*(-21*I*tan(d*x + c)^8 + 72*tan(d*x + c)^7 + 28*I*tan(d*x + c)^6 + 1
68*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 + 56*tan(d*x + c)^3 + 252*I*tan(d
*x + c)^2 - 168*tan(d*x + c))/(a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-21i\tan(dx+c)^8 + 72\tan(dx+c)^7 + 28i\tan(dx+c)^6 + 168\tan(dx+c)^5 + 210i\tan(dx+c)^4 +}{168\,a^3d}$$

input
```
integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/168*(-21*I*tan(d*x + c)^8 + 72*tan(d*x + c)^7 + 28*I*tan(d*x + c)^6 + 1
68*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 + 56*tan(d*x + c)^3 + 252*I*tan(d
*x + c)^2 - 168*tan(d*x + c))/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.26

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\cos(c+d\,x)^8\,91\mathrm{i}+128\,\sin(c+d\,x)\,\cos(c+d\,x)^7+64\,\sin(c+d\,x)\,\cos(c+d\,x)^5+48\,\sin(c+d\,x)\,\cos}{168\,a^3\,d\cos(c+d\,x)^8}$$

input
```
int(1/(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
(48*cos(c + d*x)^3*sin(c + d*x) - 72*cos(c + d*x)*sin(c + d*x) + 64*cos(c
+ d*x)^5*sin(c + d*x) + 128*cos(c + d*x)^7*sin(c + d*x) - cos(c + d*x)^2*1
12i + cos(c + d*x)^8*91i + 21i)/(168*a^3*d*cos(c + d*x)^8)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.95

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{-128\cos(dx+c)\sin(dx+c)^7+448\cos(dx+c)\sin(dx+c)^5-560\cos(dx+c)\sin(dx+c)^3+168\cos}{168a^3d\left(\sin(dx+c)^8-4\sin(dx+\right.}$$

input
```
int(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - 128*cos(c + d*x)*sin(c + d*x)**7 + 448*cos(c + d*x)*sin(c + d*x)**5 -
560*cos(c + d*x)*sin(c + d*x)**3 + 168*cos(c + d*x)*sin(c + d*x) - 35*sin(
c + d*x)**8*i + 140*sin(c + d*x)**6*i - 210*sin(c + d*x)**4*i + 252*sin(c
+ d*x)**2*i - 126*i)/(168*a**3*d*(sin(c + d*x)**8 - 4*sin(c + d*x)**6 + 6*
sin(c + d*x)**4 - 4*sin(c + d*x)**2 + 1))
```

3.132 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{2i(a-ia\tan(c+dx))^5}{5a^8d} - \frac{i(a-ia\tan(c+dx))^6}{6a^9d}$$

output
```
2/5*I*(a-I*a*tan(d*x+c))^5/a^8/d-1/6*I*(a-I*a*tan(d*x+c))^6/a^9/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{(7+5i\tan(c+dx))(i+\tan(c+dx))^5}{30a^3d}$$

input
```
Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((7 + (5*I)*Tan[c + d*x])*(I + Tan[c + d*x])^5)/(30*a^3*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^{10}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)d(ia\tan(c+dx))}{a^9d}$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{i\int \left(2a(a-ia\tan(c+dx))^4-(a-ia\tan(c+dx))^5\right)d(ia\tan(c+dx))}{a^9d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{i\left(\frac{1}{6}(a-ia\tan(c+dx))^6-\frac{2}{5}a(a-ia\tan(c+dx))^5\right)}{a^9d}$$

input
```
Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*((-2*a*(a - I*a*Tan[c + d*x])^5)/5 + (a - I*a*Tan[c + d*x])^6/6))/(a
^9*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{32 i \left(6 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{15 d \,a^{3} \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{6}}$	36
derivativedivides	$-\frac{i \left(i \tan \left(d x +c \right)-\frac{\tan \left(d x +c \right)^{6}}{6}-\frac{3 i \tan \left(d x +c \right)^{5}}{5}+\frac{\tan \left(d x +c \right)^{4}}{2}-\frac{2 i \tan \left(d x +c \right)^{3}}{3}+\frac{3 \tan \left(d x +c \right)^{2}}{2}\right)}{a^{3} d}$	72
default	$-\frac{i \left(i \tan \left(d x +c \right)-\frac{\tan \left(d x +c \right)^{6}}{6}-\frac{3 i \tan \left(d x +c \right)^{5}}{5}+\frac{\tan \left(d x +c \right)^{4}}{2}-\frac{2 i \tan \left(d x +c \right)^{3}}{3}+\frac{3 \tan \left(d x +c \right)^{2}}{2}\right)}{a^{3} d}$	72

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
32/15*I*(6*exp(2*I*(d*x+c))+1)/d/a^3/(exp(2*I*(d*x+c))+1)^6
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(43) = 86$.

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx =$$
$$-\frac{32\left(-6i\,e^{(2i\,dx+2i\,c)}-i\right)}{15\left(a^3de^{(12i\,dx+12i\,c)}+6\,a^3de^{(10i\,dx+10i\,c)}+15\,a^3de^{(8i\,dx+8i\,c)}+20\,a^3de^{(6i\,dx+6i\,c)}+15\,a^3de^{(4i\,dx+4i\,c)}+6\,a^3 \right.}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-32/15*(-6*I*e^(2*I*d*x + 2*I*c) - I)/(a^3*d*e^(12*I*d*x + 12*I*c) + 6*a^3
*d*e^(10*I*d*x + 10*I*c) + 15*a^3*d*e^(8*I*d*x + 8*I*c) + 20*a^3*d*e^(6*I*
d*x + 6*I*c) + 15*a^3*d*e^(4*I*d*x + 4*I*c) + 6*a^3*d*e^(2*I*d*x + 2*I*c)
+ a^3*d)
```

Sympy [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^{10}(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sec(c + d*x)**10/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan
(c + d*x) + I), x)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{5i\,\tan\left(dx+c\right)^{6} - 18\,\tan\left(dx+c\right)^{5} - 15i\,\tan\left(dx+c\right)^{4} - 20\,\tan\left(dx+c\right)^{3} - 45i\,\tan\left(dx+c\right)^{2} + 30\,\tan}{30\,a^{3}d}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/30*(5*I*tan(d*x + c)^6 - 18*tan(d*x + c)^5 - 15*I*tan(d*x + c)^4 - 20*ta
n(d*x + c)^3 - 45*I*tan(d*x + c)^2 + 30*tan(d*x + c))/(a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx =$$
$$-\frac{-5i\,\tan\left(dx+c\right)^{6} + 18\,\tan\left(dx+c\right)^{5} + 15i\,\tan\left(dx+c\right)^{4} + 20\,\tan\left(dx+c\right)^{3} + 45i\,\tan\left(dx+c\right)^{2} - 30}{30\,a^{3}d}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/30*(-5*I*tan(d*x + c)^6 + 18*tan(d*x + c)^5 + 15*I*tan(d*x + c)^4 + 20*
tan(d*x + c)^3 + 45*I*tan(d*x + c)^2 - 30*tan(d*x + c))/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.07

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\sin\left(c+d\,x\right)\,\left(-30\cos\left(c+d\,x\right)^5+\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)\,45\mathrm{i}+20\cos\left(c+d\,x\right)^3\sin\left(c+d\,x\right)^2+\mathrm{c}\right.}{30\,a^3\,d\cos\left(c+d\,x\right)^6}$$

input
```
int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
-(sin(c + d*x)*(18*cos(c + d*x)*sin(c + d*x)^4 + cos(c + d*x)^4*sin(c + d*
x)*45i - 30*cos(c + d*x)^5 - sin(c + d*x)^5*5i + cos(c + d*x)^2*sin(c + d*
x)^3*15i + 20*cos(c + d*x)^3*sin(c + d*x)^2))/(30*a^3*d*cos(c + d*x)^6)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.24

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{-32\cos\left(dx+c\right)\sin\left(dx+c\right)^5+80\cos\left(dx+c\right)\sin\left(dx+c\right)^3-30\cos\left(dx+c\right)\sin\left(dx+c\right)-5\sin\left(dx+\right.}{30a^3d\left(\sin\left(dx+c\right)^6-3\sin\left(dx+c\right)^4+3\sin\left(dx+c\right)^2-\right.}$$

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - 32*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(c + d*x)**3 - 30
*cos(c + d*x)*sin(c + d*x) - 5*sin(c + d*x)**6*i + 15*sin(c + d*x)**4*i -
45*sin(c + d*x)**2*i + 30*i)/(30*a**3*d*(sin(c + d*x)**6 - 3*sin(c + d*x)*
*4 + 3*sin(c + d*x)**2 - 1))
```

3.133 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3} dx = \frac{i(a-ia\tan(c+dx))^4}{4a^7d}$$

output
```
1/4*I*(a-I*a*tan(d*x+c))^4/a^7/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3} dx = \frac{\tan(c+dx)\left(4-6i\tan(c+dx)-4\tan^2(c+dx)+i\tan^3(c+dx)\right)}{4a^3d}$$

input
```
Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(Tan[c + d*x]*(4 - (6*I)*Tan[c + d*x] - 4*Tan[c + d*x]^2 + I*Tan[c + d*x]^
3))/(4*a^3*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^8}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^3d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{i(a-ia\tan(c+dx))^4}{4a^7d}$$

input
```
Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/4)*(a - I*a*Tan[c + d*x])^4)/(a^7*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{i(\tan(dx+c)+i)^4}{4a^3d}$	21
default	$\frac{i(\tan(dx+c)+i)^4}{4a^3d}$	21
risch	$\frac{4i}{d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}$	23

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*I/a^3/d*(tan(d*x+c)+I)^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(21) = 42.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.56

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{4i}{a^3de^{(8i\,dx+8i\,c)}+4\,a^3de^{(6i\,dx+6i\,c)}+6\,a^3de^{(4i\,dx+4i\,c)}+4\,a^3de^{(2i\,dx+2i\,c)}+a^3d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
4*I/(a^3*d*e^(8*I*d*x + 8*I*c) + 4*a^3*d*e^(6*I*d*x + 6*I*c) + 6*a^3*d*e^(
4*I*d*x + 4*I*c) + 4*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^8(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sec(c + d*x)**8/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(
c + d*x) + I), x)/a**3
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(21) = 42$.

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-i\tan(dx+c)^4+4\tan(dx+c)^3+6i\tan(dx+c)^2-4\tan(dx+c)}{4a^3d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/4*(-I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 4*tan(d*
x + c))/(a^3*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(21) = 42$.

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-i\tan(dx+c)^4 + 4\tan(dx+c)^3 + 6i\tan(dx+c)^2 - 4\tan(dx+c)}{4a^3d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/4*(-I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 4*tan(d*
x + c))/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.85

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\sin(c+d\,x)\,\left(-4\cos(c+d\,x)^3 + \cos(c+d\,x)^2\,\sin(c+d\,x)\,6\mathrm{i} + 4\,\cos(c+d\,x)\,\sin(c+d\,x)^2 - \sin(c\right.}{4\,a^3\,d\cos(c+d\,x)^4}$$

input
```
int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
-(sin(c + d*x)*(4*cos(c + d*x)*sin(c + d*x)^2 + cos(c + d*x)^2*sin(c + d*x
)*6i - 4*cos(c + d*x)^3 - sin(c + d*x)^3*1i))/(4*a^3*d*cos(c + d*x)^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.15

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{-8\cos(dx+c)\sin(dx+c)^3+4\cos(dx+c)\sin(dx+c)+\sin(dx+c)^4 i+6\sin(dx+c)^2 i-6i}{4a^3d\left(\sin(dx+c)^4-2\sin(dx+c)^2+1\right)}$$

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - 8*cos(c + d*x)*sin(c + d*x)**3 + 4*cos(c + d*x)*sin(c + d*x) + sin(c +
 d*x)**4*i + 6*sin(c + d*x)**2*i - 6*i)/(4*a**3*d*(sin(c + d*x)**4 - 2*sin
(c + d*x)**2 + 1))
```

3.134 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=\frac{4x}{a^3}+\frac{4i\log(\cos(c+dx))}{a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{i\tan^2(c+dx)}{2a^3d}$$

output
```
4*x/a^3+4*I*ln(cos(d*x+c))/a^3/d-3*tan(d*x+c)/a^3/d+1/2*I*tan(d*x+c)^2/a^3
/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=\frac{-8i\log(i-\tan(c+dx))-6\tan(c+dx)+i\tan^2(c+dx)}{2a^3d}$$

input
```
Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-8*I)*Log[I - Tan[c + d*x]] - 6*Tan[c + d*x] + I*Tan[c + d*x]^2)/(2*a^3*
d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{(a-ia\tan(c+dx))^2}{i\tan(c+dx)a+a}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(\frac{4a^2}{i\tan(c+dx)a+a}+i\tan(c+dx)a-3a\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{1}{2}a^2\tan^2(c+dx)-3ia^2\tan(c+dx)+4a^2\log(a+ia\tan(c+dx))\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*(4*a^2*Log[a + I*a*Tan[c + d*x]] - (3*I)*a^2*Tan[c + d*x] - (a^2*Tan
[c + d*x]^2)/2))/(a^5*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$-\frac{3\tan(dx+c)}{a^3d}+\frac{i\tan(dx+c)^2}{2a^3d}+\frac{4\arctan(\tan(dx+c))}{a^3d}-\frac{2i\ln\left(1+\tan(dx+c)^2\right)}{a^3d}$	68
default	$-\frac{3\tan(dx+c)}{a^3d}+\frac{i\tan(dx+c)^2}{2a^3d}+\frac{4\arctan(\tan(dx+c))}{a^3d}-\frac{2i\ln\left(1+\tan(dx+c)^2\right)}{a^3d}$	68
risch	$\frac{8x}{a^3}+\frac{8c}{a^3d}-\frac{2i\left(2\,e^{2i(dx+c)}+3\right)}{d\,a^3\left(e^{2i(dx+c)}+1\right)^2}+\frac{4i\ln\left(e^{2i(dx+c)}+1\right)}{a^3d}$	73

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-3*tan(d*x+c)/a^3/d+1/2*I*tan(d*x+c)^2/a^3/d+4/a^3/d*arctan(tan(d*x+c))-2*
I/a^3/d*ln(1+tan(d*x+c)^2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(52) = 104$.

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.95

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{2\left(4\,dxe^{(4i\,dx+4i\,c)}+4\,dx+2\,(4\,dx-i)e^{(2i\,dx+2i\,c)}-2\left(-i\,e^{(4i\,dx+4i\,c)}-2i\,e^{(2i\,dx+2i\,c)}-i\right)\log\left(e^{(2i\,dx+2i\,c)}+\right.\right.}{a^3de^{(4i\,dx+4i\,c)}+2\,a^3de^{(2i\,dx+2i\,c)}+a^3d}$$

input

```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output

```
2*(4*d*x*e^(4*I*d*x + 4*I*c) + 4*d*x + 2*(4*d*x - I)*e^(2*I*d*x + 2*I*c) -
 2*(-I*e^(4*I*d*x + 4*I*c) - 2*I*e^(2*I*d*x + 2*I*c) - I)*log(e^(2*I*d*x +
 2*I*c) + 1) - 3*I)/(a^3*d*e^(4*I*d*x + 4*I*c) + 2*a^3*d*e^(2*I*d*x + 2*I*
c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^6(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input

```
integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**3,x)
```

output

```
I*Integral(sec(c + d*x)**6/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(
c + d*x) + I), x)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\frac{i\tan(dx+c)^2-6\tan(dx+c)}{a^3} - \frac{8i\log(i\tan(dx+c)+1)}{a^3}}{2\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*((I*tan(d*x + c)^2 - 6*tan(d*x + c))/a^3 - 8*I*log(I*tan(d*x + c) + 1)
/a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{4i\log\left(\tan\left(dx+c\right)-i\right)}{a^3d} - \frac{-i\,a^3d\tan\left(dx+c\right)^2+6\,a^3d\tan\left(dx+c\right)}{2\,a^6d^2}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-4*I*log(tan(d*x + c) - I)/(a^3*d) - 1/2*(-I*a^3*d*tan(d*x + c)^2 + 6*a^3*
d*tan(d*x + c))/(a^6*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,8\mathrm{i}+6\tan(c+d\,x)-\tan(c+d\,x)^2\,1\mathrm{i}}{2\,a^3\,d}$$

input
```
int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
-(log(tan(c + d*x) - 1i)*8i + 6*tan(c + d*x) - tan(c + d*x)^2*1i)/(2*a^3*d
)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 299, normalized size of antiderivative = 5.16

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{18\cos(dx+c)\sin(dx+c) - 8\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6 - 6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 i - 15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4 + 20\tan\left(\frac{dx}{2}+\right.\right.}{}$$

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x)
```

output
```
(18*cos(c + d*x)*sin(c + d*x) - 8*log(tan((c + d*x)/2)**6 - 6*tan((c + d*x
)/2)**5*i - 15*tan((c + d*x)/2)**4 + 20*tan((c + d*x)/2)**3*i + 15*tan((c
+ d*x)/2)**2 - 6*tan((c + d*x)/2)*i - 1)*sin(c + d*x)**2*i + 8*log(tan((c
+ d*x)/2)**6 - 6*tan((c + d*x)/2)**5*i - 15*tan((c + d*x)/2)**4 + 20*tan((
c + d*x)/2)**3*i + 15*tan((c + d*x)/2)**2 - 6*tan((c + d*x)/2)*i - 1)*i +
24*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*i - 24*log(tan((c + d*x)/2) -
 1)*i + 24*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*i - 24*log(tan((c + d
*x)/2) + 1)*i + 3*sin(c + d*x)**2*i - 6*i)/(6*a**3*d*(sin(c + d*x)**2 - 1)
)
```

3.135 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 48

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{x}{a^3} - \frac{i\log(\cos(c+dx))}{a^3d} + \frac{2i}{a^3d(1+i\tan(c+dx))}$$

output
```
-x/a^3-I*ln(cos(d*x+c))/a^3/d+2*I/a^3/d/(1+I*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{i\left(-\log(i-\tan(c+dx)) - \frac{2a}{a+ia\tan(c+dx)}\right)}{a^3d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*(-Log[I - Tan[c + d*x]] - (2*a)/(a + I*a*Tan[c + d*x])))/(a^3*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^4}{(a+ia\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int \frac{a-ia\tan(c+dx)}{(i\tan(c+dx)a+a)^2}d(ia\tan(c+dx))}{a^3d}$$
$$\Big\downarrow \text{49}$$
$$-\frac{i\int \left(\frac{2a}{(i\tan(c+dx)a+a)^2}+\frac{1}{-i\tan(c+dx)a-a}\right)d(ia\tan(c+dx))}{a^3d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{i\left(-\frac{2a}{a+ia\tan(c+dx)}-\log(a+ia\tan(c+dx))\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*(-Log[a + I*a*Tan[c + d*x]] - (2*a)/(a + I*a*Tan[c + d*x])))/(a^3*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{2}{a^3 d(-i+\tan(dx+c))}+\frac{i\ln\left(1+\tan(dx+c)^2\right)}{2a^3 d}-\frac{\arctan(\tan(dx+c))}{a^3 d}$	56
default	$\frac{2}{a^3 d(-i+\tan(dx+c))}+\frac{i\ln\left(1+\tan(dx+c)^2\right)}{2a^3 d}-\frac{\arctan(\tan(dx+c))}{a^3 d}$	56
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{a^3 d}-\frac{2x}{a^3}-\frac{2c}{a^3 d}-\frac{i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^3 d}$	56

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
2/a^3/d/(-I+tan(d*x+c))+1/2*I/a^3/d*ln(1+tan(d*x+c)^2)-1/a^3/d*arctan(tan(
d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= -\frac{\left(2\,dxe^{(2i\,dx+2i\,c)}+i\,e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\right)e^{(-2i\,dx-2i\,c)}}{a^3d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-(2*d*x*e^(2*I*d*x + 2*I*c) + I*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c
) + 1) - I)*e^(-2*I*d*x - 2*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^4(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sec(c + d*x)**4/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(
c + d*x) + I), x)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\frac{4(-i\tan(dx+c)-1)}{2i\,a^3\tan(dx+c)^2+4\,a^3\tan(dx+c)-2i\,a^3}-\frac{i\log(i\tan(dx+c)+1)}{a^3}}{d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-(4*(-I*tan(d*x + c) - 1)/(2*I*a^3*tan(d*x + c)^2 + 4*a^3*tan(d*x + c) - 2
*I*a^3) - I*log(I*tan(d*x + c) + 1)/a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\log(\tan(dx+c)-i)}{a^3d} + \frac{2}{a^3d(\tan(dx+c)-i)}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
I*log(tan(d*x + c) - I)/(a^3*d) + 2/(a^3*d*(tan(d*x + c) - I))
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\ln(\tan(c+d\,x)-\mathrm{i})\;1\mathrm{i}}{a^3\,d} + \frac{2\mathrm{i}}{a^3\,d\,\left(1+\tan(c+d\,x)\;1\mathrm{i}\right)}$$

input
```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
(log(tan(c + d*x) - 1i)*1i)/(a^3*d) + 2i/(a^3*d*(tan(c + d*x)*1i + 1))
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x)
```

output

```
( - 408*int(cos(c + d*x)/(4*cos(c + d*x)*sin(c + d*x)**3*i - 3*cos(c + d*x
)*sin(c + d*x)*i - 4*sin(c + d*x)**4 + 5*sin(c + d*x)**2 - 1),x)*d + 432*i
nt(cos(c + d*x)/(4*cos(c + d*x)*sin(c + d*x)**3 - 3*cos(c + d*x)*sin(c + d
*x) + 4*sin(c + d*x)**4*i - 5*sin(c + d*x)**2*i + i),x)*d*i + 24*int(cos(c
 + d*x)/(4*cos(c + d*x)*sin(c + d*x)**2*i - cos(c + d*x)*i - 4*sin(c + d*x
)**3 + 3*sin(c + d*x)),x)*d*i - 1368*int(sin(c + d*x)**4/(4*cos(c + d*x)*s
in(c + d*x)**3*i - 3*cos(c + d*x)*sin(c + d*x)*i - 4*sin(c + d*x)**4 + 5*s
in(c + d*x)**2 - 1),x)*d + 1440*int(sin(c + d*x)**4/(4*cos(c + d*x)*sin(c
+ d*x)**3 - 3*cos(c + d*x)*sin(c + d*x) + 4*sin(c + d*x)**4*i - 5*sin(c +
d*x)**2*i + i),x)*d*i + 1632*int(sin(c + d*x)**3/(4*cos(c + d*x)*sin(c + d
*x)**3*i - 3*cos(c + d*x)*sin(c + d*x)*i - 4*sin(c + d*x)**4 + 5*sin(c + d
*x)**2 - 1),x)*d*i + 1632*int(sin(c + d*x)**3/(4*cos(c + d*x)*sin(c + d*x)
**3 - 3*cos(c + d*x)*sin(c + d*x) + 4*sin(c + d*x)**4*i - 5*sin(c + d*x)**
2*i + i),x)*d + 168*int(sin(c + d*x)**3/(4*cos(c + d*x)*sin(c + d*x)**2*i
- cos(c + d*x)*i - 4*sin(c + d*x)**3 + 3*sin(c + d*x)),x)*d + 1836*int(sin
(c + d*x)**2/(4*cos(c + d*x)*sin(c + d*x)**3*i - 3*cos(c + d*x)*sin(c + d*
x)*i - 4*sin(c + d*x)**4 + 5*sin(c + d*x)**2 - 1),x)*d - 1944*int(sin(c +
d*x)**2/(4*cos(c + d*x)*sin(c + d*x)**3 - 3*cos(c + d*x)*sin(c + d*x) + 4*
sin(c + d*x)**4*i - 5*sin(c + d*x)**2*i + i),x)*d*i - 1224*int(sin(c + d*x
)/(4*cos(c + d*x)*sin(c + d*x)**3*i - 3*cos(c + d*x)*sin(c + d*x)*i - 4...
```

3.136 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i}{2ad(a+ia\tan(c+dx))^2}$$

output
```
1/2*I/a/d/(a+I*a*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{i}{2a^3d(-i+\tan(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(-1/2*I)/(a^3*d*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^2}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{1}{(i\tan(c+dx)a+a)^3}d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$\frac{i}{2ad(a+ia\tan(c+dx))^2}$$

input
```
Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(I/2)/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{i}{2ad(a+ia\tan(dx+c))^2}$	24
default	$\frac{i}{2ad(a+ia\tan(dx+c))^2}$	24
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{4a^3d}+\frac{i\mathrm{e}^{-4i(dx+c)}}{8a^3d}$	38

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*I/a/d/(a+I*a*tan(d*x+c))^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=\frac{\left(2i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{8\,a^3d}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/8*(2*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x - 4*I*c)/(a^3*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(19) = 38$.

Time = 0.88 (sec) , antiderivative size = 153, normalized size of antiderivative = 5.67

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \begin{cases} -\frac{i\tan(c+dx)\sec^2(c+dx)}{8a^3d\tan^3(c+dx)-24ia^3d\tan^2(c+dx)-24a^3d\tan(c+dx)+8ia^3d} - \frac{3\sec^2(c+dx)}{8a^3d\tan^3(c+dx)-24ia^3d\tan^2(c+dx)-24a^3d\tan(c+dx)+8ia^3d} \\ \frac{x\sec^2(c)}{(ia\tan(c)+a)^3} \end{cases}$$

input
```
integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise((-I*tan(c + d*x)*sec(c + d*x)**2/(8*a**3*d*tan(c + d*x)**3 - 24*
I*a**3*d*tan(c + d*x)**2 - 24*a**3*d*tan(c + d*x) + 8*I*a**3*d) - 3*sec(c
+ d*x)**2/(8*a**3*d*tan(c + d*x)**3 - 24*I*a**3*d*tan(c + d*x)**2 - 24*a**
3*d*tan(c + d*x) + 8*I*a**3*d), Ne(d, 0)), (x*sec(c)**2/(I*a*tan(c) + a)**
3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i}{2\,(i\,a\tan(dx+c)+a)^2ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*I/((I*a*tan(d*x + c) + a)^2*a*d)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{i}{2\,a^3d(\tan\left(dx+c\right)-i)^2}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*I/(a^3*d*(tan(d*x + c) - I)^2)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{1\mathrm{i}}{2\,a^3\,d\,\left(\tan\left(c+d\,x\right)-\mathrm{i}\right)^2}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
-1i/(2*a^3*d*(tan(c + d*x) - 1i)^2)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{12\left(\int \frac{\sin(dx+c)^3}{4\cos(dx+c)\sin(dx+c)^2i-\cos(dx+c)i-4\sin(dx+c)^3+3\sin(dx+c)}dx\right)d-9\left(\int \frac{\sin(dx+c)}{4\cos(dx+c)\sin(dx+c)^2i-\cos(dx+c)i-4\sin(dx+c}\right.}{}$$

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x)
```

output
```
(12*int(sin(c + d*x)**3/(4*cos(c + d*x)*sin(c + d*x)**2*i - cos(c + d*x)*i
 - 4*sin(c + d*x)**3 + 3*sin(c + d*x)),x)*d - 9*int(sin(c + d*x)/(4*cos(c
+ d*x)*sin(c + d*x)**2*i - cos(c + d*x)*i - 4*sin(c + d*x)**3 + 3*sin(c +
d*x)),x)*d - 12*int((cos(c + d*x)*sin(c + d*x)**2)/(4*cos(c + d*x)*sin(c +
 d*x)**2 - cos(c + d*x) + 4*sin(c + d*x)**3*i - 3*sin(c + d*x)*i),x)*d - 2
*log(tan((c + d*x)/2)**6 - 6*tan((c + d*x)/2)**5*i - 15*tan((c + d*x)/2)**
4 + 20*tan((c + d*x)/2)**3*i + 15*tan((c + d*x)/2)**2 - 6*tan((c + d*x)/2)
*i - 1)*i + 9*log(tan((c + d*x)/2)**2 + 1)*i - log(tan((c + d*x)/2)**6*i +
 6*tan((c + d*x)/2)**5 - 15*tan((c + d*x)/2)**4*i - 20*tan((c + d*x)/2)**3
 + 15*tan((c + d*x)/2)**2*i + 6*tan((c + d*x)/2) - i)*i - 6*d*x)/(3*a**3*d
)
```

3.137 $\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = \frac{x}{8a^3}+\frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}$$

output
```
1/8*x/a^3+1/6*I/d/(a+I*a*tan(d*x+c))^3+1/8*I/a/d/(a+I*a*tan(d*x+c))^2+1/8*
I/d/(a^3+I*a^3*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = \frac{-10-9i\tan(c+dx)+3\tan^2(c+dx)+3\arctan(\tan(c+dx))(-i+\tan(c+dx))^3}{24a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(-3),x]
```

output
```
(-10 - (9*I)*Tan[c + d*x] + 3*Tan[c + d*x]^2 + 3*ArcTan[Tan[c + d*x]]*(-I
+ Tan[c + d*x])^3)/(24*a^3*d*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 3960, 3042, 3960, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a+ia\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3960}$$
$$\frac{\int \frac{1}{(i\tan(c+dx)a+a)^2}dx}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \frac{1}{(i\tan(c+dx)a+a)^2}dx}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}$$
$$\Big\downarrow \text{3960}$$
$$\frac{\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}$$
$$\Big\downarrow \text{3960}$$
$$\frac{\frac{\frac{\int 1dx}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow 24$$

$$\frac{\frac{\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(-3),x]
```

output
```
(I/6)/(d*(a + I*a*Tan[c + d*x])^3) + ((I/4)/(d*(a + I*a*Tan[c + d*x])^2) +
 (x/(2*a) + (I/2)/(d*(a + I*a*Tan[c + d*x])))/(2*a))/(2*a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{x}{8a^3}+\frac{3i\mathrm{e}^{-2i(dx+c)}}{16a^3d}+\frac{3i\mathrm{e}^{-4i(dx+c)}}{32a^3d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{48a^3d}$	62
derivativedivides	$\frac{\arctan(\tan(dx+c))}{8a^3d}-\frac{i}{8d\,a^3(-i+\tan(dx+c))^2}-\frac{1}{6d\,a^3(-i+\tan(dx+c))^3}+\frac{1}{8a^3d(-i+\tan(dx+c))}$	75
default	$\frac{\arctan(\tan(dx+c))}{8a^3d}-\frac{i}{8d\,a^3(-i+\tan(dx+c))^2}-\frac{1}{6d\,a^3(-i+\tan(dx+c))^3}+\frac{1}{8a^3d(-i+\tan(dx+c))}$	75
norman	$\frac{\frac{x}{8a}+\frac{7\tan(dx+c)}{8ad}+\frac{\tan(dx+c)^3}{3ad}+\frac{\tan(dx+c)^5}{8ad}+\frac{3x\tan(dx+c)^2}{8a}+\frac{3x\tan(dx+c)^4}{8a}+\frac{x\tan(dx+c)^6}{8a}+\frac{5i}{12ad}-\frac{i\tan(dx+c)^2}{4ad}}{a^2\left(1+\tan(dx+c)^2\right)^3}$	138

input
```
int(1/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*x/a^3+3/16*I/a^3/d*exp(-2*I*(d*x+c))+3/32*I/a^3/d*exp(-4*I*(d*x+c))+1/
48*I/a^3/d*exp(-6*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(12\,dxe^{(6i\,dx+6i\,c)}+18i\,e^{(4i\,dx+4i\,c)}+9i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^3d}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/96*(12*d*x*e^(6*I*d*x + 6*I*c) + 18*I*e^(4*I*d*x + 4*I*c) + 9*I*e^(2*I*d
*x + 2*I*c) + 2*I)*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.76

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = \begin{cases} \frac{(4608ia^6d^2e^{10ic}e^{-2idx}+2304ia^6d^2e^{8ic}e^{-4idx}+512ia^6d^2e^{6ic}e^{-6idx})e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic}\neq 0 \\ x\left(\frac{(e^{6ic}+3e^{4ic}+3e^{2ic}+1)e^{-6ic}}{8a^3}-\frac{1}{8a^3}\right) & \text{otherwise} \end{cases} + \frac{x}{8a^3}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise(((4608*I*a**6*d**2*exp(10*I*c)*exp(-2*I*d*x) + 2304*I*a**6*d**2*
exp(8*I*c)*exp(-4*I*d*x) + 512*I*a**6*d**2*exp(6*I*c)*exp(-6*I*d*x))*exp(-
12*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*((exp(6*I*c)
+ 3*exp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-6*I*c)/(8*a**3) - 1/(8*a**3)), Tru
e)) + x/(8*a**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\log\left(\tan\left(dx+c\right)+i\right)}{16\,a^3d} - \frac{i\log\left(\tan\left(dx+c\right)-i\right)}{16\,a^3d} - \frac{i\left(3i\tan\left(dx+c\right)^2+9\tan\left(dx+c\right)-10i\right)}{24\,a^3d(\tan\left(dx+c\right)-i)^3}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/16*I*log(tan(d*x + c) + I)/(a^3*d) - 1/16*I*log(tan(d*x + c) - I)/(a^3*d
) - 1/24*I*(3*I*tan(d*x + c)^2 + 9*tan(d*x + c) - 10*I)/(a^3*d*(tan(d*x +
c) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.57

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = \frac{x}{8\,a^3} - \frac{\frac{\tan(c+d\,x)^2\,1\mathrm{i}}{8} + \frac{3\tan(c+d\,x)}{8} - \frac{5}{12}\mathrm{i}}{a^3\,d\,(1+\tan\left(c+d\,x\right)\,1\mathrm{i})^3}$$

input
```
int(1/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
x/(8*a^3) - ((3*tan(c + d*x))/8 + (tan(c + d*x)^2*1i)/8 - 5i/12)/(a^3*d*(t
an(c + d*x)*1i + 1)^3)
```

Reduce [F]

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\int \frac{1}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx}{a^3}$$

input
```
int(1/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - int(1/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x
))/a**3
```

3.138 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{5x}{32a^3}+\frac{ia}{16d(a+ia\tan(c+dx))^4}+\frac{i}{12d(a+ia\tan(c+dx))^3}-\frac{i}{32d\left(a^3-ia^3\tan(c+dx)\right)}+\frac{3ia^3}{32d\left(a^3+ia^3\tan(c+dx)\right)^2}+\frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}$$

output
```
5/32*x/a^3+1/16*I*a/d/(a+I*a*tan(d*x+c))^4+1/12*I/d/(a+I*a*tan(d*x+c))^3-1
/32*I/d/(a^3-I*a^3*tan(d*x+c))+3/32*I*a^3/d/(a^3+I*a^3*tan(d*x+c))^2+1/8*I
/d/(a^3+I*a^3*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{ia^3\left(\frac{5i\arctan(\tan(c+dx))}{32a^6}+\frac{1}{32a^5(a-ia\tan(c+dx))}-\frac{1}{16a^2(a+ia\tan(c+dx))^4}-\frac{1}{12a^3(a+ia\tan(c+dx))^3}-\frac{3}{32a^4(a+ia\tan(c+dx)}\right.}{d}$$

input
```
Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*a^3*((((5*I)/32)*ArcTan[Tan[c + d*x]])/a^6 + 1/(32*a^5*(a - I*a*Tan[
c + d*x])) - 1/(16*a^2*(a + I*a*Tan[c + d*x])^4) - 1/(12*a^3*(a + I*a*Tan[
c + d*x])^3) - 3/(32*a^4*(a + I*a*Tan[c + d*x])^2) - 1/(8*a^5*(a + I*a*Tan
[c + d*x]))))/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^5}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^3\int \left(\frac{1}{32a^5(a-ia\tan(c+dx))^2}+\frac{1}{8a^5(i\tan(c+dx)a+a)^2}+\frac{3}{16a^4(i\tan(c+dx)a+a)^3}+\frac{1}{4a^3(i\tan(c+dx)a+a)^4}+\frac{1}{4a^2(i\tan(c+dx)a+a)^5}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^3\left(\frac{5i\arctan(\tan(c+dx))}{32a^6}+\frac{1}{32a^5(a-ia\tan(c+dx))}-\frac{1}{8a^5(a+ia\tan(c+dx))}-\frac{3}{32a^4(a+ia\tan(c+dx))^2}-\frac{1}{12a^3(a+ia\tan(c+dx))^3}-\right.}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*a^3*((((5*I)/32)*ArcTan[Tan[c + d*x]])/a^6 + 1/(32*a^5*(a - I*a*Tan[
c + d*x])) - 1/(16*a^2*(a + I*a*Tan[c + d*x])^4) - 1/(12*a^3*(a + I*a*Tan[
c + d*x])^3) - 3/(32*a^4*(a + I*a*Tan[c + d*x])^2) - 1/(8*a^5*(a + I*a*Tan
[c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.67

method	result
risch	$\frac{5x}{32a^3}+\frac{5i\mathrm{e}^{-4i(dx+c)}}{64a^3d}+\frac{5i\mathrm{e}^{-6i(dx+c)}}{192a^3d}+\frac{i\mathrm{e}^{-8i(dx+c)}}{256a^3d}+\frac{9i\cos(2dx+2c)}{64a^3d}+\frac{11\sin(2dx+2c)}{64a^3d}$
derivativedivides	$\frac{-\frac{5i\ln(-i+\tan(dx+c))}{64}+\frac{i}{16(-i+\tan(dx+c))^4}-\frac{3i}{32(-i+\tan(dx+c))^2}-\frac{1}{12(-i+\tan(dx+c))^3}+\frac{1}{-8i+8\tan(dx+c)}+\frac{5i\ln(\tan(dx+c)-}{64}}{d\,a^3}$
default	$\frac{-\frac{5i\ln(-i+\tan(dx+c))}{64}+\frac{i}{16(-i+\tan(dx+c))^4}-\frac{3i}{32(-i+\tan(dx+c))^2}-\frac{1}{12(-i+\tan(dx+c))^3}+\frac{1}{-8i+8\tan(dx+c)}+\frac{5i\ln(\tan(dx+c)-}{64}}{d\,a^3}$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
5/32*x/a^3+5/64*I/a^3/d*exp(-4*I*(d*x+c))+5/192*I/a^3/d*exp(-6*I*(d*x+c))+
1/256*I/a^3/d*exp(-8*I*(d*x+c))+9/64*I/a^3/d*cos(2*d*x+2*c)+11/64/a^3/d*si
n(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.52

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(120\,dxe^{(8i\,dx+8i\,c)}-12i\,e^{(10i\,dx+10i\,c)}+120i\,e^{(6i\,dx+6i\,c)}+60i\,e^{(4i\,dx+4i\,c)}+20i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-8i\,dx-8i\,c)}}{768\,a^3d}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/768*(120*d*x*e^(8*I*d*x + 8*I*c) - 12*I*e^(10*I*d*x + 10*I*c) + 120*I*e^
(6*I*d*x + 6*I*c) + 60*I*e^(4*I*d*x + 4*I*c) + 20*I*e^(2*I*d*x + 2*I*c) +
3*I)*e^(-8*I*d*x - 8*I*c)/(a^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.54

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \begin{cases} \frac{(-100663296ia^{12}d^4e^{22ic}e^{2idx}+1006632960ia^{12}d^4e^{18ic}e^{-2idx}+503316480ia^{12}d^4e^{16ic}e^{-4idx}+167772160ia^{12}d^4e^{14ic}e^{-6idx}+25165824ia^{12}d}{6442450944a^{15}d^5} \\ x\left(\frac{(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1)e^{-8ic}}{32a^3}-\frac{5}{32a^3}\right) \end{cases} + \frac{5x}{32a^3}$$

input
```
integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise(((-100663296*I*a**12*d**4*exp(22*I*c)*exp(2*I*d*x) + 1006632960*
I*a**12*d**4*exp(18*I*c)*exp(-2*I*d*x) + 503316480*I*a**12*d**4*exp(16*I*c
)*exp(-4*I*d*x) + 167772160*I*a**12*d**4*exp(14*I*c)*exp(-6*I*d*x) + 25165
824*I*a**12*d**4*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(6442450944*a**15
*d**5), Ne(a**15*d**5*exp(20*I*c), 0)), (x*((exp(10*I*c) + 5*exp(8*I*c) +
10*exp(6*I*c) + 10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-8*I*c)/(32*a**3) -
5/(32*a**3)), True)) + 5*x/(32*a**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.71

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{5i\,\log\left(\tan\left(dx+c\right)+i\right)}{64\,a^3d} - \frac{5i\,\log\left(\tan\left(dx+c\right)-i\right)}{64\,a^3d} - \frac{i\left(15i\,\tan\left(dx+c\right)^4+45\,\tan\left(dx+c\right)^3-35i\,\tan\left(dx+c\right)^2+15\,\tan\left(dx+c\right)-32i\right)}{96\,a^3d(\tan\left(dx+c\right)+i)(\tan\left(dx+c\right)-i)^4}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
5/64*I*log(tan(d*x + c) + I)/(a^3*d) - 5/64*I*log(tan(d*x + c) - I)/(a^3*d
) - 1/96*I*(15*I*tan(d*x + c)^4 + 45*tan(d*x + c)^3 - 35*I*tan(d*x + c)^2
+ 15*tan(d*x + c) - 32*I)/(a^3*d*(tan(d*x + c) + I)*(tan(d*x + c) - I)^4)
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.86

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{5\,x}{32\,a^3} + \frac{\frac{1}{3\,a^3}+\frac{35\tan(c+d\,x)^2}{96\,a^3}-\frac{5\tan(c+d\,x)^4}{32\,a^3}+\frac{\tan(c+d\,x)\,5\mathrm{i}}{32\,a^3}+\frac{\tan(c+d\,x)^3\,15\mathrm{i}}{32\,a^3}}{d\,\left(-\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4 3\mathrm{i}+2\tan\left(c+d\,x\right)^3+\tan\left(c+d\,x\right)^2 2\mathrm{i}+3\tan\left(c+d\,x\right)-\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(5*x)/(32*a^3) + ((tan(c + d*x)*5i)/(32*a^3) + 1/(3*a^3) + (35*tan(c + d*x
)^2)/(96*a^3) + (tan(c + d*x)^3*15i)/(32*a^3) - (5*tan(c + d*x)^4)/(32*a^3
))/(d*(3*tan(c + d*x) + tan(c + d*x)^2*2i + 2*tan(c + d*x)^3 + tan(c + d*x
)^4*3i - tan(c + d*x)^5 - 1i))
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\int \frac{\cos(dx+c)^2}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx}{a^3}$$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - int(cos(c + d*x)**2/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c +
 d*x)*i - 1),x))/a**3
```

3.139 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 209

$$
\begin{aligned}
\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = {} & \frac{21x}{128a^3}+\frac{ia^2}{40d(a+ia\tan(c+dx))^5} \\
& + \frac{i}{16d(a+ia\tan(c+dx))^3}+\frac{3ia^5}{64d\,(a^2+ia^2\tan(c+dx))^4} \\
& - \frac{3i}{64d\,(a^3-ia^3\tan(c+dx))} \\
& + \frac{15i}{128d\,(a^3+ia^3\tan(c+dx))} \\
& - \frac{ia^5}{128d\,(a^4-ia^4\tan(c+dx))^2} \\
& + \frac{5ia^5}{64d\,(a^4+ia^4\tan(c+dx))^2}
\end{aligned}
$$

output
```
21/128*x/a^3+1/40*I*a^2/d/(a+I*a*tan(d*x+c))^5+1/16*I/d/(a+I*a*tan(d*x+c))
^3+3/64*I*a^5/d/(a^2+I*a^2*tan(d*x+c))^4-3/64*I/d/(a^3-I*a^3*tan(d*x+c))+1
5/128*I/d/(a^3+I*a^3*tan(d*x+c))-1/128*I*a^5/d/(a^4-I*a^4*tan(d*x+c))^2+5/
64*I*a^5/d/(a^4+I*a^4*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.76

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\sec^7(c+dx)(-1050\cos(c+dx)-469\cos(3(c+dx))+105\cos(5(c+dx))+6\cos(7(c+dx))-350i\sin}{5120a^3}$$

input
```
Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^7*(-1050*Cos[c + d*x] - 469*Cos[3*(c + d*x)] + 105*Cos[5*(c
+ d*x)] + 6*Cos[7*(c + d*x)] - (350*I)*Sin[c + d*x] + (840*I)*ArcTan[Tan[c
 + d*x]]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) - (189*I)*Sin[3*(c + d*x)
] + (175*I)*Sin[5*(c + d*x)] + (14*I)*Sin[7*(c + d*x)]))/(5120*a^3*d*(-I +
 Tan[c + d*x])^5*(I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sec(c+dx)^4(a+ia\tan(c+dx))^3}dx$$
$$\Big\downarrow \; 3968$$
$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^6}d(ia\tan(c+dx))}{d}$$
$$\Big\downarrow \; 54$$

$$-\frac{ia^5\int\left(\frac{3}{64a^7(a-ia\tan(c+dx))^2}+\frac{15}{128a^7(i\tan(c+dx)a+a)^2}+\frac{1}{64a^6(a-ia\tan(c+dx))^3}+\frac{5}{32a^6(i\tan(c+dx)a+a)^3}+\frac{3}{16a^5(i\tan(c+dx)a}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^5\left(\frac{21i\arctan(\tan(c+dx))}{128a^8}+\frac{3}{64a^7(a-ia\tan(c+dx))}-\frac{15}{128a^7(a+ia\tan(c+dx))}+\frac{1}{128a^6(a-ia\tan(c+dx))^2}-\frac{5}{64a^6(a+ia\tan(c+dx))^2}\right.}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*a^5*((((21*I)/128)*ArcTan[Tan[c + d*x]])/a^8 + 1/(128*a^6*(a - I*a*T
an[c + d*x])^2) + 3/(64*a^7*(a - I*a*Tan[c + d*x])) - 1/(40*a^3*(a + I*a*T
an[c + d*x])^5) - 3/(64*a^4*(a + I*a*Tan[c + d*x])^4) - 1/(16*a^5*(a + I*a
*Tan[c + d*x])^3) - 5/(64*a^6*(a + I*a*Tan[c + d*x])^2) - 15/(128*a^7*(a +
 I*a*Tan[c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{-\frac{21i\ln(-i+\tan(dx+c))}{256}+\frac{3i}{64(-i+\tan(dx+c))^4}-\frac{5i}{64(-i+\tan(dx+c))^2}+\frac{1}{40(-i+\tan(dx+c))^5}-\frac{1}{16(-i+\tan(dx+c))^3}+\frac{15}{128(-i+\tan}}{d\,a^3}$
default	$\frac{-\frac{21i\ln(-i+\tan(dx+c))}{256}+\frac{3i}{64(-i+\tan(dx+c))^4}-\frac{5i}{64(-i+\tan(dx+c))^2}+\frac{1}{40(-i+\tan(dx+c))^5}-\frac{1}{16(-i+\tan(dx+c))^3}+\frac{15}{128(-i+\tan}}{d\,a^3}$
risch	$\frac{21x}{128a^3}+\frac{7ie^{-6i(dx+c)}}{256a^3d}+\frac{7ie^{-8i(dx+c)}}{1024a^3d}+\frac{ie^{-10i(dx+c)}}{1280a^3d}+\frac{17i\cos(4dx+4c)}{256a^3d}+\frac{9\sin(4dx+4c)}{128a^3d}+\frac{7i\cos(2dx+2c}{64a^3d}$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(-21/256*I*ln(-I+tan(d*x+c))+3/64*I/(-I+tan(d*x+c))^4-5/64*I/(-I+t
an(d*x+c))^2+1/40/(-I+tan(d*x+c))^5-1/16/(-I+tan(d*x+c))^3+15/128/(-I+tan(
d*x+c))+1/128*I/(tan(d*x+c)+I)^2+21/256*I*ln(tan(d*x+c)+I)+3/64/(tan(d*x+c
)+I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.47

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(840\,dxe^{(10i\,dx+10i\,c)}-10i\,e^{(14i\,dx+14i\,c)}-140i\,e^{(12i\,dx+12i\,c)}+700i\,e^{(8i\,dx+8i\,c)}+350i\,e^{(6i\,dx+6i\,c)}+140i\,e^{(4i\,d}}{5120\,a^3d}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/5120*(840*d*x*e^(10*I*d*x + 10*I*c) - 10*I*e^(14*I*d*x + 14*I*c) - 140*I
*e^(12*I*d*x + 12*I*c) + 700*I*e^(8*I*d*x + 8*I*c) + 350*I*e^(6*I*d*x + 6*
I*c) + 140*I*e^(4*I*d*x + 4*I*c) + 35*I*e^(2*I*d*x + 2*I*c) + 4*I)*e^(-10*
I*d*x - 10*I*c)/(a^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.40

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \begin{cases} \frac{(-11258999068426240ia^{18}d^6e^{34ic}e^{4idx}-157625986957967360ia^{18}d^6e^{32ic}e^{2idx}+788129934789836800ia^{18}d^6e^{28ic}e^{-2idx}+394064967394918}{5764607} \\ x\left(\frac{(e^{14ic}+7e^{12ic}+21e^{10ic}+35e^{8ic}+35e^{6ic}+21e^{4ic}+7e^{2ic}+1)e^{-10ic}}{128a^3}-\frac{21}{128a^3}\right) \end{cases}$$
$$+\frac{21x}{128a^3}$$

input
```
integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise(((-11258999068426240*I*a**18*d**6*exp(34*I*c)*exp(4*I*d*x) - 157
625986957967360*I*a**18*d**6*exp(32*I*c)*exp(2*I*d*x) + 788129934789836800
*I*a**18*d**6*exp(28*I*c)*exp(-2*I*d*x) + 394064967394918400*I*a**18*d**6*
exp(26*I*c)*exp(-4*I*d*x) + 157625986957967360*I*a**18*d**6*exp(24*I*c)*ex
p(-6*I*d*x) + 39406496739491840*I*a**18*d**6*exp(22*I*c)*exp(-8*I*d*x) + 4
503599627370496*I*a**18*d**6*exp(20*I*c)*exp(-10*I*d*x))*exp(-30*I*c)/(576
4607523034234880*a**21*d**7), Ne(a**21*d**7*exp(30*I*c), 0)), (x*((exp(14*
I*c) + 7*exp(12*I*c) + 21*exp(10*I*c) + 35*exp(8*I*c) + 35*exp(6*I*c) + 21
*exp(4*I*c) + 7*exp(2*I*c) + 1)*exp(-10*I*c)/(128*a**3) - 21/(128*a**3)),
True)) + 21*x/(128*a**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.59

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{21i\,\log\left(\tan\left(dx+c\right)+i\right)}{256\,a^3d} - \frac{21i\,\log\left(\tan\left(dx+c\right)-i\right)}{256\,a^3d} - \frac{i\left(105i\,\tan\left(dx+c\right)^6+315\,\tan\left(dx+c\right)^5-140i\,\tan\left(dx+c\right)^4+420\,\tan\left(dx+c\right)^3-469i\,\tan\left(dx+c\right)^2 \ldots\right.}{640\,a^3d(\tan\left(dx+c\right)+i)^2(\tan\left(dx+c\right)-i)^5}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
21/256*I*log(tan(d*x + c) + I)/(a^3*d) - 21/256*I*log(tan(d*x + c) - I)/(a
^3*d) - 1/640*I*(105*I*tan(d*x + c)^6 + 315*tan(d*x + c)^5 - 140*I*tan(d*x
 + c)^4 + 420*tan(d*x + c)^3 - 469*I*tan(d*x + c)^2 - 7*tan(d*x + c) - 176
*I)/(a^3*d*(tan(d*x + c) + I)^2*(tan(d*x + c) - I)^5)
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.83

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{21\,x}{128\,a^3} + \frac{\frac{7\tan(c+d\,x)}{640\,a^3}+\frac{11\mathrm{i}}{40\,a^3}+\frac{\tan(c+d\,x)^2\,469\mathrm{i}}{640\,a^3}-\frac{21\tan(c+d\,x)^3}{32\,a^3}+\frac{\tan(c+d\,x)^4\,7\mathrm{i}}{32\,a^3}-\frac{63\tan(c+d\,x)^5}{128\,a^3}-\ldots}{d\,\left(-\tan\left(c+d\,x\right)^7\,1\mathrm{i}-3\tan\left(c+d\,x\right)^6+\tan\left(c+d\,x\right)^5\,1\mathrm{i}-5\tan\left(c+d\,x\right)^4+\tan\left(c+d\,x\right)^3\,5\mathrm{i}-\tan \ldots\right)}$$

input
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(21*x)/(128*a^3) + ((7*tan(c + d*x))/(640*a^3) + 11i/(40*a^3) + (tan(c + d
*x)^2*469i)/(640*a^3) - (21*tan(c + d*x)^3)/(32*a^3) + (tan(c + d*x)^4*7i)
/(32*a^3) - (63*tan(c + d*x)^5)/(128*a^3) - (tan(c + d*x)^6*21i)/(128*a^3)
)/(d*(tan(c + d*x)*3i - tan(c + d*x)^2 + tan(c + d*x)^3*5i - 5*tan(c + d*x
)^4 + tan(c + d*x)^5*1i - 3*tan(c + d*x)^6 - tan(c + d*x)^7*1i + 1))
```

Reduce [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\int \frac{\cos(dx+c)^4}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx}{a^3}$$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - int(cos(c + d*x)**4/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c +
 d*x)*i - 1),x))/a**3
```

3.140 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 119

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{7\text{arctanh}(\sin(c+dx))}{8a^3d} - \frac{7i\sec^5(c+dx)}{15a^3d} + \frac{7\sec(c+dx)\tan(c+dx)}{8a^3d} + \frac{7\sec^3(c+dx)\tan(c+dx)}{12a^3d} - \frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

output
```
7/8*arctanh(sin(d*x+c))/a^3/d-7/15*I*sec(d*x+c)^5/a^3/d+7/8*sec(d*x+c)*tan
(d*x+c)/a^3/d+7/12*sec(d*x+c)^3*tan(d*x+c)/a^3/d-2/3*I*sec(d*x+c)^7/a/d/(a
+I*a*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\sec^8(c+dx)(\cos(3(c+dx))+i\sin(3(c+dx)))\left(448+1680i\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)\cos^5(c+\cdots\right.}{960a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^8*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)])*(448 + (1680*I)*Ar
cTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]^5 + 640*Cos[2*(c + d*x)]
- (150*I)*Sin[2*(c + d*x)] + (105*I)*Sin[4*(c + d*x)]))/(960*a^3*d*(-I + T
an[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3981, 3042, 3982, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^9}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{7\int \frac{\sec^7(c+dx)}{i\tan(c+dx)a+a}dx}{3a^2}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int \frac{\sec(c+dx)^7}{i\tan(c+dx)a+a}dx}{3a^2}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3982}$$

$$\frac{7\left(\frac{\int \sec^5(c+dx)dx}{a}-\frac{i\sec^5(c+dx)}{5ad}\right)}{3a^2}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{a}-\frac{i\sec^5(c+dx)}{5ad}\right)}{3a^2}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7\left(\frac{\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}\right)}{3a^2}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}\right)}{3a^2}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7\left(\frac{\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}\right)}{3a^2}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}\right)}{3a^2}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{7\left(\frac{\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}\right)}{3a^2}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

input
```
Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(((-2*I)/3)*Sec[c + d*x]^7)/(a*d*(a + I*a*Tan[c + d*x])^2) + (7*(((-1/5*I)
*Sec[c + d*x]^5)/(a*d) + ((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTan
h[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4)/a))/(3*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{i\left(105\,\mathrm{e}^{9i(dx+c)}+490\,\mathrm{e}^{7i(dx+c)}+896\,\mathrm{e}^{5i(dx+c)}+790\,\mathrm{e}^{3i(dx+c)}-105\,\mathrm{e}^{i(dx+c)}\right)}{60d\,a^{3}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}-\frac{7\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8a^{3}d}+\frac{7\ln\left(\mathrm{e}^{i(dx+c)}\right.}{8a^{3}d}$
derivativedivides	$-\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}+\frac{2\left(\frac{1}{16}+\frac{13i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(-\frac{3}{8}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{2\left(-\frac{5}{16}+\frac{11i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{3}{4}+\frac{7i}{24}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{8}$
default	$-\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}+\frac{2\left(\frac{1}{16}+\frac{13i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(-\frac{3}{8}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{2\left(-\frac{5}{16}+\frac{11i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{3}{4}+\frac{7i}{24}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{8}$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*I/d/a^3/(exp(2*I*(d*x+c))+1)^5*(105*exp(9*I*(d*x+c))+490*exp(7*I*(d*
x+c))+896*exp(5*I*(d*x+c))+790*exp(3*I*(d*x+c))-105*exp(I*(d*x+c)))-7/8/a^
3/d*ln(exp(I*(d*x+c))-I)+7/8/a^3/d*ln(exp(I*(d*x+c))+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(103) = 206.

Time = 0.09 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.34

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{105\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-1}{120\,(a^3d}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/120*(105*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x
+ 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*
x + I*c) + I) - 105*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^
(6*I*d*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*lo
g(e^(I*d*x + I*c) - I) - 210*I*e^(9*I*d*x + 9*I*c) - 980*I*e^(7*I*d*x + 7*
I*c) - 1792*I*e^(5*I*d*x + 5*I*c) - 1580*I*e^(3*I*d*x + 3*I*c) + 210*I*e^(
I*d*x + I*c))/(a^3*d*e^(10*I*d*x + 10*I*c) + 5*a^3*d*e^(8*I*d*x + 8*I*c) +
 10*a^3*d*e^(6*I*d*x + 6*I*c) + 10*a^3*d*e^(4*I*d*x + 4*I*c) + 5*a^3*d*e^(
2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^9(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sec(c + d*x)**9/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(
c + d*x) + I), x)/a**3
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(103) = 206$.

Time = 0.05 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.87

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\frac{16\left(-\frac{15i\sin(dx+c)}{\cos(dx+c)+1}+\frac{320\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{390i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{400\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{960\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{390i\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{360\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{15i\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-136}{-120ia^3+\frac{600ia^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1200ia^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1200ia^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{600ia^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{120ia^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}}{8d}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/8*(16*(-15*I*sin(d*x + c)/(cos(d*x + c) + 1) + 320*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 390*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 400*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 + 960*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 390
*I*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 360*sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 + 15*I*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 136)/(-120*I*a^3 + 600
*I*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1200*I*a^3*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 1200*I*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 600*I
*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 120*I*a^3*sin(d*x + c)^10/(cos(
d*x + c) + 1)^10) + 7*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 - 7*log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.38

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\frac{105\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^3} - \frac{105\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^3} + \frac{2\left(15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+360i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-390\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-960i\tan\cdots\right.}{\cdots}}{120\,d}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/120*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 105*log(tan(1/2*d*x + 1/2*c
) - 1)/a^3 + 2*(15*tan(1/2*d*x + 1/2*c)^9 + 360*I*tan(1/2*d*x + 1/2*c)^8 -
 390*tan(1/2*d*x + 1/2*c)^7 - 960*I*tan(1/2*d*x + 1/2*c)^6 + 400*I*tan(1/2
*d*x + 1/2*c)^4 + 390*tan(1/2*d*x + 1/2*c)^3 - 320*I*tan(1/2*d*x + 1/2*c)^
2 - 15*tan(1/2*d*x + 1/2*c) + 136*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a^3))
/d
```

Mupad [B] (verification not implemented)

Time = 3.23 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.26

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^3\,d} + \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 6\mathrm{i}-\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 16\mathrm{i}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 20\mathrm{i}}{3}+\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-\frac{\mathrm{ta}}{}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^5}$$

input
```
int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
(7*atanh(tan(c/2 + (d*x)/2)))/(4*a^3*d) + ((13*tan(c/2 + (d*x)/2)^3)/2 - (
tan(c/2 + (d*x)/2)^2*16i)/3 - tan(c/2 + (d*x)/2)/4 + (tan(c/2 + (d*x)/2)^4
*20i)/3 - tan(c/2 + (d*x)/2)^6*16i - (13*tan(c/2 + (d*x)/2)^7)/2 + tan(c/2
 + (d*x)/2)^8*6i + tan(c/2 + (d*x)/2)^9/4 + 34i/15)/(a^3*d*(tan(c/2 + (d*x
)/2)^2 - 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.34

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{-105\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4+210\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+}{}$$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - 105*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 + 210*cos(c
 + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 - 105*cos(c + d*x)*log(t
an((c + d*x)/2) - 1) + 105*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c +
d*x)**4 - 210*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 + 105
*cos(c + d*x)*log(tan((c + d*x)/2) + 1) + 8*cos(c + d*x)*sin(c + d*x)**4*i
 - 105*cos(c + d*x)*sin(c + d*x)**3 - 16*cos(c + d*x)*sin(c + d*x)**2*i +
15*cos(c + d*x)*sin(c + d*x) + 8*cos(c + d*x)*i + 160*sin(c + d*x)**2*i -
136*i)/(120*cos(c + d*x)*a**3*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.141 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{5\text{arctanh}(\sin(c+dx))}{2a^3d} - \frac{5i\sec^3(c+dx)}{3a^3d} + \frac{5\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

output
```
5/2*arctanh(sin(d*x+c))/a^3/d-5/3*I*sec(d*x+c)^3/a^3/d+5/2*sec(d*x+c)*tan(
d*x+c)/a^3/d-2*I*sec(d*x+c)^5/a/d/(a+I*a*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{60\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right) - i\sec^3(c+dx)(20+24\cos(2(c+dx))-9i\sin(2(c+dx)))}{12a^3d}$$

input
```
Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(60*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] - I*Sec[c + d*x]^3*(20 + 24*Cos[
2*(c + d*x)] - (9*I)*Sin[2*(c + d*x)]))/(12*a^3*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3981, 3042, 3982, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^7}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{5\int \frac{\sec^5(c+dx)}{i\tan(c+dx)a+a}dx}{a^2}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{\sec(c+dx)^5}{i\tan(c+dx)a+a}dx}{a^2}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3982}$$

$$\frac{5\left(\frac{\int \sec^3(c+dx)dx}{a}-\frac{i\sec^3(c+dx)}{3ad}\right)}{a^2}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a}-\frac{i\sec^3(c+dx)}{3ad}\right)}{a^2}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{5\left(\frac{\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{i\sec^3(c+dx)}{3ad}\right)}{a^2}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{i\sec^3(c+dx)}{3ad}\right)}{a^2}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{5\left(\frac{\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{i\sec^3(c+dx)}{3ad}\right)}{a^2}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

input
```
Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-2*I)*Sec[c + d*x]^5)/(a*d*(a + I*a*Tan[c + d*x])^2) + (5*(((-1/3*I)*Sec
[c + d*x]^3)/(a*d) + (ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c +
d*x])/(2*d))/a))/a^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{i\left(15\,{\mathrm e}^{5 i (d x+c)}+40\,{\mathrm e}^{3 i (d x+c)}+33\,{\mathrm e}^{i (d x+c)}\right)}{3 d\,a^{3} \left({\mathrm e}^{2 i (d x+c)}+1\right)^{3}}-\frac{5 \ln \left({\mathrm e}^{i (d x+c)}-i\right)}{2 a^{3} d}+\frac{5 \ln \left({\mathrm e}^{i (d x+c)}+i\right)}{2 a^{3} d}$
derivativedivides	$\frac{-\frac{i}{3 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2 \left(-\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2 \left(-\frac{3}{4}+\frac{7 i}{4}\right)}{\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1}-\frac{5 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{i}{3 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2 \left(\frac{3}{4}-\frac{i}{4}\right)}{(\tan (\frac{d x}{2}+\frac{c}{2})+1}}{a^{3} d}$
default	$\frac{-\frac{i}{3 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2 \left(-\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2 \left(-\frac{3}{4}+\frac{7 i}{4}\right)}{\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1}-\frac{5 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{i}{3 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2 \left(\frac{3}{4}-\frac{i}{4}\right)}{(\tan (\frac{d x}{2}+\frac{c}{2})+1}}{a^{3} d}$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/d/a^3/(exp(2*I*(d*x+c))+1)^3*(15*exp(5*I*(d*x+c))+40*exp(3*I*(d*x+c
))+33*exp(I*(d*x+c)))-5/2/a^3/d*ln(exp(I*(d*x+c))-I)+5/2/a^3/d*ln(exp(I*(d
*x+c))+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(81) = 162$.

Time = 0.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.96

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{15\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2}\right.}{6\left(a^3de^{(6i\,dx+6i\,c)}+3\,a^3de^{(4i\,dx+4i\,c)}+3\,a^3de\right.}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/6*(15*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*
c) + 1)*log(e^(I*d*x + I*c) + I) - 15*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x
+ 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 30*I*e^(5
*I*d*x + 5*I*c) - 80*I*e^(3*I*d*x + 3*I*c) - 66*I*e^(I*d*x + I*c))/(a^3*d*
e^(6*I*d*x + 6*I*c) + 3*a^3*d*e^(4*I*d*x + 4*I*c) + 3*a^3*d*e^(2*I*d*x + 2
*I*c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^7(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sec(c + d*x)**7/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(
c + d*x) + I), x)/a**3
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(81) = 162$.

Time = 0.05 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.31

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\frac{4\left(-\frac{9i\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{9i\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+22\right)}{6i\,a^3-\frac{18i\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18i\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6i\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{5\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}-\frac{5\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{2\,d}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(4*(-9*I*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 18*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 9*I*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 + 22)/(6*I*a^3 - 18*I*a^3*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 18*I*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6*I*a^3*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) + 5*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/
a^3 - 5*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\frac{15\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^3}-\frac{15\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^3}-\frac{2\left(9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-18i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+48i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3a^3}}{6\,d}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 15*log(tan(1/2*d*x + 1/2*c) -
1)/a^3 - 2*(9*tan(1/2*d*x + 1/2*c)^5 - 18*I*tan(1/2*d*x + 1/2*c)^4 + 48*I*
tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) - 22*I)/((tan(1/2*d*x + 1/
2*c)^2 - 1)^3*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.45

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d} + \frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a^3}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,16{}\mathrm{i}}{a^3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,6{}\mathrm{i}}{a^3}+\frac{22{}\mathrm{i}}{3\,a^3}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

input
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
(5*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) + ((tan(c/2 + (d*x)/2)^4*6i)/a^3 - (
tan(c/2 + (d*x)/2)^2*16i)/a^3 - (3*tan(c/2 + (d*x)/2)^5)/a^3 + 22i/(3*a^3)
 + (3*tan(c/2 + (d*x)/2))/a^3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d
*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.92

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{-15\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2+15\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+15\cos(d}{}$$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - 15*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 + 15*cos(c +
 d*x)*log(tan((c + d*x)/2) - 1) + 15*cos(c + d*x)*log(tan((c + d*x)/2) + 1
)*sin(c + d*x)**2 - 15*cos(c + d*x)*log(tan((c + d*x)/2) + 1) - 10*cos(c +
 d*x)*sin(c + d*x)**2*i + 9*cos(c + d*x)*sin(c + d*x) + 10*cos(c + d*x)*i
- 24*sin(c + d*x)**2*i + 22*i)/(6*cos(c + d*x)*a**3*d*(sin(c + d*x)**2 - 1
))
```

3.142 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=-\frac{3\text{arctanh}(\sin(c+dx))}{a^3d}+\frac{3i\sec(c+dx)}{a^3d}+\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

output
```
-3*arctanh(sin(d*x+c))/a^3/d+3*I*sec(d*x+c)/a^3/d+2*I*sec(d*x+c)^3/a/d/(a+
I*a*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.66

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=\frac{\sec^3(c+dx)(i\cos(dx)-\sin(dx))^3\left(6\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)(\cos(3c)+i\sin(3c))+(\cos(2c-\right.}{a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^3*(I*Cos[d*x] - Sin[d*x])^3*(6*ArcTanh[Sin[c] + Cos[c]*Tan[(
d*x)/2]]*(Cos[3*c] + I*Sin[3*c]) + (Cos[2*c - d*x] + I*Sin[2*c - d*x])*(-5
*I + Tan[c + d*x])))/(a^3*d*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3981, 3042, 3982, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}-\frac{3\int \frac{\sec^3(c+dx)}{i\tan(c+dx)a+a}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}-\frac{3\int \frac{\sec(c+dx)^3}{i\tan(c+dx)a+a}dx}{a^2}$$

$$\Big\downarrow \text{3982}$$

$$\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}-\frac{3\left(\frac{\int \sec(c+dx)dx}{a}-\frac{i\sec(c+dx)}{ad}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}-\frac{3\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{i\sec(c+dx)}{ad}\right)}{a^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}-\frac{3\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}\right)}{a^2}$$

input
```
Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(-3*(ArcTanh[Sin[c + d*x]]/(a*d) - (I*Sec[c + d*x])/(a*d)))/a^2 + ((2*I)*S
ec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

method	result	size
derivativedivides	$\frac{\frac{8}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3d}$	86
default	$\frac{\frac{8}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3d}$	86
risch	$\frac{4i\mathrm{e}^{-i(dx+c)}}{a^3d}+\frac{2i\mathrm{e}^{i(dx+c)}}{d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^3d}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^3d}$	93

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
2/d/a^3*(4/(-I+tan(1/2*d*x+1/2*c))-1/2*I/(tan(1/2*d*x+1/2*c)-1)+3/2*ln(tan
(1/2*d*x+1/2*c)-1)+1/2*I/(tan(1/2*d*x+1/2*c)+1)-3/2*ln(tan(1/2*d*x+1/2*c)+
1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.72

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{3\left(e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(2i\,dx+2}}{a^3de^{(3i\,dx+3i\,c)}+a^3de^{(i\,dx+i\,c)}}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-(3*(e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*log(e^(I*d*x + I*c) + I) - 3*(
e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*log(e^(I*d*x + I*c) - I) - 6*I*e^(2
*I*d*x + 2*I*c) - 4*I)/(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^5(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sec(c + d*x)**5/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(
c + d*x) + I), x)/a**3
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(59) = 118$.

Time = 0.14 (sec) , antiderivative size = 319, normalized size of antiderivative = 4.91

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{6(\cos(3dx+3c)+\cos(dx+c)+i\sin(3dx+3c)+i\sin(dx+c))\arctan(\cos(dx+c),\sin(dx+c)+$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
(6*(cos(3*d*x + 3*c) + cos(d*x + c) + I*sin(3*d*x + 3*c) + I*sin(d*x + c))
*arctan2(cos(d*x + c), sin(d*x + c) + 1) + 6*(cos(3*d*x + 3*c) + cos(d*x +
 c) + I*sin(3*d*x + 3*c) + I*sin(d*x + c))*arctan2(cos(d*x + c), -sin(d*x
+ c) + 1) + 3*(I*cos(3*d*x + 3*c) + I*cos(d*x + c) - sin(3*d*x + 3*c) - si
n(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3*
(-I*cos(3*d*x + 3*c) - I*cos(d*x + c) + sin(3*d*x + 3*c) + sin(d*x + c))*l
og(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 12*cos(2*d*x +
2*c) + 12*I*sin(2*d*x + 2*c) + 8)/((-2*I*a^3*cos(3*d*x + 3*c) - 2*I*a^3*co
s(d*x + c) + 2*a^3*sin(3*d*x + 3*c) + 2*a^3*sin(d*x + c))*d)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3} - \frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^3} - \frac{2\left(4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-5\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)a^3}}{d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-(3*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 3*log(tan(1/2*d*x + 1/2*c) - 1)/a^
3 - 2*(4*tan(1/2*d*x + 1/2*c)^2 - I*tan(1/2*d*x + 1/2*c) - 5)/((tan(1/2*d*
x + 1/2*c)^3 - I*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + I)*a^3))/
d
```

Mupad [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.62

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{6\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d} - \frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 8\mathrm{i}}{a^3} - \frac{10\mathrm{i}}{a^3}}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 1\mathrm{i} - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}+1\right)}$$

input
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
- (6*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - ((tan(c/2 + (d*x)/2)^2*8i)/a^3 -
 10i/a^3 + (2*tan(c/2 + (d*x)/2))/a^3)/(d*(tan(c/2 + (d*x)/2)*1i - tan(c/2
 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

Reduce [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x)
```

output
```
(8160*cos(c + d*x)*int(cos(c + d*x)/(4*cos(c + d*x)*sin(c + d*x)**4*i - 5*
cos(c + d*x)*sin(c + d*x)**2*i + cos(c + d*x)*i - 4*sin(c + d*x)**5 + 7*si
n(c + d*x)**3 - 3*sin(c + d*x)),x)*d*i + 696*cos(c + d*x)*int(cos(c + d*x)
/(4*cos(c + d*x)*sin(c + d*x)**3*i - 3*cos(c + d*x)*sin(c + d*x)*i - 4*sin
(c + d*x)**4 + 5*sin(c + d*x)**2 - 1),x)*d - 720*cos(c + d*x)*int(cos(c +
d*x)/(4*cos(c + d*x)*sin(c + d*x)**3 - 3*cos(c + d*x)*sin(c + d*x) + 4*sin
(c + d*x)**4*i - 5*sin(c + d*x)**2*i + i),x)*d*i - 12048*cos(c + d*x)*int(
sin(c + d*x)**5/(4*cos(c + d*x)*sin(c + d*x)**4*i - 5*cos(c + d*x)*sin(c +
 d*x)**2*i + cos(c + d*x)*i - 4*sin(c + d*x)**5 + 7*sin(c + d*x)**3 - 3*si
n(c + d*x)),x)*d + 27360*cos(c + d*x)*int(sin(c + d*x)**4/(4*cos(c + d*x)*
sin(c + d*x)**4*i - 5*cos(c + d*x)*sin(c + d*x)**2*i + cos(c + d*x)*i - 4*
sin(c + d*x)**5 + 7*sin(c + d*x)**3 - 3*sin(c + d*x)),x)*d*i + 2352*cos(c
+ d*x)*int(sin(c + d*x)**4/(4*cos(c + d*x)*sin(c + d*x)**3*i - 3*cos(c + d
*x)*sin(c + d*x)*i - 4*sin(c + d*x)**4 + 5*sin(c + d*x)**2 - 1),x)*d - 240
0*cos(c + d*x)*int(sin(c + d*x)**4/(4*cos(c + d*x)*sin(c + d*x)**3 - 3*cos
(c + d*x)*sin(c + d*x) + 4*sin(c + d*x)**4*i - 5*sin(c + d*x)**2*i + i),x)
*d*i + 38940*cos(c + d*x)*int(sin(c + d*x)**3/(4*cos(c + d*x)*sin(c + d*x)
**4*i - 5*cos(c + d*x)*sin(c + d*x)**2*i + cos(c + d*x)*i - 4*sin(c + d*x)
**5 + 7*sin(c + d*x)**3 - 3*sin(c + d*x)),x)*d - 2784*cos(c + d*x)*int(sin
(c + d*x)**3/(4*cos(c + d*x)*sin(c + d*x)**3*i - 3*cos(c + d*x)*sin(c +...
```

3.143 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 32

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3} dx = \frac{i\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^3}$$

output
```
1/3*I*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^3
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3} dx = \frac{i\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/3)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sec(c+dx)^3}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \ 3969$$

$$\frac{i\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^3}$$

input
```
Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/3)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{i e^{-3i(dx+c)}}{3a^3d}$	19
orering	$\frac{i\sec(dx+c)^3}{3d(a+ia\tan(dx+c))^3}$	29
derivativedivides	$\frac{-\frac{8}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{a^3d}$	57
default	$\frac{-\frac{8}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{a^3d}$	57

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*I/a^3/d*exp(-3*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\,e^{(-3i\,dx-3i\,c)}}{3\,a^3d}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/3*I*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(26) = 52.

Time = 0.87 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.50

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \begin{cases} -\frac{\sec^3(c+dx)}{3a^3d\tan^3(c+dx)-9ia^3d\tan^2(c+dx)-9a^3d\tan(c+dx)+3ia^3d} & \text{for } d\neq 0 \\ \frac{x\sec^3(c)}{(ia\tan(c)+a)^3} & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise((-sec(c + d*x)**3/(3*a**3*d*tan(c + d*x)**3 - 9*I*a**3*d*tan(c +
 d*x)**2 - 9*a**3*d*tan(c + d*x) + 3*I*a**3*d), Ne(d, 0)), (x*sec(c)**3/(I
*a*tan(c) + a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\cos(3\,dx+3\,c)+\sin(3\,dx+3\,c)}{3\,a^3d}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/3*(I*cos(3*d*x + 3*c) + sin(3*d*x + 3*c))/(a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{3\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.12

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 3\mathrm{i}-\mathrm{i}\right)}{3\,a^3\,d\,\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 1\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,3\mathrm{i}+1\right)}$$

input
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
-(2*(tan(c/2 + (d*x)/2)^2*3i - 1i))/(3*a^3*d*(tan(c/2 + (d*x)/2)*3i - 3*ta
n(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x)
```

output

```
( - 3*int(cos(c + d*x)/(4*cos(c + d*x)*sin(c + d*x)**2*i - cos(c + d*x)*i
- 4*sin(c + d*x)**3 + 3*sin(c + d*x)),x)*d*i - 24*int(sin(c + d*x)**3/(4*c
os(c + d*x)*sin(c + d*x)**2*i - cos(c + d*x)*i - 4*sin(c + d*x)**3 + 3*sin
(c + d*x)),x)*d + 12*int(( - sin(c + d*x)**3)/(4*cos(c + d*x)*sin(c + d*x)
**2 - cos(c + d*x) + 4*sin(c + d*x)**3*i - 3*sin(c + d*x)*i),x)*d*i - 9*in
t(( - sin(c + d*x))/(4*cos(c + d*x)*sin(c + d*x)**2 - cos(c + d*x) + 4*sin
(c + d*x)**3*i - 3*sin(c + d*x)*i),x)*d*i + 12*int(( - cos(c + d*x)*sin(c
+ d*x)**2)/(4*cos(c + d*x)*sin(c + d*x)**2 - cos(c + d*x) + 4*sin(c + d*x)
**3*i - 3*sin(c + d*x)*i),x)*d + 18*int(sin(c + d*x)/(4*cos(c + d*x)*sin(c
 + d*x)**2*i - cos(c + d*x)*i - 4*sin(c + d*x)**3 + 3*sin(c + d*x)),x)*d +
 12*int((cos(c + d*x)*sin(c + d*x)**2)/(4*cos(c + d*x)*sin(c + d*x)**2*i -
 cos(c + d*x)*i - 4*sin(c + d*x)**3 + 3*sin(c + d*x)),x)*d*i + 12*int((cos
(c + d*x)*sin(c + d*x)**2)/(4*cos(c + d*x)*sin(c + d*x)**2 - cos(c + d*x)
+ 4*sin(c + d*x)**3*i - 3*sin(c + d*x)*i),x)*d - 3*int(1/(4*cos(c + d*x)*s
in(c + d*x)**2*i - cos(c + d*x)*i - 4*sin(c + d*x)**3 + 3*sin(c + d*x)),x)
*d*i + 2*log(tan((c + d*x)/2)**6 - 6*tan((c + d*x)/2)**5*i - 15*tan((c + d
*x)/2)**4 + 20*tan((c + d*x)/2)**3*i + 15*tan((c + d*x)/2)**2 - 6*tan((c +
 d*x)/2)*i - 1)*i - 12*log(tan((c + d*x)/2)**2 + 1)*i + 2*log(tan((c + d*x
)/2)**6*i + 6*tan((c + d*x)/2)**5 - 15*tan((c + d*x)/2)**4*i - 20*tan((c +
 d*x)/2)**3 + 15*tan((c + d*x)/2)**2*i + 6*tan((c + d*x)/2) - i)*i + 9*...
```

3.144 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3} + \frac{2i\sec(c+dx)}{15ad(a+ia\tan(c+dx))^2} + \frac{2i\sec(c+dx)}{15d\left(a^3+ia^3\tan(c+dx)\right)}$$

output
```
1/5*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^3+2/15*I*sec(d*x+c)/a/d/(a+I*a*tan(d
*x+c))^2+2/15*I*sec(d*x+c)/d/(a^3+I*a^3*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.55

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\sec^3(c+dx)(5+9\cos(2(c+dx))+6i\sin(2(c+dx)))}{30a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
-1/30*(Sec[c + d*x]^3*(5 + 9*Cos[2*(c + d*x)] + (6*I)*Sin[2*(c + d*x)]))/(
a^3*d*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3983, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3983}$$
$$\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}$$
$$\Big\downarrow \text{3042}$$
$$\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}$$
$$\Big\downarrow \text{3983}$$
$$\frac{2\left(\frac{\int \frac{\sec(c+dx)}{i\tan(c+dx)a+a}dx}{3a}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}\right)}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}$$
$$\Big\downarrow \text{3042}$$
$$\frac{2\left(\frac{\int \frac{\sec(c+dx)}{i\tan(c+dx)a+a}dx}{3a}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}\right)}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}$$
$$\Big\downarrow \text{3969}$$
$$\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}+\frac{2\left(\frac{i\sec(c+dx)}{3ad(a+ia\tan(c+dx))}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}\right)}{5a}$$

input
```
Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/5)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^3) + (2*(((I/3)*Sec[c + d*x
])/(d*(a + I*a*Tan[c + d*x])^2) + ((I/3)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c
 + d*x]))))/(5*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{i\mathrm{e}^{-i(dx+c)}}{4a^3d}+\frac{i\mathrm{e}^{-3i(dx+c)}}{6a^3d}+\frac{i\mathrm{e}^{-5i(dx+c)}}{20a^3d}$	56
derivativedivides	$\frac{-\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{8}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{16}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{a^3d}$	90
default	$\frac{-\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{8}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{16}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{a^3d}$	90

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*I/a^3/d*exp(-I*(d*x+c))+1/6*I/a^3/d*exp(-3*I*(d*x+c))+1/20*I/a^3/d*exp
(-5*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.42

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(15i\,e^{(4i\,dx+4i\,c)}+10i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{60\,a^3d}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/60*(15*I*e^(4*I*d*x + 4*I*c) + 10*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d
*x - 5*I*c)/(a^3*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(82) = 164.

Time = 0.86 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.23

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \begin{cases} \frac{2\tan^2(c+dx)\sec(c+dx)}{15a^3d\tan^3(c+dx)-45ia^3d\tan^2(c+dx)-45a^3d\tan(c+dx)+15ia^3d} - \frac{6i\tan(c+dx)\sec(c+dx)}{15a^3d\tan^3(c+dx)-45ia^3d\tan^2(c+dx)-45a^3d\tan(c+dx)+15ia^3d} \\ \frac{x\sec(c)}{(ia\tan(c)+a)^3} \end{cases}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise((2*tan(c + d*x)**2*sec(c + d*x)/(15*a**3*d*tan(c + d*x)**3 - 45*
I*a**3*d*tan(c + d*x)**2 - 45*a**3*d*tan(c + d*x) + 15*I*a**3*d) - 6*I*tan
(c + d*x)*sec(c + d*x)/(15*a**3*d*tan(c + d*x)**3 - 45*I*a**3*d*tan(c + d*
x)**2 - 45*a**3*d*tan(c + d*x) + 15*I*a**3*d) - 7*sec(c + d*x)/(15*a**3*d*
tan(c + d*x)**3 - 45*I*a**3*d*tan(c + d*x)**2 - 45*a**3*d*tan(c + d*x) + 1
5*I*a**3*d), Ne(d, 0)), (x*sec(c)/(I*a*tan(c) + a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{3i\,\cos\left(5\,dx+5\,c\right)+10i\,\cos\left(3\,dx+3\,c\right)+15i\,\cos\left(dx+c\right)+3\,\sin\left(5\,dx+5\,c\right)+10\,\sin\left(3\,dx+3\,c\right)+15}{60\,a^3d}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/60*(3*I*cos(5*d*x + 5*c) + 10*I*cos(3*d*x + 3*c) + 15*I*cos(d*x + c) + 3
*sin(5*d*x + 5*c) + 10*sin(3*d*x + 3*c) + 15*sin(d*x + c))/(a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-30i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+20i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\right)}{15\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^5}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
2/15*(15*tan(1/2*d*x + 1/2*c)^4 - 30*I*tan(1/2*d*x + 1/2*c)^3 - 40*tan(1/2
*d*x + 1/2*c)^2 + 20*I*tan(1/2*d*x + 1/2*c) + 7)/(a^3*d*(tan(1/2*d*x + 1/2
*c) - I)^5)
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.36

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{2\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 15\mathrm{i}+30\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 40\mathrm{i}-20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+7\mathrm{i}\right)}{15\,a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 1\mathrm{i}+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 10\mathrm{i}-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,5\mathrm{i}+\right.}$$

input
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
(2*(30*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*40i - 20*tan(c/2 + (d*x
)/2) + tan(c/2 + (d*x)/2)^4*15i + 7i))/(15*a^3*d*(tan(c/2 + (d*x)/2)*5i -
10*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*10i + 5*tan(c/2 + (d*x)/2)^
4 + tan(c/2 + (d*x)/2)^5*1i + 1))
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - 3*int(cos(c + d*x)/(4*cos(c + d*x)*sin(c + d*x)**2*i - cos(c + d*x)*i
- 4*sin(c + d*x)**3 + 3*sin(c + d*x)),x)*d*i - 24*int(sin(c + d*x)**3/(4*c
os(c + d*x)*sin(c + d*x)**2*i - cos(c + d*x)*i - 4*sin(c + d*x)**3 + 3*sin
(c + d*x)),x)*d + 3*int(sin(c + d*x)**2/(4*cos(c + d*x)*sin(c + d*x)**2*i
- cos(c + d*x)*i - 4*sin(c + d*x)**3 + 3*sin(c + d*x)),x)*d*i + 12*int(( -
 sin(c + d*x)**3)/(4*cos(c + d*x)*sin(c + d*x)**2 - cos(c + d*x) + 4*sin(c
 + d*x)**3*i - 3*sin(c + d*x)*i),x)*d*i - 9*int(( - sin(c + d*x))/(4*cos(c
 + d*x)*sin(c + d*x)**2 - cos(c + d*x) + 4*sin(c + d*x)**3*i - 3*sin(c + d
*x)*i),x)*d*i + 12*int(( - cos(c + d*x)*sin(c + d*x)**2)/(4*cos(c + d*x)*s
in(c + d*x)**2 - cos(c + d*x) + 4*sin(c + d*x)**3*i - 3*sin(c + d*x)*i),x)
*d + 18*int(sin(c + d*x)/(4*cos(c + d*x)*sin(c + d*x)**2*i - cos(c + d*x)*
i - 4*sin(c + d*x)**3 + 3*sin(c + d*x)),x)*d + 12*int((cos(c + d*x)*sin(c
+ d*x)**2)/(4*cos(c + d*x)*sin(c + d*x)**2*i - cos(c + d*x)*i - 4*sin(c +
d*x)**3 + 3*sin(c + d*x)),x)*d*i + 12*int((cos(c + d*x)*sin(c + d*x)**2)/(
4*cos(c + d*x)*sin(c + d*x)**2 - cos(c + d*x) + 4*sin(c + d*x)**3*i - 3*si
n(c + d*x)*i),x)*d - 3*int(1/(4*cos(c + d*x)*sin(c + d*x)**2*i - cos(c + d
*x)*i - 4*sin(c + d*x)**3 + 3*sin(c + d*x)),x)*d*i + 2*log(tan((c + d*x)/2
)**6 - 6*tan((c + d*x)/2)**5*i - 15*tan((c + d*x)/2)**4 + 20*tan((c + d*x)
/2)**3*i + 15*tan((c + d*x)/2)**2 - 6*tan((c + d*x)/2)*i - 1)*i - 12*log(t
an((c + d*x)/2)**2 + 1)*i + 2*log(tan((c + d*x)/2)**6*i + 6*tan((c + d*...
```

3.145 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 101

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{12\sin(c+dx)}{35a^3d} - \frac{4\sin^3(c+dx)}{35a^3d} + \frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3} + \frac{8i\cos^3(c+dx)}{35d\,(a^3+ia^3\tan(c+dx))}$$

output
```
12/35*sin(d*x+c)/a^3/d-4/35*sin(d*x+c)^3/a^3/d+1/7*I*cos(d*x+c)/d/(a+I*a*t
an(d*x+c))^3+8/35*I*cos(d*x+c)^3/d/(a^3+I*a^3*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\sec^3(c+dx)(35+84\cos(2(c+dx))-15\cos(4(c+dx))+56i\sin(2(c+dx))-20i\sin(4(c+dx)))}{280a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
-1/280*(Sec[c + d*x]^3*(35 + 84*Cos[2*(c + d*x)] - 15*Cos[4*(c + d*x)] + (
56*I)*Sin[2*(c + d*x)] - (20*I)*Sin[4*(c + d*x)]))/(a^3*d*(-I + Tan[c + d*
x])^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3042, 3983, 3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{4\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^2}dx}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3981}$$

$$\frac{4\left(\frac{3\int \cos^3(c+dx)dx}{5a^2}+\frac{2i\cos^3(c+dx)}{5d(a^2+ia^2\tan(c+dx))}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{3\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{5a^2}+\frac{2i\cos^3(c+dx)}{5d(a^2+ia^2\tan(c+dx))}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3113}$$

$$\frac{4\left(-\frac{3\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d(a^2+ia^2\tan(c+dx))}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{4\left(-\frac{3\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d(a^2+ia^2\tan(c+dx))}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}$$

input
```
Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/7)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^3) + (4*((-3*(-Sin[c + d*x]
 + Sin[c + d*x]^3/3))/(5*a^2*d) + (((2*I)/5)*Cos[c + d*x]^3)/(d*(a^2 + I*a
^2*Tan[c + d*x]))))/(7*a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.84

method	result
risch	$\frac{i\mathrm{e}^{-3i(dx+c)}}{8a^3d}+\frac{i\mathrm{e}^{-5i(dx+c)}}{20a^3d}+\frac{i\mathrm{e}^{-7i(dx+c)}}{112a^3d}+\frac{3i\cos(dx+c)}{16a^3d}+\frac{5\sin(dx+c)}{16a^3d}$
derivativedivides	$\frac{\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{9i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{17i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{8}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{38}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{}{2(-i+\mathrm{ta}}}{a^3d}$
default	$\frac{\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{9i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{17i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{8}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{38}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{}{2(-i+\mathrm{ta}}}{a^3d}$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*I/a^3/d*exp(-3*I*(d*x+c))+1/20*I/a^3/d*exp(-5*I*(d*x+c))+1/112*I/a^3/d
*exp(-7*I*(d*x+c))+3/16*I/a^3/d*cos(d*x+c)+5/16*sin(d*x+c)/a^3/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(-35i\,e^{(8i\,dx+8i\,c)}+140i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+28i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-7i\,dx-7i\,c)}}{560\,a^3d}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/560*(-35*I*e^(8*I*d*x + 8*I*c) + 140*I*e^(6*I*d*x + 6*I*c) + 70*I*e^(4*I
*d*x + 4*I*c) + 28*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a^3*
d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(87) = 174$.

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.95

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \begin{cases} \frac{\left(-71680ia^{12}d^4e^{17ic}e^{idx}+286720ia^{12}d^4e^{15ic}e^{-idx}+143360ia^{12}d^4e^{13ic}e^{-3idx}+57344ia^{12}d^4e^{11ic}e^{-5idx}+10240ia^{12}d^4e^{9ic}e^{-7idx}\right)e^{-16ic}}{1146880a^{15}d^5} \\ \frac{x\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-7ic}}{16a^3} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise(((-71680*I*a**12*d**4*exp(17*I*c)*exp(I*d*x) + 286720*I*a**12*d*
*4*exp(15*I*c)*exp(-I*d*x) + 143360*I*a**12*d**4*exp(13*I*c)*exp(-3*I*d*x)
 + 57344*I*a**12*d**4*exp(11*I*c)*exp(-5*I*d*x) + 10240*I*a**12*d**4*exp(9
*I*c)*exp(-7*I*d*x))*exp(-16*I*c)/(1146880*a**15*d**5), Ne(a**15*d**5*exp(
16*I*c), 0)), (x*(exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*c) + 4*exp(2*I*c)
+ 1)*exp(-7*I*c)/(16*a**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.18

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{35}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)} + \frac{525\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1960i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-4025\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+4480i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3143\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^7}}{280\,d}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) + I)) + (525*tan(1/2*d*x + 1/2*c)^6 -
 1960*I*tan(1/2*d*x + 1/2*c)^5 - 4025*tan(1/2*d*x + 1/2*c)^4 + 4480*I*tan(
1/2*d*x + 1/2*c)^3 + 3143*tan(1/2*d*x + 1/2*c)^2 - 1176*I*tan(1/2*d*x + 1/
2*c) - 243)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^7))/d
```

Mupad [B] (verification not implemented)

Time = 3.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.33

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx =$$
$$-\frac{\left(35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 105\mathrm{i}-175\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 105\mathrm{i}-7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\right.}{35\,a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)\,\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,1\mathrm{i}\right)^7}$$

input
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
-((43*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*77i - 7*tan(c/2 + (d*x)/2)
^3 + tan(c/2 + (d*x)/2)^4*105i - 175*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x
)/2)^6*105i + 35*tan(c/2 + (d*x)/2)^7 - 13i)*2i)/(35*a^3*d*(tan(c/2 + (d*x
)/2) + 1i)*(tan(c/2 + (d*x)/2)*1i + 1)^7)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\int \frac{\cos(dx+c)}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx}{a^3}$$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - int(cos(c + d*x)/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c + d*
x)*i - 1),x))/a**3
```

3.146 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 121

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{10\sin(c+dx)}{21a^3d} - \frac{20\sin^3(c+dx)}{63a^3d} + \frac{2\sin^5(c+dx)}{21a^3d} + \frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3} + \frac{4i\cos^5(c+dx)}{21d\,(a^3+ia^3\tan(c+dx))}$$

output
```
10/21*sin(d*x+c)/a^3/d-20/63*sin(d*x+c)^3/a^3/d+2/21*sin(d*x+c)^5/a^3/d+1/
9*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^3+4/21*I*cos(d*x+c)^5/d/(a^3+I*a^3*t
an(d*x+c))
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.81

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\sec^3(c+dx)(-210-567\cos(2(c+dx))+162\cos(4(c+dx))+7\cos(6(c+dx))-378i\sin(2(c+dx))}{2016a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^3*(-210 - 567*Cos[2*(c + d*x)] + 162*Cos[4*(c + d*x)] + 7*Co
s[6*(c + d*x)] - (378*I)*Sin[2*(c + d*x)] + (216*I)*Sin[4*(c + d*x)] + (14
*I)*Sin[6*(c + d*x)]))/(2016*a^3*d*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 3983, 3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{2\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^2}dx}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3981}$$

$$\frac{2\left(\frac{5\int \cos^5(c+dx)dx}{7a^2}+\frac{2i\cos^5(c+dx)}{7d(a^2+ia^2\tan(c+dx))}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{5\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx}{7a^2}+\frac{2i\cos^5(c+dx)}{7d(a^2+ia^2\tan(c+dx))}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3113}$$

$$\frac{2\left(-\frac{5\int\left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d(a^2+ia^2\tan(c+dx))}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{5\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d(a^2+ia^2\tan(c+dx))}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}$$

input
```
Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/9)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3) + (2*((-5*(-Sin[c + d*
x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]^5/5))/(7*a^2*d) + (((2*I)/7)*Cos[
c + d*x]^5)/(d*(a^2 + I*a^2*Tan[c + d*x]))))/(3*a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

method	result
risch	$\frac{3ie^{-5i(dx+c)}}{64a^3d}+\frac{3ie^{-7i(dx+c)}}{224a^3d}+\frac{ie^{-9i(dx+c)}}{576a^3d}+\frac{9i\cos(dx+c)}{64a^3d}+\frac{21\sin(dx+c)}{64a^3d}+\frac{19i\cos(3dx+3c)}{192a^3d}+\frac{7\sin(3dx-}{64a^3d}$
derivativedivides	$\frac{-\frac{i}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{7}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{46i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{}{2(-i+\mathrm{ta}}}{}$
default	$\frac{-\frac{i}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{7}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{46i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{}{2(-i+\mathrm{ta}}}{}$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
3/64*I/a^3/d*exp(-5*I*(d*x+c))+3/224*I/a^3/d*exp(-7*I*(d*x+c))+1/576*I/a^3
/d*exp(-9*I*(d*x+c))+9/64*I/a^3/d*cos(d*x+c)+21/64*sin(d*x+c)/a^3/d+19/192
*I/a^3/d*cos(3*d*x+3*c)+7/64/a^3/d*sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(-21i\,e^{(12i\,dx+12i\,c)}-378i\,e^{(10i\,dx+10i\,c)}+945i\,e^{(8i\,dx+8i\,c)}+420i\,e^{(6i\,dx+6i\,c)}+189i\,e^{(4i\,dx+4i\,c)}+54i\,e^{(2i\,dx+2}\right.}{4032\,a^3d}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4032*(-21*I*e^(12*I*d*x + 12*I*c) - 378*I*e^(10*I*d*x + 10*I*c) + 945*I*
e^(8*I*d*x + 8*I*c) + 420*I*e^(6*I*d*x + 6*I*c) + 189*I*e^(4*I*d*x + 4*I*c
) + 54*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(-9*I*d*x - 9*I*c)/(a^3*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(105) = 210$.

Time = 0.38 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.19

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \begin{cases} \frac{(-811748818944ia^{18}d^6e^{28ic}e^{3idx}-14611478740992ia^{18}d^6e^{26ic}e^{idx}+36528696852480ia^{18}d^6e^{24ic}e^{-idx}+16234976378880ia^{18}d^6e^{22ic}e^{-3idx}}{155855773237248a^{21}d^7} \\ \frac{x(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1)e^{-9ic}}{64a^3} \end{cases}$$

input
```
integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise(((-811748818944*I*a**18*d**6*exp(28*I*c)*exp(3*I*d*x) - 14611478
740992*I*a**18*d**6*exp(26*I*c)*exp(I*d*x) + 36528696852480*I*a**18*d**6*e
xp(24*I*c)*exp(-I*d*x) + 16234976378880*I*a**18*d**6*exp(22*I*c)*exp(-3*I*
d*x) + 7305739370496*I*a**18*d**6*exp(20*I*c)*exp(-5*I*d*x) + 208735410585
6*I*a**18*d**6*exp(18*I*c)*exp(-7*I*d*x) + 270582939648*I*a**18*d**6*exp(1
6*I*c)*exp(-9*I*d*x))*exp(-25*I*c)/(155855773237248*a**21*d**7), Ne(a**21*
d**7*exp(25*I*c), 0)), (x*(exp(12*I*c) + 6*exp(10*I*c) + 15*exp(8*I*c) + 2
0*exp(6*I*c) + 15*exp(4*I*c) + 6*exp(2*I*c) + 1)*exp(-9*I*c)/(64*a**3), Tr
ue))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.41

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\frac{21\left(21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+36i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-19\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+\frac{3591\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-19656i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-56196\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+95760i\tan\left(\frac{1}{2}\right.}{}}{2016\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2016*(21*(21*tan(1/2*d*x + 1/2*c)^2 + 36*I*tan(1/2*d*x + 1/2*c) - 19)/(a
^3*(tan(1/2*d*x + 1/2*c) + I)^3) + (3591*tan(1/2*d*x + 1/2*c)^8 - 19656*I*
tan(1/2*d*x + 1/2*c)^7 - 56196*tan(1/2*d*x + 1/2*c)^6 + 95760*I*tan(1/2*d*
x + 1/2*c)^5 + 107730*tan(1/2*d*x + 1/2*c)^4 - 79464*I*tan(1/2*d*x + 1/2*c
)^3 - 38484*tan(1/2*d*x + 1/2*c)^2 + 10944*I*tan(1/2*d*x + 1/2*c) + 1615)/
(a^3*(tan(1/2*d*x + 1/2*c) - I)^9))/d
```

Mupad [B] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.55

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\left(63\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}189\mathrm{i}-273\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}63\mathrm{i}-378\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+\right.}{63\,a^3\,d\left(\tan\left(\right.\right.}$$

input
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
((51*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*39i + 235*tan(c/2 + (d*x)/2
)^3 + tan(c/2 + (d*x)/2)^4*450i - 306*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*
x)/2)^6*294i - 378*tan(c/2 + (d*x)/2)^7 - tan(c/2 + (d*x)/2)^8*63i - 273*t
an(c/2 + (d*x)/2)^9 - tan(c/2 + (d*x)/2)^10*189i + 63*tan(c/2 + (d*x)/2)^1
1 - 19i)*2i)/(63*a^3*d*(tan(c/2 + (d*x)/2) + 1i)^3*(tan(c/2 + (d*x)/2)*1i
+ 1)^9)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\int \frac{\cos(dx+c)^3}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx}{a^3}$$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - int(cos(c + d*x)**3/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c +
 d*x)*i - 1),x))/a**3
```

3.147 $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= \frac{56\sin(c+dx)}{99a^3d}-\frac{56\sin^3(c+dx)}{99a^3d}+\frac{56\sin^5(c+dx)}{165a^3d}\\&\quad-\frac{8\sin^7(c+dx)}{99a^3d}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}\\&\quad+\frac{16i\cos^7(c+dx)}{99d\,(a^3+ia^3\tan(c+dx))}\end{aligned}$$

output
```
56/99*sin(d*x+c)/a^3/d-56/99*sin(d*x+c)^3/a^3/d+56/165*sin(d*x+c)^5/a^3/d-
8/99*sin(d*x+c)^7/a^3/d+1/11*I*cos(d*x+c)^5/d/(a+I*a*tan(d*x+c))^3+16/99*I
*cos(d*x+c)^7/d/(a^3+I*a^3*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.86

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$=\frac{\sec^3(c+dx)(-5775-16632\cos(2(c+dx))+5940\cos(4(c+dx))+440\cos(6(c+dx))+27\cos(8(c+}{63360a^3d(-i+\tan}$$

input
```
Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^3*(-5775 - 16632*Cos[2*(c + d*x)] + 5940*Cos[4*(c + d*x)] +
440*Cos[6*(c + d*x)] + 27*Cos[8*(c + d*x)] - (11088*I)*Sin[2*(c + d*x)] +
(7920*I)*Sin[4*(c + d*x)] + (880*I)*Sin[6*(c + d*x)] + (72*I)*Sin[8*(c + d
*x)]))/(63360*a^3*d*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 3983, 3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sec(c+dx)^5(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3983}$$

$$\frac{8\int \frac{\cos^5(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{8\int \frac{1}{\sec(c+dx)^5(i\tan(c+dx)a+a)^2}dx}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{ 3981}$$

$$\frac{8\left(\frac{7\int \cos^7(c+dx)dx}{9a^2}+\frac{2i\cos^7(c+dx)}{9d(a^2+ia^2\tan(c+dx))}\right)}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{8\left(\frac{7\int \sin\left(c+dx+\frac{\pi}{2}\right)^7dx}{9a^2}+\frac{2i\cos^7(c+dx)}{9d(a^2+ia^2\tan(c+dx))}\right)}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 3113$$

$$\frac{8\left(-\frac{7\int(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1)d(-\sin(c+dx))}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d(a^2+ia^2\tan(c+dx))}\right)}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{8\left(-\frac{7\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d(a^2+ia^2\tan(c+dx))}\right)}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}$$

input
```
Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/11)*Cos[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^3) + (8*((-7*(-Sin[c + d
*x] + Sin[c + d*x]^3 - (3*Sin[c + d*x]^5)/5 + Sin[c + d*x]^7/7))/(9*a^2*d)
 + (((2*I)/9)*Cos[c + d*x]^7)/(d*(a^2 + I*a^2*Tan[c + d*x]))))/(11*a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12

method	result
risch	$\frac{i e^{-7 i (d x+c)}}{64 a^{3} d}+\frac{i e^{-9 i (d x+c)}}{288 a^{3} d}+\frac{i e^{-11 i (d x+c)}}{2816 a^{3} d}+\frac{7 i \cos (d x+c)}{64 a^{3} d}+\frac{21 \sin (d x+c)}{64 a^{3} d}+\frac{11 i \cos (5 d x+5 c)}{256 a^{3} d}+\frac{57 \sin (5 d x \cdots}{1280 a^{3} \cdots}$
derivativedivides	$\frac{217 i}{6\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{6}}+\frac{303 i}{64\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{169 i}{8\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{i}{16\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+i\right)^{4}}-\frac{5 i}{32\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+i\right)^{2}}-\frac{\cdots}{11\left(-i \cdots\right.}$
default	$\frac{217 i}{6\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{6}}+\frac{303 i}{64\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{169 i}{8\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{i}{16\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+i\right)^{4}}-\frac{5 i}{32\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+i\right)^{2}}-\frac{\cdots}{11\left(-i \cdots\right.}$

input
```
int(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/64*I/a^3/d*exp(-7*I*(d*x+c))+1/288*I/a^3/d*exp(-9*I*(d*x+c))+1/2816*I/a^
3/d*exp(-11*I*(d*x+c))+7/64*I/a^3/d*cos(d*x+c)+21/64*sin(d*x+c)/a^3/d+11/2
56*I/a^3/d*cos(5*d*x+5*c)+57/1280/a^3/d*sin(5*d*x+5*c)+31/384*I/a^3/d*cos(
3*d*x+3*c)+13/128/a^3/d*sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.77

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(-99i\,e^{(16i\,dx+16i\,c)} - 1320i\,e^{(14i\,dx+14i\,c)} - 13860i\,e^{(12i\,dx+12i\,c)} + 27720i\,e^{(10i\,dx+10i\,c)} + 11550i\,e^{(8i\,dx+8i\,c)} + \right.}{126720\,a^3d}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/126720*(-99*I*e^(16*I*d*x + 16*I*c) - 1320*I*e^(14*I*d*x + 14*I*c) - 138
60*I*e^(12*I*d*x + 12*I*c) + 27720*I*e^(10*I*d*x + 10*I*c) + 11550*I*e^(8*
I*d*x + 8*I*c) + 5544*I*e^(6*I*d*x + 6*I*c) + 1980*I*e^(4*I*d*x + 4*I*c) +
 440*I*e^(2*I*d*x + 2*I*c) + 45*I)*e^(-11*I*d*x - 11*I*c)/(a^3*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(122) = 244.

Time = 0.46 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.40

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \begin{cases} \frac{\left(-626985510622986240ia^{24}d^8e^{41ic}e^{5idx} - 8359806808306483200ia^{24}d^8e^{39ic}e^{3idx} - 8777797148721807360 0ia^{24}d^8e^{37ic}e^{idx} + 1755559429744\right.}{} \\ \frac{x\left(e^{16ic}+8e^{14ic}+28e^{12ic}+56e^{10ic}+70e^{8ic}+56e^{6ic}+28e^{4ic}+8e^{2ic}+1\right)e^{-11ic}}{256a^3} \end{cases}$$

input
```
integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise(((-626985510622986240*I*a**24*d**8*exp(41*I*c)*exp(5*I*d*x) - 83
59806808306483200*I*a**24*d**8*exp(39*I*c)*exp(3*I*d*x) - 8777797148721807
3600*I*a**24*d**8*exp(37*I*c)*exp(I*d*x) + 175555942974436147200*I*a**24*d
**8*exp(35*I*c)*exp(-I*d*x) + 73148309572681728000*I*a**24*d**8*exp(33*I*c
)*exp(-3*I*d*x) + 35111188594887229440*I*a**24*d**8*exp(31*I*c)*exp(-5*I*d
*x) + 12539710212459724800*I*a**24*d**8*exp(29*I*c)*exp(-7*I*d*x) + 278660
2269435494400*I*a**24*d**8*exp(27*I*c)*exp(-9*I*d*x) + 284993413919539200*
I*a**24*d**8*exp(25*I*c)*exp(-11*I*d*x))*exp(-36*I*c)/(802541453597422387 2
00*a**27*d**9), Ne(a**27*d**9*exp(36*I*c), 0)), (x*(exp(16*I*c) + 8*exp(14
*I*c) + 28*exp(12*I*c) + 56*exp(10*I*c) + 70*exp(8*I*c) + 56*exp(6*I*c) +
28*exp(4*I*c) + 8*exp(2*I*c) + 1)*exp(-11*I*c)/(256*a**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.60

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\frac{33\left(555\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1920i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2710\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1760i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+463\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^5}+\frac{108405\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-784080i}{}}{}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/63360*(33*(555*tan(1/2*d*x + 1/2*c)^4 + 1920*I*tan(1/2*d*x + 1/2*c)^3 -
2710*tan(1/2*d*x + 1/2*c)^2 - 1760*I*tan(1/2*d*x + 1/2*c) + 463)/(a^3*(tan
(1/2*d*x + 1/2*c) + I)^5) + (108405*tan(1/2*d*x + 1/2*c)^10 - 784080*I*tan
(1/2*d*x + 1/2*c)^9 - 2901195*tan(1/2*d*x + 1/2*c)^8 + 6652800*I*tan(1/2*d
*x + 1/2*c)^7 + 10407474*tan(1/2*d*x + 1/2*c)^6 - 11435424*I*tan(1/2*d*x +
 1/2*c)^5 - 8949270*tan(1/2*d*x + 1/2*c)^4 + 4899840*I*tan(1/2*d*x + 1/2*c
)^3 + 1816265*tan(1/2*d*x + 1/2*c)^2 - 411664*I*tan(1/2*d*x + 1/2*c) - 472
79)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^11))/d
```

Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.98

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(\frac{\cos(7\,c+7\,d\,x)}{64}+\frac{\cos(9\,c+9\,d\,x)}{288}+\frac{\cos(11\,c+11\,d\,x)}{2816}-\frac{\sin(7\,c+7\,d\,x)\,1\mathrm{i}}{64}-\frac{\sin(9\,c+9\,d\,x)\,1\mathrm{i}}{288}-\frac{\sin(11\,c+11\,d\,x)\,1\mathrm{i}}{2816}+\frac{\sqrt{224}\,\cos(5\,c+5\,d}{12}\right.}{a^3\,d}$$

input
```
int(cos(c + d*x)^5/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
((cos(7*c + 7*d*x)/64 + cos(9*c + 9*d*x)/288 + cos(11*c + 11*d*x)/2816 - (
sin(7*c + 7*d*x)*1i)/64 - (sin(9*c + 9*d*x)*1i)/288 - (sin(11*c + 11*d*x)*
1i)/2816 + (224^(1/2)*cos(5*c + atanh(57/55)*1i + 5*d*x)*1i)/1280 + (560^(
1/2)*cos(3*c + atanh(39/31)*1i + 3*d*x)*1i)/384 + (2^(1/2)*cos(c + atanh(3
)*1i + d*x)*7i)/32)*1i)/(a^3*d)
```

Reduce [F]

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\int \frac{\cos(dx+c)^5}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx}{a^3}$$

input
```
int(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - int(cos(c + d*x)**5/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c +
 d*x)*i - 1),x))/a**3
```

3.148 $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{4i(a-ia\tan(c+dx))^7}{7a^{11}d} - \frac{i(a-ia\tan(c+dx))^8}{2a^{12}d} + \frac{i(a-ia\tan(c+dx))^9}{9a^{13}d}$$

output
```
4/7*I*(a-I*a*tan(d*x+c))^7/a^11/d-1/2*I*(a-I*a*tan(d*x+c))^8/a^12/d+1/9*I*
(a-I*a*tan(d*x+c))^9/a^13/d
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.54

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{(i+\tan(c+dx))^7\left(-23-35i\tan(c+dx)+14\tan^2(c+dx)\right)}{126a^4d}$$

input
```
Integrate[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I + Tan[c + d*x])^7*(-23 - (35*I)*Tan[c + d*x] + 14*Tan[c + d*x]^2))/(12
6*a^4*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{14}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^6(i\tan(c+dx)a+a)^2d(ia\tan(c+dx))}{a^{13}d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left((a-ia\tan(c+dx))^8-4a(a-ia\tan(c+dx))^7+4a^2(a-ia\tan(c+dx))^6\right)d(ia\tan(c+dx))}{a^{13}d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{4}{7}a^2(a-ia\tan(c+dx))^7-\frac{1}{9}(a-ia\tan(c+dx))^9+\frac{1}{2}a(a-ia\tan(c+dx))^8\right)}{a^{13}d}$$

input
```
Int[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-I)*((-4*a^2*(a - I*a*Tan[c + d*x])^7)/7 + (a*(a - I*a*Tan[c + d*x])^8)/
2 - (a - I*a*Tan[c + d*x])^9/9))/(a^13*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

method	result
risch	$\frac{128 i \left(36 \,{\mathrm e}^{4 i \left(d x +c \right)}+9 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{63 d \,a^{4} \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{9}}$
derivativedivides	$\frac{\tan \left(d x +c \right)+\frac{\tan \left(d x +c \right)^{9}}{9}+\frac{i \tan \left(d x +c \right)^{8}}{2}-\frac{4 \tan \left(d x +c \right)^{7}}{7}+\frac{2 i \tan \left(d x +c \right)^{6}}{3}-2 \tan \left(d x +c \right)^{5}-i \tan \left(d x +c \right)^{4}-\frac{4 \tan \left(d x +c \right)^{3}}{3}-2 i \mathrm{ta}}{a^{4} d}$
default	$\frac{\tan \left(d x +c \right)+\frac{\tan \left(d x +c \right)^{9}}{9}+\frac{i \tan \left(d x +c \right)^{8}}{2}-\frac{4 \tan \left(d x +c \right)^{7}}{7}+\frac{2 i \tan \left(d x +c \right)^{6}}{3}-2 \tan \left(d x +c \right)^{5}-i \tan \left(d x +c \right)^{4}-\frac{4 \tan \left(d x +c \right)^{3}}{3}-2 i \mathrm{ta}}{a^{4} d}$

input
```
int(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
128/63*I*(36*exp(4*I*(d*x+c))+9*exp(2*I*(d*x+c))+1)/d/a^4/(exp(2*I*(d*x+c)
)+1)^9
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(64) = 128$.

Time = 0.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.05

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{128\left(-36i\,e^{(4i\,dx+4i\,c)}-9i\,e^{(2i}\right.}{63\left(a^4de^{(18i\,dx+18i\,c)}+9\,a^4de^{(16i\,dx+16i\,c)}+36\,a^4de^{(14i\,dx+14i\,c)}+84\,a^4de^{(12i\,dx+12i\,c)}+126\,a^4de^{(10i\,dx+10i\,c)}\right.}$$

input
```
integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-128/63*(-36*I*e^(4*I*d*x + 4*I*c) - 9*I*e^(2*I*d*x + 2*I*c) - I)/(a^4*d*e
^(18*I*d*x + 18*I*c) + 9*a^4*d*e^(16*I*d*x + 16*I*c) + 36*a^4*d*e^(14*I*d*
x + 14*I*c) + 84*a^4*d*e^(12*I*d*x + 12*I*c) + 126*a^4*d*e^(10*I*d*x + 10*
I*c) + 126*a^4*d*e^(8*I*d*x + 8*I*c) + 84*a^4*d*e^(6*I*d*x + 6*I*c) + 36*a
^4*d*e^(4*I*d*x + 4*I*c) + 9*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F]

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^{14}(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**14/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**14/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
 + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{14\,\tan\left(dx+c\right)^{9}+63i\,\tan\left(dx+c\right)^{8}-72\,\tan\left(dx+c\right)^{7}+84i\,\tan\left(dx+c\right)^{6}-252\,\tan\left(dx+c\right)^{5}-126i}{126\,a^{4}d}$$

input
```
integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/126*(14*tan(d*x + c)^9 + 63*I*tan(d*x + c)^8 - 72*tan(d*x + c)^7 + 84*I*
tan(d*x + c)^6 - 252*tan(d*x + c)^5 - 126*I*tan(d*x + c)^4 - 168*tan(d*x +
 c)^3 - 252*I*tan(d*x + c)^2 + 126*tan(d*x + c))/(a^4*d)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{14\,\tan\left(dx+c\right)^{9}+63i\,\tan\left(dx+c\right)^{8}-72\,\tan\left(dx+c\right)^{7}+84i\,\tan\left(dx+c\right)^{6}-252\,\tan\left(dx+c\right)^{5}-126i}{126\,a^{4}d}$$

input
```
integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/126*(14*tan(d*x + c)^9 + 63*I*tan(d*x + c)^8 - 72*tan(d*x + c)^7 + 84*I*
tan(d*x + c)^6 - 252*tan(d*x + c)^5 - 126*I*tan(d*x + c)^4 - 168*tan(d*x +
 c)^3 - 252*I*tan(d*x + c)^2 + 126*tan(d*x + c))/(a^4*d)
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.46

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\cos(c+d\,x)^9\,105\mathrm{i}+128\,\sin(c+d\,x)\,\cos(c+d\,x)^8+64\,\sin(c+d\,x)\,\cos(c+d\,x)^6+48\,\sin(c+d\,x)\,\mathrm{c}}{126\,a^4\,d\cos}$$

input
```
int(1/(cos(c + d*x)^14*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
(cos(c + d*x)*63i + 14*sin(c + d*x) - 128*cos(c + d*x)^2*sin(c + d*x) + 48
*cos(c + d*x)^4*sin(c + d*x) + 64*cos(c + d*x)^6*sin(c + d*x) + 128*cos(c
+ d*x)^8*sin(c + d*x) - cos(c + d*x)^3*168i + cos(c + d*x)^9*105i)/(126*a^
4*d*cos(c + d*x)^9)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.24

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{-7\cos(dx+c)\sin(dx+c)^8 i+28\cos(dx+c)\sin(dx+c)^6 i-42\cos(dx+c)\sin(dx+c)^4 i+196\cos}{126\cos(dx+c)\,a^4d\,(\sin(d}$$

input
```
int(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x)
```

output
```
( - 7*cos(c + d*x)*sin(c + d*x)**8*i + 28*cos(c + d*x)*sin(c + d*x)**6*i -
 42*cos(c + d*x)*sin(c + d*x)**4*i + 196*cos(c + d*x)*sin(c + d*x)**2*i -
112*cos(c + d*x)*i + 128*sin(c + d*x)**9 - 576*sin(c + d*x)**7 + 1008*sin(
c + d*x)**5 - 672*sin(c + d*x)**3 + 126*sin(c + d*x))/(126*cos(c + d*x)*a*
*4*d*(sin(c + d*x)**8 - 4*sin(c + d*x)**6 + 6*sin(c + d*x)**4 - 4*sin(c +
d*x)**2 + 1))
```

3.149 $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx=\frac{i(a-ia\tan(c+dx))^6}{3a^{10}d}-\frac{i(a-ia\tan(c+dx))^7}{7a^{11}d}$$

output
```
1/3*I*(a-I*a*tan(d*x+c))^6/a^10/d-1/7*I*(a-I*a*tan(d*x+c))^7/a^11/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx=\frac{(i+\tan(c+dx))^6(-4i+3\tan(c+dx))}{21a^4d}$$

input
```
Integrate[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I + Tan[c + d*x])^6*(-4*I + 3*Tan[c + d*x]))/(21*a^4*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{12}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^5(i\tan(c+dx)a+a)d(ia\tan(c+dx))}{a^{11}d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(2a(a-ia\tan(c+dx))^5-(a-ia\tan(c+dx))^6\right)d(ia\tan(c+dx))}{a^{11}d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{1}{7}(a-ia\tan(c+dx))^7-\frac{1}{3}a(a-ia\tan(c+dx))^6\right)}{a^{11}d}$$

input
```
Int[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-I)*(-1/3*(a*(a - I*a*Tan[c + d*x])^6) + (a - I*a*Tan[c + d*x])^7/7))/(a
^11*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{64i\left(7\,{\mathrm e}^{2i(dx+c)}+1\right)}{21d\,a^{4}\left({\mathrm e}^{2i(dx+c)}+1\right)^{7}}$	36
derivativedivides	$\frac{\tan(dx+c)+\frac{\tan(dx+c)^{7}}{7}+\frac{2i\tan(dx+c)^{6}}{3}-\tan(dx+c)^{5}-\frac{5\tan(dx+c)^{3}}{3}-2i\tan(dx+c)^{2}}{a^{4}d}$	67
default	$\frac{\tan(dx+c)+\frac{\tan(dx+c)^{7}}{7}+\frac{2i\tan(dx+c)^{6}}{3}-\tan(dx+c)^{5}-\frac{5\tan(dx+c)^{3}}{3}-2i\tan(dx+c)^{2}}{a^{4}d}$	67

input
```
int(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
64/21*I*(7*exp(2*I*(d*x+c))+1)/d/a^4/(exp(2*I*(d*x+c))+1)^7
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(43) = 86$.

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.31

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{64\left(-7i\,e^{(2i\,dx+2i\,c)}-i\right)}{21\left(a^4de^{(14i\,dx+14i\,c)}+7\,a^4de^{(12i\,dx+12i\,c)}+21\,a^4de^{(10i\,dx+10i\,c)}+35\,a^4de^{(8i\,dx+8i\,c)}+35\,a^4de^{(6i\,dx+6i\,c)}+2\right.}$$

input
```
integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-64/21*(-7*I*e^(2*I*d*x + 2*I*c) - I)/(a^4*d*e^(14*I*d*x + 14*I*c) + 7*a^4
*d*e^(12*I*d*x + 12*I*c) + 21*a^4*d*e^(10*I*d*x + 10*I*c) + 35*a^4*d*e^(8*
I*d*x + 8*I*c) + 35*a^4*d*e^(6*I*d*x + 6*I*c) + 21*a^4*d*e^(4*I*d*x + 4*I*
c) + 7*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F]

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^{12}(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**12/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**12/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
 + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{3\,\tan\left(dx+c\right)^7+14i\,\tan\left(dx+c\right)^6-21\,\tan\left(dx+c\right)^5-35\,\tan\left(dx+c\right)^3-42i\,\tan\left(dx+c\right)^2+21\,\tan}{21\,a^4d}$$

input
```
integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/21*(3*tan(d*x + c)^7 + 14*I*tan(d*x + c)^6 - 21*tan(d*x + c)^5 - 35*tan(
d*x + c)^3 - 42*I*tan(d*x + c)^2 + 21*tan(d*x + c))/(a^4*d)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{3\,\tan\left(dx+c\right)^7+14i\,\tan\left(dx+c\right)^6-21\,\tan\left(dx+c\right)^5-35\,\tan\left(dx+c\right)^3-42i\,\tan\left(dx+c\right)^2+21\,\tan}{21\,a^4d}$$

input
```
integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/21*(3*tan(d*x + c)^7 + 14*I*tan(d*x + c)^6 - 21*tan(d*x + c)^5 - 35*tan(
d*x + c)^3 - 42*I*tan(d*x + c)^2 + 21*tan(d*x + c))/(a^4*d)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\sin(c+d\,x)\,\left(21\cos(c+d\,x)^6-\cos(c+d\,x)^5\,\sin(c+d\,x)\,42\mathrm{i}-35\cos(c+d\,x)^4\sin(c+d\,x)^2-21\cos\right.}{21\,a^4\,d\cos(c+d\,x)^7}$$

input
```
int(1/(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
(sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^5*14i - cos(c + d*x)^5*sin(c + d*
x)*42i + 21*cos(c + d*x)^6 + 3*sin(c + d*x)^6 - 21*cos(c + d*x)^2*sin(c +
d*x)^4 - 35*cos(c + d*x)^4*sin(c + d*x)^2))/(21*a^4*d*cos(c + d*x)^7)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.67

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{4\cos(dx+c)\sin(dx+c)^6\,i-12\cos(dx+c)\sin(dx+c)^4\,i-30\cos(dx+c)\sin(dx+c)^2\,i+24\cos(dx}{21\cos(dx+c)\,a^4d\,\left(\sin(dx+c)^6-3\sin(dx+c\right.}$$

input
```
int(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(4*cos(c + d*x)*sin(c + d*x)**6*i - 12*cos(c + d*x)*sin(c + d*x)**4*i - 30
*cos(c + d*x)*sin(c + d*x)**2*i + 24*cos(c + d*x)*i + 32*sin(c + d*x)**7 -
 112*sin(c + d*x)**5 + 98*sin(c + d*x)**3 - 21*sin(c + d*x))/(21*cos(c + d
*x)*a**4*d*(sin(c + d*x)**6 - 3*sin(c + d*x)**4 + 3*sin(c + d*x)**2 - 1))
```

3.150 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i(a-ia\tan(c+dx))^5}{5a^9d}$$

output
```
1/5*I*(a-I*a*tan(d*x+c))^5/a^9/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 58 vs. 2(27) = 54.

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\tan(c+dx)\left(5-10i\tan(c+dx)-10\tan^2(c+dx)+5i\tan^3(c+dx)+\tan^4(c+dx)\right)}{5a^4d}$$

input
```
Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(Tan[c + d*x]*(5 - (10*I)*Tan[c + d*x] - 10*Tan[c + d*x]^2 + (5*I)*Tan[c +
 d*x]^3 + Tan[c + d*x]^4))/(5*a^4*d)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{3042, 3968, 17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{10}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^4d(ia\tan(c+dx))}{a^9d}$$

$$\Big\downarrow \text{17}$$

$$\frac{i(a-ia\tan(c+dx))^5}{5a^9d}$$

input
```
Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/5)*(a - I*a*Tan[c + d*x])^5)/(a^9*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{(\tan(dx+c)+i)^5}{5a^4d}$	20
default	$\frac{(\tan(dx+c)+i)^5}{5a^4d}$	20
risch	$\frac{32i}{5d\,a^4\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}$	23

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/5/a^4/d*(tan(d*x+c)+I)^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(21) = 42$.

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.11

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{32i}{5\left(a^4de^{(10i\,dx+10i\,c)}+5\,a^4de^{(8i\,dx+8i\,c)}+10\,a^4de^{(6i\,dx+6i\,c)}+10\,a^4de^{(4i\,dx+4i\,c)}+5\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
32/5*I/(a^4*d*e^(10*I*d*x + 10*I*c) + 5*a^4*d*e^(8*I*d*x + 8*I*c) + 10*a^4
*d*e^(6*I*d*x + 6*I*c) + 10*a^4*d*e^(4*I*d*x + 4*I*c) + 5*a^4*d*e^(2*I*d*x
 + 2*I*c) + a^4*d)
```

Sympy [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^{10}(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**10/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
 + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(21) = 42$.

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\tan\left(dx+c\right)^5 + 5i\tan\left(dx+c\right)^4 - 10\tan\left(dx+c\right)^3 - 10i\tan\left(dx+c\right)^2 + 5\tan\left(dx+c\right)}{5\,a^4 d}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/5*(tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 - 10*tan(d*x + c)^3 - 10*I*tan(d*
x + c)^2 + 5*tan(d*x + c))/(a^4*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(21) = 42$.

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\tan\left(dx+c\right)^5 + 5i\tan\left(dx+c\right)^4 - 10\tan\left(dx+c\right)^3 - 10i\tan\left(dx+c\right)^2 + 5\tan\left(dx+c\right)}{5\,a^4 d}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/5*(tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 - 10*tan(d*x + c)^3 - 10*I*tan(d*
x + c)^2 + 5*tan(d*x + c))/(a^4*d)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\sin(c+dx)\,\left(5\cos(c+dx)^4-\cos(c+dx)^3\,\sin(c+dx)\,10\mathrm{i}-10\cos(c+dx)^2\sin(c+dx)^2+\cos(c-\right.}{5\,a^4\,d\cos(c+dx)^5}$$

input
```
int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
(sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^3*5i - cos(c + d*x)^3*sin(c + d*x
)*10i + 5*cos(c + d*x)^4 + sin(c + d*x)^4 - 10*cos(c + d*x)^2*sin(c + d*x)
^2))/(5*a^4*d*cos(c + d*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 4.07

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{7\cos(dx+c)\sin(dx+c)^4i+6\cos(dx+c)\sin(dx+c)^2i-8\cos(dx+c)\,i+16\sin(dx+c)^5-20\sin(}{5\cos(dx+c)\,a^4d\left(\sin(dx+c)^4-2\sin(dx+c)^2+1\right)}$$

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(7*cos(c + d*x)*sin(c + d*x)**4*i + 6*cos(c + d*x)*sin(c + d*x)**2*i - 8*c
os(c + d*x)*i + 16*sin(c + d*x)**5 - 20*sin(c + d*x)**3 + 5*sin(c + d*x))/
(5*cos(c + d*x)*a**4*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.151 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{8x}{a^4}+\frac{8i\log(\cos(c+dx))}{a^4d}-\frac{4\tan(c+dx)}{a^4d}-\frac{i(a-ia\tan(c+dx))^2}{a^6d}-\frac{i(a-ia\tan(c+dx))^3}{3a^7d}$$

output
```
8*x/a^4+8*I*ln(cos(d*x+c))/a^4/d-4*tan(d*x+c)/a^4/d-I*(a-I*a*tan(d*x+c))^2
/a^6/d-1/3*I*(a-I*a*tan(d*x+c))^3/a^7/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{-24i\log(i-\tan(c+dx))-21\tan(c+dx)+6i\tan^2(c+dx)+\tan^3(c+dx)}{3a^4d}$$

input
```
Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-24*I)*Log[I - Tan[c + d*x]] - 21*Tan[c + d*x] + (6*I)*Tan[c + d*x]^2 +
Tan[c + d*x]^3)/(3*a^4*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^8}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int \frac{(a-ia\tan(c+dx))^3}{i\tan(c+dx)a+a}d(ia\tan(c+dx))}{a^7d}$$
$$\Big\downarrow \text{49}$$
$$-\frac{i\int \left(\frac{8a^3}{i\tan(c+dx)a+a}-4a^2-2(a-ia\tan(c+dx))a-(a-ia\tan(c+dx))^2\right)d(ia\tan(c+dx))}{a^7d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{i\left(-4ia^3\tan(c+dx)+8a^3\log(a+ia\tan(c+dx))+a(a-ia\tan(c+dx))^2+\frac{1}{3}(a-ia\tan(c+dx))^3\right)}{a^7d}$$

input
```
Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-I)*(8*a^3*Log[a + I*a*Tan[c + d*x]] - (4*I)*a^3*Tan[c + d*x] + a*(a - I
*a*Tan[c + d*x])^2 + (a - I*a*Tan[c + d*x])^3/3))/(a^7*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{7\tan(dx+c)}{a^4d}+\frac{\tan(dx+c)^3}{3a^4d}+\frac{2i\tan(dx+c)^2}{a^4d}+\frac{8\arctan(\tan(dx+c))}{a^4d}-\frac{4i\ln\left(1+\tan(dx+c)^2\right)}{a^4d}$	84
default	$-\frac{7\tan(dx+c)}{a^4d}+\frac{\tan(dx+c)^3}{3a^4d}+\frac{2i\tan(dx+c)^2}{a^4d}+\frac{8\arctan(\tan(dx+c))}{a^4d}-\frac{4i\ln\left(1+\tan(dx+c)^2\right)}{a^4d}$	84
risch	$\frac{16x}{a^4}+\frac{16c}{a^4d}-\frac{4i\left(6\,\mathrm{e}^{4i(dx+c)}+15\,\mathrm{e}^{2i(dx+c)}+11\right)}{3d\,a^4\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{8i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^4d}$	84

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-7*tan(d*x+c)/a^4/d+1/3/a^4/d*tan(d*x+c)^3+2*I/a^4/d*tan(d*x+c)^2+8/a^4/d*
arctan(tan(d*x+c))-4*I/a^4/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.73

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{4\left(12\,dxe^{(6i\,dx+6i\,c)}+12\,dx+6\,(6\,dx-i)e^{(4i\,dx+4i\,c)}+3\,(12\,dx-5i)e^{(2i\,dx+2i\,c)}-6\left(-i\,e^{(6i\,dx+6i\,c)}-3i\,e^{(4i}\right.\right.}{3\left(a^4de^{(6i\,dx+6i\,c)}+3\,a^4de^{(4i\,dx+4i\,c)}+3\,a^4de^{(2i\,dx+2i\,c)}+\right.}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
4/3*(12*d*x*e^(6*I*d*x + 6*I*c) + 12*d*x + 6*(6*d*x - I)*e^(4*I*d*x + 4*I*
c) + 3*(12*d*x - 5*I)*e^(2*I*d*x + 2*I*c) - 6*(-I*e^(6*I*d*x + 6*I*c) - 3*
I*e^(4*I*d*x + 4*I*c) - 3*I*e^(2*I*d*x + 2*I*c) - I)*log(e^(2*I*d*x + 2*I*
c) + 1) - 11*I)/(a^4*d*e^(6*I*d*x + 6*I*c) + 3*a^4*d*e^(4*I*d*x + 4*I*c) +
 3*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^8(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**8/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
+ d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.59

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{\tan(dx+c)^3+6i\tan(dx+c)^2-21\tan(dx+c)}{a^4}-\frac{24i\log(i\tan(dx+c)+1)}{a^4}}{3\,d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/3*((tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 21*tan(d*x + c))/a^4 - 24*I*lo
g(I*tan(d*x + c) + 1)/a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80

$$\begin{aligned}&\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx\\&=-\frac{8i\,\log\left(\tan\left(dx+c\right)-i\right)}{a^4d}\\&\quad+\frac{a^8d^2\tan\left(dx+c\right)^3+6i\,a^8d^2\tan\left(dx+c\right)^2-21\,a^8d^2\tan\left(dx+c\right)}{3\,a^{12}d^3}\end{aligned}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
-8*I*log(tan(d*x + c) - I)/(a^4*d) + 1/3*(a^8*d^2*tan(d*x + c)^3 + 6*I*a^8
*d^2*tan(d*x + c)^2 - 21*a^8*d^2*tan(d*x + c))/(a^12*d^3)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\frac{7\tan(c+d\,x)}{a^4}-\frac{\tan(c+d\,x)^3}{3\,a^4}+\frac{\ln(\tan(c+d\,x)-\mathrm{i})\,8\mathrm{i}}{a^4}-\frac{\tan(c+d\,x)^2\,2\mathrm{i}}{a^4}}{d}$$

input
```
int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
-((log(tan(c + d*x) - 1i)*8i)/a^4 + (7*tan(c + d*x))/a^4 - (tan(c + d*x)^2
*2i)/a^4 - tan(c + d*x)^3/(3*a^4))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 413, normalized size of antiderivative = 4.59

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{-6\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8-8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7 i-28\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+56\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 i+70\tan\left(\right.\right.}{}$$

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x)
```

output
```
( - 6*cos(c + d*x)*log(tan((c + d*x)/2)**8 - 8*tan((c + d*x)/2)**7*i - 28*
tan((c + d*x)/2)**6 + 56*tan((c + d*x)/2)**5*i + 70*tan((c + d*x)/2)**4 -
56*tan((c + d*x)/2)**3*i - 28*tan((c + d*x)/2)**2 + 8*tan((c + d*x)/2)*i +
 1)*sin(c + d*x)**2*i + 6*cos(c + d*x)*log(tan((c + d*x)/2)**8 - 8*tan((c
+ d*x)/2)**7*i - 28*tan((c + d*x)/2)**6 + 56*tan((c + d*x)/2)**5*i + 70*ta
n((c + d*x)/2)**4 - 56*tan((c + d*x)/2)**3*i - 28*tan((c + d*x)/2)**2 + 8*
tan((c + d*x)/2)*i + 1)*i + 24*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(
c + d*x)**2*i - 24*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*i + 24*cos(c + d
*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*i - 24*cos(c + d*x)*log(tan(
(c + d*x)/2) + 1)*i + 2*cos(c + d*x)*sin(c + d*x)**2*i - 8*cos(c + d*x)*i
- 22*sin(c + d*x)**3 + 21*sin(c + d*x))/(3*cos(c + d*x)*a**4*d*(sin(c + d*
x)**2 - 1))
```

3.152 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{4x}{a^4}-\frac{4i\log(\cos(c+dx))}{a^4d}+\frac{\tan(c+dx)}{a^4d}+\frac{4i}{d\,(a^4+ia^4\tan(c+dx))}$$

output
```
-4*x/a^4-4*I*ln(cos(d*x+c))/a^4/d+tan(d*x+c)/a^4/d+4*I/d/(a^4+I*a^4*tan(d*
x+c))
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{i\left(-4\log(i-\tan(c+dx))+i\tan(c+dx)+\frac{4i}{-i+\tan(c+dx)}\right)}{a^4d}$$

input
```
Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-I)*(-4*Log[I - Tan[c + d*x]] + I*Tan[c + d*x] + (4*I)/(-I + Tan[c + d*x
])))/(a^4*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^6}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int \frac{(a-ia\tan(c+dx))^2}{(i\tan(c+dx)a+a)^2}d(ia\tan(c+dx))}{a^5d}$$
$$\Big\downarrow \text{49}$$
$$-\frac{i\int \left(\frac{4a^2}{(i\tan(c+dx)a+a)^2}-\frac{4a}{i\tan(c+dx)a+a}+1\right)d(ia\tan(c+dx))}{a^5d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{i\left(-\frac{4a^2}{a+ia\tan(c+dx)}+ia\tan(c+dx)-4a\log(a+ia\tan(c+dx))\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-I)*(-4*a*Log[a + I*a*Tan[c + d*x]] + I*a*Tan[c + d*x] - (4*a^2)/(a + I*
a*Tan[c + d*x])))/(a^5*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{\tan(dx+c)+\frac{4}{-i+\tan(dx+c)}+4i\ln(-i+\tan(dx+c))}{d\,a^4}$	41
default	$\frac{\tan(dx+c)+\frac{4}{-i+\tan(dx+c)}+4i\ln(-i+\tan(dx+c))}{d\,a^4}$	41
risch	$\frac{2i\mathrm{e}^{-2i(dx+c)}}{a^4d}-\frac{8x}{a^4}-\frac{8c}{a^4d}+\frac{2i}{d\,a^4\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{4i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^4d}$	78

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^4*(tan(d*x+c)+4/(-I+tan(d*x+c))+4*I*ln(-I+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.62

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2\left(4\,dxe^{(4i\,dx+4i\,c)}+2\,(2\,dx-i)e^{(2i\,dx+2i\,c)}+2\left(i\,e^{(4i\,dx+4i\,c)}+i\,e^{(2i\,dx+2i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\right)}{a^4de^{(4i\,dx+4i\,c)}+a^4de^{(2i\,dx+2i\,c)}}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-2*(4*d*x*e^(4*I*d*x + 4*I*c) + 2*(2*d*x - I)*e^(2*I*d*x + 2*I*c) + 2*(I*e
^(4*I*d*x + 4*I*c) + I*e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) -
 I)/(a^4*d*e^(4*I*d*x + 4*I*c) + a^4*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^6(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**6/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
+ d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.51

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{4\left(\tan(dx+c)^2-2i\tan(dx+c)-1\right)}{a^4\tan(dx+c)^3-3i\,a^4\tan(dx+c)^2-3\,a^4\tan(dx+c)+i\,a^4}+\frac{4i\log(i\tan(dx+c)+1)}{a^4}+\frac{\tan(dx+c)}{a^4}}{d}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
(4*(tan(d*x + c)^2 - 2*I*tan(d*x + c) - 1)/(a^4*tan(d*x + c)^3 - 3*I*a^4*t
an(d*x + c)^2 - 3*a^4*tan(d*x + c) + I*a^4) + 4*I*log(I*tan(d*x + c) + 1)/
a^4 + tan(d*x + c)/a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{4i\,\log\left(\tan\left(dx+c\right)-i\right)}{a^4d} + \frac{\tan\left(dx+c\right)}{a^4d} + \frac{4}{a^4d(\tan\left(dx+c\right)-i)}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
4*I*log(tan(d*x + c) - I)/(a^4*d) + tan(d*x + c)/(a^4*d) + 4/(a^4*d*(tan(d
*x + c) - I))
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,4\mathrm{i}}{a^4\,d} + \frac{\tan(c+d\,x)}{a^4\,d} + \frac{4\mathrm{i}}{a^4\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

input
```
int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
(log(tan(c + d*x) - 1i)*4i)/(a^4*d) + tan(c + d*x)/(a^4*d) + 4i/(a^4*d*(ta
n(c + d*x)*1i + 1))
```

Reduce [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(39616*cos(c + d*x)*int(cos(c + d*x)/(8*cos(c + d*x)*sin(c + d*x)**5*i - 1
2*cos(c + d*x)*sin(c + d*x)**3*i + 4*cos(c + d*x)*sin(c + d*x)*i - 8*sin(c
 + d*x)**6 + 16*sin(c + d*x)**4 - 9*sin(c + d*x)**2 + 1),x)*d - 39680*cos(
c + d*x)*int(cos(c + d*x)/(8*cos(c + d*x)*sin(c + d*x)**5 - 12*cos(c + d*x
)*sin(c + d*x)**3 + 4*cos(c + d*x)*sin(c + d*x) + 8*sin(c + d*x)**6*i - 16
*sin(c + d*x)**4*i + 9*sin(c + d*x)**2*i - i),x)*d*i - 1920*cos(c + d*x)*i
nt(cos(c + d*x)/(8*cos(c + d*x)*sin(c + d*x)**4*i - 8*cos(c + d*x)*sin(c +
 d*x)**2*i + cos(c + d*x)*i - 8*sin(c + d*x)**5 + 12*sin(c + d*x)**3 - 4*s
in(c + d*x)),x)*d*i - 125440*cos(c + d*x)*int(sin(c + d*x)**6/(8*cos(c + d
*x)*sin(c + d*x)**5*i - 12*cos(c + d*x)*sin(c + d*x)**3*i + 4*cos(c + d*x)
*sin(c + d*x)*i - 8*sin(c + d*x)**6 + 16*sin(c + d*x)**4 - 9*sin(c + d*x)*
*2 + 1),x)*d + 125440*cos(c + d*x)*int(sin(c + d*x)**6/(8*cos(c + d*x)*sin
(c + d*x)**5 - 12*cos(c + d*x)*sin(c + d*x)**3 + 4*cos(c + d*x)*sin(c + d*
x) + 8*sin(c + d*x)**6*i - 16*sin(c + d*x)**4*i + 9*sin(c + d*x)**2*i - i)
,x)*d*i + 278016*cos(c + d*x)*int(sin(c + d*x)**5/(8*cos(c + d*x)*sin(c +
d*x)**5*i - 12*cos(c + d*x)*sin(c + d*x)**3*i + 4*cos(c + d*x)*sin(c + d*x
)*i - 8*sin(c + d*x)**6 + 16*sin(c + d*x)**4 - 9*sin(c + d*x)**2 + 1),x)*d
*i + 278016*cos(c + d*x)*int(sin(c + d*x)**5/(8*cos(c + d*x)*sin(c + d*x)*
*5 - 12*cos(c + d*x)*sin(c + d*x)**3 + 4*cos(c + d*x)*sin(c + d*x) + 8*sin
(c + d*x)**6*i - 16*sin(c + d*x)**4*i + 9*sin(c + d*x)**2*i - i),x)*d +...
```

3.153 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 29

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\tan(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)^2}$$

output
```
tan(d*x+c)/d/(a^2+I*a^2*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i(i+\tan(c+dx))^2}{4a^4d(-i+\tan(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/4)*(I + Tan[c + d*x])^2)/(a^4*d*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 38}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^4}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int \frac{a-ia\tan(c+dx)}{(i\tan(c+dx)a+a)^3}d(ia\tan(c+dx))}{a^3d}$$
$$\Big\downarrow \text{38}$$
$$\frac{\tan(c+dx)}{a^2d(a+ia\tan(c+dx))^2}$$

input
```
Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
Tan[c + d*x]/(a^2*d*(a + I*a*Tan[c + d*x])^2)
```

Defintions of rubi rules used

rule 38
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{i\mathrm{e}^{-4i(dx+c)}}{4a^4d}$	19
orering	$\frac{i\sec(dx+c)^4}{4d(a+ia\tan(dx+c))^4}$	29
derivativedivides	$\frac{-\frac{i}{(-i+\tan(dx+c))^2}-\frac{1}{-i+\tan(dx+c)}}{a^4d}$	36
default	$\frac{-\frac{i}{(-i+\tan(dx+c))^2}-\frac{1}{-i+\tan(dx+c)}}{a^4d}$	36

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*I/a^4/d*exp(-4*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.59

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx=\frac{i\,e^{(-4i\,dx-4i\,c)}}{4\,a^4d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/4*I*e^(-4*I*d*x - 4*I*c)/(a^4*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(24) = 48.

Time = 1.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.28

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} \frac{i\sec^4(c+dx)}{4a^4d\tan^4(c+dx)-16ia^4d\tan^3(c+dx)-24a^4d\tan^2(c+dx)+16ia^4d\tan(c+dx)+4a^4d} & \text{for } d\neq 0 \\ \frac{x\sec^4(c)}{(ia\tan(c)+a)^4} & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise((I*sec(c + d*x)**4/(4*a**4*d*tan(c + d*x)**4 - 16*I*a**4*d*tan(c
 + d*x)**3 - 24*a**4*d*tan(c + d*x)**2 + 16*I*a**4*d*tan(c + d*x) + 4*a**4
*d), Ne(d, 0)), (x*sec(c)**4/(I*a*tan(c) + a)**4, True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(27) = 54.

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.28

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\tan(dx+c)^2 - i\tan(dx+c)}{\left(a^4\tan(dx+c)^3 - 3i\,a^4\tan(dx+c)^2 - 3\,a^4\tan(dx+c) + i\,a^4\right)d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
-(tan(d*x + c)^2 - I*tan(d*x + c))/((a^4*tan(d*x + c)^3 - 3*I*a^4*tan(d*x
+ c)^2 - 3*a^4*tan(d*x + c) + I*a^4)*d)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\tan{(dx+c)}}{a^4d(\tan{(dx+c)}-i)^2}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
-tan(d*x + c)/(a^4*d*(tan(d*x + c) - I)^2)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\tan(c+d\,x)}{a^4\,d\,(\tan{(c+d\,x)}-\mathrm{i})^2}$$

input
```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
-tan(c + d*x)/(a^4*d*(tan(c + d*x) - 1i)^2)
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x)
```

output

```
(32*int(sin(c + d*x)**4/(8*cos(c + d*x)*sin(c + d*x)**3*i - 4*cos(c + d*x)
*sin(c + d*x)*i - 8*sin(c + d*x)**4 + 8*sin(c + d*x)**2 - 1),x)*d + 32*int
(sin(c + d*x)**2/(8*cos(c + d*x)*sin(c + d*x)**3 - 4*cos(c + d*x)*sin(c +
d*x) + 8*sin(c + d*x)**4*i - 8*sin(c + d*x)**2*i + i),x)*d*i - 32*int((cos
(c + d*x)*sin(c + d*x)**3)/(8*cos(c + d*x)*sin(c + d*x)**3 - 4*cos(c + d*x
)*sin(c + d*x) + 8*sin(c + d*x)**4*i - 8*sin(c + d*x)**2*i + i),x)*d + 16*
int((cos(c + d*x)*sin(c + d*x))/(8*cos(c + d*x)*sin(c + d*x)**3 - 4*cos(c
+ d*x)*sin(c + d*x) + 8*sin(c + d*x)**4*i - 8*sin(c + d*x)**2*i + i),x)*d
- 2*log(tan((c + d*x)/2)**8 - 8*tan((c + d*x)/2)**7*i - 28*tan((c + d*x)/2
)**6 + 56*tan((c + d*x)/2)**5*i + 70*tan((c + d*x)/2)**4 - 56*tan((c + d*x
)/2)**3*i - 28*tan((c + d*x)/2)**2 + 8*tan((c + d*x)/2)*i + 1)*i + 12*log(
tan((c + d*x)/2)**2 + 1)*i - log(tan((c + d*x)/2)**8*i + 8*tan((c + d*x)/2
)**7 - 28*tan((c + d*x)/2)**6*i - 56*tan((c + d*x)/2)**5 + 70*tan((c + d*x
)/2)**4*i + 56*tan((c + d*x)/2)**3 - 28*tan((c + d*x)/2)**2*i - 8*tan((c +
 d*x)/2) + i)*i - 8*d*x)/(4*a**4*d)
```

3.154 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx=\frac{i}{3ad(a+ia\tan(c+dx))^3}$$

output
```
1/3*I/a/d/(a+I*a*tan(d*x+c))^3
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx=-\frac{1}{3a^4d(-i+\tan(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
-1/3*1/(a^4*d*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^2}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int \frac{1}{(i\tan(c+dx)a+a)^4}d(ia\tan(c+dx))}{ad}$$
$$\Big\downarrow \text{17}$$
$$\frac{i}{3ad(a+ia\tan(c+dx))^3}$$

input
```
Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(I/3)/(a*d*(a + I*a*Tan[c + d*x])^3)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{i}{3ad(a+ia\tan(dx+c))^3}$	24
default	$\frac{i}{3ad(a+ia\tan(dx+c))^3}$	24
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^4d}+\frac{i\mathrm{e}^{-4i(dx+c)}}{8a^4d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{24a^4d}$	56

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*I/a/d/(a+I*a*tan(d*x+c))^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx=\frac{\left(3i\,e^{(4i\,dx+4i\,c)}+3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-6i\,dx-6i\,c)}}{24\,a^4d}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/24*(3*I*e^(4*I*d*x + 4*I*c) + 3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-6*I*d*x -
 6*I*c)/(a^4*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(19) = 38$.

Time = 1.33 (sec) , antiderivative size = 272, normalized size of antiderivative = 10.07

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} -\frac{i\tan^2(c+dx)\sec^2(c+dx)}{24a^4d\tan^4(c+dx)-96ia^4d\tan^3(c+dx)-144a^4d\tan^2(c+dx)+96ia^4d\tan(c+dx)+24a^4d} - \frac{4\tan(c+dx}{24a^4d\tan^4(c+dx)-96ia^4d\tan^3(c+dx)-14} \\ \frac{x\sec^2(c)}{(ia\tan(c)+a)^4} \end{cases}$$

input
```
integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise((-I*tan(c + d*x)**2*sec(c + d*x)**2/(24*a**4*d*tan(c + d*x)**4 -
 96*I*a**4*d*tan(c + d*x)**3 - 144*a**4*d*tan(c + d*x)**2 + 96*I*a**4*d*ta
n(c + d*x) + 24*a**4*d) - 4*tan(c + d*x)*sec(c + d*x)**2/(24*a**4*d*tan(c
+ d*x)**4 - 96*I*a**4*d*tan(c + d*x)**3 - 144*a**4*d*tan(c + d*x)**2 + 96*
I*a**4*d*tan(c + d*x) + 24*a**4*d) + 7*I*sec(c + d*x)**2/(24*a**4*d*tan(c
+ d*x)**4 - 96*I*a**4*d*tan(c + d*x)**3 - 144*a**4*d*tan(c + d*x)**2 + 96*
I*a**4*d*tan(c + d*x) + 24*a**4*d), Ne(d, 0)), (x*sec(c)**2/(I*a*tan(c) +
a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i}{3\,(i\,a\tan(dx+c)+a)^3ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/3*I/((I*a*tan(d*x + c) + a)^3*a*d)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{1}{3\,a^4 d(\tan(dx+c)-i)^3}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3/(a^4*d*(tan(d*x + c) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{1}{3\,a^4\,d\,(\tan(c+d\,x)-\mathrm{i})^3}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
-1/(3*a^4*d*(tan(c + d*x) - 1i)^3)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x)
```

output

```
(32*int(sin(c + d*x)**4/(8*cos(c + d*x)*sin(c + d*x)**3*i - 4*cos(c + d*x)
*sin(c + d*x)*i - 8*sin(c + d*x)**4 + 8*sin(c + d*x)**2 - 1),x)*d + 28*int
(sin(c + d*x)**2/(8*cos(c + d*x)*sin(c + d*x)**3 - 4*cos(c + d*x)*sin(c +
d*x) + 8*sin(c + d*x)**4*i - 8*sin(c + d*x)**2*i + i),x)*d*i - 32*int((cos
(c + d*x)*sin(c + d*x)**3)/(8*cos(c + d*x)*sin(c + d*x)**3 - 4*cos(c + d*x
)*sin(c + d*x) + 8*sin(c + d*x)**4*i - 8*sin(c + d*x)**2*i + i),x)*d + 16*
int((cos(c + d*x)*sin(c + d*x))/(8*cos(c + d*x)*sin(c + d*x)**3 - 4*cos(c
+ d*x)*sin(c + d*x) + 8*sin(c + d*x)**4*i - 8*sin(c + d*x)**2*i + i),x)*d
- 2*log(tan((c + d*x)/2)**8 - 8*tan((c + d*x)/2)**7*i - 28*tan((c + d*x)/2
)**6 + 56*tan((c + d*x)/2)**5*i + 70*tan((c + d*x)/2)**4 - 56*tan((c + d*x
)/2)**3*i - 28*tan((c + d*x)/2)**2 + 8*tan((c + d*x)/2)*i + 1)*i + 12*log(
tan((c + d*x)/2)**2 + 1)*i - log(tan((c + d*x)/2)**8*i + 8*tan((c + d*x)/2
)**7 - 28*tan((c + d*x)/2)**6*i - 56*tan((c + d*x)/2)**5 + 70*tan((c + d*x
)/2)**4*i + 56*tan((c + d*x)/2)**3 - 28*tan((c + d*x)/2)**2*i - 8*tan((c +
 d*x)/2) + i)*i - 8*d*x)/(4*a**4*d)
```

3.155 $\int \frac{1}{(a+ia\tan(c+dx))^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 116

$$\int \frac{1}{(a+ia\tan(c+dx))^4} dx = \frac{x}{16a^4} + \frac{i}{8d(a+ia\tan(c+dx))^4} + \frac{i}{12ad(a+ia\tan(c+dx))^3} + \frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2} + \frac{i}{16d\left(a^4+ia^4\tan(c+dx)\right)}$$

output
```
1/16*x/a^4+1/8*I/d/(a+I*a*tan(d*x+c))^4+1/12*I/a/d/(a+I*a*tan(d*x+c))^3+1/
16*I/d/(a^2+I*a^2*tan(d*x+c))^2+1/16*I/d/(a^4+I*a^4*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = -\frac{ia\left(\frac{i\arctan(\tan(c+dx))}{16a^5}-\frac{1}{8a(a+ia\tan(c+dx))^4}-\frac{1}{12a^2(a+ia\tan(c+dx))^3}-\frac{1}{16a^3(a+ia\tan(c+dx))^2}-\frac{1}{16a^4(a+ia\tan(c+dx))}\right)}{d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(-4),x]
```

output
```
((-I)*a*(((I/16)*ArcTan[Tan[c + d*x]])/a^5 - 1/(8*a*(a + I*a*Tan[c + d*x])
^4) - 1/(12*a^2*(a + I*a*Tan[c + d*x])^3) - 1/(16*a^3*(a + I*a*Tan[c + d*x
])^2) - 1/(16*a^4*(a + I*a*Tan[c + d*x]))))/d
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3960, 3042, 3960, 3042, 3960, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3960}$$
$$\frac{\int \frac{1}{(i\tan(c+dx)a+a)^3}dx}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$
$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{(i\tan(c+dx)a+a)^3}dx}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3960}$$

$$\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^2}dx}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^2}dx}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3960}$$

$$\frac{\frac{\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3960}$$

$$\frac{\frac{\frac{\frac{\int 1dx}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{\frac{\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(-4),x]
```

output
```
(I/8)/(d*(a + I*a*Tan[c + d*x])^4) + ((I/6)/(d*(a + I*a*Tan[c + d*x])^3) +
 ((I/4)/(d*(a + I*a*Tan[c + d*x])^2) + (x/(2*a) + (I/2)/(d*(a + I*a*Tan[c
+ d*x])))/(2*a))/(2*a))/(2*a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

method	result
risch	$\frac{x}{16a^4}+\frac{i e^{-2i(dx+c)}}{8a^4d}+\frac{3i e^{-4i(dx+c)}}{32a^4d}+\frac{i e^{-6i(dx+c)}}{24a^4d}+\frac{i e^{-8i(dx+c)}}{128a^4d}$
derivativedivides	$\frac{i}{8d\,a^4(-i+\tan(dx+c))^4}+\frac{\arctan(\tan(dx+c))}{16a^4d}-\frac{i}{16d\,a^4(-i+\tan(dx+c))^2}-\frac{1}{12d\,a^4(-i+\tan(dx+c))^3}+\frac{}{16d\,a^4(}$
default	$\frac{i}{8d\,a^4(-i+\tan(dx+c))^4}+\frac{\arctan(\tan(dx+c))}{16a^4d}-\frac{i}{16d\,a^4(-i+\tan(dx+c))^2}-\frac{1}{12d\,a^4(-i+\tan(dx+c))^3}+\frac{}{16d\,a^4(}$
norman	$\frac{\frac{x}{16a}+\frac{15\tan(dx+c)}{16ad}+\frac{5\tan(dx+c)^3}{48ad}+\frac{11\tan(dx+c)^5}{48ad}+\frac{\tan(dx+c)^7}{16ad}+\frac{x\tan(dx+c)^2}{4a}+\frac{3x\tan(dx+c)^4}{8a}+\frac{x\tan(dx+c)^6}{4a}+\frac{x\tan(dx+c}{16a}}{a^3\left(1+\tan(dx+c)^2\right)^4}$

input
```
int(1/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/16*x/a^4+1/8*I/a^4/d*exp(-2*I*(d*x+c))+3/32*I/a^4/d*exp(-4*I*(d*x+c))+1/
24*I/a^4/d*exp(-6*I*(d*x+c))+1/128*I/a^4/d*exp(-8*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(24\,dxe^{(8i\,dx+8i\,c)}+48i\,e^{(6i\,dx+6i\,c)}+36i\,e^{(4i\,dx+4i\,c)}+16i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-8i\,dx-8i\,c)}}{384\,a^4d}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/384*(24*d*x*e^(8*I*d*x + 8*I*c) + 48*I*e^(6*I*d*x + 6*I*c) + 36*I*e^(4*I
*d*x + 4*I*c) + 16*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-8*I*d*x - 8*I*c)/(a^4*
d)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.63

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} \frac{(98304ia^{12}d^3e^{18ic}e^{-2idx}+73728ia^{12}d^3e^{16ic}e^{-4idx}+32768ia^{12}d^3e^{14ic}e^{-6idx}+6144ia^{12}d^3e^{12ic}e^{-8idx})e^{-20ic}}{786432a^{16}d^4} & \text{for } a^{16}d^4e^{20ic}\neq 0 \\ x\left(\frac{(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1)e^{-8ic}}{16a^4}-\frac{1}{16a^4}\right) & \text{otherwise} \end{cases} + \frac{x}{16a^4}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((98304*I*a**12*d**3*exp(18*I*c)*exp(-2*I*d*x) + 73728*I*a**12*d
**3*exp(16*I*c)*exp(-4*I*d*x) + 32768*I*a**12*d**3*exp(14*I*c)*exp(-6*I*d*
x) + 6144*I*a**12*d**3*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(786432*a**
16*d**4), Ne(a**16*d**4*exp(20*I*c), 0)), (x*((exp(8*I*c) + 4*exp(6*I*c) +
 6*exp(4*I*c) + 4*exp(2*I*c) + 1)*exp(-8*I*c)/(16*a**4) - 1/(16*a**4)), Tr
ue)) + x/(16*a**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a + ia\tan(c + dx))^4} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int \frac{1}{(a + ia\tan(c + dx))^4} \, dx = \frac{i \log(\tan(dx + c) + i)}{32 a^4 d} - \frac{i \log(\tan(dx + c) - i)}{32 a^4 d} + \frac{3 \tan(dx + c)^3 - 12i \tan(dx + c)^2 - 19 \tan(dx + c) + 16i}{48 a^4 d (\tan(dx + c) - i)^4}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/32*I*log(tan(d*x + c) + I)/(a^4*d) - 1/32*I*log(tan(d*x + c) - I)/(a^4*d
) + 1/48*(3*tan(d*x + c)^3 - 12*I*tan(d*x + c)^2 - 19*tan(d*x + c) + 16*I)
/(a^4*d*(tan(d*x + c) - I)^4)
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = \frac{x}{16\,a^4} - \frac{-\frac{\tan(c+d\,x)^3}{16} + \frac{\tan(c+d\,x)^2\,1\mathrm{i}}{4} + \frac{19\tan(c+d\,x)}{48} - \frac{1}{3}\mathrm{i}}{a^4\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^4}$$

input
```
int(1/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
x/(16*a^4) - ((19*tan(c + d*x))/48 + (tan(c + d*x)^2*1i)/4 - tan(c + d*x)^
3/16 - 1i/3)/(a^4*d*(tan(c + d*x)*1i + 1)^4)
```

Reduce [F]

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{1}{\tan(dx+c)^4-4\tan(dx+c)^3i-6\tan(dx+c)^2+4\tan(dx+c)i+1}dx}{a^4}$$

input
```
int(1/(a+I*a*tan(d*x+c))^4,x)
```

output
```
int(1/(tan(c + d*x)**4 - 4*tan(c + d*x)**3*i - 6*tan(c + d*x)**2 + 4*tan(c
 + d*x)*i + 1),x)/a**4
```

3.156 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 169

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{3x}{32a^4}+\frac{ia}{20d(a+ia\tan(c+dx))^5}+\frac{i}{16d(a+ia\tan(c+dx))^4}+\frac{i}{16ad(a+ia\tan(c+dx))^3}+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}-\frac{i}{64d\left(a^4-ia^4\tan(c+dx)\right)}+\frac{5i}{64d\left(a^4+ia^4\tan(c+dx)\right)}$$

output
```
3/32*x/a^4+1/20*I*a/d/(a+I*a*tan(d*x+c))^5+1/16*I/d/(a+I*a*tan(d*x+c))^4+1
/16*I/a/d/(a+I*a*tan(d*x+c))^3+1/16*I/d/(a^2+I*a^2*tan(d*x+c))^2-1/64*I/d/
(a^4-I*a^4*tan(d*x+c))+5/64*I/d/(a^4+I*a^4*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.84

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\sec^6(c+dx)(50i+100i\cos(2(c+dx))+46i\cos(4(c+dx))-4i\cos(6(c+dx))-50\sin(2(c+dx))+6}{640a^4d(-i+\tan(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^6*(50*I + (100*I)*Cos[2*(c + d*x)] + (46*I)*Cos[4*(c + d*x)]
 - (4*I)*Cos[6*(c + d*x)] - 50*Sin[2*(c + d*x)] + 60*ArcTan[Tan[c + d*x]]*
(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)]) - 31*Sin[4*(c + d*x)] + 6*Sin[6*(c
 + d*x)]))/(640*a^4*d*(-I + Tan[c + d*x])^5*(I + Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(c+dx)^2(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^6}d(ia\tan(c+dx))}{d}$$
$$\Big\downarrow \text{54}$$

$$-\frac{ia^3\int\left(\frac{1}{64a^6(a-ia\tan(c+dx))^2}+\frac{5}{64a^6(i\tan(c+dx)a+a)^2}+\frac{1}{8a^5(i\tan(c+dx)a+a)^3}+\frac{3}{16a^4(i\tan(c+dx)a+a)^4}+\frac{1}{4a^3(i\tan(c+dx)a+a}\right.}{d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{ia^3\left(\frac{3i\arctan(\tan(c+dx))}{32a^7}+\frac{1}{64a^6(a-ia\tan(c+dx))}-\frac{5}{64a^6(a+ia\tan(c+dx))}-\frac{1}{16a^5(a+ia\tan(c+dx))^2}-\frac{1}{16a^4(a+ia\tan(c+dx))^3}-\right.}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-I)*a^3*((((3*I)/32)*ArcTan[Tan[c + d*x]])/a^7 + 1/(64*a^6*(a - I*a*Tan[
c + d*x])) - 1/(20*a^2*(a + I*a*Tan[c + d*x])^5) - 1/(16*a^3*(a + I*a*Tan[
c + d*x])^4) - 1/(16*a^4*(a + I*a*Tan[c + d*x])^3) - 1/(16*a^5*(a + I*a*Ta
n[c + d*x])^2) - 5/(64*a^6*(a + I*a*Tan[c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.68

method	result
derivativedivides	$\frac{\frac{3i\ln(\tan(dx+c)+i)}{64}+\frac{1}{64\tan(dx+c)+64i}-\frac{3i\ln(-i+\tan(dx+c))}{64}+\frac{i}{16(-i+\tan(dx+c))^4}-\frac{i}{16(-i+\tan(dx+c))^2}+\frac{1}{20(-i+\tan(dx+c))}}{d\,a^4}$
default	$\frac{\frac{3i\ln(\tan(dx+c)+i)}{64}+\frac{1}{64\tan(dx+c)+64i}-\frac{3i\ln(-i+\tan(dx+c))}{64}+\frac{i}{16(-i+\tan(dx+c))^4}-\frac{i}{16(-i+\tan(dx+c))^2}+\frac{1}{20(-i+\tan(dx+c))}}{d\,a^4}$
risch	$\frac{3x}{32a^4}+\frac{5ie^{-4i(dx+c)}}{64a^4d}+\frac{5ie^{-6i(dx+c)}}{128a^4d}+\frac{3ie^{-8i(dx+c)}}{256a^4d}+\frac{ie^{-10i(dx+c)}}{640a^4d}+\frac{7i\cos(2dx+2c)}{64a^4d}+\frac{\sin(2dx+2c)}{8a^4d}$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^4*(3/64*I*ln(tan(d*x+c)+I)+1/64/(tan(d*x+c)+I)-3/64*I*ln(-I+tan(d*x+
c))+1/16*I/(-I+tan(d*x+c))^4-1/16*I/(-I+tan(d*x+c))^2+1/20/(-I+tan(d*x+c))
^5-1/16/(-I+tan(d*x+c))^3+5/64/(-I+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.51

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(120\,dxe^{(10i\,dx+10i\,c)}-10i\,e^{(12i\,dx+12i\,c)}+150i\,e^{(8i\,dx+8i\,c)}+100i\,e^{(6i\,dx+6i\,c)}+50i\,e^{(4i\,dx+4i\,c)}+15i\,e^{(2i\,dx+2i}\right.}{1280\,a^4d}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/1280*(120*d*x*e^(10*I*d*x + 10*I*c) - 10*I*e^(12*I*d*x + 12*I*c) + 150*I
*e^(8*I*d*x + 8*I*c) + 100*I*e^(6*I*d*x + 6*I*c) + 50*I*e^(4*I*d*x + 4*I*c
) + 15*I*e^(2*I*d*x + 2*I*c) + 2*I)*e^(-10*I*d*x - 10*I*c)/(a^4*d)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.53

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} \frac{(-171798691840ia^{20}d^5e^{32ic}e^{2idx}+2576980377600ia^{20}d^5e^{28ic}e^{-2idx}+1717986918400ia^{20}d^5e^{26ic}e^{-4idx}+858993459200ia^{20}d^5e^{24ic}e^{-6idx}\cdots}{21990232555520a^{24}d^6} \\ x\left(\frac{(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1)e^{-10ic}}{64a^4}-\frac{3}{32a^4}\right) \end{cases} + \frac{3x}{32a^4}$$

input
```
integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((-171798691840*I*a**20*d**5*exp(32*I*c)*exp(2*I*d*x) + 25769803
77600*I*a**20*d**5*exp(28*I*c)*exp(-2*I*d*x) + 1717986918400*I*a**20*d**5*
exp(26*I*c)*exp(-4*I*d*x) + 858993459200*I*a**20*d**5*exp(24*I*c)*exp(-6*I
*d*x) + 257698037760*I*a**20*d**5*exp(22*I*c)*exp(-8*I*d*x) + 34359738368*
I*a**20*d**5*exp(20*I*c)*exp(-10*I*d*x))*exp(-30*I*c)/(21990232555520*a**2
4*d**6), Ne(a**24*d**6*exp(30*I*c), 0)), (x*((exp(12*I*c) + 6*exp(10*I*c)
+ 15*exp(8*I*c) + 20*exp(6*I*c) + 15*exp(4*I*c) + 6*exp(2*I*c) + 1)*exp(-1
0*I*c)/(64*a**4) - 3/(32*a**4)), True)) + 3*x/(32*a**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.67

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{3i\,\log\left(\tan\left(dx+c\right)+i\right)}{64\,a^4d} - \frac{3i\,\log\left(\tan\left(dx+c\right)-i\right)}{64\,a^4d} + \frac{15\,\tan\left(dx+c\right)^5 - 60i\,\tan\left(dx+c\right)^4 - 80\,\tan\left(dx+c\right)^3 + 20i\,\tan\left(dx+c\right)^2 - 47\,\tan\left(dx+c\right) + 48i}{160\,a^4d(\tan\left(dx+c\right)+i)(\tan\left(dx+c\right)-i)^5}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
3/64*I*log(tan(d*x + c) + I)/(a^4*d) - 3/64*I*log(tan(d*x + c) - I)/(a^4*d
) + 1/160*(15*tan(d*x + c)^5 - 60*I*tan(d*x + c)^4 - 80*tan(d*x + c)^3 + 2
0*I*tan(d*x + c)^2 - 47*tan(d*x + c) + 48*I)/(a^4*d*(tan(d*x + c) + I)*(ta
n(d*x + c) - I)^5)
```

Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.53

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{3\,x}{32\,a^4} - \frac{-\frac{3\tan(c+d\,x)^5}{32} + \frac{\tan(c+d\,x)^4\,3{}\mathrm{i}}{8} + \frac{\tan(c+d\,x)^3}{2} - \frac{\tan(c+d\,x)^2\,1{}\mathrm{i}}{8} + \frac{47\tan(c+d\,x)}{160} - \frac{3}{10}{}\mathrm{i}}{a^4\,d\,{\left(\tan\left(c+d\,x\right)-\mathrm{i}\right)}^5\,\left(\tan\left(c+d\,x\right)+1{}\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(3*x)/(32*a^4) - ((47*tan(c + d*x))/160 - (tan(c + d*x)^2*1i)/8 + tan(c +
d*x)^3/2 + (tan(c + d*x)^4*3i)/8 - (3*tan(c + d*x)^5)/32 - 3i/10)/(a^4*d*(
tan(c + d*x) - 1i)^5*(tan(c + d*x) + 1i))
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\cos(dx+c)^2}{\tan(dx+c)^4-4\tan(dx+c)^3i-6\tan(dx+c)^2+4\tan(dx+c)i+1}dx}{a^4}$$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x)
```

output
```
int(cos(c + d*x)**2/(tan(c + d*x)**4 - 4*tan(c + d*x)**3*i - 6*tan(c + d*x
)**2 + 4*tan(c + d*x)*i + 1),x)/a**4
```

3.157 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 228

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = {} & \frac{7x}{64a^4}+\frac{ia^2}{48d(a+ia\tan(c+dx))^6} \\ & +\frac{3ia}{80d(a+ia\tan(c+dx))^5}+\frac{3i}{64d(a+ia\tan(c+dx))^4} \\ & -\frac{i}{256d\left(a^2-ia^2\tan(c+dx)\right)^2} \\ & +\frac{15i}{256d\left(a^2+ia^2\tan(c+dx)\right)^2} \\ & +\frac{5ia^5}{96d\left(a^3+ia^3\tan(c+dx)\right)^3} \\ & -\frac{7i}{256d\left(a^4-ia^4\tan(c+dx)\right)} \\ & +\frac{21i}{256d\left(a^4+ia^4\tan(c+dx)\right)}\end{aligned}$$

output
```
7/64*x/a^4+1/48*I*a^2/d/(a+I*a*tan(d*x+c))^6+3/80*I*a/d/(a+I*a*tan(d*x+c))
^5+3/64*I/d/(a+I*a*tan(d*x+c))^4-1/256*I/d/(a^2-I*a^2*tan(d*x+c))^2+15/256
*I/d/(a^2+I*a^2*tan(d*x+c))^2+5/96*I*a^5/d/(a^3+I*a^3*tan(d*x+c))^3-7/256*
I/d/(a^4-I*a^4*tan(d*x+c))+21/256*I/d/(a^4+I*a^4*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.72

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\sec^8(c+dx)(525i+1120i\cos(2(c+dx))+504i\cos(4(c+dx))-96i\cos(6(c+dx))-5i\cos(8(c+dx))}{76}$$

input
```
Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^8*(525*I + (1120*I)*Cos[2*(c + d*x)] + (504*I)*Cos[4*(c + d*
x)] - (96*I)*Cos[6*(c + d*x)] - (5*I)*Cos[8*(c + d*x)] - 560*Sin[2*(c + d*
x)] + 840*ArcTan[Tan[c + d*x]]*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)]) - 2
94*Sin[4*(c + d*x)] + 144*Sin[6*(c + d*x)] + 10*Sin[8*(c + d*x)]))/(7680*a
^4*d*(-I + Tan[c + d*x])^6*(I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sec(c+dx)^4(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \; 3968$$
$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^7}d(ia\tan(c+dx))}{d}$$
$$\Big\downarrow \; 54$$

$$-\frac{ia^5\int\left(\frac{7}{256a^8(a-ia\tan(c+dx))^2}+\frac{21}{256a^8(i\tan(c+dx)a+a)^2}+\frac{1}{128a^7(a-ia\tan(c+dx))^3}+\frac{15}{128a^7(i\tan(c+dx)a+a)^3}+\frac{5}{32a^6(i\tan(c+a$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^5\left(\frac{7i\arctan(\tan(c+dx))}{64a^9}+\frac{7}{256a^8(a-ia\tan(c+dx))}-\frac{21}{256a^8(a+ia\tan(c+dx))}+\frac{1}{256a^7(a-ia\tan(c+dx))^2}-\frac{15}{256a^7(a+ia\tan(c+dx))}\right.}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-I)*a^5*((((7*I)/64)*ArcTan[Tan[c + d*x]])/a^9 + 1/(256*a^7*(a - I*a*Tan
[c + d*x])^2) + 7/(256*a^8*(a - I*a*Tan[c + d*x])) - 1/(48*a^3*(a + I*a*Ta
n[c + d*x])^6) - 3/(80*a^4*(a + I*a*Tan[c + d*x])^5) - 3/(64*a^5*(a + I*a*
Tan[c + d*x])^4) - 5/(96*a^6*(a + I*a*Tan[c + d*x])^3) - 15/(256*a^7*(a +
I*a*Tan[c + d*x])^2) - 21/(256*a^8*(a + I*a*Tan[c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)    Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{\frac{i}{256(\tan(dx+c)+i)^2}+\frac{7i\ln(\tan(dx+c)+i)}{128}+\frac{7}{256(\tan(dx+c)+i)}-\frac{7i\ln(-i+\tan(dx+c))}{128}+\frac{3i}{64(-i+\tan(dx+c))^4}-\frac{i}{48(-i+\tan(dx+c))^6}}{d\,a^4}$
default	$\frac{\frac{i}{256(\tan(dx+c)+i)^2}+\frac{7i\ln(\tan(dx+c)+i)}{128}+\frac{7}{256(\tan(dx+c)+i)}-\frac{7i\ln(-i+\tan(dx+c))}{128}+\frac{3i}{64(-i+\tan(dx+c))^4}-\frac{i}{48(-i+\tan(dx+c))^6}}{d\,a^4}$
risch	$\frac{7x}{64a^4}+\frac{7ie^{-6i(dx+c)}}{192a^4d}+\frac{7ie^{-8i(dx+c)}}{512a^4d}+\frac{ie^{-10i(dx+c)}}{320a^4d}+\frac{ie^{-12i(dx+c)}}{3072a^4d}+\frac{69i\cos(4dx+4c)}{1024a^4d}+\frac{71\sin(4dx+4c)}{1024a^4d}$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^4*(1/256*I/(tan(d*x+c)+I)^2+7/128*I*ln(tan(d*x+c)+I)+7/256/(tan(d*x+
c)+I)-7/128*I*ln(-I+tan(d*x+c))+3/64*I/(-I+tan(d*x+c))^4-1/48*I/(-I+tan(d*
x+c))^6-15/256*I/(-I+tan(d*x+c))^2+3/80/(-I+tan(d*x+c))^5-5/96/(-I+tan(d*x
+c))^3+21/256/(-I+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.48

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(1680\,dxe^{(12i\,dx+12i\,c)}-15i\,e^{(16i\,dx+16i\,c)}-240i\,e^{(14i\,dx+14i\,c)}+1680i\,e^{(10i\,dx+10i\,c)}+1050i\,e^{(8i\,dx+8i\,c)}+560\right.}{15360\,a^4d}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/15360*(1680*d*x*e^(12*I*d*x + 12*I*c) - 15*I*e^(16*I*d*x + 16*I*c) - 240
*I*e^(14*I*d*x + 14*I*c) + 1680*I*e^(10*I*d*x + 10*I*c) + 1050*I*e^(8*I*d*
x + 8*I*c) + 560*I*e^(6*I*d*x + 6*I*c) + 210*I*e^(4*I*d*x + 4*I*c) + 48*I*
e^(2*I*d*x + 2*I*c) + 5*I)*e^(-12*I*d*x - 12*I*c)/(a^4*d)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.43

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \begin{cases} \frac{(-202661983231672320ia^{28}d^7e^{46ic}e^{4idx}-3242591731706757120ia^{28}d^7e^{44ic}e^{2idx}+22698142121947299840ia^{28}d^7e^{40ic}e^{-2idx}+14186338826}{} \\ x\left(\frac{(e^{16ic}+8e^{14ic}+28e^{12ic}+56e^{10ic}+70e^{8ic}+56e^{6ic}+28e^{4ic}+8e^{2ic}+1)e^{-12ic}}{256a^4}-\frac{7}{64a^4}\right) \end{cases}$$
$$+\frac{7x}{64a^4}$$

input
```
integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((-202661983231672320*I*a**28*d**7*exp(46*I*c)*exp(4*I*d*x) - 32
42591731706757120*I*a**28*d**7*exp(44*I*c)*exp(2*I*d*x) + 226981421219472
9840*I*a**28*d**7*exp(40*I*c)*exp(-2*I*d*x) + 14186338826217062400*I*a**28
*d**7*exp(38*I*c)*exp(-4*I*d*x) + 7566047373982433280*I*a**28*d**7*exp(36*
I*c)*exp(-6*I*d*x) + 2837267765243412480*I*a**28*d**7*exp(34*I*c)*exp(-8*I
*d*x) + 648518346341351424*I*a**28*d**7*exp(32*I*c)*exp(-10*I*d*x) + 67553
994410557440*I*a**28*d**7*exp(30*I*c)*exp(-12*I*d*x))*exp(-42*I*c)/(207525
870829232455680*a**32*d**8), Ne(a**32*d**8*exp(42*I*c), 0)), (x*((exp(16*I
*c) + 8*exp(14*I*c) + 28*exp(12*I*c) + 56*exp(10*I*c) + 70*exp(8*I*c) + 56
*exp(6*I*c) + 28*exp(4*I*c) + 8*exp(2*I*c) + 1)*exp(-12*I*c)/(256*a**4) -
7/(64*a**4)), True)) + 7*x/(64*a**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.58

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{7i\log\left(\tan\left(dx+c\right)+i\right)}{128\,a^4d} - \frac{7i\log\left(\tan\left(dx+c\right)-i\right)}{128\,a^4d} + \frac{105\tan\left(dx+c\right)^7 - 420i\tan\left(dx+c\right)^6 - 455\tan\left(dx+c\right)^5 - 280i\tan\left(dx+c\right)^4 - 889\tan\left(dx+c\right)^3}{960\,a^4d(\tan\left(dx+c\right)+i)^2(\tan\left(dx+c\right)-i)^6}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
7/128*I*log(tan(d*x + c) + I)/(a^4*d) - 7/128*I*log(tan(d*x + c) - I)/(a^4
*d) + 1/960*(105*tan(d*x + c)^7 - 420*I*tan(d*x + c)^6 - 455*tan(d*x + c)^
5 - 280*I*tan(d*x + c)^4 - 889*tan(d*x + c)^3 + 476*I*tan(d*x + c)^2 - 169
*tan(d*x + c) + 256*I)/(a^4*d*(tan(d*x + c) + I)^2*(tan(d*x + c) - I)^6)
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.86

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{7\,x}{64\,a^4} + \frac{\frac{\tan(c+d\,x)\,169\mathrm{i}}{960\,a^4} + \frac{4}{15\,a^4} + \frac{119\tan(c+d\,x)^2}{240\,a^4} + \frac{\tan(c+d\,x)^3\,889\mathrm{i}}{960\,a^4} - \frac{7\tan(c+d\,x)^4}{24\,a^4} + \frac{\tan(c+d\,x)^5\,91}{192\,a^4}}{d\,\left(-\tan\left(c+d\,x\right)^8\,1\mathrm{i} - 4\tan\left(c+d\,x\right)^7 + \tan\left(c+d\,x\right)^6\,4\mathrm{i} - 4\tan\left(c+d\,x\right)^5 + \tan\left(c+d\,x\right)^4\,10\mathrm{i} + 4\,\mathrm{ta}\right.}$$

input
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(7*x)/(64*a^4) + ((tan(c + d*x)*169i)/(960*a^4) + 4/(15*a^4) + (119*tan(c
+ d*x)^2)/(240*a^4) + (tan(c + d*x)^3*889i)/(960*a^4) - (7*tan(c + d*x)^4)
/(24*a^4) + (tan(c + d*x)^5*91i)/(192*a^4) - (7*tan(c + d*x)^6)/(16*a^4) -
 (tan(c + d*x)^7*7i)/(64*a^4))/(d*(4*tan(c + d*x) + tan(c + d*x)^2*4i + 4*
tan(c + d*x)^3 + tan(c + d*x)^4*10i - 4*tan(c + d*x)^5 + tan(c + d*x)^6*4i
 - 4*tan(c + d*x)^7 - tan(c + d*x)^8*1i - 1i))
```

Reduce [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\cos(dx+c)^4}{\tan(dx+c)^4-4\tan(dx+c)^3 i-6\tan(dx+c)^2+4\tan(dx+c) i+1}dx}{a^4}$$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x)
```

output
```
int(cos(c + d*x)**4/(tan(c + d*x)**4 - 4*tan(c + d*x)**3*i - 6*tan(c + d*x
)**2 + 4*tan(c + d*x)*i + 1),x)/a**4
```

3.158 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{35\text{arctanh}(\sin(c+dx))}{8a^4d} + \frac{35\sec(c+dx)\tan(c+dx)}{8a^4d} + \frac{35\sec^3(c+dx)\tan(c+dx)}{12a^4d} - \frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3} - \frac{14i\sec^5(c+dx)}{3d\left(a^4+ia^4\tan(c+dx)\right)}$$

output
```
35/8*arctanh(sin(d*x+c))/a^4/d+35/8*sec(d*x+c)*tan(d*x+c)/a^4/d+35/12*sec(
d*x+c)^3*tan(d*x+c)/a^4/d-2*I*sec(d*x+c)^7/a/d/(a+I*a*tan(d*x+c))^3-14/3*I
*sec(d*x+c)^5/d/(a^4+I*a^4*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.78

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\sec^4(c+dx)\left(896i\cos(c+dx)+3\left(128i\cos(3(c+dx))+105\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
-1/192*(Sec[c + d*x]^4*((896*I)*Cos[c + d*x] + 3*((128*I)*Cos[3*(c + d*x)]
 + 105*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 35*Cos[4*(c + d*x)]*Log[
Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 140*Cos[2*(c + d*x)]*(Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) -
105*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 35*Cos[4*(c + d*x)]*Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]] + 42*Sin[c + d*x] + 58*Sin[3*(c + d*x)])
))/(a^4*d)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 3981, 3042, 3981, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^9}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3981}$$
$$\frac{7\int \frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$
$$\Big\downarrow \text{3042}$$
$$\frac{7\int \frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^2}dx}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$
$$\Big\downarrow \text{3981}$$
$$\frac{7\left(\frac{5\int \sec^5(c+dx)dx}{3a^2}-\frac{2i\sec^5(c+dx)}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{3a^2}-\frac{2i\sec^5(c+dx)}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7\left(\frac{5\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7\left(\frac{5\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4257}$$

$$\frac{7\left(\frac{5\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

input
```
Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-2*I)*Sec[c + d*x]^7)/(a*d*(a + I*a*Tan[c + d*x])^3) + (7*((((-2*I)/3)*S
ec[c + d*x]^5)/(d*(a^2 + I*a^2*Tan[c + d*x])) + (5*((Sec[c + d*x]^3*Tan[c
+ d*x])/(4*d) + (3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*
x])/(2*d)))/4))/(3*a^2)))/a^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{i\left(105\,\mathrm{e}^{7i(dx+c)}+385\,\mathrm{e}^{5i(dx+c)}+511\,\mathrm{e}^{3i(dx+c)}+279\,\mathrm{e}^{i(dx+c)}\right)}{12d\,a^{4}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{4}}+\frac{35\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8a^{4}d}-\frac{35\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8a^{4}d}$
derivativedivides	$\frac{\frac{2\left(\frac{25}{16}-i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{2\left(\frac{1}{4}+\frac{2i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(-\frac{27}{16}-3i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{35\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8}+\frac{2\left(\frac{1}{4}-\frac{2i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}}{a^{4}d}$
default	$\frac{\frac{2\left(\frac{25}{16}-i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{2\left(\frac{1}{4}+\frac{2i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(-\frac{27}{16}-3i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{35\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8}+\frac{2\left(\frac{1}{4}-\frac{2i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}}{a^{4}d}$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/12*I/d/a^4/(exp(2*I*(d*x+c))+1)^4*(105*exp(7*I*(d*x+c))+385*exp(5*I*(d*
x+c))+511*exp(3*I*(d*x+c))+279*exp(I*(d*x+c)))+35/8/a^4/d*ln(exp(I*(d*x+c)
)+I)-35/8/a^4/d*ln(exp(I*(d*x+c))-I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.73

$$\int \frac{\sec^{9}(c+dx)}{(a+ia\tan(c+dx))^{4}}\,dx$$
$$=\frac{105\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}+4\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-105\left(e^{(8i\,dx+8i\,c)}+4\right.}{24\left(a^{4}de^{(8i\,dx+8i\,c)}+4\,a^{4}d\right.}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/24*(105*(e^(8*I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*
I*c) + 4*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 105*(e^(8*I*d
*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x
 + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 210*I*e^(7*I*d*x + 7*I*c) - 770*
I*e^(5*I*d*x + 5*I*c) - 1022*I*e^(3*I*d*x + 3*I*c) - 558*I*e^(I*d*x + I*c)
)/(a^4*d*e^(8*I*d*x + 8*I*c) + 4*a^4*d*e^(6*I*d*x + 6*I*c) + 6*a^4*d*e^(4*
I*d*x + 4*I*c) + 4*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^9(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**9/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
+ d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(117) = 234$.

Time = 0.05 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.22

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\frac{2\left(\frac{81\sin(dx+c)}{\cos(dx+c)+1}-\frac{544i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{105\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{480i\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{105\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{96i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{81\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+160i\right)}{a^4-\frac{4a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8}} - 105\log(}{24d}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/24*(2*(81*sin(d*x + c)/(cos(d*x + c) + 1) - 544*I*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 - 105*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 480*I*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 105*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 96*
I*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 81*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 + 160*I)/(a^4 - 4*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^4*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + a^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 105*log(sin(d*x + c)/(cos(
d*x + c) + 1) + 1)/a^4 + 105*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)
/d
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.14

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\frac{105\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^4} - \frac{105\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^4} - \frac{2\left(81\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 96i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 - 105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 480i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 544i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 81\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 160i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^4 a^4}}{24\,d}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/24*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^4 - 105*log(tan(1/2*d*x + 1/2*c)
 - 1)/a^4 - 2*(81*tan(1/2*d*x + 1/2*c)^7 - 96*I*tan(1/2*d*x + 1/2*c)^6 - 1
05*tan(1/2*d*x + 1/2*c)^5 + 480*I*tan(1/2*d*x + 1/2*c)^4 - 105*tan(1/2*d*x
 + 1/2*c)^3 - 544*I*tan(1/2*d*x + 1/2*c)^2 + 81*tan(1/2*d*x + 1/2*c) + 160
*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*a^4))/d
```

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.48

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{35\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4\,a^4\,d}$$
$$+ \frac{\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{4\,a^4} + \frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4\,a^4} - \frac{27\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{4\,a^4} - \frac{27\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,a^4} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 136i}{3\,a^4} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 40i}{a^4} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 8i}{a^4} - \frac{40i}{3\,a^4}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - 4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 1\right)}$$

input
```
int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
(35*atanh(tan(c/2 + (d*x)/2)))/(4*a^4*d) + ((tan(c/2 + (d*x)/2)^2*136i)/(3
*a^4) + (35*tan(c/2 + (d*x)/2)^3)/(4*a^4) - (tan(c/2 + (d*x)/2)^4*40i)/a^4
 + (35*tan(c/2 + (d*x)/2)^5)/(4*a^4) + (tan(c/2 + (d*x)/2)^6*8i)/a^4 - (27
*tan(c/2 + (d*x)/2)^7)/(4*a^4) - 40i/(3*a^4) - (27*tan(c/2 + (d*x)/2))/(4*
a^4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d
*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.62

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{192\cos(dx+c)\sin(dx+c)^2 i - 160\cos(dx+c)\,i - 105\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4 + 210\log\left(\mathrm{t}\right.}{}$$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(192*cos(c + d*x)*sin(c + d*x)**2*i - 160*cos(c + d*x)*i - 105*log(tan((c
+ d*x)/2) - 1)*sin(c + d*x)**4 + 210*log(tan((c + d*x)/2) - 1)*sin(c + d*x
)**2 - 105*log(tan((c + d*x)/2) - 1) + 105*log(tan((c + d*x)/2) + 1)*sin(c
 + d*x)**4 - 210*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 + 105*log(tan((
c + d*x)/2) + 1) - 112*sin(c + d*x)**4*i + 87*sin(c + d*x)**3 + 224*sin(c
+ d*x)**2*i - 81*sin(c + d*x) - 112*i)/(24*a**4*d*(sin(c + d*x)**4 - 2*sin
(c + d*x)**2 + 1))
```

3.159 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{15\text{arctanh}(\sin(c+dx))}{2a^4d} - \frac{15\sec(c+dx)\tan(c+dx)}{2a^4d} + \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3} + \frac{10i\sec^3(c+dx)}{d\,(a^4+ia^4\tan(c+dx))}$$

output
```
-15/2*arctanh(sin(d*x+c))/a^4/d-15/2*sec(d*x+c)*tan(d*x+c)/a^4/d+2*I*sec(d
*x+c)^5/a/d/(a+I*a*tan(d*x+c))^3+10*I*sec(d*x+c)^3/d/(a^4+I*a^4*tan(d*x+c)
)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 988 vs. 2(107) = 214.

Time = 6.50 (sec) , antiderivative size = 988, normalized size of antiderivative = 9.23

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(15*Cos[4*c]*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^4*(
Cos[d*x] + I*Sin[d*x])^4)/(2*d*(a + I*a*Tan[c + d*x])^4) - (15*Cos[4*c]*Lo
g[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^4*(Cos[d*x] + I*Si
n[d*x])^4)/(2*d*(a + I*a*Tan[c + d*x])^4) + (Cos[d*x]*Sec[c + d*x]^4*((8*I
)*Cos[3*c] - 8*Sin[3*c])*(Cos[d*x] + I*Sin[d*x])^4)/(d*(a + I*a*Tan[c + d*
x])^4) + (Sec[c]*Sec[c + d*x]^4*((4*I)*Cos[4*c] - 4*Sin[4*c])*(Cos[d*x] +
I*Sin[d*x])^4)/(d*(a + I*a*Tan[c + d*x])^4) + (((15*I)/2)*Log[Cos[c/2 + (d
*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^4*Sin[4*c]*(Cos[d*x] + I*Sin[d*x
])^4)/(d*(a + I*a*Tan[c + d*x])^4) - (((15*I)/2)*Log[Cos[c/2 + (d*x)/2] +
Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^4*Sin[4*c]*(Cos[d*x] + I*Sin[d*x])^4)/(d*
(a + I*a*Tan[c + d*x])^4) + (Sec[c + d*x]^4*(8*Cos[3*c] + (8*I)*Sin[3*c])*
(Cos[d*x] + I*Sin[d*x])^4*Sin[d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (Sec[c
+ d*x]^4*(Cos[4*c]/4 + (I/4)*Sin[4*c])*(Cos[d*x] + I*Sin[d*x])^4)/(d*(Cos[
c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c + d*x])^4) + (Sec[c
+ d*x]^4*(-1/4*Cos[4*c] - (I/4)*Sin[4*c])*(Cos[d*x] + I*Sin[d*x])^4)/(d*(C
os[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c + d*x])^4) + (4*S
ec[c + d*x]^4*(Cos[d*x] + I*Sin[d*x])^4*(Cos[4*c - (d*x)/2]/2 - Cos[4*c +
(d*x)/2]/2 + (I/2)*Sin[4*c - (d*x)/2] - (I/2)*Sin[4*c + (d*x)/2]))/(d*(Cos
[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])*(a + I*a*Tan[c
 + d*x])^4) + (4*Sec[c + d*x]^4*(Cos[d*x] + I*Sin[d*x])^4*(-1/2*Cos[4*c...
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3981, 3042, 3981, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^7}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\int\frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\int\frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^2}dx}{a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\int\sec^3(c+dx)dx}{a^2}-\frac{2i\sec^3(c+dx)}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{2i\sec^3(c+dx)}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{2i\sec^3(c+dx)}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{2i\sec^3(c+dx)}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{2i\sec^3(c+dx)}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

input
```
Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((2*I)*Sec[c + d*x]^5)/(a*d*(a + I*a*Tan[c + d*x])^3) - (5*(((-2*I)*Sec[c
+ d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x])) + (3*(ArcTanh[Sin[c + d*x]]/(2*d)
 + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/a^2))/a^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

method	result
risch	$\frac{8i\,\mathrm{e}^{-i(dx+c)}}{a^4 d}+\frac{i\left(7\,\mathrm{e}^{3i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}\right)}{d\,a^4\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{15\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2a^4 d}+\frac{15\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2a^4 d}$
derivativedivides	$\frac{\frac{16}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2\left(\frac{1}{4}+2i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{15\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\frac{2\left(\frac{1}{4}-2i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+}{a^4 d}$
default	$\frac{\frac{16}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2\left(\frac{1}{4}+2i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{15\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\frac{2\left(\frac{1}{4}-2i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+}{a^4 d}$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
8*I/a^4/d*exp(-I*(d*x+c))+I/d/a^4/(exp(2*I*(d*x+c))+1)^2*(7*exp(3*I*(d*x+c
))+9*exp(I*(d*x+c)))-15/2/a^4/d*ln(exp(I*(d*x+c))+I)+15/2/a^4/d*ln(exp(I*(
d*x+c))-I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.50

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{15\left(e^{(5i\,dx+5i\,c)}+2\,e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(5i\,dx+5i\,c)}+2\,e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)}{2\left(a^4de^{(5i\,dx+5i\,c)}+2\,a^4de^{(3i\,dx+3i\,c)}+a^4de^{(i\,dx+i}\right.}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/2*(15*(e^(5*I*d*x + 5*I*c) + 2*e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*l
og(e^(I*d*x + I*c) + I) - 15*(e^(5*I*d*x + 5*I*c) + 2*e^(3*I*d*x + 3*I*c)
+ e^(I*d*x + I*c))*log(e^(I*d*x + I*c) - I) - 30*I*e^(4*I*d*x + 4*I*c) - 5
0*I*e^(2*I*d*x + 2*I*c) - 16*I)/(a^4*d*e^(5*I*d*x + 5*I*c) + 2*a^4*d*e^(3*
I*d*x + 3*I*c) + a^4*d*e^(I*d*x + I*c))
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^7(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**7/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
+ d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(95) = 190$.

Time = 0.16 (sec) , antiderivative size = 457, normalized size of antiderivative = 4.27

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{30\left(\cos\left(5\,dx+5\,c\right)+2\,\cos\left(3\,dx+3\,c\right)+\cos\left(dx+c\right)+i\,\sin\left(5\,dx+5\,c\right)+2i\,\sin\left(3\,dx+3\,c\right)+i\,\sin\left(d\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
(30*(cos(5*d*x + 5*c) + 2*cos(3*d*x + 3*c) + cos(d*x + c) + I*sin(5*d*x +
5*c) + 2*I*sin(3*d*x + 3*c) + I*sin(d*x + c))*arctan2(cos(d*x + c), sin(d*
x + c) + 1) + 30*(cos(5*d*x + 5*c) + 2*cos(3*d*x + 3*c) + cos(d*x + c) + I
*sin(5*d*x + 5*c) + 2*I*sin(3*d*x + 3*c) + I*sin(d*x + c))*arctan2(cos(d*x
 + c), -sin(d*x + c) + 1) + 15*(I*cos(5*d*x + 5*c) + 2*I*cos(3*d*x + 3*c)
+ I*cos(d*x + c) - sin(5*d*x + 5*c) - 2*sin(3*d*x + 3*c) - sin(d*x + c))*l
og(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 15*(-I*cos(5*d*
x + 5*c) - 2*I*cos(3*d*x + 3*c) - I*cos(d*x + c) + sin(5*d*x + 5*c) + 2*si
n(3*d*x + 3*c) + sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin
(d*x + c) + 1) + 60*cos(4*d*x + 4*c) + 100*cos(2*d*x + 2*c) + 60*I*sin(4*d
*x + 4*c) + 100*I*sin(2*d*x + 2*c) + 32)/((-4*I*a^4*cos(5*d*x + 5*c) - 8*I
*a^4*cos(3*d*x + 3*c) - 4*I*a^4*cos(d*x + c) + 4*a^4*sin(5*d*x + 5*c) + 8*
a^4*sin(3*d*x + 3*c) + 4*a^4*sin(d*x + c))*d)
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx =$$
$$-\frac{\frac{15\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^4}-\frac{15\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^4}-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-8i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^4}-\frac{}{a^4\left(\tan\left(\frac{1}{2}\right.\right.}}{2\,d}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/2*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a^4 - 15*log(tan(1/2*d*x + 1/2*c) -
 1)/a^4 - 2*(tan(1/2*d*x + 1/2*c)^3 - 8*I*tan(1/2*d*x + 1/2*c)^2 + tan(1/2
*d*x + 1/2*c) + 8*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - 32/(a^4*(tan(1
/2*d*x + 1/2*c) - I)))/d
```

Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.51

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{15\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d} + \frac{\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{a^4}-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,39\mathrm{i}}{a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,17\mathrm{i}}{a^4}+\frac{24\mathrm{i}}{a^4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,1\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,2\mathrm{i}-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}+1\right)}$$

input
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
((9*tan(c/2 + (d*x)/2)^3)/a^4 - (tan(c/2 + (d*x)/2)^2*39i)/a^4 + (tan(c/2
+ (d*x)/2)^4*17i)/a^4 + 24i/a^4 - (7*tan(c/2 + (d*x)/2))/a^4)/(d*(tan(c/2
+ (d*x)/2)*1i - 2*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*2i + tan(c/2
 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*1i + 1)) - (15*atanh(tan(c/2 + (d*x)/
2)))/(a^4*d)
```

Reduce [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(914048*cos(c + d*x)**2*sin(c + d*x) - 229432*cos(c + d*x)**2*i - 634368*c
os(c + d*x)*int(cos(c + d*x)/(8*cos(c + d*x)*sin(c + d*x)**6*i - 16*cos(c
+ d*x)*sin(c + d*x)**4*i + 9*cos(c + d*x)*sin(c + d*x)**2*i - cos(c + d*x)
*i - 8*sin(c + d*x)**7 + 20*sin(c + d*x)**5 - 16*sin(c + d*x)**3 + 4*sin(c
 + d*x)),x)*sin(c + d*x)**2*d*i + 634368*cos(c + d*x)*int(cos(c + d*x)/(8*
cos(c + d*x)*sin(c + d*x)**6*i - 16*cos(c + d*x)*sin(c + d*x)**4*i + 9*cos
(c + d*x)*sin(c + d*x)**2*i - cos(c + d*x)*i - 8*sin(c + d*x)**7 + 20*sin(
c + d*x)**5 - 16*sin(c + d*x)**3 + 4*sin(c + d*x)),x)*d*i - 68096*cos(c +
d*x)*int(cos(c + d*x)/(8*cos(c + d*x)*sin(c + d*x)**5*i - 12*cos(c + d*x)*
sin(c + d*x)**3*i + 4*cos(c + d*x)*sin(c + d*x)*i - 8*sin(c + d*x)**6 + 16
*sin(c + d*x)**4 - 9*sin(c + d*x)**2 + 1),x)*sin(c + d*x)**2*d + 68096*cos
(c + d*x)*int(cos(c + d*x)/(8*cos(c + d*x)*sin(c + d*x)**5*i - 12*cos(c +
d*x)*sin(c + d*x)**3*i + 4*cos(c + d*x)*sin(c + d*x)*i - 8*sin(c + d*x)**6
 + 16*sin(c + d*x)**4 - 9*sin(c + d*x)**2 + 1),x)*d + 68096*cos(c + d*x)*i
nt(cos(c + d*x)/(8*cos(c + d*x)*sin(c + d*x)**5 - 12*cos(c + d*x)*sin(c +
d*x)**3 + 4*cos(c + d*x)*sin(c + d*x) + 8*sin(c + d*x)**6*i - 16*sin(c + d
*x)**4*i + 9*sin(c + d*x)**2*i - i),x)*sin(c + d*x)**2*d*i - 68096*cos(c +
 d*x)*int(cos(c + d*x)/(8*cos(c + d*x)*sin(c + d*x)**5 - 12*cos(c + d*x)*s
in(c + d*x)**3 + 4*cos(c + d*x)*sin(c + d*x) + 8*sin(c + d*x)**6*i - 16*si
n(c + d*x)**4*i + 9*sin(c + d*x)**2*i - i),x)*d*i - 922112*cos(c + d*x)...
```

3.160 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^4d} + \frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3} - \frac{2i\sec(c+dx)}{d\,(a^4+ia^4\tan(c+dx))}$$

output
```
arctanh(sin(d*x+c))/a^4/d+2/3*I*sec(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^3-2*I*
sec(d*x+c)/d/(a^4+I*a^4*tan(d*x+c))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 247 vs. 2(82) = 164.

Time = 0.56 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.01

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\sec^4(c+dx)(\cos(dx)+i\sin(dx))^4\left(-3\cos(4c)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(4c)\log\left(c\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^4*(Cos[d*x] + I*Sin[d*x])^4*(-3*Cos[4*c]*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]] + 3*Cos[4*c]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
] - 2*Cos[3*d*x]*Sin[c] + 6*Cos[d*x]*Sin[3*c] - (3*I)*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]]*Sin[4*c] + (3*I)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]]*Sin[4*c] + Cos[3*c]*((-6*I)*Cos[d*x] - 6*Sin[d*x]) - (6*I)*Sin[3*c]*S
in[d*x] + (2*I)*Sin[c]*Sin[3*d*x] + 2*Cos[c]*(I*Cos[3*d*x] + Sin[3*d*x]))
/(3*a^4*d*(-I + Tan[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3981, 3042, 3981, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^5}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3981}$$
$$\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{a^2}$$
$$\Big\downarrow \text{3042}$$
$$\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^2}dx}{a^2}$$
$$\Big\downarrow \text{3981}$$
$$\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{-\frac{\int \sec(c+dx)dx}{a^2}+\frac{2i\sec(c+dx)}{d(a^2+ia^2\tan(c+dx))}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{-\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}+\frac{2i\sec(c+dx)}{d(a^2+ia^2\tan(c+dx))}}{a^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{-\frac{\operatorname{arctanh}(\sin(c+dx))}{a^2d}+\frac{2i\sec(c+dx)}{d(a^2+ia^2\tan(c+dx))}}{a^2}$$

input
```
Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((2*I)/3)*Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^3) - (-(ArcTanh[Sin
[c + d*x]]/(a^2*d)) + ((2*I)*Sec[c + d*x])/(d*(a^2 + I*a^2*Tan[c + d*x])))
/a^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{16}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}}{a^{4}d}$	71
default	$\frac{-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{16}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}}{a^{4}d}$	71
risch	$-\frac{2i\mathrm{e}^{-i(dx+c)}}{a^{4}d}+\frac{2i\mathrm{e}^{-3i(dx+c)}}{3a^{4}d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^{4}d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^{4}d}$	79

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
2/d/a^4*(-1/2*ln(tan(1/2*d*x+1/2*c)-1)+1/2*ln(tan(1/2*d*x+1/2*c)+1)+4*I/(-
I+tan(1/2*d*x+1/2*c))^2-8/3/(-I+tan(1/2*d*x+1/2*c))^3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(3\,e^{(3i\,dx+3i\,c)}\log\left(e^{(i\,dx+i\,c)}+i\right)-3\,e^{(3i\,dx+3i\,c)}\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-3i\,dx-3i\,c)}}{3\,a^4d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/3*(3*e^(3*I*d*x + 3*I*c)*log(e^(I*d*x + I*c) + I) - 3*e^(3*I*d*x + 3*I*c
)*log(e^(I*d*x + I*c) - I) - 6*I*e^(2*I*d*x + 2*I*c) + 2*I)*e^(-3*I*d*x -
3*I*c)/(a^4*d)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^5(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**5/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
+ d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.72

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{-6i\arctan(\cos(dx+c),\sin(dx+c)+1) - 6i\arctan(\cos(dx+c),-\sin(dx+c)+1) + 4i\cos(3dx}{}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/6*(-6*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 6*I*arctan2(cos(d*x +
c), -sin(d*x + c) + 1) + 4*I*cos(3*d*x + 3*c) - 12*I*cos(d*x + c) + 3*log(
cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - 3*log(cos(d*x + c)
^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 4*sin(3*d*x + 3*c) - 12*sin(d*
x + c))/(a^4*d)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^4} - \frac{3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^4} + \frac{8\left(3i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}}{3\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*log(tan(1/2*d*x + 1/2*c) + 1)/a^4 - 3*log(tan(1/2*d*x + 1/2*c) - 1)
/a^4 + 8*(3*I*tan(1/2*d*x + 1/2*c) + 1)/(a^4*(tan(1/2*d*x + 1/2*c) - I)^3)
)/d
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d} - \frac{-\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4}+\frac{8\mathrm{i}}{3\,a^4}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,1\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+1\right)}$$

input
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
(2*atanh(tan(c/2 + (d*x)/2)))/(a^4*d) - (8i/(3*a^4) - (8*tan(c/2 + (d*x)/2
))/a^4)/(d*(tan(c/2 + (d*x)/2)*3i - 3*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*
x)/2)^3*1i + 1))
```

Reduce [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(496*int(cos(c + d*x)/(8*cos(c + d*x)*sin(c + d*x)**4*i - 8*cos(c + d*x)*s
in(c + d*x)**2*i + cos(c + d*x)*i - 8*sin(c + d*x)**5 + 12*sin(c + d*x)**3
 - 4*sin(c + d*x)),x)*d*i - 3552*int(sin(c + d*x)**5/(8*cos(c + d*x)*sin(c
 + d*x)**4*i - 8*cos(c + d*x)*sin(c + d*x)**2*i + cos(c + d*x)*i - 8*sin(c
 + d*x)**5 + 12*sin(c + d*x)**3 - 4*sin(c + d*x)),x)*d + 3960*int(sin(c +
d*x)**4/(8*cos(c + d*x)*sin(c + d*x)**4*i - 8*cos(c + d*x)*sin(c + d*x)**2
*i + cos(c + d*x)*i - 8*sin(c + d*x)**5 + 12*sin(c + d*x)**3 - 4*sin(c + d
*x)),x)*d*i - 96*int(sin(c + d*x)**4/(8*cos(c + d*x)*sin(c + d*x)**3*i - 4
*cos(c + d*x)*sin(c + d*x)*i - 8*sin(c + d*x)**4 + 8*sin(c + d*x)**2 - 1),
x)*d + 5600*int(sin(c + d*x)**3/(8*cos(c + d*x)*sin(c + d*x)**4*i - 8*cos(
c + d*x)*sin(c + d*x)**2*i + cos(c + d*x)*i - 8*sin(c + d*x)**5 + 12*sin(c
 + d*x)**3 - 4*sin(c + d*x)),x)*d - 3960*int(sin(c + d*x)**2/(8*cos(c + d*
x)*sin(c + d*x)**4*i - 8*cos(c + d*x)*sin(c + d*x)**2*i + cos(c + d*x)*i -
 8*sin(c + d*x)**5 + 12*sin(c + d*x)**3 - 4*sin(c + d*x)),x)*d*i - 480*int
(( - cos(c + d*x))/(8*cos(c + d*x)*sin(c + d*x)**4 - 8*cos(c + d*x)*sin(c
+ d*x)**2 + cos(c + d*x) + 8*sin(c + d*x)**5*i - 12*sin(c + d*x)**3*i + 4*
sin(c + d*x)*i),x)*d - 3232*int(( - sin(c + d*x)**5)/(8*cos(c + d*x)*sin(c
 + d*x)**4 - 8*cos(c + d*x)*sin(c + d*x)**2 + cos(c + d*x) + 8*sin(c + d*x
)**5*i - 12*sin(c + d*x)**3*i + 4*sin(c + d*x)*i),x)*d*i - 3840*int(( - si
n(c + d*x)**4)/(8*cos(c + d*x)*sin(c + d*x)**4 - 8*cos(c + d*x)*sin(c +...
```

3.161 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4} + \frac{i\sec^3(c+dx)}{15ad(a+ia\tan(c+dx))^3}$$

output
```
1/5*I*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^4+1/15*I*sec(d*x+c)^3/a/d/(a+I*a*t
an(d*x+c))^3
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\sec^3(c+dx)(-4i+\tan(c+dx))}{15a^4d(-i+\tan(c+dx))^4}$$

input
```
Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
-1/15*(Sec[c + d*x]^3*(-4*I + Tan[c + d*x]))/(a^4*d*(-I + Tan[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^3}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{5a}+\frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^3}dx}{5a}+\frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3969}$$

$$\frac{i\sec^3(c+dx)}{15ad(a+ia\tan(c+dx))^3}+\frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}$$

input
```
Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/5)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^4) + ((I/15)*Sec[c + d*x]
^3)/(a*d*(a + I*a*Tan[c + d*x])^3)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56

method	result	size
risch	$\frac{i\,\mathrm{e}^{-3 i (d x+c)}}{6 a^{4} d}+\frac{i\,\mathrm{e}^{-5 i (d x+c)}}{10 a^{4} d}$	38
derivativedivides	$\frac{\frac{2}{-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)}-\frac{28}{3\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{3}}-\frac{8 i}{\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{16}{5\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{6 i}{\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{2}}}{a^{4} d}$	90
default	$\frac{\frac{2}{-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)}-\frac{28}{3\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{3}}-\frac{8 i}{\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{16}{5\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{6 i}{\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{2}}}{a^{4} d}$	90

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*I/a^4/d*exp(-3*I*(d*x+c))+1/10*I/a^4/d*exp(-5*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.44

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(5i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{30\,a^4d}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/30*(5*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*x - 5*I*c)/(a^4*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(54) = 108.

Time = 1.33 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.68

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \begin{cases} -\frac{\tan(c+dx)\sec^3(c+dx)}{15a^4d\tan^4(c+dx)-60ia^4d\tan^3(c+dx)-90a^4d\tan^2(c+dx)+60ia^4d\tan(c+dx)+15a^4d} + \frac{4i\sec^3(}{15a^4d\tan^4(c+dx)-60ia^4d\tan^3(c+dx)-90a} \\ \frac{x\sec^3(c)}{(ia\tan(c)+a)^4} \end{cases}$$

input
```
integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise((-tan(c + d*x)*sec(c + d*x)**3/(15*a**4*d*tan(c + d*x)**4 - 60*I
*a**4*d*tan(c + d*x)**3 - 90*a**4*d*tan(c + d*x)**2 + 60*I*a**4*d*tan(c +
d*x) + 15*a**4*d) + 4*I*sec(c + d*x)**3/(15*a**4*d*tan(c + d*x)**4 - 60*I*
a**4*d*tan(c + d*x)**3 - 90*a**4*d*tan(c + d*x)**2 + 60*I*a**4*d*tan(c + d
*x) + 15*a**4*d), Ne(d, 0)), (x*sec(c)**3/(I*a*tan(c) + a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{3i\,\cos\left(5\,dx+5\,c\right)+5i\,\cos\left(3\,dx+3\,c\right)+3\,\sin\left(5\,dx+5\,c\right)+5\,\sin\left(3\,dx+3\,c\right)}{30\,a^4d}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/30*(3*I*cos(5*d*x + 5*c) + 5*I*cos(3*d*x + 3*c) + 3*sin(5*d*x + 5*c) + 5
*sin(3*d*x + 3*c))/(a^4*d)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-15i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-25\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{15\,a^4d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^5}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
2/15*(15*tan(1/2*d*x + 1/2*c)^4 - 15*I*tan(1/2*d*x + 1/2*c)^3 - 25*tan(1/2
*d*x + 1/2*c)^2 + 5*I*tan(1/2*d*x + 1/2*c) + 4)/(a^4*d*(tan(1/2*d*x + 1/2*
c) - I)^5)
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.96

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{2\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 15i+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 25i-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+4i\right)}{15\,a^4\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 1i+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 10i-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,5i+\right.}$$

input
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
(2*(15*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*25i - 5*tan(c/2 + (d*x)
/2) + tan(c/2 + (d*x)/2)^4*15i + 4i))/(15*a^4*d*(tan(c/2 + (d*x)/2)*5i - 1
0*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*10i + 5*tan(c/2 + (d*x)/2)^4
 + tan(c/2 + (d*x)/2)^5*1i + 1))
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x)
```

output

```
( - 2*int(cos(c + d*x)/(8*cos(c + d*x)*sin(c + d*x)**3*i - 4*cos(c + d*x)*
sin(c + d*x)*i - 8*sin(c + d*x)**4 + 8*sin(c + d*x)**2 - 1),x)*d - 16*int(
sin(c + d*x)**4/(8*cos(c + d*x)*sin(c + d*x)**3*i - 4*cos(c + d*x)*sin(c +
 d*x)*i - 8*sin(c + d*x)**4 + 8*sin(c + d*x)**2 - 1),x)*d - 16*int(sin(c +
 d*x)**2/(8*cos(c + d*x)*sin(c + d*x)**3 - 4*cos(c + d*x)*sin(c + d*x) + 8
*sin(c + d*x)**4*i - 8*sin(c + d*x)**2*i + i),x)*d*i + 16*int((cos(c + d*x
)*sin(c + d*x)**3)/(8*cos(c + d*x)*sin(c + d*x)**3 - 4*cos(c + d*x)*sin(c
+ d*x) + 8*sin(c + d*x)**4*i - 8*sin(c + d*x)**2*i + i),x)*d - 8*int((cos(
c + d*x)*sin(c + d*x))/(8*cos(c + d*x)*sin(c + d*x)**3 - 4*cos(c + d*x)*si
n(c + d*x) + 8*sin(c + d*x)**4*i - 8*sin(c + d*x)**2*i + i),x)*d - 2*int(1
/(8*cos(c + d*x)*sin(c + d*x)**3*i - 4*cos(c + d*x)*sin(c + d*x)*i - 8*sin
(c + d*x)**4 + 8*sin(c + d*x)**2 - 1),x)*d + log(tan((c + d*x)/2)**8 - 8*t
an((c + d*x)/2)**7*i - 28*tan((c + d*x)/2)**6 + 56*tan((c + d*x)/2)**5*i +
 70*tan((c + d*x)/2)**4 - 56*tan((c + d*x)/2)**3*i - 28*tan((c + d*x)/2)**
2 + 8*tan((c + d*x)/2)*i + 1)*i - 8*log(tan((c + d*x)/2)**2 + 1)*i + log(t
an((c + d*x)/2)**8*i + 8*tan((c + d*x)/2)**7 - 28*tan((c + d*x)/2)**6*i -
56*tan((c + d*x)/2)**5 + 70*tan((c + d*x)/2)**4*i + 56*tan((c + d*x)/2)**3
 - 28*tan((c + d*x)/2)**2*i - 8*tan((c + d*x)/2) + i)*i + 6*d*x)/(2*a**4*d
)
```

3.162 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 132

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}+\frac{3i\sec(c+dx)}{35ad(a+ia\tan(c+dx))^3}+\frac{2i\sec(c+dx)}{35d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{2i\sec(c+dx)}{35d\left(a^4+ia^4\tan(c+dx)\right)}$$

output
```
1/7*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^4+3/35*I*sec(d*x+c)/a/d/(a+I*a*tan(d
*x+c))^3+2/35*I*sec(d*x+c)/d/(a^2+I*a^2*tan(d*x+c))^2+2/35*I*sec(d*x+c)/d/
(a^4+I*a^4*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.55

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\sec^4(c+dx)(28\cos(c+dx)+20\cos(3(c+dx))+7i\sin(c+dx)+15i\sin(3(c+dx)))}{140a^4d(-i+\tan(c+dx))^4}$$

input
```
Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/140)*Sec[c + d*x]^4*(28*Cos[c + d*x] + 20*Cos[3*(c + d*x)] + (7*I)*Sin
[c + d*x] + (15*I)*Sin[3*(c + d*x)]))/(a^4*d*(-I + Tan[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\left(\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}\right)}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}\right)}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{\sec(c+dx)}{i\tan(c+dx)a+a}dx}{3a}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}\right)}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}\right)}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{\sec(c+dx)}{i\tan(c+dx)a+a}dx}{3a}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}\right)}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}\right)}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3969}$$

$$\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}+\frac{3\left(\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}+\frac{2\left(\frac{i\sec(c+dx)}{3ad(a+ia\tan(c+dx))}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}\right)}{5a}\right)}{7a}$$

input
```
Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/7)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (3*(((I/5)*Sec[c + d*x
])/(d*(a + I*a*Tan[c + d*x])^3) + (2*(((I/3)*Sec[c + d*x])/(d*(a + I*a*Tan
[c + d*x])^2) + ((I/3)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x]))))/(5*a))
)/(7*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.56

method	result
risch	$\frac{i e^{-i(dx+c)}}{8a^4d}+\frac{i e^{-3i(dx+c)}}{8a^4d}+\frac{3i e^{-5i(dx+c)}}{40a^4d}+\frac{i e^{-7i(dx+c)}}{56a^4d}$
derivativedivides	$\frac{\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{12}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{16i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{16}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{72}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}\right.\right.}}{a^4d}$
default	$\frac{\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{12}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{16i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{16}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{72}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}\right.\right.}}{a^4d}$

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8*I/a^4/d*exp(-I*(d*x+c))+1/8*I/a^4/d*exp(-3*I*(d*x+c))+3/40*I/a^4/d*exp
(-5*I*(d*x+c))+1/56*I/a^4/d*exp(-7*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.39

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(35i\,e^{(6i\,dx+6i\,c)}+35i\,e^{(4i\,dx+4i\,c)}+21i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-7i\,dx-7i\,c)}}{280\,a^4d}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/280*(35*I*e^(6*I*d*x + 6*I*c) + 35*I*e^(4*I*d*x + 4*I*c) + 21*I*e^(2*I*d
*x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a^4*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(112) = 224$.

Time = 1.41 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.68

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \begin{cases} \frac{2\tan^3(c+dx)\sec(c+dx)}{35a^4d\tan^4(c+dx)-140ia^4d\tan^3(c+dx)-210a^4d\tan^2(c+dx)+140ia^4d\tan(c+dx)+35a^4d} - \frac{8i\tan^2(c+}{35a^4d\tan^4(c+dx)-140ia^4d\tan^3(c+dx)-2} \\ \frac{x\sec(c)}{(ia\tan(c)+a)^4} \end{cases}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise((2*tan(c + d*x)**3*sec(c + d*x)/(35*a**4*d*tan(c + d*x)**4 - 140
*I*a**4*d*tan(c + d*x)**3 - 210*a**4*d*tan(c + d*x)**2 + 140*I*a**4*d*tan(
c + d*x) + 35*a**4*d) - 8*I*tan(c + d*x)**2*sec(c + d*x)/(35*a**4*d*tan(c
+ d*x)**4 - 140*I*a**4*d*tan(c + d*x)**3 - 210*a**4*d*tan(c + d*x)**2 + 14
0*I*a**4*d*tan(c + d*x) + 35*a**4*d) - 13*tan(c + d*x)*sec(c + d*x)/(35*a*
*4*d*tan(c + d*x)**4 - 140*I*a**4*d*tan(c + d*x)**3 - 210*a**4*d*tan(c + d
*x)**2 + 140*I*a**4*d*tan(c + d*x) + 35*a**4*d) + 12*I*sec(c + d*x)/(35*a*
*4*d*tan(c + d*x)**4 - 140*I*a**4*d*tan(c + d*x)**3 - 210*a**4*d*tan(c + d
*x)**2 + 140*I*a**4*d*tan(c + d*x) + 35*a**4*d), Ne(d, 0)), (x*sec(c)/(I*a
*tan(c) + a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.69

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{5i\,\cos(7\,dx+7\,c)+21i\,\cos(5\,dx+5\,c)+35i\,\cos(3\,dx+3\,c)+35i\,\cos(dx+c)+5\,\sin(7\,dx+7\,c)+2}{280\,a^4d}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/280*(5*I*cos(7*d*x + 7*c) + 21*I*cos(5*d*x + 5*c) + 35*I*cos(3*d*x + 3*c
) + 35*I*cos(d*x + c) + 5*sin(7*d*x + 7*c) + 21*sin(5*d*x + 5*c) + 35*sin(
3*d*x + 3*c) + 35*sin(d*x + c))/(a^4*d)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.75

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{2\left(35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-105i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-210\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+210i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+147\right.}{35\,a^4d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^7}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
2/35*(35*tan(1/2*d*x + 1/2*c)^6 - 105*I*tan(1/2*d*x + 1/2*c)^5 - 210*tan(1
/2*d*x + 1/2*c)^4 + 210*I*tan(1/2*d*x + 1/2*c)^3 + 147*tan(1/2*d*x + 1/2*c
)^2 - 49*I*tan(1/2*d*x + 1/2*c) - 12)/(a^4*d*(tan(1/2*d*x + 1/2*c) - I)^7)
```

Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.48

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{e^{-c\,1i-d\,x\,1i}\,1i}{8}+\frac{e^{-c\,3i-d\,x\,3i}\,1i}{8}+\frac{e^{-c\,5i-d\,x\,5i}\,3i}{40}+\frac{e^{-c\,7i-d\,x\,7i}\,1i}{56}}{a^4\,d}$$

input
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
((exp(- c*1i - d*x*1i)*1i)/8 + (exp(- c*3i - d*x*3i)*1i)/8 + (exp(- c*5i -
 d*x*5i)*3i)/40 + (exp(- c*7i - d*x*7i)*1i)/56)/(a^4*d)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x)
```

output

```
( - 2*int(cos(c + d*x)/(8*cos(c + d*x)*sin(c + d*x)**3*i - 4*cos(c + d*x)*
sin(c + d*x)*i - 8*sin(c + d*x)**4 + 8*sin(c + d*x)**2 - 1),x)*d - 16*int(
sin(c + d*x)**4/(8*cos(c + d*x)*sin(c + d*x)**3*i - 4*cos(c + d*x)*sin(c +
 d*x)*i - 8*sin(c + d*x)**4 + 8*sin(c + d*x)**2 - 1),x)*d - 16*int(sin(c +
 d*x)**2/(8*cos(c + d*x)*sin(c + d*x)**3 - 4*cos(c + d*x)*sin(c + d*x) + 8
*sin(c + d*x)**4*i - 8*sin(c + d*x)**2*i + i),x)*d*i + 16*int((cos(c + d*x
)*sin(c + d*x)**3)/(8*cos(c + d*x)*sin(c + d*x)**3 - 4*cos(c + d*x)*sin(c
+ d*x) + 8*sin(c + d*x)**4*i - 8*sin(c + d*x)**2*i + i),x)*d - 2*int((cos(
c + d*x)*sin(c + d*x)**2)/(8*cos(c + d*x)*sin(c + d*x)**3 - 4*cos(c + d*x)
*sin(c + d*x) + 8*sin(c + d*x)**4*i - 8*sin(c + d*x)**2*i + i),x)*d*i - 8*
int((cos(c + d*x)*sin(c + d*x))/(8*cos(c + d*x)*sin(c + d*x)**3 - 4*cos(c
+ d*x)*sin(c + d*x) + 8*sin(c + d*x)**4*i - 8*sin(c + d*x)**2*i + i),x)*d
- 2*int(1/(8*cos(c + d*x)*sin(c + d*x)**3*i - 4*cos(c + d*x)*sin(c + d*x)*
i - 8*sin(c + d*x)**4 + 8*sin(c + d*x)**2 - 1),x)*d + log(tan((c + d*x)/2)
**8 - 8*tan((c + d*x)/2)**7*i - 28*tan((c + d*x)/2)**6 + 56*tan((c + d*x)/
2)**5*i + 70*tan((c + d*x)/2)**4 - 56*tan((c + d*x)/2)**3*i - 28*tan((c +
d*x)/2)**2 + 8*tan((c + d*x)/2)*i + 1)*i - 8*log(tan((c + d*x)/2)**2 + 1)*
i + log(tan((c + d*x)/2)**8*i + 8*tan((c + d*x)/2)**7 - 28*tan((c + d*x)/2
)**6*i - 56*tan((c + d*x)/2)**5 + 70*tan((c + d*x)/2)**4*i + 56*tan((c + d
*x)/2)**3 - 28*tan((c + d*x)/2)**2*i - 8*tan((c + d*x)/2) + i)*i + 6*d*...
```

3.163 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 134

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{4\sin(c+dx)}{21a^4d} - \frac{4\sin^3(c+dx)}{63a^4d} + \frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4} + \frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3} + \frac{8i\cos^3(c+dx)}{63d\left(a^4+ia^4\tan(c+dx)\right)}$$

output
```
4/21*sin(d*x+c)/a^4/d-4/63*sin(d*x+c)^3/a^4/d+1/9*I*cos(d*x+c)/d/(a+I*a*ta
n(d*x+c))^4+5/63*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^3+8/63*I*cos(d*x+c)^3
/d/(a^4+I*a^4*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.71

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{i\sec^4(c+dx)(-168\cos(c+dx) - 180\cos(3(c+dx)) + 28\cos(5(c+dx)) - 42i\sin(c+dx) - 135i\sin}{1008a^4d(-i+\tan(c+dx))^4}$$

input
```
Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-1/1008*I)*Sec[c + d*x]^4*(-168*Cos[c + d*x] - 180*Cos[3*(c + d*x)] + 28
*Cos[5*(c + d*x)] - (42*I)*Sin[c + d*x] - (135*I)*Sin[3*(c + d*x)] + (35*I
)*Sin[5*(c + d*x)]))/(a^4*d*(-I + Tan[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {3042, 3983, 3042, 3983, 3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(c+dx)(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3983}$$
$$\frac{5\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$
$$\Big\downarrow \text{3042}$$
$$\frac{5\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^3}dx}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$
$$\Big\downarrow \text{3983}$$
$$\frac{5\left(\frac{4\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$
$$\Big\downarrow \text{3042}$$
$$\frac{5\left(\frac{4\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^2}dx}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$
$$\Big\downarrow \text{3981}$$

$$\frac{5\left(\frac{4\left(\frac{3\int \cos^3(c+dx)dx}{5a^2}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{5\left(\frac{4\left(\frac{3\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{5a^2}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \; 3113$$

$$\frac{5\left(\frac{4\left(-\frac{3\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \; 2009$$

$$\frac{5\left(\frac{4\left(-\frac{3\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$

input
```
Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/9)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (5*(((I/7)*Cos[c + d*x
])/(d*(a + I*a*Tan[c + d*x])^3) + (4*((-3*(-Sin[c + d*x] + Sin[c + d*x]^3/
3))/(5*a^2*d) + (((2*I)/5)*Cos[c + d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x])))
)/(7*a)))/(9*a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

method	result
risch	$\frac{5i\mathrm{e}^{-3i(dx+c)}}{48a^4d}+\frac{i\mathrm{e}^{-5i(dx+c)}}{16a^4d}+\frac{5i\mathrm{e}^{-7i(dx+c)}}{224a^4d}+\frac{i\mathrm{e}^{-9i(dx+c)}}{288a^4d}+\frac{i\cos(dx+c)}{8a^4d}+\frac{3\sin(dx+c)}{16a^4d}$
derivativedivides	$\frac{\frac{86i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{49i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{49i}{8\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{16}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}-\frac{}{7(-i+}}{a^4d}$
default	$\frac{\frac{86i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{49i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{49i}{8\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{16}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}-\frac{}{7(-i+}}{a^4d}$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
5/48*I/a^4/d*exp(-3*I*(d*x+c))+1/16*I/a^4/d*exp(-5*I*(d*x+c))+5/224*I/a^4/
d*exp(-7*I*(d*x+c))+1/288*I/a^4/d*exp(-9*I*(d*x+c))+1/8*I/a^4/d*cos(d*x+c)
+3/16*sin(d*x+c)/a^4/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.55

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(-63i\,e^{(10i\,dx+10i\,c)}+315i\,e^{(8i\,dx+8i\,c)}+210i\,e^{(6i\,dx+6i\,c)}+126i\,e^{(4i\,dx+4i\,c)}+45i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{(-9i\,dx-9i\,c}}{2016\,a^4d}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/2016*(-63*I*e^(10*I*d*x + 10*I*c) + 315*I*e^(8*I*d*x + 8*I*c) + 210*I*e^
(6*I*d*x + 6*I*c) + 126*I*e^(4*I*d*x + 4*I*c) + 45*I*e^(2*I*d*x + 2*I*c) +
 7*I)*e^(-9*I*d*x - 9*I*c)/(a^4*d)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.72

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \begin{cases} \frac{(-1585446912ia^{20}d^5e^{26ic}e^{idx}+7927234560ia^{20}d^5e^{24ic}e^{-idx}+5284823040ia^{20}d^5e^{22ic}e^{-3idx}+3170893824ia^{20}d^5e^{20ic}e^{-5idx}+1132462080i}{50734301184a^{24}d^6} \\ \frac{x(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1)e^{-9ic}}{32a^4} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((-1585446912*I*a**20*d**5*exp(26*I*c)*exp(I*d*x) + 7927234560*I
*a**20*d**5*exp(24*I*c)*exp(-I*d*x) + 5284823040*I*a**20*d**5*exp(22*I*c)*
exp(-3*I*d*x) + 3170893824*I*a**20*d**5*exp(20*I*c)*exp(-5*I*d*x) + 113246
2080*I*a**20*d**5*exp(18*I*c)*exp(-7*I*d*x) + 176160768*I*a**20*d**5*exp(1
6*I*c)*exp(-9*I*d*x))*exp(-25*I*c)/(50734301184*a**24*d**6), Ne(a**24*d**6
*exp(25*I*c), 0)), (x*(exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*c) + 10*exp
(4*I*c) + 5*exp(2*I*c) + 1)*exp(-9*I*c)/(32*a**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.08

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$=\frac{\frac{63}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}+\frac{1953\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-9450i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-25998\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+42210i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+46368\,\tan\left(\frac{1}{2}\,dx+\right.}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^9}}{1008\,d}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/1008*(63/(a^4*(tan(1/2*d*x + 1/2*c) + I)) + (1953*tan(1/2*d*x + 1/2*c)^8
 - 9450*I*tan(1/2*d*x + 1/2*c)^7 - 25998*tan(1/2*d*x + 1/2*c)^6 + 42210*I*
tan(1/2*d*x + 1/2*c)^5 + 46368*tan(1/2*d*x + 1/2*c)^4 - 33054*I*tan(1/2*d*
x + 1/2*c)^3 - 15858*tan(1/2*d*x + 1/2*c)^2 + 4374*I*tan(1/2*d*x + 1/2*c)
+ 703)/(a^4*(tan(1/2*d*x + 1/2*c) - I)^9))/d
```

Mupad [B] (verification not implemented)

Time = 4.59 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.20

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$=\frac{\left(63\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 252\mathrm{i}-588\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 672\mathrm{i}+378\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\mathrm{t}\right.}{63\,a^4\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)\,\left(1+\mathrm{t}\right.}$$

input
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
((372*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*288i - 97*tan(c/2 + (d*x
)/2) + tan(c/2 + (d*x)/2)^4*168i + 378*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d
*x)/2)^6*672i - 588*tan(c/2 + (d*x)/2)^7 - tan(c/2 + (d*x)/2)^8*252i + 63*
tan(c/2 + (d*x)/2)^9 + 20i)*2i)/(63*a^4*d*(tan(c/2 + (d*x)/2) + 1i)*(tan(c
/2 + (d*x)/2)*1i + 1)^9)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\cos(dx+c)}{\tan(dx+c)^4-4\tan(dx+c)^3i-6\tan(dx+c)^2+4\tan(dx+c)i+1}dx}{a^4}$$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x)
```

output
```
int(cos(c + d*x)/(tan(c + d*x)**4 - 4*tan(c + d*x)**3*i - 6*tan(c + d*x)**
2 + 4*tan(c + d*x)*i + 1),x)/a**4
```

3.164 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{10\sin(c+dx)}{33a^4d} - \frac{20\sin^3(c+dx)}{99a^4d} + \frac{2\sin^5(c+dx)}{33a^4d} + \frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4} + \frac{7i\cos^3(c+dx)}{99ad(a+ia\tan(c+dx))^3} + \frac{4i\cos^5(c+dx)}{33d\,(a^4+ia^4\tan(c+dx))}$$

output
```
10/33*sin(d*x+c)/a^4/d-20/99*sin(d*x+c)^3/a^4/d+2/33*sin(d*x+c)^5/a^4/d+1/
11*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^4+7/99*I*cos(d*x+c)^3/a/d/(a+I*a*ta
n(d*x+c))^3+4/33*I*cos(d*x+c)^5/d/(a^4+I*a^4*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{i\sec^4(c+dx)(-924\cos(c+dx) - 1188\cos(3(c+dx)) + 308\cos(5(c+dx)) + 12\cos(7(c+dx)) - 23}{6336a^4d(-i+\tan(c+dx}$$

input
```
Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-1/6336*I)*Sec[c + d*x]^4*(-924*Cos[c + d*x] - 1188*Cos[3*(c + d*x)] + 3
08*Cos[5*(c + d*x)] + 12*Cos[7*(c + d*x)] - (231*I)*Sin[c + d*x] - (891*I)
*Sin[3*(c + d*x)] + (385*I)*Sin[5*(c + d*x)] + (21*I)*Sin[7*(c + d*x)]))/(
a^4*d*(-I + Tan[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3983, 3042, 3983, 3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^3}dx}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\left(\frac{2\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{2\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^2}dx}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3981}$$

$$\frac{7\left(\frac{2\left(\frac{5\int \cos^5(c+dx)dx}{7a^2}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{2\left(\frac{5\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx}{7a^2}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3113}$$

$$\frac{7\left(\frac{2\left(-\frac{5\int\left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{7\left(\frac{2\left(-\frac{5\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}$$

input
```
Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/11)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^4) + (7*(((I/9)*Cos[c +
d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3) + (2*((-5*(-Sin[c + d*x] + (2*Sin[c +
 d*x]^3)/3 - Sin[c + d*x]^5/5))/(7*a^2*d) + (((2*I)/7)*Cos[c + d*x]^5)/(d*
(a^2 + I*a^2*Tan[c + d*x]))))/(3*a)))/(11*a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.88

method	result
risch	$\frac{7i\mathrm{e}^{-5i(dx+c)}}{128a^4d}+\frac{3i\mathrm{e}^{-7i(dx+c)}}{128a^4d}+\frac{7i\mathrm{e}^{-9i(dx+c)}}{1152a^4d}+\frac{i\mathrm{e}^{-11i(dx+c)}}{1408a^4d}+\frac{7i\cos(dx+c)}{64a^4d}+\frac{7\sin(dx+c)}{32a^4d}+\frac{17i\cos(3dx+}{192a^4d}$
derivativedivides	$\frac{-\frac{i}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{2}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16i}+\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{67i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{}{(-i+\mathrm{ta}}}{}$
default	$\frac{-\frac{i}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{2}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16i}+\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{67i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{}{(-i+\mathrm{ta}}}{}$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
7/128*I/a^4/d*exp(-5*I*(d*x+c))+3/128*I/a^4/d*exp(-7*I*(d*x+c))+7/1152*I/a
^4/d*exp(-9*I*(d*x+c))+1/1408*I/a^4/d*exp(-11*I*(d*x+c))+7/64*I/a^4/d*cos(
d*x+c)+7/32*sin(d*x+c)/a^4/d+17/192*I/a^4/d*cos(3*d*x+3*c)+3/32/a^4/d*sin(
3*d*x+3*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.62

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$=\frac{\left(-33i\,e^{(14i\,dx+14i\,c)}-693i\,e^{(12i\,dx+12i\,c)}+2079i\,e^{(10i\,dx+10i\,c)}+1155i\,e^{(8i\,dx+8i\,c)}+693i\,e^{(6i\,dx+6i\,c)}+297i\,e^{(}}{12672\,a^4d}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/12672*(-33*I*e^(14*I*d*x + 14*I*c) - 693*I*e^(12*I*d*x + 12*I*c) + 2079*
I*e^(10*I*d*x + 10*I*c) + 1155*I*e^(8*I*d*x + 8*I*c) + 693*I*e^(6*I*d*x +
6*I*c) + 297*I*e^(4*I*d*x + 4*I*c) + 77*I*e^(2*I*d*x + 2*I*c) + 9*I)*e^(-1
1*I*d*x - 11*I*c)/(a^4*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(136) = 272.

Time = 0.48 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.92

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \begin{cases} \frac{(-167196136166129664ia^{28}d^7e^{39ic}e^{3idx} - 35111188594887229 44ia^{28}d^7e^{37ic}e^{idx} + 105333565784661688 32ia^{28}d^7e^{35ic}e^{-idx} + 5851864765814 \cdots}{} \\ \frac{x(e^{14ic}+7e^{12ic}+21e^{10ic}+35e^{8ic}+35e^{6ic}+21e^{4ic}+7e^{2ic}+1)e^{-11ic}}{128a^4} \end{cases}$$

input
```
integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((-167196136166129664*I*a**28*d**7*exp(39*I*c)*exp(3*I*d*x) - 35
11118859488722944*I*a**28*d**7*exp(37*I*c)*exp(I*d*x) + 105333565784661688
32*I*a**28*d**7*exp(35*I*c)*exp(-I*d*x) + 5851864765814538240*I*a**28*d**7
*exp(33*I*c)*exp(-3*I*d*x) + 3511118859488722944*I*a**28*d**7*exp(31*I*c)*
exp(-5*I*d*x) + 1504765225495166976*I*a**28*d**7*exp(29*I*c)*exp(-7*I*d*x)
 + 390124317720969216*I*a**28*d**7*exp(27*I*c)*exp(-9*I*d*x) + 45598946227
126272*I*a**28*d**7*exp(25*I*c)*exp(-11*I*d*x))*exp(-36*I*c)/(642033162877
93790976*a**32*d**8), Ne(a**32*d**8*exp(36*I*c), 0)), (x*(exp(14*I*c) + 7*
exp(12*I*c) + 21*exp(10*I*c) + 35*exp(8*I*c) + 35*exp(6*I*c) + 21*exp(4*I*
c) + 7*exp(2*I*c) + 1)*exp(-11*I*c)/(128*a**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.26

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\frac{33\left(12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+21i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-11\right)}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+\frac{5940\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-39501i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-141075\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+313236i\tan}{}}{}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3168*(33*(12*tan(1/2*d*x + 1/2*c)^2 + 21*I*tan(1/2*d*x + 1/2*c) - 11)/(a
^4*(tan(1/2*d*x + 1/2*c) + I)^3) + (5940*tan(1/2*d*x + 1/2*c)^10 - 39501*I
*tan(1/2*d*x + 1/2*c)^9 - 141075*tan(1/2*d*x + 1/2*c)^8 + 313236*I*tan(1/2
*d*x + 1/2*c)^7 + 479556*tan(1/2*d*x + 1/2*c)^6 - 516054*I*tan(1/2*d*x + 1
/2*c)^5 - 397914*tan(1/2*d*x + 1/2*c)^4 + 214500*I*tan(1/2*d*x + 1/2*c)^3
+ 79024*tan(1/2*d*x + 1/2*c)^2 - 17765*I*tan(1/2*d*x + 1/2*c) - 2155)/(a^4
*(tan(1/2*d*x + 1/2*c) - I)^11))/d
```

Mupad [B] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.38

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx =$$
$$-\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{269\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}-\frac{1307\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{64}+\frac{1307\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{64}-\frac{1099\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{32}+\frac{203\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{32}\right.}{99\,a^4\,d\,(\cos}$$

input
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
-(cos(c/2 + (d*x)/2)*((cos((3*c)/2 + (3*d*x)/2)*231i)/16 - (cos((5*c)/2 +
(5*d*x)/2)*231i)/16 + cos((7*c)/2 + (7*d*x)/2)*33i - cos((9*c)/2 + (9*d*x)
/2)*5i + (cos((11*c)/2 + (11*d*x)/2)*3i)/16 - (cos((13*c)/2 + (13*d*x)/2)*
3i)/16 + (269*sin(c/2 + (d*x)/2))/16 - (1307*sin((3*c)/2 + (3*d*x)/2))/64
+ (1307*sin((5*c)/2 + (5*d*x)/2))/64 - (1099*sin((7*c)/2 + (7*d*x)/2))/32
+ (203*sin((9*c)/2 + (9*d*x)/2))/32 - (21*sin((11*c)/2 + (11*d*x)/2))/64 +
 (21*sin((13*c)/2 + (13*d*x)/2))/64)*2i)/(99*a^4*d*(cos(c/2 + (d*x)/2) + s
in(c/2 + (d*x)/2)*1i)^11*(cos(c/2 + (d*x)/2)*1i + sin(c/2 + (d*x)/2))^3)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\cos(dx+c)^3}{\tan(dx+c)^4-4\tan(dx+c)^3 i-6\tan(dx+c)^2+4\tan(dx+c)i+1}dx}{a^4}$$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x)
```

output
```
int(cos(c + d*x)**3/(tan(c + d*x)**4 - 4*tan(c + d*x)**3*i - 6*tan(c + d*x
)**2 + 4*tan(c + d*x)*i + 1),x)/a**4
```

3.165 $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 174

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= \frac{56\sin(c+dx)}{143a^4d}-\frac{56\sin^3(c+dx)}{143a^4d}\\&\quad+\frac{168\sin^5(c+dx)}{715a^4d}-\frac{8\sin^7(c+dx)}{143a^4d}\\&\quad+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}+\frac{9i\cos^5(c+dx)}{143ad(a+ia\tan(c+dx))^3}\\&\quad+\frac{16i\cos^7(c+dx)}{143d\,(a^4+ia^4\tan(c+dx))}\end{aligned}$$

output
```
56/143*sin(d*x+c)/a^4/d-56/143*sin(d*x+c)^3/a^4/d+168/715*sin(d*x+c)^5/a^4
/d-8/143*sin(d*x+c)^7/a^4/d+1/13*I*cos(d*x+c)^5/d/(a+I*a*tan(d*x+c))^4+9/1
43*I*cos(d*x+c)^5/a/d/(a+I*a*tan(d*x+c))^3+16/143*I*cos(d*x+c)^7/d/(a^4+I*
a^4*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.80

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{i\sec^4(c+dx)(-24024\cos(c+dx)-34320\cos(3(c+dx))+11440\cos(5(c+dx))+780\cos(7(c+dx$$

input
```
Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-1/183040*I)*Sec[c + d*x]^4*(-24024*Cos[c + d*x] - 34320*Cos[3*(c + d*x)
] + 11440*Cos[5*(c + d*x)] + 780*Cos[7*(c + d*x)] + 44*Cos[9*(c + d*x)] -
(6006*I)*Sin[c + d*x] - (25740*I)*Sin[3*(c + d*x)] + (14300*I)*Sin[5*(c +
d*x)] + (1365*I)*Sin[7*(c + d*x)] + (99*I)*Sin[9*(c + d*x)]))/(a^4*d*(-I +
 Tan[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3983, 3042, 3983, 3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^5(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\int \frac{\cos^5(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{13a}+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\int \frac{1}{\sec(c+dx)^5(i\tan(c+dx)a+a)^3}dx}{13a}+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \ 3983$$

$$\frac{9\left(\frac{8\int \frac{\cos^5(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}\right)}{13a}+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{9\left(\frac{8\int \frac{1}{\sec(c+dx)^5(i\tan(c+dx)a+a)^2}dx}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}\right)}{13a}+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \ 3981$$

$$\frac{9\left(\frac{8\left(\frac{7\int \cos^7(c+dx)dx}{9a^2}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}\right)}{13a}+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{9\left(\frac{8\left(\frac{7\int \sin\left(c+dx+\frac{\pi}{2}\right)^7dx}{9a^2}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}\right)}{13a}+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \ 3113$$

$$\frac{9\left(\frac{8\left(-\frac{7\int \left(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}\right)}{13a}+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \ 2009$$

$$\frac{9\left(\frac{8\left(-\frac{7\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}\right)}{13a}+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}$$

input
```
Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/13)*Cos[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^4) + (9*(((I/11)*Cos[c +
 d*x]^5)/(d*(a + I*a*Tan[c + d*x])^3) + (8*((-7*(-Sin[c + d*x] + Sin[c + d
*x]^3 - (3*Sin[c + d*x]^5)/5 + Sin[c + d*x]^7/7))/(9*a^2*d) + (((2*I)/9)*C
os[c + d*x]^7)/(d*(a^2 + I*a^2*Tan[c + d*x]))))/(11*a)))/(13*a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99

method	result
risch	$\frac{3i\,\mathrm{e}^{-7i(dx+c)}}{128a^4d}+\frac{i\,\mathrm{e}^{-9i(dx+c)}}{128a^4d}+\frac{9i\,\mathrm{e}^{-11i(dx+c)}}{5632a^4d}+\frac{i\,\mathrm{e}^{-13i(dx+c)}}{6656a^4d}+\frac{3i\cos(dx+c)}{32a^4d}+\frac{15\sin(dx+c)}{64a^4d}+\frac{25i\cos(5dx+}{512a^4d}$
derivativedivides	$-\frac{11i}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}+\frac{825i}{128\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{12}}-\frac{1375i}{32\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{62i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-$
default	$-\frac{11i}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}+\frac{825i}{128\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{12}}-\frac{1375i}{32\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{62i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-$

input
```
int(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
3/128*I/a^4/d*exp(-7*I*(d*x+c))+1/128*I/a^4/d*exp(-9*I*(d*x+c))+9/5632*I/a
^4/d*exp(-11*I*(d*x+c))+1/6656*I/a^4/d*exp(-13*I*(d*x+c))+3/32*I/a^4/d*cos
(d*x+c)+15/64*sin(d*x+c)/a^4/d+25/512*I/a^4/d*cos(5*d*x+5*c)+127/2560/a^4/
d*sin(5*d*x+5*c)+39/512*I/a^4/d*cos(3*d*x+3*c)+45/512/a^4/d*sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.68

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$=\frac{\left(-143i\,e^{(18i\,dx+18i\,c)}-2145i\,e^{(16i\,dx+16i\,c)}-25740i\,e^{(14i\,dx+14i\,c)}+60060i\,e^{(12i\,dx+12i\,c)}+30030i\,e^{(10i\,dx+10i\,c}\right.}{366080\,a^4}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/366080*(-143*I*e^(18*I*d*x + 18*I*c) - 2145*I*e^(16*I*d*x + 16*I*c) - 25
740*I*e^(14*I*d*x + 14*I*c) + 60060*I*e^(12*I*d*x + 12*I*c) + 30030*I*e^(1
0*I*d*x + 10*I*c) + 18018*I*e^(8*I*d*x + 8*I*c) + 8580*I*e^(6*I*d*x + 6*I*
c) + 2860*I*e^(4*I*d*x + 4*I*c) + 585*I*e^(2*I*d*x + 2*I*c) + 55*I)*e^(-13
*I*d*x - 13*I*c)/(a^4*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(153) = 306$.

Time = 0.55 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.11

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \begin{cases} \frac{(-1688246017625898163896320ia^{36}d^9e^{54ic}e^{5idx} - 2532369026438847245844480 0ia^{36}d^9e^{52ic}e^{3idx} - 3038842831726616695013376 00ia^{36}d^9e^{\cdots}}{} \\ \frac{x(e^{18ic}+9e^{16ic}+36e^{14ic}+84e^{12ic}+126e^{10ic}+126e^{8ic}+84e^{6ic}+36e^{4ic}+9e^{2ic}+1)e^{-13ic}}{512a^4} \end{cases}$$

input
```
integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((-1688246017625898163896320*I*a**36*d**9*exp(54*I*c)*exp(5*I*d*
x) - 25323690264388472458444800*I*a**36*d**9*exp(52*I*c)*exp(3*I*d*x) - 30
388428317266166950133760 0*I*a**36*d**9*exp(50*I*c)*exp(I*d*x) + 7090633274
02877228836454400*I*a**36*d**9*exp(48*I*c)*exp(-I*d*x) + 3545316637014386 1
4418227200*I*a**36*d**9*exp(46*I*c)*exp(-3*I*d*x) + 212718998220863168650 9
36320*I*a**36*d**9*exp(44*I*c)*exp(-5*I*d*x) + 101294761057553889833779200
*I*a**36*d**9*exp(42*I*c)*exp(-7*I*d*x) + 33764920352517963277926400*I*a**
36*d**9*exp(40*I*c)*exp(-9*I*d*x) + 6906460981196856125030400*I*a**36*d**9
*exp(38*I*c)*exp(-11*I*d*x) + 649325391394576216883200*I*a**36*d**9*exp(36
*I*c)*exp(-13*I*d*x))*exp(-49*I*c)/(43219098051222992995745792 00*a**40*d**
10), Ne(a**40*d**10*exp(49*I*c), 0)), (x*(exp(18*I*c) + 9*exp(16*I*c) + 36
*exp(14*I*c) + 84*exp(12*I*c) + 126*exp(10*I*c) + 126*exp(8*I*c) + 84*exp(
6*I*c) + 36*exp(4*I*c) + 9*exp(2*I*c) + 1)*exp(-13*I*c)/(512*a**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.43

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{143\left(115\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+405i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-575\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-375i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+98\right)}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)^5}+\frac{166595\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-1409265i\,\mathrm{ta}}{}}{}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/91520*(143*(115*tan(1/2*d*x + 1/2*c)^4 + 405*I*tan(1/2*d*x + 1/2*c)^3 -
575*tan(1/2*d*x + 1/2*c)^2 - 375*I*tan(1/2*d*x + 1/2*c) + 98)/(a^4*(tan(1/
2*d*x + 1/2*c) + I)^5) + (166595*tan(1/2*d*x + 1/2*c)^12 - 1409265*I*tan(1
/2*d*x + 1/2*c)^11 - 6232655*tan(1/2*d*x + 1/2*c)^10 + 17535375*I*tan(1/2*
d*x + 1/2*c)^9 + 34610004*tan(1/2*d*x + 1/2*c)^8 - 49771722*I*tan(1/2*d*x
+ 1/2*c)^7 - 53349582*tan(1/2*d*x + 1/2*c)^6 + 42730974*I*tan(1/2*d*x + 1/
2*c)^5 + 25431835*tan(1/2*d*x + 1/2*c)^4 - 10954229*I*tan(1/2*d*x + 1/2*c)
^3 - 3278067*tan(1/2*d*x + 1/2*c)^2 + 614627*I*tan(1/2*d*x + 1/2*c) + 6009
4)/(a^4*(tan(1/2*d*x + 1/2*c) - I)^13))/d
```

Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.51

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{15049\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128}-\frac{4513\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{32}+\frac{4513\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{32}-\frac{15461\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{64}+\frac{3941\,\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{64}\right.}{}$$

input
```
int(cos(c + d*x)^5/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(cos(c/2 + (d*x)/2)*((cos((3*c)/2 + (3*d*x)/2)*3003i)/32 - (cos((5*c)/2 +
(5*d*x)/2)*3003i)/32 + (cos((7*c)/2 + (7*d*x)/2)*7293i)/32 - (cos((9*c)/2
+ (9*d*x)/2)*1533i)/32 + (cos((11*c)/2 + (11*d*x)/2)*103i)/32 - (cos((13*c
)/2 + (13*d*x)/2)*103i)/32 + (cos((15*c)/2 + (15*d*x)/2)*11i)/64 - (cos((1
7*c)/2 + (17*d*x)/2)*11i)/64 + (15049*sin(c/2 + (d*x)/2))/128 - (4513*sin(
(3*c)/2 + (3*d*x)/2))/32 + (4513*sin((5*c)/2 + (5*d*x)/2))/32 - (15461*sin
((7*c)/2 + (7*d*x)/2))/64 + (3941*sin((9*c)/2 + (9*d*x)/2))/64 - (183*sin(
(11*c)/2 + (11*d*x)/2))/32 + (183*sin((13*c)/2 + (13*d*x)/2))/32 - (99*sin
((15*c)/2 + (15*d*x)/2))/256 + (99*sin((17*c)/2 + (17*d*x)/2))/256)*2i)/(7
15*a^4*d*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)*1i)^13*(cos(c/2 + (d*x)/
2)*1i + sin(c/2 + (d*x)/2))^5)
```

Reduce [F]

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\cos(dx+c)^5}{\tan(dx+c)^4-4\tan(dx+c)^3i-6\tan(dx+c)^2+4\tan(dx+c)i+1}dx}{a^4}$$

input
```
int(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x)
```

output
```
int(cos(c + d*x)**5/(tan(c + d*x)**4 - 4*tan(c + d*x)**3*i - 6*tan(c + d*x
)**2 + 4*tan(c + d*x)*i + 1),x)/a**4
```

3.166 $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 134

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{192x}{a^8} - \frac{192i\log(\cos(c+dx))}{a^8d} + \frac{129\tan(c+dx)}{a^8d} - \frac{36i\tan^2(c+dx)}{a^8d} - \frac{10\tan^3(c+dx)}{a^8d} + \frac{2i\tan^4(c+dx)}{a^8d} + \frac{\tan^5(c+dx)}{5a^8d} + \frac{64i}{d\,(a^8+ia^8\tan(c+dx))}$$

output
```
-192*x/a^8-192*I*ln(cos(d*x+c))/a^8/d+129*tan(d*x+c)/a^8/d-36*I*tan(d*x+c)
^2/a^8/d-10*tan(d*x+c)^3/a^8/d+2*I*tan(d*x+c)^4/a^8/d+1/5*tan(d*x+c)^5/a^8
/d+64*I/d/(a^8+I*a^8*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.73

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{i\Big(-960\log(i-\tan(c+dx)) + 645i\tan(c+dx) + 180\tan^2(c+dx) - 50i\tan^3(c+dx) - 10\tan^4(c+}{5a^8d}$$

input
```
Integrate[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/5*I)*(-960*Log[I - Tan[c + d*x]] + (645*I)*Tan[c + d*x] + 180*Tan[c +
 d*x]^2 - (50*I)*Tan[c + d*x]^3 - 10*Tan[c + d*x]^4 + I*Tan[c + d*x]^5 + (
320*I)/(-I + Tan[c + d*x])))/(a^8*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(c+dx)^{14}}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{i\int \frac{(a-ia\tan(c+dx))^6}{(i\tan(c+dx)a+a)^2}d(ia\tan(c+dx))}{a^{13}d}$$

$$\Big\downarrow \; 49$$

$$-\frac{i\int \left(\frac{64a^6}{(i\tan(c+dx)a+a)^2}-\frac{192a^5}{i\tan(c+dx)a+a}+\tan^4(c+dx)a^4+8i\tan^3(c+dx)a^4-30\tan^2(c+dx)a^4-72i\tan(c+
}{a^{13}d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{i\left(-\frac{64a^6}{a+ia\tan(c+dx)}+\frac{1}{5}ia^5\tan^5(c+dx)-2a^5\tan^4(c+dx)-10ia^5\tan^3(c+dx)+36a^5\tan^2(c+dx)+129ia^5\tan
}{a^{13}d}$$

input
```
Int[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*(-192*a^5*Log[a + I*a*Tan[c + d*x]] + (129*I)*a^5*Tan[c + d*x] + 36*
a^5*Tan[c + d*x]^2 - (10*I)*a^5*Tan[c + d*x]^3 - 2*a^5*Tan[c + d*x]^4 + (I
/5)*a^5*Tan[c + d*x]^5 - (64*a^6)/(a + I*a*Tan[c + d*x])))/(a^13*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

method	result
risch	$\frac{32i\mathrm{e}^{-2i(dx+c)}}{a^8 d}-\frac{384x}{a^8}-\frac{384c}{a^8 d}+\frac{16i\left(50\,\mathrm{e}^{8i(dx+c)}+220\,\mathrm{e}^{6i(dx+c)}+370\,\mathrm{e}^{4i(dx+c)}+285\,\mathrm{e}^{2i(dx+c)}+87\right)}{5d\,a^8\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}-\frac{192i\ln(\mathrm{e}}{}$
derivativedivides	$\frac{129\tan(dx+c)}{a^8 d}+\frac{\tan(dx+c)^5}{5a^8 d}+\frac{2i\tan(dx+c)^4}{a^8 d}-\frac{10\tan(dx+c)^3}{a^8 d}-\frac{36i\tan(dx+c)^2}{a^8 d}-\frac{192\arctan(\tan(dx+c))}{a^8 d}$
default	$\frac{129\tan(dx+c)}{a^8 d}+\frac{\tan(dx+c)^5}{5a^8 d}+\frac{2i\tan(dx+c)^4}{a^8 d}-\frac{10\tan(dx+c)^3}{a^8 d}-\frac{36i\tan(dx+c)^2}{a^8 d}-\frac{192\arctan(\tan(dx+c))}{a^8 d}$

input
```
int(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
32*I/a^8/d*exp(-2*I*(d*x+c))-384*x/a^8-384/a^8/d*c+16/5*I*(50*exp(8*I*(d*x
+c))+220*exp(6*I*(d*x+c))+370*exp(4*I*(d*x+c))+285*exp(2*I*(d*x+c))+87)/d/
a^8/(exp(2*I*(d*x+c))+1)^5-192*I/a^8/d*ln(exp(2*I*(d*x+c))+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(122) = 244$.

Time = 0.12 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{16\,(120\,dxe^{(12i\,dx+12i\,c)} + 60\,(10\,dx-i)e^{(10i\,dx+10i\,c)} + 30\,(40\,dx-9i)e^{(8i\,dx+8i\,c)} + 10\,(120\,dx-47i)e^{(6i}}{5\,(a^8de^{(12i}}$$

input
```
integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-16/5*(120*d*x*e^(12*I*d*x + 12*I*c) + 60*(10*d*x - I)*e^(10*I*d*x + 10*I*
c) + 30*(40*d*x - 9*I)*e^(8*I*d*x + 8*I*c) + 10*(120*d*x - 47*I)*e^(6*I*d*
x + 6*I*c) + 5*(120*d*x - 77*I)*e^(4*I*d*x + 4*I*c) + (120*d*x - 137*I)*e^
(2*I*d*x + 2*I*c) + 60*(I*e^(12*I*d*x + 12*I*c) + 5*I*e^(10*I*d*x + 10*I*c
) + 10*I*e^(8*I*d*x + 8*I*c) + 10*I*e^(6*I*d*x + 6*I*c) + 5*I*e^(4*I*d*x +
 4*I*c) + I*e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) - 10*I)/(a^8
*d*e^(12*I*d*x + 12*I*c) + 5*a^8*d*e^(10*I*d*x + 10*I*c) + 10*a^8*d*e^(8*I
*d*x + 8*I*c) + 10*a^8*d*e^(6*I*d*x + 6*I*c) + 5*a^8*d*e^(4*I*d*x + 4*I*c)
 + a^8*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F]

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\int \frac{\sec^{14}(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

input
```
integrate(sec(d*x+c)**14/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Integral(sec(c + d*x)**14/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(
c + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x
)**3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.71

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\frac{320\left(\tan(dx+c)^6-6i\tan(dx+c)^5-15\tan(dx+c)^4+20i\tan(dx+c)^3+15\tan(dx+c)^2-6i\tan(dx+c)-1\right)}{a^8\tan(dx+c)^7-7i\,a^8\tan(dx+c)^6-21\,a^8\tan(dx+c)^5+35i\,a^8\tan(dx+c)^4+35\,a^8\tan(dx+c)^3-21i\,a^8\tan(dx+c)^2-7\,a^8\tan(dx+c)+i\,a^8}+\tan(}{5\,d}$$

input
```
integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/5*(320*(tan(d*x + c)^6 - 6*I*tan(d*x + c)^5 - 15*tan(d*x + c)^4 + 20*I*t
an(d*x + c)^3 + 15*tan(d*x + c)^2 - 6*I*tan(d*x + c) - 1)/(a^8*tan(d*x + c
)^7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*x +
c)^4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x + c
) + I*a^8) + (tan(d*x + c)^5 + 10*I*tan(d*x + c)^4 - 50*tan(d*x + c)^3 - 1
80*I*tan(d*x + c)^2 + 645*tan(d*x + c))/a^8 + 960*I*log(I*tan(d*x + c) + 1
)/a^8)/d
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.91

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{192i\log(\tan(dx+c)-i)}{a^8d} + \frac{64}{a^8d(\tan(dx+c)-i)} + \frac{a^{32}d^4\tan(dx+c)^5+10i\,a^{32}d^4\tan(dx+c)^4-50\,a^{32}d^4\tan(dx+c)^3-180i\,a^{32}d^4\tan(dx+c)^2+645}{5\,a^{40}d^5}$$

input
```
integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
192*I*log(tan(d*x + c) - I)/(a^8*d) + 64/(a^8*d*(tan(d*x + c) - I)) + 1/5*
(a^32*d^4*tan(d*x + c)^5 + 10*I*a^32*d^4*tan(d*x + c)^4 - 50*a^32*d^4*tan(
d*x + c)^3 - 180*I*a^32*d^4*tan(d*x + c)^2 + 645*a^32*d^4*tan(d*x + c))/(a
^40*d^5)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.78

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\frac{129\tan(c+d\,x)}{a^8}-\frac{10\tan(c+d\,x)^3}{a^8}+\frac{\tan(c+d\,x)^5}{5\,a^8}+\frac{\ln(\tan(c+d\,x)-\mathrm{i})\,192\mathrm{i}}{a^8}+\frac{64\mathrm{i}}{a^8\,(1+\tan(c+d\,x)\,1\mathrm{i})}-\frac{\tan(c+d\,x)^2\,36\mathrm{i}}{a^8}+\frac{\tan(c+d\,x)^4\,2\mathrm{i}}{a^8}}{d}$$

input
```
int(1/(cos(c + d*x)^14*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
((log(tan(c + d*x) - 1i)*192i)/a^8 + (129*tan(c + d*x))/a^8 + 64i/(a^8*(ta
n(c + d*x)*1i + 1)) - (tan(c + d*x)^2*36i)/a^8 - (10*tan(c + d*x)^3)/a^8 +
 (tan(c + d*x)^4*2i)/a^8 + tan(c + d*x)^5/(5*a^8))/d
```

Reduce [F]

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x)
```

output

```
(2932671499603968000*cos(c + d*x)**2*sin(c + d*x)**3 + 33527599364736000*c
os(c + d*x)**2*sin(c + d*x)**2*i - 3306424808337408000*cos(c + d*x)**2*sin
(c + d*x) - 42264021742848000*cos(c + d*x)**2*i + 151572263731200*cos(c +
d*x)*int(cos(c + d*x)/(128*cos(c + d*x)*sin(c + d*x)**13*i - 576*cos(c + d
*x)*sin(c + d*x)**11*i + 1040*cos(c + d*x)*sin(c + d*x)**9*i - 952*cos(c +
 d*x)*sin(c + d*x)**7*i + 456*cos(c + d*x)*sin(c + d*x)**5*i - 104*cos(c +
 d*x)*sin(c + d*x)**3*i + 8*cos(c + d*x)*sin(c + d*x)*i - 128*sin(c + d*x)
**14 + 640*sin(c + d*x)**12 - 1312*sin(c + d*x)**10 + 1408*sin(c + d*x)**8
 - 833*sin(c + d*x)**6 + 259*sin(c + d*x)**4 - 35*sin(c + d*x)**2 + 1),x)*
sin(c + d*x)**4*d*i + 448710222151680000*cos(c + d*x)*int(cos(c + d*x)/(12
8*cos(c + d*x)*sin(c + d*x)**13*i - 576*cos(c + d*x)*sin(c + d*x)**11*i +
1040*cos(c + d*x)*sin(c + d*x)**9*i - 952*cos(c + d*x)*sin(c + d*x)**7*i +
 456*cos(c + d*x)*sin(c + d*x)**5*i - 104*cos(c + d*x)*sin(c + d*x)**3*i +
 8*cos(c + d*x)*sin(c + d*x)*i - 128*sin(c + d*x)**14 + 640*sin(c + d*x)**
12 - 1312*sin(c + d*x)**10 + 1408*sin(c + d*x)**8 - 833*sin(c + d*x)**6 +
259*sin(c + d*x)**4 - 35*sin(c + d*x)**2 + 1),x)*sin(c + d*x)**4*d - 30314
4527462400*cos(c + d*x)*int(cos(c + d*x)/(128*cos(c + d*x)*sin(c + d*x)**1
3*i - 576*cos(c + d*x)*sin(c + d*x)**11*i + 1040*cos(c + d*x)*sin(c + d*x)
**9*i - 952*cos(c + d*x)*sin(c + d*x)**7*i + 456*cos(c + d*x)*sin(c + d*x)
**5*i - 104*cos(c + d*x)*sin(c + d*x)**3*i + 8*cos(c + d*x)*sin(c + d*x...
```

3.167 $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{80x}{a^8}+\frac{80i\log(\cos(c+dx))}{a^8d}-\frac{31\tan(c+dx)}{a^8d} +\frac{4i\tan^2(c+dx)}{a^8d}+\frac{\tan^3(c+dx)}{3a^8d} +\frac{16i}{d\,(a^4+ia^4\tan(c+dx))^2}-\frac{80i}{d\,(a^8+ia^8\tan(c+dx))}$$

output
```
80*x/a^8+80*I*ln(cos(d*x+c))/a^8/d-31*tan(d*x+c)/a^8/d+4*I*tan(d*x+c)^2/a^
8/d+1/3*tan(d*x+c)^3/a^8/d+16*I/d/(a^4+I*a^4*tan(d*x+c))^2-80*I/d/(a^8+I*a
^8*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{i\left(-93i\tan(c+dx)-12\tan^2(c+dx)+i\tan^3(c+dx)+48\left(5\log(i-\tan(c+dx))+\frac{-4-5i\tan(c+dx)}{(-i+\tan(c+dx))^2}\right)\right)}{3a^8d}$$

input
```
Integrate[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/3*I)*((-93*I)*Tan[c + d*x] - 12*Tan[c + d*x]^2 + I*Tan[c + d*x]^3 + 4
8*(5*Log[I - Tan[c + d*x]] + (-4 - (5*I)*Tan[c + d*x])/(-I + Tan[c + d*x])
^2)))/(a^8*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{12}}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{(a-ia\tan(c+dx))^5}{(i\tan(c+dx)a+a)^3}d(ia\tan(c+dx))}{a^{11}d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(\frac{32a^5}{(i\tan(c+dx)a+a)^3}-\frac{80a^4}{(i\tan(c+dx)a+a)^2}+\frac{80a^3}{i\tan(c+dx)a+a}+\tan^2(c+dx)a^2+8i\tan(c+dx)a^2-31a^2\right)d(ia\tan(c}{a^{11}d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{16a^5}{(a+ia\tan(c+dx))^2}+\frac{80a^4}{a+ia\tan(c+dx)}+\frac{1}{3}ia^3\tan^3(c+dx)-4a^3\tan^2(c+dx)-31ia^3\tan(c+dx)+80a^3\log(a-\right.}{a^{11}d}$$

input
```
Int[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*(80*a^3*Log[a + I*a*Tan[c + d*x]] - (31*I)*a^3*Tan[c + d*x] - 4*a^3*
Tan[c + d*x]^2 + (I/3)*a^3*Tan[c + d*x]^3 - (16*a^5)/(a + I*a*Tan[c + d*x]
)^2 + (80*a^4)/(a + I*a*Tan[c + d*x])))/(a^11*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{32 i \mathrm{e}^{-2 i (d x+c)}}{a^{8} d}+\frac{4 i \mathrm{e}^{-4 i (d x+c)}}{a^{8} d}+\frac{160 x}{a^{8}}+\frac{160 c}{a^{8} d}-\frac{4 i \left(36 \,\mathrm{e}^{4 i (d x+c)}+81 \,\mathrm{e}^{2 i (d x+c)}+47\right)}{3 d \,a^{8} \left(\mathrm{e}^{2 i (d x+c)}+1\right)^{3}}+\frac{80 i \ln \left(\mathrm{e}^{2 i (d x+c)}+1\right)}{a^{8} d}$
derivativedivides	$-\frac{31 \tan (d x+c)}{a^{8} d}+\frac{\tan (d x+c)^{3}}{3 a^{8} d}+\frac{4 i \tan (d x+c)^{2}}{a^{8} d}-\frac{80}{a^{8} d (-i+\tan (d x+c))}-\frac{16 i}{a^{8} d (-i+\tan (d x+c))^{2}}+\frac{80 \arctan}{}$
default	$-\frac{31 \tan (d x+c)}{a^{8} d}+\frac{\tan (d x+c)^{3}}{3 a^{8} d}+\frac{4 i \tan (d x+c)^{2}}{a^{8} d}-\frac{80}{a^{8} d (-i+\tan (d x+c))}-\frac{16 i}{a^{8} d (-i+\tan (d x+c))^{2}}+\frac{80 \arctan}{}$

input
```
int(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
-32*I/a^8/d*exp(-2*I*(d*x+c))+4*I/a^8/d*exp(-4*I*(d*x+c))+160*x/a^8+160/a^
8/d*c-4/3*I*(36*exp(4*I*(d*x+c))+81*exp(2*I*(d*x+c))+47)/d/a^8/(exp(2*I*(d
*x+c))+1)^3+80*I/a^8/d*ln(exp(2*I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.58

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{4\left(120\,dxe^{(10i\,dx+10i\,c)}+60\,(6\,dx-i)e^{(8i\,dx+8i\,c)}+30\,(12\,dx-5i)e^{(6i\,dx+6i\,c)}+10\,(12\,dx-11i)e^{(4i\,dx+4i\,c)}\cdots\right.}{3\left(a^8de^{(10i\,dx+10i\,c)}+3\,a^8de^{(8i\,dx+8\cdots}\right.}$$

input
```
integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
4/3*(120*d*x*e^(10*I*d*x + 10*I*c) + 60*(6*d*x - I)*e^(8*I*d*x + 8*I*c) +
30*(12*d*x - 5*I)*e^(6*I*d*x + 6*I*c) + 10*(12*d*x - 11*I)*e^(4*I*d*x + 4*
I*c) - 60*(-I*e^(10*I*d*x + 10*I*c) - 3*I*e^(8*I*d*x + 8*I*c) - 3*I*e^(6*I
*d*x + 6*I*c) - I*e^(4*I*d*x + 4*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) - 15*I
*e^(2*I*d*x + 2*I*c) + 3*I)/(a^8*d*e^(10*I*d*x + 10*I*c) + 3*a^8*d*e^(8*I*
d*x + 8*I*c) + 3*a^8*d*e^(6*I*d*x + 6*I*c) + a^8*d*e^(4*I*d*x + 4*I*c))
```

Sympy [F]

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\int \frac{\sec^{12}(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

input
```
integrate(sec(d*x+c)**12/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Integral(sec(c + d*x)**12/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(
c + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x
)**3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.68

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$

$$-\frac{\frac{48\left(5\tan(dx+c)^6-29i\tan(dx+c)^5-70\tan(dx+c)^4+90i\tan(dx+c)^3+65\tan(dx+c)^2-25i\tan(dx+c)-4\right)}{a^8\tan(dx+c)^7-7i\,a^8\tan(dx+c)^6-21\,a^8\tan(dx+c)^5+35i\,a^8\tan(dx+c)^4+35\,a^8\tan(dx+c)^3-21i\,a^8\tan(dx+c)^2-7\,a^8\tan(dx+c)+i\,a^8}-\cdots}{3\,d}$$

input
```
integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/3*(48*(5*tan(d*x + c)^6 - 29*I*tan(d*x + c)^5 - 70*tan(d*x + c)^4 + 90*
I*tan(d*x + c)^3 + 65*tan(d*x + c)^2 - 25*I*tan(d*x + c) - 4)/(a^8*tan(d*x
 + c)^7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*
x + c)^4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x
 + c) + I*a^8) - (tan(d*x + c)^3 + 12*I*tan(d*x + c)^2 - 93*tan(d*x + c))/
a^8 + 240*I*log(I*tan(d*x + c) + 1)/a^8)/d
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.79

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{80i\log\left(\tan\left(dx+c\right)-i\right)}{a^8d}-\frac{16\left(5\tan\left(dx+c\right)-4i\right)}{a^8d(\tan\left(dx+c\right)-i)^2} + \frac{a^{16}d^2\tan\left(dx+c\right)^3+12i\,a^{16}d^2\tan\left(dx+c\right)^2-93\,a^{16}d^2\tan\left(dx+c\right)}{3\,a^{24}d^3}$$

input
```
integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-80*I*log(tan(d*x + c) - I)/(a^8*d) - 16*(5*tan(d*x + c) - 4*I)/(a^8*d*(ta
n(d*x + c) - I)^2) + 1/3*(a^16*d^2*tan(d*x + c)^3 + 12*I*a^16*d^2*tan(d*x
+ c)^2 - 93*a^16*d^2*tan(d*x + c))/(a^24*d^3)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\tan(c+d\,x)^3}{3\,a^8\,d} - \frac{31\tan(c+d\,x)}{a^8\,d} + \frac{\tan(c+d\,x)^2\,4\mathrm{i}}{a^8\,d} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,80\mathrm{i}}{a^8\,d} - \frac{\frac{64}{a^8}+\frac{\tan(c+d\,x)\,80\mathrm{i}}{a^8}}{d\,\left(\tan\left(c+d\,x\right)^2\,1\mathrm{i}+2\tan\left(c+d\,x\right)-\mathrm{i}\right)}$$

input
```
int(1/(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
(tan(c + d*x)^2*4i)/(a^8*d) - (31*tan(c + d*x))/(a^8*d) - (log(tan(c + d*x
) - 1i)*80i)/(a^8*d) + tan(c + d*x)^3/(3*a^8*d) - ((tan(c + d*x)*80i)/a^8
+ 64/a^8)/(d*(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i))
```

Reduce [F]

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x)
```

output
```
( - 49927274496*cos(c + d*x)**2*sin(c + d*x) + 6335894016*cos(c + d*x)**2*
i - 1317550522368*cos(c + d*x)*int(cos(c + d*x)/(128*cos(c + d*x)*sin(c +
d*x)**11*i - 448*cos(c + d*x)*sin(c + d*x)**9*i + 592*cos(c + d*x)*sin(c +
 d*x)**7*i - 360*cos(c + d*x)*sin(c + d*x)**5*i + 96*cos(c + d*x)*sin(c +
d*x)**3*i - 8*cos(c + d*x)*sin(c + d*x)*i - 128*sin(c + d*x)**12 + 512*sin
(c + d*x)**10 - 800*sin(c + d*x)**8 + 608*sin(c + d*x)**6 - 225*sin(c + d*
x)**4 + 34*sin(c + d*x)**2 - 1),x)*sin(c + d*x)**2*d + 1317550522368*cos(c
 + d*x)*int(cos(c + d*x)/(128*cos(c + d*x)*sin(c + d*x)**11*i - 448*cos(c
+ d*x)*sin(c + d*x)**9*i + 592*cos(c + d*x)*sin(c + d*x)**7*i - 360*cos(c
+ d*x)*sin(c + d*x)**5*i + 96*cos(c + d*x)*sin(c + d*x)**3*i - 8*cos(c + d
*x)*sin(c + d*x)*i - 128*sin(c + d*x)**12 + 512*sin(c + d*x)**10 - 800*sin
(c + d*x)**8 + 608*sin(c + d*x)**6 - 225*sin(c + d*x)**4 + 34*sin(c + d*x)
**2 - 1),x)*d + 33622769664*cos(c + d*x)*int(cos(c + d*x)/(128*cos(c + d*x
)*sin(c + d*x)**10*i - 384*cos(c + d*x)*sin(c + d*x)**8*i + 416*cos(c + d*
x)*sin(c + d*x)**6*i - 192*cos(c + d*x)*sin(c + d*x)**4*i + 33*cos(c + d*x
)*sin(c + d*x)**2*i - cos(c + d*x)*i - 128*sin(c + d*x)**11 + 448*sin(c +
d*x)**9 - 592*sin(c + d*x)**7 + 360*sin(c + d*x)**5 - 96*sin(c + d*x)**3 +
 8*sin(c + d*x)),x)*sin(c + d*x)**2*d*i - 33622769664*cos(c + d*x)*int(cos
(c + d*x)/(128*cos(c + d*x)*sin(c + d*x)**10*i - 384*cos(c + d*x)*sin(c +
d*x)**8*i + 416*cos(c + d*x)*sin(c + d*x)**6*i - 192*cos(c + d*x)*sin(c...
```

3.168 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{8x}{a^8}-\frac{8i\log(\cos(c+dx))}{a^8d}+\frac{\tan(c+dx)}{a^8d}+\frac{16i}{3a^5d(a+ia\tan(c+dx))^3}-\frac{16i}{d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{24i}{d\left(a^8+ia^8\tan(c+dx)\right)}$$

output
```
-8*x/a^8-8*I*ln(cos(d*x+c))/a^8/d+tan(d*x+c)/a^8/d+16/3*I/a^5/d/(a+I*a*tan
(d*x+c))^3-16*I/d/(a^4+I*a^4*tan(d*x+c))^2+24*I/d/(a^8+I*a^8*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{i\left(-8a\log(i-\tan(c+dx))+ia\tan(c+dx)+\frac{8a(-5i+12\tan(c+dx)+9i\tan^2(c+dx))}{3(-i+\tan(c+dx))^3}\right)}{a^9d}$$

input
```
Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*(-8*a*Log[I - Tan[c + d*x]] + I*a*Tan[c + d*x] + (8*a*(-5*I + 12*Tan
[c + d*x] + (9*I)*Tan[c + d*x]^2))/(3*(-I + Tan[c + d*x])^3)))/(a^9*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{10}}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{(a-ia\tan(c+dx))^4}{(i\tan(c+dx)a+a)^4}d(ia\tan(c+dx))}{a^9d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(\frac{16a^4}{(i\tan(c+dx)a+a)^4}-\frac{32a^3}{(i\tan(c+dx)a+a)^3}+\frac{24a^2}{(i\tan(c+dx)a+a)^2}-\frac{8a}{i\tan(c+dx)a+a}+1\right)d(ia\tan(c+dx))}{a^9d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{16a^4}{3(a+ia\tan(c+dx))^3}+\frac{16a^3}{(a+ia\tan(c+dx))^2}-\frac{24a^2}{a+ia\tan(c+dx)}+ia\tan(c+dx)-8a\log(a+ia\tan(c+dx))\right)}{a^9d}$$

input
```
Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*(-8*a*Log[a + I*a*Tan[c + d*x]] + I*a*Tan[c + d*x] - (16*a^4)/(3*(a
+ I*a*Tan[c + d*x])^3) + (16*a^3)/(a + I*a*Tan[c + d*x])^2 - (24*a^2)/(a +
 I*a*Tan[c + d*x])))/(a^9*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.59

method	result	si
derivativedivides	$\frac{\tan(dx+c)+\frac{24}{-i+\tan(dx+c)}+8i\ln(-i+\tan(dx+c))-\frac{16}{3(-i+\tan(dx+c))^3}+\frac{16i}{(-i+\tan(dx+c))^2}}{d\,a^8}$	68
default	$\frac{\tan(dx+c)+\frac{24}{-i+\tan(dx+c)}+8i\ln(-i+\tan(dx+c))-\frac{16}{3(-i+\tan(dx+c))^3}+\frac{16i}{(-i+\tan(dx+c))^2}}{d\,a^8}$	68
risch	$\frac{6i\mathrm{e}^{-2i(dx+c)}}{a^8d}-\frac{2i\mathrm{e}^{-4i(dx+c)}}{a^8d}+\frac{2i\mathrm{e}^{-6i(dx+c)}}{3a^8d}-\frac{16x}{a^8}-\frac{16c}{a^8d}+\frac{2i}{d\,a^8\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{8i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^8d}$	1

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^8*(tan(d*x+c)+24/(-I+tan(d*x+c))+8*I*ln(-I+tan(d*x+c))-16/3/(-I+tan(
d*x+c))^3+16*I/(-I+tan(d*x+c))^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{2\left(24\,dxe^{(8i\,dx+8i\,c)}+12\,(2\,dx-i)e^{(6i\,dx+6i\,c)}+12\,\left(i\,e^{(8i\,dx+8i\,c)}+i\,e^{(6i\,dx+6i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-6i}{3\left(a^8de^{(8i\,dx+8i\,c)}+a^8de^{(6i\,dx+6i\,c)}\right)}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-2/3*(24*d*x*e^(8*I*d*x + 8*I*c) + 12*(2*d*x - I)*e^(6*I*d*x + 6*I*c) + 12
*(I*e^(8*I*d*x + 8*I*c) + I*e^(6*I*d*x + 6*I*c))*log(e^(2*I*d*x + 2*I*c) +
 1) - 6*I*e^(4*I*d*x + 4*I*c) + 2*I*e^(2*I*d*x + 2*I*c) - I)/(a^8*d*e^(8*I
*d*x + 8*I*c) + a^8*d*e^(6*I*d*x + 6*I*c))
```

Sympy [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\int \frac{\sec^{10}(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

input
```
integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Integral(sec(c + d*x)**10/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(
c + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x
)**3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.65

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\frac{8\left(9\tan(dx+c)^6-48i\tan(dx+c)^5-107\tan(dx+c)^4+128i\tan(dx+c)^3+87\tan(dx+c)^2-32i\tan(dx+c)-5\right)}{a^8\tan(dx+c)^7-7i\,a^8\tan(dx+c)^6-21\,a^8\tan(dx+c)^5+35i\,a^8\tan(dx+c)^4+35\,a^8\tan(dx+c)^3-21i\,a^8\tan(dx+c)^2-7\,a^8\tan(dx+c)+i\,a^8}+24i}{3\,d}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/3*(8*(9*tan(d*x + c)^6 - 48*I*tan(d*x + c)^5 - 107*tan(d*x + c)^4 + 128*
I*tan(d*x + c)^3 + 87*tan(d*x + c)^2 - 32*I*tan(d*x + c) - 5)/(a^8*tan(d*x
 + c)^7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*
x + c)^4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x
 + c) + I*a^8) + 24*I*log(I*tan(d*x + c) + 1)/a^8 + 3*tan(d*x + c)/a^8)/d
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.59

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{8i\log\left(\tan\left(dx+c\right)-i\right)}{a^8d}+\frac{\tan\left(dx+c\right)}{a^8d}+\frac{8\left(9\tan\left(dx+c\right)^2-12i\tan\left(dx+c\right)-5\right)}{3\,a^8d(\tan\left(dx+c\right)-i)^3}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
8*I*log(tan(d*x + c) - I)/(a^8*d) + tan(d*x + c)/(a^8*d) + 8/3*(9*tan(d*x
+ c)^2 - 12*I*tan(d*x + c) - 5)/(a^8*d*(tan(d*x + c) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$=\frac{\tan(c+d\,x)}{a^8\,d}-\frac{\frac{32\tan(c+d\,x)}{a^8}-\frac{40\mathrm{i}}{3\,a^8}+\frac{\tan(c+d\,x)^2\,24\mathrm{i}}{a^8}}{d\,\left(-\tan\left(c+d\,x\right)^3\,1\mathrm{i}-3\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,3\mathrm{i}+1\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,8\mathrm{i}}{a^8\,d}$$

input
```
int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
(log(tan(c + d*x) - 1i)*8i)/(a^8*d) - ((32*tan(c + d*x))/a^8 - 40i/(3*a^8)
 + (tan(c + d*x)^2*24i)/a^8)/(d*(tan(c + d*x)*3i - 3*tan(c + d*x)^2 - tan(
c + d*x)^3*1i + 1)) + tan(c + d*x)/(a^8*d)
```

Reduce [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx=\text{too large to display}$$

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x)
```

output
```
(13653760*cos(c + d*x)*int(cos(c + d*x)/(128*cos(c + d*x)*sin(c + d*x)**9*
i - 320*cos(c + d*x)*sin(c + d*x)**7*i + 272*cos(c + d*x)*sin(c + d*x)**5*
i - 88*cos(c + d*x)*sin(c + d*x)**3*i + 8*cos(c + d*x)*sin(c + d*x)*i - 12
8*sin(c + d*x)**10 + 384*sin(c + d*x)**8 - 416*sin(c + d*x)**6 + 192*sin(c
 + d*x)**4 - 33*sin(c + d*x)**2 + 1),x)*d - 13655040*cos(c + d*x)*int(cos(
c + d*x)/(128*cos(c + d*x)*sin(c + d*x)**9 - 320*cos(c + d*x)*sin(c + d*x)
**7 + 272*cos(c + d*x)*sin(c + d*x)**5 - 88*cos(c + d*x)*sin(c + d*x)**3 +
 8*cos(c + d*x)*sin(c + d*x) + 128*sin(c + d*x)**10*i - 384*sin(c + d*x)**
8*i + 416*sin(c + d*x)**6*i - 192*sin(c + d*x)**4*i + 33*sin(c + d*x)**2*i
 - i),x)*d*i - 161280*cos(c + d*x)*int(cos(c + d*x)/(128*cos(c + d*x)*sin(
c + d*x)**8*i - 256*cos(c + d*x)*sin(c + d*x)**6*i + 160*cos(c + d*x)*sin(
c + d*x)**4*i - 32*cos(c + d*x)*sin(c + d*x)**2*i + cos(c + d*x)*i - 128*s
in(c + d*x)**9 + 320*sin(c + d*x)**7 - 272*sin(c + d*x)**5 + 88*sin(c + d*
x)**3 - 8*sin(c + d*x)),x)*d*i - 774144000*cos(c + d*x)*int(sin(c + d*x)**
10/(128*cos(c + d*x)*sin(c + d*x)**9*i - 320*cos(c + d*x)*sin(c + d*x)**7*
i + 272*cos(c + d*x)*sin(c + d*x)**5*i - 88*cos(c + d*x)*sin(c + d*x)**3*i
 + 8*cos(c + d*x)*sin(c + d*x)*i - 128*sin(c + d*x)**10 + 384*sin(c + d*x)
**8 - 416*sin(c + d*x)**6 + 192*sin(c + d*x)**4 - 33*sin(c + d*x)**2 + 1),
x)*d + 774144000*cos(c + d*x)*int(sin(c + d*x)**10/(128*cos(c + d*x)*sin(c
 + d*x)**9 - 320*cos(c + d*x)*sin(c + d*x)**7 + 272*cos(c + d*x)*sin(c ...
```

3.169 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 43

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i(a-ia\tan(c+dx))^4}{8d\left(a^3+ia^3\tan(c+dx)\right)^4}$$

output
```
1/8*I*(a-I*a*tan(d*x+c))^4/d/(a^3+I*a^3*tan(d*x+c))^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i(i+\tan(c+dx))^4}{8a^8d(-i+\tan(c+dx))^4}$$

input
```
Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/8)*(I + Tan[c + d*x])^4)/(a^8*d*(-I + Tan[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^8}{(a+ia\tan(c+dx))^8}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int \frac{(a-ia\tan(c+dx))^3}{(i\tan(c+dx)a+a)^5}d(ia\tan(c+dx))}{a^7d}$$
$$\Big\downarrow \text{48}$$
$$\frac{i(a-ia\tan(c+dx))^4}{8a^8d(a+ia\tan(c+dx))^4}$$

input
```
Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/8)*(a - I*a*Tan[c + d*x])^4)/(a^8*d*(a + I*a*Tan[c + d*x])^4)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.44

method	result	size
risch	$\frac{i\mathrm{e}^{-8i(dx+c)}}{8a^{8}d}$	19
orering	$\frac{i\sec(dx+c)^{8}}{8d(a+ia\tan(dx+c))^{8}}$	29
derivativedivides	$\frac{\frac{2i}{(-i+\tan(dx+c))^{4}}+\frac{4}{(-i+\tan(dx+c))^{3}}-\frac{3i}{(-i+\tan(dx+c))^{2}}-\frac{1}{-i+\tan(dx+c)}}{a^{8}d}$	63
default	$\frac{\frac{2i}{(-i+\tan(dx+c))^{4}}+\frac{4}{(-i+\tan(dx+c))^{3}}-\frac{3i}{(-i+\tan(dx+c))^{2}}-\frac{1}{-i+\tan(dx+c)}}{a^{8}d}$	63

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/8*I/a^8/d*exp(-8*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.40

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx=\frac{i\,e^{(-8i\,dx-8i\,c)}}{8\,a^8d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/8*I*e^(-8*I*d*x - 8*I*c)/(a^8*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(34) = 68$.

Time = 10.21 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.72

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \begin{cases} \frac{i\sec^8(c+dx)}{8a^8d\tan^8(c+dx)-64ia^8d\tan^7(c+dx)-224a^8d\tan^6(c+dx)+448ia^8d\tan^5(c+dx)+560a^8d\tan^4(c+dx)-448ia^8d\tan^3(c+dx)-224a^8d\tan^2} \\ \frac{x\sec^8(c)}{(ia\tan(c)+a)^8} \end{cases}$$

input
```
integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise((I*sec(c + d*x)**8/(8*a**8*d*tan(c + d*x)**8 - 64*I*a**8*d*tan(c
 + d*x)**7 - 224*a**8*d*tan(c + d*x)**6 + 448*I*a**8*d*tan(c + d*x)**5 + 5
60*a**8*d*tan(c + d*x)**4 - 448*I*a**8*d*tan(c + d*x)**3 - 224*a**8*d*tan(
c + d*x)**2 + 64*I*a**8*d*tan(c + d*x) + 8*a**8*d), Ne(d, 0)), (x*sec(c)**
8/(I*a*tan(c) + a)**8, True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(35) = 70$.

Time = 0.05 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.67

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$
$$-\frac{\tan(dx+c)^6 - 3i\tan(dx+c)^5 - 4\tan(dx+c)^4 + 4i\tan(dx+c)^3 + 3\tan}{(a^8\tan(dx+c)^7 - 7i\,a^8\tan(dx+c)^6 - 21\,a^8\tan(dx+c)^5 + 35i\,a^8\tan(dx+c)^4 + 35\,a^8\tan(dx+c}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-(tan(d*x + c)^6 - 3*I*tan(d*x + c)^5 - 4*tan(d*x + c)^4 + 4*I*tan(d*x + c
)^3 + 3*tan(d*x + c)^2 - I*tan(d*x + c))/((a^8*tan(d*x + c)^7 - 7*I*a^8*ta
n(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*x + c)^4 + 35*a^8*ta
n(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x + c) + I*a^8)*d)
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{\tan{(dx+c)}^3-\tan{(dx+c)}}{a^8d(\tan{(dx+c)}-i)^4}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-(tan(d*x + c)^3 - tan(d*x + c))/(a^8*d*(tan(d*x + c) - I)^4)
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= -\frac{\tan(c+d\,x)\,\left(\tan(c+d\,x)^2\,1\mathrm{i}-\mathrm{i}\right)}{a^8\,d\,\left(\tan{(c+d\,x)}^4\,1\mathrm{i}+4\tan{(c+d\,x)}^3-\tan{(c+d\,x)}^2\,6\mathrm{i}-4\tan{(c+d\,x)}+1\mathrm{i}\right)}$$

input
```
int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
-(tan(c + d*x)*(tan(c + d*x)^2*1i - 1i))/(a^8*d*(4*tan(c + d*x)^3 - tan(c
+ d*x)^2*6i - 4*tan(c + d*x) + tan(c + d*x)^4*1i + 1i))
```

Reduce [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x)
```

output
```
(1024*int(sin(c + d*x)**8/(128*cos(c + d*x)*sin(c + d*x)**7*i - 192*cos(c
+ d*x)*sin(c + d*x)**5*i + 80*cos(c + d*x)*sin(c + d*x)**3*i - 8*cos(c + d
*x)*sin(c + d*x)*i - 128*sin(c + d*x)**8 + 256*sin(c + d*x)**6 - 160*sin(c
 + d*x)**4 + 32*sin(c + d*x)**2 - 1),x)*d + 2048*int(sin(c + d*x)**6/(128*
cos(c + d*x)*sin(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c
 + d*x)*sin(c + d*x)**3 - 8*cos(c + d*x)*sin(c + d*x) + 128*sin(c + d*x)**
8*i - 256*sin(c + d*x)**6*i + 160*sin(c + d*x)**4*i - 32*sin(c + d*x)**2*i
 + i),x)*d*i - 1280*int(sin(c + d*x)**4/(128*cos(c + d*x)*sin(c + d*x)**7
- 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(c + d*x)**3 - 8*c
os(c + d*x)*sin(c + d*x) + 128*sin(c + d*x)**8*i - 256*sin(c + d*x)**6*i +
 160*sin(c + d*x)**4*i - 32*sin(c + d*x)**2*i + i),x)*d*i + 256*int(sin(c
+ d*x)**2/(128*cos(c + d*x)*sin(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x
)**5 + 80*cos(c + d*x)*sin(c + d*x)**3 - 8*cos(c + d*x)*sin(c + d*x) + 128
*sin(c + d*x)**8*i - 256*sin(c + d*x)**6*i + 160*sin(c + d*x)**4*i - 32*si
n(c + d*x)**2*i + i),x)*d*i - 1024*int((cos(c + d*x)*sin(c + d*x)**7)/(128
*cos(c + d*x)*sin(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(
c + d*x)*sin(c + d*x)**3 - 8*cos(c + d*x)*sin(c + d*x) + 128*sin(c + d*x)*
*8*i - 256*sin(c + d*x)**6*i + 160*sin(c + d*x)**4*i - 32*sin(c + d*x)**2*
i + i),x)*d + 1536*int((cos(c + d*x)*sin(c + d*x)**5)/(128*cos(c + d*x)*si
n(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(...
```

3.170 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 81

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{4i}{5a^3d(a+ia\tan(c+dx))^5} + \frac{i}{3a^5d(a+ia\tan(c+dx))^3} - \frac{i}{d\left(a^2+ia^2\tan(c+dx)\right)^4}$$

output
```
4/5*I/a^3/d/(a+I*a*tan(d*x+c))^5+1/3*I/a^5/d/(a+I*a*tan(d*x+c))^3-I/d/(a^2
+I*a^2*tan(d*x+c))^4
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.54

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2-5i\tan(c+dx)-5\tan^2(c+dx)}{15a^8d(-i+\tan(c+dx))^5}$$

input
```
Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(2 - (5*I)*Tan[c + d*x] - 5*Tan[c + d*x]^2)/(15*a^8*d*(-I + Tan[c + d*x])^
5)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sec(c+dx)^6}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \ 3968$$

$$-\frac{i\int \frac{(a-ia\tan(c+dx))^2}{(i\tan(c+dx)a+a)^6}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \ 53$$

$$-\frac{i\int \left(\frac{4a^2}{(i\tan(c+dx)a+a)^6}-\frac{4a}{(i\tan(c+dx)a+a)^5}+\frac{1}{(i\tan(c+dx)a+a)^4}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \ 2009$$

$$-\frac{i\left(-\frac{4a^2}{5(a+ia\tan(c+dx))^5}+\frac{a}{(a+ia\tan(c+dx))^4}-\frac{1}{3(a+ia\tan(c+dx))^3}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*((-4*a^2)/(5*(a + I*a*Tan[c + d*x])^5) + a/(a + I*a*Tan[c + d*x])^4
- 1/(3*(a + I*a*Tan[c + d*x])^3)))/(a^5*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.60

method	result	size
derivativedivides	$\frac{-\frac{i}{(-i+\tan(dx+c))^4}+\frac{4}{5(-i+\tan(dx+c))^5}-\frac{1}{3(-i+\tan(dx+c))^3}}{a^8 d}$	49
default	$\frac{-\frac{i}{(-i+\tan(dx+c))^4}+\frac{4}{5(-i+\tan(dx+c))^5}-\frac{1}{3(-i+\tan(dx+c))^3}}{a^8 d}$	49
risch	$\frac{i\mathrm{e}^{-6i(dx+c)}}{24a^8 d}+\frac{i\mathrm{e}^{-8i(dx+c)}}{16a^8 d}+\frac{i\mathrm{e}^{-10i(dx+c)}}{40a^8 d}$	56

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/a^8/d*(-I/(-I+tan(d*x+c))^4+4/5/(-I+tan(d*x+c))^5-1/3/(-I+tan(d*x+c))^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.51

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(10i\,e^{(4i\,dx+4i\,c)}+15i\,e^{(2i\,dx+2i\,c)}+6i\right)e^{(-10i\,dx-10i\,c)}}{240\,a^8d}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/240*(10*I*e^(4*I*d*x + 4*I*c) + 15*I*e^(2*I*d*x + 2*I*c) + 6*I)*e^(-10*I
*d*x - 10*I*c)/(a^8*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(65) = 130$.

Time = 10.18 (sec) , antiderivative size = 466, normalized size of antiderivative = 5.75

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \begin{cases} -\frac{i\tan^2(c+dx)\sec^6(c+dx)}{240a^8d\tan^8(c+dx)-1920ia^8d\tan^7(c+dx)-6720a^8d\tan^6(c+dx)+13440ia^8d\tan^5(c+dx)+16800a^8d\tan^4(c+dx)-13440ia^8d\tan^3(c+d} \\ \frac{x\sec^6(c)}{(ia\tan(c)+a)^8} \end{cases}$$

input
```
integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise((-I*tan(c + d*x)**2*sec(c + d*x)**6/(240*a**8*d*tan(c + d*x)**8
- 1920*I*a**8*d*tan(c + d*x)**7 - 6720*a**8*d*tan(c + d*x)**6 + 13440*I*a*
*8*d*tan(c + d*x)**5 + 16800*a**8*d*tan(c + d*x)**4 - 13440*I*a**8*d*tan(c
 + d*x)**3 - 6720*a**8*d*tan(c + d*x)**2 + 1920*I*a**8*d*tan(c + d*x) + 24
0*a**8*d) - 8*tan(c + d*x)*sec(c + d*x)**6/(240*a**8*d*tan(c + d*x)**8 - 1
920*I*a**8*d*tan(c + d*x)**7 - 6720*a**8*d*tan(c + d*x)**6 + 13440*I*a**8*
d*tan(c + d*x)**5 + 16800*a**8*d*tan(c + d*x)**4 - 13440*I*a**8*d*tan(c +
d*x)**3 - 6720*a**8*d*tan(c + d*x)**2 + 1920*I*a**8*d*tan(c + d*x) + 240*a
**8*d) + 31*I*sec(c + d*x)**6/(240*a**8*d*tan(c + d*x)**8 - 1920*I*a**8*d*
tan(c + d*x)**7 - 6720*a**8*d*tan(c + d*x)**6 + 13440*I*a**8*d*tan(c + d*x
)**5 + 16800*a**8*d*tan(c + d*x)**4 - 13440*I*a**8*d*tan(c + d*x)**3 - 672
0*a**8*d*tan(c + d*x)**2 + 1920*I*a**8*d*tan(c + d*x) + 240*a**8*d), Ne(d,
 0)), (x*sec(c)**6/(I*a*tan(c) + a)**8, True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(65) = 130$.

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.74

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{5\,\tan\left(dx+c\right)^4 - 5i\,\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)^2 - i\,\tan}{15\left(a^8\tan\left(dx+c\right)^7 - 7i\,a^8\tan\left(dx+c\right)^6 - 21\,a^8\tan\left(dx+c\right)^5 + 35i\,a^8\tan\left(dx+c\right)^4 + 35\,a^8\tan\left(dx-\right.\right.}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/15*(5*tan(d*x + c)^4 - 5*I*tan(d*x + c)^3 + 3*tan(d*x + c)^2 - I*tan(d*
x + c) + 2)/((a^8*tan(d*x + c)^7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x
 + c)^5 + 35*I*a^8*tan(d*x + c)^4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d
*x + c)^2 - 7*a^8*tan(d*x + c) + I*a^8)*d)
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.47

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{5\,\tan\left(dx+c\right)^2+5i\,\tan\left(dx+c\right)-2}{15\,a^8d(\tan\left(dx+c\right)-i)^5}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-1/15*(5*tan(d*x + c)^2 + 5*I*tan(d*x + c) - 2)/(a^8*d*(tan(d*x + c) - I)^
5)
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{-\tan(c+d\,x)^2\,5\mathrm{i}+5\tan(c+d\,x)+2\mathrm{i}}{15\,a^8\,d\,\left(\tan\left(c+d\,x\right)^5\,1\mathrm{i}+5\tan\left(c+d\,x\right)^4-\tan\left(c+d\,x\right)^3\,10\mathrm{i}-10\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,5\mathrm{i}+1\right.}$$

input
```
int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
(5*tan(c + d*x) - tan(c + d*x)^2*5i + 2i)/(15*a^8*d*(tan(c + d*x)*5i - 10*
tan(c + d*x)^2 - tan(c + d*x)^3*10i + 5*tan(c + d*x)^4 + tan(c + d*x)^5*1i
 + 1))
```

Reduce [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x)
```

output
```
(1024*int(sin(c + d*x)**8/(128*cos(c + d*x)*sin(c + d*x)**7*i - 192*cos(c
+ d*x)*sin(c + d*x)**5*i + 80*cos(c + d*x)*sin(c + d*x)**3*i - 8*cos(c + d
*x)*sin(c + d*x)*i - 128*sin(c + d*x)**8 + 256*sin(c + d*x)**6 - 160*sin(c
 + d*x)**4 + 32*sin(c + d*x)**2 - 1),x)*d + 2048*int(sin(c + d*x)**6/(128*
cos(c + d*x)*sin(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c
 + d*x)*sin(c + d*x)**3 - 8*cos(c + d*x)*sin(c + d*x) + 128*sin(c + d*x)**
8*i - 256*sin(c + d*x)**6*i + 160*sin(c + d*x)**4*i - 32*sin(c + d*x)**2*i
 + i),x)*d*i - 1280*int(sin(c + d*x)**4/(128*cos(c + d*x)*sin(c + d*x)**7
- 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(c + d*x)**3 - 8*c
os(c + d*x)*sin(c + d*x) + 128*sin(c + d*x)**8*i - 256*sin(c + d*x)**6*i +
 160*sin(c + d*x)**4*i - 32*sin(c + d*x)**2*i + i),x)*d*i + 248*int(sin(c
+ d*x)**2/(128*cos(c + d*x)*sin(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x
)**5 + 80*cos(c + d*x)*sin(c + d*x)**3 - 8*cos(c + d*x)*sin(c + d*x) + 128
*sin(c + d*x)**8*i - 256*sin(c + d*x)**6*i + 160*sin(c + d*x)**4*i - 32*si
n(c + d*x)**2*i + i),x)*d*i - 1024*int((cos(c + d*x)*sin(c + d*x)**7)/(128
*cos(c + d*x)*sin(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(
c + d*x)*sin(c + d*x)**3 - 8*cos(c + d*x)*sin(c + d*x) + 128*sin(c + d*x)*
*8*i - 256*sin(c + d*x)**6*i + 160*sin(c + d*x)**4*i - 32*sin(c + d*x)**2*
i + i),x)*d + 1536*int((cos(c + d*x)*sin(c + d*x)**5)/(128*cos(c + d*x)*si
n(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(...
```

3.171 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx=\frac{i}{3a^2d(a+ia\tan(c+dx))^6}-\frac{i}{5a^3d(a+ia\tan(c+dx))^5}$$

output
```
1/3*I/a^2/d/(a+I*a*tan(d*x+c))^6-1/5*I/a^3/d/(a+I*a*tan(d*x+c))^5
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx=-\frac{2i+3\tan(c+dx)}{15a^8d(-i+\tan(c+dx))^6}$$

input
```
Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
-1/15*(2*I + 3*Tan[c + d*x])/(a^8*d*(-I + Tan[c + d*x])^6)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^4}{(a+ia\tan(c+dx))^8}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int \frac{a-ia\tan(c+dx)}{(i\tan(c+dx)a+a)^7}d(ia\tan(c+dx))}{a^3d}$$
$$\Big\downarrow \text{53}$$
$$-\frac{i\int \left(\frac{2a}{(i\tan(c+dx)a+a)^7}-\frac{1}{(i\tan(c+dx)a+a)^6}\right)d(ia\tan(c+dx))}{a^3d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{i\left(\frac{1}{5(a+ia\tan(c+dx))^5}-\frac{a}{3(a+ia\tan(c+dx))^6}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*(-1/3*a/(a + I*a*Tan[c + d*x])^6 + 1/(5*(a + I*a*Tan[c + d*x])^5)))/
(a^3*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{-\frac{i}{3(-i+\tan(dx+c))^6}-\frac{1}{5(-i+\tan(dx+c))^5}}{a^8 d}$	36
default	$\frac{-\frac{i}{3(-i+\tan(dx+c))^6}-\frac{1}{5(-i+\tan(dx+c))^5}}{a^8 d}$	36
risch	$\frac{i\mathrm{e}^{-4i(dx+c)}}{64a^8d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{24a^8d}+\frac{3i\mathrm{e}^{-8i(dx+c)}}{64a^8d}+\frac{i\mathrm{e}^{-10i(dx+c)}}{40a^8d}+\frac{i\mathrm{e}^{-12i(dx+c)}}{192a^8d}$	92

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/a^8/d*(-1/3*I/(-I+tan(d*x+c))^6-1/5/(-I+tan(d*x+c))^5)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(15i\,e^{(8i\,dx+8i\,c)}+40i\,e^{(6i\,dx+6i\,c)}+45i\,e^{(4i\,dx+4i\,c)}+24i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-12i\,dx-12i\,c)}}{960\,a^8d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/960*(15*I*e^(8*I*d*x + 8*I*c) + 40*I*e^(6*I*d*x + 6*I*c) + 45*I*e^(4*I*d
*x + 4*I*c) + 24*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-12*I*d*x - 12*I*c)/(a^8*
d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 774 vs. 2(42) = 84.

Time = 10.45 (sec) , antiderivative size = 774, normalized size of antiderivative = 14.07

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise((I*tan(c + d*x)**4*sec(c + d*x)**4/(960*a**8*d*tan(c + d*x)**8 -
 7680*I*a**8*d*tan(c + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 53760*I*a*
*8*d*tan(c + d*x)**5 + 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*d*tan(c
 + d*x)**3 - 26880*a**8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d*x) + 9
60*a**8*d) + 8*tan(c + d*x)**3*sec(c + d*x)**4/(960*a**8*d*tan(c + d*x)**8
 - 7680*I*a**8*d*tan(c + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 53760*I*
a**8*d*tan(c + d*x)**5 + 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*d*tan
(c + d*x)**3 - 26880*a**8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d*x) +
 960*a**8*d) - 30*I*tan(c + d*x)**2*sec(c + d*x)**4/(960*a**8*d*tan(c + d*
x)**8 - 7680*I*a**8*d*tan(c + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 537
60*I*a**8*d*tan(c + d*x)**5 + 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*
d*tan(c + d*x)**3 - 26880*a**8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d
*x) + 960*a**8*d) - 72*tan(c + d*x)*sec(c + d*x)**4/(960*a**8*d*tan(c + d*
x)**8 - 7680*I*a**8*d*tan(c + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 537
60*I*a**8*d*tan(c + d*x)**5 + 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*
d*tan(c + d*x)**3 - 26880*a**8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d
*x) + 960*a**8*d) + 129*I*sec(c + d*x)**4/(960*a**8*d*tan(c + d*x)**8 - 76
80*I*a**8*d*tan(c + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 53760*I*a**8*
d*tan(c + d*x)**5 + 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*d*tan(c +
d*x)**3 - 26880*a**8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d*x) + 9...
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(43) = 86$.

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.20

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$

$$-\frac{3\,\tan\left(dx+c\right)^2 - i\,\tan\left(dx+c\right) + 2}{15\,\left(a^8\tan\left(dx+c\right)^7 - 7i\,a^8\tan\left(dx+c\right)^6 - 21\,a^8\tan\left(dx+c\right)^5 + 35i\,a^8\tan\left(dx+c\right)^4 + 35\,a^8\tan\left(dx-\right.\right.}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/15*(3*tan(d*x + c)^2 - I*tan(d*x + c) + 2)/((a^8*tan(d*x + c)^7 - 7*I*a
^8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*x + c)^4 + 35*a
^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x + c) + I*a^8)*
d)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.51

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{3\tan(dx+c)+2i}{15\,a^8d(\tan(dx+c)-i)^6}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-1/15*(3*tan(d*x + c) + 2*I)/(a^8*d*(tan(d*x + c) - I)^6)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$
$$-\frac{-2+\tan(c+d\,x)\,3\mathrm{i}}{15\,a^8\,d\,\left(\tan(c+d\,x)^6\,1\mathrm{i}+6\tan(c+d\,x)^5-\tan(c+d\,x)^4\,15\mathrm{i}-20\tan(c+d\,x)^3+\tan(c+d\,x)^2\,15\mathrm{i}\right.}$$

input
```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
-(tan(c + d*x)*3i - 2)/(15*a^8*d*(6*tan(c + d*x) + tan(c + d*x)^2*15i - 20
*tan(c + d*x)^3 - tan(c + d*x)^4*15i + 6*tan(c + d*x)^5 + tan(c + d*x)^6*1
i - 1i))
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x)
```

output
```
(1024*int(sin(c + d*x)**8/(128*cos(c + d*x)*sin(c + d*x)**7*i - 192*cos(c
+ d*x)*sin(c + d*x)**5*i + 80*cos(c + d*x)*sin(c + d*x)**3*i - 8*cos(c + d
*x)*sin(c + d*x)*i - 128*sin(c + d*x)**8 + 256*sin(c + d*x)**6 - 160*sin(c
 + d*x)**4 + 32*sin(c + d*x)**2 - 1),x)*d + 2048*int(sin(c + d*x)**6/(128*
cos(c + d*x)*sin(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c
 + d*x)*sin(c + d*x)**3 - 8*cos(c + d*x)*sin(c + d*x) + 128*sin(c + d*x)**
8*i - 256*sin(c + d*x)**6*i + 160*sin(c + d*x)**4*i - 32*sin(c + d*x)**2*i
 + i),x)*d*i - 1272*int(sin(c + d*x)**4/(128*cos(c + d*x)*sin(c + d*x)**7
- 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(c + d*x)**3 - 8*c
os(c + d*x)*sin(c + d*x) + 128*sin(c + d*x)**8*i - 256*sin(c + d*x)**6*i +
 160*sin(c + d*x)**4*i - 32*sin(c + d*x)**2*i + i),x)*d*i + 240*int(sin(c
+ d*x)**2/(128*cos(c + d*x)*sin(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x
)**5 + 80*cos(c + d*x)*sin(c + d*x)**3 - 8*cos(c + d*x)*sin(c + d*x) + 128
*sin(c + d*x)**8*i - 256*sin(c + d*x)**6*i + 160*sin(c + d*x)**4*i - 32*si
n(c + d*x)**2*i + i),x)*d*i - 1024*int((cos(c + d*x)*sin(c + d*x)**7)/(128
*cos(c + d*x)*sin(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(
c + d*x)*sin(c + d*x)**3 - 8*cos(c + d*x)*sin(c + d*x) + 128*sin(c + d*x)*
*8*i - 256*sin(c + d*x)**6*i + 160*sin(c + d*x)**4*i - 32*sin(c + d*x)**2*
i + i),x)*d + 1536*int((cos(c + d*x)*sin(c + d*x)**5)/(128*cos(c + d*x)*si
n(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(...
```

3.172 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i}{7ad(a+ia\tan(c+dx))^7}$$

output
```
1/7*I/a/d/(a+I*a*tan(d*x+c))^7
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{1}{7a^8d(-i+\tan(c+dx))^7}$$

input
```
Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
-1/7*1/(a^8*d*(-I + Tan[c + d*x])^7)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^2}{(a+ia\tan(c+dx))^8}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int \frac{1}{(i\tan(c+dx)a+a)^8}d(ia\tan(c+dx))}{ad}$$
$$\Big\downarrow \text{17}$$
$$\frac{i}{7ad(a+ia\tan(c+dx))^7}$$

input
```
Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(I/7)/(a*d*(a + I*a*Tan[c + d*x])^7)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{i}{7ad(a+ia\tan(dx+c))^7}$
default	$\frac{i}{7ad(a+ia\tan(dx+c))^7}$
risch	$\frac{ie^{-2i(dx+c)}}{128a^8d}+\frac{3ie^{-4i(dx+c)}}{128a^8d}+\frac{5ie^{-6i(dx+c)}}{128a^8d}+\frac{5ie^{-8i(dx+c)}}{128a^8d}+\frac{3ie^{-10i(dx+c)}}{128a^8d}+\frac{ie^{-12i(dx+c)}}{128a^8d}+\frac{ie^{-14i(dx+c)}}{896a^8d}$

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/7*I/a/d/(a+I*a*tan(d*x+c))^7
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(21) = 42.

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.15

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(7i\,e^{(12i\,dx+12i\,c)}+21i\,e^{(10i\,dx+10i\,c)}+35i\,e^{(8i\,dx+8i\,c)}+35i\,e^{(6i\,dx+6i\,c)}+21i\,e^{(4i\,dx+4i\,c)}+7i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(}}{896\,a^8d}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/896*(7*I*e^(12*I*d*x + 12*I*c) + 21*I*e^(10*I*d*x + 10*I*c) + 35*I*e^(8*
I*d*x + 8*I*c) + 35*I*e^(6*I*d*x + 6*I*c) + 21*I*e^(4*I*d*x + 4*I*c) + 7*I
*e^(2*I*d*x + 2*I*c) + I)*e^(-14*I*d*x - 14*I*c)/(a^8*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1081 vs. $2(19) = 38$.

Time = 10.04 (sec) , antiderivative size = 1081, normalized size of antiderivative = 40.04

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise((-I*tan(c + d*x)**6*sec(c + d*x)**2/(896*a**8*d*tan(c + d*x)**8
- 7168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50176*I*a
**8*d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8*d*tan(
c + d*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c + d*x) +
896*a**8*d) - 8*tan(c + d*x)**5*sec(c + d*x)**2/(896*a**8*d*tan(c + d*x)**
8 - 7168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50176*I
*a**8*d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8*d*ta
n(c + d*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c + d*x)
+ 896*a**8*d) + 29*I*tan(c + d*x)**4*sec(c + d*x)**2/(896*a**8*d*tan(c + d
*x)**8 - 7168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50
176*I*a**8*d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8
*d*tan(c + d*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c +
d*x) + 896*a**8*d) + 64*tan(c + d*x)**3*sec(c + d*x)**2/(896*a**8*d*tan(c
+ d*x)**8 - 7168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 +
 50176*I*a**8*d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a
**8*d*tan(c + d*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c
 + d*x) + 896*a**8*d) - 99*I*tan(c + d*x)**2*sec(c + d*x)**2/(896*a**8*d*t
an(c + d*x)**8 - 7168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)
**6 + 50176*I*a**8*d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 5017
6*I*a**8*d*tan(c + d*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i}{7\,(i\,a\tan\left(dx+c\right)+a)^7ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/7*I/((I*a*tan(d*x + c) + a)^7*a*d)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{1}{7\,a^8d(\tan\left(dx+c\right)-i)^7}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-1/7/(a^8*d*(tan(d*x + c) - I)^7)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{1}{7\,a^8\,d\,(\tan\left(c+d\,x\right)-\mathrm{i})^7}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
-1/(7*a^8*d*(tan(c + d*x) - 1i)^7)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x)
```

output
```
(1024*int(sin(c + d*x)**8/(128*cos(c + d*x)*sin(c + d*x)**7*i - 192*cos(c
+ d*x)*sin(c + d*x)**5*i + 80*cos(c + d*x)*sin(c + d*x)**3*i - 8*cos(c + d
*x)*sin(c + d*x)*i - 128*sin(c + d*x)**8 + 256*sin(c + d*x)**6 - 160*sin(c
 + d*x)**4 + 32*sin(c + d*x)**2 - 1),x)*d + 2040*int(sin(c + d*x)**6/(128*
cos(c + d*x)*sin(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c
 + d*x)*sin(c + d*x)**3 - 8*cos(c + d*x)*sin(c + d*x) + 128*sin(c + d*x)**
8*i - 256*sin(c + d*x)**6*i + 160*sin(c + d*x)**4*i - 32*sin(c + d*x)**2*i
 + i),x)*d*i - 1256*int(sin(c + d*x)**4/(128*cos(c + d*x)*sin(c + d*x)**7
- 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(c + d*x)**3 - 8*c
os(c + d*x)*sin(c + d*x) + 128*sin(c + d*x)**8*i - 256*sin(c + d*x)**6*i +
 160*sin(c + d*x)**4*i - 32*sin(c + d*x)**2*i + i),x)*d*i + 232*int(sin(c
+ d*x)**2/(128*cos(c + d*x)*sin(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x
)**5 + 80*cos(c + d*x)*sin(c + d*x)**3 - 8*cos(c + d*x)*sin(c + d*x) + 128
*sin(c + d*x)**8*i - 256*sin(c + d*x)**6*i + 160*sin(c + d*x)**4*i - 32*si
n(c + d*x)**2*i + i),x)*d*i - 1024*int((cos(c + d*x)*sin(c + d*x)**7)/(128
*cos(c + d*x)*sin(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(
c + d*x)*sin(c + d*x)**3 - 8*cos(c + d*x)*sin(c + d*x) + 128*sin(c + d*x)*
*8*i - 256*sin(c + d*x)**6*i + 160*sin(c + d*x)**4*i - 32*sin(c + d*x)**2*
i + i),x)*d + 1536*int((cos(c + d*x)*sin(c + d*x)**5)/(128*cos(c + d*x)*si
n(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(...
```

3.173 $\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 229

$$\begin{aligned}\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx &= \frac{x}{256a^8}+\frac{i}{16d(a+ia\tan(c+dx))^8} \\ &+\frac{i}{28ad(a+ia\tan(c+dx))^7}+\frac{i}{48a^2d(a+ia\tan(c+dx))^6} \\ &+\frac{i}{80a^3d(a+ia\tan(c+dx))^5} \\ &+\frac{i}{128d\left(a^2+ia^2\tan(c+dx)\right)^4} \\ &+\frac{i}{192a^2d\left(a^2+ia^2\tan(c+dx)\right)^3} \\ &+\frac{i}{256d\left(a^4+ia^4\tan(c+dx)\right)^2} \\ &+\frac{i}{256d\left(a^8+ia^8\tan(c+dx)\right)}\end{aligned}$$

output

```
1/256*x/a^8+1/16*I/d/(a+I*a*tan(d*x+c))^8+1/28*I/a/d/(a+I*a*tan(d*x+c))^7+
1/48*I/a^2/d/(a+I*a*tan(d*x+c))^6+1/80*I/a^3/d/(a+I*a*tan(d*x+c))^5+1/128*
I/d/(a^2+I*a^2*tan(d*x+c))^4+1/192*I/a^2/d/(a^2+I*a^2*tan(d*x+c))^3+1/256*
I/d/(a^4+I*a^4*tan(d*x+c))^2+1/256*I/d/(a^8+I*a^8*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\sec^8(c+dx)(7350+12544\cos(2(c+dx))+7840\cos(4(c+dx))+3840\cos(6(c+dx))+1194\cos(8(c$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(-8),x]
```

output
```
((I/215040)*Sec[c + d*x]^8*(7350 + 12544*Cos[2*(c + d*x)] + 7840*Cos[4*(c
+ d*x)] + 3840*Cos[6*(c + d*x)] + 1194*Cos[8*(c + d*x)] + (3136*I)*Sin[2*(
c + d*x)] + (3920*I)*Sin[4*(c + d*x)] + (2880*I)*Sin[6*(c + d*x)] + (1089*
I)*Sin[8*(c + d*x)] + 840*ArcTan[Tan[c + d*x]]*((-I)*Cos[8*(c + d*x)] + Si
n[8*(c + d*x)])))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.12, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.133$, Rules used = {3042, 3960, 3042, 3960, 3042, 3960, 3042, 3960, 3042, 3960, 3042, 3960, 3042, 3960, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3960}$$

$$\frac{\int \frac{1}{(i\tan(c+dx)a+a)^7}dx}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{(i\tan(c+dx)a+a)^7}dx}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3960}$$

$$\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^6}dx}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^6}dx}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3960}$$

$$\frac{\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^5}dx}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^5}dx}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3960}$$

$$\frac{\frac{\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^4}dx}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^4}dx}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3960}$$

$$\frac{\frac{\frac{\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^3}dx}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^3}dx}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \; 3960$$

$$\frac{\frac{\frac{\frac{\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^2}dx}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\frac{\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^2}dx}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \; 3960$$

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\cdots}}{2a}$$

$$\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \; 3960$$

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{\frac{\int 1dx}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\cdots}}{2a}$$

$$\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \; 24$$

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\cdots}}{2a}$$

$$\frac{i}{16d(a+ia\tan(c+dx))^8}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(-8),x]
```

output
```
(I/16)/(d*(a + I*a*Tan[c + d*x])^8) + ((I/14)/(d*(a + I*a*Tan[c + d*x])^7)
 + ((I/12)/(d*(a + I*a*Tan[c + d*x])^6) + ((I/10)/(d*(a + I*a*Tan[c + d*x]
)^5) + ((I/8)/(d*(a + I*a*Tan[c + d*x])^4) + ((I/6)/(d*(a + I*a*Tan[c + d*
x])^3) + ((I/4)/(d*(a + I*a*Tan[c + d*x])^2) + (x/(2*a) + (I/2)/(d*(a + I*
a*Tan[c + d*x])))/(2*a))/(2*a))/(2*a))/(2*a))/(2*a))/(2*a))/(2*a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.66

method	result
risch	$\frac{x}{256a^8}+\frac{i\mathrm{e}^{-2i(dx+c)}}{64a^8d}+\frac{7i\mathrm{e}^{-4i(dx+c)}}{256a^8d}+\frac{7i\mathrm{e}^{-6i(dx+c)}}{192a^8d}+\frac{35i\mathrm{e}^{-8i(dx+c)}}{1024a^8d}+\frac{7i\mathrm{e}^{-10i(dx+c)}}{320a^8d}+\frac{7i\mathrm{e}^{-12i(dx+c)}}{768a^8d}+$
derivativedivides	$\frac{\arctan(\tan(dx+c))}{256a^8d}+\frac{i}{16d\,a^8(-i+\tan(dx+c))^8}+\frac{i}{128d\,a^8(-i+\tan(dx+c))^4}-\frac{i}{48d\,a^8(-i+\tan(dx+c))^6}-\frac{}{256d}$
default	$\frac{\arctan(\tan(dx+c))}{256a^8d}+\frac{i}{16d\,a^8(-i+\tan(dx+c))^8}+\frac{i}{128d\,a^8(-i+\tan(dx+c))^4}-\frac{i}{48d\,a^8(-i+\tan(dx+c))^6}-\frac{}{256d}$
norman	$\frac{255\tan(dx+c)}{256ad}+\frac{x}{256a}+\frac{7x\tan(dx+c)^4}{64a}+\frac{x\tan(dx+c)^2}{32a}-\frac{1117\tan(dx+c)^3}{256ad}+\frac{7x\tan(dx+c)^6}{32a}+\frac{961\tan(dx+c)^7}{8960ad}+\frac{5053\tan(dx+c)^9}{26880ad}$

input
```
int(1/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/256*x/a^8+1/64*I/a^8/d*exp(-2*I*(d*x+c))+7/256*I/a^8/d*exp(-4*I*(d*x+c))
+7/192*I/a^8/d*exp(-6*I*(d*x+c))+35/1024*I/a^8/d*exp(-8*I*(d*x+c))+7/320*I
/a^8/d*exp(-10*I*(d*x+c))+7/768*I/a^8/d*exp(-12*I*(d*x+c))+1/448*I/a^8/d*e
xp(-14*I*(d*x+c))+1/4096*I/a^8/d*exp(-16*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.48

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(1680\,dxe^{(16i\,dx+16i\,c)}+6720i\,e^{(14i\,dx+14i\,c)}+11760i\,e^{(12i\,dx+12i\,c)}+15680i\,e^{(10i\,dx+10i\,c)}+14700i\,e^{(8i\,dx+8i\,c)}\right.}{430080\,a^8 d}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/430080*(1680*d*x*e^(16*I*d*x + 16*I*c) + 6720*I*e^(14*I*d*x + 14*I*c) +
11760*I*e^(12*I*d*x + 12*I*c) + 15680*I*e^(10*I*d*x + 10*I*c) + 14700*I*e^
(8*I*d*x + 8*I*c) + 9408*I*e^(6*I*d*x + 6*I*c) + 3920*I*e^(4*I*d*x + 4*I*c
) + 960*I*e^(2*I*d*x + 2*I*c) + 105*I)*e^(-16*I*d*x - 16*I*c)/(a^8*d)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.42

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx = \begin{cases} \frac{(2269814212194729840ia^{56}d^7e^{70ic}e^{-2idx}+3972174871340777472 0ia^{56}d^7e^{68ic}e^{-4idx}+52962331617877032960ia^{56}d^7e^{66ic}e^{-6idx}+4965218}{} \\ x\left(\frac{(e^{16ic}+8e^{14ic}+28e^{12ic}+56e^{10ic}+70e^{8ic}+56e^{6ic}+28e^{4ic}+8e^{2ic}+1)e^{-16ic}}{256a^8}-\frac{1}{256a^8}\right) \end{cases} + \frac{x}{256a^8}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((2269814212194729840*I*a**56*d**7*exp(70*I*c)*exp(-2*I*d*x) +
3972174871340777472O*I*a**56*d**7*exp(68*I*c)*exp(-4*I*d*x) + 529623316178
77032960*I*a**56*d**7*exp(66*I*c)*exp(-6*I*d*x) + 49652185891759718400*I*a
**56*d**7*exp(64*I*c)*exp(-8*I*d*x) + 31777398970726219776*I*a**56*d**7*ex
p(62*I*c)*exp(-10*I*d*x) + 13240582904469258240*I*a**56*d**7*exp(60*I*c)*e
xp(-12*I*d*x) + 3242591731706757120*I*a**56*d**7*exp(58*I*c)*exp(-14*I*d*x
) + 354658470655426560*I*a**56*d**7*exp(56*I*c)*exp(-16*I*d*x))*exp(-72*I*
c)/(1452681095804627189760*a**64*d**8), Ne(a**64*d**8*exp(72*I*c), 0)), (x
*((exp(16*I*c) + 8*exp(14*I*c) + 28*exp(12*I*c) + 56*exp(10*I*c) + 70*exp(
8*I*c) + 56*exp(6*I*c) + 28*exp(4*I*c) + 8*exp(2*I*c) + 1)*exp(-16*I*c)/(2
56*a**8) - 1/(256*a**8)), True)) + x/(256*a**8)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.54

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\log(\tan(dx+c)+i)}{512\,a^8d} - \frac{i\log(\tan(dx+c)-i)}{512\,a^8d} + \frac{105\tan(dx+c)^7 - 840i\tan(dx+c)^6 - 2975\tan(dx+c)^5 + 6160i\tan(dx+c)^4 + 8351\tan(dx+}{26880\,a^8d(\tan(dx+c)-i)^8}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/512*I*log(tan(d*x + c) + I)/(a^8*d) - 1/512*I*log(tan(d*x + c) - I)/(a^8
*d) + 1/26880*(105*tan(d*x + c)^7 - 840*I*tan(d*x + c)^6 - 2975*tan(d*x +
c)^5 + 6160*I*tan(d*x + c)^4 + 8351*tan(d*x + c)^3 - 8008*I*tan(d*x + c)^2
 - 5993*tan(d*x + c) + 4096*I)/(a^8*d*(tan(d*x + c) - I)^8)
```

Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx = \frac{x}{256\,a^8} - \frac{\frac{\tan(c+d\,x)\,5993\mathrm{i}}{26880\,a^8}+\frac{16}{105\,a^8}-\frac{143\tan(c+d\,x)^2}{480\,a^8}-\frac{\tan(c+d\,x)^3\,1193\mathrm{i}}{3840\,a^8}+\frac{11\tan(c+d\,x)^4}{48\,a^8}+\frac{\tan(c+d}{768\,}}{d\,\left(\tan(c+d\,x)^8\,1\mathrm{i}+8\tan(c+d\,x)^7-\tan(c+d\,x)^6\,28\mathrm{i}-56\tan(c+d\,x)^5+\tan(c+d\,x)^4\,70\mathrm{i}+56\right.}$$

input
```
int(1/(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
x/(256*a^8) - ((tan(c + d*x)*5993i)/(26880*a^8) + 16/(105*a^8) - (143*tan(
c + d*x)^2)/(480*a^8) - (tan(c + d*x)^3*1193i)/(3840*a^8) + (11*tan(c + d*
x)^4)/(48*a^8) + (tan(c + d*x)^5*85i)/(768*a^8) - tan(c + d*x)^6/(32*a^8)
- (tan(c + d*x)^7*1i)/(256*a^8))/(d*(56*tan(c + d*x)^3 - tan(c + d*x)^2*28
i - 8*tan(c + d*x) + tan(c + d*x)^4*70i - 56*tan(c + d*x)^5 - tan(c + d*x)
^6*28i + 8*tan(c + d*x)^7 + tan(c + d*x)^8*1i + 1i))
```

Reduce [F]

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx = \frac{\int \frac{1}{\tan(dx+c)^8-8\tan(dx+c)^7i-28\tan(dx+c)^6+56\tan(dx+c)^5i+70\tan(dx+c)^4-56\tan(dx+c)^3i-28\tan(dx+c)^2+8\tan(dx+c)i+1}dx}{a^8}$$

input
```
int(1/(a+I*a*tan(d*x+c))^8,x)
```

output
```
int(1/(tan(c + d*x)**8 - 8*tan(c + d*x)**7*i - 28*tan(c + d*x)**6 + 56*tan
(c + d*x)**5*i + 70*tan(c + d*x)**4 - 56*tan(c + d*x)**3*i - 28*tan(c + d*
x)**2 + 8*tan(c + d*x)*i + 1),x)/a**8
```

3.174 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 278

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= \frac{5x}{512a^8}+\frac{ia}{36d(a+ia\tan(c+dx))^9}\\
&\quad+\frac{i}{32d(a+ia\tan(c+dx))^8}+\frac{3i}{112ad(a+ia\tan(c+dx))^7}\\
&\quad+\frac{i}{48a^2d(a+ia\tan(c+dx))^6}\\
&\quad+\frac{i}{64a^3d(a+ia\tan(c+dx))^5}\\
&\quad+\frac{7i}{768a^5d(a+ia\tan(c+dx))^3}\\
&\quad+\frac{3i}{256d\left(a^2+ia^2\tan(c+dx)\right)^4}\\
&\quad+\frac{i}{128d\left(a^4+ia^4\tan(c+dx)\right)^2}\\
&\quad-\frac{i}{1024d\left(a^8-ia^8\tan(c+dx)\right)}\\
&\quad+\frac{9i}{1024d\left(a^8+ia^8\tan(c+dx)\right)}\end{aligned}$$

output
```
5/512*x/a^8+1/36*I*a/d/(a+I*a*tan(d*x+c))^9+1/32*I/d/(a+I*a*tan(d*x+c))^8+
3/112*I/a/d/(a+I*a*tan(d*x+c))^7+1/48*I/a^2/d/(a+I*a*tan(d*x+c))^6+1/64*I/
a^3/d/(a+I*a*tan(d*x+c))^5+7/768*I/a^5/d/(a+I*a*tan(d*x+c))^3+3/256*I/d/(a
^2+I*a^2*tan(d*x+c))^4+1/128*I/d/(a^4+I*a^4*tan(d*x+c))^2-1/1024*I/d/(a^8-
I*a^8*tan(d*x+c))+9/1024*I/d/(a^8+I*a^8*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.68

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\sec^{10}(c+dx)(2520\arctan(\tan(c+dx))(\cos(8(c+dx))+i\sin(8(c+dx)))+i(7938+14112\cos(2(c+d}{}$$

input
```
Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(Sec[c + d*x]^10*(2520*ArcTan[Tan[c + d*x]]*(Cos[8*(c + d*x)] + I*Sin[8*(c
 + d*x)]) + I*(7938 + 14112*Cos[2*(c + d*x)] + 10080*Cos[4*(c + d*x)] + 64
80*Cos[6*(c + d*x)] + 2462*Cos[8*(c + d*x)] - 112*Cos[10*(c + d*x)] + (352
8*I)*Sin[2*(c + d*x)] + (5040*I)*Sin[4*(c + d*x)] + (4860*I)*Sin[6*(c + d*
x)] + (2147*I)*Sin[8*(c + d*x)] - (140*I)*Sin[10*(c + d*x)])))/(258048*a^8
*d*(-I + Tan[c + d*x])^9*(I + Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{10}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^3\int \left(\frac{1}{1024a^{10}(a-ia\tan(c+dx))^2}+\frac{9}{1024a^{10}(i\tan(c+dx)a+a)^2}+\frac{1}{64a^9(i\tan(c+dx)a+a)^3}+\frac{7}{256a^8(i\tan(c+dx)a+a)^4}+\frac{3}{64a^7(i\tan(c+dx)a+a)^5}\right.}{}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^3\left(\frac{5i\arctan(\tan(c+dx))}{512a^{11}}+\frac{1}{1024a^{10}(a-ia\tan(c+dx))}-\frac{9}{1024a^{10}(a+ia\tan(c+dx))}-\frac{1}{128a^9(a+ia\tan(c+dx))^2}-\frac{7}{768a^8(a+ia\tan(c+dx))^3}\right.}{}$$

input
```
Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*a^3*((((5*I)/512)*ArcTan[Tan[c + d*x]])/a^11 + 1/(1024*a^10*(a - I*a
*Tan[c + d*x])) - 1/(36*a^2*(a + I*a*Tan[c + d*x])^9) - 1/(32*a^3*(a + I*a
*Tan[c + d*x])^8) - 3/(112*a^4*(a + I*a*Tan[c + d*x])^7) - 1/(48*a^5*(a +
I*a*Tan[c + d*x])^6) - 1/(64*a^6*(a + I*a*Tan[c + d*x])^5) - 3/(256*a^7*(a
 + I*a*Tan[c + d*x])^4) - 7/(768*a^8*(a + I*a*Tan[c + d*x])^3) - 1/(128*a^
9*(a + I*a*Tan[c + d*x])^2) - 9/(1024*a^10*(a + I*a*Tan[c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.61

method	result
derivativedivides	$\frac{-\frac{5i\ln(-i+\tan(dx+c))}{1024}+\frac{3i}{256(-i+\tan(dx+c))^4}+\frac{i}{32(-i+\tan(dx+c))^8}-\frac{i}{48(-i+\tan(dx+c))^6}-\frac{i}{128(-i+\tan(dx+c))^2}+\frac{1}{36(-i+\tan}$
default	$\frac{-\frac{5i\ln(-i+\tan(dx+c))}{1024}+\frac{3i}{256(-i+\tan(dx+c))^4}+\frac{i}{32(-i+\tan(dx+c))^8}-\frac{i}{48(-i+\tan(dx+c))^6}-\frac{i}{128(-i+\tan(dx+c))^2}+\frac{1}{36(-i+\tan}$
risch	$\frac{5x}{512a^8}+\frac{15ie^{-4i(dx+c)}}{512a^8d}+\frac{35ie^{-6i(dx+c)}}{1024a^8d}+\frac{63ie^{-8i(dx+c)}}{2048a^8d}+\frac{21ie^{-10i(dx+c)}}{1024a^8d}+\frac{5ie^{-12i(dx+c)}}{512a^8d}+\frac{45ie^{-14i(dx}}{14336a^8d}$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^8*(-5/1024*I*ln(-I+tan(d*x+c))+3/256*I/(-I+tan(d*x+c))^4+1/32*I/(-I+
tan(d*x+c))^8-1/48*I/(-I+tan(d*x+c))^6-1/128*I/(-I+tan(d*x+c))^2+1/36/(-I+
tan(d*x+c))^9-3/112/(-I+tan(d*x+c))^7+1/64/(-I+tan(d*x+c))^5-7/768/(-I+tan
(d*x+c))^3+9/1024/(-I+tan(d*x+c))+5/1024*I*ln(tan(d*x+c)+I)+1/1024/(tan(d*
x+c)+I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.47

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(5040\,dxe^{(18i\,dx+18i\,c)}-252i\,e^{(20i\,dx+20i\,c)}+11340i\,e^{(16i\,dx+16i\,c)}+15120i\,e^{(14i\,dx+14i\,c)}+17640i\,e^{(12i\,dx+12i\,c}}{}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/516096*(5040*d*x*e^(18*I*d*x + 18*I*c) - 252*I*e^(20*I*d*x + 20*I*c) + 1
1340*I*e^(16*I*d*x + 16*I*c) + 15120*I*e^(14*I*d*x + 14*I*c) + 17640*I*e^(
12*I*d*x + 12*I*c) + 15876*I*e^(10*I*d*x + 10*I*c) + 10584*I*e^(8*I*d*x +
8*I*c) + 5040*I*e^(6*I*d*x + 6*I*c) + 1620*I*e^(4*I*d*x + 4*I*c) + 315*I*e
^(2*I*d*x + 2*I*c) + 28*I)*e^(-18*I*d*x - 18*I*c)/(a^8*d)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.42

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \begin{cases} \frac{(-24956871191993266341966344435584ia^{72}d^9e^{92ic}e^{2idx}+112305920363969698538848549601280ia^{72}d^9e^{88ic}e^{-2idx}+1497412271519595980\cdots}{\cdots} \\ x\left(\frac{(e^{20ic}+10e^{18ic}+45e^{16ic}+120e^{14ic}+210e^{12ic}+252e^{10ic}+210e^{8ic}+120e^{6ic}+45e^{4ic}+10e^{2ic}+1)e^{-18ic}}{1024a^8}-\frac{5}{512a^8}\right) \end{cases} + \frac{5x}{512a^8}$$

input
```
integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-24956871191993266341966344355584*I*a**72*d**9*exp(92*I*c)*exp(
2*I*d*x) + 112305920363969698538848549601280*I*a**72*d**9*exp(88*I*c)*exp(
-2*I*d*x) + 149741227151959598051798066135040*I*a**72*d**9*exp(86*I*c)*exp
(-4*I*d*x) + 174698098343952864393764410490880*I*a**72*d**9*exp(84*I*c)*ex
p(-6*I*d*x) + 1572282885095557577954387969441792*I*a**72*d**9*exp(82*I*c)*e
xp(-8*I*d*x) + 104818859006371718636258646294528*I*a**72*d**9*exp(80*I*c)*
exp(-10*I*d*x) + 49913742383986532683932688711680*I*a**72*d**9*exp(78*I*c)
*exp(-12*I*d*x) + 16043702909138528362692649943040*I*a**72*d**9*exp(76*I*c
)*exp(-14*I*d*x) + 3119608898999158292745793044480*I*a**72*d**9*exp(74*I*c
)*exp(-16*I*d*x) + 277298568799925181577403826176*I*a**72*d**9*exp(72*I*c)
*exp(-18*I*d*x))*exp(-90*I*c)/(5111167220120220946834707324076032*a**80*d*
*10), Ne(a**80*d**10*exp(90*I*c), 0)), (x*((exp(20*I*c) + 10*exp(18*I*c) +
 45*exp(16*I*c) + 120*exp(14*I*c) + 210*exp(12*I*c) + 252*exp(10*I*c) + 21
0*exp(8*I*c) + 120*exp(6*I*c) + 45*exp(4*I*c) + 10*exp(2*I*c) + 1)*exp(-18
*I*c)/(1024*a**8) - 5/(512*a**8)), True)) + 5*x/(512*a**8)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.55

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{5i\,\log\left(\tan\left(dx+c\right)+i\right)}{1024\,a^8d} - \frac{5i\,\log\left(\tan\left(dx+c\right)-i\right)}{1024\,a^8d} + \frac{315\,\tan\left(dx+c\right)^9 - 2520i\,\tan\left(dx+c\right)^8 - 8610\,\tan\left(dx+c\right)^7 + 15960i\,\tan\left(dx+c\right)^6 + 16128\,\tan\left(dx\right.}{32256\,a^8d(\tan\left(dx+c\right.}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
5/1024*I*log(tan(d*x + c) + I)/(a^8*d) - 5/1024*I*log(tan(d*x + c) - I)/(a
^8*d) + 1/32256*(315*tan(d*x + c)^9 - 2520*I*tan(d*x + c)^8 - 8610*tan(d*x
 + c)^7 + 15960*I*tan(d*x + c)^6 + 16128*tan(d*x + c)^5 - 5544*I*tan(d*x +
 c)^4 + 7074*tan(d*x + c)^3 - 11736*I*tan(d*x + c)^2 - 9019*tan(d*x + c) +
 5120*I)/(a^8*d*(tan(d*x + c) + I)*(tan(d*x + c) - I)^9)
```

Mupad [B] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{5\,x}{512\,a^8} + \frac{\frac{163\tan(c+d\,x)^2}{448\,a^8} - \frac{10}{63\,a^8} - \frac{\tan(c+d\,x)\,9019\mathrm{i}}{32256\,a^8} + \frac{\tan(c+d\,x)^3\,393\mathrm{i}}{1792\,a^8} + \frac{11\tan(c+d\,x)^4}{64\,a^8} + \frac{\tan(c+d\,x)^5\,1\mathrm{i}}{2\,a^8} - }{d\,\left(\tan\left(c+d\,x\right)^{10}\,1\mathrm{i} + 8\tan\left(c+d\,x\right)^9 - \tan\left(c+d\,x\right)^8\,27\mathrm{i} - 48\tan\left(c+d\,x\right)^7 + \tan\left(c+d\,x\right)^6\,42\mathrm{i} + \mathrm{ta}\right.}$$

input
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(5*x)/(512*a^8) + ((163*tan(c + d*x)^2)/(448*a^8) - 10/(63*a^8) - (tan(c +
 d*x)*9019i)/(32256*a^8) + (tan(c + d*x)^3*393i)/(1792*a^8) + (11*tan(c +
d*x)^4)/(64*a^8) + (tan(c + d*x)^5*1i)/(2*a^8) - (95*tan(c + d*x)^6)/(192*
a^8) - (tan(c + d*x)^7*205i)/(768*a^8) + (5*tan(c + d*x)^8)/(64*a^8) + (ta
n(c + d*x)^9*5i)/(512*a^8))/(d*(48*tan(c + d*x)^3 - tan(c + d*x)^2*27i - 8
*tan(c + d*x) + tan(c + d*x)^4*42i + tan(c + d*x)^6*42i - 48*tan(c + d*x)^
7 - tan(c + d*x)^8*27i + 8*tan(c + d*x)^9 + tan(c + d*x)^10*1i + 1i))
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\int \frac{\cos(dx+c)^2}{\tan(dx+c)^8-8\tan(dx+c)^7 i-28\tan(dx+c)^6+56\tan(dx+c)^5 i+70\tan(dx+c)^4-56\tan(dx+c)^3 i-28\tan(dx+c)^2+8\tan(dx+c) i+1}dx}{a^8}$$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x)
```

output
```
int(cos(c + d*x)**2/(tan(c + d*x)**8 - 8*tan(c + d*x)**7*i - 28*tan(c + d*
x)**6 + 56*tan(c + d*x)**5*i + 70*tan(c + d*x)**4 - 56*tan(c + d*x)**3*i -
 28*tan(c + d*x)**2 + 8*tan(c + d*x)*i + 1),x)/a**8
```

3.175 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 333

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= \frac{33x}{2048a^8}+\frac{ia^2}{80d(a+ia\tan(c+dx))^{10}} \\ &\quad+\frac{ia}{48d(a+ia\tan(c+dx))^9}+\frac{3i}{128d(a+ia\tan(c+dx))^8} \\ &\quad+\frac{5i}{224ad(a+ia\tan(c+dx))^7} \\ &\quad+\frac{5i}{256a^2d(a+ia\tan(c+dx))^6} \\ &\quad+\frac{21i}{1280a^3d(a+ia\tan(c+dx))^5} \\ &\quad+\frac{3i}{256a^5d(a+ia\tan(c+dx))^3} \\ &\quad+\frac{7i}{512d\left(a^2+ia^2\tan(c+dx)\right)^4} \\ &\quad-\frac{i}{4096d\left(a^4-ia^4\tan(c+dx)\right)^2} \\ &\quad+\frac{45i}{4096d\left(a^4+ia^4\tan(c+dx)\right)^2} \\ &\quad-\frac{11i}{4096d\left(a^8-ia^8\tan(c+dx)\right)} \\ &\quad+\frac{55i}{4096d\left(a^8+ia^8\tan(c+dx)\right)}\end{aligned}$$

output
```
33/2048*x/a^8+1/80*I*a^2/d/(a+I*a*tan(d*x+c))^10+1/48*I*a/d/(a+I*a*tan(d*x
+c))^9+3/128*I/d/(a+I*a*tan(d*x+c))^8+5/224*I/a/d/(a+I*a*tan(d*x+c))^7+5/2
56*I/a^2/d/(a+I*a*tan(d*x+c))^6+21/1280*I/a^3/d/(a+I*a*tan(d*x+c))^5+3/256
*I/a^5/d/(a+I*a*tan(d*x+c))^3+7/512*I/d/(a^2+I*a^2*tan(d*x+c))^4-1/4096*I/
d/(a^4-I*a^4*tan(d*x+c))^2+45/4096*I/d/(a^4+I*a^4*tan(d*x+c))^2-11/4096*I/
d/(a^8-I*a^8*tan(d*x+c))+55/4096*I/d/(a^8+I*a^8*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.62

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\sec^{12}(c+dx)(48510i+88704i\cos(2(c+dx))+69300i\cos(4(c+dx))+52800i\cos(6(c+dx))+21538i}{}$$

input
```
Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(Sec[c + d*x]^12*(48510*I + (88704*I)*Cos[2*(c + d*x)] + (69300*I)*Cos[4*(
c + d*x)] + (52800*I)*Cos[6*(c + d*x)] + (21538*I)*Cos[8*(c + d*x)] - (224
0*I)*Cos[10*(c + d*x)] - (84*I)*Cos[12*(c + d*x)] - 22176*Sin[2*(c + d*x)]
 - 34650*Sin[4*(c + d*x)] - 39600*Sin[6*(c + d*x)] + 27720*ArcTan[Tan[c +
d*x]]*(Cos[8*(c + d*x)] + I*Sin[8*(c + d*x)]) - 18073*Sin[8*(c + d*x)] + 2
800*Sin[10*(c + d*x)] + 126*Sin[12*(c + d*x)]))/(1720320*a^8*d*(-I + Tan[c
 + d*x])^10*(I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sec(c+dx)^4(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{11}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \; 54$$

$$-\frac{ia^5\int \Big(\frac{11}{4096a^{12}(a-ia\tan(c+dx))^2}+\frac{55}{4096a^{12}(i\tan(c+dx)a+a)^2}+\frac{1}{2048a^{11}(a-ia\tan(c+dx))^3}+\frac{45}{2048a^{11}(i\tan(c+dx)a+a)^3}+\frac{}{256a^{10}}}{}$$

$$\Big\downarrow \; 2009$$

$$-\frac{ia^5\Big(\frac{33i\arctan(\tan(c+dx))}{2048a^{13}}+\frac{11}{4096a^{12}(a-ia\tan(c+dx))}-\frac{55}{4096a^{12}(a+ia\tan(c+dx))}+\frac{1}{4096a^{11}(a-ia\tan(c+dx))^2}-\frac{45}{4096a^{11}(a+ia}}{}$$

input
```
Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*a^5*((((33*I)/2048)*ArcTan[Tan[c + d*x]])/a^13 + 1/(4096*a^11*(a - I
*a*Tan[c + d*x])^2) + 11/(4096*a^12*(a - I*a*Tan[c + d*x])) - 1/(80*a^3*(a
 + I*a*Tan[c + d*x])^10) - 1/(48*a^4*(a + I*a*Tan[c + d*x])^9) - 3/(128*a^
5*(a + I*a*Tan[c + d*x])^8) - 5/(224*a^6*(a + I*a*Tan[c + d*x])^7) - 5/(25
6*a^7*(a + I*a*Tan[c + d*x])^6) - 21/(1280*a^8*(a + I*a*Tan[c + d*x])^5) -
 7/(512*a^9*(a + I*a*Tan[c + d*x])^4) - 3/(256*a^10*(a + I*a*Tan[c + d*x])
^3) - 45/(4096*a^11*(a + I*a*Tan[c + d*x])^2) - 55/(4096*a^12*(a + I*a*Tan
[c + d*x]))))/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{-\frac{33 i \ln(-i+\tan(dx+c))}{4096}+\frac{7i}{512(-i+\tan(dx+c))^4}+\frac{3i}{128(-i+\tan(dx+c))^8}-\frac{i}{80(-i+\tan(dx+c))^{10}}-\frac{5i}{256(-i+\tan(dx+c))^6}-\frac{}{4096(-i}}{}$
default	$\frac{-\frac{33 i \ln(-i+\tan(dx+c))}{4096}+\frac{7i}{512(-i+\tan(dx+c))^4}+\frac{3i}{128(-i+\tan(dx+c))^8}-\frac{i}{80(-i+\tan(dx+c))^{10}}-\frac{5i}{256(-i+\tan(dx+c))^6}-\frac{}{4096(-i}}{}$
risch	$\frac{33x}{2048a^8}+\frac{33i e^{-6i(dx+c)}}{1024a^8 d}+\frac{231i e^{-8i(dx+c)}}{8192a^8 d}+\frac{99i e^{-10i(dx+c)}}{5120a^8 d}+\frac{165i e^{-12i(dx+c)}}{16384a^8 d}+\frac{55i e^{-14i(dx+c)}}{14336a^8 d}+\frac{33ie^{-1}}{327}$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^8*(-33/4096*I*ln(-I+tan(d*x+c))+7/512*I/(-I+tan(d*x+c))^4+3/128*I/(-
I+tan(d*x+c))^8-1/80*I/(-I+tan(d*x+c))^10-5/256*I/(-I+tan(d*x+c))^6-45/409
6*I/(-I+tan(d*x+c))^2+1/48/(-I+tan(d*x+c))^9-5/224/(-I+tan(d*x+c))^7+21/12
80/(-I+tan(d*x+c))^5-3/256/(-I+tan(d*x+c))^3+55/4096/(-I+tan(d*x+c))+1/409
6*I/(tan(d*x+c)+I)^2+33/4096*I*ln(tan(d*x+c)+I)+11/4096/(tan(d*x+c)+I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.46

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(55440\,dxe^{(20i\,dx+20i\,c)} - 210i\,e^{(24i\,dx+24i\,c)} - 5040i\,e^{(22i\,dx+22i\,c)} + 92400i\,e^{(18i\,dx+18i\,c)} + 103950i\,e^{(16i\,dx+16i\,c)}\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/3440640*(55440*d*x*e^(20*I*d*x + 20*I*c) - 210*I*e^(24*I*d*x + 24*I*c) -
 5040*I*e^(22*I*d*x + 22*I*c) + 92400*I*e^(18*I*d*x + 18*I*c) + 103950*I*e
^(16*I*d*x + 16*I*c) + 110880*I*e^(14*I*d*x + 14*I*c) + 97020*I*e^(12*I*d*
x + 12*I*c) + 66528*I*e^(10*I*d*x + 10*I*c) + 34650*I*e^(8*I*d*x + 8*I*c)
+ 13200*I*e^(6*I*d*x + 6*I*c) + 3465*I*e^(4*I*d*x + 4*I*c) + 560*I*e^(2*I*
d*x + 2*I*c) + 42*I)*e^(-20*I*d*x - 20*I*c)/(a^8*d)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.39

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \begin{cases} \frac{\left(-11433487528543532372369386809707411904921600ia^{88}d^{11}e^{114ic}e^{4idx} - 27440370068504477693686528343297788571811840 0ia^{88}d^{11}e^{112\cdots}\right.}{} \\ x\left(\frac{\left(e^{24ic}+12e^{22ic}+66e^{20ic}+220e^{18ic}+495e^{16ic}+792e^{14ic}+924e^{12ic}+792e^{10ic}+495e^{8ic}+220e^{6ic}+66e^{4ic}+12e^{2ic}+1\right)e^{-20ic}}{4096a^8} - \frac{33}{2048a^8}\right) \end{cases} + \frac{33x}{2048a^8}$$

input
```
integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-11433487528543532372369386809707411904921600*I*a**88*d**11*ex
p(114*I*c)*exp(4*I*d*x) - 274403700685044776936865283432977885718118400*I*
a**88*d**11*exp(112*I*c)*exp(2*I*d*x) + 503073451255915424384253019627126l
238165504000*I*a**88*d**11*exp(108*I*c)*exp(-2*I*d*x) + 565957632662904852
43228464708051688929361920 00*I*a**88*d**11*exp(106*I*c)*exp(-4*I*d*x) + 60
3688141507098509261103623552551348579860480 0*I*a**88*d**11*exp(104*I*c)*ex
p(-6*I*d*x) + 528227123818711195603465670608482430007377920 0*I*a**88*d**11
*exp(102*I*c)*exp(-8*I*d*x) + 3622128849042591055566621741315308091479162 8
80*I*a**88*d**11*exp(100*I*c)*exp(-10*I*d*x) + 188652544220968284144094882
3601722964312064000*I*a**88*d**11*exp(98*I*c)*exp(-12*I*d*x) + 71867635893
7022034834647170895894462595072000*I*a**88*d**11*exp(96*I*c)*exp(-14*I*d*x
) + 188652544220968284144094882360172296431206400*I*a**88*d**11*exp(94*I*c
)*exp(-16*I*d*x) + 30489300076116086326318364825886431746457600*I*a**88*d*
*11*exp(92*I*c)*exp(-18*I*d*x) + 228669750570870647447387736194148238098 43
20*I*a**88*d**11*exp(90*I*c)*exp(-20*I*d*x))*exp(-110*I*c)/(18732625966765
72343889000334902462366502354944 00*a**96*d**12), Ne(a**96*d**12*exp(110*I*
c), 0)), (x*((exp(24*I*c) + 12*exp(22*I*c) + 66*exp(20*I*c) + 220*exp(18*I
*c) + 495*exp(16*I*c) + 792*exp(14*I*c) + 924*exp(12*I*c) + 792*exp(10*I*c
) + 495*exp(8*I*c) + 220*exp(6*I*c) + 66*exp(4*I*c) + 12*exp(2*I*c) + 1)*e
xp(-20*I*c)/(4096*a**8) - 33/(2048*a**8)), True)) + 33*x/(2048*a**8)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.52

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{33i\log(\tan(dx+c)+i)}{4096\,a^8d} - \frac{33i\log(\tan(dx+c)-i)}{4096\,a^8d}$$
$$+\frac{3465\tan(dx+c)^{11} - 27720i\tan(dx+c)^{10} - 91245\tan(dx+c)^9 + 147840i\tan(dx+c)^8 + 82698\,\mathrm{t}}{}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
33/4096*I*log(tan(d*x + c) + I)/(a^8*d) - 33/4096*I*log(tan(d*x + c) - I)/
(a^8*d) + 1/215040*(3465*tan(d*x + c)^11 - 27720*I*tan(d*x + c)^10 - 91245
*tan(d*x + c)^9 + 147840*I*tan(d*x + c)^8 + 82698*tan(d*x + c)^7 + 114576*
I*tan(d*x + c)^6 + 255222*tan(d*x + c)^5 - 190080*I*tan(d*x + c)^4 - 21395
*tan(d*x + c)^3 - 72776*I*tan(d*x + c)^2 - 66953*tan(d*x + c) + 34816*I)/(
a^8*d*(tan(d*x + c) + I)^2*(tan(d*x + c) - I)^10)
```

Mupad [B] (verification not implemented)

Time = 2.86 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.88

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{33\,x}{2048\,a^8}$$
$$-\frac{\frac{\tan(c+d\,x)\,66953\mathrm{i}}{215040\,a^8} + \frac{17}{105\,a^8} - \frac{9097\tan(c+d\,x)^2}{26880\,a^8} + \frac{\tan(c+d\,x)^3\,4279\mathrm{i}}{43008\,a^8} - \frac{99\tan(c+d\,x)^4}{112\,a^8} -}{d\,\left(\tan(c+d\,x)^{12}\,1\mathrm{i} + 8\tan(c+d\,x)^{11} - \tan(c+d\,x)^{10}\,26\mathrm{i} - 40\tan(c+d\,x)^9 + \tan(c+d\,x)^8\,15\mathrm{i} -\right.}$$

input
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(33*x)/(2048*a^8) - ((tan(c + d*x)*66953i)/(215040*a^8) + 17/(105*a^8) - (
9097*tan(c + d*x)^2)/(26880*a^8) + (tan(c + d*x)^3*4279i)/(43008*a^8) - (9
9*tan(c + d*x)^4)/(112*a^8) - (tan(c + d*x)^5*42537i)/(35840*a^8) + (341*t
an(c + d*x)^6)/(640*a^8) - (tan(c + d*x)^7*1969i)/(5120*a^8) + (11*tan(c +
 d*x)^8)/(16*a^8) + (tan(c + d*x)^9*869i)/(2048*a^8) - (33*tan(c + d*x)^10
)/(256*a^8) - (tan(c + d*x)^11*33i)/(2048*a^8))/(d*(40*tan(c + d*x)^3 - ta
n(c + d*x)^2*26i - 8*tan(c + d*x) + tan(c + d*x)^4*15i + 48*tan(c + d*x)^5
 + tan(c + d*x)^6*84i - 48*tan(c + d*x)^7 + tan(c + d*x)^8*15i - 40*tan(c
+ d*x)^9 - tan(c + d*x)^10*26i + 8*tan(c + d*x)^11 + tan(c + d*x)^12*1i +
1i))
```

Reduce [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\int \frac{\cos(dx+c)^4}{\tan(dx+c)^8-8\tan(dx+c)^7 i-28\tan(dx+c)^6+56\tan(dx+c)^5 i+70\tan(dx+c)^4-56\tan(dx+c)^3 i-28\tan(dx+c)^2+8\tan(dx+c) i+1}dx}{a^8}$$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x)
```

output
```
int(cos(c + d*x)**4/(tan(c + d*x)**8 - 8*tan(c + d*x)**7*i - 28*tan(c + d*
x)**6 + 56*tan(c + d*x)**5*i + 70*tan(c + d*x)**4 - 56*tan(c + d*x)**3*i -
 28*tan(c + d*x)**2 + 8*tan(c + d*x)*i + 1),x)/a**8
```

3.176 $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 205

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{1155\text{arctanh}(\sin(c+dx))}{8a^8d} + \frac{1155\sec(c+dx)\tan(c+dx)}{8a^8d} + \frac{385\sec^3(c+dx)\tan(c+dx)}{4a^8d} + \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{22i\sec^9(c+dx)}{3a^3d(a+ia\tan(c+dx))^5} - \frac{66i\sec^7(c+dx)}{a^2d\,(a^2+ia^2\tan(c+dx))^3} - \frac{154i\sec^5(c+dx)}{d\,(a^8+ia^8\tan(c+dx))}$$

output
```
1155/8*arctanh(sin(d*x+c))/a^8/d+1155/8*sec(d*x+c)*tan(d*x+c)/a^8/d+385/4*
sec(d*x+c)^3*tan(d*x+c)/a^8/d+2/3*I*sec(d*x+c)^11/a/d/(a+I*a*tan(d*x+c))^7
-22/3*I*sec(d*x+c)^9/a^3/d/(a+I*a*tan(d*x+c))^5-66*I*sec(d*x+c)^7/a^2/d/(a
^2+I*a^2*tan(d*x+c))^3-154*I*sec(d*x+c)^5/d/(a^8+I*a^8*tan(d*x+c))
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1704 vs. $2(205) = 410$.

Time = 7.19 (sec) , antiderivative size = 1704, normalized size of antiderivative = 8.31

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(-1155*Cos[8*c]*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^
8*(Cos[d*x] + I*Sin[d*x])^8)/(8*d*(a + I*a*Tan[c + d*x])^8) + (1155*Cos[8*
c]*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*(Cos[d*x] +
 I*Sin[d*x])^8)/(8*d*(a + I*a*Tan[c + d*x])^8) + (Cos[3*d*x]*Sec[c + d*x]^
8*(((32*I)/3)*Cos[5*c] - (32*Sin[5*c])/3)*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a
 + I*a*Tan[c + d*x])^8) + (Cos[d*x]*Sec[c + d*x]^8*((-160*I)*Cos[7*c] + 16
0*Sin[7*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) - (((1
155*I)/8)*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[
8*c]*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (((1155*I)/
8)*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[8*c]*(C
os[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c]*Sec[c + d*
x]^8*(((-236*I)/3)*Cos[8*c] + (236*Sin[8*c])/3)*(Cos[d*x] + I*Sin[d*x])^8)
/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(-160*Cos[7*c] - (160*I)*S
in[7*c])*(Cos[d*x] + I*Sin[d*x])^8*Sin[d*x])/(d*(a + I*a*Tan[c + d*x])^8)
 + (Sec[c + d*x]^8*((32*Cos[5*c])/3 + ((32*I)/3)*Sin[5*c])*(Cos[d*x] + I*Si
n[d*x])^8*Sin[3*d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(Cos[
8*c]/16 + (I/16)*Sin[8*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(Cos[c/2 + (d*x)/
2] - Sin[c/2 + (d*x)/2])^4*(a + I*a*Tan[c + d*x])^8) - ((1/96 + I/96)*Sec[
c + d*x]^8*((-407*I)*Cos[(15*c)/2] + 343*Cos[(17*c)/2] + 407*Sin[(15*c)/2]
 + (343*I)*Sin[(17*c)/2])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(Cos[c/2] - Sin...
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 3981, 3042, 3981, 3042, 3981, 3042, 3981, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^{13}}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{ 3981}$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\int \frac{\sec^{11}(c+dx)}{(i\tan(c+dx)a+a)^6}dx}{3a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\int \frac{\sec(c+dx)^{11}}{(i\tan(c+dx)a+a)^6}dx}{3a^2}$$

$$\Big\downarrow \text{ 3981}$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\int \frac{\sec^9(c+dx)}{(i\tan(c+dx)a+a)^4}dx}{a^2}\right)}{3a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\int \frac{\sec(c+dx)^9}{(i\tan(c+dx)a+a)^4}dx}{a^2}\right)}{3a^2}$$

$$\Big\downarrow \text{ 3981}$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\left(\frac{7\int\frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}\right)}{a^2}\right)}{3a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\left(\frac{7\int\frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^2}dx}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}\right)}{a^2}\right)}{3a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\left(\frac{7\left(\frac{5\int\sec^5(c+dx)dx}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}\right)}{a^2}\right)}{3a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\left(\frac{7\left(\frac{5\int\csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}\right)}{a^2}\right)}{3a^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\left(\frac{7\left(\frac{5\left(\frac{3}{4}\int\sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}\right)}{a^2}\right)}{3a^2}}{}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\left(\frac{7\left(\frac{5\left(\frac{3}{4}\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}\right)}{a^2}\right)}{3a^2}$$

$$\Big\downarrow \textbf{4255}$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\left(\frac{7\left(\frac{5\left(\frac{3}{4}\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}-\frac{2i\sec^7(c+}{ad(a+ia\tan(}\right.}{a^2}\right.}{3a^2}$$

$\Big\downarrow$ 3042

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\left(\frac{7\left(\frac{5\left(\frac{3}{4}\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}-\frac{2i\sec}{ad(a+ia}}{a^2}}{3a^2}$$

$\Big\downarrow$ 4257

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\left(\frac{7\left(\frac{5\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}-\frac{2i}{ad(a+}}{a^2}}{3a^2}$$

input
```
Int[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(((2*I)/3)*Sec[c + d*x]^11)/(a*d*(a + I*a*Tan[c + d*x])^7) - (11*(((2*I)*S
ec[c + d*x]^9)/(a*d*(a + I*a*Tan[c + d*x])^5) - (9*(((-2*I)*Sec[c + d*x]^7
)/(a*d*(a + I*a*Tan[c + d*x])^3) + (7*((((-2*I)/3)*Sec[c + d*x]^5)/(d*(a^2
 + I*a^2*Tan[c + d*x])) + (5*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(Ar
cTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/(3*a^2
)))/a^2))/a^2))/(3*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{160 i \mathrm{e}^{-i(dx+c)}}{a^{8} d}+\frac{32 i \mathrm{e}^{-3 i(dx+c)}}{3 a^{8} d}-\frac{i\left(1545 \,\mathrm{e}^{7 i(dx+c)}+5153 \,\mathrm{e}^{5 i(dx+c)}+5855 \,\mathrm{e}^{3 i(dx+c)}+2295 \,\mathrm{e}^{i(dx+c)}\right)}{12 d \,a^{8} \left(\mathrm{e}^{2 i(dx+c)}+1\right)^{4}}-1155 \ln$
derivativedivides	$\frac{2\left(\frac{1}{4}-\frac{4 i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{121}{16}-2 i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{123}{16}+38 i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{1155 \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2\left(\frac{121}{16}-2 i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}$
default	$\frac{2\left(\frac{1}{4}-\frac{4 i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{121}{16}-2 i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{123}{16}+38 i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{1155 \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2\left(\frac{121}{16}-2 i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}$

input
```
int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
-160*I/a^8/d*exp(-I*(d*x+c))+32/3*I/a^8/d*exp(-3*I*(d*x+c))-1/12*I/d/a^8/(
exp(2*I*(d*x+c))+1)^4*(1545*exp(7*I*(d*x+c))+5153*exp(5*I*(d*x+c))+5855*ex
p(3*I*(d*x+c))+2295*exp(I*(d*x+c)))-1155/8/a^8/d*ln(exp(I*(d*x+c))-I)+1155
/8/a^8/d*ln(exp(I*(d*x+c))+I)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.30

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{3465 \left(e^{(11i\,dx+11i\,c)}+4\,e^{(9i\,dx+9i\,c)}+6\,e^{(7i\,dx+7i\,c)}+4\,e^{(5i\,dx+5i\,c)}+e^{(3i\,dx+3i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3465\,(e^{(}}{2}$$

input
```
integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/24*(3465*(e^(11*I*d*x + 11*I*c) + 4*e^(9*I*d*x + 9*I*c) + 6*e^(7*I*d*x +
 7*I*c) + 4*e^(5*I*d*x + 5*I*c) + e^(3*I*d*x + 3*I*c))*log(e^(I*d*x + I*c)
 + I) - 3465*(e^(11*I*d*x + 11*I*c) + 4*e^(9*I*d*x + 9*I*c) + 6*e^(7*I*d*x
 + 7*I*c) + 4*e^(5*I*d*x + 5*I*c) + e^(3*I*d*x + 3*I*c))*log(e^(I*d*x + I*
c) - I) - 6930*I*e^(10*I*d*x + 10*I*c) - 25410*I*e^(8*I*d*x + 8*I*c) - 337
26*I*e^(6*I*d*x + 6*I*c) - 18414*I*e^(4*I*d*x + 4*I*c) - 2816*I*e^(2*I*d*x
 + 2*I*c) + 256*I)/(a^8*d*e^(11*I*d*x + 11*I*c) + 4*a^8*d*e^(9*I*d*x + 9*I
*c) + 6*a^8*d*e^(7*I*d*x + 7*I*c) + 4*a^8*d*e^(5*I*d*x + 5*I*c) + a^8*d*e^
(3*I*d*x + 3*I*c))
```

Sympy [F]

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\int \frac{\sec^{13}(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

input
```
integrate(sec(d*x+c)**13/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Integral(sec(c + d*x)**13/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(
c + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x
)**3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 786 vs. $2(179) = 358$.

Time = 0.23 (sec) , antiderivative size = 786, normalized size of antiderivative = 3.83

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-(6930*(cos(11*d*x + 11*c) + 4*cos(9*d*x + 9*c) + 6*cos(7*d*x + 7*c) + 4*c
os(5*d*x + 5*c) + cos(3*d*x + 3*c) + I*sin(11*d*x + 11*c) + 4*I*sin(9*d*x
+ 9*c) + 6*I*sin(7*d*x + 7*c) + 4*I*sin(5*d*x + 5*c) + I*sin(3*d*x + 3*c))
*arctan2(cos(d*x + c), sin(d*x + c) + 1) + 6930*(cos(11*d*x + 11*c) + 4*co
s(9*d*x + 9*c) + 6*cos(7*d*x + 7*c) + 4*cos(5*d*x + 5*c) + cos(3*d*x + 3*c
) + I*sin(11*d*x + 11*c) + 4*I*sin(9*d*x + 9*c) + 6*I*sin(7*d*x + 7*c) + 4
*I*sin(5*d*x + 5*c) + I*sin(3*d*x + 3*c))*arctan2(cos(d*x + c), -sin(d*x +
 c) + 1) + 3465*(I*cos(11*d*x + 11*c) + 4*I*cos(9*d*x + 9*c) + 6*I*cos(7*d
*x + 7*c) + 4*I*cos(5*d*x + 5*c) + I*cos(3*d*x + 3*c) - sin(11*d*x + 11*c)
 - 4*sin(9*d*x + 9*c) - 6*sin(7*d*x + 7*c) - 4*sin(5*d*x + 5*c) - sin(3*d*
x + 3*c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3465
*(-I*cos(11*d*x + 11*c) - 4*I*cos(9*d*x + 9*c) - 6*I*cos(7*d*x + 7*c) - 4*
I*cos(5*d*x + 5*c) - I*cos(3*d*x + 3*c) + sin(11*d*x + 11*c) + 4*sin(9*d*x
 + 9*c) + 6*sin(7*d*x + 7*c) + 4*sin(5*d*x + 5*c) + sin(3*d*x + 3*c))*log(
cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 13860*cos(10*d*x +
 10*c) + 50820*cos(8*d*x + 8*c) + 67452*cos(6*d*x + 6*c) + 36828*cos(4*d*x
 + 4*c) + 5632*cos(2*d*x + 2*c) + 13860*I*sin(10*d*x + 10*c) + 50820*I*sin
(8*d*x + 8*c) + 67452*I*sin(6*d*x + 6*c) + 36828*I*sin(4*d*x + 4*c) + 5632
*I*sin(2*d*x + 2*c) - 512)/((-48*I*a^8*cos(11*d*x + 11*c) - 192*I*a^8*cos(
9*d*x + 9*c) - 288*I*a^8*cos(7*d*x + 7*c) - 192*I*a^8*cos(5*d*x + 5*c) ...
```

Giac [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.95

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\frac{3465\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^8} - \frac{3465\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^8} - \frac{1024\left(6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-15i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-7\right)}{a^8\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^3} - \frac{2\left(369\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/24*(3465*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 - 3465*log(tan(1/2*d*x + 1/2*
c) - 1)/a^8 - 1024*(6*tan(1/2*d*x + 1/2*c)^2 - 15*I*tan(1/2*d*x + 1/2*c) -
 7)/(a^8*(tan(1/2*d*x + 1/2*c) - I)^3) - 2*(369*tan(1/2*d*x + 1/2*c)^7 - 1
728*I*tan(1/2*d*x + 1/2*c)^6 - 393*tan(1/2*d*x + 1/2*c)^5 + 5568*I*tan(1/2
*d*x + 1/2*c)^4 - 393*tan(1/2*d*x + 1/2*c)^3 - 5696*I*tan(1/2*d*x + 1/2*c)
^2 + 369*tan(1/2*d*x + 1/2*c) + 1856*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*a^
8))/d
```

Mupad [B] (verification not implemented)

Time = 4.78 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.68

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\frac{33847\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{6a^8} - \frac{12041\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3a^8} - \frac{3585\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{a^8} + \frac{3505\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{4a^8} + \frac{4293\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^8}}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}1i - 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9 7i + 13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7 18i - \right.}$$
$$+ \frac{1155\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4a^8 d}$$

input
```
int(1/(cos(c + d*x)^13*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
((tan(c/2 + (d*x)/2)^2*27565i)/(12*a^8) - (12041*tan(c/2 + (d*x)/2)^3)/(3*
a^8) - (tan(c/2 + (d*x)/2)^4*4575i)/a^8 + (33847*tan(c/2 + (d*x)/2)^5)/(6*
a^8) + (tan(c/2 + (d*x)/2)^6*25993i)/(6*a^8) - (3585*tan(c/2 + (d*x)/2)^7)
/a^8 - (tan(c/2 + (d*x)/2)^8*5639i)/(3*a^8) + (3505*tan(c/2 + (d*x)/2)^9)/
(4*a^8) + (tan(c/2 + (d*x)/2)^10*1147i)/(4*a^8) - 1360i/(3*a^8) + (4293*ta
n(c/2 + (d*x)/2))/(4*a^8))/(d*(tan(c/2 + (d*x)/2)*3i - 7*tan(c/2 + (d*x)/2
)^2 - tan(c/2 + (d*x)/2)^3*13i + 18*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)
/2)^5*22i - 22*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^7*18i + 13*tan(c/
2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^9*7i - 3*tan(c/2 + (d*x)/2)^10 - tan(c
/2 + (d*x)/2)^11*1i + 1)) + (1155*atanh(tan(c/2 + (d*x)/2)))/(4*a^8*d)
```

Reduce [F]

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x)
```

output

```
( - 456672017154048*cos(c + d*x)**2*sin(c + d*x)**3 - 4982530650240*cos(c
+ d*x)**2*sin(c + d*x)**2*i + 514842517856256*cos(c + d*x)**2*sin(c + d*x)
 + 6279604284096*cos(c + d*x)**2*i - 8168122220544*cos(c + d*x)*int(cos(c
+ d*x)/(128*cos(c + d*x)*sin(c + d*x)**12*i - 512*cos(c + d*x)*sin(c + d*x
)**10*i + 800*cos(c + d*x)*sin(c + d*x)**8*i - 608*cos(c + d*x)*sin(c + d*
x)**6*i + 225*cos(c + d*x)*sin(c + d*x)**4*i - 34*cos(c + d*x)*sin(c + d*x
)**2*i + cos(c + d*x)*i - 128*sin(c + d*x)**13 + 576*sin(c + d*x)**11 - 10
40*sin(c + d*x)**9 + 952*sin(c + d*x)**7 - 456*sin(c + d*x)**5 + 104*sin(c
 + d*x)**3 - 8*sin(c + d*x)),x)*sin(c + d*x)**4*d*i + 16336244441088*cos(c
 + d*x)*int(cos(c + d*x)/(128*cos(c + d*x)*sin(c + d*x)**12*i - 512*cos(c
+ d*x)*sin(c + d*x)**10*i + 800*cos(c + d*x)*sin(c + d*x)**8*i - 608*cos(c
 + d*x)*sin(c + d*x)**6*i + 225*cos(c + d*x)*sin(c + d*x)**4*i - 34*cos(c
+ d*x)*sin(c + d*x)**2*i + cos(c + d*x)*i - 128*sin(c + d*x)**13 + 576*sin
(c + d*x)**11 - 1040*sin(c + d*x)**9 + 952*sin(c + d*x)**7 - 456*sin(c + d
*x)**5 + 104*sin(c + d*x)**3 - 8*sin(c + d*x)),x)*sin(c + d*x)**2*d*i - 81
68122220544*cos(c + d*x)*int(cos(c + d*x)/(128*cos(c + d*x)*sin(c + d*x)**
12*i - 512*cos(c + d*x)*sin(c + d*x)**10*i + 800*cos(c + d*x)*sin(c + d*x)
**8*i - 608*cos(c + d*x)*sin(c + d*x)**6*i + 225*cos(c + d*x)*sin(c + d*x)
**4*i - 34*cos(c + d*x)*sin(c + d*x)**2*i + cos(c + d*x)*i - 128*sin(c + d
*x)**13 + 576*sin(c + d*x)**11 - 1040*sin(c + d*x)**9 + 952*sin(c + d*x...
```

3.177 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 183

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{63\text{arctanh}(\sin(c+dx))}{2a^8d} - \frac{63\sec(c+dx)\tan(c+dx)}{2a^8d} + \frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7} - \frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{42i\sec^5(c+dx)}{5a^2d\,(a^2+ia^2\tan(c+dx))^3} + \frac{42i\sec^3(c+dx)}{d\,(a^8+ia^8\tan(c+dx))}$$

output
```
-63/2*arctanh(sin(d*x+c))/a^8/d-63/2*sec(d*x+c)*tan(d*x+c)/a^8/d+2/5*I*sec
(d*x+c)^9/a/d/(a+I*a*tan(d*x+c))^7-6/5*I*sec(d*x+c)^7/a^3/d/(a+I*a*tan(d*x
+c))^5+42/5*I*sec(d*x+c)^5/a^2/d/(a^2+I*a^2*tan(d*x+c))^3+42*I*sec(d*x+c)^
3/d/(a^8+I*a^8*tan(d*x+c))
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1244 vs. $2(183) = 366$.

Time = 6.85 (sec) , antiderivative size = 1244, normalized size of antiderivative = 6.80

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(63*Cos[8*c]*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*(
Cos[d*x] + I*Sin[d*x])^8)/(2*d*(a + I*a*Tan[c + d*x])^8) - (63*Cos[8*c]*Lo
g[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*(Cos[d*x] + I*Si
n[d*x])^8)/(2*d*(a + I*a*Tan[c + d*x])^8) + (Cos[5*d*x]*Sec[c + d*x]^8*(((
8*I)/5)*Cos[3*c] - (8*Sin[3*c])/5)*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*
Tan[c + d*x])^8) + (Cos[3*d*x]*Sec[c + d*x]^8*((-8*I)*Cos[5*c] + 8*Sin[5*c
])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (Cos[d*x]*Sec
[c + d*x]^8*((48*I)*Cos[7*c] - 48*Sin[7*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*
(a + I*a*Tan[c + d*x])^8) + (Sec[c]*Sec[c + d*x]^8*((8*I)*Cos[8*c] - 8*Sin
[8*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (((63*I)/
2)*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[8*c]*(C
os[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) - (((63*I)/2)*Log[Co
s[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[8*c]*(Cos[d*x] +
 I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(48*Cos[7*c
] + (48*I)*Sin[7*c])*(Cos[d*x] + I*Sin[d*x])^8*Sin[d*x])/(d*(a + I*a*Tan[c
 + d*x])^8) + (Sec[c + d*x]^8*(-8*Cos[5*c] - (8*I)*Sin[5*c])*(Cos[d*x] + I
*Sin[d*x])^8*Sin[3*d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*((
8*Cos[3*c])/5 + ((8*I)/5)*Sin[3*c])*(Cos[d*x] + I*Sin[d*x])^8*Sin[5*d*x])/
(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(Cos[8*c]/4 + (I/4)*Sin[8*c
])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/...
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3981, 3042, 3981, 3042, 3981, 3042, 3981, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{11}}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\int \frac{\sec^9(c+dx)}{(i\tan(c+dx)a+a)^6}dx}{5a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\int \frac{\sec(c+dx)^9}{(i\tan(c+dx)a+a)^6}dx}{5a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\left(\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^5}-\frac{7\int \frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^4}dx}{3a^2}\right)}{5a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\left(\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^5}-\frac{7\int \frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^4}dx}{3a^2}\right)}{5a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\left(\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^5}-\frac{7\left(\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\int\frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{a^2}\right)}{3a^2}\right)}{5a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\left(\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^5}-\frac{7\left(\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\int\frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^2}dx}{a^2}\right)}{3a^2}\right)}{5a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\left(\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^5}-\frac{7\left(\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\int\sec^3(c+dx)dx}{a^2}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}\right)}{3a^2}\right)}{5a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\left(\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^5}-\frac{7\left(\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}\right)}{3a^2}\right)}{5a^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\left(\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^5}-\frac{7\left(\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{2i\sec^3(c+dx)}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}\right)}{3a^2}\right)}{5a^2}}{}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\left(\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^5}-\frac{7\left(\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{2i\sec^3(c+dx)}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}\right)}{3a^2}\right)}{5a^2}}{}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\left(\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^5}-\frac{7\left(\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{2i\sec^3(c+dx)}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}\right)}{3a^2}\right)}{5a^2}}{}$$

input
```
Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(((2*I)/5)*Sec[c + d*x]^9)/(a*d*(a + I*a*Tan[c + d*x])^7) - (9*((((2*I)/3)
*Sec[c + d*x]^7)/(a*d*(a + I*a*Tan[c + d*x])^5) - (7*(((2*I)*Sec[c + d*x]^
5)/(a*d*(a + I*a*Tan[c + d*x])^3) - (5*(((-2*I)*Sec[c + d*x]^3)/(d*(a^2 +
I*a^2*Tan[c + d*x])) + (3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan
[c + d*x])/(2*d)))/a^2))/a^2))/(3*a^2)))/(5*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.78

method	result
risch	$\frac{48 i \,{\mathrm e}^{-i(dx+c)}}{a^{8} d}-\frac{8 i \,{\mathrm e}^{-3 i(dx+c)}}{a^{8} d}+\frac{8 i \,{\mathrm e}^{-5 i(dx+c)}}{5 a^{8} d}+\frac{i\left(15 \,{\mathrm e}^{3 i(dx+c)}+17 \,{\mathrm e}^{i(dx+c)}\right)}{d \,a^{8} \left({\mathrm e}^{2 i(dx+c)}+1\right)^{2}}+\frac{63 \ln\left({\mathrm e}^{i(dx+c)}-i\right)}{2 a^{8} d}-\frac{63 \ln\left({\mathrm e}^{i(d}\right.}{2 a^{8}}$
derivativedivides	$\frac{\frac{2\left(\frac{1}{4}-4 i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{63 \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{32 i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{128 i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{256}{5\left(-i+\tan\left(\frac{dx}{2}+\right.\right.}}{a^{8} d}$
default	$\frac{\frac{2\left(\frac{1}{4}-4 i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{63 \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{32 i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{128 i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{256}{5\left(-i+\tan\left(\frac{dx}{2}+\right.\right.}}{a^{8} d}$

input
```
int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
48*I/a^8/d*exp(-I*(d*x+c))-8*I/a^8/d*exp(-3*I*(d*x+c))+8/5*I/a^8/d*exp(-5*
I*(d*x+c))+I/d/a^8/(exp(2*I*(d*x+c))+1)^2*(15*exp(3*I*(d*x+c))+17*exp(I*(d
*x+c)))+63/2/a^8/d*ln(exp(I*(d*x+c))-I)-63/2/a^8/d*ln(exp(I*(d*x+c))+I)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.99

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{315\left(e^{(9i\,dx+9i\,c)}+2\,e^{(7i\,dx+7i\,c)}+e^{(5i\,dx+5i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-315\left(e^{(9i\,dx+9i\,c)}+2\,e^{(7i\,dx+7i\,c)}+e^{(5i\,dx}\right.}{10\left(a^8de^{(9i\,dx+9i\,c)}+2\,a^8de^{(}\right.}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-1/10*(315*(e^(9*I*d*x + 9*I*c) + 2*e^(7*I*d*x + 7*I*c) + e^(5*I*d*x + 5*I
*c))*log(e^(I*d*x + I*c) + I) - 315*(e^(9*I*d*x + 9*I*c) + 2*e^(7*I*d*x +
7*I*c) + e^(5*I*d*x + 5*I*c))*log(e^(I*d*x + I*c) - I) - 630*I*e^(8*I*d*x
+ 8*I*c) - 1050*I*e^(6*I*d*x + 6*I*c) - 336*I*e^(4*I*d*x + 4*I*c) + 48*I*e
^(2*I*d*x + 2*I*c) - 16*I)/(a^8*d*e^(9*I*d*x + 9*I*c) + 2*a^8*d*e^(7*I*d*x
 + 7*I*c) + a^8*d*e^(5*I*d*x + 5*I*c))
```

Sympy [F]

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\int \frac{\sec^{11}(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

input
```
integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Integral(sec(c + d*x)**11/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(
c + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x
)**3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(157) = 314$.

Time = 0.25 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.90

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
(630*(cos(9*d*x + 9*c) + 2*cos(7*d*x + 7*c) + cos(5*d*x + 5*c) + I*sin(9*d
*x + 9*c) + 2*I*sin(7*d*x + 7*c) + I*sin(5*d*x + 5*c))*arctan2(cos(d*x + c
), sin(d*x + c) + 1) + 630*(cos(9*d*x + 9*c) + 2*cos(7*d*x + 7*c) + cos(5*
d*x + 5*c) + I*sin(9*d*x + 9*c) + 2*I*sin(7*d*x + 7*c) + I*sin(5*d*x + 5*c
))*arctan2(cos(d*x + c), -sin(d*x + c) + 1) + 315*(I*cos(9*d*x + 9*c) + 2*
I*cos(7*d*x + 7*c) + I*cos(5*d*x + 5*c) - sin(9*d*x + 9*c) - 2*sin(7*d*x +
 7*c) - sin(5*d*x + 5*c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x
+ c) + 1) + 315*(-I*cos(9*d*x + 9*c) - 2*I*cos(7*d*x + 7*c) - I*cos(5*d*x
+ 5*c) + sin(9*d*x + 9*c) + 2*sin(7*d*x + 7*c) + sin(5*d*x + 5*c))*log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 1260*cos(8*d*x + 8*c)
 + 2100*cos(6*d*x + 6*c) + 672*cos(4*d*x + 4*c) - 96*cos(2*d*x + 2*c) + 12
60*I*sin(8*d*x + 8*c) + 2100*I*sin(6*d*x + 6*c) + 672*I*sin(4*d*x + 4*c) -
 96*I*sin(2*d*x + 2*c) + 32)/((-20*I*a^8*cos(9*d*x + 9*c) - 40*I*a^8*cos(7
*d*x + 7*c) - 20*I*a^8*cos(5*d*x + 5*c) + 20*a^8*sin(9*d*x + 9*c) + 40*a^8
*sin(7*d*x + 7*c) + 20*a^8*sin(5*d*x + 5*c))*d)
```

Giac [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.90

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$

$$-\frac{\frac{315\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^8} - \frac{315\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^8} - \frac{10\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-16i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+16i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^8} - \frac{64(\cdots}{\cdots}}{10\,d}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-1/10*(315*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 - 315*log(tan(1/2*d*x + 1/2*c
) - 1)/a^8 - 10*(tan(1/2*d*x + 1/2*c)^3 - 16*I*tan(1/2*d*x + 1/2*c)^2 + ta
n(1/2*d*x + 1/2*c) + 16*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^8) - 64*(10*t
an(1/2*d*x + 1/2*c)^4 - 45*I*tan(1/2*d*x + 1/2*c)^3 - 85*tan(1/2*d*x + 1/2
*c)^2 + 55*I*tan(1/2*d*x + 1/2*c) + 13)/(a^8*(tan(1/2*d*x + 1/2*c) - I)^5)
)/d
```

Mupad [B] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.55

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{63\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^8\,d}$$
$$+\frac{\frac{1223\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{a^8}-\frac{1109\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a^8}+\frac{309\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{a^8}-\frac{431\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^8}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 440}{5\,a^8}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9 1\mathrm{i}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 12\mathrm{i}-20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 26\mathrm{i}+2\right.}$$

input
```
int(1/(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
((1223*tan(c/2 + (d*x)/2)^3)/a^8 - (tan(c/2 + (d*x)/2)^2*4407i)/(5*a^8) +
(tan(c/2 + (d*x)/2)^4*7351i)/(5*a^8) - (1109*tan(c/2 + (d*x)/2)^5)/a^8 - (
tan(c/2 + (d*x)/2)^6*761i)/a^8 + (309*tan(c/2 + (d*x)/2)^7)/a^8 + (tan(c/2
 + (d*x)/2)^8*65i)/a^8 + 496i/(5*a^8) - (431*tan(c/2 + (d*x)/2))/a^8)/(d*(
tan(c/2 + (d*x)/2)*5i - 12*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*20i
 + 26*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*26i - 20*tan(c/2 + (d*x)
/2)^6 - tan(c/2 + (d*x)/2)^7*12i + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x
)/2)^9*1i + 1)) - (63*atanh(tan(c/2 + (d*x)/2)))/(a^8*d)
```

Reduce [F]

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x)
```

output

```
(10155807232*cos(c + d*x)**2*sin(c + d*x) - 1271664240*cos(c + d*x)**2*i -
 6837540864*cos(c + d*x)*int(cos(c + d*x)/(128*cos(c + d*x)*sin(c + d*x)**
10*i - 384*cos(c + d*x)*sin(c + d*x)**8*i + 416*cos(c + d*x)*sin(c + d*x)*
*6*i - 192*cos(c + d*x)*sin(c + d*x)**4*i + 33*cos(c + d*x)*sin(c + d*x)**
2*i - cos(c + d*x)*i - 128*sin(c + d*x)**11 + 448*sin(c + d*x)**9 - 592*si
n(c + d*x)**7 + 360*sin(c + d*x)**5 - 96*sin(c + d*x)**3 + 8*sin(c + d*x))
,x)*sin(c + d*x)**2*d*i + 6837540864*cos(c + d*x)*int(cos(c + d*x)/(128*co
s(c + d*x)*sin(c + d*x)**10*i - 384*cos(c + d*x)*sin(c + d*x)**8*i + 416*c
os(c + d*x)*sin(c + d*x)**6*i - 192*cos(c + d*x)*sin(c + d*x)**4*i + 33*co
s(c + d*x)*sin(c + d*x)**2*i - cos(c + d*x)*i - 128*sin(c + d*x)**11 + 448
*sin(c + d*x)**9 - 592*sin(c + d*x)**7 + 360*sin(c + d*x)**5 - 96*sin(c +
d*x)**3 + 8*sin(c + d*x)),x)*d*i - 196331520*cos(c + d*x)*int(cos(c + d*x)
/(128*cos(c + d*x)*sin(c + d*x)**9*i - 320*cos(c + d*x)*sin(c + d*x)**7*i
+ 272*cos(c + d*x)*sin(c + d*x)**5*i - 88*cos(c + d*x)*sin(c + d*x)**3*i +
 8*cos(c + d*x)*sin(c + d*x)*i - 128*sin(c + d*x)**10 + 384*sin(c + d*x)**
8 - 416*sin(c + d*x)**6 + 192*sin(c + d*x)**4 - 33*sin(c + d*x)**2 + 1),x)
*sin(c + d*x)**2*d + 196331520*cos(c + d*x)*int(cos(c + d*x)/(128*cos(c +
d*x)*sin(c + d*x)**9*i - 320*cos(c + d*x)*sin(c + d*x)**7*i + 272*cos(c +
d*x)*sin(c + d*x)**5*i - 88*cos(c + d*x)*sin(c + d*x)**3*i + 8*cos(c + d*x
)*sin(c + d*x)*i - 128*sin(c + d*x)**10 + 384*sin(c + d*x)**8 - 416*sin...
```

3.178 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^8d}+\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7} - \frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{2i\sec^3(c+dx)}{3a^2d\,(a^2+ia^2\tan(c+dx))^3}-\frac{2i\sec(c+dx)}{d\,(a^8+ia^8\tan(c+dx))}$$

output
```
arctanh(sin(d*x+c))/a^8/d+2/7*I*sec(d*x+c)^7/a/d/(a+I*a*tan(d*x+c))^7-2/5*
I*sec(d*x+c)^5/a^3/d/(a+I*a*tan(d*x+c))^5+2/3*I*sec(d*x+c)^3/a^2/d/(a^2+I*
a^2*tan(d*x+c))^3-2*I*sec(d*x+c)/d/(a^8+I*a^8*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.95

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\sec^8(c+dx)\left(70i\cos\left(\frac{1}{2}(c+dx)\right)-42i\cos\left(\frac{3}{2}(c+dx)\right)-210i\cos\left(\frac{5}{2}(c+dx)\right)+30i\cos\left(\frac{7}{2}(c+dx)\right)-1\right.}{}$$

input
```
Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(Sec[c + d*x]^8*((70*I)*Cos[(c + d*x)/2] - (42*I)*Cos[(3*(c + d*x))/2] - (
210*I)*Cos[(5*(c + d*x))/2] + (30*I)*Cos[(7*(c + d*x))/2] - 105*Cos[(7*(c
+ d*x))/2]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*Cos[(7*(c + d*x)
)/2]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 70*Sin[(c + d*x)/2] - 42*S
in[(3*(c + d*x))/2] + 210*Sin[(5*(c + d*x))/2] + 30*Sin[(7*(c + d*x))/2] -
 (105*I)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2] + (
105*I)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2])*(Cos
[(9*(c + d*x))/2] + I*Sin[(9*(c + d*x))/2]))/(105*a^8*d*(-I + Tan[c + d*x]
)^8)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3981, 3042, 3981, 3042, 3981, 3042, 3981, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^9}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{\int \frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^6}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{\int \frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^6}dx}{a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^5}-\frac{\int\frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^4}dx}{a^2}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^5}-\frac{\int\frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^4}dx}{a^2}}{a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^5}-\frac{\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{\int\frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{a^2}}{a^2}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^5}-\frac{\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{\int\frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^2}dx}{a^2}}{a^2}}{a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^5}-\frac{\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{-\frac{\int\sec(c+dx)dx}{a^2}+\frac{2i\sec(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}}{a^2}}{a^2}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^5}-\frac{\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{-\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}+\frac{2i\sec(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}}{a^2}}{a^2}}{a^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^5}-\frac{\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{-\frac{\text{arctanh}(\sin(c+dx))}{a^2d}+\frac{2i\sec(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}}{a^2}}{a^2}}{a^2}$$

input
```
Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(((2*I)/7)*Sec[c + d*x]^7)/(a*d*(a + I*a*Tan[c + d*x])^7) - ((((2*I)/5)*Se
c[c + d*x]^5)/(a*d*(a + I*a*Tan[c + d*x])^5) - ((((2*I)/3)*Sec[c + d*x]^3)
/(a*d*(a + I*a*Tan[c + d*x])^3) - (-(ArcTanh[Sin[c + d*x]]/(a^2*d)) + ((2*
I)*Sec[c + d*x])/(d*(a^2 + I*a^2*Tan[c + d*x])))/a^2)/a^2)/a^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{2 i \mathrm{e}^{-i(dx+c)}}{a^{8} d}+\frac{2 i \mathrm{e}^{-3 i(dx+c)}}{3 a^{8} d}-\frac{2 i \mathrm{e}^{-5 i(dx+c)}}{5 a^{8} d}+\frac{2 i \mathrm{e}^{-7 i(dx+c)}}{7 a^{8} d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^{8} d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^{8} d}$
derivativedivides	$\frac{\frac{128 i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}+\frac{16 i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{128 i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{256}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}+\frac{896}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{}{3(-i+\mathrm{tar}}}{a^{8} d}$
default	$\frac{\frac{128 i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}+\frac{16 i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{128 i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{256}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}+\frac{896}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{}{3(-i+\mathrm{tar}}}{a^{8} d}$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
-2*I/a^8/d*exp(-I*(d*x+c))+2/3*I/a^8/d*exp(-3*I*(d*x+c))-2/5*I/a^8/d*exp(-
5*I*(d*x+c))+2/7*I/a^8/d*exp(-7*I*(d*x+c))-1/a^8/d*ln(exp(I*(d*x+c))-I)+1/
a^8/d*ln(exp(I*(d*x+c))+I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.63

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(105\,e^{(7i\,dx+7i\,c)}\log\left(e^{(i\,dx+i\,c)}+i\right)-105\,e^{(7i\,dx+7i\,c)}\log\left(e^{(i\,dx+i\,c)}-i\right)-210i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}-\right.}{105\,a^8d}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/105*(105*e^(7*I*d*x + 7*I*c)*log(e^(I*d*x + I*c) + I) - 105*e^(7*I*d*x +
 7*I*c)*log(e^(I*d*x + I*c) - I) - 210*I*e^(6*I*d*x + 6*I*c) + 70*I*e^(4*I
*d*x + 4*I*c) - 42*I*e^(2*I*d*x + 2*I*c) + 30*I)*e^(-7*I*d*x - 7*I*c)/(a^8
*d)
```

Sympy [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\int \frac{\sec^9(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

input
```
integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Integral(sec(c + d*x)**9/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(c
 + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x)
**3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.19

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{-210i\,\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)-210i\,\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right)+60i\,\cos}{}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/210*(-210*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 210*I*arctan2(cos(
d*x + c), -sin(d*x + c) + 1) + 60*I*cos(7*d*x + 7*c) - 84*I*cos(5*d*x + 5*
c) + 140*I*cos(3*d*x + 3*c) - 420*I*cos(d*x + c) + 105*log(cos(d*x + c)^2
+ sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - 105*log(cos(d*x + c)^2 + sin(d*x
+ c)^2 - 2*sin(d*x + c) + 1) + 60*sin(7*d*x + 7*c) - 84*sin(5*d*x + 5*c) +
 140*sin(3*d*x + 3*c) - 420*sin(d*x + c))/(a^8*d)
```

Giac [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.79

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\frac{105\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^8}-\frac{105\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^8}-\frac{16\left(-105i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-175\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+490i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+294}{a^8\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^7}}{105\,d}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/105*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 - 105*log(tan(1/2*d*x + 1/2*c
) - 1)/a^8 - 16*(-105*I*tan(1/2*d*x + 1/2*c)^5 - 175*tan(1/2*d*x + 1/2*c)^
4 + 490*I*tan(1/2*d*x + 1/2*c)^3 + 294*tan(1/2*d*x + 1/2*c)^2 - 133*I*tan(
1/2*d*x + 1/2*c) - 19)/(a^8*(tan(1/2*d*x + 1/2*c) - I)^7))/d
```

Mupad [B] (verification not implemented)

Time = 4.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.33

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^8\,d} + \frac{\frac{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a^8} - \frac{224\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3\,a^8} + \frac{304\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{15\,a^8} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 224\mathrm{i}}{5\,a^8} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3\,a^8}}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 1\mathrm{i} - 7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 21\mathrm{i} + 35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 35\mathrm{i} -\right.}$$

input
```
int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
(2*atanh(tan(c/2 + (d*x)/2)))/(a^8*d) + ((tan(c/2 + (d*x)/2)^2*224i)/(5*a^
8) - (224*tan(c/2 + (d*x)/2)^3)/(3*a^8) - (tan(c/2 + (d*x)/2)^4*80i)/(3*a^
8) + (16*tan(c/2 + (d*x)/2)^5)/a^8 - 304i/(105*a^8) + (304*tan(c/2 + (d*x)
/2))/(15*a^8))/(d*(tan(c/2 + (d*x)/2)*7i - 21*tan(c/2 + (d*x)/2)^2 - tan(c
/2 + (d*x)/2)^3*35i + 35*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*21i -
 7*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^7*1i + 1))
```

Reduce [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x)
```

output

```
(8128*int(cos(c + d*x)/(128*cos(c + d*x)*sin(c + d*x)**8*i - 256*cos(c + d
*x)*sin(c + d*x)**6*i + 160*cos(c + d*x)*sin(c + d*x)**4*i - 32*cos(c + d*
x)*sin(c + d*x)**2*i + cos(c + d*x)*i - 128*sin(c + d*x)**9 + 320*sin(c +
d*x)**7 - 272*sin(c + d*x)**5 + 88*sin(c + d*x)**3 - 8*sin(c + d*x)),x)*d*
i - 982016*int(sin(c + d*x)**9/(128*cos(c + d*x)*sin(c + d*x)**8*i - 256*c
os(c + d*x)*sin(c + d*x)**6*i + 160*cos(c + d*x)*sin(c + d*x)**4*i - 32*co
s(c + d*x)*sin(c + d*x)**2*i + cos(c + d*x)*i - 128*sin(c + d*x)**9 + 320*
sin(c + d*x)**7 - 272*sin(c + d*x)**5 + 88*sin(c + d*x)**3 - 8*sin(c + d*x
)),x)*d + 1040256*int(sin(c + d*x)**8/(128*cos(c + d*x)*sin(c + d*x)**8*i
- 256*cos(c + d*x)*sin(c + d*x)**6*i + 160*cos(c + d*x)*sin(c + d*x)**4*i
- 32*cos(c + d*x)*sin(c + d*x)**2*i + cos(c + d*x)*i - 128*sin(c + d*x)**9
 + 320*sin(c + d*x)**7 - 272*sin(c + d*x)**5 + 88*sin(c + d*x)**3 - 8*sin(
c + d*x)),x)*d*i - 7168*int(sin(c + d*x)**8/(128*cos(c + d*x)*sin(c + d*x)
**7*i - 192*cos(c + d*x)*sin(c + d*x)**5*i + 80*cos(c + d*x)*sin(c + d*x)*
*3*i - 8*cos(c + d*x)*sin(c + d*x)*i - 128*sin(c + d*x)**8 + 256*sin(c + d
*x)**6 - 160*sin(c + d*x)**4 + 32*sin(c + d*x)**2 - 1),x)*d + 2488320*int(
sin(c + d*x)**7/(128*cos(c + d*x)*sin(c + d*x)**8*i - 256*cos(c + d*x)*sin
(c + d*x)**6*i + 160*cos(c + d*x)*sin(c + d*x)**4*i - 32*cos(c + d*x)*sin(
c + d*x)**2*i + cos(c + d*x)*i - 128*sin(c + d*x)**9 + 320*sin(c + d*x)**7
 - 272*sin(c + d*x)**5 + 88*sin(c + d*x)**3 - 8*sin(c + d*x)),x)*d - 20...
```

3.179 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx=\frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8}+\frac{i\sec^7(c+dx)}{63ad(a+ia\tan(c+dx))^7}$$

output
```
1/9*I*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^8+1/63*I*sec(d*x+c)^7/a/d/(a+I*a*t
an(d*x+c))^7
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx=-\frac{\sec^7(c+dx)(-8i+\tan(c+dx))}{63a^8d(-i+\tan(c+dx))^8}$$

input
```
Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
-1/63*(Sec[c + d*x]^7*(-8*I + Tan[c + d*x]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^7}{(a+ia\tan(c+dx))^8}dx$$
$$\Big\downarrow \text{3983}$$
$$\frac{\int \frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^7}dx}{9a}+\frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^7}dx}{9a}+\frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8}$$
$$\Big\downarrow \text{3969}$$
$$\frac{i\sec^7(c+dx)}{63ad(a+ia\tan(c+dx))^7}+\frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8}$$

input
```
Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/9)*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^8) + ((I/63)*Sec[c + d*x]
^7)/(a*d*(a + I*a*Tan[c + d*x])^7)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56

method	result
risch	$\frac{i e^{-7 i (d x+c)}}{14 a^{8} d}+\frac{i e^{-9 i (d x+c)}}{18 a^{8} d}$
derivativedivides	$\frac{\frac{2}{-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)}-\frac{1856}{7\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{7}}+\frac{272}{\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{256}{9\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{9}}-\frac{128 i}{\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{8}}+\frac{992 i}{3\left(-i+\tan\left(\frac{d}{2}\right.\right.}}{a^{8} d}$
default	$\frac{\frac{2}{-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)}-\frac{1856}{7\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{7}}+\frac{272}{\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{256}{9\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{9}}-\frac{128 i}{\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{8}}+\frac{992 i}{3\left(-i+\tan\left(\frac{d}{2}\right.\right.}}{a^{8} d}$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/14*I/a^8/d*exp(-7*I*(d*x+c))+1/18*I/a^8/d*exp(-9*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.44

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(9i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{(-9i\,dx-9i\,c)}}{126\,a^8d}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/126*(9*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(-9*I*d*x - 9*I*c)/(a^8*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(54) = 108$.

Time = 10.09 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.57

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \begin{cases} -\frac{\tan(c+dx)\sec^7(c+dx)}{63a^8d\tan^8(c+dx)-504ia^8d\tan^7(c+dx)-1764a^8d\tan^6(c+dx)+3528ia^8d\tan^5(c+dx)+4410a^8d\tan^4(c+dx)-3528ia^8d\tan^3(c+dx)-17} \\ \frac{x\sec^7(c)}{(ia\tan(c)+a)^8} \end{cases}$$

input
```
integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise((-tan(c + d*x)*sec(c + d*x)**7/(63*a**8*d*tan(c + d*x)**8 - 504*
I*a**8*d*tan(c + d*x)**7 - 1764*a**8*d*tan(c + d*x)**6 + 3528*I*a**8*d*tan
(c + d*x)**5 + 4410*a**8*d*tan(c + d*x)**4 - 3528*I*a**8*d*tan(c + d*x)**3
 - 1764*a**8*d*tan(c + d*x)**2 + 504*I*a**8*d*tan(c + d*x) + 63*a**8*d) +
8*I*sec(c + d*x)**7/(63*a**8*d*tan(c + d*x)**8 - 504*I*a**8*d*tan(c + d*x)
**7 - 1764*a**8*d*tan(c + d*x)**6 + 3528*I*a**8*d*tan(c + d*x)**5 + 4410*a
**8*d*tan(c + d*x)**4 - 3528*I*a**8*d*tan(c + d*x)**3 - 1764*a**8*d*tan(c
+ d*x)**2 + 504*I*a**8*d*tan(c + d*x) + 63*a**8*d), Ne(d, 0)), (x*sec(c)**
7/(I*a*tan(c) + a)**8, True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{7i\,\cos(9\,dx+9\,c)+9i\,\cos(7\,dx+7\,c)+7\,\sin(9\,dx+9\,c)+9\,\sin(7\,dx+7\,c)}{126\,a^8d}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/126*(7*I*cos(9*d*x + 9*c) + 9*I*cos(7*d*x + 7*c) + 7*sin(9*d*x + 9*c) +
9*sin(7*d*x + 7*c))/(a^8*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(56) = 112.

Time = 0.79 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.84

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2\left(63\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-63i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-483\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+315i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+693\right.}{63\,a^8d\left(\tan\left(\frac{1}{2}\,dx+\right.\right.}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
2/63*(63*tan(1/2*d*x + 1/2*c)^8 - 63*I*tan(1/2*d*x + 1/2*c)^7 - 483*tan(1/
2*d*x + 1/2*c)^6 + 315*I*tan(1/2*d*x + 1/2*c)^5 + 693*tan(1/2*d*x + 1/2*c)
^4 - 189*I*tan(1/2*d*x + 1/2*c)^3 - 225*tan(1/2*d*x + 1/2*c)^2 + 9*I*tan(1
/2*d*x + 1/2*c) + 8)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^9)
```

Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.54

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2\left(\frac{e^{-c\,7i-d\,x\,7i}\,9i}{4}+\frac{e^{-c\,9i-d\,x\,9i}\,7i}{4}\right)}{63\,a^8\,d}$$

input
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
(2*((exp(- c*7i - d*x*7i)*9i)/4 + (exp(- c*9i - d*x*9i)*7i)/4))/(63*a^8*d)
```

Reduce [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x)
```

output
```
( - 4*int(cos(c + d*x)/(128*cos(c + d*x)*sin(c + d*x)**7*i - 192*cos(c + d
*x)*sin(c + d*x)**5*i + 80*cos(c + d*x)*sin(c + d*x)**3*i - 8*cos(c + d*x)
*sin(c + d*x)*i - 128*sin(c + d*x)**8 + 256*sin(c + d*x)**6 - 160*sin(c +
d*x)**4 + 32*sin(c + d*x)**2 - 1),x)*d - 512*int(sin(c + d*x)**8/(128*cos(
c + d*x)*sin(c + d*x)**7*i - 192*cos(c + d*x)*sin(c + d*x)**5*i + 80*cos(c
 + d*x)*sin(c + d*x)**3*i - 8*cos(c + d*x)*sin(c + d*x)*i - 128*sin(c + d*
x)**8 + 256*sin(c + d*x)**6 - 160*sin(c + d*x)**4 + 32*sin(c + d*x)**2 - 1
),x)*d - 1024*int(sin(c + d*x)**6/(128*cos(c + d*x)*sin(c + d*x)**7 - 192*
cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(c + d*x)**3 - 8*cos(c +
 d*x)*sin(c + d*x) + 128*sin(c + d*x)**8*i - 256*sin(c + d*x)**6*i + 160*s
in(c + d*x)**4*i - 32*sin(c + d*x)**2*i + i),x)*d*i + 640*int(sin(c + d*x)
**4/(128*cos(c + d*x)*sin(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 +
 80*cos(c + d*x)*sin(c + d*x)**3 - 8*cos(c + d*x)*sin(c + d*x) + 128*sin(c
 + d*x)**8*i - 256*sin(c + d*x)**6*i + 160*sin(c + d*x)**4*i - 32*sin(c +
d*x)**2*i + i),x)*d*i - 128*int(sin(c + d*x)**2/(128*cos(c + d*x)*sin(c +
d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(c + d*x)*
*3 - 8*cos(c + d*x)*sin(c + d*x) + 128*sin(c + d*x)**8*i - 256*sin(c + d*x
)**6*i + 160*sin(c + d*x)**4*i - 32*sin(c + d*x)**2*i + i),x)*d*i + 512*in
t((cos(c + d*x)*sin(c + d*x)**7)/(128*cos(c + d*x)*sin(c + d*x)**7 - 192*c
os(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(c + d*x)**3 - 8*cos(c...
```

3.180 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 138

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8} + \frac{i\sec^5(c+dx)}{33ad(a+ia\tan(c+dx))^7} + \frac{2i\sec^5(c+dx)}{231a^2d(a+ia\tan(c+dx))^6} + \frac{2i\sec^5(c+dx)}{1155a^3d(a+ia\tan(c+dx))^5}$$

output
```
1/11*I*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^8+1/33*I*sec(d*x+c)^5/a/d/(a+I*a*
tan(d*x+c))^7+2/231*I*sec(d*x+c)^5/a^2/d/(a+I*a*tan(d*x+c))^6+2/1155*I*sec
(d*x+c)^5/a^3/d/(a+I*a*tan(d*x+c))^5
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.53

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\sec^8(c+dx)(440\cos(c+dx)+168\cos(3(c+dx))+55i\sin(c+dx)+63i\sin(3(c+dx)))}{4620a^8d(-i+\tan(c+dx))^8}$$

input
```
Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/4620)*Sec[c + d*x]^8*(440*Cos[c + d*x] + 168*Cos[3*(c + d*x)] + (55*I)
*Sin[c + d*x] + (63*I)*Sin[3*(c + d*x)]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\int \frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^7}dx}{11a}+\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^7}dx}{11a}+\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\left(\frac{2\int \frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^6}dx}{9a}+\frac{i\sec^5(c+dx)}{9d(a+ia\tan(c+dx))^7}\right)}{11a}+\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2\int\frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^6}dx}{9a}+\frac{i\sec^5(c+dx)}{9d(a+ia\tan(c+dx))^7}\right)}{11a}+\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\left(\frac{2\left(\frac{\int\frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^5}dx}{7a}+\frac{i\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^6}\right)}{9a}+\frac{i\sec^5(c+dx)}{9d(a+ia\tan(c+dx))^7}\right)}{11a}+\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2\left(\frac{\int\frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^5}dx}{7a}+\frac{i\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^6}\right)}{9a}+\frac{i\sec^5(c+dx)}{9d(a+ia\tan(c+dx))^7}\right)}{11a}+\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3969}$$

$$\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}+\frac{3\left(\frac{i\sec^5(c+dx)}{9d(a+ia\tan(c+dx))^7}+\frac{2\left(\frac{i\sec^5(c+dx)}{35ad(a+ia\tan(c+dx))^5}+\frac{i\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^6}\right)}{9a}\right)}{11a}$$

input
```
Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/11)*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^8) + (3*(((I/9)*Sec[c +
d*x]^5)/(d*(a + I*a*Tan[c + d*x])^7) + (2*(((I/7)*Sec[c + d*x]^5)/(d*(a +
I*a*Tan[c + d*x])^6) + ((I/35)*Sec[c + d*x]^5)/(a*d*(a + I*a*Tan[c + d*x])
^5)))/(9*a)))/(11*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

method	result
risch	$\frac{i e^{-5i(dx+c)}}{40a^8 d}+\frac{3i e^{-7i(dx+c)}}{56a^8 d}+\frac{i e^{-9i(dx+c)}}{24a^8 d}+\frac{i e^{-11i(dx+c)}}{88a^8 d}$
derivativedivides	$\frac{\frac{584i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{576i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{1864}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{4752}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{}{(-i+\tan}}{a}$
default	$\frac{\frac{584i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{576i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{1864}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{4752}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{}{(-i+\tan}}{a}$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/40*I/a^8/d*exp(-5*I*(d*x+c))+3/56*I/a^8/d*exp(-7*I*(d*x+c))+1/24*I/a^8/d
*exp(-9*I*(d*x+c))+1/88*I/a^8/d*exp(-11*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.38

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(231i\,e^{(6i\,dx+6i\,c)}+495i\,e^{(4i\,dx+4i\,c)}+385i\,e^{(2i\,dx+2i\,c)}+105i\right)e^{(-11i\,dx-11i\,c)}}{9240\,a^8d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/9240*(231*I*e^(6*I*d*x + 6*I*c) + 495*I*e^(4*I*d*x + 4*I*c) + 385*I*e^(2
*I*d*x + 2*I*c) + 105*I)*e^(-11*I*d*x - 11*I*c)/(a^8*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(119) = 238.

Time = 9.84 (sec) , antiderivative size = 620, normalized size of antiderivative = 4.49

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise((2*tan(c + d*x)**3*sec(c + d*x)**5/(1155*a**8*d*tan(c + d*x)**8
- 9240*I*a**8*d*tan(c + d*x)**7 - 32340*a**8*d*tan(c + d*x)**6 + 64680*I*a
**8*d*tan(c + d*x)**5 + 80850*a**8*d*tan(c + d*x)**4 - 64680*I*a**8*d*tan(
c + d*x)**3 - 32340*a**8*d*tan(c + d*x)**2 + 9240*I*a**8*d*tan(c + d*x) +
1155*a**8*d) - 16*I*tan(c + d*x)**2*sec(c + d*x)**5/(1155*a**8*d*tan(c + d
*x)**8 - 9240*I*a**8*d*tan(c + d*x)**7 - 32340*a**8*d*tan(c + d*x)**6 + 64
680*I*a**8*d*tan(c + d*x)**5 + 80850*a**8*d*tan(c + d*x)**4 - 64680*I*a**8
*d*tan(c + d*x)**3 - 32340*a**8*d*tan(c + d*x)**2 + 9240*I*a**8*d*tan(c +
d*x) + 1155*a**8*d) - 61*tan(c + d*x)*sec(c + d*x)**5/(1155*a**8*d*tan(c +
 d*x)**8 - 9240*I*a**8*d*tan(c + d*x)**7 - 32340*a**8*d*tan(c + d*x)**6 +
64680*I*a**8*d*tan(c + d*x)**5 + 80850*a**8*d*tan(c + d*x)**4 - 64680*I*a*
*8*d*tan(c + d*x)**3 - 32340*a**8*d*tan(c + d*x)**2 + 9240*I*a**8*d*tan(c
+ d*x) + 1155*a**8*d) + 152*I*sec(c + d*x)**5/(1155*a**8*d*tan(c + d*x)**8
 - 9240*I*a**8*d*tan(c + d*x)**7 - 32340*a**8*d*tan(c + d*x)**6 + 64680*I*
a**8*d*tan(c + d*x)**5 + 80850*a**8*d*tan(c + d*x)**4 - 64680*I*a**8*d*tan
(c + d*x)**3 - 32340*a**8*d*tan(c + d*x)**2 + 9240*I*a**8*d*tan(c + d*x) +
 1155*a**8*d), Ne(d, 0)), (x*sec(c)**5/(I*a*tan(c) + a)**8, True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.70

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{105i\,\cos(11\,dx+11\,c)+385i\,\cos(9\,dx+9\,c)+495i\,\cos(7\,dx+7\,c)+231i\,\cos(5\,dx+5\,c)+105\,\sin(}{9240\,a^8d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/9240*(105*I*cos(11*d*x + 11*c) + 385*I*cos(9*d*x + 9*c) + 495*I*cos(7*d*
x + 7*c) + 231*I*cos(5*d*x + 5*c) + 105*sin(11*d*x + 11*c) + 385*sin(9*d*x
 + 9*c) + 495*sin(7*d*x + 7*c) + 231*sin(5*d*x + 5*c))/(a^8*d)
```

Giac [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{2\left(1155\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-3465i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-13860\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+23100i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
2/1155*(1155*tan(1/2*d*x + 1/2*c)^10 - 3465*I*tan(1/2*d*x + 1/2*c)^9 - 138
60*tan(1/2*d*x + 1/2*c)^8 + 23100*I*tan(1/2*d*x + 1/2*c)^7 + 37422*tan(1/2
*d*x + 1/2*c)^6 - 32802*I*tan(1/2*d*x + 1/2*c)^5 - 27060*tan(1/2*d*x + 1/2
*c)^4 + 11220*I*tan(1/2*d*x + 1/2*c)^3 + 4895*tan(1/2*d*x + 1/2*c)^2 - 517
*I*tan(1/2*d*x + 1/2*c) - 152)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^11)
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.46

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\frac{\mathrm{e}^{-c\,5\mathrm{i}-d\,x\,5\mathrm{i}}\,1\mathrm{i}}{40}+\frac{\mathrm{e}^{-c\,7\mathrm{i}-d\,x\,7\mathrm{i}}\,3\mathrm{i}}{56}+\frac{\mathrm{e}^{-c\,9\mathrm{i}-d\,x\,9\mathrm{i}}\,1\mathrm{i}}{24}+\frac{\mathrm{e}^{-c\,11\mathrm{i}-d\,x\,11\mathrm{i}}\,1\mathrm{i}}{88}}{a^8\,d}$$

input
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
((exp(- c*5i - d*x*5i)*1i)/40 + (exp(- c*7i - d*x*7i)*3i)/56 + (exp(- c*9i
 - d*x*9i)*1i)/24 + (exp(- c*11i - d*x*11i)*1i)/88)/(a^8*d)
```

Reduce [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x)
```

output
```
( - 4*int(cos(c + d*x)/(128*cos(c + d*x)*sin(c + d*x)**7*i - 192*cos(c + d
*x)*sin(c + d*x)**5*i + 80*cos(c + d*x)*sin(c + d*x)**3*i - 8*cos(c + d*x)
*sin(c + d*x)*i - 128*sin(c + d*x)**8 + 256*sin(c + d*x)**6 - 160*sin(c +
d*x)**4 + 32*sin(c + d*x)**2 - 1),x)*d - 512*int(sin(c + d*x)**8/(128*cos(
c + d*x)*sin(c + d*x)**7*i - 192*cos(c + d*x)*sin(c + d*x)**5*i + 80*cos(c
 + d*x)*sin(c + d*x)**3*i - 8*cos(c + d*x)*sin(c + d*x)*i - 128*sin(c + d*
x)**8 + 256*sin(c + d*x)**6 - 160*sin(c + d*x)**4 + 32*sin(c + d*x)**2 - 1
),x)*d - 1024*int(sin(c + d*x)**6/(128*cos(c + d*x)*sin(c + d*x)**7 - 192*
cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(c + d*x)**3 - 8*cos(c +
 d*x)*sin(c + d*x) + 128*sin(c + d*x)**8*i - 256*sin(c + d*x)**6*i + 160*s
in(c + d*x)**4*i - 32*sin(c + d*x)**2*i + i),x)*d*i + 640*int(sin(c + d*x)
**4/(128*cos(c + d*x)*sin(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 +
 80*cos(c + d*x)*sin(c + d*x)**3 - 8*cos(c + d*x)*sin(c + d*x) + 128*sin(c
 + d*x)**8*i - 256*sin(c + d*x)**6*i + 160*sin(c + d*x)**4*i - 32*sin(c +
d*x)**2*i + i),x)*d*i - 128*int(sin(c + d*x)**2/(128*cos(c + d*x)*sin(c +
d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(c + d*x)*
*3 - 8*cos(c + d*x)*sin(c + d*x) + 128*sin(c + d*x)**8*i - 256*sin(c + d*x
)**6*i + 160*sin(c + d*x)**4*i - 32*sin(c + d*x)**2*i + i),x)*d*i + 512*in
t((cos(c + d*x)*sin(c + d*x)**7)/(128*cos(c + d*x)*sin(c + d*x)**7 - 192*c
os(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(c + d*x)**3 - 8*cos(c...
```

3.181 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 213

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}+\frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7}\\ &\quad+\frac{20i\sec^3(c+dx)}{1287a^2d(a+ia\tan(c+dx))^6}\\ &\quad+\frac{20i\sec^3(c+dx)}{3003a^3d(a+ia\tan(c+dx))^5}\\ &\quad+\frac{8i\sec^3(c+dx)}{3003d\left(a^2+ia^2\tan(c+dx)\right)^4}\\ &\quad+\frac{8i\sec^3(c+dx)}{9009a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}\end{aligned}$$

output
```
1/13*I*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^8+5/143*I*sec(d*x+c)^3/a/d/(a+I*a
*tan(d*x+c))^7+20/1287*I*sec(d*x+c)^3/a^2/d/(a+I*a*tan(d*x+c))^6+20/3003*I
*sec(d*x+c)^3/a^3/d/(a+I*a*tan(d*x+c))^5+8/3003*I*sec(d*x+c)^3/d/(a^2+I*a^
2*tan(d*x+c))^4+8/9009*I*sec(d*x+c)^3/a^2/d/(a^2+I*a^2*tan(d*x+c))^3
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.45

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{i\sec^8(c+dx)(11440\cos(c+dx)+6552\cos(3(c+dx))+1848\cos(5(c+dx))+1430i\sin(c+dx)+245}{144144a^8d(-i+\tan(c+dx))^8}$$

input
```
Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/144144)*Sec[c + d*x]^8*(11440*Cos[c + d*x] + 6552*Cos[3*(c + d*x)] + 1
848*Cos[5*(c + d*x)] + (1430*I)*Sin[c + d*x] + (2457*I)*Sin[3*(c + d*x)] +
 (1155*I)*Sin[5*(c + d*x)]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sec(c+dx)^3}{(a+ia\tan(c+dx))^8}dx$$
$$\Big\downarrow \; 3983$$
$$\frac{5\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^7}dx}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$
$$\Big\downarrow \; 3042$$

$$\frac{5\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^7}dx}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{5\left(\frac{4\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^6}dx}{11a}+\frac{i\sec^3(c+dx)}{11d(a+ia\tan(c+dx))^7}\right)}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{4\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^6}dx}{11a}+\frac{i\sec^3(c+dx)}{11d(a+ia\tan(c+dx))^7}\right)}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{5\left(\frac{4\left(\frac{\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^5}dx}{3a}+\frac{i\sec^3(c+dx)}{9d(a+ia\tan(c+dx))^6}\right)}{11a}+\frac{i\sec^3(c+dx)}{11d(a+ia\tan(c+dx))^7}\right)}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{4\left(\frac{\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^5}dx}{3a}+\frac{i\sec^3(c+dx)}{9d(a+ia\tan(c+dx))^6}\right)}{11a}+\frac{i\sec^3(c+dx)}{11d(a+ia\tan(c+dx))^7}\right)}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{5\left(\frac{4\left(\frac{\frac{2\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^4}dx}{7a}+\frac{i\sec^3(c+dx)}{7d(a+ia\tan(c+dx))^5}}{3a}+\frac{i\sec^3(c+dx)}{9d(a+ia\tan(c+dx))^6}\right)}{11a}+\frac{i\sec^3(c+dx)}{11d(a+ia\tan(c+dx))^7}\right)}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{4\left(\frac{\frac{2\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^4}dx}{7a}+\frac{i\sec^3(c+dx)}{7d(a+ia\tan(c+dx))^5}}{3a}+\frac{i\sec^3(c+dx)}{9d(a+ia\tan(c+dx))^6}\right)}{11a}+\frac{i\sec^3(c+dx)}{11d(a+ia\tan(c+dx))^7}\right)}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \; 3983$$

$$\frac{5\left(\frac{4\left(\frac{\frac{2\left(\frac{\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{5a}+\frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}\right)}{7a}+\frac{i\sec^3(c+dx)}{7d(a+ia\tan(c+dx))^5}}{3a}+\frac{i\sec^3(c+dx)}{9d(a+ia\tan(c+dx))^6}\right)}{11a}+\frac{i\sec^3(c+dx)}{11d(a+ia\tan(c+dx))^7}\right)}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \; 3042$$

$$\frac{5\left(\frac{4\left(\frac{\frac{2\left(\frac{\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^3}dx}{5a}+\frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}\right)}{7a}+\frac{i\sec^3(c+dx)}{7d(a+ia\tan(c+dx))^5}}{3a}+\frac{i\sec^3(c+dx)}{9d(a+ia\tan(c+dx))^6}\right)}{11a}+\frac{i\sec^3(c+dx)}{11d(a+ia\tan(c+dx))^7}\right)}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \textit{3969}$$

$$\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}+\frac{5\left(\frac{i\sec^3(c+dx)}{11d(a+ia\tan(c+dx))^7}+\frac{4\left(\frac{i\sec^3(c+dx)}{9d(a+ia\tan(c+dx))^6}+\frac{\frac{i\sec^3(c+dx)}{7d(a+ia\tan(c+dx))^5}+\frac{2\left(\frac{i\sec^3(c+dx)}{15ad(a+ia\tan(c+dx))^3}+\frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}\right)}{7a}}{3a}\right)}{11a}\right)}{13a}$$

input
```
Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/13)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^8) + (5*(((I/11)*Sec[c +
 d*x]^3)/(d*(a + I*a*Tan[c + d*x])^7) + (4*(((I/9)*Sec[c + d*x]^3)/(d*(a +
 I*a*Tan[c + d*x])^6) + (((I/7)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^
5) + (2*(((I/5)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^4) + ((I/15)*Sec
[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^3)))/(7*a))/(3*a)))/(11*a)))/(13*
a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.52

method	result
risch	$\frac{i e^{-3i(dx+c)}}{96a^8d}+\frac{i e^{-5i(dx+c)}}{32a^8d}+\frac{5i e^{-7i(dx+c)}}{112a^8d}+\frac{5i e^{-9i(dx+c)}}{144a^8d}+\frac{5i e^{-11i(dx+c)}}{352a^8d}+\frac{i e^{-13i(dx+c)}}{416a^8d}$
derivativedivides	$\frac{480}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{864i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{4544}{11\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}}-\frac{200i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{14i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{}{13(-i+}$
default	$\frac{480}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{864i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{4544}{11\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}}-\frac{200i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{14i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{}{13(-i+}$

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/96*I/a^8/d*exp(-3*I*(d*x+c))+1/32*I/a^8/d*exp(-5*I*(d*x+c))+5/112*I/a^8/
d*exp(-7*I*(d*x+c))+5/144*I/a^8/d*exp(-9*I*(d*x+c))+5/352*I/a^8/d*exp(-11*
I*(d*x+c))+1/416*I/a^8/d*exp(-13*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.35

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(3003i\,e^{(10i\,dx+10i\,c)}+9009i\,e^{(8i\,dx+8i\,c)}+12870i\,e^{(6i\,dx+6i\,c)}+10010i\,e^{(4i\,dx+4i\,c)}+4095i\,e^{(2i\,dx+2i\,c)}+693i\right)}{288288\,a^8d}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/288288*(3003*I*e^(10*I*d*x + 10*I*c) + 9009*I*e^(8*I*d*x + 8*I*c) + 1287
0*I*e^(6*I*d*x + 6*I*c) + 10010*I*e^(4*I*d*x + 4*I*c) + 4095*I*e^(2*I*d*x
+ 2*I*c) + 693*I)*e^(-13*I*d*x - 13*I*c)/(a^8*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 928 vs. 2(189) = 378.

Time = 10.42 (sec) , antiderivative size = 928, normalized size of antiderivative = 4.36

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise((-8*tan(c + d*x)**5*sec(c + d*x)**3/(9009*a**8*d*tan(c + d*x)**8
 - 72072*I*a**8*d*tan(c + d*x)**7 - 252252*a**8*d*tan(c + d*x)**6 + 504504
*I*a**8*d*tan(c + d*x)**5 + 630630*a**8*d*tan(c + d*x)**4 - 504504*I*a**8*
d*tan(c + d*x)**3 - 252252*a**8*d*tan(c + d*x)**2 + 72072*I*a**8*d*tan(c +
 d*x) + 9009*a**8*d) + 64*I*tan(c + d*x)**4*sec(c + d*x)**3/(9009*a**8*d*t
an(c + d*x)**8 - 72072*I*a**8*d*tan(c + d*x)**7 - 252252*a**8*d*tan(c + d*
x)**6 + 504504*I*a**8*d*tan(c + d*x)**5 + 630630*a**8*d*tan(c + d*x)**4 -
504504*I*a**8*d*tan(c + d*x)**3 - 252252*a**8*d*tan(c + d*x)**2 + 72072*I*
a**8*d*tan(c + d*x) + 9009*a**8*d) + 236*tan(c + d*x)**3*sec(c + d*x)**3/(
9009*a**8*d*tan(c + d*x)**8 - 72072*I*a**8*d*tan(c + d*x)**7 - 252252*a**8
*d*tan(c + d*x)**6 + 504504*I*a**8*d*tan(c + d*x)**5 + 630630*a**8*d*tan(c
 + d*x)**4 - 504504*I*a**8*d*tan(c + d*x)**3 - 252252*a**8*d*tan(c + d*x)*
*2 + 72072*I*a**8*d*tan(c + d*x) + 9009*a**8*d) - 544*I*tan(c + d*x)**2*se
c(c + d*x)**3/(9009*a**8*d*tan(c + d*x)**8 - 72072*I*a**8*d*tan(c + d*x)**
7 - 252252*a**8*d*tan(c + d*x)**6 + 504504*I*a**8*d*tan(c + d*x)**5 + 6306
30*a**8*d*tan(c + d*x)**4 - 504504*I*a**8*d*tan(c + d*x)**3 - 252252*a**8*
d*tan(c + d*x)**2 + 72072*I*a**8*d*tan(c + d*x) + 9009*a**8*d) - 911*tan(c
 + d*x)*sec(c + d*x)**3/(9009*a**8*d*tan(c + d*x)**8 - 72072*I*a**8*d*tan(
c + d*x)**7 - 252252*a**8*d*tan(c + d*x)**6 + 504504*I*a**8*d*tan(c + d*x)
**5 + 630630*a**8*d*tan(c + d*x)**4 - 504504*I*a**8*d*tan(c + d*x)**3 -...
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.66

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{693i\cos(13\,dx+13\,c)+4095i\cos(11\,dx+11\,c)+10010i\cos(9\,dx+9\,c)+12870i\cos(7\,dx+7\,c)+9}{}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/288288*(693*I*cos(13*d*x + 13*c) + 4095*I*cos(11*d*x + 11*c) + 10010*I*c
os(9*d*x + 9*c) + 12870*I*cos(7*d*x + 7*c) + 9009*I*cos(5*d*x + 5*c) + 300
3*I*cos(3*d*x + 3*c) + 693*sin(13*d*x + 13*c) + 4095*sin(11*d*x + 11*c) +
10010*sin(9*d*x + 9*c) + 12870*sin(7*d*x + 7*c) + 9009*sin(5*d*x + 5*c) +
3003*sin(3*d*x + 3*c))/(a^8*d)
```

Giac [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.83

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{2\left(9009\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-45045i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-183183\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+435435i\,\tan\left(\frac{1}{2}\,dx\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
2/9009*(9009*tan(1/2*d*x + 1/2*c)^12 - 45045*I*tan(1/2*d*x + 1/2*c)^11 - 1
83183*tan(1/2*d*x + 1/2*c)^10 + 435435*I*tan(1/2*d*x + 1/2*c)^9 + 810810*t
an(1/2*d*x + 1/2*c)^8 - 1051050*I*tan(1/2*d*x + 1/2*c)^7 - 1076790*tan(1/2
*d*x + 1/2*c)^6 + 785070*I*tan(1/2*d*x + 1/2*c)^5 + 451165*tan(1/2*d*x + 1
/2*c)^4 - 171457*I*tan(1/2*d*x + 1/2*c)^3 - 51675*tan(1/2*d*x + 1/2*c)^2 +
 7111*I*tan(1/2*d*x + 1/2*c) + 1240)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^13)
```

Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.75

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\frac{\cos(3\,c+3\,d\,x)^3\,5i}{36}+\frac{5\,\sin(3\,c+3\,d\,x)\,\cos(3\,c+3\,d\,x)^2}{36}-\frac{\cos(3\,c+3\,d\,x)\,3i}{32}+\frac{\cos(5\,c+5\,d\,x)\,1i}{32}+\frac{\cos(7\,c+7\,d\,x)\,5i}{112}+\frac{\cos(11\,c+11\,d\,x)\,5i}{352}+}{a^8\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
((cos(5*c + 5*d*x)*1i)/32 - (cos(3*c + 3*d*x)*3i)/32 + (cos(7*c + 7*d*x)*5
i)/112 + (cos(11*c + 11*d*x)*5i)/352 + (cos(13*c + 13*d*x)*1i)/416 - (7*si
n(3*c + 3*d*x))/288 + sin(5*c + 5*d*x)/32 + (5*sin(7*c + 7*d*x))/112 + (5*
sin(11*c + 11*d*x))/352 + sin(13*c + 13*d*x)/416 + (cos(3*c + 3*d*x)^3*5i)
/36 + (5*cos(3*c + 3*d*x)^2*sin(3*c + 3*d*x))/36)/(a^8*d)
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x)
```

output
```
( - 4*int(cos(c + d*x)/(128*cos(c + d*x)*sin(c + d*x)**7*i - 192*cos(c + d
*x)*sin(c + d*x)**5*i + 80*cos(c + d*x)*sin(c + d*x)**3*i - 8*cos(c + d*x)
*sin(c + d*x)*i - 128*sin(c + d*x)**8 + 256*sin(c + d*x)**6 - 160*sin(c +
d*x)**4 + 32*sin(c + d*x)**2 - 1),x)*d - 512*int(sin(c + d*x)**8/(128*cos(
c + d*x)*sin(c + d*x)**7*i - 192*cos(c + d*x)*sin(c + d*x)**5*i + 80*cos(c
 + d*x)*sin(c + d*x)**3*i - 8*cos(c + d*x)*sin(c + d*x)*i - 128*sin(c + d*
x)**8 + 256*sin(c + d*x)**6 - 160*sin(c + d*x)**4 + 32*sin(c + d*x)**2 - 1
),x)*d - 1024*int(sin(c + d*x)**6/(128*cos(c + d*x)*sin(c + d*x)**7 - 192*
cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(c + d*x)**3 - 8*cos(c +
 d*x)*sin(c + d*x) + 128*sin(c + d*x)**8*i - 256*sin(c + d*x)**6*i + 160*s
in(c + d*x)**4*i - 32*sin(c + d*x)**2*i + i),x)*d*i + 640*int(sin(c + d*x)
**4/(128*cos(c + d*x)*sin(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 +
 80*cos(c + d*x)*sin(c + d*x)**3 - 8*cos(c + d*x)*sin(c + d*x) + 128*sin(c
 + d*x)**8*i - 256*sin(c + d*x)**6*i + 160*sin(c + d*x)**4*i - 32*sin(c +
d*x)**2*i + i),x)*d*i - 128*int(sin(c + d*x)**2/(128*cos(c + d*x)*sin(c +
d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(c + d*x)*
*3 - 8*cos(c + d*x)*sin(c + d*x) + 128*sin(c + d*x)**8*i - 256*sin(c + d*x
)**6*i + 160*sin(c + d*x)**4*i - 32*sin(c + d*x)**2*i + i),x)*d*i + 512*in
t((cos(c + d*x)*sin(c + d*x)**7)/(128*cos(c + d*x)*sin(c + d*x)**7 - 192*c
os(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(c + d*x)**3 - 8*cos(c...
```

3.182 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 269

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}+\frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7}\\&\quad+\frac{14i\sec(c+dx)}{715a^2d(a+ia\tan(c+dx))^6}\\&\quad+\frac{14i\sec(c+dx)}{1287a^3d(a+ia\tan(c+dx))^5}\\&\quad+\frac{8i\sec(c+dx)}{1287d\left(a^2+ia^2\tan(c+dx)\right)^4}\\&\quad+\frac{8i\sec(c+dx)}{2145a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}\\&\quad+\frac{16i\sec(c+dx)}{6435d\left(a^4+ia^4\tan(c+dx)\right)^2}\\&\quad+\frac{16i\sec(c+dx)}{6435d\left(a^8+ia^8\tan(c+dx)\right)}\end{aligned}$$

output
```
1/15*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^8+7/195*I*sec(d*x+c)/a/d/(a+I*a*tan
(d*x+c))^7+14/715*I*sec(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^6+14/1287*I*sec(d*
x+c)/a^3/d/(a+I*a*tan(d*x+c))^5+8/1287*I*sec(d*x+c)/d/(a^2+I*a^2*tan(d*x+c
))^4+8/2145*I*sec(d*x+c)/a^2/d/(a^2+I*a^2*tan(d*x+c))^3+16/6435*I*sec(d*x+
c)/d/(a^4+I*a^4*tan(d*x+c))^2+16/6435*I*sec(d*x+c)/d/(a^8+I*a^8*tan(d*x+c)
)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.43

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{i\sec^8(c+dx)(28600\cos(c+dx)+19656\cos(3(c+dx))+9240\cos(5(c+dx))+3432\cos(7(c+dx))+}{411840a^8d(-i+\tan(c}$$

input
```
Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/411840)*Sec[c + d*x]^8*(28600*Cos[c + d*x] + 19656*Cos[3*(c + d*x)] +
9240*Cos[5*(c + d*x)] + 3432*Cos[7*(c + d*x)] + (3575*I)*Sin[c + d*x] + (7
371*I)*Sin[3*(c + d*x)] + (5775*I)*Sin[5*(c + d*x)] + (3003*I)*Sin[7*(c +
d*x)]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^7}dx}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^7}dx}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\left(\frac{6\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^6}dx}{13a}+\frac{i\sec(c+dx)}{13d(a+ia\tan(c+dx))^7}\right)}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{6\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^6}dx}{13a}+\frac{i\sec(c+dx)}{13d(a+ia\tan(c+dx))^7}\right)}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\left(\frac{6\left(\frac{5\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^5}dx}{11a}+\frac{i\sec(c+dx)}{11d(a+ia\tan(c+dx))^6}\right)}{13a}+\frac{i\sec(c+dx)}{13d(a+ia\tan(c+dx))^7}\right)}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{6\left(\frac{5\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^5}dx}{11a}+\frac{i\sec(c+dx)}{11d(a+ia\tan(c+dx))^6}\right)}{13a}+\frac{i\sec(c+dx)}{13d(a+ia\tan(c+dx))^7}\right)}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^4}dx}{9a}+\frac{i\sec(c+dx)}{9d(a+ia\tan(c+dx))^5}\right)}{11a}+\frac{i\sec(c+dx)}{11d(a+ia\tan(c+dx))^6}\right)}{13a}+\frac{i\sec(c+dx)}{13d(a+ia\tan(c+dx))^7}\right)}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$\Big\downarrow$ 3042

$$\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^4}dx}{9a}+\frac{i\sec(c+dx)}{9d(a+ia\tan(c+dx))^5}\right)}{11a}+\frac{i\sec(c+dx)}{11d(a+ia\tan(c+dx))^6}\right)}{13a}+\frac{i\sec(c+dx)}{13d(a+ia\tan(c+dx))^7}\right)}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$\Big\downarrow$ 3983

$$\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\left(\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}\right)}{9a}+\frac{i\sec(c+dx)}{9d(a+ia\tan(c+dx))^5}\right)}{11a}+\frac{i\sec(c+dx)}{11d(a+ia\tan(c+dx))^6}\right)}{13a}+\frac{i\sec(c+dx)}{13d(a+ia\tan(c+dx))^7}\right)}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$\Big\downarrow$ 3042

$$\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\left(\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}\right)}{9a}+\frac{i\sec(c+dx)}{9d(a+ia\tan(c+dx))^5}\right)}{11a}+\frac{i\sec(c+dx)}{11d(a+ia\tan(c+dx))^6}\right)}{13a}+\frac{i\sec(c+dx)}{13d(a+ia\tan(c+dx))^7}\right)}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$\Big\downarrow$ 3983

$$\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\left(\frac{3\left(\frac{2\int\frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}\right)}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}\right)}{9a}+\frac{i\sec(c+dx)}{9d(a+ia\tan(c+dx))^5}\right)}{11a}+\frac{i\sec(c+dx)}{11d(a+ia\tan(c+dx))^6}\right)}{13a}+\frac{}{13d(a}\right)}{15a}$$

$$\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \; 3042$$

$$\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\left(\frac{3\left(\frac{2\int\frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}\right)}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}\right)}{9a}+\frac{i\sec(c+dx)}{9d(a+ia\tan(c+dx))^5}\right)}{11a}+\frac{i\sec(c+dx)}{11d(a+ia\tan(c+dx))^6}\right)}{13a}+\frac{}{13d(a}\right)}{15a}$$

$$\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \; 3983$$

$$7\left(\frac{6\left(\frac{5\left(\frac{4\left(\frac{3\left(\frac{2\left(\frac{\int \frac{\sec(c+dx)}{i\tan(c+dx)a+a}dx}{3a}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}\right)}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}\right)}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}\right)}{9a}+\frac{i\sec(c+dx)}{9d(a+ia\tan(c+dx))^5}\right)}{11a}+\frac{i\sec(c}{11d(a+ia\,\mathrm{ta}}}{13a}\right.$$

↓ 3042

$$7\left(\frac{6\left(\frac{5\left(\frac{4\left(\frac{3\left(\frac{2\left(\frac{\int \frac{\sec(c+dx)}{i\tan(c+dx)a+a}dx}{3a}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}\right)}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}\right)}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}\right)}{9a}+\frac{i\sec(c+dx)}{9d(a+ia\tan(c+dx))^5}\right)}{11a}+\frac{i\sec(c}{11d(a+ia\tan}}{13a}\right.$$

$$\Big\downarrow \text{3969}$$

$$\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}+\frac{7\left(\frac{i\sec(c+dx)}{13d(a+ia\tan(c+dx))^7}+\frac{6\left(\frac{i\sec(c+dx)}{11d(a+ia\tan(c+dx))^6}+\frac{5\left(\frac{i\sec(c+dx)}{9d(a+ia\tan(c+dx))^5}+\frac{4\left(\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}+\frac{3\left(\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}+\frac{2\left(\frac{i\sec(c+}{3ad(a+ia\tan}\right.}{}\right.}{7a}\right.}{9a}\right.}{11a}\right.}{13a}\right.}{15a}$$

input
```
Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/15)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (7*(((I/13)*Sec[c + d
*x])/(d*(a + I*a*Tan[c + d*x])^7) + (6*(((I/11)*Sec[c + d*x])/(d*(a + I*a*
Tan[c + d*x])^6) + (5*(((I/9)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^5) +
 (4*(((I/7)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (3*(((I/5)*Sec[c
+ d*x])/(d*(a + I*a*Tan[c + d*x])^3) + (2*(((I/3)*Sec[c + d*x])/(d*(a + I*
a*Tan[c + d*x])^2) + ((I/3)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x]))))/(
5*a)))/(7*a)))/(9*a)))/(11*a)))/(13*a)))/(15*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.54

method	result
risch	$\frac{i e^{-i(dx+c)}}{128a^8d}+\frac{7i e^{-3i(dx+c)}}{384a^8d}+\frac{21i e^{-5i(dx+c)}}{640a^8d}+\frac{5i e^{-7i(dx+c)}}{128a^8d}+\frac{35i e^{-9i(dx+c)}}{1152a^8d}+\frac{21i e^{-11i(dx+c)}}{1408a^8d}+\frac{7i e^{-13i(d}}{1664a^8}$
derivativedivides	$-\frac{196}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{29792}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}-\frac{2128}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}-\frac{3584i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{12}}+\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{14}}+\frac{}{5(-}$
default	$-\frac{196}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{29792}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}-\frac{2128}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}-\frac{3584i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{12}}+\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{14}}+\frac{}{5(-}$

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/128*I/a^8/d*exp(-I*(d*x+c))+7/384*I/a^8/d*exp(-3*I*(d*x+c))+21/640*I/a^8
/d*exp(-5*I*(d*x+c))+5/128*I/a^8/d*exp(-7*I*(d*x+c))+35/1152*I/a^8/d*exp(-
9*I*(d*x+c))+21/1408*I/a^8/d*exp(-11*I*(d*x+c))+7/1664*I/a^8/d*exp(-13*I*(
d*x+c))+1/1920*I/a^8/d*exp(-15*I*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.36

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{(6435i\,e^{(14i\,dx+14i\,c)} + 15015i\,e^{(12i\,dx+12i\,c)} + 27027i\,e^{(10i\,dx+10i\,c)} + 32175i\,e^{(8i\,dx+8i\,c)} + 25025i\,e^{(6i\,dx+6i\,c)} +}{823680\,a^8d}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/823680*(6435*I*e^(14*I*d*x + 14*I*c) + 15015*I*e^(12*I*d*x + 12*I*c) + 2
7027*I*e^(10*I*d*x + 10*I*c) + 32175*I*e^(8*I*d*x + 8*I*c) + 25025*I*e^(6*
I*d*x + 6*I*c) + 12285*I*e^(4*I*d*x + 4*I*c) + 3465*I*e^(2*I*d*x + 2*I*c)
+ 429*I)*e^(-15*I*d*x - 15*I*c)/(a^8*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1221 vs. $2(238) = 476$.

Time = 10.26 (sec) , antiderivative size = 1221, normalized size of antiderivative = 4.54

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise((16*tan(c + d*x)**7*sec(c + d*x)/(6435*a**8*d*tan(c + d*x)**8 -
51480*I*a**8*d*tan(c + d*x)**7 - 180180*a**8*d*tan(c + d*x)**6 + 360360*I*
a**8*d*tan(c + d*x)**5 + 450450*a**8*d*tan(c + d*x)**4 - 360360*I*a**8*d*t
an(c + d*x)**3 - 180180*a**8*d*tan(c + d*x)**2 + 51480*I*a**8*d*tan(c + d*
x) + 6435*a**8*d) - 128*I*tan(c + d*x)**6*sec(c + d*x)/(6435*a**8*d*tan(c
+ d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 180180*a**8*d*tan(c + d*x)**6
 + 360360*I*a**8*d*tan(c + d*x)**5 + 450450*a**8*d*tan(c + d*x)**4 - 36036
0*I*a**8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c + d*x)**2 + 51480*I*a**8*
d*tan(c + d*x) + 6435*a**8*d) - 456*tan(c + d*x)**5*sec(c + d*x)/(6435*a**
8*d*tan(c + d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 180180*a**8*d*tan(c
 + d*x)**6 + 360360*I*a**8*d*tan(c + d*x)**5 + 450450*a**8*d*tan(c + d*x)*
*4 - 360360*I*a**8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c + d*x)**2 + 514
80*I*a**8*d*tan(c + d*x) + 6435*a**8*d) + 960*I*tan(c + d*x)**4*sec(c + d*
x)/(6435*a**8*d*tan(c + d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 180180*
a**8*d*tan(c + d*x)**6 + 360360*I*a**8*d*tan(c + d*x)**5 + 450450*a**8*d*t
an(c + d*x)**4 - 360360*I*a**8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c + d
*x)**2 + 51480*I*a**8*d*tan(c + d*x) + 6435*a**8*d) + 1350*tan(c + d*x)**3
*sec(c + d*x)/(6435*a**8*d*tan(c + d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**
7 - 180180*a**8*d*tan(c + d*x)**6 + 360360*I*a**8*d*tan(c + d*x)**5 + 4504
50*a**8*d*tan(c + d*x)**4 - 360360*I*a**8*d*tan(c + d*x)**3 - 180180*a*...
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.67

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{429i\,\cos(15\,dx+15\,c)+3465i\,\cos(13\,dx+13\,c)+12285i\,\cos(11\,dx+11\,c)+25025i\,\cos(9\,dx+9\,c)+\cdots}{}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/823680*(429*I*cos(15*d*x + 15*c) + 3465*I*cos(13*d*x + 13*c) + 12285*I*c
os(11*d*x + 11*c) + 25025*I*cos(9*d*x + 9*c) + 32175*I*cos(7*d*x + 7*c) +
27027*I*cos(5*d*x + 5*c) + 15015*I*cos(3*d*x + 3*c) + 6435*I*cos(d*x + c)
+ 429*sin(15*d*x + 15*c) + 3465*sin(13*d*x + 13*c) + 12285*sin(11*d*x + 11
*c) + 25025*sin(9*d*x + 9*c) + 32175*sin(7*d*x + 7*c) + 27027*sin(5*d*x +
5*c) + 15015*sin(3*d*x + 3*c) + 6435*sin(d*x + c))/(a^8*d)
```

Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.75

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{2\left(6435\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14}-45045i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-210210\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+630630i\tan\left(\frac{1}{2}\,dx\right.\right.}{}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
2/6435*(6435*tan(1/2*d*x + 1/2*c)^14 - 45045*I*tan(1/2*d*x + 1/2*c)^13 - 2
10210*tan(1/2*d*x + 1/2*c)^12 + 630630*I*tan(1/2*d*x + 1/2*c)^11 + 1414413
*tan(1/2*d*x + 1/2*c)^10 - 2357355*I*tan(1/2*d*x + 1/2*c)^9 - 3063060*tan(
1/2*d*x + 1/2*c)^8 + 3063060*I*tan(1/2*d*x + 1/2*c)^7 + 2407405*tan(1/2*d*
x + 1/2*c)^6 - 1444443*I*tan(1/2*d*x + 1/2*c)^5 - 668850*tan(1/2*d*x + 1/2
*c)^4 + 222950*I*tan(1/2*d*x + 1/2*c)^3 + 54915*tan(1/2*d*x + 1/2*c)^2 - 7
845*I*tan(1/2*d*x + 1/2*c) - 952)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^15)
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.83

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{2\left(2\sin\left(\frac{c}{4}+\frac{d\,x}{4}\right)^2-1\right)\left(-\frac{\sin(c+d\,x)^2\,44779\mathrm{i}}{32}+\frac{32175\,\sin(c+d\,x)}{128}-\frac{\sin(2\,c+2\,d\,x)^2\,26075\mathrm{i}}{16}-\frac{3575\,\sin(2\,c+2\,d\,x)}{8}+\frac{\sin\left(\frac{c}{2}+\right.}{}\right.}{}$$

input
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
(2*(2*sin(c/4 + (d*x)/4)^2 - 1)*((32175*sin(c + d*x))/128 - (3575*sin(2*c
+ 2*d*x))/8 + (84227*sin(3*c + 3*d*x))/128 - 754*sin(4*c + 4*d*x) + (11152
7*sin(5*c + 5*d*x))/128 - (7187*sin(6*c + 6*d*x))/8 + (121427*sin(7*c + 7*
d*x))/128 - (sin(2*c + 2*d*x)^2*26075i)/16 + (sin(c/2 + (d*x)/2)^2*114583i
)/64 - (sin(3*c + 3*d*x)^2*57925i)/32 + (sin((3*c)/2 + (3*d*x)/2)^2*116585
i)/64 + (sin((5*c)/2 + (5*d*x)/2)^2*119315i)/64 + (sin((7*c)/2 + (7*d*x)/2
)^2*122285i)/64 - (sin(c + d*x)^2*44779i)/32 - 952i))/(6435*a^8*d*(sin((15
*c)/2 + (15*d*x)/2)*1i - 2*sin((15*c)/4 + (15*d*x)/4)^2 + 1))
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x)
```

output
```
( - 4*int(cos(c + d*x)/(128*cos(c + d*x)*sin(c + d*x)**7*i - 192*cos(c + d
*x)*sin(c + d*x)**5*i + 80*cos(c + d*x)*sin(c + d*x)**3*i - 8*cos(c + d*x)
*sin(c + d*x)*i - 128*sin(c + d*x)**8 + 256*sin(c + d*x)**6 - 160*sin(c +
d*x)**4 + 32*sin(c + d*x)**2 - 1),x)*d - 512*int(sin(c + d*x)**8/(128*cos(
c + d*x)*sin(c + d*x)**7*i - 192*cos(c + d*x)*sin(c + d*x)**5*i + 80*cos(c
 + d*x)*sin(c + d*x)**3*i - 8*cos(c + d*x)*sin(c + d*x)*i - 128*sin(c + d*
x)**8 + 256*sin(c + d*x)**6 - 160*sin(c + d*x)**4 + 32*sin(c + d*x)**2 - 1
),x)*d - 1024*int(sin(c + d*x)**6/(128*cos(c + d*x)*sin(c + d*x)**7 - 192*
cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(c + d*x)**3 - 8*cos(c +
 d*x)*sin(c + d*x) + 128*sin(c + d*x)**8*i - 256*sin(c + d*x)**6*i + 160*s
in(c + d*x)**4*i - 32*sin(c + d*x)**2*i + i),x)*d*i + 640*int(sin(c + d*x)
**4/(128*cos(c + d*x)*sin(c + d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 +
 80*cos(c + d*x)*sin(c + d*x)**3 - 8*cos(c + d*x)*sin(c + d*x) + 128*sin(c
 + d*x)**8*i - 256*sin(c + d*x)**6*i + 160*sin(c + d*x)**4*i - 32*sin(c +
d*x)**2*i + i),x)*d*i - 128*int(sin(c + d*x)**2/(128*cos(c + d*x)*sin(c +
d*x)**7 - 192*cos(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(c + d*x)*
*3 - 8*cos(c + d*x)*sin(c + d*x) + 128*sin(c + d*x)**8*i - 256*sin(c + d*x
)**6*i + 160*sin(c + d*x)**4*i - 32*sin(c + d*x)**2*i + i),x)*d*i + 512*in
t((cos(c + d*x)*sin(c + d*x)**7)/(128*cos(c + d*x)*sin(c + d*x)**7 - 192*c
os(c + d*x)*sin(c + d*x)**5 + 80*cos(c + d*x)*sin(c + d*x)**3 - 8*cos(c...
```

3.183 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 271

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= \frac{192\sin(c+dx)}{12155a^8d}-\frac{64\sin^3(c+dx)}{12155a^8d}\\
&\quad+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}+\frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7}\\
&\quad+\frac{24i\cos(c+dx)}{1105a^2d(a+ia\tan(c+dx))^6}\\
&\quad+\frac{168i\cos(c+dx)}{12155a^3d(a+ia\tan(c+dx))^5}\\
&\quad+\frac{112i\cos(c+dx)}{12155d\left(a^2+ia^2\tan(c+dx)\right)^4}\\
&\quad+\frac{16i\cos(c+dx)}{2431a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}\\
&\quad+\frac{128i\cos^3(c+dx)}{12155d\left(a^8+ia^8\tan(c+dx)\right)}\end{aligned}$$

output
```
192/12155*sin(d*x+c)/a^8/d-64/12155*sin(d*x+c)^3/a^8/d+1/17*I*cos(d*x+c)/d
/(a+I*a*tan(d*x+c))^8+3/85*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^7+24/1105*I
*cos(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^6+168/12155*I*cos(d*x+c)/a^3/d/(a+I*a
*tan(d*x+c))^5+112/12155*I*cos(d*x+c)/d/(a^2+I*a^2*tan(d*x+c))^4+16/2431*I
*cos(d*x+c)/a^2/d/(a^2+I*a^2*tan(d*x+c))^3+128/12155*I*cos(d*x+c)^3/d/(a^8
+I*a^8*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.51

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{i\sec^8(c+dx)(-194480\cos(c+dx)-148512\cos(3(c+dx))-89760\cos(5(c+dx))-58344\cos(7(c$$

input
```
Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/3111680*I)*Sec[c + d*x]^8*(-194480*Cos[c + d*x] - 148512*Cos[3*(c + d
*x)] - 89760*Cos[5*(c + d*x)] - 58344*Cos[7*(c + d*x)] + 5720*Cos[9*(c + d
*x)] - (24310*I)*Sin[c + d*x] - (55692*I)*Sin[3*(c + d*x)] - (56100*I)*Sin
[5*(c + d*x)] - (51051*I)*Sin[7*(c + d*x)] + (6435*I)*Sin[9*(c + d*x)]))/(
a^8*d*(-I + Tan[c + d*x])^8)
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^7}dx}{17a}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^7}dx}{17a}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\left(\frac{8\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^6}dx}{15a}+\frac{i\cos(c+dx)}{15d(a+ia\tan(c+dx))^7}\right)}{17a}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\left(\frac{8\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^6}dx}{15a}+\frac{i\cos(c+dx)}{15d(a+ia\tan(c+dx))^7}\right)}{17a}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\left(\frac{8\left(\frac{7\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^5}dx}{13a}+\frac{i\cos(c+dx)}{13d(a+ia\tan(c+dx))^6}\right)}{15a}+\frac{i\cos(c+dx)}{15d(a+ia\tan(c+dx))^7}\right)}{17a}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\left(\frac{8\left(\frac{7\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^5}dx}{13a}+\frac{i\cos(c+dx)}{13d(a+ia\tan(c+dx))^6}\right)}{15a}+\frac{i\cos(c+dx)}{15d(a+ia\tan(c+dx))^7}\right)}{17a}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\left(\frac{8\left(\frac{7\left(\frac{6\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^4}dx}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right)}{13a}+\frac{i\cos(c+dx)}{13d(a+ia\tan(c+dx))^6}\right)}{15a}+\frac{i\cos(c+dx)}{15d(a+ia\tan(c+dx))^7}\right)}{17a}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9\left(\frac{8\left(\frac{7\left(\frac{6\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^4}dx}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right)}{13a}+\frac{i\cos(c+dx)}{13d(a+ia\tan(c+dx))^6}\right)}{15a}+\frac{i\cos(c+dx)}{15d(a+ia\tan(c+dx))^7}\right)}{17a}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{ 3983}$$

$$\frac{9\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{5\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}\right)}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right)}{13a}+\frac{i\cos(c+dx)}{13d(a+ia\tan(c+dx))^6}\right)}{15a}+\frac{i\cos(c+dx)}{15d(a+ia\tan(c+dx))^7}\right)}{17a}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{5\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^3}dx}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}\right)}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right)}{13a}+\frac{i\cos(c+dx)}{13d(a+ia\tan(c+dx))^6}\right)}{15a}+\frac{i\cos(c+dx)}{15d(a+ia\tan(c+dx))}\right.}{17a}$$

$$\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$9\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\int\frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}\right)}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right)}{13a}+\frac{i\cos(c+dx)}{13d(a+ia\tan(c+dx))^6}\right)}{15a}+\frac{}{15d(}\right.$$

$$17a$$

$$\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \ 3042$$

$$\frac{9\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^2}dx}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}\right)}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right)}{13a}+\frac{i\cos(c+dx)}{13d(a+ia\tan(c+dx))^6}\right)}{15a}\right.}{17a}$$

$$\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\downarrow 3981$$

$$9\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\left(\frac{3\int\cos^3(c+dx)dx}{5a^2}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}\right)}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right)}{13a}+\frac{i\cos(c}{13d(a+ia\,\mathrm{ta}}\right.}{15a}\right.$$

$\downarrow$ 3042

$$9\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\left(\frac{3\int\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{5a^2}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}\right)}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right)}{13a}+\frac{i\,c}{13d(a+}}{15a}$$

↓ 3113

$$9\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\left(-\frac{3\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}\right)}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx)}}{13a}}{15a}\right.$$

$\downarrow$ 2009

$$9\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\left(-\frac{3\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}\right)}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right)}{13a}}{15a}\right.$$

input
```
Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/17)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (9*(((I/15)*Cos[c + d
*x])/(d*(a + I*a*Tan[c + d*x])^7) + (8*(((I/13)*Cos[c + d*x])/(d*(a + I*a*
Tan[c + d*x])^6) + (7*(((I/11)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^5)
+ (6*(((I/9)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (5*(((I/7)*Cos[c
 + d*x])/(d*(a + I*a*Tan[c + d*x])^3) + (4*((-3*(-Sin[c + d*x] + Sin[c + d
*x]^3/3))/(5*a^2*d) + (((2*I)/5)*Cos[c + d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d
*x]))))/(7*a)))/(9*a)))/(11*a)))/(13*a)))/(15*a)))/(17*a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.65

method	result
risch	$\frac{3ie^{-3i(dx+c)}}{128a^8d}+\frac{21ie^{-5i(dx+c)}}{640a^8d}+\frac{9ie^{-7i(dx+c)}}{256a^8d}+\frac{7ie^{-9i(dx+c)}}{256a^8d}+\frac{21ie^{-11i(dx+c)}}{1408a^8d}+\frac{9ie^{-13i(dx+c)}}{1664a^8d}+\frac{3ie^{-15i}}{2560}$
derivativedivides	$\frac{\frac{1793i}{128\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{13313i}{8\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{1568i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{14}}-\frac{7937i}{32\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{16}}-\frac{}{2(}}{}$
default	$\frac{\frac{1793i}{128\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{13313i}{8\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{1568i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{14}}-\frac{7937i}{32\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{16}}-\frac{}{2(}}{}$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
3/128*I/a^8/d*exp(-3*I*(d*x+c))+21/640*I/a^8/d*exp(-5*I*(d*x+c))+9/256*I/a
^8/d*exp(-7*I*(d*x+c))+7/256*I/a^8/d*exp(-9*I*(d*x+c))+21/1408*I/a^8/d*exp
(-11*I*(d*x+c))+9/1664*I/a^8/d*exp(-13*I*(d*x+c))+3/2560*I/a^8/d*exp(-15*I
*(d*x+c))+1/8704*I/a^8/d*exp(-17*I*(d*x+c))+1/64*I/a^8/d*cos(d*x+c)+5/256*
sin(d*x+c)/a^8/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.44

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(-12155i\,e^{(18i\,dx+18i\,c)}+109395i\,e^{(16i\,dx+16i\,c)}+145860i\,e^{(14i\,dx+14i\,c)}+204204i\,e^{(12i\,dx+12i\,c)}+218790i\,e^{(1}\right.}{622}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/6223360*(-12155*I*e^(18*I*d*x + 18*I*c) + 109395*I*e^(16*I*d*x + 16*I*c)
 + 145860*I*e^(14*I*d*x + 14*I*c) + 204204*I*e^(12*I*d*x + 12*I*c) + 21879
0*I*e^(10*I*d*x + 10*I*c) + 170170*I*e^(8*I*d*x + 8*I*c) + 92820*I*e^(6*I*
d*x + 6*I*c) + 33660*I*e^(4*I*d*x + 4*I*c) + 7293*I*e^(2*I*d*x + 2*I*c) +
715*I)*e^(-17*I*d*x - 17*I*c)/(a^8*d)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.35

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \begin{cases} \frac{(-14350091149820134393118720 0ia^{72}d^9e^{82ic}e^{idx}+12915082034838120953806848 00ia^{72}d^9e^{80ic}e^{-idx}+17220109379784161271742464 00ia^{7}} {} \\ \frac{x(e^{18ic}+9e^{16ic}+36e^{14ic}+84e^{12ic}+126e^{10ic}+126e^{8ic}+84e^{6ic}+36e^{4ic}+9e^{2ic}+1)e^{-17ic}}{512a^8} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-143500911498201343931187200*I*a**72*d**9*exp(82*I*c)*exp(I*d*
x) + 129150820348381209538068480 0*I*a**72*d**9*exp(80*I*c)*exp(-I*d*x) + 1
7220109379784161271742464 00*I*a**72*d**9*exp(78*I*c)*exp(-3*I*d*x) + 24108
15313169782578043944960*I*a**72*d**9*exp(76*I*c)*exp(-5*I*d*x) + 258301640
6967624190761369600*I*a**72*d**9*exp(74*I*c)*exp(-7*I*d*x) + 2009012760974
818815036620800*I*a**72*d**9*exp(72*I*c)*exp(-9*I*d*x) + 10958251423499011
71838156800*I*a**72*d**9*exp(70*I*c)*exp(-11*I*d*x) + 3973871395334806447 3
2518400*I*a**72*d**9*exp(68*I*c)*exp(-13*I*d*x) + 861005468989208063587123
20*I*a**72*d**9*exp(66*I*c)*exp(-15*I*d*x) + 8441230088129490819481600*I*a
**72*d**9*exp(64*I*c)*exp(-17*I*d*x))*exp(-81*I*c)/(7347246668707908809276
7846400*a**80*d**10), Ne(a**80*d**10*exp(81*I*c), 0)), (x*(exp(18*I*c) + 9
*exp(16*I*c) + 36*exp(14*I*c) + 84*exp(12*I*c) + 126*exp(10*I*c) + 126*exp
(8*I*c) + 84*exp(6*I*c) + 36*exp(4*I*c) + 9*exp(2*I*c) + 1)*exp(-17*I*c)/(
512*a**8), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.92

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\frac{12155}{a^8\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)} + \frac{6211205\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{16}-55791450i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{15}-303072770\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{14}+1091397450i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/3111680*(12155/(a^8*(tan(1/2*d*x + 1/2*c) + I)) + (6211205*tan(1/2*d*x +
 1/2*c)^16 - 55791450*I*tan(1/2*d*x + 1/2*c)^15 - 303072770*tan(1/2*d*x +
1/2*c)^14 + 1091397450*I*tan(1/2*d*x + 1/2*c)^13 + 2909561798*tan(1/2*d*x
+ 1/2*c)^12 - 5901218466*I*tan(1/2*d*x + 1/2*c)^11 - 9405145178*tan(1/2*d*
x + 1/2*c)^10 + 11877161010*I*tan(1/2*d*x + 1/2*c)^9 + 12017308160*tan(1/2
*d*x + 1/2*c)^8 - 9710430158*I*tan(1/2*d*x + 1/2*c)^7 - 6263238566*tan(1/2
*d*x + 1/2*c)^6 + 3172666718*I*tan(1/2*d*x + 1/2*c)^5 + 1247921210*tan(1/2
*d*x + 1/2*c)^4 - 365303990*I*tan(1/2*d*x + 1/2*c)^3 - 77883902*tan(1/2*d*
x + 1/2*c)^2 + 10498214*I*tan(1/2*d*x + 1/2*c) + 982907)/(a^8*(tan(1/2*d*x
 + 1/2*c) - I)^17))/d
```

Mupad [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.97

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{152329\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{128}-\frac{41121\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{32}+\frac{41121\sin\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{32}-\frac{96165\sin\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{64}+\frac{96165\sin\left(\frac{9c}{2}+9\right.}{64}\right.}{}$$

input
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(cos(c/2 + (d*x)/2)*((cos((3*c)/2 + (3*d*x)/2)*12155i)/16 - (cos((5*c)/2 +
 (5*d*x)/2)*12155i)/16 + (cos((7*c)/2 + (7*d*x)/2)*21437i)/16 - (cos((9*c)
/2 + (9*d*x)/2)*21437i)/16 + (cos((11*c)/2 + (11*d*x)/2)*27047i)/16 - (cos
((13*c)/2 + (13*d*x)/2)*27047i)/16 + (cos((15*c)/2 + (15*d*x)/2)*61387i)/3
2 - (cos((17*c)/2 + (17*d*x)/2)*715i)/32 + (152329*sin(c/2 + (d*x)/2))/128
 - (41121*sin((3*c)/2 + (3*d*x)/2))/32 + (41121*sin((5*c)/2 + (5*d*x)/2))/
32 - (96165*sin((7*c)/2 + (7*d*x)/2))/64 + (96165*sin((9*c)/2 + (9*d*x)/2)
)/64 - (55095*sin((11*c)/2 + (11*d*x)/2))/32 + (55095*sin((13*c)/2 + (13*d
*x)/2))/32 - (491811*sin((15*c)/2 + (15*d*x)/2))/256 + (6435*sin((17*c)/2
+ (17*d*x)/2))/256)*2i)/(12155*a^8*d*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)
/2)*1i)^17*(cos(c/2 + (d*x)/2)*1i + sin(c/2 + (d*x)/2)))
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\int \frac{\cos(dx+c)}{\tan(dx+c)^8-8\tan(dx+c)^7i-28\tan(dx+c)^6+56\tan(dx+c)^5i+70\tan(dx+c)^4-56\tan(dx+c)^3i-28\tan(dx+c)^2+8\tan(dx+c)i+1}dx}{a^8}$$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x)
```

output
```
int(cos(c + d*x)/(tan(c + d*x)**8 - 8*tan(c + d*x)**7*i - 28*tan(c + d*x)*
*6 + 56*tan(c + d*x)**5*i + 70*tan(c + d*x)**4 - 56*tan(c + d*x)**3*i - 28
*tan(c + d*x)**2 + 8*tan(c + d*x)*i + 1),x)/a**8
```

3.184 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 301

$$
\begin{aligned}
\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = {} & \frac{160\sin(c+dx)}{4199a^8d} - \frac{320\sin^3(c+dx)}{12597a^8d} + \frac{32\sin^5(c+dx)}{4199a^8d} \\
& + \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} + \frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7} \\
& + \frac{22i\cos^3(c+dx)}{969a^2d(a+ia\tan(c+dx))^6} \\
& + \frac{66i\cos^3(c+dx)}{4199a^3d(a+ia\tan(c+dx))^5} \\
& + \frac{48i\cos^3(c+dx)}{4199d\left(a^2+ia^2\tan(c+dx)\right)^4} \\
& + \frac{112i\cos^3(c+dx)}{12597a^2d\left(a^2+ia^2\tan(c+dx)\right)^3} \\
& + \frac{64i\cos^5(c+dx)}{4199d\left(a^8+ia^8\tan(c+dx)\right)}
\end{aligned}
$$

output
```
160/4199*sin(d*x+c)/a^8/d-320/12597*sin(d*x+c)^3/a^8/d+32/4199*sin(d*x+c)^
5/a^8/d+1/19*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^8+11/323*I*cos(d*x+c)^3/a
/d/(a+I*a*tan(d*x+c))^7+22/969*I*cos(d*x+c)^3/a^2/d/(a+I*a*tan(d*x+c))^6+6
6/4199*I*cos(d*x+c)^3/a^3/d/(a+I*a*tan(d*x+c))^5+48/4199*I*cos(d*x+c)^3/d/
(a^2+I*a^2*tan(d*x+c))^4+112/12597*I*cos(d*x+c)^3/a^2/d/(a^2+I*a^2*tan(d*x
+c))^3+64/4199*I*cos(d*x+c)^5/d/(a^8+I*a^8*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.53

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{i\sec^8(c+dx)(-739024\cos(c+dx)-604656\cos(3(c+dx))-426360\cos(5(c+dx))-369512\cos(7(c+dx))}{}$$

input
```
Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/12899328*I)*Sec[c + d*x]^8*(-739024*Cos[c + d*x] - 604656*Cos[3*(c +
d*x)] - 426360*Cos[5*(c + d*x)] - 369512*Cos[7*(c + d*x)] + 65208*Cos[9*(c
 + d*x)] + 1768*Cos[11*(c + d*x)] - (92378*I)*Sin[c + d*x] - (226746*I)*Si
n[3*(c + d*x)] - (266475*I)*Sin[5*(c + d*x)] - (323323*I)*Sin[7*(c + d*x)]
 + (73359*I)*Sin[9*(c + d*x)] + (2431*I)*Sin[11*(c + d*x)]))/(a^8*d*(-I +
Tan[c + d*x])^8)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.06, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{11\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^7}dx}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^7}dx}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{11\left(\frac{10\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^6}dx}{17a}+\frac{i\cos^3(c+dx)}{17d(a+ia\tan(c+dx))^7}\right)}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11\left(\frac{10\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^6}dx}{17a}+\frac{i\cos^3(c+dx)}{17d(a+ia\tan(c+dx))^7}\right)}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{11\left(\frac{10\left(\frac{3\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^5}dx}{5a}+\frac{i\cos^3(c+dx)}{15d(a+ia\tan(c+dx))^6}\right)}{17a}+\frac{i\cos^3(c+dx)}{17d(a+ia\tan(c+dx))^7}\right)}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11\left(\frac{10\left(\frac{3\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^5}dx}{5a}+\frac{i\cos^3(c+dx)}{15d(a+ia\tan(c+dx))^6}\right)}{17a}+\frac{i\cos^3(c+dx)}{17d(a+ia\tan(c+dx))^7}\right)}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \color{blue}{3983}$$

$$\frac{11\left(\frac{10\left(\frac{3\left(\frac{8\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^4}dx}{13a}+\frac{i\cos^3(c+dx)}{13d(a+ia\tan(c+dx))^5}\right)}{5a}+\frac{i\cos^3(c+dx)}{15d(a+ia\tan(c+dx))^6}\right)}{17a}+\frac{i\cos^3(c+dx)}{17d(a+ia\tan(c+dx))^7}\right)}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{11\left(\frac{10\left(\frac{3\left(\frac{8\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^4}dx}{13a}+\frac{i\cos^3(c+dx)}{13d(a+ia\tan(c+dx))^5}\right)}{5a}+\frac{i\cos^3(c+dx)}{15d(a+ia\tan(c+dx))^6}\right)}{17a}+\frac{i\cos^3(c+dx)}{17d(a+ia\tan(c+dx))^7}\right)}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \color{blue}{3983}$$

$$\frac{11\left(\frac{10\left(\frac{3\left(\frac{8\left(\frac{7\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}\right)}{13a}+\frac{i\cos^3(c+dx)}{13d(a+ia\tan(c+dx))^5}\right)}{5a}+\frac{i\cos^3(c+dx)}{15d(a+ia\tan(c+dx))^6}\right)}{17a}+\frac{i\cos^3(c+dx)}{17d(a+ia\tan(c+dx))^7}\right)}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$\Big\downarrow$ 3042

$$\frac{11\left(\frac{10\left(\frac{3\left(\frac{8\left(\frac{7\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^3}dx}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}\right)}{13a}+\frac{i\cos^3(c+dx)}{13d(a+ia\tan(c+dx))^5}\right)}{5a}+\frac{i\cos^3(c+dx)}{15d(a+ia\tan(c+dx))^6}\right)}{17a}+\frac{i\cos^3(c+dx}{17d(a+ia\tan(c+}\right)}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$\Big\downarrow$ 3983

$$\frac{11\left(\frac{10\left(\frac{3\left(\frac{8\left(\frac{7\left(\frac{2\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}\right)}{13a}+\frac{i\cos^3(c+dx)}{13d(a+ia\tan(c+dx))^5}\right)}{5a}+\frac{i\cos^3(c+dx)}{15d(a+ia\tan(c+dx))^6}\right)}{17a}+\frac{}{1}\right)}{19a}$$

$$\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \; 3042$$

$$11\left(\frac{10\left(\frac{3\left(\frac{8\left(\frac{7\left(\frac{2\int\frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^2}dx}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}\right)}{13a}+\frac{i\cos^3(c+dx)}{13d(a+ia\tan(c+dx))^5}\right)}{5a}+\frac{i\cos^3(c+dx)}{15d(a+ia\tan(c+dx}}{17a}\right.$$

$$\frac{\quad}{19a}$$

$$\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$\downarrow$ 3981

$$11\left(\frac{10\left(\frac{3\left(\frac{8\left(\frac{7\left(\frac{2\left(\frac{5\int\cos^5(c+dx)dx}{7a^2}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}\right)}{13a}+\frac{i\cos^3(c+dx)}{13d(a+ia\tan(c+dx))^5}\right)}{5a}+\frac{i\,\mathrm{co}}{15d(a+}}{17a}\right.$$

$\downarrow$ 3042

$$11\left(\frac{10\left(\frac{3\left(\frac{8\left(\frac{7\left(\frac{2\left(\frac{5\int\sin\left(c+dx+\frac{\pi}{2}\right)^5dx}{7a^2}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}\right)}{13a}+\frac{i\cos^3(c+dx)}{13d(a+ia\tan(c+dx))^5}\right)}{5a}+\frac{}{15a}}{17a}\right.$$

↓ 3113

$$11\left(\frac{10\left(\frac{3\left(\frac{8\left(\frac{7\left(\frac{2\left(-\frac{5\int\left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}\right)}{13a}+\cdots\right)}{5a}\cdots\right)}{17a}\cdots\right.$$

$\downarrow$ 2009

$$11\left(\frac{10\left(\frac{3\left(\frac{8\left(\frac{7\left(\frac{2\left(-\frac{5\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}\right)}{13a}+\frac{}{13d(}\right.}{5a}\right.}{17a}\right.$$

input
```
Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/19)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^8) + (11*(((I/17)*Cos[c
+ d*x]^3)/(d*(a + I*a*Tan[c + d*x])^7) + (10*(((I/15)*Cos[c + d*x]^3)/(d*(
a + I*a*Tan[c + d*x])^6) + (3*(((I/13)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c +
 d*x])^5) + (8*(((I/11)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^4) + (7*
(((I/9)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3) + (2*((-5*(-Sin[c + d
*x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]^5/5))/(7*a^2*d) + (((2*I)/7)*Cos
[c + d*x]^5)/(d*(a^2 + I*a^2*Tan[c + d*x]))))/(3*a)))/(11*a)))/(13*a)))/(5
*a)))/(17*a)))/(19*a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.70

method	result
risch	$\frac{33ie^{-5i(dx+c)}}{1024a^8d}+\frac{33ie^{-7i(dx+c)}}{1024a^8d}+\frac{77ie^{-9i(dx+c)}}{3072a^8d}+\frac{15ie^{-11i(dx+c)}}{1024a^8d}+\frac{165ie^{-13i(dx+c)}}{26624a^8d}+\frac{11ie^{-15i(dx+c)}}{6144a^8d}+$
derivativedivides	$\frac{\frac{8856i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{14}}-\frac{1}{768\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{3}}+\frac{3}{256\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{7181i}{512\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{i}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{2}}+}{}$
default	$\frac{\frac{8856i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{14}}-\frac{1}{768\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{3}}+\frac{3}{256\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{7181i}{512\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{i}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{2}}+}{}$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
33/1024*I/a^8/d*exp(-5*I*(d*x+c))+33/1024*I/a^8/d*exp(-7*I*(d*x+c))+77/307
2*I/a^8/d*exp(-9*I*(d*x+c))+15/1024*I/a^8/d*exp(-11*I*(d*x+c))+165/26624*I
/a^8/d*exp(-13*I*(d*x+c))+11/6144*I/a^8/d*exp(-15*I*(d*x+c))+11/34816*I/a^
8/d*exp(-17*I*(d*x+c))+1/38912*I/a^8/d*exp(-19*I*(d*x+c))+11/512*I/a^8/d*c
os(d*x+c)+33/1024*sin(d*x+c)/a^8/d+41/1536*I/a^8/d*cos(3*d*x+3*c)+83/3072/
a^8/d*sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.47

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(-4199i\,e^{(22i\,dx+22i\,c)}-138567i\,e^{(20i\,dx+20i\,c)}+692835i\,e^{(18i\,dx+18i\,c)}+692835i\,e^{(16i\,dx+16i\,c)}+831402i\,e^{(14\cdots}\right.}{}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/25798656*(-4199*I*e^(22*I*d*x + 22*I*c) - 138567*I*e^(20*I*d*x + 20*I*c)
 + 692835*I*e^(18*I*d*x + 18*I*c) + 692835*I*e^(16*I*d*x + 16*I*c) + 83140
2*I*e^(14*I*d*x + 14*I*c) + 831402*I*e^(12*I*d*x + 12*I*c) + 646646*I*e^(1
0*I*d*x + 10*I*c) + 377910*I*e^(8*I*d*x + 8*I*c) + 159885*I*e^(6*I*d*x + 6
*I*c) + 46189*I*e^(4*I*d*x + 4*I*c) + 8151*I*e^(2*I*d*x + 2*I*c) + 663*I)*
e^(-19*I*d*x - 19*I*c)/(a^8*d)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.45

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$

$$= \begin{cases} \frac{(-6279106898588469469113471576881812733952ia^{88}d^{11}e^{103ic}e^{3idx}-2072105276534194924807445620370998202220416ia^{88}d^{11}e^{101ic}e^{idx}+ \\ \frac{x(e^{22ic}+11e^{20ic}+55e^{18ic}+165e^{16ic}+330e^{14ic}+462e^{12ic}+462e^{10ic}+330e^{8ic}+165e^{6ic}+55e^{4ic}+11e^{2ic}+1)e^{-19ic}}{2048a^8} \end{cases}$$

input
```
integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-627910689858846946911347157688181273395 2*I*a**88*d**11*exp(10
3*I*c)*exp(3*I*d*x) - 2072105276534194924807445620370998202 20416*I*a**88*d
**11*exp(101*I*c)*exp(I*d*x) + 10360526382670974624037228101854991 01102080
*I*a**88*d**11*exp(99*I*c)*exp(-I*d*x) + 103605263826709746240372281018549
9101102080*I*a**88*d**11*exp(97*I*c)*exp(-3*I*d*x) + 124326316592051695488
44673722225989213 22496*I*a**88*d**11*exp(95*I*c)*exp(-5*I*d*x) + 124326316
592051695488446737222259892132 2496*I*a**88*d**11*exp(93*I*c)*exp(-7*I*d*x)
 + 966982462382624298243474622839799161028608*I*a**88*d**11*exp(91*I*c)*ex
p(-9*I*d*x) + 565119620872962252220212441919363146055680*I*a**88*d**11*exp
(89*I*c)*exp(-11*I*d*x) + 239089070369330183631628340812038254100480*I*a**
88*d**11*exp(87*I*c)*exp(-13*I*d*x) + 690701758844731641602481873456999400
73472*I*a**88*d**11*exp(85*I*c)*exp(-15*I*d*x) + 121888545678482054400437 9
776688224718848*I*a**88*d**11*exp(83*I*c)*exp(-17*I*d*x) + 99143793135607
41267021270910866020 10624*I*a**88*d**11*exp(81*I*c)*exp(-19*I*d*x))*exp(-1
00*I*c)/(3857883278492755641823316936836185743740 1088*a**96*d**12), Ne(a**
96*d**12*exp(100*I*c), 0)), (x*(exp(22*I*c) + 11*exp(20*I*c) + 55*exp(18*I
*c) + 165*exp(16*I*c) + 330*exp(14*I*c) + 462*exp(12*I*c) + 462*exp(10*I*c
) + 330*exp(8*I*c) + 165*exp(6*I*c) + 55*exp(4*I*c) + 11*exp(2*I*c) + 1)*e
xp(-19*I*c)/(2048*a**8), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\frac{4199\left(18\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+33i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-17\right)}{a^8\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)^3}+\frac{12823746\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{18}-140368371i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}-879644311\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/6449664*(4199*(18*tan(1/2*d*x + 1/2*c)^2 + 33*I*tan(1/2*d*x + 1/2*c) - 1
7)/(a^8*(tan(1/2*d*x + 1/2*c) + I)^3) + (12823746*tan(1/2*d*x + 1/2*c)^18
- 140368371*I*tan(1/2*d*x + 1/2*c)^17 - 879644311*tan(1/2*d*x + 1/2*c)^16
+ 3693272440*I*tan(1/2*d*x + 1/2*c)^15 + 11467502592*tan(1/2*d*x + 1/2*c)^
14 - 27403194676*I*tan(1/2*d*x + 1/2*c)^13 - 51919375300*tan(1/2*d*x + 1/2
*c)^12 + 79183835016*I*tan(1/2*d*x + 1/2*c)^11 + 98304418212*tan(1/2*d*x +
 1/2*c)^10 - 99750226290*I*tan(1/2*d*x + 1/2*c)^9 - 82860874122*tan(1/2*d*
x + 1/2*c)^8 + 56110430792*I*tan(1/2*d*x + 1/2*c)^7 + 30766700912*tan(1/2*
d*x + 1/2*c)^6 - 13462452660*I*tan(1/2*d*x + 1/2*c)^5 - 4616712644*tan(1/2
*d*x + 1/2*c)^4 + 1197851960*I*tan(1/2*d*x + 1/2*c)^3 + 226248618*tan(1/2*
d*x + 1/2*c)^2 - 27911475*I*tan(1/2*d*x + 1/2*c) - 2143959)/(a^8*(tan(1/2*
d*x + 1/2*c) - I)^19))/d
```

Mupad [B] (verification not implemented)

Time = 6.93 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.02

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$
$$-\frac{2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{46189\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{64}-\frac{46189\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{64}-\frac{20995\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{16}+\frac{20995\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{16}-\frac{221255\,c}{}\right.}{}$$

input
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
-(2*cos(c/2 + (d*x)/2)*((46189*cos((5*c)/2 + (5*d*x)/2))/64 - (46189*cos((
3*c)/2 + (3*d*x)/2))/64 - (20995*cos((7*c)/2 + (7*d*x)/2))/16 + (20995*cos
((9*c)/2 + (9*d*x)/2))/16 - (221255*cos((11*c)/2 + (11*d*x)/2))/128 + (221
255*cos((13*c)/2 + (13*d*x)/2))/128 - (66861*cos((15*c)/2 + (15*d*x)/2))/3
2 + (2093*cos((17*c)/2 + (17*d*x)/2))/32 - (221*cos((19*c)/2 + (19*d*x)/2)
)/128 + (221*cos((21*c)/2 + (21*d*x)/2))/128 + (sin(c/2 + (d*x)/2)*309861i
)/256 - (sin((3*c)/2 + (3*d*x)/2)*665911i)/512 + (sin((5*c)/2 + (5*d*x)/2)
*665911i)/512 - (sin((7*c)/2 + (7*d*x)/2)*194821i)/128 + (sin((9*c)/2 + (9
*d*x)/2)*194821i)/128 - (sin((11*c)/2 + (11*d*x)/2)*1825043i)/1024 + (sin(
(13*c)/2 + (13*d*x)/2)*1825043i)/1024 - (sin((15*c)/2 + (15*d*x)/2)*107418
3i)/512 + (sin((17*c)/2 + (17*d*x)/2)*37895i)/512 - (sin((19*c)/2 + (19*d*
x)/2)*2431i)/1024 + (sin((21*c)/2 + (21*d*x)/2)*2431i)/1024))/(12597*a^8*d
*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)*1i)^19*(cos(c/2 + (d*x)/2)*1i +
sin(c/2 + (d*x)/2))^3)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$=\frac{\int \frac{\cos(dx+c)^3}{\tan(dx+c)^8-8\tan(dx+c)^7 i-28\tan(dx+c)^6+56\tan(dx+c)^5 i+70\tan(dx+c)^4-56\tan(dx+c)^3 i-28\tan(dx+c)^2+8\tan(dx+c) i+1}dx}{a^8}$$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x)
```

output
```
int(cos(c + d*x)**3/(tan(c + d*x)**8 - 8*tan(c + d*x)**7*i - 28*tan(c + d*
x)**6 + 56*tan(c + d*x)**5*i + 70*tan(c + d*x)**4 - 56*tan(c + d*x)**3*i -
 28*tan(c + d*x)**2 + 8*tan(c + d*x)*i + 1),x)/a**8
```

3.185 $\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 123

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = -\frac{6ae^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}+\frac{6ae^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2ae(e\sec(c+dx))^{5/2}\sin(c+dx)}{5d}$$

output
```
-6/5*a*e^4*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec
(d*x+c))^(1/2)+2/7*I*a*(e*sec(d*x+c))^(7/2)/d+6/5*a*e^3*(e*sec(d*x+c))^(1/
2)*sin(d*x+c)/d+2/5*a*e*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.97 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.27

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \frac{aee^{-idx}(e\sec(c+dx))^{5/2}(\cos(dx)-i\sin(dx))(\cos(c+3dx)+i\sin(c+3dx))\left(-36i\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*e*(e*Sec[c + d*x])^(5/2)*(Cos[d*x] - I*Sin[d*x])*(Cos[c + 3*d*x] + I*Si
n[c + 3*d*x])*(-36*I - (28*I)*Cos[2*(c + d*x)] + ((7*I)*(1 + E^((2*I)*(c +
 d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((
2*I)*(c + d*x)) + 7*Sec[c + d*x]*Sin[3*(c + d*x)] + 27*Tan[c + d*x]))/(70*
d*E^(I*d*x))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3967, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int (e\sec(c+dx))^{7/2}dx+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{5}e^2\int (e\sec(c+dx))^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{ 4255}$$

$$a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{ 3042}$$

$$a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{ 4258}$$

$$a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{ 3042}$$

$$a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{ 3119}$$

$$a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}$$

input
```
Int[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(((2*I)/7)*a*(e*Sec[c + d*x])^(7/2))/d + a*((2*e*(e*Sec[c + d*x])^(5/2)*Si
n[c + d*x])/(5*d) + (3*e^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos
[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])
/d))/5)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(106) = 212.

Time = 33.58 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.86

method	result
default	$\frac{2a\sqrt{e\sec(dx+c)}\left(21\sin(dx+c)+7\tan(dx+c)+7\sec(dx+c)\tan(dx+c)+5i\left(\sec(dx+c)^3+\sec(dx+c)^2\right)+21i\left(-\cos(dx+c)^2-2\cos(d\right.\right.}{}$
parts	$-\frac{2a\sqrt{e\sec(dx+c)}\,e^3\left(-3\sin(dx+c)-\tan(dx+c)-\sec(dx+c)\tan(dx+c)+i\left(3\cos(dx+c)^2+6\cos(dx+c)+3\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(}{\cos(d}}\right.}{}$

input
```
int((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/35*a/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(21*sin(d*x+c)+7*tan(d*x+c)+7
*sec(d*x+c)*tan(d*x+c)+5*I*(sec(d*x+c)^3+sec(d*x+c)^2)+21*I*(-cos(d*x+c)^2
-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)
-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+21*I*(cos(d*x+c)^2+2*cos(d*x+c)+1
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)
*(1/(cos(d*x+c)+1))^(1/2))*e^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.69

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(21i\,ae^3e^{(7i\,dx+7i\,c)}+77i\,ae^3e^{(5i\,dx+5i\,c)}+23i\,ae^3e^{(3i\,dx+3i\,c)}+7i\,ae^3e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\right.}}{35\left(de^{(6i\,c}\right.}$$

input
```
integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2/35*(sqrt(2)*(21*I*a*e^3*e^(7*I*d*x + 7*I*c) + 77*I*a*e^3*e^(5*I*d*x + 5
*I*c) + 23*I*a*e^3*e^(3*I*d*x + 3*I*c) + 7*I*a*e^3*e^(I*d*x + I*c))*sqrt(e
/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 21*sqrt(2)*(I*a*e^3*
e^(6*I*d*x + 6*I*c) + 3*I*a*e^3*e^(4*I*d*x + 4*I*c) + 3*I*a*e^3*e^(2*I*d*x
 + 2*I*c) + I*a*e^3)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, e^(I*d*x + I*c))))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) +
 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx=\text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(7/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx=\int (e\sec(dx+c))^{\frac{7}{2}}(i\,a\tan(dx+c)+a)\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a), x)
```

Giac [F]

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \int (e\sec(dx+c))^{\frac{7}{2}}(i\,a\tan(dx+c)+a)\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}(a+a\tan(c+d\,x)\;1\mathrm{i})\,dx$$

input
```
int((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \frac{\sqrt{e}\,a\,e^{3}\left(2\sqrt{\sec(dx+c)}\,\sec(dx+c)^{3}i+7\left(\int\sqrt{\sec(dx+c)}\,\sec(dx+c)^{3}dx\right)d\right)}{7d}$$

input
```
int((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
(sqrt(e)*a*e**3*(2*sqrt(sec(c + d*x))*sec(c + d*x)**3*i + 7*int(sqrt(sec(c
 + d*x))*sec(c + d*x)**3,x)*d))/(7*d)
```

3.186 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 94

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \frac{2ae^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d} + \frac{2ia(e\sec(c+dx))^{5/2}}{5d} + \frac{2ae(e\sec(c+dx))^{3/2}\sin(c+dx)}{3d}$$

output
```
2/3*a*e^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d
*x+c))^(1/2)/d+2/5*I*a*(e*sec(d*x+c))^(5/2)/d+2/3*a*e*(e*sec(d*x+c))^(3/2)
*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \frac{a(e\sec(c+dx))^{5/2}\left(6i+10\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))\right)}{15d}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*(e*Sec[c + d*x])^(5/2)*(6*I + 10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)
/2, 2] + 5*Sin[2*(c + d*x)]))/(15*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3967, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int (e\sec(c+dx))^{5/2}dx+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{3}e^2\int \sqrt{e\sec(c+dx)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}$$

input
```
Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(((2*I)/5)*a*(e*Sec[c + d*x])^(5/2))/d + a*((2*e^2*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d) + (2*e*(e*Sec[c + d*x])^
(3/2)*Sin[c + d*x])/(3*d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 32.66 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09

method	result
default	$\frac{a\left(\frac{2\tan(dx+c)}{3}+\frac{2i\sec(dx+c)^2}{5}-\frac{2i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)}{3}\right)e^2\sqrt{e\sec(dx+c)}}{d}$
parts	$\frac{a\left(-\frac{2i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)}{3}+\frac{2\tan(dx+c)}{3}\right)e^2\sqrt{e\sec(dx+c)}}{d}+\frac{2ia(e\sec(d}{5d}$

input
```
int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a/d*(2/3*tan(d*x+c)+2/5*I*sec(d*x+c)^2-2/3*I*(cos(d*x+c)+1)*(1/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d
*x+c)),I))*e^2*(e*sec(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.64

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx =$$
$$-\frac{2\left(\sqrt{2}\left(5i\,ae^2e^{(4i\,dx+4i\,c)}-12i\,ae^2e^{(2i\,dx+2i\,c)}-5i\,ae^2\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+5\sqrt{2}\left(i\,ae^2e^{(4i\,dx+4i\,c)}+2\right.}{15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2/15*(sqrt(2)*(5*I*a*e^2*e^(4*I*d*x + 4*I*c) - 12*I*a*e^2*e^(2*I*d*x + 2*
I*c) - 5*I*a*e^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c
) + 5*sqrt(2)*(I*a*e^2*e^(4*I*d*x + 4*I*c) + 2*I*a*e^2*e^(2*I*d*x + 2*I*c)
 + I*a*e^2)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(d*e^(4*I
*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = ia\left(\int \left(-i(e\sec{(c+dx)})^{\frac{5}{2}}\right)\,dx + \int (e\sec{(c+dx)})^{\frac{5}{2}}\tan{(c+dx)}\,dx\right)$$

input
```
integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*(e*sec(c + d*x))**(5/2), x) + Integral((e*sec(c + d*x))**
(5/2)*tan(c + d*x), x))
```

Maxima [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \int (e\sec(dx+c))^{\frac{5}{2}}(i\,a\tan(dx+c)+a)\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a), x)
```

Giac [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \int (e\sec(dx+c))^{\frac{5}{2}}(i\,a\tan(dx+c)+a)\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/2}(a + ia\tan(c+dx))\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}(a+a\tan(c+d\,x)\ 1\mathrm{i})\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \frac{\sqrt{e}\,a\,e^2\left(2\sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,i+5\left(\int\sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)d\right)}{5d}$$

input
```
int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
(sqrt(e)*a*e**2*(2*sqrt(sec(c + d*x))*sec(c + d*x)**2*i + 5*int(sqrt(sec(c
 + d*x))*sec(c + d*x)**2,x)*d))/(5*d)
```

3.187 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 90

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = -\frac{2ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2ia(e\sec(c+dx))^{3/2}}{3d} + \frac{2ae\sqrt{e\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
-2*a*e^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec(d
*x+c))^(1/2)+2/3*I*a*(e*sec(d*x+c))^(3/2)/d+2*a*e*(e*sec(d*x+c))^(1/2)*sin
(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.92 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \frac{2aee^{-2idx}\sqrt{e\sec(c+dx)}(\cos(c+3dx)+i\sin(c+3dx))\left(-2i+i\sqrt{1+e^{2i(c+dx)}}\,\mathrm{Hype}\right.}{3d}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(2*a*e*Sqrt[e*Sec[c + d*x]]*(Cos[c + 3*d*x] + I*Sin[c + 3*d*x])*(-2*I + I*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(
c + d*x))] + Tan[c + d*x]))/(3*d*E^((2*I)*d*x))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3967, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int (e\sec(c+dx))^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(((2*I)/3)*a*(e*Sec[c + d*x])^(3/2))/d + a*((-2*e^2*EllipticE[(c + d*x)/2,
 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x
]]*Sin[c + d*x])/d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(81) = 162$.

Time = 7.31 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.18

method	result
default	$\frac{2a\left(3i\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}+3i\left(-\cos(dx+c)^2-2\cos\right.\right.}{3d(\cos(dx+}$
parts	$\frac{2a\left(i\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}+i\left(-\cos(dx+c)^2-2\cos(d\right.\right.}{d(\cos(dx+c)+1)}$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/3*a/d*(3*I*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+3*I*(-c
os(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*
(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+I+I*sec(d*x+c)+3*sin(d
*x+c))*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.29

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(3i\,aee^{(3i\,dx+3i\,c)}+i\,aee^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+3\,\sqrt{2}\left(i\,aee^{(2i\,dx+2i\,c)}+i\,ae\right)\sqrt{e}\text{weierstra}}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(2)*(3*I*a*e*e^(3*I*d*x + 3*I*c) + I*a*e*e^(I*d*x + I*c))*sqrt(e
/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 3*sqrt(2)*(I*a*e*e^(
2*I*d*x + 2*I*c) + I*a*e)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, e^(I*d*x + I*c))))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = ia\left(\int \left(-i(e\sec(c+dx))^{\frac{3}{2}}\right)\,dx + \int (e\sec(c+dx))^{\frac{3}{2}}\tan(c+dx)\,dx\right)$$

input
```
integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*(e*sec(c + d*x))**(3/2), x) + Integral((e*sec(c + d*x))**
(3/2)*tan(c + d*x), x))
```

Maxima [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \int (e\sec(dx+c))^{\frac{3}{2}}\,(i\,a\tan(dx+c)+a)\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a), x)
```

Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \int (e\sec(dx+c))^{\frac{3}{2}}\,(i\,a\tan(dx+c)+a)\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a + ia\tan(c+dx))\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}\,(a+a\tan(c+d\,x)\,1\mathrm{i})\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \frac{\sqrt{e}\,ae\left(2\sqrt{\sec(dx+c)}\,\sec(dx+c)\,i+3\left(\int\sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)d\right)}{3d}$$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
(sqrt(e)*a*e*(2*sqrt(sec(c + d*x))*sec(c + d*x)*i + 3*int(sqrt(sec(c + d*x
))*sec(c + d*x),x)*d))/(3*d)
```

3.188 $\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = \frac{2ia\sqrt{e\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}$$

output
```
2*I*a*(e*sec(d*x+c))^(1/2)/d+2*a*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+
1/2*c,2^(1/2))*(e*sec(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = \frac{2a\left(i+\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)\sqrt{e\sec(c+dx)}}{d}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

output
```
(2*a*(I + Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[e*Sec[c + d*x
]])/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sqrt{e\sec(c+dx)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4258}$$

$$a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}}{d}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

output
```
((2*I)*a*Sqrt[e*Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 5.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.37

method	result	s
default	$\frac{ia\left(2-2\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\cos(dx+c)+1)\operatorname{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)\right)\sqrt{e\sec(dx+c)}}{d}$	8
parts	$-\frac{2ia(\cos(dx+c)+1)\sqrt{e\sec(dx+c)}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)}{d}+\frac{2ia\sqrt{e\sec(dx+c)}}{d}$	9

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
I*a/d*(2-2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)+1)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I))*(e*sec(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = -\frac{2\left(-i\sqrt{2}a\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+i\sqrt{2}a\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{d}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2*(-I*sqrt(2)*a*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)
 + I*sqrt(2)*a*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/d
```

Sympy [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = ia\left(\int\left(-i\sqrt{e\sec{(c+dx)}}\right)dx + \int\sqrt{e\sec{(c+dx)}}\tan{(c+dx)}\,dx\right)$$

input
```
integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*sqrt(e*sec(c + d*x)), x) + Integral(sqrt(e*sec(c + d*x))*
tan(c + d*x), x))
```

Maxima [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = \int \sqrt{e\sec(dx+c)}(i\,a\tan(dx+c)+a)\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[2,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{1,[1]%%%}]%%},[1,0]%%%
}+%%%{%%%
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = \frac{2\,a\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+1\mathrm{i}\right)\,\sqrt{\frac{e}{\cos(c+d\,x)}}}{d}$$

input
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i),x)
```

output
```
(2*a*(cos(c + d*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2) + 1i)*(e/cos(c + d*x)
)^(1/2))/d
```

Reduce [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = \frac{\sqrt{e}\,a\left(2\sqrt{\sec\left(dx+c\right)}\,i+\left(\int\sqrt{\sec\left(dx+c\right)}dx\right)d\right)}{d}$$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
(sqrt(e)*a*(2*sqrt(sec(c + d*x))*i + int(sqrt(sec(c + d*x)),x)*d))/d
```

3.189 $\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx=-\frac{2ia}{d\sqrt{e\sec(c+dx)}}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

output
```
-2*I*a/d/(e*sec(d*x+c))^(1/2)+2*a*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/
cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.66 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx=-\frac{4iae^{2i(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)}{3d\sqrt{1+e^{2i(c+dx)}}\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])/Sqrt[e*Sec[c + d*x]],x]
```

output
```
(((-4*I)/3)*a*E^((2*I)*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*
I)*(c + d*x))])/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3967, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \frac{1}{\sqrt{e\sec(c+dx)}}dx-\frac{2ia}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2ia}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}$$

input
```
Int[(a + I*a*Tan[c + d*x])/Sqrt[e*Sec[c + d*x]],x]
```

output
```
((-2*I)*a)/(d*Sqrt[e*Sec[c + d*x]]) + (2*a*EllipticE[(c + d*x)/2, 2])/(d*S
qrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(55) = 110$.

Time = 4.98 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.23

method	result
parts	$\frac{2a\left(i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(-\cos(dx+c)-2-\sec(dx+c))+i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx}{\cos(dx+}}\right.}{d(\cos(dx+c)+1)\sqrt{e\sec(dx+c)}}$
risch	$-\frac{2ia\sqrt{2}}{d\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\left(-2i\,\mathrm{EllipticE}\left(\sqrt{-i\left(\mathrm{e}^{i(dx}\right.}\right.\right.}{\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx+c)}}}}{d\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\left(\mathrm{e}^{2i(dx+c)}+\right.}$
default	$\frac{a\left(i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}\,\mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,(-4\cos(dx+c)-8-4\sec(dx+c))+i\sqrt{\frac{c}{}}\right.}{}$

input
```
int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(-cos
(d*x+c)-2-sec(d*x+c))+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)+2+sec(d*x+c))+
sin(d*x+c))-2*I*a/d/(e*sec(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.45

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx = \frac{2i\sqrt{2}a\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{d\sqrt{e}}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x +
 I*c)))/(d*sqrt(e))
```

Sympy [F]

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx = ia\left(\int \left(-\frac{i}{\sqrt{e\sec{(c+dx)}}}\right)dx + \int \frac{\tan{(c+dx)}}{\sqrt{e\sec{(c+dx)}}}\,dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(1/2),x)
```

output
```
I*a*(Integral(-I/sqrt(e*sec(c + d*x)), x) + Integral(tan(c + d*x)/sqrt(e*s
ec(c + d*x)), x))
```

Maxima [F]

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{i\,a\tan{(dx+c)}+a}{\sqrt{e\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)/sqrt(e*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{i\,a\tan{(dx+c)}+a}{\sqrt{e\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)/sqrt(e*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{a+a\tan(c+d\,x)\,1\mathrm{i}}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx=\frac{\sqrt{e}\,a\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx+\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)}{\sec(dx+c)}dx\right)i\right)}{e}$$

input
```
int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*a*(int(sqrt(sec(c + d*x))/sec(c + d*x),x) + int((sqrt(sec(c + d*x
))*tan(c + d*x))/sec(c + d*x),x)*i))/e
```

3.190 $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2ia}{3d(e\sec(c+dx))^{3/2}} + \frac{2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2} + \frac{2a\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}$$

output
```
-2/3*I*a/d/(e*sec(d*x+c))^(3/2)+2/3*a*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2
*d*x+1/2*c,2^(1/2))*(e*sec(d*x+c))^(1/2)/d/e^2+2/3*a*sin(d*x+c)/d/e/(e*sec
(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx = \frac{2a\left(-i\cos(c+dx)+\frac{\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{\sqrt{\cos(c+dx)}}+\sin(c+dx)\right)}{3de\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(2*a*((-I)*Cos[c + d*x] + EllipticF[(c + d*x)/2, 2]/Sqrt[Cos[c + d*x]] + S
in[c + d*x]))/(3*d*e*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3967, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \frac{1}{(e\sec(c+dx))^{3/2}}dx-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(((-2*I)/3)*a)/(d*(e*Sec[c + d*x])^(3/2)) + a*((2*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3
*d*e*Sqrt[e*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 6.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06

method	result
default	$\frac{a\left(-\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)(-1-\sec(dx+c))}{3}-\frac{2i\cos(dx+c)}{3}+\frac{2\sin(dx+c)}{3}\right)}{d\sqrt{e\sec(dx+c)}\,e}$
parts	$\frac{a\left(-\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(\sec(dx+c)+1)}{3}+\frac{2\sin(dx+c)}{3}\right)}{d\sqrt{e\sec(dx+c)}\,e}-\frac{2ia}{3d(e\sec(dx+c))^{\frac{3}{2}}}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}a\sqrt{2}}{3de\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}+\frac{2\sqrt{-i(\mathrm{e}^{i(dx+c)}+i)}\sqrt{i(\mathrm{e}^{i(dx+c)}-i)}\sqrt{i\mathrm{e}^{i(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{-i(\mathrm{e}^{i(dx+c)}+i)},\frac{\sqrt{2}}{2}\right)a\sqrt{e\,\mathrm{e}^{i(dx+c)}(\mathrm{e}^{2i(dx+c)}+}}{3d\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx+c)}}\,e\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}$

input
```
int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a/d*(-2/3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ell
ipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(-1-sec(d*x+c))-2/3*I*cos(d*x+c)+2/3*s
in(d*x+c))/(e*sec(d*x+c))^(1/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.79

$$\int \frac{a + ia\tan(c + dx)}{(e\sec(c + dx))^{3/2}}\,dx = \frac{\sqrt{2}\left(-i\,ae^{(2i\,dx+2i\,c)} - i\,a\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)} - 2i\,\sqrt{2}a\sqrt{e}\text{weierstrassPInve}}{3\,de^2}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-I*a*e^(2*I*d*x + 2*I*c) - I*a)*sqrt(e/(e^(2*I*d*x + 2*I*c)
+ 1))*e^(1/2*I*d*x + 1/2*I*c) - 2*I*sqrt(2)*a*sqrt(e)*weierstrassPInverse(
-4, 0, e^(I*d*x + I*c)))/(d*e^2)
```

Sympy [F]

$$\int \frac{a + ia\tan(c + dx)}{(e\sec(c + dx))^{3/2}}\,dx = ia\left(\int \left(-\frac{i}{(e\sec{(c + dx)})^{\frac{3}{2}}}\right)dx + \int \frac{\tan{(c + dx)}}{(e\sec{(c + dx)})^{\frac{3}{2}}}\,dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(3/2),x)
```

output
```
I*a*(Integral(-I/(e*sec(c + d*x))**(3/2), x) + Integral(tan(c + d*x)/(e*se
c(c + d*x))**(3/2), x))
```

Maxima [F]

$$\int \frac{a + ia\tan(c + dx)}{(e\sec(c + dx))^{3/2}}\,dx = \int \frac{i\,a\tan{(dx + c)} + a}{(e\sec{(dx + c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{a+a\tan(c+d\,x)\ 1\mathrm{i}}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{e}\,a\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx + \left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)}{\sec(dx+c)^2}dx\right)i\right)}{e^2}$$

input
```
int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*a*(int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x) + int((sqrt(sec(c +
d*x))*tan(c + d*x))/sec(c + d*x)**2,x)*i))/e**2
```

3.191 $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2ia}{5d(e\sec(c+dx))^{5/2}} + \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2a\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}$$

output
```
-2/5*I*a/d/(e*sec(d*x+c))^(5/2)+6/5*a*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)/d/e^2/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+2/5*a*sin(d*x+c)/d/e/(e*sec(
d*x+c))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{a\left(2+2\cos(2(c+dx))-2\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3i\sin(2(c+dx))\right)(\cdots}{5de^2\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(5/2),x]
```

output
```
-1/5*(a*(2 + 2*Cos[2*(c + d*x)] - 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] - (3*I)*Sin[2*(c + d*x)])*(
-I + Tan[c + d*x]))/(d*e^2*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3967, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \frac{1}{(e\sec(c+dx))^{5/2}}dx-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(((-2*I)/5)*a)/(d*(e*Sec[c + d*x])^(5/2)) + a*((6*EllipticE[(c + d*x)/2, 2
])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*Sin[c + d*x])/(5
*d*e*(e*Sec[c + d*x])^(3/2)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(83) = 166$.

Time = 10.00 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.19

method	result
parts	$\frac{2a\left(\sin(dx+c)\left(\cos(dx+c)^2+\cos(dx+c)+3\right)-3i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\cos(dx+c)+2+\sec(dx+c))\,\mathrm{EllipticF}(i(\csc(dx+c)-c}{5d(\cos(dx+c)+1)\sqrt{e\sec(}$
default	$\frac{2a\left(\sin(dx+c)\left(\cos(dx+c)^2+\cos(dx+c)+3\right)+i\left(-\cos(dx+c)^3-\cos(dx+c)^2\right)+3i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticE}(i(\csc(dx+}{5d(\cos(dx}$
risch	$-\frac{i\left(\mathrm{e}^{2i(dx+c)}+7\right)a\sqrt{2}}{10d\,e^2\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{3i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\sqrt{i\mathrm{e}^{i(dx+c)}}\left(-2i\,\mathrm{EllipticE}\left(\sqrt{-i}\right.\right.}{\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(d}}}}{5d\,e^2\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{e}{\mathrm{e}^{2i}}}}$

input
```
int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*a/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)/e^2*(sin(d*x+c)*(cos(d*x+c)^2+
cos(d*x+c)+3)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)+2+sec(d*x+c))*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)+3*I*(1
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+2+sec
(d*x+c))*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I))-2/5*I*a/d/(e*sec(d*x+c))^
(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.14

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\left(12i\sqrt{2}a\sqrt{e}e^{(i\,dx+i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c}\right.\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/10*(12*I*sqrt(2)*a*sqrt(e)*e^(I*d*x + I*c)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-I*a*e^(4*I*d*x + 4*I*c)
 + 4*I*a*e^(2*I*d*x + 2*I*c) + 5*I*a)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^
(1/2*I*d*x + 1/2*I*c))*e^(-I*d*x - I*c)/(d*e^3)
```

Sympy [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = ia\left(\int\left(-\frac{i}{(e\sec(c+dx))^{\frac{5}{2}}}\right)dx+\int\frac{\tan(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(5/2),x)
```

output
```
I*a*(Integral(-I/(e*sec(c + d*x))**(5/2), x) + Integral(tan(c + d*x)/(e*se
c(c + d*x))**(5/2), x))
```

Maxima [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{a+a\tan(c+d\,x)\,1\mathrm{i}}{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{e}\,a\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx + \left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)}{\sec(dx+c)^3}dx\right)i\right)}{e^3}$$

input
```
int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*a*(int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x) + int((sqrt(sec(c +
d*x))*tan(c + d*x))/sec(c + d*x)**3,x)*i))/e**3
```

3.192 $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 125

$$\begin{aligned}\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx &= -\frac{2ia}{7d(e\sec(c+dx))^{7/2}} \\ &+ \frac{10a\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21de^4} \\ &+ \frac{2a\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}} + \frac{10a\sin(c+dx)}{21de^3\sqrt{e\sec(c+dx)}}\end{aligned}$$

output
```
-2/7*I*a/d/(e*sec(d*x+c))^(7/2)+10/21*a*cos(d*x+c)^(1/2)*InverseJacobiAM(1
/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x+c))^(1/2)/d/e^4+2/7*a*sin(d*x+c)/d/e/(e*s
ec(d*x+c))^(5/2)+10/21*a*sin(d*x+c)/d/e^3/(e*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = \frac{a\sqrt{e\sec(c+dx)}(\cos(c+dx)+i\sin(c+dx))\left(-14i\cos(c+dx)+2i\cos(3(c+\right.}{}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(a*Sqrt[e*Sec[c + d*x]]*(Cos[c + d*x] + I*Sin[c + d*x])*((-14*I)*Cos[c + d
*x] + (2*I)*Cos[3*(c + d*x)] + 20*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*(Cos[c + d*x] - I*Sin[c + d*x]) + 5*Sin[c + d*x] + 5*Sin[3*(c + d*x)]
))/(42*d*e^4)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3967, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \frac{1}{(e\sec(c+dx))^{7/2}}dx-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{5\left(\frac{\int\sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5\left(\frac{\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(((-2*I)/7)*a)/(d*(e*Sec[c + d*x])^(7/2)) + a*((2*Sin[c + d*x])/(7*d*e*(e*
Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c +
d*x]])))/(7*e^2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 10.94 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.93

method	result
default	$-\frac{2a\left(\sin(dx+c)\left(-3\cos(dx+c)^2-5\right)+i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(5+5\sec(dx+c))+3i\cos$
parts	$-\frac{2a\left(\sin(dx+c)\left(-3\cos(dx+c)^2-5\right)+i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(5+5\sec(dx+c))\right)}{21d\sqrt{e\sec(dx+c)}\,e^3}-$

input
```
int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*a/d/(e*sec(d*x+c))^(1/2)/e^3*(sin(d*x+c)*(-3*cos(d*x+c)^2-5)+I*(1/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x
+c)-cot(d*x+c)),I)*(5+5*sec(d*x+c))+3*I*cos(d*x+c)^3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\left(-40i\sqrt{2}a\sqrt{e}e^{(2i\,dx+2i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-3i\,ae^{(6i\,dx\right.\right.}}{8}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/84*(-40*I*sqrt(2)*a*sqrt(e)*e^(2*I*d*x + 2*I*c)*weierstrassPInverse(-4,
0, e^(I*d*x + I*c)) + sqrt(2)*(-3*I*a*e^(6*I*d*x + 6*I*c) - 19*I*a*e^(4*I*
d*x + 4*I*c) - 9*I*a*e^(2*I*d*x + 2*I*c) + 7*I*a)*sqrt(e/(e^(2*I*d*x + 2*I
*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-2*I*d*x - 2*I*c)/(d*e^4)
```

Sympy [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = ia\left(\int \left(-\frac{i}{(e\sec{(c+dx)})^{\frac{7}{2}}}\right)dx + \int \frac{\tan{(c+dx)}}{(e\sec{(c+dx)})^{\frac{7}{2}}}\,dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(7/2),x)
```

output
```
I*a*(Integral(-I/(e*sec(c + d*x))**(7/2), x) + Integral(tan(c + d*x)/(e*se
c(c + d*x))**(7/2), x))
```

Maxima [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{i\,a\tan{(dx+c)}+a}{(e\sec{(dx+c)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{i\,a\tan{(dx+c)}+a}{(e\sec{(dx+c)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + ia\tan(c + dx)}{(e\sec(c + dx))^{7/2}}\,dx = \int \frac{a + a\tan(c + d\,x)\ 1\mathrm{i}}{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(7/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{a + ia\tan(c + dx)}{(e\sec(c + dx))^{7/2}}\,dx = \frac{\sqrt{e}\,a\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx + \left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)}{\sec(dx+c)^4}dx\right) i\right)}{e^4}$$

input
```
int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x)
```

output
```
(sqrt(e)*a*(int(sqrt(sec(c + d*x))/sec(c + d*x)**4,x) + int((sqrt(sec(c +
d*x))*tan(c + d*x))/sec(c + d*x)**4,x)*i))/e**4
```

3.193 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 138

$$\begin{aligned}\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx &= \\ &-\frac{14a^2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14ia^2(e\sec(c+dx))^{3/2}}{15d} \\ &+\frac{14a^2e\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}{5d}\end{aligned}$$

output
```
-14/5*a^2*e^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*
sec(d*x+c))^(1/2)+14/15*I*a^2*(e*sec(d*x+c))^(3/2)/d+14/5*a^2*e*(e*sec(d*x
+c))^(1/2)*sin(d*x+c)/d+2/5*I*(e*sec(d*x+c))^(3/2)*(a^2+I*a^2*tan(d*x+c))/
d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.99 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.93

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx = \frac{(e\sec(c+dx))^{3/2}\left(-\frac{14i\sqrt{2}\left(3\sqrt{1+e^{2i(c+dx)}}-e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)}{\left(-1+e^{2ic}\right)\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}}\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((e*Sec[c + d*x])^(3/2)*(((-14*I)*Sqrt[2]*(3*Sqrt[1 + E^((2*I)*(c + d*x))]
 - E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((
2*I)*(c + d*x))]))/((-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*
(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]) + (Csc[c]*Sec[c + d*x]^(5/2)*(
Cos[2*c] - I*Sin[2*c])*(36*Cos[d*x] + 27*Cos[2*c + d*x] + 21*Cos[2*c + 3*d
*x] - (20*I)*Sin[d*x] + (20*I)*Sin[2*c + d*x]))/2)*(a + I*a*Tan[c + d*x])^
2)/(15*d*Sec[c + d*x]^(7/2)*(Cos[d*x] + I*Sin[d*x])^2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3979, 3042, 3967, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int (a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{7}{5}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{5}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{7}{5}a\left(a\int (e\sec(c+dx))^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{5}a\left(a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}+\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)$$

input
```
Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(7*a*((((2*I)/3)*a*(e*Sec[c + d*x])^(3/2))/d + a*((-2*e^2*EllipticE[(c + d
*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c
 + d*x]]*Sin[c + d*x])/d)))/5 + (((2*I)/5)*(e*Sec[c + d*x])^(3/2)*(a^2 + I
*a^2*Tan[c + d*x]))/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 13.82 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.59

method	result
default	$\frac{2a^2\left(21\sin(dx+c)-3\tan(dx+c)-3\sec(dx+c)\tan(dx+c)+10i+10i\sec(dx+c)+21i\left(-\cos(dx+c)^2-2\cos(dx+c)-1\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right.}{}$
parts	$\frac{2a^2\left(i\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}+i\left(-\cos(dx+c)^2-2\cos(\right.\right.}{d(\cos(dx+c)+1)}$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/15*a^2/d*(21*sin(d*x+c)-3*tan(d*x+c)-3*sec(d*x+c)*tan(d*x+c)+10*I+10*I*s
ec(d*x+c)+21*I*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+21*I
*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE
(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2))*(e*sec(d*x+c))^(1/
2)/(cos(d*x+c)+1)*e
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx =$$
$$-\frac{2\left(\sqrt{2}\left(21i\,a^2ee^{(5i\,dx+5i\,c)}+16i\,a^2ee^{(3i\,dx+3i\,c)}+7i\,a^2ee^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+21\sqrt{2}\left(i\,a^2ee^{(4i}\right.\right.}{15\left(de^{(4i\,dx+4i\,c)}+2\,d\right.}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/15*(sqrt(2)*(21*I*a^2*e*e^(5*I*d*x + 5*I*c) + 16*I*a^2*e*e^(3*I*d*x + 3
*I*c) + 7*I*a^2*e*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/
2*I*d*x + 1/2*I*c) + 21*sqrt(2)*(I*a^2*e*e^(4*I*d*x + 4*I*c) + 2*I*a^2*e*e
^(2*I*d*x + 2*I*c) + I*a^2*e)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, e^(I*d*x + I*c))))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x +
 2*I*c) + d)
```

Sympy [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int\left(-(e\sec(c+dx))^{\frac{3}{2}}\right)dx + \int (e\sec(c+dx))^{\frac{3}{2}}\tan^2(c+dx)\,dx + \int\left(-2i(e\sec(c+dx))^{\frac{3}{2}}\tan(c+dx)\right)dx\right)$$

input
```
integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(-(e*sec(c + d*x))**(3/2), x) + Integral((e*sec(c + d*x))**
(3/2)*tan(c + d*x)**2, x) + Integral(-2*I*(e*sec(c + d*x))**(3/2)*tan(c +
d*x), x))
```

Maxima [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx = \int (e\sec{(dx+c)})^{\frac{3}{2}}\,(i\,a\tan{(dx+c)}+a)^2\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^2, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx = \int (e\sec{(dx+c)})^{\frac{3}{2}}\,(i\,a\tan{(dx+c)}+a)^2\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx = \int \left(\frac{e}{\cos{(c+d\,x)}}\right)^{3/2}\,(a+a\tan(c+d\,x)\,1\mathrm{i})^2\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx = \frac{\sqrt{e}\,a^2e\left(4\sqrt{\sec(dx+c)}\,\sec(dx+c)\,i-3\left(\int\sqrt{\sec(dx+c)}\,\sec(dx+c)\tan(dx+c\right.\right.}{3d}$$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(sqrt(e)*a**2*e*(4*sqrt(sec(c + d*x))*sec(c + d*x)*i - 3*int(sqrt(sec(c +
d*x))*sec(c + d*x)*tan(c + d*x)**2,x)*d + 3*int(sqrt(sec(c + d*x))*sec(c +
 d*x),x)*d))/(3*d)
```

3.194 $\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 106

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx = \frac{10ia^2\sqrt{e\sec(c+dx)}}{3d} + \frac{10a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d} + \frac{2i\sqrt{e\sec(c+dx)}(a^2+ia^2\tan(c+dx))}{3d}$$

output
```
10/3*I*a^2*(e*sec(d*x+c))^(1/2)/d+10/3*a^2*cos(d*x+c)^(1/2)*InverseJacobiA
M(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x+c))^(1/2)/d+2/3*I*(e*sec(d*x+c))^(1/2)
*(a^2+I*a^2*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx = \frac{2a^2(e\sec(c+dx))^{3/2}\left(6i\cos(c+dx)+5\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\sin(c+dx)\right)}{3de}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(2*a^2*(e*Sec[c + d*x])^(3/2)*((6*I)*Cos[c + d*x] + 5*Cos[c + d*x]^(3/2)*E
llipticF[(c + d*x)/2, 2] - Sin[c + d*x]))/(3*d*e)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3979, 3042, 3967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{5}{3}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx+\frac{2i(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx+\frac{2i(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{5}{3}a\left(a\int \sqrt{e\sec(c+dx)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\left(a\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5}{3}a\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{5}{3}a\left(\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)$$

input
```
Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(5*a*(((2*I)*a*Sqrt[e*Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d))/3 + (((2*I)/3)*Sqrt[e*Sec[c + d
*x]]*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 10.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

method	result
default	$\frac{a^{2}\left(-\frac{2\tan\left(dx+c\right)}{3}+4i+\frac{10i\left(\cos\left(dx+c\right)+1\right)\sqrt{\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\,\sqrt{\frac{1}{\cos\left(dx+c\right)+1}}\,\operatorname{EllipticF}\left(i\left(\cot\left(dx+c\right)-\csc\left(dx+c\right)\right),i\right)}{3}\right)\sqrt{e\sec\left(dx+c\right)}}{d}$
parts	$-\frac{2ia^{2}\left(\cos\left(dx+c\right)+1\right)\sqrt{e\sec\left(dx+c\right)}\,\sqrt{\frac{1}{\cos\left(dx+c\right)+1}}\,\sqrt{\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\,\operatorname{EllipticF}\left(i\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right),i\right)}{d}+\frac{4ia^{2}\sqrt{e\sec\left(dx+c\right)}}{d}$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(-2/3*tan(d*x+c)+4*I+10/3*I*(cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I))*
(e*sec(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx = -\frac{2\left(\sqrt{2}\left(-7i\,a^2e^{(2i\,dx+2i\,c)}-5i\,a^2\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+5\,\sqrt{2}\left(i\,a^2e^{(2i\,dx+2i\,c)}+i\,a^2\right)\sqrt{e}\mathrm{weierstrassP}\right.}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/3*(sqrt(2)*(-7*I*a^2*e^(2*I*d*x + 2*I*c) - 5*I*a^2)*sqrt(e/(e^(2*I*d*x
+ 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 5*sqrt(2)*(I*a^2*e^(2*I*d*x + 2*I
*c) + I*a^2)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(d*e^(2*
I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int\left(-\sqrt{e\sec\left(c+dx\right)}\right)dx+\int\sqrt{e\sec\left(c+dx\right)}\tan^2\left(c+dx\right)dx + \int\left(-2i\sqrt{e\sec\left(c+dx\right)}\tan\left(c+dx\right)\right)dx\right)$$

input
```
integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(-sqrt(e*sec(c + d*x)), x) + Integral(sqrt(e*sec(c + d*x))*
tan(c + d*x)**2, x) + Integral(-2*I*sqrt(e*sec(c + d*x))*tan(c + d*x), x))
```

Maxima [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx = \int \sqrt{e\sec\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)^2\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[2,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{1,[1]%%%}]%%},[1,0]%%%
}+%%%{%%%
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx = \int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{\sqrt{e}\,a^2\left(4\sqrt{\sec\left(dx+c\right)}\,i+\left(\int\sqrt{\sec\left(dx+c\right)}dx\right)d-\left(\int\sqrt{\sec\left(dx+c\right)}\,\tan\left(dx+c\right)^2dx\right)d\right)}{d}$$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(sqrt(e)*a**2*(4*sqrt(sec(c + d*x))*i + int(sqrt(sec(c + d*x)),x)*d - int(
sqrt(sec(c + d*x))*tan(c + d*x)**2,x)*d))/d
```

3.195 $\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 107

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = \frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{6a^2\sqrt{e\sec(c+dx)}\sin(c+dx)}{de} - \frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}$$

output
```
6*a^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec(d*x+
c))^(1/2)-6*a^2*(e*sec(d*x+c))^(1/2)*sin(d*x+c)/d/e-4*I*(a^2+I*a^2*tan(d*x
+c))/d/(e*sec(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.23

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{2i\sqrt{2}a^2e^{2i(c+dx)}\left(-\sqrt{1+e^{2i(c+dx)}}+\left(1+e^{2i(c+dx)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)}{d\sqrt{\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)^{3/2}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^2/Sqrt[e*Sec[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[2]*a^2*E^((2*I)*(c + d*x))*(-Sqrt[1 + E^((2*I)*(c + d*x))] +
(1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
d*x))]))/(d*Sqrt[(e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2
*I)*(c + d*x)))^(3/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3977, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3977$$

$$-\frac{3a^2\int (e\sec(c+dx))^{3/2}dx}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3a^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}$$

$\Big\downarrow$ 4255

$$-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}$$

$\Big\downarrow$ 4258

$$-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}$$

$\Big\downarrow$ 3119

$$-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^2/Sqrt[e*Sec[c + d*x]],x]
```

output
```
(-3*a^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*S
ec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/e^2 - ((4*I)*(
a^2 + I*a^2*Tan[c + d*x]))/(d*Sqrt[e*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(99) = 198$.

Time = 9.87 (sec) , antiderivative size = 379, normalized size of antiderivative = 3.54

method	result
parts	$\frac{2a^2\left(i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(-\cos(dx+c)-2-\sec(dx+c))+i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx-}}\right.}{d(\cos(dx+c)+1)\sqrt{e\sec(dx+c)}}$
default	$\frac{a^2\left(i\ln\left(\frac{4\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}+4\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}-2\cos(dx+c)+2}{\cos(dx+c)+1}\right)-i\ln\left(\frac{2\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}+2\sqrt{-\frac{\cos(dx+c}{(\cos(dx+c)}}}{\cos(dx+c)+1}\right.\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^2/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)*(I*(1/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(-c
os(d*x+c)-2-sec(d*x+c))+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)+2+sec(d*x+c)
)+sin(d*x+c))-4*I*a^2/(e*sec(d*x+c))^(1/2)/d+2*a^2/d/(cos(d*x+c)+1)/(e*sec
(d*x+c))^(1/2)*(sin(d*x+c)-tan(d*x+c)-2*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+2+sec(d*x+c))*EllipticF(I*(csc(d*x+
c)-cot(d*x+c)),I)+2*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)+2+sec(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.64

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{2\left(-i\sqrt{2}a^2\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}-3i\sqrt{2}a^2\sqrt{e}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e\right.\right.\right.}{de}$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2*(-I*sqrt(2)*a^2*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*
c) - 3*I*sqrt(2)*a^2*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, e^(I*d*x + I*c))))/(d*e)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = -a^2\left(\int\left(-\frac{1}{\sqrt{e\sec(c+dx)}}\right)dx + \int\frac{\tan^2(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx + \int\left(-\frac{2i\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(1/2),x)
```

output
```
-a**2*(Integral(-1/sqrt(e*sec(c + d*x)), x) + Integral(tan(c + d*x)**2/sqr
t(e*sec(c + d*x)), x) + Integral(-2*I*tan(c + d*x)/sqrt(e*sec(c + d*x)), x
))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{\sqrt{e\sec(dx+c)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/sqrt(e*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{\sqrt{e\sec(dx+c)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/sqrt(e*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = \frac{\sqrt{e}\,a^2\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx-\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^2}{\sec(dx+c)}dx\right)+2\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)}{\sec(dx+c)}dx\right)i\right)}{e}$$

input
```
int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*a**2*(int(sqrt(sec(c + d*x))/sec(c + d*x),x) - int((sqrt(sec(c +
d*x))*tan(c + d*x)**2)/sec(c + d*x),x) + 2*int((sqrt(sec(c + d*x))*tan(c +
 d*x))/sec(c + d*x),x)*i))/e
```

3.196 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2} - \frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}$$

output
```
-2/3*a^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*
x+c))^(1/2)/d/e^2-4/3*I*(a^2+I*a^2*tan(d*x+c))/d/(e*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sec^2(c+dx)\left(2i\cos(c+dx)+\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(c+dx)-i\sin(c+dx))\right)(}{3d(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))^2}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*a^2*Sec[c + d*x]^2*((2*I)*Cos[c + d*x] + Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]))*(Cos[c + 3*d*x] + I*Sin[c
 + 3*d*x]))/(3*d*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3977, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3977}$$

$$-\frac{a^2\int \sqrt{e\sec(c+dx)}dx}{3e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$-\frac{a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])
/(3*d*e^2) - (((4*I)/3)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(3
/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 10.62 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.20

method	result
default	$\frac{a^{2}\left(-\frac{2 i \sqrt{\frac{1}{\cos(dx+c)+1}}\, \operatorname{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\sec(dx+c)+1)}{3}-\frac{4 i\cos(dx+c)}{3}+\frac{4\sin(dx+c)}{3}\right)}{d\sqrt{e\sec(dx+c)}\, e}$
parts	$\frac{a^{2}\left(-\frac{2 i \sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\, \operatorname{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(\sec(dx+c)+1)}{3}+\frac{2\sin(dx+c)}{3}\right)}{d\sqrt{e\sec(dx+c)}\, e}-\frac{4 i a^{2}}{3 d (e\sec(dx+c))^{\frac{3}{2}}}-\frac{a^{2}(}{}$
risch	$-\frac{2 i \mathrm{e}^{i(dx+c)} a^{2}\sqrt{2}}{3 d e \sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{2\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\, \operatorname{EllipticF}\left(\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)a^{2}\sqrt{e\,\mathrm{e}^{i(dx+c)}\left(\mathrm{e}^{2i(dx-\right.}}}{3 d\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx+c)}}\, e\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}$

input
```
int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2/d*(-2/3*I*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c))
,I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(sec(d*x+c)+1)-4/3*I*cos(d*x+c)+4/3*
sin(d*x+c))/(e*sec(d*x+c))^(1/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(-i\sqrt{2}a^{2}\sqrt{e}\,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(i\,a^{2}e^{(2i\,dx+2i\,c)}+i\,a^{2}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}\right.}{3\,de^{2}}$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(-I*sqrt(2)*a^2*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) +
 sqrt(2)*(I*a^2*e^(2*I*d*x + 2*I*c) + I*a^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) +
 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^2)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = -a^2\left(\int\left(-\frac{1}{(e\sec(c+dx))^{\frac{3}{2}}}\right)dx + \int \frac{\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx + \int\left(-\frac{2i\tan(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(3/2),x)
```

output
```
-a**2*(Integral(-1/(e*sec(c + d*x))**(3/2), x) + Integral(tan(c + d*x)**2/
(e*sec(c + d*x))**(3/2), x) + Integral(-2*I*tan(c + d*x)/(e*sec(c + d*x))*
*(3/2), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + ia\tan(c + dx))^2}{(e\sec(c + dx))^{3/2}}\,dx = \int \frac{(a + a\tan(c + d\,x)\ 1\mathrm{i})^2}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a + ia\tan(c + dx))^2}{(e\sec(c + dx))^{3/2}}\,dx = \frac{\sqrt{e}\,a^2\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx - \left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^2}{\sec(dx+c)^2}dx\right) + 2\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)}{\sec(dx+c)^2}\right.\right.}{e^2}$$

input
```
int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*a**2*(int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x) - int((sqrt(sec(c
 + d*x))*tan(c + d*x)**2)/sec(c + d*x)**2,x) + 2*int((sqrt(sec(c + d*x))*t
an(c + d*x))/sec(c + d*x)**2,x)*i))/e**2
```

3.197 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx=\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}$$

output
```
2/5*a^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e^2/cos(d*x+c)^(1/2)/(e*se
c(d*x+c))^(1/2)-4/5*I*(a^2+I*a^2*tan(d*x+c))/d/(e*sec(d*x+c))^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.89 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{i\sqrt{2}a^2\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{3/2}\left(1+e^{2i(c+dx)}\right)^{3/2}\left(3\sqrt{1+e^{2i(c+dx)}}+2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)}{15de^4}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(5/2),x]
```

output
```
((-1/15*I)*Sqrt[2]*a^2*((e*E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(3/
2)*(1 + E^((2*I)*(c + d*x)))^(3/2)*(3*Sqrt[1 + E^((2*I)*(c + d*x))] + 2*Hy
pergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(d*e^4)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3977, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{a^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a^2\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^2*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) - (((4*I)/5)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(5/
2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(75) = 150$.

Time = 13.75 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.66

method	result
default	$\frac{2a^2\left(\sin(dx+c)\left(2\cos(dx+c)^2+2\cos(dx+c)+1\right)+i\left(-2\cos(dx+c)^3-2\cos(dx+c)^2\right)+i\,\mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\sqrt{\frac{\cos}{\cos(}}\right.}{5d(\cos}$
risch	$-\frac{i\left(\mathrm{e}^{2i(dx+c)}+2\right)a^2\sqrt{2}}{5d\,e^2\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\sqrt{i\mathrm{e}^{i(dx+c)}}\left(-2i\,\mathrm{EllipticE}\left(\sqrt{-i(\right.\right.}{\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx}}}\right.}{5d\,e^2\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{e}{\mathrm{e}^{2i}}}}$
parts	$\frac{2a^2\left(\sin(dx+c)\left(\cos(dx+c)^2+\cos(dx+c)+3\right)-3i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(\cos(dx+c)+2+\sec(dx+c)\right)\mathrm{EllipticF}(i(\csc(dx+c)-\right.}{5d(\cos(dx+c)+1)\sqrt{e\sec(}}$

input
```
int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*a^2/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)/e^2*(sin(d*x+c)*(2*cos(d*x+c
)^2+2*cos(d*x+c)+1)+I*(-2*cos(d*x+c)^3-2*cos(d*x+c)^2)+I*EllipticF(I*(cot(
d*x+c)-csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)+2+sec(d*x+c))+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*(-cos(d*x+c)-2-
sec(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = \frac{2i\sqrt{2}a^2\sqrt{e}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{}}{5\,de^3}$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(2*I*sqrt(2)*a^2*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, e^(I*d*x + I*c))) + sqrt(2)*(-I*a^2*e^(3*I*d*x + 3*I*c) - I*a^2*e^(I*
d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*
e^3)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = -a^2\left(\int\left(-\frac{1}{(e\sec{(c+dx)})^{\frac{5}{2}}}\right)dx + \int\frac{\tan^2{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx + \int\left(-\frac{2i\tan{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(5/2),x)
```

output
```
-a**2*(Integral(-1/(e*sec(c + d*x))**(5/2), x) + Integral(tan(c + d*x)**2/
(e*sec(c + d*x))**(5/2), x) + Integral(-2*I*tan(c + d*x)/(e*sec(c + d*x))*
*(5/2), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = \int\frac{(i\,a\tan{(dx+c)}+a)^2}{(e\sec{(dx+c)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^2}{(e\sec\left(dx+c\right))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^2}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{e}\,a^2\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx-\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^2}{\sec(dx+c)^3}dx\right)+2\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)}{\sec(dx+c)^3}\right.\right.}{e^3}$$

input
```
int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*a**2*(int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x) - int((sqrt(sec(c
 + d*x))*tan(c + d*x)**2)/sec(c + d*x)**3,x) + 2*int((sqrt(sec(c + d*x))*t
an(c + d*x))/sec(c + d*x)**3,x)*i))/e**3
```

3.198 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \frac{2a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7de^4} + \frac{2a^2\sin(c+dx)}{7de^3\sqrt{e\sec(c+dx)}} - \frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}$$

output
```
2/7*a^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x
+c))^(1/2)/d/e^4+2/7*a^2*sin(d*x+c)/d/e^3/(e*sec(d*x+c))^(1/2)-4/7*I*(a^2+
I*a^2*tan(d*x+c))/d/(e*sec(d*x+c))^(7/2)
```

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \frac{a^2\sqrt{e\sec(c+dx)}\left(-2i-2i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c\right.\right.}{7}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(a^2*Sqrt[e*Sec[c + d*x]]*(-2*I - (2*I)*Cos[2*(c + d*x)] + 2*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]) -
Sin[2*(c + d*x)])*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)]))/(7*d*e^4*(C
os[d*x] + I*Sin[d*x])^2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3977, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{3a^2\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{3a^2\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a^2\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{3a^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{3a^2\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(3*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x
]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2) -
(((4*I)/7)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(7/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 15.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00

method	result
default	$-\frac{2a^2\left(\sin(dx+c)\left(-2\cos(dx+c)^2-1\right)+i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\operatorname{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(\sec(dx+c)+1)+2i\operatorname{co}}{7d\sqrt{e\sec(dx+c)}\,e^3}$
parts	$-\frac{2a^2\left(\sin(dx+c)\left(-3\cos(dx+c)^2-5\right)+i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\operatorname{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(5+5\sec(dx+c))\right)}{21d\sqrt{e\sec(dx+c)}\,e^3}-$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}\left(\mathrm{e}^{2i(dx+c)}+3\right)a^2\sqrt{2}}{14d\,e^3\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}+\frac{2\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\sqrt{i\mathrm{e}^{i(dx+c)}}\operatorname{EllipticF}\left(\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)a^2\sqrt{e\,\mathrm{e}^{i(dx}}}{7d\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx+c)}}\,e^3\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}$

input
```
int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*a^2/d/(e*sec(d*x+c))^(1/2)/e^3*(sin(d*x+c)*(-2*cos(d*x+c)^2-1)+I*(1/(
cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(sec(d*x+c)+1)+2*I*cos(d*x+c)^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \frac{-4i\sqrt{2}a^2\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-i\,a^2e^{(4i\,dx+4i\,c)}-4\right.}{14\,de^4}$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/14*(-4*I*sqrt(2)*a^2*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c))
 + sqrt(2)*(-I*a^2*e^(4*I*d*x + 4*I*c) - 4*I*a^2*e^(2*I*d*x + 2*I*c) - 3*I
*a^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^4)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = -a^2\left(\int\left(-\frac{1}{(e\sec(c+dx))^{\frac{7}{2}}}\right)dx + \int\frac{\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\,dx + \int\left(-\frac{2i\tan(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(7/2),x)
```

output
```
-a**2*(Integral(-1/(e*sec(c + d*x))**(7/2), x) + Integral(tan(c + d*x)**2/
(e*sec(c + d*x))**(7/2), x) + Integral(-2*I*tan(c + d*x)/(e*sec(c + d*x))*
*(7/2), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(7/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\sqrt{e}\,a^2\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx - \left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^2}{\sec(dx+c)^4}dx\right) + 2\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)}{\sec(dx+c)^4}\right.\right.}{e^4}$$

input
```
int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x)
```

output
```
(sqrt(e)*a**2*(int(sqrt(sec(c + d*x))/sec(c + d*x)**4,x) - int((sqrt(sec(c
 + d*x))*tan(c + d*x)**2)/sec(c + d*x)**4,x) + 2*int((sqrt(sec(c + d*x))*t
an(c + d*x))/sec(c + d*x)**4,x)*i))/e**4
```

3.199 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2a^2\sin(c+dx)}{9de^3(e\sec(c+dx))^{3/2}} - \frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

output
```
2/3*a^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e^4/cos(d*x+c)^(1/2)/(e*se
c(d*x+c))^(1/2)+2/9*a^2*sin(d*x+c)/d/e^3/(e*sec(d*x+c))^(3/2)-4/9*I*(a^2+I
*a^2*tan(d*x+c))/d/(e*sec(d*x+c))^(9/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.45 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \frac{ia^2\left(9-4e^{2i(c+dx)}-e^{4i(c+dx)}-\frac{8e^{2i(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}\right)}{18\sqrt{2}de^4\sqrt{\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(9/2),x]
```

output
```
((I/18)*a^2*(9 - 4*E^((2*I)*(c + d*x)) - E^((4*I)*(c + d*x)) - (8*E^((2*I)
*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/Sqrt[1
 + E^((2*I)*(c + d*x))]))/(Sqrt[2]*d*e^4*Sqrt[(e*E^(I*(c + d*x)))/(1 + E^(
(2*I)*(c + d*x)))])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3977, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{5a^2\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5a^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{5a^2\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5a^2\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5a^2\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5a^2\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{5a^2\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(9/2),x]
```

output
```
(5*a^2*((6*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*S
ec[c + d*x]]) + (2*Sin[c + d*x])/(5*d*e*(e*Sec[c + d*x])^(3/2))))/(9*e^2)
- (((4*I)/9)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(9/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(102) = 204$.

Time = 24.54 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.03

method	result
default	$\frac{2a^2\left(\sin(dx+c)\left(2\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2+\cos(dx+c)+3\right)+3i\,\mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}{}$
risch	$-\frac{i\left(e^{4i(dx+c)}+4\,e^{2i(dx+c)}+15\right)a^2\sqrt{2}}{36d\,e^4\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\left(-2i\,\mathrm{E}\right.}{\sqrt{e\,e^{3i}}}\right.}{3d\,e^4\left(e^{2i(d}\right.}$
parts	$\frac{2a^2\left(\sin(dx+c)\left(5\cos(dx+c)^4+5\cos(dx+c)^3+7\cos(dx+c)^2+7\cos(dx+c)+21\right)-21i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\cos(dx+c)+2+\right.}{45d(\cos}$

input
```
int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*a^2/d*(sin(d*x+c)*(2*cos(d*x+c)^4+2*cos(d*x+c)^3+cos(d*x+c)^2+cos(d*x+
c)+3)+3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)+2+sec(d*x+c))*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)-2*I*cos(d*x+c)
^5-2*I*cos(d*x+c)^4-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)+2+sec(d*x+c))*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I))
/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)/e^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.13

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \frac{\left(24i\sqrt{2}a^2\sqrt{e}e^{(i\,dx+i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,d}\right.\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

output
```
1/36*(24*I*sqrt(2)*a^2*sqrt(e)*e^(I*d*x + I*c)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-I*a^2*e^(6*I*d*x + 6*
I*c) - 5*I*a^2*e^(4*I*d*x + 4*I*c) + 5*I*a^2*e^(2*I*d*x + 2*I*c) + 9*I*a^2
)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-I*d*x - I
*c)/(d*e^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(9/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(9/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \frac{\sqrt{e}\,a^2\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^5}dx - \left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^2}{\sec(dx+c)^5}dx\right) + 2\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)}{\sec(dx+c)^5}\right.\right.}{e^5}$$

input
```
int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x)
```

output
```
(sqrt(e)*a**2*(int(sqrt(sec(c + d*x))/sec(c + d*x)**5,x) - int((sqrt(sec(c
 + d*x))*tan(c + d*x)**2)/sec(c + d*x)**5,x) + 2*int((sqrt(sec(c + d*x))*t
an(c + d*x))/sec(c + d*x)**5,x)*i))/e**5
```

3.200 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 147

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx = \frac{10a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33de^6} + \frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}} + \frac{10a^2\sin(c+dx)}{33de^5\sqrt{e\sec(c+dx)}} - \frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}$$

output
```
10/33*a^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d
*x+c))^(1/2)/d/e^6+2/11*a^2*sin(d*x+c)/d/e^3/(e*sec(d*x+c))^(5/2)+10/33*a^
2*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(1/2)-4/11*I*(a^2+I*a^2*tan(d*x+c))/d/(e
*sec(d*x+c))^(11/2)
```

Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx = \frac{a^2\sqrt{e\sec(c+dx)}\left(-28i-24i\cos(2(c+dx))+4i\cos(4(c+dx))+40\sqrt{\cos}\right.}{}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(11/2),x]
```

output
```
(a^2*Sqrt[e*Sec[c + d*x]]*(-28*I - (24*I)*Cos[2*(c + d*x)] + (4*I)*Cos[4*(
c + d*x)] + 40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*
x)] - I*Sin[2*(c + d*x)]) - 6*Sin[2*(c + d*x)] + 7*Sin[4*(c + d*x)])*(Cos[
2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)]))/(132*d*e^6*(Cos[d*x] + I*Sin[d*x])
^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 3977, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{7a^2\int \frac{1}{(e\sec(c+dx))^{7/2}}dx}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7a^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{7a^2\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7a^2\left(\frac{5\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{7a^2\left(\frac{5\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7a^2\left(\frac{5\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{7a^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7a^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{7a^2\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(11/2),x]
```

output
```
(7*a^2*((2*Sin[c + d*x])/(7*d*e*(e*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*S
in[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2)))/(11*e^2) - (((4*I)/1
1)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(11/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 28.62 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.87

method	result
default	$-\frac{2a^2\left(\sin(dx+c)\left(-6\cos(dx+c)^4-3\cos(dx+c)^2-5\right)+i\sqrt{\frac{1}{\cos(dx+c)+1}}\ \mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ (-5-}{33d\sqrt{e\sec(dx+c)}\,e^5}$
parts	$\frac{a^2\left(-\frac{2\sin(dx+c)\left(-7\cos(dx+c)^4-9\cos(dx+c)^2-15\right)}{77}-\frac{30i\sqrt{\frac{1}{\cos(dx+c)+1}}\ \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(\sec(dx+c)+1)}{77}\right.}{d\sqrt{e\sec(dx+c)}\,e^5}$

input
```
int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/33*a^2/d/(e*sec(d*x+c))^(1/2)/e^5*(sin(d*x+c)*(-6*cos(d*x+c)^4-3*cos(d*
x+c)^2-5)+I*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-5-5*sec(d*x+c))+6*I*cos(d*x+c)^5)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx = \frac{\left(-80i\sqrt{2}a^2\sqrt{e}e^{(2i\,dx+2i\,c)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-3i\,a^2\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x, algorithm="fricas"
)
```

output
```
1/264*(-80*I*sqrt(2)*a^2*sqrt(e)*e^(2*I*d*x + 2*I*c)*weierstrassPInverse(-
4, 0, e^(I*d*x + I*c)) + sqrt(2)*(-3*I*a^2*e^(8*I*d*x + 8*I*c) - 18*I*a^2*
e^(6*I*d*x + 6*I*c) - 56*I*a^2*e^(4*I*d*x + 4*I*c) - 30*I*a^2*e^(2*I*d*x +
 2*I*c) + 11*I*a^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I
*c))*e^(-2*I*d*x - 2*I*c)/(d*e^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x, algorithm="maxima"
)
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(11/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^2}{\left(e\sec\left(dx+c\right)\right)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx=\int \frac{\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^2}{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(11/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(11/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx=\frac{\sqrt{e}\,a^2\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^6}dx-\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^2}{\sec(dx+c)^6}dx\right)+2\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)}{\sec(dx+c)^6}\right.\right.}{e^6}$$

input
```
int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x)
```

output
```
(sqrt(e)*a**2*(int(sqrt(sec(c + d*x))/sec(c + d*x)**6,x) - int((sqrt(sec(c
 + d*x))*tan(c + d*x)**2)/sec(c + d*x)**6,x) + 2*int((sqrt(sec(c + d*x))*t
an(c + d*x))/sec(c + d*x)**6,x)*i))/e**6
```

3.201 $\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 28, antiderivative size = 202

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx = -\frac{2a^3e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{10ia^3(e\sec(c+dx))^{7/2}}{21d} + \frac{2a^3e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^3e(e\sec(c+dx))^{5/2}\sin(c+dx)}{3d} + \frac{2ia(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}{11d} + \frac{10i(e\sec(c+dx))^{7/2}\left(a^3+ia^3\tan(c+dx)\right)}{33d}$$

output

```
-2*a^3*e^4*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec
(d*x+c))^(1/2)+10/21*I*a^3*(e*sec(d*x+c))^(7/2)/d+2*a^3*e^3*(e*sec(d*x+c))
^(1/2)*sin(d*x+c)/d+2/3*a^3*e*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/d+2/11*I*a*(
e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^2/d+10/33*I*(e*sec(d*x+c))^(7/2)*(a
^3+I*a^3*tan(d*x+c))/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.57 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.80

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx =$$
$$-\frac{a^3e^3\sec^4(c+dx)\sqrt{e\sec(c+dx)}\left(-908\cos(c+dx)-858\cos(3(c+dx))-154\cos(5(c+dx))+\frac{77}{2}e^{-5i(c}\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
-1/1848*(a^3*e^3*Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(-908*Cos[c + d*x] -
858*Cos[3*(c + d*x)] - 154*Cos[5*(c + d*x)] + (77*(1 + E^((2*I)*(c + d*x))
)^(11/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/(2*E^((5*
I)*(c + d*x))) - (38*I)*Sin[c + d*x] - (451*I)*Sin[3*(c + d*x)] - (77*I)*S
in[5*(c + d*x)])*(-I + Tan[c + d*x]))/d
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3979, 3042, 3979, 3042, 3967, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^3(e\sec(c+dx))^{7/2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a+ia\tan(c+dx))^3(e\sec(c+dx))^{7/2}dx$$
$$\Big\downarrow \text{3979}$$

$$\frac{15}{11}a\int (e\sec(c+dx))^{7/2}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$\Big\downarrow$ 3042

$$\frac{15}{11}a\int (e\sec(c+dx))^{7/2}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$\Big\downarrow$ 3979

$$\frac{15}{11}a\left(\frac{11}{9}a\int (e\sec(c+dx))^{7/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}{9d}\right)+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$\Big\downarrow$ 3042

$$\frac{15}{11}a\left(\frac{11}{9}a\int (e\sec(c+dx))^{7/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}{9d}\right)+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$\Big\downarrow$ 3967

$$\frac{15}{11}a\left(\frac{11}{9}a\left(a\int (e\sec(c+dx))^{7/2}dx+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}{9d}\right)+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$\Big\downarrow$ 3042

$$\frac{15}{11}a\left(\frac{11}{9}a\left(a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx)}{9d}\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$\Big\downarrow$ 4255

$$\frac{15}{11}a\left(\frac{11}{9}a\left(a\left(\frac{3}{5}e^2\int (e\sec(c+dx))^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}\right)+\frac{2i(a}{}\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$\Big\downarrow$ 3042

$$\frac{15}{11}a\left(\frac{11}{9}a\left(a\left(\frac{3}{5}e^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}\right)\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \; 4255$$

$$\frac{15}{11}a\left(\frac{11}{9}a\left(a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/}}{5d}\right.\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \; 3042$$

$$\frac{15}{11}a\left(\frac{11}{9}a\left(a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+}{5d}\right.\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \; 4258$$

$$\frac{15}{11}a\left(\frac{11}{9}a\left(a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+}{5d}\right.\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \; 3042$$

$$\frac{15}{11}a\left(\frac{11}{9}a\left(a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+}{5d}\right.\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \; 3119$$

$$\frac{15}{11}a\left(\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}{9d}+\frac{11}{9}a\left(a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}\right.}{d\sqrt{\cos(c+d}}\right.\right.\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

input
```
Int[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(((2*I)/11)*a*(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2)/d + (15*a*(
(11*a*((((2*I)/7)*a*(e*Sec[c + d*x])^(7/2))/d + a*((2*e*(e*Sec[c + d*x])^(
5/2)*Sin[c + d*x])/(5*d) + (3*e^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(d*S
qrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c
+ d*x])/d))/5)))/9 + (((2*I)/9)*(e*Sec[c + d*x])^(7/2)*(a^2 + I*a^2*Tan[c
+ d*x]))/d))/11
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 5.88 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.35

$$\frac{2a^3\Big(\tan (dx+c)\sec (dx+c)^3\left(231+77\big(-3\cos (dx+c)^2-\cos (dx+c)-4\big)\sin (dx+c)^2\right)+132i\sec (dx}{}$$

input
```
int((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x)
```

output
```
2/231*a^3/d*(tan(d*x+c)*sec(d*x+c)^3*(231+77*(-3*cos(d*x+c)^2-cos(d*x+c)-4
)*sin(d*x+c)^2)+132*I*sec(d*x+c)^2+132*I*sec(d*x+c)^3-21*I*sec(d*x+c)^4-21
*I*sec(d*x+c)^5+231*I*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),
I)+231*I*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I))*(e*sec(d*x
+c))^(1/2)/(cos(d*x+c)+1)*e^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.56

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{2\left(\sqrt{2}\big(231i\,a^3e^3e^{(11i\,dx+11i\,c)}+1309i\,a^3e^3e^{(9i\,dx+9i\,c)}+946i\,a^3e^3e^{(7i\,dx+7i\,c)}+870i\,a^3e^3e^{(5i\,dx+5i\,c)}+407i\,a^3}{}$$

input
```
integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/231*(sqrt(2)*(231*I*a^3*e^3*e^(11*I*d*x + 11*I*c) + 1309*I*a^3*e^3*e^(9
*I*d*x + 9*I*c) + 946*I*a^3*e^3*e^(7*I*d*x + 7*I*c) + 870*I*a^3*e^3*e^(5*I
*d*x + 5*I*c) + 407*I*a^3*e^3*e^(3*I*d*x + 3*I*c) + 77*I*a^3*e^3*e^(I*d*x
+ I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 231*sq
rt(2)*(I*a^3*e^3*e^(10*I*d*x + 10*I*c) + 5*I*a^3*e^3*e^(8*I*d*x + 8*I*c) +
 10*I*a^3*e^3*e^(6*I*d*x + 6*I*c) + 10*I*a^3*e^3*e^(4*I*d*x + 4*I*c) + 5*I
*a^3*e^3*e^(2*I*d*x + 2*I*c) + I*a^3*e^3)*sqrt(e)*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(d*e^(10*I*d*x + 10*I*c) + 5*
d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c
) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(7/2)*(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx = \int (e\sec(dx+c))^{\frac{7}{2}}\,(i\,a\tan(dx+c)+a)^3\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)^3, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx = \int (e\sec(dx+c))^{\frac{7}{2}}(ia\tan(dx+c)+a)^3\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}(a+a\tan(c+d\,x)\,1\mathrm{i})^3\,dx$$

input
```
int((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx = \frac{\sqrt{e}\,a^3e^3\left(-14\sqrt{\sec(dx+c)}\,\sec(dx+c)^3\tan(dx+c)^2 i+74\sqrt{\sec(dx+c)}\,\sec(dx\right.}{}$$

input
```
int((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(sqrt(e)*a**3*e**3*( - 14*sqrt(sec(c + d*x))*sec(c + d*x)**3*tan(c + d*x)*
*2*i + 74*sqrt(sec(c + d*x))*sec(c + d*x)**3*i - 231*int(sqrt(sec(c + d*x)
)*sec(c + d*x)**3*tan(c + d*x)**2,x)*d + 77*int(sqrt(sec(c + d*x))*sec(c +
 d*x)**3,x)*d))/(77*d)
```

3.202 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 28, antiderivative size = 175

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx = \frac{26a^3e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21d} + \frac{26ia^3(e\sec(c+dx))^{5/2}}{35d} + \frac{26a^3e(e\sec(c+dx))^{3/2}\sin(c+dx)}{21d} + \frac{2ia(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}{9d} + \frac{26i(e\sec(c+dx))^{5/2}\left(a^3+ia^3\tan(c+dx)\right)}{63d}$$

output
```
26/21*a^3*e^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*s
ec(d*x+c))^(1/2)/d+26/35*I*a^3*(e*sec(d*x+c))^(5/2)/d+26/21*a^3*e*(e*sec(d
*x+c))^(3/2)*sin(d*x+c)/d+2/9*I*a*(e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^
2/d+26/63*I*(e*sec(d*x+c))^(5/2)*(a^3+I*a^3*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 2.59 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.51

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx = \frac{a^3\sec^2(c+dx)(e\sec(c+dx))^{5/2}\left(728i+1008i\cos(2(c+dx))+1560\cos^{\frac{9}{2}}(c+dx)\,\mathrm{I}\right.}{1260d}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*Sec[c + d*x]^2*(e*Sec[c + d*x])^(5/2)*(728*I + (1008*I)*Cos[2*(c + d*
x)] + 1560*Cos[c + d*x]^(9/2)*EllipticF[(c + d*x)/2, 2] - 150*Sin[2*(c + d
*x)] + 195*Sin[4*(c + d*x)]))/(1260*d)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3979, 3042, 3979, 3042, 3967, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^3(e\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^3(e\sec(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{13}{9}a\int (e\sec(c+dx))^{5/2}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13}{9}a\int (e\sec(c+dx))^{5/2}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{13}{9}a\left(\frac{9}{7}a\int (e\sec(c+dx))^{5/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}{7d}\right)+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13}{9}a\left(\frac{9}{7}a\int (e\sec(c+dx))^{5/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}{7d}\right)+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{13}{9}a\left(\frac{9}{7}a\left(a\int (e\sec(c+dx))^{5/2}dx+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}{7d}\right)+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13}{9}a\left(\frac{9}{7}a\left(a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5}}{7d}\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{13}{9}a\left(\frac{9}{7}a\left(a\left(\frac{1}{3}e^2\int \sqrt{e\sec(c+dx)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2i\left(a^2+\right.}{}\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13}{9}a\left(\frac{9}{7}a\left(a\left(\frac{1}{3}e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2i}{}\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{13}{9}a\left(\frac{9}{7}a\left(a\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\right.\right.\right.$$

$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13}{9}a\left(\frac{9}{7}a\left(a\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\right.\right.\right.$$

$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{13}{9}a\left(\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}{7d}+\frac{9}{7}a\left(a\left(\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+}}{3d}\right.\right.\right.$$

$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

input
```
Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(((2*I)/9)*a*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2)/d + (13*a*((
9*a*(((((2*I)/5)*a*(e*Sec[c + d*x])^(5/2))/d + a*((2*e^2*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d) + (2*e*(e*Sec[c + d
*x])^(3/2)*Sin[c + d*x])/(3*d))))/7 + (((2*I)/7)*(e*Sec[c + d*x])^(5/2)*(a
^2 + I*a^2*Tan[c + d*x]))/d))/9
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 5.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.77

$$-\frac{a^3\left(-\frac{26\tan(dx+c)}{21}+\frac{6\tan(dx+c)\sec(dx+c)^2}{7}+i\left(\frac{2\sec(dx+c)^4}{9}-\frac{8\sec(dx+c)^2}{5}\right)+\frac{26i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}{21}\right.}{d}$$

input
```
int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x)
```

output
```
-a^3/d*(-26/21*tan(d*x+c)+6/7*tan(d*x+c)*sec(d*x+c)^2+I*(2/9*sec(d*x+c)^4-
8/5*sec(d*x+c)^2)+26/21*I*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)
-cot(d*x+c)),I)*(cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*e^2*(e*s
ec(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.47

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{2\left(\sqrt{2}(195i\,a^3e^2e^{(8i\,dx+8i\,c)}-1158i\,a^3e^2e^{(6i\,dx+6i\,c)}-1456i\,a^3e^2e^{(4i\,dx+4i\,c)}-858i\,a^3e^2e^{(2i\,dx+2i\,c)}-195i\,a^3\right.}{315\,(}$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/315*(sqrt(2)*(195*I*a^3*e^2*e^(8*I*d*x + 8*I*c) - 1158*I*a^3*e^2*e^(6*I
*d*x + 6*I*c) - 1456*I*a^3*e^2*e^(4*I*d*x + 4*I*c) - 858*I*a^3*e^2*e^(2*I*
d*x + 2*I*c) - 195*I*a^3*e^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d
*x + 1/2*I*c) + 195*sqrt(2)*(I*a^3*e^2*e^(8*I*d*x + 8*I*c) + 4*I*a^3*e^2*e
^(6*I*d*x + 6*I*c) + 6*I*a^3*e^2*e^(4*I*d*x + 4*I*c) + 4*I*a^3*e^2*e^(2*I*
d*x + 2*I*c) + I*a^3*e^2)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*
c)))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4
*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx = \int (e\sec(dx+c))^{\frac{5}{2}}(i\,a\tan(dx+c)+a)^3\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^3, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx = \int (e\sec(dx+c))^{\frac{5}{2}}(i\,a\tan(dx+c)+a)^3\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/2}(a + ia\tan(c+dx))^3\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}(a+a\tan(c+d\,x)\;1\mathrm{i})^3\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx = \frac{\sqrt{e}\,a^3e^2\left(-10\sqrt{\sec(dx+c)}\sec(dx+c)^2\tan(dx+c)^2 i + 62\sqrt{\sec(dx+c)}\sec(dx\right.}{}$$

input
```
int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(sqrt(e)*a**3*e**2*( - 10*sqrt(sec(c + d*x))*sec(c + d*x)**2*tan(c + d*x)*
*2*i + 62*sqrt(sec(c + d*x))*sec(c + d*x)**2*i - 135*int(sqrt(sec(c + d*x)
)*sec(c + d*x)**2*tan(c + d*x)**2,x)*d + 45*int(sqrt(sec(c + d*x))*sec(c +
 d*x)**2,x)*d))/(45*d)
```

3.203 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 28, antiderivative size = 175

$$\begin{aligned}\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx= & -\frac{22a^3e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{22ia^3(e\sec(c+dx))^{3/2}}{15d}\\ & +\frac{22a^3e\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}{7d}\\ & +\frac{22i(e\sec(c+dx))^{3/2}\left(a^3+ia^3\tan(c+dx)\right)}{35d}\end{aligned}$$

output

```
-22/5*a^3*e^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*
sec(d*x+c))^(1/2)+22/15*I*a^3*(e*sec(d*x+c))^(3/2)/d+22/5*a^3*e*(e*sec(d*x
+c))^(1/2)*sin(d*x+c)/d+2/7*I*a*(e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2/
d+22/35*I*(e*sec(d*x+c))^(3/2)*(a^3+I*a^3*tan(d*x+c))/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.74

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx = \frac{a^3(e\sec(c+dx))^{3/2}(1+i\tan(c+dx))\left(-116i-308i\cos(2(c+dx))+77ie^{-2i(c+dx)}\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*(e*Sec[c + d*x])^(3/2)*(1 + I*Tan[c + d*x])*(-116*I - (308*I)*Cos[2*(
c + d*x)] + ((77*I)*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2,
 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + 77*Sec[c + d*x]*Si
n[3*(c + d*x)] + 17*Tan[c + d*x]))/(210*d)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3979, 3042, 3979, 3042, 3967, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{11}{7}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{7}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}$$

$\big\downarrow$ 3979

$$\frac{11}{7}a\left(\frac{7}{5}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\left(e\sec(c+dx)\right)^{3/2}}{5d}\right)+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}$$

$\big\downarrow$ 3042

$$\frac{11}{7}a\left(\frac{7}{5}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\left(e\sec(c+dx)\right)^{3/2}}{5d}\right)+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}$$

$\big\downarrow$ 3967

$$\frac{11}{7}a\left(\frac{7}{5}a\left(a\int (e\sec(c+dx))^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\left(e\sec(c+dx)\right)^{3/2}}{5d}\right)+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}$$

$\big\downarrow$ 3042

$$\frac{11}{7}a\left(\frac{7}{5}a\left(a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\left(e\sec(c+dx)\right)^{3}}{5d}\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}$$

$\big\downarrow$ 4255

$$\frac{11}{7}a\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\right.}{}\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}$$

$\big\downarrow$ 3042

$$\frac{11}{7}a\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i(a^2\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{11}{7}a\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i(a^2\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{7}a\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i(a\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{11}{7}a\left(\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}+\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx\right.}{d\sqrt{\cos(c+dx)}\sqrt{e\,s}}\right.\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(((2*I)/7)*a*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2)/d + (11*a*((
7*a*((((2*I)/3)*a*(e*Sec[c + d*x])^(3/2))/d + a*((-2*e^2*EllipticE[(c + d*
x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c
+ d*x]]*Sin[c + d*x])/d)))/5 + (((2*I)/5)*(e*Sec[c + d*x])^(3/2)*(a^2 + I*
a^2*Tan[c + d*x]))/d))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 8.32 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.38

method	result
default	$-\frac{2a^3\sqrt{e\sec(dx+c)}\left(-231\sin(dx+c)+63\tan(dx+c)+63\sec(dx+c)\tan(dx+c)+5i\left(3\sec(dx+c)^3+3\sec(dx+c)^2-28\sec(dx+c)-2\right.\right.}{}$
parts	$\frac{2a^3\left(i\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}+i\left(-\cos(dx+c)^2-2\cos(\right.\right.}{d(\cos(dx+c)+1)}$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/105*a^3/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(-231*sin(d*x+c)+63*tan(d
*x+c)+63*sec(d*x+c)*tan(d*x+c)+5*I*(3*sec(d*x+c)^3+3*sec(d*x+c)^2-28*sec(d
*x+c)-28)+231*I*(cos(d*x+c)^2+2*cos(d*x+c)+1)*EllipticF(I*(cot(d*x+c)-csc(
d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+231*
I*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2))*e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.19

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(231i\,a^3ee^{(7i\,dx+7i\,c)}+287i\,a^3ee^{(5i\,dx+5i\,c)}+253i\,a^3ee^{(3i\,dx+3i\,c)}+77i\,a^3ee^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,c\right.}}{105\,(de}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/105*(sqrt(2)*(231*I*a^3*e*e^(7*I*d*x + 7*I*c) + 287*I*a^3*e*e^(5*I*d*x
+ 5*I*c) + 253*I*a^3*e*e^(3*I*d*x + 3*I*c) + 77*I*a^3*e*e^(I*d*x + I*c))*s
qrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 231*sqrt(2)*(I*
a^3*e*e^(6*I*d*x + 6*I*c) + 3*I*a^3*e*e^(4*I*d*x + 4*I*c) + 3*I*a^3*e*e^(2
*I*d*x + 2*I*c) + I*a^3*e)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, e^(I*d*x + I*c))))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*
I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx = -ia^3\left(\int i(e\sec{(c+dx)})^{\frac{3}{2}}\,dx + \int \left(-3(e\sec{(c+dx)})^{\frac{3}{2}}\tan{(c+dx)}\right)\,dx + \int (e\sec{(c+dx)})^{\frac{3}{2}}\tan^3{(c+dx)}\,dx + \int \left(-3i(e\sec{(c+dx)})^{\frac{3}{2}}\tan^2{(c+dx)}\right)\,dx\right)$$

input
```
integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-I*a**3*(Integral(I*(e*sec(c + d*x))**(3/2), x) + Integral(-3*(e*sec(c + d
*x))**(3/2)*tan(c + d*x), x) + Integral((e*sec(c + d*x))**(3/2)*tan(c + d*
x)**3, x) + Integral(-3*I*(e*sec(c + d*x))**(3/2)*tan(c + d*x)**2, x))
```

Maxima [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx = \int (e\sec{(dx+c)})^{\frac{3}{2}}\,(i\,a\tan{(dx+c)}+a)^3\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^3, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx = \int (e\sec(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)^3\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}(a+a\tan(c+d\,x)\,1\mathrm{i})^3\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx = \frac{\sqrt{e}\,a^3e\left(-6\sqrt{\sec(dx+c)}\sec(dx+c)\tan(dx+c)^2 i+50\sqrt{\sec(dx+c)}\sec(dx+c\right.}{}$$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x)
```

output

```
(sqrt(e)*a**3*e*( - 6*sqrt(sec(c + d*x))*sec(c + d*x)*tan(c + d*x)**2*i +
50*sqrt(sec(c + d*x))*sec(c + d*x)*i - 63*int(sqrt(sec(c + d*x))*sec(c + d
*x)*tan(c + d*x)**2,x)*d + 21*int(sqrt(sec(c + d*x))*sec(c + d*x),x)*d))/(
21*d)
```

3.204 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 28, antiderivative size = 139

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx = \frac{6ia^3\sqrt{e\sec(c+dx)}}{d}+\frac{6a^3\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d} + \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}{5d}+\frac{6i\sqrt{e\sec(c+dx)}(a^3+ia^3\tan(c+dx))}{5d}$$

output

```
6*I*a^3*(e*sec(d*x+c))^(1/2)/d+6*a^3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*
d*x+1/2*c,2^(1/2))*(e*sec(d*x+c))^(1/2)/d+2/5*I*a*(e*sec(d*x+c))^(1/2)*(a+
I*a*tan(d*x+c))^2/d+6/5*I*(e*sec(d*x+c))^(1/2)*(a^3+I*a^3*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.57

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx = \frac{a^3\sec^2(c+dx)\sqrt{e\sec(c+dx)}\left(18i+20i\cos(2(c+dx))+30\cos^{\frac{5}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-5\,\mathrm{s}\right.}{5d}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*Sec[c + d*x]^2*Sqrt[e*Sec[c + d*x]]*(18*I + (20*I)*Cos[2*(c + d*x)] +
 30*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] - 5*Sin[2*(c + d*x)]))/(5
*d)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3979, 3042, 3979, 3042, 3967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{9}{5}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{5}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{9}{5}a\left(\frac{5}{3}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)+\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{5}a\left(\frac{5}{3}a\int\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)+\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{9}{5}a\left(\frac{5}{3}a\left(a\int\sqrt{e\sec(c+dx)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)+\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{5}a\left(\frac{5}{3}a\left(a\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)+\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{9}{5}a\left(\frac{5}{3}a\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\cdot}{3d}\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{5}a\left(\frac{5}{3}a\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+\right.}{3}\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{9}{5}a\left(\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{5}{3}a\left(\frac{2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}+\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(((2*I)/5)*a*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2)/d + (9*a*((5*a
*(((2*I)*a*Sqrt[e*Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d))/3 + (((2*I)/3)*Sqrt[e*Sec[c + d*x]]
*(a^2 + I*a^2*Tan[c + d*x]))/d))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 7.86 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.74

method	result
default	$\frac{a^3\left(-2\tan(dx+c)+8i-\frac{2i\sec(dx+c)^2}{5}+6i(\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\,\text{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\right)\sqrt{e\,\mathrm{s}}}{d}$
parts	$-\frac{2ia^3(\cos(dx+c)+1)\sqrt{e\sec(dx+c)}\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)}{d}+\frac{ia^3\sqrt{e\sec(dx+c)}\left(4\right.}{}$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3/d*(-2*tan(d*x+c)+8*I-2/5*I*sec(d*x+c)^2+6*I*(cos(d*x+c)+1)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-cs
c(d*x+c)),I))*(e*sec(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\int\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx=$$
$$-\frac{2\left(\sqrt{2}\left(-25i\,a^3e^{(4i\,dx+4i\,c)}-36i\,a^3e^{(2i\,dx+2i\,c)}-15i\,a^3\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+15\sqrt{2}\left(i\,a^3e^{(4i\,dx+4i\,c)}\right.\right.}{5\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/5*(sqrt(2)*(-25*I*a^3*e^(4*I*d*x + 4*I*c) - 36*I*a^3*e^(2*I*d*x + 2*I*c
) - 15*I*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) +
15*sqrt(2)*(I*a^3*e^(4*I*d*x + 4*I*c) + 2*I*a^3*e^(2*I*d*x + 2*I*c) + I*a^
3)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(d*e^(4*I*d*x + 4*
I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$
\begin{aligned}
&\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx \\
&= -ia^3\left(\int i\sqrt{e\sec{(c+dx)}}\,dx + \int \left(-3\sqrt{e\sec{(c+dx)}}\tan{(c+dx)}\right)\,dx\right. \\
&\quad\left.+\int \sqrt{e\sec{(c+dx)}}\tan^3{(c+dx)}\,dx + \int \left(-3i\sqrt{e\sec{(c+dx)}}\tan^2{(c+dx)}\right)\,dx\right)
\end{aligned}
$$

input
```
integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-I*a**3*(Integral(I*sqrt(e*sec(c + d*x)), x) + Integral(-3*sqrt(e*sec(c +
d*x))*tan(c + d*x), x) + Integral(sqrt(e*sec(c + d*x))*tan(c + d*x)**3, x)
 + Integral(-3*I*sqrt(e*sec(c + d*x))*tan(c + d*x)**2, x))
```

Maxima [F]

$$
\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx = \int \sqrt{e\sec{(dx+c)}}(i\,a\tan{(dx+c)}+a)^3\,dx
$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^3, x)
```

Giac [F(-2)]

Exception generated.

$$
\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx = \text{Exception raised: TypeError}
$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[2,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{1,[1]%%%}]%%},[1,0]%%%
}+%%%{%%%
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx=\int \sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)^3\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{\sqrt{e}\,a^3\left(-2\sqrt{\sec(dx+c)}\,\tan(dx+c)^2 i+38\sqrt{\sec(dx+c)}\,i+5\left(\int\sqrt{\sec(dx+c)}dx\right)d-15\left(\int\sqrt{\sec(c$$
$$5d$$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(sqrt(e)*a**3*( - 2*sqrt(sec(c + d*x))*tan(c + d*x)**2*i + 38*sqrt(sec(c +
 d*x))*i + 5*int(sqrt(sec(c + d*x)),x)*d - 15*int(sqrt(sec(c + d*x))*tan(c
 + d*x)**2,x)*d))/(5*d)
```

3.205 $\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 124

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{26ia^3}{3d\sqrt{e\sec(c+dx)}}+\frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{6a^3\tan(c+dx)}{d\sqrt{e\sec(c+dx)}}-\frac{2ia^3\tan^2(c+dx)}{3d\sqrt{e\sec(c+dx)}}$$

output
```
-26/3*I*a^3/d/(e*sec(d*x+c))^(1/2)+14*a^3*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2))/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-6*a^3*tan(d*x+c)/d/(e*sec(d*
x+c))^(1/2)-2/3*I*a^3*tan(d*x+c)^2/d/(e*sec(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.53 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.81

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx$$
$$= \frac{2a^3\sqrt{e\sec(c+dx)}(\cos(c)+i\sin(c))(-i\cos(dx)+\sin(dx))\left(-8+7\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\right.}{3de}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3/Sqrt[e*Sec[c + d*x]],x]
```

output
```
(2*a^3*Sqrt[e*Sec[c + d*x]]*(Cos[c] + I*Sin[c])*((-I)*Cos[d*x] + Sin[d*x])
*(-8 + 7*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))] - I*Tan[c + d*x]))/(3*d*e)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3979, 3042, 3977, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{7}{3}a\int \frac{(i\tan(c+dx)a+a)^2}{\sqrt{e\sec(c+dx)}}dx+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{3}a\int \frac{(i\tan(c+dx)a+a)^2}{\sqrt{e\sec(c+dx)}}dx+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3977}$$

$$\frac{7}{3}a\left(-\frac{3a^2\int (e\sec(c+dx))^{3/2}dx}{e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{3}a\left(-\frac{3a^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7}{3}a\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)}{e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{3}a\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{7}{3}a\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{3}a\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{7}{3}a\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^3/Sqrt[e*Sec[c + d*x]],x]
```

output
```
(((2*I)/3)*a*(a + I*a*Tan[c + d*x])^2)/(d*Sqrt[e*Sec[c + d*x]]) + (7*a*((-
3*a^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec
[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/e^2 - ((4*I)*(a^
2 + I*a^2*Tan[c + d*x]))/(d*Sqrt[e*Sec[c + d*x]])))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(110) = 220$.

Time = 7.26 (sec) , antiderivative size = 473, normalized size of antiderivative = 3.81

method	result
default	$\frac{2a^3\left(3i\ln\left(\frac{4\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}+4\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}-2\cos(dx+c)+2}{\cos(dx+c)+1}\right)-3i\ln\left(\frac{2\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}+2\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}}{\cos(dx+c)+1}\right.\right.}{}$
parts	$\frac{2a^3\left(i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(-\cos(dx+c)-2-\sec(dx+c))+i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right.}{d(\cos(dx+c)+1)\sqrt{e\sec(dx+c)}}$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*a^3/d*(3*I*ln(2*(2*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*(
-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-cos(d*x+c)+1)/(cos(d*x+c)+1))-3*I*ln((
2*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*(-cos(d*x+c)/(cos(d*x+
c)+1)^2)^(1/2)-cos(d*x+c)+1)/(cos(d*x+c)+1))+21*I*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*El
lipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*(-cos(d*x+c)-2-sec(d*x+c))+21*I*(1/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d*
x+c)+1)^2)^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(cos(d*x+c)+2+sec(
d*x+c))+3*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(4*sin(d*x+c)-3*tan(d*x+c))
+I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(-12*cos(d*x+c)-12-sec(d*x+c)-sec(
d*x+c)^2))/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1)
^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.99

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{2\left(\sqrt{2}\left(-9i\,a^3e^{(3i\,dx+3i\,c)}-7i\,a^3e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+21\sqrt{2}\left(-i\,a^3e^{(2i\,dx+2i\,c)}-i\,a^3\right)\sqrt{e}\cdots}{3\left(dee^{(2i\,dx+2i\,c)}+de\right)}$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(2)*(-9*I*a^3*e^(3*I*d*x + 3*I*c) - 7*I*a^3*e^(I*d*x + I*c))*sqr
t(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 21*sqrt(2)*(-I*a^
3*e^(2*I*d*x + 2*I*c) - I*a^3)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, e^(I*d*x + I*c))))/(d*e*e^(2*I*d*x + 2*I*c) + d*e)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx = -ia^3\left(\int \frac{i}{\sqrt{e\sec(c+dx)}}\,dx + \int \left(-\frac{3\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\right)dx + \int \frac{\tan^3(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx + \int \left(-\frac{3i\tan^2(c+dx)}{\sqrt{e\sec(c+dx)}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(1/2),x)
```

output
```
-I*a**3*(Integral(I/sqrt(e*sec(c + d*x)), x) + Integral(-3*tan(c + d*x)/sq
rt(e*sec(c + d*x)), x) + Integral(tan(c + d*x)**3/sqrt(e*sec(c + d*x)), x)
 + Integral(-3*I*tan(c + d*x)**2/sqrt(e*sec(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^3}{\sqrt{e\sec(dx+c)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/sqrt(e*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^3}{\sqrt{e\sec(dx+c)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/sqrt(e*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{e}\,a^3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx - \left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^3}{\sec(dx+c)}dx\right)i - 3\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^2}{\sec(dx+c)}dx\right) + 3\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(d}{\sec(dx+c)}\right.\right.}{e}$$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*a**3*(int(sqrt(sec(c + d*x))/sec(c + d*x),x) - int((sqrt(sec(c + 
d*x))*tan(c + d*x)**3)/sec(c + d*x),x)*i - 3*int((sqrt(sec(c + d*x))*tan(c
 + d*x)**2)/sec(c + d*x),x) + 3*int((sqrt(sec(c + d*x))*tan(c + d*x))/sec(
c + d*x),x)*i))/e
```

3.206 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 111

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{10ia^3\sqrt{e\sec(c+dx)}}{3de^2} - \frac{10a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2} - \frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

output
```
-10/3*I*a^3*(e*sec(d*x+c))^(1/2)/d/e^2-10/3*a^3*cos(d*x+c)^(1/2)*InverseJa
cobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x+c))^(1/2)/d/e^2-4/3*I*a*(a+I*a*ta
n(d*x+c))^2/d/(e*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.11

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2a^3\sec^2(c+dx)\left(7i\cos(c+dx)+5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(c+dx)-i\sin(c+dx))\right.}{3d(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))^3}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*a^3*Sec[c + d*x]^2*((7*I)*Cos[c + d*x] + 5*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]) + 3*Sin[c + d*x])*(Cos[c
 + 4*d*x] + I*Sin[c + 4*d*x]))/(3*d*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*S
in[d*x])^3)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3977, 3042, 3967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{5a^2\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5a^2\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3967}$$

$$-\frac{5a^2\left(a\int \sqrt{e\sec(c+dx)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5a^2\left(a\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \textbf{4258}$$

$$-\frac{5a^2\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{5a^2\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \textbf{3120}$$

$$-\frac{5a^2\left(\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(-5*a^2*(((2*I)*a*Sqrt[e*Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d))/(3*e^2) - (((4*I)/3)*a*(a +
I*a*Tan[c + d*x])^2)/(d*(e*Sec[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 7.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.04

method	result
default	$\frac{a^{3}\left(\frac{8\sin(dx+c)}{3}+i\left(-\frac{8\cos(dx+c)}{3}-2\sec(dx+c)\right)+\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(5+5\sec(dx+c))}{3}\right)}{d\sqrt{e\sec(dx+c)}\,e}$
parts	$\frac{a^{3}\left(-\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(\sec(dx+c)+1)}{3}+\frac{2\sin(dx+c)}{3}\right)}{d\sqrt{e\sec(dx+c)}\,e}-\frac{ia^{3}\left(\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^{2}}}\ln\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^3/d*(8/3*sin(d*x+c)+I*(-8/3*cos(d*x+c)-2*sec(d*x+c))+2/3*I*EllipticF(I*(
csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*(5+5*sec(d*x+c)))/(e*sec(d*x+c))^(1/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.74

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2\left(-5i\sqrt{2}a^3\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(2i\,a^3e^{(2i\,dx+2i\,c)}+5i\,a^3\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\right.}\right.}{3\,de^2}$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(-5*I*sqrt(2)*a^3*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c))
 + sqrt(2)*(2*I*a^3*e^(2*I*d*x + 2*I*c) + 5*I*a^3)*sqrt(e/(e^(2*I*d*x + 2*
I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^2)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = -ia^3\left(\int\frac{i}{(e\sec{(c+dx)})^{\frac{3}{2}}}\,dx+\int\left(-\frac{3\tan{(c+dx)}}{(e\sec{(c+dx)})^{\frac{3}{2}}}\right)dx + \int\frac{\tan^3{(c+dx)}}{(e\sec{(c+dx)})^{\frac{3}{2}}}\,dx+\int\left(-\frac{3i\tan^2{(c+dx)}}{(e\sec{(c+dx)})^{\frac{3}{2}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(3/2),x)
```

output
```
-I*a**3*(Integral(I/(e*sec(c + d*x))**(3/2), x) + Integral(-3*tan(c + d*x)
/(e*sec(c + d*x))**(3/2), x) + Integral(tan(c + d*x)**3/(e*sec(c + d*x))**
(3/2), x) + Integral(-3*I*tan(c + d*x)**2/(e*sec(c + d*x))**(3/2), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{e}\,a^3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx - \left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^3}{\sec(dx+c)^2}dx\right)i - 3\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c}{\sec(dx+c)^2}\right.\right.}{e^2}$$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*a**3*(int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x) - int((sqrt(sec(c
 + d*x))*tan(c + d*x)**3)/sec(c + d*x)**2,x)*i - 3*int((sqrt(sec(c + d*x))
*tan(c + d*x)**2)/sec(c + d*x)**2,x) + 3*int((sqrt(sec(c + d*x))*tan(c + d
*x))/sec(c + d*x)**2,x)*i))/e**2
```

3.207 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 111

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx = \frac{6ia^3}{5de^2\sqrt{e\sec(c+dx)}} - \frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

output
```
6/5*I*a^3/d/e^2/(e*sec(d*x+c))^(1/2)-6/5*a^3*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))/d/e^2/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-4/5*I*a*(a+I*a*tan(d*
x+c))^2/d/(e*sec(d*x+c))^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{4ia^3e^{2i(c+dx)}\left(1+e^{2i(c+dx)}-\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)}{5de^2\left(1+e^{2i(c+dx)}\right)\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(((-4*I)/5)*a^3*E^((2*I)*(c + d*x))*(1 + E^((2*I)*(c + d*x)) - Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])
)/(d*e^2*(1 + E^((2*I)*(c + d*x)))*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3977, 3042, 3967, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{3a^2\int \frac{i\tan(c+dx)a+a}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a^2\int \frac{i\tan(c+dx)a+a}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3967}$$

$$-\frac{3a^2\left(a\int \frac{1}{\sqrt{e\sec(c+dx)}}dx-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a^2\left(a\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \; 4258$$

$$-\frac{3a^2\left(\frac{a\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3a^2\left(\frac{a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3119$$

$$-\frac{3a^2\left(\frac{2aE\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(-3*a^2*(((-2*I)*a)/(d*Sqrt[e*Sec[c + d*x]]) + (2*a*EllipticE[(c + d*x)/2,
 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])))/(5*e^2) - (((4*I)/5)*a*
(a + I*a*Tan[c + d*x])^2)/(d*(e*Sec[c + d*x])^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(96) = 192$.

Time = 8.45 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.92

method	result
risch	$-\frac{2i\left(e^{2i(dx+c)}-3\right)a^3\sqrt{2}}{5d\,e^2\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}+\frac{3i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,\left(-2i\,\mathrm{EllipticE}\left(\sqrt{-\cdots}\right.\right.}{\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(\cdots}}}\right.}{5d\,e^2\left(e^{2i(dx+c)}+1\right)\sqrt{\cdots}}$
default	$-\frac{a^3\left(5i\ln\left(\frac{4\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}+4\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}-2\cos(dx+c)+2}{\cos(dx+c)+1}\right)-5i\ln\left(\frac{2\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}+2\sqrt{-\frac{\cos\cdots}{(\cos(d\cdots}}}{\cos(dx+c)+1}\right.\right.}{\cdots}$
parts	$\frac{2a^3\left(\sin(dx+c)\left(\cos(dx+c)^2+\cos(dx+c)+3\right)-3i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\cos(dx+c)+2+\sec(dx+c))\,\mathrm{EllipticF}(i(\csc(dx+c)-\cdots\right.}{5d(\cos(dx+c)+1)\sqrt{e\sec(\cdots}}$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*I*(exp(I*(d*x+c))^2-3)/d*a^3*2^(1/2)/e^2/(e*exp(I*(d*x+c))/(exp(I*(d*
x+c))^2+1))^(1/2)+3/5*I/d*(-2*(e*exp(I*(d*x+c))^2+e)/e/(exp(I*(d*x+c))*(e*
exp(I*(d*x+c))^2+e))^(1/2)+I*(-I*(exp(I*(d*x+c))+I))^(1/2)*2^(1/2)*(I*(exp
(I*(d*x+c))-I))^(1/2)*(I*exp(I*(d*x+c)))^(1/2)/(e*exp(I*(d*x+c))^3+e*exp(I
*(d*x+c)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2)
)+I*EllipticF((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))))*a^3*2^(1/2)/e^2
/(exp(I*(d*x+c))^2+1)/(e*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)*(e*exp
(I*(d*x+c))*(exp(I*(d*x+c))^2+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx =$$
$$-\frac{2\left(3i\sqrt{2}a^3\sqrt{e}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{2}\left(i\,a^3e^{(3i\,dx+3i\,c)}+i\,a^3e^{(i\right.}\right.}{5\,de^3}$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/5*(3*I*sqrt(2)*a^3*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, e^(I*d*x + I*c))) + sqrt(2)*(I*a^3*e^(3*I*d*x + 3*I*c) + I*a^3*e^(I*
d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*
e^3)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx = -ia^3\left(\int \frac{i}{(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx + \int \left(-\frac{3\tan{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\right)dx + \int \frac{\tan^3{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx + \int \left(-\frac{3i\tan^2{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(5/2),x)
```

output
```
-I*a**3*(Integral(I/(e*sec(c + d*x))**(5/2), x) + Integral(-3*tan(c + d*x)
/(e*sec(c + d*x))**(5/2), x) + Integral(tan(c + d*x)**3/(e*sec(c + d*x))**
(5/2), x) + Integral(-3*I*tan(c + d*x)**2/(e*sec(c + d*x))**(5/2), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^3}{(e\sec{(dx+c)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{(e\sec\left(dx+c\right))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{e}\,a^3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx - \left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^3}{\sec(dx+c)^3}dx\right)i - 3\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+}{\sec(dx+c)^3}\right.\right.}{e^3}$$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*a**3*(int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x) - int((sqrt(sec(c
 + d*x))*tan(c + d*x)**3)/sec(c + d*x)**3,x)*i - 3*int((sqrt(sec(c + d*x))
*tan(c + d*x)**2)/sec(c + d*x)**3,x) + 3*int((sqrt(sec(c + d*x))*tan(c + d
*x))/sec(c + d*x)**3,x)*i))/e**3
```

3.208 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 124

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{2a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21de^4} - \frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} - \frac{4i(a^3+ia^3\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}}$$

output
```
-2/21*a^3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d
*x+c))^(1/2)/d/e^4-2/7*I*(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(7/2)-4/21*
I*(a^3+I*a^3*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{a^3\sqrt{e\sec(c+dx)}\Big(5i+5i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(2(c+dx))-i\sin}{21de^4(\cos(dx)+i\sin(dx))^3}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(7/2),x]
```

output
```
-1/21*(a^3*Sqrt[e*Sec[c + d*x]]*(5*I + (5*I)*Cos[2*(c + d*x)] + 2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)
]) - Sin[2*(c + d*x)])*(Cos[2*c + 5*d*x] + I*Sin[2*c + 5*d*x]))/(d*e^4*(Co
s[d*x] + I*Sin[d*x])^3)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3978, 3042, 3977, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3977}$$

$$\frac{a\left(-\frac{a^2\int \sqrt{e\sec(c+dx)}dx}{3e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(-\frac{a^2\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\left(-\frac{a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(-\frac{a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{a\left(-\frac{2a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(((-2*I)/7)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(7/2)) + (a*((-2
*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3
*d*e^2) - (((4*I)/3)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(3/2)
)))/(7*e^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 11.74 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04

method	result
default	$-\frac{2a^3\left(\sin(dx+c)\left(-12\cos(dx+c)^2+1\right)+i\left(12\cos(dx+c)^3-7\cos(dx+c)\right)+i\,\mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\cdots}}{21d\sqrt{e\sec(dx+c)}\,e^3}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}\left(3\,\mathrm{e}^{2i(dx+c)}+2\right)a^3\sqrt{2}}{21d\,e^3\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{2\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\sqrt{i\mathrm{e}^{i(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)a^3\sqrt{e\,\mathrm{e}^{i(\cdots}}}{21d\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx+c)}}\,e^3\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}$
parts	$-\frac{2a^3\left(\sin(dx+c)\left(-3\cos(dx+c)^2-5\right)+i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(5+5\sec(dx+c))\right)}{21d\sqrt{e\sec(dx+c)}\,e^3}-$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*a^3/d/(e*sec(d*x+c))^(1/2)/e^3*(sin(d*x+c)*(-12*cos(d*x+c)^2+1)+I*(1
2*cos(d*x+c)^3-7*cos(d*x+c))+I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(-1-sec(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = \frac{2i\sqrt{2}a^3\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-3i\,a^3e^{(4i\,dx+4i\,c)}-5i\right.}{21\,de^4}$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/21*(2*I*sqrt(2)*a^3*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c))
+ sqrt(2)*(-3*I*a^3*e^(4*I*d*x + 4*I*c) - 5*I*a^3*e^(2*I*d*x + 2*I*c) - 2*
I*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^4)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = -ia^3\left(\int \frac{i}{(e\sec(c+dx))^{\frac{7}{2}}}\,dx+\int\left(-\frac{3\tan(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\right)dx + \int \frac{\tan^3(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\,dx+\int\left(-\frac{3i\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(7/2),x)
```

output
```
-I*a**3*(Integral(I/(e*sec(c + d*x))**(7/2), x) + Integral(-3*tan(c + d*x)
/(e*sec(c + d*x))**(7/2), x) + Integral(tan(c + d*x)**3/(e*sec(c + d*x))**
(7/2), x) + Integral(-3*I*tan(c + d*x)**2/(e*sec(c + d*x))**(7/2), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(7/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\sqrt{e}\,a^3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx - \left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^3}{\sec(dx+c)^4}dx\right)i - 3\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c}{\sec(dx+c)^4}\right.\right.}{e^4}$$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x)
```

output
```
(sqrt(e)*a**3*(int(sqrt(sec(c + d*x))/sec(c + d*x)**4,x) - int((sqrt(sec(c
 + d*x))*tan(c + d*x)**3)/sec(c + d*x)**4,x)*i - 3*int((sqrt(sec(c + d*x))
*tan(c + d*x)**2)/sec(c + d*x)**4,x) + 3*int((sqrt(sec(c + d*x))*tan(c + d
*x))/sec(c + d*x)**4,x)*i))/e**4
```

3.209 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 124

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx = \frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}} - \frac{4i(a^3+ia^3\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}}$$

output
```
2/15*a^3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e^4/cos(d*x+c)^(1/2)/(e*s
ec(d*x+c))^(1/2)-2/9*I*(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(9/2)-4/15*I*
(a^3+I*a^3*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.87 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.95

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx = -\frac{a^3e^{-2i(c+dx)}\left(11+16e^{2i(c+dx)}+5e^{4i(c+dx)}+4\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)(-i}{90de^2(e\sec(c+dx))^{5/2}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(9/2),x]
```

output
```
-1/90*(a^3*(11 + 16*E^((2*I)*(c + d*x)) + 5*E^((4*I)*(c + d*x)) + 4*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])*(-I + Tan[c + d*x])^3)/(d*e^2*E^((2*I)*(c + d*x))*(e*Sec[c + d*x])^(
5/2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3978, 3042, 3977, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{5/2}}dx}{3e^2}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{5/2}}dx}{3e^2}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3977}$$

$$\frac{a\left(\frac{a^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{a^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \textbf{4258}$$

$$\frac{a\left(\frac{a^2\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{a\left(\frac{a^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \textbf{3119}$$

$$\frac{a\left(\frac{2a^2E\left(\frac{1}{2}(c+dx)|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(9/2),x]
```

output
```
(((-2*I)/9)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(9/2)) + (a*((2*
a^2*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
d*x]]) - (((4*I)/5)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(5/2))
))/(3*e^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(108) = 216$.

Time = 18.48 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.08

method	result
default	$\frac{2a^3 (\sin(dx+c) (20\cos(dx+c)^4+20\cos(dx+c)^3+\cos(dx+c)^2+\cos(dx+c)+3)-3i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\cos(dx+c)+2+\sec}{}$
risch	$-\frac{i(5\,\mathrm{e}^{4i(dx+c)}+11\,\mathrm{e}^{2i(dx+c)}+12)a^3\sqrt{2}}{90d\,e^4\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}} - \frac{i(-\frac{2(e\,\mathrm{e}^{2i(dx+c)}+e)}{e\sqrt{\mathrm{e}^{i(dx+c)}(e\,\mathrm{e}^{2i(dx+c)}+e)}}+\frac{i\sqrt{-i(\mathrm{e}^{i(dx+c)}+i)}\,\sqrt{2}\,\sqrt{i(\mathrm{e}^{i(dx+c)}-i)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,(-2}{\sqrt{e}}}{15d\,e^4(\mathrm{e}}$
parts	$\frac{2a^3 (\sin(dx+c) (5\cos(dx+c)^4+5\cos(dx+c)^3+7\cos(dx+c)^2+7\cos(dx+c)+21)-21i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\cos(dx+c)+2+}{45d(\cos}$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*a^3/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)/e^4*(sin(d*x+c)*(20*cos(d*x
+c)^4+20*cos(d*x+c)^3+cos(d*x+c)^2+cos(d*x+c)+3)-3*I*(1/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+2+sec(d*x+c))*EllipticE(
I*(cot(d*x+c)-csc(d*x+c)),I)+3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)+2+sec(d*x+c))*EllipticF(I*(cot(d*x+c)-csc(d*
x+c)),I)+I*(-20*cos(d*x+c)^5-20*cos(d*x+c)^4+9*cos(d*x+c)^3+9*cos(d*x+c)^2
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx = \frac{12i\sqrt{2}a^3\sqrt{e}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

output
```
1/90*(12*I*sqrt(2)*a^3*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-5*I*a^3*e^(5*I*d*x + 5*I*c) - 16*I*a^
3*e^(3*I*d*x + 3*I*c) - 11*I*a^3*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I
*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx=\int \frac{(i\,a\tan{(dx+c)}+a)^3}{(e\sec{(dx+c)})^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(9/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx=\int \frac{(i\,a\tan{(dx+c)}+a)^3}{(e\sec{(dx+c)})^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx=\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(9/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx = \frac{\sqrt{e}\,a^3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^5}dx - \left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^3}{\sec(dx+c)^5}dx\right)i - 3\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c}{\sec(dx+c)^5}\right.\right.}{e^5}$$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x)
```

output
```
(sqrt(e)*a**3*(int(sqrt(sec(c + d*x))/sec(c + d*x)**5,x) - int((sqrt(sec(c
 + d*x))*tan(c + d*x)**3)/sec(c + d*x)**5,x)*i - 3*int((sqrt(sec(c + d*x))
*tan(c + d*x)**2)/sec(c + d*x)**5,x) + 3*int((sqrt(sec(c + d*x))*tan(c + d
*x))/sec(c + d*x)**5,x)*i))/e**5
```

3.210 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 155

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx = \frac{10a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77de^6} + \frac{10a^3\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} - \frac{20i(a^3+ia^3\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}}$$

output
```
10/77*a^3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d
*x+c))^(1/2)/d/e^6+10/77*a^3*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(1/2)-2/11*I*
(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(11/2)-20/77*I*(a^3+I*a^3*tan(d*x+c)
)/d/e^2/(e*sec(d*x+c))^(7/2)
```

Mathematica [A] (verified)

Time = 2.00 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx = \frac{a^3\sqrt{e\sec(c+dx)}\Big(-46i\cos(c+dx) - 22i\cos(3(c+dx)) - 15\sin(c+dx) +$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(11/2),x]
```

output
```
(a^3*Sqrt[e*Sec[c + d*x]]*((-46*I)*Cos[c + d*x] - (22*I)*Cos[3*(c + d*x)]
- 15*Sin[c + d*x] + 20*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[3
*(c + d*x)] - I*Sin[3*(c + d*x)]) - 15*Sin[3*(c + d*x)])*(Cos[3*(c + 2*d*x
)] + I*Sin[3*(c + 2*d*x)]))/(154*d*e^6*(Cos[d*x] + I*Sin[d*x])^3)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3978, 3042, 3977, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{5a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{7/2}}dx}{11e^2}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{7/2}}dx}{11e^2}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3977}$$

$$\frac{5a\left(\frac{3a^2\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5a\left(\frac{3a^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \; 4256$$

$$\frac{5a\left(\frac{3a^2\left(\frac{\int\sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{5a\left(\frac{3a^2\left(\frac{\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \; 4258$$

$$\frac{5a\left(\frac{3a^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{5a\left(\frac{3a^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \; 3120$$

$$\frac{5a\left(\frac{3a^2\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(11/2),x]
```

output
```
(((-2*I)/11)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(11/2)) + (5*a*
((3*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*
x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2) -
 (((4*I)/7)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(7/2))))/(11*e
^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 21.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

method	result
default	$-\frac{2a^3\left(\sin(dx+c)\left(-28\cos(dx+c)^4-3\cos(dx+c)^2-5\right)+28i\cos(dx+c)^5-11i\cos(dx+c)^3+5i\,\mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\right.}{77d\sqrt{e\sec(dx+c)}\,e^5}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}\left(7\,\mathrm{e}^{4i(dx+c)}+24\,\mathrm{e}^{2i(dx+c)}+37\right)a^3\sqrt{2}}{308d\,e^5\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}+\frac{10\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right.}\right.}{77d\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx+c)}}\,e^5\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{e}{\mathrm{e}^{2i}}}}$
parts	$\frac{a^3\left(-\frac{2\sin(dx+c)\left(-7\cos(dx+c)^4-9\cos(dx+c)^2-15\right)}{77}-\frac{30i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(\sec(dx+c)+1)}{77}\right.}{d\sqrt{e\sec(dx+c)}\,e^5}$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/77*a^3/d*(sin(d*x+c)*(-28*cos(d*x+c)^4-3*cos(d*x+c)^2-5)+28*I*cos(d*x+c
)^5-11*I*cos(d*x+c)^3+5*I*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(-1-sec(d*x+c)))/(e*sec(
d*x+c))^(1/2)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx = \frac{-40i\sqrt{2}a^3\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-7i\,a^3e^{(6i\,dx+6i\,c)}-\right.}{308\,de^{6}}$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x, algorithm="fricas"
)
```

output
```
1/308*(-40*I*sqrt(2)*a^3*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c
)) + sqrt(2)*(-7*I*a^3*e^(6*I*d*x + 6*I*c) - 31*I*a^3*e^(4*I*d*x + 4*I*c)
- 61*I*a^3*e^(2*I*d*x + 2*I*c) - 37*I*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1
))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x, algorithm="maxima"
)
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(11/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{(e\sec\left(dx+c\right))^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx=\int \frac{\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(11/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(11/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx=\frac{\sqrt{e}\,a^3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^6}dx-\left(\int \frac{\sqrt{\sec(dx+c)}\ \tan(dx+c)^3}{\sec(dx+c)^6}dx\right)i-3\left(\int \frac{\sqrt{\sec(dx+c)}\ \tan(dx+c}{\sec(dx+c)^6}\right.\right.}{e^6}$$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x)
```

output
```
(sqrt(e)*a**3*(int(sqrt(sec(c + d*x))/sec(c + d*x)**6,x) - int((sqrt(sec(c
 + d*x))*tan(c + d*x)**3)/sec(c + d*x)**6,x)*i - 3*int((sqrt(sec(c + d*x))
*tan(c + d*x)**2)/sec(c + d*x)**6,x) + 3*int((sqrt(sec(c + d*x))*tan(c + d
*x))/sec(c + d*x)**6,x)*i))/e**6
```

3.211 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 155

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx = \frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14a^3\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}} - \frac{28i(a^3+ia^3\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}}$$

output
```
14/39*a^3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e^6/cos(d*x+c)^(1/2)/(e*
sec(d*x+c))^(1/2)+14/117*a^3*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(3/2)-2/13*I*
(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(13/2)-28/117*I*(a^3+I*a^3*tan(d*x+c
))/d/e^2/(e*sec(d*x+c))^(9/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.17 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx = \frac{a^3\sqrt{e\sec(c+dx)}(-i\cos(3(c+dx))+\sin(3(c+dx)))\left(62+8\cos(2(c+dx)\right.}{}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(13/2),x]
```

output
```
(a^3*Sqrt[e*Sec[c + d*x]]*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(62 +
 8*Cos[2*(c + d*x)] - 54*Cos[4*(c + d*x)] + (56*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c +
d*x)) + (42*I)*Sin[2*(c + d*x)] + (63*I)*Sin[4*(c + d*x)]))/(468*d*e^7)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3978, 3042, 3977, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}dx$$
$$\Big\downarrow \text{3978}$$
$$\frac{7a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{9/2}}dx}{13e^2}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{7a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{9/2}}dx}{13e^2}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$
$$\Big\downarrow \text{3977}$$
$$\frac{7a\left(\frac{5a^2\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$
$$\Big\downarrow \text{3042}$$

$$\frac{7a\left(\frac{5a^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{7a\left(\frac{5a^2\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{7a\left(\frac{5a^2\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{7a\left(\frac{5a^2\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{7a\left(\frac{5a^2\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{7a\left(\frac{5a^2\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(13/2),x]
```

output
```
(((-2*I)/13)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(13/2)) + (7*a*
((5*a^2*((6*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*
Sec[c + d*x]]) + (2*Sin[c + d*x])/(5*d*e*(e*Sec[c + d*x])^(3/2))))/(9*e^2)
 - (((4*I)/9)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(9/2))))/(13
*e^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(135) = 270$.

Time = 30.82 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.82

method	result
default	$-\frac{2a^3\left(\sin(dx+c)\left(-36\cos(dx+c)^6-36\cos(dx+c)^5-5\cos(dx+c)^4-5\cos(dx+c)^3-7\cos(dx+c)^2-7\cos(dx+c)-21\right)-21i\sqrt{\frac{1}{\cos(dx+}}}{}$
risch	$-\frac{i(9\,\mathrm{e}^{6i(dx+c)}+41\,\mathrm{e}^{4i(dx+c)}+83\,\mathrm{e}^{2i(dx+c)}+219)a^3\sqrt{2}}{936d\,e^6\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{7i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right.}}{}\right.}{}$
parts	$\frac{2a^3\left(\sin(dx+c)\left(45\cos(dx+c)^6+45\cos(dx+c)^5+55\cos(dx+c)^4+55\cos(dx+c)^3+77\cos(dx+c)^2+77\cos(dx+c)+231\right)-231i\sqrt{\frac{}{\cos(}}}{}$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/117*a^3/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)/e^6*(sin(d*x+c)*(-36*cos(
d*x+c)^6-36*cos(d*x+c)^5-5*cos(d*x+c)^4-5*cos(d*x+c)^3-7*cos(d*x+c)^2-7*co
s(d*x+c)-21)-21*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)+2+sec(d*x+c))*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+21*I*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+2+se
c(d*x+c))*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)+I*(36*cos(d*x+c)^7+36*cos
(d*x+c)^6-13*cos(d*x+c)^5-13*cos(d*x+c)^4))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx = \frac{\left(336i\sqrt{2}a^3\sqrt{e}e^{(i\,dx+i\,c)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\right.\right.\right.}}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x, algorithm="fricas"
)
```

output
```
1/936*(336*I*sqrt(2)*a^3*sqrt(e)*e^(I*d*x + I*c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-9*I*a^3*e^(8*I*d*x
+ 8*I*c) - 50*I*a^3*e^(6*I*d*x + 6*I*c) - 124*I*a^3*e^(4*I*d*x + 4*I*c) +
34*I*a^3*e^(2*I*d*x + 2*I*c) + 117*I*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1)
)*e^(1/2*I*d*x + 1/2*I*c))*e^(-I*d*x - I*c)/(d*e^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(13/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{(e\sec\left(dx+c\right))^{\frac{13}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x, algorithm="maxima"
)
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(13/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{(e\sec\left(dx+c\right))^{\frac{13}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{13/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(13/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(13/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx = \frac{\sqrt{e}\,a^3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^7}dx - \left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^3}{\sec(dx+c)^7}dx\right)i - 3\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c}{\sec(dx+c)^7}\right.\right.}{e^7}$$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x)
```

output
```
(sqrt(e)*a**3*(int(sqrt(sec(c + d*x))/sec(c + d*x)**7,x) - int((sqrt(sec(c
 + d*x))*tan(c + d*x)**3)/sec(c + d*x)**7,x)*i - 3*int((sqrt(sec(c + d*x))
*tan(c + d*x)**2)/sec(c + d*x)**7,x) + 3*int((sqrt(sec(c + d*x))*tan(c + d
*x))/sec(c + d*x)**7,x)*i))/e**7
```

3.212 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 186

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx = \frac{2a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{11de^8} + \frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} + \frac{2a^3\sin(c+dx)}{11de^7\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{12i(a^3+ia^3\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}}$$

output
```
2/11*a^3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*
x+c))^(1/2)/d/e^8+6/55*a^3*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(5/2)+2/11*a^3*
sin(d*x+c)/d/e^7/(e*sec(d*x+c))^(1/2)-2/15*I*(a+I*a*tan(d*x+c))^3/d/(e*sec
(d*x+c))^(15/2)-12/55*I*(a^3+I*a^3*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(11/2)
```

Mathematica [A] (verified)

Time = 2.50 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.91

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx = \frac{a^3\sqrt{e\sec(c+dx)}\Big(-332i\cos(c+dx)-154i\cos(3(c+dx))+22i\cos(5(c+$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(15/2),x]
```

output
```
(a^3*Sqrt[e*Sec[c + d*x]]*((-332*I)*Cos[c + d*x] - (154*I)*Cos[3*(c + d*x)
] + (22*I)*Cos[5*(c + d*x)] - 114*Sin[c + d*x] + 240*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*(Cos[3*(c + d*x)] - I*Sin[3*(c + d*x)]) - 81*Sin[3
*(c + d*x)] + 33*Sin[5*(c + d*x)])*(Cos[3*(c + 2*d*x)] + I*Sin[3*(c + 2*d*
x)]))/(1320*d*e^8*(Cos[d*x] + I*Sin[d*x])^3)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3978, 3042, 3977, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{3a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{11/2}}dx}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\int\frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{11/2}}dx}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3977}$$

$$\frac{3a\left(\frac{7a^2\int\frac{1}{(e\sec(c+dx))^{7/2}}dx}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\left(\frac{7a^2\int\frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{3a\left(\frac{7a^2\left(\frac{5\int\frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\left(\frac{7a^2\left(\frac{5\int\frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{3a\left(\frac{7a^2\left(\frac{5\left(\frac{\int\sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\left(\frac{7a^2\left(\frac{5\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{3a\left(\frac{7a^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\left(\frac{7a^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{3a\left(\frac{7a^2\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(15/2),x]
```

output
```
(((-2*I)/15)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(15/2)) + (3*a*
((7*a^2*((2*Sin[c + d*x])/(7*d*e*(e*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*
Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2)))/(11*e^2) - (((4*I)/
11)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(11/2))))/(5*e^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 32.80 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.81

method	result
default	$\frac{a^3 \left(\frac{2 \sin(dx+c)\left(44 \cos(dx+c)^6+7 \cos(dx+c)^4+9 \cos(dx+c)^2+15\right)}{165}+\frac{2i\left(-44 \cos(dx+c)^7+15 \cos(dx+c)^5\right)}{165}+\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\ \text{EllipticF}(i(\csc(dx}{}\right.}{d\sqrt{e\sec(dx+c)}\, e^7}$
parts	$\frac{a^3 \left(\frac{2 \sin(dx+c)\left(77 \cos(dx+c)^6+91 \cos(dx+c)^4+117 \cos(dx+c)^2+195\right)}{1155}+\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\ \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \text{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)}{1155}\right.}{d\sqrt{e\sec(dx+c)}\, e^7}$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x,method=_RETURNVERBOSE)
```

output
```
a^3/d*(2/165*sin(d*x+c)*(44*cos(d*x+c)^6+7*cos(d*x+c)^4+9*cos(d*x+c)^2+15)
+2/165*I*(-44*cos(d*x+c)^7+15*cos(d*x+c)^5)+2/165*I*(1/(cos(d*x+c)+1))^(1/
2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(-15-15*sec(d*x+c)))/(e*sec(d*x+c))^(1/2)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.84

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx = \frac{\left(-480i\sqrt{2}a^3\sqrt{e}e^{(2i\,dx+2i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}(-11i\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x, algorithm="fricas"
)
```

output
```
1/2640*(-480*I*sqrt(2)*a^3*sqrt(e)*e^(2*I*d*x + 2*I*c)*weierstrassPInverse
(-4, 0, e^(I*d*x + I*c)) + sqrt(2)*(-11*I*a^3*e^(10*I*d*x + 10*I*c) - 73*I
*a^3*e^(8*I*d*x + 8*I*c) - 218*I*a^3*e^(6*I*d*x + 6*I*c) - 446*I*a^3*e^(4*
I*d*x + 4*I*c) - 235*I*a^3*e^(2*I*d*x + 2*I*c) + 55*I*a^3)*sqrt(e/(e^(2*I*
d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-2*I*d*x - 2*I*c)/(d*e^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(15/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{15}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x, algorithm="maxima"
)
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(15/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{(e\sec\left(dx+c\right))^{\frac{15}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(15/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx=\int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{15/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(15/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(15/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx=\frac{\sqrt{e}\,a^3\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^8}dx-\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^3}{\sec(dx+c)^8}dx\right)i-3\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+}{\sec(dx+c)^8}\right.\right.}{e^8}$$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x)
```

output
```
(sqrt(e)*a**3*(int(sqrt(sec(c + d*x))/sec(c + d*x)**8,x) - int((sqrt(sec(c
 + d*x))*tan(c + d*x)**3)/sec(c + d*x)**8,x)*i - 3*int((sqrt(sec(c + d*x))
*tan(c + d*x)**2)/sec(c + d*x)**8,x) + 3*int((sqrt(sec(c + d*x))*tan(c + d
*x))/sec(c + d*x)**8,x)*i))/e**8
```

3.213 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 28, antiderivative size = 215

$$\begin{aligned}\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx = & -\frac{22a^4e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{22ia^4(e\sec(c+dx))^{3/2}}{9d}\\ & +\frac{22a^4e\sqrt{e\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}{9d}\\ & +\frac{10i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)^2}{21d}\\ & +\frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}{21d}\end{aligned}$$

output
```
-22/3*a^4*e^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*
sec(d*x+c))^(1/2)+22/9*I*a^4*(e*sec(d*x+c))^(3/2)/d+22/3*a^4*e*(e*sec(d*x+
c))^(1/2)*sin(d*x+c)/d+2/9*I*a*(e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3/d
+10/21*I*(e*sec(d*x+c))^(3/2)*(a^2+I*a^2*tan(d*x+c))^2/d+22/21*I*(e*sec(d*
x+c))^(3/2)*(a^4+I*a^4*tan(d*x+c))/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.91 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.52

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx = \frac{(e\sec(c+dx))^{3/2}\left(\frac{22i\sqrt{2}e^{-i(3c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}\left(-1+e^{2ic}\right)\text{Hy}\right.}{-1+e^{2ic}}\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((e*Sec[c + d*x])^(3/2)*(((22*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I
)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))]))/(E^(I*(3*c + d*x))*(-1 + E^((2*I)*c))) + (Csc[c]*Se
c[c + d*x]^(9/2)*(Cos[4*c] - I*Sin[4*c])*(1260*Cos[d*x] + 1050*Cos[2*c + d
*x] + 742*Cos[2*c + 3*d*x] + 413*Cos[4*c + 3*d*x] + 231*Cos[4*c + 5*d*x] -
 (720*I)*Sin[d*x] + (720*I)*Sin[2*c + d*x] - (336*I)*Sin[2*c + 3*d*x] + (3
36*I)*Sin[4*c + 3*d*x]))/56)*(a + I*a*Tan[c + d*x])^4)/(9*d*Sec[c + d*x]^(
11/2)*(Cos[d*x] + I*Sin[d*x])^4)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3979, 3042, 3979, 3042, 3979, 3042, 3967, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^4(e\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^4(e\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{5}{3}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^3dx+\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^3dx+\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{5}{3}a\left(\frac{11}{7}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}\right)+\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\left(\frac{11}{7}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}\right)+\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{5}{3}a\left(\frac{11}{7}a\left(\frac{7}{5}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}\right)+\frac{2ia(\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\left(\frac{11}{7}a\left(\frac{7}{5}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}\right)+\frac{2ia(\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{5}{3}a\left(\frac{11}{7}a\left(\frac{7}{5}a\left(a\int(e\sec(c+dx))^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\left(\frac{11}{7}a\left(\frac{7}{5}a\left(a\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{5}{3}a\left(\frac{11}{7}a\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i(a^2}{}\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\left(\frac{11}{7}a\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5}{3}a\left(\frac{11}{7}a\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\left(\frac{11}{7}a\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{5}{3}a\left(\frac{11}{7}a\left(\frac{2i(a^2+ia^2\tan(c+dx))\,(e\sec(c+dx))^{3/2}}{5d}+\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c\right.}{d\sqrt{\cos(c+dx)}}\right.\right.\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((2*I)/9)*a*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3)/d + (5*a*(((
(2*I)/7)*a*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2)/d + (11*a*((7*
a*((((2*I)/3)*a*(e*Sec[c + d*x])^(3/2))/d + a*((-2*e^2*EllipticE[(c + d*x)
/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c +
d*x]]*Sin[c + d*x])/d)))/5 + (((2*I)/5)*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^
2*Tan[c + d*x]))/d))/7))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 17.10 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.25

method	result
default	$-\frac{2a^4e\sqrt{e\sec(dx+c)}\left(\tan(dx+c)\sec(dx+c)^3\left(-231\cos(dx+c)^4+91\cos(dx+c)^3+91\cos(dx+c)^2-7\cos(dx+c)-7\right)+i\left(-168-168\right.\right.}{}$
parts	$\frac{2a^4\left(i\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\operatorname{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)+i\left(-\cos(dx+c)^2-2\cos(\right.\right.}{d(\cos(dx+c)+1)}$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-2/63*a^4/d*e*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(tan(d*x+c)*sec(d*x+c)^3
*(-231*cos(d*x+c)^4+91*cos(d*x+c)^3+91*cos(d*x+c)^2-7*cos(d*x+c)-7)+I*(-16
8-168*sec(d*x+c)+36*sec(d*x+c)^2+36*sec(d*x+c)^3)+I*(231*cos(d*x+c)^2+462*
cos(d*x+c)+231)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*(-231*cos(d*x+c)^2-462*cos(d*x+c
)-231)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.16

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx =$$
$$-\frac{2\left(\sqrt{2}(231i\,a^4ee^{(9i\,dx+9i\,c)}+406i\,a^4ee^{(7i\,dx+7i\,c)}+540i\,a^4ee^{(5i\,dx+5i\,c)}+330i\,a^4ee^{(3i\,dx+3i\,c)}+77i\,a^4ee^{(i\,dx+i}\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-2/63*(sqrt(2)*(231*I*a^4*e*e^(9*I*d*x + 9*I*c) + 406*I*a^4*e*e^(7*I*d*x +
 7*I*c) + 540*I*a^4*e*e^(5*I*d*x + 5*I*c) + 330*I*a^4*e*e^(3*I*d*x + 3*I*c
) + 77*I*a^4*e*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I
*d*x + 1/2*I*c) + 231*sqrt(2)*(I*a^4*e*e^(8*I*d*x + 8*I*c) + 4*I*a^4*e*e^(
6*I*d*x + 6*I*c) + 6*I*a^4*e*e^(4*I*d*x + 4*I*c) + 4*I*a^4*e*e^(2*I*d*x +
2*I*c) + I*a^4*e)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, e^(I*d*x + I*c))))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*
d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx = a^4\left(\int (e\sec(c+dx))^{\frac{3}{2}}\,dx + \int \left(-6(e\sec(c+dx))^{\frac{3}{2}}\tan^2(c+dx)\right)dx + \int (e\sec(c+dx))^{\frac{3}{2}}\tan^4(c+dx)\,dx + \int 4i(e\sec(c+dx))^{\frac{3}{2}}\tan(c+dx)\,dx + \int \left(-4i(e\sec(c+dx))^{\frac{3}{2}}\tan^3(c+dx)\right)dx\right)$$

input
```
integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**4,x)
```

output
```
a**4*(Integral((e*sec(c + d*x))**(3/2), x) + Integral(-6*(e*sec(c + d*x))*
*(3/2)*tan(c + d*x)**2, x) + Integral((e*sec(c + d*x))**(3/2)*tan(c + d*x)
**4, x) + Integral(4*I*(e*sec(c + d*x))**(3/2)*tan(c + d*x), x) + Integral
(-4*I*(e*sec(c + d*x))**(3/2)*tan(c + d*x)**3, x))
```

Maxima [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx = \int (e\sec(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)^4\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^4, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx = \int (e\sec(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)^4\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}(a+a\tan(c+d\,x)\,1\mathrm{i})^4\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^4, x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx = \frac{\sqrt{e}\,a^4e\Big(-24\sqrt{\sec(dx+c)}\,\sec(dx+c)\tan(dx+c)^2\,i+88\sqrt{\sec(dx+c)}\,\sec(dx+$$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x)
```

output
```
(sqrt(e)*a**4*e*( - 24*sqrt(sec(c + d*x))*sec(c + d*x)*tan(c + d*x)**2*i +
 88*sqrt(sec(c + d*x))*sec(c + d*x)*i + 21*int(sqrt(sec(c + d*x))*sec(c +
d*x)*tan(c + d*x)**4,x)*d - 126*int(sqrt(sec(c + d*x))*sec(c + d*x)*tan(c
+ d*x)**2,x)*d + 21*int(sqrt(sec(c + d*x))*sec(c + d*x),x)*d))/(21*d)
```

3.214 $\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 28, antiderivative size = 183

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx = \frac{78ia^4\sqrt{e\sec(c+dx)}}{7d} + \frac{78a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7d} + \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}{7d} + \frac{26i\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)^2}{35d} + \frac{78i\sqrt{e\sec(c+dx)}\left(a^4+ia^4\tan(c+dx)\right)}{35d}$$

output
```
78/7*I*a^4*(e*sec(d*x+c))^(1/2)/d+78/7*a^4*cos(d*x+c)^(1/2)*InverseJacobiA
M(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x+c))^(1/2)/d+2/7*I*a*(e*sec(d*x+c))^(1/
2)*(a+I*a*tan(d*x+c))^3/d+26/35*I*(e*sec(d*x+c))^(1/2)*(a^2+I*a^2*tan(d*x+
c))^2/d+78/35*I*(e*sec(d*x+c))^(1/2)*(a^4+I*a^4*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 2.68 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.55

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{a^4\sec^4(c+dx)\sqrt{e\sec(c+dx)}\Big(728i+1008i\cos(2(c+dx))+280i\cos(4(c+dx))+1560\cos^{\frac{9}{2}}(c+dx)\,\mathrm{E}}{140d}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(a^4*Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(728*I + (1008*I)*Cos[2*(c + d*x)
] + (280*I)*Cos[4*(c + d*x)] + 1560*Cos[c + d*x]^(9/2)*EllipticF[(c + d*x)
/2, 2] - 150*Sin[2*(c + d*x)] - 85*Sin[4*(c + d*x)]))/(140*d)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3979, 3042, 3979, 3042, 3979, 3042, 3967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^4\sqrt{e\sec(c+dx)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int (a+ia\tan(c+dx))^4\sqrt{e\sec(c+dx)}dx$$
$$\Big\downarrow \ 3979$$
$$\frac{13}{7}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^3dx+\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$
$$\Big\downarrow \ 3042$$
$$\frac{13}{7}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^3dx+\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \; 3979$$

$$\frac{13}{7}a\left(\frac{9}{5}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}\right)+\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{13}{7}a\left(\frac{9}{5}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}\right)+\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \; 3979$$

$$\frac{13}{7}a\left(\frac{9}{5}a\left(\frac{5}{3}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)+\frac{2ia(a+i}{}\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{13}{7}a\left(\frac{9}{5}a\left(\frac{5}{3}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)+\frac{2ia(a+i}{}\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \; 3967$$

$$\frac{13}{7}a\left(\frac{9}{5}a\left(\frac{5}{3}a\left(a\int \sqrt{e\sec(c+dx)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)+\frac{2i}{}\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{13}{7}a\left(\frac{9}{5}a\left(\frac{5}{3}a\left(a\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{13}{7}a\left(\frac{9}{5}a\left(\frac{5}{3}a\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i(a^2+ia^2\tan(c+\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13}{7}a\left(\frac{9}{5}a\left(\frac{5}{3}a\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i(a^2+ia^2\,\mathrm{ta}\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{13}{7}a\left(\frac{9}{5}a\left(\frac{2i(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}{3d}+\frac{5}{3}a\left(\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+d}{d}\right.\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((2*I)/7)*a*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3)/d + (13*a*((((
2*I)/5)*a*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2)/d + (9*a*((5*a*((
(2*I)*a*Sqrt[e*Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d))/3 + (((2*I)/3)*Sqrt[e*Sec[c + d*x]]*(a
^2 + I*a^2*Tan[c + d*x]))/d))/5))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 14.79 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.65

method	result
default	$\frac{a^4\left(-\frac{34\tan(dx+c)}{7}+\frac{2\tan(dx+c)\sec(dx+c)^2}{7}+16i-\frac{8i\sec(dx+c)^2}{5}+\frac{78i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}(i(\cot(dx+c)-\right.}{d}$
parts	$-\frac{2ia^4(\cos(dx+c)+1)\sqrt{e\sec(dx+c)}\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)}{d}+\frac{a^4\left(-\frac{6\tan(dx+c)}{7}+\frac{2\right.}{}$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
a^4/d*(-34/7*tan(d*x+c)+2/7*tan(d*x+c)*sec(d*x+c)^2+16*I-8/5*I*sec(d*x+c)^
2+78/7*I*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I))*(e*sec(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.03

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx =$$
$$-\frac{2\left(\sqrt{2}\left(-365i\,a^4e^{(6i\,dx+6i\,c)}-793i\,a^4e^{(4i\,dx+4i\,c)}-663i\,a^4e^{(2i\,dx+2i\,c)}-195i\,a^4\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{35\left(de^{(6i\,dx+6i\,c)}+3\,de^{(}\right.}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-2/35*(sqrt(2)*(-365*I*a^4*e^(6*I*d*x + 6*I*c) - 793*I*a^4*e^(4*I*d*x + 4*
I*c) - 663*I*a^4*e^(2*I*d*x + 2*I*c) - 195*I*a^4)*sqrt(e/(e^(2*I*d*x + 2*I
*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 195*sqrt(2)*(I*a^4*e^(6*I*d*x + 6*I*c)
 + 3*I*a^4*e^(4*I*d*x + 4*I*c) + 3*I*a^4*e^(2*I*d*x + 2*I*c) + I*a^4)*sqrt
(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(d*e^(6*I*d*x + 6*I*c) +
3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\begin{aligned}\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx &= a^4\left(\int \sqrt{e\sec{(c+dx)}}\,dx+\int\left(-6\sqrt{e\sec{(c+dx)}}\tan^2{(c+dx)}\right)dx\right.\\ &\quad+\int \sqrt{e\sec{(c+dx)}}\tan^4{(c+dx)}\,dx+\int 4i\sqrt{e\sec{(c+dx)}}\tan{(c+dx)}\,dx\\ &\quad\left.+\int\left(-4i\sqrt{e\sec{(c+dx)}}\tan^3{(c+dx)}\right)dx\right)\end{aligned}$$

input
```
integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**4,x)
```

output
```
a**4*(Integral(sqrt(e*sec(c + d*x)), x) + Integral(-6*sqrt(e*sec(c + d*x))
*tan(c + d*x)**2, x) + Integral(sqrt(e*sec(c + d*x))*tan(c + d*x)**4, x) +
 Integral(4*I*sqrt(e*sec(c + d*x))*tan(c + d*x), x) + Integral(-4*I*sqrt(e
*sec(c + d*x))*tan(c + d*x)**3, x))
```

Maxima [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx = \int \sqrt{e\sec\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)^4\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[2,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{1,[1]%%%}]%%},[1,0]%%%
}+%%%{%%%
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx = \int \sqrt{\frac{e}{\cos(c+d\,x)}}\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^4\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^4, x)
```

Reduce [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{\sqrt{e}\,a^4\Big(-8\sqrt{\sec(dx+c)}\,\tan(dx+c)^2 i+72\sqrt{\sec(dx+c)}\,i+5\left(\int\sqrt{\sec(dx+c)}dx\right)d+5\Big(\int\sqrt{\sec(dx}}{5d}$$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x)
```

output
```
(sqrt(e)*a**4*( - 8*sqrt(sec(c + d*x))*tan(c + d*x)**2*i + 72*sqrt(sec(c +
 d*x))*i + 5*int(sqrt(sec(c + d*x)),x)*d + 5*int(sqrt(sec(c + d*x))*tan(c
+ d*x)**4,x)*d - 30*int(sqrt(sec(c + d*x))*tan(c + d*x)**2,x)*d))/(5*d)
```

3.215 $\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 178

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx &= \frac{154a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} \\ &\quad - \frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2} \\ &\quad - \frac{154a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de} \\ &\quad - \frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}} \\ &\quad - \frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}{5de^2}\end{aligned}$$

output
```
154/5*a^4*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec(
d*x+c))^(1/2)-154/15*I*a^4*(e*sec(d*x+c))^(3/2)/d/e^2-154/5*a^4*(e*sec(d*x
+c))^(1/2)*sin(d*x+c)/d/e-4*I*a*(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(1/2
)-22/5*I*(e*sec(d*x+c))^(3/2)*(a^4+I*a^4*tan(d*x+c))/d/e^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.79 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.69

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{2ia^4e^{i(c+dx)}\left(-77-176e^{2i(c+dx)}-111e^{4i(c+dx)}+77\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)\sqrt{e\sec(c+dx)}}{15de\left(1+e^{2i(c+dx)}\right)^2}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^4/Sqrt[e*Sec[c + d*x]],x]
```

output
```
(((-2*I)/15)*a^4*E^(I*(c + d*x))*(-77 - 176*E^((2*I)*(c + d*x)) - 111*E^((
4*I)*(c + d*x)) + 77*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2
, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(d*e*(1 + E^((2*I
)*(c + d*x)))^2)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3977, 3042, 3979, 3042, 3967, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{11a^2\int(e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2dx}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$-\frac{11a^2\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2dx}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \textcolor{blue}{3979}$$

$$-\frac{11a^2\left(\frac{7}{5}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx+\frac{2i(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}{5d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$-\frac{11a^2\left(\frac{7}{5}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx+\frac{2i(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}{5d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \textcolor{blue}{3967}$$

$$-\frac{11a^2\left(\frac{7}{5}a\left(a\int (e\sec(c+dx))^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}{5d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$-\frac{11a^2\left(\frac{7}{5}a\left(a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}{5d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \textcolor{blue}{4255}$$

$$-\frac{11a^2\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/}}{5d}\right.}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$-\frac{11a^2\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc(c+dx+\frac{\pi}{2})}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i(a^2+ia^2\tan(c+dx))(e\sec(c+}{5d}\right.}{e^2}$$
$$\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 4258$$

$$-\frac{11a^2\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i(a^2+ia^2\tan(c+dx))(e\sec(c+dx}{5d}\right.}{e^2}$$
$$\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{11a^2\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin(c+dx+\frac{\pi}{2})}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i(a^2+ia^2\tan(c+dx))(e\sec(c+}{5d}\right.}{e^2}$$
$$\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3119$$

$$-\frac{11a^2\left(\frac{2i(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}{5d}+\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx)}{3d}\right.\right.}{e^2}$$
$$\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^4/Sqrt[e*Sec[c + d*x]],x]
```

output
```
((-4*I)*a*(a + I*a*Tan[c + d*x])^3)/(d*Sqrt[e*Sec[c + d*x]]) - (11*a^2*((7
*a*((((2*I)/3)*a*(e*Sec[c + d*x])^(3/2))/d + a*((-2*e^2*EllipticE[(c + d*x
)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c +
 d*x]]*Sin[c + d*x])/d)))/5 + (((2*I)/5)*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a
^2*Tan[c + d*x]))/d))/e^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 497 vs. 2(155) = 310.

Time = 13.46 (sec) , antiderivative size = 498, normalized size of antiderivative = 2.80

method	result
default	$\frac{2a^4\left(30i\ln\left(\frac{4\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}+4\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}-2\cos(dx+c)+2}{\cos(dx+c)+1}\right)-30i\ln\left(\frac{2\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}+2\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}}{\cos(dx+c)+1}\right)}{}$
parts	Expression too large to display

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*a^4/d*(30*I*ln(2*(2*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2
*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-cos(d*x+c)+1)/(cos(d*x+c)+1))-30*I*l
n((2*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*(-cos(d*x+c)/(cos(d
*x+c)+1)^2)^(1/2)-cos(d*x+c)+1)/(cos(d*x+c)+1))+231*I*(-cos(d*x+c)/(cos(d*
x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*(-cos(d*x+c)-2-sec(d*x+c))+231*I*
(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(cos(d*x+c)+2
+sec(d*x+c))+3*tan(d*x+c)*sec(d*x+c)^2*(40*cos(d*x+c)^3-37*cos(d*x+c)^2+co
s(d*x+c)+1)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+20*I*(-cos(d*x+c)/(cos(d*
x+c)+1)^2)^(1/2)*(-6-6*cos(d*x+c)-sec(d*x+c)-sec(d*x+c)^2))/(cos(d*x+c)+1)
/(e*sec(d*x+c))^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.92

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx =$$
$$-\frac{2\left(\sqrt{2}\left(-111i\,a^4e^{(5i\,dx+5i\,c)}-176i\,a^4e^{(3i\,dx+3i\,c)}-77i\,a^4e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+231\sqrt{2}(-}{15\,(dee^{(4i\,dx+4i\,c)}+}$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(sqrt(2)*(-111*I*a^4*e^(5*I*d*x + 5*I*c) - 176*I*a^4*e^(3*I*d*x + 3*
I*c) - 77*I*a^4*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*
I*d*x + 1/2*I*c) + 231*sqrt(2)*(-I*a^4*e^(4*I*d*x + 4*I*c) - 2*I*a^4*e^(2*
I*d*x + 2*I*c) - I*a^4)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, e^(I*d*x + I*c))))/(d*e*e^(4*I*d*x + 4*I*c) + 2*d*e*e^(2*I*d*x + 2
*I*c) + d*e)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx = a^4\left(\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx+\int\left(-\frac{6\tan^2(c+dx)}{\sqrt{e\sec(c+dx)}}\right)dx + \int \frac{\tan^4(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx+\int\frac{4i\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx + \int\left(-\frac{4i\tan^3(c+dx)}{\sqrt{e\sec(c+dx)}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(1/2),x)
```

output
```
a**4*(Integral(1/sqrt(e*sec(c + d*x)), x) + Integral(-6*tan(c + d*x)**2/sq
rt(e*sec(c + d*x)), x) + Integral(tan(c + d*x)**4/sqrt(e*sec(c + d*x)), x)
 + Integral(4*I*tan(c + d*x)/sqrt(e*sec(c + d*x)), x) + Integral(-4*I*tan(
c + d*x)**3/sqrt(e*sec(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{(i\,a\tan{(dx+c)}+a)^4}{\sqrt{e\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/sqrt(e*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{(i\,a\tan{(dx+c)}+a)^4}{\sqrt{e\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/sqrt(e*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{e}\,a^4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx + \int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^4}{\sec(dx+c)}dx - 4\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^3}{\sec(dx+c)}dx\right)i - 6\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c}{\sec(dx+c)}\right.\right.}{e}$$

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*a**4*(int(sqrt(sec(c + d*x))/sec(c + d*x),x) + int((sqrt(sec(c +
d*x))*tan(c + d*x)**4)/sec(c + d*x),x) - 4*int((sqrt(sec(c + d*x))*tan(c +
 d*x)**3)/sec(c + d*x),x)*i - 6*int((sqrt(sec(c + d*x))*tan(c + d*x)**2)/s
ec(c + d*x),x) + 4*int((sqrt(sec(c + d*x))*tan(c + d*x))/sec(c + d*x),x)*i
))/e
```

3.216 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 146

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{10ia^4\sqrt{e\sec(c+dx)}}{de^2} - \frac{10a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{de^2} - \frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}} - \frac{2i\sqrt{e\sec(c+dx)}(a^4+ia^4\tan(c+dx))}{de^2}$$

output
```
-10*I*a^4*(e*sec(d*x+c))^(1/2)/d/e^2-10*a^4*cos(d*x+c)^(1/2)*InverseJacobi
AM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x+c))^(1/2)/d/e^2-4/3*I*a*(a+I*a*tan(d*
x+c))^3/d/(e*sec(d*x+c))^(3/2)-2*I*(e*sec(d*x+c))^(1/2)*(a^4+I*a^4*tan(d*x
+c))/d/e^2
```

Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.89

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = \frac{a^4\sec^3(c+dx)\left(21+19\cos(2(c+dx))-30i\cos^{\frac{3}{2}}(c+dx)\,\mathrm{EllipticF}\left(\frac{1}{2}(c+\right.\right.}{3d(e\sec(c+}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(a^4*Sec[c + d*x]^3*(21 + 19*Cos[2*(c + d*x)] - (30*I)*Cos[c + d*x]^(3/2)*
EllipticF[(c + d*x)/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]) - (11*I)*Sin[2*(
c + d*x)])*((-I)*Cos[c + 5*d*x] + Sin[c + 5*d*x]))/(3*d*(e*Sec[c + d*x])^(
3/2)*(Cos[d*x] + I*Sin[d*x])^4)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3977, 3042, 3979, 3042, 3967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{3a^2\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2dx}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a^2\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2dx}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3979}$$

$$-\frac{3a^2\left(\frac{5}{3}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a^2\left(\frac{5}{3}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3967}$$

$$-\frac{3a^2\left(\frac{5}{3}a\left(a\int \sqrt{e\sec(c+dx)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a^2\left(\frac{5}{3}a\left(a\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{3a^2\left(\frac{5}{3}a\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a^2\left(\frac{5}{3}a\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{3a^2\left(\frac{2i(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}{3d}+\frac{5}{3}a\left(\frac{2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(((-4*I)/3)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(3/2)) - (3*a^
2*((5*a*(((2*I)*a*Sqrt[e*Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d))/3 + (((2*I)/3)*Sqrt[e*Sec[c
+ d*x]]*(a^2 + I*a^2*Tan[c + d*x]))/d))/e^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 12.39 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.88

method	result
default	$\frac{a^4\left(\frac{16\sin(dx+c)}{3}+\frac{2\sec(dx+c)\tan(dx+c)}{3}+i\left(-\frac{16\cos(dx+c)}{3}-8\sec(dx+c)\right)+\frac{2i\,\mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(}{\cos(d}}}{3}\right.}{d\sqrt{e\sec(dx+c)}\,e}$
parts	$\frac{a^4\left(-\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(\sec(dx+c)+1)}{3}+\frac{2\sin(dx+c)}{3}\right)}{d\sqrt{e\sec(dx+c)}\,e}+\frac{a^4\left(\frac{2\sin(dx+c)}{3}+\frac{2\sec(dx+c\right.}{}}{}$

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^4/d*(16/3*sin(d*x+c)+2/3*sec(d*x+c)*tan(d*x+c)+I*(-16/3*cos(d*x+c)-8*sec
(d*x+c))+2/3*I*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-15-15*sec(d*x+c)))/(e*sec(d*x+c))
^(1/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.88

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2\left(\sqrt{2}\left(4i\,a^4e^{(4i\,dx+4i\,c)}+21i\,a^4e^{(2i\,dx+2i\,c)}+15i\,a^4\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+15\sqrt{2}\left(-i\,a^4e^{(2i\,dx+2i\,c)}-i\right.}{3\left(de^2e^{(2i\,dx+2i\,c)}+de^2\right)}$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(2)*(4*I*a^4*e^(4*I*d*x + 4*I*c) + 21*I*a^4*e^(2*I*d*x + 2*I*c)
+ 15*I*a^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 15
*sqrt(2)*(-I*a^4*e^(2*I*d*x + 2*I*c) - I*a^4)*sqrt(e)*weierstrassPInverse(
-4, 0, e^(I*d*x + I*c)))/(d*e^2*e^(2*I*d*x + 2*I*c) + d*e^2)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = a^4\left(\int \frac{1}{\left(e\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx + \int \left(-\frac{6\tan^2\left(c+dx\right)}{\left(e\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\right)dx + \int \frac{\tan^4\left(c+dx\right)}{\left(e\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx + \int \frac{4i\tan\left(c+dx\right)}{\left(e\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx + \int \left(-\frac{4i\tan^3\left(c+dx\right)}{\left(e\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(3/2),x)
```

output
```
a**4*(Integral((e*sec(c + d*x))**(-3/2), x) + Integral(-6*tan(c + d*x)**2/
(e*sec(c + d*x))**(3/2), x) + Integral(tan(c + d*x)**4/(e*sec(c + d*x))**(
3/2), x) + Integral(4*I*tan(c + d*x)/(e*sec(c + d*x))**(3/2), x) + Integra
l(-4*I*tan(c + d*x)**3/(e*sec(c + d*x))**(3/2), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^4}{(e\sec\left(dx+c\right))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^4}{(e\sec\left(dx+c\right))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{e}\,a^4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx+\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^4}{\sec(dx+c)^2}dx-4\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^3}{\sec(dx+c)^2}dx\right.\right.}{e^2}$$

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*a**4*(int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x) + int((sqrt(sec(c
 + d*x))*tan(c + d*x)**4)/sec(c + d*x)**2,x) - 4*int((sqrt(sec(c + d*x))*t
an(c + d*x)**3)/sec(c + d*x)**2,x)*i - 6*int((sqrt(sec(c + d*x))*tan(c + d
*x)**2)/sec(c + d*x)**2,x) + 4*int((sqrt(sec(c + d*x))*tan(c + d*x))/sec(c
 + d*x)**2,x)*i))/e**2
```

3.217 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 156

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{42a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{42a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de^3} - \frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}} + \frac{28i(a^4+ia^4\tan(c+dx))}{5de^2\sqrt{e\sec(c+dx)}}$$

output

```
-42/5*a^4*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e^2/cos(d*x+c)^(1/2)/(e*
sec(d*x+c))^(1/2)+42/5*a^4*(e*sec(d*x+c))^(1/2)*sin(d*x+c)/d/e^3-4/5*I*a*(
a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(5/2)+28/5*I*(a^4+I*a^4*tan(d*x+c))/d
/e^2/(e*sec(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.77 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{4ia^4e^{2i(c+dx)}\left(7+2e^{2i(c+dx)}-7\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)}{5de^2\left(1+e^{2i(c+dx)}\right)\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(((-4*I)/5)*a^4*E^((2*I)*(c + d*x))*(7 + 2*E^((2*I)*(c + d*x)) - 7*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))]))/(d*e^2*(1 + E^((2*I)*(c + d*x)))*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3977, 3042, 3977, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{7a^2\int \frac{(i\tan(c+dx)a+a)^2}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7a^2\int\frac{(i\tan(c+dx)a+a)^2}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3977}$$

$$-\frac{7a^2\left(-\frac{3a^2\int(e\sec(c+dx))^{3/2}dx}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7a^2\left(-\frac{3a^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4255}$$

$$-\frac{7a^2\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7a^2\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{7a^2\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7a^2\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3119$$

$$-\frac{7a^2\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(((-4*I)/5)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(5/2)) - (7*a^
2*((-3*a^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[
e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/e^2 - ((4*I
)*(a^2 + I*a^2*Tan[c + d*x]))/(d*Sqrt[e*Sec[c + d*x]])))/(5*e^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(136) = 272$.

Time = 14.07 (sec) , antiderivative size = 496, normalized size of antiderivative = 3.18

method	result
default	$\frac{2a^4\left(5i\ln\left(\frac{4\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}+4\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}-2\cos(dx+c)+2}{\cos(dx+c)+1}\right)-5i\ln\left(\frac{2\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}+2\sqrt{-\frac{\cos(\ldots}{(\cos(dx\ldots}}}{\cos(dx+c)+1}\right.\right.}{}$
parts	Expression too large to display

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*a^4/d*(5*I*ln(2*(2*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*(
-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-cos(d*x+c)+1)/(cos(d*x+c)+1))-5*I*ln((
2*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*(-cos(d*x+c)/(cos(d*x+
c)+1)^2)^(1/2)-cos(d*x+c)+1)/(cos(d*x+c)+1))+21*I*(1/(cos(d*x+c)+1))^(1/2)
*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*El
lipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*(cos(d*x+c)+2+sec(d*x+c))+21*I*(1/(co
s(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(-cos(d*x+c)-2-sec(
d*x+c))+tan(d*x+c)*(8*cos(d*x+c)^3+8*cos(d*x+c)^2-16*cos(d*x+c)+5)*(-cos(d
*x+c)/(cos(d*x+c)+1)^2)^(1/2)+4*I*(5-2*cos(d*x+c)^3-2*cos(d*x+c)^2+5*cos(d
*x+c))*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2))/(cos(d*x+c)+1)/(e*sec(d*x+c))
^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)/e^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.60

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2\left(21i\sqrt{2}a^4\sqrt{e}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{2}\left(2i\,a^4e^{(3i\,dx+3i\,c)}+7i\,a\right.\right.}{5\,de^3}$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/5*(21*I*sqrt(2)*a^4*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(2*I*a^4*e^(3*I*d*x + 3*I*c) + 7*I*a^4*
e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)
)/(d*e^3)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx = a^4\left(\int \frac{1}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx + \int \left(-\frac{6\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\right)dx + \int \frac{\tan^4(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx + \int \frac{4i\tan(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx + \int \left(-\frac{4i\tan^3(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(5/2),x)
```

output
```
a**4*(Integral((e*sec(c + d*x))**(-5/2), x) + Integral(-6*tan(c + d*x)**2/
(e*sec(c + d*x))**(5/2), x) + Integral(tan(c + d*x)**4/(e*sec(c + d*x))**(
5/2), x) + Integral(4*I*tan(c + d*x)/(e*sec(c + d*x))**(5/2), x) + Integra
l(-4*I*tan(c + d*x)**3/(e*sec(c + d*x))**(5/2), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{e}\,a^4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx + \int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^4}{\sec(dx+c)^3}dx - 4\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^3}{\sec(dx+c)^3}dx\right.\right.}{e^3}$$

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*a**4*(int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x) + int((sqrt(sec(c
 + d*x))*tan(c + d*x)**4)/sec(c + d*x)**3,x) - 4*int((sqrt(sec(c + d*x))*t
an(c + d*x)**3)/sec(c + d*x)**3,x)*i - 6*int((sqrt(sec(c + d*x))*tan(c + d
*x)**2)/sec(c + d*x)**3,x) + 4*int((sqrt(sec(c + d*x))*tan(c + d*x))/sec(c
 + d*x)**3,x)*i))/e**3
```

3.218 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = \frac{10a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21de^4} - \frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} + \frac{20i(a^4+ia^4\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}}$$

output
```
10/21*a^4*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d
*x+c))^(1/2)/d/e^4-4/7*I*a*(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(7/2)+20/
21*I*(a^4+I*a^4*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 2.55 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = \frac{2a^4\sqrt{e\sec(c+dx)}\Big(2i+2i\cos(2(c+dx))+5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.}{2}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(2*a^4*Sqrt[e*Sec[c + d*x]]*(2*I + (2*I)*Cos[2*(c + d*x)] + 5*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]) +
 8*Sin[2*(c + d*x)])*(Cos[2*(c + 3*d*x)] + I*Sin[2*(c + 3*d*x)]))/(21*d*e^
4*(Cos[d*x] + I*Sin[d*x])^4)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3977, 3042, 3977, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{5a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3977}$$

$$-\frac{5a^2\left(-\frac{a^2\int \sqrt{e\sec(c+dx)}dx}{3e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5a^2\left(-\frac{a^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \; 4258$$

$$-\frac{5a^2\left(-\frac{a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5a^2\left(-\frac{a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \; 3120$$

$$-\frac{5a^2\left(-\frac{2a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(((-4*I)/7)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(7/2)) - (5*a^
2*((-2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x
]])/(3*d*e^2) - (((4*I)/3)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])
^(3/2))))/(7*e^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 17.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03

method	result
default	$\frac{2a^4\left(\sin(dx+c)\left(24\cos(dx+c)^2-16\right)+i\left(-24\cos(dx+c)^3+28\cos(dx+c)\right)+i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\cot(dx+c)-\right.}{21d\sqrt{e\sec(dx+c)}\,e^3}$
risch	$-\frac{2ie^{i(dx+c)}\left(3\,e^{2i(dx+c)}-5\right)a^4\sqrt{2}}{21d\,e^3\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}+\frac{10\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,\operatorname{EllipticF}\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)a^4\sqrt{e\,e}}{21d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\,e^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$
parts	$-\frac{2a^4\left(\sin(dx+c)\left(-3\cos(dx+c)^2-5\right)+i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(5+5\sec(dx+c))\right)}{21d\sqrt{e\sec(dx+c)}\,e^3}-$

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*a^4/d/(e*sec(d*x+c))^(1/2)/e^3*(sin(d*x+c)*(24*cos(d*x+c)^2-16)+I*(-2
4*cos(d*x+c)^3+28*cos(d*x+c))+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(5+5*sec(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx =$$
$$-\frac{2\left(5i\sqrt{2}a^4\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(3i\,a^4e^{(4i\,dx+4i\,c)}-2i\,a^4e^{(2i\,dx+2i\,c)}-5i\,a^4\right)\sqrt{\frac{\ }{e^{(2i}}}}{21\,de^4}$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-2/21*(5*I*sqrt(2)*a^4*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c))
 + sqrt(2)*(3*I*a^4*e^(4*I*d*x + 4*I*c) - 2*I*a^4*e^(2*I*d*x + 2*I*c) - 5*
I*a^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^4}{(e\sec\left(dx+c\right))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(7/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\sqrt{e}\,a^4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx+\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^4}{\sec(dx+c)^4}dx-4\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^3}{\sec(dx+c)^4}d\right.\right.}{e^4}$$

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x)
```

output
```
(sqrt(e)*a**4*(int(sqrt(sec(c + d*x))/sec(c + d*x)**4,x) + int((sqrt(sec(c
 + d*x))*tan(c + d*x)**4)/sec(c + d*x)**4,x) - 4*int((sqrt(sec(c + d*x))*t
an(c + d*x)**3)/sec(c + d*x)**4,x)*i - 6*int((sqrt(sec(c + d*x))*tan(c + d
*x)**2)/sec(c + d*x)**4,x) + 4*int((sqrt(sec(c + d*x))*tan(c + d*x))/sec(c
 + d*x)**4,x)*i))/e**4
```

3.219 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx=-\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}+\frac{4i(a^4+ia^4\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}}$$

output
```
-2/15*a^4*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e^4/cos(d*x+c)^(1/2)/(e*
sec(d*x+c))^(1/2)-4/9*I*a*(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(9/2)+4/15
*I*(a^4+I*a^4*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.98 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx=-\frac{ia^4e^{i(c+dx)}\left(2+7e^{2i(c+dx)}+5e^{4i(c+dx)}-2\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)\sqrt{e\sec}}{45de^5}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(9/2),x]
```

output
```
((-1/45*I)*a^4*E^(I*(c + d*x))*(2 + 7*E^((2*I)*(c + d*x)) + 5*E^((4*I)*(c
+ d*x)) - 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(d*e^5)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3977, 3042, 3977, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{5/2}}dx}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{5/2}}dx}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3977}$$

$$-\frac{a^2\left(\frac{a^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2\left(\frac{a^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{a^2\left(\frac{a^2\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2\left(\frac{a^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{a^2\left(\frac{2a^2E\left(\frac{1}{2}(c+dx)|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(9/2),x]
```

output
```
(((-4*I)/9)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(9/2)) - (a^2*
((2*a^2*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) - (((4*I)/5)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(5
/2))))/(3*e^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(109) = 218$.

Time = 26.88 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.16

method	result
default	$-\frac{2a^4\left(\sin(dx+c)\left(-40\cos(dx+c)^4-40\cos(dx+c)^3+16\cos(dx+c)^2+16\cos(dx+c)+3\right)+i\,\mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\sqrt{\cdots}\right.}{\cdots}$
risch	$-\frac{i(5\,\mathrm{e}^{4i(dx+c)}+2\,\mathrm{e}^{2i(dx+c)}-6)a^4\sqrt{2}}{45d\,e^4\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}+\frac{i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\sqrt{i\mathrm{e}^{i(dx+c)}}\left(-2i\,\mathrm{E}\cdots\right.}{\sqrt{e\,\mathrm{e}^{3\cdots}}}\right.}{15d\,e^4\left(\mathrm{e}^{2i(\cdots}\right.}$
parts	$\frac{2a^4\left(\sin(dx+c)\left(5\cos(dx+c)^4+5\cos(dx+c)^3+7\cos(dx+c)^2+7\cos(dx+c)+21\right)-21i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}(\cos(dx+c)+2+\cdots\right.}{45d(\cos\cdots}$

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*a^4/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)/e^4*(sin(d*x+c)*(-40*cos(d
*x+c)^4-40*cos(d*x+c)^3+16*cos(d*x+c)^2+16*cos(d*x+c)+3)+I*EllipticF(I*(co
t(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(3*cos(d*x+c)+6+3*sec(d*x+c))+I*EllipticE(I*(cot(d*x+c)-csc(d*x+c
)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-3*cos(d
*x+c)-6-3*sec(d*x+c))+I*(40*cos(d*x+c)^5+40*cos(d*x+c)^4-36*cos(d*x+c)^3-3
6*cos(d*x+c)^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = \frac{-6i\sqrt{2}a^4\sqrt{e}\,\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+}{4}$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

output
```
1/45*(-6*I*sqrt(2)*a^4*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-5*I*a^4*e^(5*I*d*x + 5*I*c) - 7*I*a^4
*e^(3*I*d*x + 3*I*c) - 2*I*a^4*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c
) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^4}{(e\sec\left(dx+c\right))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(9/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^4}{(e\sec\left(dx+c\right))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(9/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = \frac{\sqrt{e}\,a^4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^5}dx + \int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^4}{\sec(dx+c)^5}dx - 4\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^3}{\sec(dx+c)^5}dx\right.\right.}{e^5}$$

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x)
```

output
```
(sqrt(e)*a**4*(int(sqrt(sec(c + d*x))/sec(c + d*x)**5,x) + int((sqrt(sec(c
 + d*x))*tan(c + d*x)**4)/sec(c + d*x)**5,x) - 4*int((sqrt(sec(c + d*x))*t
an(c + d*x)**3)/sec(c + d*x)**5,x)*i - 6*int((sqrt(sec(c + d*x))*tan(c + d
*x)**2)/sec(c + d*x)**5,x) + 4*int((sqrt(sec(c + d*x))*tan(c + d*x))/sec(c
 + d*x)**5,x)*i))/e**5
```

3.220 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 156

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = -\frac{2a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77de^6} - \frac{2a^4\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} + \frac{4i(a^4+ia^4\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}}$$

output
```
-2/77*a^4*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d
*x+c))^(1/2)/d/e^6-2/77*a^4*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(1/2)-4/11*I*a
*(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(11/2)+4/77*I*(a^4+I*a^4*tan(d*x+c)
)/d/e^2/(e*sec(d*x+c))^(7/2)
```

Mathematica [A] (verified)

Time = 2.40 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = -\frac{a^4\sqrt{e\sec(c+dx)}\left(37i\cos(c+dx)+11i\cos(3(c+dx))-3\sin(c+dx)+4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c\right.\right.}{154de^6(\cos(dx)+}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(11/2),x]
```

output
```
-1/154*(a^4*Sqrt[e*Sec[c + d*x]]*((37*I)*Cos[c + d*x] + (11*I)*Cos[3*(c +
d*x)] - 3*Sin[c + d*x] + 4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(C
os[3*(c + d*x)] - I*Sin[3*(c + d*x)]) - 3*Sin[3*(c + d*x)])*(Cos[3*c + 7*d
*x] + I*Sin[3*c + 7*d*x]))/(d*e^6*(Cos[d*x] + I*Sin[d*x])^4)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3977, 3042, 3977, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}dx$$

$$\Big\downarrow \textbf{3977}$$

$$-\frac{a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{7/2}}dx}{11e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{7/2}}dx}{11e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3977}$$

$$-\frac{a^2\left(\frac{3a^2\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2\left(\frac{3a^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{4256}$$

$$-\frac{a^2\left(\frac{3a^2\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2\left(\frac{3a^2\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{a^2\left(\frac{3a^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2\left(\frac{3a^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{a^2\left(\frac{3a^2\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(11/2),x]
```

output
```
(((-4*I)/11)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(11/2)) - (a^
2*((3*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c +
d*x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2)
 - (((4*I)/7)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(7/2))))/(11
*e^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 31.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

method	result
default	$\frac{2a^4\left(\sin(dx+c)\left(56\cos(dx+c)^4-16\cos(dx+c)^2-1\right)-56i\cos(dx+c)^5+44i\cos(dx+c)^3+i\,\mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\sqrt{\frac{\cos}{\cos}}\cdots\right.}{77d\sqrt{e\sec(dx+c)}\,e^5}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}\left(7\,\mathrm{e}^{4i(dx+c)}+13\,\mathrm{e}^{2i(dx+c)}+4\right)a^4\sqrt{2}}{154d\,e^5\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{2\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)},\cdots\right.}{77d\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx+c)}}\,e^5\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{e\,\mathrm{e}^{i(\cdots}}{\mathrm{e}^{2i(d\cdots}}}}$
parts	$\frac{a^4\left(-\frac{2\sin(dx+c)\left(-7\cos(dx+c)^4-9\cos(dx+c)^2-15\right)}{77}-\frac{30i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(\sec(dx+c)+1)}{77}\cdots\right.}{d\sqrt{e\sec(dx+c)}\,e^5}$

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/77*a^4/d*(sin(d*x+c)*(56*cos(d*x+c)^4-16*cos(d*x+c)^2-1)-56*I*cos(d*x+c)
^5+44*I*cos(d*x+c)^3+I*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-1-sec(d*x+c)))/(e*sec(d*x
+c))^(1/2)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = \frac{4i\sqrt{2}a^4\sqrt{e}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-7i\,a^4e^{(6i\,dx+6i\,c)}-20\right.}{154\,de^6}$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x, algorithm="fricas"
)
```

output
```
1/154*(4*I*sqrt(2)*a^4*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c))
 + sqrt(2)*(-7*I*a^4*e^(6*I*d*x + 6*I*c) - 20*I*a^4*e^(4*I*d*x + 4*I*c) -
17*I*a^4*e^(2*I*d*x + 2*I*c) - 4*I*a^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
e^(1/2*I*d*x + 1/2*I*c))/(d*e^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^4}{(e\sec\left(dx+c\right))^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x, algorithm="maxima"
)
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(11/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^4}{(e\sec\left(dx+c\right))^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(11/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(11/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = \frac{\sqrt{e}\,a^4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^6}dx + \int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^4}{\sec(dx+c)^6}dx - 4\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^3}{\sec(dx+c)^6}dx\right.\right.}{e^6}$$

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x)
```

output
```
(sqrt(e)*a**4*(int(sqrt(sec(c + d*x))/sec(c + d*x)**6,x) + int((sqrt(sec(c
 + d*x))*tan(c + d*x)**4)/sec(c + d*x)**6,x) - 4*int((sqrt(sec(c + d*x))*t
an(c + d*x)**3)/sec(c + d*x)**6,x)*i - 6*int((sqrt(sec(c + d*x))*tan(c + d
*x)**2)/sec(c + d*x)**6,x) + 4*int((sqrt(sec(c + d*x))*tan(c + d*x))/sec(c
 + d*x)**6,x)*i))/e**6
```

3.221 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 156

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = \frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2a^4\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}} - \frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}} - \frac{4i(a^4+ia^4\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}}$$

output
```
2/39*a^4*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e^6/cos(d*x+c)^(1/2)/(e*s
ec(d*x+c))^(1/2)+2/117*a^4*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(3/2)-4/13*I*a*
(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(13/2)-4/117*I*(a^4+I*a^4*tan(d*x+c)
)/d/e^2/(e*sec(d*x+c))^(9/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.97 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.78

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = -\frac{ia^4e^{i(c+dx)}\left(31+59e^{2i(c+dx)}+37e^{4i(c+dx)}+9e^{6i(c+dx)}+8\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i}\right.}{468de^7}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(13/2),x]
```

output
```
((-1/468*I)*a^4*E^(I*(c + d*x))*(31 + 59*E^((2*I)*(c + d*x)) + 37*E^((4*I)
*(c + d*x)) + 9*E^((6*I)*(c + d*x)) + 8*Sqrt[1 + E^((2*I)*(c + d*x))]*Hype
rgeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/
(d*e^7)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3977, 3042, 3977, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{9/2}}dx}{13e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{9/2}}dx}{13e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{3977}$$

$$\frac{a^2\left(\frac{5a^2\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{5a^2\int\frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{a^2\left(\frac{5a^2\left(\frac{3\int\frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{5a^2\left(\frac{3\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a^2\left(\frac{5a^2\left(\frac{3\int\sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{5a^2\left(\frac{3\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{a^2\left(\frac{5a^2\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(13/2),x]
```

output
```
(((-4*I)/13)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(13/2)) + (a^
2*((5*a^2*((6*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[
e*Sec[c + d*x]]) + (2*Sin[c + d*x])/(5*d*e*(e*Sec[c + d*x])^(3/2))))/(9*e^
2) - (((4*I)/9)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(9/2))))/(
13*e^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(136) = 272$.

Time = 42.07 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.78

method	result
default	$\frac{2a^4\left(\sin(dx+c)\left(72\cos(dx+c)^6+72\cos(dx+c)^5-16\cos(dx+c)^4-16\cos(dx+c)^3+\cos(dx+c)^2+\cos(dx+c)+3\right)+3i\,\mathrm{EllipticF}(i(\cot(d\right.}{}$
risch	$-\frac{i\left(9\,\mathrm{e}^{6i(dx+c)}+28\,\mathrm{e}^{4i(dx+c)}+31\,\mathrm{e}^{2i(dx+c)}+24\right)a^4\sqrt{2}}{468d\,e^6\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\right.}{}\right.}{}$
parts	Expression too large to display

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x,method=_RETURNVERBOSE)
```

output
```
2/117*a^4/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)/e^6*(sin(d*x+c)*(72*cos(d*
x+c)^6+72*cos(d*x+c)^5-16*cos(d*x+c)^4-16*cos(d*x+c)^3+cos(d*x+c)^2+cos(d*
x+c)+3)+3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)+2+sec(d*x+c))*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)-3*I*(1/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+2+sec(d*x+c
))*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)+I*(-72*cos(d*x+c)^7-72*cos(d*x+c
)^6+52*cos(d*x+c)^5+52*cos(d*x+c)^4))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = \frac{24i\sqrt{2}a^4\sqrt{e}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x, algorithm="fricas"
)
```

output
```
1/468*(24*I*sqrt(2)*a^4*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-9*I*a^4*e^(7*I*d*x + 7*I*c) - 37*I*a
^4*e^(5*I*d*x + 5*I*c) - 59*I*a^4*e^(3*I*d*x + 3*I*c) - 31*I*a^4*e^(I*d*x
+ I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(13/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{13}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x, algorithm="maxima"
)
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(13/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx=\int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{13}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx=\int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{13/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(13/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(13/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx=\frac{\sqrt{e}\,a^4\left(\int\frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^7}dx+\int\frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^4}{\sec(dx+c)^7}dx-4\left(\int\frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^3}{\sec(dx+c)^7}dx\right.\right.}{e^7}$$

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x)
```

output

```
(sqrt(e)*a**4*(int(sqrt(sec(c + d*x))/sec(c + d*x)**7,x) + int((sqrt(sec(c
 + d*x))*tan(c + d*x)**4)/sec(c + d*x)**7,x) - 4*int((sqrt(sec(c + d*x))*t
an(c + d*x)**3)/sec(c + d*x)**7,x)*i - 6*int((sqrt(sec(c + d*x))*tan(c + d
*x)**2)/sec(c + d*x)**7,x) + 4*int((sqrt(sec(c + d*x))*tan(c + d*x))/sec(c
 + d*x)**7,x)*i))/e**7
```

3.222 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 187

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \frac{2a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33de^8} + \frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} + \frac{2a^4\sin(c+dx)}{33de^7\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{4i(a^4+ia^4\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}}$$

output
```
2/33*a^4*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*
x+c))^(1/2)/d/e^8+2/55*a^4*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(5/2)+2/33*a^4*
sin(d*x+c)/d/e^7/(e*sec(d*x+c))^(1/2)-4/15*I*a*(a+I*a*tan(d*x+c))^3/d/(e*s
ec(d*x+c))^(15/2)-4/55*I*(a^4+I*a^4*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(11/2
)
```

Mathematica [A] (verified)

Time = 3.00 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.83

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx =$$
$$-\frac{ia^4\sqrt{e\sec(c+dx)}\left(64+112\cos(2(c+dx))+48\cos(4(c+dx))-54i\sin(2(c+dx))-37i\sin(4(c+dx)\right.}{660de^8}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(15/2),x]
```

output
```
((-1/660*I)*a^4*Sqrt[e*Sec[c + d*x]]*(64 + 112*Cos[2*(c + d*x)] + 48*Cos[4
*(c + d*x)] - (54*I)*Sin[2*(c + d*x)] - (37*I)*Sin[4*(c + d*x)] + 40*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(I*Cos[4*(c + d*x)] + Sin[4*(c + d
*x)]))*(Cos[4*(c + 2*d*x)] + I*Sin[4*(c + 2*d*x)]))/(d*e^8*(Cos[d*x] + I*S
in[d*x])^4)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3977, 3042, 3977, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}dx$$
$$\Big\downarrow \text{3977}$$
$$\frac{a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{11/2}}dx}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{11/2}}dx}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3977}$$

$$\frac{a^2\left(\frac{7a^2\int \frac{1}{(e\sec(c+dx))^{7/2}}dx}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{7a^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{a^2\left(\frac{7a^2\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{7a^2\left(\frac{5\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{a^2\left(\frac{7a^2\left(\frac{5\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{7a^2\left(\frac{5\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a^2\left(\frac{7a^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{7a^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \ \text{3120}$$

$$\frac{a^2\left(\frac{7a^2\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(15/2),x]
```

output
```
(((-4*I)/15)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(15/2)) + (a^
2*((7*a^2*((2*Sin[c + d*x])/(7*d*e*(e*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (
2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2)))/(11*e^2) - (((4*I
)/11)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(11/2))))/(5*e^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 46.42 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.81

method	result
default	$\frac{a^{4}\left(\frac{2 \sin \left(d x +c \right)\left(88 \cos \left(d x +c \right)^{6}-16 \cos \left(d x +c \right)^{4}+3 \cos \left(d x +c \right)^{2}+5\right)}{165}+\frac{2 i \left(-88 \cos \left(d x +c \right)^{7}+60 \cos \left(d x +c \right)^{5}\right)}{165}+\frac{2 i \sqrt{\frac{1}{\cos \left(d x +c \right)+1}}\, \sqrt{\frac{\cos \left(d x +c \right)}{\cos \left(d x +c \right)+1}}\, \mathrm{E}\cdots}{\cdots}\right.}{d \sqrt{e \sec \left(d x +c \right)}\, e^{7}}$
risch	$-\frac{i {\mathrm e}^{i \left(d x +c \right)} \left(11 \,{\mathrm e}^{6 i \left(d x +c \right)}+47 \,{\mathrm e}^{4 i \left(d x +c \right)}+81 \,{\mathrm e}^{2 i \left(d x +c \right)}+85\right) a^{4} \sqrt{2}}{1320 d \,e^{7} \sqrt{\frac{e \,{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}}+\frac{2 \sqrt{-i \left({\mathrm e}^{i \left(d x +c \right)}+i\right)}\, \sqrt{i \left({\mathrm e}^{i \left(d x +c \right)}-i\right)}\, \sqrt{i {\mathrm e}^{i \left(d x +c \right)}}\, \operatorname{EllipticF}\left(\sqrt{-\cdots}\right.}{33 d \sqrt{e \,{\mathrm e}^{3 i \left(d x +c \right)}+e \,{\mathrm e}^{i \left(d x +c \right)}}\, e^{7} \left({\mathrm e}^{2 i \left(d \cdots\right.\right.}}$
parts	$\frac{a^{4}\left(\frac{2 \sin \left(d x +c \right)\left(77 \cos \left(d x +c \right)^{6}+91 \cos \left(d x +c \right)^{4}+117 \cos \left(d x +c \right)^{2}+195\right)}{1155}+\frac{2 i \sqrt{\frac{1}{\cos \left(d x +c \right)+1}}\, \sqrt{\frac{\cos \left(d x +c \right)}{\cos \left(d x +c \right)+1}}\, \operatorname{EllipticF}\left(i \left(\csc \left(d x +c \right)-\cot \left(d x +c \right)\right), i\right) \cdots}{1155}\right.}{d \sqrt{e \sec \left(d x +c \right)}\, e^{7}}$

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x,method=_RETURNVERBOSE)
```

output
```
a^4/d*(2/165*sin(d*x+c)*(88*cos(d*x+c)^6-16*cos(d*x+c)^4+3*cos(d*x+c)^2+5)
+2/165*I*(-88*cos(d*x+c)^7+60*cos(d*x+c)^5)+2/165*I*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2
)*(5+5*sec(d*x+c)))/(e*sec(d*x+c))^(1/2)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.66

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \frac{-80i\,\sqrt{2}a^4\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-11i\,a^4e^{(8i\,dx+8i\,c)}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x, algorithm="fricas"
)
```

output
```
1/1320*(-80*I*sqrt(2)*a^4*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*
c)) + sqrt(2)*(-11*I*a^4*e^(8*I*d*x + 8*I*c) - 58*I*a^4*e^(6*I*d*x + 6*I*c
) - 128*I*a^4*e^(4*I*d*x + 4*I*c) - 166*I*a^4*e^(2*I*d*x + 2*I*c) - 85*I*a
^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(15/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{15}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x, algorithm="maxima"
)
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(15/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{15}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(15/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{15/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(15/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(15/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \frac{\sqrt{e}\,a^4\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^8}dx + \int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^4}{\sec(dx+c)^8}dx - 4\left(\int \frac{\sqrt{\sec(dx+c)}\,\tan(dx+c)^3}{\sec(dx+c)^8}dx\right.\right.}{e^8}$$

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x)
```

output
```
(sqrt(e)*a**4*(int(sqrt(sec(c + d*x))/sec(c + d*x)**8,x) + int((sqrt(sec(c
 + d*x))*tan(c + d*x)**4)/sec(c + d*x)**8,x) - 4*int((sqrt(sec(c + d*x))*t
an(c + d*x)**3)/sec(c + d*x)**8,x)*i - 6*int((sqrt(sec(c + d*x))*tan(c + d
*x)**2)/sec(c + d*x)**8,x) + 4*int((sqrt(sec(c + d*x))*tan(c + d*x))/sec(c
 + d*x)**8,x)*i))/e**8
```

3.223 $\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 136

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = -\frac{6e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad} + \frac{6e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5ad} + \frac{2e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{5ad}$$

output
```
-6/5*e^6*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d/cos(d*x+c)^(1/2)/(e*sec
(d*x+c))^(1/2)-2/7*I*e^2*(e*sec(d*x+c))^(7/2)/a/d+6/5*e^5*(e*sec(d*x+c))^(
1/2)*sin(d*x+c)/a/d+2/5*e^3*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/a/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = \frac{e^4(e\sec(c+dx))^{3/2}\left(76+28\cos(2(c+dx))-7e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hyp}}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x]),x]
```

output
```
(e^4*(e*Sec[c + d*x])^(3/2)*(76 + 28*Cos[2*(c + d*x)] - (7*(1 + E^((2*I)*(
c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E
^((2*I)*(c + d*x)) + (7*I)*Sec[c + d*x]*Sin[3*(c + d*x)] - (13*I)*Tan[c +
d*x])*(-I + Tan[c + d*x]))/(70*a*d)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 3982, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3982}$$

$$\frac{e^2\int (e\sec(c+dx))^{7/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

$$\Big\downarrow \text{4255}$$

$$\frac{e^2\left(\frac{3}{5}e^2\int (e\sec(c+dx))^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{3}{5}e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

$$\Big\downarrow \textbf{4255}$$

$$\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

$$\Big\downarrow \textbf{4258}$$

$$\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

$$\Big\downarrow \textbf{3119}$$

$$\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

input

```
Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x]),x]
```

output

```
(((-2*I)/7)*e^2*(e*Sec[c + d*x])^(7/2))/(a*d) + (e^2*((2*e*(e*Sec[c + d*x]
)^(5/2)*Sin[c + d*x])/(5*d) + (3*e^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(
d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin
[c + d*x])/d))/5))/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.73

method	result
default	$\frac{2e^5\sqrt{e\sec(dx+c)}\left(21\sin(dx+c)+7\tan(dx+c)+7\sec(dx+c)\tan(dx+c)+5i\left(-\sec(dx+c)^3-\sec(dx+c)^2\right)+21i\left(\cos(dx+c)^2+2\cos(\right.\right.}{}$

input
```
int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/35*e^5/a/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(21*sin(d*x+c)+7*tan(d*x+
c)+7*sec(d*x+c)*tan(d*x+c)+5*I*(-sec(d*x+c)^3-sec(d*x+c)^2)+21*I*(cos(d*x+
c)^2+2*cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)+21*I*(-cos(d*x+c)^2-2*cos(d*x
+c)-1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Elliptic
E(I*(csc(d*x+c)-cot(d*x+c)),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.51

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2\left(\sqrt{2}\left(21i\,e^5e^{(7i\,dx+7i\,c)}+77i\,e^5e^{(5i\,dx+5i\,c)}+103i\,e^5e^{(3i\,dx+3i\,c)}+7i\,e^5e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+\cdots}{35\left(ade^{(6i\,dx+6i\,c)}\cdots\right.}$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2/35*(sqrt(2)*(21*I*e^5*e^(7*I*d*x + 7*I*c) + 77*I*e^5*e^(5*I*d*x + 5*I*c
) + 103*I*e^5*e^(3*I*d*x + 3*I*c) + 7*I*e^5*e^(I*d*x + I*c))*sqrt(e/(e^(2*
I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 21*sqrt(2)*(I*e^5*e^(6*I*d*
x + 6*I*c) + 3*I*e^5*e^(4*I*d*x + 4*I*c) + 3*I*e^5*e^(2*I*d*x + 2*I*c) + I
*e^5)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x +
 I*c))))/(a*d*e^(6*I*d*x + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2
*I*d*x + 2*I*c) + a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\sec(dx+c))^{\frac{11}{2}}}{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(11/2)/(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}{a+a\tan(c+d\,x)\ 1\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^5}{\tan(dx+c)i+1}dx\right)e^5}{a}$$

input
```
int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
(sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**5)/(tan(c + d*x)*i + 1),x)*
e**5)/a
```

3.224 $\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 105

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = \frac{2e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3ad} - \frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad} + \frac{2e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3ad}$$

output
```
2/3*e^4*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x
+c))^(1/2)/a/d-2/5*I*e^2*(e*sec(d*x+c))^(5/2)/a/d+2/3*e^3*(e*sec(d*x+c))^(
3/2)*sin(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.59

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = \frac{e^2(e\sec(c+dx))^{5/2}\left(-6i+10\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c\right.}{15ad}$$

input
```
Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x]),x]
```

output
```
(e^2*(e*Sec[c + d*x])^(5/2)*(-6*I + 10*Cos[c + d*x]^(5/2)*EllipticF[(c + d
*x)/2, 2] + 5*Sin[2*(c + d*x)]))/(15*a*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3982, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \text{3982}$$
$$\frac{e^2\int (e\sec(c+dx))^{5/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}$$
$$\Big\downarrow \text{3042}$$
$$\frac{e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}$$
$$\Big\downarrow \text{4255}$$
$$\frac{e^2\left(\frac{1}{3}e^2\int \sqrt{e\sec(c+dx)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}$$
$$\Big\downarrow \text{3042}$$
$$\frac{e^2\left(\frac{1}{3}e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}$$
$$\Big\downarrow \text{4258}$$

$$\frac{e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}$$

$$\Big\downarrow \text{3120}$$

$$\frac{e^2\left(\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}$$

input
```
Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x]),x]
```

output
```
(((-2*I)/5)*e^2*(e*Sec[c + d*x])^(5/2))/(a*d) + (e^2*((2*e^2*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d) + (2*e*(e*Sec[
c + d*x])^(3/2)*Sin[c + d*x])/(3*d)))/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 3.80 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

method	result
default	$-\frac{e^4\left(-\frac{2\tan(dx+c)}{3}+\frac{2i\sec(dx+c)^2}{5}+\frac{2i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)}{3}\right)\sqrt{e\sec(dx+c)}}{ad}$

input
```
int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-e^4/a/d*(-2/3*tan(d*x+c)+2/5*I*sec(d*x+c)^2+2/3*I*(cos(d*x+c)+1)*(1/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)
-cot(d*x+c)),I))*(e*sec(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.45

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2\left(\sqrt{2}\left(5i\,e^4e^{(4i\,dx+4i\,c)}+12i\,e^4e^{(2i\,dx+2i\,c)}-5i\,e^4\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+5\sqrt{2}\left(i\,e^4e^{(4i\,dx+4i\,c)}+2i\,e^4e^{(}\right.}{15\left(ade^{(4i\,dx+4i\,c)}+2\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2/15*(sqrt(2)*(5*I*e^4*e^(4*I*d*x + 4*I*c) + 12*I*e^4*e^(2*I*d*x + 2*I*c)
 - 5*I*e^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 5*
sqrt(2)*(I*e^4*e^(4*I*d*x + 4*I*c) + 2*I*e^4*e^(2*I*d*x + 2*I*c) + I*e^4)*
sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a*d*e^(4*I*d*x + 4*I
*c) + 2*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\sec(dx+c))^{\frac{9}{2}}}{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{a+a\tan(c+d\,x)\,1\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\sec(dx+c)^4}{\tan(dx+c)i+1}dx\right)e^4}{a}$$

input
```
int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
(sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**4)/(tan(c + d*x)*i + 1),x)*
e**4)/a
```

3.225 $\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 101

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad} + \frac{2e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{ad}$$

output
```
-2*e^4*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d/cos(d*x+c)^(1/2)/(e*sec(d
*x+c))^(1/2)-2/3*I*e^2*(e*sec(d*x+c))^(3/2)/a/d+2*e^3*(e*sec(d*x+c))^(1/2)
*sin(d*x+c)/a/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.69 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = \frac{2ie^3\sqrt{e\sec(c+dx)}(\cos(c)+i\sin(c))(\cos(dx)+i\sin(dx))\left(-4+\sqrt{1+e^{2i(c+dx}}\right.}{3ad}$$

input
```
Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x]),x]
```

output
```
(((2*I)/3)*e^3*Sqrt[e*Sec[c + d*x]]*(Cos[c] + I*Sin[c])*(Cos[d*x] + I*Sin[
d*x])*(-4 + Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(c + d*x))] + I*Tan[c + d*x]))/(a*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3982, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3982}$$

$$\frac{e^2\int (e\sec(c+dx))^{3/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}$$

$$\Big\downarrow \text{4255}$$

$$\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}$$

$$\Big\downarrow \text{4258}$$

$$\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}$$

input
```
Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x]),x]
```

output
```
(((-2*I)/3)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d) + (e^2*((-2*e^2*EllipticE[(c
 + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*S
ec[c + d*x]]*Sin[c + d*x])/d))/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(92) = 184$.

Time = 2.86 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.98

method	result
default	$-\frac{2e^3\left(3i\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)+3i\left(-\cos(dx+c)^2-2\right.}{3ad(\cos(}$

input
```
int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*e^3/a/d*(3*I*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)+3*
I*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)+I+I*sec(d*x+c)-3*
sin(d*x+c))*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.22

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx =$$
$$-\frac{2\left(\sqrt{2}\left(3i\,e^3e^{(3i\,dx+3i\,c)}+5i\,e^3e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+3\sqrt{2}\left(i\,e^3e^{(2i\,dx+2i\,c)}+i\,e^3\right)\sqrt{e}\text{weierstra}}{3\left(ade^{(2i\,dx+2i\,c)}+ad\right)}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(2)*(3*I*e^3*e^(3*I*d*x + 3*I*c) + 5*I*e^3*e^(I*d*x + I*c))*sqrt
(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 3*sqrt(2)*(I*e^3*e
^(2*I*d*x + 2*I*c) + I*e^3)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, e^(I*d*x + I*c))))/(a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\sec(dx+c))^{\frac{7}{2}}}{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{a+a\tan(c+d\,x)\ 1\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^3}{\tan(dx+c)i+1}dx\right)e^3}{a}$$

input
```
int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
(sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**3)/(tan(c + d*x)*i + 1),x)*
e**3)/a
```

3.226 $\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad} + \frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{ad}$$

output
```
-2*I*e^2*(e*sec(d*x+c))^(1/2)/a/d+2*e^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1
/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x+c))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = \frac{2e^2\left(-i+\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)\sqrt{e\sec(c+dx)}}{ad}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x]),x]
```

output
```
(2*e^2*(-I + Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[e*Sec[c +
d*x]])/(a*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3982, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3982}$$

$$\frac{e^2\int \sqrt{e\sec(c+dx)}dx}{a}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow \text{4258}$$

$$\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{ad}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}$$

input
```
Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-2*I)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d) + (2*e^2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(a*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{i e^{2} \left(-2+2 \operatorname{EllipticF}\left(i \left(\cot \left(d x+c\right)-\csc \left(d x+c\right)\right), i\right) \sqrt{\frac{1}{\cos \left(d x+c\right)+1}}\, \left(\cos \left(d x+c\right)+1\right) \sqrt{\frac{\cos \left(d x+c\right)}{\cos \left(d x+c\right)+1}}\right) \sqrt{e \sec \left(d x+c\right)}}{a d}$	87

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
I*e^2/a/d*(-2+2*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(e*sec(d*x+c))^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2\left(i\sqrt{2}e^2\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+i\sqrt{2}e^{\frac{5}{2}}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{ad}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(2)*e^2*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c
) + I*sqrt(2)*e^(5/2)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a*d)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = -\frac{i\int\frac{(e\sec(c+dx))^{\frac{5}{2}}}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral((e*sec(c + d*x))**(5/2)/(tan(c + d*x) - I), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{a+a\tan(c+d\,x)\ 1\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^2}{\tan(dx+c)i+1}dx\right)e^2}{a}$$

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
(sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**2)/(tan(c + d*x)*i + 1),x)*
e**2)/a
```

3.227 $\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx=\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}+\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

output
```
2*I*e^2/a/d/(e*sec(d*x+c))^(1/2)+2*e^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))/a/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx=\frac{2iee^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)\sqrt{e\sec(c+}}{ad}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x]),x]
```

output
```
((2*I)*e*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -
E^((2*I)*(c + d*x))]*Sqrt[e*Sec[c + d*x]])/(a*d*E^(I*(c + d*x)))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3982, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3982}$$

$$\frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{a}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{e^2\int \sqrt{\cos(c+dx)}dx}{a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{e^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x]),x]
```

output
```
((2*I)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]) + (2*e^2*EllipticE[(c + d*x)/2, 2])
/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(65) = 130$.

Time = 1.97 (sec) , antiderivative size = 477, normalized size of antiderivative = 6.81

method	result
default	$-\frac{e\left(i\left(4\cos(dx+c)^2+8\cos(dx+c)+4\right)\operatorname{EllipticE}(i(\cot(dx+c)-\csc(dx+c)),i)\sqrt{-\frac{\cos(dx+c)}{(\cos(dx+c)+1)^2}}\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}-i\right.}{}$

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*e/a/d*(I*(4*cos(d*x+c)^2+8*cos(d*x+c)+4)*EllipticE(I*(cot(d*x+c)-csc(
d*x+c)),I)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-I*ln((2*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+
c)+1)^2)^(1/2)+2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-cos(d*x+c)+1)/(cos(d
*x+c)+1))*cos(d*x+c)+I*(-4*cos(d*x+c)^2-8*cos(d*x+c)-4)*EllipticF(I*(cot(d
*x+c)-csc(d*x+c)),I)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*ln(2*(2*cos(d*x+c)*(-cos(d*x+
c)/(cos(d*x+c)+1)^2)^(1/2)+2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-cos(d*x+
c)+1)/(cos(d*x+c)+1))*cos(d*x+c)+I*cos(d*x+c)*(-4*cos(d*x+c)-4)*(-cos(d*x+
c)/(cos(d*x+c)+1)^2)^(1/2)-4*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+
c)*sin(d*x+c))*(e*sec(d*x+c))^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)/(
cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.37

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2\left(-i\sqrt{2}e^{\frac{3}{2}}e^{(i\,dx+i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{2}\left(-i\,ee^{(2i\,dx+2i\,c)}-\right.\right.}{ad}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2*(-I*sqrt(2)*e^(3/2)*e^(I*d*x + I*c)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-I*e*e^(2*I*d*x + 2*I*c) - I*e
)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-I*d*x - I
*c)/(a*d)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{(e\sec(c+dx))^{\frac{3}{2}}}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral((e*sec(c + d*x))**(3/2)/(tan(c + d*x) - I), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)}{\tan(dx+c)i+1}dx\right)e}{a}$$

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
(sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x))/(tan(c + d*x)*i + 1),x)*e)/
a
```

3.228 $\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 80

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3ad} + \frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}$$

output
```
2/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x+c))
^(1/2)/a/d+2/3*I*(e*sec(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = \frac{2(e\sec(c+dx))^{3/2}\left(\cos(c+dx)+\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(-i\cos(c+dx)+\sin(c+dx))\right)}{3ade(-i+\tan(c+dx))}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x]),x]
```

output
```
(2*(e*Sec[c + d*x])^(3/2)*(Cos[c + d*x] + Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*((-I)*Cos[c + d*x] + Sin[c + d*x])))/(3*a*d*e*(-I + Tan[c + d
*x]))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3983, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{\int \sqrt{e\sec(c+dx)}dx}{3a}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3a}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3a}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3ad}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x]),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*a
*d) + (((2*I)/3)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(69) = 138$.

Time = 2.80 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.01

method	result
default	$-\frac{2\sqrt{e\sec(dx+c)}\left(i\,\mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)+i\,\mathrm{EllipticF}(i(\csc(dx+c)-\cot(\right.}{3ad}$

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3/a/d*(e*sec(d*x+c))^(1/2)*(I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+I*Elli
pticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)-I*cos(d*x+c)^2-cos(d*x+c)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-2i\,\sqrt{2}\sqrt{e}e^{(2i\,dx+2i\,c)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right.\right.}{3\,ad}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(2*I*d*x + 2*I*c) + I)
*e^(1/2*I*d*x + 1/2*I*c) - 2*I*sqrt(2)*sqrt(e)*e^(2*I*d*x + 2*I*c)*weierst
rassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sqrt{e\sec(c+dx)}}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(sqrt(e*sec(c + d*x))/(tan(c + d*x) - I), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = \int \frac{\sqrt{e\sec(dx+c)}}{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = \int \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}}{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}}{\tan(dx+c)i+1}dx\right)}{a}$$

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
(sqrt(e)*int(sqrt(sec(c + d*x))/(tan(c + d*x)*i + 1),x))/a
```

3.229 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 80

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2i}{5d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}$$

output
```
6/5*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d/cos(d*x+c)^(1/2)/(e*sec(d*x+
c))^(1/2)+2/5*I/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.44 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx$$
$$= \frac{\left(4+4\cos(2(c+dx)) - 2e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+3i\sin(2(c+ \cdots\right.}{5ad\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])),x]
```

output
```
((4 + 4*Cos[2*(c + d*x)] - 2*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*
x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + (3*I)*Sin[2*
(c + d*x)])*(I + Tan[c + d*x]))/(5*a*d*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3983, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3983}$$

$$\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5a}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{3\int \sqrt{\cos(c+dx)}dx}{5a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\int \sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}dx}{5a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])),x]
```

output
```
(6*EllipticE[(c + d*x)/2, 2])/(5*a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x
]]) + ((2*I)/5)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(70) = 140$.

Time = 3.68 (sec) , antiderivative size = 407, normalized size of antiderivative = 5.09

method	result
default	$\frac{2i\tan(dx+c)\left(3\cos(dx+c)^2+6\cos(dx+c)+3\right)\text{EllipticE}(i(\cot(dx+c)-\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}{5}+\frac{2\left(3\cos(dx+c)^2+6\cos(dx+c)\right.}{}$

input
```
int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/5/a/d/(-cos(d*x+c)*sin(d*x+c)-sin(d*x+c)+I*cos(d*x+c)^2+I*cos(d*x+c))/(e
*sec(d*x+c))^(1/2)*(I*tan(d*x+c)*(3*cos(d*x+c)^2+6*cos(d*x+c)+3)*EllipticE
(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)+(3*cos(d*x+c)^2+6*cos(d*x+c)+3)*EllipticE(I*(cot(d*x+c)-csc
(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*t
an(d*x+c)*(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*EllipticF(I*(cot(d*x+c)-csc(d*x
+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(-3*cos
(d*x+c)^2-6*cos(d*x+c)-3)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*I*cos(d*x+c)*sin(d*x+c
)+2*cos(d*x+c)^2-cos(d*x+c)-3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.34

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(7i\,e^{(4i\,dx+4i\,c)}+8i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+12i\sqrt{2}\sqrt{e}e^{(3i\,dx+3i\,c)}\text{weierstrassZeta}\left(-\right.\right.}{10\,ade}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/10*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(7*I*e^(4*I*d*x + 4*I*c) +
 8*I*e^(2*I*d*x + 2*I*c) + I)*e^(1/2*I*d*x + 1/2*I*c) + 12*I*sqrt(2)*sqrt(
e)*e^(3*I*d*x + 3*I*c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e
^(I*d*x + I*c))))*e^(-3*I*d*x - 3*I*c)/(a*d*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = -\frac{i\int \frac{1}{\sqrt{e\sec(c+dx)}\tan(c+dx)-i\sqrt{e\sec(c+dx)}}\,dx}{a}$$

input
```
integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(1/(sqrt(e*sec(c + d*x))*tan(c + d*x) - I*sqrt(e*sec(c + d*x)))
, x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{\sqrt{e\sec\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)),x)
```

output
```
int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)\tan(dx+c)i+\sec(dx+c)}dx\right)}{ae}$$

input
```
int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
(sqrt(e)*int(sqrt(sec(c + d*x))/(sec(c + d*x)*tan(c + d*x)*i + sec(c + d*x
)),x))/(a*e)
```

3.230 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 114

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21ade^2} + \frac{10\sin(c+dx)}{21ade\sqrt{e\sec(c+dx)}} + \frac{2i}{7d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}$$

output
```
10/21*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x+c
))^(1/2)/a/d/e^2+10/21*sin(d*x+c)/a/d/e/(e*sec(d*x+c))^(1/2)+2/7*I/d/(e*se
c(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = -\frac{\sec^3(c+dx)\left(-14\cos(c+dx)+2\cos(3(c+dx))+20i\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(c+d}{42ad(e\sec(c+dx))^{3/2}(-i+\tan(c+dx))}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])),x]
```

output
```
-1/42*(Sec[c + d*x]^3*(-14*Cos[c + d*x] + 2*Cos[3*(c + d*x)] + (20*I)*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin[c + d*x]) +
 (5*I)*Sin[c + d*x] + (5*I)*Sin[3*(c + d*x)]))/(a*d*(e*Sec[c + d*x])^(3/2)
*(-I + Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3983, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3983}$$

$$\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{5\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])),x]
```

output
```
(5*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*a) + ((2*I)
/7)/(d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 3.77 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

method	result	siz
default	$\frac{\frac{2\sin(dx+c)\left(3\cos(dx+c)^{2}+5\right)}{21}+\frac{2i\cos(dx+c)^{3}}{7}+\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\;\mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)(5+5\sec(dx+c))}{21}}{e\sqrt{e\sec(dx+c)}\,ad}$	11

input
```
int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/21/a/d/(e*sec(d*x+c))^(1/2)*(sin(d*x+c)*(3*cos(d*x+c)^2+5)+3*I*cos(d*x+c
)^3+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF
(I*(cot(d*x+c)-csc(d*x+c)),I)*(5+5*sec(d*x+c)))/e
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-7i\,e^{(6i\,dx+6i\,c)}+9i\,e^{(4i\,dx+4i\,c)}+19i\,e^{(2i\,d\cdots}\right.\right.}{\cdots}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/84*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-7*I*e^(6*I*d*x + 6*I*c)
+ 9*I*e^(4*I*d*x + 4*I*c) + 19*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(1/2*I*d*x +
 1/2*I*c) - 40*I*sqrt(2)*sqrt(e)*e^(4*I*d*x + 4*I*c)*weierstrassPInverse(-
4, 0, e^(I*d*x + I*c)))*e^(-4*I*d*x - 4*I*c)/(a*d*e^2)
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = -\frac{i\int \frac{1}{(e\sec(c+dx))^{\frac{3}{2}}\tan(c+dx)-i(e\sec(c+dx))^{\frac{3}{2}}}\,dx}{a}$$

input
```
integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(1/((e*sec(c + d*x))**(3/2)*tan(c + d*x) - I*(e*sec(c + d*x))**
(3/2)), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{3}{2}}(ia\tan(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}\,(a+a\tan(c+d\,x)\,1\mathrm{i})}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)),x)
```

output
```
int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2\tan(dx+c)i+\sec(dx+c)^2}dx\right)}{a\,e^2}$$

input
```
int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
(sqrt(e)*int(sqrt(sec(c + d*x))/(sec(c + d*x)**2*tan(c + d*x)*i + sec(c +
d*x)**2),x))/(a*e**2)
```

3.231 $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 114

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15ade^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14\sin(c+dx)}{45ade(e\sec(c+dx))^{3/2}} + \frac{2i}{9d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}$$

output
```
14/15*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d/e^2/cos(d*x+c)^(1/2)/(e*se
c(d*x+c))^(1/2)+14/45*sin(d*x+c)/a/d/e/(e*sec(d*x+c))^(3/2)+2/9*I/d/(e*sec
(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.90 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.18

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \frac{\left(106+104\cos(2(c+dx))-2\cos(4(c+dx))-56e^{2i(c+dx)}\sqrt{}\right.}{}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])),x]
```

output
```
((106 + 104*Cos[2*(c + d*x)] - 2*Cos[4*(c + d*x)] - 56*E^((2*I)*(c + d*x))
*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))] + (70*I)*Sin[2*(c + d*x)] - (7*I)*Sin[4*(c + d*x)])*(I + Tan[c
 + d*x]))/(180*a*d*e^2*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3983, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9a}+\frac{2i}{9d(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9a}+\frac{2i}{9d(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{7\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a}+\frac{2i}{9d(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a}+\frac{2i}{9d(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{7\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a}+\frac{2i}{9d(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a}+\frac{2i}{9d(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{7\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a}+\frac{2i}{9d(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])),x]
```

output
```
(7*((6*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) + (2*Sin[c + d*x])/(5*d*e*(e*Sec[c + d*x])^(3/2))))/(9*a) + ((2*
I)/9)/(d*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(100) = 200$.

Time = 4.20 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.89

method	result
default	$\frac{-\frac{14}{15}+\frac{14 i \tan(dx+c)\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,\operatorname{EllipticE}(i(\cot(dx+c)-\csc(dx+c)),i)}{15}+\frac{14\left(\cos(dx+c)^2+2\cos\right.}{}}{}$

input
```
int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/45/a/d/(-cos(d*x+c)*sin(d*x+c)-sin(d*x+c)+I*cos(d*x+c)^2+I*cos(d*x+c))/(
e*sec(d*x+c))^(1/2)*(-21+21*I*tan(d*x+c)*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d
*x+c)-csc(d*x+c)),I)+21*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c))
,I)+21*I*tan(d*x+c)*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)
+21*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)+7*I*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)^2+cos(d*x+c)+3)+2*cos(d*x+c)^4+2*cos(d*x+c)^3+14*co
s(d*x+c)^2-7*cos(d*x+c))/e^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-9i\,e^{(8i\,dx+8i\,c)}+174i\,e^{(6i\,dx+6i\,c)}+212i\,e\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/360*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-9*I*e^(8*I*d*x + 8*I*c)
 + 174*I*e^(6*I*d*x + 6*I*c) + 212*I*e^(4*I*d*x + 4*I*c) + 34*I*e^(2*I*d*x
 + 2*I*c) + 5*I)*e^(1/2*I*d*x + 1/2*I*c) + 336*I*sqrt(2)*sqrt(e)*e^(5*I*d*
x + 5*I*c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*
c))))*e^(-5*I*d*x - 5*I*c)/(a*d*e^3)
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = -\frac{i\int \frac{1}{(e\sec(c+dx))^{\frac{5}{2}}\tan(c+dx)-i(e\sec(c+dx))^{\frac{5}{2}}}\,dx}{a}$$

input
```
integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(1/((e*sec(c + d*x))**(5/2)*tan(c + d*x) - I*(e*sec(c + d*x))**
(5/2)), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{5}{2}}(ia\tan(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}\,(a+a\tan(c+d\,x)\ 1\mathrm{i})}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)),x)
```

output
```
int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3\tan(dx+c)i+\sec(dx+c)^3}dx\right)}{a\,e^3}$$

input
```
int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
(sqrt(e)*int(sqrt(sec(c + d*x))/(sec(c + d*x)**3*tan(c + d*x)*i + sec(c +
d*x)**3),x))/(a*e**3)
```

3.232 $\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 145

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx=\frac{30\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77ade^4}+\frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}}+\frac{30\sin(c+dx)}{77ade^3\sqrt{e\sec(c+dx)}}+\frac{2i}{11d(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}$$

output

```
30/77*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x+c
))^(1/2)/a/d/e^4+18/77*sin(d*x+c)/a/d/e/(e*sec(d*x+c))^(5/2)+30/77*sin(d*x
+c)/a/d/e^3/(e*sec(d*x+c))^(1/2)+2/11*I/d/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(
d*x+c))
```

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.98

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx =$$
$$-\frac{(e\sec(c+dx))^{3/2}\left(-148\cos(c+dx)+34\cos(3(c+dx))+2\cos(5(c+dx))+240i\sqrt{\cos(c+dx)}\,\text{Ellipti}\right.}{616ade^5(-i\ }$$

input
```
Integrate[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])),x]
```

output
```
-1/616*((e*Sec[c + d*x])^(3/2)*(-148*Cos[c + d*x] + 34*Cos[3*(c + d*x)] +
2*Cos[5*(c + d*x)] + (240*I)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
(Cos[c + d*x] + I*Sin[c + d*x]) + (78*I)*Sin[c + d*x] + (87*I)*Sin[3*(c +
d*x)] + (9*I)*Sin[5*(c + d*x)]))/(a*d*e^5*(-I + Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3983, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}dx$$
$$\Big\downarrow \text{3983}$$
$$\frac{9\int \frac{1}{(e\sec(c+dx))^{7/2}}dx}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$
$$\Big\downarrow \text{3042}$$

$$\frac{9\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{9\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9\left(\frac{5\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{9\left(\frac{5\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9\left(\frac{5\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{9\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \; 3120$$

$$\frac{9\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

input

```
Int[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])),x]
```

output

```
(9*((2*Sin[c + d*x])/(7*d*e*(e*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*Sin[c
 + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2)))/(11*a) + ((2*I)/11)/(d*(
e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 10.99 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

method	result
default	$\frac{\frac{2\sin(dx+c)\left(7\cos(dx+c)^4+9\cos(dx+c)^2+15\right)}{77}+\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\ \mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ (15+15\sec(dx+c))}{77}+\frac{2i\,\mathrm{co}}{}}{ad\sqrt{e\sec(dx+c)}\,e^3}$

input
```
int(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/a/d*(2/77*sin(d*x+c)*(7*cos(d*x+c)^4+9*cos(d*x+c)^2+15)+2/77*I*(1/(cos(d
*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(15+15*sec(d*x+c))+2/11*I*cos(d*x+c)^5)/(e*sec(d*x+c))^(1/2
)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-11i\,e^{(10i\,dx+10i\,c)} - 121i\,e^{(8i\,dx+8i\,c)} + 70\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/1232*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-11*I*e^(10*I*d*x + 10*
I*c) - 121*I*e^(8*I*d*x + 8*I*c) + 70*I*e^(6*I*d*x + 6*I*c) + 226*I*e^(4*I
*d*x + 4*I*c) + 53*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(1/2*I*d*x + 1/2*I*c) -
480*I*sqrt(2)*sqrt(e)*e^(6*I*d*x + 6*I*c)*weierstrassPInverse(-4, 0, e^(I*
d*x + I*c)))*e^(-6*I*d*x - 6*I*c)/(a*d*e^4)
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = -\frac{i\int \frac{1}{(e\sec(c+dx))^{\frac{7}{2}}\tan(c+dx) - i(e\sec(c+dx))^{\frac{7}{2}}}\,dx}{a}$$

input
```
integrate(1/(e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(1/((e*sec(c + d*x))**(7/2)*tan(c + d*x) - I*(e*sec(c + d*x))**
(7/2)), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{7}{2}}(i\,a\tan(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}\,(a+a\tan(c+d\,x)\,1\mathrm{i})}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)),x)
```

output
```
int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4\tan(dx+c)i+\sec(dx+c)^4}dx\right)}{a\,e^4}$$

input
```
int(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
(sqrt(e)*int(sqrt(sec(c + d*x))/(sec(c + d*x)**4*tan(c + d*x)*i + sec(c +
d*x)**4),x))/(a*e**4)
```

3.233 $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 183

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{22e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{22e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{45a^2d} + \frac{22e^3(e\sec(c+dx))^{9/2}\sin(c+dx)}{63a^2d} - \frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\,(a^2+ia^2\tan(c+dx))}$$

output
```
-22/15*e^8*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(e
*sec(d*x+c))^(1/2)+22/15*e^7*(e*sec(d*x+c))^(1/2)*sin(d*x+c)/a^2/d+22/45*e
^5*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/a^2/d+22/63*e^3*(e*sec(d*x+c))^(9/2)*si
n(d*x+c)/a^2/d-4/7*I*e^2*(e*sec(d*x+c))^(11/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.75 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.65

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{(e\sec(c+dx))^{15/2}(\cos(dx)+i\sin(dx))^2\left(\frac{22i\sqrt{2}e^{3ic-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(\right.}{}\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((e*Sec[c + d*x])^(15/2)*(Cos[d*x] + I*Sin[d*x])^2*(((22*I)*Sqrt[2]*E^((3*
I)*c - I*d*x)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 +
E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(-1
+ E^((2*I)*c)) + (Csc[c]*Sec[c + d*x]^(9/2)*(Cos[2*c] + I*Sin[2*c])*(1260*
Cos[d*x] + 1050*Cos[2*c + d*x] + 1078*Cos[2*c + 3*d*x] + 77*Cos[4*c + 3*d*
x] + 231*Cos[4*c + 5*d*x] + (720*I)*Sin[d*x] - (720*I)*Sin[2*c + d*x]))/56
))/(45*d*Sec[c + d*x]^(11/2)*(a + I*a*Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3981, 3042, 4255, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \; 3981$$

$$\frac{11e^2\int (e\sec(c+dx))^{11/2}dx}{7a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{11e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{11/2}dx}{7a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \; 4255$$

$$\frac{11e^2\left(\frac{7}{9}e^2\int (e\sec(c+dx))^{7/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{9/2}}{9d}\right)}{7a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{11e^2\left(\frac{7}{9}e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{9/2}}{9d}\right)}{7a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \; 4255$$

$$\frac{11e^2\left(\frac{7}{9}e^2\left(\frac{3}{5}e^2\int (e\sec(c+dx))^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{9/2}}{9d}\right)}{7a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{11e^2\left(\frac{7}{9}e^2\left(\frac{3}{5}e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{9/2}}{9d}\right)}{7a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \; 4255$$

$$\frac{11e^2\left(\frac{7}{9}e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{9}}{9d}}{7a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{11e^2\left(\frac{7}{9}e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc(c+dx+\frac{\pi}{2})}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2e\sin(c+dx)(e\sec(c}{9d}}{7a^2}$$
$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{11e^2\left(\frac{7}{9}e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2e\sin(c+dx)(e\sec(c+d}{9d}}{7a^2}$$
$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{11e^2\left(\frac{7}{9}e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin(c+dx+\frac{\pi}{2})}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2e\sin(c+dx)(e\sec(c-}{9d}}{7a^2}$$
$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{11e^2\left(\frac{7}{9}e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2e\sin(c+dx)(e\sec(c+}{9d}}{7a^2}$$
$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(11*e^2*((2*e*(e*Sec[c + d*x])^(9/2)*Sin[c + d*x])/(9*d) + (7*e^2*((2*e*(e
*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d) + (3*e^2*((-2*e^2*EllipticE[(c +
d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[
c + d*x]]*Sin[c + d*x])/d))/5))/9))/(7*a^2) - (((4*I)/7)*e^2*(e*Sec[c + d*
x])^(11/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 5.67 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.41

method	result
default	$-\frac{2e^7\left(\tan(dx+c)\sec(dx+c)^3\left(-231\cos(dx+c)^4-77\cos(dx+c)^3-77\cos(dx+c)^2+35\cos(dx+c)+35\right)+90i\sec(dx+c)^2+90i\sec(dx\right.}{}$

input
```
int((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/315*e^7/a^2/d*(tan(d*x+c)*sec(d*x+c)^3*(-231*cos(d*x+c)^4-77*cos(d*x+c)
^3-77*cos(d*x+c)^2+35*cos(d*x+c)+35)+90*I*sec(d*x+c)^2+90*I*sec(d*x+c)^3+2
31*I*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(c
sc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+231*I*(-cos(d*x
+c)^2-2*cos(d*x+c)-1)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot
(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(e*sec(d*x+c))^(1/2)/(cos(d
*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.40

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\left(\sqrt{2}\left(231i\,e^7e^{(9i\,dx+9i\,c)}+1078i\,e^7e^{(7i\,dx+7i\,c)}+1980i\,e^7e^{(5i\,dx+5i\,c)}+1770i\,e^7e^{(3i\,dx+3i\,c)}+77i\,e^7e^{(i\,dx+i\,c)}\right)\right.}{315\,(}$$

input
```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
-2/315*(sqrt(2)*(231*I*e^7*e^(9*I*d*x + 9*I*c) + 1078*I*e^7*e^(7*I*d*x + 7
*I*c) + 1980*I*e^7*e^(5*I*d*x + 5*I*c) + 1770*I*e^7*e^(3*I*d*x + 3*I*c) +
77*I*e^7*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x +
 1/2*I*c) + 231*sqrt(2)*(I*e^7*e^(8*I*d*x + 8*I*c) + 4*I*e^7*e^(6*I*d*x +
6*I*c) + 6*I*e^7*e^(4*I*d*x + 4*I*c) + 4*I*e^7*e^(2*I*d*x + 2*I*c) + I*e^7
)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c
))))/(a^2*d*e^(8*I*d*x + 8*I*c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6*a^2*d*e^
(4*I*d*x + 4*I*c) + 4*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(15/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{15}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(15/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{15/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx=-\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^7}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)e^7}{a^2}$$

input
```
int((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**7)/(tan(c + d*x)**2 - 2*
tan(c + d*x)*i - 1),x)*e**7)/a**2
```

3.234 $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 152

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{6e^6\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7a^2d} + \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{7a^2d} + \frac{18e^3(e\sec(c+dx))^{7/2}\sin(c+dx)}{35a^2d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

output

```
6/7*e^6*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x
+c))^(1/2)/a^2/d+6/7*e^5*(e*sec(d*x+c))^(3/2)*sin(d*x+c)/a^2/d+18/35*e^3*(
e*sec(d*x+c))^(7/2)*sin(d*x+c)/a^2/d-4/5*I*e^2*(e*sec(d*x+c))^(9/2)/d/(a^2
+I*a^2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.56

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{e^6\sec^3(c+dx)\sqrt{e\sec(c+dx)}\left(-56i\cos(c+dx)+60\cos^{\frac{7}{2}}(c+dx)\,\text{EllipticF}\right.}{70a^2d}$$

input
```
Integrate[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(e^6*Sec[c + d*x]^3*Sqrt[e*Sec[c + d*x]]*((-56*I)*Cos[c + d*x] + 60*Cos[c
+ d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] - 5*Sin[c + d*x] + 15*Sin[3*(c + d*
x)]))/(70*a^2*d)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3981, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{9e^2\int(e\sec(c+dx))^{9/2}dx}{5a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\,(a^2+ia^2\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9e^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}dx}{5a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\,(a^2+ia^2\tan(c+dx))}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{9e^2\left(\frac{5}{7}e^2\int (e\sec(c+dx))^{5/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{7/2}}{7d}\right)}{5a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9e^2\left(\frac{5}{7}e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{7/2}}{7d}\right)}{5a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{9e^2\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\int \sqrt{e\sec(c+dx)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{7/2}}{7d}\right)}{5a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9e^2\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{7/2}}{7d}\right)}{5a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{9e^2\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{7/2}}{7d}\right)}{5a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9e^2\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{7/2}}{7d}\right)}{5a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{9e^2\left(\frac{5}{7}e^2\left(\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{7/2}}{7d}\right)}{5a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\,(a^2+ia^2\tan(c+dx))}$$

input
```
Int[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(9*e^2*((2*e*(e*Sec[c + d*x])^(7/2)*Sin[c + d*x])/(7*d) + (5*e^2*((2*e^2*S
qrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d) +
(2*e*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)))/7))/(5*a^2) - (((4*I)/5)
*e^2*(e*Sec[c + d*x])^(9/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.80

method	result
default	$-\frac{e^{6}\left(-\frac{6\tan(dx+c)}{7}+\frac{2\tan(dx+c)\sec(dx+c)^{2}}{7}+\frac{4i\sec(dx+c)^{2}}{5}+\frac{6i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}(i(\csc(dx+c)-\cot(d}{7}\right.}{a^{2}d}$

input
```
int((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-e^6/a^2/d*(-6/7*tan(d*x+c)+2/7*tan(d*x+c)*sec(d*x+c)^2+4/5*I*sec(d*x+c)^2
+6/7*I*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I))*(e*sec(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.32

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\left(\sqrt{2}\left(15i\,e^{6}e^{(6i\,dx+6i\,c)}+51i\,e^{6}e^{(4i\,dx+4i\,c)}+61i\,e^{6}e^{(2i\,dx+2i\,c)}-15i\,e^{6}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+15\sqrt{2}(\cdots}{35\left(a^{2}de^{(6i\,dx+6i\,c)}+3\,a^{2}de^{(4i\,dx+4i\,c}\cdots\right.}$$

input
```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
-2/35*(sqrt(2)*(15*I*e^6*e^(6*I*d*x + 6*I*c) + 51*I*e^6*e^(4*I*d*x + 4*I*c
) + 61*I*e^6*e^(2*I*d*x + 2*I*c) - 15*I*e^6)*sqrt(e/(e^(2*I*d*x + 2*I*c) +
 1))*e^(1/2*I*d*x + 1/2*I*c) + 15*sqrt(2)*(I*e^6*e^(6*I*d*x + 6*I*c) + 3*I
*e^6*e^(4*I*d*x + 4*I*c) + 3*I*e^6*e^(2*I*d*x + 2*I*c) + I*e^6)*sqrt(e)*we
ierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a^2*d*e^(6*I*d*x + 6*I*c) + 3*
a^2*d*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(13/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{13}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(13/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{13/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}\,dx$$

input
```
int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^6}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)e^6}{a^2}$$

input
```
int((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**6)/(tan(c + d*x)**2 - 2*
tan(c + d*x)*i - 1),x)*e**6)/a**2
```

3.235 $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 152

$$
\begin{aligned}
\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = &-\frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} \\
&+\frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^2d} \\
&+\frac{14e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^2d}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\,(a^2+ia^2\tan(c+dx))}
\end{aligned}
$$

output

```
-14/5*e^6*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(e*
sec(d*x+c))^(1/2)+14/5*e^5*(e*sec(d*x+c))^(1/2)*sin(d*x+c)/a^2/d+14/15*e^3
*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/a^2/d-4/3*I*e^2*(e*sec(d*x+c))^(7/2)/d/(a
^2+I*a^2*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.82 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2ie^5e^{i(c+dx)}\left(-47-56e^{2i(c+dx)}-21e^{4i(c+dx)}+7\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeom}}{15a^2d\left(1+e^{2i(c+dx)}\right)^2}$$

input
```
Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(((2*I)/15)*e^5*E^(I*(c + d*x))*(-47 - 56*E^((2*I)*(c + d*x)) - 21*E^((4*I
)*(c + d*x)) + 7*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/
4, 7/4, -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(a^2*d*(1 + E^((2*I)*
(c + d*x)))^2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3981, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{7e^2\int (e\sec(c+dx))^{7/2}dx}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7e^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7e^2\left(\frac{3}{5}e^2\int(e\sec(c+dx))^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7e^2\left(\frac{3}{5}e^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(7*e^2*((2*e*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d) + (3*e^2*((-2*e^2*
EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (
2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/5))/(3*a^2) - (((4*I)/3)*e^2*(e
*Sec[c + d*x])^(7/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 3.96 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.46

method	result
default	$\frac{2e^5\left(21\sin(dx+c)-3\tan(dx+c)-3\sec(dx+c)\tan(dx+c)-10i-10i\sec(dx+c)+21i\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{}\right.}{}$

input
```
int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/15*e^5/a^2/d*(21*sin(d*x+c)-3*tan(d*x+c)-3*sec(d*x+c)*tan(d*x+c)-10*I-10
*I*sec(d*x+c)+21*I*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)+2
1*I*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I))*(e*sec(d*x+c))
^(1/2)/(cos(d*x+c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.12

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\left(\sqrt{2}\left(21i\,e^5e^{(5i\,dx+5i\,c)}+56i\,e^5e^{(3i\,dx+3i\,c)}+47i\,e^5e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+21\sqrt{2}\left(i\,e^5e^{(4i\,dx+4}\right.\right.}{15\left(a^2de^{(4i\,dx+4i\,c)}+2\,a^2de^{(}\right.}$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
-2/15*(sqrt(2)*(21*I*e^5*e^(5*I*d*x + 5*I*c) + 56*I*e^5*e^(3*I*d*x + 3*I*c
) + 47*I*e^5*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d
*x + 1/2*I*c) + 21*sqrt(2)*(I*e^5*e^(4*I*d*x + 4*I*c) + 2*I*e^5*e^(2*I*d*x
 + 2*I*c) + I*e^5)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, e^(I*d*x + I*c))))/(a^2*d*e^(4*I*d*x + 4*I*c) + 2*a^2*d*e^(2*I*d*x + 2*
I*c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{11}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(11/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}{(a+a\tan(c+d\,x)\;1\mathrm{i})^2}\,dx$$

input
```
int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^5}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)e^5}{a^2}$$

input
```
int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**5)/(tan(c + d*x)**2 - 2*
tan(c + d*x)*i - 1),x)*e**5)/a**2
```

3.236 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 119

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx=\frac{10e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^2d}+\frac{10e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3a^2d}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\,(a^2+ia^2\tan(c+dx))}$$

output
```
10/3*e^4*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*
x+c))^(1/2)/a^2/d+10/3*e^3*(e*sec(d*x+c))^(3/2)*sin(d*x+c)/a^2/d-4*I*e^2*(
e*sec(d*x+c))^(5/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.56

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx=\frac{2e^3(e\sec(c+dx))^{3/2}\left(-6i\cos(c+dx)+5\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx)\right.\right.}{3a^2d}$$

input
```
Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(2*e^3*(e*Sec[c + d*x])^(3/2)*((-6*I)*Cos[c + d*x] + 5*Cos[c + d*x]^(3/2)*
EllipticF[(c + d*x)/2, 2] - Sin[c + d*x]))/(3*a^2*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3981, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{5e^2\int (e\sec(c+dx))^{5/2}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{5e^2\left(\frac{1}{3}e^2\int \sqrt{e\sec(c+dx)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5e^2\left(\frac{1}{3}e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{5e^2\left(\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(5*e^2*((2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c +
 d*x]])/(3*d) + (2*e*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)))/a^2 - ((
4*I)*e^2*(e*Sec[c + d*x])^(5/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 3.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{e^{4}\left(-\frac{2\tan(dx+c)}{3}-4i+\frac{10i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)}{3}\right)\sqrt{e\sec(dx+c)}}{a^{2}d}$	95

input
```
int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
e^4/a^2/d*(-2/3*tan(d*x+c)-4*I+10/3*I*(cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),
I))*(e*sec(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\left(\sqrt{2}\left(5i\,e^4e^{(2i\,dx+2i\,c)}+7i\,e^4\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+5\sqrt{2}\left(i\,e^4e^{(2i\,dx+2i\,c)}+i\,e^4\right)\sqrt{e}\text{weierstrassPInve}}{3\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/3*(sqrt(2)*(5*I*e^4*e^(2*I*d*x + 2*I*c) + 7*I*e^4)*sqrt(e/(e^(2*I*d*x +
 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 5*sqrt(2)*(I*e^4*e^(2*I*d*x + 2*I*
c) + I*e^4)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a^2*d*e^
(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{9}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}\,dx$$

input
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^4}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)e^4}{a^2}$$

input
```
int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**4)/(tan(c + d*x)**2 - 2*
tan(c + d*x)*i - 1),x)*e**4)/a**2
```

3.237 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 115

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{6e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{d\,(a^2+ia^2\tan(c+dx))}$$

output
```
6*e^4*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(e*sec(
d*x+c))^(1/2)-6*e^3*(e*sec(d*x+c))^(1/2)*sin(d*x+c)/a^2/d+4*I*e^2*(e*sec(d
*x+c))^(3/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2ie^3e^{-i(c+dx)}\left(-1+3\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right.\right.}{a^2d}$$

input
```
Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((2*I)*e^3*(-1 + 3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1
/2, 3/4, -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(a^2*d*E^(I*(c + d*x
)))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3981, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3981}$$
$$-\frac{3e^2\int (e\sec(c+dx))^{3/2}dx}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d\,(a^2+ia^2\tan(c+dx))}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{3e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d\,(a^2+ia^2\tan(c+dx))}$$
$$\Big\downarrow \text{4255}$$
$$-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d\,(a^2+ia^2\tan(c+dx))}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d\,(a^2+ia^2\tan(c+dx))}$$
$$\Big\downarrow \text{4258}$$

$$-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(-3*e^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*S
ec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/a^2 + ((4*I)*e
^2*(e*Sec[c + d*x])^(3/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(107) = 214$.

Time = 4.13 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.90

method	result
default	$-\frac{2\left((1-2\cos(dx+c))\sin(dx+c)+i\cos(dx+c)(-2\cos(dx+c)-2)+i\left(3\cos(dx+c)^2+6\cos(dx+c)+3\right)\operatorname{EllipticE}(i(\cot(dx+c)-\csc(dx}{}$

input
```
int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/a^2/d*((1-2*cos(d*x+c))*sin(d*x+c)+I*cos(d*x+c)*(-2*cos(d*x+c)-2)+I*(3*
cos(d*x+c)^2+6*cos(d*x+c)+3)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*(-3*cos(d*x+c)^2-6*
cos(d*x+c)-3)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(e*sec(d*x+c))^(1/2)*e^3/(cos(d*x+c
)+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx =$$
$$-\frac{2\left(-3i\sqrt{2}e^{\frac{7}{2}}e^{(i\,dx+i\,c)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{2}\left(-3i\,e^3e^{(2i\,dx+2i\right.}\right.}{a^2d}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2*(-3*I*sqrt(2)*e^(7/2)*e^(I*d*x + I*c)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-3*I*e^3*e^(2*I*d*x + 2*I*c)
 - 2*I*e^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(
-I*d*x - I*c)/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{7}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}\,dx$$

input
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^3}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)e^3}{a^2}$$

input
```
int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**3)/(tan(c + d*x)**2 - 2*
tan(c + d*x)*i - 1),x)*e**3)/a**2
```

3.238 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 90

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^2d} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
-2/3*e^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*
x+c))^(1/2)/a^2/d+4/3*I*e^2*(e*sec(d*x+c))^(1/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.12

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2(e\sec(c+dx))^{5/2}\left(-2i\cos(c+dx)+\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right.\right.}{3a^2d(-i+\tan(c+}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(2*(e*Sec[c + d*x])^(5/2)*((-2*I)*Cos[c + d*x] + Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin[c + d*x]))*(Cos[c + d*x] + I*Sin
[c + d*x]))/(3*a^2*d*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3981, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$-\frac{e^2\int \sqrt{e\sec(c+dx)}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^2d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])
/(3*a^2*d) + (((4*I)/3)*e^2*Sqrt[e*Sec[c + d*x]])/(d*(a^2 + I*a^2*Tan[c +
d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(79) = 158$.

Time = 3.48 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.82

method	result
default	$-\frac{2e^2\sqrt{e\sec(dx+c)}\left(i\,\mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)+i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c}}\right.}{3a^2d}$

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3/a^2/d*e^2*(e*sec(d*x+c))^(1/2)*(I*EllipticF(I*(cot(d*x+c)-csc(d*x+c))
,I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+
I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(
cot(d*x+c)-csc(d*x+c)),I)-2*I*cos(d*x+c)^2-2*cos(d*x+c)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx =$$
$$-\frac{2\left(-i\sqrt{2}e^{\frac{5}{2}}e^{(2i\,dx+2i\,c)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-i\,e^2e^{(2i\,dx+2i\,c)}-i\,e^2\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{(\frac{1}{2}}\right.}{3\,a^2d}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/3*(-I*sqrt(2)*e^(5/2)*e^(2*I*d*x + 2*I*c)*weierstrassPInverse(-4, 0, e^
(I*d*x + I*c)) + sqrt(2)*(-I*e^2*e^(2*I*d*x + 2*I*c) - I*e^2)*sqrt(e/(e^(2
*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-2*I*d*x - 2*I*c)/(a^2*d
)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{(e\sec(c+dx))^{\frac{5}{2}}}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral((e*sec(c + d*x))**(5/2)/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1)
, x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^2}\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^2}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)e^2}{a^2}$$

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**2)/(tan(c + d*x)**2 - 2*
tan(c + d*x)*i - 1),x)*e**2)/a**2
```

3.239 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 90

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx=\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d\sqrt{e\sec(c+dx)}\,(a^2+ia^2\tan(c+dx))}$$

output
```
2/5*e^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(e*se
c(d*x+c))^(1/2)+4/5*I*e^2/d/(e*sec(d*x+c))^(1/2)/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx=\frac{iee^{-3i(c+dx)}\left(1+e^{2i(c+dx)}+2e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4}\right.\right.}{5a^2d}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((I/5)*e*(1 + E^((2*I)*(c + d*x)) + 2*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I
)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])*Sqr
t[e*Sec[c + d*x]])/(a^2*d*E^((3*I)*(c + d*x)))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3981, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3981}$$

$$\frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5a^2}+\frac{4ie^2}{5d\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a^2}+\frac{4ie^2}{5d\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{e^2\int \sqrt{\cos(c+dx)}dx}{5a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{e^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(2*e^2*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) + (((4*I)/5)*e^2)/(d*Sqrt[e*Sec[c + d*x]]*(a^2 + I*a^2*Tan[c + d
*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(80) = 160.

Time = 3.41 (sec) , antiderivative size = 428, normalized size of antiderivative = 4.76

method	result
default	$-\frac{2\left(i\left(-\cos(dx+c)^2-2\cos(dx+c)-1\right)\sin(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticE}(i(\cot(dx+c)-\csc(dx+c)),i)+\sqrt{\frac{1}{\cos(dx+c}}\right.}{}$

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/5/a^2/d*(I*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*sin(d*x+c)*(1/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c
)),I)+(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE
(I*(cot(d*x+c)-csc(d*x+c)),I)*(-cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c))+I*
(cos(d*x+c)^2+2*cos(d*x+c)+1)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)+(1/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-
csc(d*x+c)),I)*(cos(d*x+c)^3+2*cos(d*x+c)^2+cos(d*x+c))-I*cos(d*x+c)^2*sin
(d*x+c)+cos(d*x+c)*(cos(d*x+c)^2+2*cos(d*x+c)+1))*(e*sec(d*x+c))^(1/2)*e/(
-cos(d*x+c)*sin(d*x+c)-sin(d*x+c)+I*cos(d*x+c)^2+I*cos(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(2i\sqrt{2}e^{\frac{3}{2}}e^{(3i\,dx+3i\,c)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+}\right.\right.\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/5*(2*I*sqrt(2)*e^(3/2)*e^(3*I*d*x + 3*I*c)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(2*I*e*e^(4*I*d*x + 4*I*c
) + 3*I*e*e^(2*I*d*x + 2*I*c) + I*e)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(
1/2*I*d*x + 1/2*I*c))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{(e\sec{(c+dx)})^{\frac{3}{2}}}{\tan^2{(c+dx)}-2i\tan{(c+dx)}-1}\,dx}{a^2}$$

input
```
integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral((e*sec(c + d*x))**(3/2)/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1)
, x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{(ia\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+dx)}\right)^{3/2}}{(a+a\tan(c+dx)\ 1\mathrm{i})^2}\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\sec(dx+c)}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)e}{a^2}$$

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x))/(tan(c + d*x)**2 - 2*tan
(c + d*x)*i - 1),x)*e)/a**2
```

3.240 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7a^2d} + \frac{2e\sin(c+dx)}{7a^2d\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{7d(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
2/7*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x+c))
^(1/2)/a^2/d+2/7*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(1/2)+4/7*I*e^2/d/(e*se
c(d*x+c))^(3/2)/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sec^2(c+dx)\sqrt{e\sec(c+dx)}\left(2i+2i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(2(c}{7a^2d(-i+\tan(c+dx))^2}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/7*(Sec[c + d*x]^2*Sqrt[e*Sec[c + d*x]]*(2*I + (2*I)*Cos[2*(c + d*x)] +
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2
*(c + d*x)]) - Sin[2*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3981, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{3e^2\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{3e^2\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3e^2\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{3e^2\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(3*e^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x
]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*a^2) +
(((4*I)/7)*e^2)/(d*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.53

method	result
default	$-\frac{2\sqrt{e\sec(dx+c)}\left(i\,\mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\cos(dx+c)-2i\cos(dx+c)^4+i\,\mathrm{EllipticF}(i(c}{7a^2d}$

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/7/a^2/d*(e*sec(d*x+c))^(1/2)*(I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-2*I*
cos(d*x+c)^4+I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-2*sin(d*x+c)*cos(d*x+c)^3-cos(d*x+c
)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(3i\,e^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-4i\,\sqrt{2}\sqrt{e}e^{(4i\,dx+4i\,c)}\mathrm{weierstrassPInverse}\right.}{14\,a^2d}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/14*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(3*I*e^(4*I*d*x + 4*I*c) +
 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(1/2*I*d*x + 1/2*I*c) - 4*I*sqrt(2)*sqrt(e
)*e^(4*I*d*x + 4*I*c)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-4*I
*d*x - 4*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sqrt{e\sec(c+dx)}}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sqrt(e*sec(c + d*x))/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x
)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\sqrt{e\sec\left(dx+c\right)}}{\left(i\,a\tan\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{e \sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)}{a^2}$$

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - sqrt(e)*int(sqrt(sec(c + d*x))/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1
),x))/a**2
```

3.241 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2e\sin(c+dx)}{9a^2d(e\sec(c+dx))^{3/2}}+\frac{4ie^2}{9d(e\sec(c+dx))^{5/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
2/3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(e*sec(d*
x+c))^(1/2)+2/9*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(3/2)+4/9*I*e^2/d/(e*sec
(d*x+c))^(5/2)/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.97 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\left(-\frac{8e^{4i(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2(2+8\cos(2(c+dx))+7i\sin(2(c+dx)))\right)(i\cos(2(c+dx)}{18a^2d\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(((-8*E^((4*I)*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + 2*(2 + 8*Cos[2*(c + d*x)] + (7*I)*
Sin[2*(c + d*x)]))*(I*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]))/(18*a^2*d*Sqrt
[e*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3981, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}dx$$
$$\Big\downarrow \text{3981}$$
$$\frac{5e^2\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}$$
$$\Big\downarrow \text{3042}$$

$$\frac{5e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{5e^2\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5e^2\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5e^2\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5e^2\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{5e^2\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}$$

input
```
Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(5*e^2*((6*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*S
ec[c + d*x]]) + (2*Sin[c + d*x])/(5*d*e*(e*Sec[c + d*x])^(3/2))))/(9*a^2)
+ (((4*I)/9)*e^2)/(d*(e*Sec[c + d*x])^(5/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(102) = 204$.

Time = 4.46 (sec) , antiderivative size = 480, normalized size of antiderivative = 4.14

method	result
default	$\frac{2i\sin(dx+c)\left(6\cos(dx+c)^2+12\cos(dx+c)+6\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\ \text{EllipticE}(i(\cot(dx+c)-\csc(dx+c)),i)}{9}+\frac{2\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+}}}{}$

input
```
int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/9/a^2/d/(sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)-2)+I*(2*cos(d*x+c)^3+2*cos
(d*x+c)^2-cos(d*x+c)-1))/(e*sec(d*x+c))^(1/2)*(I*sin(d*x+c)*(6*cos(d*x+c)^
2+12*cos(d*x+c)+6)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)+(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*(6*cos(
d*x+c)^3+12*cos(d*x+c)^2+3*cos(d*x+c)-6-3*sec(d*x+c))+I*sin(d*x+c)*(-6*cos
(d*x+c)^2-12*cos(d*x+c)-6)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)+(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)
*(-6*cos(d*x+c)^3-12*cos(d*x+c)^2-3*cos(d*x+c)+6+3*sec(d*x+c))+I*sin(d*x+c
)*(5*cos(d*x+c)^2-cos(d*x+c)-3)+4*cos(d*x+c)^3-2*cos(d*x+c)^2-6*cos(d*x+c)
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(15i\,e^{(6i\,dx+6i\,c)}+19i\,e^{(4i\,dx+4i\,c)}+5i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+24i\,\sqrt{2}\sqrt{e}e^{(5i\,dx+5i\,c)}\right.}{36\,a^2de}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

output
```
1/36*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(15*I*e^(6*I*d*x + 6*I*c)
+ 19*I*e^(4*I*d*x + 4*I*c) + 5*I*e^(2*I*d*x + 2*I*c) + I)*e^(1/2*I*d*x + 1
/2*I*c) + 24*I*sqrt(2)*sqrt(e)*e^(5*I*d*x + 5*I*c)*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))*e^(-5*I*d*x - 5*I*c)/(a^2*d*
e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{\sqrt{e\sec(c+dx)}\tan^2(c+dx)-2i\sqrt{e\sec(c+dx)}\tan(c+dx)-\sqrt{e\sec(c+dx)}}\,dx}{a^2}$$

input
```
integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(1/(sqrt(e*sec(c + d*x))*tan(c + d*x)**2 - 2*I*sqrt(e*sec(c + d*x
))*tan(c + d*x) - sqrt(e*sec(c + d*x))), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\sec(dx+c)}(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{\sqrt{\sec(dx+c)}\tan(dx+c)^2-2\sqrt{\sec(dx+c)}\tan(dx+c)i-\sqrt{\sec(dx+c)}}dx}{\sqrt{e}\,a^2}$$

input
```
int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(1/(sqrt(sec(c + d*x))*tan(c + d*x)**2 - 2*sqrt(sec(c + d*x))*tan(c
 + d*x)*i - sqrt(sec(c + d*x))),x))/(sqrt(e)*a**2)
```

3.242 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 150

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33a^2de^2} + \frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{33a^2de\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{11d(e\sec(c+dx))^{7/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
10/33*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x+c
))^(1/2)/a^2/d/e^2+2/11*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(5/2)+10/33*sin(
d*x+c)/a^2/d/e/(e*sec(d*x+c))^(1/2)+4/11*I*e^2/d/(e*sec(d*x+c))^(7/2)/(a^2
+I*a^2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.89

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx =$$
$$-\frac{\sec^4(c+dx)\left(28i+24i\cos(2(c+dx))-4i\cos(4(c+dx))+40\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\text{c}\right.}{132a^2d(e\sec(c+dx))^{3/2}(-i+\tan(c+}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
-1/132*(Sec[c + d*x]^4*(28*I + (24*I)*Cos[2*(c + d*x)] - (4*I)*Cos[4*(c +
d*x)] + 40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)]
+ I*Sin[2*(c + d*x)]) - 6*Sin[2*(c + d*x)] + 7*Sin[4*(c + d*x)]))/(a^2*d*(
e*Sec[c + d*x])^(3/2)*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3981, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}dx$$
$$\Big\downarrow \text{3981}$$
$$\frac{7e^2\int \frac{1}{(e\sec(c+dx))^{7/2}}dx}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}$$
$$\Big\downarrow \text{3042}$$

$$\frac{7e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{7e^2\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{7e^2\left(\frac{5\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{7e^2\left(\frac{5\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{7e^2\left(\frac{5\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{7e^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{7e^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{7e^2\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(7*e^2*((2*Sin[c + d*x])/(7*d*e*(e*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*S
in[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2)))/(11*a^2) + (((4*I)/1
1)*e^2)/(d*(e*Sec[c + d*x])^(7/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85

method	result
default	$\frac{\frac{2 \sin(dx+c)\left(6 \cos(dx+c)^4+3 \cos(dx+c)^2+5\right)}{33}+\frac{2i \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\, \sqrt{\frac{1}{\cos(dx+c)+1}}\, \operatorname{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(-5-5 \sec(dx+c))}{33}+4i \cos}{e\sqrt{e \sec(dx+c)}\, a^2 d}$

input
```
int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/33/a^2/d/(e*sec(d*x+c))^(1/2)*(sin(d*x+c)*(6*cos(d*x+c)^4+3*cos(d*x+c)^2
+5)+I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF
(I*(csc(d*x+c)-cot(d*x+c)),I)*(-5-5*sec(d*x+c))+6*I*cos(d*x+c)^5)/e
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-11i\,e^{(8i\,dx+8i\,c)}+30i\,e^{(6i\,dx+6i\,c)}+56i\,e\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

output
```
1/264*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-11*I*e^(8*I*d*x + 8*I*c
) + 30*I*e^(6*I*d*x + 6*I*c) + 56*I*e^(4*I*d*x + 4*I*c) + 18*I*e^(2*I*d*x
+ 2*I*c) + 3*I)*e^(1/2*I*d*x + 1/2*I*c) - 80*I*sqrt(2)*sqrt(e)*e^(6*I*d*x
+ 6*I*c)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-6*I*d*x - 6*I*c)
/(a^2*d*e^2)
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{(e\sec(c+dx))^{\frac{3}{2}}\tan^2(c+dx)-2i(e\sec(c+dx))^{\frac{3}{2}}\tan(c+dx)-(e\sec(c+dx))^{\frac{3}{2}}}\,dx}{a^2}$$

input
```
integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(1/((e*sec(c + d*x))**(3/2)*tan(c + d*x)**2 - 2*I*(e*sec(c + d*x)
)**(3/2)*tan(c + d*x) - (e*sec(c + d*x))**(3/2)), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{3}{2}}(ia\tan(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

Reduce [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{\sqrt{\sec(dx+c)}\sec(dx+c)\tan(dx+c)^2-2\sqrt{\sec(dx+c)}\sec(dx+c)\tan(dx+c)i-\sqrt{\sec(dx+c)}\sec(dx+c)}dx}{\sqrt{e}\,a^2e}$$

input
```
int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(1/(sqrt(sec(c + d*x))*sec(c + d*x)*tan(c + d*x)**2 - 2*sqrt(sec(c
+ d*x))*sec(c + d*x)*tan(c + d*x)*i - sqrt(sec(c + d*x))*sec(c + d*x)),x))
/(sqrt(e)*a**2*e)
```

3.243 $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 150

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{65a^2de(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{13d(e\sec(c+dx))^{9/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
42/65*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/e^2/cos(d*x+c)^(1/2)/(e*
sec(d*x+c))^(1/2)+2/13*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(7/2)+14/65*sin(d
*x+c)/a^2/d/e/(e*sec(d*x+c))^(3/2)+4/13*I*e^2/d/(e*sec(d*x+c))^(9/2)/(a^2+
I*a^2*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.66 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \frac{(\cos(2(c+dx))-i\sin(2(c+dx)))\left(88i+416i\cos(2(c+d\right.}{}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
((Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(88*I + (416*I)*Cos[2*(c + d*x)]
- (8*I)*Cos[4*(c + d*x)] - ((224*I)*E^((4*I)*(c + d*x))*Hypergeometric2F1[
1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - 356*
Sin[2*(c + d*x)] + 18*Sin[4*(c + d*x)]))/(520*a^2*d*e^2*Sqrt[e*Sec[c + d*x
]])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3981, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3981}$$
$$\frac{9e^2\int \frac{1}{(e\sec(c+dx))^{9/2}}dx}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}$$
$$\Big\downarrow \text{3042}$$

$$\frac{9e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}}dx}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{9e^2\left(\frac{7\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9e^2\left(\frac{7\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{9e^2\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9e^2\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{9e^2\left(\frac{7\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9e^2\left(\frac{7\left(\frac{3\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{9e^2\left(\frac{7\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(9*e^2*((2*Sin[c + d*x])/(9*d*e*(e*Sec[c + d*x])^(7/2)) + (7*((6*EllipticE
[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*S
in[c + d*x])/(5*d*e*(e*Sec[c + d*x])^(3/2))))/(9*e^2)))/(13*a^2) + (((4*I)
/13)*e^2)/(d*(e*Sec[c + d*x])^(9/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(132) = 264$.

Time = 4.53 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.47

method	result
default	$\frac{\frac{84i\sin(dx+c)\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\operatorname{EllipticE}(i(\cot(dx+c)-\csc(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}}{65}+\frac{42\operatorname{EllipticE}(i(\cot(dx+c)-\csc(}{}$

input
```
int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/65/a^2/d/(2*sin(d*x+c)*cos(d*x+c)*(-cos(d*x+c)-1)+2*I*cos(d*x+c)^3+2*I*c
os(d*x+c)^2-I*cos(d*x+c)-I)/(e*sec(d*x+c))^(1/2)*(42*I*sin(d*x+c)*(cos(d*x
+c)^2+2*cos(d*x+c)+1)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+21*EllipticE(I*(cot(d*x+c)-c
sc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(
2*cos(d*x+c)^3+4*cos(d*x+c)^2+cos(d*x+c)-sec(d*x+c)-2)+42*I*sin(d*x+c)*(-c
os(d*x+c)^2-2*cos(d*x+c)-1)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+21*EllipticF(I*(cot(d*
x+c)-csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*(-2*cos(d*x+c)^3-4*cos(d*x+c)^2+sec(d*x+c)-cos(d*x+c)+2)+I*sin(d*x+c)
*(9*cos(d*x+c)^4+9*cos(d*x+c)^3+35*cos(d*x+c)^2-7*cos(d*x+c)-21)+4*cos(d*x
+c)^5+4*cos(d*x+c)^4+28*cos(d*x+c)^3-14*cos(d*x+c)^2-42*cos(d*x+c))/e^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-13i\,e^{(10i\,dx+10i\,c)}+373i\,e^{(8i\,dx+8i\,c)}+4\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

output
```
1/1040*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-13*I*e^(10*I*d*x + 10*
I*c) + 373*I*e^(8*I*d*x + 8*I*c) + 474*I*e^(6*I*d*x + 6*I*c) + 118*I*e^(4*
I*d*x + 4*I*c) + 35*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(1/2*I*d*x + 1/2*I*c) +
 672*I*sqrt(2)*sqrt(e)*e^(7*I*d*x + 7*I*c)*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, e^(I*d*x + I*c))))*e^(-7*I*d*x - 7*I*c)/(a^2*d*e^3)
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{(e\sec(c+dx))^{\frac{5}{2}}\tan^2(c+dx)-2i(e\sec(c+dx))^{\frac{5}{2}}\tan(c+dx)-(e\sec(c+dx))^{\frac{5}{2}}}\,dx}{a^2}$$

input
```
integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(1/((e*sec(c + d*x))**(5/2)*tan(c + d*x)**2 - 2*I*(e*sec(c + d*x)
)**(5/2)*tan(c + d*x) - (e*sec(c + d*x))**(5/2)), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{5}{2}}(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

Reduce [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{\sqrt{\sec(dx+c)}\,\sec(dx+c)^2\tan(dx+c)^2-2\sqrt{\sec(dx+c)}\,\sec(dx+c)^2\tan(dx+c)i-\sqrt{\sec(dx+c)}\,\sec(dx+c)^2}dx}{\sqrt{e}\,a^2e^2}$$

input
```
int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(1/(sqrt(sec(c + d*x))*sec(c + d*x)**2*tan(c + d*x)**2 - 2*sqrt(sec
(c + d*x))*sec(c + d*x)**2*tan(c + d*x)*i - sqrt(sec(c + d*x))*sec(c + d*x
)**2),x))/(sqrt(e)*a**2*e**2)
```

3.244 $\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 181

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7a^2de^4} + \frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}} + \frac{6\sin(c+dx)}{35a^2de(e\sec(c+dx))^{5/2}} + \frac{2\sin(c+dx)}{7a^2de^3\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{15d(e\sec(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
2/7*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x+c))
^(1/2)/a^2/d/e^4+2/15*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(9/2)+6/35*sin(d*x
+c)/a^2/d/e/(e*sec(d*x+c))^(5/2)+2/7*sin(d*x+c)/a^2/d/e^3/(e*sec(d*x+c))^(
1/2)+4/15*I*e^2/d/(e*sec(d*x+c))^(11/2)/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.83

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{(e\sec(c+dx))^{5/2}\left(296i+228i\cos(2(c+dx))-72i\cos(4(c+dx))-4i\cos(6(c+dx))+480\sqrt{\cos(c+dx)}\right.}{1680}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
-1/1680*((e*Sec[c + d*x])^(5/2)*(296*I + (228*I)*Cos[2*(c + d*x)] - (72*I)
*Cos[4*(c + d*x)] - (4*I)*Cos[6*(c + d*x)] + 480*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) - 17*Sin[2*(c
+ d*x)] + 128*Sin[4*(c + d*x)] + 11*Sin[6*(c + d*x)]))/(a^2*d*e^6*(-I + Ta
n[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3981, 3042, 4256, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{11e^2\int \frac{1}{(e\sec(c+dx))^{11/2}}dx}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{11/2}}dx}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)\left(e\sec(c+dx)\right)^{11/2}}$$

$$\Big\downarrow \textbf{4256}$$

$$\frac{11e^2\left(\frac{9\int \frac{1}{(e\sec(c+dx))^{7/2}}dx}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)\left(e\sec(c+dx)\right)^{11/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{11e^2\left(\frac{9\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)\left(e\sec(c+dx)\right)^{11/2}}$$

$$\Big\downarrow \textbf{4256}$$

$$\frac{11e^2\left(\frac{9\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)\left(e\sec(c+dx)\right)^{11/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{11e^2\left(\frac{9\left(\frac{5\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)\left(e\sec(c+dx)\right)^{11/2}}$$

$$\Big\downarrow \textbf{4256}$$

$$\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{\int\sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \; 4258$$

$$\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}$$

input

```
Int[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output

```
(11*e^2*((2*Sin[c + d*x])/(11*d*e*(e*Sec[c + d*x])^(9/2)) + (9*((2*Sin[c +
 d*x])/(7*d*e*(e*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*
e*Sqrt[e*Sec[c + d*x]])))/(7*e^2)))/(11*e^2)))/(15*a^2) + (((4*I)/15)*e^2)
/(d*(e*Sec[c + d*x])^(11/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 17.70 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.76

method	result
default	$\frac{\frac{2 \sin(dx+c)\left(14 \cos(dx+c)^{6}+7 \cos(dx+c)^{4}+9 \cos(dx+c)^{2}+15\right)}{105}+\frac{4 i \cos(dx+c)^{7}}{15}+\frac{2 i \sqrt{\frac{1}{\cos(dx+c)+1}}\ \mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i) \sqrt{\frac{\cos(d}{\cos(dx}}}{105}}{a^{2} d \sqrt{e \sec(dx+c)}\, e^{3}}$

input
```
int(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2/d*(2/105*sin(d*x+c)*(14*cos(d*x+c)^6+7*cos(d*x+c)^4+9*cos(d*x+c)^2+1
5)+4/15*I*cos(d*x+c)^7+2/105*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF
(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(15+15*sec(d*x+c)))
/(e*sec(d*x+c))^(1/2)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.82

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-15i\,e^{(12i\,dx+12i\,c)} - 200i\,e^{(10i\,dx+10i\,c)} + \right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

output
```
1/3360*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-15*I*e^(12*I*d*x + 12*
I*c) - 200*I*e^(10*I*d*x + 10*I*c) + 245*I*e^(8*I*d*x + 8*I*c) + 592*I*e^(
6*I*d*x + 6*I*c) + 211*I*e^(4*I*d*x + 4*I*c) + 56*I*e^(2*I*d*x + 2*I*c) +
7*I)*e^(1/2*I*d*x + 1/2*I*c) - 960*I*sqrt(2)*sqrt(e)*e^(8*I*d*x + 8*I*c)*w
eierstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-8*I*d*x - 8*I*c)/(a^2*d*e^
4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{7}{2}}(ia\tan(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}(a+a\tan(c+d\,x)\;1\mathrm{i})^2}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

Reduce [F]

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{\sqrt{\sec(dx+c)}\,\sec(dx+c)^3\tan(dx+c)^2-2\sqrt{\sec(dx+c)}\,\sec(dx+c)^3\tan(dx+c)i-\sqrt{\sec(dx+c)}\,\sec(dx+c)^3}dx}{\sqrt{e}\,a^2e^3}$$

input
```
int(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(1/(sqrt(sec(c + d*x))*sec(c + d*x)**3*tan(c + d*x)**2 - 2*sqrt(sec
(c + d*x))*sec(c + d*x)**3*tan(c + d*x)*i - sqrt(sec(c + d*x))*sec(c + d*x
)**3),x))/(sqrt(e)*a**2*e**3)
```

3.245 $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 178

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{22e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d} + \frac{22e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^3d} + \frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^3d} - \frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

output
```
-22/5*e^8*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d/cos(d*x+c)^(1/2)/(e*
sec(d*x+c))^(1/2)-22/21*I*e^4*(e*sec(d*x+c))^(7/2)/a^3/d+22/5*e^7*(e*sec(d
*x+c))^(1/2)*sin(d*x+c)/a^3/d+22/15*e^5*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/a^
3/d-4/3*I*e^2*(e*sec(d*x+c))^(11/2)/a/d/(a+I*a*tan(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.17 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.72

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{e^6(e\sec(c+dx))^{3/2}\left(-556-868\cos(2(c+dx))+77e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.}{210a^3d}$$

input
```
Integrate[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
-1/210*(e^6*(e*Sec[c + d*x])^(3/2)*(-556 - 868*Cos[2*(c + d*x)] + (77*(1 +
 E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
+ d*x))])/E^((2*I)*(c + d*x)) + (203*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (1
43*I)*Tan[c + d*x])*(-I + Tan[c + d*x]))/(a^3*d)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3981, 3042, 3982, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{11e^2\int \frac{(e\sec(c+dx))^{11/2}}{i\tan(c+dx)a+a}dx}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11e^2\int \frac{(e\sec(c+dx))^{11/2}}{i\tan(c+dx)a+a}dx}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3982}$$

$$\frac{11e^2\left(\frac{e^2\int (e\sec(c+dx))^{7/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11e^2\left(\frac{e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{11e^2\left(\frac{e^2\left(\frac{3}{5}e^2\int (e\sec(c+dx))^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11e^2\left(\frac{e^2\left(\frac{3}{5}e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{11e^2\left(\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11e^2\left(\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \textbf{4258}$$

$$\frac{11e^2\left(\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{11e^2\left(\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \textbf{3119}$$

$$\frac{11e^2\left(\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

input
```
Int[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(11*e^2*((((-2*I)/7)*e^2*(e*Sec[c + d*x])^(7/2))/(a*d) + (e^2*((2*e*(e*Sec
[c + d*x])^(5/2)*Sin[c + d*x])/(5*d) + (3*e^2*((-2*e^2*EllipticE[(c + d*x)
/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c +
d*x]]*Sin[c + d*x])/d))/5))/a))/(3*a^2) - (((4*I)/3)*e^2*(e*Sec[c + d*x])^
(11/2))/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 4.78 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.38

method	result
default	$-\frac{2e^7\sqrt{e\sec(dx+c)}\left(-231\sin(dx+c)+63\tan(dx+c)+63\sec(dx+c)\tan(dx+c)+i\left(140+140\sec(dx+c)-15\sec(dx+c)^2-15\sec(dx\right.\right.}{}$

input
```
int((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/105*e^7/a^3/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(-231*sin(d*x+c)+63*t
an(d*x+c)+63*sec(d*x+c)*tan(d*x+c)+I*(140+140*sec(d*x+c)-15*sec(d*x+c)^2-1
5*sec(d*x+c)^3)+I*(231*cos(d*x+c)^2+462*cos(d*x+c)+231)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^
(1/2)+I*(-231*cos(d*x+c)^2-462*cos(d*x+c)-231)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.20

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx =$$
$$-\frac{2\left(\sqrt{2}\left(231i\,e^7e^{(7i\,dx+7i\,c)}+847i\,e^7e^{(5i\,dx+5i\,c)}+1133i\,e^7e^{(3i\,dx+3i\,c)}+637i\,e^7e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\right.}\right.}{105\left(a^3de^{(6i\,dx+\right.}}$$

input
```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
-2/105*(sqrt(2)*(231*I*e^7*e^(7*I*d*x + 7*I*c) + 847*I*e^7*e^(5*I*d*x + 5*
I*c) + 1133*I*e^7*e^(3*I*d*x + 3*I*c) + 637*I*e^7*e^(I*d*x + I*c))*sqrt(e/
(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 231*sqrt(2)*(I*e^7*e^
(6*I*d*x + 6*I*c) + 3*I*e^7*e^(4*I*d*x + 4*I*c) + 3*I*e^7*e^(2*I*d*x + 2*I
*c) + I*e^7)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(
I*d*x + I*c))))/(a^3*d*e^(6*I*d*x + 6*I*c) + 3*a^3*d*e^(4*I*d*x + 4*I*c) +
 3*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(15/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^{\frac{15}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(15/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{15/2}}{(a+a\tan(c+d\,x)\;1\mathrm{i})^3}\,dx$$

input
```
int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^7}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)e^7}{a^3}$$

input
```
int((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**7)/(tan(c + d*x)**3*i +
3*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*e**7)/a**3
```

3.246 $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 141

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{6e^6\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{a^3d} - \frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d} + \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^3d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

output
```
6*e^6*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x+c
))^(1/2)/a^3/d-18/5*I*e^4*(e*sec(d*x+c))^(5/2)/a^3/d+6*e^5*(e*sec(d*x+c))^
(3/2)*sin(d*x+c)/a^3/d-4*I*e^2*(e*sec(d*x+c))^(9/2)/a/d/(a+I*a*tan(d*x+c))
^2
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.52

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{e^4(e\sec(c+dx))^{5/2}\left(-18i-20i\cos(2(c+dx))+30\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\right.}{5a^3d}$$

input
```
Integrate[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(e^4*(e*Sec[c + d*x])^(5/2)*(-18*I - (20*I)*Cos[2*(c + d*x)] + 30*Cos[c +
d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] - 5*Sin[2*(c + d*x)]))/(5*a^3*d)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3981, 3042, 3982, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{9e^2\int \frac{(e\sec(c+dx))^{9/2}}{i\tan(c+dx)a+a}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9e^2\int \frac{(e\sec(c+dx))^{9/2}}{i\tan(c+dx)a+a}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3982}$$

$$\frac{9e^2\left(\frac{e^2\int (e\sec(c+dx))^{5/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9e^2\left(\frac{e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{9e^2\left(\frac{e^2\left(\frac{1}{3}e^2\int\sqrt{e\sec(c+dx)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{9e^2\left(\frac{e^2\left(\frac{1}{3}e^2\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \; 4258$$

$$\frac{9e^2\left(\frac{e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{9e^2\left(\frac{e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \; 3120$$

$$\frac{9e^2\left(\frac{e^2\left(\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

input
```
Int[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(9*e^2*((((-2*I)/5)*e^2*(e*Sec[c + d*x])^(5/2))/(a*d) + (e^2*((2*e^2*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d) + (2*e
*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)))/a))/a^2 - ((4*I)*e^2*(e*Sec[
c + d*x])^(9/2))/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 3.79 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

method	result
default	$-\frac{e^{6}\left(2\tan(dx+c)+8i-\frac{2i\sec(dx+c)^{2}}{5}-6i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\right)\sqrt{e}}{a^{3}d}$

input
```
int((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-e^6/a^3/d*(2*tan(d*x+c)+8*I-2/5*I*sec(d*x+c)^2-6*I*(cos(d*x+c)+1)*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c
)-csc(d*x+c)),I))*(e*sec(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.12

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2\left(\sqrt{2}\left(15i\,e^{6}e^{(4i\,dx+4i\,c)}+36i\,e^{6}e^{(2i\,dx+2i\,c)}+25i\,e^{6}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+15\sqrt{2}\left(i\,e^{6}e^{(4i\,dx+4i\,c)}+2i\,e\right.\right.}{5\left(a^{3}de^{(4i\,dx+4i\,c)}+2\,a^{3}de^{(2i\,dx+2i\,c)}+a^{3}d\right)}$$

input
```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
-2/5*(sqrt(2)*(15*I*e^6*e^(4*I*d*x + 4*I*c) + 36*I*e^6*e^(2*I*d*x + 2*I*c)
 + 25*I*e^6)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 1
5*sqrt(2)*(I*e^6*e^(4*I*d*x + 4*I*c) + 2*I*e^6*e^(2*I*d*x + 2*I*c) + I*e^6
)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a^3*d*e^(4*I*d*x +
 4*I*c) + 2*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(13/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e \sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e \sec(dx+c))^{\frac{13}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(13/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e \sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{13/2}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^3}\,dx$$

input
```
int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int \frac{(e \sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^6}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)e^6}{a^3}$$

input
```
int((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**6)/(tan(c + d*x)**3*i +
3*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*e**6)/a**3
```

3.247 $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 141

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx=\frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d}-\frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^3d}+\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}$$

output
```
14*e^6*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d/cos(d*x+c)^(1/2)/(e*sec
(d*x+c))^(1/2)+14/3*I*e^4*(e*sec(d*x+c))^(3/2)/a^3/d-14*e^5*(e*sec(d*x+c))
^(1/2)*sin(d*x+c)/a^3/d+4*I*e^2*(e*sec(d*x+c))^(7/2)/a/d/(a+I*a*tan(d*x+c)
)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.80 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.66

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx=\frac{ie^4(e\sec(c+dx))^{3/2}\left(35+33\cos(2(c+dx))-7\left(1+e^{2i(c+dx)}\right)^{3/2}\text{Hypergeo}\right.}{3a^3d}$$

input
```
Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/3)*e^4*(e*Sec[c + d*x])^(3/2)*(35 + 33*Cos[2*(c + d*x)] - 7*(1 + E^((2
*I)*(c + d*x)))^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x)
)] + (9*I)*Sin[2*(c + d*x)]))/(a^3*d)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3981, 3042, 3982, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{7e^2\int \frac{(e\sec(c+dx))^{7/2}}{i\tan(c+dx)a+a}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{7e^2\int \frac{(e\sec(c+dx))^{7/2}}{i\tan(c+dx)a+a}dx}{a^2}$$

$$\Big\downarrow \text{3982}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{7e^2\left(\frac{e^2\int (e\sec(c+dx))^{3/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{7e^2\left(\frac{e^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}\right)}{a^2}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}\right)}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}\right)}{a^2}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}\right)}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}\right)}{a^2}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}\right)}{a^2}$$

input
```
Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(-7*e^2*((((-2*I)/3)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d) + (e^2*((-2*e^2*Ell
ipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e
*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/a))/a^2 + ((4*I)*e^2*(e*Sec[c + d*
x])^(7/2))/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.62

method	result
default	$\frac{2\sqrt{e\sec(dx+c)}\,e^5\left(\sin(dx+c)(12\cos(dx+c)-9)+i\left(12\cos(dx+c)^2+12\cos(dx+c)+1+\sec(dx+c)\right)+i\left(21\cos(dx+c)^2+42\cos(dx+c)\right.\right.}{}$

input
```
int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
2/3/a^3/d*(e*sec(d*x+c))^(1/2)*e^5/(cos(d*x+c)+1)*(sin(d*x+c)*(12*cos(d*x+
c)-9)+I*(12*cos(d*x+c)^2+12*cos(d*x+c)+1+sec(d*x+c))+I*(21*cos(d*x+c)^2+42
*cos(d*x+c)+21)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+I*(-21*cos(d*x+c)^2-42*cos(d*x+c)-
21)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I
*(csc(d*x+c)-cot(d*x+c)),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2\left(\sqrt{2}\left(-21i\,e^5e^{(4i\,dx+4i\,c)}-35i\,e^5e^{(2i\,dx+2i\,c)}-12i\,e^5\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+21\sqrt{2}\left(-i\,e^5e^{(3i\,dx+3i\,c)}-\right.}{3\left(a^3de^{(3i\,dx+3i\,c)}+a^3de^{(i\,dx+}\right.}$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
-2/3*(sqrt(2)*(-21*I*e^5*e^(4*I*d*x + 4*I*c) - 35*I*e^5*e^(2*I*d*x + 2*I*c
) - 12*I*e^5)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) +
21*sqrt(2)*(-I*e^5*e^(3*I*d*x + 3*I*c) - I*e^5*e^(I*d*x + I*c))*sqrt(e)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(a^3*d*
e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^{\frac{11}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(11/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}\,dx$$

input
```
int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^5}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)e^5}{a^3}$$

input
```
int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**5)/(tan(c + d*x)**3*i +
3*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*e**5)/a**3
```

3.248 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{10ie^4\sqrt{e\sec(c+dx)}}{3a^3d} - \frac{10e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^3d} + \frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}$$

output
```
10/3*I*e^4*(e*sec(d*x+c))^(1/2)/a^3/d-10/3*e^4*cos(d*x+c)^(1/2)*InverseJac
obiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x+c))^(1/2)/a^3/d+4/3*I*e^2*(e*sec(d
*x+c))^(5/2)/a/d/(a+I*a*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.08

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{2e^4\sec^3(c+dx)\sqrt{e\sec(c+dx)}\left(-7i\cos(c+dx)+5\sqrt{\cos(c+dx)}\,\text{EllipticF}\right.}{3}$$

input
```
Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(2*e^4*Sec[c + d*x]^3*Sqrt[e*Sec[c + d*x]]*((-7*I)*Cos[c + d*x] + 5*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin[c + d*x]) + 3
*Sin[c + d*x])*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]))/(3*a^3*d*(-I +
Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3981, 3042, 3982, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}-\frac{5e^2\int \frac{(e\sec(c+dx))^{5/2}}{i\tan(c+dx)a+a}dx}{3a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}-\frac{5e^2\int \frac{(e\sec(c+dx))^{5/2}}{i\tan(c+dx)a+a}dx}{3a^2}$$

$$\Big\downarrow \text{3982}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}-\frac{5e^2\left(\frac{e^2\int \sqrt{e\sec(c+dx)}dx}{a}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}\right)}{3a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}-\frac{5e^2\left(\frac{e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}\right)}{3a^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}-\frac{5e^2\left(\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}\right)}{3a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}-\frac{5e^2\left(\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}\right)}{3a^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}-\frac{5e^2\left(\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{ad}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}\right)}{3a^2}$$

input
```
Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(-5*e^2*(((-2*I)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d) + (2*e^2*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(a*d)))/(3*a^2) + (((4*
I)/3)*e^2*(e*Sec[c + d*x])^(5/2))/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 4.01 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

method	result
default	$-\frac{2\left(i(5\cos(dx+c)+5)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)+i\left(-4\cos(dx+c)^2-3\right)-4\cos(dx+c)\sin$
	$3a^3d$

input
```
int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3/a^3/d*(I*(5*cos(d*x+c)+5)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)+I*(-4*cos(d*x+c)^2-
3)-4*cos(d*x+c)*sin(d*x+c))*(e*sec(d*x+c))^(1/2)*e^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2\left(-5i\sqrt{2}e^{\frac{9}{2}}e^{(2i\,dx+2i\,c)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-5i\,e^4e^{(2i\,dx+2i\,c)}-2i\,e^4\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+}}\right.}{3\,a^3d}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/3*(-5*I*sqrt(2)*e^(9/2)*e^(2*I*d*x + 2*I*c)*weierstrassPInverse(-4, 0,
e^(I*d*x + I*c)) + sqrt(2)*(-5*I*e^4*e^(2*I*d*x + 2*I*c) - 2*I*e^4)*sqrt(e
/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-2*I*d*x - 2*I*c)/
(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^{\frac{9}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}\,dx$$

input
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^4}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)e^4}{a^3}$$

input
```
int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**4)/(tan(c + d*x)**3*i +
3*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*e**4)/a**3
```

3.249 $\int \frac{(e \sec(c+dx))^{7/2}}{(a+ia \tan(c+dx))^3} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{(e \sec(c+dx))^{7/2}}{(a+ia \tan(c+dx))^3} \, dx = -\frac{6ie^4}{5a^3d\sqrt{e \sec(c+dx)}} - \frac{6e^4 E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e \sec(c+dx)}} + \frac{4ie^2(e \sec(c+dx))^{3/2}}{5ad(a+ia \tan(c+dx))^2}$$

output
```
-6/5*I*e^4/a^3/d/(e*sec(d*x+c))^(1/2)-6/5*e^4*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2))/a^3/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+4/5*I*e^2*(e*sec(d*x
+c))^(3/2)/a/d/(a+I*a*tan(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.55 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

$$\int \frac{(e \sec(c+dx))^{7/2}}{(a+ia \tan(c+dx))^3} \, dx = \frac{2ee^{-idx}\left(-2+\frac{6e^{2i(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}\right)(e \sec(c+dx))^{5/2}(\text{co}}{5a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(2*e*(-2 + (6*E^((2*I)*(c + d*x))*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2
*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))])*(e*Sec[c + d*x])^(5/2)*(Co
s[c + 2*d*x] + I*Sin[c + 2*d*x]))/(5*a^3*d*E^(I*d*x)*(-I + Tan[c + d*x])^3
)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3981, 3042, 3982, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{3e^2\int \frac{(e\sec(c+dx))^{3/2}}{i\tan(c+dx)a+a}dx}{5a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{3e^2\int \frac{(e\sec(c+dx))^{3/2}}{i\tan(c+dx)a+a}dx}{5a^2}$$

$$\Big\downarrow \text{3982}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{3e^2\left(\frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{a}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}\right)}{5a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{3e^2\left(\frac{e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}\right)}{5a^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{3e^2\left(\frac{e^2\int\sqrt{\cos(c+dx)}dx}{a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}\right)}{5a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{3e^2\left(\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}\right)}{5a^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{3e^2\left(\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}\right)}{5a^2}$$

input
```
Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(-3*e^2*(((2*I)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]) + (2*e^2*EllipticE[(c + d*
x)/2, 2])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])))/(5*a^2) + (((4*I
)/5)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(101) = 202$.

Time = 3.63 (sec) , antiderivative size = 446, normalized size of antiderivative = 3.84

method	result
default	$-\frac{2\left(i\left(3\cos(dx+c)^2+6\cos(dx+c)+3\right)\sin(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticE}(i(\cot(dx+c)-\csc(dx+c)),i)+\sqrt{\frac{1}{\cos(dx+c}}$

input
```
int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/5/a^3/d*(I*(3*cos(d*x+c)^2+6*cos(d*x+c)+3)*sin(d*x+c)*(1/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+
c)),I)+(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Elliptic
E(I*(cot(d*x+c)-csc(d*x+c)),I)*(3*cos(d*x+c)^3+6*cos(d*x+c)^2+3*cos(d*x+c)
)+I*(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)+(1
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(
d*x+c)-csc(d*x+c)),I)*(-3*cos(d*x+c)^3-6*cos(d*x+c)^2-3*cos(d*x+c))+I*sin(
d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)-5)+cos(d*x+c)*(2*cos(d*x+c)^2-cos(d*x+c)-
3))*e^3*(e*sec(d*x+c))^(1/2)/(-cos(d*x+c)*sin(d*x+c)-sin(d*x+c)+I*cos(d*x+
c)^2+I*cos(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2\left(3i\sqrt{2}e^{\frac{7}{2}}e^{(3i\,dx+3i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{2}\left(3i\,e^3e^{(4i\,dx+4i\,c)}\right.\right.}{5\,a^3d}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/5*(3*I*sqrt(2)*e^(7/2)*e^(3*I*d*x + 3*I*c)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(3*I*e^3*e^(4*I*d*x + 4*
I*c) + 2*I*e^3*e^(2*I*d*x + 2*I*c) - I*e^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) +
1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^{\frac{7}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}\,dx$$

input
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^3}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)e^3}{a^3}$$

input
```
int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**3)/(tan(c + d*x)**3*i +
3*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*e**3)/a**3
```

3.250 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21a^3d} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2} - \frac{2ie^2\sqrt{e\sec(c+dx)}}{21d\left(a^3+ia^3\tan(c+dx)\right)}$$

output
```
-2/21*e^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d
*x+c))^(1/2)/a^3/d+4/7*I*e^2*(e*sec(d*x+c))^(1/2)/a/d/(a+I*a*tan(d*x+c))^2
-2/21*I*e^2*(e*sec(d*x+c))^(1/2)/d/(a^3+I*a^3*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.79

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{(e\sec(c+dx))^{5/2}\left(-5i-5i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c\right.\right.}{21a^3d(-i+\tan(c+}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((e*Sec[c + d*x])^(5/2)*(-5*I - (5*I)*Cos[2*(c + d*x)] + 2*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) - Si
n[2*(c + d*x)]))/(21*a^3*d*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3981, 3042, 3983, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}-\frac{e^2\int \frac{\sqrt{e\sec(c+dx)}}{i\tan(c+dx)a+a}dx}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}-\frac{e^2\int \frac{\sqrt{e\sec(c+dx)}}{i\tan(c+dx)a+a}dx}{7a^2}$$

$$\Big\downarrow \text{3983}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}-\frac{e^2\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3a}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}\right)}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}-\frac{e^2\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3a}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}\right)}{7a^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}-\frac{e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3a}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}\right)}{7a^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}-\frac{e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}\right)}{7a^2}$$

$$\Big\downarrow \textbf{3120}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}-\frac{e^2\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3ad}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}\right)}{7a^2}$$

input
```
Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(((4*I)/7)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x])^2) - (e^2
*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3
*a*d) + (((2*I)/3)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x]))))/(7*a
^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 3.20 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.45

method	result
default	$\frac{2e^2\sqrt{e\sec(dx+c)}\left(i\,\text{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)+12i\cos(dx+c)^4+i\,\text{EllipticF}(i(}{21a^3d}$

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
2/21/a^3/d*e^2*(e*sec(d*x+c))^(1/2)*(I*EllipticF(I*(csc(d*x+c)-cot(d*x+c))
,I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+
12*I*cos(d*x+c)^4+I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+12*sin(d*x+c)*cos(d*x+c)^3-7*I
*cos(d*x+c)^2-cos(d*x+c)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.84

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(2i\sqrt{2}e^{\frac{5}{2}}e^{(4i\,dx+4i\,c)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(2i\,e^2e^{(4i\,dx+4i}\right.\right.}{21\,a^3d}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/21*(2*I*sqrt(2)*e^(5/2)*e^(4*I*d*x + 4*I*c)*weierstrassPInverse(-4, 0, e
^(I*d*x + I*c)) + sqrt(2)*(2*I*e^2*e^(4*I*d*x + 4*I*c) + 5*I*e^2*e^(2*I*d*
x + 2*I*c) + 3*I*e^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2
*I*c))*e^(-4*I*d*x - 4*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{(e\sec(c+dx))^{\frac{5}{2}}}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral((e*sec(c + d*x))**(5/2)/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2
- 3*tan(c + d*x) + I), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx=\int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx=-\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^2}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)e^2}{a^3}$$

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**2)/(tan(c + d*x)**3*i +
3*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*e**2)/a**3
```

3.251 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{9ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2} + \frac{2ie^2}{45d\sqrt{e\sec(c+dx)}\,(a^3+ia^3\tan(c+dx))}$$

output
```
2/15*e^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d/cos(d*x+c)^(1/2)/(e*s
ec(d*x+c))^(1/2)+4/9*I*e^2/a/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2+2
/45*I*e^2/d/(e*sec(d*x+c))^(1/2)/(a^3+I*a^3*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.59 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.06

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{e^{-idx}\sec^2(c+dx)(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))\left(8+8\cos(2(c+dx))+6e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\right.}{45a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
-1/45*(Sec[c + d*x]^2*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])*(8 +
8*Cos[2*(c + d*x)] + 6*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + (3*I)*Sin[2*(c +
d*x)]))/(a^3*d*E^(I*d*x)*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3981, 3042, 3983, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)}dx}{9a^2}+\frac{4ie^2}{9ad(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{e^2 \int \frac{1}{\sqrt{e \sec(c+dx)}(i \tan(c+dx)a+a)}dx}{9a^2}+\frac{4ie^2}{9ad(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3983$$

$$\frac{e^2\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5a}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{9a^2}+\frac{4ie^2}{9ad(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{e^2\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{9a^2}+\frac{4ie^2}{9ad(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 4258$$

$$\frac{e^2\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{9a^2}+\frac{4ie^2}{9ad(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{e^2\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{9a^2}+\frac{4ie^2}{9ad(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3119$$

$$\frac{e^2\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{9a^2}+\frac{4ie^2}{9ad(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(((4*I)/9)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2) + (e^2
*((6*EllipticE[(c + d*x)/2, 2])/(5*a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d
*x]]) + ((2*I)/5)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]))))/(9*a^2
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(116) = 232.

Time = 3.57 (sec) , antiderivative size = 497, normalized size of antiderivative = 3.77

method	result
default	$-\frac{2\left(-6\sin(dx+c)\cos(dx+c)\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\,\mathrm{EllipticE}(i(\csc(dx+c)-\cot(dx+c)),i)+\right.}{}$

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/45/a^3/d*(-6*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+
c)-cot(d*x+c)),I)+3*I*(2*cos(d*x+c)^4+4*cos(d*x+c)^3+cos(d*x+c)^2-2*cos(d*
x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipti
cE(I*(csc(d*x+c)-cot(d*x+c)),I)-6*sin(d*x+c)*cos(d*x+c)*(-cos(d*x+c)^2-2*c
os(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*El
lipticF(I*(csc(d*x+c)-cot(d*x+c)),I)+3*I*(-2*cos(d*x+c)^4-4*cos(d*x+c)^3-c
os(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)-sin(d*x+c)*cos(d*x+c)*
(-5*cos(d*x+c)^2+cos(d*x+c)+3)+I*cos(d*x+c)^2*(5*cos(d*x+c)^2+11*cos(d*x+c
)+6))*(e*sec(d*x+c))^(1/2)/(2*I*sin(d*x+c)*cos(d*x+c)*(-cos(d*x+c)-1)-2*co
s(d*x+c)^3-2*cos(d*x+c)^2+cos(d*x+c)+1)*e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.91

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(12i\sqrt{2}e^{\frac{3}{2}}e^{(5i\,dx+5i\,c)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx}\right.\right.\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/90*(12*I*sqrt(2)*e^(3/2)*e^(5*I*d*x + 5*I*c)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(12*I*e*e^(6*I*d*x + 6*
I*c) + 23*I*e*e^(4*I*d*x + 4*I*c) + 16*I*e*e^(2*I*d*x + 2*I*c) + 5*I*e)*sq
rt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-5*I*d*x - 5*I
*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{(e\sec(c+dx))^{\frac{3}{2}}}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral((e*sec(c + d*x))**(3/2)/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2
- 3*tan(c + d*x) + I), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^3}\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)e}{a^3}$$

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x))/(tan(c + d*x)**3*i + 3*t
an(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*e)/a**3
```

3.252 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 152

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77a^3d} + \frac{10e\sin(c+dx)}{77a^3d\sqrt{e\sec(c+dx)}} + \frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3} + \frac{20ie^2}{77d(e\sec(c+dx))^{3/2}\left(a^3+ia^3\tan(c+dx)\right)}$$

output
```
10/77*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x+c
))^(1/2)/a^3/d+10/77*e*sin(d*x+c)/a^3/d/(e*sec(d*x+c))^(1/2)+2/11*I*(e*sec
(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^3+20/77*I*e^2/d/(e*sec(d*x+c))^(3/2)/(
a^3+I*a^3*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{e \sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{i\sec^3(c+dx)\sqrt{e\sec(c+dx)}\Big(46i\cos(c+dx)+22i\cos(3(c+dx))-15\sin(c+dx)+20\sqrt{\cos(c+dx)}\,\mathrm{E}}{154a^3d(-i+\tan(c+dx))}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/154)*Sec[c + d*x]^3*Sqrt[e*Sec[c + d*x]]*((46*I)*Cos[c + d*x] + (22*I)
*Cos[3*(c + d*x)] - 15*Sin[c + d*x] + 20*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) - 15*Sin[3*(c + d*x)])
)/(a^3*d*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3983, 3042, 3981, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3983}$$
$$\frac{5\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^2}dx}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$
$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^2}dx}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3981}$$

$$\frac{5\left(\frac{3e^2\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4256}$$

$$\frac{5\left(\frac{3e^2\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3e^2\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5\left(\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{5\left(\frac{3e^2\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(((2*I)/11)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^3) + (5*((3*e^
2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*a^2) + (((4*
I)/7)*e^2)/(d*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^2*Tan[c + d*x]))))/(11*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.94

method	result
default	$-\frac{2\left(\sin(dx+c)\cos(dx+c)\left(-28\cos(dx+c)^4-3\cos(dx+c)^2-5\right)+i\cos(dx+c)^4\left(-28\cos(dx+c)^2+11\right)+i(5\cos(dx+c)+5)\sqrt{\frac{\cos(dx}{\cos(dx+}}}{77a^3d}$

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/77/a^3/d*(sin(d*x+c)*cos(d*x+c)*(-28*cos(d*x+c)^4-3*cos(d*x+c)^2-5)+I*c
os(d*x+c)^4*(-28*cos(d*x+c)^2+11)+I*(5*cos(d*x+c)+5)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/
2))*(e*sec(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(37i\,e^{(6i\,dx+6i\,c)}+61i\,e^{(4i\,dx+4i\,c)}+31i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-40i\,\sqrt{2}\sqrt{e}e^{(6i\,dx+6i}\right.}{308\,a^3d}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/308*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(37*I*e^(6*I*d*x + 6*I*c)
 + 61*I*e^(4*I*d*x + 4*I*c) + 31*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(1/2*I*d*x
 + 1/2*I*c) - 40*I*sqrt(2)*sqrt(e)*e^(6*I*d*x + 6*I*c)*weierstrassPInverse
(-4, 0, e^(I*d*x + I*c)))*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sqrt{e\sec\left(c+dx\right)}}{\tan^3\left(c+dx\right)-3i\tan^2\left(c+dx\right)-3\tan\left(c+dx\right)+i}\,dx}{a^3}$$

input
```
integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sqrt(e*sec(c + d*x))/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3
*tan(c + d*x) + I), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\sqrt{e\sec\left(dx+c\right)}}{\left(i\,a\tan\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}}{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^3}\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)}{a^3}$$

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - sqrt(e)*int(sqrt(sec(c + d*x))/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2
- 3*tan(c + d*x)*i - 1),x))/a**3
```

3.253 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 152

$$\begin{aligned}\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx &= \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14e\sin(c+dx)}{117a^3d(e\sec(c+dx))^{3/2}}\\&\quad+\frac{2i}{13d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\\&\quad+\frac{28ie^2}{117d(e\sec(c+dx))^{5/2}\left(a^3+ia^3\tan(c+dx)\right)}\end{aligned}$$

output
```
14/39*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d/cos(d*x+c)^(1/2)/(e*sec(
d*x+c))^(1/2)+14/117*e*sin(d*x+c)/a^3/d/(e*sec(d*x+c))^(3/2)+2/13*I/d/(e*s
ec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3+28/117*I*e^2/d/(e*sec(d*x+c))^(5/2)/
(a^3+I*a^3*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.00 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \frac{\sqrt{e\sec(c+dx)}(i\cos(3(c+dx))+\sin(3(c+dx)))\left(62+176\cos(2(c+dx))+114\cos(4(c+dx))-56e\right.}{468}$$

input
```
Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3),x]
```

output
```
(Sqrt[e*Sec[c + d*x]]*(I*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(62 + 176*Co
s[2*(c + d*x)] + 114*Cos[4*(c + d*x)] - 56*E^((4*I)*(c + d*x))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]
+ (126*I)*Sin[2*(c + d*x)] + (105*I)*Sin[4*(c + d*x)]))/(468*a^3*d*e)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3983, 3042, 3981, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2}dx}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2}dx}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3981}$$

$$\frac{7\left(\frac{5e^2\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{7\left(\frac{5e^2\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5e^2\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{7\left(\frac{5e^2\left(\frac{3\int\sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5e^2\left(\frac{3\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{7\left(\frac{5e^2\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3),x]
```

output
```
((2*I)/13)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3) + (7*((5*e^2*
((6*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
d*x]]) + (2*Sin[c + d*x])/(5*d*e*(e*Sec[c + d*x])^(3/2))))/(9*a^2) + (((4*
I)/9)*e^2)/(d*(e*Sec[c + d*x])^(5/2)*(a^2 + I*a^2*Tan[c + d*x]))))/(13*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(132) = 264$.

Time = 4.49 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.84

method	result
default	$-\frac{2\left(\sin(dx+c)\left(-36\cos(dx+c)^{6}-36\cos(dx+c)^{5}-5\cos(dx+c)^{4}-5\cos(dx+c)^{3}-7\cos(dx+c)^{2}-7\cos(dx+c)-21\right)+i\left(-36\cos(dx+\right.\right.}{}$

input
```
int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/117/a^3/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)*(sin(d*x+c)*(-36*cos(d*x+
c)^6-36*cos(d*x+c)^5-5*cos(d*x+c)^4-5*cos(d*x+c)^3-7*cos(d*x+c)^2-7*cos(d*
x+c)-21)+I*(-36*cos(d*x+c)^7-36*cos(d*x+c)^6+13*cos(d*x+c)^5+13*cos(d*x+c)
^4)+21*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)+2+sec(d*x+c))*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)-21*I*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+2+sec(d*x+c))
*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(219i\,e^{(8i\,dx+8i\,c)}+302i\,e^{(6i\,dx+6i\,c)}+124i\,e^{(4i\,dx+4i\,c)}+50i\,e^{(2i\,dx+2i\,c)}+9i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-\right.}{936\,a^3de}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas
")
```

output
```
1/936*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(219*I*e^(8*I*d*x + 8*I*c
) + 302*I*e^(6*I*d*x + 6*I*c) + 124*I*e^(4*I*d*x + 4*I*c) + 50*I*e^(2*I*d*
x + 2*I*c) + 9*I)*e^(1/2*I*d*x + 1/2*I*c) + 336*I*sqrt(2)*sqrt(e)*e^(7*I*d
*x + 7*I*c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I
*c))))*e^(-7*I*d*x - 7*I*c)/(a^3*d*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{1}{\sqrt{e\sec(c+dx)}\tan^3(c+dx)-3i\sqrt{e\sec(c+dx)}\tan^2(c+dx)-3\sqrt{e\sec(c+dx)}\tan(c+dx)+i\sqrt{e\sec(c+dx)}}\,dx}{a^3}$$

input
```
integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(1/(sqrt(e*sec(c + d*x))*tan(c + d*x)**3 - 3*I*sqrt(e*sec(c + d*
x))*tan(c + d*x)**2 - 3*sqrt(e*sec(c + d*x))*tan(c + d*x) + I*sqrt(e*sec(c
 + d*x))), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \int \frac{1}{\sqrt{e\sec(dx+c)}(i\,a\tan(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \int \frac{1}{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^3}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)\tan(dx+c)^3 i+3\sec(dx+c)\tan(dx+c)^2-3\sec(dx+c)\tan(dx+c)i-\sec(dx+c)}dx\right)}{a^3e}$$

input
```
int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - sqrt(e)*int(sqrt(sec(c + d*x))/(sec(c + d*x)*tan(c + d*x)**3*i + 3*sec
(c + d*x)*tan(c + d*x)**2 - 3*sec(c + d*x)*tan(c + d*x)*i - sec(c + d*x)),
x))/(a**3*e)
```

3.254 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 186

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{11a^3de^2} + \frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/2}} + \frac{2\sin(c+dx)}{11a^3de\sqrt{e\sec(c+dx)}} + \frac{2i}{15d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3} + \frac{12ie^2}{55d(e\sec(c+dx))^{7/2}\left(a^3+ia^3\tan(c+dx)\right)}$$

output
```
2/11*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x+c)
)^(1/2)/a^3/d/e^2+6/55*e*sin(d*x+c)/a^3/d/(e*sec(d*x+c))^(5/2)+2/11*sin(d*
x+c)/a^3/d/e/(e*sec(d*x+c))^(1/2)+2/15*I/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan
(d*x+c))^3+12/55*I*e^2/d/(e*sec(d*x+c))^(7/2)/(a^3+I*a^3*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.81

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \frac{\sec^5(c+dx)\left(-332\cos(c+dx)-154\cos(3(c+dx))+22\right.}{}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3),x]
```

output
```
(Sec[c + d*x]^5*(-332*Cos[c + d*x] - 154*Cos[3*(c + d*x)] + 22*Cos[5*(c +
d*x)] - (114*I)*Sin[c + d*x] + (240*I)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) - (81*I)*Sin[3*(c + d*x)
] + (33*I)*Sin[5*(c + d*x)]))/(1320*a^3*d*(e*Sec[c + d*x])^(3/2)*(-I + Tan
[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3983, 3042, 3981, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\int \frac{1}{(e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2}dx}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{(e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2}dx}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3981

$$\frac{3\left(\frac{7e^2\int \frac{1}{(e\sec(c+dx))^{7/2}}dx}{11a^2}+\frac{4ie^2}{11d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{7/2}}\right)}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{7e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{11a^2}+\frac{4ie^2}{11d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{7/2}}\right)}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4256

$$\frac{3\left(\frac{7e^2\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{7/2}}\right)}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{7e^2\left(\frac{5\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{7/2}}\right)}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4256

$$\frac{3\left(\frac{7e^2\left(\frac{5\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{7/2}}\right)}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{3\left(\frac{7e^2\left(\frac{5\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{7/2}}\right)}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \color{blue}{4258}$$

$$\frac{3\left(\frac{7e^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{7/2}}\right)}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{3\left(\frac{7e^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{7/2}}\right)}{5a}$$
$$\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{3\left(\frac{7e^2\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{7/2}}\right)}{5a}$$
$$\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3),x]
```

output
```
((2*I)/15)/(d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3) + (3*((7*e^
2*((2*Sin[c + d*x])/(7*d*e*(e*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*Sin[c
+ d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2)))/(11*a^2) + (((4*I)/11)*e^
2)/(d*(e*Sec[c + d*x])^(7/2)*(a^2 + I*a^2*Tan[c + d*x]))))/(5*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.81

method	result
default	$\frac{\frac{2\sin(dx+c)\left(44\cos(dx+c)^6+7\cos(dx+c)^4+9\cos(dx+c)^2+15\right)}{165}+\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(15+15\sec}{165}}{a^3d\sqrt{e\sec(dx+c)}\,e}$

input
```
int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3/d*(2/165*sin(d*x+c)*(44*cos(d*x+c)^6+7*cos(d*x+c)^4+9*cos(d*x+c)^2+1
5)+2/165*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elli
pticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(15+15*sec(d*x+c))+2/165*I*(44*cos(d*x+
c)^7-15*cos(d*x+c)^5))/(e*sec(d*x+c))^(1/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.74

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-55i\,e^{(10i\,dx+10i\,c)}+235i\,e^{(8i\,dx+8i\,c)}+4\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas
")
```

output
```
1/2640*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-55*I*e^(10*I*d*x + 10*
I*c) + 235*I*e^(8*I*d*x + 8*I*c) + 446*I*e^(6*I*d*x + 6*I*c) + 218*I*e^(4*
I*d*x + 4*I*c) + 73*I*e^(2*I*d*x + 2*I*c) + 11*I)*e^(1/2*I*d*x + 1/2*I*c)
- 480*I*sqrt(2)*sqrt(e)*e^(8*I*d*x + 8*I*c)*weierstrassPInverse(-4, 0, e^(
I*d*x + I*c)))*e^(-8*I*d*x - 8*I*c)/(a^3*d*e^2)
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{1}{(e\sec(c+dx))^{\frac{3}{2}}\tan^3(c+dx)-3i(e\sec(c+dx))^{\frac{3}{2}}\tan^2(c+dx)-3(e\sec(c+dx}}{a^3}$$

input
```
integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(1/((e*sec(c + d*x))**(3/2)*tan(c + d*x)**3 - 3*I*(e*sec(c + d*x
))**(3/2)*tan(c + d*x)**2 - 3*(e*sec(c + d*x))**(3/2)*tan(c + d*x) + I*(e*
sec(c + d*x))**(3/2)), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^3}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^3), x)
```

Reduce [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2\tan(dx+c)^3 i+3\sec(dx+c)^2\tan(dx+c)^2-3\sec(dx+c)^2\tan(dx+c) i-\sec(dx+c)^2}dx\right)}{a^3e^2}$$

input
```
int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - sqrt(e)*int(sqrt(sec(c + d*x))/(sec(c + d*x)**2*tan(c + d*x)**3*i + 3*
sec(c + d*x)**2*tan(c + d*x)**2 - 3*sec(c + d*x)**2*tan(c + d*x)*i - sec(c
 + d*x)**2),x))/(a**3*e**2)
```

3.255 $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 192

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{154e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{154e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d} - \frac{154e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^4d} + \frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3} + \frac{44ie^4(e\sec(c+dx))^{7/2}}{3d\,(a^4+ia^4\tan(c+dx))}$$

output
```
154/5*e^8*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/d/cos(d*x+c)^(1/2)/(e*
sec(d*x+c))^(1/2)-154/5*e^7*(e*sec(d*x+c))^(1/2)*sin(d*x+c)/a^4/d-154/15*e
^5*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/a^4/d+4*I*e^2*(e*sec(d*x+c))^(11/2)/a/d
/(a+I*a*tan(d*x+c))^3+44/3*I*e^4*(e*sec(d*x+c))^(7/2)/d/(a^4+I*a^4*tan(d*x
+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.65

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = -\frac{ie^5(e\sec(c+dx))^{5/2}\left(-1133\cos(c+dx)+77e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2}\right.\right.}{30a^4d}$$

input
```
Integrate[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-1/30*I)*e^5*(e*Sec[c + d*x])^(5/2)*(-1133*Cos[c + d*x] + (77*(1 + E^((2
*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x)
)])/E^(I*(c + d*x)) - 3*(117*Cos[3*(c + d*x)] + (33*I)*Sin[c + d*x] + (37*
I)*Sin[3*(c + d*x)])))/(a^4*d)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3981, 3042, 3981, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{ 3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{11e^2\int \frac{(e\sec(c+dx))^{11/2}}{(i\tan(c+dx)a+a)^2}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{11e^2\int\frac{(e\sec(c+dx))^{11/2}}{(i\tan(c+dx)a+a)^2}dx}{a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{11e^2\left(\frac{7e^2\int(e\sec(c+dx))^{7/2}dx}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{11e^2\left(\frac{7e^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{11e^2\left(\frac{7e^2\left(\frac{3}{5}e^2\int(e\sec(c+dx))^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{11e^2\left(\frac{7e^2\left(\frac{3}{5}e^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{11e^2\left(\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-11e^2\left(\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-11e^2\left(\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-11e^2\left(\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-11e^2\left(\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

input

```
Int[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((4*I)*e^2*(e*Sec[c + d*x])^(11/2))/(a*d*(a + I*a*Tan[c + d*x])^3) - (11*e
^2*((7*e^2*((2*e*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d) + (3*e^2*((-2*
e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])
 + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/5))/(3*a^2) - (((4*I)/3)*e^
2*(e*Sec[c + d*x])^(7/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))))/a^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 5.44 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.34

method	result
default	$\frac{2\sqrt{e\sec(dx+c)}\,e^{7}\left(\tan(dx+c)\sec(dx+c)\left(120\cos(dx+c)^{3}-111\cos(dx+c)^{2}+3\cos(dx+c)+3\right)+i\left(120\cos(dx+c)^{2}+120\cos(dx+c)+\right.}{}$

input
```
int((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
2/15/a^4/d*(e*sec(d*x+c))^(1/2)*e^7/(cos(d*x+c)+1)*(tan(d*x+c)*sec(d*x+c)*
(120*cos(d*x+c)^3-111*cos(d*x+c)^2+3*cos(d*x+c)+3)+I*(120*cos(d*x+c)^2+120
*cos(d*x+c)+20+20*sec(d*x+c))+I*(231*cos(d*x+c)^2+462*cos(d*x+c)+231)*Elli
pticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)+I*(-231*cos(d*x+c)^2-462*cos(d*x+c)-231)*EllipticE(I*(
cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.01

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx =$$
$$-\frac{2\left(\sqrt{2}\left(-231i\,e^{7}e^{(6i\,dx+6i\,c)}-616i\,e^{7}e^{(4i\,dx+4i\,c)}-517i\,e^{7}e^{(2i\,dx+2i\,c)}-120i\,e^{7}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+2\right.}{15\left(a^{4}de^{(5i\,dx+5i\,c)}\right.}$$

input
```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
-2/15*(sqrt(2)*(-231*I*e^7*e^(6*I*d*x + 6*I*c) - 616*I*e^7*e^(4*I*d*x + 4*
I*c) - 517*I*e^7*e^(2*I*d*x + 2*I*c) - 120*I*e^7)*sqrt(e/(e^(2*I*d*x + 2*I
*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 231*sqrt(2)*(-I*e^7*e^(5*I*d*x + 5*I*c
) - 2*I*e^7*e^(3*I*d*x + 3*I*c) - I*e^7*e^(I*d*x + I*c))*sqrt(e)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(a^4*d*e^(5*I*
d*x + 5*I*c) + 2*a^4*d*e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*x + I*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(15/2)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(e\sec(dx+c))^{\frac{15}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

input
```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(15/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{15/2}}{(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^4}\,dx$$

input
```
int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^7}{\tan(dx+c)^4-4\tan(dx+c)^3 i-6\tan(dx+c)^2+4\tan(dx+c)i+1}dx\right)e^7}{a^4}$$

input
```
int((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**7)/(tan(c + d*x)**4 - 4*tan
(c + d*x)**3*i - 6*tan(c + d*x)**2 + 4*tan(c + d*x)*i + 1),x)*e**7)/a**4
```

3.256 $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 157

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = -\frac{10e^6\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{a^4d} - \frac{10e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^4d} + \frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3} + \frac{12ie^4(e\sec(c+dx))^{5/2}}{d\left(a^4+ia^4\tan(c+dx)\right)}$$

output
```
-10*e^6*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x
+c))^(1/2)/a^4/d-10*e^5*(e*sec(d*x+c))^(3/2)*sin(d*x+c)/a^4/d+4/3*I*e^2*(e
*sec(d*x+c))^(9/2)/a/d/(a+I*a*tan(d*x+c))^3+12*I*e^4*(e*sec(d*x+c))^(5/2)/
d/(a^4+I*a^4*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{ie^6\sec^5(c+dx)\sqrt{e\sec(c+dx)}\Big(21+19\cos(2(c+dx))+30i\cos^{\frac{3}{2}}(c+dx)\,\mathrm{E}}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/3)*e^6*Sec[c + d*x]^5*Sqrt[e*Sec[c + d*x]]*(21 + 19*Cos[2*(c + d*x)] +
 (30*I)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin
[c + d*x]) + (11*I)*Sin[2*(c + d*x)])*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x
)]))/(a^4*d*(-I + Tan[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 3981, 3042, 3981, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{3e^2\int \frac{(e\sec(c+dx))^{9/2}}{(i\tan(c+dx)a+a)^2}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{3e^2\int \frac{(e\sec(c+dx))^{9/2}}{(i\tan(c+dx)a+a)^2}dx}{a^2}$$

$$\Big\downarrow \ 3981$$

$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{3e^2\left(\frac{5e^2\int(e\sec(c+dx))^{5/2}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{3e^2\left(\frac{5e^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \ 4255$$

$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{3e^2\left(\frac{5e^2\left(\frac{1}{3}e^2\int\sqrt{e\sec(c+dx)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{3e^2\left(\frac{5e^2\left(\frac{1}{3}e^2\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \ 4258$$

$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{3e^2\left(\frac{5e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{3e^2\left(\frac{5e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{3e^2\left(\frac{5e^2\left(\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

input
```
Int[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((4*I)/3)*e^2*(e*Sec[c + d*x])^(9/2))/(a*d*(a + I*a*Tan[c + d*x])^3) - (3
*e^2*((5*e^2*((2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*S
ec[c + d*x]])/(3*d) + (2*e*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)))/a^
2 - ((4*I)*e^2*(e*Sec[c + d*x])^(5/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))))/a^
2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 4.75 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

method	result
default	$-\frac{2\sqrt{e\sec(dx+c)}\left(\tan(dx+c)\left(-8\cos(dx+c)^2-1\right)+i(15\cos(dx+c)+15)\operatorname{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\dots\right.}{3a^4d}$

input
```
int((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-2/3/a^4/d*(e*sec(d*x+c))^(1/2)*(tan(d*x+c)*(-8*cos(d*x+c)^2-1)+I*(15*cos(
d*x+c)+15)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*(-8*cos(d*x+c)^2-12))*e^6
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx =$$
$$-\frac{2\left(\sqrt{2}\left(-15i\,e^6e^{(4i\,dx+4i\,c)}-21i\,e^6e^{(2i\,dx+2i\,c)}-4i\,e^6\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+15\sqrt{2}\left(-i\,e^6e^{(4i\,dx+4i\,c)}-\dots\right.}{3\left(a^4de^{(4i\,dx+4i\,c)}+a^4de^{(2i\,dx+2i\,c)}\right)}$$

input
```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
-2/3*(sqrt(2)*(-15*I*e^6*e^(4*I*d*x + 4*I*c) - 21*I*e^6*e^(2*I*d*x + 2*I*c
) - 4*I*e^6)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 1
5*sqrt(2)*(-I*e^6*e^(4*I*d*x + 4*I*c) - I*e^6*e^(2*I*d*x + 2*I*c))*sqrt(e)
*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a^4*d*e^(4*I*d*x + 4*I*c) +
 a^4*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(13/2)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(e\sec(dx+c))^{\frac{13}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

input
```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(13/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{13/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}\,dx$$

input
```
int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^6}{\tan(dx+c)^4-4\tan(dx+c)^3 i-6\tan(dx+c)^2+4\tan(dx+c)i+1}dx\right)e^6}{a^4}$$

input
```
int((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**6)/(tan(c + d*x)**4 - 4*tan
(c + d*x)**3*i - 6*tan(c + d*x)**2 + 4*tan(c + d*x)*i + 1),x)*e**6)/a**4
```

3.257 $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 163

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = -\frac{42e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{42e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3} - \frac{28ie^4(e\sec(c+dx))^{3/2}}{5d\left(a^4+ia^4\tan(c+dx)\right)}$$

output
```
-42/5*e^6*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/d/cos(d*x+c)^(1/2)/(e*
sec(d*x+c))^(1/2)+42/5*e^5*(e*sec(d*x+c))^(1/2)*sin(d*x+c)/a^4/d+4/5*I*e^2
*(e*sec(d*x+c))^(7/2)/a/d/(a+I*a*tan(d*x+c))^3-28/5*I*e^4*(e*sec(d*x+c))^(
3/2)/d/(a^4+I*a^4*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.54 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.65

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2ie^5e^{-3i(c+dx)}\left(-2-7e^{2i(c+dx)}+21e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)\right)\sqrt{e}}{5a^4d}$$

input
```
Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((-2*I)/5)*e^5*(-2 - 7*E^((2*I)*(c + d*x)) + 21*E^((2*I)*(c + d*x))*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c +
d*x))])*Sqrt[e*Sec[c + d*x]])/(a^4*d*E^((3*I)*(c + d*x)))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3981, 3042, 3981, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{3981}$$
$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{7e^2\int \frac{(e\sec(c+dx))^{7/2}}{(i\tan(c+dx)a+a)^2}dx}{5a^2}$$
$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{7e^2\int\frac{(e\sec(c+dx))^{7/2}}{(i\tan(c+dx)a+a)^2}dx}{5a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{7e^2\left(-\frac{3e^2\int(e\sec(c+dx))^{3/2}dx}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{5a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{7e^2\left(-\frac{3e^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{5a^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{7e^2\left(-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{5a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{7e^2\left(-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{5a^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{7e^2\left(-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{5a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-}{}\ \frac{7e^2\left(-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{5a^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{7e^2\left(-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{5a^2}$$

input
```
Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((4*I)/5)*e^2*(e*Sec[c + d*x])^(7/2))/(a*d*(a + I*a*Tan[c + d*x])^3) - (7
*e^2*((-3*e^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sq
rt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/a^2 + ((
4*I)*e^2*(e*Sec[c + d*x])^(3/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))))/(5*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(143) = 286$.

Time = 4.49 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.72

method	result
default	$-\frac{2\sqrt{e\sec(dx+c)}\left(5+21i\sin(dx+c)\left(\cos(dx+c)^{2}+2\cos(dx+c)+1\right)\operatorname{EllipticE}(i(\cot(dx+c)-\csc(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+}}$

input
```
int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-2/5/a^4/d/(-cos(d*x+c)*sin(d*x+c)-sin(d*x+c)+I*cos(d*x+c)^2+I*cos(d*x+c))
*(e*sec(d*x+c))^(1/2)*(5+21*I*(cos(d*x+c)^2+2*cos(d*x+c)+1)*sin(d*x+c)*(1/
(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)+21*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-c
sc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)^3+2*cos(d*x+c)
^2+cos(d*x+c))+21*I*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*sin(d*x+c)*(1/(cos(d*x+
c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)+21*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)
),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d
*x+c))+I*sin(d*x+c)*cos(d*x+c)*(-4*cos(d*x+c)-25)+4*cos(d*x+c)^3-17*cos(d*
x+c)^2-16*cos(d*x+c))*e^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.70

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2\left(21i\sqrt{2}e^{\frac{11}{2}}e^{(3i\,dx+3i\,c)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{2}\left(21i\,e^5e^{(4i\,dx+4}\right.\right.}{5\,a^4d}$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
-2/5*(21*I*sqrt(2)*e^(11/2)*e^(3*I*d*x + 3*I*c)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(21*I*e^5*e^(4*I*d*x +
 4*I*c) + 14*I*e^5*e^(2*I*d*x + 2*I*c) - 2*I*e^5)*sqrt(e/(e^(2*I*d*x + 2*I
*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-3*I*d*x - 3*I*c)/(a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(e\sec(dx+c))^{\frac{11}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(11/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}{(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^4}\,dx$$

input
```
int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^5}{\tan(dx+c)^4-4\tan(dx+c)^3i-6\tan(dx+c)^2+4\tan(dx+c)i+1}dx\right)e^5}{a^4}$$

input
```
int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**5)/(tan(c + d*x)**4 - 4*tan
(c + d*x)**3*i - 6*tan(c + d*x)**2 + 4*tan(c + d*x)*i + 1),x)*e**5)/a**4
```

3.258 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{10e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21a^4d} + \frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3} - \frac{20ie^4\sqrt{e\sec(c+dx)}}{21d\left(a^4+ia^4\tan(c+dx)\right)}$$

output
```
10/21*e^4*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d
*x+c))^(1/2)/a^4/d+4/7*I*e^2*(e*sec(d*x+c))^(5/2)/a/d/(a+I*a*tan(d*x+c))^3
-20/21*I*e^4*(e*sec(d*x+c))^(1/2)/d/(a^4+I*a^4*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{2e^4\sec^4(c+dx)\sqrt{e\sec(c+dx)}\left(5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(2*e^4*Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(5*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) - (2*I)*(1 + Cos[
2*(c + d*x)] + (4*I)*Sin[2*(c + d*x)]))*(Cos[2*(c + d*x)] + I*Sin[2*(c + d
*x)]))/(21*a^4*d*(-I + Tan[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3981, 3042, 3981, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{5e^2\int \frac{(e\sec(c+dx))^{5/2}}{(i\tan(c+dx)a+a)^2}dx}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{5e^2\int \frac{(e\sec(c+dx))^{5/2}}{(i\tan(c+dx)a+a)^2}dx}{7a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{5e^2\left(-\frac{e^2\int \sqrt{e\sec(c+dx)}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}\right)}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{5e^2\left(-\frac{e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}\right)}{7a^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{5e^2\left(-\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}\right)}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{5e^2\left(-\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}\right)}{7a^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{5e^2\left(-\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^2d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}\right)}{7a^2}$$

input
```
Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((4*I)/7)*e^2*(e*Sec[c + d*x])^(5/2))/(a*d*(a + I*a*Tan[c + d*x])^3) - (5
*e^2*((-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c +
d*x]])/(3*a^2*d) + (((4*I)/3)*e^2*Sqrt[e*Sec[c + d*x]])/(d*(a^2 + I*a^2*Ta
n[c + d*x]))))/(7*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 3.80 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.45

method	result
default	$\frac{2e^4\sqrt{e\sec(dx+c)}\left(5i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\cos(dx+c)\,\mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}+24i\cos(dx+c)^4+5i\sqrt{\frac{\cos(dx}{\cos(dx+}}\right.}{21a}$

input
```
int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
2/21/a^4/d*e^4*(e*sec(d*x+c))^(1/2)*(5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*cos(d*x+c)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2
)+24*I*cos(d*x+c)^4+5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot
(d*x+c)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+24*sin(d*x+c)*cos(d*x+c)^3
-28*I*cos(d*x+c)^2-16*cos(d*x+c)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.84

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2\left(5i\sqrt{2}e^{\frac{9}{2}}e^{(4i\,dx+4i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(5i\,e^4e^{(4i\,dx+4i\,c)}+2i\,e^4e^{(2i\,dx+2i\,c)}-3i\,e^4\right.\right.}{21\,a^4d}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-2/21*(5*I*sqrt(2)*e^(9/2)*e^(4*I*d*x + 4*I*c)*weierstrassPInverse(-4, 0,
e^(I*d*x + I*c)) + sqrt(2)*(5*I*e^4*e^(4*I*d*x + 4*I*c) + 2*I*e^4*e^(2*I*d
*x + 2*I*c) - 3*I*e^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/
2*I*c))*e^(-4*I*d*x - 4*I*c)/(a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(e\sec(dx+c))^{\frac{9}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^4}\,dx$$

input
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^4}{\tan(dx+c)^4-4\tan(dx+c)^3 i-6\tan(dx+c)^2+4\tan(dx+c)i+1}dx\right)e^4}{a^4}$$

input
```
int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**4)/(tan(c + d*x)**4 - 4*tan
(c + d*x)**3*i - 6*tan(c + d*x)**2 + 4*tan(c + d*x)*i + 1),x)*e**4)/a**4
```

3.259 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3} - \frac{4ie^4}{15d\sqrt{e\sec(c+dx)}\,(a^4+ia^4\tan(c+dx))}$$

output
```
-2/15*e^4*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/d/cos(d*x+c)^(1/2)/(e*
sec(d*x+c))^(1/2)+4/9*I*e^2*(e*sec(d*x+c))^(3/2)/a/d/(a+I*a*tan(d*x+c))^3-
4/15*I*e^4/d/(e*sec(d*x+c))^(1/2)/(a^4+I*a^4*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.66 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.13

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{e^3e^{-idx}\sec^4(c+dx)\sqrt{e\sec(c+dx)}\left(-7-7\cos(2(c+dx))+6e^{2i(c+dx)}\sqrt{1+\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(e^3*Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(-7 - 7*Cos[2*(c + d*x)] + 6*E^((
2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
 3/4, -E^((2*I)*(c + d*x))] + (3*I)*Sin[2*(c + d*x)])*((-I)*Cos[c + 2*d*x]
 + Sin[c + 2*d*x]))/(45*a^4*d*E^(I*d*x)*(-I + Tan[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3981, 3042, 3981, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{e^2\int \frac{(e\sec(c+dx))^{3/2}}{(i\tan(c+dx)a+a)^2}dx}{3a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{e^2\int \frac{(e\sec(c+dx))^{3/2}}{(i\tan(c+dx)a+a)^2}dx}{3a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5a^2}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{3a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a^2}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{3a^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{e^2\int\sqrt{\cos(c+dx)}dx}{5a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{3a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{3a^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{3a^2}$$

input
```
Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((4*I)/9)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^3) - (e
^2*((2*e^2*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*S
ec[c + d*x]]) + (((4*I)/5)*e^2)/(d*Sqrt[e*Sec[c + d*x]]*(a^2 + I*a^2*Tan[c
 + d*x]))))/(3*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(116) = 232$.

Time = 3.73 (sec) , antiderivative size = 501, normalized size of antiderivative = 3.80

method	result
default	$-\frac{2\left(-6\sin(dx+c)\cos(dx+c)\left(-\cos(dx+c)^2-2\cos(dx+c)-1\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \text{EllipticE}(i(\csc(dx+c)-\cot(dx+c)),i}{}$

input
```
int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-2/45/a^4/d*(-6*sin(d*x+c)*cos(d*x+c)*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x
+c)-cot(d*x+c)),I)+3*I*(-2*cos(d*x+c)^4-4*cos(d*x+c)^3-cos(d*x+c)^2+2*cos(
d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellip
ticE(I*(csc(d*x+c)-cot(d*x+c)),I)-6*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)^2+2*
cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticF(I*(csc(d*x+c)-cot(d*x+c)),I)+3*I*(2*cos(d*x+c)^4+4*cos(d*x+c)^3+c
os(d*x+c)^2-2*cos(d*x+c)-1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)-sin(d*x+c)*cos(d*x+c)*
(-10*cos(d*x+c)^2-16*cos(d*x+c)-3)+2*I*cos(d*x+c)^2*(-3+5*cos(d*x+c)^2+2*c
os(d*x+c)))*(e*sec(d*x+c))^(1/2)/(2*I*sin(d*x+c)*cos(d*x+c)*(-cos(d*x+c)-1
)-2*cos(d*x+c)^3-2*cos(d*x+c)^2+cos(d*x+c)+1)*e^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(-6i\sqrt{2}e^{\frac{7}{2}}e^{(5i\,dx+5i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx}\right.\right.\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/45*(-6*I*sqrt(2)*e^(7/2)*e^(5*I*d*x + 5*I*c)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-6*I*e^3*e^(6*I*d*x +
6*I*c) - 4*I*e^3*e^(4*I*d*x + 4*I*c) + 7*I*e^3*e^(2*I*d*x + 2*I*c) + 5*I*e
^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-5*I*d*x
 - 5*I*c)/(a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(e\sec(dx+c))^{\frac{7}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{(a+a\tan(c+d\,x)\;1\mathrm{i})^4}\,dx$$

input
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^3}{\tan(dx+c)^4-4\tan(dx+c)^3 i-6\tan(dx+c)^2+4\tan(dx+c)i+1}dx\right)e^3}{a^4}$$

input
```
int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**3)/(tan(c + d*x)**4 - 4*tan
(c + d*x)**3*i - 6*tan(c + d*x)**2 + 4*tan(c + d*x)*i + 1),x)*e**3)/a**4
```

3.260 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 163

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77a^4d} - \frac{2e^3\sin(c+dx)}{77a^4d\sqrt{e\sec(c+dx)}} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3} - \frac{4ie^4}{77d(e\sec(c+dx))^{3/2}\,(a^4+ia^4\tan(c+dx))}$$

output
```
-2/77*e^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d
*x+c))^(1/2)/a^4/d-2/77*e^3*sin(d*x+c)/a^4/d/(e*sec(d*x+c))^(1/2)+4/11*I*e
^2*(e*sec(d*x+c))^(1/2)/a/d/(a+I*a*tan(d*x+c))^3-4/77*I*e^4/d/(e*sec(d*x+c
))^(3/2)/(a^4+I*a^4*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.88

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\sec^2(c+dx)(e\sec(c+dx))^{5/2}(\cos(c+dx)+i\sin(c+dx))\left(37i\cos(c+dx)\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^2*(e*Sec[c + d*x])^(5/2)*(Cos[c + d*x] + I*Sin[c + d*x])*((3
7*I)*Cos[c + d*x] + (11*I)*Cos[3*(c + d*x)] + 3*Sin[c + d*x] - 4*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]
) + 3*Sin[3*(c + d*x)]))/(154*a^4*d*(-I + Tan[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3981, 3042, 3981, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-\frac{e^2\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^2}dx}{11a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-\frac{e^2\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^2}dx}{11a^2}$$

$$\Big\downarrow \ 3981$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{3e^2\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{3e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a^2}$$

$$\Big\downarrow \ 4256$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{3e^2\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{3e^2\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a^2}$$

$$\Big\downarrow \ 4258$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a^2}$$

$$\Big\downarrow \; 3120$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{3e^2\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a^2}$$

input
```
Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((4*I)/11)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x])^3) - (e^
2*((3*e^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c +
d*x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*a^2)
 + (((4*I)/7)*e^2)/(d*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^2*Tan[c + d*x]))))
/(11*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 3.62 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.88

method	result
default	$\frac{2\left(\sin(dx+c)\cos(dx+c)\left(56\cos(dx+c)^4-16\cos(dx+c)^2-1\right)+i\cos(dx+c)^4\left(56\cos(dx+c)^2-44\right)+i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\ \ldots\right.}{77a^4d}$

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
2/77/a^4/d*(sin(d*x+c)*cos(d*x+c)*(56*cos(d*x+c)^4-16*cos(d*x+c)^2-1)+I*co
s(d*x+c)^4*(56*cos(d*x+c)^2-44)+I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)+1)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I))*
(e*sec(d*x+c))^(1/2)*e^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.77

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(4i\sqrt{2}e^{\frac{5}{2}}e^{(6i\,dx+6i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(4i\,e^{2}e^{(6i\,dx+6i}\right.\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/154*(4*I*sqrt(2)*e^(5/2)*e^(6*I*d*x + 6*I*c)*weierstrassPInverse(-4, 0,
e^(I*d*x + I*c)) + sqrt(2)*(4*I*e^2*e^(6*I*d*x + 6*I*c) + 17*I*e^2*e^(4*I*
d*x + 4*I*c) + 20*I*e^2*e^(2*I*d*x + 2*I*c) + 7*I*e^2)*sqrt(e/(e^(2*I*d*x
+ 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-6*I*d*x - 6*I*c)/(a^4*d)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{(e\sec(c+dx))^{\frac{5}{2}}}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral((e*sec(c + d*x))**(5/2)/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 -
6*tan(c + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^4}\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^2}{\tan(dx+c)^4-4\tan(dx+c)^3 i-6\tan(dx+c)^2+4\tan(dx+c)i+1}dx\right)e^2}{a^4}$$

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**2)/(tan(c + d*x)**4 - 4*tan
(c + d*x)**3*i - 6*tan(c + d*x)**2 + 4*tan(c + d*x)*i + 1),x)*e**2)/a**4
```

3.261 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 163

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{39a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2e^3\sin(c+dx)}{117a^4d(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{13ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3} + \frac{4ie^4}{117d(e\sec(c+dx))^{5/2}\,(a^4+ia^4\tan(c+dx))}$$

output
```
2/39*e^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/d/cos(d*x+c)^(1/2)/(e*s
ec(d*x+c))^(1/2)+2/117*e^3*sin(d*x+c)/a^4/d/(e*sec(d*x+c))^(3/2)+4/13*I*e^
2/a/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3+4/117*I*e^4/d/(e*sec(d*x+c
))^(5/2)/(a^4+I*a^4*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.90 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.87

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{ie^{-idx}\sec^2(c+dx)(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))\left(28+40\cos(2(c+\right.}{234a^4d(-i+\tan}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/234)*Sec[c + d*x]^2*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])*(28
 + 40*Cos[2*(c + d*x)] + (24*E^((4*I)*(c + d*x))*Hypergeometric2F1[-1/4, 1
/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + (22*I)*Sin
[2*(c + d*x)]))/(a^4*d*E^(I*d*x)*(-I + Tan[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3981, 3042, 3981, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2}dx}{13a^2}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2}dx}{13a^2}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3981}$$

$$\frac{e^2\left(\frac{5e^2\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a^2}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{e^2\left(\frac{5e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a^2}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{e^2\left(\frac{5e^2\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a^2}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{e^2\left(\frac{5e^2\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a^2}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{e^2\left(\frac{5e^2\left(\frac{3\int\sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a^2}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{5e^2\left(\frac{3\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a^2}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{e^2\left(\frac{5e^2\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a^2}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((4*I)/13)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3) + (e^
2*((5*e^2*((6*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[
e*Sec[c + d*x]]) + (2*Sin[c + d*x])/(5*d*e*(e*Sec[c + d*x])^(3/2))))/(9*a^
2) + (((4*I)/9)*e^2)/(d*(e*Sec[c + d*x])^(5/2)*(a^2 + I*a^2*Tan[c + d*x]))
))/(13*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(143) = 286$.

Time = 3.86 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.79

method	result
default	$\frac{2\left(\sin(dx+c)\cos(dx+c)\left(72\cos(dx+c)^6+72\cos(dx+c)^5-16\cos(dx+c)^4-16\cos(dx+c)^3+\cos(dx+c)^2+\cos(dx+c)+3\right)+i\cos(dx+c}{}$

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
2/117/a^4/d*(sin(d*x+c)*cos(d*x+c)*(72*cos(d*x+c)^6+72*cos(d*x+c)^5-16*cos
(d*x+c)^4-16*cos(d*x+c)^3+cos(d*x+c)^2+cos(d*x+c)+3)+I*cos(d*x+c)^5*(72*co
s(d*x+c)^3+72*cos(d*x+c)^2-52*cos(d*x+c)-52)+I*(3*cos(d*x+c)^2+6*cos(d*x+c
)+3)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)+I*(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*EllipticF(
I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2))*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*e
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.81

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(24i\sqrt{2}e^{\frac{3}{2}}e^{(7i\,dx+7i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx\cdots}\right.\right.\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/468*(24*I*sqrt(2)*e^(3/2)*e^(7*I*d*x + 7*I*c)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(24*I*e*e^(8*I*d*x + 8
*I*c) + 55*I*e*e^(6*I*d*x + 6*I*c) + 59*I*e*e^(4*I*d*x + 4*I*c) + 37*I*e*e
^(2*I*d*x + 2*I*c) + 9*I*e)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x
 + 1/2*I*c))*e^(-7*I*d*x - 7*I*c)/(a^4*d)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{(e\sec(c+dx))^{\frac{3}{2}}}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral((e*sec(c + d*x))**(3/2)/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 -
6*tan(c + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)}{\tan(dx+c)^4-4\tan(dx+c)^3 i-6\tan(dx+c)^2+4\tan(dx+c)i+1}dx\right)e}{a^4}$$

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x))/(tan(c + d*x)**4 - 4*tan(c
+ d*x)**3*i - 6*tan(c + d*x)**2 + 4*tan(c + d*x)*i + 1),x)*e)/a**4
```

3.262 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 191

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33a^4d} + \frac{2e\sin(c+dx)}{33a^4d\sqrt{e\sec(c+dx)}} + \frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4} + \frac{14i\sqrt{e\sec(c+dx)}}{165ad(a+ia\tan(c+dx))^3} + \frac{4ie^2}{33d(e\sec(c+dx))^{3/2}\,(a^4+ia^4\tan(c+dx))}$$

output
```
2/33*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(e*sec(d*x+c)
)^(1/2)/a^4/d+2/33*e*sin(d*x+c)/a^4/d/(e*sec(d*x+c))^(1/2)+2/15*I*(e*sec(d
*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^4+14/165*I*(e*sec(d*x+c))^(1/2)/a/d/(a+I
*a*tan(d*x+c))^3+4/33*I*e^2/d/(e*sec(d*x+c))^(3/2)/(a^4+I*a^4*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\sec^4(c+dx)\sqrt{e\sec(c+dx)}\Big(40\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(4(c+dx))+i\sin(4(c+dx))}{660a^4d(-i+\tan(c}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)]) + I*(64 + 112*Cos[2*(c
 + d*x)] + 48*Cos[4*(c + d*x)] + (54*I)*Sin[2*(c + d*x)] + (37*I)*Sin[4*(c
 + d*x)])))/(660*a^4*d*(-I + Tan[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3983, 3042, 3983, 3042, 3981, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{ 3983}$$
$$\frac{7\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^3}dx}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$
$$\Big\downarrow \text{ 3042}$$

$$\frac{7\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^3}dx}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\left(\frac{5\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^2}dx}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}\right)}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^2}dx}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}\right)}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3981}$$

$$\frac{7\left(\frac{5\left(\frac{3e^2\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}\right)}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5\left(\frac{3e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}\right)}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{4256}$$

$$\frac{7\left(\frac{5\left(\frac{3e^2\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}\right)}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{7\left(\frac{5\left(\frac{3e^2\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}\right)}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \textcolor{blue}{4258}$$

$$\frac{7\left(\frac{5\left(\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}\right)}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{7\left(\frac{5\left(\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}\right)}{15a}$$

$$\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \textit{3120}$$

$$\frac{7\left(\frac{5\left(\frac{3e^2\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}\right)}{15a}$$

$$\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((2*I)/15)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^4) + (7*((((2*
I)/11)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^3) + (5*((3*e^2*((2
*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e
^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*a^2) + (((4*I)/7)
*e^2)/(d*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^2*Tan[c + d*x]))))/(11*a)))/(15
*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 3.81 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.80

method	result
default	$\frac{2\left(\sin(dx+c)\cos(dx+c)\left(88\cos(dx+c)^{6}-16\cos(dx+c)^{4}+3\cos(dx+c)^{2}+5\right)+i\cos(dx+c)^{6}\left(88\cos(dx+c)^{2}-60\right)+i(5\cos(dx+c)+5)}{165a^{4}d}$

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
2/165/a^4/d*(sin(d*x+c)*cos(d*x+c)*(88*cos(d*x+c)^6-16*cos(d*x+c)^4+3*cos(
d*x+c)^2+5)+I*cos(d*x+c)^6*(88*cos(d*x+c)^2-60)+I*(5*cos(d*x+c)+5)*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c
)-csc(d*x+c)),I))*(e*sec(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(85i\,e^{(8i\,dx+8i\,c)}+166i\,e^{(6i\,dx+6i\,c)}+128i\,e^{(4i\,dx+4i\,c)}+58i\,e^{(2i\,dx+2i\,c)}+11i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-}{1320\,a^4d}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/1320*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(85*I*e^(8*I*d*x + 8*I*c
) + 166*I*e^(6*I*d*x + 6*I*c) + 128*I*e^(4*I*d*x + 4*I*c) + 58*I*e^(2*I*d*
x + 2*I*c) + 11*I)*e^(1/2*I*d*x + 1/2*I*c) - 80*I*sqrt(2)*sqrt(e)*e^(8*I*d
*x + 8*I*c)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-8*I*d*x - 8*I
*c)/(a^4*d)
```

Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sqrt{e\sec(c+dx)}}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral(sqrt(e*sec(c + d*x))/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*t
an(c + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\sqrt{e\sec(dx+c)}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx=\int \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^4}\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx=\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}}{\tan(dx+c)^4-4\tan(dx+c)^3 i-6\tan(dx+c)^2+4\tan(dx+c)i+1}dx\right)}{a^4}$$

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(sqrt(e)*int(sqrt(sec(c + d*x))/(tan(c + d*x)**4 - 4*tan(c + d*x)**3*i - 6
*tan(c + d*x)**2 + 4*tan(c + d*x)*i + 1),x))/a**4
```

3.263 $\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx=\frac{6i2^{5/6}a\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}}{5f(1+i\tan(e+fx))^{5/6}}$$

output
```
6/5*I*2^(5/6)*a*hypergeom([-5/6, 5/6],[11/6],1/2-1/2*I*tan(f*x+e))*(d*sec(
f*x+e))^(5/3)/f/(1+I*tan(f*x+e))^(5/6)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx=\frac{3ad(d\sec(e+fx))^{2/3}\left(i\sec(e+fx)+\csc(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\sec^{2}\right.\right.}{5f}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x]),x]
```

output
```
(3*a*d*(d*Sec[e + f*x])^(2/3)*(I*Sec[e + f*x] + Csc[e + f*x]*Hypergeometri
c2F1[1/2, 5/6, 11/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]))/(5*f)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))(d\sec(e+fx))^{5/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))(d\sec(e+fx))^{5/3}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(d\sec(e+fx))^{5/3}\int (a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{11/6}dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(d\sec(e+fx))^{5/3}\int (a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{11/6}dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(d\sec(e+fx))^{5/3}\int \frac{(i\tan(e+fx)a+a)^{5/6}}{\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{5/6}a^2(d\sec(e+fx))^{5/3}\int \frac{(i\tan(e+fx)+1)^{5/6}}{2^{5/6}\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{f(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2(d\sec(e+fx))^{5/3}\int \frac{(i\tan(e+fx)+1)^{5/6}}{\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{f(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{79}$$

$$\frac{6i2^{5/6}a(d\sec(e+fx))^{5/3}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(1+i\tan(e+fx))^{5/6}}$$

input
```
Int[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x]),x]
```

output
```
(((6*I)/5)*2^(5/6)*a*Hypergeometric2F1[-5/6, 5/6, 11/6, (1 - I*Tan[e + f*x
])/2]*(d*Sec[e + f*x])^(5/3))/(f*(1 + I*Tan[e + f*x])^(5/6))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (d\sec (fx+e))^{\frac{5}{3}} (a+ia\tan (fx+e))\, dx$$

input
```
int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\, dx = \int (d\sec (fx+e))^{\frac{5}{3}} (i\, a\tan (fx+e)+a)\, dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

output
```
-1/10*(3*2^(2/3)*(5*I*a*d*e^(3*I*f*x + 3*I*e) + I*a*d*e^(I*f*x + I*e))*(d/
(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e) - 10*(f*e^(2*I*f*
x + 2*I*e) + f)*integral(1/2*I*2^(2/3)*a*d*(d/(e^(2*I*f*x + 2*I*e) + 1))^(
2/3)*e^(2/3*I*f*x + 2/3*I*e)/f, x))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx = ia\left(\int \left(-i(d\sec(e+fx))^{\frac{5}{3}}\right)\,dx + \int (d\sec(e+fx))^{\frac{5}{3}}\tan(e+fx)\,dx\right)$$

input
```
integrate((d*sec(f*x+e))**(5/3)*(a+I*a*tan(f*x+e)),x)
```

output
```
I*a*(Integral(-I*(d*sec(e + f*x))**(5/3), x) + Integral((d*sec(e + f*x))**
(5/3)*tan(e + f*x), x))
```

Maxima [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{5}{3}}(i\,a\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a), x)
```

Giac [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{5}{3}}(i\,a\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}(a+a\tan(e+f\,x)\;1\mathrm{i})\;dx$$

input
```
int((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i),x)
```

output
```
int((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i), x)
```

Reduce [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx = \frac{d^{\frac{5}{3}}a\left(3\sec(fx+e)^{\frac{5}{3}}i+5\left(\int\sec(fx+e)^{\frac{5}{3}}dx\right)f\right)}{5f}$$

input
```
int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x)
```

output
```
(d**(2/3)*a*d*(3*sec(e + f*x)**(2/3)*sec(e + f*x)*i + 5*int(sec(e + f*x)**
(2/3)*sec(e + f*x),x)*f))/(5*f)
```

3.264 $\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx = \frac{6i\sqrt[6]{2}a\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}}{f\sqrt[6]{1+i\tan(e+fx)}}$$

output
```
6*I*2^(1/6)*a*hypergeom([-1/6, 1/6],[7/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x
+e))^(1/3)/f/(1+I*tan(f*x+e))^(1/6)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx = \frac{3a\sqrt[3]{d\sec(e+fx)}\left(i+\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{f}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x]),x]
```

output
```
(3*a*(d*Sec[e + f*x])^(1/3)*(I + Cot[e + f*x]*Hypergeometric2F1[1/6, 1/2,
7/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]))/f
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))\sqrt[3]{d\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))\sqrt[3]{d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int \sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{7/6}dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int \sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{7/6}dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sqrt[3]{d\sec(e+fx)}\int \frac{\sqrt[6]{i\tan(e+fx)a+a}}{(a-ia\tan(e+fx))^{5/6}}d\tan(e+fx)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{\sqrt[6]{2}a^2\sqrt[3]{d\sec(e+fx)}\int \frac{\sqrt[6]{i\tan(e+fx)+1}}{\sqrt[6]{2}(a-ia\tan(e+fx))^{5/6}}d\tan(e+fx)}{f\sqrt[6]{1+i\tan(e+fx)}\sqrt[6]{a-ia\tan(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2 \sqrt[3]{d \sec(e+fx)} \int \frac{\sqrt[6]{i \tan(e+fx)+1}}{(a-ia\tan(e+fx))^{5/6}} d\tan(e+fx)}{f \sqrt[6]{1+i\tan(e+fx)} \sqrt[6]{a-ia\tan(e+fx)}}$$

$$\Big\downarrow \text{79}$$

$$\frac{6i\sqrt[6]{2}a\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f\sqrt[6]{1+i\tan(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x]),x]
```

output
```
((6*I)*2^(1/6)*a*Hypergeometric2F1[-1/6, 1/6, 7/6, (1 - I*Tan[e + f*x])/2]
*(d*Sec[e + f*x])^(1/3))/(f*(1 + I*Tan[e + f*x])^(1/6))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (d\sec (fx+e))^{\frac{1}{3}} (a+ia\tan (fx+e))\, dx$$

input
```
int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x)
```

Fricas [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\, dx = \int (d\sec (fx+e))^{\frac{1}{3}} (i\, a\tan (fx+e)+a)\, dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

output
```
(3*I*2^(1/3)*a*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e)
 + f*integral(-I*2^(1/3)*a*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f
*x - 2/3*I*e)/f, x))/f
```

Sympy [F]

$$\int \sqrt[3]{d \sec(e+fx)}(a+ia\tan(e+fx))\,dx = ia\left(\int \left(-i\sqrt[3]{d\sec{(e+fx)}}\right)\,dx + \int \sqrt[3]{d\sec{(e+fx)}}\tan{(e+fx)}\,dx\right)$$

input
```
integrate((d*sec(f*x+e))**(1/3)*(a+I*a*tan(f*x+e)),x)
```

output
```
I*a*(Integral(-I*(d*sec(e + f*x))**(1/3), x) + Integral((d*sec(e + f*x))**
(1/3)*tan(e + f*x), x))
```

Maxima [F]

$$\int \sqrt[3]{d \sec(e+fx)}(a+ia\tan(e+fx))\,dx = \int (d\sec{(fx+e)})^{\frac{1}{3}}\,(i\,a\tan{(fx+e)}+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \sqrt[3]{d \sec(e+fx)}(a+ia\tan(e+fx))\,dx = \int (d\sec{(fx+e)})^{\frac{1}{3}}\,(i\,a\tan{(fx+e)}+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d \sec(e+fx)}(a+ia\tan(e+fx))\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{1/3} (a + a\tan(e+f\,x)\ 1\mathrm{i})\ dx$$

input
```
int((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i),x)
```

output
```
int((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i), x)
```

Reduce [F]

$$\int \sqrt[3]{d \sec(e+fx)}(a+ia\tan(e+fx))\,dx = \frac{d^{\frac{1}{3}} a \left(3\sec(fx+e)^{\frac{1}{3}}\, i + \left(\int \sec(fx+e)^{\frac{1}{3}}\, dx\right) f\right)}{f}$$

input
```
int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x)
```

output
```
(d**(1/3)*a*(3*sec(e + f*x)**(1/3)*i + int(sec(e + f*x)**(1/3),x)*f))/f
```

3.265 $\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3i2^{5/6}a\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{f\sqrt[3]{d\sec(e+fx)}}$$

output
```
-3*I*2^(5/6)*a*hypergeom([-1/6, 1/6],[5/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(
f*x+e))^(1/6)/f/(d*sec(f*x+e))^(1/3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3a\left(i+\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{f\sqrt[3]{d\sec(e+fx)}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
```

output
```
(-3*a*(I + Cot[e + f*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[e + f*x]^2]*
Sqrt[-Tan[e + f*x]^2]))/(f*(d*Sec[e + f*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{(i\tan(e+fx)a+a)^{5/6}}{\sqrt[6]{a-ia\tan(e+fx)}}dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{(i\tan(e+fx)a+a)^{5/6}}{\sqrt[6]{a-ia\tan(e+fx)}}dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{1}{(a-ia\tan(e+fx))^{7/6}\sqrt[6]{i\tan(e+fx)a+a}}d\tan(e+fx)}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2\sqrt[6]{1+i\tan(e+fx)}\sqrt[6]{a-ia\tan(e+fx)}\int \frac{\sqrt[6]{2}}{\sqrt[6]{i\tan(e+fx)+1}(a-ia\tan(e+fx))^{7/6}}d\tan(e+fx)}{\sqrt[6]{2}f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2 \sqrt[6]{1+i\tan(e+fx)} \sqrt[6]{a-ia\tan(e+fx)} \int \frac{1}{\sqrt[6]{i\tan(e+fx)+1}(a-ia\tan(e+fx))^{7/6}}d\tan(e+fx)}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{3i2^{5/6}a\sqrt[6]{1+i\tan(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f\sqrt[3]{d\sec(e+fx)}}$$

input
```
Int[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
```

output
```
((-3*I)*2^(5/6)*a*Hypergeometric2F1[-1/6, 1/6, 5/6, (1 - I*Tan[e + f*x])/2
]*(1 + I*Tan[e + f*x])^(1/6))/(f*(d*Sec[e + f*x])^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{a+ia\tan(fx+e)}{(d\sec(fx+e))^{\frac{1}{3}}}dx$$

input
```
int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

output
```
int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx=\int \frac{i\,a\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
-(3*2^(2/3)*(I*a*e^(2*I*f*x + 2*I*e) + I*a)*(d/(e^(2*I*f*x + 2*I*e) + 1))^
(2/3)*e^(2/3*I*f*x + 2/3*I*e) - (d*f*e^(I*f*x + I*e) - d*f)*integral(-2*2^
(2/3)*(I*a*e^(2*I*f*x + 2*I*e) + I*a*e^(I*f*x + I*e) + I*a)*(d/(e^(2*I*f*x
 + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e)/(d*f*e^(3*I*f*x + 3*I*e) - 2
*d*f*e^(2*I*f*x + 2*I*e) + d*f*e^(I*f*x + I*e)), x))/(d*f*e^(I*f*x + I*e)
- d*f)
```

Sympy [F]

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = ia\left(\int \left(-\frac{i}{\sqrt[3]{d\sec{(e+fx)}}}\right)dx + \int \frac{\tan{(e+fx)}}{\sqrt[3]{d\sec{(e+fx)}}}\,dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))**(1/3),x)
```

output
```
I*a*(Integral(-I/(d*sec(e + f*x))**(1/3), x) + Integral(tan(e + f*x)/(d*se
c(e + f*x))**(1/3), x))
```

Maxima [F]

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{i\,a\tan{(fx+e)}+a}{(d\sec{(fx+e)})^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{i\,a\tan{(fx+e)}+a}{(d\sec{(fx+e)})^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx=\int \frac{a+a\tan(e+f\,x)\ 1\mathrm{i}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int((a + a*tan(e + f*x)*1i)/(d/cos(e + f*x))^(1/3),x)
```

output
```
int((a + a*tan(e + f*x)*1i)/(d/cos(e + f*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx=\frac{a\left(\left(\int \frac{\tan(fx+e)}{\sec(fx+e)^{\frac{1}{3}}}dx\right)i+\int \frac{1}{\sec(fx+e)^{\frac{1}{3}}}dx\right)}{d^{\frac{1}{3}}}$$

input
```
int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

output
```
(a*(int(tan(e + f*x)/sec(e + f*x)**(1/3),x)*i + int(1/sec(e + f*x)**(1/3),
x)))/d**(1/3)
```

3.266 $\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{3i\sqrt[6]{2}a\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{5f(d\sec(e+fx))^{5/3}}$$

output
```
-3/5*I*2^(1/6)*a*hypergeom([-5/6, 5/6],[1/6],1/2-1/2*I*tan(f*x+e))*(1+I*ta
n(f*x+e))^(5/6)/f/(d*sec(f*x+e))^(5/3)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{3a\left(i+\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{5f(d\sec(e+fx))^{5/3}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

output
```
(-3*a*(I + Cot[e + f*x]*Hypergeometric2F1[-5/6, 1/2, 1/6, Sec[e + f*x]^2]*
Sqrt[-Tan[e + f*x]^2]))/(5*f*(d*Sec[e + f*x])^(5/3))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{\sqrt[6]{i\tan(e+fx)a+a}}{(a-ia\tan(e+fx))^{5/6}}dx}{(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{\sqrt[6]{i\tan(e+fx)a+a}}{(a-ia\tan(e+fx))^{5/6}}dx}{(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{1}{(a-ia\tan(e+fx))^{11/6}(i\tan(e+fx)a+a)^{5/6}}d\tan(e+fx)}{f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}\int \frac{2^{5/6}}{(i\tan(e+fx)+1)^{5/6}(a-ia\tan(e+fx))^{11/6}}d\tan(e+fx)}{2^{5/6}f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}\int\frac{1}{(i\tan(e+fx)+1)^{5/6}(a-ia\tan(e+fx))^{11/6}}d\tan(e+fx)}{f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow 79$$

$$-\frac{3i\sqrt[6]{2}a(1+i\tan(e+fx))^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(d\sec(e+fx))^{5/3}}$$

input
```
Int[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

output
```
(((-3*I)/5)*2^(1/6)*a*Hypergeometric2F1[-5/6, 5/6, 1/6, (1 - I*Tan[e + f*x
])/2]*(1 + I*Tan[e + f*x])^(5/6))/(f*(d*Sec[e + f*x])^(5/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{a + ia\tan(fx + e)}{(d\sec(fx + e))^{\frac{5}{3}}}dx$$

input
```
int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

output
```
int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \frac{a + ia\tan(e + fx)}{(d\sec(e + fx))^{5/3}}\,dx = \int \frac{i\,a\tan(fx + e) + a}{(d\sec(fx + e))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
```

output
```
1/10*(10*d^2*f*integral(-2/5*I*2^(1/3)*a*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/
3)*e^(-2/3*I*f*x - 2/3*I*e)/(d^2*f), x) - 3*2^(1/3)*(I*a*e^(2*I*f*x + 2*I*
e) + I*a)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e))/(d^
2*f)
```

Sympy [F]

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = ia\left(\int \left(-\frac{i}{(d\sec(e+fx))^{\frac{5}{3}}}\right)dx + \int \frac{\tan(e+fx)}{(d\sec(e+fx))^{\frac{5}{3}}}\,dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))**(5/3),x)
```

output
```
I*a*(Integral(-I/(d*sec(e + f*x))**(5/3), x) + Integral(tan(e + f*x)/(d*se
c(e + f*x))**(5/3), x))
```

Maxima [F]

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{i\,a\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

Giac [F]

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{i\,a\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
```

output
```
integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + ia\tan(e + fx)}{(d\sec(e + fx))^{5/3}}\, dx = \int \frac{a + a\tan(e + f\,x)\ 1\mathrm{i}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}\, dx$$

input
```
int((a + a*tan(e + f*x)*1i)/(d/cos(e + f*x))^(5/3),x)
```

output
```
int((a + a*tan(e + f*x)*1i)/(d/cos(e + f*x))^(5/3), x)
```

Reduce [F]

$$\int \frac{a + ia\tan(e + fx)}{(d\sec(e + fx))^{5/3}}\, dx = \frac{a\left(\left(\int \frac{\tan(fx+e)}{\sec(fx+e)^{\frac{5}{3}}}dx\right) i + \int \frac{1}{\sec(fx+e)^{\frac{5}{3}}}dx\right)}{d^{\frac{5}{3}}}$$

input
```
int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

output
```
(a*(int(tan(e + f*x)/(sec(e + f*x)**(2/3)*sec(e + f*x)),x)*i + int(1/(sec(
e + f*x)**(2/3)*sec(e + f*x)),x)))/(d**(2/3)*d)
```

3.267 $\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 71

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx=\frac{12i2^{5/6}a^2\,\text{Hypergeometric2F1}\left(-\frac{11}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}}{5f(1+i\tan(e+fx))^{5/6}}$$

output
```
12/5*I*2^(5/6)*a^2*hypergeom([-11/6, 5/6],[11/6],1/2-1/2*I*tan(f*x+e))*(d*
sec(f*x+e))^(5/3)/f/(1+I*tan(f*x+e))^(5/6)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.56

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx=\frac{3ia^2(d\sec(e+fx))^{5/3}\left(i\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{6},\frac{11}{6},\sec^2(e+fx)\right)\tan(e+fx)\right.}{5f\sqrt{-\tan}}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2,x]
```

output
```
(((3*I)/5)*a^2*(d*Sec[e + f*x])^(5/3)*(I*Hypergeometric2F1[-1/2, 5/6, 11/6
, Sec[e + f*x]^2]*Tan[e + f*x] + I*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[e
 + f*x]^2]*Tan[e + f*x] + 2*Sqrt[-Tan[e + f*x]^2]))/(f*Sqrt[-Tan[e + f*x]^
2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^2(d\sec(e+fx))^{5/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^2(d\sec(e+fx))^{5/3}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(d\sec(e+fx))^{5/3}\int (a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{17/6}dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(d\sec(e+fx))^{5/3}\int (a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{17/6}dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(d\sec(e+fx))^{5/3}\int \frac{(i\tan(e+fx)a+a)^{11/6}}{\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{80}$$

$$\frac{2\ 2^{5/6}a^3(d\sec(e+fx))^{5/3}\int \frac{(i\tan(e+fx)+1)^{11/6}}{2\ 2^{5/6}\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{f(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3(d\sec(e+fx))^{5/3}\int\frac{(i\tan(e+fx)+1)^{11/6}}{\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{f(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow 79$$

$$\frac{12i2^{5/6}a^2(d\sec(e+fx))^{5/3}\text{Hypergeometric2F1}\left(-\frac{11}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(1+i\tan(e+fx))^{5/6}}$$

input
```
Int[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2,x]
```

output
```
(((12*I)/5)*2^(5/6)*a^2*Hypergeometric2F1[-11/6, 5/6, 11/6, (1 - I*Tan[e +
 f*x])/2]*(d*Sec[e + f*x])^(5/3))/(f*(1 + I*Tan[e + f*x])^(5/6))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (d\sec (fx+e))^{\frac{5}{3}} (a+ia\tan (fx+e))^2 \, dx$$

input
```
int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2 \, dx = \int (d\sec (fx+e))^{\frac{5}{3}} (i\,a\tan (fx+e)+a)^2 \, dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/80*(3*2^(2/3)*(55*I*a^2*d*e^(5*I*f*x + 5*I*e) + 26*I*a^2*d*e^(3*I*f*x +
 3*I*e) + 11*I*a^2*d*e^(I*f*x + I*e))*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*
e^(2/3*I*f*x + 2/3*I*e) - 80*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I
*e) + f)*integral(11/16*I*2^(2/3)*a^2*d*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3
)*e^(2/3*I*f*x + 2/3*I*e)/f, x))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x +
 2*I*e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/3)*(a+I*a*tan(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{5}{3}}(ia\tan(fx+e)+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{5}{3}}(ia\tan(fx+e)+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}(a+a\tan(e+f\,x)\,1\mathrm{i})^2\,dx$$

input
```
int((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)^2,x)
```

output
```
int((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)^2, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx = \frac{d^{\frac{5}{3}}a^2\left(6\sec(fx+e)^{\frac{5}{3}}i-5\left(\int\sec(fx+e)^{\frac{5}{3}}\tan(fx+e)^2\,dx\right)f+5\left(\int\sec(fx+e)^{\frac{5}{3}}\right.}{5f}$$

input
```
int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x)
```

output
```
(d**(2/3)*a**2*d*(6*sec(e + f*x)**(2/3)*sec(e + f*x)*i - 5*int(sec(e + f*x
)**(2/3)*sec(e + f*x)*tan(e + f*x)**2,x)*f + 5*int(sec(e + f*x)**(2/3)*sec
(e + f*x),x)*f))/(5*f)
```

3.268 $\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 69

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx = \frac{12i\sqrt[6]{2}a^2\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}}{f\sqrt[6]{1+i\tan(e+fx)}}$$

output
```
12*I*2^(1/6)*a^2*hypergeom([-7/6, 1/6],[7/6],1/2-1/2*I*tan(f*x+e))*(d*sec(
f*x+e))^(1/3)/f/(1+I*tan(f*x+e))^(1/6)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.48

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx = \frac{3a^2\sqrt[3]{d\sec(e+fx)}\left(2i+\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6},\frac{7}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}+\cdots\right.}{f}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2,x]
```

output
```
(3*a^2*(d*Sec[e + f*x])^(1/3)*(2*I + Cot[e + f*x]*Hypergeometric2F1[-1/2,
1/6, 7/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2] + Cot[e + f*x]*Hypergeomet
ric2F1[1/6, 1/2, 7/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]))/f
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^2\sqrt[3]{d\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^2\sqrt[3]{d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int\sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{13/6}dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int\sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{13/6}dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sqrt[3]{d\sec(e+fx)}\int\frac{(i\tan(e+fx)a+a)^{7/6}}{(a-ia\tan(e+fx))^{5/6}}d\tan(e+fx)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{2\sqrt[6]{2}a^3\sqrt[3]{d\sec(e+fx)}\int\frac{(i\tan(e+fx)+1)^{7/6}}{2\sqrt[6]{2}(a-ia\tan(e+fx))^{5/6}}d\tan(e+fx)}{f\sqrt[6]{1+i\tan(e+fx)}\sqrt[6]{a-ia\tan(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3 \sqrt[3]{d \sec(e+fx)} \int \frac{(i \tan(e+fx)+1)^{7/6}}{(a-ia\tan(e+fx))^{5/6}} d\tan(e+fx)}{f \sqrt[6]{1+i\tan(e+fx)} \sqrt[6]{a-ia\tan(e+fx)}}$$

$$\Big\downarrow \; 79$$

$$\frac{12i\sqrt[6]{2}a^2 \sqrt[3]{d \sec(e+fx)} \, \text{Hypergeometric2F1}\left(-\frac{7}{6}, \frac{1}{6}, \frac{7}{6}, \frac{1}{2}(1-i\tan(e+fx))\right)}{f \sqrt[6]{1+i\tan(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2,x]
```

output
```
((12*I)*2^(1/6)*a^2*Hypergeometric2F1[-7/6, 1/6, 7/6, (1 - I*Tan[e + f*x])
/2]*(d*Sec[e + f*x])^(1/3))/(f*(1 + I*Tan[e + f*x])^(1/6))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (d\sec (fx+e))^{\frac{1}{3}} (a+ia\tan (fx+e))^2 \, dx$$

input
```
int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2 \, dx = \int (d\sec (fx+e))^{\frac{1}{3}} (i\,a\tan (fx+e)+a)^2 \, dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/4*(3*2^(1/3)*(-9*I*a^2*e^(2*I*f*x + 2*I*e) - 7*I*a^2)*(d/(e^(2*I*f*x +
2*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e) - 4*(f*e^(2*I*f*x + 2*I*e) + f)
*integral(-7/4*I*2^(1/3)*a^2*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I
*f*x - 2/3*I*e)/f, x))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx$$
$$= -a^2\left(\int\left(-\sqrt[3]{d\sec{(e+fx)}}\right)dx + \int\sqrt[3]{d\sec{(e+fx)}}\tan^2{(e+fx)}\,dx + \int\left(-2i\sqrt[3]{d\sec{(e+fx)}}\tan{(e+fx)}\right)dx\right)$$

input
```
integrate((d*sec(f*x+e))**(1/3)*(a+I*a*tan(f*x+e))**2,x)
```

output
```
-a**2*(Integral(-(d*sec(e + f*x))**(1/3), x) + Integral((d*sec(e + f*x))**
(1/3)*tan(e + f*x)**2, x) + Integral(-2*I*(d*sec(e + f*x))**(1/3)*tan(e +
f*x), x))
```

Maxima [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}\,(i\,a\tan(fx+e)+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}\,(i\,a\tan(fx+e)+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx$$
$$=\int \left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}(a+a\tan(e+f\,x)\,1\mathrm{i})^2\,dx$$

input
```
int((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)^2,x)
```

output
```
int((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)^2, x)
```

Reduce [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx$$
$$=\frac{d^{\frac{1}{3}}a^2\left(6\sec(fx+e)^{\frac{1}{3}}i+\left(\int \sec(fx+e)^{\frac{1}{3}}dx\right)f-\left(\int \sec(fx+e)^{\frac{1}{3}}\tan(fx+e)^2dx\right)f\right)}{f}$$

input
```
int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x)
```

output
```
(d**(1/3)*a**2*(6*sec(e + f*x)**(1/3)*i + int(sec(e + f*x)**(1/3),x)*f - i
nt(sec(e + f*x)**(1/3)*tan(e + f*x)**2,x)*f))/f
```

3.269 $\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 83

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{6i2^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\left(a^2+ia^2\tan(e+fx)\right)}{f\sqrt[3]{d\sec(e+fx)}(1+i\tan(e+fx))^{5/6}}$$

output
```
-6*I*2^(5/6)*hypergeom([-5/6, -1/6],[5/6],1/2-1/2*I*tan(f*x+e))*(a^2+I*a^2
*tan(f*x+e))/f/(d*sec(f*x+e))^(1/3)/(1+I*tan(f*x+e))^(5/6)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.24

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{3a^2\left(\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},\sec^2(e+fx)\right)\tan(e+fx)+\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(e\right.\right.}{f\sqrt[3]{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

output
```
(3*a^2*(Hypergeometric2F1[-1/2, -1/6, 5/6, Sec[e + f*x]^2]*Tan[e + f*x] +
Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[e + f*x]^2]*Tan[e + f*x] - (2*I)*Sqr
t[-Tan[e + f*x]^2]))/(f*(d*Sec[e + f*x])^(1/3)*Sqrt[-Tan[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{(i\tan(e+fx)a+a)^{11/6}}{\sqrt[6]{a-ia\tan(e+fx)}}dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{(i\tan(e+fx)a+a)^{11/6}}{\sqrt[6]{a-ia\tan(e+fx)}}dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{(i\tan(e+fx)a+a)^{5/6}}{(a-ia\tan(e+fx))^{7/6}}d\tan(e+fx)}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{5/6}a^2\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))\int\frac{(i\tan(e+fx)+1)^{5/6}}{2^{5/6}(a-ia\tan(e+fx))^{7/6}}d\tan(e+fx)}{f(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{a^2\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))\int\frac{(i\tan(e+fx)+1)^{5/6}}{(a-ia\tan(e+fx))^{7/6}}d\tan(e+fx)}{f(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \; 79$$

$$-\frac{6i2^{5/6}a(a+ia\tan(e+fx))\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}}$$

input
```
Int[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

output
```
((-6*I)*2^(5/6)*a*Hypergeometric2F1[-5/6, -1/6, 5/6, (1 - I*Tan[e + f*x])/
2]*(a + I*a*Tan[e + f*x]))/(f*(d*Sec[e + f*x])^(1/3)*(1 + I*Tan[e + f*x])^
(5/6))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(a+ia\tan(fx+e))^2}{(d\sec(fx+e))^{\frac{1}{3}}}dx$$

input
```
int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

output
```
int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(i\,a\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
-1/2*(3*2^(2/3)*(4*I*a^2*e^(2*I*f*x + 2*I*e) + I*a^2*e^(I*f*x + I*e) + 5*I
*a^2)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e) - 2*(d*f
*e^(I*f*x + I*e) - d*f)*integral(-5*2^(2/3)*(I*a^2*e^(2*I*f*x + 2*I*e) + I
*a^2*e^(I*f*x + I*e) + I*a^2)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I
*f*x + 2/3*I*e)/(d*f*e^(3*I*f*x + 3*I*e) - 2*d*f*e^(2*I*f*x + 2*I*e) + d*f
*e^(I*f*x + I*e)), x))/(d*f*e^(I*f*x + I*e) - d*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = -a^2\left(\int\left(-\frac{1}{\sqrt[3]{d\sec(e+fx)}}\right)dx + \int\frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx + \int\left(-\frac{2i\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))**2/(d*sec(f*x+e))**(1/3),x)
```

output
```
-a**2*(Integral(-1/(d*sec(e + f*x))**(1/3), x) + Integral(tan(e + f*x)**2/
(d*sec(e + f*x))**(1/3), x) + Integral(-2*I*tan(e + f*x)/(d*sec(e + f*x))*
*(1/3), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(i\,a\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx=\int \frac{(i\,a\tan\left(fx+e\right)+a)^2}{(d\sec\left(fx+e\right))^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx=\int \frac{(a+a\tan(e+f\,x)\ 1\mathrm{i})^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int((a + a*tan(e + f*x)*1i)^2/(d/cos(e + f*x))^(1/3),x)
```

output
```
int((a + a*tan(e + f*x)*1i)^2/(d/cos(e + f*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{a^2\left(-\left(\int \frac{\tan(fx+e)^2}{\sec(fx+e)^{\frac{1}{3}}}dx\right)+2\left(\int \frac{\tan(fx+e)}{\sec(fx+e)^{\frac{1}{3}}}dx\right)i+\int \frac{1}{\sec(fx+e)^{\frac{1}{3}}}dx\right)}{d^{\frac{1}{3}}}$$

input
```
int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

output
```
(a**2*( - int(tan(e + f*x)**2/sec(e + f*x)**(1/3),x) + 2*int(tan(e + f*x)/
sec(e + f*x)**(1/3),x)*i + int(1/sec(e + f*x)**(1/3),x)))/d**(1/3)
```

3.270 $\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{6i\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\left(a^2+ia^2\tan(e+fx)\right)}{5f(d\sec(e+fx))^{5/3}\sqrt[6]{1+i\tan(e+fx)}}$$

output
```
-6/5*I*2^(1/6)*hypergeom([-5/6, -1/6],[1/6],1/2-1/2*I*tan(f*x+e))*(a^2+I*a
^2*tan(f*x+e))/f/(d*sec(f*x+e))^(5/3)/(1+I*tan(f*x+e))^(1/6)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \frac{3a^2\left(\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{6},\sec^2(e+fx)\right)\tan(e+fx)+\text{Hyperge}\right.}{5f(d\sec(e+fx))^{5/3}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
```

output
```
(3*a^2*(Hypergeometric2F1[-5/6, -1/2, 1/6, Sec[e + f*x]^2]*Tan[e + f*x] +
Hypergeometric2F1[-5/6, 1/2, 1/6, Sec[e + f*x]^2]*Tan[e + f*x] - (2*I)*Sqr
t[-Tan[e + f*x]^2]))/(5*f*(d*Sec[e + f*x])^(5/3)*Sqrt[-Tan[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{(i\tan(e+fx)a+a)^{7/6}}{(a-ia\tan(e+fx))^{5/6}}dx}{(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{(i\tan(e+fx)a+a)^{7/6}}{(a-ia\tan(e+fx))^{5/6}}dx}{(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{\sqrt[6]{i\tan(e+fx)a+a}}{(a-ia\tan(e+fx))^{11/6}}d\tan(e+fx)}{f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{80}$$

$$\frac{\sqrt[6]{2}a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))\int \frac{\sqrt[6]{i\tan(e+fx)+1}}{\sqrt[6]{2}(a-ia\tan(e+fx))^{11/6}}d\tan(e+fx)}{f\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))\int \frac{\sqrt[6]{i\tan(e+fx)+1}}{(a-ia\tan(e+fx))^{11/6}}d\tan(e+fx)}{f\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \; 79$$

$$-\frac{6i\sqrt[6]{2}a(a+ia\tan(e+fx))\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}}$$

input
```
Int[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
```

output
```
(((-6*I)/5)*2^(1/6)*a*Hypergeometric2F1[-5/6, -1/6, 1/6, (1 - I*Tan[e + f*
x])/2]*(a + I*a*Tan[e + f*x]))/(f*(d*Sec[e + f*x])^(5/3)*(1 + I*Tan[e + f*
x])^(1/6))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(a+ia\tan(fx+e))^2}{(d\sec(fx+e))^{\frac{5}{3}}}dx$$

input
```
int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

output
```
int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{(i\,a\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
```

output
```
1/5*(5*d^2*f*integral(1/5*I*2^(1/3)*a^2*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3
)*e^(-2/3*I*f*x - 2/3*I*e)/(d^2*f), x) - 3*2^(1/3)*(I*a^2*e^(2*I*f*x + 2*I
*e) + I*a^2)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e))/
(d^2*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = -a^2\left(\int\left(-\frac{1}{(d\sec(e+fx))^{\frac{5}{3}}}\right)dx + \int\frac{\tan^2(e+fx)}{(d\sec(e+fx))^{\frac{5}{3}}}\,dx + \int\left(-\frac{2i\tan(e+fx)}{(d\sec(e+fx))^{\frac{5}{3}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))**2/(d*sec(f*x+e))**(5/3),x)
```

output
```
-a**2*(Integral(-1/(d*sec(e + f*x))**(5/3), x) + Integral(tan(e + f*x)**2/
(d*sec(e + f*x))**(5/3), x) + Integral(-2*I*tan(e + f*x)/(d*sec(e + f*x))*
*(5/3), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \int\frac{(i\,a\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \int\frac{(i\,a\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
```

output
```
integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{(a+a\tan(e+f\,x)\ 1\mathrm{i})^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}\,dx$$

input
```
int((a + a*tan(e + f*x)*1i)^2/(d/cos(e + f*x))^(5/3),x)
```

output
```
int((a + a*tan(e + f*x)*1i)^2/(d/cos(e + f*x))^(5/3), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \frac{a^2\left(-\left(\int \frac{\tan(fx+e)^2}{\sec(fx+e)^{\frac{5}{3}}}dx\right)+2\left(\int \frac{\tan(fx+e)}{\sec(fx+e)^{\frac{5}{3}}}dx\right)i+\int \frac{1}{\sec(fx+e)^{\frac{5}{3}}}dx\right)}{d^{\frac{5}{3}}}$$

input
```
int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

output
```
(a**2*( - int(tan(e + f*x)**2/(sec(e + f*x)**(2/3)*sec(e + f*x)),x) + 2*in
t(tan(e + f*x)/(sec(e + f*x)**(2/3)*sec(e + f*x)),x)*i + int(1/(sec(e + f*
x)**(2/3)*sec(e + f*x)),x)))/(d**(2/3)*d)
```

3.271 $\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 84

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \frac{3ia\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{7}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}(1+}{5\sqrt[6]{2}f(a+ia\tan(e+fx))^2}$$

output
```
3/10*I*a*hypergeom([5/6, 7/6],[11/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^
(5/3)*(1+I*tan(f*x+e))^(7/6)*2^(5/6)/f/(a+I*a*tan(f*x+e))^2
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \frac{6de^{i(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{2}{3},\frac{5}{6},-e^{2i(e+fx)}\right)(d\sec(e+fx))^{2/3}}{a\sqrt[3]{1+e^{2i(e+fx)}}f(-i+\tan(e+fx))}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x]),x]
```

output
```
(6*d*E^(I*(e + f*x))*Hypergeometric2F1[-1/6, 2/3, 5/6, -E^((2*I)*(e + f*x)
)]*(d*Sec[e + f*x])^(2/3))/(a*(1 + E^((2*I)*(e + f*x)))^(1/3)*f*(-I + Tan[
e + f*x]))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{ 3986}$$

$$\frac{(d\sec(e+fx))^{5/3}\int \frac{(a-ia\tan(e+fx))^{5/6}}{\sqrt[6]{i\tan(e+fx)a+a}}dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(d\sec(e+fx))^{5/3}\int \frac{(a-ia\tan(e+fx))^{5/6}}{\sqrt[6]{i\tan(e+fx)a+a}}dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{ 4006}$$

$$\frac{a^2(d\sec(e+fx))^{5/3}\int \frac{1}{\sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{7/6}}d\tan(e+fx)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{ 80}$$

$$\frac{a\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}\int \frac{2\sqrt[6]{2}}{(i\tan(e+fx)+1)^{7/6}\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{2\sqrt[6]{2}f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}\int \frac{1}{(i\tan(e+fx)+1)^{7/6}\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{3i\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{7}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5\sqrt[6]{2}f(a+ia\tan(e+fx))}$$

input
```
Int[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x]),x]
```

output
```
(((3*I)/5)*Hypergeometric2F1[5/6, 7/6, 11/6, (1 - I*Tan[e + f*x])/2]*(d*Se
c[e + f*x])^(5/3)*(1 + I*Tan[e + f*x])^(1/6))/(2^(1/6)*f*(a + I*a*Tan[e +
f*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(d\sec{(fx+e)})^{\frac{5}{3}}}{a+ia\tan{(fx+e)}}dx$$

input
```
int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \int \frac{(d\sec{(fx+e)})^{\frac{5}{3}}}{i\,a\tan{(fx+e)}+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

output
```
(a*f*e^(I*f*x + I*e)*integral(-I*2^(2/3)*d*(d/(e^(2*I*f*x + 2*I*e) + 1))^(
2/3)*e^(2/3*I*f*x + 2/3*I*e)/(a*f), x) - 3*2^(2/3)*(-I*d*e^(2*I*f*x + 2*I*
e) - I*d)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e))*e^(
-I*f*x - I*e)/(a*f)
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = -\frac{i\int \frac{(d\sec(e+fx))^{\frac{5}{3}}}{\tan(e+fx)-i}\,dx}{a}$$

input
```
integrate((d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
-I*Integral((d*sec(e + f*x))**(5/3)/(tan(e + f*x) - I), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{i\,a\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/3)/(I*a*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}{a+a\tan\left(e+f\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((d/cos(e + f*x))^(5/3)/(a + a*tan(e + f*x)*1i),x)
```

output
```
int((d/cos(e + f*x))^(5/3)/(a + a*tan(e + f*x)*1i), x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \frac{d^{\frac{5}{3}}\left(\int \frac{\sec(fx+e)^{\frac{5}{3}}}{\tan(fx+e)i+1}dx\right)}{a}$$

input
```
int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
(d**(2/3)*int((sec(e + f*x)**(2/3)*sec(e + f*x))/(tan(e + f*x)*i + 1),x)*d
)/a
```

3.272 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 82

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = \frac{3ia\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{11}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}(1+i\tan(e+fx))^{11/6}}{2^{5/6}f(a+ia\tan(e+fx))^2}$$

output
```
3/2*I*a*hypergeom([1/6, 11/6],[7/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(
1/3)*(1+I*tan(f*x+e))^(11/6)*2^(1/6)/f/(a+I*a*tan(f*x+e))^2
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = -\frac{3ie^{-2i(e+fx)}\left(-1-e^{2i(e+fx)}+4e^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{7}{6},-e^{2i(e+fx)}\right)\right)\sqrt[3]{d\,\mathrm{se}}}{10af}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x]),x]
```

output
```
(((-3*I)/10)*(-1 - E^((2*I)*(e + f*x)) + 4*E^((2*I)*(e + f*x))*(1 + E^((2*
I)*(e + f*x)))^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))
])*(d*Sec[e + f*x])^(1/3))/(a*E^((2*I)*(e + f*x))*f)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int \frac{\sqrt[6]{a-ia\tan(e+fx)}}{(i\tan(e+fx)a+a)^{5/6}}dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int \frac{\sqrt[6]{a-ia\tan(e+fx)}}{(i\tan(e+fx)a+a)^{5/6}}dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sqrt[3]{d\sec(e+fx)}\int \frac{1}{(a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{11/6}}d\tan(e+fx)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\int \frac{2\ 2^{5/6}}{(i\tan(e+fx)+1)^{11/6}(a-ia\tan(e+fx))^{5/6}}d\tan(e+fx)}{2\ 2^{5/6}f\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\int\frac{1}{(i\tan(e+fx)+1)^{11/6}(a-ia\tan(e+fx))^{5/6}}d\tan(e+fx)}{f\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))}$$
$$\Big\downarrow \text{79}$$
$$\frac{3i(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{11}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{2^{5/6}f(a+ia\tan(e+fx))}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x]),x]
```

output
```
((3*I)*Hypergeometric2F1[1/6, 11/6, 7/6, (1 - I*Tan[e + f*x])/2]*(d*Sec[e
+ f*x])^(1/3)*(1 + I*Tan[e + f*x])^(5/6))/(2^(5/6)*f*(a + I*a*Tan[e + f*x]
))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{a+ia\tan(fx+e)}dx$$

input
```
int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{i\,a\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

output
```
1/10*(10*a*f*e^(2*I*f*x + 2*I*e)*integral(-2/5*I*2^(1/3)*(d/(e^(2*I*f*x +
2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a*f), x) - 3*2^(1/3)*(d/(e^(2
*I*f*x + 2*I*e) + 1))^(1/3)*(-I*e^(2*I*f*x + 2*I*e) - I)*e^(1/3*I*f*x + 1/
3*I*e))*e^(-2*I*f*x - 2*I*e)/(a*f)
```

Sympy [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = -\frac{i\int \frac{\sqrt[3]{d\sec(e+fx)}}{\tan(e+fx)-i}\,dx}{a}$$

input
```
integrate((d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
-I*Integral((d*sec(e + f*x))**(1/3)/(tan(e + f*x) - I), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{i\,a\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)/(I*a*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}{a+a\tan\left(e+f\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((d/cos(e + f*x))^(1/3)/(a + a*tan(e + f*x)*1i),x)
```

output
```
int((d/cos(e + f*x))^(1/3)/(a + a*tan(e + f*x)*1i), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = \frac{d^{\frac{1}{3}}\left(\int \frac{\sec(fx+e)^{\frac{1}{3}}}{\tan(fx+e)i+1}dx\right)}{a}$$

input
```
int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
(d**(1/3)*int(sec(e + f*x)**(1/3)/(tan(e + f*x)*i + 1),x))/a
```

3.273 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = -\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{13}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{2\sqrt[6]{2}af\sqrt[3]{d\sec(e+fx)}}$$

output
```
-3/4*I*hypergeom([-1/6, 13/6],[5/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(f*x+e))
^(1/6)*2^(5/6)/a/f/(d*sec(f*x+e))^(1/3)
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = \frac{3\left(-8e^{2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-e^{2i(e+fx)}\right)+5(5+5\cos(2(e+fx))+4i\,s\right.}{70af\sqrt[3]{d\sec(e+fx)}}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])),x]
```

output
```
(3*(-8*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(2/3)*Hypergeometric2
F1[2/3, 5/6, 11/6, -E^((2*I)*(e + f*x))] + 5*(5 + 5*Cos[2*(e + f*x)] + (4*
I)*Sin[2*(e + f*x)]))*(I + Tan[e + f*x]))/(70*a*f*(d*Sec[e + f*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(e+fx))\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(e+fx))\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{1}{\sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{7/6}}dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{1}{\sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{7/6}}dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{1}{(a-ia\tan(e+fx))^{7/6}(i\tan(e+fx)a+a)^{13/6}}d\tan(e+fx)}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{\sqrt[6]{1+i\tan(e+fx)}\sqrt[6]{a-ia\tan(e+fx)}\int \frac{4\sqrt[6]{2}}{(i\tan(e+fx)+1)^{13/6}(a-ia\tan(e+fx))^{7/6}}d\tan(e+fx)}{4\sqrt[6]{2}f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[6]{1+i\tan(e+fx)}\sqrt[6]{a-ia\tan(e+fx)}\int \frac{1}{(i\tan(e+fx)+1)^{13/6}(a-ia\tan(e+fx))^{7/6}}d\tan(e+fx)}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{3i\sqrt[6]{1+i\tan(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{13}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{2\sqrt[6]{2}af\sqrt[3]{d\sec(e+fx)}}$$

input
```
Int[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])),x]
```

output
```
(((-3*I)/2)*Hypergeometric2F1[-1/6, 13/6, 5/6, (1 - I*Tan[e + f*x])/2]*(1
+ I*Tan[e + f*x])^(1/6))/(2^(1/6)*a*f*(d*Sec[e + f*x])^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}(a+ia\tan(fx+e))}dx$$

input
```
int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}(i\,a\tan(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

output
```
-1/28*(3*2^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(7*I*e^(5*I*f*x + 5*I
*e) + 9*I*e^(4*I*f*x + 4*I*e) + 6*I*e^(3*I*f*x + 3*I*e) + 10*I*e^(2*I*f*x
+ 2*I*e) - I*e^(I*f*x + I*e) + I)*e^(2/3*I*f*x + 2/3*I*e) - 28*(a*d*f*e^(4
*I*f*x + 4*I*e) - a*d*f*e^(3*I*f*x + 3*I*e))*integral(-8/7*2^(2/3)*(d/(e^(
2*I*f*x + 2*I*e) + 1))^(2/3)*(I*e^(2*I*f*x + 2*I*e) + I*e^(I*f*x + I*e) +
I)*e^(2/3*I*f*x + 2/3*I*e)/(a*d*f*e^(3*I*f*x + 3*I*e) - 2*a*d*f*e^(2*I*f*x
 + 2*I*e) + a*d*f*e^(I*f*x + I*e)), x))/(a*d*f*e^(4*I*f*x + 4*I*e) - a*d*f
*e^(3*I*f*x + 3*I*e))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = -\frac{i\int \frac{1}{\sqrt[3]{d\sec{(e+fx)}}\tan{(e+fx)}-i\sqrt[3]{d\sec{(e+fx)}}}\,dx}{a}$$

input
```
integrate(1/(d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
-I*Integral(1/((d*sec(e + f*x))**(1/3)*tan(e + f*x) - I*(d*sec(e + f*x))**
(1/3)), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}(i\,a\tan(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}(a+a\tan(e+f\,x)\;1\mathrm{i})}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)),x)
```

output
```
int(1/((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = \frac{\int \frac{1}{\sec(fx+e)^{\frac{1}{3}}\tan(fx+e)i+\sec(fx+e)^{\frac{1}{3}}}dx}{d^{\frac{1}{3}}a}$$

input
```
int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
int(1/(sec(e + f*x)**(1/3)*tan(e + f*x)*i + sec(e + f*x)**(1/3)),x)/(d**(1
/3)*a)
```

3.274 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx = -\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{17}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{10\,2^{5/6}af(d\sec(e+fx))^{5/3}}$$

output
```
-3/20*I*hypergeom([-5/6, 17/6],[1/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(f*x+e)
)^(5/6)*2^(1/6)/a/f/(d*sec(f*x+e))^(5/3)
```

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.68

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx = -\frac{3\sec^2(e+fx)\left(-26+6\cos(2(e+fx))+\frac{128e^{2i(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{7}{6},-e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^{2/3}}+16i\sin(2(e+fx))\right)}{220af(d\sec(e+fx))^{5/3}(-i+\tan(e+fx))}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])),x]
```

output
```
(-3*Sec[e + f*x]^2*(-26 + 6*Cos[2*(e + f*x)] + (128*E^((2*I)*(e + f*x))*Hy
pergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))])/(1 + E^((2*I)*(e + f
*x)))^(2/3) + (16*I)*Sin[2*(e + f*x)]))/(220*a*f*(d*Sec[e + f*x])^(5/3)*(-
I + Tan[e + f*x]))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(e+fx))(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(e+fx))(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{1}{(a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{11/6}}dx}{(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{1}{(a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{11/6}}dx}{(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{1}{(a-ia\tan(e+fx))^{11/6}(i\tan(e+fx)a+a)^{17/6}}d\tan(e+fx)}{f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{80}$$

$$\frac{(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}\int \frac{4\ 2^{5/6}}{(i\tan(e+fx)+1)^{17/6}(a-ia\tan(e+fx))^{11/6}}d\tan(e+fx)}{4\ 2^{5/6}f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}\int \frac{1}{(i\tan(e+fx)+1)^{17/6}(a-ia\tan(e+fx))^{11/6}}d\tan(e+fx)}{f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{3i(1+i\tan(e+fx))^{5/6}\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{17}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{10\ 2^{5/6}af(d\sec(e+fx))^{5/3}}$$

input
```
Int[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])),x]
```

output
```
(((-3*I)/10)*Hypergeometric2F1[-5/6, 17/6, 1/6, (1 - I*Tan[e + f*x])/2]*(1
 + I*Tan[e + f*x])^(5/6))/(2^(5/6)*a*f*(d*Sec[e + f*x])^(5/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{1}{\left(d \sec \left(f x+e\right)\right)^{\frac{5}{3}} \left(a+i a \tan \left(f x+e\right)\right)}d x$$

input
```
int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

Fricas [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx=\int \frac{1}{\left(d \sec \left(f x+e\right)\right)^{\frac{5}{3}} \left(i\, a \tan \left(f x+e\right)+a\right)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

output
```
1/440*(440*a*d^2*f*e^(4*I*f*x + 4*I*e)*integral(-16/55*I*2^(1/3)*(d/(e^(2*
I*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a*d^2*f), x) - 3*2^(1
/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*(11*I*e^(6*I*f*x + 6*I*e) - 15*I*e
^(4*I*f*x + 4*I*e) - 31*I*e^(2*I*f*x + 2*I*e) - 5*I)*e^(1/3*I*f*x + 1/3*I*
e))*e^(-4*I*f*x - 4*I*e)/(a*d^2*f)
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx = -\frac{i\int \frac{1}{(d\sec(e+fx))^{\frac{5}{3}}\tan(e+fx)-i(d\sec(e+fx))^{\frac{5}{3}}}\,dx}{a}$$

input
```
integrate(1/(d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
-I*Integral(1/((d*sec(e + f*x))**(5/3)*tan(e + f*x) - I*(d*sec(e + f*x))**
(5/3)), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{3}}(ia\tan(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx=\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}\,\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)),x)
```

output
```
int(1/((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)), x)
```

Reduce [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx=\frac{\int \frac{1}{\sec(fx+e)^{\frac{5}{3}}\tan(fx+e)i+\sec(fx+e)^{\frac{5}{3}}}dx}{d^{\frac{5}{3}}a}$$

input
```
int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
int(1/(sec(e + f*x)**(2/3)*sec(e + f*x)*tan(e + f*x)*i + sec(e + f*x)**(2/
3)*sec(e + f*x)),x)/(d**(2/3)*a*d)
```

3.275 $\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 84

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx=\frac{3ia\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{13}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}(1}{10\sqrt[6]{2}f(a+ia\tan(e+fx))^3}$$

output
```
3/20*I*a*hypergeom([5/6, 13/6],[11/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))
^(5/3)*(1+I*tan(f*x+e))^(13/6)*2^(5/6)/f/(a+I*a*tan(f*x+e))^3
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.52

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx =$$
$$-\frac{3e^{-i(4e+5fx)}\left(1+e^{2i(e+fx)}\right)\left(1+e^{2i(e+fx)}+2e^{2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{2}{3},\frac{5}{6},-e^{2}\right.\right.}{28a^2f}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x])^2,x]
```

output
```
(-3*(1 + E^((2*I)*(e + f*x)))*(1 + E^((2*I)*(e + f*x)) + 2*E^((2*I)*(e + f
*x))*(1 + E^((2*I)*(e + f*x)))^(2/3)*Hypergeometric2F1[-1/6, 2/3, 5/6, -E^
((2*I)*(e + f*x))])*(d*Sec[e + f*x])^(5/3)*((-I)*Cos[f*x] + Sin[f*x]))/(28
*a^2*E^(I*(4*e + 5*f*x))*f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}dx$$
$$\Big\downarrow \text{3986}$$
$$\frac{(d\sec(e+fx))^{5/3}\int \frac{(a-ia\tan(e+fx))^{5/6}}{(i\tan(e+fx)a+a)^{7/6}}dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{(d\sec(e+fx))^{5/3}\int \frac{(a-ia\tan(e+fx))^{5/6}}{(i\tan(e+fx)a+a)^{7/6}}dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$
$$\Big\downarrow \text{4006}$$
$$\frac{a^2(d\sec(e+fx))^{5/3}\int \frac{1}{\sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{13/6}}d\tan(e+fx)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$
$$\Big\downarrow \text{80}$$
$$\frac{\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}\int \frac{4\sqrt[6]{2}}{(i\tan(e+fx)+1)^{13/6}\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{4\sqrt[6]{2}f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))}$$

$$\Big\downarrow \, 27$$

$$\frac{\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}\int \frac{1}{(i\tan(e+fx)+1)^{13/6}\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))}$$

$$\Big\downarrow \, 79$$

$$\frac{3i\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{13}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{10\sqrt[6]{2}af(a+ia\tan(e+fx))}$$

input
```
Int[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x])^2,x]
```

output
```
(((3*I)/10)*Hypergeometric2F1[5/6, 13/6, 11/6, (1 - I*Tan[e + f*x])/2]*(d*
Sec[e + f*x])^(5/3)*(1 + I*Tan[e + f*x])^(1/6))/(2^(1/6)*a*f*(a + I*a*Tan[
e + f*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(d\sec (fx+e))^{\frac{5}{3}}}{(a+ia\tan (fx+e))^{2}}dx$$

input
```
int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx=\int \frac{(d\sec (fx+e))^{\frac{5}{3}}}{(i\,a\tan (fx+e)+a)^{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/14*(14*a^2*f*e^(3*I*f*x + 3*I*e)*integral(-1/7*I*2^(2/3)*d*(d/(e^(2*I*f*
x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e)/(a^2*f), x) - 3*2^(2/3)*(-2
*I*d*e^(4*I*f*x + 4*I*e) - 3*I*d*e^(2*I*f*x + 2*I*e) - I*d)*(d/(e^(2*I*f*x
 + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e))*e^(-3*I*f*x - 3*I*e)/(a^2*f
)
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{\int \frac{(d\sec(e+fx))^{\frac{5}{3}}}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx}{a^2}$$

input
```
integrate((d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e))**2,x)
```

output
```
-Integral((d*sec(e + f*x))**(5/3)/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1)
, x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{(i\,a\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/3)/(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}{(a+a\tan\left(e+f\,x\right)\,1\mathrm{i})^2}\,dx$$

input
```
int((d/cos(e + f*x))^(5/3)/(a + a*tan(e + f*x)*1i)^2,x)
```

output
```
int((d/cos(e + f*x))^(5/3)/(a + a*tan(e + f*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{d^{\frac{5}{3}}\left(\int \frac{\sec(fx+e)^{\frac{5}{3}}}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)}{a^2}$$

input
```
int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

output
```
( - d**(2/3)*int((sec(e + f*x)**(2/3)*sec(e + f*x))/(tan(e + f*x)**2 - 2*t
an(e + f*x)*i - 1),x)*d)/a**2
```

3.276 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 84

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \frac{3ia\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{17}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}(1+i\tan(e+fx))^{17/6}}{2\;2^{5/6}f(a+ia\tan(e+fx))^3}$$

output
```
3/4*I*a*hypergeom([1/6, 17/6],[7/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(
1/3)*(1+I*tan(f*x+e))^(17/6)*2^(1/6)/f/(a+I*a*tan(f*x+e))^3
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \frac{3\sec^2(e+fx)\sqrt[3]{d\sec(e+fx)}\left(-2i-2i\cos(2(e+fx))+4ie^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\,\text{Hypergeometric2F1}\right.}{22a^2f(-i+\tan(e+fx))^2}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x])^2,x]
```

output
```
(3*Sec[e + f*x]^2*(d*Sec[e + f*x])^(1/3)*(-2*I - (2*I)*Cos[2*(e + f*x)] +
(4*I)*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*Hypergeometric2F
1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))] + Sin[2*(e + f*x)]))/(22*a^2*f*(-I
+ Tan[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}dx$$
$$\Big\downarrow \text{3986}$$
$$\frac{\sqrt[3]{d\sec(e+fx)}\int \frac{\sqrt[6]{a-ia\tan(e+fx)}}{(i\tan(e+fx)a+a)^{11/6}}dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sqrt[3]{d\sec(e+fx)}\int \frac{\sqrt[6]{a-ia\tan(e+fx)}}{(i\tan(e+fx)a+a)^{11/6}}dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$
$$\Big\downarrow \text{4006}$$
$$\frac{a^2\sqrt[3]{d\sec(e+fx)}\int \frac{1}{(a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{17/6}}d\tan(e+fx)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$
$$\Big\downarrow \text{80}$$
$$\frac{(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\int \frac{4\ 2^{5/6}}{(i\tan(e+fx)+1)^{17/6}(a-ia\tan(e+fx))^{5/6}}d\tan(e+fx)}{4\ 2^{5/6}f\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))}$$

$$\Big\downarrow 27$$

$$\frac{(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\int\frac{1}{(i\tan(e+fx)+1)^{17/6}(a-ia\tan(e+fx))^{5/6}}d\tan(e+fx)}{f\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))}$$

$$\Big\downarrow 79$$

$$\frac{3i(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{17}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{2\,2^{5/6}af(a+ia\tan(e+fx))}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x])^2,x]
```

output
```
(((3*I)/2)*Hypergeometric2F1[1/6, 17/6, 7/6, (1 - I*Tan[e + f*x])/2]*(d*Se
c[e + f*x])^(1/3)*(1 + I*Tan[e + f*x])^(5/6))/(2^(5/6)*a*f*(a + I*a*Tan[e
+ f*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{(a+ia\tan(fx+e))^{2}}dx$$

input
```
int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx=\int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{(i\,a\tan(fx+e)+a)^{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/44*(44*a^2*f*e^(4*I*f*x + 4*I*e)*integral(-2/11*I*2^(1/3)*(d/(e^(2*I*f*x
 + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a^2*f), x) - 3*2^(1/3)*(d/
(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*(-3*I*e^(4*I*f*x + 4*I*e) - 4*I*e^(2*I*f*
x + 2*I*e) - I)*e^(1/3*I*f*x + 1/3*I*e))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{\int \frac{\sqrt[3]{d\sec{(e+fx)}}}{\tan^2{(e+fx)}-2i\tan{(e+fx)}-1}\,dx}{a^2}$$

input
```
integrate((d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e))**2,x)
```

output
```
-Integral((d*sec(e + f*x))**(1/3)/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1)
, x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \int \frac{(d\sec{(fx+e)})^{\frac{1}{3}}}{(i\,a\tan{(fx+e)}+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)/(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}{(a+a\tan\left(e+f\,x\right)\,1\mathrm{i})^2}\,dx$$

input
```
int((d/cos(e + f*x))^(1/3)/(a + a*tan(e + f*x)*1i)^2,x)
```

output
```
int((d/cos(e + f*x))^(1/3)/(a + a*tan(e + f*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{d^{\frac{1}{3}}\left(\int \frac{\sec(fx+e)^{\frac{1}{3}}}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)}{a^2}$$

input
```
int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

output
```
( - d**(1/3)*int(sec(e + f*x)**(1/3)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i -
 1),x))/a**2
```

3.277 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = -\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{19}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{4\sqrt[6]{2}a^2f\sqrt[3]{d\sec(e+fx)}}$$

output
```
-3/8*I*hypergeom([-1/6, 19/6],[5/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(f*x+e))
^(1/6)*2^(5/6)/a^2/f/(d*sec(f*x+e))^(1/3)
```

Mathematica [A] (verified)

Time = 2.00 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.99

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = \frac{(d\sec(e+fx))^{2/3}\left(16e^{3i(e+fx)}\left(1+e^{2i(e+fx)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-e^{2i(e+fx)}\right)-10(7\cos(e+\right.}{260a^2d}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2),x]
```

output
```
((d*Sec[e + f*x])^(2/3)*(16*E^((3*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^
(2/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -E^((2*I)*(e + f*x))] - 10*(7*Cos[
e + f*x] + 5*Cos[3*(e + f*x)] + (18*I)*Cos[e + f*x]^2*Sin[e + f*x])*((-3*
I)*Cos[2*(e + f*x)] - 3*Sin[2*(e + f*x)]))/(260*a^2*d*f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{3042, 3986, 3042, 4006, 80, 27, 79\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(e+fx))^2\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(e+fx))^2\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{1}{\sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{13/6}}dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{1}{\sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{13/6}}dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{1}{(a-ia\tan(e+fx))^{7/6}(i\tan(e+fx)a+a)^{19/6}}d\tan(e+fx)}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{\sqrt[6]{1+i\tan(e+fx)}\sqrt[6]{a-ia\tan(e+fx)}\int \frac{8\sqrt[6]{2}}{(i\tan(e+fx)+1)^{19/6}(a-ia\tan(e+fx))^{7/6}}d\tan(e+fx)}{8\sqrt[6]{2}af\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\sqrt[6]{1+i\tan(e+fx)}\sqrt[6]{a-ia\tan(e+fx)}\int \frac{1}{(i\tan(e+fx)+1)^{19/6}(a-ia\tan(e+fx))^{7/6}}d\tan(e+fx)}{af\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \textbf{79}$$

$$-\frac{3i\sqrt[6]{1+i\tan(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{19}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{4\sqrt[6]{2}a^2f\sqrt[3]{d\sec(e+fx)}}$$

input
```
Int[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2),x]
```

output
```
(((-3*I)/4)*Hypergeometric2F1[-1/6, 19/6, 5/6, (1 - I*Tan[e + f*x])/2]*(1
+ I*Tan[e + f*x])^(1/6))/(2^(1/6)*a^2*f*(d*Sec[e + f*x])^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(a+ia\tan\left(fx+e\right)\right)^{2}}dx$$

input
```
int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

output
```
int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = \int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(i\,a\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas
")
```

output
```
-1/104*(3*2^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(13*I*e^(7*I*f*x + 7
*I*e) + 19*I*e^(6*I*f*x + 6*I*e) + 9*I*e^(5*I*f*x + 5*I*e) + 23*I*e^(4*I*f
*x + 4*I*e) - 5*I*e^(3*I*f*x + 3*I*e) + 5*I*e^(2*I*f*x + 2*I*e) - I*e^(I*f
*x + I*e) + I)*e^(2/3*I*f*x + 2/3*I*e) - 104*(a^2*d*f*e^(6*I*f*x + 6*I*e)
- a^2*d*f*e^(5*I*f*x + 5*I*e))*integral(-8/13*2^(2/3)*(d/(e^(2*I*f*x + 2*I
*e) + 1))^(2/3)*(I*e^(2*I*f*x + 2*I*e) + I*e^(I*f*x + I*e) + I)*e^(2/3*I*f
*x + 2/3*I*e)/(a^2*d*f*e^(3*I*f*x + 3*I*e) - 2*a^2*d*f*e^(2*I*f*x + 2*I*e)
 + a^2*d*f*e^(I*f*x + I*e)), x))/(a^2*d*f*e^(6*I*f*x + 6*I*e) - a^2*d*f*e^
(5*I*f*x + 5*I*e))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = -\frac{\int \frac{1}{\sqrt[3]{d\sec\left(e+fx\right)}\tan^2\left(e+fx\right)-2i\sqrt[3]{d\sec\left(e+fx\right)}\tan\left(e+fx\right)-\sqrt[3]{d\sec\left(e+fx\right)}}\,dx}{a^2}$$

input
```
integrate(1/(d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e))**2,x)
```

output
```
-Integral(1/((d*sec(e + f*x))**(1/3)*tan(e + f*x)**2 - 2*I*(d*sec(e + f*x)
)**(1/3)*tan(e + f*x) - (d*sec(e + f*x))**(1/3)), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = \int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(i\,a\tan\left(fx+e\right)+a\right)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}\,\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)^2),x)
```

output
```
int(1/((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = -\frac{\int \frac{1}{\sec(fx+e)^{\frac{1}{3}}\tan(fx+e)^2-2\sec(fx+e)^{\frac{1}{3}}\tan(fx+e)i-\sec(fx+e)^{\frac{1}{3}}}dx}{d^{\frac{1}{3}}a^2}$$

input
```
int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

output
```
( - int(1/(sec(e + f*x)**(1/3)*tan(e + f*x)**2 - 2*sec(e + f*x)**(1/3)*tan
(e + f*x)*i - sec(e + f*x)**(1/3)),x))/(d**(1/3)*a**2)
```

3.278 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = -\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{23}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{20\,2^{5/6}a^2f(d\sec(e+fx))^{5/3}}$$

output

```
-3/40*I*hypergeom([-5/6, 23/6],[1/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(f*x+e)
)^(5/6)*2^(1/6)/a^2/f/(d*sec(f*x+e))^(5/3)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 143 vs. 2(71) = 142.

Time = 1.74 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.01

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = \frac{3i\sec^4(e+fx)\left(-46-40\cos(2(e+fx))+6\cos(4(e+f\right.}{}$$

input

```
Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2),x]
```

output
```
(((3*I)/680)*Sec[e + f*x]^4*(-46 - 40*Cos[2*(e + f*x)] + 6*Cos[4*(e + f*x)
] + 128*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*Hypergeometric
2F1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))] - (10*I)*Sin[2*(e + f*x)] + (11*I
)*Sin[4*(e + f*x)]))/(a^2*f*(d*Sec[e + f*x])^(5/3)*(-I + Tan[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(e+fx))^2(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(e+fx))^2(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{1}{(a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{17/6}}dx}{(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{1}{(a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{17/6}}dx}{(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{1}{(a-ia\tan(e+fx))^{11/6}(i\tan(e+fx)a+a)^{23/6}}d\tan(e+fx)}{f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{80}$$

$$\frac{(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}\int \frac{8\ 2^{5/6}}{(i\tan(e+fx)+1)^{23/6}(a-ia\tan(e+fx))^{11/6}}d\tan(e+fx)}{8\ 2^{5/6}af(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}\int\frac{1}{(i\tan(e+fx)+1)^{23/6}(a-ia\tan(e+fx))^{11/6}}d\tan(e+fx)}{af(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{3i(1+i\tan(e+fx))^{5/6}\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{23}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{20\ 2^{5/6}a^2f(d\sec(e+fx))^{5/3}}$$

input
```
Int[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2),x]
```

output
```
(((-3*I)/20)*Hypergeometric2F1[-5/6, 23/6, 1/6, (1 - I*Tan[e + f*x])/2]*(1
 + I*Tan[e + f*x])^(5/6))/(2^(5/6)*a^2*f*(d*Sec[e + f*x])^(5/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{1}{\left(d \sec \left(f x+e\right)\right)^{\frac{5}{3}} \left(a+i a \tan \left(f x+e\right)\right)^{2}}d x$$

input
```
int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

output
```
int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx=\int \frac{1}{\left(d \sec \left(f x+e\right)\right)^{\frac{5}{3}} \left(i\, a \tan \left(f x+e\right)+a\right)^{2}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas
")
```

output
```
1/1360*(1360*a^2*d^2*f*e^(6*I*f*x + 6*I*e)*integral(-16/85*I*2^(1/3)*(d/(e
^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a^2*d^2*f), x) -
3*2^(1/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*(17*I*e^(8*I*f*x + 8*I*e) -
50*I*e^(6*I*f*x + 6*I*e) - 92*I*e^(4*I*f*x + 4*I*e) - 30*I*e^(2*I*f*x + 2*
I*e) - 5*I)*e^(1/3*I*f*x + 1/3*I*e))*e^(-6*I*f*x - 6*I*e)/(a^2*d^2*f)
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = -\frac{\int \frac{1}{(d\sec(e+fx))^{\frac{5}{3}}\tan^2(e+fx)-2i(d\sec(e+fx))^{\frac{5}{3}}\tan(e+fx)-(d\sec(e+fx))^{\frac{5}{3}}}\,dx}{a^2}$$

input
```
integrate(1/(d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e))**2,x)
```

output
```
-Integral(1/((d*sec(e + f*x))**(5/3)*tan(e + f*x)**2 - 2*I*(d*sec(e + f*x)
)**(5/3)*tan(e + f*x) - (d*sec(e + f*x))**(5/3)), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{3}}(i\,a\tan(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)^2),x)
```

output
```
int(1/((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)^2), x)
```

Reduce [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = -\frac{\int \frac{1}{\sec(fx+e)^{\frac{5}{3}}\tan(fx+e)^2-2\sec(fx+e)^{\frac{5}{3}}\tan(fx+e)i-\sec(fx+e)^{\frac{5}{3}}}dx}{d^{\frac{5}{3}}a^2}$$

input
```
int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

output
```
( - int(1/(sec(e + f*x)**(2/3)*sec(e + f*x)*tan(e + f*x)**2 - 2*sec(e + f*
x)**(2/3)*sec(e + f*x)*tan(e + f*x)*i - sec(e + f*x)**(2/3)*sec(e + f*x)),
x))/(d**(2/3)*a**2*d)
```

3.279 $\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^4d} + \frac{24i(a+ia\tan(c+dx))^{11/2}}{11a^5d} - \frac{12i(a+ia\tan(c+dx))^{13/2}}{13a^6d} + \frac{2i(a+ia\tan(c+dx))^{15/2}}{15a^7d}$$

output
```
-16/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^4/d+24/11*I*(a+I*a*tan(d*x+c))^(11/2)/a
^5/d-12/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^6/d+2/15*I*(a+I*a*tan(d*x+c))^(15
/2)/a^7/d
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2(-i+\tan(c+dx))^4\sqrt{a+ia\tan(c+dx)}(-1241i-2367\tan(c+dx)+1683i\tan^2(c+dx)+429\tan^3(c+dx))}{6435d}$$

input
```
Integrate[Sec[c + d*x]^8*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*(-I + Tan[c + d*x])^4*Sqrt[a + I*a*Tan[c + d*x]]*(-1241*I - 2367*Tan[c
+ d*x] + (1683*I)*Tan[c + d*x]^2 + 429*Tan[c + d*x]^3))/(6435*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^8\sqrt{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{7/2}d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(-(i\tan(c+dx)a+a)^{13/2}+6a(i\tan(c+dx)a+a)^{11/2}-12a^2(i\tan(c+dx)a+a)^{9/2}+8a^3(i\tan(c+dx)a+a)^{7/2}\right)d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{16}{9}a^3(a+ia\tan(c+dx))^{9/2}-\frac{24}{11}a^2(a+ia\tan(c+dx))^{11/2}-\frac{2}{15}(a+ia\tan(c+dx))^{15/2}+\frac{12}{13}a(a+ia\tan(c+\right.}{a^7d}$$

input
```
Int[Sec[c + d*x]^8*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*((16*a^3*(a + I*a*Tan[c + d*x])^(9/2))/9 - (24*a^2*(a + I*a*Tan[c +
d*x])^(11/2))/11 + (12*a*(a + I*a*Tan[c + d*x])^(13/2))/13 - (2*(a + I*a*T
an[c + d*x])^(15/2))/15))/(a^7*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{15}{2}}}{15}-\frac{6a(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}+\frac{12a^2(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}\right)}{d\,a^7}$	82
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{15}{2}}}{15}-\frac{6a(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}+\frac{12a^2(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}\right)}{d\,a^7}$	82

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^7*(1/15*(a+I*a*tan(d*x+c))^(15/2)-6/13*a*(a+I*a*tan(d*x+c))^(13/2)
+12/11*a^2*(a+I*a*tan(d*x+c))^(11/2)-8/9*a^3*(a+I*a*tan(d*x+c))^(9/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.32

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{256\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(16i\,e^{(15i\,dx+15i\,c)}+120i\,e^{(13i\,dx+13i\,c)}+390i\,e^{(11i\,dx+11i\,c)}+715i\,e^{(9\right.}}{6435\left(de^{(14i\,dx+14i\,c)}+7\,de^{(12i\,dx+12i\,c)}+21\,de^{(10i\,dx+10i\,c)}+35\,de^{(8i\,dx+8i\,c)}+35\,de^{(6i\,dx+6i\,c)}+21\,de^{(4i\,dx+\right.}}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-256/6435*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(15*I*d*x + 15
*I*c) + 120*I*e^(13*I*d*x + 13*I*c) + 390*I*e^(11*I*d*x + 11*I*c) + 715*I*
e^(9*I*d*x + 9*I*c))/(d*e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c)
+ 21*d*e^(10*I*d*x + 10*I*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x
+ 6*I*c) + 21*d*e^(4*I*d*x + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^8\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2i\left(429\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{15}{2}}-2970\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{13}{2}}a+7020\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{11}{2}}a^2-572\right.}{6435\,a^7d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2/6435*I*(429*(I*a*tan(d*x + c) + a)^(15/2) - 2970*(I*a*tan(d*x + c) + a)^
(13/2)*a + 7020*(I*a*tan(d*x + c) + a)^(11/2)*a^2 - 5720*(I*a*tan(d*x + c)
 + a)^(9/2)*a^3)/(a^7*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 9.40 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.05

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^8,x)
```

output
```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
40960i)/(1287*d*(exp(c*2i + d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2048i)/(6435*d*(exp(c*2i + d
*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2
i) + 1))^(1/2)*512i)/(2145*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c
*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(1287*d*(
exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(ex
p(c*2i + d*x*2i) + 1))^(1/2)*4096i)/(6435*d) - ((a - (a*(exp(c*2i + d*x*2i
)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*52736i)/(715*d*(exp(c*2i +
d*x*2i) + 1)^5) + ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*
x*2i) + 1))^(1/2)*11776i)/(195*d*(exp(c*2i + d*x*2i) + 1)^6) - ((a - (a*(e
xp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(15*d
*(exp(c*2i + d*x*2i) + 1)^7)
```

Reduce [F]

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$= \frac{\sqrt{a}\,i\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^8+17\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^8\tan(dx+c)\,dx\right)d\right)}{d}$$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*i*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**8 + 17*int(sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**8*tan(c + d*x),x)*d))/d
```

3.280 $\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^3d} + \frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^4d} - \frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^5d}$$

output
```
-8/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^3/d+8/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^4/d
-2/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^5/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}(-151+182i\tan(c+dx)+63\tan^2(c+dx))}{693d}$$

input
```
Integrate[Sec[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]]*(-151 + (182*I)*Tan[c
+ d*x] + 63*Tan[c + d*x]^2))/(693*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(c+dx)^6\sqrt{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int (a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{5/2}d(ia\tan(c+dx))}{a^5d}$$
$$\Big\downarrow \text{53}$$
$$-\frac{i\int \left((i\tan(c+dx)a+a)^{9/2}-4a(i\tan(c+dx)a+a)^{7/2}+4a^2(i\tan(c+dx)a+a)^{5/2}\right)d(ia\tan(c+dx))}{a^5d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{i\left(\frac{8}{7}a^2(a+ia\tan(c+dx))^{7/2}+\frac{2}{11}(a+ia\tan(c+dx))^{11/2}-\frac{8}{9}a(a+ia\tan(c+dx))^{9/2}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*((8*a^2*(a + I*a*Tan[c + d*x])^(7/2))/7 - (8*a*(a + I*a*Tan[c + d*x]
)^(9/2))/9 + (2*(a + I*a*Tan[c + d*x])^(11/2))/11))/(a^5*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}+\frac{4a(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}\right)}{d\,a^5}$	63
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}+\frac{4a(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}\right)}{d\,a^5}$	63

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^5*(-1/11*(a+I*a*tan(d*x+c))^(11/2)+4/9*a*(a+I*a*tan(d*x+c))^(9/2)-
4/7*a^2*(a+I*a*tan(d*x+c))^(7/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.35

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{64\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(8i\,e^{(11i\,dx+11i\,c)}+44i\,e^{(9i\,dx+9i\,c)}+99i\,e^{(7i\,dx+7i\,c)}\right)}{693\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-64/693*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(8*I*e^(11*I*d*x + 11*I*
c) + 44*I*e^(9*I*d*x + 9*I*c) + 99*I*e^(7*I*d*x + 7*I*c))/(d*e^(10*I*d*x +
 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*
I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^6\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i\left(63\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{11}{2}}-308\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{9}{2}}a+396\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}a^2\right)}{693\,a^5d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2/693*I*(63*(I*a*tan(d*x + c) + a)^(11/2) - 308*(I*a*tan(d*x + c) + a)^(9
/2)*a + 396*(I*a*tan(d*x + c) + a)^(7/2)*a^2)/(a^5*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 352, normalized size of antiderivative = 4.00

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{693\,d} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{693\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{231\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} + \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,7232\mathrm{i}}{693\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1472\mathrm{i}}{99\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4} + \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{11\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^6,x)
```

output
```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
7232i)/(693*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1
i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(693*d*(exp(c*2i + d*x*2
i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*64i)/(231*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i +
d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(693*d) - ((a -
 (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1472i
)/(99*d*(exp(c*2i + d*x*2i) + 1)^4) + ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i
)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(11*d*(exp(c*2i + d*x*2i) + 1)^
5)
```

Reduce [F]

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$=\frac{\sqrt{a}\,i\left(-2\sqrt{\tan\left(dx+c\right)i+1}\,\sec\left(dx+c\right)^6+13\left(\int\sqrt{\tan\left(dx+c\right)i+1}\,\sec\left(dx+c\right)^6\tan\left(dx+c\right)dx\right)d\right)}{d}$$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*i*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**6 + 13*int(sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**6*tan(c + d*x),x)*d))/d
```

3.281 $\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{4i(a+ia\tan(c+dx))^{5/2}}{5a^2d} + \frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^3d}$$

output
```
-4/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^2/d+2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^3/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2(-i+\tan(c+dx))^2(9i+5\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{35d}$$

input
```
Integrate[Sec[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*(-I + Tan[c + d*x])^2*(9*I + 5*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]
)/(35*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4\sqrt{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int\left(2a(i\tan(c+dx)a+a)^{3/2}-(i\tan(c+dx)a+a)^{5/2}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{4}{5}a(a+ia\tan(c+dx))^{5/2}-\frac{2}{7}(a+ia\tan(c+dx))^{7/2}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*((4*a*(a + I*a*Tan[c + d*x])^(5/2))/5 - (2*(a + I*a*Tan[c + d*x])^(7
/2))/7))/(a^3*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{2a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^{3}}$	44
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{2a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^{3}}$	44

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^3*(1/7*(a+I*a*tan(d*x+c))^(7/2)-2/5*a*(a+I*a*tan(d*x+c))^(5/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.42

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{16\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(2i\,e^{(7i\,dx+7i\,c)}+7i\,e^{(5i\,dx+5i\,c)}\right)}{35\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-16/35*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(2*I*e^(7*I*d*x + 7*I*c)
+ 7*I*e^(5*I*d*x + 5*I*c))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c
) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^4\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2i\left(5\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}-14\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}a\right)}{35\,a^3d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2/35*I*(5*(I*a*tan(d*x + c) + a)^(7/2) - 14*(I*a*tan(d*x + c) + a)^(5/2)*a
)/(a^3*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.48 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.90

$$\begin{aligned}\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= -\frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,32i}{35\,d} \\ &\quad -\frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,16i}{35\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)} \\ &\quad +\frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,128i}{35\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^2} \\ &\quad -\frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,16i}{7\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^3}\end{aligned}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^4,x)
```

output
```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
128i)/(35*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i + d*x*2i)*1i
- 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(35*d*(exp(c*2i + d*x*2i) +
 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))
^(1/2)*32i)/(35*d) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
 d*x*2i) + 1))^(1/2)*16i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3)
```

Reduce [F]

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$=\frac{\sqrt{a}\,i\left(-2\sqrt{\tan\left(dx+c\right)i+1}\,\sec\left(dx+c\right)^4+9\left(\int\sqrt{\tan\left(dx+c\right)i+1}\,\sec\left(dx+c\right)^4\tan\left(dx+c\right)dx\right)d\right)}{d}$$

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*i*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**4 + 9*int(sqrt(tan
(c + d*x)*i + 1)*sec(c + d*x)**4*tan(c + d*x),x)*d))/d
```

3.282 $\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad}$$

output
```
-2/3*I*(a+I*a*tan(d*x+c))^(3/2)/a/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad}$$

input
```
Integrate[Sec[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \sec(c+dx)^2\sqrt{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \; 3968$$
$$-\frac{i\int \sqrt{i\tan(c+dx)a+a}d(ia\tan(c+dx))}{ad}$$
$$\Big\downarrow \; 17$$
$$-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad}$$

input
```
Int[Sec[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2i(a+ia\tan(dx+c))^{\frac{3}{2}}}{3ad}$	24
default	$-\frac{2i(a+ia\tan(dx+c))^{\frac{3}{2}}}{3ad}$	24

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*(a+I*a*tan(d*x+c))^(3/2)/a/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(21) = 42.

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.59

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{4i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(3i\,dx+3i\,c)}}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-4/3*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(3*I*d*x + 3*I*c)/(d*e^
(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^2\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}{3\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2/3*I*(I*a*tan(d*x + c) + a)^(3/2)/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.83

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$= -\frac{(\cos(2c+2dx)+1+\sin(2c+2dx)\ 1\mathrm{i})\sqrt{\frac{a(\cos(2c+2dx)+1+\sin(2c+2dx)\ 1\mathrm{i})}{\cos(2c+2dx)+1}}\ 2\mathrm{i}}{3d\ (\cos(2c+2dx)+1)}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^2,x)
```

output
```
-((cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1)*((a*(cos(2*c + 2*d*x) + sin
(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2)*2i)/(3*d*(cos(2*c + 2
*d*x) + 1))
```

Reduce [F]

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$= \frac{\sqrt{a}\,i\left(-2\sqrt{\tan(dx+c)\,i+1}\ \sec(dx+c)^2+5\left(\int\sqrt{\tan(dx+c)\,i+1}\ \sec(dx+c)^2\tan(dx+c)\,dx\right)d\right)}{d}$$

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*i*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**2 + 5*int(sqrt(tan
(c + d*x)*i + 1)*sec(c + d*x)**2*tan(c + d*x),x)*d))/d
```

3.283 $\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 124

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{3i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d}+\frac{3ia}{4d\sqrt{a+ia\tan(c+dx)}}-\frac{ia^3}{2d\sqrt{a+ia\tan(c+dx)}\,(a^2-ia^2\tan(c+dx))}$$

output
```
-3/8*I*a^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/
2)/d+3/4*I*a/d/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*a^3/d/(a+I*a*tan(d*x+c))^(1/
2)/(a^2-I*a^2*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.41

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{ia\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{2d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I/2)*a*Hypergeometric2F1[-1/2, 2, 1/2, (1 + I*Tan[c + d*x])/2])/(d*Sqrt[
a + I*a*Tan[c + d*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3968, 52, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^3\left(\frac{3\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^3\left(\frac{3\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{ia^3\left(\frac{3\left(\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

$$\Big\downarrow \ 219$$

$$-\frac{ia^3\left(\frac{3\left(\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

input
```
Int[Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*a^3*(1/(2*a*(a - I*a*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]) + (3*
((I*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[
a + I*a*Tan[c + d*x]])))/(4*a)))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(98) = 196$.

Time = 4.44 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.65

method	result
default	$\frac{\left(3i\sin(dx+c)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)+(-3\cos(dx+c)-3)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \text{a}}{}$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*(3*I*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(1/2)*(
cot(d*x+c)-csc(d*x+c))/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-
1)^(1/2))+(-3*cos(d*x+c)-3)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(
1/2)*(cot(d*x+c)-csc(d*x+c))/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x
+c)^2-1)^(1/2))+I*(3*cos(d*x+c)+3)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arct
an(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))+3*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))*s
in(d*x+c)+2*I*cos(d*x+c)^2+6*cos(d*x+c)*sin(d*x+c))*(a*(1+I*tan(d*x+c)))^(
1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(91) = 182$.

Time = 0.09 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.04

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx =$$
$$-\frac{\left(3\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(i\,dx+i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/8*(3*sqrt(1/2)*d*sqrt(-a/d^2)*e^(I*d*x + I*c)*log(-4*(sqrt(2)*sqrt(1/2)
*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a
/d^2) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3*sqrt(1/2)*d*sqrt(-a/d^2)*
e^(I*d*x + I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - a*e^(I*d*x + I*c))*e^(-I
*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(4*I*d*x +
4*I*c) + I*e^(2*I*d*x + 2*I*c) + 2*I))*e^(-I*d*x - I*c)/d
```

Sympy [F]

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\cos^2\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*cos(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{i\left(3\sqrt{2}a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(3\left(i\,a\tan(dx+c)+a\right)a^2-4\,a^3\right)}{\left(i\,a\tan(dx+c)+a\right)^{\frac{3}{2}}-2\sqrt{i\,a\tan(dx+c)+a}a}\right)}{16\,ad}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/16*I*(3*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(3*(I*a*tan(d*x +
c) + a)*a^2 - 4*a^3)/((I*a*tan(d*x + c) + a)^(3/2) - 2*sqrt(I*a*tan(d*x +
c) + a)*a))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^2\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^2 dx\right)$$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**2,x)
```

3.284 $\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 197

$$\begin{aligned}\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= -\frac{35i\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d}+\frac{35ia^2}{96d(a+ia\tan(c+dx))^{3/2}}\\&\quad-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}}+\frac{35ia}{64d\sqrt{a+ia\tan(c+dx)}}\\&\quad-\frac{7ia^5}{16d(a+ia\tan(c+dx))^{3/2}\left(a^3-ia^3\tan(c+dx)\right)}\end{aligned}$$

output
```
-35/128*I*a^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^
(1/2)/d+35/96*I*a^2/d/(a+I*a*tan(d*x+c))^(3/2)-1/4*I*a^4/d/(a-I*a*tan(d*x+
c))^2/(a+I*a*tan(d*x+c))^(3/2)+35/64*I*a/d/(a+I*a*tan(d*x+c))^(1/2)-7/16*I
*a^5/d/(a+I*a*tan(d*x+c))^(3/2)/(a^3-I*a^3*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.27

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{ia^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},3,-\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{12d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Integrate[Cos[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I/12)*a^2*Hypergeometric2F1[-3/2, 3, -1/2, (1 + I*Tan[c + d*x])/2])/(d*(
a + I*a*Tan[c + d*x])^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3968, 52, 52, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{7\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \; 52$$

$$-\frac{ia^5\left(\frac{7\left(\frac{5\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(\cdots}\right.}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^5\left(\frac{7\left(\frac{5\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{\cdots}{4a(a-\cdots}\right.}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^5\left(\frac{7\left(\frac{5\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx\cdots}\right.}{8a}\right.}{d}$$

$$\Big\downarrow \; 73$$

$$-\frac{ia^5\left(\frac{7\left(\frac{5\left(\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{}{4}\right.}{d}$$

$$\Big\downarrow \ 219$$

$$-\frac{ia^5\left(\frac{7\left(\frac{5\left(\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{}{4a(a-ia\tan(c}\right.}{d}$$

input
```
Int[Cos[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*a^5*(1/(4*a*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(3/2)) +
 (7*(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2)) + (5*(-1/
3*1/(a*(a + I*a*Tan[c + d*x])^(3/2)) + ((I*ArcTan[(Sqrt[a]*Tan[c + d*x])/S
qrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a)))/(4*
a)))/(8*a)))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(158) = 316.

Time = 4.24 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.78

method	result
default	$\frac{\left(105i\sin(dx+c)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)+(-105\cos(dx+c)-105)\sqrt{-\frac{\cos(d}{\cos(dx}}}{}$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/d*(105*I*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(1/
2)*(cot(d*x+c)-csc(d*x+c))/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c
)^2-1)^(1/2))+(-105*cos(d*x+c)-105)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arc
tanh(2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)
+csc(d*x+c)^2-1)^(1/2))+I*(105*cos(d*x+c)+105)*(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))+105*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*2^(1/2))*sin(d*x+c)+sin(d*x+c)*cos(d*x+c)*(112*cos(d*x+c)^2+210)+I*cos
(d*x+c)^2*(16*cos(d*x+c)^2+70))*(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.40

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx =$$
$$-\frac{\left(105\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(3i\,dx+3i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i}}{}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/384*(105*sqrt(1/2)*d*sqrt(-a/d^2)*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*s
qrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*sqrt(-a/d^2) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 105*sqrt(1/2)*d*sqr
t(-a/d^2)*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x +
 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - a*e^(I*d*x
 + I*c))*e^(-I*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-6
*I*e^(8*I*d*x + 8*I*c) - 45*I*e^(6*I*d*x + 6*I*c) + 41*I*e^(4*I*d*x + 4*I*
c) + 88*I*e^(2*I*d*x + 2*I*c) + 8*I))*e^(-3*I*d*x - 3*I*c)/d
```

Sympy [F]

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\cos^4\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*cos(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.89

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$= \frac{i\left(105\sqrt{2}a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(105\,(i\,a\tan(dx+c)+a)^3a^2-350\,(i\,a\tan(dx+c)+a)^2a^3+224\,(i\,a\tan(dx+c)+a)a\cdots\right.}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}-4\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a+4\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^2\cdots}\right.}{768\,ad}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/768*I*(105*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
 + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(105*(I*a*tan(d
*x + c) + a)^3*a^2 - 350*(I*a*tan(d*x + c) + a)^2*a^3 + 224*(I*a*tan(d*x +
 c) + a)*a^4 + 64*a^5)/((I*a*tan(d*x + c) + a)^(7/2) - 4*(I*a*tan(d*x + c)
 + a)^(5/2)*a + 4*(I*a*tan(d*x + c) + a)^(3/2)*a^2))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^4\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

input
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^4dx\right)$$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**4,x)
```

3.285 $\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 274

$$\begin{aligned}&\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx\\&=-\frac{231i\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}d}+\frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}}\\&\quad-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}}+\frac{77ia^2}{256d(a+ia\tan(c+dx))^{3/2}}\\&\quad+\frac{231ia}{512d\sqrt{a+ia\tan(c+dx)}}-\frac{11ia^7}{48d(a+ia\tan(c+dx))^{5/2}\left(a^2-ia^2\tan(c+dx)\right)^2}\\&\quad-\frac{33ia^7}{64d(a+ia\tan(c+dx))^{5/2}\left(a^4-ia^4\tan(c+dx)\right)}\end{aligned}$$

output
```
-231/1024*I*a^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*
2^(1/2)/d+231/640*I*a^3/d/(a+I*a*tan(d*x+c))^(5/2)-1/6*I*a^6/d/(a-I*a*tan(
d*x+c))^3/(a+I*a*tan(d*x+c))^(5/2)+77/256*I*a^2/d/(a+I*a*tan(d*x+c))^(3/2)
+231/512*I*a/d/(a+I*a*tan(d*x+c))^(1/2)-11/48*I*a^7/d/(a+I*a*tan(d*x+c))^(
5/2)/(a^2-I*a^2*tan(d*x+c))^2-33/64*I*a^7/d/(a+I*a*tan(d*x+c))^(5/2)/(a^4-
I*a^4*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.19

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{ia^3 \,\text{Hypergeometric2F1}\left(-\frac{5}{2},4,-\frac{3}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{40d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I/40)*a^3*Hypergeometric2F1[-5/2, 4, -3/2, (1 + I*Tan[c + d*x])/2])/(d*(
a + I*a*Tan[c + d*x])^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3968, 52, 52, 52, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^7\int \frac{1}{(a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^7\left(\frac{11\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{12a}+\frac{1}{6a(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}}\right)}{d}$$

$$\Big\downarrow \; 52$$

$$-\frac{ia^7\left(\frac{11\left(\frac{9\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}\right)}{12a}+\frac{1}{6a(a-ia\tan(c+dx))^3(a+ia\tan \cdots}\right.}{d}$$

$$\Big\downarrow \; 52$$

$$-\frac{ia^7\left(\frac{11\left(\frac{9\left(\frac{7\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+ \cdots}\right.}{12a}\right.}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^7\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}\right)}{8a}+\frac{1}{4a(a \cdots}\right.}{12a}\right.}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^7\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+\cdots}}{8a}}{12a}\cdots\right.}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^7\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{}{2a(a}\right.}{8a}\right.}{12a}\right.}{d}$$

$\downarrow$ 73

$$-\frac{ia^7\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{\frac{\frac{\int\frac{1}{a^2\tan^2(c+dx)+2a}\,d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{\cdots}{2a(a-ia\tan(c+dx\cdots}\cdots\right)}{8a}\cdots\right)}{12a}\cdots\right)}{d\cdots}$$

$\downarrow$ 219

$$-\frac{ia^7\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+\cdots}\cdots\right)}{8a}\cdots\right)}{12a}\cdots\right)}{d}$$

input
```
Int[Cos[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*a^7*(1/(6*a*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(5/2)) +
 (11*(1/(4*a*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(5/2)) + (9*(
1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2)) + (7*(-1/5*1/(
a*(a + I*a*Tan[c + d*x])^(5/2)) + (-1/3*1/(a*(a + I*a*Tan[c + d*x])^(3/2))
 + ((I*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sq
rt[a + I*a*Tan[c + d*x]]))/(2*a))/(2*a)))/(4*a)))/(8*a)))/(12*a)))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (warning: unable to verify)

Time = 4.66 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.35

method	result
default	$\frac{\left(-3465 i \sin(dx+c) \operatorname{arctanh}\left(\frac{(\csc(dx+c)-\cot(dx+c))\sqrt{2}}{\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+(3465\cos(dx+c)+3465)\sqrt{-\frac{\cos}{\cos}}\right.}{}$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15360/d*(-3465*I*sin(d*x+c)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x
+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)+(3465*cos(d*x+c)+3465)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*
(csc(d*x+c)-cot(d*x+c))*2^(1/2))+I*(3465*cos(d*x+c)+3465)*(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)
)+3465*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*2^(1/2))*sin(d*x+c)+sin(d*x+c)*cos(d*x+c)*(2816*cos(d*x+c)
^4+3696*cos(d*x+c)^2+6930)+I*cos(d*x+c)^2*(256*cos(d*x+c)^4+528*cos(d*x+c)
^2+2310))*(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.08

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{\left(3465\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(5i\,dx+5i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/15360*(3465*sqrt(1/2)*d*sqrt(-a/d^2)*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2
)*sqrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt(-a/d^2) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3465*sqrt(1/2)*d
*sqrt(-a/d^2)*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d
*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - a*e^(I
*d*x + I*c))*e^(-I*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*(-40*I*e^(12*I*d*x + 12*I*c) - 350*I*e^(10*I*d*x + 10*I*c) - 1645*I*e^(8*
I*d*x + 8*I*c) + 1433*I*e^(6*I*d*x + 6*I*c) + 3184*I*e^(4*I*d*x + 4*I*c) +
 464*I*e^(2*I*d*x + 2*I*c) + 48*I))*e^(-5*I*d*x - 5*I*c)/d
```

Sympy [F]

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\cos^6\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*cos(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.84

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$= \frac{i\left(3465\sqrt{2}a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(3465\,(i\,a\tan(dx+c)+a)^{5}a^{2}-18480\,(i\,a\tan(dx+c)+a)^{4}a^{3}+30492\,(i\,a\tan(dx+\right.}{(i\,a\tan(dx+c)+a)^{\frac{11}{2}}-6\,(i\,a\tan(dx+c)+a)^{\frac{9}{2}}a+12}\right.}{30720\,ad}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/30720*I*(3465*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x +
 c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(3465*(I*a*t
an(d*x + c) + a)^5*a^2 - 18480*(I*a*tan(d*x + c) + a)^4*a^3 + 30492*(I*a*t
an(d*x + c) + a)^3*a^4 - 12672*(I*a*tan(d*x + c) + a)^2*a^5 - 2816*(I*a*ta
n(d*x + c) + a)*a^6 - 1536*a^7)/((I*a*tan(d*x + c) + a)^(11/2) - 6*(I*a*ta
n(d*x + c) + a)^(9/2)*a + 12*(I*a*tan(d*x + c) + a)^(7/2)*a^2 - 8*(I*a*tan
(d*x + c) + a)^(5/2)*a^3))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^6\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^6 dx\right)$$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**6,x)
```

3.286 $\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{256ia^4\sec^7(c+dx)}{3003d(a+ia\tan(c+dx))^{7/2}} + \frac{64ia^3\sec^7(c+dx)}{429d(a+ia\tan(c+dx))^{5/2}} + \frac{24ia^2\sec^7(c+dx)}{143d(a+ia\tan(c+dx))^{3/2}} + \frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

output
```
256/3003*I*a^4*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(7/2)+64/429*I*a^3*sec(d*
x+c)^7/d/(a+I*a*tan(d*x+c))^(5/2)+24/143*I*a^2*sec(d*x+c)^7/d/(a+I*a*tan(d
*x+c))^(3/2)+2/13*I*a*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$=\frac{2\sec^6(c+dx)(390\cos(c+dx)+445\cos(3(c+dx))+7i(26\sin(c+dx)+59\sin(3(c+dx))))(i\cos(4(c-}{3003d}$$

input
```
Integrate[Sec[c + d*x]^7*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*Sec[c + d*x]^6*(390*Cos[c + d*x] + 445*Cos[3*(c + d*x)] + (7*I)*(26*Sin
[c + d*x] + 59*Sin[3*(c + d*x)]))*(I*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])*
Sqrt[a + I*a*Tan[c + d*x]])/(3003*d)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3975, 3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(c+dx)^7\sqrt{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \text{3975}$$
$$\frac{12}{13}a\int \frac{\sec^7(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{12}{13}a\int \frac{\sec(c+dx)^7}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \color{blue}{3975}$$

$$\frac{12}{13}a\left(\frac{8}{11}a\int \frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{12}{13}a\left(\frac{8}{11}a\int \frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \color{blue}{3975}$$

$$\frac{12}{13}a\left(\frac{8}{11}a\left(\frac{4}{9}a\int \frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{12}{13}a\left(\frac{8}{11}a\left(\frac{4}{9}a\int \frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \color{blue}{3974}$$

$$\frac{12}{13}a\left(\frac{8}{11}a\left(\frac{8ia^2\sec^7(c+dx)}{63d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

input

```
Int[Sec[c + d*x]^7*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output

```
(((2*I)/13)*a*Sec[c + d*x]^7)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (12*a*((((2
*I)/11)*a*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (8*a*((((8*I)
/63)*a^2*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((2*I)/9)*a*S
ec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(5/2))))/11))/13
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.78

method	result
default	$\frac{\left(\frac{2i\left(-21\sec(dx+c)^5-40\sec(dx+c)^3+1024\cos(dx+c)-128\sec(dx+c)\right)}{3003}+\frac{2\tan(dx+c)\sec(dx+c)^5}{13}+\frac{80\tan(dx+c)\sec(dx+c)^3}{429}+\frac{256\sec(dx+c)\tan}{1001}\right.}{d}$

input
```
int(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/3003*I*(-21*sec(d*x+c)^5-40*sec(d*x+c)^3+1024*cos(d*x+c)-128*sec(d*
x+c))+2/13*tan(d*x+c)*sec(d*x+c)^5+80/429*tan(d*x+c)*sec(d*x+c)^3+256/1001
*sec(d*x+c)*tan(d*x+c)+2048/3003*sin(d*x+c))*(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{128\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-429i\,e^{(6i\,dx+6i\,c)}-286i\,e^{(4i\,dx+4i\,c)}-104i\,e^{(2i\,dx+2i\,c)}-16i\right)}{3003\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}\right.}$$

input
```
integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-128/3003*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-429*I*e^(6*I*d*x + 6
*I*c) - 286*I*e^(4*I*d*x + 4*I*c) - 104*I*e^(2*I*d*x + 2*I*c) - 16*I)/(d*e
^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c
) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x +
 2*I*c) + d)
```

Sympy [F]

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^7\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**7*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**7, x)
```

Maxima [F(-1)]

Timed out.

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.97

$$\begin{aligned}\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{7\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3} \\ &\quad - \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{3\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^4} \\ &\quad + \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,384\mathrm{i}}{11\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^5} \\ &\quad - \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{13\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^6}\end{aligned}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^7,x)
```

output
```
(exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
 d*x*2i) + 1))^(1/2)*128i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3) - (exp(- c*1i
- d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1
))^(1/2)*128i)/(3*d*(exp(c*2i + d*x*2i) + 1)^4) + (exp(- c*1i - d*x*1i)*(a
 - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*384
i)/(11*d*(exp(c*2i + d*x*2i) + 1)^5) - (exp(- c*1i - d*x*1i)*(a - (a*(exp(
c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(13*d*(e
xp(c*2i + d*x*2i) + 1)^6)
```

Reduce [F]

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\sqrt{a}\,i\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^7+15\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^7\tan(dx+c)\,dx\right)d\right)}{d}$$

input
```
int(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*i*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**7 + 15*int(sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**7*tan(c + d*x),x)*d))/d
```

3.287 $\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{64ia^3\sec^5(c+dx)}{315d(a+ia\tan(c+dx))^{5/2}} + \frac{16ia^2\sec^5(c+dx)}{63d(a+ia\tan(c+dx))^{3/2}} + \frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}$$

output

```
64/315*I*a^3*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(5/2)+16/63*I*a^2*sec(d*x+c
)^5/d/(a+I*a*tan(d*x+c))^(3/2)+2/9*I*a*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(
1/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.70

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2\sec^4(c+dx)(36+71\cos(2(c+dx))+55i\sin(2(c+dx)))(i\cos(3(c+dx))+\sin(3(c+dx)))\sqrt{a+ia\,\mathrm{t}}}{315d}$$

input
```
Integrate[Sec[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*Sec[c + d*x]^4*(36 + 71*Cos[2*(c + d*x)] + (55*I)*Sin[2*(c + d*x)])*(I*
Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(315*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^5\sqrt{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{9}a\int \frac{\sec^5(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{9}a\int \frac{\sec(c+dx)^5}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{9}a\left(\frac{4}{7}a\int \frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{9}a\left(\frac{4}{7}a\int \frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8}{9}a\left(\frac{8ia^2\sec^5(c+dx)}{35d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[Sec[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((2*I)/9)*a*Sec[c + d*x]^5)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (8*a*((((8*I
)/35)*a^2*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((2*I)/7)*a*
Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(3/2))))/9
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.81

method	result
default	$\frac{\left(\frac{2i\left(-5\sec(dx+c)^3+128\cos(dx+c)-16\sec(dx+c)\right)}{315}+\frac{2\tan(dx+c)\sec(dx+c)^3}{9}+\frac{32\sec(dx+c)\tan(dx+c)}{105}+\frac{256\sin(dx+c)}{315}\right)\sqrt{a(1+i\tan(dx+c))}}{d}$

input
```
int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/315*I*(-5*sec(d*x+c)^3+128*cos(d*x+c)-16*sec(d*x+c))+2/9*tan(d*x+c)
*sec(d*x+c)^3+32/105*sec(d*x+c)*tan(d*x+c)+256/315*sin(d*x+c))*(a*(1+I*tan
(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{32\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-63i\,e^{(4i\,dx+4i\,c)}-36i\,e^{(2i\,dx+2i\,c)}-8i\right)}{315\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-32/315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-63*I*e^(4*I*d*x + 4*I*
c) - 36*I*e^(2*I*d*x + 2*I*c) - 8*I)/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d
*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^5\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**5, x)
```

Maxima [F(-1)]

Timed out.

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$= \frac{32\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,36\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,63\mathrm{i}+8\mathrm{i}\right)}{315\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^4}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^5,x)
```

output
```
(32*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2
i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*36i + exp(c*4i + d*x*4i)*63i +
 8i))/(315*d*(exp(c*2i + d*x*2i) + 1)^4)
```

Reduce [F]

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$=\frac{\sqrt{a}\,i\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^5+11\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^5\tan(dx+c)\,dx\right)d\right)}{d}$$

input
```
int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*i*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**5 + 11*int(sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**5*tan(c + d*x),x)*d))/d
```

3.288 $\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 73

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{8ia^2\sec^3(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}} + \frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}$$

output
```
8/15*I*a^2*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(3/2)+2/5*I*a*sec(d*x+c)^3/d/
(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2\sec(c+dx)(\cos(2(c+dx))-i\sin(2(c+dx)))(-7i+3\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{15d}$$

input
```
Integrate[Sec[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(-2*Sec[c + d*x]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-7*I + 3*Tan[c +
 d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(15*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3\sqrt{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{5}a\int \frac{\sec^3(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\int \frac{\sec(c+dx)^3}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8ia^2\sec^3(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[Sec[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((8*I)/15)*a^2*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((2*I)
/5)*a*Sec[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\left(\frac{2i(8\cos(dx+c)-\sec(dx+c))}{15}+\frac{2\sec(dx+c)\tan(dx+c)}{5}+\frac{16\sin(dx+c)}{15}\right)\sqrt{a(1+i\tan(dx+c))}}{d}$	63

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/15*I*(8*cos(d*x+c)-sec(d*x+c))+2/5*sec(d*x+c)*tan(d*x+c)+16/15*sin(
d*x+c))*(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{8\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-5i\,e^{(2i\,dx+2i\,c)}-2i\right)}{15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-8/15*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-5*I*e^(2*I*d*x + 2*I*c)
- 2*I)/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^3\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(57) = 114.

Time = 16.07 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.04

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$= \frac{}{15\left(\cos\left(2\,dx+2\,c\right)^2+\sin\left(2\,dx+2\,c\right)^2+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{4}}\left(\left(\cos\left(4\,dx+4\,c\right)+2\,\cos\left(2\,dx+2\,c\right)+\right.\right.}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
8/15*(5*I*sqrt(2)*cos(2*d*x + 2*c) - 5*sqrt(2)*sin(2*d*x + 2*c) + 2*I*sqrt
(2))*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*((cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + I*sin(4*d*x + 4*c)
+ 2*I*sin(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + (I*cos(4*d*x + 4*c) + 2*I*cos(2*d*x + 2*c) - sin(4*d*x + 4*c)
 - 2*sin(2*d*x + 2*c) + I)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)))*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.21

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{8\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,5\mathrm{i}+2\mathrm{i}\right)\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}}{15\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^2}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^3,x)
```

output
```
(8*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*5i + 2i)*(a - (a*(exp(c*2i + d
*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(15*d*(exp(c*2i + d*x
*2i) + 1)^2)
```

Reduce [F]

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$=\frac{\sqrt{a}\,i\left(-2\sqrt{\tan\left(dx+c\right)i+1}\,\sec\left(dx+c\right)^3+7\left(\int\sqrt{\tan\left(dx+c\right)i+1}\,\sec\left(dx+c\right)^3\tan\left(dx+c\right)dx\right)d\right)}{d}$$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*i*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**3 + 7*int(sqrt(tan
(c + d*x)*i + 1)*sec(c + d*x)**3*tan(c + d*x),x)*d))/d
```

3.289 $\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 31

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2ia\sec(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

output
```
2*I*a*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2(i\cos(c+dx)+\sin(c+dx))\sqrt{a+ia\tan(c+dx)}}{d}$$

input
```
Integrate[Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*(I*Cos[c + d*x] + Sin[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \text{3974}$$
$$\frac{2ia\sec(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((2*I)*a*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.68

method	result	size
default	$-\frac{i\sqrt{a(1+i\tan(dx+c))}\,(\sin(dx+c)+i\cos(dx+c)+i)^2}{d(\cos(dx+c)+1)}$	52

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(sin(d*x+c)+I*cos(d*x+c)+I)^2/(cos(d*x+c)+
1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{d}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [F]

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\,(\tan\,(c+dx)-i)}\sec\,(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2\left(\sin\left(c+d\,x\right)+\cos\left(c+d\,x\right)\,1\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}{d}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x),x)
```

output
```
(2*(cos(c + d*x)*1i + sin(c + d*x))*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*
x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))/d
```

Reduce [F]

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$=\frac{\sqrt{a}\,i\left(-2\sqrt{\tan\left(dx+c\right)i+1}\,\sec\left(dx+c\right)+3\left(\int\sqrt{\tan\left(dx+c\right)i+1}\,\sec\left(dx+c\right)\tan\left(dx+c\right)dx\right)d\right)}{d}$$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*i*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x) + 3*int(sqrt(tan(c
+ d*x)*i + 1)*sec(c + d*x)*tan(c + d*x),x)*d))/d
```

3.290 $\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{i\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d} - \frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

output
```
1/2*I*a^(1/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1
/2))*2^(1/2)/d-I*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{ie^{-i(c+dx)}\left(1+e^{2i(c+dx)}-\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{2d}$$

input
```
Integrate[Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-1/2*I)*(1 + E^((2*I)*(c + d*x)) - Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh
[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c +
d*x)))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{3970}$$

$$\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

input
```
Int[Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*
x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(68) = 136$.

Time = 4.36 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.86

method	result
default	$\left(-\frac{i\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{(\csc(dx+c)-\cot(dx+c))\sqrt{2}}{\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)\sqrt{2}\,\sin(dx+c)}{4}-\frac{i\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{2}\,(\cos(dx+c)+1)\,\text{ar}}{4}\right.$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/(cot(d*x+c)^2-2
*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/
2))*2^(1/2)*sin(d*x+c)-1/4*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*
(cos(d*x+c)+1)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))-I*
cos(d*x+c)+1/4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*(cos(d*x+c)+1)
*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(cs
c(d*x+c)-cot(d*x+c))*2^(1/2))-1/4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arc
tan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))*2^(1/2)*sin(d*x+c))*
(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(64) = 128$.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.22

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\sqrt{2}d\sqrt{-\frac{a}{d^2}}\log\left(\frac{2\left(\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}+i\,a\right)e^{(-i\,dx-i\,c)}}{d}\right)-\sqrt{2}d\sqrt{-\frac{a}{d^2}}\log\left(-\frac{2\left(\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\cdots}\right.}{\cdots}\right.}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*d*sqrt(-a/d^2)*log(2*((d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) + I*a)*e^(-I*d*x - I*c)/d) - sqrt(2)*d
*sqrt(-a/d^2)*log(-2*((d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt(-a/d^2) - I*a)*e^(-I*d*x - I*c)/d) - 2*sqrt(2)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*(I*e^(2*I*d*x + 2*I*c) + I))/d
```

Sympy [F]

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\cos\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*cos(c + d*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(64) = 128$.

Time = 0.25 (sec) , antiderivative size = 774, normalized size of antiderivative = 9.33

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/8*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(I*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 - sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(
a) + (2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sq
rt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(
sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x...
```

Giac [F(-2)]

Exception generated.

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \cos\left(c+d\,x\right)\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)dx\right)$$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x),x)
```

3.291 $\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 154

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{5i\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}d} + \frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{5i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d} - \frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

output
```
5/16*I*a^(1/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(
1/2))*2^(1/2)/d+5/12*I*a*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-5/8*I*cos(d
*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/3*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/
2)/d
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.82

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx =$$
$$-\frac{ie^{-3i(c+dx)}\left(-3+11e^{2i(c+dx)}+16e^{4i(c+dx)}+2e^{6i(c+dx)}-15e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+d}}\right.\right.}{48d}$$

input
```
Integrate[Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-1/48*I)*(-3 + 11*E^((2*I)*(c + d*x)) + 16*E^((4*I)*(c + d*x)) + 2*E^((6
*I)*(c + d*x)) - 15*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcT
anh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^((3*I
)*(c + d*x)))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sec(c+dx)^3}dx$$
$$\Big\downarrow \text{3978}$$
$$\frac{5}{6}a\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}a\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{3983}$$

$$\frac{5}{6}a\left(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}a\left(\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{3970}$$

$$\frac{5}{6}a\left(\frac{3\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{6}a\left(\frac{3\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

input
```
Int[Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-1/3*I)*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (5*a*(((I/2)*Cos[
c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]
*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos
[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d))/(4*a)))/6
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(123) = 246$.

Time = 4.66 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.21

method	result
default	$\frac{\left(15i\sin(dx+c)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)+(-15\cos(dx+c)-15)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)-}}\right.}{}$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d*(15*I*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(1/2)
*(cot(d*x+c)-csc(d*x+c))/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^
2-1)^(1/2))+(-15*cos(d*x+c)-15)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh
(2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc
(d*x+c)^2-1)^(1/2))+I*(-15*cos(d*x+c)-15)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))-15*(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^
(1/2))*sin(d*x+c)+I*cos(d*x+c)*(4*cos(d*x+c)^2-30)+20*sin(d*x+c)*cos(d*x+c
)^2)*(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(115) = 230$.

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.59

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$= \frac{\left(15\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(2i\,dx+2i\,c)}\log\left(\frac{5\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}+i\,a\right)e^{(-i\,dx-i\,c)}}{4\,d}\right)-15\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/48*(15*sqrt(1/2)*d*sqrt(-a/d^2)*e^(2*I*d*x + 2*I*c)*log(5/4*(sqrt(2)*sqr
t(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
-a/d^2) + I*a)*e^(-I*d*x - I*c)/d) - 15*sqrt(1/2)*d*sqrt(-a/d^2)*e^(2*I*d*
x + 2*I*c)*log(-5/4*(sqrt(2)*sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - I*a)*e^(-I*d*x - I*c)/d) + sqrt(
2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(6*I*d*x + 6*I*c) - 16*I*e^(4
*I*d*x + 4*I*c) - 11*I*e^(2*I*d*x + 2*I*c) + 3*I))*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(115) = 230$.

Time = 0.27 (sec) , antiderivative size = 935, normalized size of antiderivative = 6.07

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/192*(8*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(3/4)*(I*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) - sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqr
t(a) + 12*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((-I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + 4*I*sq
rt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)
*cos(2*d*x + 2*c) - I*sqrt(2)*sin(2*d*x + 2*c) - 4*sqrt(2))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 15*(2*sqrt(2)*arctan
2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(...
```

Giac [F(-2)]

Exception generated.

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^3\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^3dx\right)$$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**3,x)
```

3.292 $\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 223

$$\begin{aligned}\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= \frac{63i\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}d}\\ &\quad+\frac{21ia\cos(c+dx)}{64d\sqrt{a+ia\tan(c+dx)}}\\ &\quad+\frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}}\\ &\quad-\frac{63i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d}\\ &\quad-\frac{21i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{80d}\\ &\quad-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}\end{aligned}$$

output

```
63/256*I*a^(1/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))
^(1/2))*2^(1/2)/d+21/64*I*a*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+9/40*I*a
*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)-63/128*I*cos(d*x+c)*(a+I*a*tan(d*
x+c))^(1/2)/d-21/80*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)/d-1/5*I*cos(d*
x+c)^5*(a+I*a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.68

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx =$$
$$-\frac{ie^{-5i(c+dx)}\left(-10-95e^{2i(c+dx)}+203e^{4i(c+dx)}+344e^{6i(c+dx)}+64e^{8i(c+dx)}+8e^{10i(c+dx)}-315e^{4i(c+dx)}\sqrt{1}\right.}{1280d}$$

input
```
Integrate[Cos[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-1/1280*I)*(-10 - 95*E^((2*I)*(c + d*x)) + 203*E^((4*I)*(c + d*x)) + 344
*E^((6*I)*(c + d*x)) + 64*E^((8*I)*(c + d*x)) + 8*E^((10*I)*(c + d*x)) - 3
15*E^((4*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((
2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^((5*I)*(c + d*x)))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3978, 3042, 3983, 3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{9}{10}a\int \frac{\cos^3(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{10}a\int \frac{1}{\sec(c+dx)^3\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$\big\downarrow$ 3983

$$\frac{9}{10}a\left(\frac{7\int \cos^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$\big\downarrow$ 3042

$$\frac{9}{10}a\left(\frac{7\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^3}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$\big\downarrow$ 3978

$$\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$\big\downarrow$ 3042

$$\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$\big\downarrow$ 3983

$$\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$\big\downarrow$ 3042

$$\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)$$

$$\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}\right.$$

$$\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}\right.$$

$$\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$$\Big\downarrow \text{3970}$$

$$\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+i}}{3d}}{8a}\right.$$

$$\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$$\Big\downarrow \text{219}$$

$$\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan}}{3d}}{8a}\right.$$

$$\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

input
```
Int[Cos[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-1/5*I)*Cos[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]])/d + (9*a*(((I/4)*Cos[
c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (7*(((-1/3*I)*Cos[c + d*x]^3*
Sqrt[a + I*a*Tan[c + d*x]])/d + (5*a*(((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a
*Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*S
qrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Ta
n[c + d*x]])/d))/(4*a)))/6))/(8*a)))/10
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 4.48 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.62

method	result
default	$\frac{\left(-315 i \sin(dx+c) \operatorname{arctanh}\left(\frac{(\csc(dx+c)-\cot(dx+c))\sqrt{2}}{\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+(315\cos(dx+c)+315)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right.}{}$

input

```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/1280/d*(-315*I*sin(d*x+c)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c
)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)+(315*cos(d*x+c)+315)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc
(d*x+c)-cot(d*x+c))*2^(1/2))+I*(-315*cos(d*x+c)-315)*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))-315
*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*2^(1/2))*sin(d*x+c)+sin(d*x+c)*cos(d*x+c)^2*(288*cos(d*x+c)^2+42
0)+I*cos(d*x+c)*(32*cos(d*x+c)^4+84*cos(d*x+c)^2-630))*(a*(1+I*tan(d*x+c))
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.20

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\left(315\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(4i\,dx+4i\,c)}\log\left(\frac{63\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}+i\,a\right)e^{(-i\,dx-i\,c)}}{64\,d}\right)-315\sqrt{\frac{1}{2}}d\sqrt{-\right.}{}$$

input

```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/1280*(315*sqrt(1/2)*d*sqrt(-a/d^2)*e^(4*I*d*x + 4*I*c)*log(63/64*(sqrt(2
)*sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt(-a/d^2) + I*a)*e^(-I*d*x - I*c)/d) - 315*sqrt(1/2)*d*sqrt(-a/d^2)*e^(
4*I*d*x + 4*I*c)*log(-63/64*(sqrt(2)*sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - I*a)*e^(-I*d*x - I*c)/d)
 + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-8*I*e^(10*I*d*x + 10*I*c) -
 64*I*e^(8*I*d*x + 8*I*c) - 344*I*e^(6*I*d*x + 6*I*c) - 203*I*e^(4*I*d*x +
 4*I*c) + 95*I*e^(2*I*d*x + 2*I*c) + 10*I))*e^(-4*I*d*x - 4*I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2220 vs. $2(168) = 336$.

Time $= 0.40$ (sec) , antiderivative size $= 2220$, normalized size of antiderivative $= 9.96$

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/5120*(20*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((-3*I*sqrt(2)*cos(4*d*x + 4*c
) - 3*sqrt(2)*sin(4*d*x + 4*c) - 8*I*sqrt(2))*cos(3/2*arctan2(sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 1)) + (3*sqrt(2)*cos(4*d*x + 4*c) - 3*I*sqrt(2)*sin
(4*d*x + 4*c) + 8*sqrt(2))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) + 1)))*sqrt(a) + 4*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(8*(-I*sqrt(2)*cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - I*sqrt(2)*sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - 2*I*sqrt(2)*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) - I*sqrt(2))*cos(5/2*arctan2(sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))) + 1)) + 5*(5*I*sqrt(2)*cos(4*d*x + 4*c) + 20*I*sqr
t(2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 5*sqrt(2)*sin(
4*d*x + 4*c) + 20*sqrt(2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) - 48*I*sqrt(2))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +...
```

Giac [F(-2)]

Exception generated.

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \cos(c+d\,x)^5\,\sqrt{a+a\tan(c+d\,x)\;1\mathrm{i}}\,dx$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^5\,dx\right)$$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**5,x)
```

3.293 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^4d}+\frac{24i(a+ia\tan(c+dx))^{13/2}}{13a^5d}-\frac{4i(a+ia\tan(c+dx))^{15/2}}{5a^6d}+\frac{2i(a+ia\tan(c+dx))^{17/2}}{17a^7d}$$

output
```
-16/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^4/d+24/13*I*(a+I*a*tan(d*x+c))^(13/2)
/a^5/d-4/5*I*(a+I*a*tan(d*x+c))^(15/2)/a^6/d+2/17*I*(a+I*a*tan(d*x+c))^(17
/2)/a^7/d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2a(1+i\tan(c+dx))^5\sqrt{a+ia\tan(c+dx)}(-1767i-3641\tan(c+dx)+2717i\tan^2(c+dx}{12155d}$$

input
```
Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*a*(1 + I*Tan[c + d*x])^5*Sqrt[a + I*a*Tan[c + d*x]]*(-1767*I - 3641*Tan
[c + d*x] + (2717*I)*Tan[c + d*x]^2 + 715*Tan[c + d*x]^3))/(12155*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{3042, 3968, 53, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec(c+dx)^8(a+ia\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \ 3968$$

$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{9/2}d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \ 53$$

$$-\frac{i\int \left(-(i\tan(c+dx)a+a)^{15/2}+6a(i\tan(c+dx)a+a)^{13/2}-12a^2(i\tan(c+dx)a+a)^{11/2}+8a^3(i\tan(c+dx)\right.}{a^7d}$$

$$\Big\downarrow \ 2009$$

$$-\frac{i\left(\frac{16}{11}a^3(a+ia\tan(c+dx))^{11/2}-\frac{24}{13}a^2(a+ia\tan(c+dx))^{13/2}-\frac{2}{17}(a+ia\tan(c+dx))^{17/2}+\frac{4}{5}a(a+ia\tan(c\right.}{a^7d}$$

input
```
Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*((16*a^3*(a + I*a*Tan[c + d*x])^(11/2))/11 - (24*a^2*(a + I*a*Tan[c
+ d*x])^(13/2))/13 + (4*a*(a + I*a*Tan[c + d*x])^(15/2))/5 - (2*(a + I*a*T
an[c + d*x])^(17/2))/17))/(a^7*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{17}{2}}}{17}-\frac{2a(a+ia\tan(dx+c))^{\frac{15}{2}}}{5}+\frac{12a^{2}(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{8a^{3}(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}\right)}{d\,a^{7}}$	82
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{17}{2}}}{17}-\frac{2a(a+ia\tan(dx+c))^{\frac{15}{2}}}{5}+\frac{12a^{2}(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{8a^{3}(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}\right)}{d\,a^{7}}$	82

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^7*(1/17*(a+I*a*tan(d*x+c))^(17/2)-2/5*a*(a+I*a*tan(d*x+c))^(15/2)+
12/13*a^2*(a+I*a*tan(d*x+c))^(13/2)-8/11*a^3*(a+I*a*tan(d*x+c))^(11/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.45

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx =$$
$$-\frac{512\sqrt{2}\left(16i\,ae^{(17i\,dx+17i\,c)}+136i\,ae^{(15i\,dx+15i\,c)}+510i\,ae^{(13i\,dx+13i\,c)}+1105i\,ae^{(11i\,dx\cdots}\right.}{12155\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx\cdots}\right.}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-512/12155*sqrt(2)*(16*I*a*e^(17*I*d*x + 17*I*c) + 136*I*a*e^(15*I*d*x + 1
5*I*c) + 510*I*a*e^(13*I*d*x + 13*I*c) + 1105*I*a*e^(11*I*d*x + 11*I*c))*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(16*I*d*x + 16*I*c) + 8*d*e^(14*I*d*
x + 14*I*c) + 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(10*I*d*x + 10*I*c) + 70
*d*e^(8*I*d*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) + 28*d*e^(4*I*d*x + 4*I*
c) + 8*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2i\left(715\,(i\,a\tan(dx+c)+a)^{\frac{17}{2}} - 4862\,(i\,a\tan(dx+c)+a)^{\frac{15}{2}}a + 11220\,(i\,a\tan(dx+c)+a\right.}{12155\,a^7d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
2/12155*I*(715*(I*a*tan(d*x + c) + a)^(17/2) - 4862*(I*a*tan(d*x + c) + a)
^(15/2)*a + 11220*(I*a*tan(d*x + c) + a)^(13/2)*a^2 - 8840*(I*a*tan(d*x +
c) + a)^(11/2)*a^3)/(a^7*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 12.43 (sec) , antiderivative size = 544, normalized size of antiderivative = 4.65

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^8,x)
```

output
```
(a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*155136i)/(2431*d*(exp(c*2i + d*x*2i) + 1)^4) - (a*(a - (a*(exp(c*2i + d*
x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*4096i)/(12155*d*(exp(c*
2i + d*x*2i) + 1)) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
 + d*x*2i) + 1))^(1/2)*3072i)/(12155*d*(exp(c*2i + d*x*2i) + 1)^2) - (a*(a
 - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512
i)/(2431*d*(exp(c*2i + d*x*2i) + 1)^3) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i
 - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*8192i)/(12155*d) - (a*(a - (a*(
exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2413568i)/
(12155*d*(exp(c*2i + d*x*2i) + 1)^5) + (a*(a - (a*(exp(c*2i + d*x*2i)*1i -
 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*270336i)/(1105*d*(exp(c*2i + d*x*
2i) + 1)^6) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*
2i) + 1))^(1/2)*11776i)/(85*d*(exp(c*2i + d*x*2i) + 1)^7) + (a*(a - (a*(ex
p(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(17*d*
(exp(c*2i + d*x*2i) + 1)^8)
```

Reduce [F]

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2\sqrt{a}\,ai\left(-\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^8+9\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^8\tan(dx\right.\right.}{d}$$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(2*sqrt(a)*a*i*( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**8 + 9*int(sqrt(t
an(c + d*x)*i + 1)*sec(c + d*x)**8*tan(c + d*x),x)*d))/d
```

3.294 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^3d} + \frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^4d} - \frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^5d}$$

output
```
-8/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^3/d+8/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^4
/d-2/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^5/d
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2ia(-i+\tan(c+dx))^4\sqrt{a+ia\tan(c+dx)}(-203+270i\tan(c+dx)+99\tan^2(c+dx))}{1287d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((2*I)/1287)*a*(-I + Tan[c + d*x])^4*Sqrt[a + I*a*Tan[c + d*x]]*(-203 + (
270*I)*Tan[c + d*x] + 99*Tan[c + d*x]^2))/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+ia\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{7/2}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left((i\tan(c+dx)a+a)^{11/2}-4a(i\tan(c+dx)a+a)^{9/2}+4a^2(i\tan(c+dx)a+a)^{7/2}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{8}{9}a^2(a+ia\tan(c+dx))^{9/2}+\frac{2}{13}(a+ia\tan(c+dx))^{13/2}-\frac{8}{11}a(a+ia\tan(c+dx))^{11/2}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*((8*a^2*(a + I*a*Tan[c + d*x])^(9/2))/9 - (8*a*(a + I*a*Tan[c + d*x]
)^(11/2))/11 + (2*(a + I*a*Tan[c + d*x])^(13/2))/13))/(a^5*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}+\frac{4a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}\right)}{d\,a^5}$	63
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}+\frac{4a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}\right)}{d\,a^5}$	63

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^5*(-1/13*(a+I*a*tan(d*x+c))^(13/2)+4/11*a*(a+I*a*tan(d*x+c))^(11/2
)-4/9*a^2*(a+I*a*tan(d*x+c))^(9/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(64) = 128$.

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.52

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{128\sqrt{2}\left(8i\,ae^{(13i\,dx+13i\,c)}+52i\,ae^{(11i\,dx+11i\,c)}+143i\,ae^{(9i\,dx+9i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{1287\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+\right.}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-128/1287*sqrt(2)*(8*I*a*e^(13*I*d*x + 13*I*c) + 52*I*a*e^(11*I*d*x + 11*I
*c) + 143*I*a*e^(9*I*d*x + 9*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^
(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c)
 + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x +
2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2i\left(99\,(i\,a\tan\left(dx+c\right)+a)^{\frac{13}{2}}-468\,(i\,a\tan\left(dx+c\right)+a)^{\frac{11}{2}}a+572\,(i\,a\tan\left(dx+c\right)+a)^{\frac{9}{2}}a^2\right)}{1287\,a^5d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-2/1287*I*(99*(I*a*tan(d*x + c) + a)^(13/2) - 468*(I*a*tan(d*x + c) + a)^(
11/2)*a + 572*(I*a*tan(d*x + c) + a)^(9/2)*a^2)/(a^5*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 4.51 (sec) , antiderivative size = 420, normalized size of antiderivative = 4.77

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{a\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{1287\,d} - \frac{a\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{1287\,d\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{a\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{429\,d\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^2} + \frac{a\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,27136\mathrm{i}}{1287\,d\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3} - \frac{a\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,58624\mathrm{i}}{1287\,d\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^4} + \frac{a\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,5120\mathrm{i}}{143\,d\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^5} - \frac{a\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{13\,d\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^6}$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^6,x)
```

output
```
(a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*27136i)/(1287*d*(exp(c*2i + d*x*2i) + 1)^3) - (a*(a - (a*(exp(c*2i + d*x
*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(1287*d*(exp(c*2i
+ d*x*2i) + 1)) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
d*x*2i) + 1))^(1/2)*128i)/(429*d*(exp(c*2i + d*x*2i) + 1)^2) - (a*(a - (a*
(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(1
287*d) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*58624i)/(1287*d*(exp(c*2i + d*x*2i) + 1)^4) + (a*(a - (a*(exp(c
*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*5120i)/(143*d*(
exp(c*2i + d*x*2i) + 1)^5) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(
exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(13*d*(exp(c*2i + d*x*2i) + 1)^6)
```

Reduce [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2\sqrt{a}\,ai\left(-\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^6+7\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^6\tan(dx\right.\right.}{d}$$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(2*sqrt(a)*a*i*( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**6 + 7*int(sqrt(t
an(c + d*x)*i + 1)*sec(c + d*x)**6*tan(c + d*x),x)*d))/d
```

3.295 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{4i(a+ia\tan(c+dx))^{7/2}}{7a^2d}+\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^3d}$$

output
```
-4/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^2/d+2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^3/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2a(-1-i\tan(c+dx))^3(11i+7\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{63d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*a*(-1 - I*Tan[c + d*x])^3*(11*I + 7*Tan[c + d*x])*Sqrt[a + I*a*Tan[c +
d*x]])/(63*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(c+dx)^4(a+ia\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{ 53}$$

$$-\frac{i\int \left(2a(i\tan(c+dx)a+a)^{5/2}-(i\tan(c+dx)a+a)^{7/2}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{i\left(\frac{4}{7}a(a+ia\tan(c+dx))^{7/2}-\frac{2}{9}(a+ia\tan(c+dx))^{9/2}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*((4*a*(a + I*a*Tan[c + d*x])^(7/2))/7 - (2*(a + I*a*Tan[c + d*x])^(9
/2))/9))/(a^3*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{2a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}\right)}{d\,a^{3}}$	44
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{2a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}\right)}{d\,a^{3}}$	44

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^3*(1/9*(a+I*a*tan(d*x+c))^(9/2)-2/7*a*(a+I*a*tan(d*x+c))^(7/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(43) = 86$.

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.66

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{32\sqrt{2}\left(2i\,ae^{(9i\,dx+9i\,c)}+9i\,ae^{(7i\,dx+7i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{63\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-32/63*sqrt(2)*(2*I*a*e^(9*I*d*x + 9*I*c) + 9*I*a*e^(7*I*d*x + 7*I*c))*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6
*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan(c+dx)-i))^{\frac{3}{2}}\sec^4(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2i\left(7\,(i\,a\tan(dx+c)+a)^{\frac{9}{2}} - 18\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a\right)}{63\,a^3d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
2/63*I*(7*(I*a*tan(d*x + c) + a)^(9/2) - 18*(I*a*tan(d*x + c) + a)^(7/2)*a
)/(a^3*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 296, normalized size of antiderivative = 5.02

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{a\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{63\,d} - \frac{a\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{63\,d\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} + \frac{a\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,160\mathrm{i}}{21\,d\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^2} - \frac{a\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,608\mathrm{i}}{63\,d\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3} + \frac{a\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,d\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^4}$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^4,x)
```

output
```
(a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*160i)/(21*d*(exp(c*2i + d*x*2i) + 1)^2) - (a*(a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(63*d*(exp(c*2i + d*x*2
i) + 1)) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i)
 + 1))^(1/2)*64i)/(63*d) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(ex
p(c*2i + d*x*2i) + 1))^(1/2)*608i)/(63*d*(exp(c*2i + d*x*2i) + 1)^3) + (a*
(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*3
2i)/(9*d*(exp(c*2i + d*x*2i) + 1)^4)
```

Reduce [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2\sqrt{a}\,ai\left(-\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^4+5\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^4\tan(dx$$

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(2*sqrt(a)*a*i*( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**4 + 5*int(sqrt(t
an(c + d*x)*i + 1)*sec(c + d*x)**4*tan(c + d*x),x)*d))/d
```

3.296 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad}$$

output
```
-2/5*I*(a+I*a*tan(d*x+c))^(5/2)/a/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad}$$

input
```
Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+ia\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (i\tan(c+dx)a+a)^{3/2}d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad}$$

input
```
Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2i(a+ia\tan(dx+c))^{\frac{5}{2}}}{5ad}$	24
default	$-\frac{2i(a+ia\tan(dx+c))^{\frac{5}{2}}}{5ad}$	24

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*I*(a+I*a*tan(d*x+c))^(5/2)/a/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(21) = 42.

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.03

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{8i\sqrt{2}a\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(5i\,dx+5i\,c)}}{5\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-8/5*I*sqrt(2)*a*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(5*I*d*x + 5*I*c)/(d*
e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan(c+dx)-i))^{\frac{3}{2}}\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2i\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}{5\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-2/5*I*(I*a*tan(d*x + c) + a)^(5/2)/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 153, normalized size of antiderivative = 5.28

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{4\,a\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(\cos(2\,c+2\,d\,x)\,7\mathrm{i}+\cos(4\,c+4\,d\,x)\,4\mathrm{i}+\cos(6\,c+6\,d\,x)\,1\mathrm{i}-5\,\sin(\dots}{5\,d\,(15\,\cos(2\,c+2\,d\,x)+6\,\cos(4\,c+4\,d\,x)+\cos(6\,c+6\dots}$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^2,x)
```

output
```
-(4*a*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x)
+ 1))^(1/2)*(cos(2*c + 2*d*x)*7i + cos(4*c + 4*d*x)*4i + cos(6*c + 6*d*x)*
1i - 5*sin(2*c + 2*d*x) - 4*sin(4*c + 4*d*x) - sin(6*c + 6*d*x) + 4i))/(5*
d*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

Reduce [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2\sqrt{a}\,ai\left(-\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^2+3\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^2\tan(dx\dots\right.\right.}{d}$$

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(2*sqrt(a)*a*i*( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**2 + 3*int(sqrt(t
an(c + d*x)*i + 1)*sec(c + d*x)**2*tan(c + d*x),x)*d))/d
```

3.297 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 97

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{ia^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d}-\frac{ia^3\sqrt{a+ia\tan(c+dx)}}{2d\left(a^2-ia^2\tan(c+dx)\right)}$$

output
```
-1/4*I*a^(3/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/
2)/d-1/2*I*a^3*(a+I*a*tan(d*x+c))^(1/2)/d/(a^2-I*a^2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{\sqrt{2}a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)(1-i\tan(c+dx))+2a\sqrt{a+ia\tan(c+dx)}}{4d(i+\tan(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(Sqrt[2]*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]*(1
- I*Tan[c + d*x]) + 2*a*Sqrt[a + I*a*Tan[c + d*x]])/(4*d*(I + Tan[c + d*x]
))
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 3968, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \ 3968$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \ 52$$

$$-\frac{ia^3\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{4a}+\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)}{d}$$

$$\Big\downarrow \ 73$$

$$-\frac{ia^3\left(\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{2a}+\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)}{d}$$

$$\Big\downarrow \ 219$$

$$-\frac{ia^3\left(\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{2\sqrt{2}a^{3/2}}+\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*a^3*(((I/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)
) + Sqrt[a + I*a*Tan[c + d*x]]/(2*a*(a - I*a*Tan[c + d*x]))))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(77) = 154$.

Time = 4.71 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.08

method	result
default	$-\frac{\left((\cos(dx+c)+1)\sin(dx+c)\operatorname{arctanh}\left(\frac{(\csc(dx+c)-\cot(dx+c))\sqrt{2}}{\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+i\operatorname{arctanh}\left(\frac{}{\sqrt{\cot(dx+c}}\right.\right.}{}$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*((cos(d*x+c)+1)*sin(d*x+c)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc
(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)+I*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+
csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)^2+cos(d*x+c))+I*(cos(d*x+c)+1)*sin(d*x+c)*arct
an(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)+cos(d*x+c)*(-cos(d*x+c)-1)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*2^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*sin(d*x+c)*co
s(d*x+c)+cos(d*x+c)*(cos(d*x+c)+1))*cos(d*x+c)*(-tan(d*x+c)+I)*(a*(1+I*tan
(d*x+c)))^(1/2)*a/(1+cos(d*x+c)+I*sin(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(72) = 144$.

Time = 0.08 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.52

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}d\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ide^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}-a^2e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{a}\right)-\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}d\log\left(-\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(1/2)*sqrt(-a^3/d^2)*d*log(-4*(sqrt(2)*sqrt(1/2)*(I*d*e^(2*I*d*x
 + 2*I*c) + I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) - a^2*e^
(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - sqrt(1/2)*sqrt(-a^3/d^2)*d*log(-4*(sq
rt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1)) - a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - sqrt(
2)*(-I*a*e^(3*I*d*x + 3*I*c) - I*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1)))/d
```

Sympy [F]

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\cos^2{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*cos(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{i\left(\sqrt{2}a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{16\,\sqrt{i\,a\tan(dx+c)+a}a^3}{4i\,a\tan(dx+c)-4\,a}\right)}{8\,ad}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/8*I*(sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))
/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 16*sqrt(I*a*tan(d*x + c
) + a)*a^3/(4*I*a*tan(d*x + c) - 4*a))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)^2\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,dx$$

input
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\begin{aligned}&\int \cos^2(c+dx)(a+ia\tan(c\\&+dx))^{3/2}\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^{2}\tan\left(dx+c\right)dx\right)i\right.\\&\left.+\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^{2}dx\right)\end{aligned}$$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**2*tan(c + d*x),x)*i
+ int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**2,x))
```

3.298 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 170

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=-\frac{15ia^{3/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}d}+\frac{15ia^2}{32d\sqrt{a+ia\tan(c+dx)}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}-\frac{5ia^5}{16d\sqrt{a+ia\tan(c+dx)}\,(a^3-ia^3\tan(c+dx))}$$

output
```
-15/64*I*a^(3/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(
1/2)/d+15/32*I*a^2/d/(a+I*a*tan(d*x+c))^(1/2)-1/4*I*a^4/d/(a-I*a*tan(d*x+c
))^2/(a+I*a*tan(d*x+c))^(1/2)-5/16*I*a^5/d/(a+I*a*tan(d*x+c))^(1/2)/(a^3-I
*a^3*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.31

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{ia^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},3,\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{4d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I/4)*a^2*Hypergeometric2F1[-1/2, 3, 1/2, (1 + I*Tan[c + d*x])/2])/(d*Sqr
t[a + I*a*Tan[c + d*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 3968, 52, 52, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{5\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

$$\Big\downarrow 52$$

$$-\frac{ia^5\left(\frac{5\left(\frac{3\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c-\cdots}}\cdots\right.}{d}$$

$$\Big\downarrow 61$$

$$-\frac{ia^5\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{\cdots}{4a(a-ia\tan(c+\cdots}\right.}{d}$$

$$\Big\downarrow 73$$

$$-\frac{ia^5\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2\sqrt{a+i\cdots}}\right.}{d}$$

$$\Big\downarrow 219$$

$$-\frac{ia^5\left(\frac{5\left(\frac{3\left(\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*a^5*(1/(4*a*(a - I*a*Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]) + (
5*(1/(2*a*(a - I*a*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]) + (3*((I*ArcT
an[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[a + I*a*
Tan[c + d*x]])))/(4*a)))/(8*a)))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 771 vs. $2(137) = 274$.

Time = 4.34 (sec) , antiderivative size = 772, normalized size of antiderivative = 4.54

method	result	size
default	Expression too large to display	772

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/64/d*cos(d*x+c)*((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(1/2)*(cot
(d*x+c)-csc(d*x+c))/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^
(1/2))*(-30*sin(d*x+c)^2-15*tan(d*x+c)*sin(d*x+c))+I*(-30*cos(d*x+c)^2-15*
cos(d*x+c)+15)*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(1/
2)*(cot(d*x+c)-csc(d*x+c))/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c
)^2-1)^(1/2))+I*(30*cos(d*x+c)+15)*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*arctanh(2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(cot(d*x+c)^2-2*cot(d*x+c)*
csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))+(-30*cos(d*x+c)^2-15*cos(d*x+c)+15)*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(co
t(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))+I*(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/
2))*(30*sin(d*x+c)^2+15*tan(d*x+c)*sin(d*x+c))+(-30*cos(d*x+c)^2-15*cos(d*
x+c)+15)*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))+(30*cos(d*x+c)+15)*sin(d*x+c)*(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*2^(1/2))+I*(30*cos(d*x+c)^2+15*cos(d*x+c)-15)*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))+40*I*si
n(d*x+c)^2*cos(d*x+c)+sin(d*x+c)*(-24*cos(d*x+c)^2+30)+I*cos(d*x+c)*(24*co
s(d*x+c)^2-30)+40*sin(d*x+c)*cos(d*x+c)^2)*a*(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(127) = 254$.

Time = 0.09 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.69

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{\left(15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(i\,dx+i\,c)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}-a^2e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{a}\right)-15\sqrt{\frac{1}{2}}\cdots}{\cdots}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/64*(15*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(I*d*x + I*c)*log(-4*(sqrt(2)*sqrt(
1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1)) - a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - 15*sqrt(1/2)*sqrt
(-a^3/d^2)*d*e^(I*d*x + I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x +
2*I*c) - I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) - a^2*e^(I*
d*x + I*c))*e^(-I*d*x - I*c)/a) - sqrt(2)*(-2*I*a*e^(6*I*d*x + 6*I*c) - 11
*I*a*e^(4*I*d*x + 4*I*c) - I*a*e^(2*I*d*x + 2*I*c) + 8*I*a)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{i\left(15\sqrt{2}a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)+\frac{4\left(15\,(ia\tan(dx+c)+a)^2a^3-50\,(ia\tan(dx+c)+a)a^4+32\,a^5\right)}{(ia\tan(dx+c)+a)^{\frac{5}{2}}-4\,(ia\tan(dx+c)+a)^{\frac{3}{2}}a+4\,\sqrt{ia\tan(dx+c)+\cdots}}\cdots}{128\,ad}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/128*I*(15*sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
+ a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(15*(I*a*tan(d*x
 + c) + a)^2*a^3 - 50*(I*a*tan(d*x + c) + a)*a^4 + 32*a^5)/((I*a*tan(d*x +
 c) + a)^(5/2) - 4*(I*a*tan(d*x + c) + a)^(3/2)*a + 4*sqrt(I*a*tan(d*x + c
) + a)*a^2))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)^4\,\left(a+a\,\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\begin{aligned}&\int \cos^4(c+dx)(a+ia\tan(c\\&+dx))^{3/2}\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^4\tan\left(dx+c\right)dx\right)i\right.\\&\left.+\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^4dx\right)\end{aligned}$$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**4*tan(c + d*x),x)*i
+ int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**4,x))
```

3.299 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 247

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{105ia^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}d} + \frac{35ia^3}{128d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}} + \frac{105ia^2}{256d\sqrt{a+ia\tan(c+dx)}} - \frac{3ia^7}{16d(a+ia\tan(c+dx))^{3/2}\left(a^2-ia^2\tan(c+dx)\right)^2} - \frac{21ia^7}{64d(a+ia\tan(c+dx))^{3/2}\left(a^4-ia^4\tan(c+dx)\right)}$$

output
```
-105/512*I*a^(3/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2
^(1/2)/d+35/128*I*a^3/d/(a+I*a*tan(d*x+c))^(3/2)-1/6*I*a^6/d/(a-I*a*tan(d*
x+c))^3/(a+I*a*tan(d*x+c))^(3/2)+105/256*I*a^2/d/(a+I*a*tan(d*x+c))^(1/2)-
3/16*I*a^7/d/(a+I*a*tan(d*x+c))^(3/2)/(a^2-I*a^2*tan(d*x+c))^2-21/64*I*a^7
/d/(a+I*a*tan(d*x+c))^(3/2)/(a^4-I*a^4*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.21

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{ia^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},4,-\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{24d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I/24)*a^3*Hypergeometric2F1[-3/2, 4, -1/2, (1 + I*Tan[c + d*x])/2])/(d*(
a + I*a*Tan[c + d*x])^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 3968, 52, 52, 52, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \ 3968$$

$$-\frac{ia^7\int \frac{1}{(a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \ 52$$

$$-\frac{ia^7\left(\frac{3\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{6a(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \; 52$$

$$-\frac{ia^7\left(\frac{3\left(\frac{7\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{6a(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \; 52$$

$$-\frac{ia^7\left(\frac{3\left(\frac{7\left(\frac{5\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{6a(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^7\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{6a(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^7\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+}}{8a}}{4a}}{d}$$

$$\Big\downarrow 73$$

$$-\frac{ia^7\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\cdots}{4a}\cdots\right)}{d}$$

$$\Big\downarrow \; 219$$

$$-\frac{ia^7\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{}{4a(a-ia\tan(c+\cdots}}{4a}\cdots\right)}{d}$$

input
```
Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*a^7*(1/(6*a*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(3/2)) +
 (3*(1/(4*a*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(3/2)) + (7*(1
/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2)) + (5*(-1/3*1/(a
*(a + I*a*Tan[c + d*x])^(3/2)) + ((I*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]
])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a)))/(4*a)))/(
8*a)))/(4*a)))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(201) = 402$.

Time = 5.06 (sec) , antiderivative size = 815, normalized size of antiderivative = 3.30

method	result	size
default	Expression too large to display	815

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1536/d*cos(d*x+c)*((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/(cot(d*x
+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c)
)*2^(1/2))*(630*sin(d*x+c)^2+315*tan(d*x+c)*sin(d*x+c))+I*(630*cos(d*x+c)^
2+315*cos(d*x+c)-315)*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan
h(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c
)-cot(d*x+c))*2^(1/2))+I*(-630*cos(d*x+c)-315)*sin(d*x+c)*(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x
+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))+(630*cos(d*x+c)^2+315*cos(
d*x+c)-315)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/(cot(d*x+c)^2-2*c
ot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2)
)+I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*2^(1/2))*(630*sin(d*x+c)^2+315*tan(d*x+c)*sin(d*x+c))+(-630*c
os(d*x+c)^2-315*cos(d*x+c)+315)*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))+(630*cos(d*x+
c)+315)*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))+I*(630*cos(d*x+c)^2+315*cos(d*x+c)-31
5)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*2^(1/2))+I*sin(d*x+c)^2*cos(d*x+c)*(384*cos(d*x+c)^2+840)+sin(
d*x+c)*(-128*cos(d*x+c)^4-504*cos(d*x+c)^2+630)+sin(d*x+c)*cos(d*x+c)^2*(3
84*cos(d*x+c)^2+840)+I*cos(d*x+c)*(128*cos(d*x+c)^4+504*cos(d*x+c)^2-63...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.26

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx =$$
$$-\frac{\left(315\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(3i\,dx+3i\,c)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}-a^2e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{a}\right)-315\right.}{}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/1536*(315*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2
)*sqrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1)) - a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - 315*sqrt(1
/2)*sqrt(-a^3/d^2)*d*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e
^(2*I*d*x + 2*I*c) - I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
 - a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - sqrt(2)*(-8*I*a*e^(10*I*d*x
+ 10*I*c) - 58*I*a*e^(8*I*d*x + 8*I*c) - 215*I*a*e^(6*I*d*x + 6*I*c) + 43*
I*a*e^(4*I*d*x + 4*I*c) + 224*I*a*e^(2*I*d*x + 2*I*c) + 16*I*a)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1)))*e^(-3*I*d*x - 3*I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.86

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{i\left(315\sqrt{2}a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(315\,(i\,a\tan(dx+c)+a)^4a^3-1680\,(i\,a\tan(dx+c)+a)^3a^4+2772\,(\cdots\right.}{(i\,a\tan(dx+c)+a)^{\frac{9}{2}}-6\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a+12\,(i\,a\cdots}\right.}{3072\,ad}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/3072*I*(315*sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c
) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(315*(I*a*tan(
d*x + c) + a)^4*a^3 - 1680*(I*a*tan(d*x + c) + a)^3*a^4 + 2772*(I*a*tan(d*
x + c) + a)^2*a^5 - 1152*(I*a*tan(d*x + c) + a)*a^6 - 256*a^7)/((I*a*tan(d
*x + c) + a)^(9/2) - 6*(I*a*tan(d*x + c) + a)^(7/2)*a + 12*(I*a*tan(d*x +
c) + a)^(5/2)*a^2 - 8*(I*a*tan(d*x + c) + a)^(3/2)*a^3))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)^6\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^6\tan(dx+c)\,dx\right)i + \int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^6\,dx\right)$$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**6*tan(c + d*x),x)*i
+ int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**6,x))
```

3.300 $\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\frac{256ia^4\sec^5(c+dx)}{1155d(a+ia\tan(c+dx))^{5/2}}+\frac{64ia^3\sec^5(c+dx)}{231d(a+ia\tan(c+dx))^{3/2}}+\frac{8ia^2\sec^5(c+dx)}{33d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

output
```
256/1155*I*a^4*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(5/2)+64/231*I*a^3*sec(d*
x+c)^5/d/(a+I*a*tan(d*x+c))^(3/2)+8/33*I*a^2*sec(d*x+c)^5/d/(a+I*a*tan(d*x
+c))^(1/2)+2/11*I*a*sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.74

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\frac{2a\sec^4(c+dx)(\cos(dx)-i\sin(dx))(i\cos(3c+2dx)+\sin(3c+2dx))(39+494\cos(2(c+dx

input
```
Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*a*Sec[c + d*x]^4*(Cos[d*x] - I*Sin[d*x])*(I*Cos[3*c + 2*d*x] + Sin[3*c
+ 2*d*x])*(39 + 494*Cos[2*(c + d*x)] + (215*I)*Sec[c + d*x]*Sin[3*(c + d*x
)] + (110*I)*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(1155*d)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3975, 3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^5(a+ia\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{11}a\int \sec^5(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{11}a\int \sec(c+dx)^5\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{11}a\left(\frac{8}{9}a\int \frac{\sec^5(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{11}a\left(\frac{8}{9}a\int \frac{\sec(c+dx)^5}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{11}a\left(\frac{8}{9}a\left(\frac{4}{7}a\int \frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{11}a\left(\frac{8}{9}a\left(\frac{4}{7}a\int \frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

$$\Big\downarrow \text{3974}$$

$$\frac{12}{11}a\left(\frac{8}{9}a\left(\frac{8ia^2\sec^5(c+dx)}{35d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

input
```
Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((2*I)/11)*a*Sec[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]])/d + (12*a*((((2*I
)/9)*a*Sec[c + d*x]^5)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (8*a*((((8*I)/35)*
a^2*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((2*I)/7)*a*Sec[c
+ d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(3/2))))/9))/11
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.68

method	result
default	$\frac{\left(\frac{2i\left(105\sec(dx+c)^5-20\sec(dx+c)^3+512\cos(dx+c)-64\sec(dx+c)\right)}{1155}+\frac{8\tan(dx+c)\sec(dx+c)^3}{33}+\frac{128\sec(dx+c)\tan(dx+c)}{385}+\frac{1024\sin(dx+c)}{1155}\right)a\sqrt{}}{d}$

input
```
int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/1155*I*(105*sec(d*x+c)^5-20*sec(d*x+c)^3+512*cos(d*x+c)-64*sec(d*x+
c))+8/33*tan(d*x+c)*sec(d*x+c)^3+128/385*sec(d*x+c)*tan(d*x+c)+1024/1155*s
in(d*x+c))*a*(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.85

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{64\sqrt{2}\left(-231i\,ae^{(6i\,dx+6i\,c)}-198i\,ae^{(4i\,dx+4i\,c)}-88i\,ae^{(2i\,dx+2i\,c)}-16i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{1155\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-64/1155*sqrt(2)*(-231*I*a*e^(6*I*d*x + 6*I*c) - 198*I*a*e^(4*I*d*x + 4*I*
c) - 88*I*a*e^(2*I*d*x + 2*I*c) - 16*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6
*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan(c+dx)-i))^{\frac{3}{2}}\sec^5(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**5, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 994 vs. 2(115) = 230.

Time = 9.30 (sec) , antiderivative size = 994, normalized size of antiderivative = 6.76

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
64/1155*(231*I*sqrt(2)*a*cos(6*d*x + 6*c) + 198*I*sqrt(2)*a*cos(4*d*x + 4*
c) + 88*I*sqrt(2)*a*cos(2*d*x + 2*c) - 231*sqrt(2)*a*sin(6*d*x + 6*c) - 19
8*sqrt(2)*a*sin(4*d*x + 4*c) - 88*sqrt(2)*a*sin(2*d*x + 2*c) + 16*I*sqrt(2
)*a)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sqrt(a)/(((4*cos(2*d*x + 2*c)^3 + (4*cos(2*d*x + 2*c) + 1)*sin(2*d*x +
 2*c)^2 + 4*I*sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + 4*(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 6*(cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
 9*cos(2*d*x + 2*c)^2 + (I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I
*cos(2*d*x + 2*c) + I)*sin(8*d*x + 8*c) + 4*(I*cos(2*d*x + 2*c)^2 + I*sin(
2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(6*d*x + 6*c) + 6*(I*cos(2*d
*x + 2*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(4*d*x +
 4*c) + 4*(I*cos(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(2*d*x + 2*
c) + 6*cos(2*d*x + 2*c) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) + (4*I*cos(2*d*x + 2*c)^3 + (4*I*cos(2*d*x + 2*c) + I)*sin(2*d
*x + 2*c)^2 - 4*sin(2*d*x + 2*c)^3 + (I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x +
 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*cos(8*d*x + 8*c) + 4*(I*cos(2*d*x + 2*
c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*cos(6*d*x + 6*c) +
 6*(I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) ...
```

Giac [F(-2)]

Exception generated.

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 4.66 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.99

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{a\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{5\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} - \frac{a\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,192\mathrm{i}}{7\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} + \frac{a\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4} - \frac{a\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{11\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^5,x)
```

output
```
(a*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
 + d*x*2i) + 1))^(1/2)*64i)/(5*d*(exp(c*2i + d*x*2i) + 1)^2) - (a*exp(- c*
1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i)
+ 1))^(1/2)*192i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3) + (a*exp(- c*1i - d*x*1
i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*64i)/(3*d*(exp(c*2i + d*x*2i) + 1)^4) - (a*exp(- c*1i - d*x*1i)*(a - (a*
(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(11*
d*(exp(c*2i + d*x*2i) + 1)^5)
```

Reduce [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2\sqrt{a}\,ai\left(-\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^5+6\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^5\tan(dx+c)\,dx\right)d\right)}{d}$$

input
```
int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(2*sqrt(a)*a*i*( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**5 + 6*int(sqrt(t
an(c + d*x)*i + 1)*sec(c + d*x)**5*tan(c + d*x),x)*d))/d
```

3.301 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{64ia^3\sec^3(c+dx)}{105d(a+ia\tan(c+dx))^{3/2}} + \frac{16ia^2\sec^3(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} + \frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

output
```
64/105*I*a^3*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(3/2)+16/35*I*a^2*sec(d*x+c
)^3/d/(a+I*a*tan(d*x+c))^(1/2)+2/7*I*a*sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/
2)/d
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2a\sec^3(c+dx)(\cos(dx)-i\sin(dx))(28+43\cos(2(c+dx))+27i\sin(2(c+dx)))(i\cos(2c+ \cdots}{105d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*a*Sec[c + d*x]^3*(Cos[d*x] - I*Sin[d*x])*(28 + 43*Cos[2*(c + d*x)] + (2
7*I)*Sin[2*(c + d*x)])*(I*Cos[2*c + d*x] + Sin[2*c + d*x])*Sqrt[a + I*a*Ta
n[c + d*x]])/(105*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3(a+ia\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{7}a\int \sec^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{7}a\int \sec(c+dx)^3\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{7}a\left(\frac{4}{5}a\int \frac{\sec^3(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{7}a\left(\frac{4}{5}a\int \frac{\sec(c+dx)^3}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8}{7}a\left(\frac{8ia^2\sec^3(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

input
```
Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((2*I)/7)*a*Sec[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (8*a*((((8*I)/
15)*a^2*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((2*I)/5)*a*Se
c[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]])))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] && !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\left(\frac{2i\left(15\sec(dx+c)^3+64\cos(dx+c)-8\sec(dx+c)\right)}{105}+\frac{16\sec(dx+c)\tan(dx+c)}{35}+\frac{128\sin(dx+c)}{105}\right)a\sqrt{a(1+i\tan(dx+c))}}{d}$	74

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/105*I*(15*sec(d*x+c)^3+64*cos(d*x+c)-8*sec(d*x+c))+16/35*sec(d*x+c)
*tan(d*x+c)+128/105*sin(d*x+c))*a*(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.81

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{16\sqrt{2}\left(-35i\,ae^{(4i\,dx+4i\,c)}-28i\,ae^{(2i\,dx+2i\,c)}-8i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{105\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-16/105*sqrt(2)*(-35*I*a*e^(4*I*d*x + 4*I*c) - 28*I*a*e^(2*I*d*x + 2*I*c)
- 8*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^
(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan(c+dx)-i))^{\frac{3}{2}}\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(86) = 172$.

Time = 0.47 (sec) , antiderivative size = 580, normalized size of antiderivative = 5.27

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
16/105*(35*I*sqrt(2)*a*cos(4*d*x + 4*c) + 28*I*sqrt(2)*a*cos(2*d*x + 2*c)
- 35*sqrt(2)*a*sin(4*d*x + 4*c) - 28*sqrt(2)*a*sin(2*d*x + 2*c) + 8*I*sqrt
(2)*a)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*sqrt(a)/(((2*cos(2*d*x + 2*c)^3 + (2*cos(2*d*x + 2*c) + 1)*sin(2*d*x
 + 2*c)^2 + 2*I*sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 5*cos(2*d*x + 2*c)^2 + (I
*cos(2*d*x + 2*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin
(4*d*x + 4*c) + 2*(I*cos(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(2*
d*x + 2*c) + 4*cos(2*d*x + 2*c) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + (2*I*cos(2*d*x + 2*c)^3 + (2*I*cos(2*d*x + 2*c) + I)
*sin(2*d*x + 2*c)^2 - 2*sin(2*d*x + 2*c)^3 + (I*cos(2*d*x + 2*c)^2 + I*sin
(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*cos(4*d*x + 4*c) + 5*I*cos(2*d
*x + 2*c)^2 - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)*sin(4*d*x + 4*c) - 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
sin(2*d*x + 2*c) + 4*I*cos(2*d*x + 2*c) + I)*sin(3/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)))*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{16\,a\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,28\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,35\mathrm{i}+8\mathrm{i}\right)}{105\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3}$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^3,x)
```

output
```
(16*a*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c
*2i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*28i + exp(c*4i + d*x*4i)*35i
 + 8i))/(105*d*(exp(c*2i + d*x*2i) + 1)^3)
```

Reduce [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\frac{2\sqrt{a}\,ai\left(-\sqrt{\tan\left(dx+c\right)i+1}\,\sec\left(dx+c\right)^3+4\left(\int\sqrt{\tan\left(dx+c\right)i+1}\,\sec\left(dx+c\right)^3\tan\left(dx\right.\right.\right.}{d}$$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(2*sqrt(a)*a*i*( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**3 + 4*int(sqrt(t
an(c + d*x)*i + 1)*sec(c + d*x)**3*tan(c + d*x),x)*d))/d
```

3.302 $\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 69

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{8ia^2\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}} + \frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

output
```
8/3*I*a^2*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+2/3*I*a*sec(d*x+c)*(a+I*a*
tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2a(\cos(c)-i\sin(c))(\cos(dx)-i\sin(dx))(-5i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{3d}$$

input
```
Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(-2*a*(Cos[c] - I*Sin[c])*(Cos[d*x] - I*Sin[d*x])*(-5*I + Tan[c + d*x])*Sq
rt[a + I*a*Tan[c + d*x]])/(3*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{3}a\int \sec(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a\int \sec(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8ia^2\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

input
```
Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((8*I)/3)*a^2*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((2*I)/3)*a
*Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\left(\frac{2i(4\cos(dx+c)+\sec(dx+c))}{3}+\frac{8\sin(dx+c)}{3}\right)\sqrt{a(1+i\tan(dx+c))}\,a}{d}$	48

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/3*I*(4*cos(d*x+c)+sec(d*x+c))+8/3*sin(d*x+c))*(a*(1+I*tan(d*x+c)))^
(1/2)*a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{4\sqrt{2}\left(-3i\,ae^{(2i\,dx+2i\,c)}-2i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-4/3*sqrt(2)*(-3*I*a*e^(2*I*d*x + 2*I*c) - 2*I*a)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\sec{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.42

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2\,a\,\sqrt{\frac{a\,\left(2\cos(c+d\,x)^2+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{2\cos(c+d\,x)^2}}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,8\mathrm{i}+\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2\,2\mathrm{i}+\sin\left(c+d\,x\right)+\sin\right.}{3\,d\cos\left(c+d\,x\right)^2}$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x),x)
```

output
```
(2*a*((a*(sin(2*c + 2*d*x)*1i + 2*cos(c + d*x)^2))/(2*cos(c + d*x)^2))^(1/
2)*(sin(c + d*x) + sin(3*c + 3*d*x) + cos(c/2 + (d*x)/2)^2*8i + cos((3*c)/
2 + (3*d*x)/2)^2*2i - 5i))/(3*d*cos(c + d*x)^2)
```

Reduce [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2\sqrt{a}\,ai\left(-\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)+2\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)\tan(dx -\right.\right.}{d}$$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(2*sqrt(a)*a*i*( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x) + 2*int(sqrt(tan(
c + d*x)*i + 1)*sec(c + d*x)*tan(c + d*x),x)*d))/d
```

3.303 $\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 31

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

output
```
-2*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-2*I)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3974}$$

$$-\frac{2ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-2*I)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Maple [A] (verified)

Time = 3.86 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{2i\cos(dx+c)\sqrt{a(1+i\tan(dx+c))}\,a}{d}$	29

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I/d*cos(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{\sqrt{2}\left(-i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
sqrt(2)*(-I*a*e^(2*I*d*x + 2*I*c) - I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
/d
```

Sympy [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int \left(ia(\tan{(c+dx)}-i)\right)^{\frac{3}{2}}\cos{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*cos(c + d*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(25) = 50$.

Time = 0.19 (sec) , antiderivative size = 201, normalized size of antiderivative = 6.48

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2\left(ia^{\frac{3}{2}} - \frac{2i\,a^{\frac{3}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{i\,a^{\frac{3}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - 1\right)^{\frac{3}{2}}}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{3}{2}}\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1} - \frac{2i\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - 1\right)}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
2*(I*a^(3/2) - 2*I*a^(3/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + I*a^(3/2)
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2)/(d*(sin(d*x + c)/(cos(d
*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)*(-2*I*
sin(d*x + c)/(cos(d*x + c) + 1) - 2*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
+ sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1))
```

Giac [F(-2)]

Exception generated.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{a\left(2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\sqrt{\frac{a\left(2\cos(c+d\,x)^2+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{2\cos(c+d\,x)^2}}\,2\mathrm{i}}{d}$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
-(a*(2*cos(c/2 + (d*x)/2)^2 - 1)*((a*(sin(2*c + 2*d*x)*1i + 2*cos(c + d*x)
^2))/(2*cos(c + d*x)^2))^(1/2)*2i)/d
```

Reduce [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)\tan(dx+c)\,dx\right)i+\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)\,dx\right)$$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)*tan(c + d*x),x)*i + i
nt(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x),x))
```

3.304 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 122

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{ia^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}d} - \frac{ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d} - \frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

output
```
1/4*I*a^(3/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1
/2))*2^(1/2)/d-1/2*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/3*I*cos(d*x
+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{iae^{-i(c+dx)}\left(4+5e^{2i(c+dx)}+e^{4i(c+dx)}-3\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{12d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-1/12*I)*a*(4 + 5*E^((2*I)*(c + d*x)) + E^((4*I)*(c + d*x)) - 3*Sqrt[1 +
 E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a
*Tan[c + d*x]])/(d*E^(I*(c + d*x)))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 3971, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3970}$$

$$\frac{1}{2}a\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}a\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-1/3*I)*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d + (a*((I*Sqrt[a]*
ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqr
t[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 734 vs. $2(97) = 194$.

Time = 4.89 (sec) , antiderivative size = 735, normalized size of antiderivative = 6.02

method	result
default	$-\frac{\cos(dx+c)\left(\operatorname{arctanh}\left(\frac{(\csc(dx+c)-\cot(dx+c))\sqrt{2}}{\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(-6\sin(dx+c)^2-3\tan(dx+c)\sin(dx+\right.\right.}{}$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/d*cos(d*x+c)*(arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*
x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(-6*sin(d*x+c)^2-3*tan(d*x+c)*sin(d*x+c))+I*(-6*cos(d*x+c)^2-3*c
os(d*x+c)+3)*tan(d*x+c)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+cs
c(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)+I*(6*cos(d*x+c)+3)*sin(d*x+c)*arctanh(1/(cot(d*x+c)^2-2*cot(
d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(-6*cos(d*x+c)^2-3*cos(d*x+c)+3)*arctanh
(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)
-cot(d*x+c))*2^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*arctan(1/2*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(6*sin(d*x+c)^2+3*tan(d*x+c)*sin(d*x+c))+(-6*cos(d*x+c)^2-3*cos(d*x+c)+
3)*tan(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)+(6*cos(d*x+c)+3)*sin(d*x+c)*arctan(1/2*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)+I*(6*cos(d*x+c)^2+3*cos(d*x+c)-3)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*2^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+6*I*sin(d*x+c)^2+1
0*I*cos(d*x+c)^2-4*cos(d*x+c)*sin(d*x+c))*a*(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(91) = 182$.

Time = 0.08 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.82

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}d\log\left(\frac{\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}+i\,a^2\right)e^{(-i\,dx-i\,c)}}{d}\right)-3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}d\log}{}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/12*(3*sqrt(1/2)*sqrt(-a^3/d^2)*d*log((sqrt(2)*sqrt(1/2)*(d*e^(2*I*d*x +
2*I*c) + d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) + I*a^2)*e^(-
I*d*x - I*c)/d) - 3*sqrt(1/2)*sqrt(-a^3/d^2)*d*log(-(sqrt(2)*sqrt(1/2)*(d*
e^(2*I*d*x + 2*I*c) + d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
- I*a^2)*e^(-I*d*x - I*c)/d) + sqrt(2)*(-I*a*e^(4*I*d*x + 4*I*c) - 5*I*a*e
^(2*I*d*x + 2*I*c) - 4*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(91) = 182$.

Time = 0.28 (sec) , antiderivative size = 884, normalized size of antiderivative = 7.25

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/48*(4*(I*sqrt(2)*a*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - sqrt(2)*a*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))
*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*
sqrt(a) + 12*(I*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) - sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sqrt(a) + 3*(2*sqrt(2)*a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) + 1) - 2*sqrt(2)*a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) -
 I*sqrt(2)*a*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 +
 sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*a*log(sqrt(...
```

Giac [F(-2)]

Exception generated.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)^3\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2} \,d x$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\begin{aligned}\int \cos^3(c+dx)(a+ia\tan(c & \\ +dx))^{3/2}\,dx &= \sqrt{a}\,a\left(\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^{3}\tan\left(dx+c\right)dx\right)i \right.\\ &\quad \left.+\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^{3}dx\right)\end{aligned}$$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**3*tan(c + d*x),x)*i
+ int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**3,x))
```

3.305 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 192

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{7ia^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}d} + \frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}} - \frac{7ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16d} - \frac{7ia\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{30d} - \frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

output
```
7/32*I*a^(3/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(
1/2))*2^(1/2)/d+7/24*I*a^2*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-7/16*I*a*
cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-7/30*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x
+c))^(1/2)/d-1/5*I*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.83

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx =$$
$$-\frac{iae^{-3i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-15+101e^{2i(c+dx)}+148e^{4i(c+dx)}+38e^{6i(c+dx)}+6e^{8i(c+dx)}-105e^{2i(c+dx)}\sqrt{1+e\right.}{240\sqrt{2}d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-1/240*I)*a*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-15
 + 101*E^((2*I)*(c + d*x)) + 148*E^((4*I)*(c + d*x)) + 38*E^((6*I)*(c + d*
x)) + 6*E^((8*I)*(c + d*x)) - 105*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*E^((3*I)*(c
+ d*x)))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {3042, 3978, 3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{7}{10}a\int \cos^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^3}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{7}{10}a\left(\frac{5}{6}a\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}a\left(\frac{5}{6}a\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3983}$$

$$\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{}}{}\right.$$

$$\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3970}$$

$$\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{ia\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3}{}\right.$$

$$\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{219}$$

$$\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c}{}\right.$$

$$\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-1/5*I)*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2))/d + (7*a*(((-1/3*I)
*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (5*a*(((I/2)*Cos[c + d*x])
/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c +
d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]
*Sqrt[a + I*a*Tan[c + d*x]])/d))/(4*a)))/6))/10
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 794 vs. $2(155) = 310$.

Time = 4.83 (sec) , antiderivative size = 795, normalized size of antiderivative = 4.14

method	result	size
default	Expression too large to display	795

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/480/d*cos(d*x+c)*(arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*
x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(210*sin(d*x+c)^2+105*tan(d*x+c)*sin(d*x+c))+I*(210*cos(d*x+c)^2
+105*cos(d*x+c)-105)*tan(d*x+c)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d
*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)+I*(-210*cos(d*x+c)-105)*sin(d*x+c)*arctanh(1/(cot(d*
x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c
))*2^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(210*cos(d*x+c)^2+105*cos(d
*x+c)-105)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)
^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
+I*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))*(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(-210*sin(d*x+c)^2-105*tan(d*x+c)*sin(d*x+c))+(210*co
s(d*x+c)^2+105*cos(d*x+c)-105)*tan(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*2^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(-210*cos(d*x+
c)-105)*sin(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)
)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*(-210*cos(d*x+c)^2-105*cos(d*x+c)+1
05)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))*(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)+I*sin(d*x+c)^2*(168*cos(d*x+c)^2-210)+sin(d*x+c)*cos
(d*x+c)*(-72*cos(d*x+c)^2+350)+sin(d*x+c)*cos(d*x+c)*(168*cos(d*x+c)^2-210
)+I*cos(d*x+c)^2*(72*cos(d*x+c)^2-350))*a*(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.43

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{\left(105\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(2i\,dx+2i\,c)}\log\left(\frac{7\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}+i\,a^2\right)e^{(-i\,dx-i\,c)}}{8\,d}\right)-10\cdots}{\cdots}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/480*(105*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(2*I*d*x + 2*I*c)*log(7/8*(sqrt(2)
*sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1)) + I*a^2)*e^(-I*d*x - I*c)/d) - 105*sqrt(1/2)*sqrt(-a^3/d^2)*
d*e^(2*I*d*x + 2*I*c)*log(-7/8*(sqrt(2)*sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) +
 d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) - I*a^2)*e^(-I*d*x -
I*c)/d) + sqrt(2)*(-6*I*a*e^(8*I*d*x + 8*I*c) - 38*I*a*e^(6*I*d*x + 6*I*c)
 - 148*I*a*e^(4*I*d*x + 4*I*c) - 101*I*a*e^(2*I*d*x + 2*I*c) + 15*I*a)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)^5\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2} \,d x$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^5\tan(dx+c)\,dx\right)i + \int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^5\,dx\right)$$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**5*tan(c + d*x),x)*i
+ int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**5,x))
```

3.306 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{16i(a+ia\tan(c+dx))^{13/2}}{13a^4d}+\frac{8i(a+ia\tan(c+dx))^{15/2}}{5a^5d} -\frac{12i(a+ia\tan(c+dx))^{17/2}}{17a^6d}+\frac{2i(a+ia\tan(c+dx))^{19/2}}{19a^7d}$$

output
```
-16/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^4/d+8/5*I*(a+I*a*tan(d*x+c))^(15/2)/a
^5/d-12/17*I*(a+I*a*tan(d*x+c))^(17/2)/a^6/d+2/19*I*(a+I*a*tan(d*x+c))^(19
/2)/a^7/d
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2a^2(-i+\tan(c+dx))^6\sqrt{a+ia\tan(c+dx)}(-2429i-5291\tan(c+dx)+4095i\tan^2(c+dx)+1105\,\mathrm{ta}}{20995d}$$

input
```
Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*(-I + Tan[c + d*x])^6*Sqrt[a + I*a*Tan[c + d*x]]*(-2429*I - 5291*T
an[c + d*x] + (4095*I)*Tan[c + d*x]^2 + 1105*Tan[c + d*x]^3))/(20995*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^8(a+ia\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{11/2}d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(-(i\tan(c+dx)a+a)^{17/2}+6a(i\tan(c+dx)a+a)^{15/2}-12a^2(i\tan(c+dx)a+a)^{13/2}+8a^3(i\tan(c+dx)\right.}{a^7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{16}{13}a^3(a+ia\tan(c+dx))^{13/2}-\frac{8}{5}a^2(a+ia\tan(c+dx))^{15/2}-\frac{2}{19}(a+ia\tan(c+dx))^{19/2}+\frac{12}{17}a(a+ia\tan(c\right.}{a^7d}$$

input
```
Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*((16*a^3*(a + I*a*Tan[c + d*x])^(13/2))/13 - (8*a^2*(a + I*a*Tan[c +
 d*x])^(15/2))/5 + (12*a*(a + I*a*Tan[c + d*x])^(17/2))/17 - (2*(a + I*a*T
an[c + d*x])^(19/2))/19))/(a^7*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{19}{2}}}{19}-\frac{6a(a+ia\tan(dx+c))^{\frac{17}{2}}}{17}+\frac{4a^2(a+ia\tan(dx+c))^{\frac{15}{2}}}{5}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}\right)}{d\,a^7}$$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
2*I/d/a^7*(1/19*(a+I*a*tan(d*x+c))^(19/2)-6/17*a*(a+I*a*tan(d*x+c))^(17/2)
+4/5*a^2*(a+I*a*tan(d*x+c))^(15/2)-8/13*a^3*(a+I*a*tan(d*x+c))^(13/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(85) = 170$.

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.62

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{1024\sqrt{2}\left(16i\,a^2e^{(19i\,dx+19i\,c)}+152i\,a^2e^{(17i\,dx+17i\,c)}+646i\,a^2e^{(15i\,dx+15i\,c)}+16\right.}{20995\left(de^{(18i\,dx+18i\,c)}+9\,de^{(16i\,dx+16i\,c)}+36\,de^{(14i\,dx+14i\,c)}+84\,de^{(12i\,dx+12i\,c)}+126\,de^{(10i\,dx+10i\,c)}+126\,de^{(\right.}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1024/20995*sqrt(2)*(16*I*a^2*e^(19*I*d*x + 19*I*c) + 152*I*a^2*e^(17*I*d*
x + 17*I*c) + 646*I*a^2*e^(15*I*d*x + 15*I*c) + 1615*I*a^2*e^(13*I*d*x + 1
3*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(18*I*d*x + 18*I*c) + 9*d*e
^(16*I*d*x + 16*I*c) + 36*d*e^(14*I*d*x + 14*I*c) + 84*d*e^(12*I*d*x + 12*
I*c) + 126*d*e^(10*I*d*x + 10*I*c) + 126*d*e^(8*I*d*x + 8*I*c) + 84*d*e^(6
*I*d*x + 6*I*c) + 36*d*e^(4*I*d*x + 4*I*c) + 9*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2i\left(1105\,(i\,a\tan\left(dx+c\right)+a)^{\frac{19}{2}}-7410\,(i\,a\tan\left(dx+c\right)+a)^{\frac{17}{2}}a+16796\,(i\,a\tan\left(dx+c\right)+\right.}{20995\,a^7d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2/20995*I*(1105*(I*a*tan(d*x + c) + a)^(19/2) - 7410*(I*a*tan(d*x + c) + a
)^(17/2)*a + 16796*(I*a*tan(d*x + c) + a)^(15/2)*a^2 - 12920*(I*a*tan(d*x
+ c) + a)^(13/2)*a^3)/(a^7*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 12.90 (sec) , antiderivative size = 626, normalized size of antiderivative = 5.35

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^8,x)
```

output
```
(a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*536576i)/(4199*d*(exp(c*2i + d*x*2i) + 1)^4) - (a^2*(a - (a*(exp(c*2i
+ d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*8192i)/(20995*d*(ex
p(c*2i + d*x*2i) + 1)) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(ex
p(c*2i + d*x*2i) + 1))^(1/2)*6144i)/(20995*d*(exp(c*2i + d*x*2i) + 1)^2) -
 (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(
1/2)*1024i)/(4199*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*(a - (a*(exp(c*2i +
 d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16384i)/(20995*d) -
(a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*10484736i)/(20995*d*(exp(c*2i + d*x*2i) + 1)^5) + (a^2*(a - (a*(exp(c*
2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*17262592i)/(2099
5*d*(exp(c*2i + d*x*2i) + 1)^6) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i
)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1129472i)/(1615*d*(exp(c*2i + d*x*2i
) + 1)^7) + (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*
2i) + 1))^(1/2)*98304i)/(323*d*(exp(c*2i + d*x*2i) + 1)^8) - (a^2*(a - (a*
(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(1
9*d*(exp(c*2i + d*x*2i) + 1)^9)
```

Reduce [F]

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\sqrt{a}\,a^2\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^8\,i-\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^8\tan(dx$$

$$d$$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x)
```

output

```
(sqrt(a)*a**2*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**8*i - int(sqrt(
tan(c + d*x)*i + 1)*sec(c + d*x)**8*tan(c + d*x)**2,x)*d + 19*int(sqrt(tan
(c + d*x)*i + 1)*sec(c + d*x)**8*tan(c + d*x),x)*d*i))/d
```

3.307 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^3d} + \frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^4d} - \frac{2i(a+ia\tan(c+dx))^{15/2}}{15a^5d}$$

output
```
-8/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^3/d+8/13*I*(a+I*a*tan(d*x+c))^(13/2)/a
^4/d-2/15*I*(a+I*a*tan(d*x+c))^(15/2)/a^5/d
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2a^2(-i+\tan(c+dx))^5\sqrt{a+ia\tan(c+dx)}(-263+374i\tan(c+dx)+143\tan^2(c+dx))}{2145d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*(-I + Tan[c + d*x])^5*Sqrt[a + I*a*Tan[c + d*x]]*(-263 + (374*I)*T
an[c + d*x] + 143*Tan[c + d*x]^2))/(2145*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+ia\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{9/2}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left((i\tan(c+dx)a+a)^{13/2}-4a(i\tan(c+dx)a+a)^{11/2}+4a^2(i\tan(c+dx)a+a)^{9/2}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{8}{11}a^2(a+ia\tan(c+dx))^{11/2}+\frac{2}{15}(a+ia\tan(c+dx))^{15/2}-\frac{8}{13}a(a+ia\tan(c+dx))^{13/2}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*((8*a^2*(a + I*a*Tan[c + d*x])^(11/2))/11 - (8*a*(a + I*a*Tan[c + d*
x])^(13/2))/13 + (2*(a + I*a*Tan[c + d*x])^(15/2))/15))/(a^5*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{15}{2}}}{15}+\frac{4a(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{4a^{2}(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}\right)}{d\,a^{5}}$$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
2*I/d/a^5*(-1/15*(a+I*a*tan(d*x+c))^(15/2)+4/13*a*(a+I*a*tan(d*x+c))^(13/2
)-4/11*a^2*(a+I*a*tan(d*x+c))^(11/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(64) = 128$.

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.73

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{256\,\sqrt{2}\left(8i\,a^2e^{(15i\,dx+15i\,c)}+60i\,a^2e^{(13i\,dx+13i\,c)}+195i\,a^2e^{(11i\,dx+11i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{2145\left(de^{(14i\,dx+14i\,c)}+7\,de^{(12i\,dx+12i\,c)}+21\,de^{(10i\,dx+10i\,c)}+35\,de^{(8i\,dx+8i\,c)}+35\,de^{(6i\,dx+6i\,c)}+21\,de^{(4i\,dx+4i\,c)}\cdots\right)}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-256/2145*sqrt(2)*(8*I*a^2*e^(15*I*d*x + 15*I*c) + 60*I*a^2*e^(13*I*d*x +
13*I*c) + 195*I*a^2*e^(11*I*d*x + 11*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))/(d*e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x
 + 10*I*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^
(4*I*d*x + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2i\left(143\,(i\,a\tan\left(dx+c\right)+a)^{\frac{15}{2}}-660\,(i\,a\tan\left(dx+c\right)+a)^{\frac{13}{2}}a+780\,(i\,a\tan\left(dx+c\right)+a)^{\frac{11}{2}}a^2\right)}{2145\,a^5d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-2/2145*I*(143*(I*a*tan(d*x + c) + a)^(15/2) - 660*(I*a*tan(d*x + c) + a)^
(13/2)*a + 780*(I*a*tan(d*x + c) + a)^(11/2)*a^2)/(a^5*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 498, normalized size of antiderivative = 5.66

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^6,x)
```

output
```
(a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*18176i)/(429*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*(a - (a*(exp(c*2i +
d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(2145*d*(exp(c
*2i + d*x*2i) + 1)) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c
*2i + d*x*2i) + 1))^(1/2)*256i)/(715*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^2*
(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2
048i)/(2145*d) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
 d*x*2i) + 1))^(1/2)*52736i)/(429*d*(exp(c*2i + d*x*2i) + 1)^4) + (a^2*(a
- (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1039
36i)/(715*d*(exp(c*2i + d*x*2i) + 1)^5) - (a^2*(a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*15616i)/(195*d*(exp(c*2i + d
*x*2i) + 1)^6) + (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
 d*x*2i) + 1))^(1/2)*256i)/(15*d*(exp(c*2i + d*x*2i) + 1)^7)
```

Reduce [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\sqrt{a}\,a^2\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^6\,i-\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^6\tan(dx$$ $$d$$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*a**2*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**6*i - int(sqrt(
tan(c + d*x)*i + 1)*sec(c + d*x)**6*tan(c + d*x)**2,x)*d + 15*int(sqrt(tan
(c + d*x)*i + 1)*sec(c + d*x)**6*tan(c + d*x),x)*d*i))/d
```

3.308 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{4i(a+ia\tan(c+dx))^{9/2}}{9a^2d}+\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^3d}$$

output
```
-4/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^2/d+2/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^3
/d
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2a^2(-i+\tan(c+dx))^4(13i+9\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{99d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*(-I + Tan[c + d*x])^4*(13*I + 9*Tan[c + d*x])*Sqrt[a + I*a*Tan[c +
 d*x]])/(99*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{3042, 3968, 53, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4(a+ia\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(2a(i\tan(c+dx)a+a)^{7/2}-(i\tan(c+dx)a+a)^{9/2}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{4}{9}a(a+ia\tan(c+dx))^{9/2}-\frac{2}{11}(a+ia\tan(c+dx))^{11/2}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*((4*a*(a + I*a*Tan[c + d*x])^(9/2))/9 - (2*(a + I*a*Tan[c + d*x])^(1
1/2))/11))/(a^3*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 100.76 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{2a(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}\right)}{d\,a^{3}}$	44
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{2a(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}\right)}{d\,a^{3}}$	44

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^3*(1/11*(a+I*a*tan(d*x+c))^(11/2)-2/9*a*(a+I*a*tan(d*x+c))^(9/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(43) = 86$.

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.93

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{64\sqrt{2}\left(2i\,a^2e^{(11i\,dx+11i\,c)}+11i\,a^2e^{(9i\,dx+9i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{99\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-64/99*sqrt(2)*(2*I*a^2*e^(11*I*d*x + 11*I*c) + 11*I*a^2*e^(9*I*d*x + 9*I*
c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*
I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d
*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2i\left(9\,(i\,a\tan\left(dx+c\right)+a)^{\frac{11}{2}}-22\,(i\,a\tan\left(dx+c\right)+a)^{\frac{9}{2}}a\right)}{99\,a^3d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2/99*I*(9*(I*a*tan(d*x + c) + a)^(11/2) - 22*(I*a*tan(d*x + c) + a)^(9/2)*
a)/(a^3*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.76 (sec) , antiderivative size = 370, normalized size of antiderivative = 6.27

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{99\,d} - \frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{99\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} + \frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{33\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} - \frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2944\mathrm{i}}{99\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} + \frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2176\mathrm{i}}{99\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4} - \frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{11\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^4,x)
```

output
```
(a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*512i)/(33*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^2*(a - (a*(exp(c*2i + d*x
*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(99*d*(exp(c*2i + d
*x*2i) + 1)) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d
*x*2i) + 1))^(1/2)*128i)/(99*d) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i
)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2944i)/(99*d*(exp(c*2i + d*x*2i) + 1
)^3) + (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*2176i)/(99*d*(exp(c*2i + d*x*2i) + 1)^4) - (a^2*(a - (a*(exp(c*
2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(11*d*(exp(
c*2i + d*x*2i) + 1)^5)
```

Reduce [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\sqrt{a}\,a^2\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^4\,i-\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^4\tan(dx\right.\right.}{d}$$

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x)
```

output

```
(sqrt(a)*a**2*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**4*i - int(sqrt(
tan(c + d*x)*i + 1)*sec(c + d*x)**4*tan(c + d*x)**2,x)*d + 11*int(sqrt(tan
(c + d*x)*i + 1)*sec(c + d*x)**4*tan(c + d*x),x)*d*i))/d
```

3.309 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad}$$

output
```
-2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad}$$

input
```
Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((-2*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+ia\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (i\tan(c+dx)a+a)^{5/2}d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad}$$

input
```
Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((-2*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2i(a+ia\tan(dx+c))^{\frac{7}{2}}}{7ad}$	24
default	$-\frac{2i(a+ia\tan(dx+c))^{\frac{7}{2}}}{7ad}$	24

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(21) = 42.

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.52

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{16i\sqrt{2}a^2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(7i\,dx+7i\,c)}}{7\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-16/7*I*sqrt(2)*a^2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(7*I*d*x + 7*I*c)/
(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c)
 + d)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (ia(\tan{(c+dx)}-i))^{\frac{5}{2}}\sec^2{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(5/2)*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2i\,(i\,a\tan{(dx+c)}+a)^{\frac{7}{2}}}{7\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-2/7*I*(I*a*tan(d*x + c) + a)^(7/2)/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 242, normalized size of antiderivative = 8.34

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,d}+\frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,48\mathrm{i}}{7\,d\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,48\mathrm{i}}{7\,d\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^2}+\frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,d\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3}$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^2,x)
```

output
```
(a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*48i)/(7*d*(exp(c*2i + d*x*2i) + 1)) - (a^2*(a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(7*d) - (a^2*(a - (a*(e
xp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*48i)/(7*d*(
exp(c*2i + d*x*2i) + 1)^2) + (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)
/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3)
```

Reduce [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\sqrt{a}\,a^2\Big(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^2\,i-\Big(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^2\tan(dx$$
$$d$$

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*a**2*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**2*i - int(sqrt(
tan(c + d*x)*i + 1)*sec(c + d*x)**2*tan(c + d*x)**2,x)*d + 7*int(sqrt(tan(
c + d*x)*i + 1)*sec(c + d*x)**2*tan(c + d*x),x)*d*i))/d
```

3.310 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{ia^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d} - \frac{ia^3\sqrt{a+ia\tan(c+dx)}}{d(a-ia\tan(c+dx))}$$

output
```
1/2*I*a^(5/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2
)/d-I*a^3*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{ia^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d} + \frac{a^2\sqrt{a+ia\tan(c+dx)}}{d(i+\tan(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(I*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]
*d) + (a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*(I + Tan[c + d*x]))
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = $\{3042, 3968, 51, 73, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{\sqrt{i\tan(c+dx)a+a}}{(a-ia\tan(c+dx))^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{51}$$

$$-\frac{ia^3\left(\frac{\sqrt{a+ia\tan(c+dx)}}{a-ia\tan(c+dx)}-\frac{1}{2}\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))\right)}{d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{ia^3\left(\frac{\sqrt{a+ia\tan(c+dx)}}{a-ia\tan(c+dx)}-\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ia^3\left(\frac{\sqrt{a+ia\tan(c+dx)}}{a-ia\tan(c+dx)}-\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}\sqrt{a}}\right)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*a^3*(((-I)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*Sqrt[a])
 + Sqrt[a + I*a*Tan[c + d*x]]/(a - I*a*Tan[c + d*x])))/d
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(73) = 146$.

Time = 6.14 (sec) , antiderivative size = 425, normalized size of antiderivative = 4.78

method	result
default	$\frac{\cos(dx+c)^2\left((\cos(dx+c)+1)\sin(dx+c)\operatorname{arctanh}\left(\frac{(\csc(dx+c)-\cot(dx+c))\sqrt{2}}{\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+i\operatorname{arctanh}\left(\right.\right.}{}$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*cos(d*x+c)^2*((cos(d*x+c)+1)*sin(d*x+c)*arctanh(1/(cot(d*x+c)^2-2*cot(
d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*c
sc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)^2+cos(d*x+c))+I*(cos(d*x+c)+1)*sin(d
*x+c)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))*(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)+cos(d*x+c)*(-cos(d*x+c)-1)*arctan(1/2*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-I*sin
(d*x+c)*cos(d*x+c)+cos(d*x+c)*(-cos(d*x+c)-1))*(a*(1+I*tan(d*x+c)))^(1/2)*
a^2*(-tan(d*x+c)+I)^2/((-1-2*cos(d*x+c))*sin(d*x+c)+I*(-1+2*cos(d*x+c)^2+c
os(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(68) = 136$.

Time = 0.12 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.65

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\sqrt{2}\sqrt{-\frac{a^5}{d^2}}d\log\left(\frac{4\left(a^3e^{(i\,dx+i\,c)}-\sqrt{-\frac{a^5}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\right)-\sqrt{2}\sqrt{-\frac{a^5}{d^2}}d\log}{}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*sqrt(-a^5/d^2)*d*log(4*(a^3*e^(I*d*x + I*c) - sqrt(-a^5/d^2)*
(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d
*x - I*c)/a^2) - sqrt(2)*sqrt(-a^5/d^2)*d*log(4*(a^3*e^(I*d*x + I*c) - sqr
t(-a^5/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1)))*e^(-I*d*x - I*c)/a^2) - 2*sqrt(2)*(I*a^2*e^(3*I*d*x + 3*I*c) + I*a^2
*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{i\left(\sqrt{2}a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)-\frac{8\sqrt{i\,a\tan(dx+c)+a}a^4}{2i\,a\tan(dx+c)-2\,a}\right)}{4\,ad}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/4*I*(sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a)
)/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 8*sqrt(I*a*tan(d*x + c
) + a)*a^4/(2*I*a*tan(d*x + c) - 2*a))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)^2\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\begin{aligned}&\int \cos^2(c+dx)(a+ia\tan(c\\&+dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^2\tan(dx+c)^2\,dx\right)\right.\\&+2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^2\tan(dx+c)\,dx\right)i\\&\left.+\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^2\,dx\right)\end{aligned}$$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**2*tan(c + d*x)
**2,x) + 2*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**2*tan(c + d*x),x)*i
+ int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**2,x))
```

3.311 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 141

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{3ia^{5/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d} - \frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2} - \frac{3ia^5\sqrt{a+ia\tan(c+dx)}}{16d\left(a^3-ia^3\tan(c+dx)\right)}$$

output
```
-3/32*I*a^(5/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1
/2)/d-1/4*I*a^4*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))^2-3/16*I*a^5
*(a+I*a*tan(d*x+c))^(1/2)/d/(a^3-I*a^3*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.38

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{ia^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},\frac{1}{2}(1+i\tan(c+dx))\right)\sqrt{a+ia\tan(c+dx)}}{4d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-1/4*I)*a^2*Hypergeometric2F1[1/2, 3, 3/2, (1 + I*Tan[c + d*x])/2]*Sqrt[
a + I*a*Tan[c + d*x]])/d
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3968, 52, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{3\int \frac{1}{(a-ia\tan(c+dx))^2\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{8a}+\frac{\sqrt{a+ia\tan(c+dx)}}{4a(a-ia\tan(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{3\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{4a}+\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)}{8a}+\frac{\sqrt{a+ia\tan(c+dx)}}{4a(a-ia\tan(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{ia^5\left(\frac{3\left(\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{2a}+\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)}{8a}+\frac{\sqrt{a+ia\tan(c+dx)}}{4a(a-ia\tan(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ia^5\left(\frac{3\left(\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{2\sqrt{2}a^{3/2}}+\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)}{8a}+\frac{\sqrt{a+ia\tan(c+dx)}}{4a(a-ia\tan(c+dx))^2}\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*a^5*(Sqrt[a + I*a*Tan[c + d*x]]/(4*a*(a - I*a*Tan[c + d*x])^2) + (3*
(((I/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) + Sqrt[a
 + I*a*Tan[c + d*x]]/(2*a*(a - I*a*Tan[c + d*x]))))/(8*a)))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(114) = 228$.

Time = 67.12 (sec) , antiderivative size = 472, normalized size of antiderivative = 3.35

method	result
default	$-\frac{\cos(dx+c)^2\left(\sin(dx+c)\cos(dx+c)(6\cos(dx+c)+6)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \operatorname{arctanh}\left(\frac{(\csc(dx+c)-\cot(dx+c))\sqrt{2}}{\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)\right.}{}$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/d*cos(d*x+c)^2*(sin(d*x+c)*cos(d*x+c)*(6*cos(d*x+c)+6)*(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(
d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))+I*(6*cos(d*x+c)^3+6*cos
(d*x+c)^2-3*cos(d*x+c)-3)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/(co
t(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d
*x+c))*2^(1/2))+I*sin(d*x+c)*cos(d*x+c)*(6*cos(d*x+c)+6)*(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))
+(-6*cos(d*x+c)^3-6*cos(d*x+c)^2+3*cos(d*x+c)+3)*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))+I*sin(d
*x+c)*cos(d*x+c)*(4*cos(d*x+c)-3)+cos(d*x+c)*(4*cos(d*x+c)^2+7*cos(d*x+c)+
3))*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*(-tan(d*x+c)+I)^2/(-sin(d*x+c)+I*cos(d*
x+c)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(106) = 212.

Time = 0.09 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.87

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}d\log\left(\frac{4\left(a^3e^{(i\,dx+i\,c)}-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\right)-3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}d\log\left(\frac{4}{-}\right.}{}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/32*(3*sqrt(1/2)*sqrt(-a^5/d^2)*d*log(4*(a^3*e^(I*d*x + I*c) - sqrt(2)*s
qrt(1/2)*sqrt(-a^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - 3*sqrt(1/2)*sqrt(-a^5/d^2)*d*log(
4*(a^3*e^(I*d*x + I*c) - sqrt(2)*sqrt(1/2)*sqrt(-a^5/d^2)*(-I*d*e^(2*I*d*x
 + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2)
- sqrt(2)*(-2*I*a^2*e^(5*I*d*x + 5*I*c) - 7*I*a^2*e^(3*I*d*x + 3*I*c) - 5*
I*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{i\left(3\sqrt{2}a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)+\frac{4\left(3\,(ia\tan(dx+c)+a)^{\frac{3}{2}}a^4-10\sqrt{ia\tan(dx+c)+a}\,a^5\right)}{(ia\tan(dx+c)+a)^2-4\,(ia\tan(dx+c)+a)a+4\,a^2}\right)}{64\,ad}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/64*I*(3*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(3*(I*a*tan(d*x +
c) + a)^(3/2)*a^4 - 10*sqrt(I*a*tan(d*x + c) + a)*a^5)/((I*a*tan(d*x + c)
+ a)^2 - 4*(I*a*tan(d*x + c) + a)*a + 4*a^2))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)^4\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2} \,d x$$

input
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\begin{aligned}\int \cos^4(c+dx)(a+ia\tan(c &\\ +dx))^{5/2}\,dx &= \sqrt{a}\,a^{2}\left(-\left(\int \sqrt{\tan \left(dx+c\right) i+1}\,\cos \left(dx+c\right)^{4} \tan \left(dx+c\right)^{2}d x\right)\right.\\ &+2\left(\int \sqrt{\tan \left(dx+c\right) i+1}\,\cos \left(dx+c\right)^{4} \tan \left(dx+c\right)d x\right) i\\ &\left.+\int \sqrt{\tan \left(dx+c\right) i+1}\,\cos \left(dx+c\right)^{4}d x\right)\end{aligned}$$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**4*tan(c + d*x)
**2,x) + 2*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**4*tan(c + d*x),x)*i
+ int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**4,x))
```

3.312 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 218

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{35ia^{5/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}d} + \frac{35ia^3}{128d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}} - \frac{7ia^7}{48d\sqrt{a+ia\tan(c+dx)}\,(a^2-ia^2\tan(c+dx))^2} - \frac{35ia^7}{192d\sqrt{a+ia\tan(c+dx)}\,(a^4-ia^4\tan(c+dx))}$$

output
```
-35/256*I*a^(5/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^
(1/2)/d+35/128*I*a^3/d/(a+I*a*tan(d*x+c))^(1/2)-1/6*I*a^6/d/(a-I*a*tan(d*x
+c))^3/(a+I*a*tan(d*x+c))^(1/2)-7/48*I*a^7/d/(a+I*a*tan(d*x+c))^(1/2)/(a^2
-I*a^2*tan(d*x+c))^2-35/192*I*a^7/d/(a+I*a*tan(d*x+c))^(1/2)/(a^4-I*a^4*ta
n(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.24

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\frac{ia^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},4,\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{8d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/8)*a^3*Hypergeometric2F1[-1/2, 4, 1/2, (1 + I*Tan[c + d*x])/2])/(d*Sqr
t[a + I*a*Tan[c + d*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3968, 52, 52, 52, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^7\int \frac{1}{(a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^7\left(\frac{7\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{12a}+\frac{1}{6a(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

$$\Big\downarrow \textcolor{blue}{52}$$

$$-\frac{ia^7\left(\frac{7\left(\frac{5\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}\right)}{12a}+\frac{1}{6a(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(}}\right.}{d}$$

$$\Big\downarrow \textcolor{blue}{52}$$

$$-\frac{ia^7\left(\frac{7\left(\frac{5\left(\frac{3\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}\right.}{12a}\right.}{d}$$

$$\Big\downarrow \textcolor{blue}{61}$$

$$-\frac{ia^7\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{}{4a(a-ia\tan(c+d}\right.}{12a}\right.}{d}$$

$$\Big\downarrow \textcolor{blue}{73}$$

$$-\frac{ia^7\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}\right)}{12a}\right.}{d}$$

$\downarrow$ 219

$$-\frac{ia^7\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}\right)}{12a}+\right.}{d}$$

input
```
Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*a^7*(1/(6*a*(a - I*a*Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]]) + (
7*(1/(4*a*(a - I*a*Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]) + (5*(1/(2*
a*(a - I*a*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]) + (3*((I*ArcTan[(Sqrt
[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[a + I*a*Tan[c +
d*x]])))/(4*a)))/(8*a)))/(12*a)))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1314 vs. $2(178) = 356$.

Time = 2.57 (sec) , antiderivative size = 1315, normalized size of antiderivative = 6.03

Expression too large to display

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
1/768/d*cos(d*x+c)^2*(I*(-420*cos(d*x+c)^3-210*cos(d*x+c)^2+315*cos(d*x+c)
+105)*tan(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))+(420*cos(d*x+c)^3+210*cos(d*x+c)^2-31
5*cos(d*x+c)-105)*tan(d*x+c)^2*arctanh(2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(co
t(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)+(-840*cos(d*x+c)^2-420*cos(d*x+c)+210)*sin(d*x+c)*tan(d
*x+c)*arctanh(2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(cot(d*x+c)^2-2*cot(d*x+c)*c
sc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*(-42
0*cos(d*x+c)^2-210*cos(d*x+c)+105)*sin(d*x+c)*tan(d*x+c)^2*arctanh(2^(1/2)
*(cot(d*x+c)-csc(d*x+c))/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^
2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*(420*cos(d*x+c)^2+210*cos
(d*x+c)-105)*sin(d*x+c)*arctanh(2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(cot(d*x+c
)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)+(-420*cos(d*x+c)^3-210*cos(d*x+c)^2+315*cos(d*x+c)+105)*arctan
h(2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+cs
c(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(-420*cos(d*x+c)^2
-210*cos(d*x+c)+105)*sin(d*x+c)*tan(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))+I*sin(d*x+c
)^2*(896*cos(d*x+c)^2-420)+I*(420*cos(d*x+c)^3+210*cos(d*x+c)^2-315*cos(d*
x+c)-105)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.42

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{\left(105\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}de^{(i\,dx+i\,c)}\log\left(\frac{4\left(a^3e^{(i\,dx+i\,c)}-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\right)-105\sqrt{\frac{1}{2}}\cdots}{\cdots}$$

input

```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output

```
-1/768*(105*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(I*d*x + I*c)*log(4*(a^3*e^(I*d*x
 + I*c) - sqrt(2)*sqrt(1/2)*sqrt(-a^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - 105*sqrt(1/2)*
sqrt(-a^5/d^2)*d*e^(I*d*x + I*c)*log(4*(a^3*e^(I*d*x + I*c) - sqrt(2)*sqrt
(1/2)*sqrt(-a^5/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - sqrt(2)*(-8*I*a^2*e^(8*I*d*x + 8*I*
c) - 46*I*a^2*e^(6*I*d*x + 6*I*c) - 125*I*a^2*e^(4*I*d*x + 4*I*c) - 39*I*a
^2*e^(2*I*d*x + 2*I*c) + 48*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-
I*d*x - I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input

```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(5/2),x)
```

output

```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.89

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{i\left(105\sqrt{2}a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)+\frac{4\left(105\,(ia\tan(dx+c)+a)^3a^4-560\,(ia\tan(dx+c)+a)^2a^5+924\,(\right.}{(ia\tan(dx+c)+a)^{\frac{7}{2}}-6\,(ia\tan(dx+c)+a)^{\frac{5}{2}}a+12\,(ia\tan(dx+c)+}\right.}{1536\,ad}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/1536*I*(105*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c
) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(105*(I*a*tan(
d*x + c) + a)^3*a^4 - 560*(I*a*tan(d*x + c) + a)^2*a^5 + 924*(I*a*tan(d*x
+ c) + a)*a^6 - 384*a^7)/((I*a*tan(d*x + c) + a)^(7/2) - 6*(I*a*tan(d*x +
c) + a)^(5/2)*a + 12*(I*a*tan(d*x + c) + a)^(3/2)*a^2 - 8*sqrt(I*a*tan(d*x
 + c) + a)*a^3))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)^6\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}\,dx$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^6\tan(dx+c)^2\,dx\right)+2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^6\tan(dx+c)\,dx\right)i+\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^6\,dx\right)$$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**6*tan(c + d*x)
**2,x) + 2*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**6*tan(c + d*x),x)*i
+ int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**6,x))
```

3.313 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{256ia^4\sec^3(c+dx)}{315d(a+ia\tan(c+dx))^{3/2}} + \frac{64ia^3\sec^3(c+dx)}{105d\sqrt{a+ia\tan(c+dx)}} + \frac{8ia^2\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{21d} + \frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

output
```
256/315*I*a^4*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(3/2)+64/105*I*a^3*sec(d*x
+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)+8/21*I*a^2*sec(d*x+c)^3*(a+I*a*tan(d*x+c)
)^(1/2)/d+2/9*I*a*sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.70

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2a^2\sec^3(c+dx)(i\cos(2c)+\sin(2c))(77+242\cos(2(c+dx))+89i\sec(c+dx)\sin(3(c+dx}{315d(\cos(dx)+i\sin(dx))^2}$$

input
```
Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(2*a^2*Sec[c + d*x]^3*(I*Cos[2*c] + Sin[2*c])*(77 + 242*Cos[2*(c + d*x)] +
 (89*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (54*I)*Tan[c + d*x])*Sqrt[a + I*a*
Tan[c + d*x]])/(315*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3975, 3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3(a+ia\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{3}a\int \sec^3(c+dx)(i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a\int \sec(c+dx)^3(i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{3}a\left(\frac{8}{7}a\int \sec^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}\right)+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a\left(\frac{8}{7}a\int \sec(c+dx)^3\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}\right)+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{3}a\left(\frac{8}{7}a\left(\frac{4}{5}a\int \frac{\sec^3(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}\right)+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a\left(\frac{8}{7}a\left(\frac{4}{5}a\int \frac{\sec(c+dx)^3}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}\right)+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3974}$$

$$\frac{4}{3}a\left(\frac{8}{7}a\left(\frac{8ia^2\sec^3(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}\right)+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

input
```
Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((2*I)/9)*a*Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d + (4*a*((((2*I
)/7)*a*Sec[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (8*a*((((8*I)/15)*a^
2*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((2*I)/5)*a*Sec[c +
d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]])))/7))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 21.63 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

method	result
default	$\frac{\left(\frac{2i\left(95\sec(dx+c)^{3}-32\sec(dx+c)+256\cos(dx+c)\right)}{315}-\frac{2\tan(dx+c)\sec(dx+c)^{3}}{9}+\frac{64\sec(dx+c)\tan(dx+c)}{105}+\frac{512\sin(dx+c)}{315}\right)\sqrt{a(1+i\tan(dx+c))}$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/315*I*(95*sec(d*x+c)^3-32*sec(d*x+c)+256*cos(d*x+c))-2/9*tan(d*x+c)
*sec(d*x+c)^3+64/105*sec(d*x+c)*tan(d*x+c)+512/315*sin(d*x+c))*(a*(1+I*tan
(d*x+c)))^(1/2)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.82

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{32\sqrt{2}\left(-105i\,a^2e^{(6i\,dx+6i\,c)}-126i\,a^2e^{(4i\,dx+4i\,c)}-72i\,a^2e^{(2i\,dx+2i\,c)}-16i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{315\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-32/315*sqrt(2)*(-105*I*a^2*e^(6*I*d*x + 6*I*c) - 126*I*a^2*e^(4*I*d*x + 4
*I*c) - 72*I*a^2*e^(2*I*d*x + 2*I*c) - 16*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x
 + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 5.13 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.05

$$\begin{aligned}\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx &= \frac{a^2\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{3\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}\\ &\quad-\frac{a^2\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,96\mathrm{i}}{5\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}\\ &\quad+\frac{a^2\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,96\mathrm{i}}{7\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}-\frac{a^2\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}\end{aligned}$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^3,x)
```

output
```
(a^2*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*
2i + d*x*2i) + 1))^(1/2)*32i)/(3*d*(exp(c*2i + d*x*2i) + 1)) - (a^2*exp(-
c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i
) + 1))^(1/2)*96i)/(5*d*(exp(c*2i + d*x*2i) + 1)^2) + (a^2*exp(- c*1i - d*
x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(
1/2)*96i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*exp(- c*1i - d*x*1i)*(a
- (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)
/(9*d*(exp(c*2i + d*x*2i) + 1)^4)
```

Reduce [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\sqrt{a}\,a^2\Big(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^3\,i-\Big(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^3\tan(dx$$
$$d$$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*a**2*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**3*i - int(sqrt(
tan(c + d*x)*i + 1)*sec(c + d*x)**3*tan(c + d*x)**2,x)*d + 9*int(sqrt(tan(
c + d*x)*i + 1)*sec(c + d*x)**3*tan(c + d*x),x)*d*i))/d
```

3.314 $\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{64ia^3\sec(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} + \frac{16ia^2\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d} + \frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

output
```
64/15*I*a^3*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+16/15*I*a^2*sec(d*x+c)*(
a+I*a*tan(d*x+c))^(1/2)/d+2/5*I*a*sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2a^2\sec^2(c+dx)(i\cos(c-dx)+\sin(c-dx))(20+23\cos(2(c+dx))+7i\sin(2(c+dx)))\sqrt{c}}{15d(\cos(dx)+i\sin(dx))^2}$$

input
```
Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(2*a^2*Sec[c + d*x]^2*(I*Cos[c - d*x] + Sin[c - d*x])*(20 + 23*Cos[2*(c +
d*x)] + (7*I)*Sin[2*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(15*d*(Cos[d*x
] + I*Sin[d*x])^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{5}a\int \sec(c+dx)(i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\int \sec(c+dx)(i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \sec(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \sec(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8}{5}a\left(\frac{8ia^2\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

input
```
Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((2*I)/5)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2))/d + (8*a*((((8*I)/
3)*a^2*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((2*I)/3)*a*Sec[c +
 d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{\left(\frac{2i(32\cos(dx+c)+11\sec(dx+c))}{15}-\frac{2\sec(dx+c)\tan(dx+c)}{5}+\frac{64\sin(dx+c)}{15}\right)\sqrt{a(1+i\tan(dx+c))}\,a^2}{d}$	66

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/15*I*(32*cos(d*x+c)+11*sec(d*x+c))-2/5*sec(d*x+c)*tan(d*x+c)+64/15*
sin(d*x+c))*(a*(1+I*tan(d*x+c)))^(1/2)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.80

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{8\sqrt{2}\left(-15i\,a^2e^{(4i\,dx+4i\,c)}-20i\,a^2e^{(2i\,dx+2i\,c)}-8i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-8/15*sqrt(2)*(-15*I*a^2*e^(4*I*d*x + 4*I*c) - 20*I*a^2*e^(2*I*d*x + 2*I*c
) - 8*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(4*I*d*x + 4*I*c) + 2*
d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (ia(\tan(c+dx)-i))^{\frac{5}{2}}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(5/2)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{8\,a^2\,e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,\left(e^{c\,2i+d\,x\,2i}\,20i+e^{c\,4i+d\,x\,4i}\,15i+8i\right)}{15\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^2}$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x),x)
```

output
```
(8*a^2*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(
c*2i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*20i + exp(c*4i + d*x*4i)*15
i + 8i))/(15*d*(exp(c*2i + d*x*2i) + 1)^2)
```

Reduce [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\sqrt{a}\,a^2\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)\,i-\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)\tan(dx+\right.\right.}{d}$$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*a**2*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)*i - int(sqrt(tan
(c + d*x)*i + 1)*sec(c + d*x)*tan(c + d*x)**2,x)*d + 5*int(sqrt(tan(c + d*
x)*i + 1)*sec(c + d*x)*tan(c + d*x),x)*d*i))/d
```

3.315 $\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{8ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

output
```
-8*I*a^2*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d+2*I*a*cos(d*x+c)*(a+I*a*tan
(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2ia^2(3\cos(c+dx)-i\sin(c+dx))\sqrt{a+ia\tan(c+dx)}}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-2*I)*a^2*(3*Cos[c + d*x] - I*Sin[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/
d
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sec(c+dx)}dx$$
$$\Big\downarrow \text{3975}$$
$$4a\int \cos(c+dx)(i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$
$$\Big\downarrow \text{3042}$$
$$4a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{\sec(c+dx)}dx+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$
$$\Big\downarrow \text{3974}$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}-\frac{8ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-8*I)*a^2*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + ((2*I)*a*Cos[c +
d*x]*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 6.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{(-2\sin(dx+c)-6i\cos(dx+c))a^2\sqrt{a(1+i\tan(dx+c))}}{d}$	41

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*sin(d*x+c)-6*I*cos(d*x+c))*a^2*(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2\sqrt{2}\left(i\,a^2e^{(2i\,dx+2i\,c)}+2i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2*sqrt(2)*(I*a^2*e^(2*I*d*x + 2*I*c) + 2*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(53) = 106.

Time = 0.22 (sec) , antiderivative size = 331, normalized size of antiderivative = 5.09

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2\left(-3i\,a^{\frac{5}{2}}-\frac{2\,a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}+\frac{9i\,a^{\frac{5}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4\,a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{9i\,a^{\frac{5}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2\,a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3i}{(c}\right.}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{5}{2}}\left(\frac{4i\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right.}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2*(-3*I*a^(5/2) - 2*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 9*I*a^(5/2)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^(5/2)*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 9*I*a^(5/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2*a^(5/2)*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*I*a^(5/2)*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 1)^(5/2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(si
n(d*x + c)/(cos(d*x + c) + 1) - 1)^(5/2)*(4*I*sin(d*x + c)/(cos(d*x + c) +
 1) - 5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 5*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 - 4*I*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + 1))
```

Giac [F(-2)]

Exception generated.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2\,a^2\,\left(\sin\left(c+d\,x\right)+\cos\left(c+d\,x\right)\,3{}\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}{d}$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
-(2*a^2*(cos(c + d*x)*3i + sin(c + d*x))*((a*(cos(2*c + 2*d*x) + sin(2*c +
 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))/d
```

Reduce [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)\tan(dx+c)^2\,dx\right) + 2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)\tan(dx+c)\,dx\right)i + \int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)\,dx\right)$$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)*tan(c + d*x)**2
,x) + 2*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)*tan(c + d*x),x)*i + int(
sqrt(tan(c + d*x)*i + 1)*cos(c + d*x),x))
```

3.316 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

output
```
-2/3*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.97

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2a^2\cos^2(c+dx)(-i\cos(c+3dx)+\sin(c+3dx))\sqrt{a+ia\tan(c+dx)}}{3d(\cos(dx)+i\sin(dx))^2}$$

input
```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(2*a^2*Cos[c + d*x]^2*((-I)*Cos[c + 3*d*x] + Sin[c + 3*d*x])*Sqrt[a + I*a*
Tan[c + d*x]])/(3*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3974}$$

$$-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((-2*I)/3)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Maple [A] (verified)

Time = 14.98 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

method	result	size
default	$\frac{\left(\frac{2\sin(dx+c)\cos(dx+c)^2}{3}-\frac{2i\cos(dx+c)^3}{3}\right)a^2\sqrt{a(1+i\tan(dx+c))}}{d}$	51

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/3*sin(d*x+c)*cos(d*x+c)^2-2/3*I*cos(d*x+c)^3)*a^2*(a*(1+I*tan(d*x+c
)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(27) = 54.

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\sqrt{2}\left(-i\,a^2e^{(4i\,dx+4i\,c)}-2i\,a^2e^{(2i\,dx+2i\,c)}-i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(2)*(-I*a^2*e^(4*I*d*x + 4*I*c) - 2*I*a^2*e^(2*I*d*x + 2*I*c) - I*
a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(27) = 54$.

Time = 0.22 (sec) , antiderivative size = 328, normalized size of antiderivative = 9.37

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2\left(i\,a^{\frac{5}{2}} - \frac{4i\,a^{\frac{5}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{6i\,a^{\frac{5}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{4i\,a^{\frac{5}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{i\,a^{\frac{5}{2}}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right.}{-3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{5}{2}}\left(\frac{2i\sin(dx+c)}{\cos(dx+c)+1} + \frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{6i\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \right.}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2*(I*a^(5/2) - 4*I*a^(5/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*I*a^(5/
2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*I*a^(5/2)*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + I*a^(5/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(-2*I*sin(
d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/
2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)^(5/2)*(-6*I*sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 - 18*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 18*I*
sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 6*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 - 6*I*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3*sin(d*x + c)^8/(cos(d*x +
c) + 1)^8 - 3))
```

Giac [F(-2)]

Exception generated.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.54

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{a^2\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(-\sin\left(c+d\,x\right)-\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,3\mathrm{i}+\cos\left(3\,c+3\,d\,x\right)}{6\,d}$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
-(a^2*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x)
+ 1))^(1/2)*(cos(c + d*x)*3i - sin(c + d*x) + cos(3*c + 3*d*x)*1i - sin(3*
c + 3*d*x)))/(6*d)
```

Reduce [F]

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^3\tan(dx+c)^2\,dx\right)+2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^3\tan(dx+c)\,dx\right)i+\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^3\,dx\right)$$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**3*tan(c + d*x)
**2,x) + 2*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**3*tan(c + d*x),x)*i
+ int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**3,x))
```

3.317 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 159

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{ia^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}d} - \frac{ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d} - \frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{6d} - \frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

output
```
1/8*I*a^(5/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1
/2))*2^(1/2)/d-1/4*I*a^2*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/6*I*a*cos
(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d-1/5*I*cos(d*x+c)^5*(a+I*a*tan(d*x+c))
^(5/2)/d
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.74

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{ia^2e^{-i(c+dx)}\left(23+34e^{2i(c+dx)}+14e^{4i(c+dx)}+3e^{6i(c+dx)}-15\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a-}}{120d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-1/120*I)*a^2*(23 + 34*E^((2*I)*(c + d*x)) + 14*E^((4*I)*(c + d*x)) + 3*
E^((6*I)*(c + d*x)) - 15*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^
((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*x)))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 3971, 3042, 3971, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{ 3971}$$

$$\frac{1}{2}a\int \cos^3(c+dx)(i\tan(c+dx)a+a)^{3/2}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{\sec(c+dx)^3}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}\right)-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}\right)-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia\tan(c+d}{3d}\right.$$
$$\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia\tan(c+d}{3d}\right.$$
$$\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3970}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia\tan}{3d}\right.$$
$$\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia\tan(c}{3d}\right.$$
$$\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-1/5*I)*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2))/d + (a*(((-1/3*I)*C
os[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d + (a*((I*Sqrt[a]*ArcTanh[(Sq
rt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (
I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d))/2))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1278 vs. $2(128) = 256$.

Time = 285.87 (sec) , antiderivative size = 1279, normalized size of antiderivative = 8.04

method	result	size
default	Expression too large to display	1279

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/120/d*cos(d*x+c)^2*(I*(-60*cos(d*x+c)^3-30*cos(d*x+c)^2+45*cos(d*x+c)+15
)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*2^(1/2))+(-60*cos(d*x+c)^3-30*cos(d*x+c)^2+45*cos(d*x+c)+15)*(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*tan(d*x+c)^2*arctanh(1/(cot(d*x+c)^2-2*co
t(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))
+(120*cos(d*x+c)^2+60*cos(d*x+c)-30)*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*tan(d*x+c)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*
x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))+I*(-60*cos(d*x+c)^2-30*co
s(d*x+c)+15)*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/(cot(
d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x
+c))*2^(1/2))+I*(-120*cos(d*x+c)^2-60*cos(d*x+c)+30)*sin(d*x+c)*(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*tan(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*2^(1/2))+(60*cos(d*x+c)^3+30*cos(d*x+c)^2-45*cos(d*x+c)-15)*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*
x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))+(60*cos(d*x+c)
^2+30*cos(d*x+c)-15)*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*tan(d*x
+c)^2*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))-160*I*sin(d
*x+c)^2*cos(d*x+c)+I*(60*cos(d*x+c)^3+30*cos(d*x+c)^2-45*cos(d*x+c)-15)*(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*tan(d*x+c)^2*arctan(1/2*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*2^(1/2))+(120*cos(d*x+c)^3+60*cos(d*x+c)^2-90*cos(d...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(120) = 240$.

Time = 0.09 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.53

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}d\log\left(\frac{\left(ia^3+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{2\,d}\right)-15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}d\,\mathrm{l}}{}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/120*(15*sqrt(1/2)*sqrt(-a^5/d^2)*d*log(1/2*(I*a^3 + sqrt(2)*sqrt(1/2)*sq
rt(-a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
)*e^(-I*d*x - I*c)/d) - 15*sqrt(1/2)*sqrt(-a^5/d^2)*d*log(1/2*(I*a^3 - sqr
t(2)*sqrt(1/2)*sqrt(-a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) + sqrt(2)*(-3*I*a^2*e^(6*I*d*x + 6*
I*c) - 14*I*a^2*e^(4*I*d*x + 4*I*c) - 34*I*a^2*e^(2*I*d*x + 2*I*c) - 23*I*
a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(120) = 240$.

Time = 0.29 (sec) , antiderivative size = 1076, normalized size of antiderivative = 6.77

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/480*(20*(I*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) - sqrt(2)*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*sqrt(a) + 12*(5*I*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - 5*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) + (I*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + I*sqrt(2)*a^2*sin(
2*d*x + 2*c)^2 + 2*I*sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2)*cos(5/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (sqrt(2)*a^2*cos(2*d*x
 + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c
) + sqrt(2)*a^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))
*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
sqrt(a) + 15*(2*sqrt(2)*a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + 1) - 2*sqrt(2)*a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)
- I*sqrt(2)*a^2*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co...
```

Giac [F(-2)]

Exception generated.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)^5\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2} \,d x$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\begin{aligned}&\int \cos^5(c+dx)(a+ia\tan(c\\&+dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(-\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^5\tan\left(dx+c\right)^2dx\right)\right.\\&+2\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^5\tan\left(dx+c\right)dx\right)i\\&\left.+\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^5dx\right)\end{aligned}$$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**5*tan(c + d*x)
**2,x) + 2*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**5*tan(c + d*x),x)*i
+ int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**5,x))
```

3.318 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 231

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{9ia^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}d} + \frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}} - \frac{9ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32d} - \frac{3ia^2\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{20d} - \frac{9ia\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{70d} - \frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

output
```
9/64*I*a^(5/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(
1/2))*2^(1/2)/d+3/16*I*a^3*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-9/32*I*a^
2*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-3/20*I*a^2*cos(d*x+c)^3*(a+I*a*tan
(d*x+c))^(1/2)/d-9/70*I*a*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2)/d-1/7*I*co
s(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2)/d
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.67

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx =$$
$$-\frac{ia^2e^{-3i(c+dx)}\left(-35+353e^{2i(c+dx)}+544e^{4i(c+dx)}+214e^{6i(c+dx)}+68e^{8i(c+dx)}+10e^{10i(c+dx)}-315e^{2i(c+dx)}\sqrt{\right.}{2240d}$$

input
```
Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-1/2240*I)*a^2*(-35 + 353*E^((2*I)*(c + d*x)) + 544*E^((4*I)*(c + d*x))
+ 214*E^((6*I)*(c + d*x)) + 68*E^((8*I)*(c + d*x)) + 10*E^((10*I)*(c + d*x
)) - 315*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1
+ E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^((3*I)*(c + d*x)
))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3978, 3042, 3978, 3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{9}{14}a\int \cos^5(c+dx)(i\tan(c+dx)a+a)^{3/2}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{14}a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{\sec(c+dx)^5}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \textbf{3978}$$

$$\frac{9}{14}a\left(\frac{7}{10}a\int \cos^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}\right)-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{9}{14}a\left(\frac{7}{10}a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^3}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}\right)-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \textbf{3978}$$

$$\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)-\frac{i\cos^5(c+dx)(a+ia\tan(c-\right.$$
$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)-\frac{i\cos^5(c+dx)(a\right.$$
$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \textbf{3983}$$

$$\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan\right.\right.$$
$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)-\frac{i\,c}{}\right.$$

$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+}{}\right.\right.$$

$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+}{}\right.\right.$$

$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3970}$$

$$\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{ia\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\right.\right.$$

$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{219}$$

$$\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{i\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i}{}\right.\right.$$

$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

input
```
Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-1/7*I)*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(5/2))/d + (9*a*(((-1/5*I)
*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2))/d + (7*a*(((-1/3*I)*Cos[c +
d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (5*a*(((I/2)*Cos[c + d*x])/(d*Sqrt[
a + I*a*Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sq
rt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a +
 I*a*Tan[c + d*x]])/d))/(4*a)))/6))/10))/14
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1342 vs. $2(188) = 376$.

Time = 3.42 (sec) , antiderivative size = 1343, normalized size of antiderivative = 5.81

Expression too large to display

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
-1/2240/d*cos(d*x+c)^2*(I*cos(d*x+c)*(-400*cos(d*x+c)^4+2184*cos(d*x+c)^2-
630)+(-1260*cos(d*x+c)^3-630*cos(d*x+c)^2+945*cos(d*x+c)+315)*(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*tan(d*x+c)^2*arctanh(2^(1/2)*(cot(d*x+c)-csc(d*x+c)
)/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))+(2520*cos(d
*x+c)^2+1260*cos(d*x+c)-630)*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*tan(d*x+c)*arctanh(2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(cot(d*x+c)^2-2*cot(d*
x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))+I*(2520*cos(d*x+c)^2+1260*cos(d*x+c
)-630)*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*tan(d*x+c)*arctan(1/2
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))+I*(-1260*cos(d*x+c)^2-630*c
os(d*x+c)+315)*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(1/
2)*(cot(d*x+c)-csc(d*x+c))/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c
)^2-1)^(1/2))+(1260*cos(d*x+c)^3+630*cos(d*x+c)^2-945*cos(d*x+c)-315)*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(co
t(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))+(-1260*cos(d*x+c
)^2-630*cos(d*x+c)+315)*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*tan(
d*x+c)^2*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))+I*(-1260
*cos(d*x+c)^3-630*cos(d*x+c)^2+945*cos(d*x+c)+315)*(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*tan(d*x+c)^2*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2
^(1/2))+I*sin(d*x+c)*tan(d*x+c)*(400*cos(d*x+c)^4-2184*cos(d*x+c)^2+630)+(
-2520*cos(d*x+c)^3-1260*cos(d*x+c)^2+1890*cos(d*x+c)+630)*(-cos(d*x+c)/...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.30

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{\left(315\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}de^{(2i\,dx+2i\,c)}\log\left(-\frac{9\left(-i\,a^3+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{16\,d}\right)-315\sqrt{\frac{1}{2}}\sqrt{-\cdots}}{}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/2240*(315*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(2*I*d*x + 2*I*c)*log(-9/16*(-I*
a^3 + sqrt(2)*sqrt(1/2)*sqrt(-a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) - 315*sqrt(1/2)*sqrt(-a^5/
d^2)*d*e^(2*I*d*x + 2*I*c)*log(-9/16*(-I*a^3 - sqrt(2)*sqrt(1/2)*sqrt(-a^5
/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I
*d*x - I*c)/d) - sqrt(2)*(-10*I*a^2*e^(10*I*d*x + 10*I*c) - 68*I*a^2*e^(8*
I*d*x + 8*I*c) - 214*I*a^2*e^(6*I*d*x + 6*I*c) - 544*I*a^2*e^(4*I*d*x + 4*
I*c) - 353*I*a^2*e^(2*I*d*x + 2*I*c) + 35*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1)))*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)^7\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\begin{aligned}
&\int \cos^7(c+dx)(a+ia\tan(c\\
&+dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^7\tan(dx+c)^2\,dx\right)\right.\\
&+2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^7\tan(dx+c)\,dx\right)i\\
&\left.+\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^7\,dx\right)
\end{aligned}$$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**7*tan(c + d*x)
**2,x) + 2*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**7*tan(c + d*x),x)*i
+ int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**7,x))
```

3.319 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{16i(a+ia\tan(c+dx))^{15/2}}{15a^4d}+\frac{24i(a+ia\tan(c+dx))^{17/2}}{17a^5d} -\frac{12i(a+ia\tan(c+dx))^{19/2}}{19a^6d}+\frac{2i(a+ia\tan(c+dx))^{21/2}}{21a^7d}$$

output
```
-16/15*I*(a+I*a*tan(d*x+c))^(15/2)/a^4/d+24/17*I*(a+I*a*tan(d*x+c))^(17/2)
/a^5/d-12/19*I*(a+I*a*tan(d*x+c))^(19/2)/a^6/d+2/21*I*(a+I*a*tan(d*x+c))^(
21/2)/a^7/d
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2a^3(-i+\tan(c+dx))^7\sqrt{a+ia\tan(c+dx)}(-3243+7365i\tan(c+dx)+5865\tan^2(c+dx}{33915d}$$

input
```
Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(2*a^3*(-I + Tan[c + d*x])^7*Sqrt[a + I*a*Tan[c + d*x]]*(-3243 + (7365*I)*
Tan[c + d*x] + 5865*Tan[c + d*x]^2 - (1615*I)*Tan[c + d*x]^3))/(33915*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec(c+dx)^8(a+ia\tan(c+dx))^{7/2}dx$$

$$\Big\downarrow \ 3968$$

$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{13/2}d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \ 53$$

$$-\frac{i\int \left(-(i\tan(c+dx)a+a)^{19/2}+6a(i\tan(c+dx)a+a)^{17/2}-12a^2(i\tan(c+dx)a+a)^{15/2}+8a^3(i\tan(c+dx)\right.}{a^7d}$$

$$\Big\downarrow \ 2009$$

$$-\frac{i\left(\frac{16}{15}a^3(a+ia\tan(c+dx))^{15/2}-\frac{24}{17}a^2(a+ia\tan(c+dx))^{17/2}-\frac{2}{21}(a+ia\tan(c+dx))^{21/2}+\frac{12}{19}a(a+ia\tan(c\right.}{a^7d}$$

input
```
Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*((16*a^3*(a + I*a*Tan[c + d*x])^(15/2))/15 - (24*a^2*(a + I*a*Tan[c
+ d*x])^(17/2))/17 + (12*a*(a + I*a*Tan[c + d*x])^(19/2))/19 - (2*(a + I*a
*Tan[c + d*x])^(21/2))/21))/(a^7*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{21}{2}}}{21}-\frac{6a(a+ia\tan(dx+c))^{\frac{19}{2}}}{19}+\frac{12a^2(a+ia\tan(dx+c))^{\frac{17}{2}}}{17}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{15}{2}}}{15}\right)}{d\,a^7}$$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
2*I/d/a^7*(1/21*(a+I*a*tan(d*x+c))^(21/2)-6/19*a*(a+I*a*tan(d*x+c))^(19/2)
+12/17*a^2*(a+I*a*tan(d*x+c))^(17/2)-8/15*a^3*(a+I*a*tan(d*x+c))^(15/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(85) = 170.

Time = 0.14 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.73

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{2048\sqrt{2}\left(16i\,a^3e^{(21i\,dx+21i\,c)}+168i\,a^3e^{(19i\,dx+19i\,c)}+798i\,a^3e^{(17i}\right.}{33915\left(de^{(20i\,dx+20i\,c)}+10\,de^{(18i\,dx+18i\,c)}+45\,de^{(16i\,dx+16i\,c)}+120\,de^{(14i\,dx+14i\,c)}+210\,de^{(12i\,dx+12i\,c)}+252\,c\right.}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-2048/33915*sqrt(2)*(16*I*a^3*e^(21*I*d*x + 21*I*c) + 168*I*a^3*e^(19*I*d*
x + 19*I*c) + 798*I*a^3*e^(17*I*d*x + 17*I*c) + 2261*I*a^3*e^(15*I*d*x + 1
5*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(20*I*d*x + 20*I*c) + 10*d*
e^(18*I*d*x + 18*I*c) + 45*d*e^(16*I*d*x + 16*I*c) + 120*d*e^(14*I*d*x + 1
4*I*c) + 210*d*e^(12*I*d*x + 12*I*c) + 252*d*e^(10*I*d*x + 10*I*c) + 210*d
*e^(8*I*d*x + 8*I*c) + 120*d*e^(6*I*d*x + 6*I*c) + 45*d*e^(4*I*d*x + 4*I*c
) + 10*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2i\left(1615\,(i\,a\tan(dx+c)+a)^{\frac{21}{2}}-10710\,(i\,a\tan(dx+c)+a)^{\frac{19}{2}}a+23940\,(i\,a\tan(dx+c)\cdots\right.}{33915\,a^7d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
2/33915*I*(1615*(I*a*tan(d*x + c) + a)^(21/2) - 10710*(I*a*tan(d*x + c) +
a)^(19/2)*a + 23940*(I*a*tan(d*x + c) + a)^(17/2)*a^2 - 18088*(I*a*tan(d*x
 + c) + a)^(15/2)*a^3)/(a^7*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 12.77 (sec) , antiderivative size = 690, normalized size of antiderivative = 5.90

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Too large to display}$$

input
```
int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x)^8,x)
```

output
```
(a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*247808i)/(969*d*(exp(c*2i + d*x*2i) + 1)^4) - (a^3*(a - (a*(exp(c*2i +
 d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16384i)/(33915*d*(ex
p(c*2i + d*x*2i) + 1)) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(ex
p(c*2i + d*x*2i) + 1))^(1/2)*4096i)/(11305*d*(exp(c*2i + d*x*2i) + 1)^2) -
 (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(
1/2)*2048i)/(6783*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^3*(a - (a*(exp(c*2i +
 d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32768i)/(33915*d) -
(a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*1943552i)/(1615*d*(exp(c*2i + d*x*2i) + 1)^5) + (a^3*(a - (a*(exp(c*2i
 + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*12019712i)/(4845*d
*(exp(c*2i + d*x*2i) + 1)^6) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1
i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*95516672i)/(33915*d*(exp(c*2i + d*x*2i)
 + 1)^7) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2
i) + 1))^(1/2)*4159488i)/(2261*d*(exp(c*2i + d*x*2i) + 1)^8) - (a^3*(a - (
a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*260096i
)/(399*d*(exp(c*2i + d*x*2i) + 1)^9) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i
 - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2048i)/(21*d*(exp(c*2i + d*x*2i
) + 1)^10)
```

Reduce [F]

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{\sqrt{a}\,a^3\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^8\,i-\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^8\tan(dx\right.\right.}{}$$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*a**3*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**8*i - int(sqrt(
tan(c + d*x)*i + 1)*sec(c + d*x)**8*tan(c + d*x)**3,x)*d*i - 3*int(sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**8*tan(c + d*x)**2,x)*d + 20*int(sqrt(tan(c
 + d*x)*i + 1)*sec(c + d*x)**8*tan(c + d*x),x)*d*i))/d
```

3.320 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^3d} + \frac{8i(a+ia\tan(c+dx))^{15/2}}{15a^4d} - \frac{2i(a+ia\tan(c+dx))^{17/2}}{17a^5d}$$

output
```
-8/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^3/d+8/15*I*(a+I*a*tan(d*x+c))^(15/2)/a
^4/d-2/17*I*(a+I*a*tan(d*x+c))^(17/2)/a^5/d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2a^3(-i+\tan(c+dx))^6\sqrt{a+ia\tan(c+dx)}(331i+494\tan(c+dx)-195i\tan^2(c+dx))}{3315d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(2*a^3*(-I + Tan[c + d*x])^6*Sqrt[a + I*a*Tan[c + d*x]]*(331*I + 494*Tan[c
 + d*x] - (195*I)*Tan[c + d*x]^2))/(3315*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+ia\tan(c+dx))^{7/2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{11/2}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left((i\tan(c+dx)a+a)^{15/2}-4a(i\tan(c+dx)a+a)^{13/2}+4a^2(i\tan(c+dx)a+a)^{11/2}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{8}{13}a^2(a+ia\tan(c+dx))^{13/2}+\frac{2}{17}(a+ia\tan(c+dx))^{17/2}-\frac{8}{15}a(a+ia\tan(c+dx))^{15/2}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*((8*a^2*(a + I*a*Tan[c + d*x])^(13/2))/13 - (8*a*(a + I*a*Tan[c + d*
x])^(15/2))/15 + (2*(a + I*a*Tan[c + d*x])^(17/2))/17))/(a^5*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{17}{2}}}{17}+\frac{4a(a+ia\tan(dx+c))^{\frac{15}{2}}}{15}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}\right)}{d\,a^5}$$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
2*I/d/a^5*(-1/17*(a+I*a*tan(d*x+c))^(17/2)+4/15*a*(a+I*a*tan(d*x+c))^(15/2
)-4/13*a^2*(a+I*a*tan(d*x+c))^(13/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(64) = 128$.

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.86

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{512\sqrt{2}\left(8i\,a^3e^{(17i\,dx+17i\,c)}+68i\,a^3e^{(15i\,dx+15i\,c)}+255i\,a^3e^{(13i\,dx+13i\,c)}\right)\sqrt{\frac{a}{e^{(2i}}}}{3315\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+6}\right.}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-512/3315*sqrt(2)*(8*I*a^3*e^(17*I*d*x + 17*I*c) + 68*I*a^3*e^(15*I*d*x +
15*I*c) + 255*I*a^3*e^(13*I*d*x + 13*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))/(d*e^(16*I*d*x + 16*I*c) + 8*d*e^(14*I*d*x + 14*I*c) + 28*d*e^(12*I*d*x
 + 12*I*c) + 56*d*e^(10*I*d*x + 10*I*c) + 70*d*e^(8*I*d*x + 8*I*c) + 56*d*
e^(6*I*d*x + 6*I*c) + 28*d*e^(4*I*d*x + 4*I*c) + 8*d*e^(2*I*d*x + 2*I*c) +
 d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{2i\left(195\,(i\,a\tan\left(dx+c\right)+a)^{\frac{17}{2}}-884\,(i\,a\tan\left(dx+c\right)+a)^{\frac{15}{2}}a+1020\,(i\,a\tan\left(dx+c\right)+a)^{\frac{13}{2}}a^2\right)}{3315\,a^5d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-2/3315*I*(195*(I*a*tan(d*x + c) + a)^(17/2) - 884*(I*a*tan(d*x + c) + a)^
(15/2)*a + 1020*(I*a*tan(d*x + c) + a)^(13/2)*a^2)/(a^5*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 562, normalized size of antiderivative = 6.39

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Too large to display}$$

input
```
int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x)^6,x)
```

output
```
(a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*56320i)/(663*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^3*(a - (a*(exp(c*2i +
d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2048i)/(3315*d*(exp(c
*2i + d*x*2i) + 1)) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c
*2i + d*x*2i) + 1))^(1/2)*512i)/(1105*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^3
*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
4096i)/(3315*d) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
+ d*x*2i) + 1))^(1/2)*205312i)/(663*d*(exp(c*2i + d*x*2i) + 1)^4) + (a^3*(
a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*54
0672i)/(1105*d*(exp(c*2i + d*x*2i) + 1)^5) - (a^3*(a - (a*(exp(c*2i + d*x*
2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1341952i)/(3315*d*(exp(c*
2i + d*x*2i) + 1)^6) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(
c*2i + d*x*2i) + 1))^(1/2)*44032i)/(255*d*(exp(c*2i + d*x*2i) + 1)^7) - (a
^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*512i)/(17*d*(exp(c*2i + d*x*2i) + 1)^8)
```

Reduce [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{\sqrt{a}\,a^3\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^6\,i-\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^6\tan(dx\right.\right.}{}$$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*a**3*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**6*i - int(sqrt(
tan(c + d*x)*i + 1)*sec(c + d*x)**6*tan(c + d*x)**3,x)*d*i - 3*int(sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**6*tan(c + d*x)**2,x)*d + 16*int(sqrt(tan(c
 + d*x)*i + 1)*sec(c + d*x)**6*tan(c + d*x),x)*d*i))/d
```

3.321 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{4i(a+ia\tan(c+dx))^{11/2}}{11a^2d}+\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^3d}$$

output
```
-4/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^2/d+2/13*I*(a+I*a*tan(d*x+c))^(13/2)/a
^3/d
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2a^3(15-11i\tan(c+dx))(-i+\tan(c+dx))^5\sqrt{a+ia\tan(c+dx)}}{143d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(2*a^3*(15 - (11*I)*Tan[c + d*x])*(-I + Tan[c + d*x])^5*Sqrt[a + I*a*Tan[c
 + d*x]])/(143*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(c+dx)^4(a+ia\tan(c+dx))^{7/2}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{9/2}d(ia\tan(c+dx))}{a^3d}$$
$$\Big\downarrow \text{53}$$
$$-\frac{i\int\left(2a(i\tan(c+dx)a+a)^{9/2}-(i\tan(c+dx)a+a)^{11/2}\right)d(ia\tan(c+dx))}{a^3d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{i\left(\frac{4}{11}a(a+ia\tan(c+dx))^{11/2}-\frac{2}{13}(a+ia\tan(c+dx))^{13/2}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*((4*a*(a + I*a*Tan[c + d*x])^(11/2))/11 - (2*(a + I*a*Tan[c + d*x])^
(13/2))/13))/(a^3*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 100.94 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{2a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}\right)}{d\,a^{3}}$	44
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{2a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}\right)}{d\,a^{3}}$	44

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^3*(1/13*(a+I*a*tan(d*x+c))^(13/2)-2/11*a*(a+I*a*tan(d*x+c))^(11/2)
)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(43) = 86$.

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.14

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{128\sqrt{2}\left(2i\,a^3e^{(13i\,dx+13i\,c)}+13i\,a^3e^{(11i\,dx+11i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{143\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+\right.}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-128/143*sqrt(2)*(2*I*a^3*e^(13*I*d*x + 13*I*c) + 13*I*a^3*e^(11*I*d*x + 1
1*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(12*I*d*x + 12*I*c) + 6*d*e
^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c)
 + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2i\left(11\,(i\,a\tan\left(dx+c\right)+a)^{\frac{13}{2}}-26\,(i\,a\tan\left(dx+c\right)+a)^{\frac{11}{2}}a\right)}{143\,a^3d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
2/143*I*(11*(I*a*tan(d*x + c) + a)^(13/2) - 26*(I*a*tan(d*x + c) + a)^(11/
2)*a)/(a^3*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 4.97 (sec) , antiderivative size = 434, normalized size of antiderivative = 7.36

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{a^3\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{143\,d} - \frac{a^3\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{143\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} + \frac{a^3\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,4480\mathrm{i}}{143\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} - \frac{a^3\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,11520\mathrm{i}}{143\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} + \frac{a^3\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,12800\mathrm{i}}{143\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4} - \frac{a^3\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,6784\mathrm{i}}{143\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5} + \frac{a^3\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{13\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}$$

input

```
int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x)^4,x)
```

output

```
(a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*4480i)/(143*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^3*(a - (a*(exp(c*2i + d
*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(143*d*(exp(c*2i
 + d*x*2i) + 1)) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
 + d*x*2i) + 1))^(1/2)*256i)/(143*d) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i
 - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*11520i)/(143*d*(exp(c*2i + d*x*
2i) + 1)^3) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*
x*2i) + 1))^(1/2)*12800i)/(143*d*(exp(c*2i + d*x*2i) + 1)^4) - (a^3*(a - (
a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*6784i)/
(143*d*(exp(c*2i + d*x*2i) + 1)^5) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i -
 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(13*d*(exp(c*2i + d*x*2i) +
 1)^6)
```

Reduce [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{\sqrt{a}\,a^3\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^4\,i-\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^4\tan(dx$$

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*a**3*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**4*i - int(sqrt(
tan(c + d*x)*i + 1)*sec(c + d*x)**4*tan(c + d*x)**3,x)*d*i - 3*int(sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**4*tan(c + d*x)**2,x)*d + 12*int(sqrt(tan(c
 + d*x)*i + 1)*sec(c + d*x)**4*tan(c + d*x),x)*d*i))/d
```

3.322 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=-\frac{2i(a+ia\tan(c+dx))^{9/2}}{9ad}$$

output
```
-2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=-\frac{2i(a+ia\tan(c+dx))^{9/2}}{9ad}$$

input
```
Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((-2*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+ia\tan(c+dx))^{7/2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int(i\tan(c+dx)a+a)^{7/2}d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$-\frac{2i(a+ia\tan(c+dx))^{9/2}}{9ad}$$

input
```
Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((-2*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2i(a+ia\tan(dx+c))^{\frac{9}{2}}}{9ad}$	24
default	$-\frac{2i(a+ia\tan(dx+c))^{\frac{9}{2}}}{9ad}$	24

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(21) = 42.

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.93

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{32i\,\sqrt{2}a^3\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(9i\,dx+9i\,c)}}{9\,(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d)}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-32/9*I*sqrt(2)*a^3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(9*I*d*x + 9*I*c)/
(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c)
 + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{2i\,(i\,a\tan\left(dx+c\right)+a)^{\frac{9}{2}}}{9\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-2/9*I*(I*a*tan(d*x + c) + a)^(9/2)/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 306, normalized size of antiderivative = 10.55

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{a^3\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,d} + \frac{a^3\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{9\,d\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{a^3\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{3\,d\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^2} + \frac{a^3\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{9\,d\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3} - \frac{a^3\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,d\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^4}$$

input
```
int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x)^2,x)
```

output
```
(a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*128i)/(9*d*(exp(c*2i + d*x*2i) + 1)) - (a^3*(a - (a*(exp(c*2i + d*x*2i
)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(9*d) - (a^3*(a - (a*(
exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(3*d*
(exp(c*2i + d*x*2i) + 1)^2) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i
)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(9*d*(exp(c*2i + d*x*2i) + 1)^3) -
 (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(
1/2)*32i)/(9*d*(exp(c*2i + d*x*2i) + 1)^4)
```

Reduce [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{\sqrt{a}\,a^3\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^2\,i-\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^2\tan(d\right.\right.}{}$$

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*a**3*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**2*i - int(sqrt(
tan(c + d*x)*i + 1)*sec(c + d*x)**2*tan(c + d*x)**3,x)*d*i - 3*int(sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**2*tan(c + d*x)**2,x)*d + 8*int(sqrt(tan(c
+ d*x)*i + 1)*sec(c + d*x)**2*tan(c + d*x),x)*d*i))/d
```

3.323 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 116

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{3i\sqrt{2}a^{7/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{d} - \frac{ia^3(a+ia\tan(c+dx))^{3/2}}{d(a-ia\tan(c+dx))}$$

output
```
3*I*2^(1/2)*a^(7/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/
d-3*I*a^3*(a+I*a*tan(d*x+c))^(1/2)/d-I*a^3*(a+I*a*tan(d*x+c))^(3/2)/d/(a-I
*a*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{ia\,\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{5/2}}{10d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-1/10*I)*a*Hypergeometric2F1[2, 5/2, 7/2, (1 + I*Tan[c + d*x])/2]*(a + I
*a*Tan[c + d*x])^(5/2))/d
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3968, 51, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+ia\tan(c+dx))^{7/2}}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \ 3968$$

$$-\frac{ia^3\int \frac{(i\tan(c+dx)a+a)^{3/2}}{(a-ia\tan(c+dx))^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \ 51$$

$$-\frac{ia^3\left(\frac{(a+ia\tan(c+dx))^{3/2}}{a-ia\tan(c+dx)}-\frac{3}{2}\int \frac{\sqrt{i\tan(c+dx)a+a}}{a-ia\tan(c+dx)}d(ia\tan(c+dx))\right)}{d}$$

$$\Big\downarrow \ 60$$

$$-\frac{ia^3\left(\frac{(a+ia\tan(c+dx))^{3/2}}{a-ia\tan(c+dx)}-\frac{3}{2}\left(2a\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))-2\sqrt{a+ia\tan(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \ 73$$

$$-\frac{ia^3\left(\frac{(a+ia\tan(c+dx))^{3/2}}{a-ia\tan(c+dx)}-\frac{3}{2}\left(4a\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}-2\sqrt{a+ia\tan(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ia^3\left(\frac{(a+ia\tan(c+dx))^{3/2}}{a-ia\tan(c+dx)}-\frac{3}{2}\left(2i\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)-2\sqrt{a+ia\tan(c+dx)}\right)\right)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*a^3*((a + I*a*Tan[c + d*x])^(3/2)/(a - I*a*Tan[c + d*x]) - (3*((2*I)
*Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]] - 2*Sqrt[a + I*a*T
an[c + d*x]]))/2))/d
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(96) = 192$.

Time = 10.76 (sec) , antiderivative size = 462, normalized size of antiderivative = 3.98

method	result
default	$-\frac{2\cos(dx+c)^3\left(i(-3\cos(dx+c)-3)\sin(dx+c)\operatorname{arctanh}\left(\frac{(\csc(dx+c)-\cot(dx+c))\sqrt{2}}{\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\operatorname{arct}\right.}{}$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*cos(d*x+c)^3*(I*(-3*cos(d*x+c)-3)*sin(d*x+c)*arctanh(1/(cot(d*x+c)^2-
2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1
/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+arctanh(1/(cot(d*x+c)^2-2*cot(d*x+
c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(3*cos(d*x+c)^2+3*cos(d*x+c))+(3*cos(d*x+c)+
3)*sin(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*cos(d*x+c)*(3*cos(d*x+c)+3)*arctan(1/2*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)+(1-cos(d*x+c))*sin(d*x+c)+I*(cos(d*x+c)^2+2*cos(d*x+c)+1))*(a*(1+I*t
an(d*x+c)))^(1/2)*a^3*(-tan(d*x+c)+I)^3/((4*cos(d*x+c)^2+2*cos(d*x+c)-1)*s
in(d*x+c)+I*(-4*cos(d*x+c)^3-2*cos(d*x+c)^2+3*cos(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(89) = 178.

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.03

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{3\sqrt{2}\sqrt{-\frac{a^7}{d^2}}d\log\left(\frac{4\left(a^4e^{(i\,dx+i\,c)}+\sqrt{-\frac{a^7}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^3}\right)-3\sqrt{2}\sqrt{-\frac{a^7}{d^2}}d\log\left(\frac{4\left(a^4e^{(i\,d}\right.}{}\right.}{2}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-1/2*(3*sqrt(2)*sqrt(-a^7/d^2)*d*log(4*(a^4*e^(I*d*x + I*c) + sqrt(-a^7/d^
2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-
I*d*x - I*c)/a^3) - 3*sqrt(2)*sqrt(-a^7/d^2)*d*log(4*(a^4*e^(I*d*x + I*c)
+ sqrt(-a^7/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1)))*e^(-I*d*x - I*c)/a^3) + 2*sqrt(2)*(I*a^3*e^(3*I*d*x + 3*I*c) +
3*I*a^3*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{i\left(3\sqrt{2}a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)+4\sqrt{ia\tan(dx+c)+a}a^4-\frac{4\sqrt{ia\tan(dx+c)+a}a^5}{ia\tan(dx+c)-a}\right)}{2ad}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-1/2*I*(3*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*sqrt(I*a*tan(d*x +
 c) + a)*a^4 - 4*sqrt(I*a*tan(d*x + c) + a)*a^5/(I*a*tan(d*x + c) - a))/(a
*d)
```

Giac [F(-2)]

Exception generated.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\int \cos\left(c+d\,x\right)^2\,\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)^{7/2}\,dx$$

input
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

Reduce [F]

$$\begin{aligned}\int \cos^2(c+dx)(a+ia\tan(c&\\ +dx))^{7/2}\,dx=\sqrt{a}\,a^3\Bigg(-\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^2\tan\left(dx+c\right)^3dx\right)i&\\ -3\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^2\tan\left(dx+c\right)^2dx\right)&\\ +3\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^2\tan\left(dx+c\right)dx\right)i&\\ +\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^2dx\Bigg)&\end{aligned}$$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
sqrt(a)*a**3*( - int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**2*tan(c + d*x)
**3,x)*i - 3*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**2*tan(c + d*x)**2,
x) + 3*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**2*tan(c + d*x),x)*i + in
t(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**2,x))
```

3.324 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 141

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{ia^{7/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}d} - \frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2} + \frac{ia^5\sqrt{a+ia\tan(c+dx)}}{8d\left(a^2-ia^2\tan(c+dx)\right)}$$

output
```
1/16*I*a^(7/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/
2)/d-1/2*I*a^5*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))^2+1/8*I*a^5*(
a+I*a*tan(d*x+c))^(1/2)/d/(a^2-I*a^2*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.38

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{ia^2\,\text{Hypergeometric2F1}\left(\frac{3}{2},3,\frac{5}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{3/2}}{12d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-1/12*I)*a^2*Hypergeometric2F1[3/2, 3, 5/2, (1 + I*Tan[c + d*x])/2]*(a +
 I*a*Tan[c + d*x])^(3/2))/d
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3968, 51, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{7/2}}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{\sqrt{i\tan(c+dx)a+a}}{(a-ia\tan(c+dx))^3}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{51}$$

$$-\frac{ia^5\left(\frac{\sqrt{a+ia\tan(c+dx)}}{2(a-ia\tan(c+dx))^2}-\frac{1}{4}\int \frac{1}{(a-ia\tan(c+dx))^2\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))\right)}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{1}{4}\left(-\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{4a}-\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)+\frac{\sqrt{a+ia\tan(c+dx)}}{2(a-ia\tan(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{ia^5\left(\frac{1}{4}\left(-\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{2a}-\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)+\frac{\sqrt{a+ia\tan(c+dx)}}{2(a-ia\tan(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ia^5\left(\frac{1}{4}\left(-\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{2\sqrt{2}a^{3/2}}-\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)+\frac{\sqrt{a+ia\tan(c+dx)}}{2(a-ia\tan(c+dx))^2}\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*a^5*(Sqrt[a + I*a*Tan[c + d*x]]/(2*(a - I*a*Tan[c + d*x])^2) + (((-1
/2*I)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - Sqrt[a +
 I*a*Tan[c + d*x]]/(2*a*(a - I*a*Tan[c + d*x])))/4))/d
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(114) = 228$.

Time = 66.06 (sec) , antiderivative size = 485, normalized size of antiderivative = 3.44

method	result
default	$-\frac{\left(\sin(dx+c)\cos(dx+c)(-2\cos(dx+c)-2)\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c))}{\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+i\left(-2\,\mathrm{co}\right.\right.}{}$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d*(sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)-2)*arctanh(2^(1/2)*(cot(d*x+c
)-csc(d*x+c))/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))
*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*(-2*cos(d*x+c)^3-2*cos(d*x+c)^2+cos(
d*x+c)+1)*arctanh(2^(1/2)*(cot(d*x+c)-csc(d*x+c))/(cot(d*x+c)^2-2*cot(d*x+
c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*
sin(d*x+c)*cos(d*x+c)*(2*cos(d*x+c)+2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*2^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(-2*cos(d*x+c)^3-
2*cos(d*x+c)^2+cos(d*x+c)+1)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*2^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*sin(d*x+c)*cos(d*x+c)*(-1
-4*cos(d*x+c))+cos(d*x+c)*(-4*cos(d*x+c)^2-3*cos(d*x+c)+1))*cos(d*x+c)^3*(
-tan(d*x+c)+I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3/(I*(2*cos(d*x+c)+1)*sin(d*
x+c)+2*cos(d*x+c)^2+cos(d*x+c)-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(106) = 212.

Time = 0.08 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.85

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}d\log\left(\frac{4\left(a^4e^{(i\,dx+i\,c)}-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^3}\right)-\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{d}}}{}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/16*(sqrt(1/2)*sqrt(-a^7/d^2)*d*log(4*(a^4*e^(I*d*x + I*c) - sqrt(2)*sqrt
(1/2)*sqrt(-a^7/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1)))*e^(-I*d*x - I*c)/a^3) - sqrt(1/2)*sqrt(-a^7/d^2)*d*log(4*(a^
4*e^(I*d*x + I*c) - sqrt(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(-I*d*e^(2*I*d*x + 2*
I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^3) + sqr
t(2)*(-2*I*a^3*e^(5*I*d*x + 5*I*c) - 3*I*a^3*e^(3*I*d*x + 3*I*c) - I*a^3*e
^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.98

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$
$$-\frac{i\left(\sqrt{2}a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left((i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^5+2\,\sqrt{i\,a\tan(dx+c)+a}a^6\right)}{(i\,a\tan(dx+c)+a)^2-4\,(i\,a\tan(dx+c)+a)a+4\,a^2}\right)}{32\,ad}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-1/32*I*(sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*((I*a*tan(d*x + c)
+ a)^(3/2)*a^5 + 2*sqrt(I*a*tan(d*x + c) + a)*a^6)/((I*a*tan(d*x + c) + a)
^2 - 4*(I*a*tan(d*x + c) + a)*a + 4*a^2))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int \cos\left(c+d\,x\right)^4\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}\,dx$$

input
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

Reduce [F]

$$\begin{aligned}&\int \cos^4(c+dx)(a+ia\tan(c\\&+dx))^{7/2}\,dx = \sqrt{a}\,a^3\left(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^4\tan(dx+c)^3\,dx\right)i\right.\\&-3\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^4\tan(dx+c)^2\,dx\right)\\&+3\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^4\tan(dx+c)\,dx\right)i\\&\left.+\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^4\,dx\right)\end{aligned}$$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
sqrt(a)*a**3*( - int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**4*tan(c + d*x)
**3,x)*i - 3*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**4*tan(c + d*x)**2,
x) + 3*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**4*tan(c + d*x),x)*i + in
t(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**4,x))
```

3.325 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 189

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{5ia^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d}-\frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3}-\frac{5ia^7\sqrt{a+ia\tan(c+dx)}}{48d\left(a^2-ia^2\tan(c+dx)\right)^2}-\frac{5ia^7\sqrt{a+ia\tan(c+dx)}}{64d\left(a^4-ia^4\tan(c+dx)\right)}$$

output
```
-5/128*I*a^(7/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(
1/2)/d-1/6*I*a^6*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))^3-5/48*I*a^
7*(a+I*a*tan(d*x+c))^(1/2)/d/(a^2-I*a^2*tan(d*x+c))^2-5/64*I*a^7*(a+I*a*ta
n(d*x+c))^(1/2)/d/(a^4-I*a^4*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.28

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{ia^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},4,\frac{3}{2},\frac{1}{2}(1+i\tan(c+dx))\right)\sqrt{a+ia\tan(c+dx)}}{8d}$$

input
```
Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-1/8*I)*a^3*Hypergeometric2F1[1/2, 4, 3/2, (1 + I*Tan[c + d*x])/2]*Sqrt[
a + I*a*Tan[c + d*x]])/d
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 3968, 52, 52, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^{7/2}}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{ia^7\int \frac{1}{(a-ia\tan(c+dx))^4\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \; 52$$

$$-\frac{ia^7\left(\frac{5\int \frac{1}{(a-ia\tan(c+dx))^3\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{12a}+\frac{\sqrt{a+ia\tan(c+dx)}}{6a(a-ia\tan(c+dx))^3}\right)}{d}$$

$$\Big\downarrow \textbf{52}$$

$$-\frac{ia^7\left(\frac{5\left(\frac{3\int \frac{1}{(a-ia\tan(c+dx))^2\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{8a}+\frac{\sqrt{a+ia\tan(c+dx)}}{4a(a-ia\tan(c+dx))^2}\right)}{12a}+\frac{\sqrt{a+ia\tan(c+dx)}}{6a(a-ia\tan(c+dx))^3}\right)}{d}$$

$$\Big\downarrow \textbf{52}$$

$$-\frac{ia^7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{4a}+\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)}{8a}+\frac{\sqrt{a+ia\tan(c+dx)}}{4a(a-ia\tan(c+dx))^2}\right)}{12a}+\frac{\sqrt{a+ia\tan(c+dx)}}{6a(a-ia\tan(c+dx))^3}\right)}{d}$$

$$\Big\downarrow \textbf{73}$$

$$-\frac{ia^7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{2a}+\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)}{8a}+\frac{\sqrt{a+ia\tan(c+dx)}}{4a(a-ia\tan(c+dx))^2}\right)}{12a}+\frac{\sqrt{a+ia\tan(c+dx)}}{6a(a-ia\tan(c+dx))^3}\right)}{d}$$

$$\Big\downarrow \textbf{219}$$

$$-\frac{ia^7\left(\frac{5\left(\frac{3\left(\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{2\sqrt{2}a^{3/2}}+\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)}{8a}+\frac{\sqrt{a+ia\tan(c+dx)}}{4a(a-ia\tan(c+dx))^2}\right)}{12a}+\frac{\sqrt{a+ia\tan(c+dx)}}{6a(a-ia\tan(c+dx))^3}\right)}{d}$$

input
```
Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*a^7*(Sqrt[a + I*a*Tan[c + d*x]]/(6*a*(a - I*a*Tan[c + d*x])^3) + (5*
(Sqrt[a + I*a*Tan[c + d*x]]/(4*a*(a - I*a*Tan[c + d*x])^2) + (3*(((I/2)*Ar
cTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) + Sqrt[a + I*a*Tan
[c + d*x]]/(2*a*(a - I*a*Tan[c + d*x]))))/(8*a)))/(12*a)))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(155) = 310$.

Time = 2.20 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.81

$$\frac{\cos(dx+c)^3\left(\left(60\cos(dx+c)^3+60\cos(dx+c)^2-15\cos(dx+c)-15\right)\sin(dx+c)\operatorname{arctanh}\left(\frac{}{\sqrt{\cot(dx+c}}\right.\right.}{}$$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
1/192/d*cos(d*x+c)^3*((60*cos(d*x+c)^3+60*cos(d*x+c)^2-15*cos(d*x+c)-15)*s
in(d*x+c)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^
(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+
I*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(c
sc(d*x+c)-cot(d*x+c))*2^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(60*cos(
d*x+c)^4+60*cos(d*x+c)^3-45*cos(d*x+c)^2-45*cos(d*x+c))+I*(60*cos(d*x+c)^3
+60*cos(d*x+c)^2-15*cos(d*x+c)-15)*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))+cos(d*x+c)
*(-60*cos(d*x+c)^3-60*cos(d*x+c)^2+45*cos(d*x+c)+45)*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))+I*s
in(d*x+c)*cos(d*x+c)*(32*cos(d*x+c)^2-50*cos(d*x+c)-15)+cos(d*x+c)*(32*cos
(d*x+c)^3+82*cos(d*x+c)^2+35*cos(d*x+c)-15))*(a*(1+I*tan(d*x+c)))^(1/2)*(-
tan(d*x+c)+I)^3*a^3/(1+cos(d*x+c)+I*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.47

$$\int\cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}d\log\left(\frac{4\left(a^4e^{(i\,dx+i\,c)}-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^3}\right)-15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}d\log\left(\right.}{}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-1/384*(15*sqrt(1/2)*sqrt(-a^7/d^2)*d*log(4*(a^4*e^(I*d*x + I*c) - sqrt(2)
*sqrt(1/2)*sqrt(-a^7/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^3) - 15*sqrt(1/2)*sqrt(-a^7/d^2)*d*l
og(4*(a^4*e^(I*d*x + I*c) - sqrt(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(-I*d*e^(2*I*
d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^
3) - sqrt(2)*(-8*I*a^3*e^(7*I*d*x + 7*I*c) - 34*I*a^3*e^(5*I*d*x + 5*I*c)
- 59*I*a^3*e^(3*I*d*x + 3*I*c) - 33*I*a^3*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.93

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{i\left(15\sqrt{2}a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(15\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a^5-80\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^6+132\,\sqrt{i\,a}\right.}{(i\,a\tan(dx+c)+a)^3-6\,(i\,a\tan(dx+c)+a)^2a+12\,(i\,a\tan(dx+c)}\right.}{768\,ad}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
1/768*I*(15*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
+ a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(15*(I*a*tan(d*x
 + c) + a)^(5/2)*a^5 - 80*(I*a*tan(d*x + c) + a)^(3/2)*a^6 + 132*sqrt(I*a*
tan(d*x + c) + a)*a^7)/((I*a*tan(d*x + c) + a)^3 - 6*(I*a*tan(d*x + c) + a
)^2*a + 12*(I*a*tan(d*x + c) + a)*a^2 - 8*a^3))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int \cos\left(c+d\,x\right)^6\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2} \,d x$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

Reduce [F]

$$\begin{aligned}
&\int \cos^6(c+dx)(a+ia\tan(c \\
&+dx))^{7/2}\,dx = \sqrt{a}\,a^3\left(-\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^6\tan\left(dx+c\right)^3dx\right)i\right. \\
&-3\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^6\tan\left(dx+c\right)^2dx\right) \\
&+3\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^6\tan\left(dx+c\right)dx\right)i \\
&\left.+\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^6dx\right)
\end{aligned}$$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
sqrt(a)*a**3*( - int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**6*tan(c + d*x)
**3,x)*i - 3*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**6*tan(c + d*x)**2,
x) + 3*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**6*tan(c + d*x),x)*i + in
t(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**6,x))
```

3.326 $\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{256ia^4\sec(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} + \frac{64ia^3\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d} + \frac{24ia^2\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{35d} + \frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

output
```
256/35*I*a^4*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+64/35*I*a^3*sec(d*x+c)*
(a+I*a*tan(d*x+c))^(1/2)/d+24/35*I*a^2*sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)
/d+2/7*I*a*sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.78

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2a^3\sec^2(c+dx)(i\cos(c-2dx)+\sin(c-2dx))(75+102\cos(2(c+dx))+19i\sec(c+dx)\sin(3(c+dx))+14i\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{35d(\cos(dx)+i\sin(dx))^3}$$

input
```
Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(2*a^3*Sec[c + d*x]^2*(I*Cos[c - 2*d*x] + Sin[c - 2*d*x])*(75 + 102*Cos[2*
(c + d*x)] + (19*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (14*I)*Tan[c + d*x])*S
qrt[a + I*a*Tan[c + d*x]])/(35*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3975, 3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{7}a\int \sec(c+dx)(i\tan(c+dx)a+a)^{5/2}dx+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a\int \sec(c+dx)(i\tan(c+dx)a+a)^{5/2}dx+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow 3975$$

$$\frac{12}{7}a\left(\frac{8}{5}a\int \sec(c+dx)(i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}\right)+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow 3042$$

$$\frac{12}{7}a\left(\frac{8}{5}a\int \sec(c+dx)(i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}\right)+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow 3975$$

$$\frac{12}{7}a\left(\frac{8}{5}a\left(\frac{4}{3}a\int \sec(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)+\frac{2ia\sec(c+dx)(a}{}\right.$$
$$\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow 3042$$

$$\frac{12}{7}a\left(\frac{8}{5}a\left(\frac{4}{3}a\int \sec(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)+\frac{2ia\sec(c+dx)(a}{}\right.$$
$$\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow 3974$$

$$\frac{12}{7}a\left(\frac{8}{5}a\left(\frac{8ia^2\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx)}{5d}\right.$$
$$\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

input
```
Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((2*I)/7)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2))/d + (12*a*((((2*I)
/5)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2))/d + (8*a*((((8*I)/3)*a^2*
Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((2*I)/3)*a*Sec[c + d*x]*S
qrt[a + I*a*Tan[c + d*x]])/d))/5))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\left(\frac{2 i \left(-5 \sec \left(d x +c \right)^{3}+128 \cos \left(d x +c \right)+54 \sec \left(d x +c \right)\right)}{35}-\frac{44 \sec \left(d x +c \right) \tan \left(d x +c \right)}{35}+\frac{256 \sin \left(d x +c \right)}{35}\right) \sqrt{a \left(1+i \tan \left(d x +c \right)\right)}\, a^{3}}{d}$	76

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/35*I*(-5*sec(d*x+c)^3+128*cos(d*x+c)+54*sec(d*x+c))-44/35*sec(d*x+c
)*tan(d*x+c)+256/35*sin(d*x+c))*(a*(1+I*tan(d*x+c)))^(1/2)*a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.78

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{16\sqrt{2}\left(-35i\,a^3e^{(6i\,dx+6i\,c)}-70i\,a^3e^{(4i\,dx+4i\,c)}-56i\,a^3e^{(2i\,dx+2i\,c)}-16i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{35\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-16/35*sqrt(2)*(-35*I*a^3*e^(6*I*d*x + 6*I*c) - 70*I*a^3*e^(4*I*d*x + 4*I*
c) - 56*I*a^3*e^(2*I*d*x + 2*I*c) - 16*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x +
2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.06

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{a^3\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{d} - \frac{a^3\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} + \frac{a^3\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,48\mathrm{i}}{5\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} - \frac{a^3\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}$$

input
```
int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x),x)
```

output
```
(a^3*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*
2i + d*x*2i) + 1))^(1/2)*16i)/d - (a^3*exp(- c*1i - d*x*1i)*(a - (a*(exp(c
*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(d*(exp(c*
2i + d*x*2i) + 1)) + (a^3*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*48i)/(5*d*(exp(c*2i + d*x*2i
) + 1)^2) - (a^3*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)
*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3)
```

Reduce [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{\sqrt{a}\,a^3\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)\,i-\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)\tan(dx\right.\right.}{}$$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*a**3*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)*i - int(sqrt(tan
(c + d*x)*i + 1)*sec(c + d*x)*tan(c + d*x)**3,x)*d*i - 3*int(sqrt(tan(c +
d*x)*i + 1)*sec(c + d*x)*tan(c + d*x)**2,x)*d + 6*int(sqrt(tan(c + d*x)*i
+ 1)*sec(c + d*x)*tan(c + d*x),x)*d*i))/d
```

3.327 $\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{64ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d} + \frac{16ia^2\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d}$$

output
```
-64/3*I*a^3*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d+16/3*I*a^2*cos(d*x+c)*(a
+I*a*tan(d*x+c))^(3/2)/d+2/3*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.57

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{2ia^3\sec(c+dx)(12+11\cos(2(c+dx))-5i\sin(2(c+dx)))\sqrt{a+ia\tan(c+dx)}}{3d}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((-2*I)/3)*a^3*Sec[c + d*x]*(12 + 11*Cos[2*(c + d*x)] - (5*I)*Sin[2*(c +
d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{7/2}}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{3}a\int \cos(c+dx)(i\tan(c+dx)a+a)^{5/2}dx+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{3}a\int \frac{(i\tan(c+dx)a+a)^{5/2}}{\sec(c+dx)}dx+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{3}a\left(4a\int \cos(c+dx)(i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}\right)+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{3}a\left(4a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{\sec(c+dx)}dx+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}\right)+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8}{3}a\left(\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}-\frac{8ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((2*I)/3)*a*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2))/d + (8*a*(((-8*I)*
a^2*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + ((2*I)*a*Cos[c + d*x]*(a
+ I*a*Tan[c + d*x])^(3/2))/d))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 7.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\left(-\frac{2i(\sec(dx+c)+22\cos(dx+c))}{3}-\frac{20\sin(dx+c)}{3}\right)\sqrt{a(1+i\tan(dx+c))}\,a^3}{d}$	50

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/3*I*(sec(d*x+c)+22*cos(d*x+c))-20/3*sin(d*x+c))*(a*(1+I*tan(d*x+c)
))^(1/2)*a^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{4\sqrt{2}\left(3i\,a^3e^{(4i\,dx+4i\,c)}+12i\,a^3e^{(2i\,dx+2i\,c)}+8i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-4/3*sqrt(2)*(3*I*a^3*e^(4*I*d*x + 4*I*c) + 12*I*a^3*e^(2*I*d*x + 2*I*c) +
 8*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(80) = 160$.

Time = 0.23 (sec) , antiderivative size = 418, normalized size of antiderivative = 4.02

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2\left(23i\,a^{\frac{7}{2}} + \frac{20\,a^{\frac{7}{2}}\sin(dx+c)}{\cos(dx+c)+1} - \frac{88i\,a^{\frac{7}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{60\,a^{\frac{7}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{130i\,a^{\frac{7}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{60\,a^{\frac{7}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right.}{-3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{7}{2}}\left(\frac{6i\,\sin(dx+c)}{\cos(dx+c)+1} - \frac{14\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{14i\,\sin(\right.}}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
2*(23*I*a^(7/2) + 20*a^(7/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 88*I*a^(7/2
)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 60*a^(7/2)*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 130*I*a^(7/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 60*a^(7
/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 88*I*a^(7/2)*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 - 20*a^(7/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 23*I*a
^(7/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(-2*I*sin(d*x + c)/(cos(d*x +
c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2)/(d*(sin(d*x + c)/
(cos(d*x + c) + 1) + 1)^(7/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)*
(-18*I*sin(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 42*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 42*I*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 - 42*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 18*I*sin(d*x
+ c)^7/(cos(d*x + c) + 1)^7 + 3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 3))
```

Giac [F(-2)]

Exception generated.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{2\,a^3\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(5\,\sin\left(c+d\,x\right)+5\,\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,35{}\mathrm{i}+\cos\left(3\,c+3\right.}{3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
-(2*a^3*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x
) + 1))^(1/2)*(cos(c + d*x)*35i + 5*sin(c + d*x) + cos(3*c + 3*d*x)*11i +
5*sin(3*c + 3*d*x)))/(3*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \sqrt{a}\,a^3\left(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)\tan(dx+c)^3\,dx\right)i - 3\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)\tan(dx+c)^2\,dx\right) + 3\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)\tan(dx+c)\,dx\right)i + \int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)\,dx\right)$$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
sqrt(a)*a**3*( - int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)*tan(c + d*x)**3
,x)*i - 3*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)*tan(c + d*x)**2,x) + 3
*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)*tan(c + d*x),x)*i + int(sqrt(ta
n(c + d*x)*i + 1)*cos(c + d*x),x))
```

3.328 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\frac{8ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}}{d}$$

output
```
8/3*I*a^2*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d-2*I*a*cos(d*x+c)^3*(a+I*
a*tan(d*x+c))^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.21

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\frac{2a^3\cos(c+dx)(i\cos(c+dx)+3\sin(c+dx))(\cos(c+4dx)+i\sin(c+4dx))\sqrt{a+ia\tan(c}{3d(\cos(dx)+i\sin(dx))^3}$$

input
```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(2*a^3*Cos[c + d*x]*(I*Cos[c + d*x] + 3*Sin[c + d*x])*(Cos[c + 4*d*x] + I*
Sin[c + 4*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*(Cos[d*x] + I*Sin[d*x])^3
)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{7/2}}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3975}$$

$$-4a\int \cos^3(c+dx)(i\tan(c+dx)a+a)^{5/2}dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-4a\int \frac{(i\tan(c+dx)a+a)^{5/2}}{\sec(c+dx)^3}dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}}{d}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}}{d}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((8*I)/3)*a^2*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d - ((2*I)*a*C
os[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 15.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2a^{3}\sqrt{a(1+i\tan(dx+c))}\,\left(-2i\cos(dx+c)^{3}+2\sin(dx+c)\cos(dx+c)^{2}+3i\cos(dx+c)\right)}{3d}$	61

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*a^3*(a*(1+I*tan(d*x+c)))^(1/2)*(-2*I*cos(d*x+c)^3+2*sin(d*x+c)*cos(d
*x+c)^2+3*I*cos(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{\sqrt{2}\left(-i\,a^3e^{(4i\,dx+4i\,c)}+i\,a^3e^{(2i\,dx+2i\,c)}+2i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(2)*(-I*a^3*e^(4*I*d*x + 4*I*c) + I*a^3*e^(2*I*d*x + 2*I*c) + 2*I*
a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(57) = 114$.

Time = 0.23 (sec) , antiderivative size = 504, normalized size of antiderivative = 7.10

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-2*(-I*a^(7/2) - 6*a^(7/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 5*I*a^(7/2)*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 24*a^(7/2)*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 10*I*a^(7/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^(7/2)*
sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 10*I*a^(7/2)*sin(d*x + c)^6/(cos(d*x
 + c) + 1)^6 + 24*a^(7/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5*I*a^(7/2
)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a^(7/2)*sin(d*x + c)^9/(cos(d*x
+ c) + 1)^9 + I*a^(7/2)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)*(-2*I*sin(d
*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2
)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(sin(d*x + c)/(cos(d*x +
c) + 1) - 1)^(7/2)*(12*I*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 24*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 42*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 - 42*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
- 24*I*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 9*sin(d*x + c)^8/(cos(d*x + c
) + 1)^8 - 12*I*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 3*sin(d*x + c)^10/(c
os(d*x + c) + 1)^10 + 3))
```

Giac [F(-2)]

Exception generated.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{a^3\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1i)}{\cos(2\,c+2\,d\,x)+1}}\,\left(\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,3i-\cos\left(3\right.\right.}{3\,d}$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
(a^3*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) +
 1))^(1/2)*(cos(c + d*x)*3i + sin(c + d*x) - cos(3*c + 3*d*x)*1i + sin(3*c
 + 3*d*x)))/(3*d)
```

Reduce [F]

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \sqrt{a}\,a^3\left(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^3\tan(dx+c)^3\,dx\right)i - 3\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^3\tan(dx+c)^2\,dx\right) + 3\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^3\tan(dx+c)\,dx\right)i + \int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^3\,dx\right)$$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
sqrt(a)*a**3*( - int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**3*tan(c + d*x)
**3,x)*i - 3*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**3*tan(c + d*x)**2,
x) + 3*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**3*tan(c + d*x),x)*i + in
t(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**3,x))
```

3.329 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

output
```
-2/5*I*a*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2)/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.

Time = 1.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\frac{2a^3\cos^3(c+dx)(-i\cos(2c+5dx)+\sin(2c+5dx))\sqrt{a+ia\tan(c+dx)}}{5d(\cos(dx)+i\sin(dx))^3}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(2*a^3*Cos[c + d*x]^3*((-I)*Cos[2*c + 5*d*x] + Sin[2*c + 5*d*x])*Sqrt[a +
I*a*Tan[c + d*x]])/(5*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{3042, 3974\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{7/2}}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3974}$$

$$-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((-2*I)/5)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(29) = 58$.

Time = 3.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.77

$$\frac{\left(-\frac{4i\cos(dx+c)^5}{5}+\frac{4\cos(dx+c)^4\sin(dx+c)}{5}+\frac{2i\cos(dx+c)^3}{5}\right)a^3\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}}{d}$$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
1/d*(-4/5*I*cos(d*x+c)^5+4/5*cos(d*x+c)^4*sin(d*x+c)+2/5*I*cos(d*x+c)^3)*a
^3*(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(27) = 54$.

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int\cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\frac{\sqrt{2}\left(-i\,a^3e^{(6i\,dx+6i\,c)}-3i\,a^3e^{(4i\,dx+4i\,c)}-3i\,a^3e^{(2i\,dx+2i\,c)}-i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{20\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/20*sqrt(2)*(-I*a^3*e^(6*I*d*x + 6*I*c) - 3*I*a^3*e^(4*I*d*x + 4*I*c) - 3
*I*a^3*e^(2*I*d*x + 2*I*c) - I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(27) = 54$.

Time = 0.28 (sec) , antiderivative size = 454, normalized size of antiderivative = 12.97

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
2*(I*a^(7/2) - 6*I*a^(7/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*I*a^(7
/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*I*a^(7/2)*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + 15*I*a^(7/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*I*
a^(7/2)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + I*a^(7/2)*sin(d*x + c)^12/
(cos(d*x + c) + 1)^12)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) +
1)^(7/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)*(-10*I*sin(d*x + c)/(
cos(d*x + c) + 1) - 20*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 50*I*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 - 25*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 100
*I*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 100*I*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + 25*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 50*I*sin(d*x + c)^9/(c
os(d*x + c) + 1)^9 + 20*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 10*I*sin(d
*x + c)^11/(cos(d*x + c) + 1)^11 + 5*sin(d*x + c)^12/(cos(d*x + c) + 1)^12
 - 5))
```

Giac [F(-2)]

Exception generated.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.20

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{a^3\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(-2\,\sin(c+d\,x)-3\,\sin(3\,c+3\,d\,x)-\sin(5\,c+5\,d\,x)+\cos(c+d\,x)}{20\,d}$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
-(a^3*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x)
+ 1))^(1/2)*(cos(c + d*x)*4i - 2*sin(c + d*x) + cos(3*c + 3*d*x)*3i + cos(
5*c + 5*d*x)*1i - 3*sin(3*c + 3*d*x) - sin(5*c + 5*d*x)))/(20*d)
```

Reduce [F]

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \sqrt{a}\,a^3\left(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^5\tan(dx+c)^3\,dx\right)i - 3\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^5\tan(dx+c)^2\,dx\right) + 3\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^5\tan(dx+c)\,dx\right)i + \int \sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^5\,dx\right)$$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
sqrt(a)*a**3*( - int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**5*tan(c + d*x)
**3,x)*i - 3*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**5*tan(c + d*x)**2,
x) + 3*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**5*tan(c + d*x),x)*i + in
t(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**5,x))
```

3.330 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 196

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{ia^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}d} - \frac{ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d} - \frac{ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{12d} - \frac{ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{10d} - \frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

output
```
1/16*I*a^(7/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(
1/2))*2^(1/2)/d-1/8*I*a^3*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/12*I*a^2
*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d-1/10*I*a*cos(d*x+c)^5*(a+I*a*tan(
d*x+c))^(5/2)/d-1/7*I*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2)/d
```

Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.67

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$
$$-\frac{ia^3e^{-i(c+dx)}\left(176+298e^{2i(c+dx)}+188e^{4i(c+dx)}+81e^{6i(c+dx)}+15e^{8i(c+dx)}-105\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1\right.\right.}{1680d}$$

input
```
Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-1/1680*I)*a^3*(176 + 298*E^((2*I)*(c + d*x)) + 188*E^((4*I)*(c + d*x))
+ 81*E^((6*I)*(c + d*x)) + 15*E^((8*I)*(c + d*x)) - 105*Sqrt[1 + E^((2*I)*
(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d
*x]])/(d*E^(I*(c + d*x)))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {3042, 3971, 3042, 3971, 3042, 3971, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{7/2}}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\int \cos^5(c+dx)(i\tan(c+dx)a+a)^{5/2}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\int \frac{(i\tan(c+dx)a+a)^{5/2}}{\sec(c+dx)^5}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\int \cos^3(c+dx)(i\tan(c+dx)a+a)^{3/2}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}\right)-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{\sec(c+dx)^3}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}\right)-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{1}{2}a\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}\right)-\frac{i\cos^5(c+dx)(a\cdots}{}\right.$$
$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{1}{2}a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}\right)-\frac{i\cos^5(c+dx)(a+ia\tan(c\cdots}{5d}\right.$$
$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia\tan(c\cdots}{3d}\right.\right.$$
$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia\tan(c\cdots}{3d}\right.\right.$$
$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3970}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{ia\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+i\right.\right.$$

$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia\tan\right.\right.$$

$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

input
```
Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-1/7*I)*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(7/2))/d + (a*(((-1/5*I)*C
os[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2))/d + (a*(((-1/3*I)*Cos[c + d*x]
^3*(a + I*a*Tan[c + d*x])^(3/2))/d + (a*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c
 + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d
*x]*Sqrt[a + I*a*Tan[c + d*x]])/d))/2))/2))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1925 vs. $2(159) = 318$.

Time = 3.98 (sec) , antiderivative size = 1926, normalized size of antiderivative = 9.83

Expression too large to display

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x)
```

output

```
-1/1680/d*cos(d*x+c)^3*((840*cos(d*x+c)^3+420*cos(d*x+c)^2-420*cos(d*x+c)-
105)*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/(cot(d*x+c)^2
-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(
1/2))*tan(d*x+c)^3+I*(840*cos(d*x+c)^4+420*cos(d*x+c)^3-840*cos(d*x+c)^2-3
15*cos(d*x+c)+105)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/(cot(d*x+c
)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*
2^(1/2))*tan(d*x+c)^3+I*(2520*cos(d*x+c)^3+1260*cos(d*x+c)^2-1260*cos(d*x+
c)-315)*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))*tan(d*x+c)+(2520*cos(d*x+c)^4+1260*co
s(d*x+c)^3-2520*cos(d*x+c)^2-945*cos(d*x+c)+315)*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^
(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*tan(d*x+c)^2+(-2520*cos(d*x+c)^3-12
60*cos(d*x+c)^2+1260*cos(d*x+c)+315)*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(
1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*tan(d*x+c)+I*(-840*cos(d*x+c)^3-420*
cos(d*x+c)^2+420*cos(d*x+c)+105)*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))*tan(d*x+c)^3
+I*sin(d*x+c)^2*(3864*cos(d*x+c)^2-630)+(-840*cos(d*x+c)^4-420*cos(d*x+c)^
3+840*cos(d*x+c)^2+315*cos(d*x+c)-105)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.32

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{105\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\,d\log\left(\frac{\left(i\,a^4+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{4\,d}\right)-105\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}}{}$$

input

```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/1680*(105*sqrt(1/2)*sqrt(-a^7/d^2)*d*log(1/4*(I*a^4 + sqrt(2)*sqrt(1/2)*
sqrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
)))*e^(-I*d*x - I*c)/d) - 105*sqrt(1/2)*sqrt(-a^7/d^2)*d*log(1/4*(I*a^4 -
sqrt(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) + sqrt(2)*(-15*I*a^3*e^(8*I*d*x
+ 8*I*c) - 81*I*a^3*e^(6*I*d*x + 6*I*c) - 188*I*a^3*e^(4*I*d*x + 4*I*c) -
298*I*a^3*e^(2*I*d*x + 2*I*c) - 176*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1253 vs. $2(149) = 298$.

Time = 0.43 (sec) , antiderivative size = 1253, normalized size of antiderivative = 6.39

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-1/6720*(20*(7*I*sqrt(2)*a^3*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) - 7*sqrt(2)*a^3*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + 3*(I*sqrt(2)*a^3*cos(2*d*x + 2*c)^2 + I*sqrt(2)*a^3*sin(2*d*x
 + 2*c)^2 + 2*I*sqrt(2)*a^3*cos(2*d*x + 2*c) + I*sqrt(2)*a^3)*cos(7/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 3*(sqrt(2)*a^3*cos(2*d*x +
2*c)^2 + sqrt(2)*a^3*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^3*cos(2*d*x + 2*c) +
 sqrt(2)*a^3)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*sqr
t(a) + 84*(5*I*sqrt(2)*a^3*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) - 5*sqrt(2)*a^3*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + (I*sqrt(2)*a^3*cos(2*d*x + 2*c)^2 + I*sqrt(2)*a^3*sin(2*d*x + 2
*c)^2 + 2*I*sqrt(2)*a^3*cos(2*d*x + 2*c) + I*sqrt(2)*a^3)*cos(5/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (sqrt(2)*a^3*cos(2*d*x + 2*c)^2
 + sqrt(2)*a^3*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^3*cos(2*d*x + 2*c) + sqrt(
2)*a^3)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) +
 105*(2*sqrt(2)*a^3*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)...
```

Giac [F(-2)]

Exception generated.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int \cos\left(c+d\,x\right)^7\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}\,dx$$

input
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

Reduce [F]

$$\begin{aligned}
&\int \cos^7(c+dx)(a+ia\tan(c\\
&+dx))^{7/2}\,dx = \sqrt{a}\,a^3\left(-\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^7\tan\left(dx+c\right)^3dx\right)i\right.\\
&-3\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^7\tan\left(dx+c\right)^2dx\right)\\
&+3\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^7\tan\left(dx+c\right)dx\right)i\\
&\left.+\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^7dx\right)
\end{aligned}$$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
sqrt(a)*a**3*( - int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**7*tan(c + d*x)
**3,x)*i - 3*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**7*tan(c + d*x)**2,
x) + 3*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**7*tan(c + d*x),x)*i + in
t(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**7,x))
```

3.331 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 268

$$\begin{aligned}\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx &= \frac{11ia^{7/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}d} \\ &+ \frac{11ia^4\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} - \frac{11ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64d} \\ &- \frac{11ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{120d} \\ &- \frac{11ia^2\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{140d} \\ &- \frac{11ia\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{126d} \\ &- \frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}\end{aligned}$$

output
```
11/128*I*a^(7/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))
^(1/2))*2^(1/2)/d+11/96*I*a^4*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-11/64*
I*a^3*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-11/120*I*a^3*cos(d*x+c)^3*(a+I
*a*tan(d*x+c))^(1/2)/d-11/140*I*a^2*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2)/
d-11/126*I*a*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2)/d-1/9*I*cos(d*x+c)^9*(a
+I*a*tan(d*x+c))^(7/2)/d
```

Mathematica [A] (verified)

Time = 3.55 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.70

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$
$$-\frac{ia^3e^{-3i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-315+4303e^{2i(c+dx)}+7034e^{4i(c+dx)}+3754e^{6i(c+dx)}+1798e^{8i(c+dx)}+530e^{10i(c}\right.}{20160\sqrt{2}d}$$

input
```
Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-1/20160*I)*a^3*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*
(-315 + 4303*E^((2*I)*(c + d*x)) + 7034*E^((4*I)*(c + d*x)) + 3754*E^((6*I
)*(c + d*x)) + 1798*E^((8*I)*(c + d*x)) + 530*E^((10*I)*(c + d*x)) + 70*E^
((12*I)*(c + d*x)) - 3465*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))
]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*E^((3*I)*(c + d*x)))
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {3042, 3978, 3042, 3978, 3042, 3978, 3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{7/2}}{\sec(c+dx)^9}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{11}{18}a\int \cos^7(c+dx)(i\tan(c+dx)a+a)^{5/2}dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{18}a\int \frac{(i\tan(c+dx)a+a)^{5/2}}{\sec(c+dx)^7}dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{11}{18}a\left(\frac{9}{14}a\int \cos^5(c+dx)(i\tan(c+dx)a+a)^{3/2}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}\right)-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{18}a\left(\frac{9}{14}a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{\sec(c+dx)^5}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}\right)-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{11}{18}a\left(\frac{9}{14}a\left(\frac{7}{10}a\int \cos^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}\right)-\frac{i\cos^7(c+dx}{}\right.$$
$$\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{18}a\left(\frac{9}{14}a\left(\frac{7}{10}a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^3}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}\right)-\frac{i\cos^7(c+dx)(a+ia\tan}{7d}\right.$$
$$\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{11}{18}a\left(\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)-\frac{i\cos^5(c+dx)(a+ia\,t}{5d}\right.\right.$$
$$\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{18}a\left(\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\int\frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)-\frac{i\cos^5(c+\right.\right.\right.$$

$$\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3983}$$

$$\frac{11}{18}a\left(\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\int\cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+}}{3d}\right.\right.\right.$$

$$\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{18}a\left(\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)\right.\right.$$

$$\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{11}{18}a\left(\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\,\mathrm{co}}{}\right.\right.\right.$$

$$\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{18}a\left(\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\,\mathrm{co}}{}\right.\right.\right.$$

$$\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3970}$$

$$\frac{11}{18}a\left(\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right.\right.\right.\right.$$

$$\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \; 219$$

$$\frac{11}{18}a\left(\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right.\right.\right.\right.$$

$$\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

input
```
Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-1/9*I)*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^(7/2))/d + (11*a*(((-1/7*I
)*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(5/2))/d + (9*a*(((-1/5*I)*Cos[c +
 d*x]^5*(a + I*a*Tan[c + d*x])^(3/2))/d + (7*a*(((-1/3*I)*Cos[c + d*x]^3*S
qrt[a + I*a*Tan[c + d*x]])/d + (5*a*(((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a*
Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sq
rt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan
[c + d*x]])/d))/(4*a)))/6))/10))/14))/18
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2004 vs. $2(219) = 438$.

Time = 6.88 (sec) , antiderivative size = 2005, normalized size of antiderivative = 7.48

Expression too large to display

input
```
int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
-1/40320/d*cos(d*x+c)^3*((27720*cos(d*x+c)^3+13860*cos(d*x+c)^2-13860*cos(
d*x+c)-3465)*sin(d*x+c)*tan(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ar
ctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d
*x+c)-cot(d*x+c))*2^(1/2))+I*(83160*cos(d*x+c)^3+41580*cos(d*x+c)^2-41580*
cos(d*x+c)-10395)*sin(d*x+c)*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*arctan(1/2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2))+I*cos(d*x+c)^2*(
-7840*cos(d*x+c)^4+50424*cos(d*x+c)^2-25410)+(83160*cos(d*x+c)^4+41580*cos
(d*x+c)^3-83160*cos(d*x+c)^2-31185*cos(d*x+c)+10395)*tan(d*x+c)^2*(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)
+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))+(-83160*cos(d*x+c)
^3-41580*cos(d*x+c)^2+41580*cos(d*x+c)+10395)*sin(d*x+c)*tan(d*x+c)*(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+
c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))+I*sin(d*x+c)^2*(
23520*cos(d*x+c)^4-151272*cos(d*x+c)^2+76230)+I*sin(d*x+c)^2*(-36960*cos(d
*x+c)^4+127512*cos(d*x+c)^2-20790)+(-27720*cos(d*x+c)^4-13860*cos(d*x+c)^3
+27720*cos(d*x+c)^2+10395*cos(d*x+c)-3465)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*
(csc(d*x+c)-cot(d*x+c))*2^(1/2))+I*(27720*cos(d*x+c)^4+13860*cos(d*x+c)^3-
27720*cos(d*x+c)^2-10395*cos(d*x+c)+3465)*tan(d*x+c)^3*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.17

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{\left(3465\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}de^{(2i\,dx+2i\,c)}\log\left(-\frac{11\left(-i\,a^4+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{32\,d}\right)-3465\sqrt{\frac{1}{2}}\cdots\right.}{}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-1/40320*(3465*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(2*I*d*x + 2*I*c)*log(-11/32*(
-I*a^4 + sqrt(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) - 3465*sqrt(1/2)*sqrt(-
a^7/d^2)*d*e^(2*I*d*x + 2*I*c)*log(-11/32*(-I*a^4 - sqrt(2)*sqrt(1/2)*sqrt
(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*
e^(-I*d*x - I*c)/d) - sqrt(2)*(-70*I*a^3*e^(12*I*d*x + 12*I*c) - 530*I*a^3
*e^(10*I*d*x + 10*I*c) - 1798*I*a^3*e^(8*I*d*x + 8*I*c) - 3754*I*a^3*e^(6*
I*d*x + 6*I*c) - 7034*I*a^3*e^(4*I*d*x + 4*I*c) - 4303*I*a^3*e^(2*I*d*x +
2*I*c) + 315*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c
)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int \cos\left(c+d\,x\right)^9\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2} \,d x$$

input
```
int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

Reduce [F]

$$\begin{aligned}&\int \cos^9(c+dx)(a+ia\tan(c\\&+dx))^{7/2}\,dx = \sqrt{a}\,a^3\left(-\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^9\tan\left(dx+c\right)^3dx\right)i\right.\\&-3\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^9\tan\left(dx+c\right)^2dx\right)\\&+3\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^9\tan\left(dx+c\right)dx\right)i\\&\left.+\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^9dx\right)\end{aligned}$$

input
```
int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
sqrt(a)*a**3*( - int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**9*tan(c + d*x)
**3,x)*i - 3*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**9*tan(c + d*x)**2,
x) + 3*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**9*tan(c + d*x),x)*i + in
t(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**9,x))
```

3.332 $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 342

$$\begin{aligned}
\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx &= \frac{195ia^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{1024\sqrt{2}d} \\
&+ \frac{65ia^4\cos(c+dx)}{512d\sqrt{a+ia\tan(c+dx)}} + \frac{39ia^4\cos^3(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{195ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{1024d} \\
&- \frac{13ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d} \\
&- \frac{13ia^3\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{168d} \\
&- \frac{65ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{924d} \\
&- \frac{5ia\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{66d} \\
&- \frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}
\end{aligned}$$

output
```
195/2048*I*a^(7/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c
))^(1/2))*2^(1/2)/d+65/512*I*a^4*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+39/
448*I*a^4*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)-195/1024*I*a^3*cos(d*x+c
)*(a+I*a*tan(d*x+c))^(1/2)/d-13/128*I*a^3*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^
(1/2)/d-13/168*I*a^3*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2)/d-65/924*I*a^2*
cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(3/2)/d-5/66*I*a*cos(d*x+c)^9*(a+I*a*tan(d
*x+c))^(5/2)/d-1/11*I*cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2)/d
```

Mathematica [A] (verified)

Time = 5.75 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.57

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$
$$-\frac{ia^3e^{-5i(c+dx)}\Big(-462-7161e^{2i(c+dx)}+47413e^{4i(c+dx)}+78800e^{6i(c+dx)}+38512e^{8i(c+dx)}+19552e^{10i(c+dx)}+}{}$$

input
```
Integrate[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-1/473088*I)*a^3*(-462 - 7161*E^((2*I)*(c + d*x)) + 47413*E^((4*I)*(c +
d*x)) + 78800*E^((6*I)*(c + d*x)) + 38512*E^((8*I)*(c + d*x)) + 19552*E^((
10*I)*(c + d*x)) + 7184*E^((12*I)*(c + d*x)) + 1624*E^((14*I)*(c + d*x)) +
 168*E^((16*I)*(c + d*x)) - 45045*E^((4*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x
]])/(d*E^((5*I)*(c + d*x)))
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.07, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.731$, Rules used = {3042, 3978, 3042, 3978, 3042, 3978, 3042, 3978, 3042, 3983, 3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{7/2}}{\sec(c+dx)^{11}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{15}{22}a\int \cos^9(c+dx)(i\tan(c+dx)a+a)^{5/2}dx-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15}{22}a\int \frac{(i\tan(c+dx)a+a)^{5/2}}{\sec(c+dx)^9}dx-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\int \cos^7(c+dx)(i\tan(c+dx)a+a)^{3/2}dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{9d}\right)-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{\sec(c+dx)^7}dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{9d}\right)-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\int \cos^5(c+dx)\sqrt{i\tan(c+dx)a+a}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{7d}\right)-\frac{i\cos^9(c+dx}{}\right.$$
$$-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^5}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{7d}\right)-\frac{i\cos^9(c+dx)(a+ia\tan}{9d}\right.$$
$$-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\int\frac{\cos^3(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}\right)-\frac{i\cos^7(c+dx)(a+ia\right.\right.\right.$$
$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\int\frac{1}{\sec(c+dx)^3\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}\right)-\frac{i\cos^7(c\right.\right.\right.$$
$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3983}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\int\cos^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^5(c+dx)\sqrt{a}}{5}\right.\right.\right.\right.$$
$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^3}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}\right.\right.\right.\right.$$
$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\int\frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i}{}\right.\right.\right.\right.$$
$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\int\frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx}}\right.\right.\right.\right.$$

$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3983}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int\cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\right.\right.\right.\right.$$

$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\,c}{4d\sqrt{a-}}\right.\right.\right.\right.$$

$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx}{}\right.}{8a}\right.\right.\right.\right.$$

$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx}{}\right.}{8a}\right.\right.\right.\right.$$

$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \ 3970$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{ia\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)\right.}{8a}\right.\right.\right.\right.$$

$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \ 219$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\right.}{8a}\right.\right.\right.\right.$$

$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

input
```
Int[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-1/11*I)*Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^(7/2))/d + (15*a*(((-1/9
*I)*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^(5/2))/d + (13*a*(((-1/7*I)*Cos[
c + d*x]^7*(a + I*a*Tan[c + d*x])^(3/2))/d + (11*a*(((-1/5*I)*Cos[c + d*x]
^5*Sqrt[a + I*a*Tan[c + d*x]])/d + (9*a*(((I/4)*Cos[c + d*x]^3)/(d*Sqrt[a
+ I*a*Tan[c + d*x]]) + (7*(((-1/3*I)*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d
*x]])/d + (5*a*(((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*(
(I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*
x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d))/(4*a)
))/6))/(8*a)))/10))/14))/18))/22
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2084 vs. $2(281) = 562$.

Time = 7.72 (sec) , antiderivative size = 2085, normalized size of antiderivative = 6.10

Expression too large to display

input
```
int(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
-1/473088/d*cos(d*x+c)^3*((360360*cos(d*x+c)^3+180180*cos(d*x+c)^2-180180*
cos(d*x+c)-45045)*sin(d*x+c)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+
c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*tan(d*x+c)^3+I*(360360*cos(d*x+c)^4+180180*cos(d*x+c)^3
-360360*cos(d*x+c)^2-135135*cos(d*x+c)+45045)*arctanh(1/(cot(d*x+c)^2-2*co
t(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))
*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*tan(d*x+c)^3+I*sin(d*x+c)^2*tan(d*x+c)
^2*(80640*cos(d*x+c)^6+160160*cos(d*x+c)^4-552552*cos(d*x+c)^2+90090)+(108
1080*cos(d*x+c)^4+540540*cos(d*x+c)^3-1081080*cos(d*x+c)^2-405405*cos(d*x+
c)+135135)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)
^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*tan(d*x+c)^2+(-1081080*cos(d*x+c)^3-540540*cos(d*x+c)^2+540540*cos(d*x+c)
+135135)*sin(d*x+c)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*
x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*tan(d*x+c)+I*sin(d*x+c)^2*(-241920*cos(d*x+c)^6-480480*cos(d*x+c
)^4+1657656*cos(d*x+c)^2-270270)+I*(-1081080*cos(d*x+c)^3-540540*cos(d*x+c
)^2+540540*cos(d*x+c)+135135)*sin(d*x+c)*arctanh(1/(cot(d*x+c)^2-2*cot(d*x
+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(csc(d*x+c)-cot(d*x+c))*2^(1/2))*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*tan(d*x+c)^2+(-360360*cos(d*x+c)^4-180180*c
os(d*x+c)^3+360360*cos(d*x+c)^2+135135*cos(d*x+c)-45045)*arctanh(1/(cot...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{\left(45045\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}de^{(4i\,dx+4i\,c)}\log\left(-\frac{195\left(-i\,a^4+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{512\,d}\right)-45045\right.}{}$$

input
```
integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-1/473088*(45045*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(4*I*d*x + 4*I*c)*log(-195/5
12*(-I*a^4 + sqrt(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) - 45045*sqrt(1/2)*s
qrt(-a^7/d^2)*d*e^(4*I*d*x + 4*I*c)*log(-195/512*(-I*a^4 - sqrt(2)*sqrt(1/
2)*sqrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1)))*e^(-I*d*x - I*c)/d) - sqrt(2)*(-168*I*a^3*e^(16*I*d*x + 16*I*c) - 1
624*I*a^3*e^(14*I*d*x + 14*I*c) - 7184*I*a^3*e^(12*I*d*x + 12*I*c) - 19552
*I*a^3*e^(10*I*d*x + 10*I*c) - 38512*I*a^3*e^(8*I*d*x + 8*I*c) - 78800*I*a
^3*e^(6*I*d*x + 6*I*c) - 47413*I*a^3*e^(4*I*d*x + 4*I*c) + 7161*I*a^3*e^(2
*I*d*x + 2*I*c) + 462*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-4*I*d*
x - 4*I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**11*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int \cos\left(c+d\,x\right)^{11}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}\,dx$$

input
```
int(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

Reduce [F]

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \sqrt{a}\,a^{3}\left(-\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^{11}\tan\left(dx+c\right)^{3}dx\right)i - 3\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^{11}\tan\left(dx+c\right)^{2}dx\right) + 3\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^{11}\tan\left(dx+c\right)dx\right)i + \int \sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^{11}dx\right)$$

input
```
int(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
sqrt(a)*a**3*( - int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**11*tan(c + d*x
)**3,x)*i - 3*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**11*tan(c + d*x)**
2,x) + 3*int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**11*tan(c + d*x),x)*i +
 int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**11,x))
```

3.333 $\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{16i(a+ia\tan(c+dx))^{7/2}}{7a^4d} + \frac{8i(a+ia\tan(c+dx))^{9/2}}{3a^5d} - \frac{12i(a+ia\tan(c+dx))^{11/2}}{11a^6d} + \frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^7d}$$

output
```
-16/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^4/d+8/3*I*(a+I*a*tan(d*x+c))^(9/2)/a^5/
d-12/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^6/d+2/13*I*(a+I*a*tan(d*x+c))^(13/2)
/a^7/d
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}(-835+1421i\tan(c+dx)+945\tan^2(c+dx)-231i\tan^3(c}{3003ad}$$

input
```
Integrate[Sec[c + d*x]^8/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]]*(-835 + (1421*I)*Tan[c
 + d*x] + 945*Tan[c + d*x]^2 - (231*I)*Tan[c + d*x]^3))/(3003*a*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^8}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{5/2}d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(-(i\tan(c+dx)a+a)^{11/2}+6a(i\tan(c+dx)a+a)^{9/2}-12a^2(i\tan(c+dx)a+a)^{7/2}+8a^3(i\tan(c+dx)a\right.}{a^7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{16}{7}a^3(a+ia\tan(c+dx))^{7/2}-\frac{8}{3}a^2(a+ia\tan(c+dx))^{9/2}-\frac{2}{13}(a+ia\tan(c+dx))^{13/2}+\frac{12}{11}a(a+ia\tan(c+\right.}{a^7d}$$

input
```
Int[Sec[c + d*x]^8/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*((16*a^3*(a + I*a*Tan[c + d*x])^(7/2))/7 - (8*a^2*(a + I*a*Tan[c + d
*x])^(9/2))/3 + (12*a*(a + I*a*Tan[c + d*x])^(11/2))/11 - (2*(a + I*a*Tan[
c + d*x])^(13/2))/13))/(a^7*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{6a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}+\frac{4a^2(a+ia\tan(dx+c))^{\frac{9}{2}}}{3}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}\right)}{d\,a^7}$	82
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{6a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}+\frac{4a^2(a+ia\tan(dx+c))^{\frac{9}{2}}}{3}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}\right)}{d\,a^7}$	82

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^7*(1/13*(a+I*a*tan(d*x+c))^(13/2)-6/11*a*(a+I*a*tan(d*x+c))^(11/2)
+4/3*a^2*(a+I*a*tan(d*x+c))^(9/2)-8/7*a^3*(a+I*a*tan(d*x+c))^(7/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.28

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{128\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(16i\,e^{(13i\,dx+13i\,c)}+104i\,e^{(11i\,dx+11i\,c)}+286i\,e^{(9i\,dx+9i\,c)}+429i\,e^{(7i\,dx+7i}\right.}{3003\left(ade^{(12i\,dx+12i\,c)}+6\,ade^{(10i\,dx+10i\,c)}+15\,ade^{(8i\,dx+8i\,c)}+20\,ade^{(6i\,dx+6i\,c)}+15\,ade^{(4i\,dx+4i\,c)}+6\,ade^{(}\right.}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-128/3003*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(13*I*d*x + 13
*I*c) + 104*I*e^(11*I*d*x + 11*I*c) + 286*I*e^(9*I*d*x + 9*I*c) + 429*I*e^
(7*I*d*x + 7*I*c))/(a*d*e^(12*I*d*x + 12*I*c) + 6*a*d*e^(10*I*d*x + 10*I*c
) + 15*a*d*e^(8*I*d*x + 8*I*c) + 20*a*d*e^(6*I*d*x + 6*I*c) + 15*a*d*e^(4*
I*d*x + 4*I*c) + 6*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^8(c+dx)}{\sqrt{ia\left(\tan(c+dx)-i\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**8/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(85) = 170$.

Time = 0.04 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.54

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx =$$

$$-\frac{2i\left(15015\sqrt{i\,a\tan\left(dx+c\right)+a}-\frac{3003\left(3\left(i\,a\tan(dx+c)+a\right)^{\frac{5}{2}}-10\left(i\,a\tan(dx+c)+a\right)^{\frac{3}{2}}a+15\sqrt{i\,a\tan(dx+c)+a}a^2\right)}{a^2}+\frac{143\,(\ldots}{\ldots}\right.}{\ldots}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2/15015*I*(15015*sqrt(I*a*tan(d*x + c) + a) - 3003*(3*(I*a*tan(d*x + c) +
 a)^(5/2) - 10*(I*a*tan(d*x + c) + a)^(3/2)*a + 15*sqrt(I*a*tan(d*x + c) +
 a)*a^2)/a^2 + 143*(35*(I*a*tan(d*x + c) + a)^(9/2) - 180*(I*a*tan(d*x + c
) + a)^(7/2)*a + 378*(I*a*tan(d*x + c) + a)^(5/2)*a^2 - 420*(I*a*tan(d*x +
 c) + a)^(3/2)*a^3 + 315*sqrt(I*a*tan(d*x + c) + a)*a^4)/a^4 - 5*(231*(I*a
*tan(d*x + c) + a)^(13/2) - 1638*(I*a*tan(d*x + c) + a)^(11/2)*a + 5005*(I
*a*tan(d*x + c) + a)^(9/2)*a^2 - 8580*(I*a*tan(d*x + c) + a)^(7/2)*a^3 + 9
009*(I*a*tan(d*x + c) + a)^(5/2)*a^4 - 6006*(I*a*tan(d*x + c) + a)^(3/2)*a
^5 + 3003*sqrt(I*a*tan(d*x + c) + a)*a^6)/a^6)/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.25 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.71

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2048\mathrm{i}}{3003\,a\,d} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{3003\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{1001\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,640\mathrm{i}}{3003\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} + \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,6784\mathrm{i}}{429\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,3456\mathrm{i}}{143\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5} + \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{13\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}$$

input
```
int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
6784i)/(429*a*d*(exp(c*2i + d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(3003*a*d*(exp(c*2i +
 d*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x
*2i) + 1))^(1/2)*256i)/(1001*a*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(e
xp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*640i)/(3003
*a*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1
i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2048i)/(3003*a*d) - ((a - (a*(exp(c*2i
+ d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*3456i)/(143*a*d*(ex
p(c*2i + d*x*2i) + 1)^5) + ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(
c*2i + d*x*2i) + 1))^(1/2)*128i)/(13*a*d*(exp(c*2i + d*x*2i) + 1)^6)
```

Reduce [F]

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{2\sqrt{a}\,i\left(-\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^8+6\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)^8\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)\tan(dx+c)^2 d+6\left(\right.}{ad\left(\tan(dx+c)^2+1\right)}$$

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*sqrt(a)*i*( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**8 + 6*int((sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**8*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*t
an(c + d*x)**2*d + 6*int((sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**8*tan(c +
 d*x))/(tan(c + d*x)**2 + 1),x)*d))/(a*d*(tan(c + d*x)**2 + 1))
```

3.334 $\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^3d}+\frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^4d}-\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^5d}$$

output
```
-8/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^3/d+8/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^4/d
-2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^5/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}(107i+110\tan(c+dx)-35i\tan^2(c+dx))}{315ad}$$

input
```
Integrate[Sec[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]*(107*I + 110*Tan[c + d
*x] - (35*I)*Tan[c + d*x]^2))/(315*a*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{3/2}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int\left((i\tan(c+dx)a+a)^{7/2}-4a(i\tan(c+dx)a+a)^{5/2}+4a^2(i\tan(c+dx)a+a)^{3/2}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{8}{5}a^2(a+ia\tan(c+dx))^{5/2}+\frac{2}{9}(a+ia\tan(c+dx))^{9/2}-\frac{8}{7}a(a+ia\tan(c+dx))^{7/2}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*((8*a^2*(a + I*a*Tan[c + d*x])^(5/2))/5 - (8*a*(a + I*a*Tan[c + d*x]
)^(7/2))/7 + (2*(a + I*a*Tan[c + d*x])^(9/2))/9))/(a^5*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}+\frac{4a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^5}$	63
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}+\frac{4a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^5}$	63

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^5*(-1/9*(a+I*a*tan(d*x+c))^(9/2)+4/7*a*(a+I*a*tan(d*x+c))^(7/2)-4/
5*a^2*(a+I*a*tan(d*x+c))^(5/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{32\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(8i\,e^{(9i\,dx+9i\,c)}+36i\,e^{(7i\,dx+7i\,c)}+63i\,e^{(5i\,dx+5i\,c)}\right)}{315\left(ade^{(8i\,dx+8i\,c)}+4\,ade^{(6i\,dx+6i\,c)}+6\,ade^{(4i\,dx+4i\,c)}+4\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-32/315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(8*I*e^(9*I*d*x + 9*I*c)
 + 36*I*e^(7*I*d*x + 7*I*c) + 63*I*e^(5*I*d*x + 5*I*c))/(a*d*e^(8*I*d*x +
8*I*c) + 4*a*d*e^(6*I*d*x + 6*I*c) + 6*a*d*e^(4*I*d*x + 4*I*c) + 4*a*d*e^(
2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^6(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**6/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(64) = 128.

Time = 0.05 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.92

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx =$$
$$-\frac{2i\left(315\sqrt{ia\tan(dx+c)+a}-\frac{42\left(3(ia\tan(dx+c)+a)^{\frac{5}{2}}-10(ia\tan(dx+c)+a)^{\frac{3}{2}}a+15\sqrt{ia\tan(dx+c)+a}\,a^2\right)}{a^2}+\frac{35(ia\tan}{}\right.}{315\,ad}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2/315*I*(315*sqrt(I*a*tan(d*x + c) + a) - 42*(3*(I*a*tan(d*x + c) + a)^(5
/2) - 10*(I*a*tan(d*x + c) + a)^(3/2)*a + 15*sqrt(I*a*tan(d*x + c) + a)*a^
2)/a^2 + (35*(I*a*tan(d*x + c) + a)^(9/2) - 180*(I*a*tan(d*x + c) + a)^(7/
2)*a + 378*(I*a*tan(d*x + c) + a)^(5/2)*a^2 - 420*(I*a*tan(d*x + c) + a)^(
3/2)*a^3 + 315*sqrt(I*a*tan(d*x + c) + a)*a^4)/a^4)/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.48

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{315\,a\,d} - \frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{315\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{105\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^2} + \frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,320\mathrm{i}}{63\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3} - \frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^4}$$

input
```
int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
320i)/(63*a*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1
i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(315*a*d*(exp(c*2i + d*x
*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i)
 + 1))^(1/2)*32i)/(105*a*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2
i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(315*a*d) -
 ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)
*32i)/(9*a*d*(exp(c*2i + d*x*2i) + 1)^4)
```

Reduce [F]

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2\sqrt{a}\,i\left(-\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^6+4\left(\int\frac{\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)^6\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)\tan(dx+c)^2 d+4\left(\cdots\right.\right.}{ad\left(\tan(dx+c)^2+1\right)}$$

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*sqrt(a)*i*( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**6 + 4*int((sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**6*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*t
an(c + d*x)**2*d + 4*int((sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**6*tan(c +
 d*x))/(tan(c + d*x)**2 + 1),x)*d))/(a*d*(tan(c + d*x)**2 + 1))
```

3.335 $\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{4i(a+ia\tan(c+dx))^{3/2}}{3a^2d}+\frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^3d}$$

output
```
-4/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^2/d+2/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^3/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2(7-3i\tan(c+dx))(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{15ad}$$

input
```
Integrate[Sec[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*(7 - (3*I)*Tan[c + d*x])*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]
)/(15*a*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^4}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int\left(2a\sqrt{i\tan(c+dx)a+a}-(i\tan(c+dx)a+a)^{3/2}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i(\frac{4}{3}a(a+ia\tan(c+dx))^{3/2}-\frac{2}{5}(a+ia\tan(c+dx))^{5/2})}{a^3d}$$

input
```
Int[Sec[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*((4*a*(a + I*a*Tan[c + d*x])^(3/2))/3 - (2*(a + I*a*Tan[c + d*x])^(5
/2))/5))/(a^3*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right)}{d\,a^{3}}$	44
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right)}{d\,a^{3}}$	44

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^3*(1/5*(a+I*a*tan(d*x+c))^(5/2)-2/3*a*(a+I*a*tan(d*x+c))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{8\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(2i\,e^{(5i\,dx+5i\,c)}+5i\,e^{(3i\,dx+3i\,c)}\right)}{15\left(ade^{(4i\,dx+4i\,c)}+2\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-8/15*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(2*I*e^(5*I*d*x + 5*I*c) +
 5*I*e^(3*I*d*x + 3*I*c))/(a*d*e^(4*I*d*x + 4*I*c) + 2*a*d*e^(2*I*d*x + 2*
I*c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^4(c+dx)}{\sqrt{ia\left(\tan(c+dx)-i\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**4/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\left(15\sqrt{i\,a\tan(dx+c)+a}-\frac{3\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}-10\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a+15\,\sqrt{i\,a\tan(dx+c)+a}a^2}{a^2}\right)}{15\,ad}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2/15*I*(15*sqrt(I*a*tan(d*x + c) + a) - (3*(I*a*tan(d*x + c) + a)^(5/2) -
 10*(I*a*tan(d*x + c) + a)^(3/2)*a + 15*sqrt(I*a*tan(d*x + c) + a)*a^2)/a^
2)/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.63

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{8\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(\cos\left(2\,c+2\,d\,x\right)\,27\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,9\mathrm{i}+\cos\left(6\,c+6\,d\,x\right)\,1\mathrm{i}-5\,\mathrm{si}}{15\,a\,d\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c\right.\right.}$$

input
```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
-(8*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) +
1))^(1/2)*(cos(2*c + 2*d*x)*27i + cos(4*c + 4*d*x)*9i + cos(6*c + 6*d*x)*1
i - 5*sin(2*c + 2*d*x) - 4*sin(4*c + 4*d*x) - sin(6*c + 6*d*x) + 19i))/(15
*a*d*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$=\frac{2\sqrt{a}\,i\left(-\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^4+2\left(\int\frac{\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)^4\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)\tan(dx+c)^2d+2\left(\right.}{ad\left(\tan(dx+c)^2+1\right)}$$

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*sqrt(a)*i*( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**4 + 2*int((sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**4*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*t
an(c + d*x)**2*d + 2*int((sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**4*tan(c +
 d*x))/(tan(c + d*x)**2 + 1),x)*d))/(a*d*(tan(c + d*x)**2 + 1))
```

3.336 $\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 27

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad}$$

output
```
-2*I*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad}$$

input
```
Integrate[Sec[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sec(c+dx)^2}{\sqrt{a+ia\tan(c+dx)}}dx$$
$$\Big\downarrow \text{ 3968}$$
$$-\frac{i\int \frac{1}{\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{ad}$$
$$\Big\downarrow \text{ 17}$$
$$-\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad}$$

input
```
Int[Sec[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$-\frac{2i\sqrt{a+ia\tan(dx+c)}}{ad}$	24
default	$-\frac{2i\sqrt{a+ia\tan(dx+c)}}{ad}$	24

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}}{ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^2(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**2/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\sqrt{i\,a\tan(dx+c)+a}}{ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2*I*sqrt(I*a*tan(d*x + c) + a)/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\sqrt{\frac{a\left(2\cos(c+d\,x)^2+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{2\cos(c+d\,x)^2}}\,2\mathrm{i}}{a\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
-(((a*(sin(2*c + 2*d*x)*1i + 2*cos(c + d*x)^2))/(2*cos(c + d*x)^2))^(1/2)*
2i)/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2\sqrt{a}\,\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^2\,i}{ad\left(\tan(dx+c)^2+1\right)}$$

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
( - 2*sqrt(a)*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**2*i)/(a*d*(tan(c + d*
x)**2 + 1))
```

3.337 $\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 150

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{5i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}d} + \frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}} + \frac{5i}{8d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^3}{2d(a+ia\tan(c+dx))^{3/2}\left(a^2-ia^2\tan(c+dx)\right)}$$

output
```
-5/16*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/a^(1
/2)/d+5/12*I*a/d/(a+I*a*tan(d*x+c))^(3/2)+5/8*I/d/(a+I*a*tan(d*x+c))^(1/2)
-1/2*I*a^3/d/(a+I*a*tan(d*x+c))^(3/2)/(a^2-I*a^2*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.34

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{ia\,\text{Hypergeometric2F1}\left(-\frac{3}{2},2,-\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{6d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Integrate[Cos[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I/6)*a*Hypergeometric2F1[-3/2, 2, -1/2, (1 + I*Tan[c + d*x])/2])/(d*(a +
 I*a*Tan[c + d*x])^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 3968, 52, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^3\left(\frac{5\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^3\left(\frac{5\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^3\left(\frac{5\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{ia^3\left(\frac{5\left(\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ia^3\left(\frac{5\left(\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{d}$$

input
```
Int[Cos[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*a^3*(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2)) + (
5*(-1/3*1/(a*(a + I*a*Tan[c + d*x])^(3/2)) + ((I*ArcTan[(Sqrt[a]*Tan[c + d
*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a)
))/(4*a)))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(118) = 236.

Time = 9.30 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.03

method	result
default	$\frac{15i\sin(dx+c)\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c)+i)}{2\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)+(15\cos(dx+c)+15)\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,(\cot(dx+c)}{2\sqrt{\cot(dx+c)^2-2\cot(dx+}}\right.}{48d(}$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d*(15*I*sin(d*x+c)*arctanh(1/2*2^(1/2)*(cot(d*x+c)-csc(d*x+c)+I)/(cot
(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))+(15*cos(d*x+c)+15
)*arctanh(1/2*2^(1/2)*(cot(d*x+c)-csc(d*x+c)+I)/(cot(d*x+c)^2-2*cot(d*x+c)
*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))+15*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)+sin(d*x+c)*cos(d*x+c)*(20*cos(d*x+c)+20)*(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)+I*cos(d*x+c)*(-4*cos(d*x+c)^2-4*cos(d*x+c)+30)*(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(a
*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(109) = 218.

Time = 0.09 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.81

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\left(-15i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(3i\,dx+3i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/48*(-15*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(3*I*d*x + 3*I*c)*log(4*(sqrt(
2)*sqrt(1/2)*(a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt(1/(a*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 15*I*sqrt(1/
2)*a*d*sqrt(1/(a*d^2))*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a*d*
e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2
)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*(-3*I*e^(6*I*d*x + 6*I*c) + 11*I*e^(4*I*d*x + 4*I*c) + 16*I*e^
(2*I*d*x + 2*I*c) + 2*I))*e^(-3*I*d*x - 3*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos^2(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**2/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\left(15\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)+\frac{4\left(15\,(ia\tan(dx+c)+a)^2a-20\,(ia\tan(dx+c)+a)a^2-8\,a^3\right)}{(ia\tan(dx+c)+a)^{\frac{5}{2}}-2\,(ia\tan(dx+c)+a)^{\frac{3}{2}}a}\right)}{96\,ad}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/96*I*(15*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
 a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(15*(I*a*tan(d*x
+ c) + a)^2*a - 20*(I*a*tan(d*x + c) + a)*a^2 - 8*a^3)/((I*a*tan(d*x + c)
+ a)^(5/2) - 2*(I*a*tan(d*x + c) + a)^(3/2)*a))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\cos(dx+c)^2\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)i+\int \frac{\sqrt{\tan(dx+c)i+1}\,\cos(dx+c)^2}{\tan(dx+c)^2+1}dx\right)}{a}$$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**2*tan(c + d*x))/(
tan(c + d*x)**2 + 1),x)*i + int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**2)
/(tan(c + d*x)**2 + 1),x)))/a
```

3.338 $\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 223

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx =& -\frac{63i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}d} \\ &+ \frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}} \\ &- \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} \\ &+ \frac{21ia}{64d(a+ia\tan(c+dx))^{3/2}} + \frac{63i}{128d\sqrt{a+ia\tan(c+dx)}} \\ &- \frac{9ia^5}{16d(a+ia\tan(c+dx))^{5/2}\,(a^3-ia^3\tan(c+dx))}\end{aligned}$$

output
```
-63/256*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/a^
(1/2)/d+63/160*I*a^2/d/(a+I*a*tan(d*x+c))^(5/2)-1/4*I*a^4/d/(a-I*a*tan(d*x
+c))^2/(a+I*a*tan(d*x+c))^(5/2)+21/64*I*a/d/(a+I*a*tan(d*x+c))^(3/2)+63/12
8*I/d/(a+I*a*tan(d*x+c))^(1/2)-9/16*I*a^5/d/(a+I*a*tan(d*x+c))^(5/2)/(a^3-
I*a^3*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.24

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{ia^2\,\text{Hypergeometric2F1}\left(-\frac{5}{2},3,-\frac{3}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{20d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I/20)*a^2*Hypergeometric2F1[-5/2, 3, -3/2, (1 + I*Tan[c + d*x])/2])/(d*(
a + I*a*Tan[c + d*x])^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3042, 3968, 52, 52, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^4\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{9\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}\right)}{d}$$

$\downarrow$ 52

$$-\frac{ia^5\left(\frac{9\left(\frac{7\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(\cdots)}\cdots\right)}{d}$$

$\downarrow$ 61

$$-\frac{ia^5\left(\frac{9\left(\frac{7\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}\right)}{8a}+\frac{\cdots}{4a(a-\cdots}\cdots\right)}{d}$$

$\downarrow$ 61

$$-\frac{ia^5\left(\frac{9\left(\frac{7\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\,\mathrm{ta}\cdots}\cdots\right)}{8a}\cdots\right)}{d}$$

$\downarrow$ 61

$$-\frac{ia^5\left(\frac{9\left(\frac{7\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{}{2a(a-ia}\right)}{8a}}{d}$$

$\downarrow$ 73

$$-\frac{ia^5\left(\frac{9\left(\frac{7\left(\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a}\right)}{8a}}{d}$$

$\downarrow$ 219

$$-\frac{ia^5\left(\frac{9\left(\frac{7\left(\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))}\right.}{8a}\right.}{d}$$

input
```
Int[Cos[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*a^5*(1/(4*a*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(5/2)) +
 (9*(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2)) + (7*(-1/
5*1/(a*(a + I*a*Tan[c + d*x])^(5/2)) + (-1/3*1/(a*(a + I*a*Tan[c + d*x])^(
3/2)) + ((I*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/
(a*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a))/(2*a)))/(4*a)))/(8*a)))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 7.74 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.40

method	result
default	$-\frac{-315 i \sin(dx+c) \operatorname{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)+i}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+315(-\cos(dx+c)-1) \operatorname{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)+i}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)-315 i\sqrt{2}\sqrt{-\frac{2\cos}{\cos(}}}{}$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1280/d*(-315*I*sin(d*x+c)*arctanh(1/2/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(cot(d*x+c)-csc(d*x+c)+I))+315*(-cos(d*x+c)-1)*arctanh(1/2/(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+I))-315*I*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*2^(1/2)+12*sin(d*x+c)*cos(d*x+c)*(-35-24*cos(d*x+c)^3-24
*cos(d*x+c)^2-35*cos(d*x+c))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*I*cos(d*
x+c)*(-315+16*cos(d*x+c)^4+16*cos(d*x+c)^3+42*cos(d*x+c)^2+42*cos(d*x+c))*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.31

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\left(-315i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(5i\,dx+5i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/1280*(-315*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(sq
rt(2)*sqrt(1/2)*(a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt(1/(a*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 315*I*sqr
t(1/2)*a*d*sqrt(1/(a*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(
a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a
*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*(-10*I*e^(10*I*d*x + 10*I*c) - 95*I*e^(8*I*d*x + 8*I*c) +
203*I*e^(6*I*d*x + 6*I*c) + 344*I*e^(4*I*d*x + 4*I*c) + 64*I*e^(2*I*d*x +
2*I*c) + 8*I))*e^(-5*I*d*x - 5*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos^4(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**4/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.86

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\left(315\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(315\,(i\,a\tan(dx+c)+a)^4a-1050\,(i\,a\tan(dx+c)+a)^3a^2+672\,(i\,a\tan(dx+c)+a)\right.}{(i\,a\tan(dx+c)+a)^{\frac{9}{2}}-4\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a+4\,(i\,a\tan}\right.}{2560\,ad}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2560*I*(315*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c
) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(315*(I*a*tan(
d*x + c) + a)^4*a - 1050*(I*a*tan(d*x + c) + a)^3*a^2 + 672*(I*a*tan(d*x +
 c) + a)^2*a^3 + 192*(I*a*tan(d*x + c) + a)*a^4 + 128*a^5)/((I*a*tan(d*x +
 c) + a)^(9/2) - 4*(I*a*tan(d*x + c) + a)^(7/2)*a + 4*(I*a*tan(d*x + c) +
a)^(5/2)*a^2))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)^4}{\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}\,dx$$

input
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\cos(dx+c)^4\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)i+\int \frac{\sqrt{\tan(dx+c)i+1}\,\cos(dx+c)^4}{\tan(dx+c)^2+1}dx\right)}{a}$$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**4*tan(c + d*x))/(
tan(c + d*x)**2 + 1),x)*i + int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**4)
/(tan(c + d*x)**2 + 1),x)))/a
```

3.339 $\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 300

$$\begin{aligned}\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx =& -\frac{429i\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{1024\sqrt{2}\sqrt{a}d} \\ &+ \frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}} \\ &- \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}} \\ &+ \frac{429ia^2}{1280d(a+ia\tan(c+dx))^{5/2}} \\ &+ \frac{143ia}{512d(a+ia\tan(c+dx))^{3/2}} \\ &+ \frac{429i}{1024d\sqrt{a+ia\tan(c+dx)}} \\ &- \frac{13ia^7}{48d(a+ia\tan(c+dx))^{7/2}\left(a^2-ia^2\tan(c+dx)\right)^2} \\ &- \frac{143ia^7}{192d(a+ia\tan(c+dx))^{7/2}\left(a^4-ia^4\tan(c+dx)\right)}\end{aligned}$$

output
```
-429/2048*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/
a^(1/2)/d+429/896*I*a^3/d/(a+I*a*tan(d*x+c))^(7/2)-1/6*I*a^6/d/(a-I*a*tan(
d*x+c))^3/(a+I*a*tan(d*x+c))^(7/2)+429/1280*I*a^2/d/(a+I*a*tan(d*x+c))^(5/
2)+143/512*I*a/d/(a+I*a*tan(d*x+c))^(3/2)+429/1024*I/d/(a+I*a*tan(d*x+c))^
(1/2)-13/48*I*a^7/d/(a+I*a*tan(d*x+c))^(7/2)/(a^2-I*a^2*tan(d*x+c))^2-143/
192*I*a^7/d/(a+I*a*tan(d*x+c))^(7/2)/(a^4-I*a^4*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.18

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{ia^3\,\text{Hypergeometric2F1}\left(-\frac{7}{2},4,-\frac{5}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{56d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Integrate[Cos[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I/56)*a^3*Hypergeometric2F1[-7/2, 4, -5/2, (1 + I*Tan[c + d*x])/2])/(d*(
a + I*a*Tan[c + d*x])^(7/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.423$, Rules used = {3042, 3968, 52, 52, 52, 61, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^6\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^7\int \frac{1}{(a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^7\left(\frac{13\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{12a}+\frac{1}{6a(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}}\right)}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^7\left(\frac{13\left(\frac{11\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}}\right)}{12a}+\frac{1}{6a(a-ia\tan(c+dx))^3(a+ia$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^7\left(\frac{13\left(\frac{11\left(\frac{9\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c-$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^7\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\right)}{8a}+\frac{}{4a(}\right.}{12a}\right.}{d}$$

$\Big\downarrow$ 61

$$-\frac{ia^7\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a-}}{8a}}{12a}}{d}$$

$\big\downarrow$ 61

$$-\frac{ia^7\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right.}{4a}\right.}{8a}\right.}{12a}\right.}{d}$$

$\downarrow$ 61

$$-\frac{ia^7\left(13\frac{\left(11\frac{\left(9\frac{\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\cdots\right)}{4a}\cdots\right)}{8a}\cdots\right)}{12a}\cdots\right)}{\cdots}$$

$$\Big\downarrow \mathbf{73}$$

$$-\frac{ia^7\left(13\left(11\left(9\left(\frac{\frac{\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx)}\right.\right.\right.\right.}{4a \quad 8a \quad 12a \quad d}$$

$\downarrow$ 219

$$-\frac{ia^7\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{\cdots}{2a(a-\cdots}}{8a}\cdots}{12a}\cdots}{d}$$

input
```
Int[Cos[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*a^7*(1/(6*a*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(7/2)) +
 (13*(1/(4*a*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(7/2)) + (11*
(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(7/2)) + (9*(-1/7*1/
(a*(a + I*a*Tan[c + d*x])^(7/2)) + (-1/5*1/(a*(a + I*a*Tan[c + d*x])^(5/2)
) + (-1/3*1/(a*(a + I*a*Tan[c + d*x])^(3/2)) + ((I*ArcTan[(Sqrt[a]*Tan[c +
 d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[a + I*a*Tan[c + d*x]]))/(2*
a))/(2*a))/(2*a)))/(4*a)))/(8*a)))/(12*a)))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 7.72 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.17

method	result
default	$\frac{45045i\sin(dx+c)\operatorname{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)+i}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+45045(\cos(dx+c)+1)\operatorname{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)+i}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+45045i\sqrt{2}\sqrt{-\frac{2\cos}{\cos(}}}{}$

input
```
int(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/215040/d*(45045*I*sin(d*x+c)*arctanh(1/2/(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(cot(d*x+c)-csc(d*x+c)+I))+45045*(cos(d*x+c)+1)*arctanh(1/2/(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+I))+45045*I*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)+52*sin(d*x+c)*cos(d*x+c)*(1155+640*cos(d*x+
c)^5+640*cos(d*x+c)^4+792*cos(d*x+c)^3+792*cos(d*x+c)^2+1155*cos(d*x+c))*(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*I*cos(d*x+c)*(45045-1280*cos(d*x+c)^6-
1280*cos(d*x+c)^5-2288*cos(d*x+c)^4-2288*cos(d*x+c)^3-6006*cos(d*x+c)^2-60
06*cos(d*x+c))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))/(cos(d*x+c)+1)/(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.05

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\left(-45045i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(7i\,dx+7i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/215040*(-45045*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(7*I*d*x + 7*I*c)*log(4
*(sqrt(2)*sqrt(1/2)*(a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt(1/(a*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 45045
*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(7*I*d*x + 7*I*c)*log(-4*(sqrt(2)*sqrt(
1/2)*(a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t(1/(a*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*(-280*I*e^(14*I*d*x + 14*I*c) - 2870*I*e^(12*I*d*x +
 12*I*c) - 16345*I*e^(10*I*d*x + 10*I*c) + 27029*I*e^(8*I*d*x + 8*I*c) + 4
9792*I*e^(6*I*d*x + 6*I*c) + 11072*I*e^(4*I*d*x + 4*I*c) + 2304*I*e^(2*I*d
*x + 2*I*c) + 240*I))*e^(-7*I*d*x - 7*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos^6(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(cos(d*x+c)**6/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**6/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.82

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\left(45045\,\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(45045\,(i\,a\tan(dx+c)+a)^6a-240240\,(i\,a\tan(dx+c)+a)^5a^2+396396\,(i\,a\tan\right.}{(i\,a\tan(dx+c)+a)^{\frac{13}{2}}-6\,(i\,a\,\mathrm{ta}}\right.}{430080\,ad}$$

input
```
integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/430080*I*(45045*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x
 + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(45045*(I*
a*tan(d*x + c) + a)^6*a - 240240*(I*a*tan(d*x + c) + a)^5*a^2 + 396396*(I*
a*tan(d*x + c) + a)^4*a^3 - 164736*(I*a*tan(d*x + c) + a)^3*a^4 - 36608*(I
*a*tan(d*x + c) + a)^2*a^5 - 19968*(I*a*tan(d*x + c) + a)*a^6 - 15360*a^7)
/((I*a*tan(d*x + c) + a)^(13/2) - 6*(I*a*tan(d*x + c) + a)^(11/2)*a + 12*(
I*a*tan(d*x + c) + a)^(9/2)*a^2 - 8*(I*a*tan(d*x + c) + a)^(7/2)*a^3))/(a*
d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^6}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(cos(c + d*x)^6/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)^6/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\cos(dx+c)^6\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)i+\int \frac{\sqrt{\tan(dx+c)i+1}\cos(dx+c)^6}{\tan(dx+c)^2+1}dx\right)}{a}$$

input
```
int(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**6*tan(c + d*x))/(
tan(c + d*x)**2 + 1),x)*i + int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**6)
/(tan(c + d*x)**2 + 1),x)))/a
```

3.340 $\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{256ia^4\sec^9(c+dx)}{6435d(a+ia\tan(c+dx))^{9/2}}+\frac{64ia^3\sec^9(c+dx)}{715d(a+ia\tan(c+dx))^{7/2}}+\frac{8ia^2\sec^9(c+dx)}{65d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

output
```
256/6435*I*a^4*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(9/2)+64/715*I*a^3*sec(d*
x+c)^9/d/(a+I*a*tan(d*x+c))^(7/2)+8/65*I*a^2*sec(d*x+c)^9/d/(a+I*a*tan(d*x
+c))^(5/2)+2/15*I*a*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$=\frac{2\sec^8(c+dx)(510\cos(c+dx)+731\cos(3(c+dx))+3i(90\sin(c+dx)+233\sin(3(c+dx))))(i\cos(4(c}{6435d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^9/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*Sec[c + d*x]^8*(510*Cos[c + d*x] + 731*Cos[3*(c + d*x)] + (3*I)*(90*Sin
[c + d*x] + 233*Sin[3*(c + d*x)]))*(I*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])
)/(6435*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3975, 3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^9}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{5}a\int \frac{\sec^9(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\int \frac{\sec(c+dx)^9}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{5}a\left(\frac{8}{13}a\int \frac{\sec^9(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\left(\frac{8}{13}a\int \frac{\sec(c+dx)^9}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{5}a\left(\frac{8}{13}a\left(\frac{4}{11}a\int \frac{\sec^9(c+dx)}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\left(\frac{8}{13}a\left(\frac{4}{11}a\int \frac{\sec(c+dx)^9}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{4}{5}a\left(\frac{8}{13}a\left(\frac{8ia^2\sec^9(c+dx)}{99d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Sec[c + d*x]^9/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((2*I)/15)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (4*a*((((
2*I)/13)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (8*a*((((8*I
)/99)*a^2*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((2*I)/11)*a
*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(7/2))))/13))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 6.71 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.76

method	result
default	$\frac{\frac{2\tan(dx+c)\sec(dx+c)^7\left(1024\cos(dx+c)^6+640\cos(dx+c)^4+504\cos(dx+c)^2+429\right)}{6435}+\frac{2i\left(1024\sec(dx+c)+128\sec(dx+c)^3+56\sec(dx+c)^5+33\sec(d\right.}{6435}}{d\sqrt{a(1+i\tan(dx+c))}}$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/6435/d/(a*(1+I*tan(d*x+c)))^(1/2)*(tan(d*x+c)*sec(d*x+c)^7*(1024*cos(d*x
+c)^6+640*cos(d*x+c)^4+504*cos(d*x+c)^2+429)+I*(1024*sec(d*x+c)+128*sec(d*
x+c)^3+56*sec(d*x+c)^5+33*sec(d*x+c)^7))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.04

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{256\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-715i\,e^{(6i\,dx+6i\,c)}-390i\,e^{(4i\,dx+4i\,c)}-120i\,e^{(2i\,dx+2i\,c)}\right.}{6435\left(ade^{(14i\,dx+14i\,c)}+7\,ade^{(12i\,dx+12i\,c)}+21\,ade^{(10i\,dx+10i\,c)}+35\,ade^{(8i\,dx+8i\,c)}+35\,ade^{(6i\,dx+6i\,c)}+21\,a\right.}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-256/6435*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-715*I*e^(6*I*d*x + 6
*I*c) - 390*I*e^(4*I*d*x + 4*I*c) - 120*I*e^(2*I*d*x + 2*I*c) - 16*I)/(a*d
*e^(14*I*d*x + 14*I*c) + 7*a*d*e^(12*I*d*x + 12*I*c) + 21*a*d*e^(10*I*d*x
+ 10*I*c) + 35*a*d*e^(8*I*d*x + 8*I*c) + 35*a*d*e^(6*I*d*x + 6*I*c) + 21*a
*d*e^(4*I*d*x + 4*I*c) + 7*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^9(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**9/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 608 vs. $2(115) = 230$.

Time = 0.26 (sec) , antiderivative size = 608, normalized size of antiderivative = 4.14

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2/6435*(-1241*I*sqrt(a) - 5194*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) +
6090*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2490*sqrt(a)*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 - 14430*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 - 33618*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 13442*I*sqrt
(a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 18590*sqrt(a)*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7 - 18590*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 13
442*I*sqrt(a)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 33618*sqrt(a)*sin(d*
x + c)^11/(cos(d*x + c) + 1)^11 + 14430*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x
 + c) + 1)^12 + 2490*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 6090*
I*sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 5194*sqrt(a)*sin(d*x + c
)^15/(cos(d*x + c) + 1)^15 + 1241*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c)
+ 1)^16)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos(
d*x + c) + 1) - 1)/((a - 8*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 56*a*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + 70*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 56*a*sin(d*x + c)^10/(cos(
d*x + c) + 1)^10 + 28*a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a*sin(d*
x + c)^14/(cos(d*x + c) + 1)^14 + a*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)
*d*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 1))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.32 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.05

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,256i}{9\,a\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^4} - \frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,768i}{11\,a\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^5} + \frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,768i}{13\,a\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^6} - \frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,256i}{15\,a\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^7}$$

input
```
int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
(exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
 d*x*2i) + 1))^(1/2)*256i)/(9*a*d*(exp(c*2i + d*x*2i) + 1)^4) - (exp(- c*1
i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*768i)/(11*a*d*(exp(c*2i + d*x*2i) + 1)^5) + (exp(- c*1i - d*x*1
i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*768i)/(13*a*d*(exp(c*2i + d*x*2i) + 1)^6) - (exp(- c*1i - d*x*1i)*(a - (
a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(
15*a*d*(exp(c*2i + d*x*2i) + 1)^7)
```

Reduce [F]

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2\sqrt{a}\,i\left(-\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^9+7\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)^9\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)\tan(dx+c)^2\,d+7\left(\right.}{ad\left(\tan(dx+c)^2+1\right)}$$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*sqrt(a)*i*( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**9 + 7*int((sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**9*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*t
an(c + d*x)**2*d + 7*int((sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**9*tan(c +
 d*x))/(tan(c + d*x)**2 + 1),x)*d))/(a*d*(tan(c + d*x)**2 + 1))
```

3.341 $\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{64ia^3\sec^7(c+dx)}{693d(a+ia\tan(c+dx))^{7/2}} + \frac{16ia^2\sec^7(c+dx)}{99d(a+ia\tan(c+dx))^{5/2}} + \frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}$$

output
```
64/693*I*a^3*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(7/2)+16/99*I*a^2*sec(d*x+c
)^7/d/(a+I*a*tan(d*x+c))^(5/2)+2/11*I*a*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^
(3/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.70

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2\sec^6(c+dx)(44+107\cos(2(c+dx))+91i\sin(2(c+dx)))(i\cos(3(c+dx))+\sin(3(c+dx)))}{693d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^7/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*Sec[c + d*x]^6*(44 + 107*Cos[2*(c + d*x)] + (91*I)*Sin[2*(c + d*x)])*(I
*Cos[3*(c + d*x)] + Sin[3*(c + d*x)]))/(693*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^7}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{11}a\int \frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{11}a\int \frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{11}a\left(\frac{4}{9}a\int \frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{11}a\left(\frac{4}{9}a\int \frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8}{11}a\left(\frac{8ia^2\sec^7(c+dx)}{63d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Sec[c + d*x]^7/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((2*I)/11)*a*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (8*a*((((
8*I)/63)*a^2*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((2*I)/9)
*a*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(5/2))))/11
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 4.91 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\frac{2\tan(dx+c)\sec(dx+c)^5\left(128\cos(dx+c)^4+80\cos(dx+c)^2+63\right)}{693}+\frac{2i\left(128\sec(dx+c)+16\sec(dx+c)^3+7\sec(dx+c)^5\right)}{693}}{d\sqrt{a(1+i\tan(dx+c))}}$	91

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/693*tan(d*x+c)*sec(d*x+c)^5*(128*cos(d*x+c)^4+80*cos(d*x+c)^2+63)+2
/693*I*(128*sec(d*x+c)+16*sec(d*x+c)^3+7*sec(d*x+c)^5))/(a*(1+I*tan(d*x+c)
))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{64\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-99i\,e^{(4i\,dx+4i\,c)}-44i\,e^{(2i\,dx+2i\,c)}-8i\right)}{693\left(ade^{(10i\,dx+10i\,c)}+5\,ade^{(8i\,dx+8i\,c)}+10\,ade^{(6i\,dx+6i\,c)}+10\,ade^{(4i\,dx+4i\,c)}+5\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-64/693*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-99*I*e^(4*I*d*x + 4*I*
c) - 44*I*e^(2*I*d*x + 2*I*c) - 8*I)/(a*d*e^(10*I*d*x + 10*I*c) + 5*a*d*e^
(8*I*d*x + 8*I*c) + 10*a*d*e^(6*I*d*x + 6*I*c) + 10*a*d*e^(4*I*d*x + 4*I*c
) + 5*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^7(c+dx)}{\sqrt{ia\left(\tan(c+dx)-i\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**7/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(86) = 172$.

Time = 0.22 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.31

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2/693*(-151*I*sqrt(a) - 542*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) + 484
*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 22*sqrt(a)*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 - 627*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
- 1452*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1452*sqrt(a)*sin(d*x
+ c)^7/(cos(d*x + c) + 1)^7 + 627*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) +
 1)^8 - 22*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 484*I*sqrt(a)*sin
(d*x + c)^10/(cos(d*x + c) + 1)^10 - 542*sqrt(a)*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 + 151*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*sqrt(s
in(d*x + c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/((a - 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 20*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10
 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d*sqrt(-2*I*sin(d*x + c)/(cos(
d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{64\,e^{-c\,1i-d\,x\,1i}\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,\left(e^{c\,2i+d\,x\,2i}\,44i+e^{c\,4i+d\,x\,4i}\,99i+8i\right)}{693\,a\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^5}$$

input
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
(64*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2
i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*44i + exp(c*4i + d*x*4i)*99i +
 8i))/(693*a*d*(exp(c*2i + d*x*2i) + 1)^5)
```

Reduce [F]

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2\sqrt{a}\,i\left(-\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^7+5\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)^7\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)\tan(dx+c)^2\,d+5\left(\right.\right.}{ad\left(\tan(dx+c)^2+1\right)}$$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*sqrt(a)*i*( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**7 + 5*int((sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**7*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*t
an(c + d*x)**2*d + 5*int((sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**7*tan(c +
 d*x))/(tan(c + d*x)**2 + 1),x)*d))/(a*d*(tan(c + d*x)**2 + 1))
```

3.342 $\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 73

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}} \, dx = \frac{8ia^2\sec^5(c+dx)}{35d(a+ia\tan(c+dx))^{5/2}} + \frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}$$

output
```
8/35*I*a^2*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(5/2)+2/7*I*a*sec(d*x+c)^5/d/
(a+I*a*tan(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}} \, dx = -\frac{2\sec^3(c+dx)(\cos(2(c+dx)) - i\sin(2(c+dx)))(-9i+5\tan(c+dx))}{35d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^5/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(-2*Sec[c + d*x]^3*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-9*I + 5*Tan[c
 + d*x]))/(35*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{7}a\int \frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{7}a\int \frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8ia^2\sec^5(c+dx)}{35d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Sec[c + d*x]^5/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((8*I)/35)*a^2*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((2*I)
/7)*a*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{\frac{16\sec(dx+c)\tan(dx+c)}{35}+\frac{16i\sec(dx+c)}{35}+\frac{2\tan(dx+c)\sec(dx+c)^{3}}{7}+\frac{2i\sec(dx+c)^{3}}{35}}{d\sqrt{a(1+i\tan(dx+c))}}$	72

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/35/d/(a*(1+I*tan(d*x+c)))^(1/2)*(8*sec(d*x+c)*tan(d*x+c)+8*I*sec(d*x+c)+
5*tan(d*x+c)*sec(d*x+c)^3+I*sec(d*x+c)^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{16\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-7i\,e^{(2i\,dx+2i\,c)}-2i\right)}{35\left(ade^{(6i\,dx+6i\,c)}+3\,ade^{(4i\,dx+4i\,c)}+3\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-16/35*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-7*I*e^(2*I*d*x + 2*I*c)
 - 2*I)/(a*d*e^(6*I*d*x + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2*
I*d*x + 2*I*c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^5(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**5/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(57) = 114.

Time = 0.18 (sec) , antiderivative size = 340, normalized size of antiderivative = 4.66

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx =$$

$$-\frac{2\left(-9i\sqrt{a}-\frac{26\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}+\frac{14i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{14\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{26\sqrt{a}\sin}{(\cos(dx}\right.}{35\left(a-\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\sqrt{-}}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2/35*(-9*I*sqrt(a) - 26*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) + 14*I*sq
rt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 14*sqrt(a)*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 - 14*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*I*s
qrt(a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 26*sqrt(a)*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7 + 9*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*sqrt(
sin(d*x + c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1)
- 1)/((a - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a*sin(d*x +
 c)^8/(cos(d*x + c) + 1)^8)*d*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) +
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{16\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,7\mathrm{i}+2\mathrm{i}\right)\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}}{35\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3}$$

input
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
(16*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*7i + 2i)*(a - (a*(exp(c*2i +
d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(35*a*d*(exp(c*2i +
d*x*2i) + 1)^3)
```

Reduce [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{2\sqrt{a}\,i\left(-\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^5+3\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)^5\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)\tan(dx+c)^2 d+3\left(\right.}{ad\left(\tan(dx+c)^2+1\right)}$$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*sqrt(a)*i*( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**5 + 3*int((sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**5*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*t
an(c + d*x)**2*d + 3*int((sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**5*tan(c +
 d*x))/(tan(c + d*x)**2 + 1),x)*d))/(a*d*(tan(c + d*x)**2 + 1))
```

3.343 $\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{2ia\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

output
```
2/3*I*a*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{2\sec(c+dx)(i+\tan(c+dx))}{3d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*Sec[c + d*x]*(I + Tan[c + d*x]))/(3*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^3}{\sqrt{a+ia\tan(c+dx)}}dx$$
$$\Big\downarrow \text{3974}$$
$$\frac{2ia\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Sec[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((2*I)/3)*a*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\frac{2i\sec(dx+c)}{3}+\frac{2\sec(dx+c)\tan(dx+c)}{3}}{d\sqrt{a(1+i\tan(dx+c))}}$	44

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/(a*(1+I*tan(d*x+c)))^(1/2)*(I*sec(d*x+c)+sec(d*x+c)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{4i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{3\left(ade^{(2i\,dx+2i\,c)}+ad\right)}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
4/3*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(a*d*e^(2*I*d*x + 2*I*c) +
 a*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^3\left(c+dx\right)}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**3/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(27) = 54$.

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 5.89

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2\left(-i\sqrt{a}-\frac{2\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}-1}}{3\left(a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2/3*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sqrt(a)*s
in(d*x + c)^3/(cos(d*x + c) + 1)^3 + I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos
(d*x + c) + 1) - 1)/((a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4)*d*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) +
1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.80

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,1\mathrm{i}+\cos\left(3\,c+3\,d\,x\right)\,1\mathrm{i})}{3\,a\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
(2*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1
))^(1/2)*(cos(c + d*x)*1i + sin(c + d*x) + cos(3*c + 3*d*x)*1i + sin(3*c +
 3*d*x)))/(3*a*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2\sqrt{a}\,i\left(-\sqrt{\tan\left(dx+c\right)i+1}\,\sec\left(dx+c\right)^3+\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)^3\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)\tan\left(dx+c\right)^2d+\left(\int\right.\right.}{ad\left(\tan\left(dx+c\right)^2+1\right)}$$

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*sqrt(a)*i*( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**3 + int((sqrt(tan(
c + d*x)*i + 1)*sec(c + d*x)**3*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*tan
(c + d*x)**2*d + int((sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**3*tan(c + d*x
))/(tan(c + d*x)**2 + 1),x)*d))/(a*d*(tan(c + d*x)**2 + 1))
```

3.344 $\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 52

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
I*2^(1/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))
/a^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2ie^{i(c+dx)}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((2*I)*E^(I*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])/(d*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3970}$$

$$\frac{2i\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d}$$

input
```
Int[Sec[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(I*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*
x]])])/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(41) = 82$.

Time = 6.55 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.08

method	result	size
default	$\frac{(-i(\csc(dx+c)-\cot(dx+c))-1)\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,(i-\cot(dx+c)+\csc(dx+c))}{2\sqrt{\csc(dx+c)^2(1-\cos(dx+c))^2-1}}\right)}{d\sqrt{a(1+i\tan(dx+c))}\,\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$	108

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-I*(csc(d*x+c)-cot(d*x+c))-1)*arctanh(1/2/(csc(d*x+c)^2*(1-cos(d*x+c)
)^2-1)^(1/2)*2^(1/2)*(I-cot(d*x+c)+csc(d*x+c)))/(a*(1+I*tan(d*x+c)))^(1/2)
/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(39) = 78$.

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.87

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= -\frac{1}{2}i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{4\left(\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-i\right)e^{(-i\,dx-i\,c)}}{d}\right)$$
$$+\frac{1}{2}i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{4\left(\left(-i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-i\right)e^{(-i\,dx-i\,c)}}{d}\right)$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*I*sqrt(2)*sqrt(1/(a*d^2))*log(-4*((I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) - I)*e^(-I*d*x - I*c)/d) +
1/2*I*sqrt(2)*sqrt(1/(a*d^2))*log(-4*((-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) - I)*e^(-I*d*x - I*c)/d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec\left(c+dx\right)}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/sqrt(I*a*tan(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2\sqrt{a}\,i\left(\sqrt{\tan\left(dx+c\right)i+1}\,\sec\left(dx+c\right)+\left(\int\frac{\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)\tan\left(dx+c\right)^2 d+\left(\int \right.\right.}{ad\left(\tan\left(dx+c\right)^2+1\right)}$$

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
( - 2*sqrt(a)*i*(sqrt(tan(c + d*x)*i + 1)*sec(c + d*x) + int((sqrt(tan(c +
 d*x)*i + 1)*sec(c + d*x)*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*tan(c + d
*x)**2*d + int((sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)*tan(c + d*x))/(tan(c
 + d*x)**2 + 1),x)*d))/(a*d*(tan(c + d*x)**2 + 1))
```

3.345 $\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 122

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}d}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}-\frac{3i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad}$$

output
```
3/8*I*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^(
1/2)/a^(1/2)/d+1/2*I*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-3/4*I*cos(d*x+c
)*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sec(c+dx)\left(3i\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)-i(1+\cos(2(c+dx))+3i\sin(2(c+dx)))\right)}{8d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(Sec[c + d*x]*((3*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*
I)*(c + d*x))]] - I*(1 + Cos[2*(c + d*x)] + (3*I)*Sin[2*(c + d*x)])))/(8*d
*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = $\{3042, 3983, 3042, 3971, 3042, 3970, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3971}$$

$$\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3970}$$

$$\frac{3\left(\frac{ia\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[Cos[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTa
nh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*
d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d))/(4*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(97) = 194$.

Time = 9.28 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.64

method	result
default	$-\frac{3i\sin(dx+c)\operatorname{arctanh}\left(\frac{(i-\cot(dx+c)+\csc(dx+c))\sqrt{2}}{2\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)+(3\cos(dx+c)+3)\operatorname{arctanh}\left(\frac{(i-\cot(dx+c)+}{2\sqrt{\cot(dx+c)^2-2\cot(dx+c}}\right.}{}$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d*(3*I*sin(d*x+c)*arctanh(1/2/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+c
sc(d*x+c)^2-1)^(1/2)*(I-cot(d*x+c)+csc(d*x+c))*2^(1/2))+(3*cos(d*x+c)+3)*a
rctanh(1/2/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(I-
cot(d*x+c)+csc(d*x+c))*2^(1/2))+3*I*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)-3*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)+I*cos(d*x+c)*(2*cos(d*x+c)-4)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-6*sin(d
*x+c)*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))/(cos(d*x+c)+1)/(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(91) = 182$.

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.01

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\left(-3i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(2i\,dx+2i\,c)}\log\left(-\frac{3\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-i\right)e^{(-i\,dx-i\,c)}}{2\,d}\right)+3i\sqrt{\frac{1}{2}}ad\sqrt{\cdots}}{}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/8*(-3*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(2*I*d*x + 2*I*c)*log(-3/2*(sqrt
(2)*sqrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*sqrt(1/(a*d^2)) - I)*e^(-I*d*x - I*c)/d) + 3*I*sqrt(1/2)*a*d*sqrt(1/
(a*d^2))*e^(2*I*d*x + 2*I*c)*log(-3/2*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x
+ 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) - I)*e^(
-I*d*x - I*c)/d) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(4*I*
d*x + 4*I*c) - I*e^(2*I*d*x + 2*I*c) + I))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos(c+dx)}{\sqrt{ia\left(\tan(c+dx)-i\right)}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(91) = 182$.

Time = 0.25 (sec) , antiderivative size = 837, normalized size of antiderivative = 6.86

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/32*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*((-I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + 2*I*sqr
t(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*
cos(2*d*x + 2*c) - I*sqrt(2)*sin(2*d*x + 2*c) - 2*sqrt(2))*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*(2*sqrt(2)*arctan2(
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) + 1) + I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) ...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\cos(dx+c)\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)i+\int \frac{\sqrt{\tan(dx+c)i+1}\,\cos(dx+c)}{\tan(dx+c)^2+1}dx\right)}{a}$$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)*tan(c + d*x))/(tan
(c + d*x)**2 + 1),x)*i + int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*x))/(tan(
c + d*x)**2 + 1),x)))/a
```

3.346 $\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 193

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{35i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}d} + \frac{35i\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} + \frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64ad} - \frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{24ad}$$

output
```
35/128*I*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*
2^(1/2)/a^(1/2)/d+35/96*I*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+1/4*I*cos(
d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)-35/64*I*cos(d*x+c)*(a+I*a*tan(d*x+c))^
(1/2)/a/d-7/24*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.61

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sec(c+dx)\left(-41i+105i\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)-43i\cos(2(c+dx))-2i\cos(4(c+dx)\right.}{384d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(Sec[c + d*x]*(-41*I + (105*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[
1 + E^((2*I)*(c + d*x))]] - (43*I)*Cos[2*(c + d*x)] - (2*I)*Cos[4*(c + d*x
)] + 133*Sin[2*(c + d*x)] + 14*Sin[4*(c + d*x)]))/(384*d*Sqrt[a + I*a*Tan[
c + d*x]])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {3042, 3983, 3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\int \cos^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^3}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{7\left(\frac{5}{6}a\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5}{6}a\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\left(\frac{5}{6}a\left(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5}{6}a\left(\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3971}$$

$$\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 3970$$

$$\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{ia\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 219$$

$$\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{i\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[Cos[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I/4)*Cos[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (7*(((-1/3*I)*Cos[
c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (5*a*(((I/2)*Cos[c + d*x])/(d*S
qrt[a + I*a*Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])
/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt
[a + I*a*Tan[c + d*x]])/d))/(4*a)))/6))/(8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(156) = 312$.

Time = 9.39 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.87

method	result
default	$-\frac{-105i \operatorname{arctanh}\left(\frac{\sqrt{2}\,(-i+\cot(dx+c)-\csc(dx+c))}{2\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)\sin(dx+c)+(-105\cos(dx+c)-105)\operatorname{arctanh}\left(\frac{\sqrt{2}\,(}{2\sqrt{\cot(dx+c)^2-}}\right.}{}$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384/d*(-105*I*arctanh(1/2*2^(1/2)*(-I+cot(d*x+c)-csc(d*x+c))/(cot(d*x+c
)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*sin(d*x+c)+(-105*cos(d*
x+c)-105)*arctanh(1/2*2^(1/2)*(-I+cot(d*x+c)-csc(d*x+c))/(cot(d*x+c)^2-2*c
ot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))+105*I*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*cos(d*x+c)*2^(1/2)-105*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)+sin(d*x+c)*cos(d*x+c)*(-112*cos(d*x+c)^2-112*cos(d*x+c)-
210)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*cos(d*x+c)*(16*cos(d*x+c)^3+16*c
os(d*x+c)^2+70*cos(d*x+c)-140)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))/(cos(d*
x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.38

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(-105i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(4i\,dx+4i\,c)}\log\left(-\frac{35\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-i\right)e^{(-i\,dx-i\,c)}}{32\,d}\right)+105i\sqrt{\frac{1}{2}}\right.}{}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/384*(-105*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(4*I*d*x + 4*I*c)*log(-35/32
*(sqrt(2)*sqrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt(1/(a*d^2)) - I)*e^(-I*d*x - I*c)/d) + 105*I*sqrt(1/2)*a*d
*sqrt(1/(a*d^2))*e^(4*I*d*x + 4*I*c)*log(-35/32*(sqrt(2)*sqrt(1/2)*(-I*d*e
^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)
) - I)*e^(-I*d*x - I*c)/d) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-8
*I*e^(8*I*d*x + 8*I*c) - 88*I*e^(6*I*d*x + 6*I*c) - 41*I*e^(4*I*d*x + 4*I*
c) + 45*I*e^(2*I*d*x + 2*I*c) + 6*I))*e^(-4*I*d*x - 4*I*c)/(a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1938 vs. $2(146) = 292$.

Time = 0.38 (sec) , antiderivative size = 1938, normalized size of antiderivative = 10.04

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/1536*(4*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((-3*I*sqrt(2)*cos(4*d*x + 4*c
) - 3*sqrt(2)*sin(4*d*x + 4*c) + 8*I*sqrt(2))*cos(3/2*arctan2(sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 1)) + (3*sqrt(2)*cos(4*d*x + 4*c) - 3*I*sqrt(2)*sin
(4*d*x + 4*c) - 8*sqrt(2))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) + 1)))*sqrt(a) + 12*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((-I*sqrt(2)*cos(4*
d*x + 4*c) - 12*I*sqrt(2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) - sqrt(2)*sin(4*d*x + 4*c) - 12*sqrt(2)*sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))) + 24*I*sqrt(2))*cos(1/2*arctan2(sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1)) + (sqrt(2)*cos(4*d*x + 4*c) + 12*sqrt(2)*cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - I*sqrt(2)*sin(4*d*x + 4*c) - 12
*I*sqrt(2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 24*sqrt(
2))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))),
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 1...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)^3}{\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}\,dx$$

input
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\cos(dx+c)^3\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)i+\int \frac{\sqrt{\tan(dx+c)i+1}\,\cos(dx+c)^3}{\tan(dx+c)^2+1}dx\right)}{a}$$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**3*tan(c + d*x))/(
tan(c + d*x)**2 + 1),x)*i + int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**3)
/(tan(c + d*x)**2 + 1),x)))/a
```

3.347 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{16i(a+ia\tan(c+dx))^{5/2}}{5a^4d} + \frac{24i(a+ia\tan(c+dx))^{7/2}}{7a^5d} - \frac{4i(a+ia\tan(c+dx))^{9/2}}{3a^6d} + \frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^7d}$$

output
```
-16/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^4/d+24/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^5
/d-4/3*I*(a+I*a*tan(d*x+c))^(9/2)/a^6/d+2/11*I*(a+I*a*tan(d*x+c))^(11/2)/a
^7/d
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{2(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}(-533i-755\tan(c+dx)+455i\tan^2(c+dx)+105\tan^3(c+}{1155a^2d}$$

input
```
Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(-2*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]*(-533*I - 755*Tan[c +
 d*x] + (455*I)*Tan[c + d*x]^2 + 105*Tan[c + d*x]^3))/(1155*a^2*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^8}{(a+ia\tan(c+dx))^{3/2}}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{3/2}d(ia\tan(c+dx))}{a^7d}$$
$$\Big\downarrow \text{53}$$
$$-\frac{i\int \left(-(i\tan(c+dx)a+a)^{9/2}+6a(i\tan(c+dx)a+a)^{7/2}-12a^2(i\tan(c+dx)a+a)^{5/2}+8a^3(i\tan(c+dx)a+\right.}{a^7d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{i\left(\frac{16}{5}a^3(a+ia\tan(c+dx))^{5/2}-\frac{24}{7}a^2(a+ia\tan(c+dx))^{7/2}-\frac{2}{11}(a+ia\tan(c+dx))^{11/2}+\frac{4}{3}a(a+ia\tan(c+\right.}{a^7d}$$

input
```
Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*((16*a^3*(a + I*a*Tan[c + d*x])^(5/2))/5 - (24*a^2*(a + I*a*Tan[c +
d*x])^(7/2))/7 + (4*a*(a + I*a*Tan[c + d*x])^(9/2))/3 - (2*(a + I*a*Tan[c
+ d*x])^(11/2))/11))/(a^7*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{2a(a+ia\tan(dx+c))^{\frac{9}{2}}}{3}+\frac{12a^2(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^7}$	82
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{2a(a+ia\tan(dx+c))^{\frac{9}{2}}}{3}+\frac{12a^2(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^7}$	82

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^7*(1/11*(a+I*a*tan(d*x+c))^(11/2)-2/3*a*(a+I*a*tan(d*x+c))^(9/2)+1
2/7*a^2*(a+I*a*tan(d*x+c))^(7/2)-8/5*a^3*(a+I*a*tan(d*x+c))^(5/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.27

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{64\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(16i\,e^{(11i\,dx+11i\,c)}+88i\,e^{(9i\,dx+9i\,c)}+198i\,e^{(7i\,dx+7i\,c)}+231i\,e^{(5i\,dx+5i\,c)}\right)}{1155\left(a^2de^{(10i\,dx+10i\,c)}+5\,a^2de^{(8i\,dx+8i\,c)}+10\,a^2de^{(6i\,dx+6i\,c)}+10\,a^2de^{(4i\,dx+4i\,c)}+5\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-64/1155*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(11*I*d*x + 11*
I*c) + 88*I*e^(9*I*d*x + 9*I*c) + 198*I*e^(7*I*d*x + 7*I*c) + 231*I*e^(5*I
*d*x + 5*I*c))/(a^2*d*e^(10*I*d*x + 10*I*c) + 5*a^2*d*e^(8*I*d*x + 8*I*c)
+ 10*a^2*d*e^(6*I*d*x + 6*I*c) + 10*a^2*d*e^(4*I*d*x + 4*I*c) + 5*a^2*d*e^
(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^8{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**8/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i\left(105\,(i\,a\tan\left(dx+c\right)+a)^{\frac{11}{2}} - 770\,(i\,a\tan\left(dx+c\right)+a)^{\frac{9}{2}}a + 1980\,(i\,a\,\mathrm{t}\right.}{1155\,a^7d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
2/1155*I*(105*(I*a*tan(d*x + c) + a)^(11/2) - 770*(I*a*tan(d*x + c) + a)^(
9/2)*a + 1980*(I*a*tan(d*x + c) + a)^(7/2)*a^2 - 1848*(I*a*tan(d*x + c) +
a)^(5/2)*a^3)/(a^7*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 5.13 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.16

$$
\begin{aligned}
\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = & -\frac{\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,1024i}{1155\,a^2\,d} - \frac{\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,512i}{1155\,a^2\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)} \\
& -\frac{\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,128i}{385\,a^2\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^2} - \frac{\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,64i}{231\,a^2\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^3} \\
& +\frac{\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,256i}{33\,a^2\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^4} - \frac{\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,64i}{11\,a^2\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^5}
\end{aligned}
$$

input
```
int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
256i)/(33*a^2*d*(exp(c*2i + d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(1155*a^2*d*(exp(c*2i
+ d*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*
x*2i) + 1))^(1/2)*128i)/(385*a^2*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*
(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(231
*a^2*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)
*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(1155*a^2*d) - ((a - (a*(exp(c
*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(11*a^2*d*
(exp(c*2i + d*x*2i) + 1)^5)
```

Reduce [F]

$$
\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^8\,i-13\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)^8\,t}{\tan(dx+c)^3 i+\tan(dx+c)^2+\tan}\right.\right.}{}
$$

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**8*i - 13*int((sqrt(t
an(c + d*x)*i + 1)*sec(c + d*x)**8*tan(c + d*x)**2)/(tan(c + d*x)**3*i + t
an(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*d - 13*int((sqrt(t
an(c + d*x)*i + 1)*sec(c + d*x)**8*tan(c + d*x)**2)/(tan(c + d*x)**3*i + t
an(c + d*x)**2 + tan(c + d*x)*i + 1),x)*d + 11*int((sqrt(tan(c + d*x)*i +
1)*sec(c + d*x)**8*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*x)**2 + ta
n(c + d*x)*i + 1),x)*tan(c + d*x)**2*d*i + 11*int((sqrt(tan(c + d*x)*i + 1
)*sec(c + d*x)**8*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan
(c + d*x)*i + 1),x)*d*i))/(a**2*d*(tan(c + d*x)**2 + 1))
```

3.348 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^3d} + \frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^4d} - \frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^5d}$$

output
```
-8/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^3/d+8/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^4/d
-2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^5/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{2(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}(-71+54i\tan(c+dx)+15\tan^2(c+dx))}{105a^2d}$$

input
```
Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(-2*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]*(-71 + (54*I)*Tan[c + d
*x] + 15*Tan[c + d*x]^2))/(105*a^2*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^6}{(a+ia\tan(c+dx))^{3/2}}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int (a-ia\tan(c+dx))^2\sqrt{i\tan(c+dx)a+a}d(ia\tan(c+dx))}{a^5d}$$
$$\Big\downarrow \text{53}$$
$$-\frac{i\int \left((i\tan(c+dx)a+a)^{5/2}-4a(i\tan(c+dx)a+a)^{3/2}+4a^2\sqrt{i\tan(c+dx)a+a}\right)d(ia\tan(c+dx))}{a^5d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{i\left(\frac{8}{3}a^2(a+ia\tan(c+dx))^{3/2}+\frac{2}{7}(a+ia\tan(c+dx))^{7/2}-\frac{8}{5}a(a+ia\tan(c+dx))^{5/2}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*((8*a^2*(a + I*a*Tan[c + d*x])^(3/2))/3 - (8*a*(a + I*a*Tan[c + d*x]
)^(5/2))/5 + (2*(a + I*a*Tan[c + d*x])^(7/2))/7))/(a^5*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{4a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right)}{d\,a^5}$	63
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{4a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right)}{d\,a^5}$	63

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^5*(-1/7*(a+I*a*tan(d*x+c))^(7/2)+4/5*a*(a+I*a*tan(d*x+c))^(5/2)-4/
3*a^2*(a+I*a*tan(d*x+c))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.23

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{16\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(8i\,e^{(7i\,dx+7i\,c)}+28i\,e^{(5i\,dx+5i\,c)}+35i\,e^{(3i\,dx+3i\,c)}\right)}{105\left(a^2de^{(6i\,dx+6i\,c)}+3\,a^2de^{(4i\,dx+4i\,c)}+3\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-16/105*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(8*I*e^(7*I*d*x + 7*I*c)
 + 28*I*e^(5*I*d*x + 5*I*c) + 35*I*e^(3*I*d*x + 3*I*c))/(a^2*d*e^(6*I*d*x
+ 6*I*c) + 3*a^2*d*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^(2*I*d*x + 2*I*c) + a^2
*d)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^6(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**6/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{2i\left(15\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}-84\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a+140\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^2\right)}{105\,a^5d}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-2/105*I*(15*(I*a*tan(d*x + c) + a)^(7/2) - 84*(I*a*tan(d*x + c) + a)^(5/2
)*a + 140*(I*a*tan(d*x + c) + a)^(3/2)*a^2)/(a^5*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.75

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,128i}{105\,a^2\,d} - \frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,64i}{105\,a^2\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)} - \frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,16i}{35\,a^2\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^2} + \frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,16i}{7\,a^2\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^3}$$

input
```
int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
16i)/(7*a^2*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1
i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(105*a^2*d*(exp(c*2i + d*
x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i
) + 1))^(1/2)*16i)/(35*a^2*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c
*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(105*a^2*
d)
```

Reduce [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^6\,i-9\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)^6\tan\cdots}{\tan(dx+c)^3 i+\tan(dx+c)^2+\tan(\cdots}\right.\right.}{}$$

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**6*i - 9*int((sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**6*tan(c + d*x)**2)/(tan(c + d*x)**3*i + ta
n(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*d - 9*int((sqrt(tan
(c + d*x)*i + 1)*sec(c + d*x)**6*tan(c + d*x)**2)/(tan(c + d*x)**3*i + tan
(c + d*x)**2 + tan(c + d*x)*i + 1),x)*d + 7*int((sqrt(tan(c + d*x)*i + 1)*
sec(c + d*x)**6*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c
 + d*x)*i + 1),x)*tan(c + d*x)**2*d*i + 7*int((sqrt(tan(c + d*x)*i + 1)*se
c(c + d*x)**6*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c +
 d*x)*i + 1),x)*d*i))/(a**2*d*(tan(c + d*x)**2 + 1))
```

3.349 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{4i\sqrt{a+ia\tan(c+dx)}}{a^2d}+\frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^3d}$$

output
```
-4*I*(a+I*a*tan(d*x+c))^(1/2)/a^2/d+2/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^3/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.65

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{2(5i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{3a^2d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(-2*(5*I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(3*a^2*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^4}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{a-ia\tan(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(\frac{2a}{\sqrt{i\tan(c+dx)a+a}}-\sqrt{i\tan(c+dx)a+a}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(4a\sqrt{a+ia\tan(c+dx)}-\frac{2}{3}(a+ia\tan(c+dx))^{3/2}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*(4*a*Sqrt[a + I*a*Tan[c + d*x]] - (2*(a + I*a*Tan[c + d*x])^(3/2))/3
))/(a^3*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-2a\sqrt{a+ia\tan(dx+c)}\right)}{d\,a^{3}}$	44
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-2a\sqrt{a+ia\tan(dx+c)}\right)}{d\,a^{3}}$	44

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^3*(1/3*(a+I*a*tan(d*x+c))^(3/2)-2*a*(a+I*a*tan(d*x+c))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(2i\,e^{(3i\,dx+3i\,c)}+3i\,e^{(i\,dx+i\,c)}\right)}{3\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-4/3*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(2*I*e^(3*I*d*x + 3*I*c) +
3*I*e^(I*d*x + I*c))/(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^4(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.67

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i\left((i\,a\tan(dx+c)+a)^{\frac{3}{2}}-6\sqrt{i\,a\tan(dx+c)+a}\,a\right)}{3\,a^3d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
2/3*I*((I*a*tan(d*x + c) + a)^(3/2) - 6*sqrt(I*a*tan(d*x + c) + a)*a)/(a^3
*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.49

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{2\left(\cos\left(2\,c+2\,d\,x\right)\,5\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+5\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}{3\,a^2\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
-(2*(cos(2*c + 2*d*x)*5i + sin(2*c + 2*d*x) + 5i)*((a*(cos(2*c + 2*d*x) +
sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))/(3*a^2*d*(cos(2*c
 + 2*d*x) + 1))
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^4\,i - 5\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)^4 \tan}{\tan(dx+c)^3 i+\tan(dx+c)^2+\tan(}\right.\right.}{}$$

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**4*i - 5*int((sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**4*tan(c + d*x)**2)/(tan(c + d*x)**3*i + ta
n(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*d - 5*int((sqrt(tan
(c + d*x)*i + 1)*sec(c + d*x)**4*tan(c + d*x)**2)/(tan(c + d*x)**3*i + tan
(c + d*x)**2 + tan(c + d*x)*i + 1),x)*d + 3*int((sqrt(tan(c + d*x)*i + 1)*
sec(c + d*x)**4*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c
 + d*x)*i + 1),x)*tan(c + d*x)**2*d*i + 3*int((sqrt(tan(c + d*x)*i + 1)*se
c(c + d*x)**4*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c +
 d*x)*i + 1),x)*d*i))/(a**2*d*(tan(c + d*x)**2 + 1))
```

3.350 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 27

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{ad\sqrt{a+ia\tan(c+dx)}}$$

output
```
2*I/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{ad\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*I)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\sec(c+dx)^2}{(a+ia\tan(c+dx))^{3/2}}dx$$
$$\Big\downarrow \ 3968$$
$$-\frac{i\int \frac{1}{(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{ad}$$
$$\Big\downarrow \ 17$$
$$\frac{2i}{ad\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*I)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2i}{ad\sqrt{a+ia\tan(dx+c)}}$	24
default	$\frac{2i}{ad\sqrt{a+ia\tan(dx+c)}}$	24

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(21) = 42.

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-i\,dx-i\,c)}}{a^2d}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(2*I*d*x + 2*I*c) + I)*e^(-
I*d*x - I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{\sqrt{i\,a\tan(dx+c)+a}ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
2*I/(sqrt(I*a*tan(d*x + c) + a)*a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.48

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(\cos\left(c+d\,x\right)^2\,2\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)\right)\,\sqrt{\frac{a\,\left(2\,\cos\left(c+d\,x\right)^2+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{2\,\cos\left(c+d\,x\right)^2}}}{a^2\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
((sin(2*c + 2*d*x) + cos(c + d*x)^2*2i)*((a*(sin(2*c + 2*d*x)*1i + 2*cos(c
 + d*x)^2))/(2*cos(c + d*x)^2))^(1/2))/(a^2*d)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\int \frac{\sec(dx+c)^2}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i+\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a}$$

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
int(sec(c + d*x)**2/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i + sqrt(tan(c
+ d*x)*i + 1)),x)/(sqrt(a)*a)
```

3.351 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 179

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{7i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}d} + \frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}} + \frac{7i}{24d(a+ia\tan(c+dx))^{3/2}} + \frac{7i}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{ia^3}{2d(a+ia\tan(c+dx))^{5/2}\,(a^2-ia^2\tan(c+dx))}$$

output
```
-7/32*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/a^(3
/2)/d+7/20*I*a/d/(a+I*a*tan(d*x+c))^(5/2)+7/24*I/d/(a+I*a*tan(d*x+c))^(3/2
)+7/16*I/a/d/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*a^3/d/(a+I*a*tan(d*x+c))^(5/2)
/(a^2-I*a^2*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.28

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{ia\,\text{Hypergeometric2F1}\left(-\frac{5}{2},2,-\frac{3}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{10d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I/10)*a*Hypergeometric2F1[-5/2, 2, -3/2, (1 + I*Tan[c + d*x])/2])/(d*(a
+ I*a*Tan[c + d*x])^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3968, 52, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^3\left(\frac{7\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}\right)}{d}$$

$$\Big\downarrow \textbf{61}$$

$$-\frac{ia^3\left(\frac{7\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}\right)}{d}$$

$$\Big\downarrow \textbf{61}$$

$$-\frac{ia^3\left(\frac{7\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+i\cdots}\cdots\right.}{d}$$

$$\Big\downarrow \textbf{61}$$

$$-\frac{ia^3\left(\frac{7\left(\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{\cdots}{2a(a-ia\cdots}\right.}{d}$$

$$\Big\downarrow \textbf{73}$$

$$-\frac{ia^3\left(\frac{7\left(\frac{\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{\cdots}{2a(a-ia\tan(c+dx))\cdots}\right.}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ia^3\left(\frac{7\left(\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+}\right.}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*a^3*(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2)) + (
7*(-1/5*1/(a*(a + I*a*Tan[c + d*x])^(5/2)) + (-1/3*1/(a*(a + I*a*Tan[c + d
*x])^(3/2)) + ((I*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)
) - 1/(a*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a))/(2*a)))/(4*a)))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(141) = 282$.

Time = 9.46 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.11

method	result
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(dx+c)-\csc(dx+c)+i)}{2\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)(-210\sin(dx+c)-105\tan(dx+c))+i\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(dx+\cdots}{2\sqrt{\cot(dx+c)^2-2\cot(dx\cdots}}\right.}{}$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/480/d/(cos(d*x+c)+1)/(-tan(d*x+c)+I)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/
a/(a*(1+I*tan(d*x+c)))^(1/2)*(arctanh(1/2*2^(1/2)*(cot(d*x+c)-csc(d*x+c)+I
)/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-210*sin(d
*x+c)-105*tan(d*x+c))+I*arctanh(1/2*2^(1/2)*(cot(d*x+c)-csc(d*x+c)+I)/(cot
(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(210*cos(d*x+c)+1
05-105*sec(d*x+c))-105*I*tan(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2
^(1/2)-105*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)+I*sin(d*x+c)*(168*
cos(d*x+c)^2+168*cos(d*x+c)-210)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(72*co
s(d*x+c)^3+72*cos(d*x+c)^2-350*cos(d*x+c)-140)*(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(130) = 260$.

Time = 0.09 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.64

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(-105i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(5i\,dx+5i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)\sqrt{\frac{}{e^{(2i}}}\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/480*(-105*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(5*I*d*x + 5*I*c)*log(4*
(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) +
105*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2
)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt(1/(a^3*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-15*I*e^(8*I*d*x + 8*I*c) + 101*I*e^(6
*I*d*x + 6*I*c) + 148*I*e^(4*I*d*x + 4*I*c) + 38*I*e^(2*I*d*x + 2*I*c) + 6
*I))*e^(-5*I*d*x - 5*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\left(\frac{105\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{\sqrt{a}} + \frac{4\left(105\,(ia\tan(dx+c)+a)^3-140\,(ia\tan(dx+c)+a)^2a-56\,(ia\tan(dx+c)+a)a^2-48\,a^3\right)}{(ia\tan(dx+c)+a)^{\frac{7}{2}}-2\,(ia\tan(dx+c)+a)^{\frac{5}{2}}a}\right)}{960\,ad}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/960*I*(105*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(
sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) + 4*(105*(I*a*tan(d
*x + c) + a)^3 - 140*(I*a*tan(d*x + c) + a)^2*a - 56*(I*a*tan(d*x + c) + a
)*a^2 - 48*a^3)/((I*a*tan(d*x + c) + a)^(7/2) - 2*(I*a*tan(d*x + c) + a)^(
5/2)*a))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\int \frac{\cos(dx+c)^2}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i+\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a}$$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
int(cos(c + d*x)**2/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i + sqrt(tan(c
+ d*x)*i + 1)),x)/(sqrt(a)*a)
```

3.352 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 252

$$
\begin{aligned}
\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = &-\frac{99i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}a^{3/2}d} \\
&+\frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} \\
&+\frac{99ia}{320d(a+ia\tan(c+dx))^{5/2}}+\frac{33i}{128d(a+ia\tan(c+dx))^{3/2}} \\
&+\frac{99i}{256ad\sqrt{a+ia\tan(c+dx)}}-\frac{11ia^5}{16d(a+ia\tan(c+dx))^{7/2}\left(a^3-ia^3\tan(c+dx)\right)}
\end{aligned}
$$

output

```
-99/512*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/a^
(3/2)/d+99/224*I*a^2/d/(a+I*a*tan(d*x+c))^(7/2)-1/4*I*a^4/d/(a-I*a*tan(d*x
+c))^2/(a+I*a*tan(d*x+c))^(7/2)+99/320*I*a/d/(a+I*a*tan(d*x+c))^(5/2)+33/1
28*I/d/(a+I*a*tan(d*x+c))^(3/2)+99/256*I/a/d/(a+I*a*tan(d*x+c))^(1/2)-11/1
6*I*a^5/d/(a+I*a*tan(d*x+c))^(7/2)/(a^3-I*a^3*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.21

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{ia^2\,\text{Hypergeometric2F1}\left(-\frac{7}{2},3,-\frac{5}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{28d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I/28)*a^2*Hypergeometric2F1[-7/2, 3, -5/2, (1 + I*Tan[c + d*x])/2])/(d*(
a + I*a*Tan[c + d*x])^(7/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3968, 52, 52, 61, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^4(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{11\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}}\right)}{d}$$

$$\Big\downarrow \ 52$$

$$-\frac{ia^5\left(\frac{11\left(\frac{9\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan}\right.}{d}$$

$$\Big\downarrow \ 61$$

$$-\frac{ia^5\left(\frac{11\left(\frac{9\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\right)}{8a}+\frac{}{4a(a}\right.}{d}$$

$$\Big\downarrow \ 61$$

$$-\frac{ia^5\left(\frac{11\left(\frac{9\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia}\right.}{8a}\right.}{d}$$

$$\Big\downarrow \ 61$$

$$-\frac{ia^5\left(\frac{11\left(\frac{9\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\right.}{8a}\right.}{d}$$

$\downarrow$ 61

$$-\frac{ia^5\left(\frac{11\left(\frac{9\left(\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{}{7a(a+ia\,\mathrm{t}}\right.}{4a}\right.}{8a}\right.}{d}$$

$\Big\downarrow$ 73

$$
-\frac{ia^5\left(\frac{11\left(\frac{9\left(\frac{\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right.}{4a}\right.}{8a}\right.}{d}
$$

$\Big\downarrow$ 219

$$-\frac{ia^5\left(\frac{11\left(\frac{9\left(\frac{\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{}{2a(a-ia\,\mathrm{t}}}{8a}}{d}$$

input

```
Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output

```
((-I)*a^5*(1/(4*a*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(7/2)) +
 (11*(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(7/2)) + (9*(-1
/7*1/(a*(a + I*a*Tan[c + d*x])^(7/2)) + (-1/5*1/(a*(a + I*a*Tan[c + d*x])^
(5/2)) + (-1/3*1/(a*(a + I*a*Tan[c + d*x])^(3/2)) + ((I*ArcTan[(Sqrt[a]*Ta
n[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[a + I*a*Tan[c + d*x]])
)/(2*a))/(2*a))/(2*a)))/(4*a)))/(8*a)))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(201) = 402$.

Time = 8.86 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.67

method	result
default	$\frac{i(6930\cos(dx+c)+3465)\sin(dx+c)\,\mathrm{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)+i}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+\left(6930\cos(dx+c)^2+3465\cos(dx+c)-3465\right)\mathrm{arctanh}\left(\frac{\cot(}{2}\right.}{}$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/17920/d*(I*(6930*cos(d*x+c)+3465)*sin(d*x+c)*arctanh(1/2/(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+I))+(6930*cos(d*x+c)^2+3465*cos
(d*x+c)-3465)*arctanh(1/2/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-c
sc(d*x+c)+I))+3465*I*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)-3465*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+sin(d*x+c)
*cos(d*x+c)*(3520*cos(d*x+c)^4+3520*cos(d*x+c)^3+5544*cos(d*x+c)^2+5544*co
s(d*x+c)-6930)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*cos(d*x+c)*(-960*cos(d
*x+c)^5-960*cos(d*x+c)^4-2376*cos(d*x+c)^3-2376*cos(d*x+c)^2+11550*cos(d*x
+c)+4620)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))/(a*(1+I*tan(d*x+c)))^(1/2)/(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(I*sin(d*x+c)*cos(d*x+c)+I*sin(d*x+c)+co
s(d*x+c)^2+cos(d*x+c))/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.25

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(-3465i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(7i\,dx+7i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)\sqrt{\frac{}{e^{(}}}\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/17920*(-3465*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(7*I*d*x + 7*I*c)*log
(4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
 + 3465*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(7*I*d*x + 7*I*c)*log(-4*(sq
rt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt(1/(a^3*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqr
t(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-70*I*e^(12*I*d*x + 12*I*c) - 805*
I*e^(10*I*d*x + 10*I*c) + 2833*I*e^(8*I*d*x + 8*I*c) + 4584*I*e^(6*I*d*x +
 6*I*c) + 1304*I*e^(4*I*d*x + 4*I*c) + 328*I*e^(2*I*d*x + 2*I*c) + 40*I))*
e^(-7*I*d*x - 7*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos^4{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.82

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\left(\frac{3465\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{\sqrt{a}} + \frac{4\left(3465\,(ia\tan(dx+c)+a)^5-11550\,(ia\tan(dx+c)+\right.}{(ia\tan(dx-}\right.}{358}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/35840*I*(3465*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a)
)/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) + 4*(3465*(I*a*t
an(d*x + c) + a)^5 - 11550*(I*a*tan(d*x + c) + a)^4*a + 7392*(I*a*tan(d*x
+ c) + a)^3*a^2 + 2112*(I*a*tan(d*x + c) + a)^2*a^3 + 1408*(I*a*tan(d*x +
c) + a)*a^4 + 1280*a^5)/((I*a*tan(d*x + c) + a)^(11/2) - 4*(I*a*tan(d*x +
c) + a)^(9/2)*a + 4*(I*a*tan(d*x + c) + a)^(7/2)*a^2))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)^4}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\int \frac{\cos(dx+c)^4}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i+\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a}$$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
int(cos(c + d*x)**4/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i + sqrt(tan(c
+ d*x)*i + 1)),x)/(sqrt(a)*a)
```

3.353 $\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 329

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{715i\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2048\sqrt{2}a^{3/2}d} + \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} + \frac{715ia^2}{1792d(a+ia\tan(c+dx))^{7/2}} + \frac{143ia}{512d(a+ia\tan(c+dx))^{5/2}} + \frac{715i}{3072d(a+ia\tan(c+dx))^{3/2}} + \frac{715i}{2048ad\sqrt{a+ia\tan(c+dx)}} - \frac{5ia^7}{16d(a+ia\tan(c+dx))^{9/2}\left(a^2-ia^2\tan(c+dx)\right)^2} - \frac{65ia^7}{64d(a+ia\tan(c+dx))^{9/2}\left(a^4-ia^4\tan(c+dx)\right)}$$

output
```
-715/4096*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/
a^(3/2)/d+715/1152*I*a^3/d/(a+I*a*tan(d*x+c))^(9/2)-1/6*I*a^6/d/(a-I*a*tan
(d*x+c))^3/(a+I*a*tan(d*x+c))^(9/2)+715/1792*I*a^2/d/(a+I*a*tan(d*x+c))^(7
/2)+143/512*I*a/d/(a+I*a*tan(d*x+c))^(5/2)+715/3072*I/d/(a+I*a*tan(d*x+c))
^(3/2)+715/2048*I/a/d/(a+I*a*tan(d*x+c))^(1/2)-5/16*I*a^7/d/(a+I*a*tan(d*x
+c))^(9/2)/(a^2-I*a^2*tan(d*x+c))^2-65/64*I*a^7/d/(a+I*a*tan(d*x+c))^(9/2)
/(a^4-I*a^4*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.47 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.16

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{ia^3\,\text{Hypergeometric2F1}\left(-\frac{9}{2},4,-\frac{7}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{72d(a+ia\tan(c+dx))^{9/2}}$$

input
```
Integrate[Cos[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I/72)*a^3*Hypergeometric2F1[-9/2, 4, -7/2, (1 + I*Tan[c + d*x])/2])/(d*(
a + I*a*Tan[c + d*x])^(9/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 3968, 52, 52, 52, 61, 61, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^6(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^7\int \frac{1}{(a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^7\left(\frac{5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{6a(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}}\right)}{d}$$

$\downarrow$ 52

$$-\frac{ia^7\left(\frac{5\left(\frac{13\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\frac{1}{6a(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}}\right)}{d}$$

$\downarrow$ 52

$$-\frac{ia^7\left(\frac{5\left(\frac{13\left(\frac{11\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\ldots\right)}{d}$$

$\downarrow$ 61

$$-\frac{ia^7\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}\right)}{8a}+\frac{1}{4a(\ldots}\right)}{4a}\ldots\right)}{d}$$

$\downarrow$ 61

$$-\frac{ia^7\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a-}}{8a}}{4a}}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^7\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}\cdots\right)}{4a}\cdots\right)}{8a}\cdots\right)}{4a}\cdots\right)}{d}\cdots$$

$\downarrow$ 61

$$-\frac{ia^7\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}\right.}{4a}\right.}{8a}\right.}{4a}\right.}{}$$

$\downarrow$ 61

$$ia^7\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a}{2a}}{4a}}{8a}}{4a}$$

$$\Big\downarrow \ 73$$

$$ia^7\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}\,d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx}}{2a}\right.}{4a}\right.}{8a}\right.}{4a}\right.$$

$\downarrow$ 219

$$ia^7\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{}{9a(a+i}\cdots\right.}{4a}\cdots\right.}{8a}\cdots\right.}{4a}\cdots\right.$$

input
```
Int[Cos[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*a^7*(1/(6*a*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(9/2)) +
 (5*(1/(4*a*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(9/2)) + (13*(
1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(9/2)) + (11*(-1/9*1/
(a*(a + I*a*Tan[c + d*x])^(9/2)) + (-1/7*1/(a*(a + I*a*Tan[c + d*x])^(7/2)
) + (-1/5*1/(a*(a + I*a*Tan[c + d*x])^(5/2)) + (-1/3*1/(a*(a + I*a*Tan[c +
 d*x])^(3/2)) + ((I*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/
2)) - 1/(a*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a))/(2*a))/(2*a))/(2*a)))/(4*a)
))/(8*a)))/(4*a)))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 9.95 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.40

method	result
default	$\frac{(-90090\cos(dx+c)-45045)\sin(dx+c)\operatorname{arctanh}\left(\frac{-\cot(dx+c)+\csc(dx+c)-i}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+i\left(90090\cos(dx+c)^2+45045\cos(dx+c)-45045\right)\operatorname{arcta}}{}$

input
```
int(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/258048/d*((-90090*cos(d*x+c)-45045)*sin(d*x+c)*arctanh(1/2*(-cot(d*x+c)+
csc(d*x+c)-I)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+I*(90090*cos(d*x+c)^2+45
045*cos(d*x+c)-45045)*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-I)/(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2))+45045*I*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)+45045*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
+I*sin(d*x+c)*cos(d*x+c)*(-35840*cos(d*x+c)^6-35840*cos(d*x+c)^5-45760*cos
(d*x+c)^4-45760*cos(d*x+c)^3-72072*cos(d*x+c)^2-72072*cos(d*x+c)+90090)*(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+cos(d*x+c)*(-7168*cos(d*x+c)^7-7168*cos(d
*x+c)^6-12480*cos(d*x+c)^5-12480*cos(d*x+c)^4-30888*cos(d*x+c)^3-30888*cos
(d*x+c)^2+150150*cos(d*x+c)+60060)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))/(a*
(1+I*tan(d*x+c)))^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(-I*cos(d*x+c)^
2+cos(d*x+c)*sin(d*x+c)-I*cos(d*x+c)+sin(d*x+c))/a
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.03

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(-45045i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(9i\,dx+9i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)\sqrt{\frac{}{e}}\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/258048*(-45045*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(9*I*d*x + 9*I*c)*l
og(4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c
)) + 45045*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(9*I*d*x + 9*I*c)*log(-4*
(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) +
sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-168*I*e^(16*I*d*x + 16*I*c) -
1974*I*e^(14*I*d*x + 14*I*c) - 13209*I*e^(12*I*d*x + 12*I*c) + 33301*I*e^(
10*I*d*x + 10*I*c) + 57632*I*e^(8*I*d*x + 8*I*c) + 17344*I*e^(6*I*d*x + 6*
I*c) + 5440*I*e^(4*I*d*x + 4*I*c) + 1136*I*e^(2*I*d*x + 2*I*c) + 112*I))*e
^(-9*I*d*x - 9*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos^6{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**6/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**6/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.79

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\left(\frac{45045\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)}{\sqrt{a}} + \frac{4\left(45045\,(i\,a\tan(dx+c)+a)^7-240240\,(i\,a\tan(dx+\right.}{}\right.}{}$$

input
```
integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/516096*I*(45045*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) + 4*(45045*(I*
a*tan(d*x + c) + a)^7 - 240240*(I*a*tan(d*x + c) + a)^6*a + 396396*(I*a*ta
n(d*x + c) + a)^5*a^2 - 164736*(I*a*tan(d*x + c) + a)^4*a^3 - 36608*(I*a*t
an(d*x + c) + a)^3*a^4 - 19968*(I*a*tan(d*x + c) + a)^2*a^5 - 15360*(I*a*t
an(d*x + c) + a)*a^6 - 14336*a^7)/((I*a*tan(d*x + c) + a)^(15/2) - 6*(I*a*
tan(d*x + c) + a)^(13/2)*a + 12*(I*a*tan(d*x + c) + a)^(11/2)*a^2 - 8*(I*a
*tan(d*x + c) + a)^(9/2)*a^3))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^6}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^6/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)^6/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\int \frac{\cos(dx+c)^6}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i+\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a}$$

input
```
int(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
int(cos(c + d*x)**6/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i + sqrt(tan(c
+ d*x)*i + 1)),x)/(sqrt(a)*a)
```

3.354 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{256ia^4\sec^{11}(c+dx)}{12155d(a+ia\tan(c+dx))^{11/2}} + \frac{64ia^3\sec^{11}(c+dx)}{1105d(a+ia\tan(c+dx))^{9/2}} + \frac{8ia^2\sec^{11}(c+dx)}{85d(a+ia\tan(c+dx))^{7/2}} + \frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

output
```
256/12155*I*a^4*sec(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(11/2)+64/1105*I*a^3*se
c(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(9/2)+8/85*I*a^2*sec(d*x+c)^11/d/(a+I*a*t
an(d*x+c))^(7/2)+2/17*I*a*sec(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(5/2)
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.73

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2\sec^9(c+dx)(i\cos(4(c+dx))+\sin(4(c+dx)))(475i-2242i\cos(2(c+d}{12155ad(-i+\tan(c+dx))\sqrt{a+}}$$

input
```
Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*Sec[c + d*x]^9*(I*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])*(475*I - (2242*I
)*Cos[2*(c + d*x)] + 1089*Sec[c + d*x]*Sin[3*(c + d*x)] + 374*Tan[c + d*x]
))/(12155*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3975, 3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{11}}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{17}a\int \frac{\sec^{11}(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{17}a\int \frac{\sec(c+dx)^{11}}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{17}a\left(\frac{8}{15}a\int \frac{\sec^{11}(c+dx)}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{17}a\left(\frac{8}{15}a\int \frac{\sec(c+dx)^{11}}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{17}a\left(\frac{8}{15}a\left(\frac{4}{13}a\int \frac{\sec^{11}(c+dx)}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{17}a\left(\frac{8}{15}a\left(\frac{4}{13}a\int \frac{\sec(c+dx)^{11}}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{12}{17}a\left(\frac{8}{15}a\left(\frac{8ia^2\sec^{11}(c+dx)}{143d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((2*I)/17)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (12*a*((
((2*I)/15)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (8*a*((((
8*I)/143)*a^2*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((2*I)
/13)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(9/2))))/15))/17
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 9.92 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91

method	result
default	$\frac{\frac{2\tan(dx+c)\sec(dx+c)^8\left(1024\cos(dx+c)^6+896\cos(dx+c)^4+792\cos(dx+c)^2+715\right)}{12155}+\frac{2i\left(1024\sec(dx+c)^2+384\sec(dx+c)^4+216\sec(dx+c)^6+143\,\mathrm{se}\right.}{12155}}{d(i\sin(dx+c)+\cos(dx+c))\sqrt{a(1+i\tan(dx+c))}\,a}$

input
```
int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/12155/d/(I*sin(d*x+c)+cos(d*x+c))/(a*(1+I*tan(d*x+c)))^(1/2)/a*(tan(d*x+
c)*sec(d*x+c)^8*(1024*cos(d*x+c)^6+896*cos(d*x+c)^4+792*cos(d*x+c)^2+715)+
I*(1024*sec(d*x+c)^2+384*sec(d*x+c)^4+216*sec(d*x+c)^6+143*sec(d*x+c)^8))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.25

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{512\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-1105i\,e^{(6i\,dx+6i\,c)}-510i\,e^{(4i\,dx+4i\,c)}-13\right.}{12155\left(a^2de^{(16i\,dx+16i\,c)}+8\,a^2de^{(14i\,dx+14i\,c)}+28\,a^2de^{(12i\,dx+12i\,c)}+56\,a^2de^{(10i\,dx+10i\,c)}+70\,a^2de^{(8i\,dx+8i\,c)}+\right.}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-512/12155*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-1105*I*e^(6*I*d*x +
 6*I*c) - 510*I*e^(4*I*d*x + 4*I*c) - 136*I*e^(2*I*d*x + 2*I*c) - 16*I)/(a
^2*d*e^(16*I*d*x + 16*I*c) + 8*a^2*d*e^(14*I*d*x + 14*I*c) + 28*a^2*d*e^(1
2*I*d*x + 12*I*c) + 56*a^2*d*e^(10*I*d*x + 10*I*c) + 70*a^2*d*e^(8*I*d*x +
 8*I*c) + 56*a^2*d*e^(6*I*d*x + 6*I*c) + 28*a^2*d*e^(4*I*d*x + 4*I*c) + 8*
a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 764 vs. $2(115) = 230$.

Time = 0.30 (sec) , antiderivative size = 764, normalized size of antiderivative = 5.20

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-2/12155*(-1767*I*sqrt(a) - 6854*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) +
 2088*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 16438*sqrt(a)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 - 5661*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 - 56984*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 13328*I*sqr
t(a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 129336*sqrt(a)*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 7514*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 -
 156468*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 156468*sqrt(a)*sin(d
*x + c)^11/(cos(d*x + c) + 1)^11 - 7514*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x
 + c) + 1)^12 - 129336*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 133
28*I*sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 56984*sqrt(a)*sin(d*x
 + c)^15/(cos(d*x + c) + 1)^15 + 5661*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x +
 c) + 1)^16 - 16438*sqrt(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 2088*I
*sqrt(a)*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 - 6854*sqrt(a)*sin(d*x + c)
^19/(cos(d*x + c) + 1)^19 + 1767*I*sqrt(a)*sin(d*x + c)^20/(cos(d*x + c) +
 1)^20)*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)^(3/2)/((a^2 - 10*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 45*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 120*a^2*sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + 210*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 252*a^2*si
n(d*x + c)^10/(cos(d*x + c) + 1)^10 + 210*a^2*sin(d*x + c)^12/(cos(d*x + c
) + 1)^12 - 120*a^2*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 45*a^2*sin(...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 7.53 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.05

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{11\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5} - \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1536\mathrm{i}}{13\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6} + \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{5\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^7} - \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{17\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^8}$$

input
```
int(1/(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
(exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
 d*x*2i) + 1))^(1/2)*512i)/(11*a^2*d*(exp(c*2i + d*x*2i) + 1)^5) - (exp(-
c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i
) + 1))^(1/2)*1536i)/(13*a^2*d*(exp(c*2i + d*x*2i) + 1)^6) + (exp(- c*1i -
 d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1)
)^(1/2)*512i)/(5*a^2*d*(exp(c*2i + d*x*2i) + 1)^7) - (exp(- c*1i - d*x*1i)
*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
512i)/(17*a^2*d*(exp(c*2i + d*x*2i) + 1)^8)
```

Reduce [F]

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^{11}\,i-19\left(\int\frac{\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)^{11}}{\tan(dx+c)^3i+\tan(dx+c)^2+\mathrm{ta}}\right.\right.}{}$$

input
```
int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x)
```

output

```
(sqrt(a)*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**11*i - 19*int((sqrt(
tan(c + d*x)*i + 1)*sec(c + d*x)**11*tan(c + d*x)**2)/(tan(c + d*x)**3*i +
 tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*d - 19*int((sqrt
(tan(c + d*x)*i + 1)*sec(c + d*x)**11*tan(c + d*x)**2)/(tan(c + d*x)**3*i
+ tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*d + 17*int((sqrt(tan(c + d*x)*i
 + 1)*sec(c + d*x)**11*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*x)**2
+ tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*d*i + 17*int((sqrt(tan(c + d*x)*i
 + 1)*sec(c + d*x)**11*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*x)**2
+ tan(c + d*x)*i + 1),x)*d*i))/(a**2*d*(tan(c + d*x)**2 + 1))
```

3.355 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{64ia^3\sec^9(c+dx)}{1287d(a+ia\tan(c+dx))^{9/2}} + \frac{16ia^2\sec^9(c+dx)}{143d(a+ia\tan(c+dx))^{7/2}} + \frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}$$

output
```
64/1287*I*a^3*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(9/2)+16/143*I*a^2*sec(d*x
+c)^9/d/(a+I*a*tan(d*x+c))^(7/2)+2/13*I*a*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c)
)^(5/2)
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2\sec^8(c+dx)(52+151\cos(2(c+dx))+135i\sin(2(c+dx)))(\cos(3(c+d}{1287ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*Sec[c + d*x]^8*(52 + 151*Cos[2*(c + d*x)] + (135*I)*Sin[2*(c + d*x)])*(
Cos[3*(c + d*x)] - I*Sin[3*(c + d*x)]))/(1287*a*d*(-I + Tan[c + d*x])*Sqrt
[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^9}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{13}a\int \frac{\sec^9(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{13}a\int \frac{\sec(c+dx)^9}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{13}a\left(\frac{4}{11}a\int \frac{\sec^9(c+dx)}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{13}a\left(\frac{4}{11}a\int \frac{\sec(c+dx)^9}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8}{13}a\left(\frac{8ia^2\sec^9(c+dx)}{99d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}$$

input

```
Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output

```
(((2*I)/13)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (8*a*((((
8*I)/99)*a^2*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((2*I)/11
)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(7/2))))/13
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 5.38 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{\frac{2\tan(dx+c)\sec(dx+c)^6\left(128\cos(dx+c)^4+112\cos(dx+c)^2+99\right)}{1287}+\frac{2i\left(128\sec(dx+c)^2+48\sec(dx+c)^4+27\sec(dx+c)^6\right)}{1287}}{d(i\sin(dx+c)+\cos(dx+c))\sqrt{a(1+i\tan(dx+c))}\,a}$	114

input

```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/1287/d/(I*sin(d*x+c)+cos(d*x+c))/(a*(1+I*tan(d*x+c)))^(1/2)/a*(tan(d*x+c
)*sec(d*x+c)^6*(128*cos(d*x+c)^4+112*cos(d*x+c)^2+99)+I*(128*sec(d*x+c)^2+
48*sec(d*x+c)^4+27*sec(d*x+c)^6))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.30

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{128\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-143i\,e^{(4i\,dx+4i\,c)}-52i\,e^{(2i\,dx+2i\,c)}-8i\right)}{1287\left(a^2de^{(12i\,dx+12i\,c)}+6\,a^2de^{(10i\,dx+10i\,c)}+15\,a^2de^{(8i\,dx+8i\,c)}+20\,a^2de^{(6i\,dx+6i\,c)}+15\,a^2de^{(4i\,dx+4i\,c)}+6\,a^2\right.}$$

input

```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output

```
-128/1287*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-143*I*e^(4*I*d*x + 4
*I*c) - 52*I*e^(2*I*d*x + 2*I*c) - 8*I)/(a^2*d*e^(12*I*d*x + 12*I*c) + 6*a
^2*d*e^(10*I*d*x + 10*I*c) + 15*a^2*d*e^(8*I*d*x + 8*I*c) + 20*a^2*d*e^(6*
I*d*x + 6*I*c) + 15*a^2*d*e^(4*I*d*x + 4*I*c) + 6*a^2*d*e^(2*I*d*x + 2*I*c
) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^9(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**9/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(86) = 172$.

Time = 0.25 (sec) , antiderivative size = 626, normalized size of antiderivative = 5.69

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-2/1287*(-203*I*sqrt(a) - 678*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*
I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1802*sqrt(a)*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 - 26*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
- 3614*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 858*I*sqrt(a)*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 - 6578*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 - 6578*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 858*I*sqrt(a)*s
in(d*x + c)^10/(cos(d*x + c) + 1)^10 - 3614*sqrt(a)*sin(d*x + c)^11/(cos(d
*x + c) + 1)^11 + 26*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 180
2*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 2*I*sqrt(a)*sin(d*x + c)
^14/(cos(d*x + c) + 1)^14 - 678*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)
^15 + 203*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*(sin(d*x + c)/(
cos(d*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)/(
(a^2 - 8*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^2*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 56*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^2*
sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 56*a^2*sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10 + 28*a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a^2*sin(d*x +
c)^14/(cos(d*x + c) + 1)^14 + a^2*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*d
*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{128\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,52\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,143\mathrm{i}+8\mathrm{i}\right)}{1287\,a^2\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^6}$$

input
```
int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
(128*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*
2i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*52i + exp(c*4i + d*x*4i)*143i
 + 8i))/(1287*a^2*d*(exp(c*2i + d*x*2i) + 1)^6)
```

Reduce [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^9\,i-15\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)^9\,\mathrm{t}}{\tan(dx+c)^3 i+\tan(dx+c)^2+\tan}\right.\right.}{}$$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**9*i - 15*int((sqrt(t
an(c + d*x)*i + 1)*sec(c + d*x)**9*tan(c + d*x)**2)/(tan(c + d*x)**3*i + t
an(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*d - 15*int((sqrt(t
an(c + d*x)*i + 1)*sec(c + d*x)**9*tan(c + d*x)**2)/(tan(c + d*x)**3*i + t
an(c + d*x)**2 + tan(c + d*x)*i + 1),x)*d + 13*int((sqrt(tan(c + d*x)*i +
1)*sec(c + d*x)**9*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*x)**2 + ta
n(c + d*x)*i + 1),x)*tan(c + d*x)**2*d*i + 13*int((sqrt(tan(c + d*x)*i + 1
)*sec(c + d*x)**9*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan
(c + d*x)*i + 1),x)*d*i))/(a**2*d*(tan(c + d*x)**2 + 1))
```

3.356 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 73

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{8ia^2\sec^7(c+dx)}{63d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}$$

output
```
8/63*I*a^2*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(7/2)+2/9*I*a*sec(d*x+c)^7/d/
(a+I*a*tan(d*x+c))^(5/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2\sec^5(c+dx)(i\cos(2(c+dx))+\sin(2(c+dx)))(-11i+7\tan(c+dx))}{63ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*Sec[c + d*x]^5*(I*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])*(-11*I + 7*Tan[c
 + d*x]))/(63*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^7}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3975}$$

$$\frac{4}{9}a\int \frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4}{9}a\int \frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3974}$$

$$\frac{8ia^2\sec^7(c+dx)}{63d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((8*I)/63)*a^2*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((2*I)
/9)*a*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 4.79 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{\frac{32 i \sec(dx+c)}{63}+\frac{32 \sec(dx+c) \tan(dx+c)}{63}+\frac{4 i \sec(dx+c)^{3}}{63}+\frac{20 \tan(dx+c) \sec(dx+c)^{3}}{63}-\frac{2 i \sec(dx+c)^{5}}{9}}{d \sqrt{a(1+i \tan(dx+c))}\, a}$	86

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/63/d/(a*(1+I*tan(d*x+c)))^(1/2)/a*(16*I*sec(d*x+c)+16*sec(d*x+c)*tan(d*x
+c)+2*I*sec(d*x+c)^3+10*tan(d*x+c)*sec(d*x+c)^3-7*I*sec(d*x+c)^5)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{32\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-9i\,e^{(2i\,dx+2i\,c)}-2i\right)}{63\left(a^2de^{(8i\,dx+8i\,c)}+4\,a^2de^{(6i\,dx+6i\,c)}+6\,a^2de^{(4i\,dx+4i\,c)}+4\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-32/63*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-9*I*e^(2*I*d*x + 2*I*c)
 - 2*I)/(a^2*d*e^(8*I*d*x + 8*I*c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6*a^2*d
*e^(4*I*d*x + 4*I*c) + 4*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^7(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**7/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(57) = 114$.

Time = 0.22 (sec) , antiderivative size = 488, normalized size of antiderivative = 6.68

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-2/63*(-11*I*sqrt(a) - 30*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 12*I*s
qrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 86*sqrt(a)*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 + 9*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 108*
sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 108*sqrt(a)*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 - 9*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 86
*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 12*I*sqrt(a)*sin(d*x + c)^1
0/(cos(d*x + c) + 1)^10 - 30*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11
 + 11*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*(sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)/((a^2
 - 6*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^2*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 20*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a^2*sin(
d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)
^10 + a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d*(-2*I*sin(d*x + c)/(cos
(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{32\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,9\mathrm{i}+2\mathrm{i}\right)\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}}{63\,a^2\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^4}$$

input
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
(32*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*9i + 2i)*(a - (a*(exp(c*2i +
d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(63*a^2*d*(exp(c*2i
+ d*x*2i) + 1)^4)
```

Reduce [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\,\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^7\,i-11\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)^7\,t}{\tan(dx+c)^3i+\tan(dx+c)^2+\tan}\right.\right.}{}$$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**7*i - 11*int((sqrt(t
an(c + d*x)*i + 1)*sec(c + d*x)**7*tan(c + d*x)**2)/(tan(c + d*x)**3*i + t
an(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*d - 11*int((sqrt(t
an(c + d*x)*i + 1)*sec(c + d*x)**7*tan(c + d*x)**2)/(tan(c + d*x)**3*i + t
an(c + d*x)**2 + tan(c + d*x)*i + 1),x)*d + 9*int((sqrt(tan(c + d*x)*i + 1
)*sec(c + d*x)**7*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan
(c + d*x)*i + 1),x)*tan(c + d*x)**2*d*i + 9*int((sqrt(tan(c + d*x)*i + 1)*
sec(c + d*x)**7*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c
 + d*x)*i + 1),x)*d*i))/(a**2*d*(tan(c + d*x)**2 + 1))
```

3.357 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2ia\sec^5(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

output
```
2/5*I*a*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(5/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2\sec^3(c+dx)(1-i\tan(c+dx))}{5ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*Sec[c + d*x]^3*(1 - I*Tan[c + d*x]))/(5*a*d*(-I + Tan[c + d*x])*Sqrt[a
+ I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3974}$$

$$\frac{2ia\sec^5(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((2*I)/5)*a*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

method	result	size
default	$-\frac{2i\sec(dx+c)^3(\sin(dx+c)+i\cos(dx+c))^2}{5d\sqrt{a(1+i\tan(dx+c))}\,a}$	51

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*I/d*sec(d*x+c)^3*(sin(d*x+c)+I*cos(d*x+c))^2/(a*(1+I*tan(d*x+c)))^(1/
2)/a
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(27) = 54.

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{8i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{5\left(a^2de^{(4i\,dx+4i\,c)}+2\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
8/5*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(a^2*d*e^(4*I*d*x + 4*I*c)
 + 2*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^5{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**5/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(27) = 54$.

Time = 0.18 (sec) , antiderivative size = 350, normalized size of antiderivative = 10.00

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\left(-i\sqrt{a}-\frac{2\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{2i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{6\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\cdots}{5\left(a^2-\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\left(-\frac{2i\sin(d\cdots}{\cos(dx+\cdots}\right.}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-2/5*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*I*sqrt(a)
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 6*sqrt(a)*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 - 6*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*I*sqrt(a)*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)/(c
os(d*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)/((
a^2 - 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.97

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{(\cos(d\,x)-\sin(d\,x)\ 1\mathrm{i})\ (\cos(c)-\sin(c)\ 1\mathrm{i})\ \sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\ \cdots}{\cos(2\,c+2\,d\,x)+1}}\ \cdots}{5\,a^2\,d\ (4\ \cos(2\,c\ \cdots}$$

input
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
((cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i)*((a*(cos(2*c + 2*d*x) + sin
(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(2*cos(2*c + 2*d*x) +
 cos(4*c + 4*d*x) - sin(2*c + 2*d*x)*2i - sin(4*c + 4*d*x)*1i + 1)*4i)/(5*
a^2*d*(4*cos(2*c + 2*d*x) + cos(4*c + 4*d*x) + 3))
```

Reduce [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^5\,i-7\left(\int\frac{\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)^5\,\tan\cdots}{\tan(dx+c)^3i+\tan(dx+c)^2+\tan(\cdots}\cdots\right.\right.}{\cdots}$$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**5*i - 7*int((sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**5*tan(c + d*x)**2)/(tan(c + d*x)**3*i + ta
n(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*d - 7*int((sqrt(tan
(c + d*x)*i + 1)*sec(c + d*x)**5*tan(c + d*x)**2)/(tan(c + d*x)**3*i + tan
(c + d*x)**2 + tan(c + d*x)*i + 1),x)*d + 5*int((sqrt(tan(c + d*x)*i + 1)*
sec(c + d*x)**5*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c
 + d*x)*i + 1),x)*tan(c + d*x)**2*d*i + 5*int((sqrt(tan(c + d*x)*i + 1)*se
c(c + d*x)**5*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c +
 d*x)*i + 1),x)*d*i))/(a**2*d*(tan(c + d*x)**2 + 1))
```

3.358 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}$$

output
```
2*I*2^(1/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))/a^(3/2)/d-2*I*sec(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.17

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{8e^{3i(c+dx)}\left(-1+\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{ad\left(1+e^{2i(c+dx)}\right)^2(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(8*E^((3*I)*(c + d*x))*(-1 + Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1
+ E^((2*I)*(c + d*x))]]))/(a*d*(1 + E^((2*I)*(c + d*x)))^2*(-I + Tan[c + d
*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 3972, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sec(c+dx)^3}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \ 3972$$

$$\frac{2\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{a}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{2\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{a}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 3970$$

$$\frac{4i\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{ad}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 219$$

$$\frac{2i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((2*I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c
+ d*x]])])/(a^(3/2)*d) - ((2*I)*Sec[c + d*x])/(a*d*Sqrt[a + I*a*Tan[c + d*
x]])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3972
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e
+ f*x])^(n + 1)/(b*f*(m - 2))), x] + Simp[2*(d^2/a)   Int[(d*Sec[e + f*x])^
(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && EqQ[m/2 + n, 0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(71) = 142$.

Time = 6.52 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.97

method	result	s
default	$-\frac{2(\cot(dx+c)-\csc(dx+c)+i)^{3}\left(i-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(i-\cot(dx+c)+\csc(dx+c))}{2\sqrt{\csc(dx+c)^{2}(1-\cos(dx+c))^{2}-1}}\right)\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}+\csc(dx+c)-\cot(dx+c)\right)}{da\sqrt{a(1+i\tan(dx+c))}\left(\csc(dx+c)^{2}(1-\cos(dx+c))^{2}-1\right)^{2}(-i+\tan(dx+c))}$	1

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/a/(a*(1+I*tan(d*x+c)))^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^2*(cot
(d*x+c)-csc(d*x+c)+I)^3*(I-2^(1/2)*arctanh(1/2/(csc(d*x+c)^2*(1-cos(d*x+c)
)^2-1)^(1/2)*2^(1/2)*(I-cot(d*x+c)+csc(d*x+c)))*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)+csc(d*x+c)-cot(d*x+c))/(-I+tan(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(67) = 134$.

Time = 0.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.28

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{-i\sqrt{2}a^2d\sqrt{\frac{1}{a^3d^2}}\log\left(-\frac{8\left(\left(iade^{(2i\,dx+2i\,c)}+i\,ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^3d^2}}-i\right)e^{(-i\,dx-i\,c)}}{ad}\right)}{}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a^2*d*sqrt(1/(a^3*d^2))*log(-8*((I*a*d*e^(2*I*d*x + 2*I*c) + I
*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x -
 I*c)/(a*d)) + I*sqrt(2)*a^2*d*sqrt(1/(a^3*d^2))*log(-8*((-I*a*d*e^(2*I*d*
x + 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) -
I)*e^(-I*d*x - I*c)/(a*d)) - 2*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
)/(a^2*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^3{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(67) = 134$.

Time = 0.26 (sec) , antiderivative size = 813, normalized size of antiderivative = 9.45

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/2*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sq
rt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(
sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^3\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^3\,i-3\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)^3\tan}{\tan(dx+c)^3i+\tan(dx+c)^2+\tan(}\right.\right.}{}$$

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**3*i - 3*int((sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**3*tan(c + d*x)**2)/(tan(c + d*x)**3*i + ta
n(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*d - 3*int((sqrt(tan
(c + d*x)*i + 1)*sec(c + d*x)**3*tan(c + d*x)**2)/(tan(c + d*x)**3*i + tan
(c + d*x)**2 + tan(c + d*x)*i + 1),x)*d + int((sqrt(tan(c + d*x)*i + 1)*se
c(c + d*x)**3*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c +
 d*x)*i + 1),x)*tan(c + d*x)**2*d*i + int((sqrt(tan(c + d*x)*i + 1)*sec(c
+ d*x)**3*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x
)*i + 1),x)*d*i))/(a**2*d*(tan(c + d*x)**2 + 1))
```

3.359 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}$$

output
```
1/4*I*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^(
1/2)/a^(3/2)/d+1/2*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(2+\frac{2e^{2i(c+dx)}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}}\right)\sec(c+dx)}{4ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((2 + (2*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])/Sqrt[
1 + E^((2*I)*(c + d*x))])*Sec[c + d*x])/(4*a*d*(-I + Tan[c + d*x])*Sqrt[a
+ I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 3983, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$
$$\Big\downarrow \text{3983}$$
$$\frac{\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}$$
$$\Big\downarrow \text{3970}$$
$$\frac{i\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{2ad}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}$$
$$\Big\downarrow \text{219}$$
$$\frac{i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])
])/(Sqrt[2]*a^(3/2)*d) + ((I/2)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(3
/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(68) = 136$.

Time = 7.80 (sec) , antiderivative size = 343, normalized size of antiderivative = 3.94

method	result
default	$-\frac{\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\,(2\cos(dx+c)-\sec(dx+c))+(-4\cos(dx+c)-2)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\operatorname{arctanh}\left(\frac{(i-\cot(dx+c)+\csc(dx+}{2\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+}}\right.}{4d}$

input

```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/4/d/(-I+tan(d*x+c))/(a*(1+I*tan(d*x+c)))^(1/2)/(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)/a/(cos(d*x+c)+1)*(2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(2*cos(d*x+c)-sec(d*x+c))+(-4*cos(d*x+c)-2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)+arctanh(1/2/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/
2)*(I-cot(d*x+c)+csc(d*x+c))*2^(1/2))*(2*sin(d*x+c)+tan(d*x+c))+2*I*sin(d*
x+c)*((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)-2*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2))+I*arctanh(1/2/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x
+c)^2-1)^(1/2)*(I-cot(d*x+c)+csc(d*x+c))*2^(1/2))*(-2*cos(d*x+c)-1+sec(d*x
+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(64) = 128$.

Time = 0.09 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.83

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(2i\,dx+2i\,c)}\log\left(\frac{\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,ade^{(2i\,dx+2i\,c)}+i\,ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^3d^2}}\right.}{ad}\right.\right.}{}$$

input

```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/4*(I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(2*I*d*x + 2*I*c)*log((sqrt(2)*
sqrt(1/2)*(I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*sqrt(1/(a^3*d^2)) + I)*e^(-I*d*x - I*c)/(a*d)) - I*sqrt(1/2)*a^2*d*s
qrt(1/(a^3*d^2))*e^(2*I*d*x + 2*I*c)*log((sqrt(2)*sqrt(1/2)*(-I*a*d*e^(2*I
*d*x + 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2))
 + I)*e^(-I*d*x - I*c)/(a*d)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
(I*e^(2*I*d*x + 2*I*c) + I))*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec{(dx+c)}}{(i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\int \frac{\sec(dx+c)}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i+\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a}$$

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
int(sec(c + d*x)/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i + sqrt(tan(c + d
*x)*i + 1)),x)/(sqrt(a)*a)
```

3.360 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 157

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{15i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}a^{3/2}d} + \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{15i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d}$$

output
```
15/64*I*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2
^(1/2)/a^(3/2)/d+1/4*I*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(3/2)+5/16*I*cos(d*
x+c)/a/d/(a+I*a*tan(d*x+c))^(1/2)-15/32*I*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1
/2)/a^2/d
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.76

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sec(c+dx)\left(\frac{30e^{2i(c+dx)}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 2(-9+6\cos(2(c+dx))+1\right.}{64ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(Sec[c + d*x]*((30*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x)
)]])/Sqrt[1 + E^((2*I)*(c + d*x))] - 2*(-9 + 6*Cos[2*(c + d*x)] + (10*I)*S
in[2*(c + d*x)])))/(64*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3983, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{5\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{8a} + \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{8a} + \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3983$$

$$\frac{5\left(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{5\left(\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3971$$

$$\frac{5\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{5\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3970$$

$$\frac{5\left(\frac{3\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{5\left(\frac{3\left(\frac{i\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I/4)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (5*(((I/2)*Cos[c +
 d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Se
c[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c
+ d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d))/(4*a)))/(8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(126) = 252$.

Time = 9.35 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.38

method	result
default	$-\frac{\operatorname{arctanh}\left(\frac{(i-\cot(dx+c)+\csc(dx+c))\sqrt{2}}{2\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)(-30\sin(dx+c)-15\tan(dx+c))+i\operatorname{arctanh}\left(\frac{(i-\cot(dx+}{2\sqrt{\cot(dx+c)^2-2\cot(dx}}\right.}{}$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64/d/(cos(d*x+c)+1)/(-tan(d*x+c)+I)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/
a/(a*(1+I*tan(d*x+c)))^(1/2)*(arctanh(1/2/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d
*x+c)+csc(d*x+c)^2-1)^(1/2)*(I-cot(d*x+c)+csc(d*x+c))*2^(1/2))*(-30*sin(d*
x+c)-15*tan(d*x+c))+I*arctanh(1/2/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+cs
c(d*x+c)^2-1)^(1/2)*(I-cot(d*x+c)+csc(d*x+c))*2^(1/2))*(30*cos(d*x+c)+15-1
5*sec(d*x+c))-30*I*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
+2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-30*cos(d*x+c)+15*sec(d*x+c
))+I*sin(d*x+c)*(-40*cos(d*x+c)+20)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(-2
4*cos(d*x+c)^2+36*cos(d*x+c)+30)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(118) = 236$.

Time = 0.09 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.72

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(-15i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(4i\,dx+4i\,c)}\log\left(-\frac{15\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,ade^{(2i\,dx+2i\,c)}+i\,ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}}}\right.}{16\,ad}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/64*(-15*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(4*I*d*x + 4*I*c)*log(-15/
16*(sqrt(2)*sqrt(1/2)*(I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x - I*c)/(a*d)) + 15*I*sq
rt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(4*I*d*x + 4*I*c)*log(-15/16*(sqrt(2)*sq
rt(1/2)*(-I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x - I*c)/(a*d)) + sqrt(2)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*(-8*I*e^(6*I*d*x + 6*I*c) + I*e^(4*I*d*x + 4*I*c) +
11*I*e^(2*I*d*x + 2*I*c) + 2*I))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1821 vs. $2(118) = 236$.

Time = 0.30 (sec) , antiderivative size = 1821, normalized size of antiderivative = 11.60

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/256*(36*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((-I*sqrt(2)*cos(4*d*x + 4*c)
- sqrt(2)*sin(4*d*x + 4*c))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1)) + (sqrt(2)*cos(4*d*x + 4*c) - I*sqrt(2)*sin(4*d*x + 4*c))*sin(3/2
*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 4*(cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)^(1/4)*((7*I*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*sin(4*d*x +
 4*c) + 8*I*sqrt(2))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
) - (7*sqrt(2)*cos(4*d*x + 4*c) - 7*I*sqrt(2)*sin(4*d*x + 4*c) + 8*sqrt(2)
)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 15*(2*
sqrt(2)*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x ...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}}\,dx$$

input
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\int \frac{\cos(dx+c)}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i+\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a}$$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
int(cos(c + d*x)/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i + sqrt(tan(c + d
*x)*i + 1)),x)/(sqrt(a)*a)
```

3.361 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 233

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{105i\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{256\sqrt{2}a^{3/2}d} + \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{128ad\sqrt{a+ia\tan(c+dx)}} + \frac{3i\cos^3(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{105i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{256a^2d} - \frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d}$$

output

```
105/512*I*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))
*2^(1/2)/a^(3/2)/d+1/6*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(3/2)+35/128*I*
cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(1/2)+3/16*I*cos(d*x+c)^3/a/d/(a+I*a*tan
(d*x+c))^(1/2)-105/256*I*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^2/d-7/32*I*
cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)/a^2/d
```

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.62

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sec(c+dx)\left(\frac{630e^{2i(c+dx)}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 2(158\cos(2(c+dx)) + 8\operatorname{co}\right.}{1536ad(-i+\tan(c+dx))\sqrt{}}$$

input
```
Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(Sec[c + d*x]*((630*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x
))]])/Sqrt[1 + E^((2*I)*(c + d*x))] - 2*(158*Cos[2*(c + d*x)] + 8*Cos[4*(c
 + d*x)] + (3*I)*(55*I + 86*Sin[2*(c + d*x)] + 8*Sin[4*(c + d*x)]))))/(153
6*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3983, 3042, 3983, 3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\int \frac{\cos^3(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a} + \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{\sec(c+dx)^3\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\left(\frac{7\int \cos^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{7\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^3}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{3\left(\frac{7\left(\frac{5}{6}a\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{7\left(\frac{5}{6}a\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3971}$$

$$\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx}{4d\sqrt{a+ia\tan(c}}\right.}{4a}$$
$$\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx}{4d\sqrt{a+ia\tan(c}}\right.}{4a}$$
$$\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3970}$$

$$\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\right.}{4a}$$

$$\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{}{4d}\right.}{4a}$$

$$\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I/6)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (3*(((I/4)*Cos[c
 + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (7*(((-1/3*I)*Cos[c + d*x]^3*S
qrt[a + I*a*Tan[c + d*x]])/d + (5*a*(((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a*
Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sq
rt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan
[c + d*x]])/d))/(4*a)))/6))/(8*a)))/(4*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(190) = 380$.

Time = 9.94 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.77

method	result
default	$\frac{(630\cos(dx+c)+315)\sin(dx+c)\operatorname{arctanh}\left(\frac{-i+\cot(dx+c)-\csc(dx+c)}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+i\left(-630\cos(dx+c)^2-315\cos(dx+c)+315\right)\operatorname{arctanh}\left(\frac{-i+\cot}{2\sqrt{}}\right.}{}$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1536/d*((630*cos(d*x+c)+315)*sin(d*x+c)*arctanh(1/2/(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(-I+cot(d*x+c)-csc(d*x+c)))+I*(-630*cos(d*x+c)^2-315*cos(d*
x+c)+315)*arctanh(1/2/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-I+cot(d*x+c)-cs
c(d*x+c)))-630*I*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*2^(1/2)+(-630*cos(d*x+c)^2+315)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)+I*sin(d*x+c)*cos(d*x+c)*(-384*cos(d*x+c)^3-384*cos(d*x+c)^2-840*cos(d
*x+c)+420)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+cos(d*x+c)*(-128*cos(d*x+c)^
4-128*cos(d*x+c)^3-504*cos(d*x+c)^2+756*cos(d*x+c)+630)*(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2))/(a*(1+I*tan(d*x+c)))^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)/(-I*cos(d*x+c)^2+cos(d*x+c)*sin(d*x+c)-I*cos(d*x+c)+sin(d*x+c))/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.25

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(-315i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(6i\,dx+6i\,c)}\log\left(-\frac{105\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,ade^{(2i\,dx+2i\,c)}+i\,ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2}}}\right.}{128\,ad}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/1536*(-315*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(6*I*d*x + 6*I*c)*log(-
105/128*(sqrt(2)*sqrt(1/2)*(I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x - I*c)/(a*d)) + 31
5*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(6*I*d*x + 6*I*c)*log(-105/128*(sq
rt(2)*sqrt(1/2)*(-I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x - I*c)/(a*d)) + sqrt(2)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*(-16*I*e^(10*I*d*x + 10*I*c) - 224*I*e^(8*I*
d*x + 8*I*c) - 43*I*e^(6*I*d*x + 6*I*c) + 215*I*e^(4*I*d*x + 4*I*c) + 58*I
*e^(2*I*d*x + 2*I*c) + 8*I))*e^(-6*I*d*x - 6*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos^3(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2632 vs. $2(178) = 356$.

Time = 0.41 (sec) , antiderivative size = 2632, normalized size of antiderivative = 11.30

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/6144*(8*(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1
/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(
6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(3/4)*((-4*I*sqrt(2)*cos(6*d*x + 6*c
) - 45*I*sqrt(2)*cos(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - 4*
sqrt(2)*sin(6*d*x + 6*c) - 45*sqrt(2)*sin(2/3*arctan2(sin(6*d*x + 6*c), co
s(6*d*x + 6*c))) + 8*I*sqrt(2))*cos(3/2*arctan2(sin(1/3*arctan2(sin(6*d*x
+ 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x +
6*c))) + 1)) + (4*sqrt(2)*cos(6*d*x + 6*c) + 45*sqrt(2)*cos(2/3*arctan2(si
n(6*d*x + 6*c), cos(6*d*x + 6*c))) - 4*I*sqrt(2)*sin(6*d*x + 6*c) - 45*I*s
qrt(2)*sin(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - 8*sqrt(2))*s
in(3/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1
/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)))*sqrt(a) + 12*(cos(1
/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*
d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos
(6*d*x + 6*c))) + 1)^(1/4)*(((-I*sqrt(2)*cos(6*d*x + 6*c) - sqrt(2)*sin(6*
d*x + 6*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + (-I*s
qrt(2)*cos(6*d*x + 6*c) - sqrt(2)*sin(6*d*x + 6*c))*sin(1/3*arctan2(sin(6*
d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*(-I*sqrt(2)*cos(6*d*x + 6*c) - sqrt(2
)*sin(6*d*x + 6*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) -
 I*sqrt(2)*cos(6*d*x + 6*c) - sqrt(2)*sin(6*d*x + 6*c))*cos(5/2*arctan2...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\int \frac{\cos(dx+c)^3}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i+\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a}$$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
int(cos(c + d*x)**3/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i + sqrt(tan(c
+ d*x)*i + 1)),x)/(sqrt(a)*a)
```

3.362 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 146

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{32i(a+ia\tan(c+dx))^{5/2}}{5a^5d} + \frac{64i(a+ia\tan(c+dx))^{7/2}}{7a^6d} - \frac{16i(a+ia\tan(c+dx))^{9/2}}{3a^7d} + \frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^8d} - \frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^9d}$$

output
```
-32/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^5/d+64/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^6
/d-16/3*I*(a+I*a*tan(d*x+c))^(9/2)/a^7/d+16/11*I*(a+I*a*tan(d*x+c))^(11/2)
/a^8/d-2/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^9/d
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}(9683i+16700\tan(c+dx)-14}{15015a^3d}$$

input
```
Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(2*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]*(9683*I + 16700*Tan[c
+ d*x] - (14210*I)*Tan[c + d*x]^2 - 6300*Tan[c + d*x]^3 + (1155*I)*Tan[c +
 d*x]^4))/(15015*a^3*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{10}}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)^{3/2}d(ia\tan(c+dx))}{a^9d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left((i\tan(c+dx)a+a)^{11/2}-8a(i\tan(c+dx)a+a)^{9/2}+24a^2(i\tan(c+dx)a+a)^{7/2}-32a^3(i\tan(c+dx)a\right.}{a^9d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{32}{5}a^4(a+ia\tan(c+dx))^{5/2}-\frac{64}{7}a^3(a+ia\tan(c+dx))^{7/2}+\frac{16}{3}a^2(a+ia\tan(c+dx))^{9/2}+\frac{2}{13}(a+ia\tan(c+\right.}{a^9d}$$

input
```
Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*((32*a^4*(a + I*a*Tan[c + d*x])^(5/2))/5 - (64*a^3*(a + I*a*Tan[c +
d*x])^(7/2))/7 + (16*a^2*(a + I*a*Tan[c + d*x])^(9/2))/3 - (16*a*(a + I*a*
Tan[c + d*x])^(11/2))/11 + (2*(a + I*a*Tan[c + d*x])^(13/2))/13))/(a^9*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}+\frac{8a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{8a^2(a+ia\tan(dx+c))^{\frac{9}{2}}}{3}+\frac{32a^3(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{16a^4(a+ia\tan(dx+c}{5}\right.}{d\,a^9}$
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}+\frac{8a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{8a^2(a+ia\tan(dx+c))^{\frac{9}{2}}}{3}+\frac{32a^3(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{16a^4(a+ia\tan(dx+c}{5}\right.}{d\,a^9}$

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^9*(-1/13*(a+I*a*tan(d*x+c))^(13/2)+8/11*a*(a+I*a*tan(d*x+c))^(11/2
)-8/3*a^2*(a+I*a*tan(d*x+c))^(9/2)+32/7*a^3*(a+I*a*tan(d*x+c))^(7/2)-16/5*
a^4*(a+I*a*tan(d*x+c))^(5/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.20

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{128\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(128i\,e^{(13i\,dx+13i\,c)}+832i\,e^{(11i\,dx+11i\,c)}+2288i\,e^{(9i\,dx+9i\,c)}+3432i\,e^{(7i\,dx+7i\,c)}+30\right.}{15015\left(a^3de^{(12i\,dx+12i\,c)}+6\,a^3de^{(10i\,dx+10i\,c)}+15\,a^3de^{(8i\,dx+8i\,c)}+20\,a^3de^{(6i\,dx+6i\,c)}+15\,a^3de^{(4i\,dx+4i\,c)}+6\,a\right.}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-128/15015*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(128*I*e^(13*I*d*x +
13*I*c) + 832*I*e^(11*I*d*x + 11*I*c) + 2288*I*e^(9*I*d*x + 9*I*c) + 3432*
I*e^(7*I*d*x + 7*I*c) + 3003*I*e^(5*I*d*x + 5*I*c))/(a^3*d*e^(12*I*d*x + 1
2*I*c) + 6*a^3*d*e^(10*I*d*x + 10*I*c) + 15*a^3*d*e^(8*I*d*x + 8*I*c) + 20
*a^3*d*e^(6*I*d*x + 6*I*c) + 15*a^3*d*e^(4*I*d*x + 4*I*c) + 6*a^3*d*e^(2*I
*d*x + 2*I*c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.64

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$
$$-\frac{2i\left(1155\,(i\,a\tan\left(dx+c\right)+a)^{\frac{13}{2}}-10920\,(i\,a\tan\left(dx+c\right)+a)^{\frac{11}{2}}a+40040\,(i\,a\tan\left(dx+c\right)+a)^{\frac{9}{2}}a^2-68\right.}{15015\,a^9d}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-2/15015*I*(1155*(I*a*tan(d*x + c) + a)^(13/2) - 10920*(I*a*tan(d*x + c) +
 a)^(11/2)*a + 40040*(I*a*tan(d*x + c) + a)^(9/2)*a^2 - 68640*(I*a*tan(d*x
 + c) + a)^(7/2)*a^3 + 48048*(I*a*tan(d*x + c) + a)^(5/2)*a^4)/(a^9*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.97

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,16384i}{15015\,a^3\,d} - \frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,8192i}{15015\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)} - \frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,2048i}{5005\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^2} - \frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,1024i}{3003\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^3} - \frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,128i}{429\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^4} + \frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,1792i}{143\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^5} - \frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,128i}{13\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^6}$$

input

```
int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output

```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
1792i)/(143*a^3*d*(exp(c*2i + d*x*2i) + 1)^5) - ((a - (a*(exp(c*2i + d*x*2
i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*8192i)/(15015*a^3*d*(exp(c
*2i + d*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
+ d*x*2i) + 1))^(1/2)*2048i)/(5005*a^3*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a
 - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*102
4i)/(3003*a^3*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(429*a^3*d*(exp(c*2i +
 d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d
*x*2i) + 1))^(1/2)*16384i)/(15015*a^3*d) - ((a - (a*(exp(c*2i + d*x*2i)*1i
 - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(13*a^3*d*(exp(c*2i + d*x
*2i) + 1)^6)
```

Reduce [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\int \frac{\sec(dx+c)^{10}}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - int(sec(c + d*x)**10/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqr
t(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x))/(sqrt
(a)*a**2)
```

3.363 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{16i(a+ia\tan(c+dx))^{3/2}}{3a^4d}+\frac{24i(a+ia\tan(c+dx))^{5/2}}{5a^5d}-\frac{12i(a+ia\tan(c+dx))^{7/2}}{7a^6d}+\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^7d}$$

output
```
-16/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^4/d+24/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^5
/d-12/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^6/d+2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^
7/d
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2(1+i\tan(c+dx))\sqrt{a+ia\tan(c+dx)}(-319i-321\tan(c+dx)+165i}{315a^3d}$$

input
```
Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(2*(1 + I*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]*(-319*I - 321*Tan[c + d
*x] + (165*I)*Tan[c + d*x]^2 + 35*Tan[c + d*x]^3))/(315*a^3*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^8}{(a+ia\tan(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int (a-ia\tan(c+dx))^3\sqrt{i\tan(c+dx)a+a}d(ia\tan(c+dx))}{a^7d}$$
$$\Big\downarrow \text{53}$$
$$-\frac{i\int \left(-(i\tan(c+dx)a+a)^{7/2}+6a(i\tan(c+dx)a+a)^{5/2}-12a^2(i\tan(c+dx)a+a)^{3/2}+8a^3\sqrt{i\tan(c+dx)a}\right.}{a^7d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{i(\frac{16}{3}a^3(a+ia\tan(c+dx))^{3/2}-\frac{24}{5}a^2(a+ia\tan(c+dx))^{5/2}-\frac{2}{9}(a+ia\tan(c+dx))^{9/2}+\frac{12}{7}a(a+ia\tan(c+a}{a^7d}$$

input
```
Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*((16*a^3*(a + I*a*Tan[c + d*x])^(3/2))/3 - (24*a^2*(a + I*a*Tan[c +
d*x])^(5/2))/5 + (12*a*(a + I*a*Tan[c + d*x])^(7/2))/7 - (2*(a + I*a*Tan[c
 + d*x])^(9/2))/9))/(a^7*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{6a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{12a^{2}(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{8a^{3}(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right)}{d\,a^{7}}$	82
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{6a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{12a^{2}(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{8a^{3}(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right)}{d\,a^{7}}$	82

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^7*(1/9*(a+I*a*tan(d*x+c))^(9/2)-6/7*a*(a+I*a*tan(d*x+c))^(7/2)+12/
5*a^2*(a+I*a*tan(d*x+c))^(5/2)-8/3*a^3*(a+I*a*tan(d*x+c))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.15

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{32\,\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(16i\,e^{(9i\,dx+9i\,c)}+72i\,e^{(7i\,dx+7i\,c)}+126i\,e^{(5i\,dx+5i\,c)}+105i\,e^{(3i\,dx+3i\,c)}\right)}{315\left(a^3de^{(8i\,dx+8i\,c)}+4\,a^3de^{(6i\,dx+6i\,c)}+6\,a^3de^{(4i\,dx+4i\,c)}+4\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-32/315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(9*I*d*x + 9*I*c
) + 72*I*e^(7*I*d*x + 7*I*c) + 126*I*e^(5*I*d*x + 5*I*c) + 105*I*e^(3*I*d*
x + 3*I*c))/(a^3*d*e^(8*I*d*x + 8*I*c) + 4*a^3*d*e^(6*I*d*x + 6*I*c) + 6*a
^3*d*e^(4*I*d*x + 4*I*c) + 4*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^8(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**8/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i\left(35\,(i\,a\tan{(dx+c)}+a)^{\frac{9}{2}} - 270\,(i\,a\tan{(dx+c)}+a)^{\frac{7}{2}}a + 756\,(i\,a\tan\right.}{315\,a^7d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2/315*I*(35*(I*a*tan(d*x + c) + a)^(9/2) - 270*(I*a*tan(d*x + c) + a)^(7/2
)*a + 756*(I*a*tan(d*x + c) + a)^(5/2)*a^2 - 840*(I*a*tan(d*x + c) + a)^(3
/2)*a^3)/(a^7*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.62

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{315\,a^3\,d} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{315\,a^3\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{105\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{63\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} + \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}$$

input
```
int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
32i)/(9*a^3*d*(exp(c*2i + d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)*1
i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(315*a^3*d*(exp(c*2i + d
*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2
i) + 1))^(1/2)*64i)/(105*a^3*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp
(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(63*a^3*
d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/
(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(315*a^3*d)
```

Reduce [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\int \frac{\sec(dx+c)^8}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - int(sec(c + d*x)**8/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt
(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x))/(sqrt(
a)*a**2)
```

3.364 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{8i\sqrt{a+ia\tan(c+dx)}}{a^3d} + \frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^4d} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^5d}$$

output
```
-8*I*(a+I*a*tan(d*x+c))^(1/2)/a^3/d+8/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^4/d-2
/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^5/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i\sqrt{a+ia\tan(c+dx)}(-43+14i\tan(c+dx)+3\tan^2(c+dx))}{15a^3d}$$

input
```
Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((2*I)/15)*Sqrt[a + I*a*Tan[c + d*x]]*(-43 + (14*I)*Tan[c + d*x] + 3*Tan[
c + d*x]^2))/(a^3*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{(a-ia\tan(c+dx))^2}{\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(\frac{4a^2}{\sqrt{i\tan(c+dx)a+a}}-4\sqrt{i\tan(c+dx)a+a}a+(i\tan(c+dx)a+a)^{3/2}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(8a^2\sqrt{a+ia\tan(c+dx)}+\frac{2}{5}(a+ia\tan(c+dx))^{5/2}-\frac{8}{3}a(a+ia\tan(c+dx))^{3/2}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*(8*a^2*Sqrt[a + I*a*Tan[c + d*x]] - (8*a*(a + I*a*Tan[c + d*x])^(3/2
))/3 + (2*(a + I*a*Tan[c + d*x])^(5/2))/5))/(a^5*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{4a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-4a^2\sqrt{a+ia\tan(dx+c)}\right)}{d\,a^5}$	63
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{4a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-4a^2\sqrt{a+ia\tan(dx+c)}\right)}{d\,a^5}$	63

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^5*(-1/5*(a+I*a*tan(d*x+c))^(5/2)+4/3*a*(a+I*a*tan(d*x+c))^(3/2)-4*
a^2*(a+I*a*tan(d*x+c))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.08

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{8\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(8i\,e^{(5i\,dx+5i\,c)}+20i\,e^{(3i\,dx+3i\,c)}+15i\,e^{(i\,dx+i\,c)}\right)}{15\left(a^3de^{(4i\,dx+4i\,c)}+2\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-8/15*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(8*I*e^(5*I*d*x + 5*I*c) +
 20*I*e^(3*I*d*x + 3*I*c) + 15*I*e^(I*d*x + I*c))/(a^3*d*e^(4*I*d*x + 4*I*
c) + 2*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^6{(c+dx)}}{(ia\,(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**6/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2i\left(3\,(i\,a\tan{(dx+c)}+a)^{\frac{5}{2}}-20\,(i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}a+60\,\sqrt{i\,a\tan{(dx+c)}+a}a^2\right)}{15\,a^5d}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-2/15*I*(3*(I*a*tan(d*x + c) + a)^(5/2) - 20*(I*a*tan(d*x + c) + a)^(3/2)*
a + 60*sqrt(I*a*tan(d*x + c) + a)*a^2)/(a^5*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.80

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{4\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,321\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,132\mathrm{i}+\cos\left(6\,c+6\,d\,x\right)\,23\mathrm{i}+35\right.}{15\,a^3\,d\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(\right.\right.}$$

input
```
int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
-(4*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) +
1))^(1/2)*(cos(2*c + 2*d*x)*321i + cos(4*c + 4*d*x)*132i + cos(6*c + 6*d*x
)*23i + 35*sin(2*c + 2*d*x) + 28*sin(4*c + 4*d*x) + 7*sin(6*c + 6*d*x) + 2
12i))/(15*a^3*d*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*
x) + 10))
```

Reduce [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\int \frac{\sec(dx+c)^6}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - int(sec(c + d*x)**6/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt
(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x))/(sqrt(
a)*a**2)
```

3.365 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 55

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{4i}{a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{2i\sqrt{a+ia\tan(c+dx)}}{a^3d}$$

output
```
4*I/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+2*I*(a+I*a*tan(d*x+c))^(1/2)/a^3/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{6i-2\tan(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(6*I - 2*Tan[c + d*x])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^4}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{a-ia\tan(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(\frac{2a}{(i\tan(c+dx)a+a)^{3/2}}-\frac{1}{\sqrt{i\tan(c+dx)a+a}}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{4a}{\sqrt{a+ia\tan(c+dx)}}-2\sqrt{a+ia\tan(c+dx)}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*((-4*a)/Sqrt[a + I*a*Tan[c + d*x]] - 2*Sqrt[a + I*a*Tan[c + d*x]]))/
(a^3*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{2i\left(\sqrt{a+ia\tan(dx+c)}+\frac{2a}{\sqrt{a+ia\tan(dx+c)}}\right)}{d\,a^3}$	42
default	$\frac{2i\left(\sqrt{a+ia\tan(dx+c)}+\frac{2a}{\sqrt{a+ia\tan(dx+c)}}\right)}{d\,a^3}$	42

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^3*((a+I*a*tan(d*x+c))^(1/2)+2*a/(a+I*a*tan(d*x+c))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-2i\,e^{(2i\,dx+2i\,c)}-i\right)e^{(-i\,dx-i\,c)}}{a^3d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(2*I*d*x + 2*I*c) - I
)*e^(-I*d*x - I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^4(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i\left(\frac{\sqrt{i\,a\tan(dx+c)+a}}{a^2}+\frac{2}{\sqrt{i\,a\tan(dx+c)+a}a}\right)}{ad}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2*I*(sqrt(I*a*tan(d*x + c) + a)/a^2 + 2/(sqrt(I*a*tan(d*x + c) + a)*a))/(a
*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2\left(\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+2\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1}{\cos(2\,c+2\,d\,x)+1}}}{a^3\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
(2*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 2i)*((a*(cos(2*c + 2*d*x) + s
in(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))/(a^3*d)
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\int \frac{\sec(dx+c)^4}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - int(sec(c + d*x)**4/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt
(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x))/(sqrt(
a)*a**2)
```

3.366 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i}{3ad(a+ia\tan(c+dx))^{3/2}}$$

output
```
2/3*I/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i}{3ad(a+ia\tan(c+dx))^{3/2}}$$

input
```
Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((2*I)/3)/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = $\{3042, 3968, 17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sec(c+dx)^2}{(a+ia\tan(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{ 3968}$$
$$-\frac{i\int \frac{1}{(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{ad}$$
$$\Big\downarrow \text{ 17}$$
$$\frac{2i}{3ad(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((2*I)/3)/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2i}{3ad(a+ia\tan(dx+c))^{\frac{3}{2}}}$	24
default	$\frac{2i}{3ad(a+ia\tan(dx+c))^{\frac{3}{2}}}$	24

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(21) = 42.

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.10

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(4i\,dx+4i\,c)}+2i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{6\,a^3d}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(4*I*d*x + 4*I*c) + 2*I
*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i}{3\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2/3*I/((I*a*tan(d*x + c) + a)^(3/2)*a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2\mathrm{i}}{3\,a\,d\,(a+a\tan(c+d\,x)\,\,1\mathrm{i})^{3/2}}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
2i/(3*a*d*(a + a*tan(c + d*x)*1i)^(3/2))
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\int \frac{\sec(dx+c)^2}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - int(sec(c + d*x)**2/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt
(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x))/(sqrt(
a)*a**2)
```

3.367 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 208

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= -\frac{9i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d} \\ &+ \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} + \frac{9i}{40d(a+ia\tan(c+dx))^{5/2}} \\ &+ \frac{3i}{16ad(a+ia\tan(c+dx))^{3/2}} + \frac{9i}{32a^2d\sqrt{a+ia\tan(c+dx)}} \\ &- \frac{ia^3}{2d(a+ia\tan(c+dx))^{7/2}\left(a^2-ia^2\tan(c+dx)\right)}\end{aligned}$$

output
```
-9/64*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/a^(5
/2)/d+9/28*I*a/d/(a+I*a*tan(d*x+c))^(7/2)+9/40*I/d/(a+I*a*tan(d*x+c))^(5/2
)+3/16*I/a/d/(a+I*a*tan(d*x+c))^(3/2)+9/32*I/a^2/d/(a+I*a*tan(d*x+c))^(1/2
)-1/2*I*a^3/d/(a+I*a*tan(d*x+c))^(7/2)/(a^2-I*a^2*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.25

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{ia\,\text{Hypergeometric2F1}\left(-\frac{7}{2},2,-\frac{5}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{14d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/14)*a*Hypergeometric2F1[-7/2, 2, -5/2, (1 + I*Tan[c + d*x])/2])/(d*(a
+ I*a*Tan[c + d*x])^(7/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 3968, 52, 61, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^3\left(\frac{9\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\right)}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^3\left(\frac{9\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\right)}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^3\left(\frac{9\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+i}\right.}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^3\left(\frac{9\left(\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{}{2a}\right.}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^3\left(\frac{9\left(\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c\cdots}{4a}\cdots}{d}$$

$\Big\downarrow$ 73

$$-\frac{ia^3\left(\frac{9\left(\frac{\frac{\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}\cdots}{d}$$

$\Big\downarrow$ 219

$$-\frac{ia^3\left(\frac{9\left(\frac{\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{\cdots}{2a(a-ia\,\mathrm{ta}\cdots}\cdots}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*a^3*(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(7/2)) + (
9*(-1/7*1/(a*(a + I*a*Tan[c + d*x])^(7/2)) + (-1/5*1/(a*(a + I*a*Tan[c + d
*x])^(5/2)) + (-1/3*1/(a*(a + I*a*Tan[c + d*x])^(3/2)) + ((I*ArcTan[(Sqrt[
a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[a + I*a*Tan[c + d
*x]]))/(2*a))/(2*a))/(2*a)))/(4*a)))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(164) = 328$.

Time = 10.18 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.16

method	result
default	$\frac{i\left(1260\cos(dx+c)^2+630\cos(dx+c)-315\right)\sin(dx+c)\operatorname{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)+i}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+\left(1260\cos(dx+c)^3+630\cos(dx+c)^2-945\right.}{}$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2240/d*(I*(1260*cos(d*x+c)^2+630*cos(d*x+c)-315)*sin(d*x+c)*arctanh(1/2/
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+I))+(1260*cos(d*
x+c)^3+630*cos(d*x+c)^2-945*cos(d*x+c)-315)*arctanh(1/2/(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+I))+I*(630*cos(d*x+c)^2-315)*2^(1/
2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-630*sin(d*x+c)*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+sin(d*x+c)*cos(d*x+c)*(720*cos(d*x+c)
^3+720*cos(d*x+c)^2-1680*cos(d*x+c)-420)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)+I*cos(d*x+c)*(-400*cos(d*x+c)^4-400*cos(d*x+c)^3+2184*cos(d*x+c)^2+924*c
os(d*x+c)-630)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))/(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)/(I*sin(d*x+c)*cos(d*x+c)*(2*cos(d*x+c)+2)+2*cos(d*x+c)^3+2*cos
(d*x+c)^2-cos(d*x+c)-1)/(a*(1+I*tan(d*x+c)))^(1/2)/a^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(151) = 302.

Time = 0.09 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.47

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(-315i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(7i\,dx+7i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2i\,dx+2i\,c)}+a^3d\right)\sqrt{\frac{}{e^{(2i}}}\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/2240*(-315*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(7*I*d*x + 7*I*c)*log(4
*(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) +
 315*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(7*I*d*x + 7*I*c)*log(-4*(sqrt(
2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(1/(a^5*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-35*I*e^(10*I*d*x + 10*I*c) + 353*I*e
^(8*I*d*x + 8*I*c) + 544*I*e^(6*I*d*x + 6*I*c) + 214*I*e^(4*I*d*x + 4*I*c)
 + 68*I*e^(2*I*d*x + 2*I*c) + 10*I))*e^(-7*I*d*x - 7*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos^2{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(cos(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.84

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\left(\frac{4\left(315\,(i\,a\tan(dx+c)+a)^4-420\,(i\,a\tan(dx+c)+a)^3a-168\,(i\,a\tan(dx+c)+a)^2a^2-144\,(i\,a\tan(dx\right.}{(i\,a\tan(dx+c)+a)^{\frac{9}{2}}a-2\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a^2}}{4480\,ad}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/4480*I*(4*(315*(I*a*tan(d*x + c) + a)^4 - 420*(I*a*tan(d*x + c) + a)^3*a
 - 168*(I*a*tan(d*x + c) + a)^2*a^2 - 144*(I*a*tan(d*x + c) + a)*a^3 - 160
*a^4)/((I*a*tan(d*x + c) + a)^(9/2)*a - 2*(I*a*tan(d*x + c) + a)^(7/2)*a^2
) + 315*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(
2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(3/2))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\int \frac{\cos(dx+c)^2}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - int(cos(c + d*x)**2/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt
(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x))/(sqrt(
a)*a**2)
```

3.368 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 281

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= -\frac{143i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}a^{5/2}d} \\ &+ \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} \\ &+ \frac{143ia}{448d(a+ia\tan(c+dx))^{7/2}} + \frac{143i}{640d(a+ia\tan(c+dx))^{5/2}} \\ &+ \frac{143i}{768ad(a+ia\tan(c+dx))^{3/2}} + \frac{143i}{512a^2d\sqrt{a+ia\tan(c+dx)}} \\ &- \frac{13ia^5}{16d(a+ia\tan(c+dx))^{9/2}\,(a^3-ia^3\tan(c+dx))}\end{aligned}$$

output
```
-143/1024*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/
a^(5/2)/d+143/288*I*a^2/d/(a+I*a*tan(d*x+c))^(9/2)-1/4*I*a^4/d/(a-I*a*tan(
d*x+c))^2/(a+I*a*tan(d*x+c))^(9/2)+143/448*I*a/d/(a+I*a*tan(d*x+c))^(7/2)+
143/640*I/d/(a+I*a*tan(d*x+c))^(5/2)+143/768*I/a/d/(a+I*a*tan(d*x+c))^(3/2
)+143/512*I/a^2/d/(a+I*a*tan(d*x+c))^(1/2)-13/16*I*a^5/d/(a+I*a*tan(d*x+c)
)^(9/2)/(a^3-I*a^3*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.19

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{ia^2\,\text{Hypergeometric2F1}\left(-\frac{9}{2},3,-\frac{7}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{36d(a+ia\tan(c+dx))^{9/2}}$$

input
```
Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/36)*a^2*Hypergeometric2F1[-9/2, 3, -7/2, (1 + I*Tan[c + d*x])/2])/(d*(
a + I*a*Tan[c + d*x])^(9/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {3042, 3968, 52, 52, 61, 61, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^4(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{13\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}}\right)}{d}$$

$$\Big\downarrow 52$$

$$-\frac{ia^5\left(\frac{13\left(\frac{11\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\,\mathrm{ta}}\right.}{d}$$

$$\Big\downarrow 61$$

$$-\frac{ia^5\left(\frac{13\left(\frac{11\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}\right)}{8a}+\frac{}{4a(}\right.}{d}$$

$$\Big\downarrow 61$$

$$-\frac{ia^5\left(\frac{13\left(\frac{11\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia}\right.}{8a}\right.}{d}$$

$$\Big\downarrow 61$$

$$-\frac{ia^5\left(\frac{13\left(\frac{11\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\cdots\right)}{8a}\cdots\right)}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\cdots\right)}{4a}\cdots\right)}{8a}\cdots\right)}{d}\cdots$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{\;}{7a(a+ia}}{2a}\right)}{4a}\right)}{8a}\right)}{d}$$

$\downarrow$ 73

$$-\frac{ia^5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/}}\right.}{4a}\right.}{8a}\right.}{d}$$

$\downarrow$ 219

$$-\frac{ia^5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{\dots}{9a(a+ia\tan \dots}\dots\right)}{4a}\dots\right)}{8a}\dots\right)}{d}\dots$$

input
```
Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*a^5*(1/(4*a*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(9/2)) +
 (13*(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(9/2)) + (11*(-
1/9*1/(a*(a + I*a*Tan[c + d*x])^(9/2)) + (-1/7*1/(a*(a + I*a*Tan[c + d*x])
^(7/2)) + (-1/5*1/(a*(a + I*a*Tan[c + d*x])^(5/2)) + (-1/3*1/(a*(a + I*a*T
an[c + d*x])^(3/2)) + ((I*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]
*a^(3/2)) - 1/(a*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a))/(2*a))/(2*a))/(2*a)))
/(4*a)))/(8*a)))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(224) = 448$.

Time = 9.48 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.74

method	result
default	$-\frac{i\left(180180\cos(dx+c)^2+90090\cos(dx+c)-45045\right)\sin(dx+c)\operatorname{arctanh}\left(\frac{-\cot(dx+c)+\csc(dx+c)-i}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+\left(180180\cos(dx+c)^3+90090\,c\right.}{}$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/322560/d*(I*(180180*cos(d*x+c)^2+90090*cos(d*x+c)-45045)*sin(d*x+c)*arc
tanh(1/2*(-cot(d*x+c)+csc(d*x+c)-I)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+(1
80180*cos(d*x+c)^3+90090*cos(d*x+c)^2-135135*cos(d*x+c)-45045)*arctanh(1/2
*(-cot(d*x+c)+csc(d*x+c)-I)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+I*(-90090*
cos(d*x+c)^2+45045)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+90090*sin
(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+sin(d*x+c)
*cos(d*x+c)*(-58240*cos(d*x+c)^5-58240*cos(d*x+c)^4-102960*cos(d*x+c)^3-10
2960*cos(d*x+c)^2+240240*cos(d*x+c)+60060)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)+I*cos(d*x+c)*(22400*cos(d*x+c)^6+22400*cos(d*x+c)^5+57200*cos(d*x+c)^4
+57200*cos(d*x+c)^3-312312*cos(d*x+c)^2-132132*cos(d*x+c)+90090)*(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(I*sin(d*x+c
)*cos(d*x+c)*(2*cos(d*x+c)+2)+2*cos(d*x+c)^3+2*cos(d*x+c)^2-cos(d*x+c)-1)/
(a*(1+I*tan(d*x+c)))^(1/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.16

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(-45045i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(9i\,dx+9i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2i\,dx+2i\,c)}+a^3d\right)\sqrt{\cdot}\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/322560*(-45045*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(9*I*d*x + 9*I*c)*l
og(4*(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c
)) + 45045*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(9*I*d*x + 9*I*c)*log(-4*
(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) +
sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-630*I*e^(14*I*d*x + 14*I*c) -
8505*I*e^(12*I*d*x + 12*I*c) + 42709*I*e^(10*I*d*x + 10*I*c) + 69392*I*e^(
8*I*d*x + 8*I*c) + 26752*I*e^(6*I*d*x + 6*I*c) + 10144*I*e^(4*I*d*x + 4*I*
c) + 2480*I*e^(2*I*d*x + 2*I*c) + 280*I))*e^(-9*I*d*x - 9*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos^4(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(cos(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.81

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\left(\frac{4\left(45045\,(i\,a\tan(dx+c)+a)^6-150150\,(i\,a\tan(dx+c)+a)^5a+96096\,(i\,a\tan(dx+c)+a)^4a^2+27456\,(\right.}{(i\,a\tan(dx+c)+a)^{\frac{13}{2}}a-4\,(i\,a\tan(dx+c)}\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/645120*I*(4*(45045*(I*a*tan(d*x + c) + a)^6 - 150150*(I*a*tan(d*x + c) +
 a)^5*a + 96096*(I*a*tan(d*x + c) + a)^4*a^2 + 27456*(I*a*tan(d*x + c) + a
)^3*a^3 + 18304*(I*a*tan(d*x + c) + a)^2*a^4 + 16640*(I*a*tan(d*x + c) + a
)*a^5 + 17920*a^6)/((I*a*tan(d*x + c) + a)^(13/2)*a - 4*(I*a*tan(d*x + c)
+ a)^(11/2)*a^2 + 4*(I*a*tan(d*x + c) + a)^(9/2)*a^3) + 45045*sqrt(2)*log(
-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*
a*tan(d*x + c) + a)))/a^(3/2))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^4}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\int \frac{\cos(dx+c)^4}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - int(cos(c + d*x)**4/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt
(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x))/(sqrt(
a)*a**2)
```

3.369 $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\begin{aligned}\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= \frac{256ia^4\sec^{13}(c+dx)}{20995d(a+ia\tan(c+dx))^{13/2}}\\ &+\frac{64ia^3\sec^{13}(c+dx)}{1615d(a+ia\tan(c+dx))^{11/2}}\\ &+\frac{24ia^2\sec^{13}(c+dx)}{323d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}\end{aligned}$$

output
```
256/20995*I*a^4*sec(d*x+c)^13/d/(a+I*a*tan(d*x+c))^(13/2)+64/1615*I*a^3*se
c(d*x+c)^13/d/(a+I*a*tan(d*x+c))^(11/2)+24/323*I*a^2*sec(d*x+c)^13/d/(a+I*
a*tan(d*x+c))^(9/2)+2/19*I*a*sec(d*x+c)^13/d/(a+I*a*tan(d*x+c))^(7/2)
```

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.76

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sec^{12}(c+dx)(798\cos(c+dx)+1631\cos(3(c+dx))+13i(38\sin(c+dx)\cdots}{20995a^2d(-i+\tan(c+dx))^2\sqrt{\cdots}}$$

input
```
Integrate[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(Sec[c + d*x]^12*(798*Cos[c + d*x] + 1631*Cos[3*(c + d*x)] + (13*I)*(38*Si
n[c + d*x] + 123*Sin[3*(c + d*x)]))*((-2*I)*Cos[4*(c + d*x)] - 2*Sin[4*(c
+ d*x)]))/(20995*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3975, 3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{13}}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{19}a\int \frac{\sec^{13}(c+dx)}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{19}a\int \frac{\sec(c+dx)^{13}}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{19}a\left(\frac{8}{17}a\int \frac{\sec^{13}(c+dx)}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{19}a\left(\frac{8}{17}a\int \frac{\sec(c+dx)^{13}}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{19}a\left(\frac{8}{17}a\left(\frac{4}{15}a\int\frac{\sec^{13}(c+dx)}{(i\tan(c+dx)a+a)^{11/2}}dx+\frac{2ia\sec^{13}(c+dx)}{15d(a+ia\tan(c+dx))^{11/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{19}a\left(\frac{8}{17}a\left(\frac{4}{15}a\int\frac{\sec(c+dx)^{13}}{(i\tan(c+dx)a+a)^{11/2}}dx+\frac{2ia\sec^{13}(c+dx)}{15d(a+ia\tan(c+dx))^{11/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{12}{19}a\left(\frac{8}{17}a\left(\frac{8ia^2\sec^{13}(c+dx)}{195d(a+ia\tan(c+dx))^{13/2}}+\frac{2ia\sec^{13}(c+dx)}{15d(a+ia\tan(c+dx))^{11/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Int[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((2*I)/19)*a*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (12*a*((
((2*I)/17)*a*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (8*a*((((
8*I)/195)*a^2*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(13/2)) + (((2*I)
/15)*a*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(11/2))))/17))/19
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 25.75 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

method	result
default	$\frac{\frac{2\tan(dx+c)\sec(dx+c)^9\left(1024\cos(dx+c)^6+1152\cos(dx+c)^4+1144\cos(dx+c)^2+1105\right)}{20995}+\frac{2i\left(1024\sec(dx+c)^3+640\sec(dx+c)^5+440\sec(dx+c)^7+32\right.}{20995}}{d\left(2i\sin(dx+c)\cos(dx+c)+2\cos(dx+c)^2-1\right)\sqrt{a(1+i\tan(dx+c))}\,a^2}$

input
```
int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/20995/d/(2*I*sin(d*x+c)*cos(d*x+c)+2*cos(d*x+c)^2-1)/(a*(1+I*tan(d*x+c))
)^(1/2)/a^2*(tan(d*x+c)*sec(d*x+c)^9*(1024*cos(d*x+c)^6+1152*cos(d*x+c)^4+
1144*cos(d*x+c)^2+1105)+I*(1024*sec(d*x+c)^3+640*sec(d*x+c)^5+440*sec(d*x+
c)^7+325*sec(d*x+c)^9))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.35

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{1024\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-1615i\,e^{(6i\,dx+6i\,c)}-646i\,e^{(4i\right.}}{20995\left(a^3de^{(18i\,dx+18i\,c)}+9\,a^3de^{(16i\,dx+16i\,c)}+36\,a^3de^{(14i\,dx+14i\,c)}+84\,a^3de^{(12i\,dx+12i\,c)}+126\,a^3de^{(10i\,dx+10i\,c}\right.}$$

input
```
integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1024/20995*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-1615*I*e^(6*I*d*x
+ 6*I*c) - 646*I*e^(4*I*d*x + 4*I*c) - 152*I*e^(2*I*d*x + 2*I*c) - 16*I)/(
a^3*d*e^(18*I*d*x + 18*I*c) + 9*a^3*d*e^(16*I*d*x + 16*I*c) + 36*a^3*d*e^(
14*I*d*x + 14*I*c) + 84*a^3*d*e^(12*I*d*x + 12*I*c) + 126*a^3*d*e^(10*I*d*
x + 10*I*c) + 126*a^3*d*e^(8*I*d*x + 8*I*c) + 84*a^3*d*e^(6*I*d*x + 6*I*c)
 + 36*a^3*d*e^(4*I*d*x + 4*I*c) + 9*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**13/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(115) = 230$.

Time $= 0.37$ (sec) , antiderivative size $= 902$, normalized size of antiderivative $= 6.14$

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-2/20995*(-2429*I*sqrt(a) - 8850*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) -
 5122*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 45190*sqrt(a)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 - 12924*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 - 152478*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 40470*I*s
qrt(a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 397594*sqrt(a)*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 - 50065*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^
8 - 722228*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 19380*I*sqrt(a)*s
in(d*x + c)^10/(cos(d*x + c) + 1)^10 - 936700*sqrt(a)*sin(d*x + c)^11/(cos
(d*x + c) + 1)^11 - 936700*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 +
 19380*I*sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 722228*sqrt(a)*si
n(d*x + c)^15/(cos(d*x + c) + 1)^15 + 50065*I*sqrt(a)*sin(d*x + c)^16/(cos
(d*x + c) + 1)^16 - 397594*sqrt(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 +
 40470*I*sqrt(a)*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 - 152478*sqrt(a)*si
n(d*x + c)^19/(cos(d*x + c) + 1)^19 + 12924*I*sqrt(a)*sin(d*x + c)^20/(cos
(d*x + c) + 1)^20 - 45190*sqrt(a)*sin(d*x + c)^21/(cos(d*x + c) + 1)^21 +
5122*I*sqrt(a)*sin(d*x + c)^22/(cos(d*x + c) + 1)^22 - 8850*sqrt(a)*sin(d*
x + c)^23/(cos(d*x + c) + 1)^23 + 2429*I*sqrt(a)*sin(d*x + c)^24/(cos(d*x
+ c) + 1)^24)*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(sin(d*x + c)/(c
os(d*x + c) + 1) - 1)^(5/2)/((a^3 - 12*a^3*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 66*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 220*a^3*sin(d*x + c...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.05

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{e^{-c\,1i-d\,x\,1i}\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,1024i}{13\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^6} - \frac{e^{-c\,1i-d\,x\,1i}\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,1024i}{5\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^7} + \frac{e^{-c\,1i-d\,x\,1i}\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,3072i}{17\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^8} - \frac{e^{-c\,1i-d\,x\,1i}\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,1024i}{19\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^9}$$

input
```
int(1/(cos(c + d*x)^13*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
(exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
 d*x*2i) + 1))^(1/2)*1024i)/(13*a^3*d*(exp(c*2i + d*x*2i) + 1)^6) - (exp(-
 c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2
i) + 1))^(1/2)*1024i)/(5*a^3*d*(exp(c*2i + d*x*2i) + 1)^7) + (exp(- c*1i -
 d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1)
)^(1/2)*3072i)/(17*a^3*d*(exp(c*2i + d*x*2i) + 1)^8) - (exp(- c*1i - d*x*1
i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*1024i)/(19*a^3*d*(exp(c*2i + d*x*2i) + 1)^9)
```

Reduce [F]

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\int \frac{\sec(dx+c)^{13}}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - int(sec(c + d*x)**13/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqr
t(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x))/(sqrt
(a)*a**2)
```

3.370 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{64ia^3\sec^{11}(c+dx)}{2145d(a+ia\tan(c+dx))^{11/2}} + \frac{16ia^2\sec^{11}(c+dx)}{195d(a+ia\tan(c+dx))^{9/2}} + \frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}$$

output
```
64/2145*I*a^3*sec(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(11/2)+16/195*I*a^2*sec(d
*x+c)^11/d/(a+I*a*tan(d*x+c))^(9/2)+2/15*I*a*sec(d*x+c)^11/d/(a+I*a*tan(d*
x+c))^(7/2)
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sec^{10}(c+dx)(60+203\cos(2(c+dx))+187i\sin(2(c+dx)))(-2i\cos(3(c}{2145a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx}}$$

input
```
Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(Sec[c + d*x]^10*(60 + 203*Cos[2*(c + d*x)] + (187*I)*Sin[2*(c + d*x)])*((
-2*I)*Cos[3*(c + d*x)] - 2*Sin[3*(c + d*x)]))/(2145*a^2*d*(-I + Tan[c + d*
x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{11}}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{15}a\int \frac{\sec^{11}(c+dx)}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{15}a\int \frac{\sec(c+dx)^{11}}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{15}a\left(\frac{4}{13}a\int \frac{\sec^{11}(c+dx)}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{15}a\left(\frac{4}{13}a\int \frac{\sec(c+dx)^{11}}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8}{15}a\left(\frac{8ia^2\sec^{11}(c+dx)}{143d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((2*I)/15)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (8*a*(((
(8*I)/143)*a^2*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((2*I
)/13)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(9/2))))/15
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 6.64 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{\frac{2\tan(dx+c)\sec(dx+c)^{7}\left(128\cos(dx+c)^{4}+144\cos(dx+c)^{2}+143\right)}{2145}+\frac{2i\left(128\sec(dx+c)^{3}+80\sec(dx+c)^{5}+55\sec(dx+c)^{7}\right)}{2145}}{d\left(2i\sin(dx+c)\cos(dx+c)+2\cos(dx+c)^{2}-1\right)\sqrt{a(1+i\tan(dx+c))}\,a^{2}}$	125

input
```
int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/2145/d/(2*I*sin(d*x+c)*cos(d*x+c)+2*cos(d*x+c)^2-1)/(a*(1+I*tan(d*x+c)))
^(1/2)/a^2*(tan(d*x+c)*sec(d*x+c)^7*(128*cos(d*x+c)^4+144*cos(d*x+c)^2+143
)+I*(128*sec(d*x+c)^3+80*sec(d*x+c)^5+55*sec(d*x+c)^7))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.44

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{256\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-195i\,e^{(4i\,dx+4i\,c)}-60i\,e^{(2i\,dx+2i\,c)}-8i\right)}{2145\left(a^{3}de^{(14i\,dx+14i\,c)}+7\,a^{3}de^{(12i\,dx+12i\,c)}+21\,a^{3}de^{(10i\,dx+10i\,c)}+35\,a^{3}de^{(8i\,dx+8i\,c)}+35\,a^{3}de^{(6i\,dx+6i\,c)}+21\right.}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-256/2145*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-195*I*e^(4*I*d*x + 4
*I*c) - 60*I*e^(2*I*d*x + 2*I*c) - 8*I)/(a^3*d*e^(14*I*d*x + 14*I*c) + 7*a
^3*d*e^(12*I*d*x + 12*I*c) + 21*a^3*d*e^(10*I*d*x + 10*I*c) + 35*a^3*d*e^(
8*I*d*x + 8*I*c) + 35*a^3*d*e^(6*I*d*x + 6*I*c) + 21*a^3*d*e^(4*I*d*x + 4*
I*c) + 7*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 764 vs. $2(86) = 172$.

Time = 0.39 (sec) , antiderivative size = 764, normalized size of antiderivative = 6.95

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-2/2145*(-263*I*sqrt(a) - 830*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 76
0*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4270*sqrt(a)*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 - 1085*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 - 11576*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2000*I*sqrt(a)*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 - 23000*sqrt(a)*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 - 2470*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 33540*
sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 33540*sqrt(a)*sin(d*x + c)^1
1/(cos(d*x + c) + 1)^11 + 2470*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1
)^12 - 23000*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 2000*I*sqrt(a
)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 11576*sqrt(a)*sin(d*x + c)^15/(c
os(d*x + c) + 1)^15 + 1085*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16
 - 4270*sqrt(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 + 760*I*sqrt(a)*sin(
d*x + c)^18/(cos(d*x + c) + 1)^18 - 830*sqrt(a)*sin(d*x + c)^19/(cos(d*x +
 c) + 1)^19 + 263*I*sqrt(a)*sin(d*x + c)^20/(cos(d*x + c) + 1)^20)*(sin(d*
x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)
^(5/2)/((a^3 - 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 45*a^3*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 - 120*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 + 210*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 252*a^3*sin(d*x + c)^10/(
cos(d*x + c) + 1)^10 + 210*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 120
*a^3*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 45*a^3*sin(d*x + c)^16/(co...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{256\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,60\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,195\mathrm{i}+8\mathrm{i}\right)}{2145\,a^3\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^7}$$

input
```
int(1/(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
(256*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*
2i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*60i + exp(c*4i + d*x*4i)*195i
 + 8i))/(2145*a^3*d*(exp(c*2i + d*x*2i) + 1)^7)
```

Reduce [F]

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\int \frac{\sec(dx+c)^{11}}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - int(sec(c + d*x)**11/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqr
t(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x))/(sqrt
(a)*a**2)
```

3.371 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 73

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{8ia^2\sec^9(c+dx)}{99d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}$$

output
```
8/99*I*a^2*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(9/2)+2/11*I*a*sec(d*x+c)^9/d
/(a+I*a*tan(d*x+c))^(7/2)
```

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{2\sec^7(c+dx)(\cos(2(c+dx))-i\sin(2(c+dx)))(-13i+9\tan(c+dx))}{99a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(2*Sec[c + d*x]^7*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-13*I + 9*Tan[c
 + d*x]))/(99*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^9}{(a+ia\tan(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3975}$$
$$\frac{4}{11}a\int \frac{\sec^9(c+dx)}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{4}{11}a\int \frac{\sec(c+dx)^9}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}$$
$$\Big\downarrow \text{3974}$$
$$\frac{8ia^2\sec^9(c+dx)}{99d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((8*I)/99)*a^2*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((2*I)
/11)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 5.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

method	result	size
default	$\frac{\frac{64i\sec(dx+c)}{99}+\frac{64\sec(dx+c)\tan(dx+c)}{99}+\frac{8i\sec(dx+c)^3}{99}+\frac{40\tan(dx+c)\sec(dx+c)^3}{99}-\frac{46i\sec(dx+c)^5}{99}-\frac{2\tan(dx+c)\sec(dx+c)^5}{11}}{d\sqrt{a(1+i\tan(dx+c))}\,a^2}$	102

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/99/d/(a*(1+I*tan(d*x+c)))^(1/2)/a^2*(32*I*sec(d*x+c)+32*sec(d*x+c)*tan(d
*x+c)+4*I*sec(d*x+c)^3+20*tan(d*x+c)*sec(d*x+c)^3-23*I*sec(d*x+c)^5-9*tan(
d*x+c)*sec(d*x+c)^5)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(57) = 114$.

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{64\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-11i\,e^{(2i\,dx+2i\,c)}-2i\right)}{99\left(a^3de^{(10i\,dx+10i\,c)}+5\,a^3de^{(8i\,dx+8i\,c)}+10\,a^3de^{(6i\,dx+6i\,c)}+10\,a^3de^{(4i\,dx+4i\,c)}+5\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-64/99*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-11*I*e^(2*I*d*x + 2*I*c
) - 2*I)/(a^3*d*e^(10*I*d*x + 10*I*c) + 5*a^3*d*e^(8*I*d*x + 8*I*c) + 10*a
^3*d*e^(6*I*d*x + 6*I*c) + 10*a^3*d*e^(4*I*d*x + 4*I*c) + 5*a^3*d*e^(2*I*d
*x + 2*I*c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(57) = 114$.

Time = 0.27 (sec) , antiderivative size = 626, normalized size of antiderivative = 8.58

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-2/99*(-13*I*sqrt(a) - 34*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 46*I*s
qrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 174*sqrt(a)*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 - 54*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 39
4*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 22*I*sqrt(a)*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 - 550*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
 550*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 22*I*sqrt(a)*sin(d*x +
c)^10/(cos(d*x + c) + 1)^10 - 394*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) +
1)^11 + 54*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 174*sqrt(a)*s
in(d*x + c)^13/(cos(d*x + c) + 1)^13 + 46*I*sqrt(a)*sin(d*x + c)^14/(cos(d
*x + c) + 1)^14 - 34*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 + 13*I*
sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*(sin(d*x + c)/(cos(d*x + c)
 + 1) + 1)^(5/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(5/2)/((a^3 - 8*a^3
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^3*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 - 56*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^3*sin(d*x + c)
^8/(cos(d*x + c) + 1)^8 - 56*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 2
8*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a^3*sin(d*x + c)^14/(cos(d
*x + c) + 1)^14 + a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*d*(-2*I*sin(d
*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2
))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.65 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{64\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,11\mathrm{i}+2\mathrm{i}\right)\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}}{99\,a^3\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^5}$$

input
```
int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
(64*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*11i + 2i)*(a - (a*(exp(c*2i +
 d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(99*a^3*d*(exp(c*2i
 + d*x*2i) + 1)^5)
```

Reduce [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\int \frac{\sec(dx+c)^9}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - int(sec(c + d*x)**9/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt
(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x))/(sqrt(
a)*a**2)
```

3.372 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2ia\sec^7(c+dx)}{7d(a+ia\tan(c+dx))^{7/2}}$$

output
```
2/7*I*a*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(7/2)
```

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.63

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2\sec^5(c+dx)(i+\tan(c+dx))}{7a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(-2*Sec[c + d*x]^5*(I + Tan[c + d*x]))/(7*a^2*d*(-I + Tan[c + d*x])^2*Sqrt
[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(c+dx)^7}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 3974$$

$$\frac{2ia\sec^7(c+dx)}{7d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((2*I)/7)*a*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(29) = 58.

Time = 4.89 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.14

method	result	size
default	$\frac{\frac{8i\sec(dx+c)}{7}+\frac{8\sec(dx+c)\tan(dx+c)}{7}-\frac{6i\sec(dx+c)^3}{7}-\frac{2\tan(dx+c)\sec(dx+c)^3}{7}}{d\sqrt{a(1+i\tan(dx+c))}\,a^2}$	75

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/7/d/(a*(1+I*tan(d*x+c)))^(1/2)/a^2*(4*I*sec(d*x+c)+4*sec(d*x+c)*tan(d*x+
c)-3*I*sec(d*x+c)^3-tan(d*x+c)*sec(d*x+c)^3)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(27) = 54.

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.11

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{16i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{7\,(a^3de^{(6i\,dx+6i\,c)}+3\,a^3de^{(4i\,dx+4i\,c)}+3\,a^3de^{(2i\,dx+2i\,c)}+a^3d)}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
16/7*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(a^3*d*e^(6*I*d*x + 6*I*c
) + 3*a^3*d*e^(4*I*d*x + 4*I*c) + 3*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^7{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**7/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(27) = 54$.

Time = 0.21 (sec) , antiderivative size = 488, normalized size of antiderivative = 13.94

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-2/7*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*I*sqrt(a)
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 10*sqrt(a)*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 5*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*sqrt(a)
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 20*sqrt(a)*sin(d*x + c)^7/(cos(d*x
+ c) + 1)^7 + 5*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 10*sqrt(a)
*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 4*I*sqrt(a)*sin(d*x + c)^10/(cos(d*
x + c) + 1)^10 - 2*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + I*sqrt(
a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*(sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)^(5/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(5/2)/((a^3 - 6*a^3*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 - 20*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8/(c
os(d*x + c) + 1)^8 - 6*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*sin
(d*x + c)^12/(cos(d*x + c) + 1)^12)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{e^{-c\,4i-d\,x\,4i}\,\sqrt{a+\frac{a\,\sin(c+d\,x)\,1i}{\cos(c+d\,x)}}\,2i}{7\,a^3\,d\,\cos\left(c+d\,x\right)^3}$$

input
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
(exp(- c*4i - d*x*4i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^(1/2)*2i)/(7*
a^3*d*cos(c + d*x)^3)
```

Reduce [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\int \frac{\sec(dx+c)^7}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - int(sec(c + d*x)**7/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt
(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x))/(sqrt(
a)*a**2)
```

3.373 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 123

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{4i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} - \frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{4i\sec(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}}$$

output
```
4*I*2^(1/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))/a^(5/2)/d-2/3*I*sec(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^(3/2)-4*I*sec(d*x+c
)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.67

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2\sec(c+dx)\left(7i-6i\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)+\tan(c+dx)\right)}{3a^2d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(-2*Sec[c + d*x]*(7*I - (6*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1
 + E^((2*I)*(c + d*x))]] + Tan[c + d*x]))/(3*a^2*d*Sqrt[a + I*a*Tan[c + d*
x]])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 3972, 3042, 3972, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3972}$$

$$\frac{2\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^{3/2}}dx}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3972}$$

$$\frac{2\left(\frac{2\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{a}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}\right)}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{2\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{a}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}\right)}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3970}$$

$$\frac{2\left(\frac{4i\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{ad}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}\right)}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(\frac{2i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}\right)}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((-2*I)/3)*Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + (2*(((2*I
)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x
]])])/(a^(3/2)*d) - ((2*I)*Sec[c + d*x])/(a*d*Sqrt[a + I*a*Tan[c + d*x]]))
)/a
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3972
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e
+ f*x])^(n + 1)/(b*f*(m - 2))), x] + Simp[2*(d^2/a)   Int[(d*Sec[e + f*x])^
(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && EqQ[m/2 + n, 0] && LtQ[n, -1]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(102) = 204$.

Time = 7.00 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.89

method	result
default	$\frac{2(-i(\csc(dx+c)-\cot(dx+c))-1)^5 \left(-7i+\frac{24\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(i-\cot(dx+c)+\csc(dx+c))}{2\sqrt{\csc(dx+c)^2(1-\cos(dx+c))^2-1}}\right)\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}{\cos(dx+c)+1}+7\csc(dx+c)^3(1-\right.}{3d\,a^2\sqrt{a(1+i\tan(dx+c))}\left(\csc(dx+c)^2(1-\cos(dx+c))^2-1\right)^4(-i+\mathrm{ta}}$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(-I*(csc(d*x+c)-cot(d*x+c))-1)^5/a^2/(a*(1+I*tan(d*x+c)))^(1/2)/(csc
(d*x+c)^2*(1-cos(d*x+c))^2-1)^4*(-7*I+24*arctanh(1/2/(csc(d*x+c)^2*(1-cos(
d*x+c))^2-1)^(1/2)*2^(1/2)*(I-cot(d*x+c)+csc(d*x+c)))*cos(d*x+c)/(cos(d*x+
c)+1)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+7*csc(d*x+c)^3*(1-cos(d*x+c))^3-9
*csc(d*x+c)+9*cot(d*x+c)+9*I*csc(d*x+c)^2*(1-cos(d*x+c))^2)/(-I+tan(d*x+c)
)^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(96) = 192$.

Time = 0.08 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.20

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2\left(3\sqrt{2}\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{1}{a^5d^2}}\log\left(-\frac{16\left(\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-i\right)e^{(-i\,dx-i\,c)}}{a^2d}\right)+3\right.}{}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(1/(a^5*d^2))*
log(-16*((I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + 3*sqrt(2)*(-I*
a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(1/(a^5*d^2))*log(-16*((-I*a^2*d*
e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a
^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*(3*I*e^(2*I*d*x + 2*I*c) + 4*I))/(a^3*d*e^(2*I*d*x + 2*I*c) +
a^3*d)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^5(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**5/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(96) = 192$.

Time = 0.32 (sec) , antiderivative size = 1074, normalized size of antiderivative = 8.73

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/3*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((-3*I*sqrt(2)*cos(2*d*x + 2*c) + 3*sqrt(2)*sin(2*d*x + 2*c) - 4*I*s
qrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (3*sqrt
(2)*cos(2*d*x + 2*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2))*sin(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) - 3*(2*(sqrt(2)*c
os(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c
) + sqrt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*(sqrt
(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - (I
*sqrt(2)*cos(2*d*x + 2*c)^2 + I*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*I*sqrt(2)*c
os(2*d*x + 2*c) + I*sqrt(2))*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^5\,(a+a\tan(c+d\,x)\;1\mathrm{i})^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\int \frac{\sec(dx+c)^5}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - int(sec(c + d*x)**5/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt
(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x))/(sqrt(
a)*a**2)
```

3.374 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}a^{5/2}d}+\frac{i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}$$

output
```
-1/2*I*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^
(1/2)/a^(5/2)/d+I*sec(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.73

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{ie^{-\frac{1}{2}i(2c+dx)}\left(-1-e^{2i(c+dx)}+e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right.\right.}{2a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan}}$$

input
```
Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/2)*(-1 - E^((2*I)*(c + d*x)) + E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(
c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]^3*(Cos[c +
 (d*x)/2] + I*Sin[c + (d*x)/2]))/(a^2*d*E^((I/2)*(2*c + d*x))*(-I + Tan[c
+ d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.47, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 3982, 3042, 3983, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^3}{(a+ia\tan(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3982}$$
$$\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{a}$$
$$\Big\downarrow \text{3983}$$
$$\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\left(\frac{\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\left(\frac{\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{a}$$

$$\Big\downarrow \text{3970}$$

$$\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\left(\frac{i\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{2ad}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\left(\frac{i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{a}$$

input
```
Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((2*I)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) - (2*(((I/2)*ArcTa
nh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*
a^(3/2)*d) + ((I/2)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(3/2))))/a
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(71) = 142$.

Time = 7.18 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.02

method	result
default	$\frac{-2i\cos(dx+c)\,\mathrm{arctanh}\left(\frac{(i-\cot(dx+c)+\csc(dx+c))\sqrt{2}}{2\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)-i\sin(dx+c)\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}+2\sin(dx+c)\,\mathrm{arctanl}}{2d(-i\cos(dx+c)+\sin(dx+c)-i)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,a^2\sqrt{a}}$

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d/(-I*cos(d*x+c)+sin(d*x+c)-I)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a^2/
(a*(1+I*tan(d*x+c)))^(1/2)*(-2*I*cos(d*x+c)*arctanh(1/2/(cot(d*x+c)^2-2*co
t(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(I-cot(d*x+c)+csc(d*x+c))*2^(1/2
))-I*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*sin(d*x+c)*
arctanh(1/2/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(I
-cot(d*x+c)+csc(d*x+c))*2^(1/2))+2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(67) = 134$.

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.85

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(i\sqrt{2}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(2i\,dx+2i\,c)}\log\left(\frac{2\left(\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-i\right)}{a^2d}\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/4*(I*sqrt(2)*a^3*d*sqrt(1/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*log(2*((I*a^2*d
*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(
a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) - I*sqrt(2)*a^3*d*sqrt(1/(a^5*d^2
))*e^(2*I*d*x + 2*I*c)*log(2*((-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^
2*d)) - 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(2*I*d*x + 2*I*c
) - I))*e^(-2*I*d*x - 2*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^3(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(67) = 134$.

Time = 0.32 (sec) , antiderivative size = 827, normalized size of antiderivative = 9.62

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/8*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((-I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*cos(2*d*x + 2*c
) - I*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)))*sqrt(a) - (2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
- 1) - I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*log(sqrt(c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^3\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\int \frac{\sec(dx+c)^3}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - int(sec(c + d*x)**3/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt
(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x))/(sqrt(
a)*a**2)
```

3.375 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 122

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{3i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}} + \frac{3i\sec(c+dx)}{16ad(a+ia\tan(c+dx))^{3/2}}$$

output
```
3/32*I*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^
(1/2)/a^(5/2)/d+1/4*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(5/2)+3/16*I*sec(d*x
+c)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{i\sec^3(c+dx)\left(7+3e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)+7\cos(2(c+dx))+3i\sin(2(c+dx}{32a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-1/32*I)*Sec[c + d*x]^3*(7 + 3*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c
+ d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] + 7*Cos[2*(c + d*x)] + (3*
I)*Sin[2*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*
x]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 3983, 3042, 3983, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3983}$$
$$\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}$$
$$\Big\downarrow \text{3983}$$
$$\frac{3\left(\frac{\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}$$
$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3970}$$

$$\frac{3\left(\frac{i\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{2ad}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3\left(\frac{i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/4)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (3*(((I/2)*ArcTanh
[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*a^
(3/2)*d) + ((I/2)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(3/2))))/(8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(97) = 194$.

Time = 7.97 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.39

method	result
default	$-\frac{i\,\mathrm{arctanh}\left(\frac{(i-\cot(dx+c)+\csc(dx+c))\sqrt{2}}{2\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)(12\sin(dx+c)+6\tan(dx+c)-3\sec(dx+c)\tan(dx+c))+\mathrm{arctanh}\Big(}{}$

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d/(1+I*tan(d*x+c))^2/(a*(1+I*tan(d*x+c)))^(1/2)/(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)/a^2/(cos(d*x+c)+1)*(I*arctanh(1/2/(cot(d*x+c)^2-2*cot(d*x+c
)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(I-cot(d*x+c)+csc(d*x+c))*2^(1/2))*(12*
sin(d*x+c)+6*tan(d*x+c)-3*sec(d*x+c)*tan(d*x+c))+arctanh(1/2/(cot(d*x+c)^2
-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(I-cot(d*x+c)+csc(d*x+c))*2
^(1/2))*(12*cos(d*x+c)+6-9*sec(d*x+c)-3*sec(d*x+c)^2)+3*2^(1/2)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(-4*sin(d*x+c)+sec(d*x+c)*tan(d*x+c))+3*I*2^(1/
2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(4*cos(d*x+c)-3*sec(d*x+c))+6*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(4*sin(d*x+c)+tan(d*x+c))-2*I*(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(7+12*cos(d*x+c)-2*sec(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(91) = 182$.

Time = 0.08 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.19

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(-3i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(4i\,dx+4i\,c)}\log\left(-\frac{3\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}}}\right.}{8\,a^2d}\right.\right.}{}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/32*(-3*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(4*I*d*x + 4*I*c)*log(-3/8*
(sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + 3*I*
sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(4*I*d*x + 4*I*c)*log(-3/8*(sqrt(2)*sq
rt(1/2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + sqrt(2)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*(5*I*e^(4*I*d*x + 4*I*c) + 7*I*e^(2*I*d*x + 2*
I*c) + 2*I))*e^(-4*I*d*x - 4*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\int \frac{\sec(dx+c)}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - int(sec(c + d*x)/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt(ta
n(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x))/(sqrt(a)*
a**2)
```

3.376 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 192

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{35i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}a^{5/2}d} + \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{192a^2d\sqrt{a+ia\tan(c+dx)}} - \frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{128a^3d}$$

output

```
35/256*I*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*
2^(1/2)/a^(5/2)/d+1/6*I*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(5/2)+7/48*I*cos(d
*x+c)/a/d/(a+I*a*tan(d*x+c))^(3/2)+35/192*I*cos(d*x+c)/a^2/d/(a+I*a*tan(d*
x+c))^(1/2)-35/128*I*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^3/d
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.74

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\sec^3(c+dx)\left(-125-105e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right.}{768a^2d(-i+\tan(c-}$$

input
```
Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/768)*Sec[c + d*x]^3*(-125 - 105*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*
(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 85*Cos[2*(c + d*x)] +
 40*Cos[4*(c + d*x)] + (7*I)*Sin[2*(c + d*x)] + (56*I)*Sin[4*(c + d*x)]))/
(a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3983}$$
$$\frac{7\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{7\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^{3/2}}dx}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\left(\frac{5\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\left(\frac{5\left(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5\left(\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3971}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3970}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/6)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (7*(((I/4)*Cos[c +
 d*x])/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (5*(((I/2)*Cos[c + d*x])/(d*Sqrt
[a + I*a*Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(S
qrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a
+ I*a*Tan[c + d*x]])/d))/(4*a)))/(8*a)))/(12*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(155) = 310.

Time = 9.64 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.26

method	result
default	$-\frac{i \operatorname{arctanh}\left(\frac{(i-\cot(dx+c)+\csc(dx+c))\sqrt{2}}{2\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)(420\sin(dx+c)+210\tan(dx+c)-105\sec(dx+c)\tan(dx+c))+\operatorname{arct}}{}$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/768/d/(1+I*tan(d*x+c))^2/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)/a^2/(a*(1+I*tan(d*x+c)))^(1/2)*(I*arctanh(1/2/(cot(d*x+c)^2-2*cot(d*x+
c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(I-cot(d*x+c)+csc(d*x+c))*2^(1/2))*(42
0*sin(d*x+c)+210*tan(d*x+c)-105*sec(d*x+c)*tan(d*x+c))+arctanh(1/2/(cot(d*
x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(I-cot(d*x+c)+csc(d*x
+c))*2^(1/2))*(420*cos(d*x+c)+210-315*sec(d*x+c)-105*sec(d*x+c)^2)+2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-420*sin(d*x+c)+105*sec(d*x+c)*tan(
d*x+c))+I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(420*cos(d*x+c)-315
*sec(d*x+c))+tan(d*x+c)*(-448*cos(d*x+c)^2+392*cos(d*x+c)+210)*(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)+I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(320*cos(d*x+
c)^2-520*cos(d*x+c)-490+140*sec(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.51

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(-105i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(6i\,dx+6i\,c)}\log\left(-\frac{35\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+}}}\right.}{64\,a^2d}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/768*(-105*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(6*I*d*x + 6*I*c)*log(-3
5/64*(sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) +
 105*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(6*I*d*x + 6*I*c)*log(-35/64*(s
qrt(2)*sqrt(1/2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + sqrt(
2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-48*I*e^(8*I*d*x + 8*I*c) + 39*I*e^(
6*I*d*x + 6*I*c) + 125*I*e^(4*I*d*x + 4*I*c) + 46*I*e^(2*I*d*x + 2*I*c) +
8*I))*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(cos(c + d*x)/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2297 vs. $2(145) = 290$.

Time = 0.33 (sec) , antiderivative size = 2297, normalized size of antiderivative = 11.96

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/3072*(544*(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(
1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin
(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(3/4)*((-I*sqrt(2)*cos(6*d*x + 6*c)
 - sqrt(2)*sin(6*d*x + 6*c))*cos(3/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6
*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c
))) + 1)) + (sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2)*sin(6*d*x + 6*c))*sin(3/
2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*ar
ctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)))*sqrt(a) + 12*(cos(1/3*ar
ctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x +
 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*
x + 6*c))) + 1)^(1/4)*(29*((I*sqrt(2)*cos(6*d*x + 6*c) + sqrt(2)*sin(6*d*x
 + 6*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + (I*sqrt(
2)*cos(6*d*x + 6*c) + sqrt(2)*sin(6*d*x + 6*c))*sin(1/3*arctan2(sin(6*d*x
+ 6*c), cos(6*d*x + 6*c)))^2 + 2*(I*sqrt(2)*cos(6*d*x + 6*c) + sqrt(2)*sin
(6*d*x + 6*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + I*sq
rt(2)*cos(6*d*x + 6*c) + sqrt(2)*sin(6*d*x + 6*c))*cos(5/2*arctan2(sin(1/3
*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x +
 6*c), cos(6*d*x + 6*c))) + 1)) + (19*I*sqrt(2)*cos(6*d*x + 6*c) + 19*sqrt
(2)*sin(6*d*x + 6*c) - 16*I*sqrt(2))*cos(1/2*arctan2(sin(1/3*arctan2(sin(6
*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

input
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\int \frac{\cos(dx+c)}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - int(cos(c + d*x)/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt(ta
n(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x))/(sqrt(a)*
a**2)
```

3.377 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 270

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}} \, dx = \frac{1155 i \operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4096\sqrt{2}a^{5/2}d} + \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11 i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} + \frac{385 i\cos(c+dx)}{2048a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{33 i\cos^3(c+dx)}{256a^2d\sqrt{a+ia\tan(c+dx)}} - \frac{1155 i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4096a^3d} - \frac{77 i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{512a^3d}$$

output
```
1155/8192*I*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))*2^(1/2)/a^(5/2)/d+1/8*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(5/2)+11/96*I
*cos(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^(3/2)+385/2048*I*cos(d*x+c)/a^2/d/(a+
I*a*tan(d*x+c))^(1/2)+33/256*I*cos(d*x+c)^3/a^2/d/(a+I*a*tan(d*x+c))^(1/2)
-1155/4096*I*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^3/d-77/512*I*cos(d*x+c)
^3*(a+I*a*tan(d*x+c))^(1/2)/a^3/d
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.61

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\sec^3(c+dx)\left(-3325-3465e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/24576)*Sec[c + d*x]^3*(-3325 - 3465*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2
*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 1605*Cos[2*(c + d
*x)] + 1800*Cos[4*(c + d*x)] + 80*Cos[6*(c + d*x)] + (1111*I)*Sin[2*(c + d
*x)] + (2552*I)*Sin[4*(c + d*x)] + (176*I)*Sin[6*(c + d*x)]))/(a^2*d*(-I +
 Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{11\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^{3/2}}dx}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3983}$$

$$\frac{11\left(\frac{3\int \frac{\cos^3(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{11\left(\frac{3\int \frac{1}{\sec(c+dx)^3\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3983}$$

$$\frac{11\left(\frac{3\left(\frac{7\int \cos^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{11\left(\frac{3\left(\frac{7\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^3}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3978}$$

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3983$$

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3971$$

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+}}\right.}{4a}\right.}{16a}$$

$$\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+}}\right.}{4a}\right.}{16a}$$

$$\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3970}$$

$$11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{ia\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}}{4a}\right.}{16a}$$

$$\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{i\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{}{4a}}{4a}\right.}{16a}$$

$$\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/8)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (11*(((I/6)*Cos[
c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (3*(((I/4)*Cos[c + d*x]^3)/
(d*Sqrt[a + I*a*Tan[c + d*x]]) + (7*(((-1/3*I)*Cos[c + d*x]^3*Sqrt[a + I*a
*Tan[c + d*x]])/d + (5*a*(((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x
]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*
Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])
/d))/(4*a)))/6))/(8*a)))/(4*a)))/(16*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(221) = 442$.

Time = 9.09 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.78

method	result
default	$-\frac{i\left(13860\cos(dx+c)^2+6930\cos(dx+c)-3465\right)\sin(dx+c)\operatorname{arctanh}\left(\frac{i-\cot(dx+c)+\csc(dx+c)}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+\left(13860\cos(dx+c)^3+6930\cos(dx+c\right.}{}$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24576/d*(I*(13860*cos(d*x+c)^2+6930*cos(d*x+c)-3465)*sin(d*x+c)*arctanh
(1/2/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(I-cot(d*x+c)+csc(d*x+c)))+(13860*
cos(d*x+c)^3+6930*cos(d*x+c)^2-10395*cos(d*x+c)-3465)*arctanh(1/2/(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(I-cot(d*x+c)+csc(d*x+c)))+sin(d*x+c)*(-13860*c
os(d*x+c)^2+3465)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*cos(d*x+c
)*(13860*cos(d*x+c)^2-10395)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+
sin(d*x+c)*cos(d*x+c)*(-5632*cos(d*x+c)^4-5632*cos(d*x+c)^3-14784*cos(d*x+
c)^2+12936*cos(d*x+c)+6930)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*cos(d*x+c
)*(2560*cos(d*x+c)^5+2560*cos(d*x+c)^4+10560*cos(d*x+c)^3-17160*cos(d*x+c)
^2-16170*cos(d*x+c)+4620)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))/(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)/(I*sin(d*x+c)*cos(d*x+c)*(2*cos(d*x+c)+2)+2*cos(d*x
+c)^3+2*cos(d*x+c)^2-cos(d*x+c)-1)/(a*(1+I*tan(d*x+c)))^(1/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.15

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(-3465i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(8i\,dx+8i\,c)}\log\left(-\frac{1155\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{}{e^{(2i}}}\right.}{2048\,a^2d}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/24576*(-3465*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(8*I*d*x + 8*I*c)*log
(-1155/2048*(sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^
2*d)) + 3465*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(8*I*d*x + 8*I*c)*log(-
1155/2048*(sqrt(2)*sqrt(1/2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2
*d)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-128*I*e^(12*I*d*x + 12*
I*c) - 2176*I*e^(10*I*d*x + 10*I*c) + 247*I*e^(8*I*d*x + 8*I*c) + 3325*I*e
^(6*I*d*x + 6*I*c) + 1358*I*e^(4*I*d*x + 4*I*c) + 376*I*e^(2*I*d*x + 2*I*c
) + 48*I))*e^(-8*I*d*x - 8*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos^3{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(cos(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3789 vs. $2(207) = 414$.

Time = 0.43 (sec) , antiderivative size = 3789, normalized size of antiderivative = 14.03

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/98304*(4*(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(
1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin
(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(3/4)*(15*((-I*sqrt(2)*cos(8*d*x +
8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d
*x + 8*c)))^2 + (-I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*s
in(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*(-I*sqrt(2)*cos(
8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c),
 cos(8*d*x + 8*c))) - I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c
))*cos(7/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), c
os(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + (55*I*sqrt(2)*
cos(8*d*x + 8*c) + 960*I*sqrt(2)*cos(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d
*x + 8*c))) - 1296*I*sqrt(2)*cos(1/2*arctan2(sin(8*d*x + 8*c), cos(8*d*x +
 8*c))) + 55*sqrt(2)*sin(8*d*x + 8*c) + 960*sqrt(2)*sin(3/4*arctan2(sin(8*
d*x + 8*c), cos(8*d*x + 8*c))) - 1296*sqrt(2)*sin(1/2*arctan2(sin(8*d*x +
8*c), cos(8*d*x + 8*c))) + 128*I*sqrt(2))*cos(3/2*arctan2(sin(1/4*arctan2(
sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), co
s(8*d*x + 8*c))) + 1)) + 15*((sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d
*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + (sqrt(
2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(1/4*arctan2(sin(8*d*
x + 8*c), cos(8*d*x + 8*c)))^2 + 2*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^3}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\int \frac{\cos(dx+c)^3}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a^2}$$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - int(cos(c + d*x)**3/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt
(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x))/(sqrt(
a)*a**2)
```

3.378 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 146

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{32i(a+ia\tan(c+dx))^{3/2}}{3a^5d} + \frac{64i(a+ia\tan(c+dx))^{5/2}}{5a^6d} - \frac{48i(a+ia\tan(c+dx))^{7/2}}{7a^7d} + \frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^8d} - \frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^9d}$$

output
```
-32/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^5/d+64/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^6
/d-48/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^7/d+16/9*I*(a+I*a*tan(d*x+c))^(9/2)/a
^8/d-2/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^9/d
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.55

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}(5419-6396i\tan(c+dx)-453}{3465a^4d}$$

input
```
Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(2*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]*(5419 - (6396*I)*Tan[c +
 d*x] - 4530*Tan[c + d*x]^2 + (1820*I)*Tan[c + d*x]^3 + 315*Tan[c + d*x]^4
))/(3465*a^4*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{10}}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^4\sqrt{i\tan(c+dx)a+a}d(ia\tan(c+dx))}{a^9d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left((i\tan(c+dx)a+a)^{9/2}-8a(i\tan(c+dx)a+a)^{7/2}+24a^2(i\tan(c+dx)a+a)^{5/2}-32a^3(i\tan(c+dx)a\right.}{a^9d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{32}{3}a^4(a+ia\tan(c+dx))^{3/2}-\frac{64}{5}a^3(a+ia\tan(c+dx))^{5/2}+\frac{48}{7}a^2(a+ia\tan(c+dx))^{7/2}+\frac{2}{11}(a+ia\tan(c\right.}{a^9d}$$

input
```
Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*((32*a^4*(a + I*a*Tan[c + d*x])^(3/2))/3 - (64*a^3*(a + I*a*Tan[c +
d*x])^(5/2))/5 + (48*a^2*(a + I*a*Tan[c + d*x])^(7/2))/7 - (16*a*(a + I*a*
Tan[c + d*x])^(9/2))/9 + (2*(a + I*a*Tan[c + d*x])^(11/2))/11))/(a^9*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}+\frac{8a(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{24a^2(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{32a^3(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{16a^4(a+ia\tan(dx+c}{3}\right.}{d\,a^9}$
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}+\frac{8a(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{24a^2(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{32a^3(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{16a^4(a+ia\tan(dx+c}{3}\right.}{d\,a^9}$

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^9*(-1/11*(a+I*a*tan(d*x+c))^(11/2)+8/9*a*(a+I*a*tan(d*x+c))^(9/2)-
24/7*a^2*(a+I*a*tan(d*x+c))^(7/2)+32/5*a^3*(a+I*a*tan(d*x+c))^(5/2)-16/3*a
^4*(a+I*a*tan(d*x+c))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{64\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(128i\,e^{(11i\,dx+11i\,c)}+704i\,e^{(9i\,dx+9i\,c)}+1584i\,e^{(7i\,dx+7i\,c)}+1848i\,e^{(5i\,dx+5i\,c)}+1155i\,e^{(3i}\right.}{3465\left(a^4de^{(10i\,dx+10i\,c)}+5\,a^4de^{(8i\,dx+8i\,c)}+10\,a^4de^{(6i\,dx+6i\,c)}+10\,a^4de^{(4i\,dx+4i\,c)}+5\,a^4de^{(2i\,dx+2i\,c)}+a^{}\right.}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-64/3465*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(128*I*e^(11*I*d*x + 11
*I*c) + 704*I*e^(9*I*d*x + 9*I*c) + 1584*I*e^(7*I*d*x + 7*I*c) + 1848*I*e^
(5*I*d*x + 5*I*c) + 1155*I*e^(3*I*d*x + 3*I*c))/(a^4*d*e^(10*I*d*x + 10*I*
c) + 5*a^4*d*e^(8*I*d*x + 8*I*c) + 10*a^4*d*e^(6*I*d*x + 6*I*c) + 10*a^4*d
*e^(4*I*d*x + 4*I*c) + 5*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.64

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx =$$

$$-\frac{2i\left(315\,(i\,a\tan\left(dx+c\right)+a)^{\frac{11}{2}}-3080\,(i\,a\tan\left(dx+c\right)+a)^{\frac{9}{2}}a+11880\,(i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}a^{2}-2217\right.}{3465\,a^{9}d}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-2/3465*I*(315*(I*a*tan(d*x + c) + a)^(11/2) - 3080*(I*a*tan(d*x + c) + a)
^(9/2)*a + 11880*(I*a*tan(d*x + c) + a)^(7/2)*a^2 - 22176*(I*a*tan(d*x + c
) + a)^(5/2)*a^3 + 18480*(I*a*tan(d*x + c) + a)^(3/2)*a^4)/(a^9*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 5.45 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.53

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{\sqrt{a-\frac{a\left(e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,8192\mathrm{i}}{3465\,a^4\,d} - \frac{\sqrt{a-\frac{a\left(e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,4096\mathrm{i}}{3465\,a^4\,d\,\left(e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{\sqrt{a-\frac{a\left(e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{1155\,a^4\,d\,\left(e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^2} - \frac{\sqrt{a-\frac{a\left(e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{693\,a^4\,d\,\left(e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3} - \frac{\sqrt{a-\frac{a\left(e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{99\,a^4\,d\,\left(e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^4} + \frac{\sqrt{a-\frac{a\left(e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{11\,a^4\,d\,\left(e^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^5}$$

input
```
int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
64i)/(11*a^4*d*(exp(c*2i + d*x*2i) + 1)^5) - ((a - (a*(exp(c*2i + d*x*2i)*
1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*4096i)/(3465*a^4*d*(exp(c*2i
+ d*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*
x*2i) + 1))^(1/2)*1024i)/(1155*a^4*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (
a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(
693*a^4*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i -
1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(99*a^4*d*(exp(c*2i + d*x*2i)
 + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*8192i)/(3465*a^4*d)
```

Reduce [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*( - 24*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**10*i - 17*int(( - s
qrt(tan(c + d*x)*i + 1)*sec(c + d*x)**10*tan(c + d*x)**4)/(tan(c + d*x)**5
*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c
 + d*x)*i - 1),x)*tan(c + d*x)**2*d - 17*int(( - sqrt(tan(c + d*x)*i + 1)*
sec(c + d*x)**10*tan(c + d*x)**4)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 -
 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d + 48
*int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**10*tan(c + d*x)**2)/(tan(c
 + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2
 - 3*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*d + 48*int(( - sqrt(tan(c + d*
x)*i + 1)*sec(c + d*x)**10*tan(c + d*x)**2)/(tan(c + d*x)**5*i + 3*tan(c +
 d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1)
,x)*d - 14*int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**10*tan(c + d*x))
/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c +
d*x)**2 - 3*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*d*i - 14*int(( - sqrt(t
an(c + d*x)*i + 1)*sec(c + d*x)**10*tan(c + d*x))/(tan(c + d*x)**5*i + 3*t
an(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*
i - 1),x)*d*i + int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**10)/(tan(c
+ d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2
- 3*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*d + int(( - sqrt(tan(c + d*x)*i
 + 1)*sec(c + d*x)**10)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(...
```

3.379 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 113

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{16i\sqrt{a+ia\tan(c+dx)}}{a^4d} + \frac{8i(a+ia\tan(c+dx))^{3/2}}{a^5d} - \frac{12i(a+ia\tan(c+dx))^{5/2}}{5a^6d} + \frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^7d}$$

output
```
-16*I*(a+I*a*tan(d*x+c))^(1/2)/a^4/d+8*I*(a+I*a*tan(d*x+c))^(3/2)/a^5/d-12
/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^6/d+2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^7/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2\sqrt{a+ia\tan(c+dx)}(-177i - 71\tan(c+dx) + 27i\tan^2(c+dx) + 5\tan^3}{35a^4d}$$

input
```
Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(2*Sqrt[a + I*a*Tan[c + d*x]]*(-177*I - 71*Tan[c + d*x] + (27*I)*Tan[c + d
*x]^2 + 5*Tan[c + d*x]^3))/(35*a^4*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(c+dx)^8}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{i\int \frac{(a-ia\tan(c+dx))^3}{\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \; 53$$

$$-\frac{i\int \left(\frac{8a^3}{\sqrt{i\tan(c+dx)a+a}}-12\sqrt{i\tan(c+dx)a+a}a^2+6(i\tan(c+dx)a+a)^{3/2}a-(i\tan(c+dx)a+a)^{5/2}\right)d(ia\,\text{ta}}{a^7d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{i\Big(16a^3\sqrt{a+ia\tan(c+dx)}-8a^2(a+ia\tan(c+dx))^{3/2}-\frac{2}{7}(a+ia\tan(c+dx))^{7/2}+\frac{12}{5}a(a+ia\tan(c+dx)}{a^7d}$$

input
```
Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*(16*a^3*Sqrt[a + I*a*Tan[c + d*x]] - 8*a^2*(a + I*a*Tan[c + d*x])^(3
/2) + (12*a*(a + I*a*Tan[c + d*x])^(5/2))/5 - (2*(a + I*a*Tan[c + d*x])^(7
/2))/7))/(a^7*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{6a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+4a^{2}(a+ia\tan(dx+c))^{\frac{3}{2}}-8a^{3}\sqrt{a+ia\tan(dx+c)}\right)}{d\,a^{7}}$	82
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{6a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+4a^{2}(a+ia\tan(dx+c))^{\frac{3}{2}}-8a^{3}\sqrt{a+ia\tan(dx+c)}\right)}{d\,a^{7}}$	82

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^7*(1/7*(a+I*a*tan(d*x+c))^(7/2)-6/5*a*(a+I*a*tan(d*x+c))^(5/2)+4*a
^2*(a+I*a*tan(d*x+c))^(3/2)-8*a^3*(a+I*a*tan(d*x+c))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{16\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(16i\,e^{(7i\,dx+7i\,c)}+56i\,e^{(5i\,dx+5i\,c)}+70i\,e^{(3i\,dx+3i\,c)}+35i\,e^{(i\,dx+i\,c)}\right)}{35\left(a^4de^{(6i\,dx+6i\,c)}+3\,a^4de^{(4i\,dx+4i\,c)}+3\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-16/35*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(7*I*d*x + 7*I*c)
 + 56*I*e^(5*I*d*x + 5*I*c) + 70*I*e^(3*I*d*x + 3*I*c) + 35*I*e^(I*d*x + I
*c))/(a^4*d*e^(6*I*d*x + 6*I*c) + 3*a^4*d*e^(4*I*d*x + 4*I*c) + 3*a^4*d*e^
(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i\left(5\,(i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}-42\,(i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}a+140\,(i\,a\tan\,(d\right.}{35\,a^7d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
2/35*I*(5*(I*a*tan(d*x + c) + a)^(7/2) - 42*(I*a*tan(d*x + c) + a)^(5/2)*a
 + 140*(I*a*tan(d*x + c) + a)^(3/2)*a^2 - 280*sqrt(I*a*tan(d*x + c) + a)*a
^3)/(a^7*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.14

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{35\,a^4\,d}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{35\,a^4\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,96\mathrm{i}}{35\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}$$

input
```
int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
- ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*256i)/(35*a^4*d) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
 d*x*2i) + 1))^(1/2)*128i)/(35*a^4*d*(exp(c*2i + d*x*2i) + 1)) - ((a - (a*
(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*96i)/(35*
a^4*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*
1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(7*a^4*d*(exp(c*2i + d*x*2i) + 1)
^3)
```

Reduce [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*( - 16*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**8*i - 13*int(( - sq
rt(tan(c + d*x)*i + 1)*sec(c + d*x)**8*tan(c + d*x)**4)/(tan(c + d*x)**5*i
 + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c +
 d*x)*i - 1),x)*tan(c + d*x)**2*d - 13*int(( - sqrt(tan(c + d*x)*i + 1)*se
c(c + d*x)**8*tan(c + d*x)**4)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*
tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d + 36*in
t(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**8*tan(c + d*x)**2)/(tan(c + d
*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3
*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*d + 36*int(( - sqrt(tan(c + d*x)*i
 + 1)*sec(c + d*x)**8*tan(c + d*x)**2)/(tan(c + d*x)**5*i + 3*tan(c + d*x)
**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d
 - 10*int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**8*tan(c + d*x))/(tan(
c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**
2 - 3*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*d*i - 10*int(( - sqrt(tan(c +
 d*x)*i + 1)*sec(c + d*x)**8*tan(c + d*x))/(tan(c + d*x)**5*i + 3*tan(c +
d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),
x)*d*i + int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**8)/(tan(c + d*x)**
5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(
c + d*x)*i - 1),x)*tan(c + d*x)**2*d + int(( - sqrt(tan(c + d*x)*i + 1)*se
c(c + d*x)**8)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)*...
```

3.380 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{8i}{a^3d\sqrt{a+ia\tan(c+dx)}} + \frac{8i\sqrt{a+ia\tan(c+dx)}}{a^4d} - \frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^5d}$$

output
```
8*I/a^3/d/(a+I*a*tan(d*x+c))^(1/2)+8*I*(a+I*a*tan(d*x+c))^(1/2)/a^4/d-2/3*
I*(a+I*a*tan(d*x+c))^(3/2)/a^5/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.58

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i(23+10i\tan(c+dx)+\tan^2(c+dx))}{3a^3d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((2*I)/3)*(23 + (10*I)*Tan[c + d*x] + Tan[c + d*x]^2))/(a^3*d*Sqrt[a + I*
a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{(a-ia\tan(c+dx))^2}{(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(\frac{4a^2}{(i\tan(c+dx)a+a)^{3/2}}-\frac{4a}{\sqrt{i\tan(c+dx)a+a}}+\sqrt{i\tan(c+dx)a+a}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{8a^2}{\sqrt{a+ia\tan(c+dx)}}-8a\sqrt{a+ia\tan(c+dx)}+\frac{2}{3}(a+ia\tan(c+dx))^{3/2}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*((-8*a^2)/Sqrt[a + I*a*Tan[c + d*x]] - 8*a*Sqrt[a + I*a*Tan[c + d*x]
] + (2*(a + I*a*Tan[c + d*x])^(3/2))/3))/(a^5*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+4a\sqrt{a+ia\tan(dx+c)}+\frac{4a^{2}}{\sqrt{a+ia\tan(dx+c)}}\right)}{d\,a^{5}}$	63
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+4a\sqrt{a+ia\tan(dx+c)}+\frac{4a^{2}}{\sqrt{a+ia\tan(dx+c)}}\right)}{d\,a^{5}}$	63

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^5*(-1/3*(a+I*a*tan(d*x+c))^(3/2)+4*a*(a+I*a*tan(d*x+c))^(1/2)+4*a^
2/(a+I*a*tan(d*x+c))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-8i\,e^{(4i\,dx+4i\,c)}-12i\,e^{(2i\,dx+2i\,c)}-3i\right)}{3\left(a^4de^{(3i\,dx+3i\,c)}+a^4de^{(i\,dx+i\,c)}\right)}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-4/3*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-8*I*e^(4*I*d*x + 4*I*c) -
 12*I*e^(2*I*d*x + 2*I*c) - 3*I)/(a^4*d*e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d
*x + I*c))
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec^6(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Integral(sec(c + d*x)**6/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i\left(\frac{12}{\sqrt{i\,a\tan(dx+c)+a}\,a^2}-\frac{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}-12\sqrt{i\,a\tan(dx+c)+a}\,a}{a^4}\right)}{3\,ad}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
2/3*I*(12/(sqrt(I*a*tan(d*x + c) + a)*a^2) - ((I*a*tan(d*x + c) + a)^(3/2)
 - 12*sqrt(I*a*tan(d*x + c) + a)*a)/a^4)/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.31

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,23\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,3\mathrm{i}\right.}{3\,a^4\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
(2*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1
))^(1/2)*(cos(2*c + 2*d*x)*23i + cos(4*c + 4*d*x)*3i + 7*sin(2*c + 2*d*x)
+ 3*sin(4*c + 4*d*x) + 20i))/(3*a^4*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*( - 4*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**6*i - 18*int((sqrt(t
an(c + d*x)*i + 1)*sec(c + d*x)**6*tan(c + d*x)**4)/(tan(c + d*x)**5*i + 3
*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x
)*i - 1),x)*tan(c + d*x)**2*d - 18*int((sqrt(tan(c + d*x)*i + 1)*sec(c + d
*x)**6*tan(c + d*x)**4)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c +
 d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d + 52*int((sqrt
(tan(c + d*x)*i + 1)*sec(c + d*x)**6*tan(c + d*x)**3)/(tan(c + d*x)**5*i +
 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d
*x)*i - 1),x)*tan(c + d*x)**2*d*i + 52*int((sqrt(tan(c + d*x)*i + 1)*sec(c
 + d*x)**6*tan(c + d*x)**3)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan
(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d*i + 48*int
((sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**6*tan(c + d*x)**2)/(tan(c + d*x)*
*5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan
(c + d*x)*i - 1),x)*tan(c + d*x)**2*d + 48*int((sqrt(tan(c + d*x)*i + 1)*s
ec(c + d*x)**6*tan(c + d*x)**2)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2
*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d - 9*in
t((sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**6*tan(c + d*x))/(tan(c + d*x)**5
*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c
 + d*x)*i - 1),x)*tan(c + d*x)**2*d*i - 9*int((sqrt(tan(c + d*x)*i + 1)*se
c(c + d*x)**6*tan(c + d*x))/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*...
```

3.381 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{4i}{3a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{2i}{a^3d\sqrt{a+ia\tan(c+dx)}}$$

output
```
4/3*I/a^2/d/(a+I*a*tan(d*x+c))^(3/2)-2*I/a^3/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{i\left(-\frac{4a}{3(a+ia\tan(c+dx))^{3/2}} + \frac{2}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^3d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*((-4*a)/(3*(a + I*a*Tan[c + d*x])^(3/2)) + 2/Sqrt[a + I*a*Tan[c + d*
x]]))/(a^3*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^4}{(a+ia\tan(c+dx))^{7/2}}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int \frac{a-ia\tan(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{a^3d}$$
$$\Big\downarrow \text{53}$$
$$-\frac{i\int \left(\frac{2a}{(i\tan(c+dx)a+a)^{5/2}}-\frac{1}{(i\tan(c+dx)a+a)^{3/2}}\right)d(ia\tan(c+dx))}{a^3d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{i\left(\frac{2}{\sqrt{a+ia\tan(c+dx)}}-\frac{4a}{3(a+ia\tan(c+dx))^{3/2}}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*((-4*a)/(3*(a + I*a*Tan[c + d*x])^(3/2)) + 2/Sqrt[a + I*a*Tan[c + d*
x]]))/(a^3*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2i\left(\frac{2a}{3(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{1}{\sqrt{a+ia\tan(dx+c)}}\right)}{d\,a^{3}}$	44
default	$\frac{2i\left(\frac{2a}{3(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{1}{\sqrt{a+ia\tan(dx+c)}}\right)}{d\,a^{3}}$	44

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^3*(2/3*a/(a+I*a*tan(d*x+c))^(3/2)-1/(a+I*a*tan(d*x+c))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-2i\,e^{(4i\,dx+4i\,c)}-i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{3\,a^4d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(4*I*d*x + 4*I*c) -
I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^4*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec^4(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Integral(sec(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{2i\,(3i\,a\tan(dx+c)+a)}{3\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^3d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-2/3*I*(3*I*a*tan(d*x + c) + a)/((I*a*tan(d*x + c) + a)^(3/2)*a^3*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^4\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x)
```

output

```
(sqrt(a)*( - 2*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**4*i - 5*int((sqrt(ta
n(c + d*x)*i + 1)*sec(c + d*x)**4*tan(c + d*x)**4)/(tan(c + d*x)**5*i + 3*
tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)
*i - 1),x)*tan(c + d*x)**2*d - 5*int((sqrt(tan(c + d*x)*i + 1)*sec(c + d*x
)**4*tan(c + d*x)**4)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d
*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d + 14*int((sqrt(t
an(c + d*x)*i + 1)*sec(c + d*x)**4*tan(c + d*x)**3)/(tan(c + d*x)**5*i + 3
*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x
)*i - 1),x)*tan(c + d*x)**2*d*i + 14*int((sqrt(tan(c + d*x)*i + 1)*sec(c +
 d*x)**4*tan(c + d*x)**3)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c
 + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d*i + 12*int((
sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**4*tan(c + d*x)**2)/(tan(c + d*x)**5
*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c
 + d*x)*i - 1),x)*tan(c + d*x)**2*d + 12*int((sqrt(tan(c + d*x)*i + 1)*sec
(c + d*x)**4*tan(c + d*x)**2)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*t
an(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d - int((s
qrt(tan(c + d*x)*i + 1)*sec(c + d*x)**4)/(tan(c + d*x)**5*i + 3*tan(c + d*
x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)
*tan(c + d*x)**2*d - int((sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**4)/(tan(c
 + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)...
```

3.382 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i}{5ad(a+ia\tan(c+dx))^{5/2}}$$

output
```
2/5*I/a/d/(a+I*a*tan(d*x+c))^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i}{5ad(a+ia\tan(c+dx))^{5/2}}$$

input
```
Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((2*I)/5)/(a*d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^2}{(a+ia\tan(c+dx))^{7/2}}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int \frac{1}{(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{ad}$$
$$\Big\downarrow \text{17}$$
$$\frac{2i}{5ad(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((2*I)/5)/(a*d*(a + I*a*Tan[c + d*x])^(5/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2i}{5ad(a+ia\tan(dx+c))^{\frac{5}{2}}}$	24
default	$\frac{2i}{5ad(a+ia\tan(dx+c))^{\frac{5}{2}}}$	24

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*I/a/d/(a+I*a*tan(d*x+c))^(5/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(21) = 42$.

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.48

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(6i\,dx+6i\,c)}+3i\,e^{(4i\,dx+4i\,c)}+3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-5i\,dx-5i}}{20\,a^4d}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/20*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(6*I*d*x + 6*I*c) + 3*
I*e^(4*I*d*x + 4*I*c) + 3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-5*I*d*x - 5*I*c)/
(a^4*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec^2{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Integral(sec(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i}{5\,(i\,a\tan{(dx+c)}+a)^{\frac{5}{2}}ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
2/5*I/((I*a*tan(d*x + c) + a)^(5/2)*a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2\mathrm{i}}{5\,a\,d\,(a+a\tan(c+d\,x)\,\,1\mathrm{i})^{5/2}}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
2i/(5*a*d*(a + a*tan(c + d*x)*1i)^(5/2))
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*( - 6*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**2*i + int(( - sqrt(t
an(c + d*x)*i + 1)*sec(c + d*x)**2*tan(c + d*x)**4)/(tan(c + d*x)**5*i + 3
*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x
)*i - 1),x)*tan(c + d*x)**2*d + int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d
*x)**2*tan(c + d*x)**4)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c +
 d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d - 2*int(( - sq
rt(tan(c + d*x)*i + 1)*sec(c + d*x)**2*tan(c + d*x))/(tan(c + d*x)**5*i +
3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*
x)*i - 1),x)*tan(c + d*x)**2*d*i - 2*int(( - sqrt(tan(c + d*x)*i + 1)*sec(
c + d*x)**2*tan(c + d*x))/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c
 + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d*i - int(( -
sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**2)/(tan(c + d*x)**5*i + 3*tan(c + d
*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x
)*tan(c + d*x)**2*d - int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**2)/(t
an(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x
)**2 - 3*tan(c + d*x)*i - 1),x)*d - 2*int((sqrt(tan(c + d*x)*i + 1)*sec(c
+ d*x)**2*tan(c + d*x)**4)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(
c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*tan(c + d*x)*
*2*d - 2*int((sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**2*tan(c + d*x)**4)/(t
an(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + ...
```

3.383 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 237

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{11i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} + \frac{11i}{56d(a+ia\tan(c+dx))^{7/2}} + \frac{11i}{80ad(a+ia\tan(c+dx))^{5/2}} + \frac{11i}{96a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{11i}{64a^3d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^3}{2d(a+ia\tan(c+dx))^{9/2}\,(a^2-ia^2\tan(c+dx))}$$

output
```
-11/128*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/a^
(7/2)/d+11/36*I*a/d/(a+I*a*tan(d*x+c))^(9/2)+11/56*I/d/(a+I*a*tan(d*x+c))^
(7/2)+11/80*I/a/d/(a+I*a*tan(d*x+c))^(5/2)+11/96*I/a^2/d/(a+I*a*tan(d*x+c)
)^(3/2)+11/64*I/a^3/d/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*a^3/d/(a+I*a*tan(d*x+
c))^(9/2)/(a^2-I*a^2*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.22

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{ia\,\text{Hypergeometric2F1}\left(-\frac{9}{2},2,-\frac{7}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{18d(a+ia\tan(c+dx))^{9/2}}$$

input
```
Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((I/18)*a*Hypergeometric2F1[-9/2, 2, -7/2, (1 + I*Tan[c + d*x])/2])/(d*(a
+ I*a*Tan[c + d*x])^(9/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3968, 52, 61, 61, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^3\left(\frac{11\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}\right)}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^3\left(\frac{11\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}\right)}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^3\left(\frac{11\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+\cdots}\cdots}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^3\left(\frac{11\left(\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\cdots}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^3\left(\frac{11\left(\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{\cdots}{9a(a\cdots}\cdots\right)}{4a}\cdots\right)}{d\cdots}$$

$\downarrow$ 61

$$-\frac{ia^3\left(\frac{11\left(\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan\cdots}\cdots}{2a}\cdots\right)}{4a}\cdots\right)}{d\cdots}$$

$\downarrow$ 73

$$-\frac{ia^3\left(\frac{11\left(\frac{\frac{\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\right.}{4a}}{d}$$

$$\Big\downarrow \color{blue}{219}$$

$$-\frac{ia^3\left(\frac{11\left(\frac{\frac{\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan(c\right.}{4a}}{d}$$

input

```
Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*a^3*(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(9/2)) + (
11*(-1/9*1/(a*(a + I*a*Tan[c + d*x])^(9/2)) + (-1/7*1/(a*(a + I*a*Tan[c +
d*x])^(7/2)) + (-1/5*1/(a*(a + I*a*Tan[c + d*x])^(5/2)) + (-1/3*1/(a*(a +
I*a*Tan[c + d*x])^(3/2)) + ((I*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sq
rt[2]*a^(3/2)) - 1/(a*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a))/(2*a))/(2*a))/(2
*a)))/(4*a)))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(187) = 374$.

Time = 9.44 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.92

method	result
default	$\frac{3465\tan(dx+c)\sec(dx+c)^2\left(1-8\cos(dx+c)^3-4\cos(dx+c)^2+4\cos(dx+c)\right)\operatorname{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)+i}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+3465i\operatorname{arctanh}\left(\mathrm{co}\right.}{}$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/40320/d/(-I+tan(d*x+c))^3/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)/(a*(1+I*tan(d*x+c)))^(1/2)*(3465*tan(d*x+c)*sec(d*x+c)^2*(1-8*cos(d*x+
c)^3-4*cos(d*x+c)^2+4*cos(d*x+c))*arctanh(1/2/(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(cot(d*x+c)-csc(d*x+c)+I))+3465*I*arctanh(1/2/(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+I))*(4+8*cos(d*x+c)-8*sec(d*x+c)-3*se
c(d*x+c)^2+sec(d*x+c)^3)+3465*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*tan(d*x+c)*(sec(d*x+c)^2-4)+3465*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(3*sec(d*x+c)^2-4)+154*I*tan(d*x+c)*sec(d*x+c)*(45+80*cos(d*x+c)^4+
80*cos(d*x+c)^3-276*cos(d*x+c)^2-96*cos(d*x+c))*(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)+2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-11352+3920*cos(d*x+c)^3+39
20*cos(d*x+c)^2-25212*cos(d*x+c)+12705*sec(d*x+c)+2310*sec(d*x+c)^2))/a^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.33

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-3465i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(9i\,dx+9i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^4de^{(2i\,dx+2i\,c)}+a^4d\right)\sqrt{\frac{}{e^{(2}}}\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/40320*(-3465*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(9*I*d*x + 9*I*c)*log
(4*(sqrt(2)*sqrt(1/2)*(a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
 + 3465*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(9*I*d*x + 9*I*c)*log(-4*(sq
rt(2)*sqrt(1/2)*(a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt(1/(a^7*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqr
t(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-315*I*e^(12*I*d*x + 12*I*c) + 430
3*I*e^(10*I*d*x + 10*I*c) + 7034*I*e^(8*I*d*x + 8*I*c) + 3754*I*e^(6*I*d*x
 + 6*I*c) + 1798*I*e^(4*I*d*x + 4*I*c) + 530*I*e^(2*I*d*x + 2*I*c) + 70*I)
)*e^(-9*I*d*x - 9*I*c)/(a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.82

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{i\left(\frac{4\left(3465\,(i\,a\tan(dx+c)+a)^5-4620\,(i\,a\tan(dx+c)+a)^4a-1848\,(i\,a\tan(dx+c)+a)^3a^2-1584\,(i\,a\tan\right.}{(i\,a\tan(dx+c)+a)^{\frac{11}{2}}a^2-2\,(i\,a\tan(dx+c)+a)^{\frac{9}{2}}}\right.}{8064}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
1/80640*I*(4*(3465*(I*a*tan(d*x + c) + a)^5 - 4620*(I*a*tan(d*x + c) + a)^
4*a - 1848*(I*a*tan(d*x + c) + a)^3*a^2 - 1584*(I*a*tan(d*x + c) + a)^2*a^
3 - 1760*(I*a*tan(d*x + c) + a)*a^4 - 2240*a^5)/((I*a*tan(d*x + c) + a)^(1
1/2)*a^2 - 2*(I*a*tan(d*x + c) + a)^(9/2)*a^3) + 3465*sqrt(2)*log(-(sqrt(2
)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*
x + c) + a)))/a^(5/2))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{7/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(7/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\cos(dx+c)^2\tan(dx+c)}{\tan(dx+c)^5 i+3\tan(dx+c)^4-2\tan(dx+c)^3 i+2\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)i-\left(\right.\right.}{a^4}$$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*(int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**2*tan(c + d*x))/(tan
(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)*
*2 - 3*tan(c + d*x)*i - 1),x)*i - int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*
x)**2)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*ta
n(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)))/a**4
```

3.384 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 310

$$\begin{aligned}
\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = &-\frac{195i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{1024\sqrt{2}a^{7/2}d} \\
&+\frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} \\
&-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}} \\
&+\frac{65ia}{192d(a+ia\tan(c+dx))^{9/2}}+\frac{195i}{896d(a+ia\tan(c+dx))^{7/2}} \\
&+\frac{39i}{256ad(a+ia\tan(c+dx))^{5/2}}+\frac{65i}{512a^2d(a+ia\tan(c+dx))^{3/2}} \\
&+\frac{195i}{1024a^3d\sqrt{a+ia\tan(c+dx)}} \\
&-\frac{15ia^5}{16d(a+ia\tan(c+dx))^{11/2}\left(a^3-ia^3\tan(c+dx)\right)}
\end{aligned}$$

output
```
-195/2048*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/
a^(7/2)/d+195/352*I*a^2/d/(a+I*a*tan(d*x+c))^(11/2)-1/4*I*a^4/d/(a-I*a*tan
(d*x+c))^2/(a+I*a*tan(d*x+c))^(11/2)+65/192*I*a/d/(a+I*a*tan(d*x+c))^(9/2)
+195/896*I/d/(a+I*a*tan(d*x+c))^(7/2)+39/256*I/a/d/(a+I*a*tan(d*x+c))^(5/2
)+65/512*I/a^2/d/(a+I*a*tan(d*x+c))^(3/2)+195/1024*I/a^3/d/(a+I*a*tan(d*x+
c))^(1/2)-15/16*I*a^5/d/(a+I*a*tan(d*x+c))^(11/2)/(a^3-I*a^3*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.67 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.17

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{ia^2\,\text{Hypergeometric2F1}\left(-\frac{11}{2},3,-\frac{9}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{44d(a+ia\tan(c+dx))^{11/2}}$$

input
```
Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((I/44)*a^2*Hypergeometric2F1[-11/2, 3, -9/2, (1 + I*Tan[c + d*x])/2])/(d*
(a + I*a*Tan[c + d*x])^(11/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 3968, 52, 52, 61, 61, 61, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(c+dx)^4(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{13/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \; 52$$

$$-\frac{ia^5\left(\frac{15\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{13/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}}\right)}{d}$$

$$\Big\downarrow \; 52$$

$$-\frac{ia^5\left(\frac{15\left(\frac{13\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{13/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia}\right.}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^5\left(\frac{15\left(\frac{13\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{11a(a+ia\tan(c+dx))^{11/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}}\right)}{8a}+\right.}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^5\left(\frac{15\left(\frac{13\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}}{2a}-\frac{1}{11a(a+ia\tan(c+dx))^{11/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+}\right)}{8a}}{d}$$

$\downarrow$ 61

$$-\frac{ia^5\left(\frac{15\left(\frac{13\left(\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}}{2a}-\frac{1}{11a(a+ia\tan(c+dx))^{11/2}}\right)}{4a}\right)}{8a}}{d}$$

$\downarrow$ 61

$$-\frac{ia^5\left(\frac{15\left(\frac{13\left(\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}}{2a}-\cdots}{4a}\cdots}{8a}\cdots\right)}{d}$$

$\Big\downarrow$ 61

$$-\frac{ia^5\left(\frac{15\left(\frac{13\left(\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-}{2a}\right.}{4a}\right.}{8a}\right.}{}$$

$\downarrow$ 61

$$-\frac{ia^5\left(\frac{15\left(\frac{13\left(\frac{\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+i\cdots}}{2a}}{2a}\cdots\right)}{4a}\cdots\right)}{8a}\cdots\right)}{\cdots}$$

$\downarrow$ 73

$$-\frac{ia^5\left(15\frac{\left(13\frac{\frac{\frac{\frac{\frac{\int\frac{1}{a^2\tan^2(c+dx)+2a}\,d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7}\cdots}}{2a}\cdots}{2a}\cdots\right)\cdots}{4a}\cdots\right)\cdots}{8a}\cdots}{\cdots}$$

↓ 219

$$-\frac{ia^5\left(\frac{15\left(\frac{13\left(\frac{\frac{\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{}{9a(a+ia\,\mathrm{t}}}{2a}\right.}{4a}\right.}{8a}\right.}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*a^5*(1/(4*a*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(11/2))
+ (15*(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(11/2)) + (13*
(-1/11*1/(a*(a + I*a*Tan[c + d*x])^(11/2)) + (-1/9*1/(a*(a + I*a*Tan[c + d
*x])^(9/2)) + (-1/7*1/(a*(a + I*a*Tan[c + d*x])^(7/2)) + (-1/5*1/(a*(a + I
*a*Tan[c + d*x])^(5/2)) + (-1/3*1/(a*(a + I*a*Tan[c + d*x])^(3/2)) + ((I*A
rcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[a + I
*a*Tan[c + d*x]]))/(2*a))/(2*a))/(2*a))/(2*a))/(2*a)))/(4*a)))/(8*a)))/d
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(247) = 494$.

Time = 10.43 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.62

method	result
default	$\frac{45045 \tan(dx+c) \sec(dx+c)^2 \left(-1+8\cos(dx+c)^3+4\cos(dx+c)^2-4\cos(dx+c)\right) \operatorname{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)+i}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+45045 i \operatorname{arctanh}}{}$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/473088/d/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(a*(1+I*tan(d
*x+c)))^(1/2)/(-tan(d*x+c)+I)^3*(45045*tan(d*x+c)*sec(d*x+c)^2*(-1+8*cos(d
*x+c)^3+4*cos(d*x+c)^2-4*cos(d*x+c))*arctanh(1/2/(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+I))+45045*I*arctanh(1/2/(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+I))*(-4-8*cos(d*x+c)+8*sec(d*x+c)
+3*sec(d*x+c)^2-sec(d*x+c)^3)+45045*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*tan(d*x+c)*(-sec(d*x+c)^2+4)+45045*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(-3*sec(d*x+c)^2+4)+14*I*tan(d*x+c)*sec(d*x+c)*(-6435-5760*
cos(d*x+c)^6-5760*cos(d*x+c)^5-11440*cos(d*x+c)^4-11440*cos(d*x+c)^3+39468
*cos(d*x+c)^2+13728*cos(d*x+c))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(147576-18816*cos(d*x+c)^5-18816*cos(d*x+c)^
4-50960*cos(d*x+c)^3-50960*cos(d*x+c)^2+327756*cos(d*x+c)-165165*sec(d*x+c
)-30030*sec(d*x+c)^2))/a^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.09

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-45045i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(11i\,dx+11i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^4de^{(2i\,dx+2i\,c)}+a^4d\right)\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/473088*(-45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(11*I*d*x + 11*I*c)
*log(4*(sqrt(2)*sqrt(1/2)*(a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I
*c)) + 45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(11*I*d*x + 11*I*c)*log
(-4*(sqrt(2)*sqrt(1/2)*(a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)
) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-462*I*e^(16*I*d*x + 16*I*c
) - 7161*I*e^(14*I*d*x + 14*I*c) + 47413*I*e^(12*I*d*x + 12*I*c) + 78800*I
*e^(10*I*d*x + 10*I*c) + 38512*I*e^(8*I*d*x + 8*I*c) + 19552*I*e^(6*I*d*x
+ 6*I*c) + 7184*I*e^(4*I*d*x + 4*I*c) + 1624*I*e^(2*I*d*x + 2*I*c) + 168*I
))*e^(-11*I*d*x - 11*I*c)/(a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.80

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{i\left(\frac{4\left(45045\,(i\,a\tan(dx+c)+a)^7-150150\,(i\,a\tan(dx+c)+a)^6a+96096\,(i\,a\tan(dx+c)+a)^5a^2+27456\,(\ldots\right.}{(i\,a\tan(dx+c)+a)^{\frac{15}{2}}a^2-4\ldots}\right.}{\ldots}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
1/946176*I*(4*(45045*(I*a*tan(d*x + c) + a)^7 - 150150*(I*a*tan(d*x + c) +
 a)^6*a + 96096*(I*a*tan(d*x + c) + a)^5*a^2 + 27456*(I*a*tan(d*x + c) + a
)^4*a^3 + 18304*(I*a*tan(d*x + c) + a)^3*a^4 + 16640*(I*a*tan(d*x + c) + a
)^2*a^5 + 17920*(I*a*tan(d*x + c) + a)*a^6 + 21504*a^7)/((I*a*tan(d*x + c)
 + a)^(15/2)*a^2 - 4*(I*a*tan(d*x + c) + a)^(13/2)*a^3 + 4*(I*a*tan(d*x +
c) + a)^(11/2)*a^4) + 45045*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d
*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(5/2))/(a*
d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^4}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}}\,dx$$

input
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(7/2), x)
```

Reduce [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\cos(dx+c)^4\tan(dx+c)}{\tan(dx+c)^5 i+3\tan(dx+c)^4-2\tan(dx+c)^3 i+2\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)i-\left(\right.\right.}{a^4}$$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*(int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**4*tan(c + d*x))/(tan
(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)*
*2 - 3*tan(c + d*x)*i - 1),x)*i - int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*
x)**4)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*ta
n(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)))/a**4
```

3.385 $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{64ia^3\sec^{13}(c+dx)}{3315d(a+ia\tan(c+dx))^{13/2}}+\frac{16ia^2\sec^{13}(c+dx)}{255d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}$$

output
```
64/3315*I*a^3*sec(d*x+c)^13/d/(a+I*a*tan(d*x+c))^(13/2)+16/255*I*a^2*sec(d
*x+c)^13/d/(a+I*a*tan(d*x+c))^(11/2)+2/17*I*a*sec(d*x+c)^13/d/(a+I*a*tan(d
*x+c))^(9/2)
```

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=-\frac{2\sec^{12}(c+dx)(68+263\cos(2(c+dx))+247i\sin(2(c+dx)))(\cos(3(c+dx))-i\sin(3(c+dx)))}{3315a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(-2*Sec[c + d*x]^12*(68 + 263*Cos[2*(c + d*x)] + (247*I)*Sin[2*(c + d*x)])
*(Cos[3*(c + d*x)] - I*Sin[3*(c + d*x)]))/(3315*a^3*d*(-I + Tan[c + d*x])^
3*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{13}}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{17}a\int \frac{\sec^{13}(c+dx)}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{17}a\int \frac{\sec(c+dx)^{13}}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{17}a\left(\frac{4}{15}a\int \frac{\sec^{13}(c+dx)}{(i\tan(c+dx)a+a)^{11/2}}dx+\frac{2ia\sec^{13}(c+dx)}{15d(a+ia\tan(c+dx))^{11/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{17}a\left(\frac{4}{15}a\int \frac{\sec(c+dx)^{13}}{(i\tan(c+dx)a+a)^{11/2}}dx+\frac{2ia\sec^{13}(c+dx)}{15d(a+ia\tan(c+dx))^{11/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8}{17}a\left(\frac{8ia^2\sec^{13}(c+dx)}{195d(a+ia\tan(c+dx))^{13/2}}+\frac{2ia\sec^{13}(c+dx)}{15d(a+ia\tan(c+dx))^{11/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}$$

input
```
Int[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((2*I)/17)*a*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (8*a*(((
(8*I)/195)*a^2*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(13/2)) + (((2*I
)/15)*a*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(11/2))))/17
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 10.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.30

method	result	size
default	$\frac{\frac{2\tan(dx+c)\sec(dx+c)^{8}\left(128\cos(dx+c)^{4}+176\cos(dx+c)^{2}+195\right)}{3315}+\frac{2i\left(128\sec(dx+c)^{4}+112\sec(dx+c)^{6}+91\sec(dx+c)^{8}\right)}{3315}}{d\sqrt{a(1+i\tan(dx+c))}\,a^{3}\left(4i\cos(dx+c)^{2}\sin(dx+c)+4\cos(dx+c)^{3}-i\sin(dx+c)-3\cos(dx+c)\right)}$	143

input
```
int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/3315/d/(a*(1+I*tan(d*x+c)))^(1/2)/a^3/(4*I*cos(d*x+c)^2*sin(d*x+c)+4*cos
(d*x+c)^3-I*sin(d*x+c)-3*cos(d*x+c))*(tan(d*x+c)*sec(d*x+c)^8*(128*cos(d*x
+c)^4+176*cos(d*x+c)^2+195)+I*(128*sec(d*x+c)^4+112*sec(d*x+c)^6+91*sec(d*
x+c)^8))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(86) = 172.

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.57

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx =$$

$$-\frac{512\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-255i\,e^{(4i\,dx+4i\,c)}-68i\,e^{(2i\,dx+2\right.}}{3315\left(a^{4}de^{(16i\,dx+16i\,c)}+8\,a^{4}de^{(14i\,dx+14i\,c)}+28\,a^{4}de^{(12i\,dx+12i\,c)}+56\,a^{4}de^{(10i\,dx+10i\,c)}+70\,a^{4}de^{(8i\,dx+8i\,c)}+\right.}$$

input
```
integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-512/3315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-255*I*e^(4*I*d*x + 4
*I*c) - 68*I*e^(2*I*d*x + 2*I*c) - 8*I)/(a^4*d*e^(16*I*d*x + 16*I*c) + 8*a
^4*d*e^(14*I*d*x + 14*I*c) + 28*a^4*d*e^(12*I*d*x + 12*I*c) + 56*a^4*d*e^(
10*I*d*x + 10*I*c) + 70*a^4*d*e^(8*I*d*x + 8*I*c) + 56*a^4*d*e^(6*I*d*x +
6*I*c) + 28*a^4*d*e^(4*I*d*x + 4*I*c) + 8*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*
d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**13/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(86) = 172$.

Time = 0.50 (sec) , antiderivative size = 902, normalized size of antiderivative = 8.20

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-2/3315*(-331*I*sqrt(a) - 998*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 18
38*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 7522*sqrt(a)*sin(d*x +
c)^3/(cos(d*x + c) + 1)^3 - 4836*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 - 27882*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8954*I*sqrt(a)*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 68926*sqrt(a)*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7 - 12631*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 1250
52*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 10540*I*sqrt(a)*sin(d*x +
 c)^10/(cos(d*x + c) + 1)^10 - 168980*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c
) + 1)^11 - 168980*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 10540*I
*sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 125052*sqrt(a)*sin(d*x +
c)^15/(cos(d*x + c) + 1)^15 + 12631*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c
) + 1)^16 - 68926*sqrt(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 + 8954*I*s
qrt(a)*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 - 27882*sqrt(a)*sin(d*x + c)^
19/(cos(d*x + c) + 1)^19 + 4836*I*sqrt(a)*sin(d*x + c)^20/(cos(d*x + c) +
1)^20 - 7522*sqrt(a)*sin(d*x + c)^21/(cos(d*x + c) + 1)^21 + 1838*I*sqrt(a
)*sin(d*x + c)^22/(cos(d*x + c) + 1)^22 - 998*sqrt(a)*sin(d*x + c)^23/(cos
(d*x + c) + 1)^23 + 331*I*sqrt(a)*sin(d*x + c)^24/(cos(d*x + c) + 1)^24)*(
sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(sin(d*x + c)/(cos(d*x + c) + 1
) - 1)^(7/2)/((a^4 - 12*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 66*a^4*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 - 220*a^4*sin(d*x + c)^6/(cos(d*x + ...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.70 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{512\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,68\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,255\mathrm{i}+8\mathrm{i}\right)}{3315\,a^4\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^8}$$

input
```
int(1/(cos(c + d*x)^13*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
(512*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*
2i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*68i + exp(c*4i + d*x*4i)*255i
 + 8i))/(3315*a^4*d*(exp(c*2i + d*x*2i) + 1)^8)
```

Reduce [F]

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x)
```

output

```
(sqrt(a)*( - 36*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**13*i - 23*int(( - s
qrt(tan(c + d*x)*i + 1)*sec(c + d*x)**13*tan(c + d*x)**4)/(tan(c + d*x)**5
*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c
 + d*x)*i - 1),x)*tan(c + d*x)**2*d - 23*int(( - sqrt(tan(c + d*x)*i + 1)*
sec(c + d*x)**13*tan(c + d*x)**4)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 -
 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d + 66
*int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**13*tan(c + d*x)**2)/(tan(c
 + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2
 - 3*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*d + 66*int(( - sqrt(tan(c + d*
x)*i + 1)*sec(c + d*x)**13*tan(c + d*x)**2)/(tan(c + d*x)**5*i + 3*tan(c +
 d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1)
,x)*d - 20*int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**13*tan(c + d*x))
/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c +
d*x)**2 - 3*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*d*i - 20*int(( - sqrt(t
an(c + d*x)*i + 1)*sec(c + d*x)**13*tan(c + d*x))/(tan(c + d*x)**5*i + 3*t
an(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*
i - 1),x)*d*i + int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**13)/(tan(c
+ d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2
- 3*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*d + int(( - sqrt(tan(c + d*x)*i
 + 1)*sec(c + d*x)**13)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(...
```

3.386 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 73

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{8ia^2\sec^{11}(c+dx)}{143d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}$$

output
```
8/143*I*a^2*sec(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(11/2)+2/13*I*a*sec(d*x+c)^
11/d/(a+I*a*tan(d*x+c))^(9/2)
```

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{2i\sec^9(c+dx)(\cos(2(c+dx))-i\sin(2(c+dx)))(-15i+11\tan(c+dx))}{143a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((-2*I)/143)*Sec[c + d*x]^9*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-15*
I + 11*Tan[c + d*x]))/(a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*
x]])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{11}}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{13}a\int \frac{\sec^{11}(c+dx)}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{13}a\int \frac{\sec(c+dx)^{11}}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8ia^2\sec^{11}(c+dx)}{143d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}$$

input
```
Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((8*I)/143)*a^2*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((2
*I)/13)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(9/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [A] (verified)

Time = 5.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.55

method	result
default	$\frac{\frac{128 i \sec(dx+c)}{143}+\frac{128 \sec(dx+c) \tan(dx+c)}{143}+\frac{16 i \sec(dx+c)^{3}}{143}+\frac{80 \tan(dx+c) \sec(dx+c)^{3}}{143}-\frac{136 i \sec(dx+c)^{5}}{143}-\frac{80 \tan(dx+c) \sec(dx+c)^{5}}{143}+\frac{2 i \sec(dx}{13}}{d\, a^{3} \sqrt{a(1+i \tan(dx+c))}}$

input
```
int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/143/d/a^3/(a*(1+I*tan(d*x+c)))^(1/2)*(64*I*sec(d*x+c)+64*sec(d*x+c)*tan(
d*x+c)+8*I*sec(d*x+c)^3+40*tan(d*x+c)*sec(d*x+c)^3-68*I*sec(d*x+c)^5-40*ta
n(d*x+c)*sec(d*x+c)^5+11*I*sec(d*x+c)^7)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(57) = 114$.

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.81

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{128\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-13i\,e^{(2i\,dx+2i\,c)}-2i\right)}{143\left(a^4de^{(12i\,dx+12i\,c)}+6\,a^4de^{(10i\,dx+10i\,c)}+15\,a^4de^{(8i\,dx+8i\,c)}+20\,a^4de^{(6i\,dx+6i\,c)}+15\,a^4de^{(4i\,dx+4i\,c)}+6\,a^4d\right.}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-128/143*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-13*I*e^(2*I*d*x + 2*I
*c) - 2*I)/(a^4*d*e^(12*I*d*x + 12*I*c) + 6*a^4*d*e^(10*I*d*x + 10*I*c) +
15*a^4*d*e^(8*I*d*x + 8*I*c) + 20*a^4*d*e^(6*I*d*x + 6*I*c) + 15*a^4*d*e^(
4*I*d*x + 4*I*c) + 6*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 764 vs. $2(57) = 114$.

Time = 0.38 (sec) , antiderivative size = 764, normalized size of antiderivative = 10.47

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-2/143*(-15*I*sqrt(a) - 38*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 88*I*
sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 278*sqrt(a)*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 - 213*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 -
920*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 272*I*sqrt(a)*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 - 1848*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 - 182*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2548*sqrt(a)*sin(
d*x + c)^9/(cos(d*x + c) + 1)^9 - 2548*sqrt(a)*sin(d*x + c)^11/(cos(d*x +
c) + 1)^11 + 182*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 1848*sq
rt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 272*I*sqrt(a)*sin(d*x + c)^1
4/(cos(d*x + c) + 1)^14 - 920*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)^1
5 + 213*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 278*sqrt(a)*sin(
d*x + c)^17/(cos(d*x + c) + 1)^17 + 88*I*sqrt(a)*sin(d*x + c)^18/(cos(d*x
+ c) + 1)^18 - 38*sqrt(a)*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 + 15*I*sqr
t(a)*sin(d*x + c)^20/(cos(d*x + c) + 1)^20)*(sin(d*x + c)/(cos(d*x + c) +
1) + 1)^(7/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)/((a^4 - 10*a^4*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 45*a^4*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4 - 120*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 210*a^4*sin(d*x + c)
^8/(cos(d*x + c) + 1)^8 - 252*a^4*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
210*a^4*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 120*a^4*sin(d*x + c)^14/(c
os(d*x + c) + 1)^14 + 45*a^4*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 10...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{128\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,13\mathrm{i}+2\mathrm{i}\right)\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}}{143\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}$$

input
```
int(1/(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
(128*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*13i + 2i)*(a - (a*(exp(c*2i
+ d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(143*a^4*d*(exp(c*
2i + d*x*2i) + 1)^6)
```

Reduce [F]

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x)
```

output

```
(sqrt(a)*( - 28*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**11*i - 19*int(( - s
qrt(tan(c + d*x)*i + 1)*sec(c + d*x)**11*tan(c + d*x)**4)/(tan(c + d*x)**5
*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c
 + d*x)*i - 1),x)*tan(c + d*x)**2*d - 19*int(( - sqrt(tan(c + d*x)*i + 1)*
sec(c + d*x)**11*tan(c + d*x)**4)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 -
 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d + 54
*int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**11*tan(c + d*x)**2)/(tan(c
 + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2
 - 3*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*d + 54*int(( - sqrt(tan(c + d*
x)*i + 1)*sec(c + d*x)**11*tan(c + d*x)**2)/(tan(c + d*x)**5*i + 3*tan(c +
 d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1)
,x)*d - 16*int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**11*tan(c + d*x))
/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c +
d*x)**2 - 3*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*d*i - 16*int(( - sqrt(t
an(c + d*x)*i + 1)*sec(c + d*x)**11*tan(c + d*x))/(tan(c + d*x)**5*i + 3*t
an(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*
i - 1),x)*d*i + int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**11)/(tan(c
+ d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2
- 3*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*d + int(( - sqrt(tan(c + d*x)*i
 + 1)*sec(c + d*x)**11)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(...
```

3.387 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{2ia\sec^9(c+dx)}{9d(a+ia\tan(c+dx))^{9/2}}$$

output
```
2/9*I*a*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(9/2)
```

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{2i\sec^7(c+dx)(i+\tan(c+dx))}{9a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((2*I)/9)*Sec[c + d*x]^7*(I + Tan[c + d*x]))/(a^3*d*(-I + Tan[c + d*x])^3
*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^9}{(a+ia\tan(c+dx))^{7/2}}dx$$
$$\Big\downarrow \text{3974}$$
$$\frac{2ia\sec^9(c+dx)}{9d(a+ia\tan(c+dx))^{9/2}}$$

input
```
Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((2*I)/9)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(29) = 58$.

Time = 5.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.46

method	result	size
default	$\frac{\frac{16i\sec(dx+c)}{9}+\frac{16\sec(dx+c)\tan(dx+c)}{9}-\frac{16i\sec(dx+c)^{3}}{9}-\frac{8\tan(dx+c)\sec(dx+c)^{3}}{9}+\frac{2i\sec(dx+c)^{5}}{9}}{d\,a^{3}\sqrt{a(1+i\tan(dx+c))}}$	86

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/9/d/a^3/(a*(1+I*tan(d*x+c)))^(1/2)*(8*I*sec(d*x+c)+8*sec(d*x+c)*tan(d*x+
c)-8*I*sec(d*x+c)^3-4*tan(d*x+c)*sec(d*x+c)^3+I*sec(d*x+c)^5)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(27) = 54$.

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.54

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{32i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{9\left(a^4de^{(8i\,dx+8i\,c)}+4\,a^4de^{(6i\,dx+6i\,c)}+6\,a^4de^{(4i\,dx+4i\,c)}+4\,a^4de^{(2i\,dx+2i\,c)}+a^4\right.}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
32/9*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(a^4*d*e^(8*I*d*x + 8*I*c
) + 4*a^4*d*e^(6*I*d*x + 6*I*c) + 6*a^4*d*e^(4*I*d*x + 4*I*c) + 4*a^4*d*e^
(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(27) = 54$.

Time = 0.26 (sec) , antiderivative size = 626, normalized size of antiderivative = 17.89

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-2/9*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 6*I*sqrt(a)
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 14*sqrt(a)*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 14*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 42*sqrt(a
)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*I*sqrt(a)*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 - 70*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 70*sqrt(
a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 14*I*sqrt(a)*sin(d*x + c)^10/(cos
(d*x + c) + 1)^10 - 42*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 14*
I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 14*sqrt(a)*sin(d*x + c)^
13/(cos(d*x + c) + 1)^13 + 6*I*sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^
14 - 2*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 + I*sqrt(a)*sin(d*x +
 c)^16/(cos(d*x + c) + 1)^16)*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)/((a^4 - 8*a^4*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 28*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 56*a^4*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^4*sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 - 56*a^4*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a^4*sin(d*x + c
)^12/(cos(d*x + c) + 1)^12 - 8*a^4*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 +
 a^4*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*d*(-2*I*sin(d*x + c)/(cos(d*x
+ c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{e^{-c\,5i-d\,x\,5i}\,\sqrt{a+\frac{a\,\sin(c+d\,x)\,1i}{\cos(c+d\,x)}}\,2i}{9\,a^4\,d\cos\left(c+d\,x\right)^4}$$

input
```
int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
(exp(- c*5i - d*x*5i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^(1/2)*2i)/(9*
a^4*d*cos(c + d*x)^4)
```

Reduce [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{too large to display}$$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*( - 20*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**9*i - 15*int(( - sq
rt(tan(c + d*x)*i + 1)*sec(c + d*x)**9*tan(c + d*x)**4)/(tan(c + d*x)**5*i
 + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c +
 d*x)*i - 1),x)*tan(c + d*x)**2*d - 15*int(( - sqrt(tan(c + d*x)*i + 1)*se
c(c + d*x)**9*tan(c + d*x)**4)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*
tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d + 42*in
t(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**9*tan(c + d*x)**2)/(tan(c + d
*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3
*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*d + 42*int(( - sqrt(tan(c + d*x)*i
 + 1)*sec(c + d*x)**9*tan(c + d*x)**2)/(tan(c + d*x)**5*i + 3*tan(c + d*x)
**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d
 - 12*int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**9*tan(c + d*x))/(tan(
c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**
2 - 3*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*d*i - 12*int(( - sqrt(tan(c +
 d*x)*i + 1)*sec(c + d*x)**9*tan(c + d*x))/(tan(c + d*x)**5*i + 3*tan(c +
d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),
x)*d*i + int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**9)/(tan(c + d*x)**
5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(
c + d*x)*i - 1),x)*tan(c + d*x)**2*d + int(( - sqrt(tan(c + d*x)*i + 1)*se
c(c + d*x)**9)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)*...
```

3.388 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 160

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{8i\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d} - \frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}} - \frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{8i\sec(c+dx)}{a^3d\sqrt{a+ia\tan(c+dx)}}$$

output
```
8*I*2^(1/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))/a^(7/2)/d-2/5*I*sec(d*x+c)^5/a/d/(a+I*a*tan(d*x+c))^(5/2)-4/3*I*sec(d*x
+c)^3/a^2/d/(a+I*a*tan(d*x+c))^(3/2)-8*I*sec(d*x+c)/a^3/d/(a+I*a*tan(d*x+c
))^(1/2)
```

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.81

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{128e^{7i(c+dx)}\left(-23-35e^{2i(c+dx)}-15e^{4i(c+dx)}+15\left(1+e^{2i(c+dx)}\right)^{5/2}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{15a^3d\left(1+e^{2i(c+dx)}\right)^6(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(-128*E^((7*I)*(c + d*x))*(-23 - 35*E^((2*I)*(c + d*x)) - 15*E^((4*I)*(c +
 d*x)) + 15*(1 + E^((2*I)*(c + d*x)))^(5/2)*ArcTanh[Sqrt[1 + E^((2*I)*(c +
 d*x))]]))/(15*a^3*d*(1 + E^((2*I)*(c + d*x)))^6*(-I + Tan[c + d*x])^3*Sqr
t[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 3972, 3042, 3972, 3042, 3972, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^7}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3972}$$

$$\frac{2\int \frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx}{a}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\int \frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^{5/2}}dx}{a}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3972}$$

$$\frac{2\left(\frac{2\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}\right)}{a}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2\left(\frac{2\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^{3/2}}dx}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}\right)}{a}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3972}$$

$$\frac{2\left(\frac{2\left(\frac{2\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{a}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}\right)}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}\right)}{a}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2\left(\frac{2\left(\frac{2\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{a}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}\right)}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}\right)}{a}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3970}$$

$$\frac{2\left(\frac{2\left(\frac{4i\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{ad}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}\right)}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}\right)}{a}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow 219$$

$$\frac{2\left(\frac{2\left(\frac{2i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}\right)}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}\right)}{a}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((-2*I)/5)*Sec[c + d*x]^5)/(a*d*(a + I*a*Tan[c + d*x])^(5/2)) + (2*((((-2
*I)/3)*Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + (2*(((2*I)*Sqr
t[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])
/(a^(3/2)*d) - ((2*I)*Sec[c + d*x])/(a*d*Sqrt[a + I*a*Tan[c + d*x]])))/a))
/a
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3972
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e
+ f*x])^(n + 1)/(b*f*(m - 2))), x] + Simp[2*(d^2/a)   Int[(d*Sec[e + f*x])^
(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && EqQ[m/2 + n, 0] && LtQ[n, -1]
```

Maple [A] (warning: unable to verify)

Time = 7.23 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.47

method	result
default	$\frac{8i\tan(dx+c)\operatorname{arctanh}\left(\frac{(i-\cot(dx+c)+\csc(dx+c))\sqrt{2}}{2\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-\frac{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\operatorname{arctanh}\left(\frac{}{2\sqrt{\cot(dx+c}}\right.}{}}{d\,a^3\sqrt{a(1+i\tan(dx+}}$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(8*I*tan(d*x+c)*arctanh(1/2/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(
d*x+c)^2-1)^(1/2)*(I-cot(d*x+c)+csc(d*x+c))*2^(1/2))*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)-2/15*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2/(cot(d*x
+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(I-cot(d*x+c)+csc(d*x+
c))*2^(1/2))*(-60-60*sec(d*x+c))-2/15*I*(76*sec(d*x+c)-3*sec(d*x+c)^3)-32/
15*sec(d*x+c)*tan(d*x+c))/a^3/(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(125) = 250.

Time = 0.11 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.04

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx =$$

$$-\frac{4\left(15\sqrt{2}\left(i\,a^4de^{(4i\,dx+4i\,c)}+2i\,a^4de^{(2i\,dx+2i\,c)}+i\,a^4d\right)\sqrt{\frac{1}{a^7d^2}}\log\left(-\frac{32\left(\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{a^7}\right.}{a^3d}\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-4/15*(15*sqrt(2)*(I*a^4*d*e^(4*I*d*x + 4*I*c) + 2*I*a^4*d*e^(2*I*d*x + 2*
I*c) + I*a^4*d)*sqrt(1/(a^7*d^2))*log(-32*((I*a^3*d*e^(2*I*d*x + 2*I*c) +
I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*
x - I*c)/(a^3*d)) + 15*sqrt(2)*(-I*a^4*d*e^(4*I*d*x + 4*I*c) - 2*I*a^4*d*e
^(2*I*d*x + 2*I*c) - I*a^4*d)*sqrt(1/(a^7*d^2))*log(-32*((-I*a^3*d*e^(2*I*
d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)
) - I)*e^(-I*d*x - I*c)/(a^3*d)) + 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*(15*I*e^(4*I*d*x + 4*I*c) + 35*I*e^(2*I*d*x + 2*I*c) + 23*I))/(a^4*d*
e^(4*I*d*x + 4*I*c) + 2*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec^7(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Integral(sec(c + d*x)**7/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1164 vs. $2(125) = 250$.

Time = 0.33 (sec) , antiderivative size = 1164, normalized size of antiderivative = 7.28

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-2/15*(15*(2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*
sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 1) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2
 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - 1) + (-I*sqrt(2)*cos(2*d*x + 2*c)^2 - I*sqrt(2)*sin(2*d*x
+ 2*c)^2 - 2*I*sqrt(2)*cos(2*d*x + 2*c) - I*sqrt(2))*log(sqrt(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 1) + (I*sqrt(2)*cos(2*d*x + 2*c)^2 + I*sqrt(2)*sin(2*d*x + 2*c)
^2 + 2*I*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2))*log(sqrt(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^7\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{7/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

Reduce [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*( - 14*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**7*i - 77*int((sqrt(
tan(c + d*x)*i + 1)*sec(c + d*x)**7*tan(c + d*x)**4)/(tan(c + d*x)**5*i +
3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*
x)*i - 1),x)*tan(c + d*x)**2*d - 77*int((sqrt(tan(c + d*x)*i + 1)*sec(c +
d*x)**7*tan(c + d*x)**4)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c
+ d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d + 224*int((sq
rt(tan(c + d*x)*i + 1)*sec(c + d*x)**7*tan(c + d*x)**3)/(tan(c + d*x)**5*i
 + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c +
 d*x)*i - 1),x)*tan(c + d*x)**2*d*i + 224*int((sqrt(tan(c + d*x)*i + 1)*se
c(c + d*x)**7*tan(c + d*x)**3)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*
tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d*i + 210
*int((sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**7*tan(c + d*x)**2)/(tan(c + d
*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3
*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*d + 210*int((sqrt(tan(c + d*x)*i +
 1)*sec(c + d*x)**7*tan(c + d*x)**2)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**
4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d -
 48*int((sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**7*tan(c + d*x))/(tan(c + d
*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3
*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*d*i - 48*int((sqrt(tan(c + d*x)*i
+ 1)*sec(c + d*x)**7*tan(c + d*x))/(tan(c + d*x)**5*i + 3*tan(c + d*x)*...
```

3.389 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{3i\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d} - \frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}} + \frac{6i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}}$$

output
```
-3*I*2^(1/2)*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/
2))/a^(7/2)/d-2*I*sec(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^(5/2)+6*I*sec(d*x+c)
/a^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{16e^{5i(c+dx)}\left(-1-3e^{2i(c+dx)}+3e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\text{arctanh}\left(\sqrt{1+e^{2i(c+d}\right.\right.}{a^3d\left(1+e^{2i(c+dx)}\right)^4(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(16*E^((5*I)*(c + d*x))*(-1 - 3*E^((2*I)*(c + d*x)) + 3*E^((2*I)*(c + d*x)
)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(
a^3*d*(1 + E^((2*I)*(c + d*x)))^4*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c
 + d*x]])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.38, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 3982, 3042, 3982, 3042, 3983, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3982}$$

$$\frac{6\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx}{a}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^{5/2}}dx}{a}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3982}$$

$$\frac{6\left(\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{a}\right)}{a}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\left(\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{a}\right)}{a}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{6\left(\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\left(\frac{\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{a}\right)}{a}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\left(\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\left(\frac{\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{a}\right)}{a}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3970}$$

$$\frac{6\left(\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\left(\frac{i\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{2ad}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{a}\right)}{a}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{6\left(\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\left(\frac{i\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{a}\right)}{a}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-2*I)*Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^(5/2)) + (6*(((2*I)*Se
c[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) - (2*(((I/2)*ArcTanh[(Sqrt[
a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*a^(3/2)*d
) + ((I/2)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(3/2))))/a))/a
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(102) = 204.

Time = 7.26 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.21

method	result
default	$\frac{2\cos(dx+c)+4+2\sec(dx+c)-3\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{\cos(dx+c)+1}}\,(\sin(dx+c)+\tan(dx+c))\,\operatorname{arctanh}\left(\frac{(i-\cot(dx+c)+\csc(dx+c))\sqrt{2}}{2\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+}}\right.}{d(-i\cos(dx+c)+\sin(}$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(-I*cos(d*x+c)+sin(d*x+c)-I)/a^3/(a*(1+I*tan(d*x+c)))^(1/2)*(2*cos(d*x
+c)+4+2*sec(d*x+c)-3*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(sin(d*x
+c)+tan(d*x+c))*arctanh(1/2/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+
c)^2-1)^(1/2)*(I-cot(d*x+c)+csc(d*x+c))*2^(1/2))+3*I*2^(1/2)*(cos(d*x+c)+1
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2/(cot(d*x+c)^2-2*cot(d*x
+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(I-cot(d*x+c)+csc(d*x+c))*2^(1/2))+I*
(-2*sin(d*x+c)+2*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(96) = 192.

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-3i\sqrt{2}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(2i\,dx+2i\,c)}\log\left(-\frac{12\left(\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{}{a}}\right.}{a^3d}\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/2*(-3*I*sqrt(2)*a^4*d*sqrt(1/(a^7*d^2))*e^(2*I*d*x + 2*I*c)*log(-12*((I*
a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t(1/(a^7*d^2)) + I)*e^(-I*d*x - I*c)/(a^3*d)) + 3*I*sqrt(2)*a^4*d*sqrt(1/(
a^7*d^2))*e^(2*I*d*x + 2*I*c)*log(-12*((-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a
^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) + I)*e^(-I*d*x -
 I*c)/(a^3*d)) - 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-3*I*e^(2*I*
d*x + 2*I*c) - I))*e^(-2*I*d*x - 2*I*c)/(a^4*d)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec^5{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Integral(sec(c + d*x)**5/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^5\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{7/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

Reduce [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x)
```

output

```
(sqrt(a)*( - 16*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**5*i - 63*int((sqrt(
tan(c + d*x)*i + 1)*sec(c + d*x)**5*tan(c + d*x)**4)/(tan(c + d*x)**5*i +
3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*
x)*i - 1),x)*tan(c + d*x)**2*d - 63*int((sqrt(tan(c + d*x)*i + 1)*sec(c +
d*x)**5*tan(c + d*x)**4)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c
+ d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d + 306*int((sq
rt(tan(c + d*x)*i + 1)*sec(c + d*x)**5*tan(c + d*x)**3)/(tan(c + d*x)**5*i
 + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c +
 d*x)*i - 1),x)*tan(c + d*x)**2*d*i + 306*int((sqrt(tan(c + d*x)*i + 1)*se
c(c + d*x)**5*tan(c + d*x)**3)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*
tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d*i + 144
*int((sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**5*tan(c + d*x)**2)/(tan(c + d
*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3
*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*d + 144*int((sqrt(tan(c + d*x)*i +
 1)*sec(c + d*x)**5*tan(c + d*x)**2)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**
4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d -
 62*int((sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**5*tan(c + d*x))/(tan(c + d
*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3
*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*d*i - 62*int((sqrt(tan(c + d*x)*i
+ 1)*sec(c + d*x)**5*tan(c + d*x))/(tan(c + d*x)**5*i + 3*tan(c + d*x)*...
```

3.390 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 125

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}a^{7/2}d} + \frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}} - \frac{i\sec(c+dx)}{8a^2d(a+ia\tan(c+dx))^{3/2}}$$

output
```
-1/16*I*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2
^(1/2)/a^(7/2)/d+1/2*I*sec(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(5/2)-1/8*I*sec(d
*x+c)/a^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.96

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{i\sec^3(c+dx)\left(-3+e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)-3\,\text{co}\right.}{16a^3d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+a}}$$

input
```
Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((I/16)*Sec[c + d*x]^3*(-3 + E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*
x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 3*Cos[2*(c + d*x)] + I*Sin[2
*(c + d*x)]))/(a^3*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 3982, 3042, 3983, 3042, 3983, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^3}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3982}$$

$$\frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}}-\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}}-\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx}{3a}$$

$$\Big\downarrow \text{3983}$$

$$\frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}}-\frac{2\left(\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}}-\frac{2\left(\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{3a}$$

$$\Big\downarrow \; 3983$$

$$\frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}}-\frac{2\left(\frac{3\left(\frac{\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{3a}$$

$$\Big\downarrow \; 3042$$

$$\frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}}-\frac{2\left(\frac{3\left(\frac{\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{3a}$$

$$\Big\downarrow \; 3970$$

$$\frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}}-\frac{2\left(\frac{3\left(\frac{i\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{2ad}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{3a}$$

$$\Big\downarrow \; 219$$

$$\frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}}-\frac{2\left(\frac{3\left(\frac{i\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{3a}$$

input
```
Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((2*I)/3)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(5/2)) - (2*(((I/4)*S
ec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (3*(((I/2)*ArcTanh[(Sqrt[a
]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*a^(3/2)*d)
 + ((I/2)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(3/2))))/(8*a)))/(3*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(100) = 200$.

Time = 8.44 (sec) , antiderivative size = 469, normalized size of antiderivative = 3.75

method	result
default	$\frac{\tan(dx+c)\sec(dx+c)^2\left(-1+8\cos(dx+c)^3+4\cos(dx+c)^2-4\cos(dx+c)\right)\operatorname{arctanh}\left(\frac{(i-\cot(dx+c)+\csc(dx+c))\sqrt{2}}{2\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-}}\right.}{}$

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/d/(-tan(d*x+c)+I)^3/(a*(1+I*tan(d*x+c)))^(1/2)/(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)/a^3/(cos(d*x+c)+1)*(tan(d*x+c)*sec(d*x+c)^2*(-1+8*cos(d*x+c)^
3+4*cos(d*x+c)^2-4*cos(d*x+c))*arctanh(1/2/(cot(d*x+c)^2-2*cot(d*x+c)*csc(
d*x+c)+csc(d*x+c)^2-1)^(1/2)*(I-cot(d*x+c)+csc(d*x+c))*2^(1/2))+I*arctanh(
1/2/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(I-cot(d*x
+c)+csc(d*x+c))*2^(1/2))*(-4-8*cos(d*x+c)+8*sec(d*x+c)+3*sec(d*x+c)^2-sec(
d*x+c)^3)+I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(8*sin(d*x+c)-4*s
ec(d*x+c)*tan(d*x+c))+2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(8*cos(
d*x+c)-8*sec(d*x+c)+sec(d*x+c)^3)+I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-1
6*sin(d*x+c)+4*tan(d*x+c)+12*sec(d*x+c)*tan(d*x+c))+(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(-16*cos(d*x+c)+4+20*sec(d*x+c)+2*sec(d*x+c)^2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(94) = 188$.

Time = 0.08 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.14

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(4i\,dx+4i\,c)}\log\left(-\frac{\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right.}{4\,a^3d}\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/16*(-I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(4*I*d*x + 4*I*c)*log(-1/4*(s
qrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) + I)*e^(-I*d*x - I*c)/(a^3*d)) + I*sqrt
(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(4*I*d*x + 4*I*c)*log(-1/4*(sqrt(2)*sqrt(1
/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt(1/(a^7*d^2)) + I)*e^(-I*d*x - I*c)/(a^3*d)) + sqrt(2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*(I*e^(4*I*d*x + 4*I*c) + 3*I*e^(2*I*d*x + 2*I*c) +
 2*I))*e^(-4*I*d*x - 4*I*c)/(a^4*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec^3(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Integral(sec(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(94) = 188$.

Time = 0.30 (sec) , antiderivative size = 977, normalized size of antiderivative = 7.82

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-1/64*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(3/4)*((-I*sqrt(2)*cos(4*d*x + 4*c) - sqrt(2)*sin(4*d*x + 4*c))*cos(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*cos(4*d*x + 4*
c) - I*sqrt(2)*sin(4*d*x + 4*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)))*sqrt(a) + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*((-I*sqrt(2)*cos(4*d*x + 4*c) - sqrt(2)*sin(4*
d*x + 4*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sq
rt(2)*cos(4*d*x + 4*c) - I*sqrt(2)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) - (2*sqrt(2)*arctan2((cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{7/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{\int \frac{\sec(dx+c)^3}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^3 i+3\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-3\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{a}\,a^3}$$

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
( - int(sec(c + d*x)**3/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3*i + 3*sq
rt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 3*sqrt(tan(c + d*x)*i + 1)*tan(c
+ d*x)*i - sqrt(tan(c + d*x)*i + 1)),x))/(sqrt(a)*a**3)
```

3.391 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 157

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{5i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}} + \frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}} + \frac{5i\sec(c+dx)}{64a^2d(a+ia\tan(c+dx))^{3/2}}$$

output
```
5/128*I*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2
^(1/2)/a^(7/2)/d+1/6*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(7/2)+5/48*I*sec(d*
x+c)/a/d/(a+I*a*tan(d*x+c))^(5/2)+5/64*I*sec(d*x+c)/a^2/d/(a+I*a*tan(d*x+c
))^(3/2)
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.76

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{\sec^3(c+dx)\left(52+\frac{30e^{4i(c+dx)}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}}+82\cos(2(c+dx))+50i\sin(2(c+dx))\right)}{384a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
-1/384*(Sec[c + d*x]^3*(52 + (30*E^((4*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((
2*I)*(c + d*x))]])/Sqrt[1 + E^((2*I)*(c + d*x))] + 82*Cos[2*(c + d*x)] + (
50*I)*Sin[2*(c + d*x)]))/(a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c +
 d*x]])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{5\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx}{12a}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx}{12a}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{5\left(\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{12a}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{12a}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{5\left(\frac{3\left(\frac{\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{12a}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3\left(\frac{\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{12a}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3970}$$

$$\frac{5\left(\frac{3\left(\frac{i\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{2ad}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{12a}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{5\left(\frac{3\left(\frac{i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{12a}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((I/6)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (5*(((I/4)*Sec[c +
 d*x])/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (3*(((I/2)*ArcTanh[(Sqrt[a]*Sec[
c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*a^(3/2)*d) + ((I
/2)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(3/2))))/(8*a)))/(12*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(126) = 252$.

Time = 8.06 (sec) , antiderivative size = 471, normalized size of antiderivative = 3.00

method	result
default	$-\frac{\tan(dx+c)\sec(dx+c)^2\left(120\cos(dx+c)^3+60\cos(dx+c)^2-60\cos(dx+c)-15\right)\operatorname{arctanh}\left(\frac{(i-\cot(dx+c)+\csc(dx+c))\sqrt{2}}{2\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx}}\right.}{}$

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384/d/(-tan(d*x+c)+I)^3/(a*(1+I*tan(d*x+c)))^(1/2)/(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)/a^3/(cos(d*x+c)+1)*(tan(d*x+c)*sec(d*x+c)^2*(120*cos(d*x+c)
^3+60*cos(d*x+c)^2-60*cos(d*x+c)-15)*arctanh(1/2/(cot(d*x+c)^2-2*cot(d*x+c
)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(I-cot(d*x+c)+csc(d*x+c))*2^(1/2))+I*ar
ctanh(1/2/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(I-c
ot(d*x+c)+csc(d*x+c))*2^(1/2))*(-120*cos(d*x+c)-60+120*sec(d*x+c)+45*sec(d
*x+c)^2-15*sec(d*x+c)^3)+I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
20*sin(d*x+c)-60*sec(d*x+c)*tan(d*x+c))+2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(120*cos(d*x+c)-120*sec(d*x+c)+15*sec(d*x+c)^3)+I*(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(-240*sin(d*x+c)-100*tan(d*x+c)+20*sec(d*x+c)*tan(d*
x+c))+(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-240*cos(d*x+c)-164+76*sec(d*x+c
)+30*sec(d*x+c)^2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(118) = 236$.

Time = 0.11 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.77

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-15i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(6i\,dx+6i\,c)}\log\left(-\frac{5\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c}}}\cdots\right.}{32\,a^3d}\cdots\right.\right.}{\cdots}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/384*(-15*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(6*I*d*x + 6*I*c)*log(-5/
32*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a^3*d)) + 1
5*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(6*I*d*x + 6*I*c)*log(-5/32*(sqrt(
2)*sqrt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a^3*d)) + sqrt(2)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(33*I*e^(6*I*d*x + 6*I*c) + 59*I*e^(4*I*d
*x + 4*I*c) + 34*I*e^(2*I*d*x + 2*I*c) + 8*I))*e^(-6*I*d*x - 6*I*c)/(a^4*d
)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Integral(sec(c + d*x)/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec(dx+c)}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{7/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*( - 10*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)*i - int(( - sqrt(tan
(c + d*x)*i + 1)*sec(c + d*x)*tan(c + d*x)**4)/(tan(c + d*x)**5*i + 3*tan(
c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i -
 1),x)*tan(c + d*x)**2*d - int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)*t
an(c + d*x)**4)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3
*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d + 6*int(( - sqrt(tan(c
 + d*x)*i + 1)*sec(c + d*x)*tan(c + d*x)**2)/(tan(c + d*x)**5*i + 3*tan(c
+ d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1
),x)*tan(c + d*x)**2*d + 6*int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)*t
an(c + d*x)**2)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3
*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d - 4*int(( - sqrt(tan(c
 + d*x)*i + 1)*sec(c + d*x)*tan(c + d*x))/(tan(c + d*x)**5*i + 3*tan(c + d
*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x
)*tan(c + d*x)**2*d*i - 4*int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)*ta
n(c + d*x))/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i +
 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*d*i - int(( - sqrt(tan(c + d
*x)*i + 1)*sec(c + d*x))/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c
+ d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2
*d - int(( - sqrt(tan(c + d*x)*i + 1)*sec(c + d*x))/(tan(c + d*x)**5*i + 3
*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2 - 3*tan(c + ...
```

3.392 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 227

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx &= \frac{315i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2048\sqrt{2}a^{7/2}d} \\ &+ \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}} + \frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}} \\ &+ \frac{21i\cos(c+dx)}{256a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{105i\cos(c+dx)}{1024a^3d\sqrt{a+ia\tan(c+dx)}} \\ &- \frac{315i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2048a^4d}\end{aligned}$$

output
```
315/4096*I*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
)*2^(1/2)/a^(7/2)/d+1/8*I*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(7/2)+3/32*I*cos
(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(5/2)+21/256*I*cos(d*x+c)/a^2/d/(a+I*a*tan(
d*x+c))^(3/2)+105/1024*I*cos(d*x+c)/a^3/d/(a+I*a*tan(d*x+c))^(1/2)-315/204
8*I*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^4/d
```

Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.62

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx =$$
$$-\frac{\sec^3(c+dx)\left(420+\frac{630e^{4i(c+dx)}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}}+826\cos(2(c+dx))-224\cos(4(c+dx))+474i\sin\right.}{4096a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
-1/4096*(Sec[c + d*x]^3*(420 + (630*E^((4*I)*(c + d*x))*ArcTanh[Sqrt[1 + E
^((2*I)*(c + d*x))]])/Sqrt[1 + E^((2*I)*(c + d*x))] + 826*Cos[2*(c + d*x)]
 - 224*Cos[4*(c + d*x)] + (474*I)*Sin[2*(c + d*x)] - (288*I)*Sin[4*(c + d*
x)]))/(a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(c+dx)(a+ia\tan(c+dx))^{7/2}}dx$$
$$\Big\downarrow \text{3983}$$
$$\frac{9\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$
$$\Big\downarrow \text{3042}$$

$$\frac{9\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^{5/2}}dx}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\left(\frac{7\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}\right)}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\left(\frac{7\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^{3/2}}dx}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}\right)}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\left(\frac{7\left(\frac{5\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}\right)}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\left(\frac{7\left(\frac{5\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}\right)}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}\right)}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}\right)}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \textbf{3971}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}\right)}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}\right)}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \textbf{3970}$$

$$9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i}{6d(a+i}\right.$$

$$\overline{16a}$$

$$\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{i\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(}{6d(a+ia\tan}}{16a}$$

$$\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((I/8)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (9*(((I/6)*Cos[c +
 d*x])/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (7*(((I/4)*Cos[c + d*x])/(d*(a +
 I*a*Tan[c + d*x])^(3/2)) + (5*(((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a*Tan[c
 + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a
+ I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c +
d*x]])/d))/(4*a)))/(8*a)))/(12*a)))/(16*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(184) = 368$.

Time = 10.00 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.17

method	result
default	$-\frac{\tan(dx+c)\sec(dx+c)^2\left(2520\cos(dx+c)^3+1260\cos(dx+c)^2-1260\cos(dx+c)-315\right)\operatorname{arctanh}\left(\frac{(i-\cot(dx+c)+\csc(dx+c)}{2\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)}}\right.}{}$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4096/d/(cos(d*x+c)+1)/(-tan(d*x+c)+I)^3/(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)/a^3/(a*(1+I*tan(d*x+c)))^(1/2)*(tan(d*x+c)*sec(d*x+c)^2*(2520*cos(d*x+
c)^3+1260*cos(d*x+c)^2-1260*cos(d*x+c)-315)*arctanh(1/2/(cot(d*x+c)^2-2*co
t(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2)*(I-cot(d*x+c)+csc(d*x+c))*2^(1/2
))+I*arctanh(1/2/(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/
2)*(I-cot(d*x+c)+csc(d*x+c))*2^(1/2))*(-2520*cos(d*x+c)-1260+2520*sec(d*x+
c)+945*sec(d*x+c)^2-315*sec(d*x+c)^3)+I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(2520*sin(d*x+c)-1260*sec(d*x+c)*tan(d*x+c))+2^(1/2)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(2520*cos(d*x+c)-2520*sec(d*x+c)+315*sec(d*x+c)
^3)+I*tan(d*x+c)*sec(d*x+c)*(2304*cos(d*x+c)^3-2736*cos(d*x+c)^2-2100*cos(
d*x+c)+420)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1792*cos(d*x+c)^2-3248*cos(d*x+c)-3444+1596*sec(d*x+c)+630*sec(d*
x+c)^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.32

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-315i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(8i\,dx+8i\,c)}\log\left(-\frac{315\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{}{e^{(2i\,dx}}}\right.}{1024\,a^3d}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/4096*(-315*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(8*I*d*x + 8*I*c)*log(-
315/1024*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a^3*d
)) + 315*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(8*I*d*x + 8*I*c)*log(-315/
1024*(sqrt(2)*sqrt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a^3*d))
+ sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-128*I*e^(10*I*d*x + 10*I*c)
+ 197*I*e^(8*I*d*x + 8*I*c) + 535*I*e^(6*I*d*x + 6*I*c) + 298*I*e^(4*I*d*x
 + 4*I*c) + 104*I*e^(2*I*d*x + 2*I*c) + 16*I))*e^(-8*I*d*x - 8*I*c)/(a^4*d
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2779 vs. $2(172) = 344$.

Time $= 0.36$ (sec) , antiderivative size $= 2779$, normalized size of antiderivative $= 12.24$

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-1/16384*(4*(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(
1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin
(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(3/4)*(325*((-I*sqrt(2)*cos(8*d*x +
 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*
d*x + 8*c)))^2 + (-I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*
sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*(-I*sqrt(2)*cos
(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c)
, cos(8*d*x + 8*c))) - I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*
c))*cos(7/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))),
cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + 643*(-I*sqrt(
2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(3/2*arctan2(sin(1/4*ar
ctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*
c), cos(8*d*x + 8*c))) + 1)) + 325*((sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*
sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 +
 (sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(1/4*arctan2(s
in(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*(sqrt(2)*cos(8*d*x + 8*c) - I*sq
rt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)
)) + sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(7/2*arctan
2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(si
n(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + 643*(sqrt(2)*cos(8*d*x + 8*c...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}}\,dx$$

input
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(7/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\cos(dx+c)\tan(dx+c)}{\tan(dx+c)^5 i+3\tan(dx+c)^4-2\tan(dx+c)^3 i+2\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)i-\left(\right.\right.}{a^4}$$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*(int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)*tan(c + d*x))/(tan(c
+ d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)**2
- 3*tan(c + d*x)*i - 1),x)*i - int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*x))
/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c +
d*x)**2 - 3*tan(c + d*x)*i - 1),x)))/a**4
```

3.393 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 307

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{3003i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16384\sqrt{2}a^{7/2}d} + \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} + \frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{1001i\cos(c+dx)}{8192a^3d\sqrt{a+ia\tan(c+dx)}} + \frac{429i\cos^3(c+dx)}{5120a^3d\sqrt{a+ia\tan(c+dx)}} - \frac{3003i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16384a^4d} - \frac{1001i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{10240a^4d}$$

output
```
3003/32768*I*arctanh(1/2*a^(1/2)*sec(d*x+c)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/
2))*2^(1/2)/a^(7/2)/d+1/10*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(7/2)+13/16
0*I*cos(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^(5/2)+143/1920*I*cos(d*x+c)^3/a^2/
d/(a+I*a*tan(d*x+c))^(3/2)+1001/8192*I*cos(d*x+c)/a^3/d/(a+I*a*tan(d*x+c))
^(1/2)+429/5120*I*cos(d*x+c)^3/a^3/d/(a+I*a*tan(d*x+c))^(1/2)-3003/16384*I
*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^4/d-1001/10240*I*cos(d*x+c)^3*(a+I*
a*tan(d*x+c))^(1/2)/a^4/d
```

Mathematica [A] (verified)

Time = 2.96 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.57

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx =$$

$$-\frac{\left(42140+20048e^{-2i(c+dx)}+71190e^{2i(c+dx)}+5856e^{-4i(c+dx)}-48640e^{4i(c+dx)}+768e^{-6i(c+dx)}-2560e^{6i(c+\cdots}\right.}{491520a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
-1/491520*((42140 + 20048/E^((2*I)*(c + d*x)) + 71190*E^((2*I)*(c + d*x))
+ 5856/E^((4*I)*(c + d*x)) - 48640*E^((4*I)*(c + d*x)) + 768/E^((6*I)*(c +
 d*x)) - 2560*E^((6*I)*(c + d*x)) + (90090*E^((4*I)*(c + d*x))*ArcTanh[Sqr
t[1 + E^((2*I)*(c + d*x))]])/Sqrt[1 + E^((2*I)*(c + d*x))])*Sec[c + d*x]^3
)/(a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.654$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{13\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx}{20a}+\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^{5/2}}dx}{20a}+\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{13\left(\frac{11\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}\right)}{20a}+\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13\left(\frac{11\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^{3/2}}dx}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}\right)}{20a}+\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{13\left(\frac{11\left(\frac{3\int \frac{\cos^3(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}\right)}{20a}+\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13\left(\frac{11\left(\frac{3\int \frac{1}{\sec(c+dx)^3\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}\right)}{20a}+\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\int \cos^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}\right)}{20a}+\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^3}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}\right)}{20a}+\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))}\right.}{20a}$$

$$\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(\cdots}{8d(a+ia\tan\cdots}\right.}{20a}$$

$$\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \; 3983$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx}{6d(a+ia\tan(c+d\cdots}\right.}{16a}\right.}{20a}$$

$$\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}}{20a}$$

$$\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3971}$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+}{4d\sqrt{a+ia\tan}}\right)}{4a}\right)}{16a}}{20a}$$

$$\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \ 3042$$

$$13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+\cdots}{4d\sqrt{a+ia\tan\cdots}}\cdots}{4a}\cdots}{16a}\cdots}{20a}\cdots\right.$$

$$\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \ 3970$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}}{8a}}{4a}}{16a}}{20a}$$

$$\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{i\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}\right)}{4a}\right)}{16a}\right)}{20a}$$

$$\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((I/10)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (13*(((I/8)*Cos
[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (11*(((I/6)*Cos[c + d*x]^3
)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (3*(((I/4)*Cos[c + d*x]^3)/(d*Sqrt[a
+ I*a*Tan[c + d*x]]) + (7*(((-1/3*I)*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d
*x]])/d + (5*a*(((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*(
(I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*
x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d))/(4*a)
))/6))/(8*a)))/(4*a)))/(16*a)))/(20*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(252) = 504$.

Time = 9.76 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.80

method	result
default	$\frac{\left(360360 \cos(dx+c)^{3}+180180 \cos(dx+c)^{2}-180180 \cos(dx+c)-45045\right) \sin(dx+c) \operatorname{arctanh}\left(\frac{-i+\cot(dx+c)-\csc(dx+c)}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+i\left(-360360\right.}{}$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/491520/d*((360360*cos(d*x+c)^3+180180*cos(d*x+c)^2-180180*cos(d*x+c)-450
45)*sin(d*x+c)*arctanh(1/2/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-I+cot(d*x+
c)-csc(d*x+c)))+I*(-360360*cos(d*x+c)^4-180180*cos(d*x+c)^3+360360*cos(d*x
+c)^2+135135*cos(d*x+c)-45045)*arctanh(1/2/(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(-I+cot(d*x+c)-csc(d*x+c)))+I*sin(d*x+c)*cos(d*x+c)*(-360360*cos(d*x+c
)^2+180180)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(-360360*cos(d*x+
c)^4+360360*cos(d*x+c)^2-45045)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)+I*sin(d*x+c)*cos(d*x+c)*(-106496*cos(d*x+c)^5-106496*cos(d*x+c)^4-32947
2*cos(d*x+c)^3+391248*cos(d*x+c)^2+300300*cos(d*x+c)-60060)*(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)+cos(d*x+c)*(-57344*cos(d*x+c)^6-57344*cos(d*x+c)^5-25
6256*cos(d*x+c)^4+464464*cos(d*x+c)^3+492492*cos(d*x+c)^2-228228*cos(d*x+c
)-90090)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))/(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)/((4*cos(d*x+c)^3+4*cos(d*x+c)^2-cos(d*x+c)-1)*sin(d*x+c)+I*cos(d*x+c
)*(-4*cos(d*x+c)^3-4*cos(d*x+c)^2+3*cos(d*x+c)+3))/(a*(1+I*tan(d*x+c)))^(1
/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.05

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-45045i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(10i\,dx+10i\,c)}\log\left(-\frac{3003\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\cdots}\right.}{8192\,a\cdots}\right.\right.}{\cdots}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/491520*(-45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(10*I*d*x + 10*I*c)
*log(-3003/8192*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)
/(a^3*d)) + 45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(10*I*d*x + 10*I*c
)*log(-3003/8192*(sqrt(2)*sqrt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*
d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*
c)/(a^3*d)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-1280*I*e^(14*I*d
*x + 14*I*c) - 25600*I*e^(12*I*d*x + 12*I*c) + 11275*I*e^(10*I*d*x + 10*I*
c) + 56665*I*e^(8*I*d*x + 8*I*c) + 31094*I*e^(6*I*d*x + 6*I*c) + 12952*I*e
^(4*I*d*x + 4*I*c) + 3312*I*e^(2*I*d*x + 2*I*c) + 384*I))*e^(-10*I*d*x - 1
0*I*c)/(a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5821 vs. $2(236) = 472$.

Time = 0.54 (sec) , antiderivative size = 5821, normalized size of antiderivative = 18.96

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-1/1966080*(40*(cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2
 + sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/5*
arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)^(3/4)*((79*(-I*sqrt(
2)*cos(10*d*x + 10*c) - sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10
*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 79*(-I*sqrt(2)*cos(10*d*x + 10*c) -
 sqrt(2)*sin(10*d*x + 10*c))*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*
x + 10*c)))^2 + 837*(-I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10
*d*x + 10*c)))^2 - I*sqrt(2)*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*
x + 10*c)))^2 - 2*I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x
 + 10*c))) - I*sqrt(2))*cos(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 1
0*c))) + 158*(-I*sqrt(2)*cos(10*d*x + 10*c) - sqrt(2)*sin(10*d*x + 10*c))*
cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 837*(sqrt(2)*co
s(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sqrt(2)*sin(1/5
*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*sqrt(2)*cos(1/5*ar
ctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + sqrt(2))*sin(4/5*arctan2(
sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 79*I*sqrt(2)*cos(10*d*x + 10*c)
 - 79*sqrt(2)*sin(10*d*x + 10*c))*cos(7/2*arctan2(sin(1/5*arctan2(sin(10*d
*x + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(
10*d*x + 10*c))) + 1)) + (-49*I*sqrt(2)*cos(10*d*x + 10*c) - 1155*I*sqrt(2
)*cos(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 3264*I*sqr...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}}\,dx$$

input
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(7/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\sqrt{a}\left(\left(\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\cos(dx+c)^3\tan(dx+c)}{\tan(dx+c)^5i+3\tan(dx+c)^4-2\tan(dx+c)^3i+2\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)i-\left(\right.\right.\right.}{a^4}$$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
(sqrt(a)*(int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**3*tan(c + d*x))/(tan
(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*tan(c + d*x)*
*2 - 3*tan(c + d*x)*i - 1),x)*i - int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*
x)**3)/(tan(c + d*x)**5*i + 3*tan(c + d*x)**4 - 2*tan(c + d*x)**3*i + 2*ta
n(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)))/a**4
```

3.394 $\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 398

$$\begin{aligned}\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx &= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\\ &- \frac{ia^{3/2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\ &+ \frac{ia^{3/2}e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\ &- \frac{ia^{3/2}e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}\left(\sqrt{a}+\cos(c+dx)\left(\sqrt{a}-i\sqrt{a}\tan(c+dx)\right)\right)}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output
```
I*a*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*a^(3/2)*e^(3/2)*
arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(
1/2))*sec(d*x+c)*2^(1/2)/d/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/
2)+1/2*I*a^(3/2)*e^(3/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)
/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)/d/(a-I*a*tan(d*x+c))^(1/
2)/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*a^(3/2)*e^(3/2)*arctanh(2^(1/2)*e^(1/2)*
(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)/(a^(1/2)+cos(d*x+c)*(a^(1/2)
-I*a^(1/2)*tan(d*x+c))))*sec(d*x+c)*2^(1/2)/d/(a-I*a*tan(d*x+c))^(1/2)/(a+
I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.94

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{e\sqrt{e\sec(c+dx)}(\cos(c)-i\sin(c))\left(\operatorname{arctanh}\left(\frac{\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right.\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(e*Sqrt[e*Sec[c + d*x]]*(Cos[c] - I*Sin[c])*(ArcTanh[(Sqrt[1 + I*Cos[c] -
Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan
[(d*x)/2]])]*Cos[c + d*x]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] -
 Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + Sqrt[-1 + I*Cos[c] + Sin[c]]*(Sqrt[-1 -
I*Cos[c] - Sin[c]]*(I*Cos[d*x] + Sin[d*x])*Sqrt[I - Tan[(d*x)/2]] - ArcTan
h[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c]
 - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]*Sqrt[1 - I*Cos[c] + Sin[c
]]*Sqrt[I + Tan[(d*x)/2]]))*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[-1 - I*Cos
[c] - Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 3979, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3980}$$

$$\frac{ae\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{ae\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3976}$$

$$\frac{2ia^2e^3\sec(c+dx)\int \frac{\cos(c+dx)(a-ia\tan(c+dx))}{e(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{826}$$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\int \frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-\cdots}{e}}\cdots}{2e}}{2e}\cdots\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+d}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{217}$$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan}}{\sqrt{e\sec(}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c}}{\sqrt{e\sec(c+dx}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\cos(c+\right.}}{2\sqrt{2}\sqrt{a}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+i}}$$

$$\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\cos(c+dx)(\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia}}$$

$$\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-i}{e}}}{2\sqrt{2}\sqrt{a}e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(}$$

$$\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(I*a*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + ((2*I)*a^2*e
^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sq
rt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt
[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*S
qrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I
*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x
])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a -
I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*
x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[a - I*a*Tan
[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 7.92 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.67

method	result
default	$-\frac{e\sqrt{e\sec(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\left(-i\cos(dx+c)\,\operatorname{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)-1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)-i\cos(dx+c)\,\operatorname{arctanh}\left(\frac{\cot(dx+c)-\csc(dx}{2\sqrt{\frac{1}{\cos(dx+c)-}}}\right.\right.}{2d(}$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*e*(e*sec(d*x+c))^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*(-I*cos(d*x+c)*ar
ctanh(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))-I*cos(d*x+c)
*arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))+2*I*(1/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)*arctanh(1/2*(cot(d*x+c)-csc(d*x+
c)-1)/(1/(cos(d*x+c)+1))^(1/2))+cos(d*x+c)*arctanh(1/2/(1/(cos(d*x+c)+1))^
(1/2)*(cot(d*x+c)-csc(d*x+c)+1))-2*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)
)/(-I*cos(d*x+c)+sin(d*x+c)-I)/(1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.05

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{4i\,e\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+\sqrt{\frac{i\,ae^3}{d^2}}d\log\left(\frac{2\left(\left(ee^{(2i\,dx+2i\,c)}\right.\right.}{}\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/2*(4*I*e*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) +
 1))*e^(1/2*I*d*x + 1/2*I*c) + sqrt(I*a*e^3/d^2)*d*log(2*((e*e^(2*I*d*x +
2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c)
+ 1))*e^(1/2*I*d*x + 1/2*I*c) + I*sqrt(I*a*e^3/d^2)*d)/e) - sqrt(I*a*e^3/d
^2)*d*log(2*((e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*sqrt(I*a*e^
3/d^2)*d)/e) + sqrt(-I*a*e^3/d^2)*d*log(2*((e*e^(2*I*d*x + 2*I*c) + e)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*
d*x + 1/2*I*c) + I*sqrt(-I*a*e^3/d^2)*d)/e) - sqrt(-I*a*e^3/d^2)*d*log(2*(
(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2
*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*sqrt(-I*a*e^3/d^2)*d)/e)
)/d
```

Sympy [F]

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \int (e\sec{(c+dx)})^{\frac{3}{2}}\sqrt{ia\,(\tan{(c+dx)}-i)}\,dx$$

input
```
integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((e*sec(c + d*x))**(3/2)*sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1870 vs. $2(300) = 600$.

Time = 0.44 (sec) , antiderivative size = 1870, normalized size of antiderivative = 4.70

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
-8*(2*(sqrt(2)*e*cos(2*d*x + 2*c) + I*sqrt(2)*e*sin(2*d*x + 2*c) + sqrt(2)
*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*
(sqrt(2)*e*cos(2*d*x + 2*c) + I*sqrt(2)*e*sin(2*d*x + 2*c) + sqrt(2)*e)*ar
ctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt
(2)*e*cos(2*d*x + 2*c) + I*sqrt(2)*e*sin(2*d*x + 2*c) + sqrt(2)*e)*arctan2
(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*e*
cos(2*d*x + 2*c) + I*sqrt(2)*e*sin(2*d*x + 2*c) + sqrt(2)*e)*arctan2(sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 2*(-I*sqrt(2)*e*co
s(2*d*x + 2*c) + sqrt(2)*e*sin(2*d*x + 2*c) - I*sqrt(2)*e)*arctan2(sqrt(2)
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1) - 2*(I*sqrt(2)*e*cos(2*d*x + 2*c) - sqrt(2)*e*sin(2*d*x + 2*c) +
I*sqrt(2)*e)*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(...
```

Giac [F(-2)]

Exception generated.

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx=\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx=\frac{2\sqrt{e}\,\sqrt{a}\,ei\left(-\sqrt{\sec\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,\sec\left(dx+c\right)+2\left(\int \sqrt{s\right.}\right.}{d}$$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*sqrt(e)*sqrt(a)*e*i*( - sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec
(c + d*x) + 2*int(sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)
*tan(c + d*x),x)*d))/d
```

3.395 $\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 241

$$\begin{aligned}
&\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx \\
&= \frac{i\sqrt{2}\sqrt{a}\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d} \\
&\quad - \frac{i\sqrt{2}\sqrt{a}\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d} \\
&\quad + \frac{i\sqrt{2}\sqrt{a}\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}\left(\sqrt{a}+\cos(c+dx)\left(\sqrt{a}+i\sqrt{a}\tan(c+dx)\right)\right)}\right)}{d}
\end{aligned}$$

output
```
I*2^(1/2)*a^(1/2)*e^(1/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2
)/a^(1/2)/(e*sec(d*x+c))^(1/2))/d-I*2^(1/2)*a^(1/2)*e^(1/2)*arctan(1+2^(1/
2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))/d+I*2^(1
/2)*a^(1/2)*e^(1/2)*arctanh(2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*se
c(d*x+c))^(1/2)/(a^(1/2)+cos(d*x+c)*(a^(1/2)+I*a^(1/2)*tan(d*x+c))))/d
```

Mathematica [A] (warning: unable to verify)

Time = 1.70 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.15

$$\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{2e\left(\operatorname{arctanh}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{1+i\cos(c)-\sin(c)}-\operatorname{arctanh}}{d\sqrt{e\sec(c+dx)}\sqrt{1}}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(-2*e*(ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[
-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[
c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqr
t[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])
]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*Sqrt[I + Tan[(
d*x)/2]]*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[1 + Cos[
2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.38, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}dx$$

$$\Big\downarrow \text{3976}$$

$$-\frac{4iae^2\int \frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{d}$$

$$\Big\downarrow \; 826$$

$$-\frac{4iae^2\left(\frac{\int \frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}$$

$$\Big\downarrow \; 1476$$

$$-\frac{4iae^2\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d}{2e}}{2e}\right.}{d}$$

$$\Big\downarrow \; 1082$$

$$-\frac{4iae^2\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\int\right.}{d}$$

$$\Big\downarrow \; 217$$

$$-\frac{4iae^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}$$

$$\Big\downarrow \; 1479$$

$$-\frac{4iae^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}$$

$$\Big\downarrow \; 25$$

$$-\frac{4iae^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4iae^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\sqrt{}}{2\sqrt{2}\sqrt{a}e}\right.}{d}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{4iae^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-4*I)*a*e^2*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/
(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (S
qrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]
/(Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]
*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*
Tan[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e
]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a
*Tan[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 7.78 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.85

method	result
default	$-\frac{\sqrt{e\sec(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,\cos(dx+c)\left(i\,\mathrm{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+i\,\mathrm{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)-1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+\mathrm{arct}\right.}{d(-\sin(dx+c)+i\cos(dx+c)+i)\sqrt{\frac{1}{\cos(dx+c)+1}}}$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(e*sec(d*x+c))^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*cos(d*x+c)*(I*arctanh
(1/2/(1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))+I*arctanh(1/2*(co
t(d*x+c)-csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))+arctanh(1/2/(1/(cos(d*x+c
)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))-arctanh(1/2*(cot(d*x+c)-csc(d*x+c)-
1)/(1/(cos(d*x+c)+1))^(1/2)))/(-sin(d*x+c)+I*cos(d*x+c)+I)/(1/(cos(d*x+c)+
1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.34

$$
\begin{aligned}
&\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx \\
&= \frac{1}{2}\sqrt{\frac{4i\,ae}{d^2}}\log\left(2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)} + d\sqrt{\frac{4i\,ae}{d^2}}\right) \\
&\quad - \frac{1}{2}\sqrt{\frac{4i\,ae}{d^2}}\log\left(2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)} - d\sqrt{\frac{4i\,ae}{d^2}}\right) \\
&\quad - \frac{1}{2}\sqrt{-\frac{4i\,ae}{d^2}}\log\left(2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)} + d\sqrt{-\frac{4i\,ae}{d^2}}\right) \\
&\quad + \frac{1}{2}\sqrt{-\frac{4i\,ae}{d^2}}\log\left(2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)} - d\sqrt{-\frac{4i\,ae}{d^2}}\right)
\end{aligned}
$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/2*sqrt(4*I*a*e/d^2)*log(2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2
*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(1/2*I*d*x + 1/2*I*c) +
d*sqrt(4*I*a*e/d^2)) - 1/2*sqrt(4*I*a*e/d^2)*log(2*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(
1/2*I*d*x + 1/2*I*c) - d*sqrt(4*I*a*e/d^2)) - 1/2*sqrt(-4*I*a*e/d^2)*log(2
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2
*I*d*x + 2*I*c) + 1)*e^(1/2*I*d*x + 1/2*I*c) + d*sqrt(-4*I*a*e/d^2)) + 1/2
*sqrt(-4*I*a*e/d^2)*log(2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I
*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(1/2*I*d*x + 1/2*I*c) - d*
sqrt(-4*I*a*e/d^2))
```

Sympy [F]

$$\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{e\sec{(c+dx)}}\sqrt{ia\left(\tan{(c+dx)}-i\right)}\,dx$$

input
```
integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(e*sec(c + d*x))*sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1400 vs. $2(175) = 350$.

Time = 0.36 (sec) , antiderivative size = 1400, normalized size of antiderivative = 5.81

$$\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
1/4*(-2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqr
t(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, sqr
t(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2
*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) - 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1) - 2*sqrt(2)*arctan2(sqrt(2)*sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))), sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 1) + 2*sqrt(2)*arctan2(-sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))), -sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + 1) + I*sqrt(2)*log(2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*...
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx=\int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx$$
$$=\frac{2\sqrt{e}\,\sqrt{a}\,i\left(-\sqrt{\sec\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}+\left(\int \sqrt{\sec\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,\tan\left(dx+c\right)dx\right.\right.}{d}$$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*sqrt(e)*sqrt(a)*i*( - sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1) + int
(sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*d))/d
```

3.396 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

output
```
-2*I*(a+I*a*tan(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Sec[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}dx$$
$$\Big\downarrow \text{3969}$$
$$-\frac{2i\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

input
```
Int[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Sec[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Maple [A] (verified)

Time = 8.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

method	result	size
orering	$-\frac{2i\sqrt{a+ia\tan(dx+c)}}{d\sqrt{e\sec(dx+c)}}$	31
default	$-\frac{2i\sqrt{a(1+i\tan(dx+c))}}{d\sqrt{e\sec(dx+c)}}$	32
risch	$-\frac{2i\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}}{\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	59

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I*(a+I*a*tan(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(28) = 56.

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx = \frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{de}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-I*
e^(2*I*d*x + 2*I*c) - I)*e^(1/2*I*d*x + 1/2*I*c)/(d*e)
```

Sympy [F]

$$\int \frac{\sqrt{a + ia\tan(c + dx)}}{\sqrt{e\sec(c + dx)}}\,dx = \int \frac{\sqrt{ia\,(\tan{(c + dx)} - i)}}{\sqrt{e\sec{(c + dx)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))/sqrt(e*sec(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{a + ia\tan(c + dx)}}{\sqrt{e\sec(c + dx)}}\,dx = -\frac{2i\,\sqrt{a}\sqrt{-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - 1}}{d\sqrt{e}\sqrt{-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - 1}}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
-2*I*sqrt(a)*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 1)/(d*sqrt(e)*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - 1))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}}{\sec(dx+c)}dx\right)}{e}$$

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x)
```

output

```
(sqrt(e)*sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1))/sec(c +
 d*x),x))/e
```

3.397 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx=\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

output
```
4/3*I*a*(e*sec(d*x+c))^(1/2)/d/e^2/(a+I*a*tan(d*x+c))^(1/2)-2/3*I*(a+I*a*t
an(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx=\frac{2(i+2\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

input
```
Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(I + 2*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*(e*Sec[c + d*x])^
(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

input
```
Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(((4*I)/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - ((
(2*I)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 6.46 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{\left(\frac{2i\cos(dx+c)}{3}+\frac{4\sin(dx+c)}{3}\right)\sqrt{a(1+i\tan(dx+c))}}{d\sqrt{e\sec(dx+c)}\,e}$	51
risch	$-\frac{i\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,(-2\cos(dx+c)+4i\sin(dx+c))}{3e\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	80

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/3*I*cos(d*x+c)+4/3*sin(d*x+c))*(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*
x+c))^(1/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-i\,e^{(4i\,dx+4i\,c)}+2i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{3\,de^2}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
1/3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-
I*e^(4*I*d*x + 4*I*c) + 2*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-1/2*I*d*x - 1/2
*I*c)/(d*e^2)
```

Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\left(e\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))/(e*sec(c + d*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(-i\,\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3i\,\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,\sin\left(\frac{1}{2}\,d\right.}{3\,de^{\frac{3}{2}}}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output
```
1/3*sqrt(a)*(-I*cos(3/2*d*x + 3/2*c) + 3*I*cos(1/2*d*x + 1/2*c) + sin(3/2*
d*x + 3/2*c) + 3*sin(1/2*d*x + 1/2*c))/(d*e^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}+2\,\sin\left(2\,c+2\right.\right.}{3\,d\,e^2}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(3/2),x)
```

output
```
((e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/
(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*1i + 2*sin(2*c + 2*d*x) +
1i))/(3*d*e^2)
```

Reduce [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}}{\sec(dx+c)^2}dx\right)}{e^2}$$

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1))/sec(c +
 d*x)**2,x))/e**2
```

3.398 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 122

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{8ia}{15de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{15de^2\sqrt{e\sec(c+dx)}}$$

output
```
8/15*I*a/d/e^2/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-2/5*I*(a+I*a*
tan(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(5/2)-16/15*I*(a+I*a*tan(d*x+c))^(1/2)/
d/e^2/(e*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{i(-15+\cos(2(c+dx))-4i\sin(2(c+dx)))\sqrt{a+ia\tan(c+dx)}}{15de^2\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(5/2),x]
```

output
```
((I/15)*(-15 + Cos[2*(c + d*x)] - (4*I)*Sin[2*(c + d*x)])*Sqrt[a + I*a*Tan
[c + d*x]])/(d*e^2*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3978, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{4a\left(\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}$$

input
```
Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(((-2*I)/5)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(5/2)) + (4*a*
(((2*I)/3)/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3
)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*Sqrt[e*Sec[c + d*x]])))/(5*e^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 6.59 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{2\left(i\cos(dx+c)^2+4\cos(dx+c)\sin(dx+c)-8i\right)\sqrt{a(1+i\tan(dx+c))}}{15d\sqrt{e\sec(dx+c)}\,e^2}$	62
risch	$-\frac{i\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,(30-2\cos(2dx+2c)+8i\sin(2dx+2c))}{30e^2\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	87

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*(I*cos(d*x+c)^2+4*cos(d*x+c)*sin(d*x+c)-8*I)*(a*(1+I*tan(d*x+c)))^(
1/2)/(e*sec(d*x+c))^(1/2)/e^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(6i\,dx+6i\,c)}-33i\,e^{(4i\,dx+4i\,c)}-25i\,e^{(2i\,dx+2i\,c)}\ldots\right.}{30\,de^3}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
1/30*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(
-3*I*e^(6*I*d*x + 6*I*c) - 33*I*e^(4*I*d*x + 4*I*c) - 25*I*e^(2*I*d*x + 2*
I*c) + 5*I)*e^(-3/2*I*d*x - 3/2*I*c)/(d*e^3)
```

Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{ia(\tan(c+dx)-i)}}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))/(e*sec(c + d*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(5i\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3i\cos\left(\frac{5}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right.\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
1/30*sqrt(a)*(5*I*cos(3/2*d*x + 3/2*c) - 3*I*cos(5/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) - 30*I*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 5*sin(3/2*d*x + 3/2*c) + 3*sin(5/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 30*sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))))/(d*e^(5/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(4\,\sin\left(c+d\,x\right)+4\,\sin\left(3\,c+3\,d\,x\right.}{30\,d\,e^3}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(5/2),x)
```

output
```
((e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/
(cos(2*c + 2*d*x) + 1))^(1/2)*(4*sin(c + d*x) - cos(c + d*x)*29i + cos(3*c
 + 3*d*x)*1i + 4*sin(3*c + 3*d*x)))/(30*d*e^3)
```

Reduce [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}}{\sec(dx+c)^3}dx\right)}{e^3}$$

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1))/sec(c +
 d*x)**3,x))/e**3
```

3.399 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 164

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{12ia}{35de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{32ia\sqrt{e\sec(c+dx)}}{35de^4\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{35de^2(e\sec(c+dx))^{3/2}}$$

output
```
12/35*I*a/d/e^2/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+32/35*I*a*(e
*sec(d*x+c))^(1/2)/d/e^4/(a+I*a*tan(d*x+c))^(1/2)-2/7*I*(a+I*a*tan(d*x+c))
^(1/2)/d/(e*sec(d*x+c))^(7/2)-16/35*I*(a+I*a*tan(d*x+c))^(1/2)/d/e^2/(e*se
c(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{(35i\cos(c+dx)+i\cos(3(c+dx))+70\sin(c+dx)+6\sin(3(c+dx)))\sqrt{a-}}{70de^3\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(((35*I)*Cos[c + d*x] + I*Cos[3*(c + d*x)] + 70*Sin[c + d*x] + 6*Sin[3*(c
+ d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(70*d*e^3*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3978, 3042, 3983, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{6a\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6a\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{6a\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6a\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{6a\left(\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6a\left(\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{6a\left(\frac{4\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

input
```
Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(((-2*I)/7)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(7/2)) + (6*a*
(((2*I)/5)/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (4*((((
4*I)/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((2*
I)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))))/(5*a)))/(7*
e^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 6.57 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{2\sqrt{a(1+i\tan(dx+c))}\left(i\cos(dx+c)^{3}+6\sin(dx+c)\cos(dx+c)^{2}+8i\cos(dx+c)+16\sin(dx+c)\right)}{35d\sqrt{e\sec(dx+c)}\,e^{3}}$	79

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/35/d*(a*(1+I*tan(d*x+c)))^(1/2)*(I*cos(d*x+c)^3+6*sin(d*x+c)*cos(d*x+c)^
2+8*I*cos(d*x+c)+16*sin(d*x+c))/(e*sec(d*x+c))^(1/2)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-5i\,e^{(8i\,dx+8i\,c)} - 40i\,e^{(6i\,dx+6i\,c)} + 70i\,e^{(4i\,dx+4i\,c)} \right.}{140\,de^4}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x, algorithm="fric
as")
```

output
```
1/140*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
(-5*I*e^(8*I*d*x + 8*I*c) - 40*I*e^(6*I*d*x + 6*I*c) + 70*I*e^(4*I*d*x + 4
*I*c) + 112*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(-5/2*I*d*x - 5/2*I*c)/(d*e^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\sqrt{a}\left(7i\,\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-5i\,\cos\left(\frac{7}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right.\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x, algorithm="maxi
ma")
```

output
```
1/140*sqrt(a)*(7*I*cos(5/2*d*x + 5/2*c) - 5*I*cos(7/5*arctan2(sin(5/2*d*x
+ 5/2*c), cos(5/2*d*x + 5/2*c))) - 35*I*cos(3/5*arctan2(sin(5/2*d*x + 5/2*
c), cos(5/2*d*x + 5/2*c))) + 105*I*cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), c
os(5/2*d*x + 5/2*c))) + 7*sin(5/2*d*x + 5/2*c) + 5*sin(7/5*arctan2(sin(5/2
*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 35*sin(3/5*arctan2(sin(5/2*d*x + 5
/2*c), cos(5/2*d*x + 5/2*c))) + 105*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c),
cos(5/2*d*x + 5/2*c))))/(d*e^(7/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,36\mathrm{i}+\cos\left(4\,c+4\right.\right.}{140\,d\,e^4}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(7/2),x)
```

output
```
((e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/
(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*36i + cos(4*c + 4*d*x)*1i
+ 76*sin(2*c + 2*d*x) + 6*sin(4*c + 4*d*x) + 35i))/(140*d*e^4)
```

Reduce [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}}{\sec(dx+c)^4}dx\right)}{e^4}$$

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x)
```

output
```
(sqrt(e)*sqrt(a)*int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1))/sec(c +
 d*x)**4,x))/e**4
```

3.400 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 369

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{7ia^{3/2}e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d} - \frac{7ia^{3/2}e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d} + \frac{7ia^{3/2}e^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}(\sqrt{a}+\cos(c+dx)(\sqrt{a}+i\sqrt{a}\tan(c+dx)))}\right)}{8\sqrt{2}d} + \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{7iae^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d} + \frac{ia(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{3d}$$

output
```
7/16*I*a^(3/2)*e^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a
^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2)/d-7/16*I*a^(3/2)*e^(5/2)*arctan(1+2^(
1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2
)/d+7/16*I*a^(3/2)*e^(5/2)*arctanh(2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2
)/(e*sec(d*x+c))^(1/2)/(a^(1/2)+cos(d*x+c)*(a^(1/2)+I*a^(1/2)*tan(d*x+c)))
)*2^(1/2)/d+7/12*I*a^2*(e*sec(d*x+c))^(5/2)/d/(a+I*a*tan(d*x+c))^(1/2)-7/8
*I*a*e^2*(e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d+1/3*I*a*(e*sec(d*
x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 3.73 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.02

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{a(e\sec(c+dx))^{5/2}\left(2i\sqrt{1+\cos(2c)+i\sin(2c)}(-9+7\cos(2c+2dx)+14i\sin(2c+2dx))\sqrt{i-\tan\left(\frac{dx}{2}\right)}\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
-1/96*(a*(e*Sec[c + d*x])^(5/2)*((2*I)*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*(-9
 + 7*Cos[2*c + 2*d*x] + (14*I)*Sin[2*c + 2*d*x])*Sqrt[I - Tan[(d*x)/2]] +
84*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 -
 I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]^3*Sqrt[-1 - I*Co
s[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]] - 84*Arc
Tanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos
[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]^3*Sqrt[1 - I*Cos[c] +
Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])*Sqrt[a + I*a*
Tan[c + d*x]])/(d*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]])
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.26, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3979, 3042, 3979, 3042, 3982, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{7}{6}a\int (e\sec(c+dx))^{5/2}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{6}a\int (e\sec(c+dx))^{5/2}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{7}{6}a\left(\frac{3}{4}a\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{5/2}}{2d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{6}a\left(\frac{3}{4}a\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{5/2}}{2d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{3982}$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(\frac{e^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}\right)+\frac{ia(e\sec(c+\ldots}{2d\sqrt{a+ia\,\mathrm{t}\ldots}}\right.$$
$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(\frac{e^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}\right)+\frac{ia(e\sec(c+\ldots}{2d\sqrt{a+ia\,\mathrm{t}\ldots}}\right.$$
$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{3976}$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(-\frac{2ie^4\int \frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}\right)+\ldots\right.$$
$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{826}$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\int \frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\frac{ie^2\sqrt{\ldots}}{\ldots}\right.\right.$$
$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{1476}$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c\ldots}{e}}\ldots}{2e}}{2e}\ldots\right.}{d}\ldots\right.\right.$$
$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \; 1082$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+\cdots\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\cdots\right.}{d}\cdots\right.\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \; 217$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)\cdots}}{\sqrt{e\sec(c+d\cdots}}}{2e}\cdots\right.}{d}\cdots\right.\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \; 1479$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+\cdots}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(\cdots}{\cdots}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}\cdots}\cdots\right.}{d}\cdots\right.\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \; 25$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan\cdots}{e}\right.}\cdots}{2\sqrt{2}\sqrt{a}\sqrt{e}}\cdots\right)}{d}\cdots\right.\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow 27$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+\cdots}{e}}\cdots}{2\sqrt{2}\sqrt{a}e}\cdots\right)}{d}\cdots\right.\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow 1103$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan\cdots\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\cdots\right)}{d}\cdots\right.\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

input
```
Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I/3)*a*(e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])/d + (7*a*(((I/
2)*a*(e*Sec[c + d*x])^(5/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*a*(((-2*I
)*e^4*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]
*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*S
qrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2
]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a
+ I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c +
d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a
 + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c +
 d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/d - (I*e^2*Sqrt[e*Sec[c + d*x
]]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)))/4))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 7.58 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.86

method	result
default	$e^2\sqrt{e\sec(dx+c)}\,a\sqrt{a(1+i\tan(dx+c))}\left(-21i\cos(dx+c)\,\operatorname{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)-21\cos(dx+c)\,\operatorname{arctanh}\left(\frac{\cot(dx+c)-\operatorname{cs}}{2\sqrt{\frac{1}{\cos(dx}}}\right.\right.$

input
```
int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d*e^2*(e*sec(d*x+c))^(1/2)*a*(a*(1+I*tan(d*x+c)))^(1/2)/(-sin(d*x+c)+
I*cos(d*x+c)+I)/(1/(cos(d*x+c)+1))^(1/2)*(-21*I*cos(d*x+c)*arctanh(1/2/(1/
(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))-21*cos(d*x+c)*arctanh(1/2
/(1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))-21*I*cos(d*x+c)*arcta
nh(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))+21*cos(d*x+c)*a
rctanh(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))-2*I*(1/(cos
(d*x+c)+1))^(1/2)*(-21*sin(d*x+c)-14*tan(d*x+c)+8*sec(d*x+c)*tan(d*x+c))+2
*(1/(cos(d*x+c)+1))^(1/2)*(7+21*cos(d*x+c)-22*sec(d*x+c)-8*sec(d*x+c)^2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 644 vs. 2(267) = 534.

Time = 0.10 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.75

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
1/12*((-21*I*a*e^2*e^(5*I*d*x + 5*I*c) + 18*I*a*e^2*e^(3*I*d*x + 3*I*c) +
7*I*a*e^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*
I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 6*sqrt(49/64*I*a^3*e^5/d^2)
*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/7*(7*(a*e^2*e
^(2*I*d*x + 2*I*c) + a*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2
*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 8*sqrt(49/64*I*a^3*e^5/d^2
)*d)/(a*e^2)) - 6*sqrt(49/64*I*a^3*e^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e
^(2*I*d*x + 2*I*c) + d)*log(2/7*(7*(a*e^2*e^(2*I*d*x + 2*I*c) + a*e^2)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*
d*x + 1/2*I*c) - 8*sqrt(49/64*I*a^3*e^5/d^2)*d)/(a*e^2)) - 6*sqrt(-49/64*I
*a^3*e^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/
7*(7*(a*e^2*e^(2*I*d*x + 2*I*c) + a*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 8*sqrt(-49/64
*I*a^3*e^5/d^2)*d)/(a*e^2)) + 6*sqrt(-49/64*I*a^3*e^5/d^2)*(d*e^(4*I*d*x +
 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/7*(7*(a*e^2*e^(2*I*d*x + 2*I*
c) + a*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c)
+ 1))*e^(1/2*I*d*x + 1/2*I*c) - 8*sqrt(-49/64*I*a^3*e^5/d^2)*d)/(a*e^2)))/
(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3005 vs. $2(267) = 534$.

Time = 0.58 (sec) , antiderivative size = 3005, normalized size of antiderivative = 8.14

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output
```
-192*(336*a*e^2*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 28
8*a*e^2*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 112*a*e^2*c
os(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 336*I*a*e^2*sin(11/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 288*I*a*e^2*sin(7/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 112*I*a*e^2*sin(3/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 42*(sqrt(2)*a*e^2*cos(6*d*x + 6*c) + 3*s
qrt(2)*a*e^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a*e^2*cos(2*d*x + 2*c) + I*sqrt(
2)*a*e^2*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 3*I*sqrt(
2)*a*e^2*sin(2*d*x + 2*c) + sqrt(2)*a*e^2)*arctan2(sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1) + 42*(sqrt(2)*a*e^2*cos(6*d*x + 6*c) +
3*sqrt(2)*a*e^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a*e^2*cos(2*d*x + 2*c) + I*sq
rt(2)*a*e^2*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 3*I*sq
rt(2)*a*e^2*sin(2*d*x + 2*c) + sqrt(2)*a*e^2)*arctan2(sqrt(2)*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 42*(sqrt(2)*a*e^2*cos(6*d*x + 6*c
) + 3*sqrt(2)*a*e^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a*e^2*cos(2*d*x + 2*c) +
I*sqrt(2)*a*e^2*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 3*
I*sqrt(2)*a*e^2*sin(2*d*x + 2*c) + sqrt(2)*a*e^2)*arctan2(sqrt(2)*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arcta...
```

Giac [F(-2)]

Exception generated.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{\sqrt{e}\,\sqrt{a}\,a\,e^2 i\Big(-2\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)^2+7\Big(\int\sqrt{\sec(dx}}{d}$$

input
```
int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*sqrt(a)*a*e**2*i*( - 2*sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1
)*sec(c + d*x)**2 + 7*int(sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec(
c + d*x)**2*tan(c + d*x),x)*d))/d
```

3.401 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 447

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{5ia^{5/2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia^{5/2}e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{5ia^{5/2}e^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}\left(\sqrt{a}+\cos(c+dx)\left(\sqrt{a}-i\sqrt{a}\tan(c+dx)\right)\right)}\right)\sec(c+dx)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{2d}$$

output
```
5/4*I*a^2*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)-5/8*I*a^(5/2)*e^
(3/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x
+c))^(1/2))*sec(d*x+c)*2^(1/2)/d/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c
))^(1/2)+5/8*I*a^(5/2)*e^(3/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))
^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)/d/(a-I*a*tan(d*x+c
))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-5/8*I*a^(5/2)*e^(3/2)*arctanh(2^(1/2)*e^
(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)/(a^(1/2)+cos(d*x+c)*(a
^(1/2)-I*a^(1/2)*tan(d*x+c))))*sec(d*x+c)*2^(1/2)/d/(a-I*a*tan(d*x+c))^(1/
2)/(a+I*a*tan(d*x+c))^(1/2)+1/2*I*a*(e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)
)^(1/2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 3.22 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.84

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{\cos^3(c+dx)(e\sec(c+dx))^{3/2}(\cos(dx)-i\sin(dx))\left(2\sec^2(c+dx)(i\cos(c)+\sin(\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(Cos[c + d*x]^3*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] - I*Sin[d*x])*(2*Sec[c +
d*x]^2*(I*Cos[c] + Sin[c]) + 5*Sec[c + d*x]*(I*Cos[2*c + d*x] + Sin[2*c +
d*x]) + (5*(ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(
Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] -
 Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]
]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)
/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*(Cos[2*c]
 - I*Sin[2*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[
-1 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]]))*(a + I*a*Tan[c + d*x])^(3
/2))/(4*d)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.02, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3979, 3042, 3979, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{5}{4}a\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{4}a\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{5}{4}a\left(\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{4}a\left(\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3980}$$

$$\frac{5}{4}a\left(\frac{ae\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5}{4}a\left(\frac{ae\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{ 3976}$$

$$\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\int\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{ 826}$$

$$\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\int\frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+}{}}}{}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c-}}\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \; 1082$$

$$\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int\frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int\frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan}}{\sqrt{a}\sqrt{e\sec(}}\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \; 217$$

$$\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\sqrt{}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \; 1479$$

$$\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-i}}{\sqrt{e\,\mathrm{se}}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\right.}}{}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{}}\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \; 25$$

$$\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\ta}}{\sqrt{e\sec(c+}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\cos(\right.}}{2\sqrt{}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a}}\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \ 27$$

$$\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c}}{\sqrt{e\sec(c+dx}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+d}{}}}{2\sqrt{2}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia}}\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \ 1103$$

$$\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+d}}\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I/2)*a*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/d + (5*a*((I*a
*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + ((2*I)*a^2*e^3*(
(-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e
*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e]*
Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[
a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*T
an[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]/
(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*
Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]
/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[a - I*a*Tan[c +
 d*x]]*Sqrt[a + I*a*Tan[c + d*x]])))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 7.57 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.64

method	result
default	$-\frac{e\sqrt{e\sec(dx+c)}\,a\sqrt{a(1+i\tan(dx+c))}\left(5i\cos(dx+c)\operatorname{arctanh}\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+5\cos(dx+c)\operatorname{arctanh}\left(\frac{-\cot(dx+c)+c}{2\sqrt{\frac{1}{\cos(dx}}}\right.\right.}{}$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d*e*(e*sec(d*x+c))^(1/2)*a*(a*(1+I*tan(d*x+c)))^(1/2)/(-I*cos(d*x+c)+
sin(d*x+c)-I)/(1/(cos(d*x+c)+1))^(1/2)*(5*I*cos(d*x+c)*arctanh(1/2/(1/(cos
(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+5*cos(d*x+c)*arctanh(1/2/(1/
(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+5*I*cos(d*x+c)*arctanh(1
/2*(-cot(d*x+c)+csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))-5*cos(d*x+c)*arcta
nh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))+2*I*(1/(cos(d*
x+c)+1))^(1/2)*(5*sin(d*x+c)-2*tan(d*x+c))-2*(1/(cos(d*x+c)+1))^(1/2)*(7+5
*cos(d*x+c)+2*sec(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.20

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
1/2*((9*I*a*e*e^(2*I*d*x + 2*I*c) + 5*I*a*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + sqrt(25/1
6*I*a^3*e^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/5*(5*(a*e*e^(2*I*d*x +
2*I*c) + a*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c
) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 4*I*sqrt(25/16*I*a^3*e^3/d^2)*d)/(a*e))
- sqrt(25/16*I*a^3*e^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/5*(5*(a*e*e^
(2*I*d*x + 2*I*c) + a*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*
d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - 4*I*sqrt(25/16*I*a^3*e^3/d^2)
*d)/(a*e)) + sqrt(-25/16*I*a^3*e^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/
5*(5*(a*e*e^(2*I*d*x + 2*I*c) + a*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 4*I*sqrt(-25/16*I
*a^3*e^3/d^2)*d)/(a*e)) - sqrt(-25/16*I*a^3*e^3/d^2)*(d*e^(2*I*d*x + 2*I*c
) + d)*log(2/5*(5*(a*e*e^(2*I*d*x + 2*I*c) + a*e)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - 4*I*
sqrt(-25/16*I*a^3*e^3/d^2)*d)/(a*e)))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2367 vs. $2(331) = 662$.

Time = 0.44 (sec) , antiderivative size = 2367, normalized size of antiderivative = 5.30

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output
```
32*(144*a*e*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 80*a*e*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 144*I*a*e*sin(5/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 80*I*a*e*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 10*(sqrt(2)*a*e*cos(4*d*x + 4*c) + 2*s
qrt(2)*a*e*cos(2*d*x + 2*c) + I*sqrt(2)*a*e*sin(4*d*x + 4*c) + 2*I*sqrt(2)
*a*e*sin(2*d*x + 2*c) + sqrt(2)*a*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - 10*(sqrt(2)*a*e*cos(4*d*x + 4*c) + 2*sqrt(2
)*a*e*cos(2*d*x + 2*c) + I*sqrt(2)*a*e*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a*e*
sin(2*d*x + 2*c) + sqrt(2)*a*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 1) - 10*(sqrt(2)*a*e*cos(4*d*x + 4*c) + 2*sqrt(2)*a*
e*cos(2*d*x + 2*c) + I*sqrt(2)*a*e*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a*e*sin(
2*d*x + 2*c) + sqrt(2)*a*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1) - 10*(sqrt(2)*a*e*cos(4*d*x + 4*c) + 2*sqrt(2)*a*e*cos
(2*d*x + 2*c) + I*sqrt(2)*a*e*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a*e*sin(2*d*x
 + 2*c) + sqrt(2)*a*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 1) + 10*(-I*sqrt(2)*a*e*cos(4*d*x + 4*c) - 2*I*sqrt(2)*a*e...
```

Giac [F(-2)]

Exception generated.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{\sqrt{e}\,\sqrt{a}\,aei\Big(-2\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)+5\Big(\int\sqrt{\sec(dx+c)}$$
$$d$$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*sqrt(a)*a*e*i*( - 2*sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*s
ec(c + d*x) + 5*int(sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec(c + d*
x)*tan(c + d*x),x)*d))/d
```

3.402 $\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 278

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{3ia^{3/2}\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d} - \frac{3ia^{3/2}\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d} + \frac{3ia^{3/2}\sqrt{e}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}(\sqrt{a}+\cos(c+dx)(\sqrt{a}+i\sqrt{a}\tan(c+dx)))}\right)}{\sqrt{2}d} + \frac{ia\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

output
```
3/2*I*a^(3/2)*e^(1/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^
(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2)/d-3/2*I*a^(3/2)*e^(1/2)*arctan(1+2^(1/
2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2)/
d+3/2*I*a^(3/2)*e^(1/2)*arctanh(2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(
e*sec(d*x+c))^(1/2)/(a^(1/2)+cos(d*x+c)*(a^(1/2)+I*a^(1/2)*tan(d*x+c))))*2
^(1/2)/d+I*a*(e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 2.82 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.22

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{ae\left(i\sec(c+dx)\sqrt{1+\cos(2c)+i\sin(2c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}-3\text{arctanh}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\cdots\right.}{\cdots}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(a*e*(I*Sec[c + d*x]*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]
] - 3*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-
1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c
]]*Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + 3*ArcTanh[(Sqrt[1
+ I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]
*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] +
 Sin[c]]*Sqrt[I + Tan[(d*x)/2]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec
[c + d*x]]*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.34, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3979, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{3}{2}a\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3976}$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\int \frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{d}$$

$$\Big\downarrow \text{826}$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\int \frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}$$

$$\Big\downarrow \text{1476}$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\sqrt{}}{2e}}{2e}\right.}{d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{}{a^2}}{}\right.}{d}$$

$$\Big\downarrow \text{217}$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}$$

$$\Big\downarrow \ 1479$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+}{e}\right)}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right)}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right)}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right)}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i}}{}}{2\sqrt{2}\sqrt{a}e}\right)}{d}$$

$$\Big\downarrow \ 1103$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d} - \frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e} - \frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-6*I)*a^2*e^2*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]]
)/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 +
(Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]]
)]/(Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[
e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*
a*Tan[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt
[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I
*a*Tan[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/d + (I*a*Sqrt[e*Sec
[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

Maple [A] (verified)

Time = 7.56 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.96

method	result
default	$\frac{\sqrt{e\sec(dx+c)}\,a\sqrt{a(1+i\tan(dx+c))}\,\Bigg(3i\cos(dx+c)\,\mathrm{arctanh}\Bigg(\frac{\cot(dx+c)-\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\Bigg)+3i\cos(dx+c)\,\mathrm{arctanh}\Bigg(\frac{\cot(dx+c)-\csc(dx+c}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}}{2d(-}$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(e*sec(d*x+c))^(1/2)*a*(a*(1+I*tan(d*x+c)))^(1/2)*(3*I*cos(d*x+c)*ar
ctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))+3*I*cos(d*x+
c)*arctanh(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))+2*I*(1/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*cos(d*x+c)*arctanh(1/2/(1/(cos(d*x+c)+1
))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))-3*cos(d*x+c)*arctanh(1/2*(cot(d*x+c)-c
sc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))+2*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))
^(1/2))/(-I*cos(d*x+c)+sin(d*x+c)-I)/(1/(cos(d*x+c)+1))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(204) = 408.

Time = 0.09 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.51

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{4i\,a\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}+\sqrt{\frac{9i\,a^3e}{d^2}}d\log\left(\frac{2\left(3\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(\cdots}}}\right.}{3a}\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
1/2*(4*I*a*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) +
 1))*e^(3/2*I*d*x + 3/2*I*c) + sqrt(9*I*a^3*e/d^2)*d*log(2/3*(3*(a*e^(2*I*
d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2
*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + sqrt(9*I*a^3*e/d^2)*d)/a) - sqrt(9*I
*a^3*e/d^2)*d*log(2/3*(3*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - s
qrt(9*I*a^3*e/d^2)*d)/a) - sqrt(-9*I*a^3*e/d^2)*d*log(2/3*(3*(a*e^(2*I*d*x
 + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*
c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + sqrt(-9*I*a^3*e/d^2)*d)/a) + sqrt(-9*I*
a^3*e/d^2)*d*log(2/3*(3*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - sq
rt(-9*I*a^3*e/d^2)*d)/a))/d
```

Sympy [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx = \int \sqrt{e\sec{(c+dx)}}(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(e*sec(c + d*x))*(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1871 vs. $2(204) = 408$.

Time = 0.42 (sec) , antiderivative size = 1871, normalized size of antiderivative = 6.73

$$\int \sqrt{e \sec(c + dx)}(a + ia \tan(c + dx))^{3/2} \, dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output
```
-8*(6*(sqrt(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)
*a)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*
(sqrt(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*ar
ctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt
(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*arctan2
(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2)*a*
cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*arctan2(sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(-I*sqrt(2)*a*co
s(2*d*x + 2*c) + sqrt(2)*a*sin(2*d*x + 2*c) - I*sqrt(2)*a)*arctan2(sqrt(2)
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1) + 6*(I*sqrt(2)*a*cos(2*d*x + 2*c) - sqrt(2)*a*sin(2*d*x + 2*c) +
I*sqrt(2)*a)*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(...
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx = \int \sqrt{\frac{e}{\cos(c+d\,x)}}\,(a+a\tan(c+d\,x)\;1\mathrm{i})^{3/2}\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{\sqrt{e}\,\sqrt{a}\,ai\left(-2\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}+3\left(\int\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\text{ t}\right.\right.}{d}$$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*sqrt(a)*a*i*( - 2*sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1) + 3
*int(sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*d))/d
```

3.403 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 398

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx &= \frac{i\sqrt{2}a^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&\quad-\frac{i\sqrt{2}a^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&\quad+\frac{i\sqrt{2}a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}(\sqrt{a}+\cos(c+dx)(\sqrt{a}-i\sqrt{a}\tan(c+dx)))}\right)\sec(c+dx)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&\quad-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}\end{aligned}$$

output
```
I*2^(1/2)*a^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2
)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d/e^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I
*a*tan(d*x+c))^(1/2)-I*2^(1/2)*a^(5/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan
(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d/e^(1/2)/(a-I*a*t
an(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+I*2^(1/2)*a^(5/2)*arctanh(2^(1/2
)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)/(a^(1/2)+cos(d*x+c
)*(a^(1/2)-I*a^(1/2)*tan(d*x+c))))*sec(d*x+c)/d/e^(1/2)/(a-I*a*tan(d*x+c))
^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-4*I*a*(a+I*a*tan(d*x+c))^(1/2)/d/(e*sec(d*
x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.57 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.01

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx = \frac{2ae(\cos(dx)-i\sin(dx))\left(\operatorname{arctanh}\left(\frac{\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{-1-\right.}{}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(3/2)/Sqrt[e*Sec[c + d*x]],x]
```

output
```
(2*a*e*(Cos[d*x] - I*Sin[d*x])*(ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[
I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*
Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]]*((-I)*Cos[2*c] -
Sin[2*c])*Sqrt[I + Tan[(d*x)/2]] + Sqrt[-1 + I*Cos[c] + Sin[c]]*(2*Sqrt[-1
 - I*Cos[c] - Sin[c]]*(Cos[c] - I*Sin[c])*Sqrt[I - Tan[(d*x)/2]] + ArcTanh
[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c]
- Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*(I*Cos[2*c]
 + Sin[2*c])*Sqrt[I + Tan[(d*x)/2]]))*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan
[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[-1
 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 3977, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3977$$

$$-\frac{a^2 \int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{e^2}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a^2 \int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{e^2}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3980$$

$$-\frac{a^2\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a^2\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3976$$

$$-\frac{4ia^3e\sec(c+dx)\int \frac{\cos(c+dx)(a-ia\tan(c+dx))}{e(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 826$$

$$-\frac{4ia^3e\sec(c+dx)\left(\frac{\int \frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 1476$$

$$-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-}{}}}{2e}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 1082$$

$$-\frac{4ia^3 e\sec(c+dx)\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 217$$

$$-\frac{4ia^3 e\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\,t}}{\sqrt{e\sec}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 1479$$

$$-\frac{4ia^3 e\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(}}{\sqrt{e\sec(c+d}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\cos(c+\right.}}{2\sqrt{2}\sqrt{}}}{}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+}}$$
$$\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$-\frac{4ia^3 e\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+d}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\cos(c+dx)\right.}}{2\sqrt{2}\sqrt{a}\sqrt{}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia}}$$
$$\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-}{e}}}{2\sqrt{2}\sqrt{a}e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan}}$$

$$\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 1103$$

$$-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(3/2)/Sqrt[e*Sec[c + d*x]],x]
```

output
```
((-4*I)*a^3*e*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/
(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (S
qrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]
/(Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]
*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*
Tan[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e
]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a
*Tan[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[
a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((4*I)*a*Sqrt[a + I*a*
Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Maple [A] (warning: unable to verify)

Time = 7.57 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.72

method	result
default	$-\frac{\left(-\csc(dx+c)^2(1-\cos(dx+c))^2+1\right)a\sqrt{a(1+i\tan(dx+c))}\left(-4i+\operatorname{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{\frac{1}{\cos(dx+c)+1}}-\operatorname{arctanh}\left(\right.\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)*a*(a*(1+I*tan(d*x+c)))^(1/2)/(e*se
c(d*x+c))^(1/2)*(-4*I+arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc
(d*x+c)+1))*(1/(cos(d*x+c)+1))^(1/2)-arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*
(-cot(d*x+c)+csc(d*x+c)+1))*(1/(cos(d*x+c)+1))^(1/2)+I*arctanh(1/2/(1/(cos
(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))*(1/(cos(d*x+c)+1))^(1/2)+I*ar
ctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))*(1/(cos(d*x
+c)+1))^(1/2)+4*csc(d*x+c)-4*cot(d*x+c))*(-tan(d*x+c)+I)/(cot(d*x+c)-csc(d
*x+c)+I)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.16

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx =$$

$$-\frac{de\sqrt{\frac{4i\,a^3}{d^2e}}\log\left(\frac{2\,\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+i\,de\sqrt{\frac{4i\,a^3}{d^2e}}}{a}\right)-de\sqrt{\frac{4i\,a^3}{d^2e}}\log\left(\frac{2\,\left(ae^{(2i\,dx+2i\,c)}\right.}{}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
-1/2*(d*e*sqrt(4*I*a^3/(d^2*e))*log((2*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x
+ 1/2*I*c) + I*d*e*sqrt(4*I*a^3/(d^2*e)))/a) - d*e*sqrt(4*I*a^3/(d^2*e))*l
og((2*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e
/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*d*e*sqrt(4*I*a^3/(
d^2*e)))/a) + d*e*sqrt(-4*I*a^3/(d^2*e))*log((2*(a*e^(2*I*d*x + 2*I*c) + a
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1
/2*I*d*x + 1/2*I*c) + I*d*e*sqrt(-4*I*a^3/(d^2*e)))/a) - d*e*sqrt(-4*I*a^3
/(d^2*e))*log((2*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*d*e*sqr
t(-4*I*a^3/(d^2*e)))/a) + 8*(I*a*e^(2*I*d*x + 2*I*c) + I*a)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I
*c))/(d*e)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}{\sqrt{e\sec(c+dx)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(1/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)/sqrt(e*sec(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1462 vs. $2(300) = 600$.

Time = 0.37 (sec) , antiderivative size = 1462, normalized size of antiderivative = 3.67

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx=\text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
1/4*(2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 1) + 2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1) + 2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 1) + 2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 2*sqrt(2)*a*arctan2(sqrt(2)*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
 + 2*sqrt(2)*a*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2
)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + I*sqrt(2)*a*log(2*sqrt(2)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + ia\tan(c + dx))^{3/2}}{\sqrt{e\sec(c + dx)}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx=\frac{\sqrt{e}\,\sqrt{a}\,a\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)}{\sec(dx+c)}dx\right)i+\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}}{\sec(dx+c)}dx\right)}{e}$$

input
```
int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*sqrt(a)*a*(int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c
 + d*x))/sec(c + d*x),x)*i + int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i +
 1))/sec(c + d*x),x)))/e
```

3.404 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 38

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

output
```
-2/3*I*(a+I*a*tan(d*x+c))^(3/2)/d/(e*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3969}$$

$$-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Maple [A] (verified)

Time = 7.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
orering	$-\frac{2i(a+ia\tan(dx+c))^{\frac{3}{2}}}{3d(e\sec(dx+c))^{\frac{3}{2}}}$	31
default	$\frac{\left(\frac{2\sin(dx+c)}{3}-\frac{2i\cos(dx+c)}{3}\right)\sqrt{a(1+i\tan(dx+c))}\, a}{d\sqrt{e\sec(dx+c)}\, e}$	52
risch	$-\frac{2ia\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\mathrm{e}^{i(dx+c)}}{3e\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	72

input
```
int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*(a+I*a*tan(d*x+c))^(3/2)/d/(e*sec(d*x+c))^(3/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(28) = 56.

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{2\left(-i\,ae^{(3i\,dx+3i\,c)}-i\,ae^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{3\,de^2}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
2/3*(-I*a*e^(3*I*d*x + 3*I*c) - I*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*
e^2)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}{(e\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)/(e*sec(c + d*x))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2i\,a^{\frac{3}{2}}\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}{3\,de^{\frac{3}{2}}\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output
```
-2/3*I*a^(3/2)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - 1)^(3/2)/(d*e^(3/2)*(-sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\,a\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)}{\sec(dx+c)^2}dx\right)i+\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i-}}{\sec(dx+c)^2}\right.}{e^2}$$

input
```
int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*sqrt(a)*a*(int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c
 + d*x))/sec(c + d*x)**2,x)*i + int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*
i + 1))/sec(c + d*x)**2,x)))/e**2
```

3.405 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{5de^2\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}$$

output
```
-4/5*I*a*(a+I*a*tan(d*x+c))^(1/2)/d/e^2/(e*sec(d*x+c))^(1/2)-2/5*I*(a+I*a*
tan(d*x+c))^(3/2)/d/(e*sec(d*x+c))^(5/2)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.04

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2a(\cos(dx)-i\sin(dx))(\cos(c+2dx)+i\sin(c+2dx))(3i+2\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{5de(e\sec(c+dx))^{3/2}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*a*(Cos[d*x] - I*Sin[d*x])*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x])*(3*I + 2
*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(5*d*e*(e*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3978}$$
$$\frac{2a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{2a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}$$
$$\Big\downarrow \text{3969}$$
$$-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{5de^2\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(((-4*I)/5)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*Sqrt[e*Sec[c + d*x]]) - (
((2*I)/5)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 6.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{2\left(i\cos(dx+c)^{2}-\cos(dx+c)\sin(dx+c)+2i\right)a\sqrt{a(1+i\tan(dx+c))}}{5d\sqrt{e\sec(dx+c)}\,e^{2}}$	63
risch	$-\frac{ia\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\left(\mathrm{e}^{2i(dx+c)}+5\right)}{5e^{2}\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	74

input
```
int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/d*(I*cos(d*x+c)^2-cos(d*x+c)*sin(d*x+c)+2*I)*a*(a*(1+I*tan(d*x+c)))^(
1/2)/(e*sec(d*x+c))^(1/2)/e^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\left(-i\,ae^{(4i\,dx+4i\,c)} - 6i\,ae^{(2i\,dx+2i\,c)} - 5i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\right.}}{5\,de^3}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
1/5*(-I*a*e^(4*I*d*x + 4*I*c) - 6*I*a*e^(2*I*d*x + 2*I*c) - 5*I*a)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x
+ 1/2*I*c)/(d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.73

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\left(-i\,a\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right) - 5i\,a\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right) + 5\,a\sin\right.}{5\,de^{\frac{5}{2}}}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
1/5*(-I*a*cos(5/2*d*x + 5/2*c) - 5*I*a*cos(1/2*d*x + 1/2*c) + a*sin(5/2*d*
x + 5/2*c) + 5*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/(d*e^(5/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + ia \tan(c + dx))^{3/2}}{(e \sec(c + dx))^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26

$$\int \frac{(a + ia \tan(c + dx))^{3/2}}{(e \sec(c + dx))^{5/2}} \, dx = -\frac{a \sqrt{\frac{e}{\cos(c+d\,x)}} \sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}} \left(-\sin\left(c+d\,x\right)-\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,11\mathrm{i}+\cos\left(3\right.\right.}{10\,d\,e^3}$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(5/2),x)
```

output
```
-(a*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1
))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*11i - sin(c + d*x) + cos(3*
c + 3*d*x)*1i - sin(3*c + 3*d*x)))/(10*d*e^3)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\,a\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)}{\sec(dx+c)^3}dx\right)i+\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}}{\sec(dx+c)^3}dx\right)}{e^3}$$

input
```
int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*sqrt(a)*a*(int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c
 + d*x))/sec(c + d*x)**3,x)*i + int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*
i + 1))/sec(c + d*x)**3,x)))/e**3
```

3.406 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 125

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{16ia^2\sqrt{e\sec(c+dx)}}{21de^4\sqrt{a+ia\tan(c+dx)}} - \frac{8ia\sqrt{a+ia\tan(c+dx)}}{21de^2(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}$$

output
```
16/21*I*a^2*(e*sec(d*x+c))^(1/2)/d/e^4/(a+I*a*tan(d*x+c))^(1/2)-8/21*I*a*(
a+I*a*tan(d*x+c))^(1/2)/d/e^2/(e*sec(d*x+c))^(3/2)-2/7*I*(a+I*a*tan(d*x+c)
)^(3/2)/d/(e*sec(d*x+c))^(7/2)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{a(\cos(dx)-i\sin(dx))(-7i+9i\cos(2(c+dx))+12\sin(2(c+dx)))(\cos(c}{21de^3\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(a*(Cos[d*x] - I*Sin[d*x])*(-7*I + (9*I)*Cos[2*(c + d*x)] + 12*Sin[2*(c +
d*x)])*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(21
*d*e^3*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3978, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{4a\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{4a\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{4a\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(((-2*I)/7)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(7/2)) + (4*
a*((((4*I)/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) -
 (((2*I)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))))/(7*e^
2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 6.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.64

method	result	size
default	$-\frac{2 a \sqrt{a \left(1+i \tan \left(d x +c \right)\right)}\, \left(3 i \cos \left(d x +c \right)^{3}-3 \sin \left(d x +c \right) \cos \left(d x +c \right)^{2}-4 i \cos \left(d x +c \right)-8 \sin \left(d x +c \right)\right)}{21 d \sqrt{e \sec \left(d x +c \right)}\, e^{3}}$	80
risch	$-\frac{i a \sqrt{\frac{a\,\mathrm{e}^{2 i \left(d x +c \right)}}{\mathrm{e}^{2 i \left(d x +c \right)}+1}}\, \left(3\,\mathrm{e}^{3 i \left(d x +c \right)}-7 \cos \left(d x +c \right)+35 i \sin \left(d x +c \right)\right)}{42 e^{3} \sqrt{\frac{e\,\mathrm{e}^{i \left(d x +c \right)}}{\mathrm{e}^{2 i \left(d x +c \right)}+1}}\, d}$	92

input
```
int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21/d*a*(a*(1+I*tan(d*x+c)))^(1/2)*(3*I*cos(d*x+c)^3-3*sin(d*x+c)*cos(d*
x+c)^2-4*I*cos(d*x+c)-8*sin(d*x+c))/(e*sec(d*x+c))^(1/2)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\left(-3i\,ae^{(6i\,dx+6i\,c)}-17i\,ae^{(4i\,dx+4i\,c)}+7i\,ae^{(2i\,dx+2i\,c)}+21i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{}}{42\,de^4}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x, algorithm="fric
as")
```

output
```
1/42*(-3*I*a*e^(6*I*d*x + 6*I*c) - 17*I*a*e^(4*I*d*x + 4*I*c) + 7*I*a*e^(2
*I*d*x + 2*I*c) + 21*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I
*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.67

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\left(-3i\,a\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)-14i\,a\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+21i\,a\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\right.}{42\,de^{\frac{7}{2}}}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x, algorithm="maxi
ma")
```

output
```
1/42*(-3*I*a*cos(7/2*d*x + 7/2*c) - 14*I*a*cos(3/2*d*x + 3/2*c) + 21*I*a*c
os(1/2*d*x + 1/2*c) + 3*a*sin(7/2*d*x + 7/2*c) + 14*a*sin(3/2*d*x + 3/2*c)
 + 21*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/(d*e^(7/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x, algorithm="giac
")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{a\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(\cos\left(2\,c+2\,d\,x\right)\,4\mathrm{i}-\cos\left(4\,c\right.}{84\,d\,e^4}$$

input

```
int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(7/2),x)
```

output

```
(a*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1)
)/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*4i - cos(4*c + 4*d*x)*3i
 + 38*sin(2*c + 2*d*x) + 3*sin(4*c + 4*d*x) + 7i))/(84*d*e^4)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\,a\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)}{\sec(dx+c)^4}dx\right)i+\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i}}{\sec(dx+c)^4}\right.}{e^4}$$

input

```
int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x)
```

output

```
(sqrt(e)*sqrt(a)*a*(int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c
 + d*x))/sec(c + d*x)**4,x)*i + int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*
i + 1))/sec(c + d*x)**4,x)))/e**4
```

3.407 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 167

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{16ia^2}{45de^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{4ia\sqrt{a+ia\tan(c+dx)}}{15de^2(e\sec(c+dx))^{5/2}} - \frac{32ia\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

output
```
16/45*I*a^2/d/e^4/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-4/15*I*a*(
a+I*a*tan(d*x+c))^(1/2)/d/e^2/(e*sec(d*x+c))^(5/2)-32/45*I*a*(a+I*a*tan(d*
x+c))^(1/2)/d/e^4/(e*sec(d*x+c))^(1/2)-2/9*I*(a+I*a*tan(d*x+c))^(3/2)/d/(e
*sec(d*x+c))^(9/2)
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{a(\cos(dx)-i\sin(dx))(-81i\cos(c+dx)+5i\cos(3(c+dx))-54\sin(c+ }{90de^4\sqrt{e\,s}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(9/2),x]
```

output
```
(a*(Cos[d*x] - I*Sin[d*x])*((-81*I)*Cos[c + d*x] + (5*I)*Cos[3*(c + d*x)]
- 54*Sin[c + d*x] + 10*Sin[3*(c + d*x)])*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x
])*Sqrt[a + I*a*Tan[c + d*x]])/(90*d*e^4*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3978, 3042, 3978, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{2a\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx}{3e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx}{3e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{2a\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{2a\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2a\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \textbf{3969}$$

$$\frac{2a\left(\frac{4a\left(\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

input

```
Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(9/2),x]
```

output

```
(((-2*I)/9)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(9/2)) + (2*
a*((((-2*I)/5)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(5/2)) + (4
*a*(((2*I)/3)/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I
)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*Sqrt[e*Sec[c + d*x]])))/(5*e^2)))/(3
*e^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 6.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{2a\sqrt{a(1+i\tan(dx+c))}\left(5i\cos(dx+c)^{4}-5\sin(dx+c)\cos(dx+c)^{3}-2i\cos(dx+c)^{2}-8\cos(dx+c)\sin(dx+c)+16i\right)}{45d\sqrt{e\sec(dx+c)}\,e^{4}}$	90
risch	$-\frac{ia\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\left(5\,\mathrm{e}^{4i(dx+c)}+135+12\cos(2dx+2c)+42i\sin(2dx+2c)\right)}{180e^{4}\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	99

input
```
int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45/d*a*(a*(1+I*tan(d*x+c)))^(1/2)*(5*I*cos(d*x+c)^4-5*sin(d*x+c)*cos(d*
x+c)^3-2*I*cos(d*x+c)^2-8*cos(d*x+c)*sin(d*x+c)+16*I)/(e*sec(d*x+c))^(1/2)
/e^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.62

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{\left(-5i\,ae^{(8i\,dx+8i\,c)} - 32i\,ae^{(6i\,dx+6i\,c)} - 162i\,ae^{(4i\,dx+4i\,c)} - 120i\,ae^{(2i\,dx+2i\,c)} + \right.}{180\,de^5}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x, algorithm="fric
as")
```

output
```
1/180*(-5*I*a*e^(8*I*d*x + 8*I*c) - 32*I*a*e^(6*I*d*x + 6*I*c) - 162*I*a*e
^(4*I*d*x + 4*I*c) - 120*I*a*e^(2*I*d*x + 2*I*c) + 15*I*a)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-3/2*I*d*x - 3/2*I
*c)/(d*e^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{\left(-5i\,a\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+15i\,a\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-27i\,a\cos\left(\frac{5}{3}\,\arctan\left(\sin\left(\right.\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x, algorithm="maxi
ma")
```

output
```
1/180*(-5*I*a*cos(9/2*d*x + 9/2*c) + 15*I*a*cos(3/2*d*x + 3/2*c) - 27*I*a*
cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 135*I*a*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 5*a*sin(9/2*d*
x + 9/2*c) + 15*a*sin(3/2*d*x + 3/2*c) + 27*a*sin(5/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 135*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/(d*e^(9/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 2.89 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.75

$$\int \frac{(a + ia\tan(c + dx))^{3/2}}{(e\sec(c + dx))^{9/2}}\,dx = -\frac{a\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1i)}{\cos(2\,c+2\,d\,x)+1}}\,(-42\,\sin(c+d\,x) - 47\,\sin(3\,c+3\,d\,x) - 5\,\sin(5\,c+5\,d\,x)}{360\,d\,e^5}$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(9/2),x)
```

output
```
-(a*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1
))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*282i - 42*sin(c + d*x) + co
s(3*c + 3*d*x)*17i + cos(5*c + 5*d*x)*5i - 47*sin(3*c + 3*d*x) - 5*sin(5*c
 + 5*d*x)))/(360*d*e^5)
```

Reduce [F]

$$\int \frac{(a + ia\tan(c + dx))^{3/2}}{(e\sec(c + dx))^{9/2}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\,a\left(\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)}{\sec(dx+c)^5}dx\right)i + \int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}}{\sec(dx+c)^5}\right.}{e^5}$$

input
```
int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x)
```

output
```
(sqrt(e)*sqrt(a)*a*(int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c
 + d*x))/sec(c + d*x)**5,x)*i + int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*
i + 1))/sec(c + d*x)**5,x)))/e**5
```

3.408 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 488

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{15ia^{7/2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{15ia^{7/2}e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{15ia^{7/2}e^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}\left(\sqrt{a}+\cos(c+dx)\left(\sqrt{a}-i\sqrt{a}\tan(c+dx)\right)\right)}\right)\sec(c+dx)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{4d}$$
$$+\frac{ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}{3d}$$

output
```
15/8*I*a^3*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)-15/16*I*a^(7/2)
*e^(3/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(
d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)/d/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*
x+c))^(1/2)+15/16*I*a^(7/2)*e^(3/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*
x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)/d/(a-I*a*tan(
d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-15/16*I*a^(7/2)*e^(3/2)*arctanh(2^(
1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)/(a^(1/2)+cos(d*
x+c)*(a^(1/2)-I*a^(1/2)*tan(d*x+c))))*sec(d*x+c)*2^(1/2)/d/(a-I*a*tan(d*x+
c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+3/4*I*a^2*(e*sec(d*x+c))^(3/2)*(a+I*a*t
an(d*x+c))^(1/2)/d+1/3*I*a*(e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 2.81 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.79

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\cos^4(c+dx)(e\sec(c+dx))^{3/2}\left(\frac{1}{6}\sec^3(c+dx)(63+79\cos(2(c+dx))+34i\sin(2(\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(Cos[c + d*x]^4*(e*Sec[c + d*x])^(3/2)*((Sec[c + d*x]^3*(63 + 79*Cos[2*(c
+ d*x)] + (34*I)*Sin[2*(c + d*x)])*(I*Cos[3*c + d*x] + Sin[3*c + d*x]))/6
+ (15*(ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[
-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[
c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqr
t[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])
]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*(Cos[3*c] - I*
Sin[3*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[-1 +
I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]]))*(a + I*a*Tan[c + d*x])^(5/2))/
(8*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.03, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3979, 3042, 3979, 3042, 3979, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{3}{2}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^{3/2}dx+\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^{3/2}dx+\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{3}{2}a\left(\frac{5}{4}a\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\left(\frac{5}{4}a\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{1}{2}a\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{1}{2}a\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3980}$$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{ae\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c}{}\right.\ldots+\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{ae\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c}{}\right.\ldots+\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3976}$$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\int\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{(}{}\right.\ldots+\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{826}$$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\int \frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+}}{}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}}{\sqrt{a}}\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{217}$$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx)\cdots}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx\cdots}}{2e}\cdots}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\cdots\right.\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 1479$$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{\cdots}}{\cdots}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx\cdots}}{\sqrt{e}\sqrt{e\sec(c+dx\cdots}}\cdots\right.}}{\cdots}\cdots}{d\sqrt{a-ia\tan(c+\cdots}}\cdots\right.\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 25$$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{\cdots}}{\sqrt{\cdots}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{\cdots}}{\sqrt{e}\sqrt{e\sec(c+dx)}}\cdots\right.}}{\cdots}\cdots}{d\sqrt{a-ia\tan(c+dx\cdots}}\cdots\right.\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow 27$$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-\cdots}}{\sqrt{e\sec\cdots}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\cdots}}{\cdots}\cdots\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a\cdots}}\cdots\right)\cdots\right)$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow 1103$$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cdots\right)}{2\sqrt{2}\cdots}\cdots\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(\cdots}}\cdots\right)\cdots\right)$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/3)*a*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2))/d + (3*a*(((
I/2)*a*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/d + (5*a*((I*a*(
e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + ((2*I)*a^2*e^3*((-
(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*S
ec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sq
rt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]
*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan
[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]/(S
qrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Ta
n[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]/(
2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[a - I*a*Tan[c + d
*x]]*Sqrt[a + I*a*Tan[c + d*x]])))/4))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 7.69 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.65

method	result
default	$\frac{e\sqrt{e\sec(dx+c)}\,a^2\sqrt{a(1+i\tan(dx+c))}\left(-45i\cos(dx+c)\operatorname{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+45\cos(dx+c)\operatorname{arctanh}\left(\frac{\cot(dx+c)-\csc}{2\sqrt{\frac{1}{\cos(dx}}}\right.\right.}{}$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d*e*(e*sec(d*x+c))^(1/2)*a^2*(a*(1+I*tan(d*x+c)))^(1/2)/(-sin(d*x+c)+
I*cos(d*x+c)+I)/(1/(cos(d*x+c)+1))^(1/2)*(-45*I*cos(d*x+c)*arctanh(1/2/(1/
(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))+45*cos(d*x+c)*arctanh(1/2
/(1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))-45*I*cos(d*x+c)*arcta
nh(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))-45*cos(d*x+c)*a
rctanh(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))-2*I*(1/(cos
(d*x+c)+1))^(1/2)*(8*sec(d*x+c)*tan(d*x+c)-45*sin(d*x+c)+34*tan(d*x+c))-2*
(1/(cos(d*x+c)+1))^(1/2)*(79+45*cos(d*x+c)+26*sec(d*x+c)-8*sec(d*x+c)^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.30

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
1/12*((113*I*a^2*e*e^(4*I*d*x + 4*I*c) + 126*I*a^2*e*e^(2*I*d*x + 2*I*c) +
 45*I*a^2*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c)
 + 1))*e^(1/2*I*d*x + 1/2*I*c) + 6*sqrt(225/64*I*a^5*e^3/d^2)*(d*e^(4*I*d*
x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/15*(15*(a^2*e*e^(2*I*d*x +
 2*I*c) + a^2*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*
I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 8*I*sqrt(225/64*I*a^5*e^3/d^2)*d)/(a^
2*e)) - 6*sqrt(225/64*I*a^5*e^3/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d
*x + 2*I*c) + d)*log(2/15*(15*(a^2*e*e^(2*I*d*x + 2*I*c) + a^2*e)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x +
 1/2*I*c) - 8*I*sqrt(225/64*I*a^5*e^3/d^2)*d)/(a^2*e)) + 6*sqrt(-225/64*I*
a^5*e^3/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/1
5*(15*(a^2*e*e^(2*I*d*x + 2*I*c) + a^2*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 8*I*sqrt(-22
5/64*I*a^5*e^3/d^2)*d)/(a^2*e)) - 6*sqrt(-225/64*I*a^5*e^3/d^2)*(d*e^(4*I*
d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/15*(15*(a^2*e*e^(2*I*d*x
 + 2*I*c) + a^2*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x +
2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - 8*I*sqrt(-225/64*I*a^5*e^3/d^2)*d)/
(a^2*e)))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3005 vs. $2(362) = 724$.

Time = 0.47 (sec) , antiderivative size = 3005, normalized size of antiderivative = 6.16

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
192*(1808*a^2*e*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 201
6*a^2*e*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 720*a^2*e*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1808*I*a^2*e*sin(9/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2016*I*a^2*e*sin(5/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 720*I*a^2*e*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 90*(sqrt(2)*a^2*e*cos(6*d*x + 6*c) + 3*
sqrt(2)*a^2*e*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*e*cos(2*d*x + 2*c) + I*sqrt
(2)*a^2*e*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 3*I*sqrt
(2)*a^2*e*sin(2*d*x + 2*c) + sqrt(2)*a^2*e)*arctan2(sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 90*(sqrt(2)*a^2*e*cos(6*d*x + 6*c) +
 3*sqrt(2)*a^2*e*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*e*cos(2*d*x + 2*c) + I*s
qrt(2)*a^2*e*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 3*I*s
qrt(2)*a^2*e*sin(2*d*x + 2*c) + sqrt(2)*a^2*e)*arctan2(sqrt(2)*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 90*(sqrt(2)*a^2*e*cos(6*d*x + 6*
c) + 3*sqrt(2)*a^2*e*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*e*cos(2*d*x + 2*c) +
 I*sqrt(2)*a^2*e*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 3
*I*sqrt(2)*a^2*e*sin(2*d*x + 2*c) + sqrt(2)*a^2*e)*arctan2(sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arct...
```

Giac [F(-2)]

Exception generated.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}(a+a\tan(c+d\,x)\,1\mathrm{i})^{5/2}\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\sqrt{e}\,\sqrt{a}\,a^2e\Big(-2\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\sec(dx+c)\,i-\Big(\int\sqrt{\sec(dx+}$$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*sqrt(a)*a**2*e*( - 2*sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*
sec(c + d*x)*i - int(sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec(c + d
*x)*tan(c + d*x)**2,x)*d + 6*int(sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i +
1)*sec(c + d*x)*tan(c + d*x),x)*d*i))/d
```

3.409 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 327

$$\begin{aligned}\int \sqrt{e\sec(c+dx)}(a&+ia\tan(c+dx))^{5/2}\,dx = \frac{21ia^{5/2}\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d}\\
&-\frac{21ia^{5/2}\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d}\\
&+\frac{21ia^{5/2}\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}\left(\sqrt{a}+\cos(c+dx)\left(\sqrt{a}+i\sqrt{a}\tan(c+dx)\right)\right)}\right)}{4\sqrt{2}d}\\
&+\frac{7ia^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d}\\
&+\frac{ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d}\end{aligned}$$

output
```
21/8*I*a^(5/2)*e^(1/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a
^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2)/d-21/8*I*a^(5/2)*e^(1/2)*arctan(1+2^(
1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2
)/d+21/8*I*a^(5/2)*e^(1/2)*arctanh(2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2
)/(e*sec(d*x+c))^(1/2)/(a^(1/2)+cos(d*x+c)*(a^(1/2)+I*a^(1/2)*tan(d*x+c)))
)*2^(1/2)/d+7/4*I*a^2*(e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d+1/2*
I*a*(e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 3.70 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.18

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{a^2\sqrt{e\sec(c+dx)}(\cos(2dx)+i\sin(2dx))\sqrt{a+ia\tan(c+dx)}\left(21\text{arctanh}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\right.}{}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
-1/4*(a^2*Sqrt[e*Sec[c + d*x]]*(Cos[2*d*x] + I*Sin[2*d*x])*Sqrt[a + I*a*Ta
n[c + d*x]]*(21*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]
])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]*Sqr
t[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2
]] - 21*ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt
[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]*Sqrt[1 - I*
Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + Sqr
t[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]]*(-9*I + 2*Tan[c + d*x]
)))/(d*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*(Cos[d*x] + I*Sin[d*x])^2*Sqrt[I -
Tan[(d*x)/2]])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.28, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 3979, 3042, 3979, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{7}{4}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^{3/2}dx+\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^{3/2}dx+\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{7}{4}a\left(\frac{3}{2}a\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\left(\frac{3}{2}a\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3976}$$

$$\frac{7}{4}a\left(\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\int\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{d}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 826

$$\frac{7}{4}a\left(\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\int\frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(i}{a^2+\cos^2(c+dx)(i}}{}\right.}{d}\right.$$
$$\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 1476

$$\frac{7}{4}a\left(\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}}{}\right.}{}\right.$$
$$\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 1082

$$\frac{7}{4}a\left(\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\int\frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int\frac{}{-\frac{\cos(c+}{}}}{}}{2e}\right.}{}\right.$$
$$\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 217$$

$$\frac{7}{4}a\left(\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\right.}{d}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 1479$$

$$\frac{7}{4}a\left(\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\right.}{}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 25$$

$$\frac{7}{4}a\left(\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\right.}{}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow 27$$

$$\frac{7}{4}a\left(\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\right.}{}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow 1103$$

$$\frac{7}{4}a\left(\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\right.}{}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/2)*a*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2))/d + (7*a*(((-6
*I)*a^2*e^2*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(S
qrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqr
t[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(
Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*S
qrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Ta
n[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*
Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*T
an[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/d + (I*a*Sqrt[e*Sec[c +
 d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

Maple [A] (verified)

Time = 7.80 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.88

method	result
default	$\frac{\sqrt{e\sec(dx+c)}\, a^2 \sqrt{a(1+i\tan(dx+c))} \left(21i\cos(dx+c)\,\mathrm{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+21\cos(dx+c)\,\mathrm{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)}{2\sqrt{\frac{1}{\cos(dx+c)}}}\right.\right.}{}$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*(e*sec(d*x+c))^(1/2)*a^2*(a*(1+I*tan(d*x+c)))^(1/2)/(-I*cos(d*x+c)+s
in(d*x+c)-I)/(1/(cos(d*x+c)+1))^(1/2)*(21*I*cos(d*x+c)*arctanh(1/2/(1/(cos
(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))+21*cos(d*x+c)*arctanh(1/2/(1/
(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))+21*I*cos(d*x+c)*arctanh(1
/2*(cot(d*x+c)-csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))-21*cos(d*x+c)*arcta
nh(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))+2*I*(1/(cos(d*x
+c)+1))^(1/2)*(11*sin(d*x+c)+2*tan(d*x+c))+2*(1/(cos(d*x+c)+1))^(1/2)*(9+1
1*cos(d*x+c)-2*sec(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(235) = 470.

Time = 0.09 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.61

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
1/2*((11*I*a^2*e^(3*I*d*x + 3*I*c) + 7*I*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2
*I*c) + sqrt(441/16*I*a^5*e/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/21*(21*
(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(
e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 4*sqrt(441/16*I*a^5*e/
d^2)*d)/a^2) - sqrt(441/16*I*a^5*e/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/
21*(21*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*s
qrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - 4*sqrt(441/16*I
*a^5*e/d^2)*d)/a^2) - sqrt(-441/16*I*a^5*e/d^2)*(d*e^(2*I*d*x + 2*I*c) + d
)*log(2/21*(21*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 4*sqrt(
-441/16*I*a^5*e/d^2)*d)/a^2) + sqrt(-441/16*I*a^5*e/d^2)*(d*e^(2*I*d*x + 2
*I*c) + d)*log(2/21*(21*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)
- 4*sqrt(-441/16*I*a^5*e/d^2)*d)/a^2))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2431 vs. $2(235) = 470$.

Time = 0.45 (sec) , antiderivative size = 2431, normalized size of antiderivative = 7.43

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
32*(176*a^2*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 112*a^2
*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 176*I*a^2*sin(7/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 112*I*a^2*sin(3/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 42*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2
*sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(
2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 1) - 42*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt
(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a^
2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1) - 42*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*
a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*si
n(2*d*x + 2*c) + sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) - 42*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*c
os(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*sin(2*d
*x + 2*c) + sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 1) - 42*(-I*sqrt(2)*a^2*cos(4*d*x + 4*c) - 2*I*sqrt(2)*a...
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx=\int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx=\frac{\sqrt{e}\,\sqrt{a}\,a^{2}\left(-2\sqrt{\sec\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,i-\left(\int\sqrt{\sec\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,\mathrm{t}\right.\right.}{d}$$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*sqrt(a)*a**2*( - 2*sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*i
- int(sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2,x)*d + 4
*int(sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*d*i))/d
```

3.410 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 437

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx = \frac{5ia^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{5ia^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{5ia^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}\left(\sqrt{a}+\cos(c+dx)\left(\sqrt{a}-i\sqrt{a}\tan(c+dx)\right)\right)}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

output
```
5/2*I*a^(7/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e
*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)/d/e^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/
(a+I*a*tan(d*x+c))^(1/2)-5/2*I*a^(7/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan
(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)/d/e^(1/2)/
(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+5/2*I*a^(7/2)*arctanh(2^
(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)/(a^(1/2)+cos(d
*x+c)*(a^(1/2)-I*a^(1/2)*tan(d*x+c))))*sec(d*x+c)*2^(1/2)/d/e^(1/2)/(a-I*a
*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-10*I*a^2*(a+I*a*tan(d*x+c))^(1
/2)/d/(e*sec(d*x+c))^(1/2)+I*a*(a+I*a*tan(d*x+c))^(3/2)/d/(e*sec(d*x+c))^(
1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.12 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.82

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx = \frac{e^2(a+ia\tan(c+dx))^{5/2}\left(\frac{5\operatorname{arctanh}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{1-i\cos(c)+\mathrm{si}}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i-\tan(}}\right.}{}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(5/2)/Sqrt[e*Sec[c + d*x]],x]
```

output
```
(e^2*(a + I*a*Tan[c + d*x])^(5/2)*((5*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]
*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/
2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*(Cos[3*c] - I*Sin[3*c])*Sqrt[I + Tan[(d*
x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]]) - (5*ArcTanh
[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c]
+ Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 + I*Cos[c] - Sin[c]]*(Cos[3*c] -
 I*Sin[3*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I
- Tan[(d*x)/2]]) - (Cos[2*c] - I*Sin[2*c])*(9*I + Tan[c + d*x])))/(d*(e*Se
c[c + d*x])^(5/2)*(Cos[d*x] + I*Sin[d*x])^2)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3979, 3042, 3977, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{5}{2}a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{\sqrt{e\sec(c+dx)}}dx+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{2}a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{\sqrt{e\sec(c+dx)}}dx+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3977}$$

$$\frac{5}{2}a\left(-\frac{a^2\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{e^2}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{2}a\left(-\frac{a^2\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{e^2}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3980}$$

$$\frac{5}{2}a\left(-\frac{a^2\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{2}a\left(-\frac{a^2\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3976}$$

$$\frac{5}{2}a\left(-\frac{4ia^3e\sec(c+dx)\int\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{826}$$

$$\frac{5}{2}a\left(-\frac{4ia^3e\sec(c+dx)\left(\frac{\int\frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{5}{2}a\left(-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\cos(c-}}{}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c}}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{5}{2}a\left(-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia}}{\sqrt{a}\sqrt{e\sec}}\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 217$$

$$\frac{5}{2}a\left(-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 1479$$

$$\frac{5}{2}a\left(-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-}}{\sqrt{e\,s}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\right.}}{}\right.}{d\sqrt{a-ia\tan(c+dx)}}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{5}{2}a\left(-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\,t}}{\sqrt{e\sec(c}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\cos\right.}}{2}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a}}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\big\downarrow 27$$

$$\frac{5}{2}a\left(-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(}}{\sqrt{e\sec(c+d}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\cos(c+}}{2\sqrt{2}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+i}}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\big\downarrow 1103$$

$$\frac{5}{2}a\left(-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+}}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(5/2)/Sqrt[e*Sec[c + d*x]],x]
```

output
```
(I*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*Sqrt[e*Sec[c + d*x]]) + (5*a*(((-4*I
)*a^3*e*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[
a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]
*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt
[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[
a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c
+ d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt
[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c
 + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[a - I*
a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((4*I)*a*Sqrt[a + I*a*Tan[c
+ d*x]])/(d*Sqrt[e*Sec[c + d*x]])))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Maple [A] (warning: unable to verify)

Time = 7.94 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.70

method	result
default	$\frac{a^2\sqrt{a(1+i\tan(dx+c))}\left(16\sin(dx+c)-2\tan(dx+c)+i(-16\cos(dx+c)-18-2\sec(dx+c))-5(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\ \mathrm{arctanl}\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*a^2*(a*(1+I*tan(d*x+c)))^(1/2)/(1+cos(d*x+c)+I*sin(d*x+c))/(e*sec(d*
x+c))^(1/2)*(16*sin(d*x+c)-2*tan(d*x+c)+I*(-16*cos(d*x+c)-18-2*sec(d*x+c))
-5*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*(-cot(d*x+c)+csc(d*
x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))-5*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2
)*arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+5*I*(co
s(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*
(-cot(d*x+c)+csc(d*x+c)+1))-5*I*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*ar
ctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.11

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx =$$

$$-\frac{\sqrt{\frac{25i a^5}{d^2e}}de\log\left(\frac{2\left(5\left(a^2e^{(2i\,dx+2i\,c)}+a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+i\sqrt{\frac{25i\,a^5}{d^2e}}de\right)}{5\,a^2}\right)-\sqrt{\frac{25i\,a^5}{d^2e}}de\log\left(\frac{2\left(5\cdots\right)}{\cdots}\right)\cdots}{\cdots}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
-1/2*(sqrt(25*I*a^5/(d^2*e))*d*e*log(2/5*(5*(a^2*e^(2*I*d*x + 2*I*c) + a^2
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1
/2*I*d*x + 1/2*I*c) + I*sqrt(25*I*a^5/(d^2*e))*d*e)/a^2) - sqrt(25*I*a^5/(
d^2*e))*d*e*log(2/5*(5*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) -
 I*sqrt(25*I*a^5/(d^2*e))*d*e)/a^2) + sqrt(-25*I*a^5/(d^2*e))*d*e*log(2/5*
(5*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + I*sqrt(-25*I*a^5/(d
^2*e))*d*e)/a^2) - sqrt(-25*I*a^5/(d^2*e))*d*e*log(2/5*(5*(a^2*e^(2*I*d*x
+ 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I
*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*sqrt(-25*I*a^5/(d^2*e))*d*e)/a^2) +
4*(4*I*a^2*e^(2*I*d*x + 2*I*c) + 5*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + ia\tan(c + dx))^{5/2}}{\sqrt{e\sec(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2013 vs. $2(331) = 662$.

Time = 0.33 (sec) , antiderivative size = 2013, normalized size of antiderivative = 4.61

$$\int \frac{(a + ia\tan(c + dx))^{5/2}}{\sqrt{e\sec(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
8*(10*(sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqr
t(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1
) + 10*(sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sq
rt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 1) + 10*(sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(2*d*x + 2*c) +
sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 1) + 10*(sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(2*d*x + 2*c) +
 sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 1) - 10*(-I*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*sin(2*d*x + 2*c
) - I*sqrt(2)*a^2)*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt
(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 10*(I*sqrt(2)*a^2*cos(2*d*x +
 2*c) - sqrt(2)*a^2*sin(2*d*x + 2*c) + I*sqrt(2)*a^2)*arctan2(-sqrt(2)*sin
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{5/2}}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx=\frac{\sqrt{e}\,\sqrt{a}\,a^2\left(-\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2}{\sec(dx+c)}dx\right)+2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(d}}{\sec(d}\right.\right.}{e}$$

input
```
int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*sqrt(a)*a**2*( - int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)
*tan(c + d*x)**2)/sec(c + d*x),x) + 2*int((sqrt(sec(c + d*x))*sqrt(tan(c +
 d*x)*i + 1)*tan(c + d*x))/sec(c + d*x),x)*i + int((sqrt(sec(c + d*x))*sqr
t(tan(c + d*x)*i + 1))/sec(c + d*x),x)))/e
```

3.411 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 280

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx &= -\frac{i\sqrt{2}a^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}}\\
&+\frac{i\sqrt{2}a^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}}\\
&-\frac{i\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}\left(\sqrt{a}+\cos(c+dx)\left(\sqrt{a}+i\sqrt{a}\tan(c+dx)\right)\right)}\right)}{de^{3/2}}\\
&-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}\end{aligned}$$

output

```
-I*2^(1/2)*a^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/
2)/(e*sec(d*x+c))^(1/2))/d/e^(3/2)+I*2^(1/2)*a^(5/2)*arctan(1+2^(1/2)*e^(1
/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))/d/e^(3/2)-I*2^(
1/2)*a^(5/2)*arctanh(2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c
))^(1/2)/(a^(1/2)+cos(d*x+c)*(a^(1/2)+I*a^(1/2)*tan(d*x+c))))/d/e^(3/2)-4/
3*I*a*(a+I*a*tan(d*x+c))^(3/2)/d/(e*sec(d*x+c))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.55 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.22

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{e\left(-\frac{4}{3}i\cos(dx)(\cos(c)-i\sin(c))+\frac{4}{3}(\cos(c)-i\sin(c))\sin(dx)+\frac{2\left(\operatorname{arctan}\right.}{}\right.}{}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(e*(((-4*I)/3)*Cos[d*x]*(Cos[c] - I*Sin[c]) + (4*(Cos[c] - I*Sin[c])*Sin[d
*x])/3 + (2*(ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/
(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c]
- Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c
]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x
)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*(Cos[2*c
] - I*Sin[2*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*S
qrt[I - Tan[(d*x)/2]]))*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^
(5/2)*(Cos[d*x] + I*Sin[d*x])^2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.33, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3977, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \ 3977$$

$$-\frac{a^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{e^2}-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{e^2}-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3976$$

$$\frac{4ia^3\int \frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{d}-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 826$$

$$\frac{4ia^3\left(\frac{\int \frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 1476$$

$$\frac{4ia^3\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}}{2e}\right)}{d}-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 1082$$

$$\frac{4ia^3\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\int \frac{a-\cdots}{a^2+\cdots}\right)}{d}-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 217$$

$$\frac{4ia^3\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 1479$$

$$\frac{4ia^3\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right)}}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}\right)}{d}$$

$$\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{4ia^3\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right)}d}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right)}{d}$$

$$\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{4ia^3\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan}}{\sqrt{e}}}{2\sqrt{2}\sqrt{a}e}\right)}{d}$$

$$\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 1103$$

$$\frac{4ia^3\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}-}{2e}\right.}{d}$$

$$\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

input

```
Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(3/2),x]
```

output

```
((4*I)*a^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sq
rt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt
[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(S
qrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sq
rt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan
[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*S
qrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Ta
n[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/d - (((4*I)/3)*a*(a + I*
a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

Maple [A] (warning: unable to verify)

Time = 7.65 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.06

method	result
default	$\left(-\frac{i\sqrt{\frac{1}{\cos(dx+c)+1}}\,(\cos(dx+c)-\sin(dx+c)+1)\,\operatorname{arctanh}\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)}{2}+\frac{i\sqrt{\frac{1}{\cos(dx+c)+1}}\,(\cos(dx+c)+\sin(dx+c)+1)\,\operatorname{arctanh}\left(\right.}{2}\right.$

input
```
int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)-sin(d*x+c)+1)*arctanh(1/2
/(1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+1/2*I*(1/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)+sin(d*x+c)+1)*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1
)/(1/(cos(d*x+c)+1))^(1/2))-4/3*I*cos(d*x+c)-1/2*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)+sin(d*x+c)+1)*arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c
)+csc(d*x+c)+1))-1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)-sin(d*x+c)+1)*ar
ctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))+4/3*sin(d*x
+c))*a^2*(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2)/e
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(204) = 408.

Time = 0.12 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.80

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx =$$

$$-\frac{3\,de^2\sqrt{\frac{4i\,a^5}{d^2e^3}}\log\left(\frac{de^2\sqrt{\frac{4i\,a^5}{d^2e^3}}+2\left(a^2e^{(2i\,dx+2i\,c)}+a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{a^2}\right)-3\,de^2\sqrt{\frac{4i\,a^5}{d^2e^3}}\log\left(-\frac{de^2}{}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
-1/6*(3*d*e^2*sqrt(4*I*a^5/(d^2*e^3))*log((d*e^2*sqrt(4*I*a^5/(d^2*e^3)) +
 2*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/a^2) - 3*d*e^2*sqrt(
4*I*a^5/(d^2*e^3))*log(-(d*e^2*sqrt(4*I*a^5/(d^2*e^3)) - 2*(a^2*e^(2*I*d*x
 + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*
I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/a^2) - 3*d*e^2*sqrt(-4*I*a^5/(d^2*e^3)
)*log((d*e^2*sqrt(-4*I*a^5/(d^2*e^3)) + 2*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2
*I*d*x + 1/2*I*c))/a^2) + 3*d*e^2*sqrt(-4*I*a^5/(d^2*e^3))*log(-(d*e^2*sqr
t(-4*I*a^5/(d^2*e^3)) - 2*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c
))/a^2) + 8*(I*a^2*e^(3*I*d*x + 3*I*c) + I*a^2*e^(I*d*x + I*c))*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1
/2*I*c))/(d*e^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1492 vs. $2(204) = 408$.

Time = 0.33 (sec) , antiderivative size = 1492, normalized size of antiderivative = 5.33

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output
```
-1/12*(-6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 1) - 6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1) - 6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 6*sqrt(2)*a^2*arctan2(sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) + 6*sqrt(2)*a^2*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*I*sqrt(2)*a^2*
log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + ia\tan(c + dx))^{5/2}}{(e\sec(c + dx))^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\,a^2\left(-\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2}{\sec(dx+c)^2}dx\right)+2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(d}}{\sec(dx}\right.\right.}{e^2}$$

input
```
int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*sqrt(a)*a**2*( - int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)
*tan(c + d*x)**2)/sec(c + d*x)**2,x) + 2*int((sqrt(sec(c + d*x))*sqrt(tan(
c + d*x)*i + 1)*tan(c + d*x))/sec(c + d*x)**2,x)*i + int((sqrt(sec(c + d*x
))*sqrt(tan(c + d*x)*i + 1))/sec(c + d*x)**2,x)))/e**2
```

3.412 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 38

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{5/2}}{5d(e\sec(c+dx))^{5/2}}$$

output
```
-2/5*I*(a+I*a*tan(d*x+c))^(5/2)/d/(e*sec(d*x+c))^(5/2)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{5/2}}{5d(e\sec(c+dx))^{5/2}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3969}$$
$$-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5d(e\sec(c+dx))^{5/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Maple [A] (verified)

Time = 8.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
orering	$-\frac{2i(a+ia\tan(dx+c))^{\frac{5}{2}}}{5d(e\sec(dx+c))^{\frac{5}{2}}}$	31
default	$\frac{2\sqrt{a(1+i\tan(dx+c))}\,a^2\left(-2i\cos(dx+c)^2+2\cos(dx+c)\sin(dx+c)+i\right)}{5d\sqrt{e\sec(dx+c)}\,e^2}$	65
risch	$-\frac{2ia^2\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{2i(dx+c)}}{5e^2\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	74

input
```
int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*I*(a+I*a*tan(d*x+c))^(5/2)/d/(e*sec(d*x+c))^(5/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(28) = 56.

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{2\left(-i\,a^2e^{(4i\,dx+4i\,c)}-i\,a^2e^{(2i\,dx+2i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{5\,de^3}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
2/5*(-I*a^2*e^(4*I*d*x + 4*I*c) - I*a^2*e^(2*I*d*x + 2*I*c))*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*
I*c)/(d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2i\,a^{\frac{5}{2}}\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}}{5\,de^{\frac{5}{2}}\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
-2/5*I*a^(5/2)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - 1)^(5/2)/(d*e^(5/2)*(-sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - 1)^(5/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.74

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{a^2\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(-\sin\left(c+d\,x\right)-\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,1\mathrm{i}+\cos\left(3\right.\right.}{5\,d\,e^3}$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(5/2),x)
```

output
```
-(a^2*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
 1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*1i - sin(c + d*x) + cos(3
*c + 3*d*x)*1i - sin(3*c + 3*d*x)))/(5*d*e^3)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\,a^2\left(-\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2}{\sec(dx+c)^3}dx\right)+2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(d}}{\sec(d}\right.\right.}{e^3}$$

input
```
int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*sqrt(a)*a**2*( - int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)
*tan(c + d*x)**2)/sec(c + d*x)**3,x) + 2*int((sqrt(sec(c + d*x))*sqrt(tan(
c + d*x)*i + 1)*tan(c + d*x))/sec(c + d*x)**3,x)*i + int((sqrt(sec(c + d*x
))*sqrt(tan(c + d*x)*i + 1))/sec(c + d*x)**3,x)))/e**3
```

3.413 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx=-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}$$

output
```
-4/21*I*a*(a+I*a*tan(d*x+c))^(3/2)/d/e^2/(e*sec(d*x+c))^(3/2)-2/7*I*(a+I*a
*tan(d*x+c))^(5/2)/d/(e*sec(d*x+c))^(7/2)
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{2a^2(\cos(2(c+2dx))+i\sin(2(c+2dx)))(5i+2\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{21de^2(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))^2}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(-2*a^2*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)])*(5*I + 2*Tan[c + d*x])
*Sqrt[a + I*a*Tan[c + d*x]])/(21*d*e^2*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] +
I*Sin[d*x])^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}dx$$
$$\Big\downarrow \text{3978}$$
$$\frac{2a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{2a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}$$
$$\Big\downarrow \text{3969}$$
$$-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(((-4*I)/21)*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*e^2*(e*Sec[c + d*x])^(3/2)
) - (((2*I)/7)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(7/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 6.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

method	result	size
default	$-\frac{2\sqrt{a(1+i\tan(dx+c))}\, a^2\left(6i\cos(dx+c)^3-6\sin(dx+c)\cos(dx+c)^2-i\cos(dx+c)-2\sin(dx+c)\right)}{21d\sqrt{e\sec(dx+c)}\, e^3}$	82
risch	$-\frac{i a^2\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\left(3\,e^{3i(dx+c)}+7\,e^{i(dx+c)}\right)}{21e^3\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, d}$	88

input
```
int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21/d*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*(6*I*cos(d*x+c)^3-6*sin(d*x+c)*cos(
d*x+c)^2-I*cos(d*x+c)-2*sin(d*x+c))/(e*sec(d*x+c))^(1/2)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\left(-3i\,a^2e^{(5i\,dx+5i\,c)}-10i\,a^2e^{(3i\,dx+3i\,c)}-7i\,a^2e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx-}}}}{21\,de^4}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x, algorithm="fric
as")
```

output
```
1/21*(-3*I*a^2*e^(5*I*d*x + 5*I*c) - 10*I*a^2*e^(3*I*d*x + 3*I*c) - 7*I*a^
2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x +
2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*e^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\left(-7i\,a^2\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\,a^2\cos\left(\frac{7}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\right.\right.\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x, algorithm="maxi
ma")
```

output
```
1/21*(-7*I*a^2*cos(3/2*d*x + 3/2*c) - 3*I*a^2*cos(7/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 7*a^2*sin(3/2*d*x + 3/2*c) + 3*a^2*sin(
7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/(d*e^(7/
2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{a^2\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1{}\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,10{}\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,3{}\mathrm{i}-10\,\sin\left(2\,c+2\,d\right.\right.}{42\,d\,e^4}$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(7/2),x)
```

output
```
-(a^2*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
 1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*10i + cos(4*c + 4*d*x
)*3i - 10*sin(2*c + 2*d*x) - 3*sin(4*c + 4*d*x) + 7i))/(42*d*e^4)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\,a^2\left(-\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2}{\sec(dx+c)^4}dx\right)+2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(d}}{\sec(dx}\right.\right.}{e^4}$$

input
```
int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x)
```

output
```
(sqrt(e)*sqrt(a)*a**2*( - int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)
*tan(c + d*x)**2)/sec(c + d*x)**4,x) + 2*int((sqrt(sec(c + d*x))*sqrt(tan(
c + d*x)*i + 1)*tan(c + d*x))/sec(c + d*x)**4,x)*i + int((sqrt(sec(c + d*x
))*sqrt(tan(c + d*x)*i + 1))/sec(c + d*x)**4,x)))/e**4
```

3.414 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 125

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = -\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}} - \frac{8ia(a+ia\tan(c+dx))^{3/2}}{45de^2(e\sec(c+dx))^{5/2}} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}$$

output
```
-16/45*I*a^2*(a+I*a*tan(d*x+c))^(1/2)/d/e^4/(e*sec(d*x+c))^(1/2)-8/45*I*a*
(a+I*a*tan(d*x+c))^(3/2)/d/e^2/(e*sec(d*x+c))^(5/2)-2/9*I*(a+I*a*tan(d*x+c
))^(5/2)/d/(e*sec(d*x+c))^(9/2)
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{a^2(9+25\cos(2(c+dx))-20i\sin(2(c+dx)))(-i\cos(2(c+2dx))+\sin(2}{45de^4\sqrt{e\sec(c+dx)}(\cos(dx)+i\sin(dx))^2}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(9/2),x]
```

output
```
(a^2*(9 + 25*Cos[2*(c + d*x)] - (20*I)*Sin[2*(c + d*x)])*((-I)*Cos[2*(c +
2*d*x)] + Sin[2*(c + 2*d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(45*d*e^4*Sqrt[e
*Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3978, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{4a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{(e\sec(c+dx))^{5/2}}dx}{9e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{(e\sec(c+dx))^{5/2}}dx}{9e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{4a\left(\frac{2a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}\right)}{9e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\left(\frac{2a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}\right)}{9e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{4a\left(-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{5de^2\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}\right)}{9e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}$$

input

```
Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(9/2),x]
```

output

```
(((-2*I)/9)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(9/2)) + (4*
a*((((-4*I)/5)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*Sqrt[e*Sec[c + d*x]])
- (((2*I)/5)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(5/2))))/(9
*e^2)
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 6.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.71

method	result	size
risch	$-\frac{i a^{2} \sqrt{\frac{a\,{\mathrm e}^{2 i (d x +c)}}{{\mathrm e}^{2 i (d x +c)}+1}}\, \left(5\,{\mathrm e}^{4 i (d x +c)}+18\,{\mathrm e}^{2 i (d x +c)}+45\right)}{90 e^{4} \sqrt{\frac{e\,{\mathrm e}^{i (d x +c)}}{{\mathrm e}^{2 i (d x +c)}+1}}\, d}$	89
default	$-\frac{2 \sqrt{a \left(1+i \tan \left(d x +c \right)\right)}\, a^{2} \left(10 i \cos \left(d x +c \right)^{4}-10 \sin \left(d x +c \right) \cos \left(d x +c \right)^{3}-i \cos \left(d x +c \right)^{2}-4 \cos \left(d x +c \right) \sin \left(d x +c \right)+8 i\right)}{45 d \sqrt{e \sec \left(d x +c \right)}\, e^{4}}$	92

input
```
int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/90*I*a^2/e^4/(e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^(1/2)*(a*exp(2*I*(
d*x+c))/(exp(2*I*(d*x+c))+1))^(1/2)*(5*exp(4*I*(d*x+c))+18*exp(2*I*(d*x+c)
)+45)/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{\left(-5i\,a^2e^{(6i\,dx+6i\,c)}-23i\,a^2e^{(4i\,dx+4i\,c)}-63i\,a^2e^{(2i\,dx+2i\,c)}-45i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i}}}}{90\,de^5}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x, algorithm="fric
as")
```

output
```
1/90*(-5*I*a^2*e^(6*I*d*x + 6*I*c) - 23*I*a^2*e^(4*I*d*x + 4*I*c) - 63*I*a
^2*e^(2*I*d*x + 2*I*c) - 45*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*e^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{\left(-5i\,a^2\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)-18i\,a^2\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-45i\,a^2\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{90\,c}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x, algorithm="maxi
ma")
```

output
```
1/90*(-5*I*a^2*cos(9/2*d*x + 9/2*c) - 18*I*a^2*cos(5/2*d*x + 5/2*c) - 45*I
*a^2*cos(1/2*d*x + 1/2*c) + 5*a^2*sin(9/2*d*x + 9/2*c) + 18*a^2*sin(5/2*d*
x + 5/2*c) + 45*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/(d*e^(9/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = -\frac{a^2\sqrt{\frac{e}{\cos(c+dx)}}\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(-18\,\sin(c+d\,x)-23\,\sin(3\,c+3\,d\,x)-5\,\sin(5\,c+5\,d\,x)}{180\,d\,e^5}$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(9/2),x)
```

output
```
-(a^2*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
 1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*108i - 18*sin(c + d*x) +
cos(3*c + 3*d*x)*23i + cos(5*c + 5*d*x)*5i - 23*sin(3*c + 3*d*x) - 5*sin(5
*c + 5*d*x)))/(180*d*e^5)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\,a^2\left(-\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2}{\sec(dx+c)^5}dx\right)+2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(d}}{\sec(d}\right.\right.}{e^5}$$

input
```
int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x)
```

output
```
(sqrt(e)*sqrt(a)*a**2*( - int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)
*tan(c + d*x)**2)/sec(c + d*x)**5,x) + 2*int((sqrt(sec(c + d*x))*sqrt(tan(
c + d*x)*i + 1)*tan(c + d*x))/sec(c + d*x)**5,x)*i + int((sqrt(sec(c + d*x
))*sqrt(tan(c + d*x)*i + 1))/sec(c + d*x)**5,x)))/e**5
```

3.415 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 169

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx = \frac{32ia^3\sqrt{e\sec(c+dx)}}{77de^6\sqrt{a+ia\tan(c+dx)}} - \frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{77de^4(e\sec(c+dx))^{3/2}} - \frac{12ia(a+ia\tan(c+dx))^{3/2}}{77de^2(e\sec(c+dx))^{7/2}} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

output
```
32/77*I*a^3*(e*sec(d*x+c))^(1/2)/d/e^6/(a+I*a*tan(d*x+c))^(1/2)-16/77*I*a^
2*(a+I*a*tan(d*x+c))^(1/2)/d/e^4/(e*sec(d*x+c))^(3/2)-12/77*I*a*(a+I*a*tan
(d*x+c))^(3/2)/d/e^2/(e*sec(d*x+c))^(7/2)-2/11*I*(a+I*a*tan(d*x+c))^(5/2)/
d/(e*sec(d*x+c))^(11/2)
```

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.72

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx = \frac{a^2(-55i\cos(c+dx)+35i\cos(3(c+dx))-22\sin(c+dx)+42\sin(3(c+$$
$$154de^5\sqrt{e\sec(c+dx)}(\cos($$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(11/2),x]
```

output
```
(a^2*((-55*I)*Cos[c + d*x] + (35*I)*Cos[3*(c + d*x)] - 22*Sin[c + d*x] + 4
2*Sin[3*(c + d*x)])*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)])*Sqrt[a + I
*a*Tan[c + d*x]])/(154*d*e^5*Sqrt[e*Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^
2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3978, 3042, 3978, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}dx$$
$$\Big\downarrow \; 3978$$
$$\frac{6a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{(e\sec(c+dx))^{7/2}}dx}{11e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$
$$\Big\downarrow \; 3042$$
$$\frac{6a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{(e\sec(c+dx))^{7/2}}dx}{11e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{6a\left(\frac{4a\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6a\left(\frac{4a\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{6a\left(\frac{4a\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6a\left(\frac{4a\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{6a\left(\frac{4a\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(11/2),x]
```

output
```
(((-2*I)/11)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(11/2)) + (
6*a*((((-2*I)/7)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(7/2))
+ (4*a*((((4*I)/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x
]]) - (((2*I)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))))/
(7*e^2)))/(11*e^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 6.47 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.57

method	result	s
risch	$-\frac{i a^{2} \sqrt{\frac{a\,{\mathrm e}^{2 i (d x +c)}}{{\mathrm e}^{2 i (d x +c)}+1}}\, \left(7 \,{\mathrm e}^{5 i (d x +c)}+33 \,{\mathrm e}^{3 i (d x +c)}+154 i \sin \left(d x +c \right)\right)}{308 e^{5} \sqrt{\frac{e \,{\mathrm e}^{i (d x +c)}}{{\mathrm e}^{2 i (d x +c)}+1}}\, d}$	9
default	$-\frac{2 \left(\left(-14 \cos \left(d x +c \right)^{4}-6 \cos \left(d x +c \right)^{2}-16\right) \sin \left(d x +c \right)+i \cos \left(d x +c \right) \left(14 \cos \left(d x +c \right)^{4}-\cos \left(d x +c \right)^{2}-8\right)\right) \sqrt{a \left(1+i \tan \left(d x +c \right)\right)}\, a^{2}}{77 d \sqrt{e \sec \left(d x +c \right)}\, e^{5}}$	9

input
```
int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/308*I*a^2/e^5/(e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^(1/2)*(a*exp(2*I*
(d*x+c))/(exp(2*I*(d*x+c))+1))^(1/2)/d*(7*exp(5*I*(d*x+c))+33*exp(3*I*(d*x
+c))+154*I*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.59

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx = \frac{\left(-7i\,a^2e^{(8i\,dx+8i\,c)} - 40i\,a^2e^{(6i\,dx+6i\,c)} - 110i\,a^2e^{(4i\,dx+4i\,c)} + 77i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+}}}}{308\,de^6}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x, algorithm="fri
cas")
```

output
```
1/308*(-7*I*a^2*e^(8*I*d*x + 8*I*c) - 40*I*a^2*e^(6*I*d*x + 6*I*c) - 110*I
*a^2*e^(4*I*d*x + 4*I*c) + 77*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(11/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

$$\int \frac{(a + ia \tan(c + dx))^{5/2}}{(e \sec(c + dx))^{11/2}}\, dx = \frac{\left(-7i\, a^2 \cos\left(\frac{11}{2}\, dx + \frac{11}{2}\, c\right) - 33i\, a^2 \cos\left(\frac{7}{2}\, dx + \frac{7}{2}\, c\right) - 77i\, a^2 \cos\left(\frac{3}{2}\, dx + \frac{3}{2}\, c\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x, algorithm="max
ima")
```

output
```
1/308*(-7*I*a^2*cos(11/2*d*x + 11/2*c) - 33*I*a^2*cos(7/2*d*x + 7/2*c) - 7
7*I*a^2*cos(3/2*d*x + 3/2*c) + 77*I*a^2*cos(1/2*d*x + 1/2*c) + 7*a^2*sin(1
1/2*d*x + 11/2*c) + 33*a^2*sin(7/2*d*x + 7/2*c) + 77*a^2*sin(3/2*d*x + 3/2
*c) + 77*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/(d*e^(11/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + ia \tan(c + dx))^{5/2}}{(e \sec(c + dx))^{11/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.79

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx = -\frac{a^2\sqrt{\frac{e}{\cos(c+dx)}}\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(-187\,\sin(2\,c+2\,d\,x)-40\,\sin(4\,c+4\,d\,x)-7\,\sin(6\,c+6}{616\,d\,e^6}$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(11/2),x)
```

output
```
-(a^2*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
 1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*33i + cos(4*c + 4*d*x
)*40i + cos(6*c + 6*d*x)*7i - 187*sin(2*c + 2*d*x) - 40*sin(4*c + 4*d*x) -
 7*sin(6*c + 6*d*x)))/(616*d*e^6)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\,a^2\left(-\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2}{\sec(dx+c)^6}dx\right)+2\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(d}{\sec(d}\right.\right.}{e^6}$$

input
```
int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x)
```

output
```
(sqrt(e)*sqrt(a)*a**2*( - int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)
*tan(c + d*x)**2)/sec(c + d*x)**6,x) + 2*int((sqrt(sec(c + d*x))*sqrt(tan(
c + d*x)*i + 1)*tan(c + d*x))/sec(c + d*x)**6,x)*i + int((sqrt(sec(c + d*x
))*sqrt(tan(c + d*x)*i + 1))/sec(c + d*x)**6,x)))/e**6
```

3.416 $\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 283

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{ie^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d}\\
&\quad-\frac{ie^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d}\\
&\quad+\frac{ie^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}\left(\sqrt{a}+\cos(c+dx)\left(\sqrt{a}+i\sqrt{a}\tan(c+dx)\right)\right)}\right)}{\sqrt{2}\sqrt{a}d}\\
&\quad-\frac{ie^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad}\end{aligned}$$

output

```
1/2*I*e^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e
*sec(d*x+c))^(1/2))*2^(1/2)/a^(1/2)/d-1/2*I*e^(5/2)*arctan(1+2^(1/2)*e^(1/
2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2)/a^(1/2)/
d+1/2*I*e^(5/2)*arctanh(2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*
x+c))^(1/2)/(a^(1/2)+cos(d*x+c)*(a^(1/2)+I*a^(1/2)*tan(d*x+c))))*2^(1/2)/a
^(1/2)/d-I*e^2*(e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Mathematica [A] (warning: unable to verify)

Time = 3.42 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.24

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{e^3\left(\sec(c+dx)\sqrt{1+\cos(2c)+i\sin(2c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}-i\operatorname{arctanh}\left(\frac{\sqrt{1-i\cos}}{\sqrt{-1-i\cos}}\right.\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(e^3*(Sec[c + d*x]*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]]
- I*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1
- I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]]
*Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + I*ArcTanh[(Sqrt[1 +
I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*S
qrt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + S
in[c]]*Sqrt[I + Tan[(d*x)/2]])*(-I + Tan[c + d*x]))/(d*Sqrt[e*Sec[c + d*x]
]*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]]*Sqrt[a + I*a*Tan[
c + d*x]])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.33, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3982, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \; 3982$$

$$\frac{e^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow \; 3042$$

$$\frac{e^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow \; 3976$$

$$-\frac{2ie^4\int \frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e\left(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2\right)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow \; 826$$

$$-\frac{2ie^4\left(\frac{\int \frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow \; 1476$$

$$-\frac{2ie^4\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\sqrt{\cdots}}{2e}}{2e}\cdots\right)}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow \; 1082$$

$$-\frac{2ie^4\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a\cdots}{a^2\cdots}}{\cdots}\cdots\right)}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow \; 217$$

$$-\frac{\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}}{\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}}-$$

$\downarrow$ 1479

$$-\frac{\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}}{\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}}$$

$\downarrow$ 25

$$-\frac{\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right)}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}}{\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}}$$

$\downarrow$ 27

$$-\frac{\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i}}{\sqrt{}}}{2\sqrt{2}\sqrt{a}e}\right.}{d}}{\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}}$$

$\downarrow$ 1103

$$-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}$$

$$\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}$$

input
```
Int[(e*Sec[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-2*I)*e^4*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(S
qrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqr
t[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(
Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*S
qrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Ta
n[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*
Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*T
an[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/d - (I*e^2*Sqrt[e*Sec[c
 + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 10.24 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.08

method	result
default	$-\frac{e^2\sqrt{e\sec(dx+c)}\left(-4\sqrt{\frac{1}{\cos(dx+c)+1}}\,(\sin(dx+c)+\tan(dx+c))+4i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}+(-\cos(dx+c)+\sin(dx+c)-1)\,a\right.}{}$

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*e^2*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*x+c)))^(1/2)/
(1/(cos(d*x+c)+1))^(1/2)*(-4*(1/(cos(d*x+c)+1))^(1/2)*(sin(d*x+c)+tan(d*x+
c))+4*I*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)+(-cos(d*x+c)+sin(d*x+c)-1)
*arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+(-cos(d*
x+c)-sin(d*x+c)-1)*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(1/(cos(d*x+c)+1
))^(1/2))+I*(cos(d*x+c)-sin(d*x+c)+1)*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-
1)/(1/(cos(d*x+c)+1))^(1/2))+I*(-cos(d*x+c)-sin(d*x+c)-1)*arctanh(1/2/(1/(
cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(209) = 418.

Time = 0.09 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.63

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{-4i\,e^2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}+\sqrt{\frac{i\,e^5}{ad^2}}ad\log\left(\frac{2\left(\left(e^2e^{(2i\,dx+2i\,c)}+\right.\right.}{}\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/2*(-4*I*e^2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c
) + 1))*e^(3/2*I*d*x + 3/2*I*c) + sqrt(I*e^5/(a*d^2))*a*d*log(2*((e^2*e^(2
*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*
x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + sqrt(I*e^5/(a*d^2))*a*d)/e^2) -
 sqrt(I*e^5/(a*d^2))*a*d*log(2*((e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1
/2*I*c) - sqrt(I*e^5/(a*d^2))*a*d)/e^2) - sqrt(-I*e^5/(a*d^2))*a*d*log(2*(
(e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(
e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + sqrt(-I*e^5/(a*d^2))*a
*d)/e^2) + sqrt(-I*e^5/(a*d^2))*a*d*log(2*((e^2*e^(2*I*d*x + 2*I*c) + e^2)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/
2*I*d*x + 1/2*I*c) - sqrt(-I*e^5/(a*d^2))*a*d)/e^2))/(a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2258 vs. $2(209) = 418$.

Time = 0.43 (sec) , antiderivative size = 2258, normalized size of antiderivative = 7.98

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
-8*(16*e^2*cos(3/2*d*x + 3/2*c) + 16*I*e^2*sin(3/2*d*x + 3/2*c) + 2*(sqrt(
2)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sq
rt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + s
qrt(2)*e^2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1) + 2*(sqrt(2)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*e^2)*arctan2(sqrt(2)*cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(sqrt(2)*e^2*cos(4/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*e^2*sin
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*e^2)*a
rctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) - 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 1) + 2*(sqrt(2)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + I*sqrt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) + sqrt(2)*e^2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(-I*sqrt(2)*e^2*cos(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*e^2*sin(4/3*arct...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\,e^2 i\left(-2\sqrt{\sec\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,\sec\left(dx+c\right)^2+\left(\int \frac{\sqrt{\sec(dx+}}{}\right.\right.}{}$$

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*sqrt(a)*e**2*i*( - 2*sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*
sec(c + d*x)**2 + int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec(c +
 d*x)**2*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*tan(c + d*x)**2*d + int((s
qrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**2*tan(c + d*x))/(
tan(c + d*x)**2 + 1),x)*d))/(a*d*(tan(c + d*x)**2 + 1))
```

3.417 $\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 361

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i\sqrt{2}\sqrt{a}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{i\sqrt{2}\sqrt{a}e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{i\sqrt{2}\sqrt{a}e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}\left(\sqrt{a}+\cos(c+dx)\left(\sqrt{a}-i\sqrt{a}\tan(c+dx)\right)\right)}\right)\sec(c+dx)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

output
```
-I*2^(1/2)*a^(1/2)*e^(3/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/
2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d/(a-I*a*tan(d*x+c))^(1/2)/(a+
I*a*tan(d*x+c))^(1/2)+I*2^(1/2)*a^(1/2)*e^(3/2)*arctan(1+2^(1/2)*e^(1/2)*(
a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d/(a-I*a*
tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-I*2^(1/2)*a^(1/2)*e^(3/2)*arcta
nh(2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)/(a^(1/2)+
cos(d*x+c)*(a^(1/2)-I*a^(1/2)*tan(d*x+c))))*sec(d*x+c)/d/(a-I*a*tan(d*x+c)
)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.84

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2e\sqrt{e\sec(c+dx)}\left(\operatorname{arctanh}\left(\frac{\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{-1-i\cos(c)-}}{d\sqrt{}}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*e*Sqrt[e*Sec[c + d*x]]*(ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - T
an[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[
-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 - I*
Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqr
t[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin
[c]])*(Cos[d*x] + I*Sin[d*x])*Sqrt[I + Tan[(d*x)/2]])/(d*Sqrt[-1 - I*Cos[c
] - Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]]*Sqrt[a + I
*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3980}$$

$$\frac{e\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{e\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 3976}$$

$$\frac{4iae^3\sec(c+dx)\int \frac{\cos(c+dx)(a-ia\tan(c+dx))}{e(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 826}$$

$$\frac{4iae^3\sec(c+dx)\left(\frac{\int \frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{4iae^3\sec(c+dx)\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia}{e}}}{2e}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{4iae^3\sec(c+dx)\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{4iae^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan}}{\sqrt{e\sec(c}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{4iae^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx}{}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia}}$$

$\downarrow$ 25

$$\frac{4iae^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a}{}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan}}$$

$\downarrow$ 27

$$\frac{4iae^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan}{e}}}{2\sqrt{2}\sqrt{a}e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c}}$$

$\downarrow$ 1103

$$\frac{4iae^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((4*I)*a*e^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(
Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sq
rt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/
(Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*
Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*T
an[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]
*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*
Tan[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[a
 - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Maple [A] (warning: unable to verify)

Time = 10.14 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.64

method	result
default	$\frac{\left(-\csc(dx+c)^{2}(1-\cos(dx+c))^{2}+1\right)\sec(dx+c)e\sqrt{e\sec(dx+c)}\,(\cos(dx+c)+1)(\cot(dx+c)-\csc(dx+c)+i)\left(i\,\mathrm{arctanh}\left(\frac{\cot(dx+c)-\mathrm{cs}}{2\sqrt{\frac{}{\cos(dx}}}\right.\right.}{4d\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{a}}$

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)*sec(d*x+c)*e*(e*sec(d*x+c))^(1/2)
*(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)*(cot(d
*x+c)-csc(d*x+c)+I)*(I*arctanh(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(1/(cos(d*x+c
)+1))^(1/2))+I*arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)
+1))+arctanh(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))-arcta
nh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.07

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{1}{2}\sqrt{\frac{4i\,e^{3}}{ad^{2}}}\log\left(\frac{2\left(ee^{(2i\,dx+2i\,c)}+e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+i}{e}\right.$$

$$-\frac{1}{2}\sqrt{\frac{4i\,e^{3}}{ad^{2}}}\log\left(\frac{2\left(ee^{(2i\,dx+2i\,c)}+e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-i\,ad\sqrt{\frac{4i\,e^{3}}{ad^{2}}}}{e}\right)$$

$$+\frac{1}{2}\sqrt{-\frac{4i\,e^{3}}{ad^{2}}}\log\left(\frac{2\left(ee^{(2i\,dx+2i\,c)}+e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+i\,ad\sqrt{-\frac{4i\,e^{3}}{ad^{2}}}}{e}\right)$$

$$-\frac{1}{2}\sqrt{-\frac{4i\,e^{3}}{ad^{2}}}\log\left(\frac{2\left(ee^{(2i\,dx+2i\,c)}+e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-i\,ad\sqrt{-\frac{4i\,e^{3}}{ad^{2}}}}{e}\right)$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/2*sqrt(4*I*e^3/(a*d^2))*log((2*(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*
I*c) + I*a*d*sqrt(4*I*e^3/(a*d^2)))/e) - 1/2*sqrt(4*I*e^3/(a*d^2))*log((2*
(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2
*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*a*d*sqrt(4*I*e^3/(a*d^2)
))/e) + 1/2*sqrt(-4*I*e^3/(a*d^2))*log((2*(e*e^(2*I*d*x + 2*I*c) + e)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d
*x + 1/2*I*c) + I*a*d*sqrt(-4*I*e^3/(a*d^2)))/e) - 1/2*sqrt(-4*I*e^3/(a*d^
2))*log((2*(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*s
qrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*a*d*sqrt(-4*I
*e^3/(a*d^2)))/e)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(c+dx))^{\frac{3}{2}}}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((e*sec(c + d*x))**(3/2)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(271) = 542$.

Time = 0.31 (sec) , antiderivative size = 726, normalized size of antiderivative = 2.01

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
-1/4*(2*I*sqrt(2)*e*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(
1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c)
 + 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e*arctan2(sqrt(2)*c
os(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e
*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) +
 1) - 2*sqrt(2)*e*arctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqr
t(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) + 2*sqrt(2)*e*arctan2(-sqrt(
2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), -sqrt(2)*cos(1/2*d*x + 1/2*c) + co
s(d*x + c) + 1) + I*sqrt(2)*e*log(2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2
*c) + 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1)*cos(d*x + c) + cos(d*x + c)^2 +
 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) - I*sqrt(2)*e*log(-2*sqrt(2)*sin(d*x + c
)*sin(1/2*d*x + 1/2*c) - 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1)*cos(d*x + c)
 + cos(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) - sqrt(2)*e*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*e*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*e*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2\sqrt{e}\,\sqrt{a}\,e\Big(-\sqrt{\sec\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,\sec\left(dx+c\right)\tan\left(dx+c\right)-3}{}$$

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*sqrt(e)*sqrt(a)*e*( - sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec(c
 + d*x)*tan(c + d*x) - 3*sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec(c
 + d*x)*i + int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)*
tan(c + d*x)**2)/(tan(c + d*x)**2 + 1),x)*tan(c + d*x)**2*d + int((sqrt(se
c(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)*tan(c + d*x)**2)/(tan(c
+ d*x)**2 + 1),x)*d))/(a*d*(tan(c + d*x)**2 + 1))
```

3.418 $\int \frac{\sqrt{e \sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{\sqrt{e \sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i\sqrt{e\sec(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

output
```
2*I*(e*sec(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{e \sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i\sqrt{e\sec(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((2*I)*Sqrt[e*Sec[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}dx$$
$$\Big\downarrow \text{3969}$$
$$\frac{2i\sqrt{e\sec(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((2*I)*Sqrt[e*Sec[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Maple [A] (verified)

Time = 11.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

method	result	size
orering	$\frac{2i\sqrt{e\sec(dx+c)}}{d\sqrt{a+ia\tan(dx+c)}}$	31
default	$\frac{2i\sqrt{e\sec(dx+c)}}{d\sqrt{a(1+i\tan(dx+c))}}$	32
risch	$\frac{2i\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}{\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	59

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I*(e*sec(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(28) = 56.

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{ad}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(I*e
^(2*I*d*x + 2*I*c) + I)*e^(-1/2*I*d*x - 1/2*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sqrt{e\sec\left(c+dx\right)}}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(e*sec(c + d*x))/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i\sqrt{e}\sqrt{-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}{\sqrt{a}d\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
2*I*sqrt(e)*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)/(sqrt(a)*d*sqrt
(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - 1))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,2\mathrm{i}}{d\,\sqrt{a+\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{\cos(c+d\,x)}}}$$

input
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
((e/cos(c + d*x))^(1/2)*2i)/(d*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^(1/2
))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2\sqrt{e}\,\sqrt{a}\,\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,(\tan(dx+c)+i)}{ad\left(\tan(dx+c)^2+1\right)}$$

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*sqrt(e)*sqrt(a)*sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*(tan(c + d*
x) + i))/(a*d*(tan(c + d*x)**2 + 1))
```

3.419 $\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 80

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{3d\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}$$

output
```
2/3*I/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-4/3*I*(a+I*a*tan(d*x
+c))^(1/2)/a/d/(e*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{-2i+4\tan(c+dx)}{3d\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[1/(Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(-2*I + 4*Tan[c + d*x])/(3*d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
((2*I)/3)/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)
*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*Sqrt[e*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 6.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

method	result	size
default	$-\frac{2(i-2\tan(dx+c))}{3d\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}}$	42
risch	$-\frac{i\left(3\,\mathrm{e}^{2i(dx+c)}-1\right)}{3\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	85

input
```
int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/3/d/(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2)*(I-2*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(4i\,dx+4i\,c)}-2i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{3\,ade}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output
```
1/3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-
3*I*e^(4*I*d*x + 4*I*c) - 2*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3/2*I*d*x - 3/2
*I*c)/(a*d*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\sec\left(c+dx\right)}\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*sec(c + d*x))*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\cos\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)+\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\sin\left(\frac{1}{3}\right.}{3\sqrt{a}d\sqrt{e}}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
1/3*(I*cos(3/2*d*x + 3/2*c) - 3*I*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))))/(sqrt(a)*d*sqrt(e))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(-2\,\sin\left(c+d\,x\right)+\cos\left(c+d\,x\right)\,1{}\mathrm{i}\right)}{3\,d\,e\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1{}\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}$$

input
```
int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
-(2*(e/cos(c + d*x))^(1/2)*(cos(c + d*x)*1i - 2*sin(c + d*x)))/(3*d*e*((a*
(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2
))
```

Reduce [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\left(-\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)}{\sec(dx+c)\tan(dx+c)^2+\sec(dx+c)}dx\right)i+\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}}{\sec(dx+c)\tan(dx+c)^2+\sec(dx+c)}dx\right)}{ae}$$

input
```
int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*sqrt(a)*( - int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(
c + d*x))/(sec(c + d*x)*tan(c + d*x)**2 + sec(c + d*x)),x)*i + int((sqrt(s
ec(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(sec(c + d*x)*tan(c + d*x)**2 + sec
(c + d*x)),x)))/(a*e)
```

3.420 $\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 121

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{5d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{16i\sqrt{e\sec(c+dx)}}{15de^2\sqrt{a+ia\tan(c+dx)}} - \frac{8i\sqrt{a+ia\tan(c+dx)}}{15ad(e\sec(c+dx))^{3/2}}$$

output
```
2/5*I/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+16/15*I*(e*sec(d*x+c
))^(1/2)/d/e^2/(a+I*a*tan(d*x+c))^(1/2)-8/15*I*(a+I*a*tan(d*x+c))^(1/2)/a/
d/(e*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.56

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i\sec^2(c+dx)(-15+\cos(2(c+dx))+4i\sin(2(c+dx)))}{15d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
((-1/15*I)*Sec[c + d*x]^2*(-15 + Cos[2*(c + d*x)] + (4*I)*Sin[2*(c + d*x)]
))/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3983, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{4\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
((2*I)/5)/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (4*((((4
*I)/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((2*I
)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))))/(5*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 6.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{\frac{2i(-\cos(dx+c)+8\sec(dx+c))}{15}+\frac{8\sin(dx+c)}{15}}{d\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}\,e}$	62

input
```
int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/d*(2/15*I*(-cos(d*x+c)+8*sec(d*x+c))+8/15*sin(d*x+c))/(a*(1+I*tan(d*x+c)
))^(1/2)/(e*sec(d*x+c))^(1/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-5i\,e^{(6i\,dx+6i\,c)}+25i\,e^{(4i\,dx+4i\,c)}\right.}{30\,ade^2}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output
```
1/30*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(
-5*I*e^(6*I*d*x + 6*I*c) + 25*I*e^(4*I*d*x + 4*I*c) + 33*I*e^(2*I*d*x + 2*
I*c) + 3*I)*e^(-5/2*I*d*x - 5/2*I*c)/(a*d*e^2)
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\sec{(c+dx)})^{\frac{3}{2}}\sqrt{ia\left(\tan{(c+dx)}-i\right)}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/((e*sec(c + d*x))**(3/2)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right) - 5i\cos\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
1/30*(3*I*cos(5/2*d*x + 5/2*c) - 5*I*cos(3/5*arctan2(sin(5/2*d*x + 5/2*c),
 cos(5/2*d*x + 5/2*c))) + 30*I*cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c))) + 3*sin(5/2*d*x + 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c))) + 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c
), cos(5/2*d*x + 5/2*c))))/(sqrt(a)*d*e^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.71

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(4\,\sin\left(2\,c+2\,d\,x\right)-\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}+15\mathrm{i}\right)}{15\,d\,e^2\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}$$

input
```
int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
((e/cos(c + d*x))^(1/2)*(4*sin(2*c + 2*d*x) - cos(2*c + 2*d*x)*1i + 15i))/
(15*d*e^2*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d
*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{\sqrt{e}\,\sqrt{a}\left(-\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)}{\sec(dx+c)^2\tan(dx+c)^2+\sec(dx+c)^2}dx\right)i+\int \frac{\sqrt{}}{\sec(d}\right.}{a\,e^2}$$

input
```
int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*sqrt(a)*( - int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(
c + d*x))/(sec(c + d*x)**2*tan(c + d*x)**2 + sec(c + d*x)**2),x)*i + int((
sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(sec(c + d*x)**2*tan(c + d*x)
**2 + sec(c + d*x)**2),x)))/(a*e**2)
```

3.421 $\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 165

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{7d(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} + \frac{16i}{35de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{12i\sqrt{a+ia\tan(c+dx)}}{35ad(e\sec(c+dx))^{5/2}} - \frac{32i\sqrt{a+ia\tan(c+dx)}}{35ade^2\sqrt{e\sec(c+dx)}}$$

output
```
2/7*I/d/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2)+16/35*I/d/e^2/(e*sec
(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-12/35*I*(a+I*a*tan(d*x+c))^(1/2)/a
/d/(e*sec(d*x+c))^(5/2)-32/35*I*(a+I*a*tan(d*x+c))^(1/2)/a/d/e^2/(e*sec(d*
x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.48

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i(17+\cos(2(c+dx))+3i\sec(c+dx)\sin(3(c+dx))+35i\tan(c+dx))}{35de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
((-1/35*I)*(17 + Cos[2*(c + d*x)] + (3*I)*Sec[c + d*x]*Sin[3*(c + d*x)] +
(35*I)*Tan[c + d*x]))/(d*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3983, 3042, 3978, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{6\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{6\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{6\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{6\left(\frac{4a\left(\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
((2*I)/7)/(d*(e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (6*((((-
2*I)/5)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(5/2)) + (4*a*(((2
*I)/3)/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)*Sq
rt[a + I*a*Tan[c + d*x]])/(a*d*Sqrt[e*Sec[c + d*x]])))/(5*e^2)))/(7*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 6.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.42

method	result	size
default	$-\frac{2\left(-6\cos(dx+c)\sin(dx+c)+i\cos(dx+c)^{2}-16\tan(dx+c)+8i\right)}{35d\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}\,e^{2}}$	70

input
```
int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/35/d/(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2)/e^2*(-6*cos(d*x+c)
*sin(d*x+c)+I*cos(d*x+c)^2-16*tan(d*x+c)+8*I)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.61

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-7i\,e^{(8i\,dx+8i\,c)}-112i\,e^{(6i\,dx+6i\,c}\right.}{140\,ade}$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output
```
1/140*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
(-7*I*e^(8*I*d*x + 8*I*c) - 112*I*e^(6*I*d*x + 6*I*c) - 70*I*e^(4*I*d*x +
4*I*c) + 40*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-7/2*I*d*x - 7/2*I*c)/(a*d*e^3
)
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\sec(c+dx))^{\frac{5}{2}}\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/((e*sec(c + d*x))**(5/2)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.08

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{5i\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)-7i\cos\left(\frac{5}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
1/140*(5*I*cos(7/2*d*x + 7/2*c) - 7*I*cos(5/7*arctan2(sin(7/2*d*x + 7/2*c)
, cos(7/2*d*x + 7/2*c))) + 35*I*cos(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(
7/2*d*x + 7/2*c))) - 105*I*cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d
*x + 7/2*c))) + 5*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7
/2*c), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), c
os(7/2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*
d*x + 7/2*c))))/(sqrt(a)*d*e^(5/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.61

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(-\sin\left(c+d\,x\right)-\frac{3\,\sin(3\,c+3\,d\,x)}{35}+\frac{\cos(c+d\,x)\,1{}\mathrm{i}}{2}+\frac{\cos(3\,c+3\,d\,x)\,1{}\mathrm{i}}{70}\right)}{d\,e^3\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1{}\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}}$$

input
```
int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
-((e/cos(c + d*x))^(1/2)*((cos(c + d*x)*1i)/2 - sin(c + d*x) + (cos(3*c +
3*d*x)*1i)/70 - (3*sin(3*c + 3*d*x))/35))/(d*e^3*((a*(cos(2*c + 2*d*x) + s
in(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\left(-\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)}{\sec(dx+c)^3\tan(dx+c)^2+\sec(dx+c)^3}dx\right)i+\int \frac{\sqrt{}}{\sec(d}\right.}{a\,e^3}$$

input
```
int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*sqrt(a)*( - int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(
c + d*x))/(sec(c + d*x)**3*tan(c + d*x)**2 + sec(c + d*x)**3),x)*i + int((
sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(sec(c + d*x)**3*tan(c + d*x)
**2 + sec(c + d*x)**3),x)))/(a*e**3)
```

3.422 $\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 206

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{32i}{105de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{256i\sqrt{e\sec(c+dx)}}{315de^4\sqrt{a+ia\tan(c+dx)}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{63ad(e\sec(c+dx))^{7/2}} - \frac{128i\sqrt{a+ia\tan(c+dx)}}{315ade^2(e\sec(c+dx))^{3/2}}$$

output
```
2/9*I/d/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2)+32/105*I/d/e^2/(e*se
c(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+256/315*I*(e*sec(d*x+c))^(1/2)/d/
e^4/(a+I*a*tan(d*x+c))^(1/2)-16/63*I*(a+I*a*tan(d*x+c))^(1/2)/a/d/(e*sec(d
*x+c))^(7/2)-128/315*I*(a+I*a*tan(d*x+c))^(1/2)/a/d/e^2/(e*sec(d*x+c))^(3/
2)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.42

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{e\sec(c+dx)}(945i-84i\cos(2(c+dx))-5i\cos(4(c+dx}{1260de^4\sqrt{a+ia\tan(}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(Sqrt[e*Sec[c + d*x]]*(945*I - (84*I)*Cos[2*(c + d*x)] - (5*I)*Cos[4*(c +
d*x)] + 336*Sin[2*(c + d*x)] + 40*Sin[4*(c + d*x)]))/(1260*d*e^4*Sqrt[a +
I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3983, 3042, 3978, 3042, 3983, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \; 3983$$

$$\frac{8\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{7/2}}dx}{9a}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{8\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{7/2}}dx}{9a}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{8\left(\frac{6a\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)}{9a}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\left(\frac{6a\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)}{9a}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{8\left(\frac{6a\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)}{9a}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\left(\frac{6a\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)}{9a}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{8\left(\frac{6a\left(\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)}{9a}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\left(\frac{6a\left(\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)}{9a}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{8\left(\frac{6a\left(\frac{4\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)}{9a}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
((2*I)/9)/(d*(e*Sec[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (8*((((-
2*I)/7)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(7/2)) + (6*a*(((2
*I)/5)/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (4*((((4*I)
/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((2*I)/3
)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))))/(5*a)))/(7*e^2)
))/(9*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 6.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.43

method	result	size
default	$\frac{\frac{16\sin(dx+c)\cos(dx+c)^{2}}{63}-\frac{2i\cos(dx+c)^{3}}{63}+\frac{128\sin(dx+c)}{315}-\frac{32i\cos(dx+c)}{315}+\frac{256i\sec(dx+c)}{315}}{d\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}\,e^{3}}$	88

input
```
int(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
2/315/d/(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2)/e^3*(40*sin(d*x+c)
*cos(d*x+c)^2-5*I*cos(d*x+c)^3+64*sin(d*x+c)-16*I*cos(d*x+c)+128*I*sec(d*x
+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.54

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-45i\,e^{(10i\,dx+10i\,c)}-465i\,e^{(8i\,dx\cdots}\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output
```
1/2520*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))
*(-45*I*e^(10*I*d*x + 10*I*c) - 465*I*e^(8*I*d*x + 8*I*c) + 1470*I*e^(6*I*
d*x + 6*I*c) + 2142*I*e^(4*I*d*x + 4*I*c) + 287*I*e^(2*I*d*x + 2*I*c) + 35
*I)*e^(-9/2*I*d*x - 9/2*I*c)/(a*d*e^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{35i\,\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)-45i\,\cos\left(\frac{7}{9}\,\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
1/2520*(35*I*cos(9/2*d*x + 9/2*c) - 45*I*cos(7/9*arctan2(sin(9/2*d*x + 9/2
*c), cos(9/2*d*x + 9/2*c))) + 252*I*cos(5/9*arctan2(sin(9/2*d*x + 9/2*c),
cos(9/2*d*x + 9/2*c))) - 420*I*cos(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9
/2*d*x + 9/2*c))) + 1890*I*cos(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d
*x + 9/2*c))) + 35*sin(9/2*d*x + 9/2*c) + 45*sin(7/9*arctan2(sin(9/2*d*x +
 9/2*c), cos(9/2*d*x + 9/2*c))) + 252*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c)
, cos(9/2*d*x + 9/2*c))) + 420*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9
/2*d*x + 9/2*c))) + 1890*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
 + 9/2*c))))/(sqrt(a)*d*e^(7/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.53

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(336\,\sin\left(2\,c+2\,d\,x\right)-\cos\left(4\,c+4\,d\,x\right)\,5\mathrm{i}-\cos\right.}{1260\,d\,e^4\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1-}{\cos(2\,c+2}}}$$

input
```
int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
((e/cos(c + d*x))^(1/2)*(336*sin(2*c + 2*d*x) - cos(4*c + 4*d*x)*5i - cos(
2*c + 2*d*x)*84i + 40*sin(4*c + 4*d*x) + 945i))/(1260*d*e^4*((a*(cos(2*c +
 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\left(-\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)}{\sec(dx+c)^4\tan(dx+c)^2+\sec(dx+c)^4}dx\right)i+\int \frac{\sqrt{}}{\sec(dx}\right.}{a\,e^4}$$

input
```
int(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*sqrt(a)*( - int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(
c + d*x))/(sec(c + d*x)**4*tan(c + d*x)**2 + sec(c + d*x)**4),x)*i + int((
sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(sec(c + d*x)**4*tan(c + d*x)
**2 + sec(c + d*x)**4),x)))/(a*e**4)
```

3.423 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 403

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx &= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} \\ &- \frac{3ie^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\ &+ \frac{3ie^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\ &- \frac{3ie^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}\left(\sqrt{a}+\cos(c+dx)\left(\sqrt{a}-i\sqrt{a}\tan(c+dx)\right)\right)}\right)\sec(c+dx)}{\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output
```
-I*e^2*(e*sec(d*x+c))^(3/2)/a/d/(a+I*a*tan(d*x+c))^(1/2)-3/2*I*e^(7/2)*arc
tan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2
))*sec(d*x+c)*2^(1/2)/a^(1/2)/d/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)
)^(1/2)+3/2*I*e^(7/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^
(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)/a^(1/2)/d/(a-I*a*tan(d*x+c)
)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-3/2*I*e^(7/2)*arctanh(2^(1/2)*e^(1/2)*(a-
I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)/(a^(1/2)+cos(d*x+c)*(a^(1/2)-I*
a^(1/2)*tan(d*x+c))))*sec(d*x+c)*2^(1/2)/a^(1/2)/d/(a-I*a*tan(d*x+c))^(1/2
)/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.53 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.84

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{e(e\sec(c+dx))^{5/2}\left(-i\cos(c+dx)+\sin(c+dx)+\frac{3\cos(c+dx)(\cos(c)+i\sin(c))}{}\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(e*(e*Sec[c + d*x])^(5/2)*((-I)*Cos[c + d*x] + Sin[c + d*x] + (3*Cos[c + d
*x]*(Cos[c] + I*Sin[c])*(ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan
[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1
 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 - I*Co
s[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[
I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c
]])*(Cos[d*x] + I*Sin[d*x])^2*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c]
- Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])))/(d*(a + I
*a*Tan[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.15, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3981, 3042, 3979, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 3981$$

$$\frac{3e^2\int(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3e^2\int(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 3979$$

$$\frac{3e^2\left(\frac{1}{2}a\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3e^2\left(\frac{1}{2}a\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 3980$$

$$\frac{3e^2\left(\frac{ae\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3e^2\left(\frac{ae\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 3976$$

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\int\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e\left(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2\right)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 826$$

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\int\frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\,\mathrm{s}}{d\sqrt{a+}}\right.}{a^2}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow 1476$$

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-}{e}}}{2e}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{a^2}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow 1082$$

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int\frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int\frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+d}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{a^2}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow 217$$

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c}}{\sqrt{e\sec(c+}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{a^2}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{1479}$$

$$3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)}{}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{}}}{}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia}}\right.$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-}{}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\,\mathrm{ta}}}\right.}{a^2}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\,\mathrm{ta}}{e}}}{2\sqrt{2}\sqrt{a}e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+}}\right.}{a^2}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 1103$$

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{a^2}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-4*I)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*
e^2*((I*a*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + ((2*I)*
a^2*e^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[
a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]
*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt
[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[
a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c
+ d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt
[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c
 + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[a - I*
a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])))/a^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 10.37 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.72

method	result
default	$-\frac{\sqrt{e\sec(dx+c)}\, e^{3}\left(i(3\sin(dx+c)+3\cos(dx+c)+3)\operatorname{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+(-3\sin(dx+c)+3\cos(dx+c)+3)\operatorname{arctanh}\right.}{}$

input
```
int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*(e*sec(d*x+c))^(1/2)*e^3/(cos(d*x+c)+1)/(a*(1+I*tan(d*x+c)))^(1/2)/
(1/(cos(d*x+c)+1))^(1/2)/a*(I*(3*sin(d*x+c)+3*cos(d*x+c)+3)*arctanh(1/2/(1
/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))+(-3*sin(d*x+c)+3*cos(d*x
+c)+3)*arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))+I*(
3*sin(d*x+c)-3*cos(d*x+c)-3)*arctanh(1/2*(cot(d*x+c)-csc(d*x+c)-1)/(1/(cos
(d*x+c)+1))^(1/2))+(3*sin(d*x+c)+3*cos(d*x+c)+3)*arctanh(1/2*(cot(d*x+c)-c
sc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))+I*(1/(cos(d*x+c)+1))^(1/2)*(4+4*sec
(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.20

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{-4i\,e^3\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-\sqrt{\frac{9i\,e^7}{a^3d^2}}a^2d\log\left(-\frac{2\left(i\sqrt{\frac{9i\,e^7}{a^3d^2}}\right.}{}\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
1/2*(-4*I*e^3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c
) + 1))*e^(1/2*I*d*x + 1/2*I*c) - sqrt(9*I*e^7/(a^3*d^2))*a^2*d*log(-2/3*(
I*sqrt(9*I*e^7/(a^3*d^2))*a^2*d - 3*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x
 + 1/2*I*c))/e^3) + sqrt(9*I*e^7/(a^3*d^2))*a^2*d*log(-2/3*(-I*sqrt(9*I*e^
7/(a^3*d^2))*a^2*d - 3*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/
e^3) - sqrt(-9*I*e^7/(a^3*d^2))*a^2*d*log(-2/3*(I*sqrt(-9*I*e^7/(a^3*d^2))
*a^2*d - 3*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^3) + sqrt(
-9*I*e^7/(a^3*d^2))*a^2*d*log(-2/3*(-I*sqrt(-9*I*e^7/(a^3*d^2))*a^2*d - 3*
(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(
e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^3))/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1817 vs. $2(305) = 610$.

Time $= 0.39$ (sec) , antiderivative size $= 1817$, normalized size of antiderivative $= 4.51$

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output
```
-8*(6*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/
2*d*x + 1/2*c) + 1) + 6*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) +
 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 6*sqrt(2)*e^3*arctan2(sqrt(2)*cos
(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 6*sqrt(2)*e^3*a
rctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1
) + 3*I*sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
 3*I*sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*
I*sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*I*s
qrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 16*e^3*c
os(1/2*d*x + 1/2*c) + 16*I*e^3*sin(1/2*d*x + 1/2*c) - 6*(-I*sqrt(2)*e^3*co
s(2*d*x + 2*c) + sqrt(2)*e^3*sin(2*d*x + 2*c) - I*sqrt(2)*e^3)*arctan2(sqr
t(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqrt(2)*cos(1/2*d*x + 1/2*c) + c
os(d*x + c) + 1) - 6*(I*sqrt(2)*e^3*cos(2*d*x + 2*c) - sqrt(2)*e^3*sin(2*d
*x + 2*c) + I*sqrt(2)*e^3)*arctan2(-sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x
 + c), -sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) + 3*(2*sqrt(2)*e^
3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2*d*x + 1/2*c...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2\sqrt{e}\,\sqrt{a}\,e^{3}\left(-\sqrt{\sec\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,\sec\left(dx+c\right)^{3}i-2\left(\int \frac{\sqrt{\sec}}{}\right.\right.}{}$$

input
```
int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(2*sqrt(e)*sqrt(a)*e**3*( - sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*se
c(c + d*x)**3*i - 2*int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec(c
 + d*x)**3*tan(c + d*x)**2)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c +
 d*x)*i + 1),x)*tan(c + d*x)**2*d - 2*int((sqrt(sec(c + d*x))*sqrt(tan(c +
 d*x)*i + 1)*sec(c + d*x)**3*tan(c + d*x)**2)/(tan(c + d*x)**3*i + tan(c +
 d*x)**2 + tan(c + d*x)*i + 1),x)*d + int((sqrt(sec(c + d*x))*sqrt(tan(c +
 d*x)*i + 1)*sec(c + d*x)**3*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*
x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*d*i + int((sqrt(sec(c + d*x
))*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**3*tan(c + d*x))/(tan(c + d*x)**3
*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*d*i))/(a**2*d*(tan(c + d*x)*
*2 + 1))
```

3.424 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 283

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{i\sqrt{2}e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{i\sqrt{2}e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{i\sqrt{2}e^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}\left(\sqrt{a}+\cos(c+dx)\left(\sqrt{a}+i\sqrt{a}\tan(c+dx)\right)\right)}\right)}{a^{3/2}d} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

output
```
-I*2^(1/2)*e^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/
2)/(e*sec(d*x+c))^(1/2))/a^(3/2)/d+I*2^(1/2)*e^(5/2)*arctan(1+2^(1/2)*e^(1
/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))/a^(3/2)/d-I*2^(
1/2)*e^(5/2)*arctanh(2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c
))^(1/2)/(a^(1/2)+cos(d*x+c)*(a^(1/2)+I*a^(1/2)*tan(d*x+c))))/a^(3/2)/d+4*
I*e^2*(e*sec(d*x+c))^(1/2)/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.68 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.19

$$\int \frac{(e \sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{e(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))^2\left(\cos(dx)(4i\cos(c)-4\sin(c))+\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(e*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])^2*(Cos[d*x]*((4*I)*Cos[c
] - 4*Sin[c]) + 4*(Cos[c] + I*Sin[c])*Sin[d*x] + (2*(ArcTanh[(Sqrt[1 - I*C
os[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt
[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[
c]] - ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-
1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]
]*Sqrt[-1 + I*Cos[c] + Sin[c]])*(Cos[2*c] + I*Sin[2*c])*Sqrt[I + Tan[(d*x)
/2]])/(Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]])))/(d*(a + I
*a*Tan[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.34, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3981, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e \sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(e \sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \ 3981$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}-\frac{e^2\int\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}-\frac{e^2\int\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{a^2}$$

$$\Big\downarrow \ 3976$$

$$\frac{4ie^4\int\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{ad}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 826$$

$$\frac{4ie^4\left(\frac{\int\frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{ad}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 1476$$

$$\frac{4ie^4\left(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan}}{\sqrt{e}}}{2e}}{2e}\right.}{ad}\\ \frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 1082$$

$$\frac{4ie^4\left(\frac{\frac{\int\frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int\frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-c}{a^2+c}}{}\right.}{ad}\\ \frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 217$$

$$\frac{4ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{ad}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{4ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right)}\cdots}{2\sqrt{2}\sqrt{a}\sqrt{e}}\cdots\right)}{ad}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{4ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right)}d\cdots}{2\sqrt{2}\sqrt{a}\sqrt{e}}\cdots\right)}{ad}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan\cdots}}{\sqrt{e\,\mathrm{s}\cdots}}}{2\sqrt{2}\sqrt{a}e}\cdots\right)}{ad}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{4ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}-}{2e}\right.}{ad}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((4*I)*e^4*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sq
rt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt
[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(S
qrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sq
rt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan
[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*S
qrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Ta
n[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/(a*d) + ((4*I)*e^2*Sqrt[
e*Sec[c + d*x]])/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 10.57 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.97

method	result
default	$\frac{\sqrt{e\sec(dx+c)}\, e^2 \left(i(\cos(dx+c)+\sin(dx+c)+1)\operatorname{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)-1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+i(-\cos(dx+c)+\sin(dx+c)-1)\operatorname{arctanh}\left(\frac{\cot(dx}{2}\right.\right.}{}$

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(e*sec(d*x+c))^(1/2)*e^2*(I*(cos(d*x+c)+sin(d*x+c)+1)*arctanh(1/2*(c
ot(d*x+c)-csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))+I*(-cos(d*x+c)+sin(d*x+c
)-1)*arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1))+8*I*(
cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)+(cos(d*x+c)-sin(d*x+c)+1)*arctanh(1
/2*(cot(d*x+c)-csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))+(cos(d*x+c)+sin(d*x
+c)+1)*arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)))/(c
os(d*x+c)+1)/a/(a*(1+I*tan(d*x+c)))^(1/2)/(1/(cos(d*x+c)+1))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(209) = 418.

Time = 0.11 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.90

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
-1/2*(a^2*d*sqrt(4*I*e^5/(a^3*d^2))*e^(I*d*x + I*c)*log((a^2*d*sqrt(4*I*e^
5/(a^3*d^2)) + 2*(e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^2) -
 a^2*d*sqrt(4*I*e^5/(a^3*d^2))*e^(I*d*x + I*c)*log(-(a^2*d*sqrt(4*I*e^5/(a
^3*d^2)) - 2*(e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^2) - a^2
*d*sqrt(-4*I*e^5/(a^3*d^2))*e^(I*d*x + I*c)*log((a^2*d*sqrt(-4*I*e^5/(a^3*
d^2)) + 2*(e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^2) + a^2*d*
sqrt(-4*I*e^5/(a^3*d^2))*e^(I*d*x + I*c)*log(-(a^2*d*sqrt(-4*I*e^5/(a^3*d^
2)) - 2*(e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^2) + 8*(-I*e^
2*e^(2*I*d*x + 2*I*c) - I*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e
^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-I*d*x - I*c)/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(209) = 418$.

Time = 0.36 (sec) , antiderivative size = 778, normalized size of antiderivative = 2.75

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output
```
-1/4*(2*I*sqrt(2)*e^2*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*si
n(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e^2*arctan2(sqrt(2)*cos(1/2*d*x + 1/
2*c) + 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e^2*arctan2(sqr
t(2)*cos(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqr
t(2)*e^2*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x +
1/2*c) + 1) + 2*sqrt(2)*e^2*arctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x
 + c), sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) - 2*sqrt(2)*e^2*ar
ctan2(-sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), -sqrt(2)*cos(1/2*d*x +
 1/2*c) + cos(d*x + c) + 1) + I*sqrt(2)*e^2*log(2*sqrt(2)*sin(d*x + c)*sin
(1/2*d*x + 1/2*c) + 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1)*cos(d*x + c) + co
s(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) - I*sqrt(2)*e^2*log(-2*sqr
t(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) - 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) -
 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) + sqr
t(2)*e^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*e^
2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*e^2*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\sqrt{e}\,\sqrt{a}\,e^2\left(\sqrt{\sec\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,\sec\left(dx+c\right)^2 i+\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)^2\tan(dx+}{\tan(dx+c)^3 i+\tan(dx+c)^2+\tan(dx+c)i+1}\right.\right.}{a^2 d\left(\tan\left(dx+c\right)^2+1\right)}$$

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
( - 2*sqrt(e)*sqrt(a)*e**2*(sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*se
c(c + d*x)**2*i + int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec(c +
 d*x)**2*tan(c + d*x)**2)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d
*x)*i + 1),x)*tan(c + d*x)**2*d + int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x
)*i + 1)*sec(c + d*x)**2*tan(c + d*x)**2)/(tan(c + d*x)**3*i + tan(c + d*x
)**2 + tan(c + d*x)*i + 1),x)*d))/(a**2*d*(tan(c + d*x)**2 + 1))
```

3.425 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 38

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2i(e\sec(c+dx))^{3/2}}{3d(a+ia\tan(c+dx))^{3/2}}$$

output
```
2/3*I*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2i(e\sec(c+dx))^{3/2}}{3d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((2*I)/3)*(e*Sec[c + d*x])^(3/2))/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}dx$$
$$\Big\downarrow \text{3969}$$
$$\frac{2i(e\sec(c+dx))^{3/2}}{3d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((2*I)/3)*(e*Sec[c + d*x])^(3/2))/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Maple [A] (verified)

Time = 10.67 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
orering	$\frac{2i(e\sec(dx+c))^{\frac{3}{2}}}{3d(a+ia\tan(dx+c))^{\frac{3}{2}}}$	31
default	$\frac{2ie\sqrt{e\sec(dx+c)}}{d(3i\sin(dx+c)+3\cos(dx+c))a\sqrt{a(1+i\tan(dx+c))}}$	56
risch	$\frac{2ie\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\mathrm{e}^{-i(dx+c)}}{3a\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	72

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(28) = 56$.

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2\left(i\,ee^{(2i\,dx+2i\,c)}+i\,e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{3\,a^2d}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
2/3*(I*e*e^(2*I*d*x + 2*I*c) + I*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt
(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-3/2*I*d*x - 3/2*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(e\sec{(c+dx)})^{\frac{3}{2}}}{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((e*sec(c + d*x))**(3/2)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(28) = 56.

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i\,e^{\frac{3}{2}}\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}{3\,a^{\frac{3}{2}}d\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output
```
2/3*I*e^(3/2)*(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2)/(a^(3/2)*d*
(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - 1)^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^{3/2}}\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2\sqrt{e}\,\sqrt{a}\,e\left(\sqrt{\sec\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,\sec\left(dx+c\right)i+\left(\int\frac{\sqrt{\sec(dx+}}{\tan(dx+c)^3}\right.\right.}{a^2d\,(\mathrm{t}}$$

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(2*sqrt(e)*sqrt(a)*e*(sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec(c +
d*x)*i + int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x))/(t
an(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)**
2*d + int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x))/(tan(
c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*d))/(a**2*d*(tan(
c + d*x)**2 + 1))
```

3.426 $\int \frac{\sqrt{e \sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 80

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}+\frac{4i\sqrt{e\sec(c+dx)}}{5ad\sqrt{a+ia\tan(c+dx)}}$$

output
```
2/5*I*(e*sec(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^(3/2)+4/5*I*(e*sec(d*x+c))
^(1/2)/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2\sqrt{e\sec(c+dx)}(3+2i\tan(c+dx))}{5ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*Sqrt[e*Sec[c + d*x]]*(3 + (2*I)*Tan[c + d*x]))/(5*a*d*(-I + Tan[c + d*x
])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{2\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{5a}+\frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{5a}+\frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{4i\sqrt{e\sec(c+dx)}}{5ad\sqrt{a+ia\tan(c+dx)}}+\frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((2*I)/5)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((4*I
)/5)*Sqrt[e*Sec[c + d*x]])/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 8.70 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{2i\left(i\sin(dx+c)\cos(dx+c)-\cos(dx+c)^2-2\right)\sqrt{e\sec(dx+c)}}{5da\sqrt{a(1+i\tan(dx+c))}}$	62

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*I/d*(I*sin(d*x+c)*cos(d*x+c)-cos(d*x+c)^2-2)*(e*sec(d*x+c))^(1/2)/a/(
a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(5i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{5\,a^2d}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
1/5*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(5
*I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-5/2*I*d*x - 5/2*
I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{e\sec{(c+dx)}}}{(ia\,(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(e*sec(c + d*x))/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{e}\left(i\,\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5i\,\cos\left(\frac{1}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right.\right.\right.\right.}{5}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output
```
1/5*sqrt(e)*(I*cos(5/2*d*x + 5/2*c) + 5*I*cos(1/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c))) + sin(5/2*d*x + 5/2*c) + 5*sin(1/5*arctan2(si
n(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))/(a^(3/2)*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+5{}\mathrm{i}\right)}{5\,a\,d\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1{}\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}$$

input
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
((e/cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 5i))/(5*
a*d*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) +
1))^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2\sqrt{e}\,\sqrt{a}\left(2\sqrt{\sec\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,\tan\left(dx+c\right)+\sqrt{\sec\left(dx+c\right)}\right.}{}$$

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(2*sqrt(e)*sqrt(a)*(2*sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c +
d*x) + sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*i - 3*int((sqrt(sec(c +
 d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c +
 d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*d*i - 3*int((sqrt(sec(c
+ d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x))/(tan(c + d*x)**3*i + tan(c
+ d*x)**2 + tan(c + d*x)*i + 1),x)*d*i))/(3*a**2*d*(tan(c + d*x)**2 + 1))
```

3.427 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 121

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{7d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}} + \frac{8i}{21ad\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{21a^2d\sqrt{e\sec(c+dx)}}$$

output
```
2/7*I/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2)+8/21*I/a/d/(e*sec(d*
x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-16/21*I*(a+I*a*tan(d*x+c))^(1/2)/a^2/
d/(e*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\sec^2(c+dx)(-7+9\cos(2(c+dx))+12i\sin(2(c+dx)))}{21ad\sqrt{e\sec(c+dx)}(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

output
```
-1/21*(Sec[c + d*x]^2*(-7 + 9*Cos[2*(c + d*x)] + (12*I)*Sin[2*(c + d*x)]))
/(a*d*Sqrt[e*Sec[c + d*x]]*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3983, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{4\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{4\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3969}$$

$$\frac{4\left(\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

output
```
((2*I)/7)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)) + (4*(((2*
I)/3)/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)*Sqr
t[a + I*a*Tan[c + d*x]])/(a*d*Sqrt[e*Sec[c + d*x]])))/(7*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 6.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{\frac{2i\cos(dx+c)^2}{7}+\frac{2\cos(dx+c)\sin(dx+c)}{7}-\frac{8i}{21}+\frac{16\tan(dx+c)}{21}}{d\sqrt{e\sec(dx+c)}\,a\sqrt{a(1+i\tan(dx+c))}}$	70

input
```
int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
2/21/d/(e*sec(d*x+c))^(1/2)/a/(a*(1+I*tan(d*x+c)))^(1/2)*(3*I*cos(d*x+c)^2
+3*cos(d*x+c)*sin(d*x+c)-4*I+8*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-21i\,e^{(6i\,dx+6i\,c)}-7i\,e^{(4i\,dx+4i\,c)}\right.}{42\,a^2de}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fr
icas")
```

output
```
1/42*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(
-21*I*e^(6*I*d*x + 6*I*c) - 7*I*e^(4*I*d*x + 4*I*c) + 17*I*e^(2*I*d*x + 2*
I*c) + 3*I)*e^(-7/2*I*d*x - 7/2*I*c)/(a^2*d*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{\sqrt{e\sec{(c+dx)}}\,(ia\,(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(1/(sqrt(e*sec(c + d*x))*(I*a*(tan(c + d*x) - I))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{3i\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+14i\cos\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\mathrm{co}\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="ma
xima")
```

output
```
1/42*(3*I*cos(7/2*d*x + 7/2*c) + 14*I*cos(3/7*arctan2(sin(7/2*d*x + 7/2*c)
, cos(7/2*d*x + 7/2*c))) - 21*I*cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(
7/2*d*x + 7/2*c))) + 3*sin(7/2*d*x + 7/2*c) + 14*sin(3/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 21*sin(1/7*arctan2(sin(7/2*d*x + 7/2
*c), cos(7/2*d*x + 7/2*c))))/(a^(3/2)*d*sqrt(e))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(35\,\sin\left(c+d\,x\right)+3\,\sin\left(3\,c+3\,d\,x\right)-\cos\left(c+\right.\right.}{42\,a\,d\,e\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d}{\cos(2\,c+2\,d\,x)+1}}}$$

input
```
int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
((e/cos(c + d*x))^(1/2)*(35*sin(c + d*x) - cos(c + d*x)*7i + cos(3*c + 3*d
*x)*3i + 3*sin(3*c + 3*d*x)))/(42*a*d*e*((a*(cos(2*c + 2*d*x) + sin(2*c +
2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\,\left(-\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c}{\sec(dx+c)\tan(dx+c)^3 i+\sec(dx+c)\tan(dx+c)^2+\sec(dx+c)\tan}\right.\right.}{}$$

input
```
int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*sqrt(a)*( - int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(
c + d*x))/(sec(c + d*x)*tan(c + d*x)**3*i + sec(c + d*x)*tan(c + d*x)**2 +
 sec(c + d*x)*tan(c + d*x)*i + sec(c + d*x)),x)*i + int((sqrt(sec(c + d*x)
)*sqrt(tan(c + d*x)*i + 1))/(sec(c + d*x)*tan(c + d*x)**3*i + sec(c + d*x)
*tan(c + d*x)**2 + sec(c + d*x)*tan(c + d*x)*i + sec(c + d*x)),x)))/(a**2*
e)
```

3.428 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 165

$$
\begin{aligned}
\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx &= \frac{2i}{9d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}} \\
&+ \frac{4i}{15ad(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{32i\sqrt{e\sec(c+dx)}}{45ade^2\sqrt{a+ia\tan(c+dx)}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{45a^2d(e\sec(c+dx))^{3/2}}
\end{aligned}
$$

output
```
2/9*I/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2)+4/15*I/a/d/(e*sec(d*
x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+32/45*I*(e*sec(d*x+c))^(1/2)/a/d/e^2/
(a+I*a*tan(d*x+c))^(1/2)-16/45*I*(a+I*a*tan(d*x+c))^(1/2)/a^2/d/(e*sec(d*x
+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.61

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\sec^3(c+dx)(-81\cos(c+dx)+5\cos(3(c+dx))-54i\sin(c+dx)+10i\sin(3(c+dx)))}{90ad(e\sec(c+dx))^{3/2}(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

output
```
-1/90*(Sec[c + d*x]^3*(-81*Cos[c + d*x] + 5*Cos[3*(c + d*x)] - (54*I)*Sin[
c + d*x] + (10*I)*Sin[3*(c + d*x)]))/(a*d*(e*Sec[c + d*x])^(3/2)*(-I + Tan
[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3983, 3042, 3983, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{2\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \textbf{3983}$$

$$\frac{2\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \textbf{3978}$$

$$\frac{2\left(\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2\left(\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \textbf{3969}$$

$$\frac{2\left(\frac{4\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

output
```
((2*I)/9)/(d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)) + (2*(((
2*I)/5)/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (4*((((4*I
)/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((2*I)/
3)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))))/(5*a)))/(3*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 6.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\frac{2i\cos(dx+c)^3}{9}+\frac{2\sin(dx+c)\cos(dx+c)^2}{9}-\frac{4i\cos(dx+c)}{45}+\frac{16\sin(dx+c)}{45}+\frac{32i\sec(dx+c)}{45}}{d\sqrt{e\sec(dx+c)}\,ea\sqrt{a(1+i\tan(dx+c))}}$	91

input
```
int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
2/45/d/(e*sec(d*x+c))^(1/2)/e/a/(a*(1+I*tan(d*x+c)))^(1/2)*(5*I*cos(d*x+c)
^3+5*sin(d*x+c)*cos(d*x+c)^2-2*I*cos(d*x+c)+8*sin(d*x+c)+16*I*sec(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.61

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-15i\,e^{(8i\,dx+8i\,c)}+120i\,e^{(6i\,dx}\right.}{180\,a}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fr
icas")
```

output
```
1/180*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
(-15*I*e^(8*I*d*x + 8*I*c) + 120*I*e^(6*I*d*x + 6*I*c) + 162*I*e^(4*I*d*x
+ 4*I*c) + 32*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-9/2*I*d*x - 9/2*I*c)/(a^2*d
*e^2)
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{(e\sec{(c+dx)})^{\frac{3}{2}}\,(ia\,(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(1/((e*sec(c + d*x))**(3/2)*(I*a*(tan(c + d*x) - I))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.08

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{5i\,\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+27i\,\cos\left(\frac{5}{9}\,\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="ma
xima")
```

output
```
1/180*(5*I*cos(9/2*d*x + 9/2*c) + 27*I*cos(5/9*arctan2(sin(9/2*d*x + 9/2*c
), cos(9/2*d*x + 9/2*c))) - 15*I*cos(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos
(9/2*d*x + 9/2*c))) + 135*I*cos(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*
d*x + 9/2*c))) + 5*sin(9/2*d*x + 9/2*c) + 27*sin(5/9*arctan2(sin(9/2*d*x +
 9/2*c), cos(9/2*d*x + 9/2*c))) + 15*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c),
 cos(9/2*d*x + 9/2*c))) + 135*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/
2*d*x + 9/2*c))))/(a^(3/2)*d*e^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.68

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,12\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,5\mathrm{i}+42\right.}{180\,a\,d\,e^2\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1}{\cos(2\,c+}}}$$

input
```
int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
((e/cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*12i + cos(4*c + 4*d*x)*5i + 42*s
in(2*c + 2*d*x) + 5*sin(4*c + 4*d*x) + 135i))/(180*a*d*e^2*((a*(cos(2*c +
2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\left(-\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(\cdots}{\sec(dx+c)^2\tan(dx+c)^3 i+\sec(dx+c)^2\tan(dx+c)^2+\sec(dx\cdots}\right.\right.}{}$$

input
```
int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*sqrt(a)*( - int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(
c + d*x))/(sec(c + d*x)**2*tan(c + d*x)**3*i + sec(c + d*x)**2*tan(c + d*x
)**2 + sec(c + d*x)**2*tan(c + d*x)*i + sec(c + d*x)**2),x)*i + int((sqrt(
sec(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(sec(c + d*x)**2*tan(c + d*x)**3*i
 + sec(c + d*x)**2*tan(c + d*x)**2 + sec(c + d*x)**2*tan(c + d*x)*i + sec(
c + d*x)**2),x)))/(a**2*e**2)
```

3.429 $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 209

$$\begin{aligned}\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx &= \frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\\ &+\frac{16i}{77ad(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\\ &+\frac{128i}{385ade^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\ &-\frac{96i\sqrt{a+ia\tan(c+dx)}}{385a^2d(e\sec(c+dx))^{5/2}}-\frac{256i\sqrt{a+ia\tan(c+dx)}}{385a^2de^2\sqrt{e\sec(c+dx)}}\end{aligned}$$

output
```
2/11*I/d/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2)+16/77*I/a/d/(e*sec(
d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2)+128/385*I/a/d/e^2/(e*sec(d*x+c))^(1
/2)/(a+I*a*tan(d*x+c))^(1/2)-96/385*I*(a+I*a*tan(d*x+c))^(1/2)/a^2/d/(e*se
c(d*x+c))^(5/2)-256/385*I*(a+I*a*tan(d*x+c))^(1/2)/a^2/d/e^2/(e*sec(d*x+c)
)^(1/2)
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.48

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{(e\sec(c+dx))^{3/2}(-385+660\cos(2(c+dx))+21\cos(4(c+dx))+880i\sin(2(c+dx))+56i\sin(4(c+dx)))}{1540ade^4(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

output
```
-1/1540*((e*Sec[c + d*x])^(3/2)*(-385 + 660*Cos[2*(c + d*x)] + 21*Cos[4*(c
 + d*x)] + (880*I)*Sin[2*(c + d*x)] + (56*I)*Sin[4*(c + d*x)]))/(a*d*e^4*(
-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3983, 3042, 3983, 3042, 3978, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{8\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{i\tan(c+dx)a+a}}dx}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{i\tan(c+dx)a+a}}dx}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{8\left(\frac{6\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\left(\frac{6\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{8\left(\frac{6\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\left(\frac{6\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{8\left(\frac{6\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{8\left(\frac{6\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3969}$$

$$\frac{8\left(\frac{6\left(\frac{4a\left(\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

output
```
((2*I)/11)/(d*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)) + (8*((
(2*I)/7)/(d*(e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (6*((((-2
*I)/5)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(5/2)) + (4*a*(((2*
I)/3)/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)*Sqr
t[a + I*a*Tan[c + d*x]])/(a*d*Sqrt[e*Sec[c + d*x]])))/(5*e^2)))/(7*a)))/(1
1*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 6.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.49

method	result	size
default	$-\frac{2\left(i\sin(dx+c)\left(56\cos(dx+c)^{2}+192\right)+21\cos(dx+c)^{3}+144\cos(dx+c)-128\sec(dx+c)\right)}{385d(\sin(dx+c)-i\cos(dx+c))\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}\,a\,e^{2}}$	103

input
```
int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/385/d/(sin(d*x+c)-I*cos(d*x+c))/(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c
))^(1/2)/a/e^2*(I*sin(d*x+c)*(56*cos(d*x+c)^2+192)+21*cos(d*x+c)^3+144*cos
(d*x+c)-128*sec(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.53

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-77i\,e^{(10i\,dx+10i\,c)}-1617i\,e^{(8}\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fr
icas")
```

output
```
1/3080*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))
*(-77*I*e^(10*I*d*x + 10*I*c) - 1617*I*e^(8*I*d*x + 8*I*c) - 770*I*e^(6*I*
d*x + 6*I*c) + 990*I*e^(4*I*d*x + 4*I*c) + 255*I*e^(2*I*d*x + 2*I*c) + 35*
I)*e^(-11/2*I*d*x - 11/2*I*c)/(a^2*d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.08

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{35i\cos\left(\frac{11}{2}dx+\frac{11}{2}c\right)+220i\cos\left(\frac{7}{11}\arctan\left(\sin\left(\frac{11}{2}dx+\right.\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="ma
xima")
```

output
```
1/3080*(35*I*cos(11/2*d*x + 11/2*c) + 220*I*cos(7/11*arctan2(sin(11/2*d*x
+ 11/2*c), cos(11/2*d*x + 11/2*c))) - 77*I*cos(5/11*arctan2(sin(11/2*d*x +
 11/2*c), cos(11/2*d*x + 11/2*c))) + 770*I*cos(3/11*arctan2(sin(11/2*d*x +
 11/2*c), cos(11/2*d*x + 11/2*c))) - 1540*I*cos(1/11*arctan2(sin(11/2*d*x
+ 11/2*c), cos(11/2*d*x + 11/2*c))) + 35*sin(11/2*d*x + 11/2*c) + 220*sin(
7/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 77*sin(5/1
1*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 770*sin(3/11*
arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 1540*sin(1/11*a
rctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))/(a^(3/2)*d*e^(5/2
))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.61

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(2310\,\sin\left(c+d\,x\right)+297\,\sin\left(3\,c+3\,d\,x\right)+35\right.}{3080\,a\,c}$$

input
```
int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
((e/cos(c + d*x))^(1/2)*(2310*sin(c + d*x) - cos(c + d*x)*770i + cos(3*c +
 3*d*x)*143i + cos(5*c + 5*d*x)*35i + 297*sin(3*c + 3*d*x) + 35*sin(5*c +
5*d*x)))/(3080*a*d*e^3*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(
cos(2*c + 2*d*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\left(-\left(\int \frac{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(}{\sec(dx+c)^3\tan(dx+c)^3 i+\sec(dx+c)^3\tan(dx+c)^2+\sec(dx-}\right.\right.}{}$$

input
```
int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*sqrt(a)*( - int((sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(
c + d*x))/(sec(c + d*x)**3*tan(c + d*x)**3*i + sec(c + d*x)**3*tan(c + d*x
)**2 + sec(c + d*x)**3*tan(c + d*x)*i + sec(c + d*x)**3),x)*i + int((sqrt(
sec(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(sec(c + d*x)**3*tan(c + d*x)**3*i
 + sec(c + d*x)**3*tan(c + d*x)**2 + sec(c + d*x)**3*tan(c + d*x)*i + sec(
c + d*x)**3),x)))/(a**2*e**3)
```

3.430 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 325

$$
\begin{aligned}
\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx &= -\frac{5ie^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d} \\
&+\frac{5ie^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d} \\
&-\frac{5ie^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}\left(\sqrt{a}+\cos(c+dx)\left(\sqrt{a}+i\sqrt{a}\tan(c+dx)\right)\right)}\right)}{\sqrt{2}a^{5/2}d} \\
&+\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}+\frac{5ie^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{a^3d}
\end{aligned}
$$

output
```
-5/2*I*e^(9/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(
e*sec(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d+5/2*I*e^(9/2)*arctan(1+2^(1/2)*e^(1
/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2)/a^(5/2)
/d-5/2*I*e^(9/2)*arctanh(2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d
*x+c))^(1/2)/(a^(1/2)+cos(d*x+c)*(a^(1/2)+I*a^(1/2)*tan(d*x+c))))*2^(1/2)/
a^(5/2)/d+4*I*e^2*(e*sec(d*x+c))^(5/2)/a/d/(a+I*a*tan(d*x+c))^(3/2)+5*I*e^
4*(e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^3/d
```

Mathematica [A] (warning: unable to verify)

Time = 5.05 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.14

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{e^2(e\sec(c+dx))^{5/2}(\cos(dx)+i\sin(dx))^3\left(\cos(dx)(8i\cos(2c)-8\sin(2c))\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(e^2*(e*Sec[c + d*x])^(5/2)*(Cos[d*x] + I*Sin[d*x])^3*(Cos[d*x]*((8*I)*Cos
[2*c] - 8*Sin[2*c]) + Sec[c + d*x]*(I*Cos[3*c] - Sin[3*c]) + 8*(Cos[2*c] +
 I*Sin[2*c])*Sin[d*x] + (5*(ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I -
Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt
[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 + I
*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sq
rt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Si
n[c]])*(Cos[3*c] + I*Sin[3*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[1 + Cos[2*c]
+ I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]])))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.31, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 3981, 3042, 3982, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\int\frac{(e\sec(c+dx))^{5/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\int\frac{(e\sec(c+dx))^{5/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{a^2}$$

$$\Big\downarrow \text{3982}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\left(\frac{e^2\int\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\left(\frac{e^2\int\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}\right)}{a^2}$$

$$\Big\downarrow \text{3976}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\left(-\frac{2ie^4\int\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e\left(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2\right)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}\right)}{a^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\left(-\frac{2ie^4\left(\frac{\int\frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\frac{ie^2\sqrt{a+ia\tan(c-}}{}\right.}{a^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\left(-\frac{2ie^4\left(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a}{e}}}{2e}}{2e}\right.}{d}\right.}{a^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\left(-\frac{2ie^4\left(\frac{\frac{\int\frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int\frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int}{}\right.}{d}\right.}{a^2}$$

$$\Big\downarrow \; 217$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\right.}{a^2}$$

$$\Big\downarrow \; 1479$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} - 5e^2\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}\right.$$

$$\Big\downarrow \ 25$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} - 5e^2\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}\right.$$

$$\Big\downarrow \ 27$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} - \frac{5e^2\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d}{2\sqrt{2}\sqrt{a}e}\right)}{d}\right)}{a^2}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} - \frac{5e^2\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right)}{d}\right)}{a^2}$$

input
```
Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((4*I)*e^2*(e*Sec[c + d*x])^(5/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) - (5
*e^2*(((-2*I)*e^4*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x
]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1
+ (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x
]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqr
t[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a +
I*a*Tan[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sq
rt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a +
 I*a*Tan[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/d - (I*e^2*Sqrt[e
*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)))/a^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 11.13 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.97

method	result
default	$-\frac{\sqrt{e\sec(dx+c)}\, e^{4} \left(4\sqrt{\frac{1}{\cos(dx+c)+1}}\,(\sin(dx+c)+\tan(dx+c))-36i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}+(-5\sin(dx+c)+5\cos(dx+c)+5\right.}{}$

input
```
int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*(e*sec(d*x+c))^(1/2)*e^4/(cos(d*x+c)+1)/(a*(1+I*tan(d*x+c)))^(1/2)/
(1/(cos(d*x+c)+1))^(1/2)/a^2*(4*(1/(cos(d*x+c)+1))^(1/2)*(sin(d*x+c)+tan(d
*x+c))-36*I*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)+(-5*sin(d*x+c)+5*cos(d
*x+c)+5)*arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+
(5*sin(d*x+c)+5*cos(d*x+c)+5)*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(1/(c
os(d*x+c)+1))^(1/2))+I*(5*sin(d*x+c)+5*cos(d*x+c)+5)*arctanh(1/2/(1/(cos(d
*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+I*(5*sin(d*x+c)-5*cos(d*x+c)-5
)*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(243) = 486.

Time = 0.12 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.66

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
-1/2*(sqrt(25*I*e^9/(a^5*d^2))*a^3*d*e^(I*d*x + I*c)*log(2/5*(sqrt(25*I*e^
9/(a^5*d^2))*a^3*d + 5*(e^4*e^(2*I*d*x + 2*I*c) + e^4)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/
e^4) - sqrt(25*I*e^9/(a^5*d^2))*a^3*d*e^(I*d*x + I*c)*log(-2/5*(sqrt(25*I*
e^9/(a^5*d^2))*a^3*d - 5*(e^4*e^(2*I*d*x + 2*I*c) + e^4)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)
)/e^4) - sqrt(-25*I*e^9/(a^5*d^2))*a^3*d*e^(I*d*x + I*c)*log(2/5*(sqrt(-25
*I*e^9/(a^5*d^2))*a^3*d + 5*(e^4*e^(2*I*d*x + 2*I*c) + e^4)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I
*c))/e^4) + sqrt(-25*I*e^9/(a^5*d^2))*a^3*d*e^(I*d*x + I*c)*log(-2/5*(sqrt
(-25*I*e^9/(a^5*d^2))*a^3*d - 5*(e^4*e^(2*I*d*x + 2*I*c) + e^4)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1
/2*I*c))/e^4) + 4*(-5*I*e^4*e^(2*I*d*x + 2*I*c) - 4*I*e^4)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*
c))*e^(-I*d*x - I*c)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e \sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2449 vs. $2(243) = 486$.

Time = 0.37 (sec) , antiderivative size = 2449, normalized size of antiderivative = 7.54

$$\int \frac{(e \sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
-(64*e^4*cos(1/2*d*x + 1/2*c)^2 + 64*e^4*sin(1/2*d*x + 1/2*c)^2 + 16*e^4 -
 10*(-I*sqrt(2)*e^4*cos(3/2*d*x + 3/2*c) - I*sqrt(2)*e^4*cos(1/2*d*x + 1/2
*c) - sqrt(2)*e^4*sin(3/2*d*x + 3/2*c) + sqrt(2)*e^4*sin(1/2*d*x + 1/2*c))
*arctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqrt(2)*cos(1/2*d*x
+ 1/2*c) + cos(d*x + c) + 1) - 10*(I*sqrt(2)*e^4*cos(3/2*d*x + 3/2*c) + I*
sqrt(2)*e^4*cos(1/2*d*x + 1/2*c) + sqrt(2)*e^4*sin(3/2*d*x + 3/2*c) - sqrt
(2)*e^4*sin(1/2*d*x + 1/2*c))*arctan2(-sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(
d*x + c), -sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) - (10*sqrt(2)*
e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 1) + 10*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, -sqrt(2)*
sin(1/2*d*x + 1/2*c) + 1) + 10*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1
/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 10*sqrt(2)*e^4*arctan2(sqrt
(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - 5*I*sqr
t(2)*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 5*I*sqrt(2
)*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 5*I*sqrt(2)*e
^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 5*I*sqrt(2)*e^4*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^4}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx\right)e^4}{\sqrt{a}\,a^2}$$

input
```
int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**4)/(sqrt(tan(c + d*x)*i
+ 1)*tan(c + d*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(ta
n(c + d*x)*i + 1)),x)*e**4)/(sqrt(a)*a**2)
```

3.431 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 405

$$
\begin{aligned}
\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx &= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} \\
&+ \frac{i\sqrt{2}e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{i\sqrt{2}e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{i\sqrt{2}e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}(\sqrt{a}+\cos(c+dx)(\sqrt{a}-i\sqrt{a}\tan(c+dx)))}\right)\sec(c+dx)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
\end{aligned}
$$

output

```
4/3*I*e^2*(e*sec(d*x+c))^(3/2)/a/d/(a+I*a*tan(d*x+c))^(3/2)+I*2^(1/2)*e^(7
/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c
))^(1/2))*sec(d*x+c)/a^(3/2)/d/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))
^(1/2)-I*2^(1/2)*e^(7/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)
/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/a^(3/2)/d/(a-I*a*tan(d*x+c))^(1/
2)/(a+I*a*tan(d*x+c))^(1/2)+I*2^(1/2)*e^(7/2)*arctanh(2^(1/2)*e^(1/2)*(a-I
*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)/(a^(1/2)+cos(d*x+c)*(a^(1/2)-I*a
^(1/2)*tan(d*x+c))))*sec(d*x+c)/a^(3/2)/d/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*
tan(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.88 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.88

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{e(e\sec(c+dx))^{5/2}(\cos(dx)+i\sin(dx))^3\left(\frac{4}{3}i\cos(2dx)(\cos(c)+i\sin(c))+\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(e*(e*Sec[c + d*x])^(5/2)*(Cos[d*x] + I*Sin[d*x])^3*(((4*I)/3)*Cos[2*d*x]*
(Cos[c] + I*Sin[c]) + (4*(Cos[c] + I*Sin[c])*Sin[2*d*x])/3 - (2*(ArcTanh[(
Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] +
Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*C
os[c] - Sin[c]] - ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/
2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos
[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*(Cos[2*c] + I*Sin[2*c])*Sqrt[I
 + Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c
]]*Sqrt[I - Tan[(d*x)/2]])))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 3981, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \ \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{e^2\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{a^2}$$

$$\Big\downarrow \ \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{e^2\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{a^2}$$

$$\Big\downarrow \ \text{3980}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{e^3\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{a^2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{e^3\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{a^2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ \text{3976}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4ie^5\sec(c+dx)\int\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ \text{826}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4ie^5\sec(c+dx)\left(\frac{\int\frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ \text{1476}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4ie^5\sec(c+dx)\left(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\,\mathrm{t}\cdots}{e}}\cdots}{2e}}{2e}\cdots\right.}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ \text{1082}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4ie^5\sec(c+dx)\left(\frac{\frac{\int\frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int\frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\right.}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{217}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4ie^5\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c}}{\sqrt{e\sec(c+}}}{2e}\right.}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4ie^5\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+d}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\cos(c+dx)\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}\right.}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+i}}$$

$$\Big\downarrow \text{25}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4ie^5\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\,t}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-4ie^5\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan}{e}}}{2\sqrt{2}\sqrt{a}e}\right.}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c}}$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-4ie^5\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((4*I)/3)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
- ((4*I)*e^5*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(
Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sq
rt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/
(Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*
Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*T
an[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]
*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*
Tan[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(a*d*Sqrt
[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 10.96 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.77

method	result
default	$-\frac{\left(i(-3\sin(dx+c)-3\cos(dx+c))\operatorname{arctanh}\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+i(3\sin(dx+c)-3\cos(dx+c))\operatorname{arctanh}\left(\frac{-\cot(dx+c)+\csc(dx}{2\sqrt{\frac{1}{\cos(dx+c)+}}}\right.\right.}{}$

input
```
int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/d*(I*(-3*sin(d*x+c)-3*cos(d*x+c))*arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2
)*(-cot(d*x+c)+csc(d*x+c)+1))+I*(3*sin(d*x+c)-3*cos(d*x+c))*arctanh(1/2*(-
cot(d*x+c)+csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))+4*I*(1/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)+(3*sin(d*x+c)-3*cos(d*x+c))*arctanh(1/2/(1/(cos(d*x+c)+1)
)^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+(3*sin(d*x+c)+3*cos(d*x+c))*arctanh(1/
2*(-cot(d*x+c)+csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))-4*(cos(d*x+c)+1)*(1
/(cos(d*x+c)+1))^(1/2))*(e*sec(d*x+c))^(1/2)*e^3/(-I*cos(d*x+c)+sin(d*x+c)
-I)/a^2/(a*(1+I*tan(d*x+c)))^(1/2)/(1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.34

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
-1/6*(3*a^3*d*sqrt(4*I*e^7/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*log((I*a^3*d*sqr
t(4*I*e^7/(a^5*d^2)) + 2*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)
)/e^3) - 3*a^3*d*sqrt(4*I*e^7/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*log((-I*a^3*d
*sqrt(4*I*e^7/(a^5*d^2)) + 2*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*
I*c))/e^3) + 3*a^3*d*sqrt(-4*I*e^7/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*log((I*a
^3*d*sqrt(-4*I*e^7/(a^5*d^2)) + 2*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x +
 1/2*I*c))/e^3) - 3*a^3*d*sqrt(-4*I*e^7/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*log
((-I*a^3*d*sqrt(-4*I*e^7/(a^5*d^2)) + 2*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I
*d*x + 1/2*I*c))/e^3) + 8*(-I*e^3*e^(2*I*d*x + 2*I*c) - I*e^3)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/
2*I*c))*e^(-2*I*d*x - 2*I*c)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1466 vs. $2(305) = 610$.

Time = 0.40 (sec) , antiderivative size = 1466, normalized size of antiderivative = 3.62

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
1/12*(6*I*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + 1) + 6*I*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 6*I*sqrt(2)*e^3
*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) - 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 1) + 6*I*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) - 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 6*sqrt(2)*e^3*arctan2(sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))), sqrt(2)*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 1) + 6*sqrt(2)*e^3*arctan2(-sqrt(2)*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))), -sqrt(2)*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 1) + 3*I*sqrt(2)*e^3*log(2*sqrt(2)*sin(2/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*(sqrt(2)*cos(1/3*arctan2(si...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^{5/2}}\,dx$$

input
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx=-\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^3}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx\right)e^3}{\sqrt{a}\,a^2}$$

input
```
int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**3)/(sqrt(tan(c + d*x)*i
+ 1)*tan(c + d*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(ta
n(c + d*x)*i + 1)),x)*e**3)/(sqrt(a)*a**2)
```

3.432 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 38

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{2i(e\sec(c+dx))^{5/2}}{5d(a+ia\tan(c+dx))^{5/2}}$$

output
```
2/5*I*(e*sec(d*x+c))^(5/2)/d/(a+I*a*tan(d*x+c))^(5/2)
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{2i(e\sec(c+dx))^{5/2}}{5d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((2*I)/5)*(e*Sec[c + d*x])^(5/2))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3969}$$
$$\frac{2i(e\sec(c+dx))^{5/2}}{5d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((2*I)/5)*(e*Sec[c + d*x])^(5/2))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Maple [A] (verified)

Time = 11.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
orering	$\frac{2i(e\sec(dx+c))^{\frac{5}{2}}}{5d(a+ia\tan(dx+c))^{\frac{5}{2}}}$	31
default	$\frac{2i\sec(dx+c)^{2}e^{2}\sqrt{e\sec(dx+c)}}{5d(-1-i\tan(dx+c))^{2}a^{2}\sqrt{a(1+i\tan(dx+c))}}$	59

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*I*(e*sec(d*x+c))^(5/2)/d/(a+I*a*tan(d*x+c))^(5/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(28) = 56.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.87

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2\left(i\,e^{2}e^{(2i\,dx+2i\,c)}+i\,e^{2}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{5\,a^{3}d}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
2/5*(I*e^2*e^(2*I*d*x + 2*I*c) + I*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-5/2*I*d*x - 5/2*I*c)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i\,e^{\frac{5}{2}}\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}}{5\,a^{\frac{5}{2}}d\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
2/5*I*e^(5/2)*(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2)/(a^(5/2)*d*
(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - 1)^(5/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^{5/2}}\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)^2}{\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)\,i-\sqrt{\tan(dx+c)\,i+1}}dx\right)e^2}{\sqrt{a}\,a^2}$$

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x)**2)/(sqrt(tan(c + d*x)*i
+ 1)*tan(c + d*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(ta
n(c + d*x)*i + 1)),x)*e**2)/(sqrt(a)*a**2)
```

3.433 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 80

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}}+\frac{4i(e\sec(c+dx))^{3/2}}{21ad(a+ia\tan(c+dx))^{3/2}}$$

output
```
2/7*I*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(5/2)+4/21*I*(e*sec(d*x+c)
)^(3/2)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{2(e\sec(c+dx))^{3/2}(-5i+2\tan(c+dx))}{21a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(2*(e*Sec[c + d*x])^(3/2)*(-5*I + 2*Tan[c + d*x]))/(21*a^2*d*(-I + Tan[c +
 d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{ 3983}$$
$$\frac{2\int \frac{(e\sec(c+dx))^{3/2}}{(i\tan(c+dx)a+a)^{3/2}}dx}{7a}+\frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{2\int \frac{(e\sec(c+dx))^{3/2}}{(i\tan(c+dx)a+a)^{3/2}}dx}{7a}+\frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}}$$
$$\Big\downarrow \text{ 3969}$$
$$\frac{4i(e\sec(c+dx))^{3/2}}{21ad(a+ia\tan(c+dx))^{3/2}}+\frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((2*I)/7)*(e*Sec[c + d*x])^(3/2))/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((4
*I)/21)*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 9.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

method	result	size
default	$-\frac{2i\left(6i\cos(dx+c)^2\sin(dx+c)-6\cos(dx+c)^3+2i\sin(dx+c)+\cos(dx+c)\right)\sqrt{e\sec(dx+c)}\,e}{21d\,a^2\sqrt{a(1+i\tan(dx+c))}}$	79

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*I/d*(6*I*cos(d*x+c)^2*sin(d*x+c)-6*cos(d*x+c)^3+2*I*sin(d*x+c)+cos(d
*x+c))*(e*sec(d*x+c))^(1/2)*e/a^2/(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(7i\,ee^{(4i\,dx+4i\,c)}+10i\,ee^{(2i\,dx+2i\,c)}+3i\,e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{7}{2}i\,dx\right.}}{21\,a^3d}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
1/21*(7*I*e*e^(4*I*d*x + 4*I*c) + 10*I*e*e^(2*I*d*x + 2*I*c) + 3*I*e)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-7/2*I*
d*x - 7/2*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec{(c+dx)})^{\frac{3}{2}}}{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((e*sec(c + d*x))**(3/2)/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(3i\,e\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+7i\,e\cos\left(\frac{3}{7}\,\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right.\right.\right.\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
1/21*(3*I*e*cos(7/2*d*x + 7/2*c) + 7*I*e*cos(3/7*arctan2(sin(7/2*d*x + 7/2
*c), cos(7/2*d*x + 7/2*c))) + 3*e*sin(7/2*d*x + 7/2*c) + 7*e*sin(3/7*arcta
n2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(e)/(a^(5/2)*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.28

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{e\,\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(7\,\sin\left(c+d\,x\right)+3\,\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,7{}\mathrm{i}+\cos\left(3\right.\right.}{21\,a^2\,d\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1{}\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}$$

input
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
(e*(e/cos(c + d*x))^(1/2)*(cos(c + d*x)*7i + 7*sin(c + d*x) + cos(3*c + 3*
d*x)*3i + 3*sin(3*c + 3*d*x)))/(21*a^2*d*((a*(cos(2*c + 2*d*x) + sin(2*c +
 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,\sec(dx+c)}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx\right)e}{\sqrt{a}\,a^2}$$

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - sqrt(e)*int((sqrt(sec(c + d*x))*sec(c + d*x))/(sqrt(tan(c + d*x)*i + 1
)*tan(c + d*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c
 + d*x)*i + 1)),x)*e)/(sqrt(a)*a**2)
```

3.434 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 121

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}} + \frac{8i\sqrt{e\sec(c+dx)}}{45ad(a+ia\tan(c+dx))^{3/2}} + \frac{16i\sqrt{e\sec(c+dx)}}{45a^2d\sqrt{a+ia\tan(c+dx)}}$$

output
```
2/9*I*(e*sec(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^(5/2)+8/45*I*(e*sec(d*x+c)
)^(1/2)/a/d/(a+I*a*tan(d*x+c))^(3/2)+16/45*I*(e*sec(d*x+c))^(1/2)/a^2/d/(a
+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{i\sec^2(c+dx)\sqrt{e\sec(c+dx)}(9+25\cos(2(c+dx))+20i\sin(2(c+dx)))}{45a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-1/45*I)*Sec[c + d*x]^2*Sqrt[e*Sec[c + d*x]]*(9 + 25*Cos[2*(c + d*x)] +
(20*I)*Sin[2*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c
+ d*x]])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3983, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{4\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^{3/2}}dx}{9a}+\frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^{3/2}}dx}{9a}+\frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{4\left(\frac{2\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{5a}+\frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}\right)}{9a}+\frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{2\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{5a}+\frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}\right)}{9a}+\frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{4\left(\frac{4i\sqrt{e\sec(c+dx)}}{5ad\sqrt{a+ia\tan(c+dx)}}+\frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}\right)}{9a}+\frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((2*I)/9)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (4*(((
(2*I)/5)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((4*I)/
5)*Sqrt[e*Sec[c + d*x]])/(a*d*Sqrt[a + I*a*Tan[c + d*x]])))/(9*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 9.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{2i\left(10i\sin(dx+c)\cos(dx+c)^{3}-10\cos(dx+c)^{4}+4i\sin(dx+c)\cos(dx+c)+\cos(dx+c)^{2}-8\right)\sqrt{e\sec(dx+c)}}{45d\,a^{2}\sqrt{a(1+i\tan(dx+c))}}$	87

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*I/d*(10*I*sin(d*x+c)*cos(d*x+c)^3-10*cos(d*x+c)^4+4*I*sin(d*x+c)*cos
(d*x+c)+cos(d*x+c)^2-8)*(e*sec(d*x+c))^(1/2)/a^2/(a*(1+I*tan(d*x+c)))^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(45i\,e^{(6i\,dx+6i\,c)}+63i\,e^{(4i\,dx+4i\,c)}+23i\,e^{(2i\,dx+2i\,c)}\right.}{90\,a^{3}d}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
1/90*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(
45*I*e^(6*I*d*x + 6*I*c) + 63*I*e^(4*I*d*x + 4*I*c) + 23*I*e^(2*I*d*x + 2*
I*c) + 5*I)*e^(-9/2*I*d*x - 9/2*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{\sqrt{e\sec{(c+dx)}}}{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sqrt(e*sec(c + d*x))/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{\sqrt{e}\left(5i\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+18i\cos\left(\frac{5}{9}\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\right.\right.\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
1/90*sqrt(e)*(5*I*cos(9/2*d*x + 9/2*c) + 18*I*cos(5/9*arctan2(sin(9/2*d*x
+ 9/2*c), cos(9/2*d*x + 9/2*c))) + 45*I*cos(1/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) + 5*sin(9/2*d*x + 9/2*c) + 18*sin(5/9*arctan2(s
in(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 45*sin(1/9*arctan2(sin(9/2*d
*x + 9/2*c), cos(9/2*d*x + 9/2*c))))/(a^(5/2)*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,18\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,5\mathrm{i}+18\,\sin\left(2\,c+2\,d\,x\right)\right.}{90\,a^2\,d\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}}$$

input
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
((e/cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*18i + cos(4*c + 4*d*x)*5i + 18*s
in(2*c + 2*d*x) + 5*sin(4*c + 4*d*x) + 45i))/(90*a^2*d*((a*(cos(2*c + 2*d*
x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx\right)}{\sqrt{a}\,a^2}$$

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - sqrt(e)*int(sqrt(sec(c + d*x))/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*
*2 - 2*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1))
,x))/(sqrt(a)*a**2)
```

3.435 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 162

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{2i}{11d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}+\frac{12i}{77ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}+\frac{16i}{77a^2d\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{32i\sqrt{a+ia\tan(c+dx)}}{77a^3d\sqrt{e\sec(c+dx)}}$$

output
```
2/11*I/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2)+12/77*I/a/d/(e*sec(
d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2)+16/77*I/a^2/d/(e*sec(d*x+c))^(1/2)/
(a+I*a*tan(d*x+c))^(1/2)-32/77*I*(a+I*a*tan(d*x+c))^(1/2)/a^3/d/(e*sec(d*x
+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\sec^3(c+dx)(-55\cos(c+dx)+35\cos(3(c+dx))-22i\sin(c+dx)+42i\sin(3(c+dx)))}{154a^2d\sqrt{e\sec(c+dx)}(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

output
```
((I/154)*Sec[c + d*x]^3*(-55*Cos[c + d*x] + 35*Cos[3*(c + d*x)] - (22*I)*S
in[c + d*x] + (42*I)*Sin[3*(c + d*x)]))/(a^2*d*Sqrt[e*Sec[c + d*x]]*(-I +
Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{6\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^{3/2}}dx}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^{3/2}}dx}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{6\left(\frac{4\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\left(\frac{4\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{6\left(\frac{4\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\left(\frac{4\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{6\left(\frac{4\left(\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

output
```
((2*I)/11)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)) + (6*(((2
*I)/7)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)) + (4*(((2*I)/
3)/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)*Sqrt[a
 + I*a*Tan[c + d*x]])/(a*d*Sqrt[e*Sec[c + d*x]])))/(7*a)))/(11*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 6.63 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\frac{4 i \cos\left(d x +c \right)^{4}}{11}+\frac{4 \sin\left(d x +c \right) \cos\left(d x +c \right)^{3}}{11}-\frac{2 i \cos\left(d x +c \right)^{2}}{77}+\frac{12 \cos\left(d x +c \right) \sin\left(d x +c \right)}{77}-\frac{16 i}{77}+\frac{32 \tan\left(d x +c \right)}{77}}{d \sqrt{e \sec\left(d x +c \right)}\, a^{2} \sqrt{a \left(1+i \tan\left(d x +c \right)\right)}}$	97

input
```
int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
2/77/d/(e*sec(d*x+c))^(1/2)/a^2/(a*(1+I*tan(d*x+c)))^(1/2)*(14*I*cos(d*x+c
)^4+14*sin(d*x+c)*cos(d*x+c)^3-I*cos(d*x+c)^2+6*cos(d*x+c)*sin(d*x+c)-8*I+
16*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.55

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-77i\,e^{(8i\,dx+8i\,c)}+110i\,e^{(4i\,dx+4}\right.}{308\,a^3de}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fr
icas")
```

output
```
1/308*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
(-77*I*e^(8*I*d*x + 8*I*c) + 110*I*e^(4*I*d*x + 4*I*c) + 40*I*e^(2*I*d*x +
 2*I*c) + 7*I)*e^(-11/2*I*d*x - 11/2*I*c)/(a^3*d*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{\sqrt{e\sec{(c+dx)}}\,(ia\,(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(1/(sqrt(e*sec(c + d*x))*(I*a*(tan(c + d*x) - I))**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{7i\cos\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)+33i\cos\left(\frac{7}{11}\arctan\left(\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right.\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="ma
xima")
```

output
```
1/308*(7*I*cos(11/2*d*x + 11/2*c) + 33*I*cos(7/11*arctan2(sin(11/2*d*x + 1
1/2*c), cos(11/2*d*x + 11/2*c))) + 77*I*cos(3/11*arctan2(sin(11/2*d*x + 11
/2*c), cos(11/2*d*x + 11/2*c))) - 77*I*cos(1/11*arctan2(sin(11/2*d*x + 11/
2*c), cos(11/2*d*x + 11/2*c))) + 7*sin(11/2*d*x + 11/2*c) + 33*sin(7/11*ar
ctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 77*sin(3/11*arcta
n2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 77*sin(1/11*arctan2(
sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))/(a^(5/2)*d*sqrt(e))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,(154\,\sin\left(c+d\,x\right)+33\,\sin\left(3\,c+3\,d\,x\right)+7\,\sin\left(\right.}{308\,a^2\,d\,e\,\sqrt{\frac{a\,(\cos(2\,c+2\,}{\cos}}}$$

input
```
int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
((e/cos(c + d*x))^(1/2)*(154*sin(c + d*x) + cos(3*c + 3*d*x)*33i + cos(5*c
 + 5*d*x)*7i + 33*sin(3*c + 3*d*x) + 7*sin(5*c + 5*d*x)))/(308*a^2*d*e*((a
*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/
2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx =$$
$$-\frac{\int \frac{1}{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}}dx}{\sqrt{e}\,\sqrt{a}\,a^2}$$

input
```
int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - int(1/(sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2
*sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(sec(c +
 d*x))*sqrt(tan(c + d*x)*i + 1)),x))/(sqrt(e)*sqrt(a)*a**2)
```

3.436 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 206

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}} + \frac{16i}{117ad(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}} + \frac{32i}{195a^2d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{256i\sqrt{e\sec(c+dx)}}{585a^2de^2\sqrt{a+ia\tan(c+dx)}} - \frac{128i\sqrt{a+ia\tan(c+dx)}}{585a^3d(e\sec(c+dx))^{3/2}}$$

output
```
2/13*I/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2)+16/117*I/a/d/(e*sec
(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2)+32/195*I/a^2/d/(e*sec(d*x+c))^(3/2
)/(a+I*a*tan(d*x+c))^(1/2)+256/585*I*(e*sec(d*x+c))^(1/2)/a^2/d/e^2/(a+I*a
*tan(d*x+c))^(1/2)-128/585*I*(a+I*a*tan(d*x+c))^(1/2)/a^3/d/(e*sec(d*x+c))
^(3/2)
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.52

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sec^4(c+dx)(-351i-1300i\cos(2(c+dx))+75i\cos(4(c}{2340a^2d(e\sec(c+dx))^{3/2}(-i+\tan(}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

output
```
(Sec[c + d*x]^4*(-351*I - (1300*I)*Cos[2*(c + d*x)] + (75*I)*Cos[4*(c + d*
x)] + 1040*Sin[2*(c + d*x)] - 120*Sin[4*(c + d*x)]))/(2340*a^2*d*(e*Sec[c
+ d*x])^(3/2)*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 3983$$

$$\frac{8\int \frac{1}{(e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^{3/2}}dx}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{8\int \frac{1}{(e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^{3/2}}dx}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3983$$

$$\frac{8\left(\frac{2\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{8\left(\frac{2\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3983}$$

$$\frac{8\left(\frac{2\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{8\left(\frac{2\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3978}$$

$$\frac{8\left(\frac{2\left(\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\left(\frac{2\left(\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{8\left(\frac{2\left(\frac{4\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

output
```
((2*I)/13)/(d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)) + (8*((
(2*I)/9)/(d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)) + (2*(((2
*I)/5)/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (4*((((4*I)
/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((2*I)/3
)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))))/(5*a)))/(3*a)))
/(13*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 6.61 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{\frac{2i\sin(dx+c)\left(-120\cos(dx+c)^2+320\right)}{585}-\frac{10\cos(dx+c)^3}{39}+\frac{160\cos(dx+c)}{117}-\frac{256\sec(dx+c)}{585}}{d\left(2\cos(dx+c)\sin(dx+c)-2i\cos(dx+c)^2+i\right)\sqrt{e\sec(dx+c)}\,e\,a^2\sqrt{a(1+i\tan(dx+c))}}$	115

input
```
int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
2/585/d/(2*cos(d*x+c)*sin(d*x+c)-2*I*cos(d*x+c)^2+I)/(e*sec(d*x+c))^(1/2)/
e/a^2/(a*(1+I*tan(d*x+c)))^(1/2)*(I*sin(d*x+c)*(-120*cos(d*x+c)^2+320)-75*
cos(d*x+c)^3+400*cos(d*x+c)-128*sec(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.54

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-195i\,e^{(10i\,dx+10i\,c)}+2145i\,e^{\cdots}\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fr
icas")
```

output
```
1/4680*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))
*(-195*I*e^(10*I*d*x + 10*I*c) + 2145*I*e^(8*I*d*x + 8*I*c) + 3042*I*e^(6*
I*d*x + 6*I*c) + 962*I*e^(4*I*d*x + 4*I*c) + 305*I*e^(2*I*d*x + 2*I*c) + 4
5*I)*e^(-13/2*I*d*x - 13/2*I*c)/(a^3*d*e^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{45i\cos\left(\frac{13}{2}dx+\frac{13}{2}c\right)+260i\cos\left(\frac{9}{13}\arctan\left(\sin\left(\frac{13}{2}dx+\right.\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="ma
xima")
```

output
```
1/4680*(45*I*cos(13/2*d*x + 13/2*c) + 260*I*cos(9/13*arctan2(sin(13/2*d*x
+ 13/2*c), cos(13/2*d*x + 13/2*c))) + 702*I*cos(5/13*arctan2(sin(13/2*d*x
+ 13/2*c), cos(13/2*d*x + 13/2*c))) - 195*I*cos(3/13*arctan2(sin(13/2*d*x
+ 13/2*c), cos(13/2*d*x + 13/2*c))) + 2340*I*cos(1/13*arctan2(sin(13/2*d*x
 + 13/2*c), cos(13/2*d*x + 13/2*c))) + 45*sin(13/2*d*x + 13/2*c) + 260*sin
(9/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 702*sin(5
/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 195*sin(3/1
3*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 2340*sin(1/13
*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))))/(a^(5/2)*d*e^(3
/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.66

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,507\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,260\mathrm{i}+\dots\right.}{4\dots}$$

input
```
int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
((e/cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*507i + cos(4*c + 4*d*x)*260i + c
os(6*c + 6*d*x)*45i + 897*sin(2*c + 2*d*x) + 260*sin(4*c + 4*d*x) + 45*sin
(6*c + 6*d*x) + 2340i))/(4680*a^2*d*e^2*((a*(cos(2*c + 2*d*x) + sin(2*c +
2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{\int \frac{1}{\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)\tan(dx+c)^2-2\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\sec(dx+c)\tan(dx+c)i-\sqrt{\sec(dx+c)}\,\sqrt{\tan(dx+c)i+1}}}{\sqrt{e}\,\sqrt{a}\,a^2e}$$

input
```
int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - int(1/(sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)*tan(c
+ d*x)**2 - 2*sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)*tan
(c + d*x)*i - sqrt(sec(c + d*x))*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)),x)
)/(sqrt(e)*sqrt(a)*a**2*e)
```

3.437 $\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 86

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{3i2^{2/3}a\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{6},\frac{13}{6},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{7/3}}{7d(a+ia\tan(c+dx))^{3/2}}$$

output
```
3/7*I*2^(2/3)*a*hypergeom([1/3, 7/6],[13/6],1/2-1/2*I*tan(d*x+c))*(e*sec(d
*x+c))^(7/3)*(1+I*tan(d*x+c))^(1/3)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.37

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx=$$
$$-\frac{3i\sqrt[3]{2}ee^{i(c+dx)}\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{4/3}\left(4+\left(1+e^{2i(c+dx)}\right)^{5/6}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{5}{3},-e^{2i(c+dx)}\right)\right)}{5d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(e*Sec[c + d*x])^(7/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((-3*I)/5)*2^(1/3)*e*E^(I*(c + d*x))*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(
c + d*x))))^(4/3)*(4 + (1 + E^((2*I)*(c + d*x)))^(5/6)*Hypergeometric2F1[2
/3, 5/6, 5/3, -E^((2*I)*(c + d*x))]))/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(e\sec(c+dx))^{7/3}\int (a-ia\tan(c+dx))^{7/6}(i\tan(c+dx)a+a)^{2/3}dx}{(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{7/6}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e\sec(c+dx))^{7/3}\int (a-ia\tan(c+dx))^{7/6}(i\tan(c+dx)a+a)^{2/3}dx}{(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{7/6}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(e\sec(c+dx))^{7/3}\int \frac{\sqrt[6]{a-ia\tan(c+dx)}}{\sqrt[3]{i\tan(c+dx)a+a}}d\tan(c+dx)}{d(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{7/6}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2\sqrt[3]{1+i\tan(c+dx)}(e\sec(c+dx))^{7/3}\int \frac{\sqrt[3]{2}\sqrt[6]{a-ia\tan(c+dx)}}{\sqrt[3]{i\tan(c+dx)+1}}d\tan(c+dx)}{\sqrt[3]{2}d(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2 \sqrt[3]{1+i\tan(c+dx)}(e\sec(c+dx))^{7/3} \int \frac{\sqrt[6]{a-ia\tan(c+dx)}}{\sqrt[3]{i\tan(c+dx)+1}}d\tan(c+dx)}{d(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 79$$

$$\frac{3i2^{2/3}a\sqrt[3]{1+i\tan(c+dx)}(e\sec(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{6},\frac{13}{6},\frac{1}{2}(1-i\tan(c+dx))\right)}{7d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[(e*Sec[c + d*x])^(7/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((3*I)/7)*2^(2/3)*a*Hypergeometric2F1[1/3, 7/6, 13/6, (1 - I*Tan[c + d*x]
)/2]*(e*Sec[c + d*x])^(7/3)*(1 + I*Tan[c + d*x])^(1/3))/(d*(a + I*a*Tan[c
+ d*x])^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(e\sec(dx+c))^{\frac{7}{3}}}{\sqrt{a+ia\tan(dx+c)}}dx$$

input
```
int((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{7}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/5*(-6*I*2^(5/6)*e^2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2
*I*c) + 1))^(1/3)*e^(4/3*I*d*x + 4/3*I*c) + 5*a*d*integral(-2/5*I*2^(5/6)*
e^2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*
e^(1/3*I*d*x + 1/3*I*c)/(a*d), x))/(a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(7/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{7}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((e*sec(d*x + c))^(7/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/3}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int((e/cos(c + d*x))^(7/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(7/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{2e^{\frac{7}{3}}\sqrt{a}\,i\left(-3\sec\left(dx+c\right)^{\frac{7}{3}}\sqrt{\tan\left(dx+c\right)i+1}+\left(\int\frac{\sec(dx+c)^{\frac{7}{3}}\sqrt{\tan(dx+c)i+1}\,\mathrm{ta}}{\tan(dx+c)^{2}+1}\right.\right.}{3ad\left(\tan\left(dx+\right.\right.}$$

input
```
int((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*e**(1/3)*sqrt(a)*e**2*i*( - 3*sec(c + d*x)**(1/3)*sqrt(tan(c + d*x)*i +
 1)*sec(c + d*x)**2 + int((sec(c + d*x)**(1/3)*sqrt(tan(c + d*x)*i + 1)*se
c(c + d*x)**2*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*tan(c + d*x)**2*d + i
nt((sec(c + d*x)**(1/3)*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)**2*tan(c + d
*x))/(tan(c + d*x)**2 + 1),x)*d))/(3*a*d*(tan(c + d*x)**2 + 1))
```

3.438 $\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 86

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i\sqrt[3]{2}a\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{5/3}(}{5d(a+ia\tan(c+dx))^{3/2}}$$

output
```
3/5*I*2^(1/3)*a*hypergeom([2/3, 5/6],[11/6],1/2-1/2*I*tan(d*x+c))*(e*sec(d
*x+c))^(5/3)*(1+I*tan(d*x+c))^(2/3)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.35

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i2^{2/3}ee^{i(c+dx)}\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\left(-2+\sqrt[6]{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\right.\right.}{d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((3*I)*2^(2/3)*e*E^(I*(c + d*x))*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d
*x))))^(2/3)*(-2 + (1 + E^((2*I)*(c + d*x)))^(1/6)*Hypergeometric2F1[1/6,
1/3, 4/3, -E^((2*I)*(c + d*x))]))/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(e\sec(c+dx))^{5/3}\int (a-ia\tan(c+dx))^{5/6}\sqrt[3]{i\tan(c+dx)a+a}dx}{(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{5/6}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e\sec(c+dx))^{5/3}\int (a-ia\tan(c+dx))^{5/6}\sqrt[3]{i\tan(c+dx)a+a}dx}{(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{5/6}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(e\sec(c+dx))^{5/3}\int \frac{1}{\sqrt[6]{a-ia\tan(c+dx)}(i\tan(c+dx)a+a)^{2/3}}d\tan(c+dx)}{d(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{5/6}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2(1+i\tan(c+dx))^{2/3}(e\sec(c+dx))^{5/3}\int \frac{2^{2/3}}{(i\tan(c+dx)+1)^{2/3}\sqrt[6]{a-ia\tan(c+dx)}}d\tan(c+dx)}{2^{2/3}d(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2(1+i\tan(c+dx))^{2/3}(e\sec(c+dx))^{5/3}\int\frac{1}{(i\tan(c+dx)+1)^{2/3}\sqrt[6]{a-ia\tan(c+dx)}}d\tan(c+dx)}{d(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{79}$$

$$\frac{3i\sqrt[3]{2}a(1+i\tan(c+dx))^{2/3}(e\sec(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(c+dx))\right)}{5d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[(e*Sec[c + d*x])^(5/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((3*I)/5)*2^(1/3)*a*Hypergeometric2F1[2/3, 5/6, 11/6, (1 - I*Tan[c + d*x]
)/2]*(e*Sec[c + d*x])^(5/3)*(1 + I*Tan[c + d*x])^(2/3))/(d*(a + I*a*Tan[c
+ d*x])^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(e\sec (dx+c))^{\frac{5}{3}}}{\sqrt{a+ia\tan (dx+c)}}dx$$

input
```
int((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec (dx+c))^{\frac{5}{3}}}{\sqrt{i\,a\tan (dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
-(6*2^(1/6)*(I*e*e^(2*I*d*x + 2*I*c) + I*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*e^(2/3*I*d*x + 2/3*I*c) - a*d*inte
gral(2^(1/6)*(I*e*e^(2*I*d*x + 2*I*c) + I*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*e^(-4/3*I*d*x - 4/3*I*c)/(a*d), x
))/(a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(5/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((e*sec(d*x + c))^(5/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/3}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int((e/cos(c + d*x))^(5/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(5/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2e^{\frac{5}{3}}\sqrt{a}\,i\left(-3\sec\left(dx+c\right)^{\frac{5}{3}}\sqrt{\tan\left(dx+c\right)i+1}-\left(\int \frac{\sec(dx+c)^{\frac{5}{3}}\sqrt{\tan(dx+c)i+1}\,\mathrm{ta}}{\tan(dx+c)^2+1}\right.\right.}{3ad\left(\tan\left(dx+\right.\right.}$$

input
```
int((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*e**(2/3)*sqrt(a)*e*i*( - 3*sec(c + d*x)**(2/3)*sqrt(tan(c + d*x)*i + 1)
*sec(c + d*x) - int((sec(c + d*x)**(2/3)*sqrt(tan(c + d*x)*i + 1)*sec(c +
d*x)*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*tan(c + d*x)**2*d - int((sec(c
 + d*x)**(2/3)*sqrt(tan(c + d*x)*i + 1)*sec(c + d*x)*tan(c + d*x))/(tan(c
+ d*x)**2 + 1),x)*d))/(3*a*d*(tan(c + d*x)**2 + 1))
```

3.439 $\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 86

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3ia\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{6},\frac{4}{3},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{2/3}(1+}{2\sqrt[6]{2}d(a+ia\tan(c+dx))^{3/2}}$$

output
```
3/4*I*a*hypergeom([1/3, 7/6],[4/3],1/2-1/2*I*tan(d*x+c))*(e*sec(d*x+c))^(2
/3)*(1+I*tan(d*x+c))^(7/6)*2^(5/6)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.35

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i\sqrt[6]{2}\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\sqrt[6]{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{5}{6},-e^{2i(c+dx)}\right)}{d\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

input
```
Integrate[(e*Sec[c + d*x])^(2/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((3*I)*2^(1/6)*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^(2/3)*(1 +
E^((2*I)*(c + d*x)))^(1/6)*Hypergeometric2F1[-1/6, 1/6, 5/6, -E^((2*I)*(c
+ d*x))])/(d*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(e\sec(c+dx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(c+dx)}}{\sqrt[6]{i\tan(c+dx)a+a}}dx}{\sqrt[3]{a-ia\tan(c+dx)}\sqrt[3]{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e\sec(c+dx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(c+dx)}}{\sqrt[6]{i\tan(c+dx)a+a}}dx}{\sqrt[3]{a-ia\tan(c+dx)}\sqrt[3]{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(e\sec(c+dx))^{2/3}\int \frac{1}{(a-ia\tan(c+dx))^{2/3}(i\tan(c+dx)a+a)^{7/6}}d\tan(c+dx)}{d\sqrt[3]{a-ia\tan(c+dx)}\sqrt[3]{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a\sqrt[6]{1+i\tan(c+dx)}(e\sec(c+dx))^{2/3}\int \frac{2\sqrt[6]{2}}{(i\tan(c+dx)+1)^{7/6}(a-ia\tan(c+dx))^{2/3}}d\tan(c+dx)}{2\sqrt[6]{2}d\sqrt[3]{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sqrt[6]{1+i\tan(c+dx)}(e\sec(c+dx))^{2/3}\int \frac{1}{(i\tan(c+dx)+1)^{7/6}(a-ia\tan(c+dx))^{2/3}}d\tan(c+dx)}{d\sqrt[3]{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{79}$$

$$\frac{3i\sqrt[6]{1+i\tan(c+dx)}(e\sec(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{6},\frac{4}{3},\frac{1}{2}(1-i\tan(c+dx))\right)}{2\sqrt[6]{2}d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[(e*Sec[c + d*x])^(2/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((3*I)/2)*Hypergeometric2F1[1/3, 7/6, 4/3, (1 - I*Tan[c + d*x])/2]*(e*Sec
[c + d*x])^(2/3)*(1 + I*Tan[c + d*x])^(1/6))/(2^(1/6)*d*Sqrt[a + I*a*Tan[c
 + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(e \sec (dx + c))^{\frac{2}{3}}}{\sqrt{a + ia \tan (dx + c)}} dx$$

input
```
int((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{(e \sec(c + dx))^{2/3}}{\sqrt{a + ia \tan(c + dx)}} dx = \int \frac{(e \sec (dx + c))^{\frac{2}{3}}}{\sqrt{i\, a \tan (dx + c) + a}} dx$$

input
```
integrate((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
-(3*2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1)
)^(2/3)*(-I*e^(4*I*d*x + 4*I*c) - 2*I*e^(2*I*d*x + 2*I*c) - I)*e^(2/3*I*d*
x + 2/3*I*c) - (a*d*e^(3*I*d*x + 3*I*c) - 2*a*d*e^(2*I*d*x + 2*I*c) + a*d*
e^(I*d*x + I*c))*integral(2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^
(2*I*d*x + 2*I*c) + 1))^(2/3)*(I*e^(4*I*d*x + 4*I*c) + 7*I*e^(3*I*d*x + 3*
I*c) + 5*I*e^(2*I*d*x + 2*I*c) + 7*I*e^(I*d*x + I*c) + 4*I)*e^(2/3*I*d*x +
 2/3*I*c)/(a*d*e^(4*I*d*x + 4*I*c) - 3*a*d*e^(3*I*d*x + 3*I*c) + 3*a*d*e^(
2*I*d*x + 2*I*c) - a*d*e^(I*d*x + I*c)), x))/(a*d*e^(3*I*d*x + 3*I*c) - 2*
a*d*e^(2*I*d*x + 2*I*c) + a*d*e^(I*d*x + I*c))
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(c+dx))^{\frac{2}{3}}}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(2/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((e*sec(c + d*x))**(2/3)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [F]

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{2}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((e*sec(d*x + c))^(2/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{2/3}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int((e/cos(c + d*x))^(2/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(2/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2e^{\frac{2}{3}}\sqrt{a}\,i\left(-3\sec\left(dx+c\right)^{\frac{2}{3}}\sqrt{\tan\left(dx+c\right)i+1}-4\left(\int\frac{\sec(dx+c)^{\frac{2}{3}}\sqrt{\tan(dx+c)i+1}}{\tan(dx+c)^{2}+1}\right.\right.}{3ad\left(\tan\left(dx\right.\right.}$$

input
```
int((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*e**(2/3)*sqrt(a)*i*( - 3*sec(c + d*x)**(2/3)*sqrt(tan(c + d*x)*i + 1) -
 4*int((sec(c + d*x)**(2/3)*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x))/(tan(c
+ d*x)**2 + 1),x)*tan(c + d*x)**2*d - 4*int((sec(c + d*x)**(2/3)*sqrt(tan(
c + d*x)*i + 1)*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*d))/(3*a*d*(tan(c +
 d*x)**2 + 1))
```

3.440 $\int \frac{\sqrt[3]{e \sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 84

$$\int \frac{\sqrt[3]{e \sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3ia \operatorname{Hypergeometric2F1}\left(\frac{1}{6},\frac{4}{3},\frac{7}{6},\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt[3]{e\sec(c+dx)}(1+i\tan(c+dx))^{4/3}}{\sqrt[3]{2}d(a+ia\tan(c+dx))^{3/2}}$$

output
```
3/2*I*a*hypergeom([1/6, 4/3],[7/6],1/2-1/2*I*tan(d*x+c))*(e*sec(d*x+c))^(1
/3)*(1+I*tan(d*x+c))^(4/3)*2^(2/3)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt[3]{e \sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3\left(8i - \frac{2ie^{2i(c+dx)} \operatorname{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{5}{3},-e^{2i(c+dx)}\right)}{\sqrt[6]{1+e^{2i(c+dx)}}}\right)\sqrt[3]{e\sec(c+dx)}}{16d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(e*Sec[c + d*x])^(1/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(3*(8*I - ((2*I)*E^((2*I)*(c + d*x))*Hypergeometric2F1[2/3, 5/6, 5/3, -E^(
(2*I)*(c + d*x))])/(1 + E^((2*I)*(c + d*x)))^(1/6))*(e*Sec[c + d*x])^(1/3)
)/(16*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sqrt[3]{e\sec(c+dx)}\int \frac{\sqrt[6]{a-ia\tan(c+dx)}}{\sqrt[3]{i\tan(c+dx)a+a}}dx}{\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{e\sec(c+dx)}\int \frac{\sqrt[6]{a-ia\tan(c+dx)}}{\sqrt[3]{i\tan(c+dx)a+a}}dx}{\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sqrt[3]{e\sec(c+dx)}\int \frac{1}{(a-ia\tan(c+dx))^{5/6}(i\tan(c+dx)a+a)^{4/3}}d\tan(c+dx)}{d\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a\sqrt[3]{1+i\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}\int\frac{2\sqrt[3]{2}}{(i\tan(c+dx)+1)^{4/3}(a-ia\tan(c+dx))^{5/6}}d\tan(c+dx)}{2\sqrt[3]{2}d\sqrt[6]{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a\sqrt[3]{1+i\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}\int\frac{1}{(i\tan(c+dx)+1)^{4/3}(a-ia\tan(c+dx))^{5/6}}d\tan(c+dx)}{d\sqrt[6]{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{3i\sqrt[3]{1+i\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{4}{3},\frac{7}{6},\frac{1}{2}(1-i\tan(c+dx))\right)}{\sqrt[3]{2}d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[(e*Sec[c + d*x])^(1/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((3*I)*Hypergeometric2F1[1/6, 4/3, 7/6, (1 - I*Tan[c + d*x])/2]*(e*Sec[c +
 d*x])^(1/3)*(1 + I*Tan[c + d*x])^(1/3))/(2^(1/3)*d*Sqrt[a + I*a*Tan[c + d
*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(e \sec (dx + c))^{\frac{1}{3}}}{\sqrt{a + ia \tan (dx + c)}} dx$$

input
```
int((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{e \sec(c + dx)}}{\sqrt{a + ia \tan(c + dx)}} dx = \int \frac{(e \sec (dx + c))^{\frac{1}{3}}}{\sqrt{i\, a \tan (dx + c) + a}} dx$$

input
```
integrate((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/4*(4*a*d*e^(I*d*x + I*c)*integral(-1/4*I*2^(5/6)*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(1/3*I*d*x + 1/3*I*c)/(a*
d), x) - 3*2^(5/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*
c) + 1))^(1/3)*(-I*e^(2*I*d*x + 2*I*c) - I)*e^(1/3*I*d*x + 1/3*I*c))*e^(-I
*d*x - I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sqrt[3]{e\sec{(c+dx)}}}{\sqrt{ia\,(\tan{(c+dx)}-i)}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(1/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((e*sec(c + d*x))**(1/3)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec{(dx+c)})^{\frac{1}{3}}}{\sqrt{i\,a\tan{(dx+c)}+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((e*sec(d*x + c))^(1/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{1/3}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int((e/cos(c + d*x))^(1/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(1/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{2e^{\frac{1}{3}}\sqrt{a}\,i\left(-3\sec\left(dx+c\right)^{\frac{1}{3}}\sqrt{\tan\left(dx+c\right)i+1}-5\left(\int\frac{\sec(dx+c)^{\frac{1}{3}}\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)}{\tan(dx+c)^{2}+1}dx\right)\tan\left(dx+c\right)^{2}d}{3ad\left(\tan\left(dx+c\right)^{2}+1\right)}$$

input
```
int((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*e**(1/3)*sqrt(a)*i*( - 3*sec(c + d*x)**(1/3)*sqrt(tan(c + d*x)*i + 1) -
 5*int((sec(c + d*x)**(1/3)*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x))/(tan(c
+ d*x)**2 + 1),x)*tan(c + d*x)**2*d - 5*int((sec(c + d*x)**(1/3)*sqrt(tan(
c + d*x)*i + 1)*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*d))/(3*a*d*(tan(c +
 d*x)**2 + 1))
```

3.441 $\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 84

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{3ia\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{5}{3},\frac{5}{6},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{5/3}}{2^{2/3}d\sqrt[3]{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}$$

output
```
-3/2*I*a*hypergeom([-1/6, 5/3],[5/6],1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c)
)^(5/3)*2^(1/3)/d/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.13

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{12i-\frac{30ie^{2i(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{4}{3},-e^{2i(c+dx)}\right)}{\left(1+e^{2i(c+dx)}\right)^{5/6}}}{16d\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(12*I - ((30*I)*E^((2*I)*(c + d*x))*Hypergeometric2F1[1/6, 1/3, 4/3, -E^((
2*I)*(c + d*x))])/(1 + E^((2*I)*(c + d*x)))^(5/6))/(16*d*(e*Sec[c + d*x])^
(1/3)*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}\int \frac{1}{\sqrt[6]{a-ia\tan(c+dx)}(i\tan(c+dx)a+a)^{2/3}}dx}{\sqrt[3]{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}\int \frac{1}{\sqrt[6]{a-ia\tan(c+dx)}(i\tan(c+dx)a+a)^{2/3}}dx}{\sqrt[3]{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}\int \frac{1}{(a-ia\tan(c+dx))^{7/6}(i\tan(c+dx)a+a)^{5/3}}d\tan(c+dx)}{d\sqrt[3]{e\sec(c+dx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a(1+i\tan(c+dx))^{2/3}\sqrt[6]{a-ia\tan(c+dx)}\int \frac{2\ 2^{2/3}}{(i\tan(c+dx)+1)^{5/3}(a-ia\tan(c+dx))^{7/6}}d\tan(c+dx)}{2\ 2^{2/3}d\sqrt{a+ia\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(1+i\tan(c+dx))^{2/3}\sqrt[6]{a-ia\tan(c+dx)}\int\frac{1}{(i\tan(c+dx)+1)^{5/3}(a-ia\tan(c+dx))^{7/6}}d\tan(c+dx)}{d\sqrt{a+ia\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}}$$

$$\Big\downarrow \; 79$$

$$-\frac{3i(1+i\tan(c+dx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{5}{3},\frac{5}{6},\frac{1}{2}(1-i\tan(c+dx))\right)}{2^{2/3}d\sqrt{a+ia\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}}$$

input
```
Int[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
((-3*I)*Hypergeometric2F1[-1/6, 5/3, 5/6, (1 - I*Tan[c + d*x])/2]*(1 + I*T
an[c + d*x])^(2/3))/(2^(2/3)*d*(e*Sec[c + d*x])^(1/3)*Sqrt[a + I*a*Tan[c +
 d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{1}{\left(e \sec \left(dx+c\right)\right)^{\frac{1}{3}} \sqrt{a+ia \tan \left(dx+c\right)}}dx$$

input
```
int(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{e \sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\left(e \sec \left(dx+c\right)\right)^{\frac{1}{3}} \sqrt{i\,a \tan \left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output
```
-1/8*(3*2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c)
+ 1))^(2/3)*(4*I*e^(6*I*d*x + 6*I*c) + 9*I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*
I*d*x + 2*I*c) + I)*e^(2/3*I*d*x + 2/3*I*c) - 8*(a*d*e*e^(4*I*d*x + 4*I*c)
 - a*d*e*e^(2*I*d*x + 2*I*c))*integral(-15/16*2^(1/6)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(3*I*e^(4*I*d*x + 4*I*c)
 + 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(2/3*I*d*x + 2/3*I*c)/(a*d*e*e^(6*I*d*x
+ 6*I*c) - 2*a*d*e*e^(4*I*d*x + 4*I*c) + a*d*e*e^(2*I*d*x + 2*I*c)), x))/(
a*d*e*e^(4*I*d*x + 4*I*c) - a*d*e*e^(2*I*d*x + 2*I*c))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{\sqrt[3]{e\sec{(c+dx)}}\sqrt{ia\left(\tan{(c+dx)}-i\right)}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(1/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/((e*sec(c + d*x))**(1/3)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(1/((e*sec(d*x + c))^(1/3)*sqrt(I*a*tan(d*x + c) + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{1/3}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(1/3)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
int(1/((e/cos(c + d*x))^(1/3)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\tan(dx+c)i+1}}{\sec(dx+c)^{\frac{1}{3}}\tan(dx+c)^2+\sec(dx+c)^{\frac{1}{3}}}dx - \left(\int \frac{\sqrt{\tan(dx+c)i+1}\tan(dx+c)}{\sec(dx+c)^{\frac{1}{3}}\tan(dx+c)^2+\sec(dx+c)^{\frac{1}{3}}}dx\right)i\right)}{e^{\frac{1}{3}}a}$$

input
```
int(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output

```
(sqrt(a)*(int(sqrt(tan(c + d*x)*i + 1)/(sec(c + d*x)**(1/3)*tan(c + d*x)**
2 + sec(c + d*x)**(1/3)),x) - int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x))/
(sec(c + d*x)**(1/3)*tan(c + d*x)**2 + sec(c + d*x)**(1/3)),x)*i))/(e**(1/
3)*a)
```

3.442 $\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 86

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{3ia\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{13}{6},\frac{1}{3},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{13/6}}{8\sqrt[6]{2}d(e\sec(c+dx))^{4/3}(a+ia\tan(c+dx))^{3/2}}$$

output
```
-3/16*I*a*hypergeom([-2/3, 13/6],[1/3],1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+
c))^(13/6)*2^(5/6)/d/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{3i\sec^2(c+dx)\left(3+3\cos(2(c+dx))-55\sqrt[6]{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{5}{6},-e^{2i(c+dx)}\right)+11\right.}{112d(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(4/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(((-3*I)/112)*Sec[c + d*x]^2*(3 + 3*Cos[2*(c + d*x)] - 55*(1 + E^((2*I)*(c
 + d*x)))^(1/6)*Hypergeometric2F1[-1/6, 1/6, 5/6, -E^((2*I)*(c + d*x))] +
(11*I)*Sin[2*(c + d*x)]))/(d*(e*Sec[c + d*x])^(4/3)*Sqrt[a + I*a*Tan[c + d
*x]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(a-ia\tan(c+dx))^{2/3}(a+ia\tan(c+dx))^{2/3}\int \frac{1}{(a-ia\tan(c+dx))^{2/3}(i\tan(c+dx)a+a)^{7/6}}dx}{(e\sec(c+dx))^{4/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(c+dx))^{2/3}(a+ia\tan(c+dx))^{2/3}\int \frac{1}{(a-ia\tan(c+dx))^{2/3}(i\tan(c+dx)a+a)^{7/6}}dx}{(e\sec(c+dx))^{4/3}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{2/3}(a+ia\tan(c+dx))^{2/3}\int \frac{1}{(a-ia\tan(c+dx))^{5/3}(i\tan(c+dx)a+a)^{13/6}}d\tan(c+dx)}{d(e\sec(c+dx))^{4/3}}$$

$$\Big\downarrow \text{80}$$

$$\frac{\sqrt[6]{1+i\tan(c+dx)}(a-ia\tan(c+dx))^{2/3}\sqrt{a+ia\tan(c+dx)}\int \frac{4\sqrt[6]{2}}{(i\tan(c+dx)+1)^{13/6}(a-ia\tan(c+dx))^{5/3}}d\tan(c+dx)}{4\sqrt[6]{2}d(e\sec(c+dx))^{4/3}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\sqrt[6]{1+i\tan(c+dx)}(a-ia\tan(c+dx))^{2/3}\sqrt{a+ia\tan(c+dx)}\int \frac{1}{(i\tan(c+dx)+1)^{13/6}(a-ia\tan(c+dx))^{5/3}}d\tan(c+dx)}{d(e\sec(c+dx))^{4/3}}$$

$$\Big\downarrow \textbf{79}$$

$$-\frac{3i\sqrt[6]{1+i\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{13}{6},\frac{1}{3},\frac{1}{2}(1-i\tan(c+dx))\right)}{8\sqrt[6]{2}ad(e\sec(c+dx))^{4/3}}$$

input
```
Int[1/((e*Sec[c + d*x])^(4/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(((-3*I)/8)*Hypergeometric2F1[-2/3, 13/6, 1/3, (1 - I*Tan[c + d*x])/2]*(1
+ I*Tan[c + d*x])^(1/6)*Sqrt[a + I*a*Tan[c + d*x]])/(2^(1/6)*a*d*(e*Sec[c
+ d*x])^(4/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{1}{\left(e \sec \left(dx+c\right)\right)^{\frac{4}{3}} \sqrt{a+ia \tan \left(dx+c\right)}}dx$$

input
```
int(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\left(e \sec \left(dx+c\right)\right)^{\frac{4}{3}} \sqrt{i\,a \tan \left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output
```
-1/112*(3*2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c
) + 1))^(2/3)*(7*I*e^(8*I*d*x + 8*I*c) - 14*I*e^(7*I*d*x + 7*I*c) - 38*I*e
^(6*I*d*x + 6*I*c) - 20*I*e^(5*I*d*x + 5*I*c) - 101*I*e^(4*I*d*x + 4*I*c)
+ 2*I*e^(3*I*d*x + 3*I*c) - 60*I*e^(2*I*d*x + 2*I*c) + 8*I*e^(I*d*x + I*c)
 - 4*I)*e^(2/3*I*d*x + 2/3*I*c) - 112*(a*d*e^2*e^(5*I*d*x + 5*I*c) - 2*a*d
*e^2*e^(4*I*d*x + 4*I*c) + a*d*e^2*e^(3*I*d*x + 3*I*c))*integral(-55/112*2
^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(2/
3)*(-I*e^(4*I*d*x + 4*I*c) - 7*I*e^(3*I*d*x + 3*I*c) - 5*I*e^(2*I*d*x + 2*
I*c) - 7*I*e^(I*d*x + I*c) - 4*I)*e^(2/3*I*d*x + 2/3*I*c)/(a*d*e^2*e^(4*I*
d*x + 4*I*c) - 3*a*d*e^2*e^(3*I*d*x + 3*I*c) + 3*a*d*e^2*e^(2*I*d*x + 2*I*
c) - a*d*e^2*e^(I*d*x + I*c)), x))/(a*d*e^2*e^(5*I*d*x + 5*I*c) - 2*a*d*e^
2*e^(4*I*d*x + 4*I*c) + a*d*e^2*e^(3*I*d*x + 3*I*c))
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\sec{(c+dx)})^{\frac{4}{3}}\sqrt{ia\left(\tan{(c+dx)}-i\right)}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(4/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/((e*sec(c + d*x))**(4/3)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Maxima [F]

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\sec{(dx+c)})^{\frac{4}{3}}\sqrt{i\,a\tan{(dx+c)}+a}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(1/((e*sec(d*x + c))^(4/3)*sqrt(I*a*tan(d*x + c) + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{4/3}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(4/3)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
int(1/((e/cos(c + d*x))^(4/3)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\tan(dx+c)i+1}}{\sec(dx+c)^{\frac{4}{3}}\tan(dx+c)^{2}+\sec(dx+c)^{\frac{4}{3}}}dx - \left(\int \frac{\sqrt{\tan(dx+c)i}}{\sec(dx+c)^{\frac{4}{3}}\tan(dx}\right.\right.}{e^{\frac{4}{3}}a}$$

input
```
int(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int(sqrt(tan(c + d*x)*i + 1)/(sec(c + d*x)**(1/3)*sec(c + d*x)*t
an(c + d*x)**2 + sec(c + d*x)**(1/3)*sec(c + d*x)),x) - int((sqrt(tan(c +
d*x)*i + 1)*tan(c + d*x))/(sec(c + d*x)**(1/3)*sec(c + d*x)*tan(c + d*x)**
2 + sec(c + d*x)**(1/3)*sec(c + d*x)),x)*i))/(e**(1/3)*a*e)
```

3.443 $\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 437

$$\begin{aligned}\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx &= \frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}} \\ &- \frac{5x(d\sec(e+fx))^{2/3}}{72\ 2^{2/3}a^{5/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} \\ &+ \frac{5i\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)(d\sec(e+fx))^{2/3}}{12\ 2^{2/3}\sqrt{3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} \\ &- \frac{5i\log(\cos(e+fx))(d\sec(e+fx))^{2/3}}{72\ 2^{2/3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} \\ &- \frac{5i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)(d\sec(e+fx))^{2/3}}{24\ 2^{2/3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} \\ &+ \frac{5i(d\sec(e+fx))^{2/3}}{24f\sqrt[3]{a+ia\tan(e+fx)}\left(a^2+ia^2\tan(e+fx)\right)}\end{aligned}$$

output
```
1/4*I*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(7/3)-5/144*x*(d*sec(f*x+e
))^(2/3)*2^(1/3)/a^(5/3)/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)
+5/72*I*arctan(1/3*(a^(1/3)+2^(2/3)*(a-I*a*tan(f*x+e))^(1/3))*3^(1/2)/a^(1
/3))*(d*sec(f*x+e))^(2/3)*2^(1/3)*3^(1/2)/a^(5/3)/f/(a-I*a*tan(f*x+e))^(1/
3)/(a+I*a*tan(f*x+e))^(1/3)-5/144*I*ln(cos(f*x+e))*(d*sec(f*x+e))^(2/3)*2^
(1/3)/a^(5/3)/f/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)-5/48*I*l
n(2^(1/3)*a^(1/3)-(a-I*a*tan(f*x+e))^(1/3))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a
^(5/3)/f/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)+5/24*I*(d*sec(f
*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(1/3)/(a^2+I*a^2*tan(f*x+e))
```

Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.55

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \frac{e^{-2i(e+fx)}\left(9i+33ie^{2i(e+fx)}+24ie^{4i(e+fx)}-10e^{4i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}fx-\right.}{}$$

input
```
Integrate[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(7/3),x]
```

output
```
((9*I + (33*I)*E^((2*I)*(e + f*x)) + (24*I)*E^((4*I)*(e + f*x)) - 10*E^((4
*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*f*x - (10*I)*Sqrt[3]*E^((4*
I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*ArcTan[(1 + 2*(1 + E^((2*I)*
(e + f*x)))^(1/3))/Sqrt[3]] - (15*I)*E^((4*I)*(e + f*x))*(1 + E^((2*I)*(e
+ f*x)))^(1/3)*Log[1 - (1 + E^((2*I)*(e + f*x)))^(1/3)])*Sec[e + f*x]^2*(d
*Sec[e + f*x])^(2/3))/(144*E^((2*I)*(e + f*x))*f*(a + I*a*Tan[e + f*x])^(7
/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.57, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3986, 3042, 4005, 3042, 3968, 52, 52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}dx$$
$$\Big\downarrow \text{3986}$$
$$\frac{(d\sec(e+fx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(e+fx)}}{(i\tan(e+fx)a+a)^2}dx}{\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{(d\sec(e+fx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(e+fx)}}{(i\tan(e+fx)a+a)^2}dx}{\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$
$$\Big\downarrow \text{4005}$$
$$\frac{(d\sec(e+fx))^{2/3}\int \cos^4(e+fx)(a-ia\tan(e+fx))^{7/3}dx}{a^4\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{(d\sec(e+fx))^{2/3}\int \frac{(a-ia\tan(e+fx))^{7/3}}{\sec(e+fx)^4}dx}{a^4\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$
$$\Big\downarrow \text{3968}$$
$$\frac{ia(d\sec(e+fx))^{2/3}\int \frac{1}{(a-ia\tan(e+fx))^{2/3}(i\tan(e+fx)a+a)^3}d(-ia\tan(e+fx))}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$
$$\Big\downarrow \text{52}$$
$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{5\int \frac{1}{(a-ia\tan(e+fx))^{2/3}(i\tan(e+fx)a+a)^2}d(-ia\tan(e+fx))}{12a}+\frac{\sqrt[3]{a-ia\tan(e+fx)}}{4a(a+ia\tan(e+fx))^2}\right)}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$
$$\Big\downarrow \text{52}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{5\left(\frac{\int \frac{1}{(a-ia\tan(e+fx))^{2/3}(i\tan(e+fx)a+a)}d(-ia\tan(e+fx))}{3a}+\frac{\sqrt[3]{a-ia\tan(e+fx)}}{2a(a+ia\tan(e+fx))}\right)}{12a}+\frac{\sqrt[3]{a-ia\tan(e+}}{4a(a+ia\tan(e+fx}\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$\big\downarrow$ 69

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{5\left(\frac{\frac{3\int \frac{1}{ia\tan(e+fx)+\sqrt[3]{2}\sqrt[3]{a}}d\sqrt[3]{a-ia\tan(e+fx)}}{2\ 2^{2/3}a^{2/3}}+\frac{3\int \frac{1}{-a^2\tan^2(e+fx)-i\sqrt[3]{2}a^{4/3}\tan(e+fx)+2^{2/3}a^{2/3}}d\sqrt[3]{}}{2\sqrt[3]{2}\sqrt[3]{a}}}{3a}\right.}{12a}\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan}}$$

$\big\downarrow$ 16

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{5\left(\frac{\frac{3\int \frac{1}{-a^2\tan^2(e+fx)-i\sqrt[3]{2}a^{4/3}\tan(e+fx)+2^{2/3}a^{2/3}}d\sqrt[3]{a-ia\tan(e+fx)}}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}+ia\tan(e+fx)\right)}{2\ 2^{2/3}a^{2/3}}+}{3a}\right.}{12a}\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$\big\downarrow$ 1082

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{5\left(\frac{-\frac{3\int\frac{1}{a^2\tan^2(e+fx)-3}d\left(1-i2^{2/3}a^{2/3}\tan(e+fx)\right)}{2^{2/3}a^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}+ia\tan(e+fx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a+ia\tan(e+fx))}{2\ 2^{2/3}a^{2/3}}}{3a}+\frac{\sqrt[3]{a-i}}{2a(a+}\right.}{12a}\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$\Big\downarrow$ 217

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{5\left(\frac{-\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(e+fx)}{\sqrt{3}}\right)}{2^{2/3}a^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}+ia\tan(e+fx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a+ia\tan(e+fx))}{2\ 2^{2/3}a^{2/3}}}{3a}+\frac{\sqrt[3]{a-ia\tan(e+fx)}}{2a(a+ia\tan(e+fx))}\right.}{12a}\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(7/3),x]
```

output
```
(I*a*(d*Sec[e + f*x])^(2/3)*((a - I*a*Tan[e + f*x])^(1/3)/(4*a*(a + I*a*Ta
n[e + f*x])^2) + (5*((((-I)*Sqrt[3]*ArcTanh[(a*Tan[e + f*x])/Sqrt[3]])/(2^
(2/3)*a^(2/3)) - (3*Log[2^(1/3)*a^(1/3) + I*a*Tan[e + f*x]])/(2*2^(2/3)*a^
(2/3)) + Log[a + I*a*Tan[e + f*x]]/(2*2^(2/3)*a^(2/3)))/(3*a) + (a - I*a*T
an[e + f*x])^(1/3)/(2*a*(a + I*a*Tan[e + f*x]))))/(12*a)))/(f*(a - I*a*Tan
[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4005
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*c^m   Int[Sec[e + f*x]^(2*m)*(c +
 d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[
m, 0] || GtQ[m, n]))
```

Maple [F]

$$\int \frac{(d \sec (fx+e))^{\frac{2}{3}}}{(a+ia \tan (fx+e))^{\frac{7}{3}}}dx$$

input
```
int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x)
```

output
```
int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.21

$$\int \frac{(d \sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x, algorithm="fric
as")
```

output
```
1/48*(48*a^3*f*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(6*I*f*x + 6*I*e)*log(
-2/5*(72*I*a^3*f*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(2*I*f*x + 2*I*e) -
5*2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1)
)^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e
)) + 2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) +
 1))^(2/3)*(8*I*e^(6*I*f*x + 6*I*e) + 19*I*e^(4*I*f*x + 4*I*e) + 14*I*e^(2
*I*f*x + 2*I*e) + 3*I)*e^(2*I*f*x + 2*I*e) - 24*(-I*sqrt(3)*a^3*f + a^3*f)
*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(6*I*f*x + 6*I*e)*log(2/5*(5*2^(1/3)
*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(
e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) + 36*(sqrt(3)*a^3*f + I*a^3*f
)*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*
I*e)) - 24*(I*sqrt(3)*a^3*f + a^3*f)*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^
(6*I*f*x + 6*I*e)*log(2/5*(5*2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(
d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x +
2*I*e) - 36*(sqrt(3)*a^3*f - I*a^3*f)*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e
^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)))*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(2/3)/(a+I*a*tan(f*x+e))**(7/3),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3902 vs. $2(324) = 648$.

Time = 0.43 (sec) , antiderivative size = 3902, normalized size of antiderivative = 8.93

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x, algorithm="maxi
ma")
```

output
```
1/288*(48*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/
2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4
*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(5/6)*((I*2^(1/3)*cos(4*f*x + 4*e) +
2^(1/3)*sin(4*f*x + 4*e))*cos(5/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e)
, cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))
 + 1)) - (2^(1/3)*cos(4*f*x + 4*e) - I*2^(1/3)*sin(4*f*x + 4*e))*sin(5/3*a
rctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arcta
n2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)))*d^(2/3) + 30*(cos(1/2*arcta
n2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*
e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x +
 4*e))) + 1)^(1/3)*((-I*2^(1/3)*cos(4*f*x + 4*e) - 2^(1/3)*sin(4*f*x + 4*e
))*cos(2/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), c
os(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + (2^(1/3)*cos(4
*f*x + 4*e) - I*2^(1/3)*sin(4*f*x + 4*e))*sin(2/3*arctan2(sin(1/2*arctan2(
sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), co
s(4*f*x + 4*e))) + 1)))*d^(2/3) + 5*(-2*I*sqrt(3)*2^(1/3)*arctan2(2/3*sqrt
(3)*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arct
an2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x +
 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*cos(1/3*arctan2(sin(1/2*arctan2(sin(4
*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(...
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{2}{3}}}{(i\,a\tan(fx+e)+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x, algorithm="giac
")
```

output
```
integrate((d*sec(f*x + e))^(2/3)/(I*a*tan(f*x + e) + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}{\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{7/3}}\,dx$$

input
```
int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(7/3),x)
```

output
```
int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(7/3), x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = -\frac{d^{\frac{2}{3}}\left(\int \frac{\sec(fx+e)^{\frac{2}{3}}}{(\tan(fx+e)i+1)^{\frac{1}{3}}\tan(fx+e)^{2}-2(\tan(fx+e)i+1)^{\frac{1}{3}}\tan(fx+e)i-(\tan(fx+e)i+1)^{\frac{1}{3}}}dx\right)}{a^{\frac{7}{3}}}$$

input
```
int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x)
```

output
```
( - d**(2/3)*int(sec(e + f*x)**(2/3)/((tan(e + f*x)*i + 1)**(1/3)*tan(e +
f*x)**2 - 2*(tan(e + f*x)*i + 1)**(1/3)*tan(e + f*x)*i - (tan(e + f*x)*i +
 1)**(1/3)),x))/(a**(1/3)*a**2)
```

3.444 $\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 378

$$\begin{aligned}\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx &= \frac{i(d\sec(e+fx))^{2/3}}{2f(a+ia\tan(e+fx))^{4/3}}\\ &\quad - \frac{x(d\sec(e+fx))^{2/3}}{6\ 2^{2/3}a^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\\ &\quad + \frac{i\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)(d\sec(e+fx))^{2/3}}{2^{2/3}\sqrt{3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\\ &\quad - \frac{i\log(\cos(e+fx))(d\sec(e+fx))^{2/3}}{6\ 2^{2/3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\\ &\quad - \frac{i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)(d\sec(e+fx))^{2/3}}{2\ 2^{2/3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\end{aligned}$$

output
```
1/2*I*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(4/3)-1/12*x*(d*sec(f*x+e)
)^(2/3)*2^(1/3)/a^(2/3)/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)+
1/6*I*arctan(1/3*(a^(1/3)+2^(2/3)*(a-I*a*tan(f*x+e))^(1/3))*3^(1/2)/a^(1/3
))*(d*sec(f*x+e))^(2/3)*2^(1/3)*3^(1/2)/a^(2/3)/f/(a-I*a*tan(f*x+e))^(1/3)
/(a+I*a*tan(f*x+e))^(1/3)-1/12*I*ln(cos(f*x+e))*(d*sec(f*x+e))^(2/3)*2^(1/
3)/a^(2/3)/f/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)-1/4*I*ln(2^
(1/3)*a^(1/3)-(a-I*a*tan(f*x+e))^(1/3))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(2/
3)/f/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)
```

Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.58

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx = \frac{e^{-i(e+fx)}\left(3i+3ie^{2i(e+fx)}-2e^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}fx-2i\sqrt{3}e^{2i(e+fx)}\sqrt[3]{\cdots}\right.}{\cdots}$$

input
```
Integrate[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(4/3),x]
```

output
```
((3*I + (3*I)*E^((2*I)*(e + f*x)) - 2*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e
 + f*x)))^(1/3)*f*x - (2*I)*Sqrt[3]*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e +
 f*x)))^(1/3)*ArcTan[(1 + 2*(1 + E^((2*I)*(e + f*x)))^(1/3))/Sqrt[3]] - (3
*I)*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*Log[1 - (1 + E^((2
*I)*(e + f*x)))^(1/3)])*(d*Sec[e + f*x])^(5/3))/(12*d*E^(I*(e + f*x))*f*(a
 + I*a*Tan[e + f*x])^(4/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.53, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {3042, 3986, 3042, 4005, 3042, 3968, 52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(d\sec(e+fx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(e+fx)}}{i\tan(e+fx)a+a}dx}{\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(d\sec(e+fx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(e+fx)}}{i\tan(e+fx)a+a}dx}{\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{4005}$$

$$\frac{(d\sec(e+fx))^{2/3}\int \cos^2(e+fx)(a-ia\tan(e+fx))^{4/3}dx}{a^2\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(d\sec(e+fx))^{2/3}\int \frac{(a-ia\tan(e+fx))^{4/3}}{\sec(e+fx)^2}dx}{a^2\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{3968}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\int \frac{1}{(a-ia\tan(e+fx))^{2/3}(i\tan(e+fx)a+a)^2}d(-ia\tan(e+fx))}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{52}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{\int \frac{1}{(a-ia\tan(e+fx))^{2/3}(i\tan(e+fx)a+a)}d(-ia\tan(e+fx))}{3a}+\frac{\sqrt[3]{a-ia\tan(e+fx)}}{2a(a+ia\tan(e+fx))}\right)}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{69}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{\frac{3\int \frac{1}{ia\tan(e+fx)+\sqrt[3]{2}\sqrt[3]{a}}d\sqrt[3]{a-ia\tan(e+fx)}}{2\ 2^{2/3}a^{2/3}}+\frac{3\int \frac{1}{-a^2\tan^2(e+fx)-i\sqrt[3]{2}a^{4/3}\tan(e+fx)+2^{2/3}a^{2/3}}d\sqrt[3]{a-}}{2\sqrt[3]{2}\sqrt[3]{a}}}{3a}\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \textbf{16}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{\frac{3\int \frac{1}{-a^2\tan^2(e+fx)-i\sqrt[3]{2}a^{4/3}\tan(e+fx)+2^{2/3}a^{2/3}}d\sqrt[3]{a-ia\tan(e+fx)}}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}+ia\tan(e+fx)\right)}{2\ 2^{2/3}a^{2/3}}+\log}{3a}\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \textbf{1082}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{-\frac{3\int \frac{1}{a^2\tan^2(e+fx)-3}d\left(1-i2^{2/3}a^{2/3}\tan(e+fx)\right)}{2^{2/3}a^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}+ia\tan(e+fx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a+ia\tan(e+fx))}{2\ 2^{2/3}a^{2/3}}}{3a}+\frac{\sqrt[3]{a-ia}}{2a(a+i}\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \textbf{217}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{-\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(e+fx)}{\sqrt{3}}\right)}{2^{2/3}a^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}+ia\tan(e+fx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a+ia\tan(e+fx))}{2\ 2^{2/3}a^{2/3}}}{3a}+\frac{\sqrt[3]{a-ia\tan(e+fx)}}{2a(a+ia\tan(e+fx))}\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

input

```
Int[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(4/3),x]
```

output

```
(I*a*(d*Sec[e + f*x])^(2/3)*((((-I)*Sqrt[3]*ArcTanh[(a*Tan[e + f*x])/Sqrt[
3]])/(2^(2/3)*a^(2/3)) - (3*Log[2^(1/3)*a^(1/3) + I*a*Tan[e + f*x]])/(2*2^
(2/3)*a^(2/3)) + Log[a + I*a*Tan[e + f*x]]/(2*2^(2/3)*a^(2/3)))/(3*a) + (a
 - I*a*Tan[e + f*x])^(1/3)/(2*a*(a + I*a*Tan[e + f*x]))))/(f*(a - I*a*Tan[
e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4005
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*c^m   Int[Sec[e + f*x]^(2*m)*(c +
 d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[
m, 0] || GtQ[m, n]))
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{2}{3}}}{(a+ia\tan(fx+e))^{\frac{4}{3}}}dx$$

input
```
int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x)
```

output
```
int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.36

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x, algorithm="fric
as")
```

output
```
1/4*(4*a^2*f*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(4*I*f*x + 4*I*e)*log(-2*(6*I
*a^2*f*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(2*I*f*x + 2*I*e) - 2^(1/3)*(a/(e^(
2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f
*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) + 2^(1/3)*(a/(
e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(I*e^(
4*I*f*x + 4*I*e) + 2*I*e^(2*I*f*x + 2*I*e) + I)*e^(2*I*f*x + 2*I*e) - 2*(-
I*sqrt(3)*a^2*f + a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(4*I*f*x + 4*I*e)
*log(2*(2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e
) + 1))^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) + 3*(sqrt(3)*a
^2*f + I*a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(2*I*f*x + 2*I*e))*e^(-2*I
*f*x - 2*I*e)) - 2*(I*sqrt(3)*a^2*f + a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)
*e^(4*I*f*x + 4*I*e)*log(2*(2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d
/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2
*I*e) - 3*(sqrt(3)*a^2*f - I*a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(2*I*f
*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx = \int \frac{(d\sec{(e+fx)})^{\frac{2}{3}}}{(ia(\tan{(e+fx)}-i))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(2/3)/(a+I*a*tan(f*x+e))**(4/3),x)
```

output
```
Integral((d*sec(e + f*x))**(2/3)/(I*a*(tan(e + f*x) - I))**(4/3), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1906 vs. $2(279) = 558$.

Time = 0.31 (sec) , antiderivative size = 1906, normalized size of antiderivative = 5.04

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x, algorithm="maxi
ma")
```

output
```
-1/24*(6*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/3)*((-I*2^(1/3)*cos(2*f*x + 2*e) - 2^(1/3)*sin(2*f*x + 2*e))*cos(2/3*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + (2^(1/3)*cos(2*f*x + 2*
e) - I*2^(1/3)*sin(2*f*x + 2*e))*sin(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1)))*d^(2/3) - (-2*I*sqrt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(
2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/
3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1/3*sqrt(3), 1/3*sqrt
(3)*(2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^
(1/6)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sqrt(3)))
 - 2*I*sqrt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)) + 1/3*sqrt(3), -1/3*sqrt(3)*(2*(cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - sqrt(3))) + sqrt(3)*2^(1/3)*log(4/
3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)
*(cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + sin(1/3*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2) + 4/3*(cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*(sqrt(3)*sin(1/3*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + cos(1/3*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e) + 1))) + 4/3) - sqrt(3)*2^(1/3)*log(4/3*(cos(2...
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{2}{3}}}{(i\,a\tan(fx+e)+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x, algorithm="giac
")
```

output
```
integrate((d*sec(f*x + e))^(2/3)/(I*a*tan(f*x + e) + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}{\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{4/3}}\,dx$$

input
```
int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(4/3),x)
```

output
```
int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(4/3), x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx=\frac{d^{\frac{2}{3}}\left(\int \frac{\sec(fx+e)^{\frac{2}{3}}}{(\tan(fx+e)i+1)^{\frac{1}{3}}\tan(fx+e)i+(\tan(fx+e)i+1)^{\frac{1}{3}}}dx\right)}{a^{\frac{4}{3}}}$$

input
```
int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x)
```

output
```
(d**(2/3)*int(sec(e + f*x)**(2/3)/((tan(e + f*x)*i + 1)**(1/3)*tan(e + f*x
)*i + (tan(e + f*x)*i + 1)**(1/3)),x))/(a**(1/3)*a)
```

3.445 $\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 340

$$\begin{aligned}\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx =& -\frac{\sqrt[3]{a}x(d\sec(e+fx))^{2/3}}{2\,2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} \\ &+ \frac{i\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)(d\sec(e+fx))^{2/3}}{2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} \\ &- \frac{i\sqrt[3]{a}\log(\cos(e+fx))(d\sec(e+fx))^{2/3}}{2\,2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} \\ &- \frac{3i\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)(d\sec(e+fx))^{2/3}}{2\,2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\end{aligned}$$

output
```
-1/4*a^(1/3)*x*(d*sec(f*x+e))^(2/3)*2^(1/3)/(a-I*a*tan(f*x+e))^(1/3)/(a+I*
a*tan(f*x+e))^(1/3)+1/2*I*3^(1/2)*a^(1/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(a-I
*a*tan(f*x+e))^(1/3))*3^(1/2)/a^(1/3))*(d*sec(f*x+e))^(2/3)*2^(1/3)/f/(a-I
*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)-1/4*I*a^(1/3)*ln(cos(f*x+e))
*(d*sec(f*x+e))^(2/3)*2^(1/3)/f/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)
)^(1/3)-3/4*I*a^(1/3)*ln(2^(1/3)*a^(1/3)-(a-I*a*tan(f*x+e))^(1/3))*(d*sec(
f*x+e))^(2/3)*2^(1/3)/f/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.47

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx =$$

$$-\frac{\left(\frac{de^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{2/3}\sqrt[3]{1+e^{2i(e+fx)}}\left(2fx+2i\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1+e^{2i(e+fx)}}}{\sqrt{3}}\right)+3i\log\left(1-\sqrt[3]{1+e^{2i(e+fx)}}\right)\right)}{2\,2^{2/3}\sqrt[3]{\frac{ae^{2i(e+fx)}}{1+e^{2i(e+fx)}}}f}$$

input
```
Integrate[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(1/3),x]
```

output
```
-1/2*(((d*E^(I*(e + f*x)))/(1 + E^((2*I)*(e + f*x))))^(2/3)*(1 + E^((2*I)*
(e + f*x)))^(1/3)*(2*f*x + (2*I)*Sqrt[3]*ArcTan[(1 + 2*(1 + E^((2*I)*(e +
f*x)))^(1/3))/Sqrt[3]] + (3*I)*Log[1 - (1 + E^((2*I)*(e + f*x)))^(1/3)]))/
(2^(2/3)*((a*E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x))))^(1/3)*f)
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.45, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3973, 3042, 3962, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}dx$$

$$\Big\downarrow \; 3973$$

$$\frac{(d\sec(e+fx))^{2/3}\int\sqrt[3]{a-ia\tan(e+fx)}dx}{\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(d\sec(e+fx))^{2/3}\int\sqrt[3]{a-ia\tan(e+fx)}dx}{\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3962}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\int\frac{1}{(a-ia\tan(e+fx))^{2/3}(i\tan(e+fx)a+a)}d(-ia\tan(e+fx))}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{ 69}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{3\int\frac{1}{ia\tan(e+fx)+\sqrt[3]{2}\sqrt[3]{a}}d\sqrt[3]{a-ia\tan(e+fx)}}{2\ 2^{2/3}a^{2/3}}+\frac{3\int\frac{1}{-a^2\tan^2(e+fx)-i\sqrt[3]{2}a^{4/3}\tan(e+fx)+2^{2/3}a^{2/3}}d\sqrt[3]{a}\cdots}{2\sqrt[3]{2}\sqrt[3]{a}}\cdots\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{3\int\frac{1}{-a^2\tan^2(e+fx)-i\sqrt[3]{2}a^{4/3}\tan(e+fx)+2^{2/3}a^{2/3}}d\sqrt[3]{a-ia\tan(e+fx)}}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}+ia\tan(e+fx)\right)}{2\ 2^{2/3}a^{2/3}}\cdots\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(-\frac{3\int\frac{1}{a^2\tan^2(e+fx)-3}d\left(1-i2^{2/3}a^{2/3}\tan(e+fx)\right)}{2^{2/3}a^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}+ia\tan(e+fx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a+ia\tan(e+fx))}{2\ 2^{2/3}a^{2/3}}\right)}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(e+fx)}{\sqrt{3}}\right)}{2^{2/3}a^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}+ia\tan(e+fx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a+ia\tan(e+fx))}{2\ 2^{2/3}a^{2/3}}\right)}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(1/3),x]
```

output
```
(I*a*(((-I)*Sqrt[3]*ArcTanh[(a*Tan[e + f*x])/Sqrt[3]])/(2^(2/3)*a^(2/3)) -
 (3*Log[2^(1/3)*a^(1/3) + I*a*Tan[e + f*x]])/(2*2^(2/3)*a^(2/3)) + Log[a +
 I*a*Tan[e + f*x]]/(2*2^(2/3)*a^(2/3)))*(d*Sec[e + f*x])^(2/3))/(f*(a - I*
a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 3973
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(a/d)^(2*IntPart[n])*(a + b*Tan[e + f*x])^Frac
Part[n]*((a - b*Tan[e + f*x])^FracPart[n]/(d*Sec[e + f*x])^(2*FracPart[n]))
   Int[1/(a - b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
&& EqQ[a^2 + b^2, 0] && EqQ[Simplify[m/2 + n], 0]
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{2}{3}}}{(a+ia\tan(fx+e))^{\frac{1}{3}}}dx$$

input
```
int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x)
```

output
```
int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.08

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = \frac{1}{2}\left(i\sqrt{3}-1\right)\left(\frac{i\,d^2}{4\,af^3}\right)^{\frac{1}{3}}\log\left(2\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\right.\right.$$
$$+\frac{1}{2}\left(-i\sqrt{3}-1\right)\left(\frac{i\,d^2}{4\,af^3}\right)^{\frac{1}{3}}\log\left(2\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(e^{(2i\,fx+2i\,e)}+1\right)e^{(2i\,fx+2i\,e)}\right.\right.$$
$$+\left(\frac{i\,d^2}{4\,af^3}\right)^{\frac{1}{3}}\log\left(2\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(e^{(2i\,fx+2i\,e)}+1\right)e^{(2i\,fx+2i\,e)}-2i\,af\left(\frac{i\,d^2}{4\,af^3}\right.\right.\right.$$

input
```
integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x, algorithm="fric
as")
```

output
```
1/2*(I*sqrt(3) - 1)*(1/4*I*d^2/(a*f^3))^(1/3)*log(2*(2^(1/3)*(a/(e^(2*I*f*
x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2
*I*e) + 1)*e^(2*I*f*x + 2*I*e) + (sqrt(3)*a*f + I*a*f)*(1/4*I*d^2/(a*f^3))
^(1/3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) + 1/2*(-I*sqrt(3) - 1)*(
1/4*I*d^2/(a*f^3))^(1/3)*log(2*(2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3
)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x
 + 2*I*e) - (sqrt(3)*a*f - I*a*f)*(1/4*I*d^2/(a*f^3))^(1/3)*e^(2*I*f*x + 2
*I*e))*e^(-2*I*f*x - 2*I*e)) + (1/4*I*d^2/(a*f^3))^(1/3)*log(2*(2^(1/3)*(a
/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(
2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) - 2*I*a*f*(1/4*I*d^2/(a*f^3))^(1
/3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e))
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = \int \frac{(d\sec{(e+fx)})^{\frac{2}{3}}}{\sqrt[3]{ia\,(\tan{(e+fx)}-i)}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(2/3)/(a+I*a*tan(f*x+e))**(1/3),x)
```

output
```
Integral((d*sec(e + f*x))**(2/3)/(I*a*(tan(e + f*x) - I))**(1/3), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1753 vs. $2(251) = 502$.

Time $= 0.32$ (sec) , antiderivative size $= 1753$, normalized size of antiderivative $= 5.16$

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x, algorithm="maxi
ma")
```

output
```
1/8*(-2*I*sqrt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1)) + 1/3*sqrt(3), 1/3*sqrt(3)*(2*(cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sqrt(3))) - 2*I*sqrt(3)*2^(1/3)*
arctan2(2/3*sqrt(3)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
) + 1/3*sqrt(3), -1/3*sqrt(3)*(2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1)) - sqrt(3))) + sqrt(3)*2^(1/3)*log(4/3*(cos(2*f*x + 2*e)^2 +
 sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1))^2) + 4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)^(1/6)*(sqrt(3)*sin(1/3*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1)) + cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1))) + 4/3) - sqrt(3)*2^(1/3)*log(4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1))^2) - 4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)^(1/6)*(sqrt(3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*...
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{2}{3}}}{(i\,a\tan(fx+e)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x, algorithm="giac
")
```

output
```
integrate((d*sec(f*x + e))^(2/3)/(I*a*tan(f*x + e) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}{(a+a\tan\left(e+f\,x\right)\,1\mathrm{i})^{1/3}}\,dx$$

input
```
int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(1/3),x)
```

output
```
int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(1/3), x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = \frac{d^{\frac{2}{3}}\left(\int \frac{\sec(fx+e)^{\frac{2}{3}}}{(\tan(fx+e)i+1)^{\frac{1}{3}}}dx\right)}{a^{\frac{1}{3}}}$$

input
```
int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x)
```

output
```
(d**(2/3)*int(sec(e + f*x)**(2/3)/(tan(e + f*x)*i + 1)**(1/3),x))/a**(1/3)
```

3.446 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 37

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx=\frac{3ia(d\sec(e+fx))^{2/3}}{f\sqrt[3]{a+ia\tan(e+fx)}}$$

output
```
3*I*a*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(1/3)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx=\frac{3d^2(i+\tan(e+fx))(a+ia\tan(e+fx))^{2/3}}{f(d\sec(e+fx))^{4/3}}$$

input
```
Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3),x]
```

output
```
(3*d^2*(I + Tan[e + f*x])*(a + I*a*Tan[e + f*x])^(2/3))/(f*(d*Sec[e + f*x]
)^(4/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}dx$$

$$\Big\downarrow \text{ 3974}$$

$$\frac{3ia(d\sec(e+fx))^{2/3}}{f\sqrt[3]{a+ia\tan(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3),x]
```

output
```
((3*I)*a*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{2}{3}} (a+ia\tan(fx+e))^{\frac{2}{3}} dx$$

input
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x)
```

output
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.49

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3} dx = -\frac{3\cdot 2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(-i\,e^{(2i\,fx+2i\,e)}-i\right)}{f}$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x, algorithm="fric
as")
```

output
```
-3*2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1
))^(2/3)*(-I*e^(2*I*f*x + 2*I*e) - I)/f
```

Sympy [F]

$$\int (d\sec(e+fx))^{2/3}(a + ia\tan(e+fx))^{2/3} dx = \int (d\sec(e+fx))^{\frac{2}{3}} (ia(\tan(e+fx)-i))^{\frac{2}{3}} dx$$

input
```
integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(2/3),x)
```

output
```
Integral((d*sec(e + f*x))**(2/3)*(I*a*(tan(e + f*x) - I))**(2/3), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(29) = 58$.

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.89

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx =$$
$$-\frac{3\left(-i\cdot 2^{\frac{1}{3}}\cos\left(\frac{1}{3}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)\right)-2^{\frac{1}{3}}\sin\left(\frac{1}{3}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(\right.\right.\right.\right.}{\left(\cos\left(2\,fx+2\,e\right)^{2}+\sin\left(2\,fx+2\,e\right)^{2}+2\,\cos\left(2\,fx+2\,e\right)+1\right)^{\frac{1}{6}}f}$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x, algorithm="maxi
ma")
```

output
```
-3*(-I*2^(1/3)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) -
2^(1/3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*a^(2/3)*
d^(2/3)/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/6)*f)
```

Giac [F]

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx = \int \left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}\left(i\,a\tan\left(fx+e\right)+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x, algorithm="giac
")
```

output
```
integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.19

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx=\frac{3\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}\left(\cos\left(2\,e+2\,f\,x\right)\,1\mathrm{i}+\sin\left(2\,e+2\,f\,x\right)+1\mathrm{i}\right)\left(\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e}{\cos(2\,e+2\,f\,x)+1}\right.}{2\,f}$$

input
```
int((d/cos(e + f*x))^(2/3)*(a + a*tan(e + f*x)*1i)^(2/3),x)
```

output
```
(3*(d/cos(e + f*x))^(2/3)*(cos(2*e + 2*f*x)*1i + sin(2*e + 2*f*x) + 1i)*((
a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(2
/3))/(2*f)
```

Reduce [F]

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx=d^{\frac{2}{3}}a^{\frac{2}{3}}\left(\int \sec\left(fx+e\right)^{\frac{2}{3}}\left(\tan\left(fx+e\right)i+1\right)^{\frac{2}{3}}dx\right)$$

input
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x)
```

output
```
d**(2/3)*a**(2/3)*int(sec(e + f*x)**(2/3)*(tan(e + f*x)*i + 1)**(2/3),x)
```

3.447 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 81

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = \frac{9ia^2(d\sec(e+fx))^{2/3}}{2f\sqrt[3]{a+ia\tan(e+fx)}} + \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{4f}$$

output
```
9/2*I*a^2*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(1/3)+3/4*I*a*(d*sec(f
*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3)/f
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = -\frac{3ad(\cos(e)-i\sin(e))(\cos(fx)-i\sin(fx))(-7i+\tan(e+fx))(a+ia\tan(e+fx))^{2/3}}{4f\sqrt[3]{d\sec(e+fx)}}$$

input
```
Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3),x]
```

output
```
(-3*a*d*(Cos[e] - I*Sin[e])*(Cos[f*x] - I*Sin[f*x])*(-7*I + Tan[e + f*x])*
(a + I*a*Tan[e + f*x])^(2/3))/(4*f*(d*Sec[e + f*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{3}{2}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{2/3}dx+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{2/3}dx+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}$$

$$\Big\downarrow \text{3974}$$

$$\frac{9ia^2(d\sec(e+fx))^{2/3}}{2f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}$$

input
```
Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3),x]
```

output
```
(((9*I)/2)*a^2*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3)) +
(((3*I)/4)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3))/f
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [F]

$$\int (d\sec (fx+e))^{\frac{2}{3}} (a+ia\tan (fx+e))^{\frac{5}{3}} dx$$

input
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x)
```

output
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = -\frac{3\cdot 2^{\frac{1}{3}}\left(-4i\,ae^{(2i\,fx+2i\,e)}-3i\,a\right)\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}}{2\,f}$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x, algorithm="fric
as")
```

output
```
-3/2*2^(1/3)*(-4*I*a*e^(2*I*f*x + 2*I*e) - 3*I*a)*(a/(e^(2*I*f*x + 2*I*e)
+ 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)/f
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(5/3),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(61) = 122.

Time = 0.21 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.90

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = \frac{3\left(\left(-i\cdot 2^{\frac{1}{3}}a\cos\left(\frac{4}{3}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)\right)-2^{\frac{1}{3}}a\sin\left(\frac{4}{3}\,\mathrm{ar}\right.\right.\right.}{}$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x, algorithm="maxi
ma")
```

output
```
3/2*((-I*2^(1/3)*a*cos(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
) - 2^(1/3)*a*sin(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sq
rt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*a^(2/
3)*d^(2/3) + 4*((I*2^(1/3)*a*cos(2*f*x + 2*e)^2 + I*2^(1/3)*a*sin(2*f*x +
2*e)^2 + 2*I*2^(1/3)*a*cos(2*f*x + 2*e) + I*2^(1/3)*a)*cos(1/3*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + (2^(1/3)*a*cos(2*f*x + 2*e)^2 + 2^
(1/3)*a*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a*cos(2*f*x + 2*e) + 2^(1/3)*a)*sin
(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*a^(2/3)*d^(2/3))/((
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(7/6)*f)
```

Giac [F]

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = \int (d\sec(fx+e))^{\frac{2}{3}}(i\,a\tan(fx+e)+a)^{\frac{5}{3}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x, algorithm="giac
")
```

output
```
integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(5/3), x)
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = \frac{3\,a\left(\frac{d}{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1}\right)^{2/3}\left(\cos\left(e+f\,x\right)^2\,6{}\mathrm{i}+3\,\sin\left(2\,e+2\,f\,x\right)+1{}\mathrm{i}\right)\,\left(\frac{a\,\left(2\cos(e+f\,x)^2\right.}{2\cos}\right.}{4\,f}$$

input
```
int((d/cos(e + f*x))^(2/3)*(a + a*tan(e + f*x)*1i)^(5/3),x)
```

output
```
(3*a*(d/(2*cos(e/2 + (f*x)/2)^2 - 1))^(2/3)*(3*sin(2*e + 2*f*x) + cos(e +
f*x)^2*6i + 1i)*((a*(sin(2*e + 2*f*x)*1i + 2*cos(e + f*x)^2))/(2*cos(e + f
*x)^2))^(2/3))/(4*f)
```

Reduce [F]

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = d^{\frac{2}{3}}a^{\frac{5}{3}}\left(\left(\int \sec(fx+e)^{\frac{2}{3}}(\tan(fx+e)\,i+1)^{\frac{2}{3}}\tan(fx+e)\,dx\right)i+\int \sec(fx+e)^{\frac{2}{3}}(\mathrm{ta}$$

input
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x)
```

output
```
d**(2/3)*a**(2/3)*a*(int(sec(e + f*x)**(2/3)*(tan(e + f*x)*i + 1)**(2/3)*t
an(e + f*x),x)*i + int(sec(e + f*x)**(2/3)*(tan(e + f*x)*i + 1)**(2/3),x))
```

3.448 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 122

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx = \frac{54ia^3(d\sec(e+fx))^{2/3}}{7f\sqrt[3]{a+ia\tan(e+fx)}} + \frac{9ia^2(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{7f} + \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}}{7f}$$

output
```
54/7*I*a^3*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(1/3)+9/7*I*a^2*(d*se
c(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3)/f+3/7*I*a*(d*sec(f*x+e))^(2/3)*(a
+I*a*tan(f*x+e))^(5/3)/f
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.82

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx = \frac{3a^2(d\sec(e+fx))^{5/3}(i\cos(e-fx)+\sin(e-fx))(21+23\cos(2(e+fx))+5i\sin\cdots}{14df(\cos(fx)+i\sin(fx))^2}$$

input
```
Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(8/3),x]
```

output
```
(3*a^2*(d*Sec[e + f*x])^(5/3)*(I*Cos[e - f*x] + Sin[e - f*x])*(21 + 23*Cos
[2*(e + f*x)] + (5*I)*Sin[2*(e + f*x)])*(a + I*a*Tan[e + f*x])^(2/3))/(14*
d*f*(Cos[f*x] + I*Sin[f*x])^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{7}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{5/3}dx+\frac{3ia(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{7f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{5/3}dx+\frac{3ia(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{7f}$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{7}a\left(\frac{3}{2}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{2/3}dx+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}\right)+\frac{3ia(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{7f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a\left(\frac{3}{2}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{2/3}dx+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}\right)+\frac{3ia(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{7f}$$

$$\Big\downarrow \text{3974}$$

$$\frac{12}{7}a\left(\frac{9ia^2(d\sec(e+fx))^{2/3}}{2f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}\right)+\frac{3ia(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{7f}$$

input
```
Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(8/3),x]
```

output
```
(((3*I)/7)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3))/f + (12*
a*((((9*I)/2)*a^2*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3))
 + (((3*I)/4)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3))/f))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [F]

$$\int (d\sec (fx+e))^{\frac{2}{3}} (a+ia\tan (fx+e))^{\frac{8}{3}} dx$$

input
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x)
```

output
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx = -\frac{6\cdot 2^{\frac{1}{3}}\left(-14i\,a^2e^{(4i\,fx+4i\,e)}-21i\,a^2e^{(2i\,fx+2i\,e)}-9i\,a^2\right)\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}e^{(2i\,fx+2i\,e)}}{7\left(fe^{(4i\,fx+4i\,e)}+fe^{(2i\,fx+2i\,e)}\right)}$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x, algorithm="fric
as")
```

output
```
-6/7*2^(1/3)*(-14*I*a^2*e^(4*I*f*x + 4*I*e) - 21*I*a^2*e^(2*I*f*x + 2*I*e)
 - 9*I*a^2)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) +
1))^(2/3)*e^(2*I*f*x + 2*I*e)/(f*e^(4*I*f*x + 4*I*e) + f*e^(2*I*f*x + 2*I*
e))
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(8/3),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(92) = 184$.

Time = 0.25 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.30

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx = \frac{6\left(7\left(-i\cdot 2^{\frac{1}{3}}a^2\cos\left(\frac{4}{3}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)\right)-2^{\frac{1}{3}}a^2\sin\left(\frac{4}{3}\ldots\right.\right.\right.}{\ldots}$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x, algorithm="maxi
ma")
```

output
```
6/7*(7*(-I*2^(1/3)*a^2*cos(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)) - 2^(1/3)*a^2*sin(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)))*sqrt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*a^(2/3)*d^(2/3) + 2*(I*2^(1/3)*a^2*cos(7/3*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1)) + 2^(1/3)*a^2*sin(7/3*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1)) + 7*(I*2^(1/3)*a^2*cos(2*f*x + 2*e)^2 + I*2^(1/3)*a^2*sin(
2*f*x + 2*e)^2 + 2*I*2^(1/3)*a^2*cos(2*f*x + 2*e) + I*2^(1/3)*a^2)*cos(1/3
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 7*(2^(1/3)*a^2*cos(2*f
*x + 2*e)^2 + 2^(1/3)*a^2*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a^2*cos(2*f*x + 2
*e) + 2^(1/3)*a^2)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
))*a^(2/3)*d^(2/3))/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)^(7/6)*f)
```

Giac [F]

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx=\int (d\sec(fx+e))^{\frac{2}{3}}(ia\tan(fx+e)+a)^{\frac{8}{3}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x, algorithm="giac
")
```

output
```
integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(8/3), x)
```

Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx=\frac{3a^2\left(\frac{d}{\cos(e+fx)}\right)^{2/3}\left(\frac{a(\cos(2e+2fx)+1+\sin(2e+2fx)\,1\mathrm{i})}{\cos(2e+2fx)+1}\right)^{2/3}(\cos(2e+2fx)\,44\mathrm{i}+\cos(4}{14f\,(\cos(2e+2fx)+}$$

input
```
int((d/cos(e + f*x))^(2/3)*(a + a*tan(e + f*x)*1i)^(8/3),x)
```

output
```
(3*a^2*(d/cos(e + f*x))^(2/3)*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i
+ 1))/(cos(2*e + 2*f*x) + 1))^(2/3)*(cos(2*e + 2*f*x)*44i + cos(4*e + 4*f*
x)*9i + 16*sin(2*e + 2*f*x) + 9*sin(4*e + 4*f*x) + 35i))/(14*f*(cos(2*e +
2*f*x) + 1))
```

Reduce [F]

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx = d^{\frac{2}{3}}a^{\frac{8}{3}}\left(-\left(\int \sec(fx+e)^{\frac{2}{3}}(\tan(fx+e)\,i+1)^{\frac{2}{3}}\tan(fx+e)^2\,dx\right)+2\left(\int \sec(fx+e\right.\right.$$

input
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x)
```

output
```
d**(2/3)*a**(2/3)*a**2*( - int(sec(e + f*x)**(2/3)*(tan(e + f*x)*i + 1)**(
2/3)*tan(e + f*x)**2,x) + 2*int(sec(e + f*x)**(2/3)*(tan(e + f*x)*i + 1)**
(2/3)*tan(e + f*x),x)*i + int(sec(e + f*x)**(2/3)*(tan(e + f*x)*i + 1)**(2
/3),x))
```

3.449 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 163

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx = \frac{486ia^4(d\sec(e+fx))^{2/3}}{35f\sqrt[3]{a+ia\tan(e+fx)}} + \frac{81ia^3(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{35f} + \frac{27ia^2(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}}{35f} + \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}}{10f}$$

output
```
486/35*I*a^4*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(1/3)+81/35*I*a^3*(
d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3)/f+27/35*I*a^2*(d*sec(f*x+e))^
(2/3)*(a+I*a*tan(f*x+e))^(5/3)/f+3/10*I*a*(d*sec(f*x+e))^(2/3)*(a+I*a*tan(
f*x+e))^(8/3)/f
```

Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.71

$$\int (d\sec(e+fx))^{2/3}(a +ia\tan(e+fx))^{11/3}\,dx = \frac{3a^3(d\sec(e+fx))^{5/3}(i\cos(e-2fx)+\sin(e-2fx))(364+442\cos(2(e+fx))+}{140df(\cos(fx)+}$$

input
```
Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(11/3),x]
```

output
```
(3*a^3*(d*Sec[e + f*x])^(5/3)*(I*Cos[e - 2*f*x] + Sin[e - 2*f*x])*(364 + 4
42*Cos[2*(e + f*x)] + (59*I)*Sec[e + f*x]*Sin[3*(e + f*x)] + (45*I)*Tan[e
+ f*x])*(a + I*a*Tan[e + f*x])^(2/3))/(140*d*f*(Cos[f*x] + I*Sin[f*x])^3)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3975, 3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{11/3}(d\sec(e+fx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{11/3}(d\sec(e+fx))^{2/3}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{9}{5}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{8/3}dx+\frac{3ia(a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}}{10f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{5}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{8/3}dx+\frac{3ia(a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}}{10f}$$

$$\Big\downarrow \text{3975}$$

$$\frac{9}{5}a\left(\frac{12}{7}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{5/3}dx+\frac{3ia(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{7f}\right)+\frac{3ia(a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}}{10f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{5}a\left(\frac{12}{7}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{5/3}dx+\frac{3ia(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{7f}\right)+\frac{3ia(a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}}{10f}$$

$$\Big\downarrow \text{3975}$$

$$\frac{9}{5}a\left(\frac{12}{7}a\left(\frac{3}{2}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{2/3}dx+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}\right)+\frac{3ia(a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}}{10f}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{5}a\left(\frac{12}{7}a\left(\frac{3}{2}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{2/3}dx+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}\right)+\frac{3ia(a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}}{10f}\right.$$

$$\Big\downarrow \text{3974}$$

$$\frac{9}{5}a\left(\frac{12}{7}a\left(\frac{9ia^2(d\sec(e+fx))^{2/3}}{2f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}\right)+\frac{3ia(a+ia\tan(e+fx))^{5}}{7f}\right.\frac{3ia(a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}}{10f}$$

input
```
Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(11/3),x]
```

output
```
(((3*I)/10)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(8/3))/f + (9*
a*((((3*I)/7)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3))/f + (
12*a*((((9*I)/2)*a^2*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/
3)) + (((3*I)/4)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3))/f)
)/7))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{2}{3}} (a+ia\tan(fx+e))^{\frac{11}{3}} dx$$

input
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x)
```

output
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx =$$

$$-\frac{6\cdot 2^{\frac{1}{3}}\left(-140i\,a^3e^{(6i\,fx+6i\,e)}-315i\,a^3e^{(4i\,fx+4i\,e)}-270i\,a^3e^{(2i\,fx+2i\,e)}-81i\,a^3\right)\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right.}{35\left(fe^{(6i\,fx+6i\,e)}+2\,fe^{(4i\,fx+4i\,e)}+fe^{(2i\,fx+2i\,e)}\right)}$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x, algorithm="fri
cas")
```

output
```
-6/35*2^(1/3)*(-140*I*a^3*e^(6*I*f*x + 6*I*e) - 315*I*a^3*e^(4*I*f*x + 4*I
*e) - 270*I*a^3*e^(2*I*f*x + 2*I*e) - 81*I*a^3)*(a/(e^(2*I*f*x + 2*I*e) +
1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2*I*f*x + 2*I*e)/(f*e^(6*
I*f*x + 6*I*e) + 2*f*e^(4*I*f*x + 4*I*e) + f*e^(2*I*f*x + 2*I*e))
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(11/3),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(123) = 246$.

Time = 0.24 (sec) , antiderivative size = 983, normalized size of antiderivative = 6.03

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x, algorithm="max
ima")
```

output
```
6/35*(7*(-2*I*2^(1/3)*a^3*cos(10/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1)) - 2*2^(1/3)*a^3*sin(10/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1)) + 15*(-I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 - I*2^(1/3)*a^3*sin(2*f*
x + 2*e)^2 - 2*I*2^(1/3)*a^3*cos(2*f*x + 2*e) - I*2^(1/3)*a^3)*cos(4/3*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 15*(2^(1/3)*a^3*cos(2*f*x
+ 2*e)^2 + 2^(1/3)*a^3*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a^3*cos(2*f*x + 2*e)
 + 2^(1/3)*a^3)*sin(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*
sqrt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*a^(
2/3)*d^(2/3) + 20*(3*(I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + I*2^(1/3)*a^3*sin
(2*f*x + 2*e)^2 + 2*I*2^(1/3)*a^3*cos(2*f*x + 2*e) + I*2^(1/3)*a^3)*cos(7/
3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 7*(I*2^(1/3)*a^3*cos(
2*f*x + 2*e)^4 + I*2^(1/3)*a^3*sin(2*f*x + 2*e)^4 + 4*I*2^(1/3)*a^3*cos(2*
f*x + 2*e)^3 + 6*I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 4*I*2^(1/3)*a^3*cos(2*
f*x + 2*e) + I*2^(1/3)*a^3 + 2*(I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 2*I*2^(
1/3)*a^3*cos(2*f*x + 2*e) + I*2^(1/3)*a^3)*sin(2*f*x + 2*e)^2)*cos(1/3*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 3*(2^(1/3)*a^3*cos(2*f*x +
 2*e)^2 + 2^(1/3)*a^3*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a^3*cos(2*f*x + 2*e)
+ 2^(1/3)*a^3)*sin(7/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) +
7*(2^(1/3)*a^3*cos(2*f*x + 2*e)^4 + 2^(1/3)*a^3*sin(2*f*x + 2*e)^4 + 4*2^(
1/3)*a^3*cos(2*f*x + 2*e)^3 + 6*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 4*2^(1...
```

Giac [F]

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx=\int (d\sec{(fx+e)})^{\frac{2}{3}}\,(i\,a\tan{(fx+e)}+a)^{\frac{11}{3}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x, algorithm="gia
c")
```

output
```
integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(11/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.31 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.86

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx=\frac{\left(-\frac{d}{2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1}\right)^{2/3}\left(2\sin\left(2\,e+2\,f\,x\right)^2+\sin\left(4\,e+4\,f\,x\right)\,1\mathrm{i}-1\right)\left(\frac{a^3\left(a-\frac{a\sin\ldots}{2\sin\left(\ldots\right.}\right.}{\ldots}\right.}{\ldots}$$

input
```
int((d/cos(e + f*x))^(2/3)*(a + a*tan(e + f*x)*1i)^(11/3),x)
```

output
```
((-d/(2*sin(e/2 + (f*x)/2)^2 - 1))^(2/3)*(sin(4*e + 4*f*x)*1i + 2*sin(2*e
+ 2*f*x)^2 - 1)*((a^3*(a - (a*sin(e + f*x)*1i)/(2*sin(e/2 + (f*x)/2)^2 - 1
))^(2/3)*243i)/(35*f) + (a^3*(a - (a*sin(e + f*x)*1i)/(2*sin(e/2 + (f*x)/2
)^2 - 1))^(2/3)*(sin(2*e + 2*f*x)*1i - 2*sin(e + f*x)^2 + 1)*162i)/(7*f) +
 (a^3*(a - (a*sin(e + f*x)*1i)/(2*sin(e/2 + (f*x)/2)^2 - 1))^(2/3)*(sin(4*
e + 4*f*x)*1i - 2*sin(2*e + 2*f*x)^2 + 1)*27i)/f + (a^3*(a - (a*sin(e + f*
x)*1i)/(2*sin(e/2 + (f*x)/2)^2 - 1))^(2/3)*(sin(6*e + 6*f*x)*1i - 2*sin(3*
e + 3*f*x)^2 + 1)*12i)/f))/(4*(sin(e + f*x)^2 - 1))
```

Reduce [F]

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx = d^{\frac{2}{3}}a^{\frac{11}{3}}\left(-\left(\int \sec(fx+e)^{\frac{2}{3}}(\tan(fx+e)\,i+1)^{\frac{2}{3}}\tan(fx+e)^{3}\,dx\right)i-3\left(\int \sec(fx\right.\right.$$

input
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x)
```

output
```
d**(2/3)*a**(2/3)*a**3*( - int(sec(e + f*x)**(2/3)*(tan(e + f*x)*i + 1)**(
2/3)*tan(e + f*x)**3,x)*i - 3*int(sec(e + f*x)**(2/3)*(tan(e + f*x)*i + 1)
**(2/3)*tan(e + f*x)**2,x) + 3*int(sec(e + f*x)**(2/3)*(tan(e + f*x)*i + 1
)**(2/3)*tan(e + f*x),x)*i + int(sec(e + f*x)**(2/3)*(tan(e + f*x)*i + 1)*
*(2/3),x))
```

3.450 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = \frac{i2^{5+\frac{m}{2}}a^5\,\text{Hypergeometric2F1}\left(-4-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx)$$

output
```
I*2^(5+1/2*m)*a^5*hypergeom([1/2*m, -4-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+
c))*(e*sec(d*x+c))^m/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 210 vs. 2(86) = 172.

Time = 4.89 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.44

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = \frac{a^5(e\sec(c+dx))^m\left(\frac{i(16(8+6m+m^2)-12m(4+m)\sec^2(c+dx)+m(2+m)\sec^4(c+dx))}{8+6m+m^2}+5\cot(c+dx)\,\text{Hypergeometric2F}$$

input
```
Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*(e*Sec[c + d*x])^m*((I*(16*(8 + 6*m + m^2) - 12*m*(4 + m)*Sec[c + d*x
]^2 + m*(2 + m)*Sec[c + d*x]^4))/(8 + 6*m + m^2) + 5*Cot[c + d*x]*Hypergeo
metric2F1[-3/2, m/2, (2 + m)/2, Sec[c + d*x]^2]*Sqrt[-Tan[c + d*x]^2] + 10
*Cot[c + d*x]*Hypergeometric2F1[-1/2, m/2, (2 + m)/2, Sec[c + d*x]^2]*Sqrt
[-Tan[c + d*x]^2] + Cot[c + d*x]*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Se
c[c + d*x]^2]*Sqrt[-Tan[c + d*x]^2]))/(d*m)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^5(e\sec(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^5(e\sec(c+dx))^mdx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+10}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+10}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^6 2^{\frac{m}{2}+4}(1+i\tan(c+dx))^{-m/2}(a-ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m \int \left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{\frac{m+8}{2}} (a-ia\tan(}{d}$$

$$\Big\downarrow \text{79}$$

$$\frac{ia^5 2^{\frac{m}{2}+5}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m \text{Hypergeometric2F1}\left(\frac{1}{2}(-m-8),\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx)\right)}{dm}$$

input
```
Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(I*2^(5 + m/2)*a^5*Hypergeometric2F1[(-8 - m)/2, m/2, (2 + m)/2, (1 - I*Ta
n[c + d*x])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\sec(dx+c))^m (a+ia\tan(dx+c))^5\,dx$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x)
```

output
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = \int (i\,a\tan(dx+c)+a)^5(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
integral(32*a^5*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(10*I*
d*x + 10*I*c)/(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d
*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx$$
$$= ia^5\left(\int \left(-i(e\sec\left(c+dx\right))^m\right)\,dx + \int 5(e\sec\left(c+dx\right))^m\tan\left(c+dx\right)dx + \int \left(-10(e\sec\left(c+dx\right))^m\tan^3\left(c+dx\right)\right)\,dx + \int (e\sec\left(c+dx\right))^m\tan^5\left(c+dx\right)dx + \int 10i(e\sec\left(c+dx\right))^m\tan^2\left(c+dx\right)dx + \int \left(-5i(e\sec\left(c+dx\right))^m\tan^4\left(c+dx\right)\right)\,dx\right)$$

input
```
integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**5,x)
```

output
```
I*a**5*(Integral(-I*(e*sec(c + d*x))**m, x) + Integral(5*(e*sec(c + d*x))*
*m*tan(c + d*x), x) + Integral(-10*(e*sec(c + d*x))**m*tan(c + d*x)**3, x)
 + Integral((e*sec(c + d*x))**m*tan(c + d*x)**5, x) + Integral(10*I*(e*sec
(c + d*x))**m*tan(c + d*x)**2, x) + Integral(-5*I*(e*sec(c + d*x))**m*tan(
c + d*x)**4, x))
```

Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = \int (i\,a\tan\left(dx+c\right)+a)^5(e\sec\left(dx+c\right))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^5*(e*sec(d*x + c))^m, x)
```

Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = \int (i\,a\tan(dx+c)+a)^5(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^5*(e*sec(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^m (a+a\tan(c+d\,x)\ 1\mathrm{i})^5\,dx$$

input
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^5, x)
```

Reduce [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = \frac{e^m a^5 \left(\sec(dx+c)^m\tan(dx+c)^4\,i\,m^2+2\sec(dx+c)^m\tan(dx+c)^4\,im-10\sec(dx+c)^m\tan(dx+c)\right.}{}$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x)
```

output
```
(e**m*a**5*(sec(c + d*x)**m*tan(c + d*x)**4*i*m**2 + 2*sec(c + d*x)**m*tan
(c + d*x)**4*i*m - 10*sec(c + d*x)**m*tan(c + d*x)**2*i*m**2 - 44*sec(c +
d*x)**m*tan(c + d*x)**2*i*m + 5*sec(c + d*x)**m*i*m**2 + 50*sec(c + d*x)**
m*i*m + 128*sec(c + d*x)**m*i + int(sec(c + d*x)**m,x)*d*m**3 + 6*int(sec(
c + d*x)**m,x)*d*m**2 + 8*int(sec(c + d*x)**m,x)*d*m + 5*int(sec(c + d*x)*
*m*tan(c + d*x)**4,x)*d*m**3 + 30*int(sec(c + d*x)**m*tan(c + d*x)**4,x)*d
*m**2 + 40*int(sec(c + d*x)**m*tan(c + d*x)**4,x)*d*m - 10*int(sec(c + d*x
)**m*tan(c + d*x)**2,x)*d*m**3 - 60*int(sec(c + d*x)**m*tan(c + d*x)**2,x)
*d*m**2 - 80*int(sec(c + d*x)**m*tan(c + d*x)**2,x)*d*m))/(d*m*(m**2 + 6*m
 + 8))
```

3.451 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx = \frac{i2^{3+\frac{m}{2}}a^3\,\text{Hypergeometric2F1}\left(-2-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx)}{dm}$$

output
```
I*2^(3+1/2*m)*a^3*hypergeom([1/2*m, -2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+
c))*(e*sec(d*x+c))^m/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.71

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx = -\frac{ia^3(e\sec(c+dx))^m\left(-3i(2+m)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(c+dx)\right)\tan(c+dx)-i(2+\right.}{dm(2+}$$

input
```
Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*a^3*(e*Sec[c + d*x])^m*((-3*I)*(2 + m)*Hypergeometric2F1[-1/2, m/2,
(2 + m)/2, Sec[c + d*x]^2]*Tan[c + d*x] - I*(2 + m)*Hypergeometric2F1[1/2,
 m/2, (2 + m)/2, Sec[c + d*x]^2]*Tan[c + d*x] + (-8 - 4*m + m*Sec[c + d*x]
^2)*Sqrt[-Tan[c + d*x]^2]))/(d*m*(2 + m)*Sqrt[-Tan[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^3(e\sec(c+dx))^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (a+ia\tan(c+dx))^3(e\sec(c+dx))^mdx$$

$$\Big\downarrow \ 3986$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+6}{2}}dx$$

$$\Big\downarrow \ 3042$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+6}{2}}dx$$

$$\Big\downarrow \ 4006$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \ 80$$

$$\frac{a^4 2^{\frac{m}{2}+2}(1+i\tan(c+dx))^{-m/2}(a-ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int \left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{\frac{m+4}{2}}(a-ia\tan(}{d}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{ia^3 2^{\frac{m}{2}+3}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m \text{Hypergeometric2F1}\left(\frac{1}{2}(-m-4), \frac{m}{2}, \frac{m+2}{2}, \frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

input
```
Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(I*2^(3 + m/2)*a^3*Hypergeometric2F1[(-4 - m)/2, m/2, (2 + m)/2, (1 - I*Ta
n[c + d*x])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\sec (dx + c))^m (a + ia\tan (dx + c))^3 \, dx$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x)
```

output
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3 \, dx = \int (i\,a\tan (dx + c) + a)^3 (e\sec (dx + c))^m \, dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(8*a^3*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(6*I*d*
x + 6*I*c)/(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2
*I*c) + 1), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx$$
$$= -ia^3\left(\int i(e\sec(c+dx))^m\,dx + \int(-3(e\sec(c+dx))^m\tan(c+dx))\,dx + \int(e\sec(c+dx))^m\tan^3(c+dx)\,dx + \int(-3i(e\sec(c+dx))^m\tan^2(c+dx))\,dx\right)$$

input
```
integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-I*a**3*(Integral(I*(e*sec(c + d*x))**m, x) + Integral(-3*(e*sec(c + d*x))
**m*tan(c + d*x), x) + Integral((e*sec(c + d*x))**m*tan(c + d*x)**3, x) +
Integral(-3*I*(e*sec(c + d*x))**m*tan(c + d*x)**2, x))
```

Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx = \int (ia\tan(dx+c)+a)^3(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3*(e*sec(d*x + c))^m, x)
```

Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx = \int (i\,a\tan\left(dx+c\right)+a)^3(e\sec\left(dx+c\right))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3*(e*sec(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx = \int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^m\,\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)^3\,dx$$

input
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx = \frac{e^m a^3\left(-\sec\left(dx+c\right)^m\tan\left(dx+c\right)^2 im+3\sec\left(dx+c\right)^m im+8\sec\left(dx+c\right)^m i+\left(\int\sec\left(dx+c\right)^m dx\right)d}{}$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x)
```

output
```
(e**m*a**3*( - sec(c + d*x)**m*tan(c + d*x)**2*i*m + 3*sec(c + d*x)**m*i*m
 + 8*sec(c + d*x)**m*i + int(sec(c + d*x)**m,x)*d*m**2 + 2*int(sec(c + d*x
)**m,x)*d*m - 3*int(sec(c + d*x)**m*tan(c + d*x)**2,x)*d*m**2 - 6*int(sec(
c + d*x)**m*tan(c + d*x)**2,x)*d*m))/(d*m*(m + 2))
```

3.452 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \frac{i2^{2+\frac{m}{2}}a^2\,\text{Hypergeometric2F1}\left(-1-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx)}{dm}$$

output
```
I*2^(2+1/2*m)*a^2*hypergeom([1/2*m, -1-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+
c))*(e*sec(d*x+c))^m/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.33

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \frac{a^2(e\sec(c+dx))^m\left(2i+\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}\right.}{dm}$$

input
```
Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(a^2*(e*Sec[c + d*x])^m*(2*I + Cot[c + d*x]*Hypergeometric2F1[-1/2, m/2, (
2 + m)/2, Sec[c + d*x]^2]*Sqrt[-Tan[c + d*x]^2] + Cot[c + d*x]*Hypergeomet
ric2F1[1/2, m/2, (2 + m)/2, Sec[c + d*x]^2]*Sqrt[-Tan[c + d*x]^2]))/(d*m)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.33, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^2(e\sec(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^2(e\sec(c+dx))^m dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+4}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+4}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a2^{\frac{m}{2}-1}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int \left(\frac{1}{2}-\frac{1}{2}i\tan(c+dx)\right)^{\frac{m-2}{2}}(i\tan(c+dx}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{i2^{m/2}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{m+4}{2}-\frac{m}{2}}(e\sec(c+dx))^m \operatorname{Hypergeometric2F1}\left(\frac{2-m}{2},\frac{m+4}{2},\frac{m+6}{2}\right.}{d(m+4)}$$

input
```
Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*2^(m/2)*Hypergeometric2F1[(2 - m)/2, (4 + m)/2, (6 + m)/2, (1 + I*Ta
n[c + d*x])/2]*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(-1/2*m + (4 + m)
/2))/(d*(4 + m)*(1 - I*Tan[c + d*x])^(m/2))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\sec (dx+c))^{m} (a+ia\tan (dx+c))^{2} dx$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

output
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \int (i\,a\tan (dx+c)+a)^2(e\sec (dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(4*a^2*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(4*I*d*
x + 4*I*c)/(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\begin{aligned}\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx \\ = -a^2\bigg(\int \left(-(e\sec (c+dx))^m\right)\,dx + \int (e\sec (c+dx))^m\tan^2(c+dx)\,dx \\ + \int \left(-2i(e\sec (c+dx))^m\tan (c+dx)\right)\,dx\bigg)\end{aligned}$$

input
```
integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(-(e*sec(c + d*x))**m, x) + Integral((e*sec(c + d*x))**m*ta
n(c + d*x)**2, x) + Integral(-2*I*(e*sec(c + d*x))**m*tan(c + d*x), x))
```

Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx=\int (i\,a\tan(dx+c)+a)^2(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2*(e*sec(d*x + c))^m, x)
```

Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx=\int (i\,a\tan(dx+c)+a)^2(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2*(e*sec(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx$$
$$=\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^m\,\left(a+a\,\tan(c+d\,x)\;1\mathrm{i}\right)^2\,dx$$

input
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{e^m a^2\left(2\sec\left(dx+c\right)^m i+\left(\int\sec\left(dx+c\right)^m dx\right)dm-\left(\int\sec\left(dx+c\right)^m\tan\left(dx+c\right)^2dx\right)dm\right)}{dm}$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

output
```
(e**m*a**2*(2*sec(c + d*x)**m*i + int(sec(c + d*x)**m,x)*d*m - int(sec(c +
 d*x)**m*tan(c + d*x)**2,x)*d*m))/(d*m)
```

3.453 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \frac{i2^{1+\frac{m}{2}}a\,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m/2}}{dm}$$

output
```
I*2^(1+1/2*m)*a*hypergeom([-1/2*m, 1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c))*
(e*sec(d*x+c))^m/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \frac{a(e\sec(c+dx))^m\left(i+\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}\right)}{dm}$$

input
```
Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*(e*Sec[c + d*x])^m*(I + Cot[c + d*x]*Hypergeometric2F1[1/2, m/2, (2 + m
)/2, Sec[c + d*x]^2]*Sqrt[-Tan[c + d*x]^2]))/(d*m)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))(e\sec(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))(e\sec(c+dx))^m dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+2}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+2}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a2^{\frac{m}{2}-1}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int \left(\frac{1}{2}-\frac{1}{2}i\tan(c+dx)\right)^{\frac{m-2}{2}}(i\tan(c+dx}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{i2^{m/2}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{m+2}{2}-\frac{m}{2}}(e\sec(c+dx))^m \operatorname{Hypergeometric2F1}\left(\frac{2-m}{2},\frac{m+2}{2},\frac{m+4}{2}\right.}{d(m+2)}$$

input
```
Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*2^(m/2)*Hypergeometric2F1[(2 - m)/2, (2 + m)/2, (4 + m)/2, (1 + I*Ta
n[c + d*x])/2]*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(-1/2*m + (2 + m)
/2))/(d*(2 + m)*(1 - I*Tan[c + d*x])^(m/2))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\sec(dx+c))^m (a+ia\tan(dx+c))\,dx$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

output
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \int (i\,a\tan(dx+c)+a)(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
integral(2*a*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(2*I*d*x
+ 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = ia\left(\int \left(-i(e\sec(c+dx))^m\right)dx + \int (e\sec(c+dx))^m\tan(c+dx)\,dx\right)$$

input
```
integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*(e*sec(c + d*x))**m, x) + Integral((e*sec(c + d*x))**m*ta
n(c + d*x), x))
```

Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \int (i\,a\tan\left(dx+c\right)+a)(e\sec\left(dx+c\right))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \int (i\,a\tan\left(dx+c\right)+a)(e\sec\left(dx+c\right))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^m\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)\,dx$$

input
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx$$
$$= \frac{e^m a\left(\sec\left(dx+c\right)^m i+\left(\int \sec\left(dx+c\right)^m dx\right) dm\right)}{dm}$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

output
```
(e**m*a*(sec(c + d*x)**m*i + int(sec(c + d*x)**m,x)*d*m))/(d*m)
```

3.454 $\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \frac{i2^{-1+\frac{m}{2}}\,\text{Hypergeometric2F1}\left(2-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-}}{adm}$$

output
```
I*2^(-1+1/2*m)*hypergeom([1/2*m, 2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c))*
(e*sec(d*x+c))^m/a/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.77

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = -\frac{i2^{-1+m}e^{-i(c+2dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m\left(1+e^{2i(c+dx)}\right)^m\text{Hypergeometric2F1}\left(\frac{1}{2}(-2+m),-1+m,\frac{m}{2},-e^{2i(c+d}\right.}{d(-2+m)(a+ia\tan(c+dx))}$$

input
```
Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*2^(-1 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*
I)*(c + d*x)))^m*Hypergeometric2F1[(-2 + m)/2, -1 + m, m/2, -E^((2*I)*(c +
 d*x))]*Sec[c + d*x]^(1 - m)*(e*Sec[c + d*x])^m*(Cos[d*x] + I*Sin[d*x]))/(
d*E^(I*(c + 2*d*x))*(-2 + m)*(a + I*a*Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-2}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-2}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{\frac{m}{2}-2}(1+i\tan(c+dx))^{-m/2}(a-ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int \left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{\frac{m-4}{2}}(a-ia\tan(c}{d}$$

$$\Big\downarrow \text{79}$$

$$\frac{i2^{\frac{m}{2}-1}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m \operatorname{Hypergeometric2F1}\left(\frac{4-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{adm}$$

input
```
Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

output
```
(I*2^(-1 + m/2)*Hypergeometric2F1[(4 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[c
+ d*x])/2]*(e*Sec[c + d*x])^m)/(a*d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(e\sec(dx+c))^m}{a+ia\tan(dx+c)}dx$$

input
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

output
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

Fricas [F]

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\sec(dx+c))^m}{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
integral(1/2*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*(e^(2*I*d*x
 + 2*I*c) + 1)*e^(-2*I*d*x - 2*I*c)/a, x)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{(e\sec(c+dx))^m}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral((e*sec(c + d*x))**m/(tan(c + d*x) - I), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\sec(dx+c))^m}{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{a+a\tan(c+d\,x)\ 1\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \frac{e^m\left(\int \frac{\sec(dx+c)^m}{\tan(dx+c)i+1}dx\right)}{a}$$

input
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

output
```
(e**m*int(sec(c + d*x)**m/(tan(c + d*x)*i + 1),x))/a
```

3.455 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \frac{i2^{-2+\frac{m}{2}}\,\text{Hypergeometric2F1}\left(3-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-}}{a^2dm}$$

output
```
I*2^(-2+1/2*m)*hypergeom([1/2*m, 3-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c))*
(e*sec(d*x+c))^m/a^2/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.81

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = -\frac{i2^{-2+m}e^{-2i(c+2dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m\left(1+e^{2i(c+dx)}\right)^m\text{Hypergeometric2F1}\left(\frac{1}{2}(-4+m),-2+m,\frac{1}{2}(-2+m\right.}{d(-4+m)(a+ia\tan(c+dx))^2}$$

input
```
Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*2^(-2 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*
I)*(c + d*x)))^m*Hypergeometric2F1[(-4 + m)/2, -2 + m, (-2 + m)/2, -E^((2*
I)*(c + d*x))]*Sec[c + d*x]^(2 - m)*(e*Sec[c + d*x])^m*(Cos[d*x] + I*Sin[d
*x])^2)/(d*E^((2*I)*(c + 2*d*x))*(-4 + m)*(a + I*a*Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-4}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-4}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{\frac{m}{2}-3}(1+i\tan(c+dx))^{-m/2}(a-ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int \left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{\frac{m-6}{2}}(a-ia\tan(c}{ad}$$

$$\Big\downarrow \text{79}$$

$$\frac{i2^{\frac{m}{2}-2}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m \text{Hypergeometric2F1}\left(\frac{6-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{a^2dm}$$

input
```
Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(I*2^(-2 + m/2)*Hypergeometric2F1[(6 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[c
+ d*x])/2]*(e*Sec[c + d*x])^m)/(a^2*d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(e\sec(dx+c))^m}{(a+ia\tan(dx+c))^2}dx$$

input
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

output
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

Fricas [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^m}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(1/4*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*(e^(4*I*d*x
 + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*e^(-4*I*d*x - 4*I*c)/a^2, x)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{(e\sec(c+dx))^m}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral((e*sec(c + d*x))**m/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)
/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^m}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}\,dx$$

input
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = -\frac{e^m\left(\int \frac{\sec(dx+c)^m}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)}{a^2}$$

input
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - e**m*int(sec(c + d*x)**m/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x))/
a**2
```

3.456 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \frac{i2^{-3+\frac{m}{2}}\,\text{Hypergeometric2F1}\left(4-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-}}{a^3dm}$$

output
```
I*2^(-3+1/2*m)*hypergeom([1/2*m, 4-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c))*
(e*sec(d*x+c))^m/a^3/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.78

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \frac{2^{-3+m}e^{-3i(c+2dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m\left(1+e^{2i(c+dx)}\right)^m\text{Hypergeometric2F1}\left(\frac{1}{2}(-6+m),-3+m,\frac{1}{2}(-4+m),-\right.}{a^3d(-6+m)(-i+\tan(c+dx))^3}$$

input
```
Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(2^(-3 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I)*(c
 + d*x)))^m*Hypergeometric2F1[(-6 + m)/2, -3 + m, (-4 + m)/2, -E^((2*I)*(c
 + d*x))]*Sec[c + d*x]^(3 - m)*(e*Sec[c + d*x])^m*(Cos[d*x] + I*Sin[d*x])^
3)/(a^3*d*E^((3*I)*(c + 2*d*x))*(-6 + m)*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-6}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-6}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{\frac{m}{2}-4}(1+i\tan(c+dx))^{-m/2}(a-ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int \left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{\frac{m-8}{2}}(a-ia\tan(c}{a^2d}$$

$$\Big\downarrow 79$$

$$\frac{i2^{\frac{m}{2}-3}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m \text{Hypergeometric2F1}\left(\frac{8-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{a^3dm}$$

input
```
Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(I*2^(-3 + m/2)*Hypergeometric2F1[(8 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[c
+ d*x])/2]*(e*Sec[c + d*x])^m)/(a^3*d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(e\sec(dx+c))^m}{(a+ia\tan(dx+c))^3}dx$$

input
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x)
```

output
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x)
```

Fricas [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^m}{(i\,a\tan(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(1/8*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*(e^(6*I*d*x
 + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-6*I*d*x
 - 6*I*c)/a^3, x)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{(e\sec(c+dx))^m}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral((e*sec(c + d*x))**m/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*
tan(c + d*x) + I), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^m}{(ia\tan(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^3}\,dx$$

input
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx=-\frac{e^m\left(\int \frac{\sec(dx+c)^m}{\tan(dx+c)^3i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)}{a^3}$$

input
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - e**m*int(sec(c + d*x)**m/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*ta
n(c + d*x)*i - 1),x))/a**3
```

3.457 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 107

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx = \frac{i2^{\frac{7+m}{2}}a\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-5-m),\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+}{dm}$$

output
```
I*2^(7/2+1/2*m)*a*hypergeom([1/2*m, -5/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*
x+c))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(-5/2-1/2*m)*(a+I*a*tan(d*x+c))^(5
/2)/d/m
```

Mathematica [A] (verified)

Time = 3.00 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.74

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{i2^{\frac{7}{2}+m}e^{3i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+m}\text{Hypergeometric2F1}\left(\frac{7}{2}+m,\frac{7+m}{2},\frac{9+m}{2},-e^{2i(c+d}\right.}{d(7+m)(\cos(dx)+i\sin(dx))^{7/2}}$$

input
```
Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*2^(7/2 + m)*E^((3*I)*(c + 2*d*x))*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(
1 + E^((2*I)*(c + d*x))))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^(1/2 + m)*Hy
pergeometric2F1[7/2 + m, (7 + m)/2, (9 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c
 + d*x]^(-7/2 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(7/2))/(d*(7
+ m)*(Cos[d*x] + I*Sin[d*x])^(7/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{7/2}(e\sec(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^{7/2}(e\sec(c+dx))^m dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+7}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+7}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^4 2^{\frac{m+5}{2}}(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{m+1}{2}-\frac{m}{2}}(e\sec(c+dx))^m\int\left(\frac{1}{2}i\tan\right.}{d}$$

$$\Big\downarrow \text{79}$$

$$\frac{ia^3 2^{\frac{m+5}{2}+1}(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(a+ia\tan(c+dx))^{\frac{m+1}{2}-\frac{m}{2}}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-m-\right.}{dm}$$

input
```
Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(I*2^(1 + (5 + m)/2)*a^3*Hypergeometric2F1[(-5 - m)/2, m/2, (2 + m)/2, (1
- I*Tan[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((-1 - m)/2)*
(a + I*a*Tan[c + d*x])^(-1/2*m + (1 + m)/2))/(d*m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\sec (dx+c))^{m} (a+ia\tan (dx+c))^{\frac{7}{2}}dx$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x)
```

Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx=\int (i\,a\tan (dx+c)+a)^{\frac{7}{2}}(e\sec (dx+c))^{m}\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
integral(8*sqrt(2)*a^3*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(7*I*d*x + 7*I*c)/(e^(6*I*d*x + 6*I*c)
+ 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}(e\sec\left(dx+c\right))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(7/2)*(e*sec(d*x + c))^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx=\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^m\,\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)^{7/2}\,dx$$

input
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx=\frac{e^m\sqrt{a}\,a^3\left(-2\sec\left(dx+c\right)^m\sqrt{\tan\left(dx+c\right)i+1}\,i-\left(\int\sec\left(dx+c\right)^m\sqrt{\tan\left(dx+c\right)i+1}\,\tan\right.\right.}{}$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
(e**m*sqrt(a)*a**3*( - 2*sec(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*i - int(
sec(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3,x)*d*i - 3*int(se
c(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2,x)*d + 2*int(sec(c
+ d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*d*i*m + 4*int(sec(c + d
*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*d*i))/d
```

3.458 $\int (e\sec(c+dx))^m (a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 107

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx = \frac{i2^{\frac{5+m}{2}}a\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3-m),\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+}{dm}$$

output
```
I*2^(5/2+1/2*m)*a*hypergeom([1/2*m, -3/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*
x+c))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(-3/2-1/2*m)*(a+I*a*tan(d*x+c))^(3
/2)/d/m
```

Mathematica [A] (verified)

Time = 3.24 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.74

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{i2^{\frac{5}{2}+m}e^{2i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+m}\text{Hypergeometric2F1}\left(\frac{5}{2}+m,\frac{5+m}{2},\frac{7+m}{2},-e^{2i(c+d}\right.}{d(5+m)(\cos(dx)+i\sin(dx))^{5/2}}$$

input
```
Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*2^(5/2 + m)*E^((2*I)*(c + 2*d*x))*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(
1 + E^((2*I)*(c + d*x))))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^(1/2 + m)*Hy
pergeometric2F1[5/2 + m, (5 + m)/2, (7 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c
 + d*x]^(-5/2 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(5/2))/(d*(5
+ m)*(Cos[d*x] + I*Sin[d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^m\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^m dx$$

$$\Big\downarrow \; 3986$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+5}{2}}dx$$

$$\Big\downarrow \; 3042$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+5}{2}}dx$$

$$\Big\downarrow \; 4006$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \; 80$$

$$\frac{a^3 2^{\frac{m+3}{2}}(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{m+1}{2}-\frac{m}{2}}(e\sec(c+dx))^m\int\left(\frac{1}{2}i\tan\right.}{d}$$

$$\Big\downarrow \text{79}$$

$$\frac{ia^2 2^{\frac{m+3}{2}+1}(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(a+ia\tan(c+dx))^{\frac{m+1}{2}-\frac{m}{2}}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-m-\right.}{dm}$$

input
```
Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(I*2^(1 + (3 + m)/2)*a^2*Hypergeometric2F1[(-3 - m)/2, m/2, (2 + m)/2, (1
- I*Tan[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((-1 - m)/2)*
(a + I*a*Tan[c + d*x])^(-1/2*m + (1 + m)/2))/(d*m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\sec(dx+c))^m (a+ia\tan(dx+c))^{\frac{5}{2}}\, dx$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x)
```

Fricas [F]

$$\int (e\sec(c+dx))^m (a+ia\tan(c+dx))^{5/2}\, dx = \int (i\,a\tan(dx+c)+a)^{\frac{5}{2}} (e\sec(dx+c))^m\, dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(4*sqrt(2)*a^2*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(5*I*d*x + 5*I*c)/(e^(4*I*d*x + 4*I*c)
+ 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)*(e*sec(d*x + c))^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^m (a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}\,dx$$

input
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx = \frac{e^m\sqrt{a}\,a^2\Big(-2\sec(dx+c)^m\sqrt{\tan(dx+c)\,i+1}\,i-\Big(\int\sec(dx+c)^m\sqrt{\tan(dx+c)\,i+1}\ \tan$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
(e**m*sqrt(a)*a**2*( - 2*sec(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*i - int(
sec(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2,x)*d + 2*int(sec(
c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*d*i*m + 3*int(sec(c +
 d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*d*i))/d
```

3.459 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 107

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx=\frac{i2^{\frac{3+m}{2}}a\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1-m),\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+\ldots}{dm}$$

output
```
I*2^(3/2+1/2*m)*a*hypergeom([1/2*m, -1/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*
x+c))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(-1/2-1/2*m)*(a+I*a*tan(d*x+c))^(1
/2)/d/m
```

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.74

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{i2^{\frac{3}{2}+m}e^{i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+m}\text{Hypergeometric2F1}\left(\frac{3}{2}+m,\frac{3+m}{2},\frac{5+m}{2},-e^{2i(c+dx)}\ldots\right.}{d(3+m)(\cos(dx)+i\sin(dx))^{3/2}}$$

input
```
Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*2^(3/2 + m)*E^(I*(c + 2*d*x))*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x))))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^(1/2 + m)*Hyperg
eometric2F1[3/2 + m, (3 + m)/2, (5 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d
*x]^(-3/2 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(3/2))/(d*(3 + m)
*(Cos[d*x] + I*Sin[d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^m dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+3}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+3}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a2^{\frac{m}{2}-1}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int\left(\frac{1}{2}-\frac{1}{2}i\tan(c+dx)\right)^{\frac{m-2}{2}}(i\tan(c+dx}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{i2^{m/2}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{m+3}{2}-\frac{m}{2}}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{2-m}{2},\frac{m+3}{2},\frac{m+5}{2}\right.}{d(m+3)}$$

input
```
Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*2^(m/2)*Hypergeometric2F1[(2 - m)/2, (3 + m)/2, (5 + m)/2, (1 + I*Ta
n[c + d*x])/2]*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(-1/2*m + (3 + m)
/2))/(d*(3 + m)*(1 - I*Tan[c + d*x])^(m/2))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\sec(dx+c))^m (a+ia\tan(dx+c))^{\frac{3}{2}} dx$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x)
```

Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(2*sqrt(2)*a*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(3*I*d*x + 3*I*c)/(e^(2*I*d*x + 2*I*c) +
1), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx = \int (e\sec(c+dx))^m\,(ia(\tan(c+dx)-i))^{\frac{3}{2}}\,dx$$

input
```
integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((e*sec(c + d*x))**m*(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)*(e*sec(d*x + c))^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^m (a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}\,dx$$

input
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2e^m\sqrt{a}\,ai\left(-\sec(dx+c)^m\sqrt{\tan(dx+c)\,i+1}+\left(\int \sec(dx+c)^m\sqrt{\tan(dx+c)\,i+1}\,\tan\right.\right.}{d}$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(2*e**m*sqrt(a)*a*i*( - sec(c + d*x)**m*sqrt(tan(c + d*x)*i + 1) + int(sec
(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*d*m + int(sec(c + d*
x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*d))/d
```

3.460 $\int (e\sec(c+dx))^m \sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 107

$$\int (e\sec(c+dx))^m \sqrt{a+ia\tan(c+dx)}\,dx = \frac{i2^{\frac{1+m}{2}} a \,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{\frac{1-m}{2}}}{dm\sqrt{a+ia\tan(c+dx)}}$$

output
```
I*2^(1/2+1/2*m)*a*hypergeom([1/2*m, 1/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x
+c))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(1/2-1/2*m)/d/m/(a+I*a*tan(d*x+c))^
(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.63

$$\int (e\sec(c+dx))^m \sqrt{a+ia\tan(c+dx)}\,dx = -\frac{i2^{\frac{1}{2}+m}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+m}\text{Hypergeometric2F1}\left(\frac{1}{2}+m,\frac{1+m}{2},\frac{3+m}{2},-e^{2i(c+dx)}\right)\sec}{d(1+m)\sqrt{\cos(dx)+i\sin(dx)}}$$

input
```
Integrate[(e*Sec[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*2^(1/2 + m)*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)
)))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^(1/2 + m)*Hypergeometric2F1[1/2 +
m, (1 + m)/2, (3 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-1/2 - m)*(e*
Sec[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]])/(d*(1 + m)*Sqrt[Cos[d*x] + I*S
in[d*x]])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^m dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+1}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+1}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a2^{\frac{m}{2}-1}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int\left(\frac{1}{2}-\frac{1}{2}i\tan(c+dx)\right)^{\frac{m-2}{2}}(i\tan(c+dx}{d}$$

$$\Big\downarrow 79$$

$$-\frac{i2^{m/2}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{m+1}{2}-\frac{m}{2}}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(\frac{2-m}{2},\frac{m+1}{2},\frac{m+3}{2}\right.}{d(m+1)}$$

input
```
Int[(e*Sec[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*2^(m/2)*Hypergeometric2F1[(2 - m)/2, (1 + m)/2, (3 + m)/2, (1 + I*Ta
n[c + d*x])/2]*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(-1/2*m + (1 + m)
/2))/(d*(1 + m)*(1 - I*Tan[c + d*x])^(m/2))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e \sec (dx + c))^m \sqrt{a + ia \tan (dx + c)}dx$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int (e \sec(c+dx))^m \sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan(dx+c)+a}(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(2)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \int (e\sec{(c+dx)})^m\sqrt{ia(\tan{(c+dx)}-i)}\,dx$$

input
```
integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((e*sec(c + d*x))**m*sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [F]

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan{(dx+c)}+a}(e\sec{(dx+c)})^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx=\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^m\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx$$
$$=\frac{e^m\sqrt{a}\,i\left(-2\sec\left(dx+c\right)^m\sqrt{\tan\left(dx+c\right)i+1}+2\left(\int\sec\left(dx+c\right)^m\sqrt{\tan\left(dx+c\right)i+1}\,\tan\left(dx+c\right)dx\right)\right.}{d}$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(e**m*sqrt(a)*i*( - 2*sec(c + d*x)**m*sqrt(tan(c + d*x)*i + 1) + 2*int(sec
(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*d*m + int(sec(c + d*
x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*d))/d
```

3.461 $\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 107

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i2^{\frac{1}{2}(-1+m)}a\,\text{Hypergeometric2F1}\left(\frac{3-m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))}{dm(a+ia\tan(c+dx))^{3/2}}$$

output
```
I*2^(-1/2+1/2*m)*a*hypergeom([1/2*m, 3/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*
x+c))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(3/2-1/2*m)/d/m/(a+I*a*tan(d*x+c))
^(3/2)
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.63

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i2^{-\frac{1}{2}+m}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{-\frac{1}{2}+m}\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+m),-\frac{1}{2}+m,\frac{1+m}{2},-e^{2i(c+}\right.}{d\sqrt{e^{idx}}(-1+m)\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(e*Sec[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*2^(-1/2 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-1/2 + m)*
(1 + E^((2*I)*(c + d*x)))^(-1/2 + m)*Hypergeometric2F1[(-1 + m)/2, -1/2 +
m, (1 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(1/2 - m)*(e*Sec[c + d*x]
)^m*Sqrt[Cos[d*x] + I*Sin[d*x]])/(d*Sqrt[E^(I*d*x)]*(-1 + m)*Sqrt[a + I*a*
Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-1}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-1}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+$$
$$d$$

$$\Big\downarrow \text{80}$$

$$\frac{a2^{\frac{m-3}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{m-1}{2}-\frac{m}{2}}(e\sec(c+dx))^m\int\left(\frac{1}{2}i\tan(c+\right.}{d}$$

$$\Big\downarrow \text{79}$$

$$\frac{i2^{\frac{m-3}{2}+1}(1+i\tan(c+dx))^{\frac{1-m}{2}}(a+ia\tan(c+dx))^{\frac{m-1}{2}-\frac{m}{2}}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{3-m}{2},\frac{m}{2},\frac{m+2}{2},\right.}{dm}$$

input
```
Int[(e*Sec[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(I*2^(1 + (-3 + m)/2)*Hypergeometric2F1[(3 - m)/2, m/2, (2 + m)/2, (1 - I*
Tan[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2)*(a +
I*a*Tan[c + d*x])^((-1 + m)/2 - m/2))/(d*m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(e\sec(dx+c))^m}{\sqrt{a+ia\tan(dx+c)}}dx$$

input
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^m}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(1/2*sqrt(2)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(-I*d*x - I*c)/
a, x)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec{(c+dx)})^m}{\sqrt{ia\,(\tan{(c+dx)}-i)}}\,dx$$

input
```
integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((e*sec(c + d*x))**m/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [F]

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec{(dx+c)})^m}{\sqrt{i\,a\tan{(dx+c)}+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^m/sqrt(I*a*tan(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2e^m\sqrt{a}\,i\left(-\sec\left(dx+c\right)^m\sqrt{\tan\left(dx+c\right)i+1}+\left(\int\frac{\sec(dx+c)^m\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)\tan\left(dx+c\right)^2dm\right.}{}$$

input
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*e**m*sqrt(a)*i*( - sec(c + d*x)**m*sqrt(tan(c + d*x)*i + 1) + int((sec(
c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x))/(tan(c + d*x)**2 + 1),x
)*tan(c + d*x)**2*d*m - 2*int((sec(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*ta
n(c + d*x))/(tan(c + d*x)**2 + 1),x)*tan(c + d*x)**2*d + int((sec(c + d*x)
**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*d*m -
2*int((sec(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x))/(tan(c + d*x
)**2 + 1),x)*d))/(a*d*(tan(c + d*x)**2 + 1))
```

3.462 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 107

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i2^{\frac{1}{2}(-3+m)}a\,\text{Hypergeometric2F1}\left(\frac{5-m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec}{dm(a+ia\tan(c+dx))^{5/2}}$$

output
```
I*2^(-3/2+1/2*m)*a*hypergeom([1/2*m, 5/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*
x+c))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(5/2-1/2*m)/d/m/(a+I*a*tan(d*x+c))
^(5/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.95 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.66

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{i2^{-\frac{3}{2}+m}e^{-2i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^3\text{Hypergeometric2F1}\left(1,1-\frac{m}{2},\frac{1}{2}(-1+m),-e^{2i(}\right.}{d(-3+m)(a+ia\tan(c+dx))^{3/2}}$$

input
```
Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*2^(-3/2 + m)*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x
))))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^3*Hypergeometric2F1[1, 1 - m/2, (
-1 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(3/2 - m)*(e*Sec[c + d*x])^m
*(Cos[d*x] + I*Sin[d*x])^(3/2))/(d*E^((2*I)*(c + 2*d*x))*(-3 + m)*(a + I*a
*Tan[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}dx$$
$$\Big\downarrow \text{3986}$$
$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-3}{2}}dx$$
$$\Big\downarrow \text{3042}$$
$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-3}{2}}dx$$
$$\Big\downarrow \text{4006}$$
$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$
$$\Big\downarrow \text{80}$$

$$\frac{2^{\frac{m-5}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{m-1}{2}-\frac{m}{2}}(e\sec(c+dx))^m\int\left(\frac{1}{2}i\tan(c+\right.}{d}$$

$$\Big\downarrow \text{79}$$

$$\frac{i2^{\frac{m-5}{2}+1}(1+i\tan(c+dx))^{\frac{1-m}{2}}(a+ia\tan(c+dx))^{\frac{m-1}{2}-\frac{m}{2}}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(\frac{5-m}{2},\frac{m}{2},\frac{m+2}{2},\right.}{adm}$$

input
```
Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(I*2^(1 + (-5 + m)/2)*Hypergeometric2F1[(5 - m)/2, m/2, (2 + m)/2, (1 - I*
Tan[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2)*(a +
I*a*Tan[c + d*x])^((-1 + m)/2 - m/2))/(a*d*m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(e\sec(dx+c))^m}{(a+ia\tan(dx+c))^{\frac{3}{2}}}dx$$

input
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x)
```

Fricas [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(e\sec(dx+c))^m}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(1/4*sqrt(2)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c
) + 1)*e^(-3*I*d*x - 3*I*c)/a^2, x)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(e\sec(c+dx))^m}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((e*sec(c + d*x))**m/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(e\sec(dx+c))^m}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^{3/2}}\,dx$$

input
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(e\sec\left(dx+c\right))^m}{(a+ia\tan\left(dx+c\right))^{\frac{3}{2}}}dx$$

input
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x)
```

3.463 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 107

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{i2^{\frac{1}{2}(-5+m)}a\,\text{Hypergeometric2F1}\left(\frac{7-m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m \cdots}{dm(a+ia\tan(c+dx))^{7/2}}$$

output
```
I*2^(-5/2+1/2*m)*a*hypergeom([1/2*m, 7/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*
x+c))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(7/2-1/2*m)/d/m/(a+I*a*tan(d*x+c))
^(7/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.36 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.66

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx=$$
$$-\frac{i2^{-\frac{5}{2}+m}e^{-3i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^4\text{Hypergeometric2F1}\left(1,1-\frac{m}{2},\frac{1}{2}(-3+m),-e^{2i(\cdots}\right.}{d(-5+m)(a+ia\tan(c+dx))^{5/2}}$$

input
```
Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*2^(-5/2 + m)*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x
))))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^4*Hypergeometric2F1[1, 1 - m/2, (
-3 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(5/2 - m)*(e*Sec[c + d*x])^m
*(Cos[d*x] + I*Sin[d*x])^(5/2))/(d*E^((3*I)*(c + 2*d*x))*(-5 + m)*(a + I*a
*Tan[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-5}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-5}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{\frac{m-7}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{m-1}{2}-\frac{m}{2}}(e\sec(c+dx))^m\int\left(\frac{1}{2}i\tan(c+d\right.}{ad}$$

$$\Big\downarrow 79$$

$$\frac{i2^{\frac{m-7}{2}+1}(1+i\tan(c+dx))^{\frac{1-m}{2}}(a+ia\tan(c+dx))^{\frac{m-1}{2}-\frac{m}{2}}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{7-m}{2},\frac{m}{2},\frac{m+2}{2},\right.}{a^2dm}$$

input
```
Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(I*2^(1 + (-7 + m)/2)*Hypergeometric2F1[(7 - m)/2, m/2, (2 + m)/2, (1 - I*
Tan[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2)*(a +
I*a*Tan[c + d*x])^((-1 + m)/2 - m/2))/(a^2*d*m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(e\sec (dx+c))^m}{(a+ia\tan (dx+c))^{\frac{5}{2}}}dx$$

input
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x)
```

Fricas [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec (dx+c))^m}{(i\,a\tan (dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(1/8*sqrt(2)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c
) + 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-5*I*d*x - 5*I*c)/a^3, x)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec{(c+dx)})^m}{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((e*sec(c + d*x))**m/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec{(dx+c)})^m}{(i\,a\tan{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{e^m\left(\int \frac{\sec(dx+c)^m}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx\right)}{\sqrt{a}\,a^2}$$

input
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - e**m*int(sec(c + d*x)**m/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2
*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x))/(
sqrt(a)*a**2)
```

3.464 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 26, antiderivative size = 109

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \frac{i2^{\frac{m}{2}+n}a\,\text{Hypergeometric2F1}\left(\frac{m}{2},1-\frac{m}{2}-n,\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+d}{dm}$$

output
```
I*2^(1/2*m+n)*a*hypergeom([1/2*m, 1-1/2*m-n],[1+1/2*m],1/2-1/2*I*tan(d*x+c
))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(1-1/2*m-n)*(a+I*a*tan(d*x+c))^(-1+n)
/d/m
```

Mathematica [A] (verified)

Time = 6.24 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.51

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{m+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+n}\left(1+e^{2i(c+dx)}\right)^{m+n}\text{Hypergeometric2F1}\left(\frac{m}{2}+n,m+n,1+\frac{m}{2}+n,-e^{2i}\right.}{d(m+2n)}$$

input
```
Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(m + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
(m + n)*(1 + E^((2*I)*(c + d*x)))^(m + n)*Hypergeometric2F1[m/2 + n, m + n
, 1 + m/2 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-m - n)*(e*Sec[c + d*x]
)^m*(a + I*a*Tan[c + d*x])^n)/(d*(m + 2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^mdx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m}{2}+n}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m}{2}+n}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int(a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a2^{\frac{m}{2}-1}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int\left(\frac{1}{2}-\frac{1}{2}i\tan(c+dx)\right)^{\frac{m-2}{2}}(i\tan(c+dx}{d}$$

$$\Big\downarrow 79$$

$$-\frac{i2^{m/2}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^n(e\sec(c+dx))^m \text{Hypergeometric2F1}\left(\frac{2-m}{2},\frac{m}{2}+n,\frac{m}{2}+n+\right.}{d(m+2n)}$$

input
```
Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(m/2)*Hypergeometric2F1[(2 - m)/2, m/2 + n, 1 + m/2 + n, (1 + I*Ta
n[c + d*x])/2]*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^n)/(d*(m + 2*n)*(
1 - I*Tan[c + d*x])^(m/2))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\sec(dx+c))^m (a+ia\tan(dx+c))^n \, dx$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n \, dx = \int (e\sec(dx+c))^m (i\,a\tan(dx+c)+a)^n \, dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(I*d*n*x + I*
c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e)),
x)
```

Sympy [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n \, dx = \int (e\sec(c+dx))^m (ia(\tan(c+dx)-i))^n \, dx$$

input
```
integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**m*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^m\,(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^m*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^m\,(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^m*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^m (a+a\tan(c+d\,x)\ 1\mathrm{i})^n\,dx$$

input
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = e^m\left(\int \sec\left(dx+c\right)^m\left(\tan\left(dx+c\right)ai + a\right)^n dx\right)$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

output
```
e**m*int(sec(c + d*x)**m*(tan(c + d*x)*a*i + a)**n,x)
```

3.465 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{4i(a+ia\tan(c+dx))^{3+n}}{a^3d(3+n)} + \frac{4i(a+ia\tan(c+dx))^{4+n}}{a^4d(4+n)} - \frac{i(a+ia\tan(c+dx))^{5+n}}{a^5d(5+n)}$$

output
```
-4*I*(a+I*a*tan(d*x+c))^(3+n)/a^3/d/(3+n)+4*I*(a+I*a*tan(d*x+c))^(4+n)/a^4
/d/(4+n)-I*(a+I*a*tan(d*x+c))^(5+n)/a^5/d/(5+n)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i(a+ia\tan(c+dx))^{3+n}\left(\frac{4a^2}{3+n}-\frac{4a(a+ia\tan(c+dx))}{4+n}+\frac{(a+ia\tan(c+dx))^2}{5+n}\right)}{a^5d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*(a + I*a*Tan[c + d*x])^(3 + n)*((4*a^2)/(3 + n) - (4*a*(a + I*a*Tan[
c + d*x]))/(4 + n) + (a + I*a*Tan[c + d*x])^2/(5 + n)))/(a^5*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+ia\tan(c+dx))^ndx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{n+2}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(4a^2(i\tan(c+dx)a+a)^{n+2}-4a(i\tan(c+dx)a+a)^{n+3}+(i\tan(c+dx)a+a)^{n+4}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{4a^2(a+ia\tan(c+dx))^{n+3}}{n+3}-\frac{4a(a+ia\tan(c+dx))^{n+4}}{n+4}+\frac{(a+ia\tan(c+dx))^{n+5}}{n+5}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*((4*a^2*(a + I*a*Tan[c + d*x])^(3 + n))/(3 + n) - (4*a*(a + I*a*Tan[
c + d*x])^(4 + n))/(4 + n) + (a + I*a*Tan[c + d*x])^(5 + n)/(5 + n)))/(a^5
*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(91) = 182.

Time = 1.84 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.80

method	result
derivativedivides	$\frac{\tan(dx+c)^5 e^{n\ln(a+ia\tan(dx+c))}}{d(5+n)} + \frac{(n^2+15n+60)\tan(dx+c)e^{n\ln(a+ia\tan(dx+c))}}{d(3+n)(4+n)(5+n)} - \frac{in\tan(dx+c)^4 e^{n\ln(a+ia\tan(d}}{(nd+4d)(5+n)}$
default	$\frac{\tan(dx+c)^5 e^{n\ln(a+ia\tan(dx+c))}}{d(5+n)} + \frac{(n^2+15n+60)\tan(dx+c)e^{n\ln(a+ia\tan(dx+c))}}{d(3+n)(4+n)(5+n)} - \frac{in\tan(dx+c)^4 e^{n\ln(a+ia\tan(d}}{(nd+4d)(5+n)}$
risch	Expression too large to display

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
1/d/(5+n)*tan(d*x+c)^5*exp(n*ln(a+I*a*tan(d*x+c)))+(n^2+15*n+60)/d/(3+n)/(
4+n)/(5+n)*tan(d*x+c)*exp(n*ln(a+I*a*tan(d*x+c)))-I*n/(d*n+4*d)/(5+n)*tan(
d*x+c)^4*exp(n*ln(a+I*a*tan(d*x+c)))+2*(n^2+11*n+20)/d/(3+n)/(4+n)/(5+n)*t
an(d*x+c)^3*exp(n*ln(a+I*a*tan(d*x+c)))-I*(n^2+11*n+32)/d/(3+n)/(4+n)/(5+n
)*exp(n*ln(a+I*a*tan(d*x+c)))-2*I*n*(n+7)/d/(3+n)/(4+n)/(5+n)*tan(d*x+c)^2
*exp(n*ln(a+I*a*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(85) = 170$.

Time = 0.10 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.55

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx =$$

$$-\frac{32\left(2\left(i\,n+5i\right)e^{(8i\,dx}\cdots\right.}{dn^3+12\,dn^2+47\,dn+(dn^3+12\,dn^2+47\,dn+60\,d)e^{(10i\,dx+10i\,c)}+5\,(dn^3+12\,dn^2+47\,dn+60\,d)e\cdots}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
-32*(2*(I*n + 5*I)*e^(8*I*d*x + 8*I*c) + (I*n^2 + 9*I*n + 20*I)*e^(6*I*d*x
 + 6*I*c) + 2*I*e^(10*I*d*x + 10*I*c))*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*
x + 2*I*c) + 1))^n/(d*n^3 + 12*d*n^2 + 47*d*n + (d*n^3 + 12*d*n^2 + 47*d*n
 + 60*d)*e^(10*I*d*x + 10*I*c) + 5*(d*n^3 + 12*d*n^2 + 47*d*n + 60*d)*e^(8
*I*d*x + 8*I*c) + 10*(d*n^3 + 12*d*n^2 + 47*d*n + 60*d)*e^(6*I*d*x + 6*I*c
) + 10*(d*n^3 + 12*d*n^2 + 47*d*n + 60*d)*e^(4*I*d*x + 4*I*c) + 5*(d*n^3 +
 12*d*n^2 + 47*d*n + 60*d)*e^(2*I*d*x + 2*I*c) + 60*d)
```

Sympy [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n \sec^6(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**6, x)
```

Maxima [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n \sec(dx+c)^6\,dx$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^6, x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(85) = 170$.

Time = 0.62 (sec) , antiderivative size = 623, normalized size of antiderivative = 6.42

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
(((I*a*tan(d*x + c) + a)^n*n^4*tan(d*x + c)^5 - I*(I*a*tan(d*x + c) + a)^n
*n^4*tan(d*x + c)^4 + 10*(I*a*tan(d*x + c) + a)^n*n^3*tan(d*x + c)^5 - 6*I
*(I*a*tan(d*x + c) + a)^n*n^3*tan(d*x + c)^4 + 35*(I*a*tan(d*x + c) + a)^n
*n^2*tan(d*x + c)^5 + 4*(I*a*tan(d*x + c) + a)^n*n^3*tan(d*x + c)^3 - 11*I
*(I*a*tan(d*x + c) + a)^n*n^2*tan(d*x + c)^4 + 50*(I*a*tan(d*x + c) + a)^n
*n*tan(d*x + c)^5 + 12*(I*a*tan(d*x + c) + a)^n*n^2*tan(d*x + c)^3 - 6*I*(
I*a*tan(d*x + c) + a)^n*n*tan(d*x + c)^4 + 24*(I*a*tan(d*x + c) + a)^n*tan
(d*x + c)^5 + 12*I*(I*a*tan(d*x + c) + a)^n*n^2*tan(d*x + c)^2 + 8*(I*a*ta
n(d*x + c) + a)^n*n*tan(d*x + c)^3 + 12*I*(I*a*tan(d*x + c) + a)^n*n*tan(d
*x + c)^2 - 24*(I*a*tan(d*x + c) + a)^n*n*tan(d*x + c) - 24*I*(I*a*tan(d*x
 + c) + a)^n)/(n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120) + 2*((I*a*ta
n(d*x + c) + a)^n*n^2*tan(d*x + c)^3 - I*(I*a*tan(d*x + c) + a)^n*n^2*tan(
d*x + c)^2 + 3*(I*a*tan(d*x + c) + a)^n*n*tan(d*x + c)^3 - I*(I*a*tan(d*x
+ c) + a)^n*n*tan(d*x + c)^2 + 2*(I*a*tan(d*x + c) + a)^n*tan(d*x + c)^3 +
 2*(I*a*tan(d*x + c) + a)^n*n*tan(d*x + c) + 2*I*(I*a*tan(d*x + c) + a)^n)
/(n^3 + 6*n^2 + 11*n + 6) - I*(I*a*tan(d*x + c) + a)^(n + 1)/(a*(n + 1)))/
d
```

Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.73

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{e^{-c\,5i-d\,x\,5i}\,\left(a+\frac{a\,\sin(c+d\,x)\,1i}{\cos(c+d\,x)}\right)^n\,\left(\frac{64\,e^{c\,10i+d\,x\,10i}}{d\,(n^3\,1i+n^2\,12i+n\,47i+60i)}+\frac{e^{c\,6i+d\,x\,6i}\,(32\,n^2+288\,n+640)}{d\,(n^3\,1i+n^2\,12i+n\,47i+60i)}+\frac{e^{c\,8i+d\,x\,8i}\,(64\,n+320)}{d\,(n^3\,1i+n^2\,12i+n\,47i+60i)}\right)}{32\cos\left(c+d\,x\right)^5}$$

input
```
int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^6,x)
```

output
```
(exp(- c*5i - d*x*5i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^n*((64*exp(c*
10i + d*x*10i))/(d*(n*47i + n^2*12i + n^3*1i + 60i)) + (exp(c*6i + d*x*6i)
*(288*n + 32*n^2 + 640))/(d*(n*47i + n^2*12i + n^3*1i + 60i)) + (exp(c*8i
+ d*x*8i)*(64*n + 320))/(d*(n*47i + n^2*12i + n^3*1i + 60i))))/(32*cos(c +
 d*x)^5)
```

Reduce [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (\tan(dx+c)\,ai+a)^n \sec(dx+c)^6\,dx$$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*sec(c + d*x)**6,x)
```

3.466 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{2i(a+ia\tan(c+dx))^{2+n}}{a^2d(2+n)} + \frac{i(a+ia\tan(c+dx))^{3+n}}{a^3d(3+n)}$$

output
```
-2*I*(a+I*a*tan(d*x+c))^(2+n)/a^2/d/(2+n)+I*(a+I*a*tan(d*x+c))^(3+n)/a^3/d
/(3+n)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i\left(\frac{2a(a+ia\tan(c+dx))^{2+n}}{2+n}-\frac{(a+ia\tan(c+dx))^{3+n}}{3+n}\right)}{a^3d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*((2*a*(a + I*a*Tan[c + d*x])^(2 + n))/(2 + n) - (a + I*a*Tan[c + d*x
])^(3 + n)/(3 + n)))/(a^3*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4(a+ia\tan(c+dx))^n dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{n+1}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(2a(i\tan(c+dx)a+a)^{n+1}-(i\tan(c+dx)a+a)^{n+2}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{2a(a+ia\tan(c+dx))^{n+2}}{n+2}-\frac{(a+ia\tan(c+dx))^{n+3}}{n+3}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*((2*a*(a + I*a*Tan[c + d*x])^(2 + n))/(2 + n) - (a + I*a*Tan[c + d*x
])^(3 + n)/(3 + n)))/(a^3*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(61) = 122$.

Time = 1.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.31

method	result
derivativedivides	$\frac{\tan(dx+c)^3 e^{n\ln(a+ia\tan(dx+c))}}{d(3+n)}+\frac{(n+6)\tan(dx+c)e^{n\ln(a+ia\tan(dx+c))}}{d(2+n)(3+n)}-\frac{i(4+n)e^{n\ln(a+ia\tan(dx+c))}}{d(2+n)(3+n)}-\frac{in\tan}{}$
default	$\frac{\tan(dx+c)^3 e^{n\ln(a+ia\tan(dx+c))}}{d(3+n)}+\frac{(n+6)\tan(dx+c)e^{n\ln(a+ia\tan(dx+c))}}{d(2+n)(3+n)}-\frac{i(4+n)e^{n\ln(a+ia\tan(dx+c))}}{d(2+n)(3+n)}-\frac{in\tan}{}$
risch	Expression too large to display

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
1/d/(3+n)*tan(d*x+c)^3*exp(n*ln(a+I*a*tan(d*x+c)))+(n+6)/d/(2+n)/(3+n)*tan
(d*x+c)*exp(n*ln(a+I*a*tan(d*x+c)))-I*(4+n)/d/(2+n)/(3+n)*exp(n*ln(a+I*a*t
an(d*x+c)))-I*n/d/(2+n)/(3+n)*tan(d*x+c)^2*exp(n*ln(a+I*a*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(57) = 114$.

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.18

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{8\left((in+3i)e^{(4i\,dx+4i\,c)}+i\,e^{(6i\,dx+6i\,c)}\right)\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^n}{dn^2+5\,dn+(dn^2+5\,dn+6\,d)e^{(6i\,dx+6i\,c)}+3\,(dn^2+5\,dn+6\,d)e^{(4i\,dx+4i\,c)}+3\,(dn^2+5\,dn+6\,d)e^{(2i\,c}}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
-8*((I*n + 3*I)*e^(4*I*d*x + 4*I*c) + I*e^(6*I*d*x + 6*I*c))*(2*a*e^(2*I*d
*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n/(d*n^2 + 5*d*n + (d*n^2 + 5*d*n +
 6*d)*e^(6*I*d*x + 6*I*c) + 3*(d*n^2 + 5*d*n + 6*d)*e^(4*I*d*x + 4*I*c) +
3*(d*n^2 + 5*d*n + 6*d)*e^(2*I*d*x + 2*I*c) + 6*d)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n \sec^4(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**4, x)
```

Maxima [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n \sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^4, x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(57) = 114$.

Time = 0.49 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.17

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{\frac{(i\,a\tan(dx+c)+a)^n n^2\tan(dx+c)^3 - i\,(i\,a\tan(dx+c)+a)^n n^2\tan(dx+c)^2 + 3\,(i\,a\tan(dx+c)+a)^n n\tan(dx+c)^3 - i\,(i\,a\tan(dx+c)+a)^n n\tan(dx+}{n^3+6\,n^2+11\,n+6}}{d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
(((I*a*tan(d*x + c) + a)^n*n^2*tan(d*x + c)^3 - I*(I*a*tan(d*x + c) + a)^n
*n^2*tan(d*x + c)^2 + 3*(I*a*tan(d*x + c) + a)^n*n*tan(d*x + c)^3 - I*(I*a
*tan(d*x + c) + a)^n*n*tan(d*x + c)^2 + 2*(I*a*tan(d*x + c) + a)^n*tan(d*x
 + c)^3 + 2*(I*a*tan(d*x + c) + a)^n*n*tan(d*x + c) + 2*I*(I*a*tan(d*x + c
) + a)^n)/(n^3 + 6*n^2 + 11*n + 6) - I*(I*a*tan(d*x + c) + a)^(n + 1)/(a*(
n + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.32

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx =$$
$$-\frac{4\left(\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}\right)^n\left(n\,3\mathrm{i}+\cos\left(2\,c+2\,d\,x\right)\,15\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,6\mathrm{i}+\cos\left(6\,c+6\,d\,x\right)\right.}{d\,\left(n^2+5\,n+\right.}$$

input
```
int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^4,x)
```

output
```
-(4*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) +
1))^n*(n*3i + cos(2*c + 2*d*x)*15i + cos(4*c + 4*d*x)*6i + cos(6*c + 6*d*x
)*1i - 9*sin(2*c + 2*d*x) - 6*sin(4*c + 4*d*x) - sin(6*c + 6*d*x) + n*cos(
2*c + 2*d*x)*4i + n*cos(4*c + 4*d*x)*1i - 2*n*sin(2*c + 2*d*x) - n*sin(4*c
 + 4*d*x) + 10i))/(d*(5*n + n^2 + 6)*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*
d*x) + cos(6*c + 6*d*x) + 10))
```

Reduce [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \left(\tan\left(dx+c\right)ai+a\right)^n \sec\left(dx+c\right)^4 dx$$

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*sec(c + d*x)**4,x)
```

3.467 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 32

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx=-\frac{i(a+ia\tan(c+dx))^{1+n}}{ad(1+n)}$$

output
```
-I*(a+I*a*tan(d*x+c))^(1+n)/a/d/(1+n)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx=-\frac{i(a+ia\tan(c+dx))^{1+n}}{ad(1+n)}$$

input
```
Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(1 + n))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+ia\tan(c+dx))^ndx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (i\tan(c+dx)a+a)^nd(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$-\frac{i(a+ia\tan(c+dx))^{n+1}}{ad(n+1)}$$

input
```
Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(1 + n))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result
derivativedivides	$-\frac{i(a+ia\tan(dx+c))^{1+n}}{ad(1+n)}$
default	$-\frac{i(a+ia\tan(dx+c))^{1+n}}{ad(1+n)}$
risch	$-\frac{2ia^n\left(\mathrm{e}^{2i(dx+c)}+1\right)^{-n}\left(\mathrm{e}^{i(dx+c)}\right)^{2n}2^n\mathrm{e}^{\frac{i\left(-\operatorname{csgn}\left(\frac{ia\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}\right)^3\pi n+\operatorname{csgn}\left(\frac{ia\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}\right)^2\operatorname{csgn}(ia)\pi n+\operatorname{csgn}\left(\frac{ia\,\mathrm{e}^{2i(dx+}}{\mathrm{e}^{2i(dx+c)}}\right.}{}}}{}$

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
-I*(a+I*a*tan(d*x+c))^(1+n)/a/d/(1+n)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(28) = 56.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.88

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{2i\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^n e^{(2i\,dx+2i\,c)}}{dn+(dn+d)e^{(2i\,dx+2i\,c)}+d}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
-2*I*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(2*I*d*x + 2*
I*c)/(d*n + (d*n + d)*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan{(c+dx)}-i))^n \sec^2{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i\,(i\,a\tan{(dx+c)}+a)^{n+1}}{ad(n+1)}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
-I*(I*a*tan(d*x + c) + a)^(n + 1)/(a*d*(n + 1))
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i\,(i\,a\tan{(dx+c)}+a)^{n+1}}{ad(n+1)}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
-I*(I*a*tan(d*x + c) + a)^(n + 1)/(a*d*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.25

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx$$
$$=\frac{2\,\left(\cos\left(2\,d\,x\right)+\sin\left(2\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(2\,c\right)+\sin\left(2\,c\right)\,1{}\mathrm{i}\right)\,{\left(\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}\right)}^n}{d\,\left(n+1\right)\,\left(\cos\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}-\sin\left(2\,c+2\,d\,x\right)+1{}\mathrm{i}\right)}$$

input
```
int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^2,x)
```

output
```
(2*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2*c)*1i)*((a*(cos(2*c + 2*
d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^n)/(d*(n + 1)*(co
s(2*c + 2*d*x)*1i - sin(2*c + 2*d*x) + 1i))
```

Reduce [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx=\int \left(\tan\left(dx+c\right)ai+a\right)^{n}\sec\left(dx+c\right)^{2}dx$$

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*sec(c + d*x)**2,x)
```

3.468 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 56

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{ia\,\text{Hypergeometric2F1}\left(2,-1+n,n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-1+n}}{4d(1-n)}$$

output
```
1/4*I*a*hypergeom([2, -1+n],[n],1/2+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^(
-1+n)/d/(1-n)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{ia\,\text{Hypergeometric2F1}\left(2,-1+n,n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-1+n}}{4d(-1+n)}$$

input
```
Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/4*I)*a*Hypergeometric2F1[2, -1 + n, n, (1 + I*Tan[c + d*x])/2]*(a + I
*a*Tan[c + d*x])^(-1 + n))/(d*(-1 + n))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{(i\tan(c+dx)a+a)^{n-2}}{(a-ia\tan(c+dx))^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{78}$$

$$\frac{ia(a+ia\tan(c+dx))^{n-1}\text{Hypergeometric2F1}\left(2,n-1,n,\frac{i\tan(c+dx)a+a}{2a}\right)}{4d(1-n)}$$

input
```
Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((I/4)*a*Hypergeometric2F1[2, -1 + n, n, (a + I*a*Tan[c + d*x])/(2*a)]*(a
+ I*a*Tan[c + d*x])^(-1 + n))/(d*(1 - n))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [F]

$$\int \cos (dx + c)^2 (a + ia \tan (dx + c))^n \, dx$$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos^2(c + dx)(a + ia \tan(c + dx))^n \, dx = \int (i\, a \tan (dx + c) + a)^n \cos (dx + c)^2 \, dx$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(1/4*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(4*I
*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*e^(-2*I*d*x - 2*I*c), x)
```

Sympy [F]

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n\cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*cos(c + d*x)**2, x)
```

Maxima [F]

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \cos\left(c+d\,x\right)^2\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^n\,dx$$

input
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \left(\tan\left(dx+c\right)ai+a\right)^n\cos\left(dx+c\right)^2dx$$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*cos(c + d*x)**2,x)
```

3.469 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{ia^2\,\text{Hypergeometric2F1}\left(3,-2+n,-1+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-2+n}}{8d(2-n)}$$

output
```
1/8*I*a^2*hypergeom([3, -2+n],[-1+n],1/2+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+
c))^(-2+n)/d/(2-n)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{ia^2\,\text{Hypergeometric2F1}\left(3,-2+n,-1+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-2+n}}{8d(-2+n)}$$

input
```
Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/8*I)*a^2*Hypergeometric2F1[3, -2 + n, -1 + n, (1 + I*Tan[c + d*x])/2]
*(a + I*a*Tan[c + d*x])^(-2 + n))/(d*(-2 + n))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{(i\tan(c+dx)a+a)^{n-3}}{(a-ia\tan(c+dx))^3}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{78}$$

$$\frac{ia^2(a+ia\tan(c+dx))^{n-2}\,\text{Hypergeometric2F1}\left(3,n-2,n-1,\frac{i\tan(c+dx)a+a}{2a}\right)}{8d(2-n)}$$

input
```
Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((I/8)*a^2*Hypergeometric2F1[3, -2 + n, -1 + n, (a + I*a*Tan[c + d*x])/(2*
a)]*(a + I*a*Tan[c + d*x])^(-2 + n))/(d*(2 - n))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [F]

$$\int \cos(dx+c)^4 (a+ia\tan(dx+c))^n\,dx$$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(1/16*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(8*
I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*
d*x + 2*I*c) + 1)*e^(-4*I*d*x - 4*I*c), x)
```

Sympy [F]

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n\cos^4(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*cos(c + d*x)**4, x)
```

Maxima [F]

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

Giac [F]

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \cos\left(c+d\,x\right)^4\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^n\,dx$$

input
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \left(\tan\left(dx+c\right)ai+a\right)^n\cos\left(dx+c\right)^4dx$$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*cos(c + d*x)**4,x)
```

3.470 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{ia^3\,\text{Hypergeometric2F1}\left(4,-3+n,-2+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-3+n}}{16d(3-n)}$$

output
```
1/16*I*a^3*hypergeom([4, -3+n],[-2+n],1/2+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x
+c))^(-3+n)/d/(3-n)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{ia^3\,\text{Hypergeometric2F1}\left(4,-3+n,-2+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-3+n}}{16d(-3+n)}$$

input
```
Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/16*I)*a^3*Hypergeometric2F1[4, -3 + n, -2 + n, (1 + I*Tan[c + d*x])/2
]*(a + I*a*Tan[c + d*x])^(-3 + n))/(d*(-3 + n))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^7\int \frac{(i\tan(c+dx)a+a)^{n-4}}{(a-ia\tan(c+dx))^4}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{78}$$

$$\frac{ia^3(a+ia\tan(c+dx))^{n-3}\,\text{Hypergeometric2F1}\left(4,n-3,n-2,\frac{i\tan(c+dx)a+a}{2a}\right)}{16d(3-n)}$$

input
```
Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((I/16)*a^3*Hypergeometric2F1[4, -3 + n, -2 + n, (a + I*a*Tan[c + d*x])/(2
*a)]*(a + I*a*Tan[c + d*x])^(-3 + n))/(d*(3 - n))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [F]

$$\int \cos \left(dx+c\right)^{6} \left(a+ia\tan \left(dx+c\right)\right)^{n}dx$$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx=\int \left(i\,a\tan \left(dx+c\right)+a\right)^{n}\cos \left(dx+c\right)^{6}\,dx$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(1/64*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(12
*I*d*x + 12*I*c) + 6*e^(10*I*d*x + 10*I*c) + 15*e^(8*I*d*x + 8*I*c) + 20*e
^(6*I*d*x + 6*I*c) + 15*e^(4*I*d*x + 4*I*c) + 6*e^(2*I*d*x + 2*I*c) + 1)*e
^(-6*I*d*x - 6*I*c), x)
```

Sympy [F]

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n\cos^6(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*cos(c + d*x)**6, x)
```

Maxima [F]

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^6\,dx$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^6, x)
```

Giac [F]

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^6\,dx$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx=\int \cos\left(c+d\,x\right)^6\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^n\,dx$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx=\int \left(\tan\left(dx+c\right)ai+a\right)^n\cos\left(dx+c\right)^6dx$$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*cos(c + d*x)**6,x)
```

3.471 $\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{i2^{\frac{5}{2}+n}a\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{3}{2}-n,\frac{7}{2},\frac{1}{2}(1-i\tan(c+dx))\right)\sec^5(c+dx)(1+i\tan(c+dx))^{-\frac{3}{2}-n}(a+}{5d}$$

output
```
1/5*I*2^(5/2+n)*a*hypergeom([5/2, -3/2-n],[7/2],1/2-1/2*I*tan(d*x+c))*sec(
d*x+c)^5*(1+I*tan(d*x+c))^(-3/2-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

Mathematica [A] (warning: unable to verify)

Time = 12.49 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.62

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{5+n}e^{5i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\text{Hypergeometric2F1}\left(-\frac{3}{2},1,\frac{7}{2}+n,-e^{2i(c+dx)}\right)\sec^{-n}(c+dx)(\cos(d}{d\left(1+e^{2i(c+dx)}\right)^4(5+2n)}$$

input
```
Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(5 + n)*E^((5*I)*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^
((2*I)*(c + d*x))))^n*Hypergeometric2F1[-3/2, 1, 7/2 + n, -E^((2*I)*(c + d
*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + E^((2*I)*(c + d*x)))^4*(5 + 2*n)*S
ec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec(c+dx)^5(a+ia\tan(c+dx))^n dx$$

$$\Big\downarrow \ 3986$$

$$\frac{\sec^5(c+dx)\int (a-ia\tan(c+dx))^{5/2}(i\tan(c+dx)a+a)^{n+\frac{5}{2}}dx}{(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sec^5(c+dx)\int (a-ia\tan(c+dx))^{5/2}(i\tan(c+dx)a+a)^{n+\frac{5}{2}}dx}{(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 4006$$

$$\frac{a^2\sec^5(c+dx)\int (a-ia\tan(c+dx))^{3/2}(i\tan(c+dx)a+a)^{n+\frac{3}{2}}d\tan(c+dx)}{d(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 80$$

$$\frac{a^3 2^{n+\frac{3}{2}}\sec^5(c+dx)(1+i\tan(c+dx))^{-n-\frac{1}{2}}(a+ia\tan(c+dx))^{n-2}\int \left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n+\frac{3}{2}}(a-ia\tan(c+dx}{d(a-ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 79$$

$$\frac{ia^2 2^{n+\frac{5}{2}} \sec^5(c+dx)(1+i\tan(c+dx))^{-n-\frac{1}{2}}(a+ia\tan(c+dx))^{n-2}\,\text{Hypergeometric2F1}\left(\frac{5}{2},-n-\frac{3}{2},\frac{7}{2},\frac{1}{2}(1-i\right.}{5d}$$

input
```
Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((I/5)*2^(5/2 + n)*a^2*Hypergeometric2F1[5/2, -3/2 - n, 7/2, (1 - I*Tan[c
+ d*x])/2]*Sec[c + d*x]^5*(1 + I*Tan[c + d*x])^(-1/2 - n)*(a + I*a*Tan[c +
 d*x])^(-2 + n))/d
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \sec(dx+c)^5 (a+ia\tan(dx+c))^n dx$$

input
```
int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n \sec(dx+c)^5\,dx$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(32*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(5*I*d
*x + 5*I*c)/(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x
 + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n \sec^5(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**5, x)
```

Maxima [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\sec(dx+c)^5\,dx$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^5, x)
```

Giac [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\sec(dx+c)^5\,dx$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\cos(c+d\,x)^5}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^5,x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^5, x)
```

Reduce [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (\tan(dx+c)\,ai+a)^n \sec(dx+c)^5\,dx$$

input
```
int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*sec(c + d*x)**5,x)
```

3.472 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{i2^{\frac{3}{2}+n}a\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{1}{2}-n,\frac{5}{2},\frac{1}{2}(1-i\tan(c+dx))\right)\sec^3(c+dx)(1+i\tan(c+dx))^{-\frac{1}{2}-n}(a \cdots}{3d}$$

output
```
1/3*I*2^(3/2+n)*a*hypergeom([3/2, -1/2-n],[5/2],1/2-1/2*I*tan(d*x+c))*sec(
d*x+c)^3*(1+I*tan(d*x+c))^(-1/2-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

Mathematica [A] (warning: unable to verify)

Time = 11.82 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.62

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{3+n}e^{3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{5}{2}+n,-e^{2i(c+dx)}\right)\sec^{-n}(c+dx)(\cos(d \cdots}{d\left(1+e^{2i(c+dx)}\right)^2(3+2n)}$$

input
```
Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(3 + n)*E^((3*I)*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^
((2*I)*(c + d*x))))^n*Hypergeometric2F1[-1/2, 1, 5/2 + n, -E^((2*I)*(c + d
*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + E^((2*I)*(c + d*x)))^2*(3 + 2*n)*S
ec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(c+dx)^3(a+ia\tan(c+dx))^n dx$$

$$\Big\downarrow \; 3986$$

$$\frac{\sec^3(c+dx)\int (a-ia\tan(c+dx))^{3/2}(i\tan(c+dx)a+a)^{n+\frac{3}{2}}dx}{(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sec^3(c+dx)\int (a-ia\tan(c+dx))^{3/2}(i\tan(c+dx)a+a)^{n+\frac{3}{2}}dx}{(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4006$$

$$\frac{a^2\sec^3(c+dx)\int \sqrt{a-ia\tan(c+dx)}(i\tan(c+dx)a+a)^{n+\frac{1}{2}}d\tan(c+dx)}{d(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 80$$

$$\frac{a^22^{n+\frac{1}{2}}\sec^3(c+dx)(1+i\tan(c+dx))^{-n-\frac{1}{2}}(a+ia\tan(c+dx))^{n-1}\int \left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n+\frac{1}{2}}\sqrt{a-ia\tan(c+}{d(a-ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 79$$

$$\frac{ia2^{n+\frac{3}{2}}\sec^3(c+dx)(1+i\tan(c+dx))^{-n-\frac{1}{2}}(a+ia\tan(c+dx))^{n-1}\,\mathrm{Hypergeometric2F1}\left(\frac{3}{2},-n-\frac{1}{2},\frac{5}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{3d}$$

input
```
Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((I/3)*2^(3/2 + n)*a*Hypergeometric2F1[3/2, -1/2 - n, 5/2, (1 - I*Tan[c +
d*x])/2]*Sec[c + d*x]^3*(1 + I*Tan[c + d*x])^(-1/2 - n)*(a + I*a*Tan[c + d
*x])^(-1 + n))/d
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \sec(dx+c)^3 (a+ia\tan(dx+c))^n \,dx$$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n \,dx = \int (i\,a\tan(dx+c)+a)^n \sec(dx+c)^3 \,dx$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(8*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(3*I*d*
x + 3*I*c)/(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2
*I*c) + 1), x)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n \,dx = \int (ia(\tan(c+dx)-i))^n \sec^3(c+dx) \,dx$$

input
```
integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**3, x)
```

Maxima [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n \sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

Giac [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n \sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^n}{\cos(c+d\,x)^3}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^3,x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (\tan(dx+c)\,ai+a)^n\sec(dx+c)^3\,dx$$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*sec(c + d*x)**3,x)
```

3.473 $\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{i2^{\frac{1}{2}+n}a\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-i\tan(c+dx))\right)\sec(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia$$

$$d$$

output
```
I*2^(1/2+n)*a*hypergeom([1/2, 1/2-n],[3/2],1/2-1/2*I*tan(d*x+c))*sec(d*x+c
)*(1+I*tan(d*x+c))^(1/2-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

Mathematica [A] (verified)

Time = 6.35 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.66

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx =$$

$$-\frac{i2^{1+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{1+n}\left(1+e^{2i(c+dx)}\right)^{1+n}\text{Hypergeometric2F1}\left(\frac{1}{2}+n,1+n,\frac{3}{2}+n,-e^{2i(c+dx)}\right)\text{se}}{d(1+2n)}$$

input
```
Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(1 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
(1 + n)*(1 + E^((2*I)*(c + d*x)))^(1 + n)*Hypergeometric2F1[1/2 + n, 1 + n
, 3/2 + n, -E^((2*I)*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + 2*n)*Se
c[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = $\{3042, 3986, 3042, 4006, 80, 79\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sec(c+dx)\int \sqrt{a-ia\tan(c+dx)}(i\tan(c+dx)a+a)^{n+\frac{1}{2}}dx}{\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec(c+dx)\int \sqrt{a-ia\tan(c+dx)}(i\tan(c+dx)a+a)^{n+\frac{1}{2}}dx}{\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sec(c+dx)\int \frac{(i\tan(c+dx)a+a)^{n-\frac{1}{2}}}{\sqrt{a-ia\tan(c+dx)}}d\tan(c+dx)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2 2^{n-\frac{1}{2}}\sec(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n-1}\int \frac{\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n-\frac{1}{2}}}{\sqrt{a-ia\tan(c+dx)}}d\tan(c+dx)}{d\sqrt{a-ia\tan(c+dx)}}$$

$$\Big\downarrow 79$$

$$\frac{ia2^{n+\frac{1}{2}}\sec(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-i\tan(c\right.}{d}$$

input
```
Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*2^(1/2 + n)*a*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - I*Tan[c + d*x])
/2]*Sec[c + d*x]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d*x])^(-1
 + n))/d
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \sec(dx+c)\,(a+ia\tan(dx+c))^n\,dx$$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(2*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(I*d*x
+ I*c)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan\left(dx+c\right)+a)^n\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan\left(dx+c\right)+a)^n\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (\tan(dx+c)\,ai+a)^n \sec(dx+c)\,dx$$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*sec(c + d*x),x)
```

3.474 $\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-\frac{1}{2}+n}a\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2}-n,\frac{1}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{3}{2}-n}(\ldots}{d}$$

output
```
-I*2^(-1/2+n)*a*cos(d*x+c)*hypergeom([-1/2, 3/2-n],[1/2],1/2-1/2*I*tan(d*x
+c))*(1+I*tan(d*x+c))^(3/2-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

Mathematica [A] (verified)

Time = 12.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.66

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-1+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-1+n}\left(1+e^{2i(c+dx)}\right)^{-1+n}\text{Hypergeometric2F1}\left(-1+n,-\frac{1}{2}+n,\frac{1}{2}+n,-e^{2i(\ldots}{d(-1+2n)}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(-1 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))
^(-1 + n)*(1 + E^((2*I)*(c + d*x)))^(-1 + n)*Hypergeometric2F1[-1 + n, -1/
2 + n, 1/2 + n, -E^((2*I)*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(-1 + 2
*n)*Sec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3986}$$

$$\cos(c+dx)\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{1}{2}}}{\sqrt{a-ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(c+dx)\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{1}{2}}}{\sqrt{a-ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\cos(c+dx)\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{3}{2}}}{(a-ia\tan(c+dx))^{3/2}}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a2^{n-\frac{3}{2}}\cos(c+dx)\sqrt{a-ia\tan(c+dx)}(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^n\int \frac{\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n-\frac{3}{2}}}{(a-ia\tan(c+dx))^{3/2}}d\tan(c}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{i2^{n-\frac{1}{2}}\cos(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^n \operatorname{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2}-n,\frac{1}{2},\frac{1}{2}(1-i\tan(c\right.}{d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(-1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[-1/2, 3/2 - n, 1/2, (1 -
 I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d*x])^
n)/d
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \cos (dx + c) (a + ia \tan (dx + c))^n dx$$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos(c + dx)(a + ia \tan(c + dx))^n dx = \int (i a \tan (dx + c) + a)^n \cos (dx + c) dx$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(1/2*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(2*I
*d*x + 2*I*c) + 1)*e^(-I*d*x - I*c), x)
```

Sympy [F]

$$\int \cos(c + dx)(a + ia \tan(c + dx))^n dx = \int (ia(\tan (c + dx) - i))^n \cos (c + dx) dx$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*cos(c + d*x), x)
```

Maxima [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \cos(c+d\,x)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n\,dx$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (\tan(dx+c)\,ai+a)^n\cos(dx+c)\,dx$$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*cos(c + d*x),x)
```

3.475 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-\frac{3}{2}+n}a\cos^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{2}-n,-\frac{1}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{5}{2}-}}{3d}$$

output
```
-1/3*I*2^(-3/2+n)*a*cos(d*x+c)^3*hypergeom([-3/2, 5/2-n],[-1/2],1/2-1/2*I*
tan(d*x+c))*(1+I*tan(d*x+c))^(5/2-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

Mathematica [A] (warning: unable to verify)

Time = 13.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.62

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-3+n}e^{-3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^4\text{Hypergeometric2F1}\left(1,\frac{5}{2},-\frac{1}{2}+n,-e^{2i(c+dx)}\right)\operatorname{se}}{d(-3+2n)}$$

input
```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(-3 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))
^n*(1 + E^((2*I)*(c + d*x)))^4*Hypergeometric2F1[1, 5/2, -1/2 + n, -E^((2*
I)*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*E^((3*I)*(c + d*x))*(-3 + 2*n)
*Sec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^n}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \; 3986$$

$$\cos^3(c+dx)(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{3}{2}}}{(a-ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\cos^3(c+dx)(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{3}{2}}}{(a-ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 4006$$

$$\frac{a^2\cos^3(c+dx)(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{5}{2}}}{(a-ia\tan(c+dx))^{5/2}}d\tan(c+dx)}{d}$$

$$\Big\downarrow \; 80$$

$$\frac{2^{n-\frac{5}{2}}\cos^3(c+dx)(a-ia\tan(c+dx))^{3/2}(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n+1}\int \frac{\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n-\frac{5}{2}}}{(a-ia\tan(c+dx))^{5/2}}d\,\text{ta}}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{i2^{n-\frac{3}{2}}\cos^3(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{2}-n,-\frac{1}{2},\frac{1}{2}(1-i\right.}{3ad}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/3*I)*2^(-3/2 + n)*Cos[c + d*x]^3*Hypergeometric2F1[-3/2, 5/2 - n, -1/
2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c +
 d*x])^(1 + n))/(a*d)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \cos \left(dx+c\right)^{3} \left(a+ia\tan \left(dx+c\right)\right)^{n} dx$$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \left(i\,a\tan \left(dx+c\right)+a\right)^{n}\cos \left(dx+c\right)^{3}\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(1/8*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(6*I
*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-3*I
*d*x - 3*I*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

Giac [F]

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \cos(c+d\,x)^3\,(a+a\tan(c+d\,x)\,1\mathrm{i})^n\,dx$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (\tan(dx+c)\,ai+a)^n\cos(dx+c)^3\,dx$$

input

```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

output

```
int((tan(c + d*x)*a*i + a)**n*cos(c + d*x)**3,x)
```

3.476 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-\frac{5}{2}+n}a\cos^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{2}-n,-\frac{3}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{7}{2}-}}{5d}$$

output
```
-1/5*I*2^(-5/2+n)*a*cos(d*x+c)^5*hypergeom([-5/2, 7/2-n],[-3/2],1/2-1/2*I*
tan(d*x+c))*(1+I*tan(d*x+c))^(7/2-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

Mathematica [A] (warning: unable to verify)

Time = 14.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.62

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-5+n}e^{-5i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^6\text{Hypergeometric2F1}\left(1,\frac{7}{2},-\frac{3}{2}+n,-e^{2i(c+dx)}\right)\text{se}}{d(-5+2n)}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(-5 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))
^n*(1 + E^((2*I)*(c + d*x)))^6*Hypergeometric2F1[1, 7/2, -3/2 + n, -E^((2*
I)*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*E^((5*I)*(c + d*x))*(-5 + 2*n)
*Sec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3986}$$

$$\cos^5(c+dx)(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{5}{2}}}{(a-ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^5(c+dx)(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{5}{2}}}{(a-ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\cos^5(c+dx)(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{7}{2}}}{(a-ia\tan(c+dx))^{7/2}}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{n-\frac{7}{2}}\cos^5(c+dx)(a-ia\tan(c+dx))^{5/2}(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n+2}\int \frac{\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n-\frac{7}{2}}}{(a-ia\tan(c+dx))^{7/2}}d\,\text{ta}}{ad}$$

$$\Big\downarrow \text{79}$$

$$-\frac{i2^{n-\frac{5}{2}}\cos^5(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n+2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{2}-n,-\frac{3}{2},\frac{1}{2}(1-i\right.}{5a^2d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/5*I)*2^(-5/2 + n)*Cos[c + d*x]^5*Hypergeometric2F1[-5/2, 7/2 - n, -3/
2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c +
 d*x])^(2 + n))/(a^2*d)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \cos (dx+c)^5 (a+ia\tan (dx+c))^n dx$$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n dx = \int (i\,a\tan (dx+c)+a)^n \cos (dx+c)^5 \,dx$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(1/32*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(10
*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x + 6*I*c) + 10*e^(
4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*e^(-5*I*d*x - 5*I*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx=\int (i\,a\tan\left(dx+c\right)+a)^n\cos\left(dx+c\right)^5\,dx$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^5, x)
```

Giac [F]

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx=\int (i\,a\tan\left(dx+c\right)+a)^n\cos\left(dx+c\right)^5\,dx$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx=\int \cos\left(c+d\,x\right)^5\,\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)^n\,dx$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (\tan(dx+c)\,ai+a)^n \cos(dx+c)^5\,dx$$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*cos(c + d*x)**5,x)
```

3.477 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 96

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{\frac{9}{4}+n}a\,\text{Hypergeometric2F1}\left(\frac{5}{4},-\frac{1}{4}-n,\frac{9}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{5/2}(1}{5d}$$

output
```
1/5*I*2^(9/4+n)*a*hypergeom([5/4, -1/4-n],[9/4],1/2-1/2*I*tan(d*x+c))*(e*s
ec(d*x+c))^(5/2)*(1+I*tan(d*x+c))^(-1/4-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

Mathematica [A] (verified)

Time = 6.95 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.89

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{\frac{7}{2}+n}e^{2i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+n}\text{Hypergeometric2F1}\left(\frac{5}{4}+n,\frac{5}{2}+n,\frac{9}{4}+n,-e^{2i(c+}\right.}{d(5+4n)}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(7/2 + n)*E^((2*I)*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x))))^(1/2 + n)*(1 + E^((2*I)*(c + d*x)))^(1/2 + n)*Hyperg
eometric2F1[5/4 + n, 5/2 + n, 9/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^
(-5/2 - n)*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^n)/(d*(5 + 4*n)*(
Cos[d*x] + I*Sin[d*x])^n)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n dx$$

$$\Big\downarrow \text{ 3986}$$

$$\frac{(e\sec(c+dx))^{5/2}\int (a-ia\tan(c+dx))^{5/4}(i\tan(c+dx)a+a)^{n+\frac{5}{4}}dx}{(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e\sec(c+dx))^{5/2}\int (a-ia\tan(c+dx))^{5/4}(i\tan(c+dx)a+a)^{n+\frac{5}{4}}dx}{(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}}$$

$$\Big\downarrow \text{ 4006}$$

$$\frac{a^2(e\sec(c+dx))^{5/2}\int \sqrt[4]{a-ia\tan(c+dx)}(i\tan(c+dx)a+a)^{n+\frac{1}{4}}d\tan(c+dx)}{d(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}}$$

$$\Big\downarrow \text{ 80}$$

$$\frac{a^2 2^{n+\frac{1}{4}}(e\sec(c+dx))^{5/2}(1+i\tan(c+dx))^{-n-\frac{1}{4}}(a+ia\tan(c+dx))^{n-1}\int \left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n+\frac{1}{4}}\sqrt[4]{a-ia\tan}}{d(a-ia\tan(c+dx))^{5/4}}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{ia2^{n+\frac{9}{4}}(e\sec(c+dx))^{5/2}(1+i\tan(c+dx))^{-n-\frac{1}{4}}(a+ia\tan(c+dx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{5}{4},-n-\frac{1}{4},\frac{9}{4},\frac{1}{2}(1\right.}{5d}$$

input
```
Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((I/5)*2^(9/4 + n)*a*Hypergeometric2F1[5/4, -1/4 - n, 9/4, (1 - I*Tan[c +
d*x])/2]*(e*Sec[c + d*x])^(5/2)*(1 + I*Tan[c + d*x])^(-1/4 - n)*(a + I*a*T
an[c + d*x])^(-1 + n))/d
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e \sec (dx + c))^{\frac{5}{2}} (a + ia \tan (dx + c))^n \, dx$$

input
```
int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e \sec(c+dx))^{5/2}(a+ia \tan(c+dx))^n \, dx = \int (e \sec (dx + c))^{\frac{5}{2}} (i\,a \tan (dx + c) + a)^n \, dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(4*sqrt(2)*e^2*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))
^n*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(5/2*I*d*x + 5/2*I*c)/(e^(4*I*d*x +
 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (e \sec(c + dx))^{5/2}(a + ia \tan(c + dx))^n \, dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx = \int (e\sec\left(dx+c\right))^{\frac{5}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx = \int (e\sec\left(dx+c\right))^{\frac{5}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx = \int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{5/2}\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^n\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx = \frac{\sqrt{e}\,e^2i\Big(-2\sqrt{\sec(dx+c)}\,(\tan(dx+c)\,ai+a)^n\sec(dx+c)^2+2\Big(\int\sqrt{\sec(dx+c)}\,(\cdots}{}$$

input
```
int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
(sqrt(e)*e**2*i*( - 2*sqrt(sec(c + d*x))*(tan(c + d*x)*a*i + a)**n*sec(c +
 d*x)**2 + 2*int(sqrt(sec(c + d*x))*(tan(c + d*x)*a*i + a)**n*sec(c + d*x)
**2*tan(c + d*x),x)*d*n + 5*int(sqrt(sec(c + d*x))*(tan(c + d*x)*a*i + a)*
*n*sec(c + d*x)**2*tan(c + d*x),x)*d))/(2*d*n)
```

3.478 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 96

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx=\frac{i2^{\frac{7}{4}+n}a\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}-n,\frac{7}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{3/2}(1+\ldots}{3d}$$

output
```
1/3*I*2^(7/4+n)*a*hypergeom([3/4, 1/4-n],[7/4],1/2-1/2*I*tan(d*x+c))*(e*se
c(d*x+c))^(3/2)*(1+I*tan(d*x+c))^(1/4-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

Mathematica [A] (verified)

Time = 6.85 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.77

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{\frac{5}{2}+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{3}{2}+n}\left(1+e^{2i(c+dx)}\right)^{\frac{3}{2}+n}\text{Hypergeometric2F1}\left(\frac{3}{4}+n,\frac{3}{2}+n,\frac{7}{4}+n,-e^{2i(c+dx)}\right)\sec\ldots}{d(3+4n)}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(5/2 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))
)^(3/2 + n)*(1 + E^((2*I)*(c + d*x)))^(3/2 + n)*Hypergeometric2F1[3/4 + n,
 3/2 + n, 7/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-3/2 - n)*(e*Sec[c
+ d*x])^(3/2)*(a + I*a*Tan[c + d*x])^n)/(d*(3 + 4*n)*(Cos[d*x] + I*Sin[d*x
])^n)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(e\sec(c+dx))^{3/2}\int (a-ia\tan(c+dx))^{3/4}(i\tan(c+dx)a+a)^{n+\frac{3}{4}}dx}{(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e\sec(c+dx))^{3/2}\int (a-ia\tan(c+dx))^{3/4}(i\tan(c+dx)a+a)^{n+\frac{3}{4}}dx}{(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(e\sec(c+dx))^{3/2}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{1}{4}}}{\sqrt[4]{a-ia\tan(c+dx)}}d\tan(c+dx)}{d(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2 2^{n-\frac{1}{4}}(e\sec(c+dx))^{3/2}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^{n-1}\int \frac{\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n-\frac{1}{4}}}{\sqrt[4]{a-ia\tan(c+dx)}}d\tan(c+dx)}{d(a-ia\tan(c+dx))^{3/4}}$$

$$\Big\downarrow \text{79}$$

$$\frac{ia2^{n+\frac{7}{4}}(e\sec(c+dx))^{3/2}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^{n-1}\operatorname{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}-n,\frac{7}{4},\frac{1}{2}(1-i\right.}{3d}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((I/3)*2^(7/4 + n)*a*Hypergeometric2F1[3/4, 1/4 - n, 7/4, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^(3/2)*(1 + I*Tan[c + d*x])^(1/4 - n)*(a + I*a*Tan
[c + d*x])^(-1 + n))/d
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\sec(dx+c))^{\frac{3}{2}} (a+ia\tan(dx+c))^n\,dx$$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{\frac{3}{2}} (i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(2*sqrt(2)*e*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n
*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c)/(e^(2*I*d*x + 2
*I*c) + 1), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(c+dx))^{\frac{3}{2}} (ia(\tan(c+dx)-i))^n\,dx$$

input
```
integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(3/2)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{\frac{3}{2}}\,(ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{\frac{3}{2}}\,(ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}(a+a\tan(c+d\,x)\,1\mathrm{i})^n\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = \frac{\sqrt{e}\,ei\left(-2\sqrt{\sec(dx+c)}\,(\tan(dx+c)\,ai+a)^n\sec(dx+c)+2\left(\int\sqrt{\sec(dx+c)}\,(\mathrm{ta}\right.\right.}{}$$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
(sqrt(e)*e*i*( - 2*sqrt(sec(c + d*x))*(tan(c + d*x)*a*i + a)**n*sec(c + d*
x) + 2*int(sqrt(sec(c + d*x))*(tan(c + d*x)*a*i + a)**n*sec(c + d*x)*tan(c
 + d*x),x)*d*n + 3*int(sqrt(sec(c + d*x))*(tan(c + d*x)*a*i + a)**n*sec(c
+ d*x)*tan(c + d*x),x)*d))/(2*d*n)
```

3.479 $\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 94

$$\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{\frac{5}{4}+n}a\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4}-n,\frac{5}{4},\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt{e\sec(c+dx)}(1+i\tan(c+dx))^{\frac{3}{4}-n}(a}{d}$$

output
```
I*2^(5/4+n)*a*hypergeom([1/4, 3/4-n],[5/4],1/2-1/2*I*tan(d*x+c))*(e*sec(d*
x+c))^(1/2)*(1+I*tan(d*x+c))^(3/4-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

Mathematica [A] (verified)

Time = 6.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.81

$$\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{\frac{3}{2}+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+n}\text{Hypergeometric2F1}\left(\frac{1}{4}+n,\frac{1}{2}+n,\frac{5}{4}+n,-e^{2i(c+dx)}\right)}{d(1+4n)}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(3/2 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))
)^(1/2 + n)*(1 + E^((2*I)*(c + d*x)))^(1/2 + n)*Hypergeometric2F1[1/4 + n,
 1/2 + n, 5/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-1/2 - n)*Sqrt[e*Se
c[c + d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + 4*n)*(Cos[d*x] + I*Sin[d*x])
^n)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n dx$$

$$\Big\downarrow \; 3986$$

$$\frac{\sqrt{e\sec(c+dx)}\int \sqrt[4]{a-ia\tan(c+dx)}(i\tan(c+dx)a+a)^{n+\frac{1}{4}}dx}{\sqrt[4]{a-ia\tan(c+dx)}\sqrt[4]{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{e\sec(c+dx)}\int \sqrt[4]{a-ia\tan(c+dx)}(i\tan(c+dx)a+a)^{n+\frac{1}{4}}dx}{\sqrt[4]{a-ia\tan(c+dx)}\sqrt[4]{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 4006$$

$$\frac{a^2\sqrt{e\sec(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{3}{4}}}{(a-ia\tan(c+dx))^{3/4}}d\tan(c+dx)}{d\sqrt[4]{a-ia\tan(c+dx)}\sqrt[4]{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 80$$

$$\frac{a^2 2^{n-\frac{3}{4}}\sqrt{e\sec(c+dx)}(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^{n-1}\int \frac{\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n-\frac{3}{4}}}{(a-ia\tan(c+dx))^{3/4}}d\tan(c+dx)}{d\sqrt[4]{a-ia\tan(c+dx)}}$$

$$\Big\downarrow \text{79}$$

$$\frac{ia2^{n+\frac{5}{4}}\sqrt{e\sec(c+dx)}(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^{n-1}\operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4}-n,\frac{5}{4},\frac{1}{2}(1-i\tan$$

$$d$$

input
```
Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*2^(5/4 + n)*a*Hypergeometric2F1[1/4, 3/4 - n, 5/4, (1 - I*Tan[c + d*x])
/2]*Sqrt[e*Sec[c + d*x]]*(1 + I*Tan[c + d*x])^(3/4 - n)*(a + I*a*Tan[c + d
*x])^(-1 + n))/d
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \sqrt{e \sec (dx + c)}\, (a + ia \tan (dx + c))^n \, dx$$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^n \, dx = \int \sqrt{e \sec (dx + c)}(i\, a \tan (dx + c) + a)^n \, dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*sqr
t(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c), x)
```

Sympy [F]

$$\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^n \, dx = \int \sqrt{e \sec (c + dx)}(ia(\tan (c + dx) - i))^n \, dx$$

input
```
integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral(sqrt(e*sec(c + d*x))*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx=\int \sqrt{e\sec\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx=\int \sqrt{e\sec\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx=\int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^n\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{\sqrt{e}\,i\left(-2\sqrt{\sec\left(dx+c\right)}\left(\tan\left(dx+c\right)ai+a\right)^{n}+2\left(\int\sqrt{\sec\left(dx+c\right)}\left(\tan\left(dx+c\right)ai+a\right)^{n}\tan\left(dx+c\right)d\right.\right.}{2dn}$$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
(sqrt(e)*i*( - 2*sqrt(sec(c + d*x))*(tan(c + d*x)*a*i + a)**n + 2*int(sqrt
(sec(c + d*x))*(tan(c + d*x)*a*i + a)**n*tan(c + d*x),x)*d*n + int(sqrt(se
c(c + d*x))*(tan(c + d*x)*a*i + a)**n*tan(c + d*x),x)*d))/(2*d*n)
```

3.480 $\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 94

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{i2^{\frac{3}{4}+n}a\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4}-n,\frac{3}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{5}{4}-n}(a+ia\tan(c+}{d\sqrt{e\sec(c+dx)}}$$

output
```
-I*2^(3/4+n)*a*hypergeom([-1/4, 5/4-n],[3/4],1/2-1/2*I*tan(d*x+c))*(1+I*ta
n(d*x+c))^(5/4-n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(e*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 7.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.52

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{i2^{\frac{1}{2}+n}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-\frac{1}{2}+n}\left(1+e^{2i(c+dx)}\right)^{-\frac{1}{2}+n}\text{Hypergeometric2F1}\left(-\frac{1}{2}+n,-\frac{1}{4}+n,\frac{3}{4}+n,-e^{2i(c+dx)}\right)\text{se}}{d(-1+4n)\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^n/Sqrt[e*Sec[c + d*x]],x]
```

output
```
((-I)*2^(1/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-1/2 + n)*(
1 + E^((2*I)*(c + d*x)))^(-1/2 + n)*Hypergeometric2F1[-1/2 + n, -1/4 + n,
3/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(1/2 - n)*(a + I*a*Tan[c + d*x
])^n)/(d*(-1 + 4*n)*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sqrt[4]{a-ia\tan(c+dx)}\sqrt[4]{a+ia\tan(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{1}{4}}}{\sqrt[4]{a-ia\tan(c+dx)}}dx}{\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[4]{a-ia\tan(c+dx)}\sqrt[4]{a+ia\tan(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{1}{4}}}{\sqrt[4]{a-ia\tan(c+dx)}}dx}{\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sqrt[4]{a-ia\tan(c+dx)}\sqrt[4]{a+ia\tan(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{5}{4}}}{(a-ia\tan(c+dx))^{5/4}}d\tan(c+dx)}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a2^{n-\frac{5}{4}}\sqrt[4]{a-ia\tan(c+dx)}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^n\int\frac{\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n-\frac{5}{4}}}{(a-ia\tan(c+dx))^{5/4}}d\tan(c+dx)}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \textbf{79}$$

$$-\frac{i2^{n+\frac{3}{4}}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^n\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4}-n,\frac{3}{4},\frac{1}{2}(1-i\tan(c+dx))\right)}{d\sqrt{e\sec(c+dx)}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^n/Sqrt[e*Sec[c + d*x]],x]
```

output
```
((-I)*2^(3/4 + n)*Hypergeometric2F1[-1/4, 5/4 - n, 3/4, (1 - I*Tan[c + d*x
])/2]*(1 + I*Tan[c + d*x])^(1/4 - n)*(a + I*a*Tan[c + d*x])^n)/(d*Sqrt[e*S
ec[c + d*x]])
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(a + ia \tan(dx + c))^n}{\sqrt{e \sec(dx + c)}} dx$$

input
```
int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x)
```

output
```
int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{(a + ia \tan(c + dx))^n}{\sqrt{e \sec(c + dx)}} dx = \int \frac{(i a \tan(dx + c) + a)^n}{\sqrt{e \sec(dx + c)}} dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(1/2*sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n
*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x
 - 1/2*I*c)/e, x)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(ia(\tan{(c+dx)}-i))^n}{\sqrt{e\sec{(c+dx)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**n/(e*sec(d*x+c))**(1/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n/sqrt(e*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^n}{\sqrt{e\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n/sqrt(e*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^n}{\sqrt{e\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n/sqrt(e*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,(\tan(dx+c)ai+a)^n}{\sec(dx+c)}dx\right)}{e}$$

input
```
int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*int((sqrt(sec(c + d*x))*(tan(c + d*x)*a*i + a)**n)/sec(c + d*x),x
))/e
```

3.481 $\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 96

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{i2^{\frac{1}{4}+n}a\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4}-n,\frac{1}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{7}{4}-n}(a+ia\tan(c+d$$
$$\overline{3d(e\sec(c+dx))^{3/2}}$$

output
```
-1/3*I*2^(1/4+n)*a*hypergeom([-3/4, 7/4-n],[1/4],1/2-1/2*I*tan(d*x+c))*(1+
I*tan(d*x+c))^(7/4-n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(e*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 7.77 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.49

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{i2^{-\frac{1}{2}+n}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-\frac{3}{2}+n}\left(1+e^{2i(c+dx)}\right)^{-\frac{3}{2}+n}\text{Hypergeometric2F1}\left(-\frac{3}{2}+n,-\frac{3}{4}+n,\frac{1}{4}+n,-e^{2i(c+dx)}\right)\sec$$
$$\overline{d(-3+4n)(e\sec(c+dx))^{3/2}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(3/2),x]
```

output
```
((-I)*2^(-1/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-3/2 + n)*
(1 + E^((2*I)*(c + d*x)))^(-3/2 + n)*Hypergeometric2F1[-3/2 + n, -3/4 + n,
 1/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(3/2 - n)*(a + I*a*Tan[c + d*
x])^n)/(d*(-3 + 4*n)*(e*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{3}{4}}}{(a-ia\tan(c+dx))^{3/4}}dx}{(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{3}{4}}}{(a-ia\tan(c+dx))^{3/4}}dx}{(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{7}{4}}}{(a-ia\tan(c+dx))^{7/4}}d\tan(c+dx)}{d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a2^{n-\frac{7}{4}}(a-ia\tan(c+dx))^{3/4}(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^n\int \frac{\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n-\frac{7}{4}}}{(a-ia\tan(c+dx))^{7/4}}d\tan(c+dx)}{d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow 79$$

$$-\frac{i2^{n+\frac{1}{4}}(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^n \text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4}-n,\frac{1}{4},\frac{1}{2}(1-i\tan(c+dx))\right)}{3d(e\sec(c+dx))^{3/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(3/2),x]
```

output
```
((-1/3*I)*2^(1/4 + n)*Hypergeometric2F1[-3/4, 7/4 - n, 1/4, (1 - I*Tan[c +
 d*x])/2]*(1 + I*Tan[c + d*x])^(3/4 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(e*S
ec[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(a+ia\tan(dx+c))^n}{(e\sec(dx+c))^{\frac{3}{2}}}dx$$

input
```
int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x)
```

output
```
int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx=\int \frac{(i\,a\tan(dx+c)+a)^n}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(1/4*sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n
*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2
*I*c) + 1)*e^(-3/2*I*d*x - 3/2*I*c)/e^2, x)
```

Sympy [F]

$$\int \frac{(a + ia\tan(c + dx))^n}{(e\sec(c + dx))^{3/2}}\,dx = \int \frac{(ia(\tan{(c + dx)} - i))^n}{(e\sec{(c + dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**n/(e*sec(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n/(e*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a + ia\tan(c + dx))^n}{(e\sec(c + dx))^{3/2}}\,dx = \int \frac{(i\,a\tan{(dx + c)} + a)^n}{(e\sec{(dx + c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(a + ia\tan(c + dx))^n}{(e\sec(c + dx))^{3/2}}\,dx = \int \frac{(i\,a\tan{(dx + c)} + a)^n}{(e\sec{(dx + c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + ia\tan(c + dx))^n}{(e\sec(c + dx))^{3/2}}\,dx = \int \frac{(a + a\tan(c + d\,x)\ 1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a + ia\tan(c + dx))^n}{(e\sec(c + dx))^{3/2}}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,(\tan(dx+c)ai+a)^n}{\sec(dx+c)^2}dx\right)}{e^2}$$

input
```
int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*int((sqrt(sec(c + d*x))*(tan(c + d*x)*a*i + a)**n)/sec(c + d*x)**
2,x))/e**2
```

3.482 $\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 96

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{i2^{-\frac{1}{4}+n}a\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{9}{4}-n,-\frac{1}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{9}{4}-n}(a+ia\tan(c+dx))^{-1+n}}{5d(e\sec(c+dx))^{5/2}}$$

output
```
-1/5*I*2^(-1/4+n)*a*hypergeom([-5/4, 9/4-n],[-1/4],1/2-1/2*I*tan(d*x+c))*(
1+I*tan(d*x+c))^(9/4-n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(e*sec(d*x+c))^(5/2)
```

Mathematica [A] (verified)

Time = 8.03 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.64

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{i2^{-\frac{3}{2}+n}e^{-3i(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+n}\text{Hypergeometric2F1}\left(-\frac{5}{2}+n,-\frac{5}{4}+n,-\frac{1}{4}+n,-e^{2i(c+dx)}\right)\cdots}{de^2(-5+4n)\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(5/2),x]
```

output
```
((-I)*2^(-3/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/2 + n)*(
1 + E^((2*I)*(c + d*x)))^(1/2 + n)*Hypergeometric2F1[-5/2 + n, -5/4 + n, -
1/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(1/2 - n)*(a + I*a*Tan[c + d*x
])^n)/(d*e^2*E^((3*I)*(c + d*x))*(-5 + 4*n)*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{5}{4}}}{(a-ia\tan(c+dx))^{5/4}}dx}{(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{5}{4}}}{(a-ia\tan(c+dx))^{5/4}}dx}{(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{9}{4}}}{(a-ia\tan(c+dx))^{9/4}}d\tan(c+dx)}{d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{n-\frac{9}{4}}(a-ia\tan(c+dx))^{5/4}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^{n+1}\int \frac{\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n-\frac{9}{4}}}{(a-ia\tan(c+dx))^{9/4}}d\tan(c+dx)}{d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{i2^{n-\frac{1}{4}}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{9}{4}-n,-\frac{1}{4},\frac{1}{2}(1-i\tan(c+dx))\right)}{5ad(e\sec(c+dx))^{5/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(5/2),x]
```

output
```
((-1/5*I)*2^(-1/4 + n)*Hypergeometric2F1[-5/4, 9/4 - n, -1/4, (1 - I*Tan[c
 + d*x])/2]*(1 + I*Tan[c + d*x])^(1/4 - n)*(a + I*a*Tan[c + d*x])^(1 + n))
/(a*d*(e*Sec[c + d*x])^(5/2))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(a+ia\tan(dx+c))^n}{(e\sec(dx+c))^{\frac{5}{2}}}dx$$

input
```
int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x)
```

output
```
int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx=\int \frac{(i\,a\tan(dx+c)+a)^n}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(1/8*sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n
*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4
*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-5/2*I*d*x - 5/2*I*c)/e^3, x)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(ia(\tan(c+dx)-i))^n}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**n/(e*sec(d*x+c))**(5/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n/(e*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^n}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^n}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{\sec(dx+c)}\,(\tan(dx+c)ai+a)^n}{\sec(dx+c)^3}dx\right)}{e^3}$$

input
```
int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*int((sqrt(sec(c + d*x))*(tan(c + d*x)*a*i + a)**n)/sec(c + d*x)**
3,x))/e**3
```

3.483 $\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 269

$$\begin{aligned}\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx &= \frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)} \\ &+ \frac{4i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{1+n}}{ad\,(8-6n+n^2)} \\ &- \frac{12i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{2+n}}{a^2d(2-n)(4-n)n} \\ &+ \frac{24i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{3+n}}{a^3d(4-n)n\,(4-n^2)} \\ &- \frac{24i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{4+n}}{a^4dn\,(64-20n^2+n^4)}\end{aligned}$$

output
```
I*(e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n/d/(4-n)+4*I*(e*sec(d*x+c))^(-
4-n)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(n^2-6*n+8)-12*I*(e*sec(d*x+c))^(-4-n)*(
a+I*a*tan(d*x+c))^(2+n)/a^2/d/(2-n)/(4-n)/n+24*I*(e*sec(d*x+c))^(-4-n)*(a+
I*a*tan(d*x+c))^(3+n)/a^3/d/(4-n)/n/(-n^2+4)-24*I*(e*sec(d*x+c))^(-4-n)*(a
+I*a*tan(d*x+c))^(4+n)/a^4/d/n/(n^4-20*n^2+64)
```

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.61

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx =$$
$$-\frac{i(e\sec(c+dx))^{-n}\left(192-60n^2+3n^4+4n^2(-16+n^2)\cos(2(c+dx))+n^2(-4+n^2)\cos(4(c+dx))\right.}{8de^4(-4+n)(}$$

input
```
Integrate[(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/8*I)*(192 - 60*n^2 + 3*n^4 + 4*n^2*(-16 + n^2)*Cos[2*(c + d*x)] + n^2
*(-4 + n^2)*Cos[4*(c + d*x)] + (128*I)*n*Sin[2*(c + d*x)] - (8*I)*n^3*Sin[
2*(c + d*x)] + (16*I)*n*Sin[4*(c + d*x)] - (4*I)*n^3*Sin[4*(c + d*x)])*(a
+ I*a*Tan[c + d*x])^n)/(d*e^4*(-4 + n)*(-2 + n)*n*(2 + n)*(4 + n)*(e*Sec[c
 + d*x])^n)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3985, 3042, 3985, 3042, 3985, 3042, 3985, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-4}dx$$

$$\Big\downarrow \text{3985}$$

$$\frac{4\int (e\sec(c+dx))^{-n-4}(i\tan(c+dx)a+a)^{n+1}dx}{a(4-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-4}}{d(4-n)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int (e\sec(c+dx))^{-n-4}(i\tan(c+dx)a+a)^{n+1}dx}{a(4-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-4}}{d(4-n)}$$

$$\Big\downarrow \; 3985$$

$$\frac{4\left(\frac{3\int (e\sec(c+dx))^{-n-4}(i\tan(c+dx)a+a)^{n+2}dx}{a(2-n)}+\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-4}}{d(2-n)}\right)}{a(4-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-4}}{d(4-n)}$$

$$\Big\downarrow \; 3042$$

$$\frac{4\left(\frac{3\int (e\sec(c+dx))^{-n-4}(i\tan(c+dx)a+a)^{n+2}dx}{a(2-n)}+\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-4}}{d(2-n)}\right)}{a(4-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-4}}{d(4-n)}$$

$$\Big\downarrow \; 3985$$

$$\frac{4\left(\frac{3\left(-\frac{2\int (e\sec(c+dx))^{-n-4}(i\tan(c+dx)a+a)^{n+3}dx}{an}-\frac{i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-4}}{dn}\right)}{a(2-n)}+\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-4}}{d(2-n)}\right)}{a(4-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-4}}{d(4-n)}$$

$$\Big\downarrow \; 3042$$

$$\frac{4\left(\frac{3\left(-\frac{2\int (e\sec(c+dx))^{-n-4}(i\tan(c+dx)a+a)^{n+3}dx}{an}-\frac{i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-4}}{dn}\right)}{a(2-n)}+\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-4}}{d(2-n)}\right)}{a(4-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-4}}{d(4-n)}$$

$$\Big\downarrow \; 3985$$

$$\frac{4\left(\frac{3\left(-\frac{2\left(-\frac{\int (e\sec(c+dx))^{-n-4}(i\tan(c+dx)a+a)^{n+4}dx}{a(n+2)}-\frac{i(a+ia\tan(c+dx))^{n+3}(e\sec(c+dx))^{-n-4}}{d(n+2)}\right)}{an}-\frac{i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-4}}{dn}\right)}{a(2-n)}+\right.}{a(4-n)}$$

$$\frac{i(a+ia\tan(c+dx))^{n}(e\sec(c+dx))^{-n-4}}{d(4-n)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{4\left(\frac{3\left(-\frac{2\left(-\frac{\int (e\sec(c+dx))^{-n-4}(i\tan(c+dx)a+a)^{n+4}dx}{a(n+2)}-\frac{i(a+ia\tan(c+dx))^{n+3}(e\sec(c+dx))^{-n-4}}{d(n+2)}\right)}{an}-\frac{i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-4}}{dn}\right)}{a(2-n)}+\right.}{a(4-n)}$$

$$\frac{i(a+ia\tan(c+dx))^{n}(e\sec(c+dx))^{-n-4}}{d(4-n)}$$

$$\Big\downarrow \textbf{3969}$$

$$\frac{i(a+ia\tan(c+dx))^{n}(e\sec(c+dx))^{-n-4}}{d(4-n)}+$$

$$\frac{4\left(\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-4}}{d(2-n)}+\frac{3\left(-\frac{i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-4}}{dn}-\frac{2\left(\frac{i(a+ia\tan(c+dx))^{n+4}(e\sec(c+dx))^{-n-4}}{ad(n+2)(n+4)}-\frac{i(a+\right.}{an}}{a(2-n)}\right.}{a(4-n)}$$

input
```
Int[(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(4 - n)) + (4*((
I*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(d*(2 - n)) +
(3*(((-I)*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^(2 + n))/(d*n)
- (2*(((-I)*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^(3 + n))/(d*(
2 + n)) + (I*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^(4 + n))/(a*
d*(2 + n)*(4 + n))))/(a*n)))/(a*(2 - n))))/(a*(4 - n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3985
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}
, x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.09 (sec) , antiderivative size = 4331, normalized size of antiderivative = 16.10

method	result	size
risch	Expression too large to display	4331

input
```
int((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
-1/16*I/(n-4)/d/e^4*exp(I*(d*x+c))^n/(e^n)*a^n*exp(-1/2*I*(csgn(I*a*exp(2*
I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*Pi*n-csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I
*(d*x+c))+1))^2*csgn(I*a)*Pi*n-csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))
+1))^2*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*Pi*n+csgn(I*a*exp(2*I
*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a)*csgn(I/(exp(2*I*(d*x+c))+1)*exp(
2*I*(d*x+c)))*Pi*n-n*Pi*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+n*
Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(ex
p(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*e)*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+
c))+1))^2-n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(
I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+csgn(I*exp(2*I*(d*x+c)))*csgn(I*e
xp(I*(d*x+c)))^2*Pi*n+n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(e
xp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(e
xp(2*I*(d*x+c))+1))*csgn(I/(exp(2*I*(d*x+c))+1))-2*csgn(I*exp(2*I*(d*x+c))
)^2*csgn(I*exp(I*(d*x+c)))*Pi*n-n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c
))+1))^3+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*csgn(I/(exp(2*
I*(d*x+c))+1))+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*Pi*n-csgn(I
/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*exp(2*I*(d*x+c)))*Pi*n-cs
gn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I/(exp(2*I*(d*x+c))+1))
*Pi*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c)
))*csgn(I/(exp(2*I*(d*x+c))+1))*Pi*n+csgn(I*exp(2*I*(d*x+c)))^3*Pi*n+8*...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.25

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{(-i\,n^4-4i\,n^3+4i\,n^2+(-i\,n^4+4i\,n^3+4i\,n^2-16i\,n)e^{(8i\,dx+8i\,c)}-4\,(i\,n^4-2i\,n^3-16i\,n^2+32i\,n)e^{(6i\,c}}{dn^5-20\,dn^3+64\,dn+(dn^5-20\,dn^3+64\,dn)e^{(8i\,dx+8i\,c)}+4}$$

input
```
integrate((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

output
```
(-I*n^4 - 4*I*n^3 + 4*I*n^2 + (-I*n^4 + 4*I*n^3 + 4*I*n^2 - 16*I*n)*e^(8*I
*d*x + 8*I*c) - 4*(I*n^4 - 2*I*n^3 - 16*I*n^2 + 32*I*n)*e^(6*I*d*x + 6*I*c
) - 6*(I*n^4 - 20*I*n^2 + 64*I)*e^(4*I*d*x + 4*I*c) - 4*(I*n^4 + 2*I*n^3 -
 16*I*n^2 - 32*I*n)*e^(2*I*d*x + 2*I*c) + 16*I*n)*(2*e*e^(I*d*x + I*c)/(e^
(2*I*d*x + 2*I*c) + 1))^(-n - 4)*e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x +
 I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e))/(d*n^5 - 20*d*n^3 + 64*d*n
+ (d*n^5 - 20*d*n^3 + 64*d*n)*e^(8*I*d*x + 8*I*c) + 4*(d*n^5 - 20*d*n^3 +
64*d*n)*e^(6*I*d*x + 6*I*c) + 6*(d*n^5 - 20*d*n^3 + 64*d*n)*e^(4*I*d*x + 4
*I*c) + 4*(d*n^5 - 20*d*n^3 + 64*d*n)*e^(2*I*d*x + 2*I*c))
```

Sympy [F]

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \int (e\sec{(c+dx)})^{-n-4}\,(ia(\tan{(c+dx)}-i))^n\,dx$$

input
```
integrate((e*sec(d*x+c))**(-4-n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(-n - 4)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.62

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{(-i\,a^n n^4+4i\,a^n n^3+4i\,a^n n^2-16i\,a^n n)\cos((dx+c)(n+4))-4\,(i\,a^n n^4-2i\,a^n n^3-16i\,a^n n^2+32i\,a^n n}{}$$

input
```
integrate((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

output
```
1/16*((-I*a^n*n^4 + 4*I*a^n*n^3 + 4*I*a^n*n^2 - 16*I*a^n*n)*cos((d*x + c)*
(n + 4)) - 4*(I*a^n*n^4 - 2*I*a^n*n^3 - 16*I*a^n*n^2 + 32*I*a^n*n)*cos((d*
x + c)*(n + 2)) - 4*(I*a^n*n^4 + 2*I*a^n*n^3 - 16*I*a^n*n^2 - 32*I*a^n*n)*
cos((d*x + c)*(n - 2)) + (-I*a^n*n^4 - 4*I*a^n*n^3 + 4*I*a^n*n^2 + 16*I*a^
n*n)*cos((d*x + c)*(n - 4)) - 6*(I*a^n*n^4 - 20*I*a^n*n^2 + 64*I*a^n)*cos(
(d*x + c)*n) + (a^n*n^4 - 4*a^n*n^3 - 4*a^n*n^2 + 16*a^n*n)*sin((d*x + c)*
(n + 4)) + 4*(a^n*n^4 - 2*a^n*n^3 - 16*a^n*n^2 + 32*a^n*n)*sin((d*x + c)*(
n + 2)) + 4*(a^n*n^4 + 2*a^n*n^3 - 16*a^n*n^2 - 32*a^n*n)*sin((d*x + c)*(n
 - 2)) + (a^n*n^4 + 4*a^n*n^3 - 4*a^n*n^2 - 16*a^n*n)*sin((d*x + c)*(n - 4
)) + 6*(a^n*n^4 - 20*a^n*n^2 + 64*a^n)*sin((d*x + c)*n))/((e^(n + 4)*n^5 -
 20*e^(n + 4)*n^3 + 64*e^(n + 4)*n)*d)
```

Giac [F]

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-n-4}\,(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-n - 4)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B] (verification not implemented)

Time = 7.39 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.90

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx = \frac{\left(2\sin\left(2\,c+2\,d\,x\right)^2+\sin\left(4\,c+4\,d\,x\right)\,1\mathrm{i}-1\right)\,\left(\frac{\left(a-\frac{a\,\sin\left(c+d\,x\right)\,1\mathrm{i}}{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}\right)^n\,\left(-n^3-4\,n^2+4\,n+16\right)}{d\,\left(n^4\,1\mathrm{i}-n^2\,20\mathrm{i}+64\mathrm{i}\right)}+\frac{4\,\left(a-\frac{a\,\sin\left(c+d\,x\right)\,1\mathrm{i}}{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}\right)}{}\right.}{}$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(n + 4),x)
```

output
```
((sin(4*c + 4*d*x)*1i + 2*sin(2*c + 2*d*x)^2 - 1)*(((a - (a*sin(c + d*x)*1
i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(4*n - 4*n^2 - n^3 + 16))/(d*(n^4*1i -
n^2*20i + 64i)) + (4*(a - (a*sin(c + d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1)
)^n*(sin(2*c + 2*d*x)*1i - 2*sin(c + d*x)^2 + 1)*(16*n - 2*n^2 - n^3 + 32)
)/(d*(n^4*1i - n^2*20i + 64i)) + ((a - (a*sin(c + d*x)*1i)/(2*sin(c/2 + (d
*x)/2)^2 - 1))^n*(sin(8*c + 8*d*x)*1i - 2*sin(4*c + 4*d*x)^2 + 1)*(4*n + 4
*n^2 - n^3 - 16))/(d*(n^4*1i - n^2*20i + 64i)) + (4*(a - (a*sin(c + d*x)*1
i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(sin(6*c + 6*d*x)*1i - 2*sin(3*c + 3*d*
x)^2 + 1)*(16*n + 2*n^2 - n^3 - 32))/(d*(n^4*1i - n^2*20i + 64i)) - ((a -
(a*sin(c + d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(sin(4*c + 4*d*x)*1i -
 2*sin(2*c + 2*d*x)^2 + 1)*(6*n^4 - 120*n^2 + 384))/(d*n*(n^4*1i - n^2*20i
 + 64i))))/(16*(-e/(2*sin(c/2 + (d*x)/2)^2 - 1))^(n + 4)*(sin(c + d*x)^2 -
 1)^2)
```

Reduce [F]

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx = \frac{\int \frac{(\tan(dx+c)ai+a)^n}{\sec(dx+c)^n\sec(dx+c)^4}dx}{e^n e^4}$$

input
```
int((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*a*i + a)**n/(sec(c + d*x)**n*sec(c + d*x)**4),x)/(e**n*e
**4)
```

3.484 $\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 205

$$\begin{aligned}\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx &= \frac{i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n}{d(3-n)} \\ &\quad+\frac{3i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{1+n}}{ad(3-4n+n^2)} \\ &\quad-\frac{6i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{2+n}}{a^2d(3-n)(1-n^2)} \\ &\quad+\frac{6i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{3+n}}{a^3d(9-10n^2+n^4)}\end{aligned}$$

output
```
I*(e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n/d/(3-n)+3*I*(e*sec(d*x+c))^(-
3-n)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(n^2-4*n+3)-6*I*(e*sec(d*x+c))^(-3-n)*(a
+I*a*tan(d*x+c))^(2+n)/a^2/d/(3-n)/(-n^2+1)+6*I*(e*sec(d*x+c))^(-3-n)*(a+I
*a*tan(d*x+c))^(3+n)/a^3/d/(n^4-10*n^2+9)
```

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.58

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\frac{(e\sec(c+dx))^{-n}\left(-3in(-9+n^2)\cos(c+dx)-in(-1+n^2)\cos(3(c+dx))-6(-5+n^2+(-1+n^2)\,\mathrm{c}\right.}{4de^3(-3+n)(-1+n)(1+n)(3+n)}$$

input
```
Integrate[(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(((-3*I)*n*(-9 + n^2)*Cos[c + d*x] - I*n*(-1 + n^2)*Cos[3*(c + d*x)] - 6*(
-5 + n^2 + (-1 + n^2)*Cos[2*(c + d*x)])*Sin[c + d*x])*(a + I*a*Tan[c + d*x
])^n)/(4*d*e^3*(-3 + n)*(-1 + n)*(1 + n)*(3 + n)*(e*Sec[c + d*x])^n)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3985, 3042, 3985, 3042, 3985, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-3}dx$$
$$\Big\downarrow \; 3985$$
$$\frac{3\int (e\sec(c+dx))^{-n-3}(i\tan(c+dx)a+a)^{n+1}dx}{a(3-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-3}}{d(3-n)}$$
$$\Big\downarrow \; 3042$$
$$\frac{3\int (e\sec(c+dx))^{-n-3}(i\tan(c+dx)a+a)^{n+1}dx}{a(3-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-3}}{d(3-n)}$$

$$\Big\downarrow \text{3985}$$

$$\frac{3\left(\frac{2\int(e\sec(c+dx))^{-n-3}(i\tan(c+dx)a+a)^{n+2}dx}{a(1-n)}+\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-3}}{d(1-n)}\right)}{a(3-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-3}}{d(3-n)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2\int(e\sec(c+dx))^{-n-3}(i\tan(c+dx)a+a)^{n+2}dx}{a(1-n)}+\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-3}}{d(1-n)}\right)}{a(3-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-3}}{d(3-n)}$$

$$\Big\downarrow \text{3985}$$

$$\frac{3\left(\frac{2\left(-\frac{\int(e\sec(c+dx))^{-n-3}(i\tan(c+dx)a+a)^{n+3}dx}{a(n+1)}-\frac{i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-3}}{d(n+1)}\right)}{a(1-n)}+\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-3}}{d(1-n)}\right)}{a(3-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-3}}{d(3-n)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2\left(-\frac{\int(e\sec(c+dx))^{-n-3}(i\tan(c+dx)a+a)^{n+3}dx}{a(n+1)}-\frac{i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-3}}{d(n+1)}\right)}{a(1-n)}+\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-3}}{d(1-n)}\right)}{a(3-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-3}}{d(3-n)}$$

$$\Big\downarrow \text{3969}$$

$$\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-3}}{d(3-n)}+\frac{3\left(\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-3}}{d(1-n)}+\frac{2\left(\frac{i(a+ia\tan(c+dx))^{n+3}(e\sec(c+dx))^{-n-3}}{ad(n+1)(n+3)}-\frac{i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-3}}{d(n+1)}\right)}{a(1-n)}\right)}{a(3-n)}$$

input
```
Int[(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(3 - n)) + (3*((
I*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(d*(1 - n)) +
(2*(((-I)*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^(2 + n))/(d*(1
+ n)) + (I*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^(3 + n))/(a*d*
(1 + n)*(3 + n))))/(a*(1 - n))))/(a*(3 - n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3985
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}
, x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.84 (sec) , antiderivative size = 4982, normalized size of antiderivative = 24.30

method	result	size
risch	Expression too large to display	4982

input
```
int((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
-1/8*I/(-3+n)/d*a^n*exp(I*(d*x+c))^n/(e^n)/e^3*exp(-1/2*I*(6*c-csgn(I*a*ex
p(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*csgn(I*a)*Pi*n-csgn(I*a*exp(2*I*(d*
x+c))/(exp(2*I*(d*x+c))+1))^2*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c))
)*Pi*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*exp(2*I*(d*x
+c)))*Pi*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I/(exp(2*I
*(d*x+c))+1))*Pi*n-2*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*Pi*
n+csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*Pi*n+csgn(I*a*exp(2*I*
(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a)*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2
*I*(d*x+c)))*Pi*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*exp
(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1))*Pi*n+6*d*x+3*Pi*csgn(I*e)*csgn
(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*e)*csgn(I*e*exp(I*
(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*
(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+
c))+1))^2*csgn(I/(exp(2*I*(d*x+c))+1))-3*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I
*(d*x+c))+1))^3-n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))
*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-3*Pi*csgn(I*exp(I*(d*x+c)))
*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I/(exp(2*I*(d*x+c))+1))-
n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*cs
gn(I/(exp(2*I*(d*x+c))+1))+3*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c
))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.29

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$$

$$= \frac{\left(-i\,n^3 - 3i\,n^2 + (-i\,n^3 + 3i\,n^2 + i\,n - 3i)e^{(6i\,dx+6i\,c)} - 3\,(i\,n^3 - i\,n^2 - 9i\,n + 9i)e^{(4i\,dx+4i\,c)} - 3\,(i\,n^3 + i\right.}{dn^4 - 10\,dn^2 + (dn^4 - 10\,dn^2 + 9\,d)e^{(6i\,dx+6i\,c)} + 3\,(dn^4 - 10\,dn^2 +}$$

input
```
integrate((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

output
```
(-I*n^3 - 3*I*n^2 + (-I*n^3 + 3*I*n^2 + I*n - 3*I)*e^(6*I*d*x + 6*I*c) - 3
*(I*n^3 - I*n^2 - 9*I*n + 9*I)*e^(4*I*d*x + 4*I*c) - 3*(I*n^3 + I*n^2 - 9*
I*n - 9*I)*e^(2*I*d*x + 2*I*c) + I*n + 3*I)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d
*x + 2*I*c) + 1))^(-n - 3)*e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/
(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e))/(d*n^4 - 10*d*n^2 + (d*n^4 - 10*d
*n^2 + 9*d)*e^(6*I*d*x + 6*I*c) + 3*(d*n^4 - 10*d*n^2 + 9*d)*e^(4*I*d*x +
4*I*c) + 3*(d*n^4 - 10*d*n^2 + 9*d)*e^(2*I*d*x + 2*I*c) + 9*d)
```

Sympy [F]

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(c+dx))^{-n-3}\,(ia(\tan(c+dx)-i))^n\,dx$$

input
```
integrate((e*sec(d*x+c))**(-3-n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(-n - 3)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.69

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx = \frac{(-i\,a^n n^3+3i\,a^n n^2+i\,a^n n-3i\,a^n)\cos((dx+c)(n+3))-3\,(i\,a^n n^3-i\,a^n n^2-9i\,a^n n+9i\,a^n)\cos((dx}{}$$

input
```
integrate((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

output
```
1/8*((-I*a^n*n^3 + 3*I*a^n*n^2 + I*a^n*n - 3*I*a^n)*cos((d*x + c)*(n + 3))
 - 3*(I*a^n*n^3 - I*a^n*n^2 - 9*I*a^n*n + 9*I*a^n)*cos((d*x + c)*(n + 1))
- 3*(I*a^n*n^3 + I*a^n*n^2 - 9*I*a^n*n - 9*I*a^n)*cos((d*x + c)*(n - 1)) +
 (-I*a^n*n^3 - 3*I*a^n*n^2 + I*a^n*n + 3*I*a^n)*cos((d*x + c)*(n - 3)) + (
a^n*n^3 - 3*a^n*n^2 - a^n*n + 3*a^n)*sin((d*x + c)*(n + 3)) + 3*(a^n*n^3 -
 a^n*n^2 - 9*a^n*n + 9*a^n)*sin((d*x + c)*(n + 1)) + 3*(a^n*n^3 + a^n*n^2
- 9*a^n*n - 9*a^n)*sin((d*x + c)*(n - 1)) + (a^n*n^3 + 3*a^n*n^2 - a^n*n -
 3*a^n)*sin((d*x + c)*(n - 3)))/((e^(n + 3)*n^4 - 10*e^(n + 3)*n^2 + 9*e^(
n + 3))*d)
```

Giac [F]

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec(dx+c))^{-n-3}\,(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-n - 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B] (verification not implemented)

Time = 6.52 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.07

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx =$$
$$-\frac{\left(2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,\left(2\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2+\sin\left(3\,c+3\,d\,x\right)\,1\mathrm{i}-1\right)\,\left(\frac{\left(a-\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}\right)^n\,\left(-n^3-3\,n^2+n+\right.}{d\,\left(n^4\,1\mathrm{i}-n^2\,10\mathrm{i}+9\mathrm{i}\right)}\right.}{}$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(n + 3),x)
```

output
```
-((2*sin(c/2 + (d*x)/2)^2 - 1)*(sin(3*c + 3*d*x)*1i + 2*sin((3*c)/2 + (3*d
*x)/2)^2 - 1)*(((a - (a*sin(c + d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(
n - 3*n^2 - n^3 + 3))/(d*(n^4*1i - n^2*10i + 9i)) + ((a - (a*sin(c + d*x)*
1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(sin(6*c + 6*d*x)*1i - 2*sin(3*c + 3*d
*x)^2 + 1)*(n + 3*n^2 - n^3 - 3))/(d*(n^4*1i - n^2*10i + 9i)) + ((a - (a*s
in(c + d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(sin(2*c + 2*d*x)*1i - 2*s
in(c + d*x)^2 + 1)*(27*n - 3*n^2 - 3*n^3 + 27))/(d*(n^4*1i - n^2*10i + 9i)
) + ((a - (a*sin(c + d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(sin(4*c + 4
*d*x)*1i - 2*sin(2*c + 2*d*x)^2 + 1)*(27*n + 3*n^2 - 3*n^3 - 27))/(d*(n^4*
1i - n^2*10i + 9i))))/(8*(-e/(2*sin(c/2 + (d*x)/2)^2 - 1))^(n + 3)*(sin(c
+ d*x)^2 - 1)^2)
```

Reduce [F]

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx = \frac{\int \frac{(\tan(dx+c)ai+a)^n}{\sec(dx+c)^n\sec(dx+c)^3}dx}{e^n e^3}$$

input
```
int((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*a*i + a)**n/(sec(c + d*x)**n*sec(c + d*x)**3),x)/(e**n*e
**3)
```

3.485 $\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 148

$$\begin{aligned}\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx &= \frac{i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n}{d(2-n)}\\ &\quad-\frac{2i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^{1+n}}{ad(2-n)n}\\ &\quad+\frac{2i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^{2+n}}{a^2dn\,(4-n^2)}\end{aligned}$$

output
```
I*(e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n/d/(2-n)-2*I*(e*sec(d*x+c))^(-
2-n)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(2-n)/n+2*I*(e*sec(d*x+c))^(-2-n)*(a+I*a
*tan(d*x+c))^(2+n)/a^2/d/n/(-n^2+4)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.55

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx = -\frac{i(e\sec(c+dx))^{-n}\left(-4+n^2+n^2\cos(2(c+dx))-2in\sin(2(c+dx))\right)(a+ia\tan(c+dx))^n}{2de^2(-2+n)n(2+n)}$$

input
```
Integrate[(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/2*I)*(-4 + n^2 + n^2*Cos[2*(c + d*x)] - (2*I)*n*Sin[2*(c + d*x)])*(a
+ I*a*Tan[c + d*x])^n)/(d*e^2*(-2 + n)*n*(2 + n)*(e*Sec[c + d*x])^n)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3985, 3042, 3985, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-2}dx$$

$$\Big\downarrow \text{3985}$$

$$\frac{2\int (e\sec(c+dx))^{-n-2}(i\tan(c+dx)a+a)^{n+1}dx}{a(2-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-2}}{d(2-n)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int (e\sec(c+dx))^{-n-2}(i\tan(c+dx)a+a)^{n+1}dx}{a(2-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-2}}{d(2-n)}$$

$$\Big\downarrow \text{3985}$$

$$\frac{2\left(-\frac{\int (e\sec(c+dx))^{-n-2}(i\tan(c+dx)a+a)^{n+2}dx}{an}-\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-2}}{dn}\right)}{a(2-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-2}}{d(2-n)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(-\frac{\int (e\sec(c+dx))^{-n-2}(i\tan(c+dx)a+a)^{n+2}dx}{an}-\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-2}}{dn}\right)}{a(2-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-2}}{d(2-n)}$$

$$\Big\downarrow \text{3969}$$

$$\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-2}}{d(2-n)}+\frac{2\left(\frac{i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-2}}{adn(n+2)}-\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-2}}{dn}\right)}{a(2-n)}$$

input
```
Int[(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(2 - n)) + (2*((
(-I)*(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(d*n) + (I*
(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^(2 + n))/(a*d*n*(2 + n)))
)/(a*(2 - n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3985
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}
, x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.47 (sec) , antiderivative size = 2581, normalized size of antiderivative = 17.44

method	result	size
risch	Expression too large to display	2581

input
```
int((e*sec(d*x+c))^(-n-2)*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/(n-2)/d*a^n/e^2*exp(I*(d*x+c))^n/(e^n)*exp(-1/2*I*(n*Pi*csgn(I*e)*c
sgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I*e)*csgn(I*exp(I
*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+
1))+csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*Pi*n-csgn(I*a*exp(2*
I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*csgn(I*a)*Pi*n-csgn(I*a*exp(2*I*(d*x+c)
)/(exp(2*I*(d*x+c))+1))^2*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*Pi
*n+csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a)*csgn(I/(exp(2
*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*Pi*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*
(d*x+c)))^3*Pi*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*ex
p(2*I*(d*x+c)))*Pi*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(
I/(exp(2*I*(d*x+c))+1))*Pi*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))
*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1))*Pi*n-n*Pi*csgn(I*e*
exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I
*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I*
exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*
exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I
*(d*x+c))+1))^2*csgn(I/(exp(2*I*(d*x+c))+1))-n*Pi*csgn(I*exp(I*(d*x+c)))*c
sgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I/(exp(2*I*(d*x+c))+1))+cs
gn(I*exp(2*I*(d*x+c)))^3*Pi*n-2*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d
*x+c)))*Pi*n+csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*Pi*n+4*d...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.20

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{\left(-i\,n^2+(-i\,n^2+2i\,n)e^{(4i\,dx+4i\,c)}-2\,(i\,n^2-4i)e^{(2i\,dx+2i\,c)}-2i\,n\right)\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-n-2}e^{\left(i\,dnx+i\,cn+n\log\left(\frac{2\,e}{e^{(2}}\right.\right.}}{dn^3-4\,dn+(dn^3-4\,dn)e^{(4i\,dx+4i\,c)}+2\,(dn^3-4\,dn)e^{(2i\,dx+2i\,c)}}$$

input
```
integrate((e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

output
```
(-I*n^2 + (-I*n^2 + 2*I*n)*e^(4*I*d*x + 4*I*c) - 2*(I*n^2 - 4*I)*e^(2*I*d*
x + 2*I*c) - 2*I*n)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n -
2)*e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1
)) + n*log(a/e))/(d*n^3 - 4*d*n + (d*n^3 - 4*d*n)*e^(4*I*d*x + 4*I*c) + 2*
(d*n^3 - 4*d*n)*e^(2*I*d*x + 2*I*c))
```

Sympy [F]

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \int (e\sec{(c+dx)})^{-n-2}\,(ia(\tan{(c+dx)}-i))^n\;dx$$

input
```
integrate((e*sec(d*x+c))**(-2-n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(-n - 2)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.18

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{(-i\,a^n n^2+2i\,a^n n)\cos((dx+c)(n+2))+(-i\,a^n n^2-2i\,a^n n)\cos((dx+c)(n-2))-2\,(i\,a^n n^2-4i\,a^n)}{}$$

input
```
integrate((e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

output
```
1/4*((-I*a^n*n^2 + 2*I*a^n*n)*cos((d*x + c)*(n + 2)) + (-I*a^n*n^2 - 2*I*a
^n*n)*cos((d*x + c)*(n - 2)) - 2*(I*a^n*n^2 - 4*I*a^n)*cos((d*x + c)*n) +
(a^n*n^2 - 2*a^n*n)*sin((d*x + c)*(n + 2)) + (a^n*n^2 + 2*a^n*n)*sin((d*x
+ c)*(n - 2)) + 2*(a^n*n^2 - 4*a^n)*sin((d*x + c)*n))/((e^(n + 2)*n^3 - 4*
e^(n + 2)*n)*d)
```

Giac [F]

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \int (e\sec(dx+c))^{-n-2}(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-n - 2)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B] (verification not implemented)

Time = 6.67 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.53

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\frac{(\cos(2\,c+2\,d\,x)-\sin(2\,c+2\,d\,x)\ 1\mathrm{i})\left(\frac{\left(a+\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{\cos(c+d\,x)}\right)^n(n+2)}{d\,(n^2\,1\mathrm{i}-4\mathrm{i})}+\frac{(\cos(4\,c+4\,d\,x)+\sin(4\,c+4\,d\,x)\,1\mathrm{i})\left(a+\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{\cos(c+d\,x)}\right)^n}{d\,(n^2\,1\mathrm{i}-4\mathrm{i})}\right.}{4\left(\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{1}{2}\right)\left(\frac{e}{\cos(c+d\,x)}\right)^{n+2}}$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(n + 2),x)
```

output
```
((cos(2*c + 2*d*x) - sin(2*c + 2*d*x)*1i)*(((a + (a*sin(c + d*x)*1i)/cos(c
 + d*x))^n*(n + 2))/(d*(n^2*1i - 4i)) + ((cos(4*c + 4*d*x) + sin(4*c + 4*d
*x)*1i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^n*(n - 2))/(d*(n^2*1i - 4i)
) + ((cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i)*(2*n^2 - 8)*(a + (a*sin(c +
d*x)*1i)/cos(c + d*x))^n)/(d*n*(n^2*1i - 4i))))/(4*(cos(2*c + 2*d*x)/2 + 1
/2)*(e/cos(c + d*x))^(n + 2))
```

Reduce [F]

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx=\frac{\int \frac{(\tan(dx+c)ai+a)^n}{\sec(dx+c)^n\sec(dx+c)^2}dx}{e^ne^2}$$

input
```
int((e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*a*i + a)**n/(sec(c + d*x)**n*sec(c + d*x)**2),x)/(e**n*e
**2)
```

3.486 $\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 94

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx = \frac{i(e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n}{d(1-n)} - \frac{i(e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^{1+n}}{ad(1-n^2)}$$

output
```
I*(e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n/d/(1-n)-I*(e*sec(d*x+c))^(-1-
n)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(-n^2+1)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.62

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx = -\frac{i(e\sec(c+dx))^{-1-n}(n-i\tan(c+dx))(a+ia\tan(c+dx))^n}{d(-1+n)(1+n)}$$

input
```
Integrate[(e*Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*(e*Sec[c + d*x])^(-1 - n)*(n - I*Tan[c + d*x])*(a + I*a*Tan[c + d*x]
)^n)/(d*(-1 + n)*(1 + n))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 3985, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-1}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-1}dx$$
$$\Big\downarrow \text{3985}$$
$$\frac{\int (e\sec(c+dx))^{-n-1}(i\tan(c+dx)a+a)^{n+1}dx}{a(1-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-1}}{d(1-n)}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int (e\sec(c+dx))^{-n-1}(i\tan(c+dx)a+a)^{n+1}dx}{a(1-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-1}}{d(1-n)}$$
$$\Big\downarrow \text{3969}$$
$$\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-1}}{d(1-n)}-\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-1}}{ad(1-n)(n+1)}$$

input
```
Int[(e*Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*(e*Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(1 - n)) - (I*(e
*Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(1 - n)*(1 +
n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3985
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}
, x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.90 (sec) , antiderivative size = 2484, normalized size of antiderivative = 26.43

method	result	size
risch	Expression too large to display	2484

input
```
int((e*sec(d*x+c))^(-n-1)*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/(-1+n)/d/e*a^n/(e^n)*exp(I*(d*x+c))^n*exp(-1/2*I*(2*c-csgn(I*a*exp(
2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*csgn(I*a)*Pi*n-csgn(I*a*exp(2*I*(d*x+
c))/(exp(2*I*(d*x+c))+1))^2*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*
Pi*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*exp(2*I*(d*x+c
)))*Pi*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I/(exp(2*I*(
d*x+c))+1))*Pi*n-2*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*Pi*n+
csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*Pi*n+csgn(I*a*exp(2*I*(d
*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a)*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I
*(d*x+c)))*Pi*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*exp(2
*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1))*Pi*n+2*d*x+Pi*csgn(I*e)*csgn(I*e
*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*e)*csgn(I*e*exp(I*(d*x
+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x
+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+
1))^2*csgn(I/(exp(2*I*(d*x+c))+1))-Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+
c))+1))^3-n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(
I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*
exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I/(exp(2*I*(d*x+c))+1))-n*Pi*csg
n(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I/(ex
p(2*I*(d*x+c))+1))+Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*
I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.37

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\frac{\left((-i\,n+i)e^{(2i\,dx+2i\,c)}-i\,n-i\right)\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-n-1}e^{\left(i\,dnx+i\,cn+n\log\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)+n\log\left(\frac{a}{e}\right)\right)}}{dn^2+(dn^2-d)e^{(2i\,dx+2i\,c)}-d}$$

input
```
integrate((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

output
```
((-I*n + I)*e^(2*I*d*x + 2*I*c) - I*n - I)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*
x + 2*I*c) + 1))^(-n - 1)*e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(
e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e))/(d*n^2 + (d*n^2 - d)*e^(2*I*d*x +
2*I*c) - d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(71) = 142$.

Time = 0.54 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.62

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \begin{cases} x(e\sec{(c)})^{-n-1}\left(ia\tan{(c)}+a\right)^n & \text{for } d=0 \\ \frac{dx\tan{(c+dx)}}{2ad\tan{(c+dx)}-2iad} - \frac{idx}{2ad\tan{(c+dx)}-2iad} + \frac{1}{2ad\tan{(c+dx)}-2iad} & \text{for } n=-1 \\ \frac{\frac{ax\tan^2{(c+dx)}}{2\sec^2{(c+dx)}} + \frac{ax}{2\sec^2{(c+dx)}} + \frac{a\tan{(c+dx)}}{2d\sec^2{(c+dx)}} - \frac{ia}{2d\sec^2{(c+dx)}}}{e^2} & \text{for } n=1 \\ -\frac{in(e\sec{(c+dx)})^{-n-1}(ia\tan{(c+dx)}+a)^n}{dn^2-d} - \frac{(e\sec{(c+dx)})^{-n-1}(ia\tan{(c+dx)}+a)^n\tan{(c+dx)}}{dn^2-d} & \text{otherwise} \end{cases}$$

input
```
integrate((e*sec(d*x+c))**(-1-n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Piecewise((x*(e*sec(c))**(-n - 1)*(I*a*tan(c) + a)**n, Eq(d, 0)), (d*x*tan
(c + d*x)/(2*a*d*tan(c + d*x) - 2*I*a*d) - I*d*x/(2*a*d*tan(c + d*x) - 2*I
*a*d) + 1/(2*a*d*tan(c + d*x) - 2*I*a*d), Eq(n, -1)), ((a*x*tan(c + d*x)**
2/(2*sec(c + d*x)**2) + a*x/(2*sec(c + d*x)**2) + a*tan(c + d*x)/(2*d*sec(
c + d*x)**2) - I*a/(2*d*sec(c + d*x)**2))/e**2, Eq(n, 1)), (-I*n*(e*sec(c
+ d*x))**(-n - 1)*(I*a*tan(c + d*x) + a)**n/(d*n**2 - d) - (e*sec(c + d*x)
)**(-n - 1)*(I*a*tan(c + d*x) + a)**n*tan(c + d*x)/(d*n**2 - d), True))
```

Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.20

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{(-i\,a^n n + i\,a^n)\cos\left((dx+c)(n+1)\right) + (-i\,a^n n - i\,a^n)\cos\left((dx+c)(n-1)\right) + (a^n n - a^n)\sin\left((dx+c)(\right.}{2\left(e^{n+1}n^2 - e^{n+1}\right)d}$$

input
```
integrate((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

output
```
1/2*((-I*a^n*n + I*a^n)*cos((d*x + c)*(n + 1)) + (-I*a^n*n - I*a^n)*cos((d
*x + c)*(n - 1)) + (a^n*n - a^n)*sin((d*x + c)*(n + 1)) + (a^n*n + a^n)*si
n((d*x + c)*(n - 1)))/((e^(n + 1)*n^2 - e^(n + 1))*d)
```

Giac [F]

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec(dx+c))^{-n-1}(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-n - 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx =$$
$$-\frac{\left(\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}\right)^n\,(\sin(c+d\,x)+\sin(3\,c+3\,d\,x)+n\,\cos(c+d\,x)\,3\mathrm{i}+n\,\cos(3\,c+3\,d}{2\,d\,e\,(\cos(2\,c+2\,d\,x)+1)\,(n^2-1)\,\left(\frac{e}{\cos(c+d\,x)}\right)^n}$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(n + 1),x)
```

output
```
-(((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1)
)^n*(sin(c + d*x) + sin(3*c + 3*d*x) + n*cos(c + d*x)*3i + n*cos(3*c + 3*d
*x)*1i))/(2*d*e*(cos(2*c + 2*d*x) + 1)*(n^2 - 1)*(e/cos(c + d*x))^n)
```

Reduce [F]

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx = \frac{\int \frac{(\tan(dx+c)ai+a)^n}{\sec(dx+c)^n\sec(dx+c)}dx}{e^n e}$$

input
```
int((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*a*i + a)**n/(sec(c + d*x)**n*sec(c + d*x)),x)/(e**n*e)
```

3.487 $\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 37

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = -\frac{i(e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n}{dn}$$

output
```
-I*(a+I*a*tan(d*x+c))^n/d/n/((e*sec(d*x+c))^n)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = -\frac{i(e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n}{dn}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^n,x]
```

output
```
((-I)*(a + I*a*Tan[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^n)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n}dx$$

$$\Big\downarrow \text{3969}$$

$$-\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n}}{dn}$$

input
```
Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^n,x]
```

output
```
((-I)*(a + I*a*Tan[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^n)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Maple [A] (verified)

Time = 5.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

method	result	size
orering	$-\frac{i(a+ia\tan(dx+c))^{n}(e\sec(dx+c))^{-n}}{dn}$	36
risch	Expression too large to display	842

input
```
int((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x,method=_RETURNVERBOSE)
```

output
```
-I*(a+I*a*tan(d*x+c))^n/d/n/((e*sec(d*x+c))^n)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(33) = 66.

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.27

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^{n}\,dx=-\frac{i\,e^{\left(i\,dnx+i\,cn+n\log\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)+n\log\left(\frac{a}{e}\right)\right)}}{dn\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}}$$

input
```
integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x, algorithm="fricas")
```

output
```
-I*e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1
)) + n*log(a/e))/(d*n*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n)
```

Sympy [A] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = \begin{cases} x & \text{for } d=0\wedge n=0 \\ x(e\sec{(c)})^{-n}\,(ia\tan{(c)}+a)^n & \text{for } d=0 \\ x & \text{for } n=0 \\ -\frac{i(e\sec{(c+dx)})^{-n}(ia\tan{(c+dx)}+a)^n}{dn} & \text{otherwise} \end{cases}$$

input
```
integrate((a+I*a*tan(d*x+c))**n/((e*sec(d*x+c))**n),x)
```

output
```
Piecewise((x, Eq(d, 0) & Eq(n, 0)), (x*(I*a*tan(c) + a)**n/(e*sec(c))**n,
Eq(d, 0)), (x, Eq(n, 0)), (-I*(I*a*tan(c + d*x) + a)**n/(d*n*(e*sec(c + d*
x))**n), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(33) = 66.

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.32

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = -\frac{i\,a^n e^{\left(n\log\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)-n\log\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)\right)}}{de^n n}$$

input
```
integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x, algorithm="maxima")
```

output
```
-I*a^n*e^(n*log(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - 1) - n*log(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))/(
d*e^n*n)
```

Giac [F]

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(i\,a\tan(dx+c)+a)^n}{(e\sec(dx+c))^n}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^n}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^n,x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^n, x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = \frac{\int \frac{(\tan(dx+c)ai+a)^n}{\sec(dx+c)^n}dx}{e^n}$$

input
```
int((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x)
```

output
```
int((tan(c + d*x)*a*i + a)**n/sec(c + d*x)**n,x)/e**n
```

3.488 $\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 121

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{\frac{1+n}{2}}a\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{1-n}(1+i\tan(c+dx))^{\frac{1-}{2}}}{d(1-n)}$$

output
```
I*2^(1/2+1/2*n)*a*hypergeom([1/2-1/2*n, 1/2-1/2*n],[3/2-1/2*n],1/2-1/2*I*t
an(d*x+c))*(e*sec(d*x+c))^(1-n)*(1+I*tan(d*x+c))^(1/2-1/2*n)*(a+I*a*tan(d*
x+c))^(-1+n)/d/(1-n)
```

Mathematica [A] (verified)

Time = 3.67 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx = -\frac{e(\text{Hypergeometric2F1}(1,n,1+n,i\cos(c+dx)-\sin(c+dx))-\text{Hypergeometric2F1}(1,n,1+n,-i\,c}{dn}$$

input
```
Integrate[(e*Sec[c + d*x])^(1 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
-((e*(Hypergeometric2F1[1, n, 1 + n, I*Cos[c + d*x] - Sin[c + d*x]] - Hype
rgeometric2F1[1, n, 1 + n, (-I)*Cos[c + d*x] + Sin[c + d*x]])*(a + I*a*Tan
[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^n))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{1-n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{1-n}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{\frac{n-1}{2}}(a+ia\tan(c+dx))^{\frac{n-1}{2}}(e\sec(c+dx))^{1-n}\int (a-ia\tan(c+dx))^{\frac{1-n}{2}}(i\tan(c+dx)a+a)^{\frac{n+1}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{\frac{n-1}{2}}(a+ia\tan(c+dx))^{\frac{n-1}{2}}(e\sec(c+dx))^{1-n}\int (a-ia\tan(c+dx))^{\frac{1-n}{2}}(i\tan(c+dx)a+a)^{\frac{n+1}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{\frac{n-1}{2}}(a+ia\tan(c+dx))^{\frac{n-1}{2}}(e\sec(c+dx))^{1-n}\int (a-ia\tan(c+dx))^{\frac{1}{2}(-n-1)}(i\tan(c+dx)}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2 2^{-\frac{n}{2}-\frac{1}{2}}(1-i\tan(c+dx))^{\frac{n+1}{2}}(a-ia\tan(c+dx))^{\frac{1}{2}(-n-1)+\frac{n-1}{2}}(a+ia\tan(c+dx))^{\frac{n-1}{2}}(e\sec(c+dx))^{1-n}\int \Big(\frac{1}{2}}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{ia2^{\frac{1}{2}-\frac{n}{2}}(1-i\tan(c+dx))^{\frac{n+1}{2}}(a-ia\tan(c+dx))^{\frac{1}{2}(-n-1)+\frac{n-1}{2}}(a+ia\tan(c+dx))^{\frac{n-1}{2}+\frac{n+1}{2}}(e\sec(c+dx))^{1-n}}{d(n+1)}$$

input
```
Int[(e*Sec[c + d*x])^(1 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(1/2 - n/2)*a*Hypergeometric2F1[(1 + n)/2, (1 + n)/2, (3 + n)/2, (
1 + I*Tan[c + d*x])/2]*(e*Sec[c + d*x])^(1 - n)*(1 - I*Tan[c + d*x])^((1 +
 n)/2)*(a - I*a*Tan[c + d*x])^((-1 - n)/2 + (-1 + n)/2)*(a + I*a*Tan[c + d
*x])^((-1 + n)/2 + (1 + n)/2))/(d*(1 + n))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\sec(dx+c))^{1-n}(a+ia\tan(dx+c))^n\,dx$$

input
```
int((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-n+1}(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n + 1)*e^(I*d*n
*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(
a/e)), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(c+dx)})^{1-n}\,(ia(\tan{(c+dx)}-i))^n\,dx$$

input
```
integrate((e*sec(d*x+c))**(1-n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(1 - n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(dx+c)})^{-n+1}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
-2*(a^n*e*cos(c*n + (d*n + d)*x + c) + I*a^n*e*sin(c*n + (d*n + d)*x + c)
- 2*(I*a^n*d*e^(n + 1)*n - I*a^n*d*e^(n + 1) + (I*a^n*d*e^(n + 1)*n - I*a^
n*d*e^(n + 1))*cos(2*d*x + 2*c) - (a^n*d*e^(n + 1)*n - a^n*d*e^(n + 1))*si
n(2*d*x + 2*c))*integrate(((cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*cos
(c*n + (d*n + d)*x + c) + (sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(c*n
+ (d*n + d)*x + c))/((e^n*n - e^n)*cos(4*d*x + 4*c)^2 + 4*(e^n*n - e^n)*co
s(2*d*x + 2*c)^2 + (e^n*n - e^n)*sin(4*d*x + 4*c)^2 + 4*(e^n*n - e^n)*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(e^n*n - e^n)*sin(2*d*x + 2*c)^2 + e^n*n
 + 2*(e^n*n + 2*(e^n*n - e^n)*cos(2*d*x + 2*c) - e^n)*cos(4*d*x + 4*c) + 4
*(e^n*n - e^n)*cos(2*d*x + 2*c) - e^n), x) + 2*(a^n*d*e^(n + 1)*n - a^n*d*
e^(n + 1) + (a^n*d*e^(n + 1)*n - a^n*d*e^(n + 1))*cos(2*d*x + 2*c) - (-I*a
^n*d*e^(n + 1)*n + I*a^n*d*e^(n + 1))*sin(2*d*x + 2*c))*integrate(-((sin(4
*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(c*n + (d*n + d)*x + c) - (cos(4*d*x
+ 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(c*n + (d*n + d)*x + c))/((e^n*n - e^n
)*cos(4*d*x + 4*c)^2 + 4*(e^n*n - e^n)*cos(2*d*x + 2*c)^2 + (e^n*n - e^n)*
sin(4*d*x + 4*c)^2 + 4*(e^n*n - e^n)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4
*(e^n*n - e^n)*sin(2*d*x + 2*c)^2 + e^n*n + 2*(e^n*n + 2*(e^n*n - e^n)*cos
(2*d*x + 2*c) - e^n)*cos(4*d*x + 4*c) + 4*(e^n*n - e^n)*cos(2*d*x + 2*c) -
 e^n), x))/(-I*d*e^n*n + I*d*e^n + (-I*d*e^n*n + I*d*e^n)*cos(2*d*x + 2*c)
 + (d*e^n*n - d*e^n)*sin(2*d*x + 2*c))
```

Giac [F]

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec(dx+c))^{-n+1}(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-n + 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{1-n}(a+a\tan(c+d\,x)\ 1\mathrm{i})^n\,dx$$

input
```
int((e/cos(c + d*x))^(1 - n)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^(1 - n)*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx = \frac{\left(\int \frac{(\tan(dx+c)ai+a)^n\sec(dx+c)}{\sec(dx+c)^n}dx\right)e}{e^n}$$

input
```
int((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
(int(((tan(c + d*x)*a*i + a)**n*sec(c + d*x))/sec(c + d*x)**n,x)*e)/e**n
```

3.489 $\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 113

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{1+\frac{n}{2}}a\,\text{Hypergeometric2F1}\left(\frac{2-n}{2},-\frac{n}{2},\frac{4-n}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{2-n}(1+i\tan(c+dx))^{-n}}{d(2-n)}$$

output
```
I*2^(1+1/2*n)*a*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2-1/2*I*tan(d*x+c)
)*(e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(2-n)/((1+I*tan(d*x+c))
^(1/2*n))
```

Mathematica [A] (verified)

Time = 8.82 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx = \frac{4e^2\,\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},-\cos(2(c+dx))+i\sin(2(c+dx))\right)(e\sec(c+dx))^{-n}(\cos(2c)}{d(-2+n)(-1-i\tan(dx))}$$

input
```
Integrate[(e*Sec[c + d*x])^(2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(4*e^2*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, -Cos[2*(c + d*x)] + I*Sin[2*
(c + d*x)]]*(Cos[2*c] - I*Sin[2*c])*(I + Tan[d*x])*(a + I*a*Tan[c + d*x])^
n)/(d*(-2 + n)*(e*Sec[c + d*x])^n*(-1 - I*Tan[d*x]))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{2-n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{2-n}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{\frac{n-2}{2}}(a+ia\tan(c+dx))^{\frac{n-2}{2}}(e\sec(c+dx))^{2-n}\int (a-ia\tan(c+dx))^{\frac{2-n}{2}}(i\tan(c+dx)a+a)^{\frac{n+2}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{\frac{n-2}{2}}(a+ia\tan(c+dx))^{\frac{n-2}{2}}(e\sec(c+dx))^{2-n}\int (a-ia\tan(c+dx))^{\frac{2-n}{2}}(i\tan(c+dx)a+a)^{\frac{n+2}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{\frac{n-2}{2}}(a+ia\tan(c+dx))^{\frac{n-2}{2}}(e\sec(c+dx))^{2-n}\int (a-ia\tan(c+dx))^{-n/2}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2 2^{-n/2}(1-i\tan(c+dx))^{n/2}(a-ia\tan(c+dx))^{\frac{n-2}{2}-\frac{n}{2}}(a+ia\tan(c+dx))^{\frac{n-2}{2}}(e\sec(c+dx))^{2-n}\int \left(\frac{1}{2}-\frac{1}{2}i\tan\right.}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{ia2^{1-\frac{n}{2}}(1-i\tan(c+dx))^{n/2}(a-ia\tan(c+dx))^{\frac{n-2}{2}-\frac{n}{2}}(a+ia\tan(c+dx))^{\frac{n-2}{2}+\frac{n+2}{2}}(e\sec(c+dx))^{2-n}\text{Hyperg}}{d(n+2)}$$

input
```
Int[(e*Sec[c + d*x])^(2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(1 - n/2)*a*Hypergeometric2F1[n/2, (2 + n)/2, (4 + n)/2, (1 + I*Ta
n[c + d*x])/2]*(e*Sec[c + d*x])^(2 - n)*(1 - I*Tan[c + d*x])^(n/2)*(a - I*
a*Tan[c + d*x])^((-2 + n)/2 - n/2)*(a + I*a*Tan[c + d*x])^((-2 + n)/2 + (2
 + n)/2))/(d*(2 + n))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e \sec (dx+c))^{2-n} (a+ia \tan (dx+c))^n \, dx$$

input
```
int((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e \sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n \, dx = \int (e \sec (dx+c))^{-n+2} (i\, a \tan (dx+c)+a)^n \, dx$$

input
```
integrate((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
1/2*((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n + 2)*(I*e^(2*I*d*
x + 2*I*c) + I)*e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x
 + 2*I*c) + 1)) + n*log(a/e)) + 2*d*e^(2*I*d*x + 2*I*c)*integral(1/2*(n*e^
(2*I*d*x + 2*I*c) + n)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n
 + 2)*e^(I*d*n*x + I*c*n - 2*I*d*x + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x
 + 2*I*c) + 1)) + n*log(a/e) - 2*I*c), x))*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [F]

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(c+dx)})^{2-n}\,(ia(\tan{(c+dx)}-i))^n\,\,dx$$

input
```
integrate((e*sec(d*x+c))**(2-n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(2 - n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(dx+c)})^{-n+2}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
4*(4*a^n*e^2*cos(d*n*x + c*n) + 4*I*a^n*e^2*sin(d*n*x + c*n) - (a^n*e^2*n
- 4*a^n*e^2)*cos(c*n + (d*n + 2*d)*x + 2*c) - 4*(I*a^n*d*e^(n + 2)*n^3 - 6
*I*a^n*d*e^(n + 2)*n^2 + 8*I*a^n*d*e^(n + 2)*n + (I*a^n*d*e^(n + 2)*n^3 -
6*I*a^n*d*e^(n + 2)*n^2 + 8*I*a^n*d*e^(n + 2)*n)*cos(4*d*x + 4*c) + 2*(I*a
^n*d*e^(n + 2)*n^3 - 6*I*a^n*d*e^(n + 2)*n^2 + 8*I*a^n*d*e^(n + 2)*n)*cos(
2*d*x + 2*c) - (a^n*d*e^(n + 2)*n^3 - 6*a^n*d*e^(n + 2)*n^2 + 8*a^n*d*e^(n
 + 2)*n)*sin(4*d*x + 4*c) - 2*(a^n*d*e^(n + 2)*n^3 - 6*a^n*d*e^(n + 2)*n^2
 + 8*a^n*d*e^(n + 2)*n)*sin(2*d*x + 2*c))*integrate(((cos(6*d*x + 6*c) + 3
*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(d*n*x + c*n) + (sin(6*d*x
+ 6*c) + 3*sin(4*d*x + 4*c) + 3*sin(2*d*x + 2*c))*sin(d*n*x + c*n))/(e^n*n
^2 + (e^n*n^2 - 6*e^n*n + 8*e^n)*cos(6*d*x + 6*c)^2 + 9*(e^n*n^2 - 6*e^n*n
 + 8*e^n)*cos(4*d*x + 4*c)^2 + 9*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos(2*d*x + 2
*c)^2 + (e^n*n^2 - 6*e^n*n + 8*e^n)*sin(6*d*x + 6*c)^2 + 9*(e^n*n^2 - 6*e^
n*n + 8*e^n)*sin(4*d*x + 4*c)^2 + 18*(e^n*n^2 - 6*e^n*n + 8*e^n)*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + 9*(e^n*n^2 - 6*e^n*n + 8*e^n)*sin(2*d*x + 2*c)^
2 - 6*e^n*n + 2*(e^n*n^2 - 6*e^n*n + 3*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos(4*d
*x + 4*c) + 3*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos(2*d*x + 2*c) + 8*e^n)*cos(6*
d*x + 6*c) + 6*(e^n*n^2 - 6*e^n*n + 3*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos(2*d*
x + 2*c) + 8*e^n)*cos(4*d*x + 4*c) + 6*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos(2*d
*x + 2*c) + 6*((e^n*n^2 - 6*e^n*n + 8*e^n)*sin(4*d*x + 4*c) + (e^n*n^2 ...
```

Giac [F]

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec(dx+c))^{-n+2}\,(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-n + 2)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{2-n}\,\left(a+a\tan(c+d\,x)\,\,1\mathrm{i}\right)^n\,dx$$

input
```
int((e/cos(c + d*x))^(2 - n)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^(2 - n)*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx=\frac{\left(\int \frac{(\tan(dx+c)ai+a)^n\sec(dx+c)^2}{\sec(dx+c)^n}dx\right)e^2}{e^n}$$

input
```
int((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
(int(((tan(c + d*x)*a*i + a)**n*sec(c + d*x)**2)/sec(c + d*x)**n,x)*e**2)/
e**n
```

3.490 $\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 121

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{i2^{\frac{3+n}{2}}a\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1-n),\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{3-n}(1+i\tan(c+}{d(3-n)}$$

output
```
I*2^(3/2+1/2*n)*a*hypergeom([3/2-1/2*n, -1/2-1/2*n],[5/2-1/2*n],1/2-1/2*I*
tan(d*x+c))*(e*sec(d*x+c))^(3-n)*(1+I*tan(d*x+c))^(-1/2-1/2*n)*(a+I*a*tan(
d*x+c))^(-1+n)/d/(3-n)
```

Mathematica [A] (verified)

Time = 12.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{8e^3\,\text{Hypergeometric2F1}\left(3,\frac{3-n}{2},\frac{5-n}{2},-\cos(2(c+dx))+i\sin(2(c+dx))\right)\sec(dx)(e\sec(c+dx))^{-n}(i+\text{t}}{d(-3+n)(\cos(c)+i\sin(c))^3(-i+\tan(dx))^2}$$

input
```
Integrate[(e*Sec[c + d*x])^(3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(8*e^3*Hypergeometric2F1[3, (3 - n)/2, (5 - n)/2, -Cos[2*(c + d*x)] + I*Si
n[2*(c + d*x)]]*Sec[d*x]*(I + Tan[d*x])*(a + I*a*Tan[c + d*x])^n)/(d*(-3 +
 n)*(e*Sec[c + d*x])^n*(Cos[c] + I*Sin[c])^3*(-I + Tan[d*x])^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{3-n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{3-n}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{\frac{n-3}{2}}(a+ia\tan(c+dx))^{\frac{n-3}{2}}(e\sec(c+dx))^{3-n}\int (a-ia\tan(c+dx))^{\frac{3-n}{2}}(i\tan(c+dx)a+a)^{\frac{n+3}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{\frac{n-3}{2}}(a+ia\tan(c+dx))^{\frac{n-3}{2}}(e\sec(c+dx))^{3-n}\int (a-ia\tan(c+dx))^{\frac{3-n}{2}}(i\tan(c+dx)a+a)^{\frac{n+3}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{\frac{n-3}{2}}(a+ia\tan(c+dx))^{\frac{n-3}{2}}(e\sec(c+dx))^{3-n}\int (a-ia\tan(c+dx))^{\frac{1-n}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2 2^{\frac{1}{2}-\frac{n}{2}}(1-i\tan(c+dx))^{\frac{n-1}{2}}(a-ia\tan(c+dx))^{\frac{1-n}{2}+\frac{n-3}{2}}(a+ia\tan(c+dx))^{\frac{n-3}{2}}(e\sec(c+dx))^{3-n}\int \left(\frac{1}{2}-\frac{1}{2}i\right.}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{ia2^{\frac{3}{2}-\frac{n}{2}}(1-i\tan(c+dx))^{\frac{n-1}{2}}(a-ia\tan(c+dx))^{\frac{1-n}{2}+\frac{n-3}{2}}(a+ia\tan(c+dx))^{\frac{n-3}{2}+\frac{n+3}{2}}(e\sec(c+dx))^{3-n}\operatorname{Hyp}}{d(n+3)}$$

input
```
Int[(e*Sec[c + d*x])^(3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(3/2 - n/2)*a*Hypergeometric2F1[(-1 + n)/2, (3 + n)/2, (5 + n)/2,
(1 + I*Tan[c + d*x])/2]*(e*Sec[c + d*x])^(3 - n)*(1 - I*Tan[c + d*x])^((-1
 + n)/2)*(a - I*a*Tan[c + d*x])^((1 - n)/2 + (-3 + n)/2)*(a + I*a*Tan[c +
d*x])^((-3 + n)/2 + (3 + n)/2))/(d*(3 + n))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\sec(dx+c))^{3-n} (a+ia\tan(dx+c))^n\,dx$$

input
```
int((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-n+3} (i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
1/8*(((-I*n - I)*e^(4*I*d*x + 4*I*c) - 2*I*n*e^(2*I*d*x + 2*I*c) - I*n + I
)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n + 3)*e^(I*d*n*x + I*
c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e)) +
 8*d*e^(2*I*d*x + 2*I*c)*integral(-1/8*(n^2 + (n^2 - 1)*e^(4*I*d*x + 4*I*c
) + 2*(n^2 - 1)*e^(2*I*d*x + 2*I*c) - 1)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x
+ 2*I*c) + 1))^(-n + 3)*e^(I*d*n*x + I*c*n - 2*I*d*x + n*log(2*e*e^(I*d*x
+ I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e) - 2*I*c), x))*e^(-2*I*d*x -
 2*I*c)/d
```

Sympy [F]

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(c+dx)})^{3-n}\,(ia(\tan{(c+dx)}-i))^n\,dx$$

input
```
integrate((e*sec(d*x+c))**(3-n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(3 - n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(dx+c)})^{-n+3}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
8*(6*a^n*e^3*cos(c*n + (d*n + d)*x + c) + 6*I*a^n*e^3*sin(c*n + (d*n + d)*
x + c) - (a^n*e^3*n - 5*a^n*e^3)*cos(c*n + (d*n + 3*d)*x + 3*c) - 6*((I*a^
n*d*e^(n + 3)*n^3 - 7*I*a^n*d*e^(n + 3)*n^2 + 7*I*a^n*d*e^(n + 3)*n + 15*I
*a^n*d*e^(n + 3))*cos(c*n) + ((I*a^n*d*e^(n + 3)*n^3 - 7*I*a^n*d*e^(n + 3)
*n^2 + 7*I*a^n*d*e^(n + 3)*n + 15*I*a^n*d*e^(n + 3))*cos(c*n) - (a^n*d*e^(
n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a^n*d*e^(n +
 3))*sin(c*n))*cos(6*d*x + 6*c) + 3*((I*a^n*d*e^(n + 3)*n^3 - 7*I*a^n*d*e^
(n + 3)*n^2 + 7*I*a^n*d*e^(n + 3)*n + 15*I*a^n*d*e^(n + 3))*cos(c*n) - (a^
n*d*e^(n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a^n*d
*e^(n + 3))*sin(c*n))*cos(4*d*x + 4*c) + 3*((I*a^n*d*e^(n + 3)*n^3 - 7*I*a
^n*d*e^(n + 3)*n^2 + 7*I*a^n*d*e^(n + 3)*n + 15*I*a^n*d*e^(n + 3))*cos(c*n
) - (a^n*d*e^(n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 1
5*a^n*d*e^(n + 3))*sin(c*n))*cos(2*d*x + 2*c) - (a^n*d*e^(n + 3)*n^3 - 7*a
^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a^n*d*e^(n + 3))*sin(c*n) -
((a^n*d*e^(n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a
^n*d*e^(n + 3))*cos(c*n) - (-I*a^n*d*e^(n + 3)*n^3 + 7*I*a^n*d*e^(n + 3)*n
^2 - 7*I*a^n*d*e^(n + 3)*n - 15*I*a^n*d*e^(n + 3))*sin(c*n))*sin(6*d*x + 6
*c) - 3*((a^n*d*e^(n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*
n + 15*a^n*d*e^(n + 3))*cos(c*n) - (-I*a^n*d*e^(n + 3)*n^3 + 7*I*a^n*d*e^(
n + 3)*n^2 - 7*I*a^n*d*e^(n + 3)*n - 15*I*a^n*d*e^(n + 3))*sin(c*n))*si...
```

Giac [F]

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-n+3}(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-n + 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{3-n}\,\left(a+a\,\tan(c+d\,x)\,\,1\mathrm{i}\right)^n\,dx$$

input
```
int((e/cos(c + d*x))^(3 - n)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^(3 - n)*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx=\frac{\left(\int \frac{(\tan(dx+c)ai+a)^n\sec(dx+c)^3}{\sec(dx+c)^n}dx\right)e^3}{e^n}$$

input
```
int((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
(int(((tan(c + d*x)*a*i + a)**n*sec(c + d*x)**3)/sec(c + d*x)**n,x)*e**3)/
e**n
```

3.491 $\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 156

$$\begin{aligned}\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx &= \frac{8ia^3(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-3+n}}{d(5-n)\left(12-7n+n^2\right)} \\ &\quad+\frac{4ia^2(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-2+n}}{d\left(20-9n+n^2\right)} \\ &\quad+\frac{ia(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-1+n}}{d(5-n)}\end{aligned}$$

output
```
8*I*a^3*(e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^(-3+n)/d/(5-n)/(n^2-7*n+
12)+4*I*a^2*(e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^(-2+n)/d/(n^2-9*n+20
)+I*a*(e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(5-n)
```

Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{e^6\sec^5(c+dx)(e\sec(c+dx))^{-2n}\left(-2(-5+n)+(22-9n+n^2)\cos(2(c+dx))+i(18-9n+n^2)\sin\right.}{d(-5+n)(-4+n)(-3+n)}$$

input
```
Integrate[(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
-((e^6*Sec[c + d*x]^5*(-2*(-5 + n) + (22 - 9*n + n^2)*Cos[2*(c + d*x)] + I
*(18 - 9*n + n^2)*Sin[2*(c + d*x)])*(I*Cos[3*(c + d*x)] + Sin[3*(c + d*x)]
)*(a + I*a*Tan[c + d*x])^n)/(d*(-5 + n)*(-4 + n)*(-3 + n)*(e*Sec[c + d*x])
^(2*n)))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{6-2n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{6-2n}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{4a\int (e\sec(c+dx))^{6-2n}(i\tan(c+dx)a+a)^{n-1}dx}{5-n}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{6-2n}}{d(5-n)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\int(e\sec(c+dx))^{6-2n}(i\tan(c+dx)a+a)^{n-1}dx}{5-n}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{6-2n}}{d(5-n)}$$

$$\Big\downarrow \textbf{3975}$$

$$\frac{4a\left(\frac{2a\int(e\sec(c+dx))^{6-2n}(i\tan(c+dx)a+a)^{n-2}dx}{4-n}+\frac{ia(a+ia\tan(c+dx))^{n-2}(e\sec(c+dx))^{6-2n}}{d(4-n)}\right)}{5-n}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{6-2n}}{d(5-n)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{4a\left(\frac{2a\int(e\sec(c+dx))^{6-2n}(i\tan(c+dx)a+a)^{n-2}dx}{4-n}+\frac{ia(a+ia\tan(c+dx))^{n-2}(e\sec(c+dx))^{6-2n}}{d(4-n)}\right)}{5-n}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{6-2n}}{d(5-n)}$$

$$\Big\downarrow \textbf{3974}$$

$$\frac{4a\left(\frac{2ia^2(a+ia\tan(c+dx))^{n-3}(e\sec(c+dx))^{6-2n}}{d(3-n)(4-n)}+\frac{ia(a+ia\tan(c+dx))^{n-2}(e\sec(c+dx))^{6-2n}}{d(4-n)}\right)}{5-n}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{6-2n}}{d(5-n)}$$

input
```
Int[(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*a*(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(5 - n
)) + (4*a*(((2*I)*a^2*(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan[c + d*x])^(-
3 + n))/(d*(3 - n)*(4 - n)) + (I*a*(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan
[c + d*x])^(-2 + n))/(d*(4 - n))))/(5 - n)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [F]

$$\int (e \sec (dx + c))^{6-2n} (a + ia \tan (dx + c))^n \, dx$$

input
```
int((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.06

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{((-i\,n^2+9i\,n-20i)e^{(6i\,dx+6i\,c)}+(-i\,n^2+11i\,n-30i)e^{(4i\,dx+4i\,c)}-2\,(-i\,n+6i)e^{(2i\,dx+2i\,c)}-2i)\left(\frac{2\,ee^{(i\cdots}}{e^{(2i\,dx+\cdots}}\right.}{2\,(dn^3-12\,dn^2+47\,dn-60\,d)}$$

input
```
integrate((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
```

output
```
1/2*((-I*n^2 + 9*I*n - 20*I)*e^(6*I*d*x + 6*I*c) + (-I*n^2 + 11*I*n - 30*I
)*e^(4*I*d*x + 4*I*c) - 2*(-I*n + 6*I)*e^(2*I*d*x + 2*I*c) - 2*I)*(2*e*e^(
I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 6)*e^(I*d*n*x + I*c*n - 6*
I*d*x + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e)
- 6*I*c)/(d*n^3 - 12*d*n^2 + 47*d*n - 60*d)
```

Sympy [F]

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(c+dx))^{6-2n}\,(ia(\tan(c+dx)-i))^n\,dx$$

input
```
integrate((e*sec(d*x+c))**(6-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(6 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1067 vs. $2(140) = 280$.

Time = 1.30 (sec) , antiderivative size = 1067, normalized size of antiderivative = 6.84

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
```

output
```
-32*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/2*n)*a^n*e^6*cos(n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 2
*I*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2
*n)*a^n*e^6*sin(n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^n*
e^6*n^2 - 9*a^n*e^6*n + 20*a^n*e^6)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*cos(4*d*x + n*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1) + 4*c) - 2*(a^n*e^6*n - 5*a^n*e^6)*(cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*cos(2*d*x + n
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1) + 2*c) - (-I*a^n*e^6*n^2
+ 9*I*a^n*e^6*n - 20*I*a^n*e^6)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*sin(4*d*x + n*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1) + 4*c) - 2*(I*a^n*e^6*n - 5*I*a^n*e^6)*(cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*sin(2*d*x + n
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1) + 2*c))/(((-I*e^(2*n)*n^3
 + 12*I*e^(2*n)*n^2 - 47*I*e^(2*n)*n + 60*I*e^(2*n))*2^n*cos(10*d*x + 10*c
) - 5*(I*e^(2*n)*n^3 - 12*I*e^(2*n)*n^2 + 47*I*e^(2*n)*n - 60*I*e^(2*n))*2
^n*cos(8*d*x + 8*c) - 10*(I*e^(2*n)*n^3 - 12*I*e^(2*n)*n^2 + 47*I*e^(2*n)*
n - 60*I*e^(2*n))*2^n*cos(6*d*x + 6*c) - 10*(I*e^(2*n)*n^3 - 12*I*e^(2*n)*
n^2 + 47*I*e^(2*n)*n - 60*I*e^(2*n))*2^n*cos(4*d*x + 4*c) - 5*(I*e^(2*n)*n
^3 - 12*I*e^(2*n)*n^2 + 47*I*e^(2*n)*n - 60*I*e^(2*n))*2^n*cos(2*d*x + ...
```

Giac [F]

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-2n+6}\,(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-2*n + 6)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B] (verification not implemented)

Time = 7.67 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.04

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= (\cos(6c+6dx) - \sin(6c+6dx)\ 1\mathrm{i})\left(\frac{e}{\cos(c+dx)}\right)^{6-2n}\left(\frac{\left(a+\frac{a\sin(c+dx)\,1\mathrm{i}}{\cos(c+dx)}\right)^n}{d\,(n^3\,1\mathrm{i}-n^2\,12\mathrm{i}+n\,47\mathrm{i}-60\mathrm{i})} - \frac{(2n-12)\,(\cos(2c+2dx)+\sin(2c+2dx)\ 1\mathrm{i})\left(a+\frac{a\sin(c+dx)\,1\mathrm{i}}{\cos(c+dx)}\right)^n}{2d\,(n^3\,1\mathrm{i}-n^2\,12\mathrm{i}+n\,47\mathrm{i}-60\mathrm{i})} + \frac{(\cos(6c+6dx)+\sin(6c+6dx)\ 1\mathrm{i})\left(a+\frac{a\sin(c+dx)\,1\mathrm{i}}{\cos(c+dx)}\right)^n(n^2-9n+20)}{2d\,(n^3\,1\mathrm{i}-n^2\,12\mathrm{i}+n\,47\mathrm{i}-60\mathrm{i})} + \frac{(\cos(4c+4dx)+\sin(4c+4dx)\ 1\mathrm{i})\left(a+\frac{a\sin(c+dx)\,1\mathrm{i}}{\cos(c+dx)}\right)^n(n^2-11n+30)}{2d\,(n^3\,1\mathrm{i}-n^2\,12\mathrm{i}+n\,47\mathrm{i}-60\mathrm{i})}\right)$$

input
```
int((e/cos(c + d*x))^(6 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
(cos(6*c + 6*d*x) - sin(6*c + 6*d*x)*1i)*(e/cos(c + d*x))^(6 - 2*n)*((a +
(a*sin(c + d*x)*1i)/cos(c + d*x))^n/(d*(n*47i - n^2*12i + n^3*1i - 60i)) -
 ((2*n - 12)*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i)*(a + (a*sin(c + d*x)
*1i)/cos(c + d*x))^n)/(2*d*(n*47i - n^2*12i + n^3*1i - 60i)) + ((cos(6*c +
 6*d*x) + sin(6*c + 6*d*x)*1i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^n*(n
^2 - 9*n + 20))/(2*d*(n*47i - n^2*12i + n^3*1i - 60i)) + ((cos(4*c + 4*d*x
) + sin(4*c + 4*d*x)*1i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^n*(n^2 - 1
1*n + 30))/(2*d*(n*47i - n^2*12i + n^3*1i - 60i)))
```

Reduce [F]

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx = \frac{\left(\int \frac{(\tan(dx+c)ai+a)^n\sec(dx+c)^6}{\sec(dx+c)^{2n}}dx\right)e^6}{e^{2n}}$$

input
```
int((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
(int(((tan(c + d*x)*a*i + a)**n*sec(c + d*x)**6)/sec(c + d*x)**(2*n),x)*e*
*6)/e**(2*n)
```

3.492 $\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 103

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{\frac{5}{2}-n}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}(-3+2n),\frac{7}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{5-2n}(1-i\tan(c+$$
$$5d$$

output
```
-1/5*I*2^(5/2-n)*hypergeom([5/2, -3/2+n],[7/2],1/2+1/2*I*tan(d*x+c))*(e*se
c(d*x+c))^(5-2*n)*(1-I*tan(d*x+c))^(-5/2+n)*(a+I*a*tan(d*x+c))^n/d
```

Mathematica [A] (verified)

Time = 10.66 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.61

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{5-n}e^{5i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{5}{2},5-n,\frac{7}{2},-e^{2i(c+dx)}\right)\sec^{-}$$
$$5d$$

input
```
Integrate[(e*Sec[c + d*x])^(5 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/5*I)*2^(5 - n)*E^((5*I)*(c + d*x))*(E^(I*d*x))^n*Hypergeometric2F1[5/
2, 5 - n, 7/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-5 + n)*(e*Sec[c + d*x]
)^(5 - 2*n)*(a + I*a*Tan[c + d*x])^n)/(d*(E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x))))^n*(1 + E^((2*I)*(c + d*x)))^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{5-2n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{5-2n}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n-5)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-5)}(e\sec(c+dx))^{5-2n}\int (a-ia\tan(c+dx))^{\frac{1}{2}(5-2n)}(i\tan(c+dx)a+a)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n-5)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-5)}(e\sec(c+dx))^{5-2n}\int (a-ia\tan(c+dx))^{\frac{1}{2}(5-2n)}(i\tan(c+dx)a+a)^{5/2}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{\frac{1}{2}(2n-5)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-5)}(e\sec(c+dx))^{5-2n}\int (a-ia\tan(c+dx))^{\frac{1}{2}(3-2n)}(i\tan(\ldots}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^3 2^{\frac{3}{2}-n}(1-i\tan(c+dx))^{n-\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n-5)+\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-5)}(e\sec(c+dx))^{5-2n}\int\ldots}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{ia^2 2^{\frac{5}{2}-n}(1-i\tan(c+dx))^{n-\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n-5)+\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-5)+\frac{5}{2}}(e\sec(c+dx))^{5}}{5d}$$

input
```
Int[(e*Sec[c + d*x])^(5 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/5*I)*2^(5/2 - n)*a^2*Hypergeometric2F1[5/2, (-3 + 2*n)/2, 7/2, (1 + I
*Tan[c + d*x])/2]*(e*Sec[c + d*x])^(5 - 2*n)*(1 - I*Tan[c + d*x])^(-1/2 +
n)*(a - I*a*Tan[c + d*x])^(1/2 - n + (-5 + 2*n)/2)*(a + I*a*Tan[c + d*x])^
(5/2 + (-5 + 2*n)/2))/d
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e \sec (dx + c))^{5-2n} (a + ia \tan (dx + c))^n \, dx$$

input
```
int((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e \sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n \, dx = \int (e \sec (dx + c))^{-2\,n+5} (i\,a \tan (dx + c) + a)^n \, dx$$

input
```
integrate((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
```

output
```
integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 5)*e^(I*d
*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*lo
g(a/e)), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(c+dx)})^{5-2n}\,(ia(\tan{(c+dx)}-i))^n\,dx$$

input
```
integrate((e*sec(d*x+c))**(5-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(5 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{-2\,n+5}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
```

output
```
integrate((e*sec(d*x + c))^(-2*n + 5)*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{-2\,n+5}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-2*n + 5)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{5-2\,n}(a+a\tan(c+d\,x)\ 1\mathrm{i})^n\,dx$$

input
```
int((e/cos(c + d*x))^(5 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^(5 - 2*n)*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = \frac{\left(\int \frac{(\tan(dx+c)ai+a)^n\sec(dx+c)^5}{\sec(dx+c)^{2n}}dx\right)e^5}{e^{2n}}$$

input
```
int((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
(int(((tan(c + d*x)*a*i + a)**n*sec(c + d*x)**5)/sec(c + d*x)**(2*n),x)*e**5)/e**(2*n)
```

3.493 $\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 98

$$\begin{aligned}\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx &= \frac{2ia^2(e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^{-2+n}}{d\,(6-5n+n^2)}\\ &\quad+\frac{ia(e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^{-1+n}}{d(3-n)}\end{aligned}$$

output
```
2*I*a^2*(e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^(-2+n)/d/(n^2-5*n+6)+I*a
*(e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(3-n)
```

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx = \frac{e^4\sec^2(c+dx)(e\sec(c+dx))^{-2n}(\cos(2(c+dx))-i\sin(2(c+dx)))(a+ia\tan(c+dx))^n(-i(-4+n)+}{d(-3+n)(-2+n)}$$

input
```
Integrate[(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(e^4*Sec[c + d*x]^2*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(a + I*a*Tan[c
 + d*x])^n*((-I)*(-4 + n) + (-2 + n)*Tan[c + d*x]))/(d*(-3 + n)*(-2 + n)*(
e*Sec[c + d*x])^(2*n))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{4-2n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{4-2n}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{2a\int (e\sec(c+dx))^{4-2n}(i\tan(c+dx)a+a)^{n-1}dx}{3-n}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{4-2n}}{d(3-n)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\int (e\sec(c+dx))^{4-2n}(i\tan(c+dx)a+a)^{n-1}dx}{3-n}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{4-2n}}{d(3-n)}$$

$$\Big\downarrow \text{3974}$$

$$\frac{2ia^2(a+ia\tan(c+dx))^{n-2}(e\sec(c+dx))^{4-2n}}{d(2-n)(3-n)}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{4-2n}}{d(3-n)}$$

input
```
Int[(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((2*I)*a^2*(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^(-2 + n))/(d*
(2 - n)*(3 - n)) + (I*a*(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^
(-1 + n))/(d*(3 - n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [F]

$$\int (e \sec (dx + c))^{4-2n} (a + ia \tan (dx + c))^n \, dx$$

input
```
int((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.37

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{\left((-i\,n+3i)e^{(4i\,dx+4i\,c)}+(-i\,n+4i)e^{(2i\,dx+2i\,c)}+i\right)\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-2\,n+4}e^{\left(i\,dnx+i\,cn-4i\,dx+n\log\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right.\right.}}{2\,(dn^2-5\,dn+6\,d)}$$

input
```
integrate((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
```

output
```
1/2*((-I*n + 3*I)*e^(4*I*d*x + 4*I*c) + (-I*n + 4*I)*e^(2*I*d*x + 2*I*c) +
 I)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 4)*e^(I*d*n*x
+ I*c*n - 4*I*d*x + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) +
 n*log(a/e) - 4*I*c)/(d*n^2 - 5*d*n + 6*d)
```

Sympy [F]

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \int (e\sec{(c+dx)})^{4-2n}\,(ia(\tan{(c+dx)}-i))^n\;dx$$

input
```
integrate((e*sec(d*x+c))**(4-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(4 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 595 vs. $2(88) = 176$.

Time = 0.37 (sec) , antiderivative size = 595, normalized size of antiderivative = 6.07

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
```

output
```
8*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2
*n)*a^n*e^4*cos(n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + I*(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*a^
n*e^4*sin(n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (a^n*e^4*n
- 3*a^n*e^4)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/2*n)*cos(2*d*x + n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
) + 2*c) + (-I*a^n*e^4*n + 3*I*a^n*e^4)*(cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*sin(2*d*x + n*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1) + 2*c))/(((-I*e^(2*n)*n^2 + 5*I*e^(2*n)*n - 6
*I*e^(2*n))*2^n*cos(6*d*x + 6*c) - 3*(I*e^(2*n)*n^2 - 5*I*e^(2*n)*n + 6*I*
e^(2*n))*2^n*cos(4*d*x + 4*c) - 3*(I*e^(2*n)*n^2 - 5*I*e^(2*n)*n + 6*I*e^(
2*n))*2^n*cos(2*d*x + 2*c) + (e^(2*n)*n^2 - 5*e^(2*n)*n + 6*e^(2*n))*2^n*s
in(6*d*x + 6*c) + 3*(e^(2*n)*n^2 - 5*e^(2*n)*n + 6*e^(2*n))*2^n*sin(4*d*x
+ 4*c) + 3*(e^(2*n)*n^2 - 5*e^(2*n)*n + 6*e^(2*n))*2^n*sin(2*d*x + 2*c) +
(-I*e^(2*n)*n^2 + 5*I*e^(2*n)*n - 6*I*e^(2*n))*2^n)*d)
```

Giac [F]

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-2n+4}(ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-2*n + 4)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.78

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{4\,e^4\left(\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}\right)^n\,(4\,\sin\left(2\,c+2\,d\,x\right)+\cos\left(2\,c+2\,d\,x\right)\,4\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,1\mathrm{i}-n\,1\mathrm{i}}{d\,\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n}\,\left(4\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)+\right.}$$

input
```
int((e/cos(c + d*x))^(4 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
(4*e^4*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x)
 + 1))^n*(cos(2*c + 2*d*x)*4i - n*1i + cos(4*c + 4*d*x)*1i + 4*sin(2*c + 2
*d*x) + sin(4*c + 4*d*x) - n*cos(2*c + 2*d*x)*1i - n*sin(2*c + 2*d*x) + 3i
))/(d*(e/cos(c + d*x))^(2*n)*(4*cos(2*c + 2*d*x) + cos(4*c + 4*d*x) + 3)*(
n^2 - 5*n + 6))
```

Reduce [F]

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx = \frac{\left(\int \frac{(\tan(dx+c)ai+a)^n\sec(dx+c)^4}{\sec(dx+c)^{2n}}dx\right)e^4}{e^{2n}}$$

input
```
int((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
(int(((tan(c + d*x)*a*i + a)**n*sec(c + d*x)**4)/sec(c + d*x)**(2*n),x)*e*
*4)/e**(2*n)
```

3.494 $\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 103

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{\frac{3}{2}-n}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}(-1+2n),\frac{5}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{3-2n}(1-i\tan(c+dx)}{3d}$$

output
```
-1/3*I*2^(3/2-n)*hypergeom([3/2, -1/2+n],[5/2],1/2+1/2*I*tan(d*x+c))*(e*se
c(d*x+c))^(3-2*n)*(1-I*tan(d*x+c))^(-3/2+n)*(a+I*a*tan(d*x+c))^n/d
```

Mathematica [A] (verified)

Time = 9.49 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.61

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{3-n}e^{3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{3}{2},3-n,\frac{5}{2},-e^{2i(c+dx)}\right)\sec^{-}}{3d}$$

input
```
Integrate[(e*Sec[c + d*x])^(3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/3*I)*2^(3 - n)*E^((3*I)*(c + d*x))*(E^(I*d*x))^n*Hypergeometric2F1[3/
2, 3 - n, 5/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-3 + n)*(e*Sec[c + d*x]
)^(3 - 2*n)*(a + I*a*Tan[c + d*x])^n)/(d*(E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x))))^n*(1 + E^((2*I)*(c + d*x)))^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.36, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{3-2n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{3-2n}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n-3)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-3)}(e\sec(c+dx))^{3-2n}\int (a-ia\tan(c+dx))^{\frac{1}{2}(3-2n)}(i\tan(c+dx)a+a)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n-3)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-3)}(e\sec(c+dx))^{3-2n}\int (a-ia\tan(c+dx))^{\frac{1}{2}(3-2n)}(i\tan(c+dx)a+a)^{3/2}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{\frac{1}{2}(2n-3)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-3)}(e\sec(c+dx))^{3-2n}\int (a-ia\tan(c+dx))^{\frac{1}{2}(1-2n)}\sqrt{i\tan}}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2 2^{\frac{1}{2}-n}(1-i\tan(c+dx))^{n-\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n-3)+\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-3)}(e\sec(c+dx))^{3-2n}\int}{d}$$

$$\Big\downarrow 79$$

$$-\frac{ia2^{\frac{3}{2}-n}(1-i\tan(c+dx))^{n-\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n-3)+\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-3)+\frac{3}{2}}(e\sec(c+dx))^{3-}}{3d}$$

input
```
Int[(e*Sec[c + d*x])^(3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/3*I)*2^(3/2 - n)*a*Hypergeometric2F1[3/2, (-1 + 2*n)/2, 5/2, (1 + I*T
an[c + d*x])/2]*(e*Sec[c + d*x])^(3 - 2*n)*(1 - I*Tan[c + d*x])^(-1/2 + n)
*(a - I*a*Tan[c + d*x])^(1/2 - n + (-3 + 2*n)/2)*(a + I*a*Tan[c + d*x])^(3
/2 + (-3 + 2*n)/2))/d
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !Integ
erQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\sec(dx+c))^{3-2n} (a+ia\tan(dx+c))^n\,dx$$

input
```
int((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-2n+3} (ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
```

output
```
integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 3)*e^(I*d
*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*lo
g(a/e)), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(c+dx)})^{3-2n}\,(ia(\tan{(c+dx)}-i))^n\;dx$$

input
```
integrate((e*sec(d*x+c))**(3-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(3 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(dx+c)})^{-2\,n+3}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
```

output
```
integrate((e*sec(d*x + c))^(-2*n + 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(dx+c)})^{-2\,n+3}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-2*n + 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \int \left(\frac{e}{\cos(c+d\,x)}\right)^{3-2\,n} (a+a\tan(c+d\,x)\ 1\mathrm{i})^n\,dx$$

input
```
int((e/cos(c + d*x))^(3 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^(3 - 2*n)*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = \frac{\left(\int \frac{(\tan(dx+c)ai+a)^n\sec(dx+c)^3}{\sec(dx+c)^{2n}}dx\right)e^3}{e^{2n}}$$

input
```
int((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
(int(((tan(c + d*x)*a*i + a)**n*sec(c + d*x)**3)/sec(c + d*x)**(2*n),x)*e*
*3)/e**(2*n)
```

3.495 $\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 46

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx = \frac{ia(e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^{-1+n}}{d(1-n)}$$

output
```
I*a*(e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(1-n)
```

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{e^2(e\sec(c+dx))^{-2n}(i+\sec(c)\sec(c+dx)\sin(dx)+\tan(c))(a+ia\tan(c+dx))^n}{d(-1+n)}$$

input
```
Integrate[(e*Sec[c + d*x])^(2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
-((e^2*(I + Sec[c]*Sec[c + d*x]*Sin[d*x] + Tan[c])*(a + I*a*Tan[c + d*x])^
n)/(d*(-1 + n)*(e*Sec[c + d*x])^(2*n)))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + ia\tan(c + dx))^n (e\sec(c + dx))^{2-2n} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a + ia\tan(c + dx))^n (e\sec(c + dx))^{2-2n} dx$$

$$\Big\downarrow \text{3974}$$

$$\frac{ia(a + ia\tan(c + dx))^{n-1}(e\sec(c + dx))^{2-2n}}{d(1 - n)}$$

input
```
Int[(e*Sec[c + d*x])^(2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*a*(e*Sec[c + d*x])^(2 - 2*n)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(1 - n
))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Maple [F]

$$\int (e \sec (dx+c))^{2-2n} (a+ia\tan (dx+c))^n \, dx$$

input
```
int((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(40) = 80$.

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.37

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n \, dx = \frac{\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-2\,n+2}\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)e^{\left(i\,dnx+i\,cn-2i\,dx+n\log\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)+n\log\left(\frac{a}{e}\right)-2i\,c\right)}}{2\,(dn-d)}$$

input
```
integrate((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
```

output
```
1/2*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 2)*(-I*e^(2*I*
d*x + 2*I*c) - I)*e^(I*d*n*x + I*c*n - 2*I*d*x + n*log(2*e*e^(I*d*x + I*c)
/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e) - 2*I*c)/(d*n - d)
```

Sympy [F]

$$
\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx
= \int (e\sec(c+dx))^{2-2n}\,(ia(\tan(c+dx)-i))^n\;dx
$$

input
```
integrate((e*sec(d*x+c))**(2-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(2 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(40) = 80$.

Time = 0.20 (sec) , antiderivative size = 217, normalized size of antiderivative = 4.72

$$
\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx
= \frac{\left(-i\,a^n e^2 - \frac{2\,a^n e^2\sin(dx+c)}{\cos(dx+c)+1} + \frac{i\,a^n e^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right) e^{\left(n\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)+n\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)+n\log\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+}{(\cos(dx+c}\right.\right.}}{\left(e^{2\,n}(n-1) - \frac{e^{2\,n}(n-1)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d}
$$

input
```
integrate((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
```

output
```
(-I*a^n*e^2 - 2*a^n*e^2*sin(d*x + c)/(cos(d*x + c) + 1) + I*a^n*e^2*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2)*e^(n*log(sin(d*x + c)/(cos(d*x + c) + 1) +
1) + n*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1) + n*log(-2*I*sin(d*x + c)/
(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1) - 2*n*log(-s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))/((e^(2*n)*(n - 1) - e^(2*n)*(n -
1)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*d)
```

Giac [F]

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(dx+c)})^{-2\,n+2}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-2*n + 2)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.30

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=-\frac{e^2\,(\cos{(2\,c+2\,d\,x)}\,1\mathrm{i}+\sin{(2\,c+2\,d\,x)}+1\mathrm{i})\,\left(\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}\right)^n}{d\,\left(\cos{(2\,c+2\,d\,x)}+1\right)\,\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n}\,(n-1)}$$

input
```
int((e/cos(c + d*x))^(2 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
-(e^2*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 1i)*((a*(cos(2*c + 2*d*x)
+ sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^n)/(d*(cos(2*c + 2*d*x
) + 1)*(e/cos(c + d*x))^(2*n)*(n - 1))
```

Reduce [F]

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx = \frac{\left(\int \frac{(\tan(dx+c)ai+a)^n\sec(dx+c)^2}{\sec(dx+c)^{2n}}dx\right)e^2}{e^{2n}}$$

input
```
int((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
(int(((tan(c + d*x)*a*i + a)**n*sec(c + d*x)**2)/sec(c + d*x)**(2*n),x)*e*
*2)/e**(2*n)
```

3.496 $\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 101

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{\frac{1}{2}-n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+2n),\frac{3}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{1-2n}(1-i\tan(c+dx}{d}$$

output
```
-I*2^(1/2-n)*hypergeom([1/2, 1/2+n],[3/2],1/2+1/2*I*tan(d*x+c))*(e*sec(d*x
+c))^(1-2*n)*(1-I*tan(d*x+c))^(-1/2+n)*(a+I*a*tan(d*x+c))^n/d
```

Mathematica [A] (verified)

Time = 6.33 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.52

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{1-n}e\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{1-n}\left(1+e^{2i(c+dx)}\right)^{1-n}\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-e^{2i(c+dx)}\right)\sec^n(c+}{d}$$

input
```
Integrate[(e*Sec[c + d*x])^(1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(1 - n)*e*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))
)^(1 - n)*(1 + E^((2*I)*(c + d*x)))^(1 - n)*Hypergeometric2F1[1/2, 1 - n,
3/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^n*(a + I*a*Tan[c + d*x])^n)/(d*(e*
Sec[c + d*x])^(2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.37, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{1-2n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{1-2n}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n-1)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-1)}(e\sec(c+dx))^{1-2n}\int (a-ia\tan(c+dx))^{\frac{1}{2}(1-2n)}\sqrt{i\tan(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n-1)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-1)}(e\sec(c+dx))^{1-2n}\int (a-ia\tan(c+dx))^{\frac{1}{2}(1-2n)}\sqrt{i\tan(c+dx)a+a}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{\frac{1}{2}(2n-1)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-1)}(e\sec(c+dx))^{1-2n}\int \frac{(a-ia\tan(c+dx))^{\frac{1}{2}(-2n-1)}}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+d}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2 2^{-n-\frac{1}{2}}(1-i\tan(c+dx))^{n+\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n-1)-\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-1)}(e\sec(c+dx))^{1-2n}}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{ia2^{\frac{1}{2}-n}(1-i\tan(c+dx))^{n+\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n-1)-\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-1)+\frac{1}{2}}(e\sec(c+dx))^{1-}}{d}$$

input
```
Int[(e*Sec[c + d*x])^(1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(1/2 - n)*a*Hypergeometric2F1[1/2, (1 + 2*n)/2, 3/2, (1 + I*Tan[c
+ d*x])/2]*(e*Sec[c + d*x])^(1 - 2*n)*(1 - I*Tan[c + d*x])^(1/2 + n)*(a -
I*a*Tan[c + d*x])^(-1/2 - n + (-1 + 2*n)/2)*(a + I*a*Tan[c + d*x])^(1/2 +
(-1 + 2*n)/2))/d
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\sec(dx+c))^{1-2n} (a+ia\tan(dx+c))^n\,dx$$

input
```
int((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec(dx+c))^{-2n+1} (ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
```

output
```
integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 1)*e^(I*d
*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*lo
g(a/e)), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(c+dx)})^{1-2n}\,(ia(\tan{(c+dx)}-i))^n\,dx$$

input
```
integrate((e*sec(d*x+c))**(1-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(1 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{-2\,n+1}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
```

output
```
integrate((e*sec(d*x + c))^(-2*n + 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{-2\,n+1}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-2*n + 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{1-2\,n}(a+a\tan(c+d\,x)\ 1\mathrm{i})^n\,dx$$

input
```
int((e/cos(c + d*x))^(1 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^(1 - 2*n)*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx = \frac{\left(\int \frac{(\tan(dx+c)ai+a)^n\sec(dx+c)}{\sec(dx+c)^{2n}}dx\right)e}{e^{2n}}$$

input
```
int((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
(int(((tan(c + d*x)*a*i + a)**n*sec(c + d*x))/sec(c + d*x)**(2*n),x)*e)/e*
*(2*n)
```

3.497 $\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 65

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i\,\text{Hypergeometric2F1}\left(1,-n,1-n,\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n}{2dn}$$

output
```
-1/2*I*hypergeom([1, -n],[1-n],1/2-1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/
d/n/((e*sec(d*x+c))^(2*n))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 146 vs. 2(65) = 130.

Time = 1.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.25

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{-1-n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,1+e^{2i(c+dx)}\right)\sec^n(c+}{d(1+n)}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(2*n),x]
```

output
```
(I*2^(-1 - n)*(E^(I*d*x))^n*(1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1,
 1 + n, 2 + n, 1 + E^((2*I)*(c + d*x))]*Sec[c + d*x]^n*(a + I*a*Tan[c + d*
x])^n)/(d*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^n*(1 + n)*(e*Sec[c +
 d*x])^(2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3042, 3973, 3042, 3962, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}dx$$

$$\Big\downarrow \text{3973}$$

$$(a-ia\tan(c+dx))^n(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}\int (a-ia\tan(c+dx))^{-n}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^n(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}\int (a-ia\tan(c+dx))^{-n}dx$$

$$\Big\downarrow \text{3962}$$

$$\frac{ia(a-ia\tan(c+dx))^n(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}\int \frac{(a-ia\tan(c+dx))^{-n-1}}{i\tan(c+dx)a+a}d(-ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{78}$$

$$-\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}\text{Hypergeometric2F1}\left(1,-n,1-n,\frac{a-ia\tan(c+dx)}{2a}\right)}{2dn}$$

input
```
Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(2*n),x]
```

output
```
((-1/2*I)*Hypergeometric2F1[1, -n, 1 - n, (a - I*a*Tan[c + d*x])/(2*a)]*(a
 + I*a*Tan[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^(2*n))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 3973
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(a/d)^(2*IntPart[n])*(a + b*Tan[e + f*x])^Frac
Part[n]*((a - b*Tan[e + f*x])^FracPart[n]/(d*Sec[e + f*x])^(2*FracPart[n]))
   Int[1/(a - b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
&& EqQ[a^2 + b^2, 0] && EqQ[Simplify[m/2 + n], 0]
```

Maple [F]

$$\int (a+ia\tan (dx+c))^n (e\sec (dx+c))^{-2n}\,dx$$

input
```
int((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x)
```

output
```
int((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^n}{(e\sec{(dx+c)})^{2\,n}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x, algorithm="fricas
")
```

output
```
integral(e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*
c) + 1)) + n*log(a/e))/(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(2*
n), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \int (e\sec{(c+dx)})^{-2n}\,(ia(\tan{(c+dx)}-i))^n\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**n/((e*sec(d*x+c))**(2*n)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n/(e*sec(c + d*x))**(2*n), x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^n}{(e\sec\left(dx+c\right))^{2\,n}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x, algorithm="maxima
")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(2*n), x)
```

Giac [F]

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^n}{(e\sec\left(dx+c\right))^{2\,n}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx = \frac{\int \frac{(\tan(dx+c)ai+a)^n}{\sec(dx+c)^{2n}}dx}{e^{2n}}$$

input
```
int((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x)
```

output
```
int((tan(c + d*x)*a*i + a)**n/sec(c + d*x)**(2*n),x)/e**(2*n)
```

3.498 $\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 101

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{-\frac{1}{2}-n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(3+2n),\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{-1-2n}(1-i\tan(c+\ldots}{d}$$

output
```
I*2^(-1/2-n)*hypergeom([-1/2, 3/2+n],[1/2],1/2+1/2*I*tan(d*x+c))*(e*sec(d*
x+c))^(-1-2*n)*(1-I*tan(d*x+c))^(1/2+n)*(a+I*a*tan(d*x+c))^n/d
```

Mathematica [A] (verified)

Time = 9.88 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.55

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{-1-n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-1-n}\left(1+e^{2i(c+dx)}\right)^{-1-n}\text{Hypergeometric2F1}\left(-\frac{1}{2},-1-n,\frac{1}{2},-e^{2i(c+dx)}\right)\sec^{1+\ldots}}{d}$$

input
```
Integrate[(e*Sec[c + d*x])^(-1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*2^(-1 - n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-
1 - n)*(1 + E^((2*I)*(c + d*x)))^(-1 - n)*Hypergeometric2F1[-1/2, -1 - n,
1/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(1 + n)*(e*Sec[c + d*x])^(-1 - 2*n
)*(a + I*a*Tan[c + d*x])^n)/(d*(Cos[d*x] + I*Sin[d*x])^n)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.36, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-1}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n+1)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+1)}(e\sec(c+dx))^{-2n-1}\int \frac{(a-ia\tan(c+dx))^{\frac{1}{2}(-2n-1)}}{\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n+1)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+1)}(e\sec(c+dx))^{-2n-1}\int \frac{(a-ia\tan(c+dx))^{\frac{1}{2}(-2n-1)}}{\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{\frac{1}{2}(2n+1)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+1)}(e\sec(c+dx))^{-2n-1}\int \frac{(a-ia\tan(c+dx))^{\frac{1}{2}(-2n-3)}}{(i\tan(c+dx)a+a)^{3/2}}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a2^{-n-\frac{3}{2}}(1-i\tan(c+dx))^{n+\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n+1)-\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+1)}(e\sec(c+dx))^{-2n-1}}{d}$$

$$\Big\downarrow 79$$

$$\frac{i2^{-n-\frac{1}{2}}(1-i\tan(c+dx))^{n+\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n+1)-\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+1)-\frac{1}{2}}(e\sec(c+dx))^{-2n}}{d}$$

input
```
Int[(e*Sec[c + d*x])^(-1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*2^(-1/2 - n)*Hypergeometric2F1[-1/2, (3 + 2*n)/2, 1/2, (1 + I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^(-1 - 2*n)*(1 - I*Tan[c + d*x])^(1/2 + n)*(a - I*
a*Tan[c + d*x])^(-1/2 - n + (1 + 2*n)/2)*(a + I*a*Tan[c + d*x])^(-1/2 + (1
 + 2*n)/2))/d
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\sec (dx+c))^{-1-2n} (a+ia\tan (dx+c))^n \, dx$$

input
```
int((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n \, dx = \int (e\sec (dx+c))^{-2\,n-1} (i\,a\tan (dx+c)+a)^n \, dx$$

input
```
integrate((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="frica
s")
```

output
```
integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n - 1)*e^(I*d
*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*lo
g(a/e)), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(c+dx)})^{-2n-1}\,(ia(\tan{(c+dx)}-i))^n\,dx$$

input
```
integrate((e*sec(d*x+c))**(-1-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(-2*n - 1)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(dx+c)})^{-2\,n-1}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxim
a")
```

output
```
integrate((e*sec(d*x + c))^(-2*n - 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(dx+c)})^{-2\,n-1}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac"
)
```

output
```
integrate((e*sec(d*x + c))^(-2*n - 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n+1}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 1),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 1), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx = \frac{\int \frac{(\tan(dx+c)ai+a)^n}{\sec(dx+c)^{2n}\sec(dx+c)}dx}{e^{2n}e}$$

input
```
int((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*a*i + a)**n/(sec(c + d*x)**(2*n)*sec(c + d*x)),x)/(e**(2
*n)*e)
```

3.499 $\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 74

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i\,\text{Hypergeometric2F1}\left(2,-1-n,-n,\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{-2(1+n)}(a+ia\tan(c+dx))}{4ad(1+n)}$$

output
```
-1/4*I*hypergeom([2, -1-n],[-n],1/2-1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^(
1+n)/a/d/(1+n)/((e*sec(d*x+c))^(2+2*n))
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 151 vs. 2(74) = 148.

Time = 9.90 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.04

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-3-n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^3\text{Hypergeometric2F1}\left(2,3+n,4+n,1+e^{2i(c+dx)}\right)\sec^n(c}{de^2(3+n)}$$

input
```
Integrate[(e*Sec[c + d*x])^(-2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(-3 - n)*(E^(I*d*x))^n*(1 + E^((2*I)*(c + d*x)))^3*Hypergeometric2
F1[2, 3 + n, 4 + n, 1 + E^((2*I)*(c + d*x))]*Sec[c + d*x]^n*(a + I*a*Tan[c
 + d*x])^n)/(d*e^2*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^n*(3 + n)*(
e*Sec[c + d*x])^(2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 3986, 3042, 4005, 3042, 3968, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-2}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{n+1}(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-2(n+1)}\int \frac{(a-ia\tan(c+dx))^{-n-1}}{i\tan(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{n+1}(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-2(n+1)}\int \frac{(a-ia\tan(c+dx))^{-n-1}}{i\tan(c+dx)a+a}dx$$

$$\Big\downarrow \text{4005}$$

$$\frac{(a-ia\tan(c+dx))^{n+1}(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-2(n+1)}\int \cos^2(c+dx)(a-ia\tan(c+dx))^{-n}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(c+dx))^{n+1}(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-2(n+1)}\int\frac{(a-ia\tan(c+dx))^{-n}}{\sec(c+dx)^2}dx}{a^2}$$

$$\Big\downarrow \text{3968}$$

$$\frac{ia(a-ia\tan(c+dx))^{n+1}(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-2(n+1)}\int\frac{(a-ia\tan(c+dx))^{-n-2}}{(i\tan(c+dx)a+a)^2}d(-ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{78}$$

$$-\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-2(n+1)}\text{Hypergeometric2F1}\left(2,-n-1,-n,\frac{a-ia\tan(c+dx)}{2a}\right)}{4ad(n+1)}$$

input
```
Int[(e*Sec[c + d*x])^(-2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/4*I)*Hypergeometric2F1[2, -1 - n, -n, (a - I*a*Tan[c + d*x])/(2*a)]*(
a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(1 + n)*(e*Sec[c + d*x])^(2*(1 + n)))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4005
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*c^m   Int[Sec[e + f*x]^(2*m)*(c +
 d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[
m, 0] || GtQ[m, n]))
```

Maple [F]

$$\int (e \sec (dx+c))^{-2n-2} (a+ia \tan (dx+c))^n \, dx$$

input
```
int((e*sec(d*x+c))^(-2*n-2)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(-2*n-2)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e \sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n \, dx$$
$$= \int (e \sec (dx+c))^{-2\,n-2} (i\,a \tan (dx+c)+a)^n \, dx$$

input
```
integrate((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="frica
s")
```

output
```
integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n - 2)*e^(I*d
*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*lo
g(a/e)), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(c+dx)})^{-2n-2}\,(ia(\tan{(c+dx)}-i))^n\,\,dx$$

input
```
integrate((e*sec(d*x+c))**(-2-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(-2*n - 2)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{-2\,n-2}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxim
a")
```

output
```
integrate((e*sec(d*x + c))^(-2*n - 2)*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{-2\,n-2}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac"
)
```

output
```
integrate((e*sec(d*x + c))^(-2*n - 2)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n+2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 2), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = \frac{\int \frac{(\tan(dx+c)ai+a)^n}{\sec(dx+c)^{2n}\sec(dx+c)^2}dx}{e^{2n}e^2}$$

input
```
int((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*a*i + a)**n/(sec(c + d*x)**(2*n)*sec(c + d*x)**2),x)/(e*
*(2*n)*e**2)
```

3.500 $\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 103

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{-\frac{3}{2}-n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(5+2n),-\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{-3-2n}(1-i\tan(c+$$

$$3d$$

output
```
1/3*I*2^(-3/2-n)*hypergeom([-3/2, 5/2+n],[-1/2],1/2+1/2*I*tan(d*x+c))*(e*s
ec(d*x+c))^(-3-2*n)*(1-I*tan(d*x+c))^(3/2+n)*(a+I*a*tan(d*x+c))^n/d
```

Mathematica [A] (verified)

Time = 11.47 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.61

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{-3-n}e^{-3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{3}{2},-3-n,-\frac{1}{2},-e^{2i(c+dx)}\right.}{3d}$$

input
```
Integrate[(e*Sec[c + d*x])^(-3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((I/3)*2^(-3 - n)*(E^(I*d*x))^n*Hypergeometric2F1[-3/2, -3 - n, -1/2, -E^(
(2*I)*(c + d*x))]*Sec[c + d*x]^(3 + n)*(e*Sec[c + d*x])^(-3 - 2*n)*(a + I*
a*Tan[c + d*x])^n)/(d*E^((3*I)*(c + d*x))*(E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x))))^n*(1 + E^((2*I)*(c + d*x)))^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-3}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n+3)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+3)}(e\sec(c+dx))^{-2n-3}\int \frac{(a-ia\tan(c+dx))^{\frac{1}{2}(-2n-3)}}{(i\tan(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n+3)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+3)}(e\sec(c+dx))^{-2n-3}\int \frac{(a-ia\tan(c+dx))^{\frac{1}{2}(-2n-3)}}{(i\tan(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{\frac{1}{2}(2n+3)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+3)}(e\sec(c+dx))^{-2n-3}\int \frac{(a-ia\tan(c+dx))^{\frac{1}{2}(-2n-5)}}{(i\tan(c+dx)a+a)^{5/2}}d\tan(c+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{-n-\frac{5}{2}}(1-i\tan(c+dx))^{n+\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n+3)-\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+3)}(e\sec(c+dx))^{-2n-3}}{d}$$

$$\Big\downarrow \text{79}$$

$$\frac{i2^{-n-\frac{3}{2}}(1-i\tan(c+dx))^{n+\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n+3)-\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+3)-\frac{3}{2}}(e\sec(c+dx))^{-2n}}{3ad}$$

input
```
Int[(e*Sec[c + d*x])^(-3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((I/3)*2^(-3/2 - n)*Hypergeometric2F1[-3/2, (5 + 2*n)/2, -1/2, (1 + I*Tan[
c + d*x])/2]*(e*Sec[c + d*x])^(-3 - 2*n)*(1 - I*Tan[c + d*x])^(1/2 + n)*(a
 - I*a*Tan[c + d*x])^(-1/2 - n + (3 + 2*n)/2)*(a + I*a*Tan[c + d*x])^(-3/2
 + (3 + 2*n)/2))/(a*d)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\sec (dx+c))^{-3-2n} (a+ia\tan (dx+c))^n\, dx$$

input
```
int((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\, dx = \int (e\sec (dx+c))^{-2\,n-3} (i\,a\tan (dx+c)+a)^n\, dx$$

input
```
integrate((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="frica
s")
```

output
```
integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n - 3)*e^(I*d
*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*lo
g(a/e)), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \int (e\sec{(c+dx)})^{-2n-3}\,(ia(\tan{(c+dx)}-i))^n\,dx$$

input
```
integrate((e*sec(d*x+c))**(-3-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(-2*n - 3)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \int (e\sec{(dx+c)})^{-2\,n-3}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxim
a")
```

output
```
integrate((e*sec(d*x + c))^(-2*n - 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \int (e\sec{(dx+c)})^{-2\,n-3}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac"
)
```

output
```
integrate((e*sec(d*x + c))^(-2*n - 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n+3}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 3),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 3), x)
```

Reduce [F]

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx = \frac{\int \frac{(\tan(dx+c)ai+a)^n}{\sec(dx+c)^{2n}\sec(dx+c)^3}dx}{e^{2n}e^3}$$

input
```
int((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*a*i + a)**n/(sec(c + d*x)**(2*n)*sec(c + d*x)**3),x)/(e*
*(2*n)*e**3)
```

3.501 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 66

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx = \frac{i\,\text{Hypergeometric2F1}\left(3,n,1+n,\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{8a^2fn}$$

output
```
1/8*I*hypergeom([3, n],[1+n],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(2*n)/a^
2/f/n/((a+I*a*tan(f*x+e))^n)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 165 vs. 2(66) = 132.

Time = 8.56 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.50

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx =$$

$$-\frac{i2^{-3+n}e^{2ie}\left(e^{ifx}\right)^{-n}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n}\left(1+e^{2i(e+fx)}\right)^{3}\text{Hypergeometric2F1}\left(3,3-n,4-n,1+e^{2i(e+fx)}\right)\text{se}}{f(-3+n)}$$

input
```
Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-2 - n),x]
```

output
```
((-I)*2^(-3 + n)*E^((2*I)*e)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n
*(1 + E^((2*I)*(e + f*x)))^3*Hypergeometric2F1[3, 3 - n, 4 - n, 1 + E^((2*
I)*(e + f*x))]*Sec[e + f*x]^(2 - n)*(d*Sec[e + f*x])^(2*n)*(Cos[f*x] + I*S
in[f*x])^(2 + n)*(a + I*a*Tan[e + f*x])^(-2 - n))/((E^(I*f*x))^n*f*(-3 + n
))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3042, 3986, 3042, 4005, 3042, 3968, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{-n-2}(d\sec(e+fx))^{2n}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+ia\tan(e+fx))^{-n-2}(d\sec(e+fx))^{2n}dx$$

$$\Big\downarrow \text{ 3986}$$

$$(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \frac{(a-ia\tan(e+fx))^n}{(i\tan(e+fx)a+a)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \frac{(a-ia\tan(e+fx))^n}{(i\tan(e+fx)a+a)^2}dx$$

$$\Big\downarrow \text{ 4005}$$

$$\frac{(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \cos^4(e+fx)(a-ia\tan(e+fx))^{n+2}dx}{a^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \frac{(a-ia\tan(e+fx))^{n+2}}{\sec(e+fx)^4}dx}{a^4}$$

$$\Big\downarrow \text{3968}$$

$$\frac{ia(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \frac{(a-ia\tan(e+fx))^{n-1}}{(i\tan(e+fx)a+a)^3}d(-ia\tan(e+fx))}{f}$$

$$\Big\downarrow \text{78}$$

$$\frac{i(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\text{Hypergeometric2F1}\left(3,n,n+1,\frac{a-ia\tan(e+fx)}{2a}\right)}{8a^2fn}$$

input
```
Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-2 - n),x]
```

output
```
((I/8)*Hypergeometric2F1[3, n, 1 + n, (a - I*a*Tan[e + f*x])/(2*a)]*(d*Sec
[e + f*x])^(2*n))/(a^2*f*n*(a + I*a*Tan[e + f*x])^n)
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4005
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*c^m   Int[Sec[e + f*x]^(2*m)*(c +
 d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[
m, 0] || GtQ[m, n]))
```

Maple [F]

$$\int (d\sec (fx+e))^{2n} (a+ia\tan (fx+e))^{-n-2} \, dx$$

input
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-n-2),x)
```

output
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-n-2),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx = \int (d\sec (fx+e))^{2\,n} (i\,a\tan (fx+e)+a)^{-n-2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x, algorithm="fri
cas")
```

output
```
integral((2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^(-I*e*n +
 (-I*f*n - 2*I*f)*x - (n + 2)*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e)
 + 1)) - (n + 2)*log(a/d) - 2*I*e), x)
```

Sympy [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx = \int (d\sec{(e+fx)})^{2n}\,(ia(\tan{(e+fx)}-i))^{-n-2}\,dx$$

input
```
integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(-2-n),x)
```

output
```
Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(-n - 2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x, algorithm="max
ima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx = \int (d\sec{(fx+e)})^{2\,n}\,(i\,a\tan{(fx+e)}+a)^{-n-2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x, algorithm="gia
c")
```

output
```
integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2\,n}}{(a+a\tan\left(e+f\,x\right)\,1\mathrm{i})^{n+2}}\,dx$$

input
```
int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^(n + 2),x)
```

output
```
int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^(n + 2), x)
```

Reduce [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx$$
$$= -\frac{d^{2n}\left(\int \frac{\sec(fx+e)^{2n}}{(\tan(fx+e)ai+a)^n\tan(fx+e)^2-2(\tan(fx+e)ai+a)^n\tan(fx+e)i-(\tan(fx+e)ai+a)^n}dx\right)}{a^2}$$

input
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x)
```

output
```
( - d**(2*n)*int(sec(e + f*x)**(2*n)/((tan(e + f*x)*a*i + a)**n*tan(e + f*
x)**2 - 2*(tan(e + f*x)*a*i + a)**n*tan(e + f*x)*i - (tan(e + f*x)*a*i + a
)**n),x))/a**2
```

3.502 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 66

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx = \frac{i\,\text{Hypergeometric2F1}\left(2,n,1+n,\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{4afn}$$

output
```
1/4*I*hypergeom([2, n],[1+n],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(2*n)/a/
f/n/((a+I*a*tan(f*x+e))^n)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 165 vs. 2(66) = 132.

Time = 7.89 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.50

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx = \frac{i2^{-2+n}e^{ie}\left(e^{ifx}\right)^{-n}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n}\left(1+e^{2i(e+fx)}\right)^{2}\text{Hypergeometric2F1}\left(2,2-n,3-n,1+e^{2i(e+fx)}\right)\sec^{1-}}{f(-2+n)}$$

input
```
Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-1 - n),x]
```

output
```
(I*2^(-2 + n)*E^(I*e)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n*(1 + E
^((2*I)*(e + f*x)))^2*Hypergeometric2F1[2, 2 - n, 3 - n, 1 + E^((2*I)*(e +
 f*x))]*Sec[e + f*x]^(1 - n)*(d*Sec[e + f*x])^(2*n)*(Cos[f*x] + I*Sin[f*x]
)^(1 + n)*(a + I*a*Tan[e + f*x])^(-1 - n))/((E^(I*f*x))^n*f*(-2 + n))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3042, 3986, 3042, 4005, 3042, 3968, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{-n-1}(d\sec(e+fx))^{2n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{-n-1}(d\sec(e+fx))^{2n}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \frac{(a-ia\tan(e+fx))^n}{i\tan(e+fx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \frac{(a-ia\tan(e+fx))^n}{i\tan(e+fx)a+a}dx$$

$$\Big\downarrow \text{4005}$$

$$\frac{(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \cos^2(e+fx)(a-ia\tan(e+fx))^{n+1}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \frac{(a-ia\tan(e+fx))^{n+1}}{\sec(e+fx)^2}dx}{a^2}$$

$$\Big\downarrow \text{3968}$$

$$\frac{ia(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int\frac{(a-ia\tan(e+fx))^{n-1}}{(i\tan(e+fx)a+a)^2}d(-ia\tan(e+fx))}{f}$$

$$\Big\downarrow \text{78}$$

$$\frac{i(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\,\text{Hypergeometric2F1}\left(2,n,n+1,\frac{a-ia\tan(e+fx)}{2a}\right)}{4afn}$$

input
```
Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-1 - n),x]
```

output
```
((I/4)*Hypergeometric2F1[2, n, 1 + n, (a - I*a*Tan[e + f*x])/(2*a)]*(d*Sec
[e + f*x])^(2*n))/(a*f*n*(a + I*a*Tan[e + f*x])^n)
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4005
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*c^m   Int[Sec[e + f*x]^(2*m)*(c +
 d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[
m, 0] || GtQ[m, n]))
```

Maple [F]

$$\int (d\sec (fx+e))^{2n} (a+ia\tan (fx+e))^{-n-1}\,dx$$

input
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-n-1),x)
```

output
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-n-1),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx$$
$$=\int (d\sec (fx+e))^{2\,n} (i\,a\tan (fx+e)+a)^{-n-1}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x, algorithm="fri
cas")
```

output
```
integral((2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^(-I*e*n +
 (-I*f*n - I*f)*x - (n + 1)*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) +
 1)) - (n + 1)*log(a/d) - I*e), x)
```

Sympy [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx$$
$$=\int (d\sec{(e+fx)})^{2n}\,(ia(\tan{(e+fx)}-i))^{-n-1}\,dx$$

input
```
integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(-1-n),x)
```

output
```
Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(-n - 1), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x, algorithm="max
ima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx$$
$$=\int (d\sec{(fx+e)})^{2\,n}\,(i\,a\tan{(fx+e)}+a)^{-n-1}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x, algorithm="gia
c")
```

output
```
integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2\,n}}{\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{n+1}}\,dx$$

input
```
int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^(n + 1),x)
```

output
```
int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^(n + 1), x)
```

Reduce [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx$$
$$=\frac{d^{2n}\left(\int \frac{\sec(fx+e)^{2n}}{(\tan(fx+e)ai+a)^{n}\tan(fx+e)i+(\tan(fx+e)ai+a)^{n}}dx\right)}{a}$$

input
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x)
```

output
```
(d**(2*n)*int(sec(e + f*x)**(2*n)/((tan(e + f*x)*a*i + a)**n*tan(e + f*x)*
i + (tan(e + f*x)*a*i + a)**n),x))/a
```

3.503 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 63

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx = \frac{i\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{2fn}$$

output
```
1/2*I*hypergeom([1, n],[1+n],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(2*n)/f/
n/((a+I*a*tan(f*x+e))^n)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 150 vs. 2(63) = 126.

Time = 1.49 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.38

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx = -\frac{i2^{-1+n}\left(e^{ifx}\right)^{-n}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n}\left(1+e^{2i(e+fx)}\right)\text{Hypergeometric2F1}\left(1,1-n,2-n,1+e^{2i(e+fx)}\right)\sec^{-n}(}{f(-1+n)}$$

input
```
Integrate[(d*Sec[e + f*x])^(2*n)/(a + I*a*Tan[e + f*x])^n,x]
```

output
```
((-I)*2^(-1 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n*(1 + E^((2*
I)*(e + f*x)))*Hypergeometric2F1[1, 1 - n, 2 - n, 1 + E^((2*I)*(e + f*x))]
*(d*Sec[e + f*x])^(2*n)*(Cos[f*x] + I*Sin[f*x])^n)/((E^(I*f*x))^n*f*(-1 +
n)*Sec[e + f*x]^n*(a + I*a*Tan[e + f*x])^n)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3973, 3042, 3962, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}dx$$

$$\Big\downarrow \text{3973}$$

$$(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int (a-ia\tan(e+fx))^{n}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int (a-ia\tan(e+fx))^{n}dx$$

$$\Big\downarrow \text{3962}$$

$$\frac{ia(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \frac{(a-ia\tan(e+fx))^{n-1}}{i\tan(e+fx)a+a}d(-ia\tan(e+fx))}{f}$$

$$\Big\downarrow \text{78}$$

$$\frac{i(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\,\text{Hypergeometric2F1}\left(1,n,n+1,\frac{a-ia\tan(e+fx)}{2a}\right)}{2fn}$$

input
```
Int[(d*Sec[e + f*x])^(2*n)/(a + I*a*Tan[e + f*x])^n,x]
```

output
```
((I/2)*Hypergeometric2F1[1, n, 1 + n, (a - I*a*Tan[e + f*x])/(2*a)]*(d*Sec
[e + f*x])^(2*n))/(f*n*(a + I*a*Tan[e + f*x])^n)
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 3973
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(a/d)^(2*IntPart[n])*(a + b*Tan[e + f*x])^Frac
Part[n]*((a - b*Tan[e + f*x])^FracPart[n]/(d*Sec[e + f*x])^(2*FracPart[n]))
   Int[1/(a - b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
&& EqQ[a^2 + b^2, 0] && EqQ[Simplify[m/2 + n], 0]
```

Maple [F]

$$\int (d\sec (fx+e))^{2n} (a(1+i\tan (fx+e)))^{-n}\, dx$$

input
```
int((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x)
```

output
```
int((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx=\int \frac{(d\sec{(fx+e)})^{2\,n}}{(i\,a\tan{(fx+e)}+a)^{n}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x, algorithm="fricas
")
```

output
```
integral((2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^(-I*f*n*x
 - I*e*n - n*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1)) - n*log(a/
d)), x)
```

Sympy [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx$$
$$=\int (d\sec{(e+fx)})^{2n}\,(ia(\tan{(e+fx)}-i))^{-n}\,dx$$

input
```
integrate((d*sec(f*x+e))**(2*n)/((a+I*a*tan(f*x+e))**n),x)
```

output
```
Integral((d*sec(e + f*x))**(2*n)/(I*a*(tan(e + f*x) - I))**n, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx = \int \frac{(d\sec(fx+e))^{2n}}{(ia\tan(fx+e)+a)^n}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x, algorithm="maxima
")
```

output
```
integrate((d*sec(f*x + e))^(2*n)/(I*a*tan(f*x + e) + a)^n, x)
```

Giac [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx = \int \frac{(d\sec(fx+e))^{2n}}{(ia\tan(fx+e)+a)^n}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(2*n)/(I*a*tan(f*x + e) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2\,n}}{(a+a\tan(e+f\,x)\ 1\mathrm{i})^n}\,dx$$

input
```
int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^n,x)
```

output
```
int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^n, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx = d^{2n}\left(\int \frac{\sec(fx+e)^{2n}}{(\tan(fx+e)\,ai+a)^n}dx\right)$$

input
```
int((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x)
```

output
```
d**(2*n)*int(sec(e + f*x)**(2*n)/(tan(e + f*x)*a*i + a)**n,x)
```

3.504 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 40

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx = \frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn}$$

output
```
I*a*(d*sec(f*x+e))^(2*n)/f/n/((a+I*a*tan(f*x+e))^n)
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx = \frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn}$$

input
```
Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n),x]
```

output
```
(I*a*(d*Sec[e + f*x])^(2*n))/(f*n*(a + I*a*Tan[e + f*x])^n)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}dx$$
$$\Big\downarrow \text{3974}$$
$$\frac{ia(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn}$$

input
```
Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n),x]
```

output
```
(I*a*(d*Sec[e + f*x])^(2*n))/(f*n*(a + I*a*Tan[e + f*x])^n)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 7.04 (sec) , antiderivative size = 1261, normalized size of antiderivative = 31.52

method	result	size
risch	Expression too large to display	1261

input
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x,method=_RETURNVERBOSE)
```

output
```
-I/((exp(2*I*(f*x+e))+1)^n)/f/n*2^n*a*(d^n)^2/(a^n)*exp(-1/2*I*Pi*(-2*n*cs
gn(I*d*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*d)-2*n*csgn(I/(exp(2*
I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2-2*n*csgn(I*ex
p(I*(f*x+e)))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2-2*n*csgn(I*d*e
xp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+
e))+1))+2*n*csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I*exp
(I*(f*x+e))/(exp(2*I*(f*x+e))+1))+2*n*csgn(I*d*exp(I*(f*x+e))/(exp(2*I*(f*
x+e))+1))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*d)+csgn(I/(ex
p(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))*csgn(I*a/(exp(2*I*(f*x+e))+1)*exp(2*I*
(f*x+e)))*csgn(I*a)+2*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3+2*n*
csgn(I*d*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3+csgn(I*exp(2*I*(f*x+e)))*c
sgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))*csgn(I/(exp(2*I*(f*x+e))+1))+
csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^3-csgn(I*a/(exp(2*I*(f*x+e))
+1)*exp(2*I*(f*x+e)))^2*csgn(I*a)+2*n*csgn(I*exp(2*I*(f*x+e)))^2*csgn(I*ex
p(I*(f*x+e)))-csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^2*csgn(I/(exp(
2*I*(f*x+e))+1))-n*csgn(I*a/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^3+csgn(
I*a/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^3+csgn(I*exp(2*I*(f*x+e)))^3+n*
csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^2*csgn(I/(exp(2*I*(f*x+e))+1
))+n*csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))*csgn(I*a/(exp(2*I*(f*x+
e))+1)*exp(2*I*(f*x+e)))^2+n*csgn(I*exp(2*I*(f*x+e)))*csgn(I/(exp(2*I*(...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(36) = 72.

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.88

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx$$
$$=\frac{\left(\frac{2\,de^{(i\,fx+i\,e)}}{e^{(2i\,fx+2i\,e)}+1}\right)^{2\,n}\left(i\,e^{(2i\,fx+2i\,e)}+i\right)e^{\left(-i\,en+(-i\,fn+i\,f)x-2i\,fx-(n-1)\log\left(\frac{2\,de^{(i\,fx+i\,e)}}{e^{(2i\,fx+2i\,e)}+1}\right)-(n-1)\log\left(\frac{a}{d}\right)-i\,e\right)}}{2\,fn}$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x, algorithm="fric
as")
```

output
```
1/2*(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*(I*e^(2*I*f*x +
2*I*e) + I)*e^(-I*e*n + (-I*f*n + I*f)*x - 2*I*f*x - (n - 1)*log(2*d*e^(I*
f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1)) - (n - 1)*log(a/d) - I*e)/(f*n)
```

Sympy [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx$$
$$=\int (d\sec(e+fx))^{2n}\,(ia(\tan(e+fx)-i))^{1-n}\,dx$$

input
```
integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(1-n),x)
```

output
```
Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(1 - n), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(36) = 72.

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.42

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx$$
$$= \frac{i\,a^{-n+1}d^{2\,n}e^{\left(-n\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-n\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)-n\log\left(-\frac{2i\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)+2\,n\log\left(-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right.}}{fn}$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x, algorithm="maxi
ma")
```

output
```
I*a^(-n + 1)*d^(2*n)*e^(-n*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - n*lo
g(sin(f*x + e)/(cos(f*x + e) + 1) - 1) - n*log(-2*I*sin(f*x + e)/(cos(f*x
+ e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1) + 2*n*log(-sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - 1))/(f*n)
```

Giac [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx$$
$$= \int (d\sec(fx+e))^{2\,n}\,(i\,a\tan(fx+e)+a)^{-n+1}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x, algorithm="giac
")
```

output
```
integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n + 1), x)
```

Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.55

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx=\frac{a\left(\frac{d}{\cos(e+f\,x)}\right)^{2\,n}\,1\mathrm{i}}{f\,n\,\left(\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{2\cos(e+f\,x)^2}\right)^n}$$

input
```
int((d/cos(e + f*x))^(2*n)*(a + a*tan(e + f*x)*1i)^(1 - n),x)
```

output
```
(a*(d/cos(e + f*x))^(2*n)*1i)/(f*n*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x
)*1i + 1))/(2*cos(e + f*x)^2))^n)
```

Reduce [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx$$
$$=d^{2n}a\left(\int\frac{\sec(fx+e)^{2n}}{(\tan(fx+e)\,ai+a)^n}dx+\left(\int\frac{\sec(fx+e)^{2n}\tan(fx+e)}{(\tan(fx+e)\,ai+a)^n}dx\right)i\right)$$

input
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x)
```

output
```
d**(2*n)*a*(int(sec(e + f*x)**(2*n)/(tan(e + f*x)*a*i + a)**n,x) + int((se
c(e + f*x)**(2*n)*tan(e + f*x))/(tan(e + f*x)*a*i + a)**n,x)*i)
```

3.505 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 92

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx = \frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f(1+n)} + \frac{2ia^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn(1+n)}$$

output
```
I*a*(d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n)/f/(1+n)+2*I*a^2*(d*sec(f
*x+e))^(2*n)/f/n/(1+n)/((a+I*a*tan(f*x+e))^n)
```

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx = -\frac{a^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}(-i(2+n)+n\tan(e+fx))}{fn(1+n)}$$

input
```
Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(2 - n),x]
```

output
```
-((a^2*(d*Sec[e + f*x])^(2*n)*((-I)*(2 + n) + n*Tan[e + f*x]))/(f*n*(1 + n
)*(a + I*a*Tan[e + f*x])^n))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{2-n}(d\sec(e+fx))^{2n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{2-n}(d\sec(e+fx))^{2n}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{2a\int (d\sec(e+fx))^{2n}(i\tan(e+fx)a+a)^{1-n}dx}{n+1}+\frac{ia(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f(n+1)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\int (d\sec(e+fx))^{2n}(i\tan(e+fx)a+a)^{1-n}dx}{n+1}+\frac{ia(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f(n+1)}$$

$$\Big\downarrow \text{3974}$$

$$\frac{2ia^2(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn(n+1)}+\frac{ia(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f(n+1)}$$

input
```
Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(2 - n),x]
```

output
```
(I*a*(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n))/(f*(1 + n)) +
((2*I)*a^2*(d*Sec[e + f*x])^(2*n))/(f*n*(1 + n)*(a + I*a*Tan[e + f*x])^n)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [F]

$$\int \left(d \sec \left(f x +e \right)\right)^{2 n} \left(a +i a \tan \left(f x +e \right)\right)^{2-n}d x$$

input
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x)
```

output
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.51

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx$$
$$= \frac{\left((i\,n+i)e^{(4i\,fx+4i\,e)}+(i\,n+2i)e^{(2i\,fx+2i\,e)}+i\right)\left(\frac{2\,de^{(i\,fx+i\,e)}}{e^{(2i\,fx+2i\,e)}+1}\right)^{2\,n}e^{\left(-i\,en+(-i\,fn+2i\,f)x-4i\,fx-(n-2)\log\left(\frac{2\,de^{(i\,fx+}}{e^{(2i\,fx+2i}}\right.\right.}}{2\left(fn^2+fn\right)}$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x, algorithm="fric
as")
```

output
```
1/2*((I*n + I)*e^(4*I*f*x + 4*I*e) + (I*n + 2*I)*e^(2*I*f*x + 2*I*e) + I)*
(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^(-I*e*n + (-I*f*n
+ 2*I*f)*x - 4*I*f*x - (n - 2)*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e
) + 1)) - (n - 2)*log(a/d) - 2*I*e)/(f*n^2 + f*n)
```

Sympy [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx$$
$$=\int (d\sec{(e+fx)})^{2n}\,(ia(\tan{(e+fx)}-i))^{2-n}\,dx$$

input
```
integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(2-n),x)
```

output
```
Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(2 - n), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(84) = 168.

Time = 0.30 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.30

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx$$
$$=\frac{2^{n+1}a^2d^{2n}\cos\left(n\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)+1\right)\right)-i\cdot 2^{n+1}a^2d^{2n}\sin\left(n\arctan\left(\sin\left(2fx+\right.\right.\right.}{\left(-i\,a^nn^2-i\,a^nn+\left(\right.\right.}$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x, algorithm="maxi
ma")
```

output
```
(2^(n + 1)*a^2*d^(2*n)*cos(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1)) - I*2^(n + 1)*a^2*d^(2*n)*sin(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1)) + 2*(a^2*d^(2*n)*n + a^2*d^(2*n))*2^n*cos(-2*f*x + n*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 2*e) + 2*(-I*a^2*d^(2*n)*n - I*a^2
*d^(2*n))*2^n*sin(-2*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1) - 2*e))/((-I*a^n*n^2 - I*a^n*n + (-I*a^n*n^2 - I*a^n*n)*cos(2*f*x + 2*e
) + (a^n*n^2 + a^n*n)*sin(2*f*x + 2*e))*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/2*n)*f)
```

Giac [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx$$
$$=\int (d\sec{(fx+e)})^{2\,n}\,(i\,a\tan{(fx+e)}+a)^{-n+2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x, algorithm="giac
")
```

output
```
integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n + 2), x)
```

Mupad [B] (verification not implemented)

Time = 5.23 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.83

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx$$
$$=-\mathrm{e}^{-e\,4\mathrm{i}-f\,x\,4\mathrm{i}}\left(\frac{d}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}\right)^{2\,n}\left(\frac{\left(a-\frac{a\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}\right)^{2-n}}{2\,f\,n\,\left(n\,1\mathrm{i}+1\mathrm{i}\right)}+\frac{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,\left(a-\frac{a\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}\right)^{2-n}\,(n+2)}{2\,f\,n\,\left(n\,1\mathrm{i}+1\mathrm{i}\right)}+\frac{\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\left(a-\frac{a\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}\right)^{2-n}\,(n+1)}{2\,f\,n\,\left(n\,1\mathrm{i}+1\mathrm{i}\right)}\right)$$

input
```
int((d/cos(e + f*x))^(2*n)*(a + a*tan(e + f*x)*1i)^(2 - n),x)
```

output
```
-exp(- e*4i - f*x*4i)*(d/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^
(2*n)*((a - (a*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^
(2 - n)/(2*f*n*(n*1i + 1i)) + (exp(e*2i + f*x*2i)*(a - (a*(exp(e*2i + f*x*
2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(2 - n)*(n + 2))/(2*f*n*(n*1i +
 1i)) + (exp(e*4i + f*x*4i)*(a - (a*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(
e*2i + f*x*2i) + 1))^(2 - n)*(n + 1))/(2*f*n*(n*1i + 1i)))
```

Reduce [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx$$
$$= d^{2n}a^2\left(\int \frac{\sec(fx+e)^{2n}}{(\tan(fx+e)\,ai+a)^n}dx - \left(\int \frac{\sec(fx+e)^{2n}\tan(fx+e)^2}{(\tan(fx+e)\,ai+a)^n}dx\right) + 2\left(\int \frac{\sec(fx+e)^{2n}\tan(fx+e)}{(\tan(fx+e)\,ai+a)^n}dx\right)i\right)$$

input
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x)
```

output
```
d**(2*n)*a**2*(int(sec(e + f*x)**(2*n)/(tan(e + f*x)*a*i + a)**n,x) - int(
(sec(e + f*x)**(2*n)*tan(e + f*x)**2)/(tan(e + f*x)*a*i + a)**n,x) + 2*int
((sec(e + f*x)**(2*n)*tan(e + f*x))/(tan(e + f*x)*a*i + a)**n,x)*i)
```

3.506 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 148

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx = \frac{4ia^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f(2+3n+n^2)} + \frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}}{f(2+n)} + \frac{8ia^3(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn(2+3n+n^2)}$$

output
```
4*I*a^2*(d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n)/f/(n^2+3*n+2)+I*a*(d
*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n)/f/(2+n)+8*I*a^3*(d*sec(f*x+e))
^(2*n)/f/n/(n^2+3*n+2)/((a+I*a*tan(f*x+e))^n)
```

Mathematica [A] (verified)

Time = 2.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.87

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx$$
$$= \frac{ia^3\sec^2(e+fx)(d\sec(e+fx))^{2n}(\cos(3fx)+i\sin(3fx))\,(2(2+n)+(4+3n+n^2)\cos(2(e+fx))+in}{fn(1+n)(2+n)(\cos(fx)+i\sin(fx))^3}$$

input
```
Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(3 - n),x]
```

output
```
(I*a^3*Sec[e + f*x]^2*(d*Sec[e + f*x])^(2*n)*(Cos[3*f*x] + I*Sin[3*f*x])*(
2*(2 + n) + (4 + 3*n + n^2)*Cos[2*(e + f*x)] + I*n*(3 + n)*Sin[2*(e + f*x)
]))/(f*n*(1 + n)*(2 + n)*(Cos[f*x] + I*Sin[f*x])^3*(a + I*a*Tan[e + f*x])^
n)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{3-n}(d\sec(e+fx))^{2n}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a+ia\tan(e+fx))^{3-n}(d\sec(e+fx))^{2n}dx$$
$$\Big\downarrow \text{3975}$$
$$\frac{4a\int (d\sec(e+fx))^{2n}(i\tan(e+fx)a+a)^{2-n}dx}{n+2}+\frac{ia(a+ia\tan(e+fx))^{2-n}(d\sec(e+fx))^{2n}}{f(n+2)}$$
$$\Big\downarrow \text{3042}$$

$$\frac{4a\int (d\sec(e+fx))^{2n}(i\tan(e+fx)a+a)^{2-n}dx}{n+2}+\frac{ia(a+ia\tan(e+fx))^{2-n}(d\sec(e+fx))^{2n}}{f(n+2)}$$

$$\Big\downarrow \textbf{3975}$$

$$\frac{4a\left(\frac{2a\int (d\sec(e+fx))^{2n}(i\tan(e+fx)a+a)^{1-n}dx}{n+1}+\frac{ia(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f(n+1)}\right)}{n+2}+\frac{ia(a+ia\tan(e+fx))^{2-n}(d\sec(e+fx))^{2n}}{f(n+2)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{4a\left(\frac{2a\int (d\sec(e+fx))^{2n}(i\tan(e+fx)a+a)^{1-n}dx}{n+1}+\frac{ia(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f(n+1)}\right)}{n+2}+\frac{ia(a+ia\tan(e+fx))^{2-n}(d\sec(e+fx))^{2n}}{f(n+2)}$$

$$\Big\downarrow \textbf{3974}$$

$$\frac{4a\left(\frac{2ia^2(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn(n+1)}+\frac{ia(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f(n+1)}\right)}{n+2}+\frac{ia(a+ia\tan(e+fx))^{2-n}(d\sec(e+fx))^{2n}}{f(n+2)}$$

input
```
Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(3 - n),x]
```

output
```
(I*a*(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(2 - n))/(f*(2 + n)) +
(4*a*((I*a*(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n))/(f*(1 +
n)) + ((2*I)*a^2*(d*Sec[e + f*x])^(2*n))/(f*n*(1 + n)*(a + I*a*Tan[e + f*x
])^n)))/(2 + n)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

Maple [F]

$$\int (d\sec(fx+e))^{2n}(a+ia\tan(fx+e))^{3-n}\,dx$$

input
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x)
```

output
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.16

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx$$
$$= \frac{\left((i\,n^2+3i\,n+2i)e^{(6i\,fx+6i\,e)}+(i\,n^2+5i\,n+6i)e^{(4i\,fx+4i\,e)}-2\,(-i\,n-3i)e^{(2i\,fx+2i\,e)}+2i\right)\left(\frac{2\,de^{(i\,fx+i\,e)}}{e^{(2i\,fx+2i\,e)}+1}\right)}{2\,(fn^3+3\,fn^2+2\,fn)}$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x, algorithm="fric
as")
```

output
```
1/2*((I*n^2 + 3*I*n + 2*I)*e^(6*I*f*x + 6*I*e) + (I*n^2 + 5*I*n + 6*I)*e^(
4*I*f*x + 4*I*e) - 2*(-I*n - 3*I)*e^(2*I*f*x + 2*I*e) + 2*I)*(2*d*e^(I*f*x
 + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^(-I*e*n + (-I*f*n + 3*I*f)*x -
6*I*f*x - (n - 3)*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1)) - (n
- 3)*log(a/d) - 3*I*e)/(f*n^3 + 3*f*n^2 + 2*f*n)
```

Sympy [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx = \int (d\sec{(e+fx)})^{2n}\,(ia(\tan{(e+fx)}-i))^{3-n}\,dx$$

input
```
integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(3-n),x)
```

output
```
Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(3 - n), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x, algorithm="maxi
ma")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx = \int (d\sec(fx+e))^{2n}(ia\tan(fx+e)+a)^{-n+3}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x, algorithm="giac
")
```

output
```
integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n + 3), x)
```

Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.17

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx = -(\cos(6e+6fx)-\sin(6e+6fx)\,1\mathrm{i})\left(\frac{d}{\cos(e+fx)}\right)^{2n}\left(\frac{\left(a+\frac{a\sin(e+fx)\,1\mathrm{i}}{\cos(e+fx)}\right)^{3-n}}{fn\,(n^2\,1\mathrm{i}+n\,3\mathrm{i}+2\mathrm{i})} + \frac{(\cos(4e+4fx)+\sin(4e+4fx)\,1\mathrm{i})\left(a+\frac{a\sin(e+fx)\,1\mathrm{i}}{\cos(e+fx)}\right)^{3-n}(n^2+5n+6)}{2fn\,(n^2\,1\mathrm{i}+n\,3\mathrm{i}+2\mathrm{i})} + \frac{(\cos(6e+6fx)+\sin(6e+6fx)\,1\mathrm{i})\left(a+\frac{a\sin(e+fx)\,1\mathrm{i}}{\cos(e+fx)}\right)^{3-n}(n^2+3n+2)}{2fn\,(n^2\,1\mathrm{i}+n\,3\mathrm{i}+2\mathrm{i})} + \frac{(2n+6)\,(\cos(2e+2fx)+\sin(2e+2fx)\,1\mathrm{i})\left(a+\frac{a\sin(e+fx)\,1\mathrm{i}}{\cos(e+fx)}\right)^{3-n}}{2fn\,(n^2\,1\mathrm{i}+n\,3\mathrm{i}+2\mathrm{i})}\right)$$

input
```
int((d/cos(e + f*x))^(2*n)*(a + a*tan(e + f*x)*1i)^(3 - n),x)
```

output
```
-(cos(6*e + 6*f*x) - sin(6*e + 6*f*x)*1i)*(d/cos(e + f*x))^(2*n)*((a + (a*
sin(e + f*x)*1i)/cos(e + f*x))^(3 - n)/(f*n*(n*3i + n^2*1i + 2i)) + ((cos(
4*e + 4*f*x) + sin(4*e + 4*f*x)*1i)*(a + (a*sin(e + f*x)*1i)/cos(e + f*x))
^(3 - n)*(5*n + n^2 + 6))/(2*f*n*(n*3i + n^2*1i + 2i)) + ((cos(6*e + 6*f*x
) + sin(6*e + 6*f*x)*1i)*(a + (a*sin(e + f*x)*1i)/cos(e + f*x))^(3 - n)*(3
*n + n^2 + 2))/(2*f*n*(n*3i + n^2*1i + 2i)) + ((2*n + 6)*(cos(2*e + 2*f*x)
 + sin(2*e + 2*f*x)*1i)*(a + (a*sin(e + f*x)*1i)/cos(e + f*x))^(3 - n))/(2
*f*n*(n*3i + n^2*1i + 2i)))
```

Reduce [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx$$
$$= d^{2n}a^3\left(\int \frac{\sec(fx+e)^{2n}}{(\tan(fx+e)\,ai+a)^n}dx - \left(\int \frac{\sec(fx+e)^{2n}\tan(fx+e)^3}{(\tan(fx+e)\,ai+a)^n}dx\right)i - 3\left(\int \frac{\sec(fx+e)^{2n}\tan(fx+e)^2}{(\tan(fx+e)\,ai+a)^n}dx\right) + 3\left(\int \frac{\sec(fx+e)^{2n}\tan(fx+e)}{(\tan(fx+e)\,ai+a)^n}dx\right)i\right)$$

input
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x)
```

output
```
d**(2*n)*a**3*(int(sec(e + f*x)**(2*n)/(tan(e + f*x)*a*i + a)**n,x) - int(
(sec(e + f*x)**(2*n)*tan(e + f*x)**3)/(tan(e + f*x)*a*i + a)**n,x)*i - 3*i
nt((sec(e + f*x)**(2*n)*tan(e + f*x)**2)/(tan(e + f*x)*a*i + a)**n,x) + 3*
int((sec(e + f*x)**(2*n)*tan(e + f*x))/(tan(e + f*x)*a*i + a)**n,x)*i)
```

3.507 $\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 60

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\sec^6(c+dx)}{6d}+\frac{a\tan(c+dx)}{d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}$$

output
```
1/6*b*sec(d*x+c)^6/d+a*tan(d*x+c)/d+2/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c)^
5/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\sec^6(c+dx)}{6d}+\frac{a\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^6)/(6*d) + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c
 + d*x]^5/5))/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3967, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+b\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^6(c+dx)dx+\frac{b\sec^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^6dx+\frac{b\sec^6(c+dx)}{6d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{b\sec^6(c+dx)}{6d}-\frac{a\int \left(\tan^4(c+dx)+2\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\sec^6(c+dx)}{6d}-\frac{a\left(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^6)/(6*d) - (a*(-Tan[c + d*x] - (2*Tan[c + d*x]^3)/3 - Tan[
c + d*x]^5/5))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 14.59 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.15

method	result	size
derivativedivides	$\frac{\frac{b\tan(dx+c)^6}{6}+\frac{a\tan(dx+c)^5}{5}+\frac{b\tan(dx+c)^4}{2}+\frac{2a\tan(dx+c)^3}{3}+\frac{b\tan(dx+c)^2}{2}+a\tan(dx+c)}{d}$	69
default	$\frac{\frac{b\tan(dx+c)^6}{6}+\frac{a\tan(dx+c)^5}{5}+\frac{b\tan(dx+c)^4}{2}+\frac{2a\tan(dx+c)^3}{3}+\frac{b\tan(dx+c)^2}{2}+a\tan(dx+c)}{d}$	69
risch	$\frac{\frac{32ia\,e^{6i(dx+c)}}{3}+\frac{32b\,e^{6i(dx+c)}}{3}+16ia\,e^{4i(dx+c)}+\frac{32ia\,e^{2i(dx+c)}}{5}+\frac{16ia}{15}}{d\left(e^{2i(dx+c)}+1\right)^6}$	75

input
```
int(sec(d*x+c)^6*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/6*b*tan(d*x+c)^6+1/5*a*tan(d*x+c)^5+1/2*b*tan(d*x+c)^4+2/3*a*tan(d*
x+c)^3+1/2*b*tan(d*x+c)^2+a*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{2\left(8\,a\cos\left(dx+c\right)^{5}+4\,a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+5\,b}{30\,d\cos\left(dx+c\right)^{6}}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/30*(2*(8*a*cos(d*x + c)^5 + 4*a*cos(d*x + c)^3 + 3*a*cos(d*x + c))*sin(d
*x + c) + 5*b)/(d*cos(d*x + c)^6)
```

Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx = \begin{cases} \frac{a\left(\frac{\tan^{5}\left(c+dx\right)}{5}+\frac{2\tan^{3}\left(c+dx\right)}{3}+\tan\left(c+dx\right)\right)+\frac{b\sec^{6}\left(c+dx\right)}{6}}{d} & \text{for}\ d\neq 0 \\ x(a+b\tan\left(c\right))\sec^{6}\left(c\right) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**6*(a+b*tan(d*x+c)),x)
```

output
```
Piecewise(((a*(tan(c + d*x)**5/5 + 2*tan(c + d*x)**3/3 + tan(c + d*x)) + b
*sec(c + d*x)**6/6)/d, Ne(d, 0)), (x*(a + b*tan(c))*sec(c)**6, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.17

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{5\,b\tan\left(dx+c\right)^6+6\,a\tan\left(dx+c\right)^5+15\,b\tan\left(dx+c\right)^4+20\,a\tan\left(dx+c\right)^3+15\,b\tan\left(dx+c\right)^2+30\,a}{30\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/30*(5*b*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*b*tan(d*x + c)^4 + 20*a
*tan(d*x + c)^3 + 15*b*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.17

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{5\,b\tan\left(dx+c\right)^6+6\,a\tan\left(dx+c\right)^5+15\,b\tan\left(dx+c\right)^4+20\,a\tan\left(dx+c\right)^3+15\,b\tan\left(dx+c\right)^2+30\,a}{30\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/30*(5*b*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*b*tan(d*x + c)^4 + 20*a
*tan(d*x + c)^3 + 15*b*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{\frac{b\tan(c+d\,x)^6}{6}+\frac{a\tan(c+d\,x)^5}{5}+\frac{b\tan(c+d\,x)^4}{2}+\frac{2\,a\tan(c+d\,x)^3}{3}+\frac{b\tan(c+d\,x)^2}{2}+a\tan(c+d\,x)}{d}$$

input
```
int((a + b*tan(c + d*x))/cos(c + d*x)^6,x)
```

output
```
(a*tan(c + d*x) + (2*a*tan(c + d*x)^3)/3 + (a*tan(c + d*x)^5)/5 + (b*tan(c
 + d*x)^2)/2 + (b*tan(c + d*x)^4)/2 + (b*tan(c + d*x)^6)/6)/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.97

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{\sin(dx+c)\left(-16\cos(dx+c)\sin(dx+c)^4a+40\cos(dx+c)\sin(dx+c)^2a-30\cos(dx+c)a-5\sin\right.}{30d\left(\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-\right.}$$

input
```
int(sec(d*x+c)^6*(a+b*tan(d*x+c)),x)
```

output
```
(sin(c + d*x)*( - 16*cos(c + d*x)*sin(c + d*x)**4*a + 40*cos(c + d*x)*sin(
c + d*x)**2*a - 30*cos(c + d*x)*a - 5*sin(c + d*x)**5*b + 15*sin(c + d*x)*
*3*b - 15*sin(c + d*x)*b))/(30*d*(sin(c + d*x)**6 - 3*sin(c + d*x)**4 + 3*
sin(c + d*x)**2 - 1))
```

3.508 $\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx=\frac{b\sec^4(c+dx)}{4d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

output
```
1/4*b*sec(d*x+c)^4/d+a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx=\frac{b\sec^4(c+dx)}{4d}+\frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^4)/(4*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3967, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec(c+dx)^4(a+b\tan(c+dx))dx$$

$$\Big\downarrow \ 3967$$

$$a\int \sec^4(c+dx)dx+\frac{b\sec^4(c+dx)}{4d}$$

$$\Big\downarrow \ 3042$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{b\sec^4(c+dx)}{4d}$$

$$\Big\downarrow \ 4254$$

$$\frac{b\sec^4(c+dx)}{4d}-\frac{a\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{b\sec^4(c+dx)}{4d}-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^4)/(4*d) - (a*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 3.66 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{\frac{b\tan(dx+c)^4}{4}+\frac{a\tan(dx+c)^3}{3}+\frac{b\tan(dx+c)^2}{2}+a\tan(dx+c)}{d}$	47
default	$\frac{\frac{b\tan(dx+c)^4}{4}+\frac{a\tan(dx+c)^3}{3}+\frac{b\tan(dx+c)^2}{2}+a\tan(dx+c)}{d}$	47
risch	$\frac{4ia\,e^{4i(dx+c)}+4b\,e^{4i(dx+c)}+\frac{16ia\,e^{2i(dx+c)}}{3}+\frac{4ia}{3}}{d\left(e^{2i(dx+c)}+1\right)^4}$	62

input
```
int(sec(d*x+c)^4*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*b*tan(d*x+c)^4+1/3*a*tan(d*x+c)^3+1/2*b*tan(d*x+c)^2+a*tan(d*x+c)
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{4\left(2\,a\cos\left(dx+c\right)^3+a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+3\,b}{12\,d\cos\left(dx+c\right)^4}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(4*(2*a*cos(d*x + c)^3 + a*cos(d*x + c))*sin(d*x + c) + 3*b)/(d*cos(d
*x + c)^4)
```

Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \begin{cases} \frac{a\left(\frac{\tan^3\left(c+dx\right)}{3}+\tan\left(c+dx\right)\right)+\frac{b\sec^4\left(c+dx\right)}{4}}{d} & \text{for } d\neq 0 \\ x(a+b\tan\left(c\right))\sec^4\left(c\right) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

output
```
Piecewise(((a*(tan(c + d*x)**3/3 + tan(c + d*x)) + b*sec(c + d*x)**4/4)/d,
 Ne(d, 0)), (x*(a + b*tan(c))*sec(c)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,b\tan\left(dx+c\right)^4+4\,a\tan\left(dx+c\right)^3+6\,b\tan\left(dx+c\right)^2+12\,a\tan\left(dx+c\right)}{12\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(3*b*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*b*tan(d*x + c)^2 + 12*a*
tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,b\tan\left(dx+c\right)^4+4\,a\tan\left(dx+c\right)^3+6\,b\tan\left(dx+c\right)^2+12\,a\tan\left(dx+c\right)}{12\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/12*(3*b*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*b*tan(d*x + c)^2 + 12*a*
tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{\frac{b\tan(c+d\,x)^4}{4}+\frac{a\tan(c+d\,x)^3}{3}+\frac{b\tan(c+d\,x)^2}{2}+a\tan(c+d\,x)}{d}$$

input
```
int((a + b*tan(c + d*x))/cos(c + d*x)^4,x)
```

output
```
(a*tan(c + d*x) + (a*tan(c + d*x)^3)/3 + (b*tan(c + d*x)^2)/2 + (b*tan(c +
 d*x)^4)/4)/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.82

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{\sin(dx+c)\left(-8\cos(dx+c)\sin(dx+c)^2a+12\cos(dx+c)a-3\sin(dx+c)^3b+6\sin(dx+c)b\right)}{12d\left(\sin(dx+c)^4-2\sin(dx+c)^2+1\right)}$$

input
```
int(sec(d*x+c)^4*(a+b*tan(d*x+c)),x)
```

output
```
(sin(c + d*x)*( - 8*cos(c + d*x)*sin(c + d*x)**2*a + 12*cos(c + d*x)*a - 3
*sin(c + d*x)**3*b + 6*sin(c + d*x)*b))/(12*d*(sin(c + d*x)**4 - 2*sin(c +
 d*x)**2 + 1))
```

3.509 $\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\sec^2(c+dx)}{2d} + \frac{a\tan(c+dx)}{d}$$

output
```
1/2*b*sec(d*x+c)^2/d+a*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\sec^2(c+dx)}{2d} + \frac{a\tan(c+dx)}{d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3967, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+b\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^2(c+dx)dx+\frac{b\sec^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{b\sec^2(c+dx)}{2d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{b\sec^2(c+dx)}{2d}-\frac{a\int 1d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{24}$$

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{b\tan(dx+c)^2}{2}+a\tan(dx+c)}{d}$	25
default	$\frac{\frac{b\tan(dx+c)^2}{2}+a\tan(dx+c)}{d}$	25
risch	$\frac{2ia\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}+2ia}{d\left(e^{2i(dx+c)}+1\right)^2}$	48

input
```
int(sec(d*x+c)^2*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*b*tan(d*x+c)^2+a*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx=\frac{2\,a\cos\left(dx+c\right)\sin\left(dx+c\right)+b}{2\,d\cos\left(dx+c\right)^2}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*a*cos(d*x + c)*sin(d*x + c) + b)/(d*cos(d*x + c)^2)
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx=\begin{cases}\frac{a\tan\left(c+dx\right)+\frac{b\tan^2\left(c+dx\right)}{2}}{d} & \text{for } d\neq 0\\ x(a+b\tan\left(c\right))\sec^2\left(c\right) & \text{otherwise}\end{cases}$$

input
```
integrate(sec(d*x+c)**2*(a+b*tan(d*x+c)),x)
```

output
```
Piecewise(((a*tan(c + d*x) + b*tan(c + d*x)**2/2)/d, Ne(d, 0)), (x*(a + b*
tan(c))*sec(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx=\frac{\left(b\tan\left(dx+c\right)+a\right)^2}{2\,bd}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(b*tan(d*x + c) + a)^2/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\tan\left(dx+c\right)^2+2\,a\tan\left(dx+c\right)}{2\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(b*tan(d*x + c)^2 + 2*a*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{\tan(c+d\,x)\,\left(2\,a+b\tan(c+d\,x)\right)}{2\,d}$$

input
```
int((a + b*tan(c + d*x))/cos(c + d*x)^2,x)
```

output
```
(tan(c + d*x)*(2*a + b*tan(c + d*x)))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{\sin\left(dx+c\right)\left(-2\cos\left(dx+c\right)a-\sin\left(dx+c\right)b\right)}{2d\left(\sin\left(dx+c\right)^2-1\right)}$$

input
```
int(sec(d*x+c)^2*(a+b*tan(d*x+c)),x)
```

output
```
(sin(c + d*x)*( - 2*cos(c + d*x)*a - sin(c + d*x)*b))/(2*d*(sin(c + d*x)**
2 - 1))
```

3.510 $\int (a + b\tan(c + dx))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 17

$$\int (a + b\tan(c + dx))\,dx = ax - \frac{b\log(\cos(c + dx))}{d}$$

output
```
a*x-b*ln(cos(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (a + b\tan(c + dx))\,dx = ax - \frac{b\log(\cos(c + dx))}{d}$$

input
```
Integrate[a + b*Tan[c + d*x],x]
```

output
```
a*x - (b*Log[Cos[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax-\frac{b\log(\cos(c+dx))}{d}$$

input
```
Int[a + b*Tan[c + d*x],x]
```

output
```
a*x - (b*Log[Cos[c + d*x]])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

method	result	size
default	$ax+\frac{b\ln\left(1+\tan\left(dx+c\right)^{2}\right)}{2d}$	22
norman	$ax+\frac{b\ln\left(1+\tan\left(dx+c\right)^{2}\right)}{2d}$	22
parallelrisch	$ax+\frac{b\ln\left(1+\tan\left(dx+c\right)^{2}\right)}{2d}$	22
parts	$ax+\frac{b\ln\left(1+\tan\left(dx+c\right)^{2}\right)}{2d}$	22
derivativedivides	$\frac{\frac{b\ln\left(1+\tan\left(dx+c\right)^{2}\right)}{2}+a\arctan\left(\tan\left(dx+c\right)\right)}{d}$	29
risch	$ibx+\frac{2ibc}{d}+ax-\frac{b\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$	36

input
```
int(a+b*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+1/2*b/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int (a+b\tan(c+dx))\,dx=\frac{2\,adx-b\log\left(\frac{1}{\tan(dx+c)^{2}+1}\right)}{2\,d}$$

input
```
integrate(a+b*tan(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(2*a*d*x - b*log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int (a+b\tan(c+dx))\,dx = ax + b\left(\begin{cases} \frac{\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d} & \text{for } d\neq 0 \\ x\tan\left(c\right) & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*tan(d*x+c),x)
```

output
```
a*x + b*Piecewise((log(tan(c + d*x)**2 + 1)/(2*d), Ne(d, 0)), (x*tan(c), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int (a+b\tan(c+dx))\,dx = ax + \frac{b\log\left(\sec\left(dx+c\right)\right)}{d}$$

input
```
integrate(a+b*tan(d*x+c),x, algorithm="maxima")
```

output
```
a*x + b*log(sec(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int (a+b\tan(c+dx))\,dx = ax - \frac{b\log\left(\left|\cos\left(dx+c\right)\right|\right)}{d}$$

input
```
integrate(a+b*tan(d*x+c),x, algorithm="giac")
```

output
```
a*x - b*log(abs(cos(d*x + c)))/d
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int (a+b\tan(c+dx))\,dx = a\,x+\frac{b\,\ln\left(\tan(c+d\,x)^2+1\right)}{2\,d}$$

input
```
int(a + b*tan(c + d*x),x)
```

output
```
a*x + (b*log(tan(c + d*x)^2 + 1))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int (a+b\tan(c+dx))\,dx = \frac{\log\left(\tan\left(dx+c\right)^2+1\right)b+2adx}{2d}$$

input
```
int(a+b*tan(d*x+c),x)
```

output
```
(log(tan(c + d*x)**2 + 1)*b + 2*a*d*x)/(2*d)
```

3.511 $\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 43

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx=\frac{ax}{2}-\frac{b\cos^2(c+dx)}{2d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*a*x-1/2*b*cos(d*x+c)^2/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx=\frac{a(c+dx)}{2d}-\frac{b\cos^2(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

output
```
(a*(c + d*x))/(2*d) - (b*Cos[c + d*x]^2)/(2*d) + (a*Sin[2*(c + d*x)])/(4*d
)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3967, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(c+dx)}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos^2(c+dx)dx-\frac{b\cos^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{b\cos^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{b\cos^2(c+dx)}{2d}$$

$$\Big\downarrow \text{24}$$

$$a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{b\cos^2(c+dx)}{2d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

output
```
-1/2*(b*Cos[c + d*x]^2)/d + a*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{ax}{2}-\frac{b\cos(2dx+2c)}{4d}+\frac{a\sin(2dx+2c)}{4d}$	36
derivativedivides	$\frac{-\frac{b\cos(dx+c)^2}{2}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	41
default	$\frac{-\frac{b\cos(dx+c)^2}{2}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	41

input
```
int(cos(d*x+c)^2*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x-1/4*b/d*cos(2*d*x+2*c)+1/4*a/d*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{adx - b\cos(dx+c)^2 + a\cos(dx+c)\sin(dx+c)}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*d*x - b*cos(d*x + c)^2 + a*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*cos(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{(dx+c)a + \frac{a\tan(dx+c)-b}{\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*((d*x + c)*a + (a*tan(d*x + c) - b)/(tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{(dx+c)a}{2\,d} + \frac{a\tan\left(dx+c\right) - b}{2\left(\tan\left(dx+c\right)^2+1\right)d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(d*x + c)*a/d + 1/2*(a*tan(d*x + c) - b)/((tan(d*x + c)^2 + 1)*d)
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\,x}{2} - \frac{\cos\left(c+d\,x\right)^2\left(\frac{b}{2}-\frac{a\tan(c+d\,x)}{2}\right)}{d}$$

input
```
int(cos(c + d*x)^2*(a + b*tan(c + d*x)),x)
```

output
```
(a*x)/2 - (cos(c + d*x)^2*(b/2 - (a*tan(c + d*x))/2))/d
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{\cos\left(dx+c\right)\sin\left(dx+c\right)a+\sin\left(dx+c\right)^2b+adx}{2d}$$

input
```
int(cos(d*x+c)^2*(a+b*tan(d*x+c)),x)
```

output
```
(cos(c + d*x)*sin(c + d*x)*a + sin(c + d*x)**2*b + a*d*x)/(2*d)
```

3.512 $\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3ax}{8} - \frac{b\cos^4(c+dx)}{4d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d}$$

output
```
3/8*a*x-1/4*b*cos(d*x+c)^4/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c
)^3*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3a(c+dx)}{8d} - \frac{b\cos^4(c+dx)}{4d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

output
```
(3*a*(c + d*x))/(8*d) - (b*Cos[c + d*x]^4)/(4*d) + (a*Sin[2*(c + d*x)])/(4
*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 3967, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(c+dx)}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos^4(c+dx)dx-\frac{b\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{b\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{b\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{b\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{b\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{24}$$

$$a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{b\cos^4(c+dx)}{4d}$$

input
```
Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

output
```
-1/4*(b*Cos[c + d*x]^4)/d + a*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x
/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

Maple [A] (verified)

Time = 4.88 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{-\frac{b\cos(dx+c)^4}{4}+a\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$	52
default	$\frac{-\frac{b\cos(dx+c)^4}{4}+a\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$	52
risch	$\frac{3ax}{8}-\frac{b\cos(4dx+4c)}{32d}+\frac{a\sin(4dx+4c)}{32d}-\frac{b\cos(2dx+2c)}{8d}+\frac{a\sin(2dx+2c)}{4d}$	66

input
```
int(cos(d*x+c)^4*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*b*cos(d*x+c)^4+a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3
/8*d*x+3/8*c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx$$
$$=-\frac{2\,b\cos\left(dx+c\right)^4-3\,adx-\left(2\,a\cos\left(dx+c\right)^3+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/8*(2*b*cos(d*x + c)^4 - 3*a*d*x - (2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c
))*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\cos^4(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*cos(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,(dx+c)a + \frac{3\,a\tan(dx+c)^3+5\,a\tan(dx+c)-2\,b}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/8*(3*(d*x + c)*a + (3*a*tan(d*x + c)^3 + 5*a*tan(d*x + c) - 2*b)/(tan(d*
x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,(dx+c)a}{8\,d} + \frac{3\,a\tan(dx+c)^3+5\,a\tan(dx+c)-2\,b}{8\,\left(\tan(dx+c)^2+1\right)^2 d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
3/8*(d*x + c)*a/d + 1/8*(3*a*tan(d*x + c)^3 + 5*a*tan(d*x + c) - 2*b)/((ta
n(d*x + c)^2 + 1)^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,a\,x}{8} + \frac{\cos\left(c+d\,x\right)^4\,\left(\frac{3\,a\,\tan\left(c+d\,x\right)^3}{8}+\frac{5\,a\,\tan\left(c+d\,x\right)}{8}-\frac{b}{4}\right)}{d}$$

input
```
int(cos(c + d*x)^4*(a + b*tan(c + d*x)),x)
```

output
```
(3*a*x)/8 + (cos(c + d*x)^4*((5*a*tan(c + d*x))/8 - b/4 + (3*a*tan(c + d*x
)^3)/8))/d
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{-2\cos\left(dx+c\right)\sin\left(dx+c\right)^3 a+5\cos\left(dx+c\right)\sin\left(dx+c\right)a-2\sin\left(dx+c\right)^4 b+4\sin\left(dx+c\right)^2 b+3ad}{8d}$$

input
```
int(cos(d*x+c)^4*(a+b*tan(d*x+c)),x)
```

output
```
( - 2*cos(c + d*x)*sin(c + d*x)**3*a + 5*cos(c + d*x)*sin(c + d*x)*a - 2*s
in(c + d*x)**4*b + 4*sin(c + d*x)**2*b + 3*a*d*x)/(8*d)
```

3.513 $\int \cos^6(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \cos^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{5ax}{16} - \frac{b\cos^6(c+dx)}{6d} + \frac{5a\cos(c+dx)\sin(c+dx)}{16d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{a\cos^5(c+dx)\sin(c+dx)}{6d}$$

output
```
5/16*a*x-1/6*b*cos(d*x+c)^6/d+5/16*a*cos(d*x+c)*sin(d*x+c)/d+5/24*a*cos(d*
x+c)^3*sin(d*x+c)/d+1/6*a*cos(d*x+c)^5*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.66

$$\int \cos^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{-32b\cos^6(c+dx) + a(60c+60dx+45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx)))}{192d}$$

input
```
Integrate[Cos[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

output
```
(-32*b*Cos[c + d*x]^6 + a*(60*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(
c + d*x)] + Sin[6*(c + d*x)]))/(192*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3967, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(c+dx)}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos^6(c+dx)dx-\frac{b\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^6dx-\frac{b\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{6}\int \cos^4(c+dx)dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{b\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{b\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{b\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{b\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{b\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{24}$$

$$a\left(\frac{\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5}{6}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)\right)-\frac{b\cos^6(c+dx)}{6d}$$

input
```
Int[Cos[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

output
```
-1/6*(b*Cos[c + d*x]^6)/d + a*((Cos[c + d*x]^5*Sin[c + d*x])/(6*d) + (5*((
Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])
/(2*d)))/4))/6)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

Maple [A] (verified)

Time = 20.73 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{-\frac{b\cos(dx+c)^6}{6}+a\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}$
default	$\frac{-\frac{b\cos(dx+c)^6}{6}+a\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}$
risch	$\frac{5ax}{16}-\frac{b\cos(6dx+6c)}{192d}+\frac{a\sin(6dx+6c)}{192d}-\frac{b\cos(4dx+4c)}{32d}+\frac{3a\sin(4dx+4c)}{64d}-\frac{5b\cos(2dx+2c)}{64d}+\frac{15a\sin(2dx-}{64d}$

input
```
int(cos(d*x+c)^6*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/6*b*cos(d*x+c)^6+a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*
x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int \cos^6(c+dx)(a+b\tan(c+dx))\,dx = -\frac{8\,b\cos\left(dx+c\right)^6-15\,adx-\left(8\,a\cos\left(dx+c\right)^5+10\,a\cos\left(dx+c\right)^3+15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/48*(8*b*cos(d*x + c)^6 - 15*a*d*x - (8*a*cos(d*x + c)^5 + 10*a*cos(d*x
+ c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^6(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\cos^6(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**6*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*cos(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \cos^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{15\,(dx+c)a+\frac{15\,a\tan(dx+c)^5+40\,a\tan(dx+c)^3+33\,a\tan(dx+c)-8\,b}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{48\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/48*(15*(d*x + c)*a + (15*a*tan(d*x + c)^5 + 40*a*tan(d*x + c)^3 + 33*a*t
an(d*x + c) - 8*b)/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 +
 1))/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

$$\int \cos^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{5\,(dx+c)a}{16\,d} + \frac{15\,a\tan\left(dx+c\right)^5 + 40\,a\tan\left(dx+c\right)^3 + 33\,a\tan\left(dx+c\right) - 8\,b}{48\,\left(\tan\left(dx+c\right)^2+1\right)^3 d}$$

input
```
integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
5/16*(d*x + c)*a/d + 1/48*(15*a*tan(d*x + c)^5 + 40*a*tan(d*x + c)^3 + 33*
a*tan(d*x + c) - 8*b)/((tan(d*x + c)^2 + 1)^3*d)
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int \cos^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{5\,a\,x}{16} + \frac{\frac{5\,a\tan(c+d\,x)^5}{16} + \frac{5\,a\tan(c+d\,x)^3}{6} + \frac{11\,a\tan(c+d\,x)}{16} - \frac{b}{6}}{d\,\left(\tan\left(c+d\,x\right)^6 + 3\tan\left(c+d\,x\right)^4 + 3\tan\left(c+d\,x\right)^2 + 1\right)}$$

input
```
int(cos(c + d*x)^6*(a + b*tan(c + d*x)),x)
```

output
```
(5*a*x)/16 + ((11*a*tan(c + d*x))/16 - b/6 + (5*a*tan(c + d*x)^3)/6 + (5*a
*tan(c + d*x)^5)/16)/(d*(3*tan(c + d*x)^2 + 3*tan(c + d*x)^4 + tan(c + d*x
)^6 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \cos^6(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{8\cos(dx+c)\sin(dx+c)^5 a - 26\cos(dx+c)\sin(dx+c)^3 a + 33\cos(dx+c)\sin(dx+c)\, a + 8\sin(dx}{48d}$$

input
```
int(cos(d*x+c)^6*(a+b*tan(d*x+c)),x)
```

output
```
(8*cos(c + d*x)*sin(c + d*x)**5*a - 26*cos(c + d*x)*sin(c + d*x)**3*a + 33
*cos(c + d*x)*sin(c + d*x)*a + 8*sin(c + d*x)**6*b - 24*sin(c + d*x)**4*b
+ 24*sin(c + d*x)**2*b + 15*a*d*x)/(48*d)
```

3.514 $\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{3a\text{arctanh}(\sin(c+dx))}{8d}+\frac{b\sec^5(c+dx)}{5d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$

output
```
3/8*a*arctanh(sin(d*x+c))/d+1/5*b*sec(d*x+c)^5/d+3/8*a*sec(d*x+c)*tan(d*x+
c)/d+1/4*a*sec(d*x+c)^3*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{3a\text{arctanh}(\sin(c+dx))}{8d}+\frac{b\sec^5(c+dx)}{5d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$

input
```
Integrate[Sec[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

output
```
(3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Sec[c + d*x]^5)/(5*d) + (3*a*Sec[c
+ d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3967, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^5(a+b\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^5(c+dx)dx+\frac{b\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx+\frac{b\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{b\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{b\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{b\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{b\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{3}{4}\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{b\sec^5(c+dx)}{5d}$$

input
```
Int[Sec[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^5)/(5*d) + a*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(Ar
cTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 8.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{b}{5\cos(dx+c)^5}+a\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
default	$\frac{\frac{b}{5\cos(dx+c)^5}+a\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
risch	$\frac{-15ia\,e^{9i(dx+c)}-70ia\,e^{7i(dx+c)}+128b\,e^{5i(dx+c)}+70ia\,e^{3i(dx+c)}+15ia\,e^{i(dx+c)}}{20d\left(e^{2i(dx+c)}+1\right)^5}+\frac{3a\ln\left(e^{i(dx+c)}+i\right)}{8d}-\frac{3a\ln\left(e^{i(dx-\right.}}{8d}$

input
```
int(sec(d*x+c)^5*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*b/cos(d*x+c)^5+a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+
3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{15\,a\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\,a\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+10\left(3\,a\cos\left(dx+c\right.\right.}{80\,d\cos\left(dx+c\right)^5}$$

input
```
integrate(sec(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/80*(15*a*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*a*cos(d*x + c)^5*log(
-sin(d*x + c) + 1) + 10*(3*a*cos(d*x + c)^3 + 2*a*cos(d*x + c))*sin(d*x +
c) + 16*b)/(d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx=\int (a+b\tan(c+dx))\sec^5(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**5*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*sec(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = -\frac{5\,a\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin(dx+c)+1\right)+3\,\log\left(\sin(dx+c)-1\right)\right)-\frac{16\,b}{\cos(dx+c)^5}}{80\,d}$$

input
```
integrate(sec(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/80*(5*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(
d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 16*
b/cos(d*x + c)^5)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(66) = 132.

Time = 0.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.91

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-\right.}{}}{40\,d}$$

input
```
integrate(sec(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/40*(15*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + 2*(25*a*tan(1/2*d*x + 1/2*c)^9 - 40*b*tan(1/2*d*x + 1/2*c)
^8 - 10*a*tan(1/2*d*x + 1/2*c)^7 - 80*b*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(
1/2*d*x + 1/2*c)^3 - 25*a*tan(1/2*d*x + 1/2*c) - 8*b)/(tan(1/2*d*x + 1/2*c
)^2 - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.36

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d} - \frac{-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + b*tan(c + d*x))/cos(c + d*x)^5,x)
```

output
```
(3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((2*b)/5 + (5*a*tan(c/2 + (d*x)/2)
)/4 - (a*tan(c/2 + (d*x)/2)^3)/2 + (a*tan(c/2 + (d*x)/2)^7)/2 - (5*a*tan(c
/2 + (d*x)/2)^9)/4 + 4*b*tan(c/2 + (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^8)/
(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/
2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.68

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{-15\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4 a+30\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+}{}$$

input
```
int(sec(d*x+c)^5*(a+b*tan(d*x+c)),x)
```

output
```
( - 15*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a + 30*cos(c
 + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a - 15*cos(c + d*x)*log(
tan((c + d*x)/2) - 1)*a + 15*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c
+ d*x)**4*a - 30*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a
+ 15*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a - 8*cos(c + d*x)*sin(c + d*x
)**4*b - 15*cos(c + d*x)*sin(c + d*x)**3*a + 16*cos(c + d*x)*sin(c + d*x)*
*2*b + 25*cos(c + d*x)*sin(c + d*x)*a - 8*cos(c + d*x)*b + 8*b)/(40*cos(c
+ d*x)*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.515 $\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx=\frac{a\text{arctanh}(\sin(c+dx))}{2d}+\frac{b\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*a*arctanh(sin(d*x+c))/d+1/3*b*sec(d*x+c)^3/d+1/2*a*sec(d*x+c)*tan(d*x+
c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx=\frac{a\text{arctanh}(\sin(c+dx))}{2d}+\frac{b\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*
x]*Tan[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3967, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3(a+b\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^3(c+dx)dx+\frac{b\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{b\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b\sec^3(c+dx)}{3d}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^3)/(3*d) + a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*
Tan[c + d*x])/(2*d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{b}{3\cos(dx+c)^3}+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	50
default	$\frac{\frac{b}{3\cos(dx+c)^3}+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	50
risch	$\frac{-3ia\,e^{5i(dx+c)}+8b\,e^{3i(dx+c)}+3ia\,e^{i(dx+c)}}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$	97

input
```
int(sec(d*x+c)^3*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*b/cos(d*x+c)^3+a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan
(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{3\,a\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,a\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+6\,a\cos\left(dx+c\right)\sin\left(c\right.}{12\,d\cos\left(dx+c\right)^3}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-s
in(d*x + c) + 1) + 6*a*cos(d*x + c)*sin(d*x + c) + 4*b)/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx=\int \left(a+b\tan\left(c+dx\right)\right)\sec^3\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$= -\frac{3\,a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) - \frac{4\,b}{\cos(dx+c)^3}}{12\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/12*(3*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
log(sin(d*x + c) - 1)) - 4*b/cos(d*x + c)^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(46) = 92$.

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.90

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3\,a\,\mathrm{ta}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*b*tan(1/2*d*x + 1/2*c)^4 -
3*a*tan(1/2*d*x + 1/2*c) - 2*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.02

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{2\,b}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

input
```
int((a + b*tan(c + d*x))/cos(c + d*x)^3,x)
```

output
```
(a*atanh(tan(c/2 + (d*x)/2)))/d - ((2*b)/3 + a*tan(c/2 + (d*x)/2) - a*tan(
c/2 + (d*x)/2)^5 + 2*b*tan(c/2 + (d*x)/2)^4)/(d*(3*tan(c/2 + (d*x)/2)^2 -
3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.27

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{-3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2a+3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+3\cos(dx}{}$$

input
```
int(sec(d*x+c)^3*(a+b*tan(d*x+c)),x)
```

output
```
( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a + 3*cos(c +
 d*x)*log(tan((c + d*x)/2) - 1)*a + 3*cos(c + d*x)*log(tan((c + d*x)/2) +
1)*sin(c + d*x)**2*a - 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a - 2*cos(
c + d*x)*sin(c + d*x)**2*b - 3*cos(c + d*x)*sin(c + d*x)*a + 2*cos(c + d*x
)*b - 2*b)/(6*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.516 $\int \sec(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{d} + \frac{b\sec(c+dx)}{d}$$

output
```
a*arctanh(sin(d*x+c))/d+b*sec(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\coth^{-1}(\sin(c+dx))}{d} + \frac{b\sec(c+dx)}{d}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x]),x]
```

output
```
(a*ArcCoth[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+b\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec(c+dx)dx+\frac{b\sec(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{b\sec(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]*(a + b*Tan[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\frac{\frac{b}{\cos(dx+c)}+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	32
default	$\frac{\frac{b}{\cos(dx+c)}+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	32
risch	$\frac{2b\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}$	67

input
```
int(sec(d*x+c)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b/cos(d*x+c)+a*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\cos(dx+c)\log(\sin(dx+c)+1) - a\cos(dx+c)\log(-\sin(dx+c)+1) + 2b}{2d\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x +
c) + 1) + 2*b)/(d*cos(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx = \begin{cases} \frac{a\log(\tan(c+dx)+\sec(c+dx))+b\sec(c+dx)}{d} & \text{for } d \neq 0 \\ x(a+b\tan(c))\sec(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x)
```

output
```
Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + b*sec(c + d*x))/d, Ne(d,
0)), (x*(a + b*tan(c))*sec(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+\frac{b}{\cos(dx+c)}}{d}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
(a*log(sec(d*x + c) + tan(d*x + c)) + b/cos(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(24) = 48.

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) - 2*b/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,b}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

input
```
int((a + b*tan(c + d*x))/cos(c + d*x),x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)))/d - (2*b)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.67

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{-\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a-\cos(dx+c)\,b+b}{\cos(dx+c)\,d}$$

input
```
int(sec(d*x+c)*(a+b*tan(d*x+c)),x)
```

output
```
( - cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a + cos(c + d*x)*log(tan((c + d
*x)/2) + 1)*a - cos(c + d*x)*b + b)/(cos(c + d*x)*d)
```

3.517 $\int \cos(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx = -\frac{b\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}$$

output
```
-b*cos(d*x+c)/d+a*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx = -\frac{b\cos(c)\cos(dx)}{d}+\frac{a\cos(dx)\sin(c)}{d} + \frac{a\cos(c)\sin(dx)}{d}+\frac{b\sin(c)\sin(dx)}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x]),x]
```

output
```
-((b*Cos[c]*Cos[d*x])/d) + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d +
 (b*Sin[c]*Sin[d*x])/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3967, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(c+dx)}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos(c+dx)dx-\frac{b\cos(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\cos(c+dx)}{d}$$

$$\Big\downarrow \text{3117}$$

$$\frac{a\sin(c+dx)}{d}-\frac{b\cos(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]*(a + b*Tan[c + d*x]),x]
```

output
```
-((b*Cos[c + d*x])/d) + (a*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-b\cos(dx+c)+a\sin(dx+c)}{d}$	23
default	$\frac{-b\cos(dx+c)+a\sin(dx+c)}{d}$	23
risch	$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$	25

input
```
int(cos(d*x+c)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b*cos(d*x+c)+a*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx = -\frac{b\cos(dx+c)-a\sin(dx+c)}{d}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-(b*cos(d*x + c) - a*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*cos(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx = -\frac{b\cos(dx+c)-a\sin(dx+c)}{d}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-(b*cos(d*x + c) - a*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(24) = 48$.

Time = 0.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 5.38

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx =$$
$$-\frac{b\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2+2\,a\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)+2\,a\tan\left(\frac{1}{2}\,dx\right)\tan\left(\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx\right)^2-4\,b\tan}{d\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2+d\tan\left(\frac{1}{2}\,dx\right)^2+d\tan}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-(b*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*tan(
1/2*d*x)*tan(1/2*c)^2 - b*tan(1/2*d*x)^2 - 4*b*tan(1/2*d*x)*tan(1/2*c) - b
*tan(1/2*c)^2 - 2*a*tan(1/2*d*x) - 2*a*tan(1/2*c) + b)/(d*tan(1/2*d*x)^2*t
an(1/2*c)^2 + d*tan(1/2*d*x)^2 + d*tan(1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx = -\frac{2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

input
```
int(cos(c + d*x)*(a + b*tan(c + d*x)),x)
```

output
```
-(2*cos(c/2 + (d*x)/2)*(b*cos(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx=\frac{-\cos(dx+c)\,b+\sin(dx+c)\,a+b}{d}$$

input
```
int(cos(d*x+c)*(a+b*tan(d*x+c)),x)
```

output
```
( - cos(c + d*x)*b + sin(c + d*x)*a + b)/d
```

3.518 $\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx=-\frac{b\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

output
```
-1/3*b*cos(d*x+c)^3/d+a*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx=-\frac{b\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

output
```
-1/3*(b*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(c+dx)}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos^3(c+dx)dx-\frac{b\cos^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\cos^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{a\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}-\frac{b\cos^3(c+dx)}{3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{b\cos^3(c+dx)}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

output
```
-1/3*(b*Cos[c + d*x]^3)/d - (a*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{-\frac{b\cos(dx+c)^3}{3}+\frac{a\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$	36
default	$\frac{-\frac{b\cos(dx+c)^3}{3}+\frac{a\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$	36
risch	$-\frac{b\cos(dx+c)}{4d}+\frac{3a\sin(dx+c)}{4d}-\frac{b\cos(3dx+3c)}{12d}+\frac{a\sin(3dx+3c)}{12d}$	56

input
```
int(cos(d*x+c)^3*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*b*cos(d*x+c)^3+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx = -\frac{b\cos\left(dx+c\right)^3-\left(a\cos\left(dx+c\right)^2+2\,a\right)\sin\left(dx+c\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/3*(b*cos(d*x + c)^3 - (a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a+b\tan\left(c+dx\right)\right)\cos^3\left(c+dx\right)\,dx$$

input
```
integrate(cos(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*cos(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx = -\frac{b\cos\left(dx+c\right)^3+\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/3*(b*cos(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11886 vs. $2(40) = 80$.

Time = 2.13 (sec) , antiderivative size = 11886, normalized size of antiderivative = 270.14

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/96*(3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
 + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*pi*b*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c
)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*
d*x)^6*tan(1/2*c)^6 - 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan
(1/2*d*x)^6*tan(1/2*c)^6 + 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1
)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 6*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - t
an(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)
^6*tan(1/2*c)^6 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*pi*b*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x...
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{2\,a\,\sin\left(c+d\,x\right)}{3\,d}-\frac{b\cos\left(c+d\,x\right)^3}{3\,d}+\frac{a\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int(cos(c + d*x)^3*(a + b*tan(c + d*x)),x)
```

output
```
(2*a*sin(c + d*x))/(3*d) - (b*cos(c + d*x)^3)/(3*d) + (a*cos(c + d*x)^2*si
n(c + d*x))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{\cos\left(dx+c\right)\sin\left(dx+c\right)^2 b-\cos\left(dx+c\right)b-\sin\left(dx+c\right)^3 a+3\sin\left(dx+c\right)a+b}{3d}$$

input
```
int(cos(d*x+c)^3*(a+b*tan(d*x+c)),x)
```

output
```
(cos(c + d*x)*sin(c + d*x)**2*b - cos(c + d*x)*b - sin(c + d*x)**3*a + 3*s
in(c + d*x)*a + b)/(3*d)
```

3.519 $\int \cos^5(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 60

$$\int \cos^5(c+dx)(a+b\tan(c+dx))\,dx = -\frac{b\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}$$

output
```
-1/5*b*cos(d*x+c)^5/d+a*sin(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin(d*x+c)
^5/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \cos^5(c+dx)(a+b\tan(c+dx))\,dx = -\frac{b\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

output
```
-1/5*(b*Cos[c + d*x]^5)/d + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d
) + (a*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(c+dx)}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos^5(c+dx)dx-\frac{b\cos^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{b\cos^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{a\int \left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}-\frac{b\cos^5(c+dx)}{5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{b\cos^5(c+dx)}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

output
```
-1/5*(b*Cos[c + d*x]^5)/d - (a*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 - Sin
[c + d*x]^5/5))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

Maple [A] (verified)

Time = 9.60 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{-\frac{\cos(dx+c)^5 b}{5}+\frac{a\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}}{d}$	46
default	$\frac{-\frac{\cos(dx+c)^5 b}{5}+\frac{a\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}}{d}$	46
risch	$-\frac{b\cos(dx+c)}{8d}+\frac{5a\sin(dx+c)}{8d}-\frac{b\cos(5dx+5c)}{80d}+\frac{a\sin(5dx+5c)}{80d}-\frac{b\cos(3dx+3c)}{16d}+\frac{5a\sin(3dx+3c)}{48d}$	86

input
```
int(cos(d*x+c)^5*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/5*cos(d*x+c)^5*b+1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x
+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \cos^5(c+dx)(a+b\tan(c+dx))\,dx = -\frac{3\,b\cos\left(dx+c\right)^5-\left(3\,a\cos\left(dx+c\right)^4+4\,a\cos\left(dx+c\right)^2+8\,a\right)\sin\left(dx+c\right)}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/15*(3*b*cos(d*x + c)^5 - (3*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 8*a
)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^5(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a+b\tan\left(c+dx\right)\right)\cos^5\left(c+dx\right)\,dx$$

input
```
integrate(cos(d*x+c)**5*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*cos(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int \cos^5(c+dx)(a+b\tan(c+dx))\,dx$$
$$= -\frac{3\,b\cos\left(dx+c\right)^5-\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/15*(3*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin
(d*x + c))*a)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27424 vs. 2(54) = 108.

Time = 5.08 (sec) , antiderivative size = 27424, normalized size of antiderivative = 457.07

$$\int \cos^5(c+dx)(a+b\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/1920*(45*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2
*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*b*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan
(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*ta
n(1/2*d*x)^10*tan(1/2*c)^10 - 75*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
 - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 75*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(
1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 - 90*pi*b*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 +
1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 225*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/
2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/
2*c)^8 + 225*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1
/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/...
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \cos^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{8\,a\,\sin\left(c+d\,x\right)}{15\,d}-\frac{b\cos\left(c+d\,x\right)^5}{5\,d}+\frac{4\,a\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{15\,d}+\frac{a\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{5\,d}$$

input
```
int(cos(c + d*x)^5*(a + b*tan(c + d*x)),x)
```

output
```
(8*a*sin(c + d*x))/(15*d) - (b*cos(c + d*x)^5)/(5*d) + (4*a*cos(c + d*x)^2
*sin(c + d*x))/(15*d) + (a*cos(c + d*x)^4*sin(c + d*x))/(5*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.38

$$\int \cos^5(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{-3\cos(dx+c)\sin(dx+c)^4 b + 6\cos(dx+c)\sin(dx+c)^2 b - 3\cos(dx+c) b + 3\sin(dx+c)^5 a - 10\,\mathrm{s}}{15d}$$

input
```
int(cos(d*x+c)^5*(a+b*tan(d*x+c)),x)
```

output
```
( - 3*cos(c + d*x)*sin(c + d*x)**4*b + 6*cos(c + d*x)*sin(c + d*x)**2*b -
3*cos(c + d*x)*b + 3*sin(c + d*x)**5*a - 10*sin(c + d*x)**3*a + 15*sin(c +
 d*x)*a + 3*b)/(15*d)
```

3.520 $\int \cos^7(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \cos^7(c+dx)(a+b\tan(c+dx))\,dx=-\frac{b\cos^7(c+dx)}{7d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^7(c+dx)}{7d}$$

output
```
-1/7*b*cos(d*x+c)^7/d+a*sin(d*x+c)/d-a*sin(d*x+c)^3/d+3/5*a*sin(d*x+c)^5/d
-1/7*a*sin(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \cos^7(c+dx)(a+b\tan(c+dx))\,dx=-\frac{b\cos^7(c+dx)}{7d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^7(c+dx)}{7d}$$

input
```
Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x]),x]
```

output
```
-1/7*(b*Cos[c + d*x]^7)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/d + (3
*a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(c+dx)}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos^7(c+dx)dx-\frac{b\cos^7(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^7dx-\frac{b\cos^7(c+dx)}{7d}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{a\int \left(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}-\frac{b\cos^7(c+dx)}{7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{b\cos^7(c+dx)}{7d}$$

input
```
Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x]),x]
```

output
```
-1/7*(b*Cos[c + d*x]^7)/d - (a*(-Sin[c + d*x] + Sin[c + d*x]^3 - (3*Sin[c
+ d*x]^5)/5 + Sin[c + d*x]^7/7))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

Maple [A] (verified)

Time = 36.89 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{-\frac{b\cos(dx+c)^7}{7}+\frac{a\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}}{d}$
default	$\frac{-\frac{b\cos(dx+c)^7}{7}+\frac{a\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}}{d}$
risch	$-\frac{5b\cos(dx+c)}{64d}+\frac{35a\sin(dx+c)}{64d}-\frac{b\cos(7dx+7c)}{448d}+\frac{a\sin(7dx+7c)}{448d}-\frac{b\cos(5dx+5c)}{64d}+\frac{7a\sin(5dx+5c)}{320d}-$

input
```
int(cos(d*x+c)^7*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/7*b*cos(d*x+c)^7+1/7*a*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos
(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \cos^7(c+dx)(a+b\tan(c+dx))\,dx = -\frac{5\,b\cos\left(dx+c\right)^7-\left(5\,a\cos\left(dx+c\right)^6+6\,a\cos\left(dx+c\right)^4+8\,a\cos\left(dx+c\right)^2+16\,a\right)\sin\left(dx+c\right)}{35\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/35*(5*b*cos(d*x + c)^7 - (5*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 + 8*a
*cos(d*x + c)^2 + 16*a)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^7(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a+b\tan\left(c+dx\right)\right)\cos^7\left(c+dx\right)\,dx$$

input
```
integrate(cos(d*x+c)**7*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*cos(c + d*x)**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \cos^7(c+dx)(a+b\tan(c+dx))\,dx = -\frac{5\,b\cos\left(dx+c\right)^7+\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a}{35\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/35*(5*b*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin
(d*x + c)^3 - 35*sin(d*x + c))*a)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49370 vs. 2(68) = 136.

Time = 10.87 (sec) , antiderivative size = 49370, normalized size of antiderivative = 667.16

$$\int \cos^7(c+dx)(a+b\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/17920*(315*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 315*pi*b*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^14*tan(1/2*c)^14 - 735*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*
d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 735*pi*b*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2
*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 - 630*pi*b*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c
)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 2205*pi*b*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2
*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*
c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14
*tan(1/2*c)^12 + 2205*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2...
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18

$$\int \cos^7(c+dx)(a+b\tan(c+dx))\,dx = \frac{16\,a\,\sin(c+d\,x)}{35\,d} - \frac{b\cos(c+d\,x)^7}{7\,d} + \frac{8\,a\cos(c+d\,x)^2\,\sin(c+d\,x)}{35\,d} + \frac{6\,a\cos(c+d\,x)^4\,\sin(c+d\,x)}{35\,d} + \frac{a\cos(c+d\,x)^6\,\sin(c+d\,x)}{7\,d}$$

input
```
int(cos(c + d*x)^7*(a + b*tan(c + d*x)),x)
```

output
```
(16*a*sin(c + d*x))/(35*d) - (b*cos(c + d*x)^7)/(7*d) + (8*a*cos(c + d*x)^
2*sin(c + d*x))/(35*d) + (6*a*cos(c + d*x)^4*sin(c + d*x))/(35*d) + (a*cos
(c + d*x)^6*sin(c + d*x))/(7*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.50

$$\int \cos^7(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{5\cos(dx+c)\sin(dx+c)^6 b - 15\cos(dx+c)\sin(dx+c)^4 b + 15\cos(dx+c)\sin(dx+c)^2 b - 5\cos(dx}{35d}$$

input
```
int(cos(d*x+c)^7*(a+b*tan(d*x+c)),x)
```

output
```
(5*cos(c + d*x)*sin(c + d*x)**6*b - 15*cos(c + d*x)*sin(c + d*x)**4*b + 15
*cos(c + d*x)*sin(c + d*x)**2*b - 5*cos(c + d*x)*b - 5*sin(c + d*x)**7*a +
 21*sin(c + d*x)**5*a - 35*sin(c + d*x)**3*a + 35*sin(c + d*x)*a + 5*b)/(3
5*d)
```

3.521 $\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{ab\sec^8(c+dx)}{4d} + \frac{a^2\tan(c+dx)}{d} + \frac{(3a^2+b^2)\tan^3(c+dx)}{3d} + \frac{3(a^2+b^2)\tan^5(c+dx)}{5d} + \frac{(a^2+3b^2)\tan^7(c+dx)}{7d} + \frac{b^2\tan^9(c+dx)}{9d}$$

output
```
1/4*a*b*sec(d*x+c)^8/d+a^2*tan(d*x+c)/d+1/3*(3*a^2+b^2)*tan(d*x+c)^3/d+3/5
*(a^2+b^2)*tan(d*x+c)^5/d+1/7*(a^2+3*b^2)*tan(d*x+c)^7/d+1/9*b^2*tan(d*x+c
)^9/d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.12

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{\tan(c+dx)\left(1260a^2+1260ab\tan(c+dx)+420(3a^2+b^2)\tan^2(c+dx)+1890ab\tan^3(c+dx)+756(a\right.}{}$$

input
```
Integrate[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^2,x]
```

output
```
(Tan[c + d*x]*(1260*a^2 + 1260*a*b*Tan[c + d*x] + 420*(3*a^2 + b^2)*Tan[c
+ d*x]^2 + 1890*a*b*Tan[c + d*x]^3 + 756*(a^2 + b^2)*Tan[c + d*x]^4 + 1260
*a*b*Tan[c + d*x]^5 + 180*(a^2 + 3*b^2)*Tan[c + d*x]^6 + 315*a*b*Tan[c + d
*x]^7 + 140*b^2*Tan[c + d*x]^8))/(1260*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 475, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(c+dx)^8(a+b\tan(c+dx))^2dx$$
$$\Big\downarrow \text{3987}$$
$$\frac{\int \frac{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)^3}{b^6}d(b\tan(c+dx))}{bd}$$
$$\Big\downarrow \text{27}$$

$$\frac{\int (a+b\tan(c+dx))^2 \left(\tan^2(c+dx)b^2+b^2\right)^3 d(b\tan(c+dx))}{b^7 d}$$

$$\Big\downarrow \text{475}$$

$$\frac{\int \left(b^8\tan^8(c+dx)+b^6\left(a^2+3b^2\right)\tan^6(c+dx)+3b^6\left(a^2+b^2\right)\tan^4(c+dx)+b^6\left(3a^2+b^2\right)\tan^2(c+dx)+a^2b^6\right)}{b^7 d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2b^7\tan(c+dx)+\frac{1}{7}b^7\left(a^2+3b^2\right)\tan^7(c+dx)+\frac{3}{5}b^7\left(a^2+b^2\right)\tan^5(c+dx)+\frac{1}{3}b^7\left(3a^2+b^2\right)\tan^3(c+dx)+\frac{1}{4}a(}{b^7 d}$$

input
```
Int[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^2,x]
```

output
```
(a^2*b^7*Tan[c + d*x] + (b^7*(3*a^2 + b^2)*Tan[c + d*x]^3)/3 + (3*b^7*(a^2
 + b^2)*Tan[c + d*x]^5)/5 + (b^7*(a^2 + 3*b^2)*Tan[c + d*x]^7)/7 + (b^9*Ta
n[c + d*x]^9)/9 + (a*(b^2 + b^2*Tan[c + d*x]^2)^4)/4)/(b^7*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 475
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp
[d*n*c^(n - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Int[ExpandIntegran
d[((c + d*x)^n - d*n*c^(n - 1)*x)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
d}, x] && IGtQ[p, 0] && IGtQ[n, 0] && LeQ[n, p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 115.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin(dx+c)^3}{9\cos(dx+c)^9}+\frac{2\sin(dx+c)^3}{21\cos(dx+c)^7}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^5}+\frac{16\sin(dx+c)^3}{315\cos(dx+c)^3}\right)+\frac{ab}{4\cos(dx+c)^8}-a^2\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}\right.}{d}$
default	$\frac{b^2\left(\frac{\sin(dx+c)^3}{9\cos(dx+c)^9}+\frac{2\sin(dx+c)^3}{21\cos(dx+c)^7}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^5}+\frac{16\sin(dx+c)^3}{315\cos(dx+c)^3}\right)+\frac{ab}{4\cos(dx+c)^8}-a^2\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}\right.}{d}$
risch	$\frac{32i\left(-630iab\,e^{10i(dx+c)}+315a^2e^{10i(dx+c)}-315b^2e^{10i(dx+c)}-630iab\,e^{8i(dx+c)}+819a^2e^{8i(dx+c)}+189b^2e^{8i(dx+c)}+756a\right.}{315d\left(e^{2i(dx+c)}+1\right)}$

input
```
int(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/1
05*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+1/4*a*b/cos
(d*x+c)^8-a^2*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2
)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{315\,ab\cos(dx+c)+4\left(16\left(9\,a^2-b^2\right)\cos(dx+c)^8+8\left(9\,a^2-b^2\right)\cos(dx+c)^6+6\left(9\,a^2-b^2\right)\cos(dx+\right.}{1260\,d\cos(dx+c)^9}$$

input
```
integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/1260*(315*a*b*cos(d*x + c) + 4*(16*(9*a^2 - b^2)*cos(d*x + c)^8 + 8*(9*a
^2 - b^2)*cos(d*x + c)^6 + 6*(9*a^2 - b^2)*cos(d*x + c)^4 + 5*(9*a^2 - b^2
)*cos(d*x + c)^2 + 35*b^2)*sin(d*x + c))/(d*cos(d*x + c)^9)
```

Sympy [F]

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\sec^8(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**8*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.12

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{140\,b^2\tan(dx+c)^9 + 315\,ab\tan(dx+c)^8 + 1260\,ab\tan(dx+c)^6 + 180\,(a^2+3\,b^2)\tan(dx+c)^7 + 1890}{}$$

input
```
integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/1260*(140*b^2*tan(d*x + c)^9 + 315*a*b*tan(d*x + c)^8 + 1260*a*b*tan(d*x
 + c)^6 + 180*(a^2 + 3*b^2)*tan(d*x + c)^7 + 1890*a*b*tan(d*x + c)^4 + 756
*(a^2 + b^2)*tan(d*x + c)^5 + 1260*a*b*tan(d*x + c)^2 + 420*(3*a^2 + b^2)*
tan(d*x + c)^3 + 1260*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.31

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{140\,b^2\tan(dx+c)^9 + 315\,ab\tan(dx+c)^8 + 180\,a^2\tan(dx+c)^7 + 540\,b^2\tan(dx+c)^7 + 1260\,ab\tan(d$$

input
```
integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/1260*(140*b^2*tan(d*x + c)^9 + 315*a*b*tan(d*x + c)^8 + 180*a^2*tan(d*x
+ c)^7 + 540*b^2*tan(d*x + c)^7 + 1260*a*b*tan(d*x + c)^6 + 756*a^2*tan(d*
x + c)^5 + 756*b^2*tan(d*x + c)^5 + 1890*a*b*tan(d*x + c)^4 + 1260*a^2*tan
(d*x + c)^3 + 420*b^2*tan(d*x + c)^3 + 1260*a*b*tan(d*x + c)^2 + 1260*a^2*
tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.11

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{\tan(c+d\,x)^3\left(a^2+\frac{b^2}{3}\right)+a^2\tan(c+d\,x)+\tan(c+d\,x)^5\left(\frac{3\,a^2}{5}+\frac{3\,b^2}{5}\right)+\tan(c+d\,x)^7\left(\frac{a^2}{7}+\frac{3\,b^2}{7}\right)+\frac{b^2\,\mathrm{ta}}{}}{d}$$

input
```
int((a + b*tan(c + d*x))^2/cos(c + d*x)^8,x)
```

output
```
(tan(c + d*x)^3*(a^2 + b^2/3) + a^2*tan(c + d*x) + tan(c + d*x)^5*((3*a^2)
/5 + (3*b^2)/5) + tan(c + d*x)^7*(a^2/7 + (3*b^2)/7) + (b^2*tan(c + d*x)^9
)/9 + a*b*tan(c + d*x)^2 + (3*a*b*tan(c + d*x)^4)/2 + a*b*tan(c + d*x)^6 +
 (a*b*tan(c + d*x)^8)/4)/d
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.03

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{\sin(dx+c)\left(-315\cos(dx+c)\sin(dx+c)^7ab+1260\cos(dx+c)\sin(dx+c)^5ab-1890\cos(dx+c)\sin\right.}{}$$

input
```
int(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x)
```

output
```
(sin(c + d*x)*( - 315*cos(c + d*x)*sin(c + d*x)**7*a*b + 1260*cos(c + d*x)
*sin(c + d*x)**5*a*b - 1890*cos(c + d*x)*sin(c + d*x)**3*a*b + 1260*cos(c
+ d*x)*sin(c + d*x)*a*b + 576*sin(c + d*x)**8*a**2 - 64*sin(c + d*x)**8*b*
*2 - 2592*sin(c + d*x)**6*a**2 + 288*sin(c + d*x)**6*b**2 + 4536*sin(c + d
*x)**4*a**2 - 504*sin(c + d*x)**4*b**2 - 3780*sin(c + d*x)**2*a**2 + 420*s
in(c + d*x)**2*b**2 + 1260*a**2))/(1260*cos(c + d*x)*d*(sin(c + d*x)**8 -
4*sin(c + d*x)**6 + 6*sin(c + d*x)**4 - 4*sin(c + d*x)**2 + 1))
```

3.522 $\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{ab\sec^6(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{(2a^2+b^2)\tan^3(c+dx)}{3d}+\frac{(a^2+2b^2)\tan^5(c+dx)}{5d}+\frac{b^2\tan^7(c+dx)}{7d}$$

output
```
1/3*a*b*sec(d*x+c)^6/d+a^2*tan(d*x+c)/d+1/3*(2*a^2+b^2)*tan(d*x+c)^3/d+1/5
*(a^2+2*b^2)*tan(d*x+c)^5/d+1/7*b^2*tan(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{\tan(c+dx)\left(105a^2+105ab\tan(c+dx)+35(2a^2+b^2)\tan^2(c+dx)+105ab\tan^3(c+dx)+21(a^2+2b\right.}{105d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

output
```
(Tan[c + d*x]*(105*a^2 + 105*a*b*Tan[c + d*x] + 35*(2*a^2 + b^2)*Tan[c + d
*x]^2 + 105*a*b*Tan[c + d*x]^3 + 21*(a^2 + 2*b^2)*Tan[c + d*x]^4 + 35*a*b*
Tan[c + d*x]^5 + 15*b^2*Tan[c + d*x]^6))/(105*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 475, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)^2}{b^4}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)^2d(b\tan(c+dx))}{b^5d}$$

$$\Big\downarrow \text{475}$$

$$\frac{\int \left(b^6\tan^6(c+dx)+b^4\left(a^2+2b^2\right)\tan^4(c+dx)+b^4\left(2a^2+b^2\right)\tan^2(c+dx)+a^2b^4\right)d(b\tan(c+dx))+\frac{1}{3}a\left(b^2\tan\right.}{b^5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2b^5\tan(c+dx)+\frac{1}{5}b^5\left(a^2+2b^2\right)\tan^5(c+dx)+\frac{1}{3}b^5\left(2a^2+b^2\right)\tan^3(c+dx)+\frac{1}{3}a\left(b^2\tan^2(c+dx)+b^2\right)^3+\frac{1}{7}b^7}{b^5d}$$

input
```
Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

output
```
(a^2*b^5*Tan[c + d*x] + (b^5*(2*a^2 + b^2)*Tan[c + d*x]^3)/3 + (b^5*(a^2 +
 2*b^2)*Tan[c + d*x]^5)/5 + (b^7*Tan[c + d*x]^7)/7 + (a*(b^2 + b^2*Tan[c +
 d*x]^2)^3)/3)/(b^5*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 475
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp
[d*n*c^(n - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Int[ExpandIntegran
d[((c + d*x)^n - d*n*c^(n - 1)*x)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
d}, x] && IGtQ[p, 0] && IGtQ[n, 0] && LeQ[n, p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 30.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin(dx+c)^3}{7\cos(dx+c)^7}+\frac{4\sin(dx+c)^3}{35\cos(dx+c)^5}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^3}\right)+\frac{ab}{3\cos(dx+c)^6}-a^2\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$
default	$\frac{b^2\left(\frac{\sin(dx+c)^3}{7\cos(dx+c)^7}+\frac{4\sin(dx+c)^3}{35\cos(dx+c)^5}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^3}\right)+\frac{ab}{3\cos(dx+c)^6}-a^2\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$
risch	$\frac{16i\left(-140iab\,e^{8i(dx+c)}+70a^2e^{8i(dx+c)}-70b^2e^{8i(dx+c)}-140iab\,e^{6i(dx+c)}+175a^2e^{6i(dx+c)}+35b^2e^{6i(dx+c)}+147a^2e^{4i(d\right.}{105d\left(e^{2i(dx+c)}+1\right)^7}$

input
```
int(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/1
05*sin(d*x+c)^3/cos(d*x+c)^3)+1/3*a*b/cos(d*x+c)^6-a^2*(-8/15-1/5*sec(d*x+
c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{35\,ab\cos(dx+c)+\left(8\left(7\,a^2-b^2\right)\cos(dx+c)^6+4\left(7\,a^2-b^2\right)\cos(dx+c)^4+3\left(7\,a^2-b^2\right)\cos(dx+c)^2\right.}{105\,d\cos(dx+c)^7}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/105*(35*a*b*cos(d*x + c) + (8*(7*a^2 - b^2)*cos(d*x + c)^6 + 4*(7*a^2 -
b^2)*cos(d*x + c)^4 + 3*(7*a^2 - b^2)*cos(d*x + c)^2 + 15*b^2)*sin(d*x + c
))/(d*cos(d*x + c)^7)
```

Sympy [F]

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan{(c+dx)})^2\sec^6{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**6*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{15\,b^2\tan{(dx+c)}^7 + 35\,ab\tan{(dx+c)}^6 + 105\,ab\tan{(dx+c)}^4 + 21\,(a^2+2\,b^2)\tan{(dx+c)}^5 + 105\,ab\,\mathrm{ta}}{105\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/105*(15*b^2*tan(d*x + c)^7 + 35*a*b*tan(d*x + c)^6 + 105*a*b*tan(d*x + c
)^4 + 21*(a^2 + 2*b^2)*tan(d*x + c)^5 + 105*a*b*tan(d*x + c)^2 + 35*(2*a^2
 + b^2)*tan(d*x + c)^3 + 105*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.22

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{15\,b^2\tan{(dx+c)}^7 + 35\,ab\tan{(dx+c)}^6 + 21\,a^2\tan{(dx+c)}^5 + 42\,b^2\tan{(dx+c)}^5 + 105\,ab\tan{(dx+c)}}{105\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/105*(15*b^2*tan(d*x + c)^7 + 35*a*b*tan(d*x + c)^6 + 21*a^2*tan(d*x + c)
^5 + 42*b^2*tan(d*x + c)^5 + 105*a*b*tan(d*x + c)^4 + 70*a^2*tan(d*x + c)^
3 + 35*b^2*tan(d*x + c)^3 + 105*a*b*tan(d*x + c)^2 + 105*a^2*tan(d*x + c))
/d
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{a^2\tan(c+d\,x)+\tan(c+d\,x)^3\left(\frac{2\,a^2}{3}+\frac{b^2}{3}\right)+\tan(c+d\,x)^5\left(\frac{a^2}{5}+\frac{2\,b^2}{5}\right)+\frac{b^2\tan(c+d\,x)^7}{7}+a\,b\tan(c+d\,x)^2}{d}$$

input
```
int((a + b*tan(c + d*x))^2/cos(c + d*x)^6,x)
```

output
```
(a^2*tan(c + d*x) + tan(c + d*x)^3*((2*a^2)/3 + b^2/3) + tan(c + d*x)^5*(a
^2/5 + (2*b^2)/5) + (b^2*tan(c + d*x)^7)/7 + a*b*tan(c + d*x)^2 + a*b*tan(
c + d*x)^4 + (a*b*tan(c + d*x)^6)/3)/d
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.93

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{\sin(dx+c)\left(-35\cos(dx+c)\sin(dx+c)^5ab+105\cos(dx+c)\sin(dx+c)^3ab-105\cos(dx+c)\sin(d\right.}{105\cos(dx+c}$$

input
```
int(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x)
```

output
```
(sin(c + d*x)*( - 35*cos(c + d*x)*sin(c + d*x)**5*a*b + 105*cos(c + d*x)*s
in(c + d*x)**3*a*b - 105*cos(c + d*x)*sin(c + d*x)*a*b + 56*sin(c + d*x)**
6*a**2 - 8*sin(c + d*x)**6*b**2 - 196*sin(c + d*x)**4*a**2 + 28*sin(c + d*
x)**4*b**2 + 245*sin(c + d*x)**2*a**2 - 35*sin(c + d*x)**2*b**2 - 105*a**2
))/(105*cos(c + d*x)*d*(sin(c + d*x)**6 - 3*sin(c + d*x)**4 + 3*sin(c + d*
x)**2 - 1))
```

3.523 $\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(a^2+b^2)\,(a+b\tan(c+dx))^3}{3b^3d} - \frac{a(a+b\tan(c+dx))^4}{2b^3d} + \frac{(a+b\tan(c+dx))^5}{5b^3d}$$

output
```
1/3*(a^2+b^2)*(a+b*tan(d*x+c))^3/b^3/d-1/2*a*(a+b*tan(d*x+c))^4/b^3/d+1/5*
(a+b*tan(d*x+c))^5/b^3/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(a+b\tan(c+dx))^3\,(a^2+10b^2-3ab\tan(c+dx)+6b^2\tan^2(c+dx))}{30b^3d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

output
```
((a + b*Tan[c + d*x])^3*(a^2 + 10*b^2 - 3*a*b*Tan[c + d*x] + 6*b^2*Tan[c +
 d*x]^2))/(30*b^3*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4(a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{(a+b\tan(c+dx))^2(\tan^2(c+dx)b^2+b^2)}{b^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left((a+b\tan(c+dx))^4-2a(a+b\tan(c+dx))^3+\left(a^2+b^2\right)(a+b\tan(c+dx))^2\right)d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}\left(a^2+b^2\right)(a+b\tan(c+dx))^3+\frac{1}{5}(a+b\tan(c+dx))^5-\frac{1}{2}a(a+b\tan(c+dx))^4}{b^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

output
```
(((a^2 + b^2)*(a + b*Tan[c + d*x])^3)/3 - (a*(a + b*Tan[c + d*x])^4)/2 + (
a + b*Tan[c + d*x])^5/5)/(b^3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 8.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin(dx+c)^3}{5\cos(dx+c)^5}+\frac{2\sin(dx+c)^3}{15\cos(dx+c)^3}\right)+\frac{ab}{2\cos(dx+c)^4}-a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
default	$\frac{b^2\left(\frac{\sin(dx+c)^3}{5\cos(dx+c)^5}+\frac{2\sin(dx+c)^3}{15\cos(dx+c)^3}\right)+\frac{ab}{2\cos(dx+c)^4}-a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
risch	$\frac{4i\left(-30iab\,e^{6i(dx+c)}+15a^2e^{6i(dx+c)}-15b^2e^{6i(dx+c)}-30iab\,e^{4i(dx+c)}+35a^2e^{4i(dx+c)}+5b^2e^{4i(dx+c)}+25a^2e^{2i(dx+c)}-5\right.}{15d\left(e^{2i(dx+c)}+1\right)^5}$

input
```
int(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+1/
2*a*b/cos(d*x+c)^4-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{15\,ab\cos(dx+c)+2\left(2\left(5\,a^2-b^2\right)\cos(dx+c)^4+\left(5\,a^2-b^2\right)\cos(dx+c)^2+3\,b^2\right)\sin(dx+c)}{30\,d\cos(dx+c)^5}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/30*(15*a*b*cos(d*x + c) + 2*(2*(5*a^2 - b^2)*cos(d*x + c)^4 + (5*a^2 - b
^2)*cos(d*x + c)^2 + 3*b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\sec^4(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{6\,b^2\tan\left(dx+c\right)^5+15\,ab\tan\left(dx+c\right)^4+30\,ab\tan\left(dx+c\right)^2+10\left(a^2+b^2\right)\tan\left(dx+c\right)^3+30\,a^2\tan\left(dx\right.}{30\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/30*(6*b^2*tan(d*x + c)^5 + 15*a*b*tan(d*x + c)^4 + 30*a*b*tan(d*x + c)^2
 + 10*(a^2 + b^2)*tan(d*x + c)^3 + 30*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{6\,b^2\tan\left(dx+c\right)^5+15\,ab\tan\left(dx+c\right)^4+10\,a^2\tan\left(dx+c\right)^3+10\,b^2\tan\left(dx+c\right)^3+30\,ab\tan\left(dx+c\right)^2}{30\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/30*(6*b^2*tan(d*x + c)^5 + 15*a*b*tan(d*x + c)^4 + 10*a^2*tan(d*x + c)^3
 + 10*b^2*tan(d*x + c)^3 + 30*a*b*tan(d*x + c)^2 + 30*a^2*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{a^2\tan(c+d\,x)+\tan(c+d\,x)^3\left(\frac{a^2}{3}+\frac{b^2}{3}\right)+\frac{b^2\tan(c+d\,x)^5}{5}+a\,b\tan(c+d\,x)^2+\frac{a\,b\tan(c+d\,x)^4}{2}}{d}$$

input
```
int((a + b*tan(c + d*x))^2/cos(c + d*x)^4,x)
```

output
```
(a^2*tan(c + d*x) + tan(c + d*x)^3*(a^2/3 + b^2/3) + (b^2*tan(c + d*x)^5)/
5 + a*b*tan(c + d*x)^2 + (a*b*tan(c + d*x)^4)/2)/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.77

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{\sin(dx+c)\left(-15\cos(dx+c)\sin(dx+c)^3ab+30\cos(dx+c)\sin(dx+c)ab+20\sin(dx+c)^4a^2-4\,\mathrm{s}\right.}{30\cos(dx+c)\,d\left(\sin(dx+c)^4-2\sin(dx+c\right.}$$

input
```
int(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x)
```

output
```
(sin(c + d*x)*( - 15*cos(c + d*x)*sin(c + d*x)**3*a*b + 30*cos(c + d*x)*si
n(c + d*x)*a*b + 20*sin(c + d*x)**4*a**2 - 4*sin(c + d*x)**4*b**2 - 50*sin
(c + d*x)**2*a**2 + 10*sin(c + d*x)**2*b**2 + 30*a**2))/(30*cos(c + d*x)*d
*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.524 $\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(a+b\tan(c+dx))^3}{3bd}$$

output
```
1/3*(a+b*tan(d*x+c))^3/b/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(22) = 44.

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{a^2\tan(c+dx)}{d} + \frac{ab\tan^2(c+dx)}{d} + \frac{b^2\tan^3(c+dx)}{3d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

output
```
(a^2*Tan[c + d*x])/d + (a*b*Tan[c + d*x]^2)/d + (b^2*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3987, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sec(c+dx)^2(a+b\tan(c+dx))^2dx$$
$$\Big\downarrow \text{ 3987}$$
$$\frac{\int (a+b\tan(c+dx))^2d(b\tan(c+dx))}{bd}$$
$$\Big\downarrow \text{ 17}$$
$$\frac{(a+b\tan(c+dx))^3}{3bd}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

output
```
(a + b*Tan[c + d*x])^3/(3*b*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(20) = 40$.

Time = 2.54 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

method	result	size
derivativedivides	$\frac{\frac{b^2 \sin(dx+c)^3}{3\cos(dx+c)^3}+\frac{ab}{\cos(dx+c)^2}+\tan(dx+c)a^2}{d}$	48
default	$\frac{\frac{b^2 \sin(dx+c)^3}{3\cos(dx+c)^3}+\frac{ab}{\cos(dx+c)^2}+\tan(dx+c)a^2}{d}$	48
risch	$-\frac{2i\left(6iab\,e^{4i(dx+c)}-3a^2e^{4i(dx+c)}+3b^2e^{4i(dx+c)}+6iab\,e^{2i(dx+c)}-6a^2e^{2i(dx+c)}-3a^2+b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}$	99

input
```
int(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*b^2*sin(d*x+c)^3/cos(d*x+c)^3+a*b/cos(d*x+c)^2+tan(d*x+c)*a^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(20) = 40$.

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.50

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{3\,ab\cos\left(dx+c\right)+\left(\left(3\,a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(3*a*b*cos(d*x + c) + ((3*a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c
))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan{(c+dx)})^2\sec^2{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(b\tan{(dx+c)}+a)^3}{3\,bd}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/3*(b*tan(d*x + c) + a)^3/(b*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(20) = 40.

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b^2\tan{(dx+c)}^3+3\,ab\tan{(dx+c)}^2+3\,a^2\tan{(dx+c)}}{3\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/3*(b^2*tan(d*x + c)^3 + 3*a*b*tan(d*x + c)^2 + 3*a^2*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{a^2\tan(c+d\,x)+a\,b\tan(c+d\,x)^2+\frac{b^2\tan(c+d\,x)^3}{3}}{d}$$

input
```
int((a + b*tan(c + d*x))^2/cos(c + d*x)^2,x)
```

output
```
(a^2*tan(c + d*x) + (b^2*tan(c + d*x)^3)/3 + a*b*tan(c + d*x)^2)/d
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.59

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{\sin(dx+c)\left(-3\cos(dx+c)\sin(dx+c)\,ab+3\sin(dx+c)^2a^2-\sin(dx+c)^2b^2-3a^2\right)}{3\cos(dx+c)\,d\left(\sin(dx+c)^2-1\right)}$$

input
```
int(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x)
```

output
```
(sin(c + d*x)*( - 3*cos(c + d*x)*sin(c + d*x)*a*b + 3*sin(c + d*x)**2*a**2
 - sin(c + d*x)**2*b**2 - 3*a**2))/(3*cos(c + d*x)*d*(sin(c + d*x)**2 - 1)
)
```

3.525 $\int (a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 39

$$\int (a+b\tan(c+dx))^2\,dx=\left(a^2-b^2\right)x-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

output
```
(a^2-b^2)*x-2*a*b*ln(cos(d*x+c))/d+b^2*tan(d*x+c)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.77

$$\int (a+b\tan(c+dx))^2\,dx =\frac{-i((a+ib)^2\log(i-\tan(c+dx))-(a-ib)^2\log(i+\tan(c+dx)))+2b^2\tan(c+dx)}{2d}$$

input
```
Integrate[(a + b*Tan[c + d*x])^2,x]
```

output
```
((-I)*((a + I*b)^2*Log[I - Tan[c + d*x]] - (a - I*b)^2*Log[I + Tan[c + d*x
]]) + 2*b^2*Tan[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3958}$$

$$2ab\int \tan(c+dx)dx+x\left(a^2-b^2\right)+\frac{b^2\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$2ab\int \tan(c+dx)dx+x\left(a^2-b^2\right)+\frac{b^2\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3956}$$

$$x\left(a^2-b^2\right)-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

input
```
Int[(a + b*Tan[c + d*x])^2,x]
```

output
```
(a^2 - b^2)*x - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

method	result	size
norman	$(a^2-b^2)x+\frac{b^2\tan(dx+c)}{d}+\frac{ab\ln\left(1+\tan(dx+c)^2\right)}{d}$	43
parallelrisch	$\frac{a^2dx-b^2dx+ab\ln\left(1+\tan(dx+c)^2\right)+\tan(dx+c)b^2}{d}$	43
derivativedivides	$\frac{\tan(dx+c)b^2+ab\ln\left(1+\tan(dx+c)^2\right)+(a^2-b^2)\arctan(\tan(dx+c))}{d}$	47
default	$\frac{\tan(dx+c)b^2+ab\ln\left(1+\tan(dx+c)^2\right)+(a^2-b^2)\arctan(\tan(dx+c))}{d}$	47
parts	$a^2x+\frac{b^2(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{ab\ln\left(1+\tan(dx+c)^2\right)}{d}$	47
risch	$2iabx+a^2x-b^2x+\frac{4iabc}{d}+\frac{2ib^2}{d(e^{2i(dx+c)}+1)}-\frac{2ab\ln(e^{2i(dx+c)}+1)}{d}$	69

input
```
int((a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
(a^2-b^2)*x+b^2*tan(d*x+c)/d+a*b/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int (a+b\tan(c+dx))^2\,dx=\frac{(a^2-b^2)dx-ab\log\left(\frac{1}{\tan(dx+c)^2+1}\right)+b^2\tan(dx+c)}{d}$$

input
```
integrate((a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
((a^2 - b^2)*d*x - a*b*log(1/(tan(d*x + c)^2 + 1)) + b^2*tan(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int (a+b\tan(c+dx))^2\,dx=\begin{cases} a^2x+\frac{ab\log(\tan^2(c+dx)+1)}{d}-b^2x+\frac{b^2\tan(c+dx)}{d} & \text{for } d\neq 0\\ x(a+b\tan(c))^2 & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise((a**2*x + a*b*log(tan(c + d*x)**2 + 1)/d - b**2*x + b**2*tan(c +
 d*x)/d, Ne(d, 0)), (x*(a + b*tan(c))**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int (a+b\tan(c+dx))^2\,dx=a^2x-\frac{(dx+c-\tan(dx+c))b^2}{d}+\frac{2ab\log(\sec(dx+c))}{d}$$

input
```
integrate((a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
a^2*x - (d*x + c - tan(d*x + c))*b^2/d + 2*a*b*log(sec(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int (a+b\tan(c+dx))^2\,dx = \frac{ab\log\left(\tan\left(dx+c\right)^2+1\right)}{d} + \frac{b^2\tan\left(dx+c\right)}{d} + \frac{(a^2-b^2)(dx+c)}{d}$$

input
```
integrate((a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
a*b*log(tan(d*x + c)^2 + 1)/d + b^2*tan(d*x + c)/d + (a^2 - b^2)*(d*x + c)
/d
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.49

$$\int (a+b\tan(c+dx))^2\,dx = \frac{a^2\,\mathrm{atan}\left(\frac{a^2\tan(c+d\,x)}{a^2-b^2}-\frac{b^2\tan(c+d\,x)}{a^2-b^2}\right)}{d} - \frac{b^2\,\mathrm{atan}\left(\frac{a^2\tan(c+d\,x)}{a^2-b^2}-\frac{b^2\tan(c+d\,x)}{a^2-b^2}\right)}{d} + \frac{b^2\tan(c+d\,x)}{d} + \frac{a\,b\,\ln\left(\tan(c+d\,x)^2+1\right)}{d}$$

input
```
int((a + b*tan(c + d*x))^2,x)
```

output
```
(a^2*atan((a^2*tan(c + d*x))/(a^2 - b^2) - (b^2*tan(c + d*x))/(a^2 - b^2))
)/d - (b^2*atan((a^2*tan(c + d*x))/(a^2 - b^2) - (b^2*tan(c + d*x))/(a^2 -
 b^2)))/d + (b^2*tan(c + d*x))/d + (a*b*log(tan(c + d*x)^2 + 1))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int (a+b\tan(c+dx))^2\,dx = \frac{\log\left(\tan\left(dx+c\right)^2+1\right)ab+\tan\left(dx+c\right)b^2+a^2dx-b^2dx}{d}$$

input
```
int((a+b*tan(d*x+c))^2,x)
```

output
```
(log(tan(c + d*x)**2 + 1)*a*b + tan(c + d*x)*b**2 + a**2*d*x - b**2*d*x)/d
```

3.526 $\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{1}{2}\left(a^2+b^2\right)x - \frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{2d}$$

output
```
1/2*(a^2+b^2)*x-1/2*cos(d*x+c)^2*(b-a*tan(d*x+c))*(a+b*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2\left(a^2+b^2\right)(c+dx)-2ab\cos(2(c+dx))+\left(a^2-b^2\right)\sin(2(c+dx))}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

output
```
(2*(a^2 + b^2)*(c + d*x) - 2*a*b*Cos[2*(c + d*x)] + (a^2 - b^2)*Sin[2*(c +
 d*x)])/(4*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3987, 27, 487, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{b^4(a+b\tan(c+dx))^2}{(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^3\int \frac{(a+b\tan(c+dx))^2}{(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{487}$$

$$\frac{b^3\left(\frac{(a^2+b^2)\int \frac{1}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2}-\frac{(a+b\tan(c+dx))(b^2-ab\tan(c+dx))}{2b^2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{b^3\left(\frac{(a^2+b^2)\arctan(\tan(c+dx))}{2b^3}-\frac{(a+b\tan(c+dx))(b^2-ab\tan(c+dx))}{2b^2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

output
```
(b^3*(((a^2 + b^2)*ArcTan[Tan[c + d*x]])/(2*b^3) - ((a + b*Tan[c + d*x])*(
b^2 - a*b*Tan[c + d*x]))/(2*b^2*(b^2 + b^2*Tan[c + d*x]^2))))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 487
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n - 1)*(a*d - b*c*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] +
 Simp[(2*p + 3)*((b*c^2 + a*d^2)/(2*a*b*(p + 1)))   Int[(c + d*x)^(n - 2)*(
a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0]
 && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

method	result	size
risch	$\frac{a^2 x}{2}+\frac{b^2 x}{2}-\frac{ab\cos(2dx+2c)}{2d}+\frac{\sin(2dx+2c)a^2}{4d}-\frac{\sin(2dx+2c)b^2}{4d}$	64
derivativedivides	$\frac{b^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ab\cos(dx+c)^2+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	70
default	$\frac{b^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ab\cos(dx+c)^2+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	70

input
```
int(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x+1/2*b^2*x-1/2*a*b/d*cos(2*d*x+2*c)+1/4/d*sin(2*d*x+2*c)*a^2-1/4/
d*sin(2*d*x+2*c)*b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= -\frac{2\,ab\cos{(dx+c)}^2-(a^2+b^2)dx-(a^2-b^2)\cos{(dx+c)}\sin{(dx+c)}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*cos(d*x + c)^2 - (a^2 + b^2)*d*x - (a^2 - b^2)*cos(d*x + c)*si
n(d*x + c))/d
```

Sympy [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan{(c+dx)})^2\cos^2{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(a^2+b^2)(dx+c)-\frac{2\,ab-(a^2-b^2)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*((a^2 + b^2)*(d*x + c) - (2*a*b - (a^2 - b^2)*tan(d*x + c))/(tan(d*x +
 c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(a^2+b^2)(dx+c)}{2\,d} + \frac{a^2\tan(dx+c)-b^2\tan(dx+c)-2\,ab}{2\left(\tan(dx+c)^2+1\right)d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(a^2 + b^2)*(d*x + c)/d + 1/2*(a^2*tan(d*x + c) - b^2*tan(d*x + c) - 2
*a*b)/((tan(d*x + c)^2 + 1)*d)
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = x\left(\frac{a^2}{2}+\frac{b^2}{2}\right) - \frac{\cos(c+d\,x)^2\left(a\,b-\tan(c+d\,x)\,\left(\frac{a^2}{2}-\frac{b^2}{2}\right)\right)}{d}$$

input
```
int(cos(c + d*x)^2*(a + b*tan(c + d*x))^2,x)
```

output
```
x*(a^2/2 + b^2/2) - (cos(c + d*x)^2*(a*b - tan(c + d*x)*(a^2/2 - b^2/2)))/
d
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{\cos(dx+c)\sin(dx+c)\,a^2-\cos(dx+c)\sin(dx+c)\,b^2+2\sin(dx+c)^2\,ab+a^2dx+b^2dx}{2d}$$

input
```
int(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x)
```

output
```
(cos(c + d*x)*sin(c + d*x)*a**2 - cos(c + d*x)*sin(c + d*x)*b**2 + 2*sin(c
 + d*x)**2*a*b + a**2*d*x + b**2*d*x)/(2*d)
```

3.527 $\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{1}{8}\left(3a^2+b^2\right)x - \frac{ab\cos^2(c+dx)}{2d} + \frac{\left(3a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{8d} + \frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{4d}$$

output
```
1/8*(3*a^2+b^2)*x-1/2*a*b*cos(d*x+c)^2/d+1/8*(3*a^2-b^2)*cos(d*x+c)*sin(d*
x+c)/d+1/4*cos(d*x+c)^3*sin(d*x+c)*(a+b*tan(d*x+c))^2/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 197 vs. 2(95) = 190.

Time = 2.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.07

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$= \frac{\left(3a^2+b^2\right)\left(2ab(2a^2+b^2)-2ab(a^2+b^2)\cos(2(c+dx))+\frac{b(a^2+b^2)^2\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)}{\sqrt{-b^2}}-\frac{b(a^2+b^2)^2\log\left(\sqrt{-b\right.}}{\sqrt{-b}}\right.}{16\left(a^2\right.}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

output
```
((3*a^2 + b^2)*(2*a*b*(2*a^2 + b^2) - 2*a*b*(a^2 + b^2)*Cos[2*(c + d*x)] +
 (b*(a^2 + b^2)^2*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/Sqrt[-b^2] - (b*(a^2 +
 b^2)^2*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/Sqrt[-b^2] + (a^4 - b^4)*Sin[2*(
c + d*x)]) + 4*(a^2 + b^2)*Cos[c + d*x]^4*(b + a*Tan[c + d*x])*(a + b*Tan[
c + d*x])^3)/(16*(a^2 + b^2)^2*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.46, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3987, 27, 495, 454, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{b^6(a+b\tan(c+dx))^2}{(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^5\int \frac{(a+b\tan(c+dx))^2}{(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{495}$$

$$\frac{b^5\left(\frac{\int \frac{3a^2+2b\tan(c+dx)a+b^2}{\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2}-\frac{(a+b\tan(c+dx))\left(b^2-ab\tan(c+dx)\right)}{4b^2\left(b^2\tan^2(c+dx)+b^2\right)^2}\right)}{d}$$

$$\Big\downarrow \text{454}$$

$$\frac{b^5\left(\frac{\frac{1}{2}\left(\frac{3a^2}{b^2}+1\right)\int\frac{1}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))-\frac{2ab^2-b\left(3a^2+b^2\right)\tan(c+dx)}{2b^2\left(b^2\tan^2(c+dx)+b^2\right)}}{4b^2}-\frac{(a+b\tan(c+dx))(b^2-ab\tan(c+dx))}{4b^2(b^2\tan^2(c+dx)+b^2)^2}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{b^5\left(\frac{\frac{\left(\frac{3a^2}{b^2}+1\right)\arctan(\tan(c+dx))}{2b}-\frac{2ab^2-b\left(3a^2+b^2\right)\tan(c+dx)}{2b^2\left(b^2\tan^2(c+dx)+b^2\right)}}{4b^2}-\frac{(a+b\tan(c+dx))(b^2-ab\tan(c+dx))}{4b^2(b^2\tan^2(c+dx)+b^2)^2}\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

output
```
(b^5*(-1/4*((a + b*Tan[c + d*x])*(b^2 - a*b*Tan[c + d*x]))/(b^2*(b^2 + b^2
*Tan[c + d*x]^2)^2) + (((1 + (3*a^2)/b^2)*ArcTan[Tan[c + d*x]])/(2*b) - (2
*a*b^2 - b*(3*a^2 + b^2)*Tan[c + d*x])/(2*b^2*(b^2 + b^2*Tan[c + d*x]^2)))
/(4*b^2)))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 9.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{ab\cos(dx+c)^4}{2}+a^2\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\right.}{d}$
default	$\frac{b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{ab\cos(dx+c)^4}{2}+a^2\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\right.}{d}$
risch	$\frac{3a^2x}{8}+\frac{b^2x}{8}-\frac{ab\cos(4dx+4c)}{16d}+\frac{\sin(4dx+4c)a^2}{32d}-\frac{\sin(4dx+4c)b^2}{32d}-\frac{ab\cos(2dx+2c)}{4d}+\frac{\sin(2dx+2c)a^2}{4d}$

input
```
int(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1
/8*c)-1/2*a*b*cos(d*x+c)^4+a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+
c)+3/8*d*x+3/8*c))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.79

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{4\,ab\cos(dx+c)^4-(3\,a^2+b^2)dx-\left(2\,(a^2-b^2)\cos(dx+c)^3+(3\,a^2+b^2)\cos(dx+c)\right)\sin(dx+c)}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/8*(4*a*b*cos(d*x + c)^4 - (3*a^2 + b^2)*d*x - (2*(a^2 - b^2)*cos(d*x +
c)^3 + (3*a^2 + b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\cos^4(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(3\,a^2+b^2)(dx+c)+\frac{(3\,a^2+b^2)\tan(dx+c)^3-4\,ab+(5\,a^2-b^2)\tan(dx+c)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/8*((3*a^2 + b^2)*(d*x + c) + ((3*a^2 + b^2)*tan(d*x + c)^3 - 4*a*b + (5*
a^2 - b^2)*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(3a^2+b^2)(dx+c)}{8d} + \frac{3a^2\tan(dx+c)^3 + b^2\tan(dx+c)^3 + 5a^2\tan(dx+c) - b^2\tan(dx+c) - 4ab}{8\left(\tan(dx+c)^2+1\right)^2 d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/8*(3*a^2 + b^2)*(d*x + c)/d + 1/8*(3*a^2*tan(d*x + c)^3 + b^2*tan(d*x +
c)^3 + 5*a^2*tan(d*x + c) - b^2*tan(d*x + c) - 4*a*b)/((tan(d*x + c)^2 + 1
)^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx = x\left(\frac{3a^2}{8}+\frac{b^2}{8}\right) + \frac{\left(\frac{3a^2}{8}+\frac{b^2}{8}\right)\tan(c+dx)^3 + \left(\frac{5a^2}{8}-\frac{b^2}{8}\right)\tan(c+dx) - \frac{ab}{2}}{d\left(\tan(c+dx)^4 + 2\tan(c+dx)^2 + 1\right)}$$

input
```
int(cos(c + d*x)^4*(a + b*tan(c + d*x))^2,x)
```

output
```
x*((3*a^2)/8 + b^2/8) + (tan(c + d*x)*((5*a^2)/8 - b^2/8) - (a*b)/2 + tan(
c + d*x)^3*((3*a^2)/8 + b^2/8))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1)
)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.21

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{-2\cos(dx+c)\sin(dx+c)^3a^2+2\cos(dx+c)\sin(dx+c)^3b^2+5\cos(dx+c)\sin(dx+c)a^2-\cos(dx}{8d}$$

input
```
int(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x)
```

output
```
( - 2*cos(c + d*x)*sin(c + d*x)**3*a**2 + 2*cos(c + d*x)*sin(c + d*x)**3*b
**2 + 5*cos(c + d*x)*sin(c + d*x)*a**2 - cos(c + d*x)*sin(c + d*x)*b**2 -
4*sin(c + d*x)**4*a*b + 8*sin(c + d*x)**2*a*b + 3*a**2*d*x + b**2*d*x)/(8*
d)
```

3.528 $\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 129

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(6a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{16d} + \frac{2ab\sec^5(c+dx)}{5d} + \frac{(6a^2-b^2)\sec(c+dx)\tan(c+dx)}{16d} + \frac{(6a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{b^2\sec^5(c+dx)\tan(c+dx)}{6d}$$

output
```
1/16*(6*a^2-b^2)*arctanh(sin(d*x+c))/d+2/5*a*b*sec(d*x+c)^5/d+1/16*(6*a^2-
b^2)*sec(d*x+c)*tan(d*x+c)/d+1/24*(6*a^2-b^2)*sec(d*x+c)^3*tan(d*x+c)/d+1/
6*b^2*sec(d*x+c)^5*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.30

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{3a^2\text{arctanh}(\sin(c+dx))}{8d} - \frac{b^2\text{arctanh}(\sin(c+dx))}{16d} + \frac{2ab\sec^5(c+dx)}{5d} + \frac{3a^2\sec(c+dx)\tan(c+dx)}{8d} - \frac{b^2\sec(c+dx)\tan(c+dx)}{16d} + \frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d} - \frac{b^2\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{b^2\sec^5(c+dx)\tan(c+dx)}{6d}$$

input
```
Integrate[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

output
```
(3*a^2*ArcTanh[Sin[c + d*x]])/(8*d) - (b^2*ArcTanh[Sin[c + d*x]])/(16*d) +
 (2*a*b*Sec[c + d*x]^5)/(5*d) + (3*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) -
(b^2*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])
/(4*d) - (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) + (b^2*Sec[c + d*x]^5*Ta
n[c + d*x])/(6*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3991, 27, 3042, 3086, 15, 4159, 298, 215, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^5(a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3991}$$

$$\int \sec^5(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+\int 2ab\sec^5(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$\int \sec^5(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+2ab\int \sec^5(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^5\left(a^2+b^2\tan(c+dx)^2\right)dx+2ab\int \sec(c+dx)^5\tan(c+dx)dx$$

$$\Big\downarrow \text{3086}$$

$$\int \sec(c+dx)^5\left(a^2+b^2\tan(c+dx)^2\right)dx+\frac{2ab\int \sec^4(c+dx)d\sec(c+dx)}{d}$$

$$\Big\downarrow \text{15}$$

$$\int \sec(c+dx)^5\left(a^2+b^2\tan(c+dx)^2\right)dx+\frac{2ab\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{a^2-(a^2-b^2)\sin^2(c+dx)}{(1-\sin^2(c+dx))^4}d\sin(c+dx)}{d}+\frac{2ab\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{6}(6a^2-b^2)\int \frac{1}{(1-\sin^2(c+dx))^3}d\sin(c+dx)+\frac{b^2\sin(c+dx)}{6(1-\sin^2(c+dx))^3}}{d}+\frac{2ab\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{6}(6a^2-b^2)\left(\frac{3}{4}\int \frac{1}{(1-\sin^2(c+dx))^2}d\sin(c+dx)+\frac{\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{b^2\sin(c+dx)}{6(1-\sin^2(c+dx))^3}}{d}+\frac{2ab\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{6}(6a^2-b^2)\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{b^2\sin(c+dx)}{6(1-\sin^2(c+dx))^3}}{d}+\frac{2ab\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{6}(6a^2-b^2)\left(\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{b^2\sin(c+dx)}{6(1-\sin^2(c+dx))^3}}{d}+\frac{2ab\sec^5(c+dx)}{5d}$$

input
```
Int[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

output
```
(2*a*b*Sec[c + d*x]^5)/(5*d) + ((b^2*Sin[c + d*x])/(6*(1 - Sin[c + d*x]^2)
^3) + ((6*a^2 - b^2)*(Sin[c + d*x]/(4*(1 - Sin[c + d*x]^2)^2) + (3*(ArcTan
h[Sin[c + d*x]]/2 + Sin[c + d*x]/(2*(1 - Sin[c + d*x]^2))))/4))/6)/d
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 15.41 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin(dx+c)^3}{6\cos(dx+c)^6}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{16\cos(dx+c)^2}+\frac{\sin(dx+c)}{16}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{2ab}{5\cos(dx+c)^5}+a^2\left(-\left(-\frac{\sec(dx+}{4}\right.\right.}{d}$
default	$\frac{b^2\left(\frac{\sin(dx+c)^3}{6\cos(dx+c)^6}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{16\cos(dx+c)^2}+\frac{\sin(dx+c)}{16}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{2ab}{5\cos(dx+c)^5}+a^2\left(-\left(-\frac{\sec(dx+}{4}\right.\right.}{d}$
risch	$-\frac{i e^{i(dx+c)}\left(90a^2 e^{10i(dx+c)}-15b^2 e^{10i(dx+c)}+510a^2 e^{8i(dx+c)}-85b^2 e^{8i(dx+c)}+420a^2 e^{6i(dx+c)}+570b^2 e^{6i(dx+c)}+1536\right.}{120d\left(e^{2i(dx\right.}}$

input
```
int(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16
*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1/16*ln(sec(d*x+c)+tan(d*x+c)))
+2/5*a*b/cos(d*x+c)^5+a^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+
3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.10

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{15\left(6a^2-b^2\right)\cos(dx+c)^6\log(\sin(dx+c)+1)-15\left(6a^2-b^2\right)\cos(dx+c)^6\log(-\sin(dx+c)+1)+}{480\,d\cos(dx}$$

input
```
integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/480*(15*(6*a^2 - b^2)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 15*(6*a^2 -
 b^2)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 192*a*b*cos(d*x + c) + 10*(3
*(6*a^2 - b^2)*cos(d*x + c)^4 + 2*(6*a^2 - b^2)*cos(d*x + c)^2 + 8*b^2)*si
n(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F]

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\sec^5(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**5*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.40

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{5\,b^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)-1)\right)-30\,a^2\left(\cdots\right.}{480\,d}$$

input
```
integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/480*(5*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(si
n(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 30*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d
*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1)
 + 3*log(sin(d*x + c) - 1)) + 192*a*b/cos(d*x + c)^5)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(119) = 238$.

Time = 0.29 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.66

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{15\,(6\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(6\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(150\,a^2\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/240*(15*(6*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(6*a^2 - b
^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(150*a^2*tan(1/2*d*x + 1/2*c)^1
1 + 15*b^2*tan(1/2*d*x + 1/2*c)^11 - 480*a*b*tan(1/2*d*x + 1/2*c)^10 - 210
*a^2*tan(1/2*d*x + 1/2*c)^9 + 235*b^2*tan(1/2*d*x + 1/2*c)^9 + 480*a*b*tan
(1/2*d*x + 1/2*c)^8 + 60*a^2*tan(1/2*d*x + 1/2*c)^7 + 390*b^2*tan(1/2*d*x
+ 1/2*c)^7 - 960*a*b*tan(1/2*d*x + 1/2*c)^6 + 60*a^2*tan(1/2*d*x + 1/2*c)^
5 + 390*b^2*tan(1/2*d*x + 1/2*c)^5 + 960*a*b*tan(1/2*d*x + 1/2*c)^4 - 210*
a^2*tan(1/2*d*x + 1/2*c)^3 + 235*b^2*tan(1/2*d*x + 1/2*c)^3 - 96*a*b*tan(1
/2*d*x + 1/2*c)^2 + 150*a^2*tan(1/2*d*x + 1/2*c) + 15*b^2*tan(1/2*d*x + 1/
2*c) + 96*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 3.56 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.54

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{\left(\frac{5\,a^2}{4}+\frac{b^2}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\left(\frac{47\,b^2}{24}-\frac{7\,a^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\left(\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\right.}$$
$$+\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a^2}{4}-\frac{b^2}{8}\right)}{d}$$

input
```
int((a + b*tan(c + d*x))^2/cos(c + d*x)^5,x)
```

output
```
((4*a*b)/5 + tan(c/2 + (d*x)/2)^5*(a^2/2 + (13*b^2)/4) + tan(c/2 + (d*x)/2
)^7*(a^2/2 + (13*b^2)/4) + tan(c/2 + (d*x)/2)^11*((5*a^2)/4 + b^2/8) - tan
(c/2 + (d*x)/2)^3*((7*a^2)/4 - (47*b^2)/24) - tan(c/2 + (d*x)/2)^9*((7*a^2
)/4 - (47*b^2)/24) + tan(c/2 + (d*x)/2)*((5*a^2)/4 + b^2/8) - (4*a*b*tan(c
/2 + (d*x)/2)^2)/5 + 8*a*b*tan(c/2 + (d*x)/2)^4 - 8*a*b*tan(c/2 + (d*x)/2)
^6 + 4*a*b*tan(c/2 + (d*x)/2)^8 - 4*a*b*tan(c/2 + (d*x)/2)^10)/(d*(15*tan(
c/2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*t
an(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)
) + (atanh(tan(c/2 + (d*x)/2))*((3*a^2)/4 - b^2/8))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 530, normalized size of antiderivative = 4.11

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x)
```

output
```
( - 96*cos(c + d*x)*a*b - 90*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**6*a**
2 + 15*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**6*b**2 + 270*log(tan((c + d
*x)/2) - 1)*sin(c + d*x)**4*a**2 - 45*log(tan((c + d*x)/2) - 1)*sin(c + d*
x)**4*b**2 - 270*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**2 + 45*log(t
an((c + d*x)/2) - 1)*sin(c + d*x)**2*b**2 + 90*log(tan((c + d*x)/2) - 1)*a
**2 - 15*log(tan((c + d*x)/2) - 1)*b**2 + 90*log(tan((c + d*x)/2) + 1)*sin
(c + d*x)**6*a**2 - 15*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**6*b**2 - 27
0*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a**2 + 45*log(tan((c + d*x)/2)
 + 1)*sin(c + d*x)**4*b**2 + 270*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2
*a**2 - 45*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b**2 - 90*log(tan((c
+ d*x)/2) + 1)*a**2 + 15*log(tan((c + d*x)/2) + 1)*b**2 - 96*sin(c + d*x)*
*6*a*b - 90*sin(c + d*x)**5*a**2 + 15*sin(c + d*x)**5*b**2 + 288*sin(c + d
*x)**4*a*b + 240*sin(c + d*x)**3*a**2 - 40*sin(c + d*x)**3*b**2 - 288*sin(
c + d*x)**2*a*b - 150*sin(c + d*x)*a**2 - 15*sin(c + d*x)*b**2 + 96*a*b)/(
240*d*(sin(c + d*x)**6 - 3*sin(c + d*x)**4 + 3*sin(c + d*x)**2 - 1))
```

3.529 $\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(4a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{2ab\sec^3(c+dx)}{3d} + \frac{(4a^2-b^2)\sec(c+dx)\tan(c+dx)}{8d} + \frac{b^2\sec^3(c+dx)\tan(c+dx)}{4d}$$

output
```
1/8*(4*a^2-b^2)*arctanh(sin(d*x+c))/d+2/3*a*b*sec(d*x+c)^3/d+1/8*(4*a^2-b^
2)*sec(d*x+c)*tan(d*x+c)/d+1/4*b^2*sec(d*x+c)^3*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.24

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{a^2\text{arctanh}(\sin(c+dx))}{2d} - \frac{b^2\text{arctanh}(\sin(c+dx))}{8d} + \frac{2ab\sec^3(c+dx)}{3d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{2d} - \frac{b^2\sec(c+dx)\tan(c+dx)}{8d} + \frac{b^2\sec^3(c+dx)\tan(c+dx)}{4d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

output
```
(a^2*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2
*a*b*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (b^2*
Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3991, 27, 3042, 3086, 15, 4159, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3(a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3991}$$

$$\int \sec^3(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+\int 2ab\sec^3(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \sec^3(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+2ab\int \sec^3(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(c+dx)^3\left(a^2+b^2\tan(c+dx)^2\right)dx+2ab\int \sec(c+dx)^3\tan(c+dx)dx$$

$$\Big\downarrow \text{ 3086}$$

$$\int \sec(c+dx)^3\left(a^2+b^2\tan(c+dx)^2\right)dx+\frac{2ab\int \sec^2(c+dx)d\sec(c+dx)}{d}$$

$$\Big\downarrow \text{ 15}$$

$$\int \sec(c+dx)^3\left(a^2+b^2\tan(c+dx)^2\right)dx+\frac{2ab\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{ 4159}$$

$$\frac{\int \frac{a^2-(a^2-b^2)\sin^2(c+dx)}{(1-\sin^2(c+dx))^3}d\sin(c+dx)}{d}+\frac{2ab\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{\frac{1}{4}(4a^2-b^2)\int \frac{1}{(1-\sin^2(c+dx))^2}d\sin(c+dx)+\frac{b^2\sin(c+dx)}{4(1-\sin^2(c+dx))^2}}{d}+\frac{2ab\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\frac{1}{4}(4a^2-b^2)\left(\frac{1}{2}\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{b^2\sin(c+dx)}{4(1-\sin^2(c+dx))^2}}{d}+\frac{2ab\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{4}(4a^2-b^2)\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{b^2\sin(c+dx)}{4(1-\sin^2(c+dx))^2}}{d}+\frac{2ab\sec^3(c+dx)}{3d}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

output
```
(2*a*b*Sec[c + d*x]^3)/(3*d) + ((b^2*Sin[c + d*x])/(4*(1 - Sin[c + d*x]^2)
^2) + ((4*a^2 - b^2)*(ArcTanh[Sin[c + d*x]]/2 + Sin[c + d*x]/(2*(1 - Sin[c
 + d*x]^2))))/4)/d
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{2ab}{3\cos(dx+c)^3}+a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(d}{}\right.}{d}$
default	$\frac{b^2\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{2ab}{3\cos(dx+c)^3}+a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(d}{}\right.}{d}$
risch	$-\frac{i e^{i(dx+c)}\left(12a^2 e^{6i(dx+c)}-3b^2 e^{6i(dx+c)}+12a^2 e^{4i(dx+c)}+21b^2 e^{4i(dx+c)}+64iab\, e^{4i(dx+c)}-12a^2 e^{2i(dx+c)}-21b^2 e^{2i(dx}\right.}{12d\left(e^{2i(dx+c)}+1\right)^4}$

input
```
int(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*
sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+2/3*a*b/cos(d*x+c)^3+a^2*(1/2*se
c(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.24

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{3\,(4\,a^2-b^2)\cos{(dx+c)}^4\log{(\sin{(dx+c)}+1)} - 3\,(4\,a^2-b^2)\cos{(dx+c)}^4\log{(-\sin{(dx+c)}+1)} + 32}{48\,d\cos{(dx+c)}^4}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/48*(3*(4*a^2 - b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2 - b^
2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 32*a*b*cos(d*x + c) + 6*((4*a^2
 - b^2)*cos(d*x + c)^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan{(c+dx)})^2\sec^3{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.33

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{3\,b^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - \log{(\sin{(dx+c)}+1)} + \log{(\sin{(dx+c)}-1)}\right) - 12\,a^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\right.}{48\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/48*(3*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
 + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*a^2*(2*
sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 32*a*b/cos(d*x + c)^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(89) = 178$.

Time = 0.25 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.57

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{3\,(4\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(4\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}{}}{}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(3*(4*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2 - b^2)
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 + 3
*b^2*tan(1/2*d*x + 1/2*c)^7 - 48*a*b*tan(1/2*d*x + 1/2*c)^6 - 12*a^2*tan(1
/2*d*x + 1/2*c)^5 + 21*b^2*tan(1/2*d*x + 1/2*c)^5 + 48*a*b*tan(1/2*d*x + 1
/2*c)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 + 21*b^2*tan(1/2*d*x + 1/2*c)^3 -
16*a*b*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/
2*d*x + 1/2*c) + 16*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.23

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{\left(a^2+\frac{b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-4\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\left(\frac{7\,b^2}{4}-a^2\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+4\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\left(\frac{7\,b^2}{4}-\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\right.}$$
$$+\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^2-\frac{b^2}{4}\right)}{d}$$

input
```
int((a + b*tan(c + d*x))^2/cos(c + d*x)^3,x)
```

output
```
((4*a*b)/3 + tan(c/2 + (d*x)/2)*(a^2 + b^2/4) + tan(c/2 + (d*x)/2)^7*(a^2
+ b^2/4) - tan(c/2 + (d*x)/2)^3*(a^2 - (7*b^2)/4) - tan(c/2 + (d*x)/2)^5*(
a^2 - (7*b^2)/4) - (4*a*b*tan(c/2 + (d*x)/2)^2)/3 + 4*a*b*tan(c/2 + (d*x)/
2)^4 - 4*a*b*tan(c/2 + (d*x)/2)^6)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2
+ (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (atan
h(tan(c/2 + (d*x)/2))*(a^2 - b^2/4))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 382, normalized size of antiderivative = 3.94

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{16\cos(dx+c)\,ab-12\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4\,a^2+3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4\,b^2-}{}$$

input
```
int(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x)
```

output
```
(16*cos(c + d*x)*a*b - 12*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**2 +
 3*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*b**2 + 24*log(tan((c + d*x)/2
) - 1)*sin(c + d*x)**2*a**2 - 6*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*
b**2 - 12*log(tan((c + d*x)/2) - 1)*a**2 + 3*log(tan((c + d*x)/2) - 1)*b**
2 + 12*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a**2 - 3*log(tan((c + d*x
)/2) + 1)*sin(c + d*x)**4*b**2 - 24*log(tan((c + d*x)/2) + 1)*sin(c + d*x)
**2*a**2 + 6*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b**2 + 12*log(tan((
c + d*x)/2) + 1)*a**2 - 3*log(tan((c + d*x)/2) + 1)*b**2 - 16*sin(c + d*x)
**4*a*b - 12*sin(c + d*x)**3*a**2 + 3*sin(c + d*x)**3*b**2 + 32*sin(c + d*
x)**2*a*b + 12*sin(c + d*x)*a**2 + 3*sin(c + d*x)*b**2 - 16*a*b)/(24*d*(si
n(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.530 $\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(2a^2-b^2)\,\text{arctanh}(\sin(c+dx))}{2d} + \frac{2ab\sec(c+dx)}{d} + \frac{b^2\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*(2*a^2-b^2)*arctanh(sin(d*x+c))/d+2*a*b*sec(d*x+c)/d+1/2*b^2*sec(d*x+c
)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{a^2\coth^{-1}(\sin(c+dx))}{d} - \frac{b^2\text{arctanh}(\sin(c+dx))}{2d} + \frac{2ab\sec(c+dx)}{d} + \frac{b^2\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

output
```
(a^2*ArcCoth[Sin[c + d*x]])/d - (b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b
*Sec[c + d*x])/d + (b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3991, 27, 3042, 3086, 24, 4159, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3991}$$

$$\int \sec(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+\int 2ab\sec(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$\int \sec(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+2ab\int \sec(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)\left(a^2+b^2\tan(c+dx)^2\right)dx+2ab\int \sec(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{3086}$$

$$\int \sec(c+dx)\left(a^2+b^2\tan(c+dx)^2\right)dx+\frac{2ab\int 1d\sec(c+dx)}{d}$$

$$\Big\downarrow \text{24}$$

$$\int \sec(c+dx)\left(a^2+b^2\tan(c+dx)^2\right)dx+\frac{2ab\sec(c+dx)}{d}$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{a^2-(a^2-b^2)\sin^2(c+dx)}{(1-\sin^2(c+dx))^2}d\sin(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}$$

$$\Big\downarrow \textbf{298}$$

$$\frac{\frac{1}{2}(2a^2-b^2)\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\frac{b^2\sin(c+dx)}{2(1-\sin^2(c+dx))}}{d}+\frac{2ab\sec(c+dx)}{d}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\frac{1}{2}(2a^2-b^2)\operatorname{arctanh}(\sin(c+dx))+\frac{b^2\sin(c+dx)}{2(1-\sin^2(c+dx))}}{d}+\frac{2ab\sec(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

output
```
(2*a*b*Sec[c + d*x])/d + (((2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/2 + (b^2*S
in[c + d*x])/(2*(1 - Sin[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{b^{2}\left(\frac{\sin(dx+c)^{3}}{2\cos(dx+c)^{2}}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ab}{\cos(dx+c)}+a^{2}\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{b^{2}\left(\frac{\sin(dx+c)^{3}}{2\cos(dx+c)^{2}}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ab}{\cos(dx+c)}+a^{2}\ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$\frac{b\,e^{i(dx+c)}\left(-ib\,e^{2i(dx+c)}+4a\,e^{2i(dx+c)}+ib+4a\right)}{d\left(e^{2i(dx+c)}+1\right)^{2}}-\frac{a^{2}\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{\ln\left(e^{i(dx+c)}-i\right)b^{2}}{2d}+\frac{a^{2}\ln\left(e^{i(dx+c)}+i\right)}{d}-$

input
```
int(sec(d*x+c)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+t
an(d*x+c)))+2*a*b/cos(d*x+c)+a^2*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.57

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{(2\,a^2-b^2)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(2\,a^2-b^2)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+8\,ab\,c}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/4*((2*a^2 - b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^2 - b^2)*co
s(d*x + c)^2*log(-sin(d*x + c) + 1) + 8*a*b*cos(d*x + c) + 2*b^2*sin(d*x +
 c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx=\int\left(a+b\tan\left(c+dx\right)\right)^2\sec\left(c+dx\right)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx =$$
$$-\frac{b^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-4a^2\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right.\right.}{4d}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/4*(b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - l
og(sin(d*x + c) - 1)) - 4*a^2*log(sec(d*x + c) + tan(d*x + c)) - 8*a*b/cos
(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(57) = 114$.

Time = 0.24 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.00

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{(2a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-(2a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4ab\,\mathrm{t}\right.}{\left(\tan\right.}}{2d}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*((2*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*a^2 - b^2)*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan
(1/2*d*x + 1/2*c)^2 + b^2*tan(1/2*d*x + 1/2*c) + 4*a*b)/(tan(1/2*d*x + 1/2
*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.74

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-4\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+4\,a\,b}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$
$$+\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(2\,a^2-b^2\right)}{d}$$

input
```
int((a + b*tan(c + d*x))^2/cos(c + d*x),x)
```

output
```
(4*a*b + b^2*tan(c/2 + (d*x)/2)^3 + b^2*tan(c/2 + (d*x)/2) - 4*a*b*tan(c/2
 + (d*x)/2)^2)/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)) + (
atanh(tan(c/2 + (d*x)/2))*(2*a^2 - b^2))/d
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 3.62

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{-4\cos(dx+c)\,ab-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2\,a^2+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2\,b^2+2}{}$$

input
```
int(sec(d*x+c)*(a+b*tan(d*x+c))^2,x)
```

output
```
( - 4*cos(c + d*x)*a*b - 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**2
+ log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b**2 + 2*log(tan((c + d*x)/2)
- 1)*a**2 - log(tan((c + d*x)/2) - 1)*b**2 + 2*log(tan((c + d*x)/2) + 1)*s
in(c + d*x)**2*a**2 - log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b**2 - 2*l
og(tan((c + d*x)/2) + 1)*a**2 + log(tan((c + d*x)/2) + 1)*b**2 - 4*sin(c +
 d*x)**2*a*b - sin(c + d*x)*b**2 + 4*a*b)/(2*d*(sin(c + d*x)**2 - 1))
```

3.531 $\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx=\frac{b^2\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{2ab\cos(c+dx)}{d}+\frac{(a^2-b^2)\sin(c+dx)}{d}$$

output
```
b^2*arctanh(sin(d*x+c))/d-2*a*b*cos(d*x+c)/d+(a^2-b^2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.79

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{-2ab\cos(c+dx)+b^2\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-2*a*b*Cos[c + d*x] + b^2*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (a^2 - b^2)*Sin[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3991, 27, 3042, 3118, 4159, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3991}$$

$$\int \cos(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+\int 2ab\sin(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$\int \cos(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+2ab\int \sin(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)}dx+2ab\int \sin(c+dx)dx$$

$$\Big\downarrow \text{3118}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)}dx-\frac{2ab\cos(c+dx)}{d}$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{a^2-(a^2-b^2)\sin^2(c+dx)}{1-\sin^2(c+dx)}d\sin(c+dx)}{d}-\frac{2ab\cos(c+dx)}{d}$$

$$\Big\downarrow \text{299}$$

$$\frac{b^2\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\left(a^2-b^2\right)\sin(c+dx)}{d}-\frac{2ab\cos(c+dx)}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{(a^2-b^2)\sin(c+dx)+b^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2ab\cos(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-2*a*b*Cos[c + d*x])/d + (b^2*ArcTanh[Sin[c + d*x]] + (a^2 - b^2)*Sin[c +
 d*x])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{b^{2}\left(-\sin\left(dx+c\right)+\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)\right)-2ab\cos\left(dx+c\right)+a^{2}\sin\left(dx+c\right)}{d}$
default	$\frac{b^{2}\left(-\sin\left(dx+c\right)+\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)\right)-2ab\cos\left(dx+c\right)+a^{2}\sin\left(dx+c\right)}{d}$
risch	$-\frac{e^{i(dx+c)}ab}{d}-\frac{ia^{2}e^{i(dx+c)}}{2d}+\frac{ie^{i(dx+c)}b^{2}}{2d}-\frac{e^{-i(dx+c)}ab}{d}+\frac{ie^{-i(dx+c)}a^{2}}{2d}-\frac{ie^{-i(dx+c)}b^{2}}{2d}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{d}$

input
```
int(cos(d*x+c)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-2*a*b*cos(d*x+c)+a^2*sin(
d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.32

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{4\,ab\cos\left(dx+c\right)-b^2\log\left(\sin\left(dx+c\right)+1\right)+b^2\log\left(-\sin\left(dx+c\right)+1\right)-2\left(a^2-b^2\right)\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(4*a*b*cos(d*x + c) - b^2*log(sin(d*x + c) + 1) + b^2*log(-sin(d*x +
c) + 1) - 2*(a^2 - b^2)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan\left(c+dx\right))^2\cos\left(c+dx\right)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*cos(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right))-4\,ab\cos\left(dx+c\right)+2\,a^2\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*(b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c))
- 4*a*b*cos(d*x + c) + 2*a^2*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1100 vs. $2(47) = 94$.

Time = 0.42 (sec) , antiderivative size = 1100, normalized size of antiderivative = 23.40

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2
*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - b^2*log(2*(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(
1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)
^2*tan(1/2*c)^2 + 4*a*b*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*log(2*(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/
(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2
*d*x)^2 - b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/
2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan
(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 + 4*a^2*tan(1/2*d*x)^2*tan(1/2*c) -
4*b^2*tan(1/2*d*x)^2*tan(1/2*c) + b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^
2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^2 - b^2*log(2*(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*...
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{4\,a\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-2\,b^2\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

input
```
int(cos(c + d*x)*(a + b*tan(c + d*x))^2,x)
```

output
```
(2*b^2*atanh(tan(c/2 + (d*x)/2)))/d - (4*a*b - tan(c/2 + (d*x)/2)*(2*a^2 -
 2*b^2))/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{-2\cos(dx+c)\,ab-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b^2+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)b^2+\sin(dx+c)\,a^2-\sin(dx+c}{d}$$

input
```
int(cos(d*x+c)*(a+b*tan(d*x+c))^2,x)
```

output
```
( - 2*cos(c + d*x)*a*b - log(tan((c + d*x)/2) - 1)*b**2 + log(tan((c + d*x
)/2) + 1)*b**2 + sin(c + d*x)*a**2 - sin(c + d*x)*b**2 + 2*a*b)/d
```

3.532 $\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx=-\frac{2ab\cos^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{(a^2-b^2)\sin^3(c+dx)}{3d}$$

output
```
-2/3*a*b*cos(d*x+c)^3/d+a^2*sin(d*x+c)/d-1/3*(a^2-b^2)*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx=-\frac{2ab\cos^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}+\frac{b^2\sin^3(c+dx)}{3d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-2*a*b*Cos[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^3
)/(3*d) + (b^2*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3991, 27, 3042, 3045, 15, 4159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(c+dx))^2}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{ 3991}$$

$$\int \cos^3(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+\int 2ab\cos^2(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \cos^3(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+2ab\int \cos^2(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^3}dx+2ab\int \cos(c+dx)^2\sin(c+dx)dx$$

$$\Big\downarrow \text{ 3045}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^3}dx-\frac{2ab\int \cos^2(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{ 15}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^3}dx-\frac{2ab\cos^3(c+dx)}{3d}$$

$$\Big\downarrow \text{ 4159}$$

$$\frac{\int \left(a^2-\left(a^2-b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}-\frac{2ab\cos^3(c+dx)}{3d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2 \sin(c+dx) - \frac{1}{3}(a^2-b^2)\sin^3(c+dx)}{d} - \frac{2ab\cos^3(c+dx)}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-2*a*b*Cos[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x] - ((a^2 - b^2)*Sin[c + d
*x]^3)/3)/d
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 4.78 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	siz
derivativedivides	$\frac{\frac{b^2\sin(dx+c)^3}{3}-\frac{2ab\cos(dx+c)^3}{3}+\frac{a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$	52
default	$\frac{\frac{b^2\sin(dx+c)^3}{3}-\frac{2ab\cos(dx+c)^3}{3}+\frac{a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$	52
risch	$-\frac{ab\cos(dx+c)}{2d}+\frac{3a^2\sin(dx+c)}{4d}+\frac{\sin(dx+c)b^2}{4d}-\frac{ab\cos(3dx+3c)}{6d}+\frac{\sin(3dx+3c)a^2}{12d}-\frac{\sin(3dx+3c)b^2}{12d}$	93

input
```
int(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*b^2*sin(d*x+c)^3-2/3*a*b*cos(d*x+c)^3+1/3*a^2*(2+cos(d*x+c)^2)*si
n(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=-\frac{2\,ab\cos\left(dx+c\right)^3-\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2+2\,a^2+b^2\right)\sin\left(dx+c\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/3*(2*a*b*cos(d*x + c)^3 - ((a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 + b^2)*si
n(d*x + c))/d
```

Sympy [F]

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan{(c+dx)})^2\cos^3{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{2\,ab\cos{(dx+c)}^3 - b^2\sin{(dx+c)}^3 + \left(\sin{(dx+c)}^3 - 3\,\sin{(dx+c)}\right)a^2}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/3*(2*a*b*cos(d*x + c)^3 - b^2*sin(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(
d*x + c))*a^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11162 vs. 2(51) = 102.

Time = 14.48 (sec) , antiderivative size = 11162, normalized size of antiderivative = 202.95

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/48*(3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2
*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*pi*a*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(
1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan
(1/2*d*x)^6*tan(1/2*c)^6 + 3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1
)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c
) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2
- tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1
/2*c)^4 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(
1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*pi*a*b*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 ...
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2\left(\frac{\sin(c+d\,x)\,a^2\cos(c+d\,x)^2}{2}+\sin\left(c+d\,x\right)\,a^2-a\,b\cos\left(c+d\,x\right)^3-\frac{\sin(c+d\,x)\,b^2\cos(c+d\,x)^2}{2}+\frac{\sin(c+d\,x)\,b^2}{2}\right)}{3\,d}$$

input
```
int(cos(c + d*x)^3*(a + b*tan(c + d*x))^2,x)
```

output
```
(2*(a^2*sin(c + d*x) + (b^2*sin(c + d*x))/2 + (a^2*cos(c + d*x)^2*sin(c +
d*x))/2 - (b^2*cos(c + d*x)^2*sin(c + d*x))/2 - a*b*cos(c + d*x)^3))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{2\cos(dx+c)\sin(dx+c)^2ab-2\cos(dx+c)ab-\sin(dx+c)^3a^2+\sin(dx+c)^3b^2+3\sin(dx+c)a^2}{3d}$$

input
```
int(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x)
```

output
```
(2*cos(c + d*x)*sin(c + d*x)**2*a*b - 2*cos(c + d*x)*a*b - sin(c + d*x)**3
*a**2 + sin(c + d*x)**3*b**2 + 3*sin(c + d*x)*a**2 + 2*a*b)/(3*d)
```

3.533 $\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{2ab\cos^5(c+dx)}{5d}+\frac{a^2\sin(c+dx)}{d} - \frac{(2a^2-b^2)\sin^3(c+dx)}{3d} + \frac{(a^2-b^2)\sin^5(c+dx)}{5d}$$

output
```
-2/5*a*b*cos(d*x+c)^5/d+a^2*sin(d*x+c)/d-1/3*(2*a^2-b^2)*sin(d*x+c)^3/d+1/
5*(a^2-b^2)*sin(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{-6ab\cos^5(c+dx)+15a^2\sin(c+dx)+5(-2a^2+b^2)\sin^3(c+dx)+3(a^2-b^2)\sin^5(c+dx)}{15d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-6*a*b*Cos[c + d*x]^5 + 15*a^2*Sin[c + d*x] + 5*(-2*a^2 + b^2)*Sin[c + d*
x]^3 + 3*(a^2 - b^2)*Sin[c + d*x]^5)/(15*d)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3991, 27, 3042, 3045, 15, 4159, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3991}$$

$$\int \cos^5(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+\int 2ab\cos^4(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$\int \cos^5(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+2ab\int \cos^4(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^5}dx+2ab\int \cos(c+dx)^4\sin(c+dx)dx$$

$$\Big\downarrow \text{3045}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^5}dx-\frac{2ab\int \cos^4(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{15}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^5}dx-\frac{2ab\cos^5(c+dx)}{5d}$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \left(1-\sin^2(c+dx)\right)\left(a^2-\left(a^2-b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}-\frac{2ab\cos^5(c+dx)}{5d}$$

$$\Big\downarrow \text{290}$$

$$\frac{\int \left((a-b)(a+b)\sin^4(c+dx)-\left(2a^2-b^2\right)\sin^2(c+dx)+a^2\right)d\sin(c+dx)}{d}-\frac{2ab\cos^5(c+dx)}{5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{5}\left(a^2-b^2\right)\sin^5(c+dx)-\frac{1}{3}\left(2a^2-b^2\right)\sin^3(c+dx)+a^2\sin(c+dx)}{d}-\frac{2ab\cos^5(c+dx)}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-2*a*b*Cos[c + d*x]^5)/(5*d) + (a^2*Sin[c + d*x] - ((2*a^2 - b^2)*Sin[c +
 d*x]^3)/3 + ((a^2 - b^2)*Sin[c + d*x]^5)/5)/d
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 20.72 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{b^{2}\left(-\frac{\cos(dx+c)^{4}\sin(dx+c)}{5}+\frac{\left(2+\cos(dx+c)^{2}\right)\sin(dx+c)}{15}\right)-\frac{2ab\cos(dx+c)^{5}}{5}+\frac{a^{2}\left(\frac{8}{3}+\cos(dx+c)^{4}+\frac{4\cos(dx+c)^{2}}{3}\right)\sin(dx+c)}{5}}{d}$
default	$\frac{b^{2}\left(-\frac{\cos(dx+c)^{4}\sin(dx+c)}{5}+\frac{\left(2+\cos(dx+c)^{2}\right)\sin(dx+c)}{15}\right)-\frac{2ab\cos(dx+c)^{5}}{5}+\frac{a^{2}\left(\frac{8}{3}+\cos(dx+c)^{4}+\frac{4\cos(dx+c)^{2}}{3}\right)\sin(dx+c)}{5}}{d}$
risch	$-\frac{ab\cos(dx+c)}{4d}+\frac{5a^{2}\sin(dx+c)}{8d}+\frac{\sin(dx+c)b^{2}}{8d}-\frac{ab\cos(5dx+5c)}{40d}+\frac{\sin(5dx+5c)a^{2}}{80d}-\frac{\sin(5dx+5c)b^{2}}{80d}-\ ab$

input
```
int(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(-1/5*cos(d*x+c)^4*sin(d*x+c)+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-2
/5*a*b*cos(d*x+c)^5+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{6\,ab\cos\left(dx+c\right)^5-\left(3\left(a^2-b^2\right)\cos\left(dx+c\right)^4+\left(4\,a^2+b^2\right)\cos\left(dx+c\right)^2+8\,a^2+2\,b^2\right)\sin\left(dx+c\right)}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/15*(6*a*b*cos(d*x + c)^5 - (3*(a^2 - b^2)*cos(d*x + c)^4 + (4*a^2 + b^2
)*cos(d*x + c)^2 + 8*a^2 + 2*b^2)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^2\cos^5\left(c+dx\right)\,dx$$

input
```
integrate(cos(d*x+c)**5*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx =$$
$$-\frac{6\,ab\cos\left(dx+c\right)^5-\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a^2+\left(3\,\sin\left(dx+c\right)^5-5\,\sin}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/15*(6*a*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*s
in(d*x + c))*a^2 + (3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*b^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28204 vs. 2(75) = 150.

Time = 29.80 (sec) , antiderivative size = 28204, normalized size of antiderivative = 348.20

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/960*(45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1
)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/
2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 -
2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 60*pi*a*b*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c
)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 +
2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2
*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^1
0*tan(1/2*c)^8 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^...
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.42

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{2\left(\frac{3\sin(c+d\,x)\,a^2\cos(c+d\,x)^4}{2}+2\sin\left(c+d\,x\right)\,a^2\cos\left(c+d\,x\right)^2+4\sin\left(c+d\,x\right)\,a^2-3\,a\,b\cos\left(c+d\,x\right)^5-\frac{3\sin}{}\right.}{15\,d}$$

input
```
int(cos(c + d*x)^5*(a + b*tan(c + d*x))^2,x)
```

output
```
(2*(4*a^2*sin(c + d*x) + b^2*sin(c + d*x) + 2*a^2*cos(c + d*x)^2*sin(c + d
*x) + (3*a^2*cos(c + d*x)^4*sin(c + d*x))/2 + (b^2*cos(c + d*x)^2*sin(c +
d*x))/2 - (3*b^2*cos(c + d*x)^4*sin(c + d*x))/2 - 3*a*b*cos(c + d*x)^5))/(
15*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.47

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{-6\cos(dx+c)\sin(dx+c)^4ab+12\cos(dx+c)\sin(dx+c)^2ab-6\cos(dx+c)ab+3\sin(dx+c)^5a^2}{15d}$$

input
```
int(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x)
```

output
```
( - 6*cos(c + d*x)*sin(c + d*x)**4*a*b + 12*cos(c + d*x)*sin(c + d*x)**2*a
*b - 6*cos(c + d*x)*a*b + 3*sin(c + d*x)**5*a**2 - 3*sin(c + d*x)**5*b**2
- 10*sin(c + d*x)**3*a**2 + 5*sin(c + d*x)**3*b**2 + 15*sin(c + d*x)*a**2
+ 6*a*b)/(15*d)
```

3.534 $\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{2ab\cos^7(c+dx)}{7d}+\frac{a^2\sin(c+dx)}{d} - \frac{(3a^2-b^2)\sin^3(c+dx)}{3d} + \frac{(3a^2-2b^2)\sin^5(c+dx)}{5d} - \frac{(a^2-b^2)\sin^7(c+dx)}{7d}$$

output
```
-2/7*a*b*cos(d*x+c)^7/d+a^2*sin(d*x+c)/d-1/3*(3*a^2-b^2)*sin(d*x+c)^3/d+1/
5*(3*a^2-2*b^2)*sin(d*x+c)^5/d-1/7*(a^2-b^2)*sin(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.86

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{-30ab\cos^7(c+dx)+105a^2\sin(c+dx)-35(3a^2-b^2)\sin^3(c+dx)+21(3a^2-2b^2)\sin^5(c+dx)-15(a}{105d}$$

input
```
Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-30*a*b*Cos[c + d*x]^7 + 105*a^2*Sin[c + d*x] - 35*(3*a^2 - b^2)*Sin[c +
d*x]^3 + 21*(3*a^2 - 2*b^2)*Sin[c + d*x]^5 - 15*(a^2 - b^2)*Sin[c + d*x]^7
)/(105*d)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3991, 27, 3042, 3045, 15, 4159, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+b\tan(c+dx))^2}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \ 3991$$

$$\int \cos^7(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+\int 2ab\cos^6(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \ 27$$

$$\int \cos^7(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+2ab\int \cos^6(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^7}dx+2ab\int \cos(c+dx)^6\sin(c+dx)dx$$

$$\Big\downarrow \ 3045$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^7}dx-\frac{2ab\int \cos^6(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \ 15$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^7}dx-\frac{2ab\cos^7(c+dx)}{7d}$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \left(1-\sin^2(c+dx)\right)^2\left(a^2-\left(a^2-b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}-\frac{2ab\cos^7(c+dx)}{7d}$$

$$\Big\downarrow \text{290}$$

$$\frac{\int \left(-\left((a-b)(a+b)\sin^6(c+dx)\right)+\left(3a^2-2b^2\right)\sin^4(c+dx)-\left(3a^2-b^2\right)\sin^2(c+dx)+a^2\right)d\sin(c+dx)}{d}-\frac{2ab\cos^7(c+dx)}{7d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{7}\left(a^2-b^2\right)\sin^7(c+dx)+\frac{1}{5}\left(3a^2-2b^2\right)\sin^5(c+dx)-\frac{1}{3}\left(3a^2-b^2\right)\sin^3(c+dx)+a^2\sin(c+dx)}{d}-\frac{2ab\cos^7(c+dx)}{7d}$$

input
```
Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-2*a*b*Cos[c + d*x]^7)/(7*d) + (a^2*Sin[c + d*x] - ((3*a^2 - b^2)*Sin[c +
 d*x]^3)/3 + ((3*a^2 - 2*b^2)*Sin[c + d*x]^5)/5 - ((a^2 - b^2)*Sin[c + d*x
]^7)/7)/d
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 67.70 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^6}{7}+\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{35}\right)-\frac{2ab\cos(dx+c)^7}{7}+\frac{a^2\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+}{5}\right.}{7}}{d}$
default	$\frac{b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^6}{7}+\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{35}\right)-\frac{2ab\cos(dx+c)^7}{7}+\frac{a^2\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+}{5}\right.}{7}}{d}$
risch	$-\frac{5ab\cos(dx+c)}{32d}+\frac{35a^2\sin(dx+c)}{64d}+\frac{5\sin(dx+c)b^2}{64d}-\frac{ab\cos(7dx+7c)}{224d}+\frac{\sin(7dx+7c)a^2}{448d}-\frac{\sin(7dx+7c)b^2}{448d}-$

input
```
int(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+
c)^2)*sin(d*x+c))-2/7*a*b*cos(d*x+c)^7+1/7*a^2*(16/5+cos(d*x+c)^6+6/5*cos(
d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.88

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{30\,ab\cos\left(dx+c\right)^7-\left(15\left(a^2-b^2\right)\cos\left(dx+c\right)^6+3\left(6\,a^2+b^2\right)\cos\left(dx+c\right)^4+4\left(6\,a^2+b^2\right)\cos\left(dx+c\right.\right.}{105\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/105*(30*a*b*cos(d*x + c)^7 - (15*(a^2 - b^2)*cos(d*x + c)^6 + 3*(6*a^2
+ b^2)*cos(d*x + c)^4 + 4*(6*a^2 + b^2)*cos(d*x + c)^2 + 48*a^2 + 8*b^2)*s
in(d*x + c))/d
```

Sympy [F]

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx=\int (a+b\tan(c+dx))^2\cos^7(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**7*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx=$$
$$-\frac{30\,ab\cos(dx+c)^7+3\left(5\sin(dx+c)^7-21\sin(dx+c)^5+35\sin(dx+c)^3-35\sin(dx+c)\right)a^2-(}{105\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/105*(30*a*b*cos(d*x + c)^7 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 +
35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^2 - (15*sin(d*x + c)^7 - 42*sin(d*x
 + c)^5 + 35*sin(d*x + c)^3)*b^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52002 vs. 2(99) = 198.

Time = 54.05 (sec) , antiderivative size = 52002, normalized size of antiderivative = 486.00

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx=\text{Too large to display}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output

```
-1/26880*(945*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*ta
n(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/
2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 945*pi*a*b*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)
^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
 - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 945*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*t
an(1/2*c) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 945*pi*a*b*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c
)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 8400*pi*a*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 6615*pi*a*b*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/
2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1
/2*d*x)^14*tan(1/2*c)^12 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c)
- 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - ta...
```

Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.64

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{16\,a^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{8\,b^2\,\sin\left(c+d\,x\right)}{105\,d}+\frac{8\,a^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{6\,a^2\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{35\,d}+\frac{a^2\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)}{7\,d}+\frac{4\,b^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{105\,d}+\frac{b^2\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{35\,d}-\frac{b^2\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)}{7\,d}-\frac{2\,a\,b\cos\left(c+d\,x\right)^7}{7\,d}$$

input
```
int(cos(c + d*x)^7*(a + b*tan(c + d*x))^2,x)
```

output
```
(16*a^2*sin(c + d*x))/(35*d) + (8*b^2*sin(c + d*x))/(105*d) + (8*a^2*cos(c
 + d*x)^2*sin(c + d*x))/(35*d) + (6*a^2*cos(c + d*x)^4*sin(c + d*x))/(35*d
) + (a^2*cos(c + d*x)^6*sin(c + d*x))/(7*d) + (4*b^2*cos(c + d*x)^2*sin(c
+ d*x))/(105*d) + (b^2*cos(c + d*x)^4*sin(c + d*x))/(35*d) - (b^2*cos(c +
d*x)^6*sin(c + d*x))/(7*d) - (2*a*b*cos(c + d*x)^7)/(7*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.52

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{30\cos\left(dx+c\right)\sin\left(dx+c\right)^6 ab-90\cos\left(dx+c\right)\sin\left(dx+c\right)^4 ab+90\cos\left(dx+c\right)\sin\left(dx+c\right)^2 ab-30\,\mathrm{c}}{}$$

input
```
int(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x)
```

output
```
(30*cos(c + d*x)*sin(c + d*x)**6*a*b - 90*cos(c + d*x)*sin(c + d*x)**4*a*b
 + 90*cos(c + d*x)*sin(c + d*x)**2*a*b - 30*cos(c + d*x)*a*b - 15*sin(c +
d*x)**7*a**2 + 15*sin(c + d*x)**7*b**2 + 63*sin(c + d*x)**5*a**2 - 42*sin(
c + d*x)**5*b**2 - 105*sin(c + d*x)**3*a**2 + 35*sin(c + d*x)**3*b**2 + 10
5*sin(c + d*x)*a**2 + 30*a*b)/(105*d)
```

3.535 $\int \cos^9(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 133

$$\int \cos^9(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{2ab\cos^9(c+dx)}{9d}+\frac{a^2\sin(c+dx)}{d} - \frac{(4a^2-b^2)\sin^3(c+dx)}{3d} + \frac{3(2a^2-b^2)\sin^5(c+dx)}{5d} - \frac{(4a^2-3b^2)\sin^7(c+dx)}{7d} + \frac{(a^2-b^2)\sin^9(c+dx)}{9d}$$

output
```
-2/9*a*b*cos(d*x+c)^9/d+a^2*sin(d*x+c)/d-1/3*(4*a^2-b^2)*sin(d*x+c)^3/d+3/
5*(2*a^2-b^2)*sin(d*x+c)^5/d-1/7*(4*a^2-3*b^2)*sin(d*x+c)^7/d+1/9*(a^2-b^2
)*sin(d*x+c)^9/d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.85

$$\int \cos^9(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{-70ab\cos^9(c+dx)+315a^2\sin(c+dx)-105(4a^2-b^2)\sin^3(c+dx)+189(2a^2-b^2)\sin^5(c+dx)-45}{315d}$$

input
```
Integrate[Cos[c + d*x]^9*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-70*a*b*Cos[c + d*x]^9 + 315*a^2*Sin[c + d*x] - 105*(4*a^2 - b^2)*Sin[c +
 d*x]^3 + 189*(2*a^2 - b^2)*Sin[c + d*x]^5 - 45*(4*a^2 - 3*b^2)*Sin[c + d*
x]^7 + 35*(a^2 - b^2)*Sin[c + d*x]^9)/(315*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3991, 27, 3042, 3045, 15, 4159, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^9(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+b\tan(c+dx))^2}{\sec(c+dx)^9}dx$$
$$\Big\downarrow \text{3991}$$
$$\int \cos^9(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+\int 2ab\cos^8(c+dx)\sin(c+dx)dx$$
$$\Big\downarrow \text{27}$$
$$\int \cos^9(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+2ab\int \cos^8(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^9}dx+2ab\int \cos(c+dx)^8\sin(c+dx)dx$$

$$\Big\downarrow \text{3045}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^9}dx-\frac{2ab\int \cos^8(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{15}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^9}dx-\frac{2ab\cos^9(c+dx)}{9d}$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \left(1-\sin^2(c+dx)\right)^3\left(a^2-\left(a^2-b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}-\frac{2ab\cos^9(c+dx)}{9d}$$

$$\Big\downarrow \text{290}$$

$$\frac{\int \left((a-b)(a+b)\sin^8(c+dx)-\left(4a^2-3b^2\right)\sin^6(c+dx)+3\left(2a^2-b^2\right)\sin^4(c+dx)-\left(4a^2-b^2\right)\sin^2(c+dx)+\right.}{d}$$
$$\frac{2ab\cos^9(c+dx)}{9d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{9}\left(a^2-b^2\right)\sin^9(c+dx)-\frac{1}{7}\left(4a^2-3b^2\right)\sin^7(c+dx)+\frac{3}{5}\left(2a^2-b^2\right)\sin^5(c+dx)-\frac{1}{3}\left(4a^2-b^2\right)\sin^3(c+dx)+a^2}{d}$$
$$\frac{2ab\cos^9(c+dx)}{9d}$$

input
```
Int[Cos[c + d*x]^9*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-2*a*b*Cos[c + d*x]^9)/(9*d) + (a^2*Sin[c + d*x] - ((4*a^2 - b^2)*Sin[c +
 d*x]^3)/3 + (3*(2*a^2 - b^2)*Sin[c + d*x]^5)/5 - ((4*a^2 - 3*b^2)*Sin[c +
 d*x]^7)/7 + ((a^2 - b^2)*Sin[c + d*x]^9)/9)/d
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 186.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{a^2\left(\frac{128}{35}+\cos(dx+c)^8+\frac{8\cos(dx+c)^6}{7}+\frac{48\cos(dx+c)^4}{35}+\frac{64\cos(dx+c)^2}{35}\right)\sin(dx+c)}{9}-\frac{2ab\cos(dx+c)^9}{9}+b^2\left(-\frac{\cos(dx+c)^8\sin(dx+c}{9}\right.}{d}$
default	$\frac{\frac{a^2\left(\frac{128}{35}+\cos(dx+c)^8+\frac{8\cos(dx+c)^6}{7}+\frac{48\cos(dx+c)^4}{35}+\frac{64\cos(dx+c)^2}{35}\right)\sin(dx+c)}{9}-\frac{2ab\cos(dx+c)^9}{9}+b^2\left(-\frac{\cos(dx+c)^8\sin(dx+c}{9}\right.}{d}$
risch	$-\frac{7ab\cos(dx+c)}{64d}+\frac{63a^2\sin(dx+c)}{128d}+\frac{7\sin(dx+c)b^2}{128d}-\frac{ab\cos(9dx+9c)}{1152d}+\frac{\sin(9dx+9c)a^2}{2304d}-\frac{\sin(9dx+9c)b^2}{2304d}-$

input
```
int(cos(d*x+c)^9*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/9*a^2*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/3
5*cos(d*x+c)^2)*sin(d*x+c)-2/9*a*b*cos(d*x+c)^9+b^2*(-1/9*cos(d*x+c)^8*sin
(d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x
+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.85

$$\int \cos^9(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{70\,ab\cos\left(dx+c\right)^9-\left(35\left(a^2-b^2\right)\cos\left(dx+c\right)^8+5\left(8\,a^2+b^2\right)\cos\left(dx+c\right)^6+6\left(8\,a^2+b^2\right)\cos\left(dx+c\right.}{315\,d}$$

input
```
integrate(cos(d*x+c)^9*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/315*(70*a*b*cos(d*x + c)^9 - (35*(a^2 - b^2)*cos(d*x + c)^8 + 5*(8*a^2
+ b^2)*cos(d*x + c)^6 + 6*(8*a^2 + b^2)*cos(d*x + c)^4 + 8*(8*a^2 + b^2)*c
os(d*x + c)^2 + 128*a^2 + 16*b^2)*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^9(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**9*(a+b*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.88

$$\int \cos^9(c+dx)(a+b\tan(c+dx))^2\,dx =$$
$$-\frac{70\,ab\cos\left(dx+c\right)^9-\left(35\,\sin\left(dx+c\right)^9-180\,\sin\left(dx+c\right)^7+378\,\sin\left(dx+c\right)^5-420\,\sin\left(dx+c\right)^3+3}{31}$$

input
```
integrate(cos(d*x+c)^9*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/315*(70*a*b*cos(d*x + c)^9 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7 +
378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^2 + (35*sin(
d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*x + c)^3)
*b^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80368 vs. $2(123) = 246$.

Time = 79.01 (sec) , antiderivative size = 80368, normalized size of antiderivative = 604.27

$$\int \cos^9(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^9*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/80640*(2205*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^18*tan(1/2*c)^18 + 2205*pi*a*b*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*
x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*
x) - 1)*tan(1/2*d*x)^18*tan(1/2*c)^18 + 2205*pi*a*b*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 +
2*tan(1/2*c) - 1)*tan(1/2*d*x)^18*tan(1/2*c)^18 + 2205*pi*a*b*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^18*tan(1/2*c)^18 - 19530*pi*a*b*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^18*tan(1/2*c)^18 + 19845*pi*a*b*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^18*tan(1/2*c)^16 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)...
```

Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.44

$$\int \cos^9(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{2\left(\frac{35\,\sin(c+d\,x)\,a^2\cos(c+d\,x)^8}{2}+20\,\sin\left(c+d\,x\right)\,a^2\cos\left(c+d\,x\right)^6+24\,\sin\left(c+d\,x\right)\,a^2\cos\left(c+d\,x\right)^4+32\,\sin\right.}{}$$

input
```
int(cos(c + d*x)^9*(a + b*tan(c + d*x))^2,x)
```

output
```
(2*(64*a^2*sin(c + d*x) + 8*b^2*sin(c + d*x) + 32*a^2*cos(c + d*x)^2*sin(c
 + d*x) + 24*a^2*cos(c + d*x)^4*sin(c + d*x) + 20*a^2*cos(c + d*x)^6*sin(c
 + d*x) + (35*a^2*cos(c + d*x)^8*sin(c + d*x))/2 + 4*b^2*cos(c + d*x)^2*si
n(c + d*x) + 3*b^2*cos(c + d*x)^4*sin(c + d*x) + (5*b^2*cos(c + d*x)^6*sin
(c + d*x))/2 - (35*b^2*cos(c + d*x)^8*sin(c + d*x))/2 - 35*a*b*cos(c + d*x
)^9))/(315*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.56

$$\int \cos^9(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{-70\cos\left(dx+c\right)\sin\left(dx+c\right)^8ab+280\cos\left(dx+c\right)\sin\left(dx+c\right)^6ab-420\cos\left(dx+c\right)\sin\left(dx+c\right)^4ab+}{}$$

input
```
int(cos(d*x+c)^9*(a+b*tan(d*x+c))^2,x)
```

output
```
( - 70*cos(c + d*x)*sin(c + d*x)**8*a*b + 280*cos(c + d*x)*sin(c + d*x)**6
*a*b - 420*cos(c + d*x)*sin(c + d*x)**4*a*b + 280*cos(c + d*x)*sin(c + d*x
)**2*a*b - 70*cos(c + d*x)*a*b + 35*sin(c + d*x)**9*a**2 - 35*sin(c + d*x)
**9*b**2 - 180*sin(c + d*x)**7*a**2 + 135*sin(c + d*x)**7*b**2 + 378*sin(c
 + d*x)**5*a**2 - 189*sin(c + d*x)**5*b**2 - 420*sin(c + d*x)**3*a**2 + 10
5*sin(c + d*x)**3*b**2 + 315*sin(c + d*x)*a**2 + 70*a*b)/(315*d)
```

3.536 $\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 194

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3a^2b\sec^8(c+dx)}{8d}+\frac{a^3\tan(c+dx)}{d}+\frac{a(a^2+b^2)\tan^3(c+dx)}{d}+\frac{b^3\tan^4(c+dx)}{4d}+\frac{3a(a^2+3b^2)\tan^5(c+dx)}{5d}+\frac{b^3\tan^6(c+dx)}{2d}+\frac{a(a^2+9b^2)\tan^7(c+dx)}{7d}+\frac{3b^3\tan^8(c+dx)}{8d}+\frac{ab^2\tan^9(c+dx)}{3d}+\frac{b^3\tan^{10}(c+dx)}{10d}$$

output
```
3/8*a^2*b*sec(d*x+c)^8/d+a^3*tan(d*x+c)/d+a*(a^2+b^2)*tan(d*x+c)^3/d+1/4*b
^3*tan(d*x+c)^4/d+3/5*a*(a^2+3*b^2)*tan(d*x+c)^5/d+1/2*b^3*tan(d*x+c)^6/d+
1/7*a*(a^2+9*b^2)*tan(d*x+c)^7/d+3/8*b^3*tan(d*x+c)^8/d+1/3*a*b^2*tan(d*x+
c)^9/d+1/10*b^3*tan(d*x+c)^10/d
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.91

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\frac{1}{4}(a^2+b^2)^3(a+b\tan(c+dx))^4-\frac{6}{5}a(a^2+b^2)^2(a+b\tan(c+dx))^5+\frac{1}{2}(a^2+b^2)(5a^2+b^2)(a+b\tan(c}{}$$

input
```
Integrate[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^3,x]
```

output
```
(((a^2 + b^2)^3*(a + b*Tan[c + d*x])^4)/4 - (6*a*(a^2 + b^2)^2*(a + b*Tan[
c + d*x])^5)/5 + ((a^2 + b^2)*(5*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/2 - (4
*a*(5*a^2 + 3*b^2)*(a + b*Tan[c + d*x])^7)/7 + (3*(5*a^2 + b^2)*(a + b*Tan
[c + d*x])^8)/8 - (2*a*(a + b*Tan[c + d*x])^9)/3 + (a + b*Tan[c + d*x])^10
/10)/(b^7*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 475, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(c+dx)^8(a+b\tan(c+dx))^3dx$$
$$\Big\downarrow \text{3987}$$
$$\frac{\int \frac{(a+b\tan(c+dx))^3(\tan^2(c+dx)b^2+b^2)^3}{b^6}d(b\tan(c+dx))}{bd}$$
$$\Big\downarrow \text{27}$$

$$\frac{\int (a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)^3d(b\tan(c+dx))}{b^7d}$$
$$\Big\downarrow 475$$
$$\frac{\int \left(\tan^9(c+dx)b^9+3\tan^7(c+dx)b^9+3\tan^5(c+dx)b^9+\tan^3(c+dx)b^9+3a\tan^8(c+dx)b^8+a\left(a^2+9b^2\right)\tan\right.}{}$$
$$\Big\downarrow 2009$$
$$\frac{a^3b^7\tan(c+dx)+\frac{3}{8}a^2\left(b^2\tan^2(c+dx)+b^2\right)^4+\frac{1}{7}ab^7\left(a^2+9b^2\right)\tan^7(c+dx)+\frac{3}{5}ab^7\left(a^2+3b^2\right)\tan^5(c+dx)+}{}$$

input
```
Int[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^3,x]
```

output
```
(a^3*b^7*Tan[c + d*x] + a*b^7*(a^2 + b^2)*Tan[c + d*x]^3 + (b^10*Tan[c + d
*x]^4)/4 + (3*a*b^7*(a^2 + 3*b^2)*Tan[c + d*x]^5)/5 + (b^10*Tan[c + d*x]^6
)/2 + (a*b^7*(a^2 + 9*b^2)*Tan[c + d*x]^7)/7 + (3*b^10*Tan[c + d*x]^8)/8 +
 (a*b^9*Tan[c + d*x]^9)/3 + (b^10*Tan[c + d*x]^10)/10 + (3*a^2*(b^2 + b^2*
Tan[c + d*x]^2)^4)/8)/(b^7*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 475
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp
[d*n*c^(n - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Int[ExpandIntegran
d[((c + d*x)^n - d*n*c^(n - 1)*x)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
d}, x] && IGtQ[p, 0] && IGtQ[n, 0] && LeQ[n, p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 188.98 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{b^3\left(\frac{\sin(dx+c)^4}{10\cos(dx+c)^{10}}+\frac{3\sin(dx+c)^4}{40\cos(dx+c)^8}+\frac{\sin(dx+c)^4}{20\cos(dx+c)^6}+\frac{\sin(dx+c)^4}{40\cos(dx+c)^4}\right)+3a\,b^2\left(\frac{\sin(dx+c)^3}{9\cos(dx+c)^9}+\frac{2\sin(dx+c)^3}{21\cos(dx+c)^7}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^5}\right.}{d}$
default	$\frac{b^3\left(\frac{\sin(dx+c)^4}{10\cos(dx+c)^{10}}+\frac{3\sin(dx+c)^4}{40\cos(dx+c)^8}+\frac{\sin(dx+c)^4}{20\cos(dx+c)^6}+\frac{\sin(dx+c)^4}{40\cos(dx+c)^4}\right)+3a\,b^2\left(\frac{\sin(dx+c)^3}{9\cos(dx+c)^9}+\frac{2\sin(dx+c)^3}{21\cos(dx+c)^7}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^5}\right.}{d}$
risch	$-\frac{32\left(-30ia^3\mathrm{e}^{2i(dx+c)}-105ia\,b^2\mathrm{e}^{8i(dx+c)}-315a^2b\,\mathrm{e}^{12i(dx+c)}+105b^3\mathrm{e}^{12i(dx+c)}-135ia^3\mathrm{e}^{4i(dx+c)}-3ia^3-630a^2b\,\mathrm{e}^{10i(}\right.}{}$

input
```
int(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(1/10*sin(d*x+c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)^4/cos(d*x+c)^8+1
/20*sin(d*x+c)^4/cos(d*x+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)^4)+3*a*b^2*(1/9
*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)
^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+3/8*a^2*b/cos(d*x+c)^8-a
^3*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c
))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{84\,b^3+105\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2+8\left(16\left(3\,a^3-ab^2\right)\cos\left(dx+c\right)^9+8\left(3\,a^3-ab^2\right)\cos\left(dx+c\right)^7+6\right.}{840\,d\cos\left(dx+c\right.}$$

input
```
integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/840*(84*b^3 + 105*(3*a^2*b - b^3)*cos(d*x + c)^2 + 8*(16*(3*a^3 - a*b^2)
*cos(d*x + c)^9 + 8*(3*a^3 - a*b^2)*cos(d*x + c)^7 + 6*(3*a^3 - a*b^2)*cos
(d*x + c)^5 + 35*a*b^2*cos(d*x + c) + 5*(3*a^3 - a*b^2)*cos(d*x + c)^3)*si
n(d*x + c))/(d*cos(d*x + c)^10)
```

Sympy [F]

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\sec^8(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**8*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.91

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx
= \frac{84\,b^3\tan(dx+c)^{10} + 280\,ab^2\tan(dx+c)^9 + 315\,(a^2b+b^3)\tan(dx+c)^8 + 120\,(a^3+9\,ab^2)\tan(dx+c}{}$$

input
```
integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/840*(84*b^3*tan(d*x + c)^10 + 280*a*b^2*tan(d*x + c)^9 + 315*(a^2*b + b^
3)*tan(d*x + c)^8 + 120*(a^3 + 9*a*b^2)*tan(d*x + c)^7 + 420*(3*a^2*b + b^
3)*tan(d*x + c)^6 + 504*(a^3 + 3*a*b^2)*tan(d*x + c)^5 + 1260*a^2*b*tan(d*
x + c)^2 + 210*(9*a^2*b + b^3)*tan(d*x + c)^4 + 840*a^3*tan(d*x + c) + 840
*(a^3 + a*b^2)*tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.13

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{84\,b^3\tan\left(dx+c\right)^{10}+280\,ab^2\tan\left(dx+c\right)^{9}+315\,a^2b\tan\left(dx+c\right)^{8}+315\,b^3\tan\left(dx+c\right)^{8}+120\,a^3\tan\left(a\right.}{}$$

input
```
integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/840*(84*b^3*tan(d*x + c)^10 + 280*a*b^2*tan(d*x + c)^9 + 315*a^2*b*tan(d
*x + c)^8 + 315*b^3*tan(d*x + c)^8 + 120*a^3*tan(d*x + c)^7 + 1080*a*b^2*t
an(d*x + c)^7 + 1260*a^2*b*tan(d*x + c)^6 + 420*b^3*tan(d*x + c)^6 + 504*a
^3*tan(d*x + c)^5 + 1512*a*b^2*tan(d*x + c)^5 + 1890*a^2*b*tan(d*x + c)^4
+ 210*b^3*tan(d*x + c)^4 + 840*a^3*tan(d*x + c)^3 + 840*a*b^2*tan(d*x + c)
^3 + 1260*a^2*b*tan(d*x + c)^2 + 840*a^3*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.90

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\tan(c+d\,x)^5\left(\frac{3\,a^3}{5}+\frac{9\,a\,b^2}{5}\right)+\tan(c+d\,x)^7\left(\frac{a^3}{7}+\frac{9\,a\,b^2}{7}\right)+\tan(c+d\,x)^6\left(\frac{3\,a^2\,b}{2}+\frac{b^3}{2}\right)+\tan(c+d\,x)^4\left(\frac{9}{}\right.}{}$$

input
```
int((a + b*tan(c + d*x))^3/cos(c + d*x)^8,x)
```

output
```
(tan(c + d*x)^5*((9*a*b^2)/5 + (3*a^3)/5) + tan(c + d*x)^7*((9*a*b^2)/7 +
a^3/7) + tan(c + d*x)^6*((3*a^2*b)/2 + b^3/2) + tan(c + d*x)^4*((9*a^2*b)/
4 + b^3/4) + a^3*tan(c + d*x) + (b^3*tan(c + d*x)^10)/10 + (3*a^2*b*tan(c
+ d*x)^2)/2 + (a*b^2*tan(c + d*x)^9)/3 + a*tan(c + d*x)^3*(a^2 + b^2) + (3
*b*tan(c + d*x)^8*(a^2 + b^2))/8)/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.81

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\sin(dx+c)\left(-384\cos(dx+c)\sin(dx+c)^8 a^3+128\cos(dx+c)\sin(dx+c)^8 a\,b^2+1728\cos(dx+c)\,\mathrm{s}\right.}{}$$

input
```
int(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x)
```

output
```
(sin(c + d*x)*( - 384*cos(c + d*x)*sin(c + d*x)**8*a**3 + 128*cos(c + d*x)
*sin(c + d*x)**8*a*b**2 + 1728*cos(c + d*x)*sin(c + d*x)**6*a**3 - 576*cos
(c + d*x)*sin(c + d*x)**6*a*b**2 - 3024*cos(c + d*x)*sin(c + d*x)**4*a**3
+ 1008*cos(c + d*x)*sin(c + d*x)**4*a*b**2 + 2520*cos(c + d*x)*sin(c + d*x
)**2*a**3 - 840*cos(c + d*x)*sin(c + d*x)**2*a*b**2 - 840*cos(c + d*x)*a**
3 - 315*sin(c + d*x)**9*a**2*b + 21*sin(c + d*x)**9*b**3 + 1575*sin(c + d*
x)**7*a**2*b - 105*sin(c + d*x)**7*b**3 - 3150*sin(c + d*x)**5*a**2*b + 21
0*sin(c + d*x)**5*b**3 + 3150*sin(c + d*x)**3*a**2*b - 210*sin(c + d*x)**3
*b**3 - 1260*sin(c + d*x)*a**2*b))/(840*d*(sin(c + d*x)**10 - 5*sin(c + d*
x)**8 + 10*sin(c + d*x)**6 - 10*sin(c + d*x)**4 + 5*sin(c + d*x)**2 - 1))
```

3.537 $\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{(a^2+b^2)^2(a+b\tan(c+dx))^4}{4b^5d} - \frac{4a(a^2+b^2)(a+b\tan(c+dx))^5}{5b^5d} + \frac{(3a^2+b^2)(a+b\tan(c+dx))^6}{3b^5d} - \frac{4a(a+b\tan(c+dx))^7}{7b^5d} + \frac{(a+b\tan(c+dx))^8}{8b^5d}$$

output
```
1/4*(a^2+b^2)^2*(a+b*tan(d*x+c))^4/b^5/d-4/5*a*(a^2+b^2)*(a+b*tan(d*x+c))^
5/b^5/d+1/3*(3*a^2+b^2)*(a+b*tan(d*x+c))^6/b^5/d-4/7*a*(a+b*tan(d*x+c))^7/
b^5/d+1/8*(a+b*tan(d*x+c))^8/b^5/d
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{\frac{1}{4}(a^2+b^2)^2(a+b\tan(c+dx))^4-\frac{4}{5}a(a^2+b^2)(a+b\tan(c+dx))^5+\frac{1}{3}(3a^2+b^2)(a+b\tan(c+dx))^6-}{b^5d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

output
```
(((a^2 + b^2)^2*(a + b*Tan[c + d*x])^4)/4 - (4*a*(a^2 + b^2)*(a + b*Tan[c
+ d*x])^5)/5 + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/3 - (4*a*(a + b*Tan[
c + d*x])^7)/7 + (a + b*Tan[c + d*x])^8/8)/(b^5*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(c+dx)^6(a+b\tan(c+dx))^3dx$$
$$\Big\downarrow \text{3987}$$
$$\frac{\int \frac{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)^2}{b^4}d(b\tan(c+dx))}{bd}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int (a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)^2d(b\tan(c+dx))}{b^5d}$$

$$\Big\downarrow \textbf{476}$$

$$\frac{\int \left((a+b\tan(c+dx))^7-4a(a+b\tan(c+dx))^6+2\left(3a^2+b^2\right)(a+b\tan(c+dx))^5-4a\left(a^2+b^2\right)(a+b\tan(c-\right.}{b^5d}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{\frac{1}{3}\left(3a^2+b^2\right)(a+b\tan(c+dx))^6-\frac{4}{5}a\left(a^2+b^2\right)(a+b\tan(c+dx))^5+\frac{1}{4}\left(a^2+b^2\right)^2(a+b\tan(c+dx))^4+\frac{1}{8}(a+}{b^5d}$$

input
```
Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

output
```
(((a^2 + b^2)^2*(a + b*Tan[c + d*x])^4)/4 - (4*a*(a^2 + b^2)*(a + b*Tan[c
+ d*x])^5)/5 + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/3 - (4*a*(a + b*Tan[
c + d*x])^7)/7 + (a + b*Tan[c + d*x])^8/8)/(b^5*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 68.53 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{b^3\left(\frac{\sin(dx+c)^4}{8\cos(dx+c)^8}+\frac{\sin(dx+c)^4}{12\cos(dx+c)^6}+\frac{\sin(dx+c)^4}{24\cos(dx+c)^4}\right)+3a\,b^2\left(\frac{\sin(dx+c)^3}{7\cos(dx+c)^7}+\frac{4\sin(dx+c)^3}{35\cos(dx+c)^5}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^3}\right)+\frac{a^2b}{2\cos(dx+c)^6}-}{d}$
default	$\frac{b^3\left(\frac{\sin(dx+c)^4}{8\cos(dx+c)^8}+\frac{\sin(dx+c)^4}{12\cos(dx+c)^6}+\frac{\sin(dx+c)^4}{24\cos(dx+c)^4}\right)+3a\,b^2\left(\frac{\sin(dx+c)^3}{7\cos(dx+c)^7}+\frac{4\sin(dx+c)^3}{35\cos(dx+c)^5}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^3}\right)+\frac{a^2b}{2\cos(dx+c)^6}-}{d}$
risch	$-\frac{16\left(-56ia^3e^{2i(dx+c)}-196ia^3e^{4i(dx+c)}-210a^2b\,e^{10i(dx+c)}+70b^3e^{10i(dx+c)}-245ia^3e^{8i(dx+c)}-7ia^3-420a^2b\,e^{8i(dx+c)}\right.}{}$

input
```
int(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(1/8*sin(d*x+c)^4/cos(d*x+c)^8+1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/2
4*sin(d*x+c)^4/cos(d*x+c)^4)+3*a*b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*s
in(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+1/2*a^2*b/cos(d*
x+c)^6-a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{105\,b^3+140\,(3\,a^2b-b^3)\cos(dx+c)^2+8\,(8\,(7\,a^3-3\,ab^2)\cos(dx+c)^7+4\,(7\,a^3-3\,ab^2)\cos(dx+c)^5-}{840\,d\cos(dx+c)^8}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/840*(105*b^3 + 140*(3*a^2*b - b^3)*cos(d*x + c)^2 + 8*(8*(7*a^3 - 3*a*b^
2)*cos(d*x + c)^7 + 4*(7*a^3 - 3*a*b^2)*cos(d*x + c)^5 + 45*a*b^2*cos(d*x
+ c) + 3*(7*a^3 - 3*a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^8
)
```

Sympy [F]

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan{(c+dx)})^3\sec^6{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**6*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{105\,b^3\tan{(dx+c)}^8+360\,ab^2\tan{(dx+c)}^7+140\,(3\,a^2b+2\,b^3)\tan{(dx+c)}^6+168\,(a^3+6\,ab^2)\tan{(dx+}}{}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/840*(105*b^3*tan(d*x + c)^8 + 360*a*b^2*tan(d*x + c)^7 + 140*(3*a^2*b +
2*b^3)*tan(d*x + c)^6 + 168*(a^3 + 6*a*b^2)*tan(d*x + c)^5 + 1260*a^2*b*ta
n(d*x + c)^2 + 210*(6*a^2*b + b^3)*tan(d*x + c)^4 + 840*a^3*tan(d*x + c) +
 280*(2*a^3 + 3*a*b^2)*tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{105\,b^3\tan\left(dx+c\right)^8+360\,ab^2\tan\left(dx+c\right)^7+420\,a^2b\tan\left(dx+c\right)^6+280\,b^3\tan\left(dx+c\right)^6+168\,a^3\tan\left(d\right.}{}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/840*(105*b^3*tan(d*x + c)^8 + 360*a*b^2*tan(d*x + c)^7 + 420*a^2*b*tan(d
*x + c)^6 + 280*b^3*tan(d*x + c)^6 + 168*a^3*tan(d*x + c)^5 + 1008*a*b^2*t
an(d*x + c)^5 + 1260*a^2*b*tan(d*x + c)^4 + 210*b^3*tan(d*x + c)^4 + 560*a
^3*tan(d*x + c)^3 + 840*a*b^2*tan(d*x + c)^3 + 1260*a^2*b*tan(d*x + c)^2 +
 840*a^3*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\tan(c+d\,x)^3\left(\frac{2\,a^3}{3}+a\,b^2\right)+\tan(c+d\,x)^5\left(\frac{a^3}{5}+\frac{6\,a\,b^2}{5}\right)+\tan(c+d\,x)^6\left(\frac{a^2\,b}{2}+\frac{b^3}{3}\right)+\tan(c+d\,x)^4\left(\frac{3\,a^2}{2}\right.}{d}$$

input
```
int((a + b*tan(c + d*x))^3/cos(c + d*x)^6,x)
```

output
```
(tan(c + d*x)^3*(a*b^2 + (2*a^3)/3) + tan(c + d*x)^5*((6*a*b^2)/5 + a^3/5)
 + tan(c + d*x)^6*((a^2*b)/2 + b^3/3) + tan(c + d*x)^4*((3*a^2*b)/2 + b^3/
4) + a^3*tan(c + d*x) + (b^3*tan(c + d*x)^8)/8 + (3*a^2*b*tan(c + d*x)^2)/
2 + (3*a*b^2*tan(c + d*x)^7)/7)/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.99

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\sin(dx+c)\left(-448\cos(dx+c)\sin(dx+c)^6a^3+192\cos(dx+c)\sin(dx+c)^6ab^2+1568\cos(dx+c)\sin\right.}{}$$

input
```
int(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x)
```

output
```
(sin(c + d*x)*( - 448*cos(c + d*x)*sin(c + d*x)**6*a**3 + 192*cos(c + d*x)
*sin(c + d*x)**6*a*b**2 + 1568*cos(c + d*x)*sin(c + d*x)**4*a**3 - 672*cos
(c + d*x)*sin(c + d*x)**4*a*b**2 - 1960*cos(c + d*x)*sin(c + d*x)**2*a**3
+ 840*cos(c + d*x)*sin(c + d*x)**2*a*b**2 + 840*cos(c + d*x)*a**3 - 420*si
n(c + d*x)**7*a**2*b + 35*sin(c + d*x)**7*b**3 + 1680*sin(c + d*x)**5*a**2
*b - 140*sin(c + d*x)**5*b**3 - 2520*sin(c + d*x)**3*a**2*b + 210*sin(c +
d*x)**3*b**3 + 1260*sin(c + d*x)*a**2*b))/(840*d*(sin(c + d*x)**8 - 4*sin(
c + d*x)**6 + 6*sin(c + d*x)**4 - 4*sin(c + d*x)**2 + 1))
```

3.538 $\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx=\frac{(a^2+b^2)\,(a+b\tan(c+dx))^4}{4b^3d}-\frac{2a(a+b\tan(c+dx))^5}{5b^3d}+\frac{(a+b\tan(c+dx))^6}{6b^3d}$$

output
```
1/4*(a^2+b^2)*(a+b*tan(d*x+c))^4/b^3/d-2/5*a*(a+b*tan(d*x+c))^5/b^3/d+1/6*
(a+b*tan(d*x+c))^6/b^3/d
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx=\frac{(a+b\tan(c+dx))^4\,(a^2+15b^2-4ab\tan(c+dx)+10b^2\tan^2(c+dx))}{60b^3d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

output
```
((a + b*Tan[c + d*x])^4*(a^2 + 15*b^2 - 4*a*b*Tan[c + d*x] + 10*b^2*Tan[c
+ d*x]^2))/(60*b^3*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4(a+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)}{b^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left((a+b\tan(c+dx))^5-2a(a+b\tan(c+dx))^4+\left(a^2+b^2\right)(a+b\tan(c+dx))^3\right)d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{4}\left(a^2+b^2\right)(a+b\tan(c+dx))^4+\frac{1}{6}(a+b\tan(c+dx))^6-\frac{2}{5}a(a+b\tan(c+dx))^5}{b^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

output
```
(((a^2 + b^2)*(a + b*Tan[c + d*x])^4)/4 - (2*a*(a + b*Tan[c + d*x])^5)/5 +
 (a + b*Tan[c + d*x])^6/6)/(b^3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 15.34 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.69

method	result
derivativedivides	$\frac{b^3\left(\frac{\sin(dx+c)^4}{6\cos(dx+c)^6}+\frac{\sin(dx+c)^4}{12\cos(dx+c)^4}\right)+3a\,b^2\left(\frac{\sin(dx+c)^3}{5\cos(dx+c)^5}+\frac{2\sin(dx+c)^3}{15\cos(dx+c)^3}\right)+\frac{3a^2b}{4\cos(dx+c)^4}-a^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+}{d}$
default	$\frac{b^3\left(\frac{\sin(dx+c)^4}{6\cos(dx+c)^6}+\frac{\sin(dx+c)^4}{12\cos(dx+c)^4}\right)+3a\,b^2\left(\frac{\sin(dx+c)^3}{5\cos(dx+c)^5}+\frac{2\sin(dx+c)^3}{15\cos(dx+c)^3}\right)+\frac{3a^2b}{4\cos(dx+c)^4}-a^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+}{d}$
risch	$-\frac{4(-15ia^3e^{8i(dx+c)}+45ia\,b^2e^{8i(dx+c)}-45a^2b\,e^{8i(dx+c)}+15b^3e^{8i(dx+c)}-50ia^3e^{6i(dx+c)}+30ia\,b^2e^{6i(dx+c)}-90a^2b\,e^{6}}{15d(e}$

input
```
int(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(1/6*sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4)+3*
a*b^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+3/4*a
^2*b/cos(d*x+c)^4-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.40

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{10\,b^3+15\,(3\,a^2b-b^3)\cos{(dx+c)}^2+4\,\left(2\,(5\,a^3-3\,ab^2)\cos{(dx+c)}^5+9\,ab^2\cos{(dx+c)}+(5\,a^3-3\,ab^2}{60\,d\cos{(dx+c)}^6}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/60*(10*b^3 + 15*(3*a^2*b - b^3)*cos(d*x + c)^2 + 4*(2*(5*a^3 - 3*a*b^2)*
cos(d*x + c)^5 + 9*a*b^2*cos(d*x + c) + (5*a^3 - 3*a*b^2)*cos(d*x + c)^3)*
sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan{(c+dx)})^3\sec^4{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**4*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.31

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{10\,b^3\tan\left(dx+c\right)^6+36\,ab^2\tan\left(dx+c\right)^5+90\,a^2b\tan\left(dx+c\right)^2+15\left(3\,a^2b+b^3\right)\tan\left(dx+c\right)^4+60\,a^3\,\mathrm{ta}}{60\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/60*(10*b^3*tan(d*x + c)^6 + 36*a*b^2*tan(d*x + c)^5 + 90*a^2*b*tan(d*x +
 c)^2 + 15*(3*a^2*b + b^3)*tan(d*x + c)^4 + 60*a^3*tan(d*x + c) + 20*(a^3
+ 3*a*b^2)*tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.49

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{10\,b^3\tan\left(dx+c\right)^6+36\,ab^2\tan\left(dx+c\right)^5+45\,a^2b\tan\left(dx+c\right)^4+15\,b^3\tan\left(dx+c\right)^4+20\,a^3\tan\left(dx+c\right.}{60\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(10*b^3*tan(d*x + c)^6 + 36*a*b^2*tan(d*x + c)^5 + 45*a^2*b*tan(d*x +
 c)^4 + 15*b^3*tan(d*x + c)^4 + 20*a^3*tan(d*x + c)^3 + 60*a*b^2*tan(d*x +
 c)^3 + 90*a^2*b*tan(d*x + c)^2 + 60*a^3*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{\tan(c+d\,x)^3\left(\frac{a^3}{3}+a\,b^2\right)+\tan(c+d\,x)^4\left(\frac{3\,a^2\,b}{4}+\frac{b^3}{4}\right)+a^3\tan(c+d\,x)+\frac{b^3\tan(c+d\,x)^6}{6}+\frac{3\,a^2\,b\tan(c+d\,x)^2}{2}+}{d}$$

input
```
int((a + b*tan(c + d*x))^3/cos(c + d*x)^4,x)
```

output
```
(tan(c + d*x)^3*(a*b^2 + a^3/3) + tan(c + d*x)^4*((3*a^2*b)/4 + b^3/4) + a
^3*tan(c + d*x) + (b^3*tan(c + d*x)^6)/6 + (3*a^2*b*tan(c + d*x)^2)/2 + (3
*a*b^2*tan(c + d*x)^5)/5)/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.65

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{\sin(dx+c)\left(-40\cos(dx+c)\sin(dx+c)^4a^3+24\cos(dx+c)\sin(dx+c)^4ab^2+100\cos(dx+c)\sin(\right.}{}$$

input
```
int(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x)
```

output
```
(sin(c + d*x)*( - 40*cos(c + d*x)*sin(c + d*x)**4*a**3 + 24*cos(c + d*x)*s
in(c + d*x)**4*a*b**2 + 100*cos(c + d*x)*sin(c + d*x)**2*a**3 - 60*cos(c +
 d*x)*sin(c + d*x)**2*a*b**2 - 60*cos(c + d*x)*a**3 - 45*sin(c + d*x)**5*a
**2*b + 5*sin(c + d*x)**5*b**3 + 135*sin(c + d*x)**3*a**2*b - 15*sin(c + d
*x)**3*b**3 - 90*sin(c + d*x)*a**2*b))/(60*d*(sin(c + d*x)**6 - 3*sin(c +
d*x)**4 + 3*sin(c + d*x)**2 - 1))
```

3.539 $\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{(a+b\tan(c+dx))^4}{4bd}$$

output
```
1/4*(a+b*tan(d*x+c))^4/b/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(22) = 44.

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.59

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\tan(c+dx)\left(4a^3+6a^2b\tan(c+dx)+4ab^2\tan^2(c+dx)+b^3\tan^3(c+dx)\right)}{4d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

output
```
(Tan[c + d*x]*(4*a^3 + 6*a^2*b*Tan[c + d*x] + 4*a*b^2*Tan[c + d*x]^2 + b^3
*Tan[c + d*x]^3))/(4*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3987, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int (a+b\tan(c+dx))^3d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+b\tan(c+dx))^4}{4bd}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

output
```
(a + b*Tan[c + d*x])^4/(4*b*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(20) = 40$.

Time = 4.44 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.27

method	result
derivativedivides	$\frac{\frac{b^3\sin(dx+c)^4}{4\cos(dx+c)^4}+\frac{a\,b^2\sin(dx+c)^3}{\cos(dx+c)^3}+\frac{3a^2b}{2\cos(dx+c)^2}+a^3\tan(dx+c)}{d}$
default	$\frac{\frac{b^3\sin(dx+c)^4}{4\cos(dx+c)^4}+\frac{a\,b^2\sin(dx+c)^3}{\cos(dx+c)^3}+\frac{3a^2b}{2\cos(dx+c)^2}+a^3\tan(dx+c)}{d}$
risch	$-\frac{2\left(-ia^3\mathrm{e}^{6i(dx+c)}+3ia\,b^2\mathrm{e}^{6i(dx+c)}-3a^2b\,\mathrm{e}^{6i(dx+c)}+b^3\mathrm{e}^{6i(dx+c)}-3ia^3\mathrm{e}^{4i(dx+c)}+3ia\,b^2\mathrm{e}^{4i(dx+c)}-6a^2b\,\mathrm{e}^{4i(dx+c)}-3\right.}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}$

input
```
int(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*b^3*sin(d*x+c)^4/cos(d*x+c)^4+a*b^2*sin(d*x+c)^3/cos(d*x+c)^3+3/2
*a^2*b/cos(d*x+c)^2+a^3*tan(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(20) = 40$.

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.55

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{b^3+2\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2+4\left(ab^2\cos\left(dx+c\right)+\left(a^3-ab^2\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^4}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(b^3 + 2*(3*a^2*b - b^3)*cos(d*x + c)^2 + 4*(a*b^2*cos(d*x + c) + (a^3
 - a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{(b\tan(dx+c)+a)^4}{4\,bd}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/4*(b*tan(d*x + c) + a)^4/(b*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(20) = 40.

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.59

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b^3\tan(dx+c)^4+4\,ab^2\tan(dx+c)^3+6\,a^2b\tan(dx+c)^2+4\,a^3\tan(dx+c)}{4\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/4*(b^3*tan(d*x + c)^4 + 4*a*b^2*tan(d*x + c)^3 + 6*a^2*b*tan(d*x + c)^2
+ 4*a^3*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.50

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{a^3\tan(c+d\,x)+\frac{3\,a^2\,b\,\tan(c+d\,x)^2}{2}+a\,b^2\tan(c+d\,x)^3+\frac{b^3\tan(c+d\,x)^4}{4}}{d}$$

input
```
int((a + b*tan(c + d*x))^3/cos(c + d*x)^2,x)
```

output
```
(a^3*tan(c + d*x) + (b^3*tan(c + d*x)^4)/4 + (3*a^2*b*tan(c + d*x)^2)/2 +
a*b^2*tan(c + d*x)^3)/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 5.55

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\sin(dx+c)\left(-4\cos(dx+c)\sin(dx+c)^2a^3+4\cos(dx+c)\sin(dx+c)^2ab^2+4\cos(dx+c)a^3-6\sin\right.}{4d\left(\sin(dx+c)^4-2\sin(dx+c)^2+1\right)}$$

input
```
int(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x)
```

output
```
(sin(c + d*x)*( - 4*cos(c + d*x)*sin(c + d*x)**2*a**3 + 4*cos(c + d*x)*sin
(c + d*x)**2*a*b**2 + 4*cos(c + d*x)*a**3 - 6*sin(c + d*x)**3*a**2*b + sin
(c + d*x)**3*b**3 + 6*sin(c + d*x)*a**2*b))/(4*d*(sin(c + d*x)**4 - 2*sin(
c + d*x)**2 + 1))
```

3.540 $\int (a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 72

$$\int (a+b\tan(c+dx))^3\,dx = a(a^2-3b^2)\,x - \frac{b(3a^2-b^2)\log(\cos(c+dx))}{d} + \frac{2ab^2\tan(c+dx)}{d} + \frac{b(a+b\tan(c+dx))^2}{2d}$$

output
```
a*(a^2-3*b^2)*x-b*(3*a^2-b^2)*ln(cos(d*x+c))/d+2*a*b^2*tan(d*x+c)/d+1/2*b*
(a+b*tan(d*x+c))^2/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int (a+b\tan(c+dx))^3\,dx = \frac{(ia-b)^3\log(i-\tan(c+dx)) - (ia+b)^3\log(i+\tan(c+dx)) + 6ab^2\tan(c+dx) + b^3\tan^2(c+dx)}{2d}$$

input
```
Integrate[(a + b*Tan[c + d*x])^3,x]
```

output
```
((I*a - b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]] + 6
*a*b^2*Tan[c + d*x] + b^3*Tan[c + d*x]^2)/(2*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3963, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a+b\tan(c+dx))^3dx$$
$$\Big\downarrow \text{3963}$$
$$\int (a+b\tan(c+dx))\left(a^2+2b\tan(c+dx)a-b^2\right)dx+\frac{b(a+b\tan(c+dx))^2}{2d}$$
$$\Big\downarrow \text{3042}$$
$$\int (a+b\tan(c+dx))\left(a^2+2b\tan(c+dx)a-b^2\right)dx+\frac{b(a+b\tan(c+dx))^2}{2d}$$
$$\Big\downarrow \text{4008}$$
$$b\left(3a^2-b^2\right)\int \tan(c+dx)dx+ax\left(a^2-3b^2\right)+\frac{2ab^2\tan(c+dx)}{d}+\frac{b(a+b\tan(c+dx))^2}{2d}$$
$$\Big\downarrow \text{3042}$$
$$b\left(3a^2-b^2\right)\int \tan(c+dx)dx+ax\left(a^2-3b^2\right)+\frac{2ab^2\tan(c+dx)}{d}+\frac{b(a+b\tan(c+dx))^2}{2d}$$
$$\Big\downarrow \text{3956}$$
$$-\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+ax\left(a^2-3b^2\right)+\frac{2ab^2\tan(c+dx)}{d}+\frac{b(a+b\tan(c+dx))^2}{2d}$$

input
```
Int[(a + b*Tan[c + d*x])^3,x]
```

output
```
a*(a^2 - 3*b^2)*x - (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (2*a*b^2*Tan[c
 + d*x])/d + (b*(a + b*Tan[c + d*x])^2)/(2*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3963
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

method	result
norman	$\left(a^{3}-3 a \,b^{2}\right) x +\frac{b^{3} \tan \left(d x +c \right)^{2}}{2 d}+\frac{3 a \,b^{2} \tan \left(d x +c \right)}{d}+\frac{b \left(3 a^{2}-b^{2}\right) \ln \left(1+\tan \left(d x +c \right)^{2}\right)}{2 d}$
derivativedivides	$\frac{\frac{\tan \left(d x +c \right)^{2} b^{3}}{2}+3 \tan \left(d x +c \right) a \,b^{2}+\frac{\left(3 a^{2} b -b^{3}\right) \ln \left(1+\tan \left(d x +c \right)^{2}\right)}{2}+\left(a^{3}-3 a \,b^{2}\right) \arctan \left(\tan \left(d x +c \right)\right)}{d}$
default	$\frac{\frac{\tan \left(d x +c \right)^{2} b^{3}}{2}+3 \tan \left(d x +c \right) a \,b^{2}+\frac{\left(3 a^{2} b -b^{3}\right) \ln \left(1+\tan \left(d x +c \right)^{2}\right)}{2}+\left(a^{3}-3 a \,b^{2}\right) \arctan \left(\tan \left(d x +c \right)\right)}{d}$
parallelrisch	$\frac{2 a^{3} d x -6 a \,b^{2} d x +\tan \left(d x +c \right)^{2} b^{3}+3 \ln \left(1+\tan \left(d x +c \right)^{2}\right) a^{2} b -\ln \left(1+\tan \left(d x +c \right)^{2}\right) b^{3}+6 \tan \left(d x +c \right) a \,b^{2}}{2 d}$
parts	$a^{3} x +\frac{b^{3} \left(\frac{\tan \left(d x +c \right)^{2}}{2}-\frac{\ln \left(1+\tan \left(d x +c \right)^{2}\right)}{2}\right)}{d}+\frac{3 a^{2} b \ln \left(1+\tan \left(d x +c \right)^{2}\right)}{2 d}+\frac{3 a \,b^{2} \left(\tan \left(d x +c \right)-\arctan \left(\tan \left(d x +c \right)\right)\right)}{d}$
risch	$3 i a^{2} b x -i b^{3} x +a^{3} x -3 a \,b^{2} x +\frac{6 i b \,a^{2} c}{d}-\frac{2 i b^{3} c}{d}+\frac{2 b^{2} \left(3 i a \,{\mathrm e}^{2 i \left(d x +c \right)}+b \,{\mathrm e}^{2 i \left(d x +c \right)}+3 i a \right)}{d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{2}}-\frac{3 b \ln \left({\mathrm e}^{2 i \left(d x\right.\right.}}{d}$

input
```
int((a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
(a^3-3*a*b^2)*x+1/2*b^3*tan(d*x+c)^2/d+3*a*b^2*tan(d*x+c)/d+1/2*b*(3*a^2-b
^2)/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int (a+b\tan(c+dx))^3\,dx = \frac{b^{3} \tan \left(d x + c\right)^{2} + 6 \, a b^{2} \tan \left(d x + c\right) + 2 \, {\left(a^{3} - 3 \, a b^{2}\right)} d x - {\left(3 \, a^{2} b - b^{3}\right)} \log \left(\frac{1}{\tan \left(d x + c\right)^{2} + 1}\right)}{2 \, d}$$

input
```
integrate((a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(b^3*tan(d*x + c)^2 + 6*a*b^2*tan(d*x + c) + 2*(a^3 - 3*a*b^2)*d*x - (
3*a^2*b - b^3)*log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.31

$$\int (a+b\tan(c+dx))^3\,dx$$
$$=\begin{cases} a^3x+\frac{3a^2b\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}-3ab^2x+\frac{3ab^2\tan\left(c+dx\right)}{d}-\frac{b^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{b^3\tan^2\left(c+dx\right)}{2d} & \text{for } d\neq 0\\ x(a+b\tan\left(c\right))^3 & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*tan(d*x+c))**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) - 3*a*b**2*x +
 3*a*b**2*tan(c + d*x)/d - b**3*log(tan(c + d*x)**2 + 1)/(2*d) + b**3*tan(
c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c))**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08

$$\int (a+b\tan(c+dx))^3\,dx=a^3x-\frac{3\left(dx+c-\tan\left(dx+c\right)\right)ab^2}{d}-\frac{b^3\left(\frac{1}{\sin\left(dx+c\right)^2-1}-\log\left(\sin\left(dx+c\right)^2-1\right)\right)}{2\,d}+\frac{3\,a^2b\log\left(\sec\left(dx+c\right)\right)}{d}$$

input
```
integrate((a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
a^3*x - 3*(d*x + c - tan(d*x + c))*a*b^2/d - 1/2*b^3*(1/(sin(d*x + c)^2 -
1) - log(sin(d*x + c)^2 - 1))/d + 3*a^2*b*log(sec(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11

$$\int (a+b\tan(c+dx))^3\,dx = \frac{(a^3-3\,ab^2)(dx+c)}{d}+\frac{(3\,a^2b-b^3)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}+\frac{b^3d\tan\left(dx+c\right)^2+6\,ab^2d\tan\left(dx+c\right)}{2\,d^2}$$

input
```
integrate((a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
(a^3 - 3*a*b^2)*(d*x + c)/d + 1/2*(3*a^2*b - b^3)*log(tan(d*x + c)^2 + 1)/
d + 1/2*(b^3*d*tan(d*x + c)^2 + 6*a*b^2*d*tan(d*x + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.47

$$\int (a+b\tan(c+dx))^3\,dx = \frac{b^3\tan(c+d\,x)^2}{2\,d}+\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{3\,a^2\,b}{2}-\frac{b^3}{2}\right)}{d}+\frac{3\,a\,b^2\tan(c+d\,x)}{d}-\frac{a\,\mathrm{atan}\left(\frac{a\tan(c+d\,x)\,\left(a^2-3\,b^2\right)}{3\,a\,b^2-a^3}\right)\,\left(a^2-3\,b^2\right)}{d}$$

input
```
int((a + b*tan(c + d*x))^3,x)
```

output
```
(b^3*tan(c + d*x)^2)/(2*d) + (log(tan(c + d*x)^2 + 1)*((3*a^2*b)/2 - b^3/2
))/d + (3*a*b^2*tan(c + d*x))/d - (a*atan((a*tan(c + d*x)*(a^2 - 3*b^2))/(
3*a*b^2 - a^3))*(a^2 - 3*b^2))/d
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08

$$\int (a+b\tan(c+dx))^3\,dx$$
$$=\frac{3\log\left(\tan\left(dx+c\right)^2+1\right)a^2b-\log\left(\tan\left(dx+c\right)^2+1\right)b^3+\tan\left(dx+c\right)^2b^3+6\tan\left(dx+c\right)a\,b^2+2a^3dx}{2d}$$

input
```
int((a+b*tan(d*x+c))^3,x)
```

output
```
(3*log(tan(c + d*x)**2 + 1)*a**2*b - log(tan(c + d*x)**2 + 1)*b**3 + tan(c
 + d*x)**2*b**3 + 6*tan(c + d*x)*a*b**2 + 2*a**3*d*x - 6*a*b**2*d*x)/(2*d)
```

3.541 $\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{1}{2}a\left(a^2+3b^2\right)x - \frac{b^3\log(\cos(c+dx))}{d} - \frac{3ab^2\tan(c+dx)}{2d} - \frac{b^3\tan^2(c+dx)}{2d} + \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{2d}$$

output
```
1/2*a*(a^2+3*b^2)*x-b^3*ln(cos(d*x+c))/d-3/2*a*b^2*tan(d*x+c)/d-1/2*b^3*ta
n(d*x+c)^2/d+1/2*cos(d*x+c)*sin(d*x+c)*(a+b*tan(d*x+c))^3/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 401 vs. 2(97) = 194.

Time = 0.51 (sec) , antiderivative size = 401, normalized size of antiderivative = 4.13

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{5a^4b^2+2a^2b^4-b^6+\left(-3a^4b^2-2a^2b^4+b^6\right)\cos(2(c+dx))+2a^2b^4\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+2b^6\log}{}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

output
```
(5*a^4*b^2 + 2*a^2*b^4 - b^6 + (-3*a^4*b^2 - 2*a^2*b^4 + b^6)*Cos[2*(c + d
*x)] + 2*a^2*b^4*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 2*b^6*Log[Sqrt[-b^2] -
 b*Tan[c + d*x]] - a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 4*a^3
*(-b^2)^(3/2)*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 3*a*(-b^2)^(5/2)*Log[Sqrt
[-b^2] - b*Tan[c + d*x]] + 2*a^2*b^4*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 2*
b^6*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*T
an[c + d*x]] + 3*a*b^4*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c + d*x]] - 4*a^3
*(-b^2)^(3/2)*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + a*b*(a^4 - 2*a^2*b^2 - 3*
b^4)*Sin[2*(c + d*x)])/(4*b*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3987, 27, 495, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3 \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(c+dx))^3}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{ 3987}$$

$$\frac{\int \frac{b^4(a+b\tan(c+dx))^3}{(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b^3\int \frac{(a+b\tan(c+dx))^3}{(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 495}$$

$$\frac{b^3\left(\frac{\int \frac{(a+b\tan(c+dx))\left(a^2-b\tan(c+dx)a+2b^2\right)}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2}-\frac{(a+b\tan(c+dx))^2(b^2-ab\tan(c+dx))}{2b^2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \textbf{657}$$

$$\frac{b^3\left(\frac{\int \left(\frac{a^3+3b^2a+2b^3\tan(c+dx)}{\tan^2(c+dx)b^2+b^2}-a\right)d(b\tan(c+dx))}{2b^2}-\frac{(a+b\tan(c+dx))^2(b^2-ab\tan(c+dx))}{2b^2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{b^3\left(\frac{\frac{a\left(a^2+3b^2\right)\arctan(\tan(c+dx))}{b}-ab\tan(c+dx)+b^2\log(b^2\tan^2(c+dx)+b^2)}{2b^2}-\frac{(a+b\tan(c+dx))^2(b^2-ab\tan(c+dx))}{2b^2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

output
```
(b^3*(((a*(a^2 + 3*b^2)*ArcTan[Tan[c + d*x]])/b + b^2*Log[b^2 + b^2*Tan[c
+ d*x]^2] - a*b*Tan[c + d*x])/(2*b^2) - ((a + b*Tan[c + d*x])^2*(b^2 - a*b
*Tan[c + d*x]))/(2*b^2*(b^2 + b^2*Tan[c + d*x]^2))))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{b^3\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)+3a\,b^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{3a^2b\cos(dx+c)^2}{2}+a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\right.}{d}$
default	$\frac{b^3\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)+3a\,b^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{3a^2b\cos(dx+c)^2}{2}+a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\right.}{d}$
risch	$ib^3x+\frac{a^3x}{2}+\frac{3a\,b^2x}{2}-\frac{3\,e^{2i(dx+c)}a^2b}{8d}+\frac{e^{2i(dx+c)}b^3}{8d}-\frac{ie^{2i(dx+c)}a^3}{8d}+\frac{3ie^{2i(dx+c)}a\,b^2}{8d}-\frac{3\,e^{-2i(dx+c)}a^2b}{8d}$

input
```
int(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+3*a*b^2*(-1/2*cos(d*x+c)*sin(d
*x+c)+1/2*d*x+1/2*c)-3/2*a^2*b*cos(d*x+c)^2+a^3*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx =$$
$$-\frac{2\,b^3\log\left(-\cos\left(dx+c\right)\right)-(a^3+3\,ab^2)dx+(3\,a^2b-b^3)\cos\left(dx+c\right)^2-(a^3-3\,ab^2)\cos\left(dx+c\right)\sin\left(d\right.}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2*(2*b^3*log(-cos(d*x + c)) - (a^3 + 3*a*b^2)*d*x + (3*a^2*b - b^3)*cos
(d*x + c)^2 - (a^3 - 3*a*b^2)*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan{(c+dx)})^3\cos^2{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b^3\log\left(\tan\left(dx+c\right)^2+1\right)+(a^3+3\,ab^2)(dx+c)-\frac{3\,a^2b-b^3-(a^3-3\,ab^2)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(b^3*log(tan(d*x + c)^2 + 1) + (a^3 + 3*a*b^2)*(d*x + c) - (3*a^2*b -
b^3 - (a^3 - 3*a*b^2)*tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b^3\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d} + \frac{(a^3+3\,ab^2)(dx+c)}{2\,d} - \frac{3\,a^2b-b^3-(a^3-3\,ab^2)\tan\left(dx+c\right)}{2\left(\tan\left(dx+c\right)^2+1\right)d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*b^3*log(tan(d*x + c)^2 + 1)/d + 1/2*(a^3 + 3*a*b^2)*(d*x + c)/d - 1/2*
(3*a^2*b - b^3 - (a^3 - 3*a*b^2)*tan(d*x + c))/((tan(d*x + c)^2 + 1)*d)
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.45

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b^3\,\ln\left(\frac{1}{\cos(c+d\,x)^2}\right)}{2\,d} + \frac{b^3\cos\left(c+d\,x\right)^2}{2\,d} + \frac{a^3\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)}{2\,d} - \frac{3\,a^2\,b\cos\left(c+d\,x\right)^2}{2\,d} + \frac{3\,a\,b^2\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)}{2\,d} + \frac{a^3\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d} - \frac{3\,a\,b^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}$$

input
```
int(cos(c + d*x)^2*(a + b*tan(c + d*x))^3,x)
```

output
```
(b^3*log(1/cos(c + d*x)^2))/(2*d) + (b^3*cos(c + d*x)^2)/(2*d) + (a^3*atan
(sin(c + d*x)/cos(c + d*x)))/(2*d) - (3*a^2*b*cos(c + d*x)^2)/(2*d) + (3*a
*b^2*atan(sin(c + d*x)/cos(c + d*x)))/(2*d) + (a^3*cos(c + d*x)*sin(c + d*
x))/(2*d) - (3*a*b^2*cos(c + d*x)*sin(c + d*x))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.51

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\cos(dx+c)\sin(dx+c)\,a^3-3\cos(dx+c)\sin(dx+c)\,a\,b^2+2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)b^3-2\log\left(\tan\left(\right.\right.}{}$$

input
```
int(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x)
```

output
```
(cos(c + d*x)*sin(c + d*x)*a**3 - 3*cos(c + d*x)*sin(c + d*x)*a*b**2 + 2*l
og(tan((c + d*x)/2)**2 + 1)*b**3 - 2*log(tan((c + d*x)/2) - 1)*b**3 - 2*lo
g(tan((c + d*x)/2) + 1)*b**3 + 3*sin(c + d*x)**2*a**2*b - sin(c + d*x)**2*
b**3 + a**3*c + a**3*d*x + 3*a*b**2*c + 3*a*b**2*d*x)/(2*d)
```

3.542 $\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3}{8}a\left(a^2+b^2\right)x-\frac{3a\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{8d}+\frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{4d}$$

output
```
3/8*a*(a^2+b^2)*x-3/8*a*cos(d*x+c)^2*(b-a*tan(d*x+c))*(a+b*tan(d*x+c))/d+1
/4*cos(d*x+c)^3*sin(d*x+c)*(a+b*tan(d*x+c))^3/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 488 vs. $2(84) = 168$.

Time = 0.79 (sec) , antiderivative size = 488, normalized size of antiderivative = 5.81

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{39a^6b^2+41a^4b^4+21a^2b^6+3b^8-4b^2\left(a^2+b^2\right)^2\left(3a^2+b^2\right)\cos(2(c+dx))+b^2\left(-3a^2+b^2\right)\left(a^2+b^2\right)^2\cos}{}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

output
```
(39*a^6*b^2 + 41*a^4*b^4 + 21*a^2*b^6 + 3*b^8 - 4*b^2*(a^2 + b^2)^2*(3*a^2
 + b^2)*Cos[2*(c + d*x)] + b^2*(-3*a^2 + b^2)*(a^2 + b^2)^2*Cos[4*(c + d*x
)] - 6*a^7*Sqrt[-b^2]*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 18*a^5*(-b^2)^(3/
2)*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 6*a*b^4*(-b^2)^(3/2)*Log[Sqrt[-b^2]
- b*Tan[c + d*x]] - 18*a^3*(-b^2)^(5/2)*Log[Sqrt[-b^2] - b*Tan[c + d*x]] +
 6*a^7*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 18*a^3*b^4*Sqrt[-b^2]
*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 6*a*b^6*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*
Tan[c + d*x]] - 18*a^5*(-b^2)^(3/2)*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 8*a
^7*b*Sin[2*(c + d*x)] + 16*a^5*b^3*Sin[2*(c + d*x)] + 8*a^3*b^5*Sin[2*(c +
 d*x)] + a^7*b*Sin[4*(c + d*x)] - a^5*b^3*Sin[4*(c + d*x)] - 5*a^3*b^5*Sin
[4*(c + d*x)] - 3*a*b^7*Sin[4*(c + d*x)])/(32*b*(a^2 + b^2)^2*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.55, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3987, 27, 490, 487, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3 \, dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+b\tan(c+dx))^3}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \ 3987$$

$$\frac{\int \frac{b^6(a+b\tan(c+dx))^3}{(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \ 27$$

$$\frac{b^5\int \frac{(a+b\tan(c+dx))^3}{(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{490}$$

$$\frac{b^5\left(\frac{3a\int \frac{(a+b\tan(c+dx))^2}{\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2}+\frac{\tan(c+dx)(a+b\tan(c+dx))^3}{4b(b^2\tan^2(c+dx)+b^2)^2}\right)}{d}$$

$$\Big\downarrow \text{487}$$

$$\frac{b^5\left(\frac{3a\left(\frac{\left(a^2+b^2\right)\int \frac{1}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2}-\frac{(a+b\tan(c+dx))\left(b^2-ab\tan(c+dx)\right)}{2b^2\left(b^2\tan^2(c+dx)+b^2\right)}\right)}{4b^2}+\frac{\tan(c+dx)(a+b\tan(c+dx))^3}{4b(b^2\tan^2(c+dx)+b^2)^2}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{b^5\left(\frac{3a\left(\frac{\left(a^2+b^2\right)\arctan(\tan(c+dx))}{2b^3}-\frac{(a+b\tan(c+dx))\left(b^2-ab\tan(c+dx)\right)}{2b^2\left(b^2\tan^2(c+dx)+b^2\right)}\right)}{4b^2}+\frac{\tan(c+dx)(a+b\tan(c+dx))^3}{4b(b^2\tan^2(c+dx)+b^2)^2}\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

output
```
(b^5*((Tan[c + d*x]*(a + b*Tan[c + d*x])^3)/(4*b*(b^2 + b^2*Tan[c + d*x]^2
)^2) + (3*a*(((a^2 + b^2)*ArcTan[Tan[c + d*x]])/(2*b^3) - ((a + b*Tan[c +
d*x])*(b^2 - a*b*Tan[c + d*x]))/(2*b^2*(b^2 + b^2*Tan[c + d*x]^2))))/(4*b^
2))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 487
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n - 1)*(a*d - b*c*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] +
 Simp[(2*p + 3)*((b*c^2 + a*d^2)/(2*a*b*(p + 1)))   Int[(c + d*x)^(n - 2)*(
a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0]
 && LtQ[p, -1]
```

rule 490
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] - Simp[c*(n/(2*a*(
p + 1)))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 17.80 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{\frac{b^3 \sin(dx+c)^4}{4}+3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{3a^2 b\cos(dx+c)^4}{4}+a^3 (\frac{(\cos(dx+c)^3+\frac{3\cos(\cdots}{\cdots}}{4}\cdots}{d}$
default	$\frac{\frac{b^3 \sin(dx+c)^4}{4}+3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{3a^2 b\cos(dx+c)^4}{4}+a^3 (\frac{(\cos(dx+c)^3+\frac{3\cos(\cdots}{\cdots}}{4}\cdots}{d}$
risch	$\frac{3a^3 x}{8}+\frac{3a\,b^2 x}{8}-\frac{3b\cos(4dx+4c)a^2}{32d}+\frac{b^3\cos(4dx+4c)}{32d}+\frac{a^3\sin(4dx+4c)}{32d}-\frac{3a\sin(4dx+4c)b^2}{32d}-\frac{3b\cos(2dx+\cdots}{8d}$

input
```
int(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*b^3*sin(d*x+c)^4+3*a*b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*
x+c)*sin(d*x+c)+1/8*d*x+1/8*c)-3/4*a^2*b*cos(d*x+c)^4+a^3*(1/4*(cos(d*x+c)
^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx =$$
$$-\frac{4\,b^3\cos\left(dx+c\right)^2+2\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^4-3\left(a^3+ab^2\right)dx-\left(2\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^3+3\left(a^3\right.\right.}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/8*(4*b^3*cos(d*x + c)^2 + 2*(3*a^2*b - b^3)*cos(d*x + c)^4 - 3*(a^3 + a
*b^2)*d*x - (2*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 3*(a^3 + a*b^2)*cos(d*x +
c))*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^3\cos^4\left(c+dx\right)\,dx$$

input
```
integrate(cos(d*x+c)**4*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.31

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{3\,(a^3+ab^2)(dx+c)-\frac{4\,b^3\tan(dx+c)^2-3\,(a^3+ab^2)\tan(dx+c)^3+6\,a^2b+2\,b^3-(5\,a^3-3\,ab^2)\tan(dx+c)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/8*(3*(a^3 + a*b^2)*(d*x + c) - (4*b^3*tan(d*x + c)^2 - 3*(a^3 + a*b^2)*t
an(d*x + c)^3 + 6*a^2*b + 2*b^3 - (5*a^3 - 3*a*b^2)*tan(d*x + c))/(tan(d*x
 + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.33

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx=\frac{3\,(a^3+ab^2)(dx+c)}{8\,d}$$
$$+\frac{3\,a^3\tan\left(dx+c\right)^3+3\,ab^2\tan\left(dx+c\right)^3-4\,b^3\tan\left(dx+c\right)^2+5\,a^3\tan\left(dx+c\right)-3\,ab^2\tan\left(dx+c\right)-6}{8\left(\tan\left(dx+c\right)^2+1\right)^2d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
3/8*(a^3 + a*b^2)*(d*x + c)/d + 1/8*(3*a^3*tan(d*x + c)^3 + 3*a*b^2*tan(d*
x + c)^3 - 4*b^3*tan(d*x + c)^2 + 5*a^3*tan(d*x + c) - 3*a*b^2*tan(d*x + c
) - 6*a^2*b - 2*b^3)/((tan(d*x + c)^2 + 1)^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.30

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3\,a^3\,x}{8} - \frac{6\,a^2\,b-\tan(c+d\,x)^3\,\left(3\,a^3+3\,a\,b^2\right)+2\,b^3+\tan(c+d\,x)\,\left(3\,a\,b^2-5\,a^3\right)+4\,b^3\,\tan(c+d\,x)^2}{d\,\left(8\,\tan\left(c+d\,x\right)^4+16\,\tan\left(c+d\,x\right)^2+8\right)} + \frac{3\,a\,b^2\,x}{8}$$

input
```
int(cos(c + d*x)^4*(a + b*tan(c + d*x))^3,x)
```

output
```
(3*a^3*x)/8 - (6*a^2*b - tan(c + d*x)^3*(3*a*b^2 + 3*a^3) + 2*b^3 + tan(c
+ d*x)*(3*a*b^2 - 5*a^3) + 4*b^3*tan(c + d*x)^2)/(d*(16*tan(c + d*x)^2 + 8
*tan(c + d*x)^4 + 8)) + (3*a*b^2*x)/8
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.62

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{-2\cos\left(dx+c\right)\sin\left(dx+c\right)^3a^3+6\cos\left(dx+c\right)\sin\left(dx+c\right)^3a\,b^2+5\cos\left(dx+c\right)\sin\left(dx+c\right)a^3-3\cos}{8}$$

input
```
int(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x)
```

output
```
( - 2*cos(c + d*x)*sin(c + d*x)**3*a**3 + 6*cos(c + d*x)*sin(c + d*x)**3*a
*b**2 + 5*cos(c + d*x)*sin(c + d*x)*a**3 - 3*cos(c + d*x)*sin(c + d*x)*a*b
**2 - 6*sin(c + d*x)**4*a**2*b + 2*sin(c + d*x)**4*b**3 + 12*sin(c + d*x)*
*2*a**2*b + 3*a**3*d*x + 3*a*b**2*d*x)/(8*d)
```

3.543 $\int \cos^6(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 147

$$\begin{aligned}
&\int \cos^6(c+dx)(a+b\tan(c+dx))^3\,dx \\
&= \frac{1}{16}a\left(5a^2+3b^2\right)x-\frac{b(5a^2+b^2)\cos^2(c+dx)}{12d}+\frac{a(15a^2-b^2)\cos(c+dx)\sin(c+dx)}{48d} \\
&\quad-\frac{\cos^4(c+dx)(2b-5a\tan(c+dx))(a+b\tan(c+dx))^2}{24d} \\
&\quad+\frac{\cos^5(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{6d}
\end{aligned}$$

output
```
1/16*a*(5*a^2+3*b^2)*x-1/12*b*(5*a^2+b^2)*cos(d*x+c)^2/d+1/48*a*(15*a^2-b^
2)*cos(d*x+c)*sin(d*x+c)/d-1/24*cos(d*x+c)^4*(2*b-5*a*tan(d*x+c))*(a+b*tan
(d*x+c))^2/d+1/6*cos(d*x+c)^5*sin(d*x+c)*(a+b*tan(d*x+c))^3/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 390 vs. $2(147) = 294$.

Time = 5.63 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.65

$$\int \cos^6(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$= \frac{\frac{8b(2a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^4}{a^2+b^2} + 16\cos^6(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^4 - \frac{(-5a^3-3ab^2)\left(4\,c\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[Cos[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

output
```
((8*b*(2*a^2 + b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)/(a^2 + b^2) + 16*
Cos[c + d*x]^6*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^4 - ((-5*a^3 - 3*
a*b^2)*(4*Cos[c + d*x]^4*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^5 - (2*
Cos[c + d*x]^2*(a + b*Tan[c + d*x])^5*(-5*a^2*b + b^3 - 3*a*(a^2 - b^2)*Ta
n[c + d*x]))/(a^2 + b^2) + (b*((3*(a^2 + b^2)^4*(Log[Sqrt[-b^2] - b*Tan[c
+ d*x]] - Log[Sqrt[-b^2] + b*Tan[c + d*x]]))/Sqrt[-b^2] - 6*b*(10*a^6 + 5*
a^4*b^2 + 4*a^2*b^4 + b^6)*Tan[c + d*x] + 12*a^3*b^2*(-5*a^2 + b^2)*Tan[c
+ d*x]^2 + 2*b^3*(-15*a^4 + 10*a^2*b^2 + b^4)*Tan[c + d*x]^3 + 6*a*b^4*(-a
^2 + b^2)*Tan[c + d*x]^4))/(a^2 + b^2)))/(a^2 + b^2)^2)/(96*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.46, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3987, 27, 494, 25, 685, 675, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{ 3987}$$

$$\frac{\int \frac{b^8(a+b\tan(c+dx))^3}{\left(\tan^2(c+dx)b^2+b^2\right)^4}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b^7\int \frac{(a+b\tan(c+dx))^3}{\left(\tan^2(c+dx)b^2+b^2\right)^4}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 494}$$

$$\frac{b^7\left(\frac{\tan(c+dx)(a+b\tan(c+dx))^3}{6b\left(b^2\tan^2(c+dx)+b^2\right)^3}-\frac{\int -\frac{(a+b\tan(c+dx))^2(5a+2b\tan(c+dx))}{\left(\tan^2(c+dx)b^2+b^2\right)^3}d(b\tan(c+dx))}{6b^2}\right)}{d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b^7\left(\frac{\int \frac{(a+b\tan(c+dx))^2(5a+2b\tan(c+dx))}{\left(\tan^2(c+dx)b^2+b^2\right)^3}d(b\tan(c+dx))}{6b^2}+\frac{\tan(c+dx)(a+b\tan(c+dx))^3}{6b\left(b^2\tan^2(c+dx)+b^2\right)^3}\right)}{d}$$

$$\Big\downarrow \text{ 685}$$

$$\frac{b^7\left(\frac{\frac{\int \frac{(a+b\tan(c+dx))\left(15a^2+5b\tan(c+dx)a+4b^2\right)}{\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2}-\frac{(a+b\tan(c+dx))^2\left(2b^2-5ab\tan(c+dx)\right)}{4b^2\left(b^2\tan^2(c+dx)+b^2\right)^2}}{6b^2}+\frac{\tan(c+dx)(a+b\tan(c+dx))^3}{6b\left(b^2\tan^2(c+dx)+b^2\right)^3}\right)}{d}$$

$$\Big\downarrow \text{ 675}$$

$$\frac{b^7\left(\frac{\frac{\frac{3}{2}a\left(\frac{5a^2}{b^2}+3\right)\int \frac{1}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))-\frac{2\left(5a^2+b^2\right)}{b^2\tan^2(c+dx)+b^2}-\frac{ab\left(1-\frac{15a^2}{b^2}\right)\tan(c+dx)}{2\left(b^2\tan^2(c+dx)+b^2\right)}}{4b^2}-\frac{(a+b\tan(c+dx))^2\left(2b^2-5ab\tan(c+dx)\right)}{4b^2\left(b^2\tan^2(c+dx)+b^2\right)^2}}{6b^2}+\frac{\tan(c+d}{6b(b^2}\right.}{d}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{b^7\left(\frac{\frac{\frac{3a\left(\frac{5a^2}{b^2}+3\right)\arctan(\tan(c+dx))}{2b}-\frac{2\left(5a^2+b^2\right)}{b^2\tan^2(c+dx)+b^2}-\frac{ab\left(1-\frac{15a^2}{b^2}\right)\tan(c+dx)}{2\left(b^2\tan^2(c+dx)+b^2\right)}}{4b^2}-\frac{(a+b\tan(c+dx))^2\left(2b^2-5ab\tan(c+dx)\right)}{4b^2\left(b^2\tan^2(c+dx)+b^2\right)^2}}{6b^2}+\frac{\tan(c+dx)(a+b\tan(c+}{6b(b^2\tan^2(c+dx)+b^2}\right.}{d}$$

input
```
Int[Cos[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

output
```
(b^7*((Tan[c + d*x]*(a + b*Tan[c + d*x])^3)/(6*b*(b^2 + b^2*Tan[c + d*x]^2
)^3) + (-1/4*((a + b*Tan[c + d*x])^2*(2*b^2 - 5*a*b*Tan[c + d*x]))/(b^2*(b
^2 + b^2*Tan[c + d*x]^2)^2) + ((3*a*(3 + (5*a^2)/b^2)*ArcTan[Tan[c + d*x]]
)/(2*b) - (2*(5*a^2 + b^2))/(b^2 + b^2*Tan[c + d*x]^2) - (a*(1 - (15*a^2)/
b^2)*b*Tan[c + d*x])/(2*(b^2 + b^2*Tan[c + d*x]^2)))/(4*b^2))/(6*b^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 494
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[1/(2*a*(p +
 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1)*(c*(2*p + 3) + d*(n + 2*p
+ 3)*x), x], x] /; FreeQ[{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 0] && (Lt
Q[n, 1] || (ILtQ[n + 2*p + 3, 0] && NeQ[n, 2])) && IntQuadraticQ[a, 0, b, c
, d, n, p, x]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

rule 685
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[(d + e*x)^(m - 1)*(a + c*x^2)
^(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[m]
 || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 66.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{b^3\left(-\frac{\cos(dx+c)^4\sin(dx+c)^2}{6}-\frac{\cos(dx+c)^4}{12}\right)+3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^5}{6}+\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\right.}{d}$
default	$\frac{b^3\left(-\frac{\cos(dx+c)^4\sin(dx+c)^2}{6}-\frac{\cos(dx+c)^4}{12}\right)+3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^5}{6}+\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\right.}{d}$
risch	$\frac{5a^3x}{16}+\frac{3a\,b^2x}{16}-\frac{b\cos(6dx+6c)a^2}{64d}+\frac{b^3\cos(6dx+6c)}{192d}+\frac{a^3\sin(6dx+6c)}{192d}-\frac{a\sin(6dx+6c)b^2}{64d}-\frac{3b\cos(4dx+4c}{32d}$

input
```
int(cos(d*x+c)^6*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(-1/6*cos(d*x+c)^4*sin(d*x+c)^2-1/12*cos(d*x+c)^4)+3*a*b^2*(-1/6*
sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16
*d*x+1/16*c)-1/2*a^2*b*cos(d*x+c)^6+a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^
3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.87

$$\int \cos^6(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{12\,b^3\cos\left(dx+c\right)^4+8\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^6-3\left(5\,a^3+3\,ab^2\right)dx-\left(8\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^5+\right.}{48\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/48*(12*b^3*cos(d*x + c)^4 + 8*(3*a^2*b - b^3)*cos(d*x + c)^6 - 3*(5*a^3
 + 3*a*b^2)*d*x - (8*(a^3 - 3*a*b^2)*cos(d*x + c)^5 + 2*(5*a^3 + 3*a*b^2)*
cos(d*x + c)^3 + 3*(5*a^3 + 3*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^6(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^3\cos^6\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**6*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00

$$\int \cos^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{3\,(5\,a^3+3\,ab^2)(dx+c)+\frac{3\,(5\,a^3+3\,ab^2)\tan(dx+c)^5-12\,b^3\tan(dx+c)^2+8\,(5\,a^3+3\,ab^2)\tan(dx+c)^3-24\,a^2b-4\,b^3+3\,(11\,a^3-3\,ab^2)\,\mathrm{t}}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{48\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/48*(3*(5*a^3 + 3*a*b^2)*(d*x + c) + (3*(5*a^3 + 3*a*b^2)*tan(d*x + c)^5
- 12*b^3*tan(d*x + c)^2 + 8*(5*a^3 + 3*a*b^2)*tan(d*x + c)^3 - 24*a^2*b -
4*b^3 + 3*(11*a^3 - 3*a*b^2)*tan(d*x + c))/(tan(d*x + c)^6 + 3*tan(d*x + c
)^4 + 3*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

$$\int \cos^6(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{(5\,a^3+3\,ab^2)(dx+c)}{16\,d}$$
$$+ \frac{15\,a^3\tan{(dx+c)}^5+9\,ab^2\tan{(dx+c)}^5+40\,a^3\tan{(dx+c)}^3+24\,ab^2\tan{(dx+c)}^3-12\,b^3\tan{(dx+}}{48\,\left(\tan{(dx+c)}^2+1\right)^3d}$$

input
```
integrate(cos(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/16*(5*a^3 + 3*a*b^2)*(d*x + c)/d + 1/48*(15*a^3*tan(d*x + c)^5 + 9*a*b^2
*tan(d*x + c)^5 + 40*a^3*tan(d*x + c)^3 + 24*a*b^2*tan(d*x + c)^3 - 12*b^3
*tan(d*x + c)^2 + 33*a^3*tan(d*x + c) - 9*a*b^2*tan(d*x + c) - 24*a^2*b -
4*b^3)/((tan(d*x + c)^2 + 1)^3*d)
```

Mupad [B] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.96

$$\int \cos^6(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{5\,a^3\,x}{16} - \frac{24\,a^2\,b - \tan(c+d\,x)^3\,(40\,a^3+24\,a\,b^2) - \tan(c+d\,x)^5\,(15\,a^3+9\,a\,b^2) + 4\,b^3 + \tan(c+d\,x)\,(9\,a\,b^2 -}{d\,\left(48\tan\left(c+d\,x\right)^6+144\tan\left(c+d\,x\right)^4+144\tan\left(c+d\,x\right)^2+48\right)} + \frac{3\,a\,b^2\,x}{16}$$

input
```
int(cos(c + d*x)^6*(a + b*tan(c + d*x))^3,x)
```

output
```
(5*a^3*x)/16 - (24*a^2*b - tan(c + d*x)^3*(24*a*b^2 + 40*a^3) - tan(c + d*
x)^5*(9*a*b^2 + 15*a^3) + 4*b^3 + tan(c + d*x)*(9*a*b^2 - 33*a^3) + 12*b^3
*tan(c + d*x)^2)/(d*(144*tan(c + d*x)^2 + 144*tan(c + d*x)^4 + 48*tan(c +
d*x)^6 + 48)) + (3*a*b^2*x)/16
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.37

$$\int \cos^6(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{8\cos\left(dx+c\right)\sin\left(dx+c\right)^5a^3 - 24\cos\left(dx+c\right)\sin\left(dx+c\right)^5a\,b^2 - 26\cos\left(dx+c\right)\sin\left(dx+c\right)^3a^3 + 42\,\mathrm{c}}{}$$

input
```
int(cos(d*x+c)^6*(a+b*tan(d*x+c))^3,x)
```

output
```
(8*cos(c + d*x)*sin(c + d*x)**5*a**3 - 24*cos(c + d*x)*sin(c + d*x)**5*a*b
**2 - 26*cos(c + d*x)*sin(c + d*x)**3*a**3 + 42*cos(c + d*x)*sin(c + d*x)*
*3*a*b**2 + 33*cos(c + d*x)*sin(c + d*x)*a**3 - 9*cos(c + d*x)*sin(c + d*x
)*a*b**2 + 24*sin(c + d*x)**6*a**2*b - 8*sin(c + d*x)**6*b**3 - 72*sin(c +
 d*x)**4*a**2*b + 12*sin(c + d*x)**4*b**3 + 72*sin(c + d*x)**2*a**2*b + 15
*a**3*d*x + 9*a*b**2*d*x)/(48*d)
```

3.544 $\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 161

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3a(2a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{16d} + \frac{b(3a^2-b^2)\sec^5(c+dx)}{5d} + \frac{b^3\sec^7(c+dx)}{7d} + \frac{3a(2a^2-b^2)\sec(c+dx)\tan(c+dx)}{16d} + \frac{a(2a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{8d} + \frac{ab^2\sec^5(c+dx)\tan(c+dx)}{2d}$$

output
```
3/16*a*(2*a^2-b^2)*arctanh(sin(d*x+c))/d+1/5*b*(3*a^2-b^2)*sec(d*x+c)^5/d+
1/7*b^3*sec(d*x+c)^7/d+3/16*a*(2*a^2-b^2)*sec(d*x+c)*tan(d*x+c)/d+1/8*a*(2
*a^2-b^2)*sec(d*x+c)^3*tan(d*x+c)/d+1/2*a*b^2*sec(d*x+c)^5*tan(d*x+c)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 637 vs. $2(161) = 322$.

Time = 2.07 (sec) , antiderivative size = 637, normalized size of antiderivative = 3.96

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\sec^7(c+dx)\left(10752a^2b+1536b^3+3584(3a^2b-b^3)\cos(2(c+dx))-4410a^3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^7*(10752*a^2*b + 1536*b^3 + 3584*(3*a^2*b - b^3)*Cos[2*(c +
d*x)] - 4410*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 2205*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
1470*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 735*a
*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 210*a^3*C
os[7*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*a*b^2*Cos[7
*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3675*a*(2*a^2 - b^2
)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]) + 4410*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]] - 2205*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] + 1470*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] - 735*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] + 210*a^3*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]] - 105*a*b^2*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
+ 4340*a^3*Sin[2*(c + d*x)] + 6790*a*b^2*Sin[2*(c + d*x)] + 2800*a^3*Sin[4
*(c + d*x)] - 1400*a*b^2*Sin[4*(c + d*x)] + 420*a^3*Sin[6*(c + d*x)] - 210
*a*b^2*Sin[6*(c + d*x)]))/(35840*d)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3991, 3042, 4159, 27, 298, 215, 215, 219, 4861, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^5(a+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3991}$$

$$\int \sec^5(c+dx)\left(a^3+3b^2\tan^2(c+dx)a\right)dx+\int \sec^5(c+dx)\tan(c+dx)\left(\tan^2(c+dx)b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^5\left(a^3+3b^2\tan(c+dx)^2a\right)dx+\int \sec(c+dx)^5\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{4159}$$

$$\int \sec(c+dx)^5\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{\int \frac{a\left(a^2-\left(a^2-3b^2\right)\sin^2(c+dx)\right)}{\left(1-\sin^2(c+dx)\right)^4}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \sec(c+dx)^5\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\int \frac{a^2-\left(a^2-3b^2\right)\sin^2(c+dx)}{\left(1-\sin^2(c+dx)\right)^4}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\int \sec(c+dx)^5\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\left(\frac{1}{2}\left(2a^2-b^2\right)\int \frac{1}{\left(1-\sin^2(c+dx)\right)^3}d\sin(c+dx)+\frac{b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)^3}\right)}{d}$$

$$\Big\downarrow \text{215}$$

$$\int \sec(c+dx)^5\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\left(\frac{1}{2}\left(2a^2-b^2\right)\left(\frac{3}{4}\int \frac{1}{\left(1-\sin^2(c+dx)\right)^2}d\sin(c+dx)+\frac{\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)+\frac{b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)^3}\right)}{d}$$

$$\Big\downarrow \text{215}$$

$$\int \sec(c+dx)^5\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\left(\frac{1}{2}\left(2a^2-b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{b^2\sin(c+dx)}{2(1-\sin^2(c+dx))^3}\right)}{d}$$

$$\Big\downarrow \; 219$$

$$\int \sec(c+dx)^5\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\left(\frac{1}{2}\left(2a^2-b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{b^2\sin(c+dx)}{2(1-\sin^2(c+dx))^3}\right)}{d}$$

$$\Big\downarrow \; 4861$$

$$\frac{a\left(\frac{1}{2}\left(2a^2-b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{b^2\sin(c+dx)}{2(1-\sin^2(c+dx))^3}\right)}{d}-\frac{\int b\left(b^2+\left(3a^2-b^2\right)\cos^2(c+dx)\right)\sec^8(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{a\left(\frac{1}{2}\left(2a^2-b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{b^2\sin(c+dx)}{2(1-\sin^2(c+dx))^3}\right)}{d}-\frac{b\int\left(b^2+\left(3a^2-b^2\right)\cos^2(c+dx)\right)\sec^8(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \; 244$$

$$\frac{a\left(\frac{1}{2}\left(2a^2-b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{b^2\sin(c+dx)}{2(1-\sin^2(c+dx))^3}\right)}{d}-\frac{b\int\left(b^2\sec^8(c+dx)+\left(3a^2-b^2\right)\sec^6(c+dx)\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{a\left(\frac{1}{2}\left(2a^2-b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{b^2\sin(c+dx)}{2(1-\sin^2(c+dx))^3}\right)}{d}-\frac{b\left(-\frac{1}{5}\left(3a^2-b^2\right)\sec^5(c+dx)-\frac{1}{7}b^2\sec^7(c+dx)\right)}{d}$$

input
```
Int[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

output
```
-((b*(-1/5*((3*a^2 - b^2)*Sec[c + d*x]^5) - (b^2*Sec[c + d*x]^7)/7))/d) +
(a*((b^2*Sin[c + d*x])/(2*(1 - Sin[c + d*x]^2)^3) + ((2*a^2 - b^2)*(Sin[c
+ d*x]/(4*(1 - Sin[c + d*x]^2)^2) + (3*(ArcTanh[Sin[c + d*x]]/2 + Sin[c +
d*x]/(2*(1 - Sin[c + d*x]^2))))/4))/2))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

rule 4861
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

Maple [A] (verified)

Time = 30.78 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.54

method	result
derivativedivides	$\frac{b^{3}\left(\frac{\sin(dx+c)^{4}}{7\cos(dx+c)^{7}}+\frac{3\sin(dx+c)^{4}}{35\cos(dx+c)^{5}}+\frac{\sin(dx+c)^{4}}{35\cos(dx+c)^{3}}-\frac{\sin(dx+c)^{4}}{35\cos(dx+c)}-\frac{\left(2+\sin(dx+c)^{2}\right)\cos(dx+c)}{35}\right)+3a\,b^{2}\left(\frac{\sin(dx+c)^{3}}{6\cos(dx+c)^{6}}+\frac{\sin(d}{8\cos(}\right.}{}$
default	$\frac{b^{3}\left(\frac{\sin(dx+c)^{4}}{7\cos(dx+c)^{7}}+\frac{3\sin(dx+c)^{4}}{35\cos(dx+c)^{5}}+\frac{\sin(dx+c)^{4}}{35\cos(dx+c)^{3}}-\frac{\sin(dx+c)^{4}}{35\cos(dx+c)}-\frac{\left(2+\sin(dx+c)^{2}\right)\cos(dx+c)}{35}\right)+3a\,b^{2}\left(\frac{\sin(dx+c)^{3}}{6\cos(dx+c)^{6}}+\frac{\sin(d}{8\cos(}\right.}{}$
risch	$-\frac{e^{i(dx+c)}\left(-1400ia^{3}e^{2i(dx+c)}-2170ia^{3}e^{4i(dx+c)}-210ia^{3}+2170ia^{3}e^{8i(dx+c)}+210ia^{3}e^{12i(dx+c)}+105ia\,b^{2}-5376a^{2}b\,e\right.}{}$

input
```
int(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/3
5*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c
)^2)*cos(d*x+c))+3*a*b^2*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*sin(d*x+c)^3/c
os(d*x+c)^4+1/16*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1/16*ln(sec(d*x
+c)+tan(d*x+c)))+3/5*a^2*b/cos(d*x+c)^5+a^3*(-(-1/4*sec(d*x+c)^3-3/8*sec(d
*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{105\,(2\,a^3-ab^2)\cos\left(dx+c\right)^7\log\left(\sin\left(dx+c\right)+1\right)-105\,(2\,a^3-ab^2)\cos\left(dx+c\right)^7\log\left(-\sin\left(dx+c\right)+\right.}{}$$

input
```
integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/1120*(105*(2*a^3 - a*b^2)*cos(d*x + c)^7*log(sin(d*x + c) + 1) - 105*(2*
a^3 - a*b^2)*cos(d*x + c)^7*log(-sin(d*x + c) + 1) + 160*b^3 + 224*(3*a^2*
b - b^3)*cos(d*x + c)^2 + 70*(3*(2*a^3 - a*b^2)*cos(d*x + c)^5 + 8*a*b^2*c
os(d*x + c) + 2*(2*a^3 - a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x +
 c)^7)
```

Sympy [F]

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^3\sec^5\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**5*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.29

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{35\,ab^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-70\,a^3}{}$$

input
```
integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/1120*(35*a*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))
/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 70*a^3*(2*(3*sin(d*x + c)^3 - 5*s
in(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) + 672*a^2*b/cos(d*x + c)^5 - 32*(7*cos(d*x
 + c)^2 - 5)*b^3/cos(d*x + c)^7)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(149) = 298.

Time = 0.35 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.89

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/560*(105*(2*a^3 - a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(2*a^3
 - a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(350*a^3*tan(1/2*d*x + 1/
2*c)^13 + 105*a*b^2*tan(1/2*d*x + 1/2*c)^13 - 1680*a^2*b*tan(1/2*d*x + 1/2
*c)^12 - 840*a^3*tan(1/2*d*x + 1/2*c)^11 + 1540*a*b^2*tan(1/2*d*x + 1/2*c)
^11 + 3360*a^2*b*tan(1/2*d*x + 1/2*c)^10 - 1120*b^3*tan(1/2*d*x + 1/2*c)^1
0 + 630*a^3*tan(1/2*d*x + 1/2*c)^9 + 1085*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 5
040*a^2*b*tan(1/2*d*x + 1/2*c)^8 - 1120*b^3*tan(1/2*d*x + 1/2*c)^8 + 6720*
a^2*b*tan(1/2*d*x + 1/2*c)^6 - 2240*b^3*tan(1/2*d*x + 1/2*c)^6 - 630*a^3*t
an(1/2*d*x + 1/2*c)^5 - 1085*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3696*a^2*b*tan
(1/2*d*x + 1/2*c)^4 - 448*b^3*tan(1/2*d*x + 1/2*c)^4 + 840*a^3*tan(1/2*d*x
 + 1/2*c)^3 - 1540*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 672*a^2*b*tan(1/2*d*x +
1/2*c)^2 - 224*b^3*tan(1/2*d*x + 1/2*c)^2 - 350*a^3*tan(1/2*d*x + 1/2*c) -
 105*a*b^2*tan(1/2*d*x + 1/2*c) - 336*a^2*b + 32*b^3)/(tan(1/2*d*x + 1/2*c
)^2 - 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.63

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2-b^2\right)}{8\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,a^3}{4}+\frac{3\,a\,b^2}{8}\right)+\frac{6\,a^2\,b}{5}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{11\,a\,b^2}{2}-3\,a^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}\,\left(\frac{11\,a\,b^2}{2}-3\,a^3\right)-}{}$$

input
```
int((a + b*tan(c + d*x))^3/cos(c + d*x)^5,x)
```

output
```
(3*a*atanh(tan(c/2 + (d*x)/2))*(2*a^2 - b^2))/(8*d) - (tan(c/2 + (d*x)/2)*
((3*a*b^2)/8 + (5*a^3)/4) + (6*a^2*b)/5 + tan(c/2 + (d*x)/2)^3*((11*a*b^2)
/2 - 3*a^3) - tan(c/2 + (d*x)/2)^11*((11*a*b^2)/2 - 3*a^3) - tan(c/2 + (d*
x)/2)^13*((3*a*b^2)/8 + (5*a^3)/4) + tan(c/2 + (d*x)/2)^5*((31*a*b^2)/8 +
(9*a^3)/4) - tan(c/2 + (d*x)/2)^9*((31*a*b^2)/8 + (9*a^3)/4) - tan(c/2 + (
d*x)/2)^10*(12*a^2*b - 4*b^3) - tan(c/2 + (d*x)/2)^2*((12*a^2*b)/5 - (4*b^
3)/5) + tan(c/2 + (d*x)/2)^8*(18*a^2*b + 4*b^3) - tan(c/2 + (d*x)/2)^6*(24
*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^4*((66*a^2*b)/5 + (8*b^3)/5) - (4*b^3
)/35 + 6*a^2*b*tan(c/2 + (d*x)/2)^12)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(
c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*
tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 -
1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 809, normalized size of antiderivative = 5.02

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x)
```

output
```
( - 210*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**6*a**3 + 105*
cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**6*a*b**2 + 630*cos(c
+ d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**3 - 315*cos(c + d*x)*l
og(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a*b**2 - 630*cos(c + d*x)*log(tan
((c + d*x)/2) - 1)*sin(c + d*x)**2*a**3 + 315*cos(c + d*x)*log(tan((c + d*
x)/2) - 1)*sin(c + d*x)**2*a*b**2 + 210*cos(c + d*x)*log(tan((c + d*x)/2)
- 1)*a**3 - 105*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b**2 + 210*cos(c
+ d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**6*a**3 - 105*cos(c + d*x)*l
og(tan((c + d*x)/2) + 1)*sin(c + d*x)**6*a*b**2 - 630*cos(c + d*x)*log(tan
((c + d*x)/2) + 1)*sin(c + d*x)**4*a**3 + 315*cos(c + d*x)*log(tan((c + d*
x)/2) + 1)*sin(c + d*x)**4*a*b**2 + 630*cos(c + d*x)*log(tan((c + d*x)/2)
+ 1)*sin(c + d*x)**2*a**3 - 315*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin
(c + d*x)**2*a*b**2 - 210*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a**3 + 10
5*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a*b**2 - 336*cos(c + d*x)*sin(c +
 d*x)**6*a**2*b + 32*cos(c + d*x)*sin(c + d*x)**6*b**3 - 210*cos(c + d*x)*
sin(c + d*x)**5*a**3 + 105*cos(c + d*x)*sin(c + d*x)**5*a*b**2 + 1008*cos(
c + d*x)*sin(c + d*x)**4*a**2*b - 96*cos(c + d*x)*sin(c + d*x)**4*b**3 + 5
60*cos(c + d*x)*sin(c + d*x)**3*a**3 - 280*cos(c + d*x)*sin(c + d*x)**3*a*
b**2 - 1008*cos(c + d*x)*sin(c + d*x)**2*a**2*b + 96*cos(c + d*x)*sin(c +
d*x)**2*b**3 - 350*cos(c + d*x)*sin(c + d*x)*a**3 - 105*cos(c + d*x)*si...
```

3.545 $\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{a(4a^2-3b^2)\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{b(3a^2-b^2)\sec^3(c+dx)}{3d} + \frac{b^3\sec^5(c+dx)}{5d} + \frac{a(4a^2-3b^2)\sec(c+dx)\tan(c+dx)}{8d} + \frac{3ab^2\sec^3(c+dx)\tan(c+dx)}{4d}$$

output
```
1/8*a*(4*a^2-3*b^2)*arctanh(sin(d*x+c))/d+1/3*b*(3*a^2-b^2)*sec(d*x+c)^3/d
+1/5*b^3*sec(d*x+c)^5/d+1/8*a*(4*a^2-3*b^2)*sec(d*x+c)*tan(d*x+c)/d+3/4*a*
b^2*sec(d*x+c)^3*tan(d*x+c)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 464 vs. $2(128) = 256$.

Time = 1.52 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.62

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\sec^5(c+dx)\left(960a^2b+64b^3+320(3a^2b-b^3)\cos(2(c+dx))-300a^3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^5*(960*a^2*b + 64*b^3 + 320*(3*a^2*b - b^3)*Cos[2*(c + d*x)]
 - 300*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 225
*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 60*a^3*
Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 45*a*b^2*Cos[5
*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 150*a*(4*a^2 - 3*b^
2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]]) + 300*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]] - 225*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]] + 60*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]] - 45*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] + 240*a^3*Sin[2*(c + d*x)] + 540*a*b^2*Sin[2*(c + d*x)] + 120*a^3*Si
n[4*(c + d*x)] - 90*a*b^2*Sin[4*(c + d*x)]))/(1920*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3991, 3042, 4159, 27, 298, 215, 219, 4861, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3(a+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{ 3991}$$

$$\int \sec^3(c+dx)\left(a^3+3b^2\tan^2(c+dx)a\right)dx+\int \sec^3(c+dx)\tan(c+dx)\left(\tan^2(c+dx)b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(c+dx)^3\left(a^3+3b^2\tan(c+dx)^2a\right)dx+\int \sec(c+dx)^3\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{ 4159}$$

$$\int \sec(c+dx)^3\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{\int \frac{a\left(a^2-\left(a^2-3b^2\right)\sin^2(c+dx)\right)}{\left(1-\sin^2(c+dx)\right)^3}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\int \sec(c+dx)^3\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\int \frac{a^2-\left(a^2-3b^2\right)\sin^2(c+dx)}{\left(1-\sin^2(c+dx)\right)^3}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 298}$$

$$\int \sec(c+dx)^3\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\left(\frac{1}{4}\left(4a^2-3b^2\right)\int \frac{1}{\left(1-\sin^2(c+dx)\right)^2}d\sin(c+dx)+\frac{3b^2\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)}{d}$$

$$\Big\downarrow \text{ 215}$$

$$\int \sec(c+dx)^3\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\left(\frac{1}{4}\left(4a^2-3b^2\right)\left(\frac{1}{2}\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\frac{\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)+\frac{3b^2\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\int \sec(c+dx)^3\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\left(\frac{1}{4}\left(4a^2-3b^2\right)\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)+\frac{3b^2\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)}{d}$$

$$\Big\downarrow \text{ 4861}$$

$$\frac{a\left(\frac{1}{4}\left(4a^2-3b^2\right)\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)+\frac{3b^2\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)}{d}-\frac{\int b\left(b^2+\left(3a^2-b^2\right)\cos^2(c+dx)\right)\sec^6(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{a\left(\frac{1}{4}\left(4a^2-3b^2\right)\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)+\frac{3b^2\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)}{d}-\frac{b\int \left(b^2+\left(3a^2-b^2\right)\cos^2(c+dx)\right)\sec^6(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \; 244$$

$$\frac{a\left(\frac{1}{4}\left(4a^2-3b^2\right)\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)+\frac{3b^2\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)}{d}-\frac{b\int \left(b^2\sec^6(c+dx)+\left(3a^2-b^2\right)\sec^4(c+dx)\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{a\left(\frac{1}{4}\left(4a^2-3b^2\right)\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)+\frac{3b^2\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)}{d}-\frac{b\left(-\frac{1}{3}\left(3a^2-b^2\right)\sec^3(c+dx)-\frac{1}{5}b^2\sec^5(c+dx)\right)}{d}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

output
```
-((b*(-1/3*((3*a^2 - b^2)*Sec[c + d*x]^3) - (b^2*Sec[c + d*x]^5)/5))/d) +
(a*((3*b^2*Sin[c + d*x])/(4*(1 - Sin[c + d*x]^2)^2) + ((4*a^2 - 3*b^2)*(Ar
cTanh[Sin[c + d*x]]/2 + Sin[c + d*x]/(2*(1 - Sin[c + d*x]^2))))/4))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

rule 4861
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

Maple [A] (verified)

Time = 8.77 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.55

method	result
derivativedivides	$\frac{b^3\left(\frac{\sin(dx+c)^4}{5\cos(dx+c)^5}+\frac{\sin(dx+c)^4}{15\cos(dx+c)^3}-\frac{\sin(dx+c)^4}{15\cos(dx+c)}-\frac{\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{15}\right)+3ab^2\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx-}{8}\right.}{d}$
default	$\frac{b^3\left(\frac{\sin(dx+c)^4}{5\cos(dx+c)^5}+\frac{\sin(dx+c)^4}{15\cos(dx+c)^3}-\frac{\sin(dx+c)^4}{15\cos(dx+c)}-\frac{\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{15}\right)+3ab^2\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx-}{8}\right.}{d}$
risch	$-\frac{e^{i(dx+c)}\left(60ia^3e^{8i(dx+c)}-45iab^2e^{8i(dx+c)}+120ia^3e^{6i(dx+c)}+270iab^2e^{6i(dx+c)}-480a^2be^{6i(dx+c)}+160b^3e^{6i(dx+c)}\right.}{60d(}$

input
```
int(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1/1
5*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))+3*a*b^2*(1/4*s
in(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*
ln(sec(d*x+c)+tan(d*x+c)))+a^2*b/cos(d*x+c)^3+a^3*(1/2*sec(d*x+c)*tan(d*x+
c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.15

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{15\,(4\,a^3-3\,ab^2)\cos(dx+c)^5\log(\sin(dx+c)+1)-15\,(4\,a^3-3\,ab^2)\cos(dx+c)^5\log(-\sin(dx+c)+}{240\,d\,\mathrm{co}}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/240*(15*(4*a^3 - 3*a*b^2)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*a
^3 - 3*a*b^2)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 48*b^3 + 80*(3*a^2*b
 - b^3)*cos(d*x + c)^2 + 30*(6*a*b^2*cos(d*x + c) + (4*a^3 - 3*a*b^2)*cos(
d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.23

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{45\,ab^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-60\,a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\mathrm{lo}\right.}{240\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/240*(45*a*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin
(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 60*a^3
*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*
x + c) - 1)) + 240*a^2*b/cos(d*x + c)^3 - 16*(5*cos(d*x + c)^2 - 3)*b^3/co
s(d*x + c)^5)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(118) = 236.

Time = 0.32 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.60

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{15\left(4\,a^3-3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(4\,a^3-3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(60\,a^3\tan(\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/120*(15*(4*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^3
 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^3*tan(1/2*d*x + 1
/2*c)^9 + 45*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 360*a^2*b*tan(1/2*d*x + 1/2*c)
^8 - 120*a^3*tan(1/2*d*x + 1/2*c)^7 + 270*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 7
20*a^2*b*tan(1/2*d*x + 1/2*c)^6 - 240*b^3*tan(1/2*d*x + 1/2*c)^6 - 480*a^2
*b*tan(1/2*d*x + 1/2*c)^4 - 80*b^3*tan(1/2*d*x + 1/2*c)^4 + 120*a^3*tan(1/
2*d*x + 1/2*c)^3 - 270*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^2*b*tan(1/2*d*
x + 1/2*c)^2 - 80*b^3*tan(1/2*d*x + 1/2*c)^2 - 60*a^3*tan(1/2*d*x + 1/2*c)
 - 45*a*b^2*tan(1/2*d*x + 1/2*c) - 120*a^2*b + 16*b^3)/(tan(1/2*d*x + 1/2*
c)^2 - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.29

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(a^3+\frac{3ab^2}{4}\right)-2a^2b-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{9ab^2}{2}-2a^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{9ab^2}{2}-2a^3\right)+\tan(\cdots}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\cdots\right.\right.}$$

$$-\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{3ab^2}{4}-a^3\right)}{d}$$

input
```
int((a + b*tan(c + d*x))^3/cos(c + d*x)^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^9*((3*a*b^2)/4 + a^3) - 2*a^2*b - tan(c/2 + (d*x)/2)^3
*((9*a*b^2)/2 - 2*a^3) + tan(c/2 + (d*x)/2)^7*((9*a*b^2)/2 - 2*a^3) + tan(
c/2 + (d*x)/2)^2*(4*a^2*b - (4*b^3)/3) - tan(c/2 + (d*x)/2)^4*(8*a^2*b + (
4*b^3)/3) + tan(c/2 + (d*x)/2)^6*(12*a^2*b - 4*b^3) + (4*b^3)/15 - tan(c/2
 + (d*x)/2)*((3*a*b^2)/4 + a^3) - 6*a^2*b*tan(c/2 + (d*x)/2)^8)/(d*(5*tan(
c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*t
an(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) - (atanh(tan(c/2 + (d*x)
/2))*((3*a*b^2)/4 - a^3))/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 595, normalized size of antiderivative = 4.65

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x)
```

output
```
( - 60*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**3 + 45*co
s(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a*b**2 + 120*cos(c +
d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**3 - 90*cos(c + d*x)*log(
tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a*b**2 - 60*cos(c + d*x)*log(tan((c
+ d*x)/2) - 1)*a**3 + 45*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b**2 + 6
0*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a**3 - 45*cos(c +
 d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a*b**2 - 120*cos(c + d*x)*
log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**3 + 90*cos(c + d*x)*log(tan((
c + d*x)/2) + 1)*sin(c + d*x)**2*a*b**2 + 60*cos(c + d*x)*log(tan((c + d*x
)/2) + 1)*a**3 - 45*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a*b**2 - 120*co
s(c + d*x)*sin(c + d*x)**4*a**2*b + 16*cos(c + d*x)*sin(c + d*x)**4*b**3 -
 60*cos(c + d*x)*sin(c + d*x)**3*a**3 + 45*cos(c + d*x)*sin(c + d*x)**3*a*
b**2 + 240*cos(c + d*x)*sin(c + d*x)**2*a**2*b - 32*cos(c + d*x)*sin(c + d
*x)**2*b**3 + 60*cos(c + d*x)*sin(c + d*x)*a**3 + 45*cos(c + d*x)*sin(c +
d*x)*a*b**2 - 120*cos(c + d*x)*a**2*b + 16*cos(c + d*x)*b**3 - 120*sin(c +
 d*x)**2*a**2*b + 40*sin(c + d*x)**2*b**3 + 120*a**2*b - 16*b**3)/(120*cos
(c + d*x)*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.546 $\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{a(2a^2-3b^2)\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{b(3a^2-b^2)\sec(c+dx)}{d} + \frac{b^3\sec^3(c+dx)}{3d} + \frac{3ab^2\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*a*(2*a^2-3*b^2)*arctanh(sin(d*x+c))/d+b*(3*a^2-b^2)*sec(d*x+c)/d+1/3*b
^3*sec(d*x+c)^3/d+3/2*a*b^2*sec(d*x+c)*tan(d*x+c)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 293 vs. 2(90) = 180.

Time = 1.51 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.26

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{36a^2b-10b^3-6a(2a^2-3b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\right.}{}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

output
```
(36*a^2*b - 10*b^3 - 6*a*(2*a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + 12*a^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 18*a*b^2*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] + (9*a*b^2)/(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^2 + b^3/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + 2*b*(18*a^2
- b^2 + 2*b^2*Cos[c + d*x] + (18*a^2 - 5*b^2)*Cos[2*(c + d*x)])*Sec[c + d*
x]^3*Sin[(c + d*x)/2]^2 - (9*a*b^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
2 + b^3/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(12*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {3042, 3991, 3042, 4159, 27, 298, 219, 4861, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3991}$$

$$\int \sec(c+dx)\left(a^3+3b^2\tan^2(c+dx)a\right)dx+\int \sec(c+dx)\tan(c+dx)\left(\tan^2(c+dx)b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)\left(a^3+3b^2\tan(c+dx)^2a\right)dx+\int \sec(c+dx)\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{4159}$$

$$\int \sec(c+dx)\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{\int \frac{a\left(a^2-\left(a^2-3b^2\right)\sin^2(c+dx)\right)}{\left(1-\sin^2(c+dx)\right)^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \sec(c+dx)\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\int \frac{a^2-(a^2-3b^2)\sin^2(c+dx)}{\left(1-\sin^2(c+dx)\right)^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\int \sec(c+dx)\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\left(\frac{1}{2}\left(2a^2-3b^2\right)\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\frac{3b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$\int \sec(c+dx)\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\left(\frac{1}{2}\left(2a^2-3b^2\right)\text{arctanh}(\sin(c+dx))+\frac{3b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)}{d}$$

$$\Big\downarrow \text{4861}$$

$$\frac{a\left(\frac{1}{2}\left(2a^2-3b^2\right)\text{arctanh}(\sin(c+dx))+\frac{3b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)}{d}-\frac{\int b\left(b^2+\left(3a^2-b^2\right)\cos^2(c+dx)\right)\sec^4(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\left(\frac{1}{2}\left(2a^2-3b^2\right)\text{arctanh}(\sin(c+dx))+\frac{3b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)}{d}-\frac{b\int \left(b^2+\left(3a^2-b^2\right)\cos^2(c+dx)\right)\sec^4(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{244}$$

$$\frac{a\left(\frac{1}{2}\left(2a^2-3b^2\right)\text{arctanh}(\sin(c+dx))+\frac{3b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)}{d}-\frac{b\int \left(b^2\sec^4(c+dx)+\left(3a^2-b^2\right)\sec^2(c+dx)\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(\frac{1}{2}\left(2a^2-3b^2\right)\text{arctanh}(\sin(c+dx))+\frac{3b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)}{d}-\frac{b\left(-\left(3a^2-b^2\right)\sec(c+dx)-\frac{1}{3}b^2\sec^3(c+dx)\right)}{d}$$

input
```
Int[Sec[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

output
```
-((b*(-((3*a^2 - b^2)*Sec[c + d*x]) - (b^2*Sec[c + d*x]^3)/3))/d) + (a*(((
2*a^2 - 3*b^2)*ArcTanh[Sin[c + d*x]])/2 + (3*b^2*Sin[c + d*x])/(2*(1 - Sin
[c + d*x]^2))))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

rule 4861
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.62

method	result
derivativedivides	$\frac{b^{3}\left(\frac{\sin(dx+c)^{4}}{3\cos(dx+c)^{3}}-\frac{\sin(dx+c)^{4}}{3\cos(dx+c)}-\frac{\left(2+\sin(dx+c)^{2}\right)\cos(dx+c)}{3}\right)+3ab^{2}\left(\frac{\sin(dx+c)^{3}}{2\cos(dx+c)^{2}}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{b^{3}\left(\frac{\sin(dx+c)^{4}}{3\cos(dx+c)^{3}}-\frac{\sin(dx+c)^{4}}{3\cos(dx+c)}-\frac{\left(2+\sin(dx+c)^{2}\right)\cos(dx+c)}{3}\right)+3ab^{2}\left(\frac{\sin(dx+c)^{3}}{2\cos(dx+c)^{2}}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
risch	$-\frac{b\,\mathrm{e}^{i(dx+c)}\left(9iab\,\mathrm{e}^{4i(dx+c)}-18a^{2}\mathrm{e}^{4i(dx+c)}+6b^{2}\mathrm{e}^{4i(dx+c)}-36a^{2}\mathrm{e}^{2i(dx+c)}+4b^{2}\mathrm{e}^{2i(dx+c)}-9iab-18a^{2}+6b^{2}\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{3}}+\frac{a^{3}\ln(}{}$

input
```
int(sec(d*x+c)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2
+sin(d*x+c)^2)*cos(d*x+c))+3*a*b^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(
d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^2*b/cos(d*x+c)+a^3*ln(sec(d*x+c)
+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.37

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{3\,(2\,a^3-3\,ab^2)\cos{(dx+c)}^3\log{(\sin{(dx+c)}+1)}-3\,(2\,a^3-3\,ab^2)\cos{(dx+c)}^3\log{(-\sin{(dx+c)}+1}}{12\,d\cos{(dx+c)}^3}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(3*(2*a^3 - 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*a^3
- 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 18*a*b^2*cos(d*x + c)*s
in(d*x + c) + 4*b^3 + 12*(3*a^2*b - b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^3
)
```

Sympy [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx=\int(a+b\tan{(c+dx)})^3\sec{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.23

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx =$$

$$-\frac{9\,ab^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-12\,a^3\log\left(\sec\left(dx+c\right)+\tan\left(dx\right.\right.}{12\,d}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/12*(9*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1
) - log(sin(d*x + c) - 1)) - 12*a^3*log(sec(d*x + c) + tan(d*x + c)) - 36*
a^2*b/cos(d*x + c) + 4*(3*cos(d*x + c)^2 - 1)*b^3/cos(d*x + c)^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(84) = 168.

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.90

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$=\frac{3\left(2\,a^3-3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(2\,a^3-3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(9\,ab^2\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{6\,d}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(3*(2*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^3 - 3
*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9*a*b^2*tan(1/2*d*x + 1/2*
c)^5 - 18*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^2 -
 12*b^3*tan(1/2*d*x + 1/2*c)^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b +
 4*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.78

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(3\,a\,b^2-2\,a^3\right)}{d} - \frac{6\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(12\,a^2\,b-4\,b^3\right)-\frac{4\,b^3}{3}+3\,a\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-3\,a\,b^2\,\tan\left(\frac{c}{2}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + b*tan(c + d*x))^3/cos(c + d*x),x)
```

output
```
- (atanh(tan(c/2 + (d*x)/2))*(3*a*b^2 - 2*a^3))/d - (6*a^2*b - tan(c/2 + (
d*x)/2)^2*(12*a^2*b - 4*b^3) - (4*b^3)/3 + 3*a*b^2*tan(c/2 + (d*x)/2) + 6*
a^2*b*tan(c/2 + (d*x)/2)^4 - 3*a*b^2*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 +
 (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 364, normalized size of antiderivative = 4.04

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{-6\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2a^3+9\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c\right.}{}$$

input
```
int(sec(d*x+c)*(a+b*tan(d*x+c))^3,x)
```

output
```
( - 6*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**3 + 9*cos(
c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a*b**2 + 6*cos(c + d*x)
*log(tan((c + d*x)/2) - 1)*a**3 - 9*cos(c + d*x)*log(tan((c + d*x)/2) - 1)
*a*b**2 + 6*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**3 -
9*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a*b**2 - 6*cos(c
+ d*x)*log(tan((c + d*x)/2) + 1)*a**3 + 9*cos(c + d*x)*log(tan((c + d*x)/2
) + 1)*a*b**2 - 18*cos(c + d*x)*sin(c + d*x)**2*a**2*b + 4*cos(c + d*x)*si
n(c + d*x)**2*b**3 - 9*cos(c + d*x)*sin(c + d*x)*a*b**2 + 18*cos(c + d*x)*
a**2*b - 4*cos(c + d*x)*b**3 + 18*sin(c + d*x)**2*a**2*b - 6*sin(c + d*x)*
*2*b**3 - 18*a**2*b + 4*b**3)/(6*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.547 $\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3ab^2\text{arctanh}(\sin(c+dx))}{d} - \frac{b(3a^2-b^2)\cos(c+dx)}{d} + \frac{b^3\sec(c+dx)}{d} + \frac{a(a^2-3b^2)\sin(c+dx)}{d}$$

output
```
3*a*b^2*arctanh(sin(d*x+c))/d-b*(3*a^2-b^2)*cos(d*x+c)/d+b^3*sec(d*x+c)/d+
a*(a^2-3*b^2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.79

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\sec(c+dx)\left(-3a^2b+3b^3+(-3a^2b+b^3)\cos(2(c+dx))-6ab^2\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\right.\right.\right.\right.}{2d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]*(-3*a^2*b + 3*b^3 + (-3*a^2*b + b^3)*Cos[2*(c + d*x)] - 6*a*
b^2*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]]) + a^3*Sin[2*(c + d*x)] - 3*a*b^2*Sin[2*(c + d
*x)]))/(2*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {3042, 3991, 3042, 4147, 27, 244, 2009, 4159, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3991}$$

$$\int \cos(c+dx)\left(a^3+3b^2\tan^2(c+dx)a\right)dx+\int \sin(c+dx)\left(\tan^2(c+dx)b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^3+3b^2\tan(c+dx)^2a}{\sec(c+dx)}dx+\int \sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{4147}$$

$$\int \frac{a^3+3b^2\tan(c+dx)^2a}{\sec(c+dx)}dx+\frac{\int b\cos^2(c+dx)\left(3a^2-b^2+b^2\sec^2(c+dx)\right)d\sec(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{a^3+3b^2\tan(c+dx)^2a}{\sec(c+dx)}dx+\frac{b\int \cos^2(c+dx)\left(3a^2-b^2+b^2\sec^2(c+dx)\right)d\sec(c+dx)}{d}$$

$$\Big\downarrow \text{244}$$

$$\int \frac{a^3+3b^2\tan(c+dx)^2a}{\sec(c+dx)}dx+\frac{b\int \left(b^2+\left(3a^2-b^2\right)\cos^2(c+dx)\right)d\sec(c+dx)}{d}$$

$$\Big\downarrow \; 2009$$

$$\int \frac{a^3+3b^2\tan(c+dx)^2a}{\sec(c+dx)}dx+\frac{b\left(b^2\sec(c+dx)-\left(3a^2-b^2\right)\cos(c+dx)\right)}{d}$$

$$\Big\downarrow \; 4159$$

$$\frac{\int \frac{a\left(a^2-\left(a^2-3b^2\right)\sin^2(c+dx)\right)}{1-\sin^2(c+dx)}d\sin(c+dx)}{d}+\frac{b\left(b^2\sec(c+dx)-\left(3a^2-b^2\right)\cos(c+dx)\right)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{a\int \frac{a^2-\left(a^2-3b^2\right)\sin^2(c+dx)}{1-\sin^2(c+dx)}d\sin(c+dx)}{d}+\frac{b\left(b^2\sec(c+dx)-\left(3a^2-b^2\right)\cos(c+dx)\right)}{d}$$

$$\Big\downarrow \; 299$$

$$\frac{a\left(3b^2\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\left(a^2-3b^2\right)\sin(c+dx)\right)}{d}+\frac{b\left(b^2\sec(c+dx)-\left(3a^2-b^2\right)\cos(c+dx)\right)}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{a\left(\left(a^2-3b^2\right)\sin(c+dx)+3b^2\text{arctanh}(\sin(c+dx))\right)}{d}+\frac{b\left(b^2\sec(c+dx)-\left(3a^2-b^2\right)\cos(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

output
```
(b*(-((3*a^2 - b^2)*Cos[c + d*x]) + b^2*Sec[c + d*x]))/d + (a*(3*b^2*ArcTa
nh[Sin[c + d*x]] + (a^2 - 3*b^2)*Sin[c + d*x]))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{b^3\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)+3a\,b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-3a^2b\cos(dx+c)+a^3}{d}$
default	$\frac{b^3\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)+3a\,b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-3a^2b\cos(dx+c)+a^3}{d}$
risch	$-\frac{3\,\mathrm{e}^{i(dx+c)}a^2b}{2d}+\frac{\mathrm{e}^{i(dx+c)}b^3}{2d}-\frac{i\mathrm{e}^{i(dx+c)}a^3}{2d}+\frac{3i\mathrm{e}^{i(dx+c)}a\,b^2}{2d}-\frac{3\,\mathrm{e}^{-i(dx+c)}a^2b}{2d}+\frac{\mathrm{e}^{-i(dx+c)}b^3}{2d}+\frac{i\mathrm{e}^{-i(dx+c)}}{2d}$

input
```
int(cos(d*x+c)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+3*a*b^2*(-s
in(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-3*a^2*b*cos(d*x+c)+a^3*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.49

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{3\,ab^2\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,ab^2\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,b^3-2\left(3\,a^2b-b^3\right.}{2\,d\cos\left(dx+c\right)}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(3*a*b^2*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*a*b^2*cos(d*x + c)*log
(-sin(d*x + c) + 1) + 2*b^3 - 2*(3*a^2*b - b^3)*cos(d*x + c)^2 + 2*(a^3 -
3*a*b^2)*cos(d*x + c)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan{(c+dx)})^3\cos{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*cos(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{2\,b^3\left(\frac{1}{\cos(dx+c)}+\cos{(dx+c)}\right)+3\,ab^2(\log{(\sin{(dx+c)}+1)}-\log{(\sin{(dx+c)}-1)}-2\,\sin{(dx+c)})-6}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(2*b^3*(1/cos(d*x + c) + cos(d*x + c)) + 3*a*b^2*(log(sin(d*x + c) + 1
) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - 6*a^2*b*cos(d*x + c) + 2*a^3
*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4309 vs. $2(73) = 146$.

Time = 2.05 (sec) , antiderivative size = 4309, normalized size of antiderivative = 59.03

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/4*(3*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(
1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 3*pi
*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^4 - 6*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*
c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 6*a^2*b*arctan((tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
 + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 6*a*b^2*l
og(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/
2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*t
an(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 6*a*b^2*log(2*(tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1
/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x
)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(
1/2*c)^4 - 12*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 -
4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^3*tan(1/2*c)^3
+ 12*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^4 - 8*b^3*tan(1/2*d*x)^4*tan(1/2*c)^4
 + 12*pi*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)^3 + 24*a^2*b*arctan((tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 24*a^2*b*ar...
```

Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.59

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{6\,a\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(6\,a\,b^2-2\,a^3)-6\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(6\,a\,b^2-2\,a^3)+4\,b^3+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}$$

input
```
int(cos(c + d*x)*(a + b*tan(c + d*x))^3,x)
```

output
```
(6*a*b^2*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 2
*a^3) - 6*a^2*b - tan(c/2 + (d*x)/2)*(6*a*b^2 - 2*a^3) + 4*b^3 + 6*a^2*b*t
an(c/2 + (d*x)/2)^2)/(d*(tan(c/2 + (d*x)/2)^4 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.14

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{-3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a\,b^2+3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a\,b^2+\cos(dx+c)\sin(}{}$$

input
```
int(cos(d*x+c)*(a+b*tan(d*x+c))^3,x)
```

output
```
( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b**2 + 3*cos(c + d*x)*log(t
an((c + d*x)/2) + 1)*a*b**2 + cos(c + d*x)*sin(c + d*x)*a**3 - 3*cos(c + d
*x)*sin(c + d*x)*a*b**2 + 3*cos(c + d*x)*a**2*b - 2*cos(c + d*x)*b**3 + 3*
sin(c + d*x)**2*a**2*b - sin(c + d*x)**2*b**3 - 3*a**2*b + 2*b**3)/(cos(c
+ d*x)*d)
```

3.548 $\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{b^3\cos(c+dx)}{d}-\frac{a^2b\cos^3(c+dx)}{d}+\frac{b^3\cos^3(c+dx)}{3d}+\frac{a^3\sin(c+dx)}{d}-\frac{a(a^2-3b^2)\sin^3(c+dx)}{3d}$$

output
```
-b^3*cos(d*x+c)/d-a^2*b*cos(d*x+c)^3/d+1/3*b^3*cos(d*x+c)^3/d+a^3*sin(d*x+
c)/d-1/3*a*(a^2-3*b^2)*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{-9b(a^2+b^2)\cos(c+dx)+(-3a^2b+b^3)\cos(3(c+dx))+2a(5a^2+3b^2+(a^2-3b^2)\cos(2(c+dx)))\sin}{12d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

output
```
(-9*b*(a^2 + b^2)*Cos[c + d*x] + (-3*a^2*b + b^3)*Cos[3*(c + d*x)] + 2*a*(
5*a^2 + 3*b^2 + (a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/(12*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3991, 3042, 4159, 2009, 4857, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3991}$$

$$\int \cos^3(c+dx)\left(a^3+3b^2\tan^2(c+dx)a\right)dx+\int \cos^2(c+dx)\sin(c+dx)\left(\tan^2(c+dx)b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^3+3b^2\tan(c+dx)^2a}{\sec(c+dx)^3}dx+\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{4159}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^2}dx+\frac{\int \left(a^3-a\left(a^2-3b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^2}dx+\frac{a^3\sin(c+dx)-\frac{1}{3}a\left(a^2-3b^2\right)\sin^3(c+dx)}{d}$$

$$\Big\downarrow \text{4857}$$

$$\frac{a^3\sin(c+dx)-\frac{1}{3}a\left(a^2-3b^2\right)\sin^3(c+dx)}{d}-\frac{\int\left(\left(1-\cos^2(c+dx)\right)b^3+3a^2\cos^2(c+dx)b\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\sin(c+dx)-\frac{1}{3}a\left(a^2-3b^2\right)\sin^3(c+dx)}{d}-\frac{a^2b\cos^3(c+dx)-\frac{1}{3}b^3\cos^3(c+dx)+b^3\cos(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

output
```
-((b^3*Cos[c + d*x] + a^2*b*Cos[c + d*x]^3 - (b^3*Cos[c + d*x]^3)/3)/d) +
(a^3*Sin[c + d*x] - (a*(a^2 - 3*b^2)*Sin[c + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Maple [A] (verified)

Time = 9.71 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{-\frac{b^3\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}+a\,b^2\sin(dx+c)^3-a^2b\cos(dx+c)^3+\frac{a^3\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
default	$\frac{-\frac{b^3\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}+a\,b^2\sin(dx+c)^3-a^2b\cos(dx+c)^3+\frac{a^3\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
risch	$-\frac{3b\cos(dx+c)a^2}{4d}-\frac{3b^3\cos(dx+c)}{4d}+\frac{3a^3\sin(dx+c)}{4d}+\frac{3a\sin(dx+c)b^2}{4d}-\frac{b\cos(3dx+3c)a^2}{4d}+\frac{b^3\cos(3dx+3c)}{12d}$

input
```
int(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*b^3*(2+sin(d*x+c)^2)*cos(d*x+c)+a*b^2*sin(d*x+c)^3-a^2*b*cos(d*x
+c)^3+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{3\,b^3\cos\left(dx+c\right)+\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^3-\left(2\,a^3+3\,ab^2+\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/3*(3*b^3*cos(d*x + c) + (3*a^2*b - b^3)*cos(d*x + c)^3 - (2*a^3 + 3*a*b
^2 + (a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\cos^3(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{3a^2b\cos(dx+c)^3 - 3ab^2\sin(dx+c)^3 + (\sin(dx+c)^3 - 3\sin(dx+c))a^3 - (\cos(dx+c)^3 - 3\cos(}{3d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/3*(3*a^2*b*cos(d*x + c)^3 - 3*a*b^2*sin(d*x + c)^3 + (sin(d*x + c)^3 -
3*sin(d*x + c))*a^3 - (cos(d*x + c)^3 - 3*cos(d*x + c))*b^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24430 vs. 2(84) = 168.

Time = 121.74 (sec) , antiderivative size = 24430, normalized size of antiderivative = 277.61

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/768*(72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(
1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 105*pi*b^3*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1
)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/
2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2
*c)^6 - 105*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(
1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 72*pi*a^2*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 105*pi*
b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6
+ 72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*...
```

Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\frac{\sin(c+d\,x)\,a^3\cos(c+d\,x)^2}{3}+\frac{2\,\sin(c+d\,x)\,a^3}{3}-a^2\,b\cos\left(c+d\,x\right)^3-\sin\left(c+d\,x\right)\,a\,b^2\cos\left(c+d\,x\right)^2+\sin\left(c+d\,x\right)\,a}{d}$$

input
```
int(cos(c + d*x)^3*(a + b*tan(c + d*x))^3,x)
```

output
```
((2*a^3*sin(c + d*x))/3 - b^3*cos(c + d*x) + (b^3*cos(c + d*x)^3)/3 - a^2*
b*cos(c + d*x)^3 + (a^3*cos(c + d*x)^2*sin(c + d*x))/3 + a*b^2*sin(c + d*x
) - a*b^2*cos(c + d*x)^2*sin(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.33

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{3\cos(dx+c)\sin(dx+c)^2 a^2b-\cos(dx+c)\sin(dx+c)^2 b^3-3\cos(dx+c)a^2b-2\cos(dx+c)b^3-\mathrm{si}}{3d}$$

input
```
int(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x)
```

output
```
(3*cos(c + d*x)*sin(c + d*x)**2*a**2*b - cos(c + d*x)*sin(c + d*x)**2*b**3
 - 3*cos(c + d*x)*a**2*b - 2*cos(c + d*x)*b**3 - sin(c + d*x)**3*a**3 + 3*
sin(c + d*x)**3*a*b**2 + 3*sin(c + d*x)*a**3 + 3*a**2*b + 2*b**3)/(3*d)
```

3.549 $\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{b^3\cos^3(c+dx)}{3d}-\frac{b(3a^2-b^2)\cos^5(c+dx)}{5d}+\frac{a^3\sin(c+dx)}{d}-\frac{a(2a^2-3b^2)\sin^3(c+dx)}{3d}+\frac{a(a^2-3b^2)\sin^5(c+dx)}{5d}$$

output

```
-1/3*b^3*cos(d*x+c)^3/d-1/5*b*(3*a^2-b^2)*cos(d*x+c)^5/d+a^3*sin(d*x+c)/d-
1/3*a*(2*a^2-3*b^2)*sin(d*x+c)^3/d+1/5*a*(a^2-3*b^2)*sin(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.09

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{-9a^2b\cos^5(c+dx)+15a^3\sin(c+dx)-5a(2a^2-3b^2)\sin^3(c+dx)+3a(a^2-3b^2)\sin^5(c+dx)+b^3\cos}{15d}$$

input

```
Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

output
```
(-9*a^2*b*Cos[c + d*x]^5 + 15*a^3*Sin[c + d*x] - 5*a*(2*a^2 - 3*b^2)*Sin[c
 + d*x]^3 + 3*a*(a^2 - 3*b^2)*Sin[c + d*x]^5 + b^3*Cos[c + d*x]*(-2 + 2/Sq
rt[Cos[c + d*x]^2] - Sin[c + d*x]^2 + 3*Sin[c + d*x]^4))/(15*d)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3991, 3042, 4159, 27, 290, 2009, 4857, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3991}$$

$$\int \cos^5(c+dx)\left(a^3+3b^2\tan^2(c+dx)a\right)dx+\int \cos^4(c+dx)\sin(c+dx)\left(\tan^2(c+dx)b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^3+3b^2\tan(c+dx)^2a}{\sec(c+dx)^5}dx+\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{4159}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^4}dx+\frac{\int a\left(1-\sin^2(c+dx)\right)\left(a^2-\left(a^2-3b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^4}dx+\frac{a\int \left(1-\sin^2(c+dx)\right)\left(a^2-\left(a^2-3b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^4}dx+\frac{a\int \left(\left(a^2-3b^2\right)\sin^4(c+dx)-\left(2a^2-3b^2\right)\sin^2(c+dx)+a^2\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^4}dx+\frac{a\left(\frac{1}{5}\left(a^2-3b^2\right)\sin^5(c+dx)-\frac{1}{3}\left(2a^2-3b^2\right)\sin^3(c+dx)+a^2\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4857}$$

$$\frac{a\left(\frac{1}{5}\left(a^2-3b^2\right)\sin^5(c+dx)-\frac{1}{3}\left(2a^2-3b^2\right)\sin^3(c+dx)+a^2\sin(c+dx)\right)}{d}-\frac{\int \left(3a^2b\cos^4(c+dx)+b^3\left(1-\cos^2(c+dx)\right)\cos^2(c+dx)\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(\frac{1}{5}\left(a^2-3b^2\right)\sin^5(c+dx)-\frac{1}{3}\left(2a^2-3b^2\right)\sin^3(c+dx)+a^2\sin(c+dx)\right)}{d}-\frac{\frac{3}{5}a^2b\cos^5(c+dx)-\frac{1}{5}b^3\cos^5(c+dx)+\frac{1}{3}b^3\cos^3(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

output
```
-(((b^3*Cos[c + d*x]^3)/3 + (3*a^2*b*Cos[c + d*x]^5)/5 - (b^3*Cos[c + d*x]
^5)/5)/d) + (a*(a^2*Sin[c + d*x] - ((2*a^2 - 3*b^2)*Sin[c + d*x]^3)/3 + ((
a^2 - 3*b^2)*Sin[c + d*x]^5)/5))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Maple [A] (verified)

Time = 35.88 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{b^3\left(-\frac{\cos(dx+c)^3\sin(dx+c)^2}{5}-\frac{2\cos(dx+c)^3}{15}\right)+3a\,b^2\left(-\frac{\cos(dx+c)^4\sin(dx+c)}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right)-\frac{3a^2b\cos(dx+c}{5}}{d}$
default	$\frac{b^3\left(-\frac{\cos(dx+c)^3\sin(dx+c)^2}{5}-\frac{2\cos(dx+c)^3}{15}\right)+3a\,b^2\left(-\frac{\cos(dx+c)^4\sin(dx+c)}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right)-\frac{3a^2b\cos(dx+c}{5}}{d}$
risch	$-\frac{3b\cos(dx+c)a^2}{8d}-\frac{b^3\cos(dx+c)}{8d}+\frac{5a^3\sin(dx+c)}{8d}+\frac{3a\sin(dx+c)b^2}{8d}-\frac{3b\cos(5dx+5c)a^2}{80d}+\frac{b^3\cos(5dx+5c)}{80d}$

input
```
int(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(-1/5*cos(d*x+c)^3*sin(d*x+c)^2-2/15*cos(d*x+c)^3)+3*a*b^2*(-1/5*
cos(d*x+c)^4*sin(d*x+c)+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-3/5*a^2*b*cos(d*
x+c)^5+1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx =$$
$$-\frac{5\,b^3\cos\left(dx+c\right)^3+3\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^5-\left(3\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^4+8\,a^3+6\,ab^2+\left(4\,a^3+\right.\right.}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/15*(5*b^3*cos(d*x + c)^3 + 3*(3*a^2*b - b^3)*cos(d*x + c)^5 - (3*(a^3 -
 3*a*b^2)*cos(d*x + c)^4 + 8*a^3 + 6*a*b^2 + (4*a^3 + 3*a*b^2)*cos(d*x + c
)^2)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\cos^5(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**5*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{9\,a^2b\cos(dx+c)^5-\left(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c)\right)a^3+3\left(3\,\sin(dx+c)^5-5\right.}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/15*(9*a^2*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15
*sin(d*x + c))*a^3 + 3*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a*b^2 - (3*co
s(d*x + c)^5 - 5*cos(d*x + c)^3)*b^3)/d
```

Giac [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.32

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{2\left(\frac{3\sin(c+dx)\,a^3\cos(c+dx)^4}{2}+2\sin(c+dx)\,a^3\cos(c+dx)^2+4\sin(c+dx)\,a^3-\frac{9a^2b\cos(c+dx)^5}{2}-\frac{9\sin(c+d\ldots}{}\right.}{15\,d}$$

input
```
int(cos(c + d*x)^5*(a + b*tan(c + d*x))^3,x)
```

output
```
(2*(4*a^3*sin(c + d*x) - (5*b^3*cos(c + d*x)^3)/2 + (3*b^3*cos(c + d*x)^5)
/2 - (9*a^2*b*cos(c + d*x)^5)/2 + 2*a^3*cos(c + d*x)^2*sin(c + d*x) + (3*a
^3*cos(c + d*x)^4*sin(c + d*x))/2 + 3*a*b^2*sin(c + d*x) + (3*a*b^2*cos(c
+ d*x)^2*sin(c + d*x))/2 - (9*a*b^2*cos(c + d*x)^4*sin(c + d*x))/2))/(15*d
)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.65

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{-9\cos(dx+c)\sin(dx+c)^4a^2b+3\cos(dx+c)\sin(dx+c)^4b^3+18\cos(dx+c)\sin(dx+c)^2a^2b-\cos\ldots}{}$$

input
```
int(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x)
```

output
```
( - 9*cos(c + d*x)*sin(c + d*x)**4*a**2*b + 3*cos(c + d*x)*sin(c + d*x)**4
*b**3 + 18*cos(c + d*x)*sin(c + d*x)**2*a**2*b - cos(c + d*x)*sin(c + d*x)
**2*b**3 - 9*cos(c + d*x)*a**2*b - 2*cos(c + d*x)*b**3 + 3*sin(c + d*x)**5
*a**3 - 9*sin(c + d*x)**5*a*b**2 - 10*sin(c + d*x)**3*a**3 + 15*sin(c + d*
x)**3*a*b**2 + 15*sin(c + d*x)*a**3 + 9*a**2*b + 2*b**3)/(15*d)
```

3.550 $\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{b^3\cos^5(c+dx)}{5d} - \frac{b(3a^2-b^2)\cos^7(c+dx)}{7d} + \frac{a^3\sin(c+dx)}{d} - \frac{a(a^2-b^2)\sin^3(c+dx)}{d} + \frac{3a(a^2-2b^2)\sin^5(c+dx)}{5d} - \frac{a(a^2-3b^2)\sin^7(c+dx)}{7d}$$

output
```
-1/5*b^3*cos(d*x+c)^5/d-1/7*b*(3*a^2-b^2)*cos(d*x+c)^7/d+a^3*sin(d*x+c)/d-
a*(a^2-b^2)*sin(d*x+c)^3/d+3/5*a*(a^2-2*b^2)*sin(d*x+c)^5/d-1/7*a*(a^2-3*b
^2)*sin(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.07

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{-30a^2b\cos^7(c+dx)+b^3\cos^5(c+dx)(-9+5\cos(2(c+dx)))+4b^3\sqrt{\cos^2(c+dx)}\sec(c+dx)+2a\sin(}{70d}$$

input
```
Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^3,x]
```

output
```
(-30*a^2*b*Cos[c + d*x]^7 + b^3*Cos[c + d*x]^5*(-9 + 5*Cos[2*(c + d*x)]) +
 4*b^3*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x] + 2*a*Sin[c + d*x]*(35*a^2 - 35*(
a^2 - b^2)*Sin[c + d*x]^2 + 21*(a^2 - 2*b^2)*Sin[c + d*x]^4 - 5*(a^2 - 3*b
^2)*Sin[c + d*x]^6))/(70*d)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3991, 3042, 4159, 27, 290, 2009, 4857, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+b\tan(c+dx))^3}{\sec(c+dx)^7}dx$$
$$\Big\downarrow \text{3991}$$
$$\int \cos^7(c+dx)\left(a^3+3b^2\tan^2(c+dx)a\right)dx+\int \cos^6(c+dx)\sin(c+dx)\left(\tan^2(c+dx)b^3+3a^2b\right)dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{a^3+3b^2\tan(c+dx)^2a}{\sec(c+dx)^7}dx+\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{4159}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^6}dx+\frac{\int a\left(1-\sin^2(c+dx)\right)^2\left(a^2-\left(a^2-3b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^6}dx+\frac{a\int \left(1-\sin^2(c+dx)\right)^2\left(a^2-\left(a^2-3b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^6}dx+\frac{a\int \left(-\left(\left(a^2-3b^2\right)\sin^6(c+dx)\right)+3\left(a^2-2b^2\right)\sin^4(c+dx)-3\left(a^2-b^2\right)\sin^2(c+dx)+a^2\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^6}dx+\frac{a\left(-\frac{1}{7}\left(a^2-3b^2\right)\sin^7(c+dx)+\frac{3}{5}\left(a^2-2b^2\right)\sin^5(c+dx)-\left(a^2-b^2\right)\sin^3(c+dx)+a^2\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4857}$$

$$\frac{a\left(-\frac{1}{7}\left(a^2-3b^2\right)\sin^7(c+dx)+\frac{3}{5}\left(a^2-2b^2\right)\sin^5(c+dx)-\left(a^2-b^2\right)\sin^3(c+dx)+a^2\sin(c+dx)\right)}{d}-\frac{\int \left(3a^2b\cos^6(c+dx)+b^3\left(1-\cos^2(c+dx)\right)\cos^4(c+dx)\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(-\frac{1}{7}\left(a^2-3b^2\right)\sin^7(c+dx)+\frac{3}{5}\left(a^2-2b^2\right)\sin^5(c+dx)-\left(a^2-b^2\right)\sin^3(c+dx)+a^2\sin(c+dx)\right)}{d}-\frac{\frac{3}{7}a^2b\cos^7(c+dx)-\frac{1}{7}b^3\cos^7(c+dx)+\frac{1}{5}b^3\cos^5(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^3,x]
```

output
```
-(((b^3*Cos[c + d*x]^5)/5 + (3*a^2*b*Cos[c + d*x]^7)/7 - (b^3*Cos[c + d*x]
^7)/7)/d) + (a*(a^2*Sin[c + d*x] - (a^2 - b^2)*Sin[c + d*x]^3 + (3*(a^2 -
2*b^2)*Sin[c + d*x]^5)/5 - ((a^2 - 3*b^2)*Sin[c + d*x]^7)/7))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Maple [A] (verified)

Time = 113.41 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{a^3\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}-\frac{3a^2b\cos(dx+c)^7}{7}+3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^6}{7}+\frac{\left(\frac{8}{3}+\cos(d\right.}{}\right.}{d}$
default	$\frac{\frac{a^3\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}-\frac{3a^2b\cos(dx+c)^7}{7}+3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^6}{7}+\frac{\left(\frac{8}{3}+\cos(d\right.}{}\right.}{d}$
risch	$-\frac{15b\cos(dx+c)a^2}{64d}-\frac{3b^3\cos(dx+c)}{64d}+\frac{35a^3\sin(dx+c)}{64d}+\frac{15a\sin(dx+c)b^2}{64d}-\frac{3b\cos(7dx+7c)a^2}{448d}+\frac{b^3\cos(7dx-}{448d}$

input
```
int(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/7*a^3*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x
+c)-3/7*a^2*b*cos(d*x+c)^7+3*a*b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3
+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+b^3*(-1/7*cos(d*x+c)^5*sin(d*x
+c)^2-2/35*cos(d*x+c)^5))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.93

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx =$$
$$-\frac{7\,b^3\cos\left(dx+c\right)^5+5\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^7-\left(5\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^6+3\left(2\,a^3+ab^2\right)\cos\left(dx+\right.\right.}{35\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/35*(7*b^3*cos(d*x + c)^5 + 5*(3*a^2*b - b^3)*cos(d*x + c)^7 - (5*(a^3 -
 3*a*b^2)*cos(d*x + c)^6 + 3*(2*a^3 + a*b^2)*cos(d*x + c)^4 + 16*a^3 + 8*a
*b^2 + 4*(2*a^3 + a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**7*(a+b*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{15\,a^2b\cos\left(dx+c\right)^7+\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a^3-(1}{35\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/35*(15*a^2*b*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 3
5*sin(d*x + c)^3 - 35*sin(d*x + c))*a^3 - (15*sin(d*x + c)^7 - 42*sin(d*x
+ c)^5 + 35*sin(d*x + c)^3)*a*b^2 - (5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*
b^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101962 vs. $2(124) = 248$.

Time = 77.23 (sec) , antiderivative size = 101962, normalized size of antiderivative = 772.44

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/17920*(945*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 210*pi*b^3*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x
)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 945*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 -
2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2
*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^1
4*tan(1/2*c)^14 + 210*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 - 2205*
pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2
*c)^14 + 1995*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x
)^14*tan(1/2*c)^14 + 2205*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*...
```

Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.62

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{16\,a^3\,\sin\left(c+d\,x\right)}{35\,d}-\frac{b^3\cos\left(c+d\,x\right)^5}{5\,d}+\frac{b^3\cos\left(c+d\,x\right)^7}{7\,d}-\frac{3\,a^2\,b\cos\left(c+d\,x\right)^7}{7\,d}+\frac{8\,a^3\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{6\,a^3\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{35\,d}+\frac{a^3\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)}{7\,d}+\frac{8\,a\,b^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{4\,a\,b^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{3\,a\,b^2\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{35\,d}-\frac{3\,a\,b^2\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)}{7\,d}$$

input
```
int(cos(c + d*x)^7*(a + b*tan(c + d*x))^3,x)
```

output
```
(16*a^3*sin(c + d*x))/(35*d) - (b^3*cos(c + d*x)^5)/(5*d) + (b^3*cos(c + d
*x)^7)/(7*d) - (3*a^2*b*cos(c + d*x)^7)/(7*d) + (8*a^3*cos(c + d*x)^2*sin(
c + d*x))/(35*d) + (6*a^3*cos(c + d*x)^4*sin(c + d*x))/(35*d) + (a^3*cos(c
 + d*x)^6*sin(c + d*x))/(7*d) + (8*a*b^2*sin(c + d*x))/(35*d) + (4*a*b^2*c
os(c + d*x)^2*sin(c + d*x))/(35*d) + (3*a*b^2*cos(c + d*x)^4*sin(c + d*x))
/(35*d) - (3*a*b^2*cos(c + d*x)^6*sin(c + d*x))/(7*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.89

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{15\cos(dx+c)\sin(dx+c)^6 a^2b - 5\cos(dx+c)\sin(dx+c)^6 b^3 - 45\cos(dx+c)\sin(dx+c)^4 a^2b + 8\,\mathrm{co}}{}$$

input
```
int(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x)
```

output
```
(15*cos(c + d*x)*sin(c + d*x)**6*a**2*b - 5*cos(c + d*x)*sin(c + d*x)**6*b
**3 - 45*cos(c + d*x)*sin(c + d*x)**4*a**2*b + 8*cos(c + d*x)*sin(c + d*x)
**4*b**3 + 45*cos(c + d*x)*sin(c + d*x)**2*a**2*b - cos(c + d*x)*sin(c + d
*x)**2*b**3 - 15*cos(c + d*x)*a**2*b - 2*cos(c + d*x)*b**3 - 5*sin(c + d*x
)**7*a**3 + 15*sin(c + d*x)**7*a*b**2 + 21*sin(c + d*x)**5*a**3 - 42*sin(c
 + d*x)**5*a*b**2 - 35*sin(c + d*x)**3*a**3 + 35*sin(c + d*x)**3*a*b**2 +
35*sin(c + d*x)*a**3 + 15*a**2*b + 2*b**3)/(35*d)
```

3.551 $\int \cos^9(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 165

$$\int \cos^9(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{b^3\cos^7(c+dx)}{7d}-\frac{b(3a^2-b^2)\cos^9(c+dx)}{9d}+\frac{a^3\sin(c+dx)}{d}-\frac{a(4a^2-3b^2)\sin^3(c+dx)}{3d}+\frac{3a(2a^2-3b^2)\sin^5(c+dx)}{5d}-\frac{a(4a^2-9b^2)\sin^7(c+dx)}{7d}+\frac{a(a^2-3b^2)\sin^9(c+dx)}{9d}$$

output
```
-1/7*b^3*cos(d*x+c)^7/d-1/9*b*(3*a^2-b^2)*cos(d*x+c)^9/d+a^3*sin(d*x+c)/d-
1/3*a*(4*a^2-3*b^2)*sin(d*x+c)^3/d+3/5*a*(2*a^2-3*b^2)*sin(d*x+c)^5/d-1/7*
a*(4*a^2-9*b^2)*sin(d*x+c)^7/d+1/9*a*(a^2-3*b^2)*sin(d*x+c)^9/d
```

Mathematica [A] (verified)

Time = 4.37 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01

$$\int \cos^9(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{-210a^2b\cos^9(c+dx)+5b^3\cos^7(c+dx)(-11+7\cos(2(c+dx)))+20b^3\sqrt{\cos^2(c+dx)}\sec(c+dx)+2a}{}$$

input
```
Integrate[Cos[c + d*x]^9*(a + b*Tan[c + d*x])^3,x]
```

output
```
(-210*a^2*b*Cos[c + d*x]^9 + 5*b^3*Cos[c + d*x]^7*(-11 + 7*Cos[2*(c + d*x)
]) + 20*b^3*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x] + 2*a*Sin[c + d*x]*(315*a^2
- 105*(4*a^2 - 3*b^2)*Sin[c + d*x]^2 + 189*(2*a^2 - 3*b^2)*Sin[c + d*x]^4
- 45*(4*a^2 - 9*b^2)*Sin[c + d*x]^6 + 35*(a^2 - 3*b^2)*Sin[c + d*x]^8))/(6
30*d)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3991, 3042, 4159, 27, 290, 2009, 4857, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^9(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+b\tan(c+dx))^3}{\sec(c+dx)^9}dx$$
$$\Big\downarrow \text{3991}$$
$$\int \cos^9(c+dx)\left(a^3+3b^2\tan^2(c+dx)a\right)dx+\int \cos^8(c+dx)\sin(c+dx)\left(\tan^2(c+dx)b^3+3a^2b\right)dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{a^3+3b^2\tan(c+dx)^2a}{\sec(c+dx)^9}dx+\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^8}dx$$

$$\Big\downarrow \text{4159}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^8}dx+\frac{\int a\left(1-\sin^2(c+dx)\right)^3\left(a^2-\left(a^2-3b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^8}dx+\frac{a\int \left(1-\sin^2(c+dx)\right)^3\left(a^2-\left(a^2-3b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^8}dx+\frac{a\int \left(\left(a^2-3b^2\right)\sin^8(c+dx)-\left(4a^2-9b^2\right)\sin^6(c+dx)+3\left(2a^2-3b^2\right)\sin^4(c+dx)-\left(4a^2-3b^2\right)\sin^2(c+dx)\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^8}dx+\frac{a\left(\frac{1}{9}\left(a^2-3b^2\right)\sin^9(c+dx)-\frac{1}{7}\left(4a^2-9b^2\right)\sin^7(c+dx)+\frac{3}{5}\left(2a^2-3b^2\right)\sin^5(c+dx)-\frac{1}{3}\left(4a^2-3b^2\right)\sin^3(c+dx\right.}{d}$$

$$\Big\downarrow \text{4857}$$

$$\frac{a\left(\frac{1}{9}\left(a^2-3b^2\right)\sin^9(c+dx)-\frac{1}{7}\left(4a^2-9b^2\right)\sin^7(c+dx)+\frac{3}{5}\left(2a^2-3b^2\right)\sin^5(c+dx)-\frac{1}{3}\left(4a^2-3b^2\right)\sin^3(c+dx\right.}{d}$$
$$\frac{\int \left(3a^2b\cos^8(c+dx)+b^3\left(1-\cos^2(c+dx)\right)\cos^6(c+dx)\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(\frac{1}{9}\left(a^2-3b^2\right)\sin^9(c+dx)-\frac{1}{7}\left(4a^2-9b^2\right)\sin^7(c+dx)+\frac{3}{5}\left(2a^2-3b^2\right)\sin^5(c+dx)-\frac{1}{3}\left(4a^2-3b^2\right)\sin^3(c+dx\right.}{d}$$
$$\frac{\frac{1}{3}a^2b\cos^9(c+dx)-\frac{1}{9}b^3\cos^9(c+dx)+\frac{1}{7}b^3\cos^7(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^9*(a + b*Tan[c + d*x])^3,x]
```

output
```
-(((b^3*Cos[c + d*x]^7)/7 + (a^2*b*Cos[c + d*x]^9)/3 - (b^3*Cos[c + d*x]^9
)/9)/d) + (a*(a^2*Sin[c + d*x] - ((4*a^2 - 3*b^2)*Sin[c + d*x]^3)/3 + (3*(
2*a^2 - 3*b^2)*Sin[c + d*x]^5)/5 - ((4*a^2 - 9*b^2)*Sin[c + d*x]^7)/7 + ((
a^2 - 3*b^2)*Sin[c + d*x]^9)/9))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Maple [A] (verified)

Time = 292.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{b^3\left(-\frac{\cos(dx+c)^7\sin(dx+c)^2}{9}-\frac{2\cos(dx+c)^7}{63}\right)+3a\,b^2\left(-\frac{\cos(dx+c)^8\sin(dx+c)}{9}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c}{5}\right.}{63}\right.}{d}$
default	$\frac{b^3\left(-\frac{\cos(dx+c)^7\sin(dx+c)^2}{9}-\frac{2\cos(dx+c)^7}{63}\right)+3a\,b^2\left(-\frac{\cos(dx+c)^8\sin(dx+c)}{9}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c}{5}\right.}{63}\right.}{d}$
risch	$-\frac{21b\cos(dx+c)a^2}{128d}-\frac{3b^3\cos(dx+c)}{128d}+\frac{63a^3\sin(dx+c)}{128d}+\frac{21a\sin(dx+c)b^2}{128d}-\frac{b\cos(9dx+9c)a^2}{768d}+\frac{b^3\cos(9dx+}{2304d}$

input
```
int(cos(d*x+c)^9*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(-1/9*cos(d*x+c)^7*sin(d*x+c)^2-2/63*cos(d*x+c)^7)+3*a*b^2*(-1/9*
cos(d*x+c)^8*sin(d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d
*x+c)^2)*sin(d*x+c))-1/3*a^2*b*cos(d*x+c)^9+1/9*a^3*(128/35+cos(d*x+c)^8+8
/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.89

$$\int \cos^9(c+dx)(a+b\tan(c+dx))^3\,dx =$$
$$-\frac{45\,b^3\cos\left(dx+c\right)^7+35\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^9-\left(35\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^8+5\left(8\,a^3+3\,ab^2\right)\cos}{3}$$

input
```
integrate(cos(d*x+c)^9*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/315*(45*b^3*cos(d*x + c)^7 + 35*(3*a^2*b - b^3)*cos(d*x + c)^9 - (35*(a
^3 - 3*a*b^2)*cos(d*x + c)^8 + 5*(8*a^3 + 3*a*b^2)*cos(d*x + c)^6 + 6*(8*a
^3 + 3*a*b^2)*cos(d*x + c)^4 + 128*a^3 + 48*a*b^2 + 8*(8*a^3 + 3*a*b^2)*co
s(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^9(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**9*(a+b*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.89

$$\int \cos^9(c+dx)(a+b\tan(c+dx))^3\,dx =$$
$$-\frac{105\,a^2b\cos\left(dx+c\right)^9-\left(35\,\sin\left(dx+c\right)^9-180\,\sin\left(dx+c\right)^7+378\,\sin\left(dx+c\right)^5-420\,\sin\left(dx+c\right)^3\right.}{}$$

input
```
integrate(cos(d*x+c)^9*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/315*(105*a^2*b*cos(d*x + c)^9 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7
 + 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^3 + 3*(35
*sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*x +
c)^3)*a*b^2 - 5*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*b^3)/d
```

Giac [F(-1)]

Timed out.

$$\int \cos^9(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^9*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.57

$$\int \cos^9(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{128\,a^3\,\sin\left(c+d\,x\right)}{315\,d}-\frac{b^3\cos\left(c+d\,x\right)^7}{7\,d}+\frac{b^3\cos\left(c+d\,x\right)^9}{9\,d}-\frac{a^2\,b\cos\left(c+d\,x\right)^9}{3\,d}+\frac{64\,a^3\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{315\,d}+\frac{16\,a^3\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{105\,d}+\frac{8\,a^3\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)}{63\,d}+\frac{a^3\cos\left(c+d\,x\right)^8\,\sin\left(c+d\,x\right)}{9\,d}+\frac{16\,a\,b^2\,\sin\left(c+d\,x\right)}{105\,d}+\frac{8\,a\,b^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{105\,d}+\frac{2\,a\,b^2\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{35\,d}+\frac{a\,b^2\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)}{21\,d}-\frac{a\,b^2\cos\left(c+d\,x\right)^8\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int(cos(c + d*x)^9*(a + b*tan(c + d*x))^3,x)
```

output
```
(128*a^3*sin(c + d*x))/(315*d) - (b^3*cos(c + d*x)^7)/(7*d) + (b^3*cos(c +
 d*x)^9)/(9*d) - (a^2*b*cos(c + d*x)^9)/(3*d) + (64*a^3*cos(c + d*x)^2*sin
(c + d*x))/(315*d) + (16*a^3*cos(c + d*x)^4*sin(c + d*x))/(105*d) + (8*a^3
*cos(c + d*x)^6*sin(c + d*x))/(63*d) + (a^3*cos(c + d*x)^8*sin(c + d*x))/(
9*d) + (16*a*b^2*sin(c + d*x))/(105*d) + (8*a*b^2*cos(c + d*x)^2*sin(c + d
*x))/(105*d) + (2*a*b^2*cos(c + d*x)^4*sin(c + d*x))/(35*d) + (a*b^2*cos(c
 + d*x)^6*sin(c + d*x))/(21*d) - (a*b^2*cos(c + d*x)^8*sin(c + d*x))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.91

$$\int \cos^9(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{-105\cos(dx+c)\sin(dx+c)^8 a^2b + 35\cos(dx+c)\sin(dx+c)^8 b^3 + 420\cos(dx+c)\sin(dx+c)^6 a^2b}{}$$

input
```
int(cos(d*x+c)^9*(a+b*tan(d*x+c))^3,x)
```

output
```
( - 105*cos(c + d*x)*sin(c + d*x)**8*a**2*b + 35*cos(c + d*x)*sin(c + d*x)
**8*b**3 + 420*cos(c + d*x)*sin(c + d*x)**6*a**2*b - 95*cos(c + d*x)*sin(c
 + d*x)**6*b**3 - 630*cos(c + d*x)*sin(c + d*x)**4*a**2*b + 75*cos(c + d*x
)*sin(c + d*x)**4*b**3 + 420*cos(c + d*x)*sin(c + d*x)**2*a**2*b - 5*cos(c
 + d*x)*sin(c + d*x)**2*b**3 - 105*cos(c + d*x)*a**2*b - 10*cos(c + d*x)*b
**3 + 35*sin(c + d*x)**9*a**3 - 105*sin(c + d*x)**9*a*b**2 - 180*sin(c + d
*x)**7*a**3 + 405*sin(c + d*x)**7*a*b**2 + 378*sin(c + d*x)**5*a**3 - 567*
sin(c + d*x)**5*a*b**2 - 420*sin(c + d*x)**3*a**3 + 315*sin(c + d*x)**3*a*
b**2 + 315*sin(c + d*x)*a**3 + 105*a**2*b + 10*b**3)/(315*d)
```

3.552 $\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^5d} - \frac{a(a^2+2b^2)\tan(c+dx)}{b^4d} + \frac{(a^2+2b^2)\tan^2(c+dx)}{2b^3d} - \frac{a\tan^3(c+dx)}{3b^2d} + \frac{\tan^4(c+dx)}{4bd}$$

output
```
(a^2+b^2)^2*ln(a+b*tan(d*x+c))/b^5/d-a*(a^2+2*b^2)*tan(d*x+c)/b^4/d+1/2*(a
^2+2*b^2)*tan(d*x+c)^2/b^3/d-1/3*a*tan(d*x+c)^3/b^2/d+1/4*tan(d*x+c)^4/b/d
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{12(a^2+b^2)^2\log(a+b\tan(c+dx)) + 3b^4\sec^4(c+dx) - 12ab(a^2+2b^2)\tan(c+dx) + 6b^2(a^2+b^2)\tan^2(}{12b^5d}$$

input
```
Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

output
```
(12*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]] + 3*b^4*Sec[c + d*x]^4 - 12*a*b*
(a^2 + 2*b^2)*Tan[c + d*x] + 6*b^2*(a^2 + b^2)*Tan[c + d*x]^2 - 4*a*b^3*Ta
n[c + d*x]^3)/(12*b^5*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{\left(\tan^2(c+dx)b^2+b^2\right)^2}{b^4(a+b\tan(c+dx))}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(\tan^2(c+dx)b^2+b^2\right)^2}{a+b\tan(c+dx)}d(b\tan(c+dx))}{b^5d}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(b^3\tan^3(c+dx)-ab^2\tan^2(c+dx)+b\left(a^2+2b^2\right)\tan(c+dx)-a\left(a^2+2b^2\right)+\frac{\left(a^2+b^2\right)^2}{a+b\tan(c+dx)}\right)d(b\tan(c+dx))}{b^5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}b^2\left(a^2+2b^2\right)\tan^2(c+dx)-ab\left(a^2+2b^2\right)\tan(c+dx)+\left(a^2+b^2\right)^2\log(a+b\tan(c+dx))-\frac{1}{3}ab^3\tan^3(c+dx)}{b^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

output
```
((a^2 + b^2)^2*Log[a + b*Tan[c + d*x]] - a*b*(a^2 + 2*b^2)*Tan[c + d*x] +
(b^2*(a^2 + 2*b^2)*Tan[c + d*x]^2)/2 - (a*b^3*Tan[c + d*x]^3)/3 + (b^4*Tan
[c + d*x]^4)/4)/(b^5*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 29.98 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{-\frac{\tan(dx+c)^4 b^3}{4}+\frac{\tan(dx+c)^3 a\,b^2}{3}-\frac{\left(a^2+2b^2\right)\tan(dx+c)^2 b}{2}+a\left(a^2+2b^2\right)\tan(dx+c)}{b^4}+\frac{\left(a^4+2b^2a^2+b^4\right)\ln(a+b\tan(dx+c))}{b^5}}{d}$
default	$\frac{-\frac{-\frac{\tan(dx+c)^4 b^3}{4}+\frac{\tan(dx+c)^3 a\,b^2}{3}-\frac{\left(a^2+2b^2\right)\tan(dx+c)^2 b}{2}+a\left(a^2+2b^2\right)\tan(dx+c)}{b^4}+\frac{\left(a^4+2b^2a^2+b^4\right)\ln(a+b\tan(dx+c))}{b^5}}{d}$
risch	$\frac{-2ia^3 e^{6i(dx+c)}-2ia\,b^2 e^{6i(dx+c)}+2a^2 b\,e^{6i(dx+c)}+2b^3 e^{6i(dx+c)}-6ia^3 e^{4i(dx+c)}-10ia\,b^2 e^{4i(dx+c)}+4a^2 b\,e^{4i(dx+c)}+8b^3}{b^4 d\left(e^{2i(dx+c)}+1\right)^4}$

input
```
int(sec(d*x+c)^6/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^4*(-1/4*tan(d*x+c)^4*b^3+1/3*tan(d*x+c)^3*a*b^2-1/2*(a^2+2*b^2)*
tan(d*x+c)^2*b+a*(a^2+2*b^2)*tan(d*x+c))+(a^4+2*a^2*b^2+b^4)/b^5*ln(a+b*ta
n(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.58

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{6\left(a^4+2\,a^2b^2+b^4\right)\cos\left(dx+c\right)^4\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-6\left(a^4\right.}{}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(6*(a^4 + 2*a^2*b^2 + b^4)*cos(d*x + c)^4*log(2*a*b*cos(d*x + c)*sin(
d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 6*(a^4 + 2*a^2*b^2 + b^4)*c
os(d*x + c)^4*log(cos(d*x + c)^2) + 3*b^4 + 6*(a^2*b^2 + b^4)*cos(d*x + c)
^2 - 4*(a*b^3*cos(d*x + c) + (3*a^3*b + 5*a*b^3)*cos(d*x + c)^3)*sin(d*x +
 c))/(b^5*d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a+b*tan(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**6/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{3\,b^3\tan(dx+c)^4-4\,ab^2\tan(dx+c)^3+6\,(a^2b+2\,b^3)\tan(dx+c)^2-12\,(a^3+2\,ab^2)\tan(dx+c)}{b^4}+\frac{12\,(a^4+2\,a^2b^2+b^4)\log(b\tan(dx+c)+a)}{b^5}}{12\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*((3*b^3*tan(d*x + c)^4 - 4*a*b^2*tan(d*x + c)^3 + 6*(a^2*b + 2*b^3)*t
an(d*x + c)^2 - 12*(a^3 + 2*a*b^2)*tan(d*x + c))/b^4 + 12*(a^4 + 2*a^2*b^2
 + b^4)*log(b*tan(d*x + c) + a)/b^5)/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.20

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{(a^4+2\,a^2b^2+b^4)\log(|b\tan(dx+c)+a|)}{b^5d} + \frac{3\,b^3d^3\tan(dx+c)^4-4\,ab^2d^3\tan(dx+c)^3+6\,a^2bd^3\tan(dx+c)^2+12\,b^3d^3\tan(dx+c)^2-12\,a^3d^3\,\mathrm{ta}}{12\,b^4d^4}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
(a^4 + 2*a^2*b^2 + b^4)*log(abs(b*tan(d*x + c) + a))/(b^5*d) + 1/12*(3*b^3
*d^3*tan(d*x + c)^4 - 4*a*b^2*d^3*tan(d*x + c)^3 + 6*a^2*b*d^3*tan(d*x + c
)^2 + 12*b^3*d^3*tan(d*x + c)^2 - 12*a^3*d^3*tan(d*x + c) - 24*a*b^2*d^3*t
an(d*x + c))/(b^4*d^4)
```

Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\tan(c+d\,x)^4}{4\,b\,d} + \frac{\tan(c+d\,x)^2\,\left(\frac{1}{b}+\frac{a^2}{2\,b^3}\right)}{d} + \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}{b^5\,d} - \frac{a\tan(c+d\,x)^3}{3\,b^2\,d} - \frac{a\tan(c+d\,x)\,\left(\frac{2}{b}+\frac{a^2}{b^3}\right)}{b\,d}$$

input
```
int(1/(cos(c + d*x)^6*(a + b*tan(c + d*x))),x)
```

output
```
tan(c + d*x)^4/(4*b*d) + (tan(c + d*x)^2*(1/b + a^2/(2*b^3)))/d + (log(a +
 b*tan(c + d*x))*(a^4 + b^4 + 2*a^2*b^2))/(b^5*d) - (a*tan(c + d*x)^3)/(3*
b^2*d) - (a*tan(c + d*x)*(2/b + a^2/b^3))/(b*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 954, normalized size of antiderivative = 8.22

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^6/(a+b*tan(d*x+c)),x)
```

output
```
(12*cos(c + d*x)*sin(c + d*x)**3*a**3*b + 20*cos(c + d*x)*sin(c + d*x)**3*
a*b**3 - 12*cos(c + d*x)*sin(c + d*x)*a**3*b - 24*cos(c + d*x)*sin(c + d*x
)*a*b**3 - 12*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**4 - 24*log(tan(
(c + d*x)/2) - 1)*sin(c + d*x)**4*a**2*b**2 - 12*log(tan((c + d*x)/2) - 1)
*sin(c + d*x)**4*b**4 + 24*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**4
+ 48*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**2*b**2 + 24*log(tan((c +
 d*x)/2) - 1)*sin(c + d*x)**2*b**4 - 12*log(tan((c + d*x)/2) - 1)*a**4 - 2
4*log(tan((c + d*x)/2) - 1)*a**2*b**2 - 12*log(tan((c + d*x)/2) - 1)*b**4
- 12*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a**4 - 24*log(tan((c + d*x)
/2) + 1)*sin(c + d*x)**4*a**2*b**2 - 12*log(tan((c + d*x)/2) + 1)*sin(c +
d*x)**4*b**4 + 24*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**4 + 48*log(
tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**2*b**2 + 24*log(tan((c + d*x)/2)
+ 1)*sin(c + d*x)**2*b**4 - 12*log(tan((c + d*x)/2) + 1)*a**4 - 24*log(tan
((c + d*x)/2) + 1)*a**2*b**2 - 12*log(tan((c + d*x)/2) + 1)*b**4 + 12*log(
tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)**4*a**4 + 2
4*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)**4*a*
*2*b**2 + 12*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c +
 d*x)**4*b**4 - 24*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*s
in(c + d*x)**2*a**4 - 48*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b
- a)*sin(c + d*x)**2*a**2*b**2 - 24*log(tan((c + d*x)/2)**2*a - 2*tan((...
```

3.553 $\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{(a^2+b^2)\log(a+b\tan(c+dx))}{b^3d} - \frac{a\tan(c+dx)}{b^2d} + \frac{\tan^2(c+dx)}{2bd}$$

output
```
(a^2+b^2)*ln(a+b*tan(d*x+c))/b^3/d-a*tan(d*x+c)/b^2/d+1/2*tan(d*x+c)^2/b/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{(a^2+b^2)\log(a+b\tan(c+dx)) - ab\tan(c+dx) + \frac{1}{2}b^2\tan^2(c+dx)}{b^3d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

output
```
((a^2 + b^2)*Log[a + b*Tan[c + d*x]] - a*b*Tan[c + d*x] + (b^2*Tan[c + d*x
]^2)/2)/(b^3*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^4}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{\tan^2(c+dx)b^2+b^2}{b^2(a+b\tan(c+dx))}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\tan^2(c+dx)b^2+b^2}{a+b\tan(c+dx)}d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(-a+b\tan(c+dx)+\frac{a^2+b^2}{a+b\tan(c+dx)}\right)d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a^2+b^2)\log(a+b\tan(c+dx))-ab\tan(c+dx)+\frac{1}{2}b^2\tan^2(c+dx)}{b^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

output
```
((a^2 + b^2)*Log[a + b*Tan[c + d*x]] - a*b*Tan[c + d*x] + (b^2*Tan[c + d*x
]^2)/2)/(b^3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 7.48 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{-\frac{b\tan(dx+c)^2}{2}+a\tan(dx+c)}{b^2}+\frac{\left(a^2+b^2\right)\ln(a+b\tan(dx+c))}{b^3}}{d}$
default	$\frac{-\frac{-\frac{b\tan(dx+c)^2}{2}+a\tan(dx+c)}{b^2}+\frac{\left(a^2+b^2\right)\ln(a+b\tan(dx+c))}{b^3}}{d}$
risch	$\frac{-2ia\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{2i(dx+c)}-2ia}{b^2d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)a^2}{b^3d}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{bd}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)a^2}{b^3d}+\ln\left(\mathrm{e}^{2i}\right.$

input
```
int(sec(d*x+c)^4/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^2*(-1/2*b*tan(d*x+c)^2+a*tan(d*x+c))+(a^2+b^2)/b^3*ln(a+b*tan(d*
x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(57) = 114$.

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.98

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{(a^2+b^2)\cos(dx+c)^2\log\left(2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2\right)-(a^2+b^2)\cos(}{2\,b^3d\cos(dx+c)^2}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((a^2 + b^2)*cos(d*x + c)^2*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2
 - b^2)*cos(d*x + c)^2 + b^2) - (a^2 + b^2)*cos(d*x + c)^2*log(cos(d*x + c
)^2) - 2*a*b*cos(d*x + c)*sin(d*x + c) + b^2)/(b^3*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{b\tan(dx+c)^2-2\,a\tan(dx+c)}{b^2}+\frac{2\,(a^2+b^2)\log(b\tan(dx+c)+a)}{b^3}}{2\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*((b*tan(d*x + c)^2 - 2*a*tan(d*x + c))/b^2 + 2*(a^2 + b^2)*log(b*tan(d
*x + c) + a)/b^3)/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{(a^2+b^2)\log(|b\tan(dx+c)+a|)}{b^3d} + \frac{bd\tan(dx+c)^2-2\,ad\tan(dx+c)}{2\,b^2d^2}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
(a^2 + b^2)*log(abs(b*tan(d*x + c) + a))/(b^3*d) + 1/2*(b*d*tan(d*x + c)^2
 - 2*a*d*tan(d*x + c))/(b^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\tan(c+d\,x)^2}{2\,b\,d}+\frac{\ln(a+b\tan(c+d\,x))\,(a^2+b^2)}{b^3\,d}-\frac{a\tan(c+d\,x)}{b^2\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + b*tan(c + d*x))),x)
```

output
```
tan(c + d*x)^2/(2*b*d) + (log(a + b*tan(c + d*x))*(a^2 + b^2))/(b^3*d) - (
a*tan(c + d*x))/(b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 378, normalized size of antiderivative = 6.41

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{2\cos(dx+c)\sin(dx+c)\,ab-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2a^2-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx}{}$$

input
```
int(sec(d*x+c)^4/(a+b*tan(d*x+c)),x)
```

output
```
(2*cos(c + d*x)*sin(c + d*x)*a*b - 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x
)**2*a**2 - 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b**2 + 2*log(tan((
c + d*x)/2) - 1)*a**2 + 2*log(tan((c + d*x)/2) - 1)*b**2 - 2*log(tan((c +
d*x)/2) + 1)*sin(c + d*x)**2*a**2 - 2*log(tan((c + d*x)/2) + 1)*sin(c + d*
x)**2*b**2 + 2*log(tan((c + d*x)/2) + 1)*a**2 + 2*log(tan((c + d*x)/2) + 1
)*b**2 + 2*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d
*x)**2*a**2 + 2*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(
c + d*x)**2*b**2 - 2*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)
*a**2 - 2*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*b**2 + sin
(c + d*x)**2*b**2 - 2*b**2)/(2*b**3*d*(sin(c + d*x)**2 - 1))
```

3.554 $\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\log(a+b\tan(c+dx))}{bd}$$

output
```
ln(a+b*tan(d*x+c))/b/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\log(a+b\tan(c+dx))}{bd}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

output
```
Log[a + b*Tan[c + d*x]]/(b*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3987, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^2}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{1}{a+b\tan(c+dx)}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log(a+b\tan(c+dx))}{bd}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

output
```
Log[a + b*Tan[c + d*x]]/(b*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.56 (sec), antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\ln(a+b\tan(dx+c))}{bd}$	19
default	$\frac{\ln(a+b\tan(dx+c))}{bd}$	19
risch	$-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{bd}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{bd}$	58

input
```
int(sec(d*x+c)^2/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*tan(d*x+c))/b/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(18) = 36.

Time = 0.10 (sec), antiderivative size = 59, normalized size of antiderivative = 3.28

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-\log\left(\cos\left(dx+c\right)^2\right)}{2\,bd}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^
2) - log(cos(d*x + c)^2))/(b*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sec^2{(c+dx)}}{a+b\tan{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*tan(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\log{(b\tan{(dx+c)}+a)}}{bd}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
log(b*tan(d*x + c) + a)/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\log{(|b\tan{(dx+c)}+a|)}}{bd}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
log(abs(b*tan(d*x + c) + a))/(b*d)
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\ln\left(a+b\tan(c+d\,x)\right)}{b\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x))),x)
```

output
```
log(a + b*tan(c + d*x))/(b*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.67

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b-a\right)}{bd}$$

input
```
int(sec(d*x+c)^2/(a+b*tan(d*x+c)),x)
```

output
```
( - log(tan((c + d*x)/2) - 1) - log(tan((c + d*x)/2) + 1) + log(tan((c + d
*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a))/(b*d)
```

3.555 $\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{a(a^2+3b^2)\,x}{2\,(a^2+b^2)^2} + \frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\,(a^2+b^2)\,d}$$

output
```
1/2*a*(a^2+3*b^2)*x/(a^2+b^2)^2+b^3*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2
)^2/d+1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.54

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2a^3c+6ab^2c+4ib^3c+2a^3dx+6ab^2dx+4ib^3dx-4ib^3\arctan(\tan(c+dx))+b(a^2+b^2)\cos(2(c+dx))}{4\,(a^2+b^2)^2\,d}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

output
```
(2*a^3*c + 6*a*b^2*c + (4*I)*b^3*c + 2*a^3*d*x + 6*a*b^2*d*x + (4*I)*b^3*d
*x - (4*I)*b^3*ArcTan[Tan[c + d*x]] + b*(a^2 + b^2)*Cos[2*(c + d*x)] + 2*b
^3*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2] + a^3*Sin[2*(c + d*x)] + a*b^2
*Sin[2*(c + d*x)])/(4*(a^2 + b^2)^2*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.70, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3987, 27, 496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{b^4}{(a+b\tan(c+dx))(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^3\int \frac{1}{(a+b\tan(c+dx))(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{496}$$

$$\frac{b^3\left(\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}-\frac{\int -\frac{a^2+b\tan(c+dx)a+2b^2}{(a+b\tan(c+dx))\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2(a^2+b^2)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^3\left(\frac{\int \frac{a^2+b\tan(c+dx)a+2b^2}{(a+b\tan(c+dx))\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{657}$$

$$\frac{b^3\left(\frac{\int\left(\frac{2b^2}{(a^2+b^2)(a+b\tan(c+dx))}+\frac{a^3+3b^2a-2b^3\tan(c+dx)}{(a^2+b^2)(\tan^2(c+dx)b^2+b^2)}\right)d(b\tan(c+dx))}{2b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^3\left(\frac{\frac{a(a^2+3b^2)\arctan(\tan(c+dx))}{b(a^2+b^2)}-\frac{b^2\log(b^2\tan^2(c+dx)+b^2)}{a^2+b^2}+\frac{2b^2\log(a+b\tan(c+dx))}{a^2+b^2}}{2b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

output
```
(b^3*(((a*(a^2 + 3*b^2)*ArcTan[Tan[c + d*x]])/(b*(a^2 + b^2)) + (2*b^2*Log
[a + b*Tan[c + d*x]])/(a^2 + b^2) - (b^2*Log[b^2 + b^2*Tan[c + d*x]^2])/(a
^2 + b^2))/(2*b^2*(a^2 + b^2)) + (b^2 + a*b*Tan[c + d*x])/(2*b^2*(a^2 + b^
2)*(b^2 + b^2*Tan[c + d*x]^2))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{\frac{b^{3}\ln\left(a+b\tan\left(dx+c\right)\right)}{\left(a^{2}+b^{2}\right)^{2}}+\frac{\frac{\left(\frac{1}{2}a^{3}+\frac{1}{2}a\,b^{2}\right)\tan\left(dx+c\right)+\frac{a^{2}b}{2}+\frac{b^{3}}{2}}{1+\tan\left(dx+c\right)^{2}}-\frac{\ln\left(1+\tan\left(dx+c\right)^{2}\right)b^{3}}{2}+\frac{\left(a^{3}+3a\,b^{2}\right)\arctan\left(\tan\left(dx+c\right)\right)}{2}}{\left(a^{2}+b^{2}\right)^{2}}}{d}$
default	$\frac{\frac{b^{3}\ln\left(a+b\tan\left(dx+c\right)\right)}{\left(a^{2}+b^{2}\right)^{2}}+\frac{\frac{\left(\frac{1}{2}a^{3}+\frac{1}{2}a\,b^{2}\right)\tan\left(dx+c\right)+\frac{a^{2}b}{2}+\frac{b^{3}}{2}}{1+\tan\left(dx+c\right)^{2}}-\frac{\ln\left(1+\tan\left(dx+c\right)^{2}\right)b^{3}}{2}+\frac{\left(a^{3}+3a\,b^{2}\right)\arctan\left(\tan\left(dx+c\right)\right)}{2}}{\left(a^{2}+b^{2}\right)^{2}}}{d}$
risch	$\frac{2ixb}{4iab-2a^{2}+2b^{2}}-\frac{xa}{4iab-2a^{2}+2b^{2}}-\frac{i\,{\mathrm e}^{2i\left(dx+c\right)}}{8\left(-ib+a\right)d}+\frac{i\,{\mathrm e}^{-2i\left(dx+c\right)}}{8\left(ib+a\right)d}-\frac{2ib^{3}x}{a^{4}+2b^{2}a^{2}+b^{4}}-\frac{2ib^{3}c}{d\left(a^{4}+2b^{2}a^{2}+b^{4}\right)}+\frac{b^{3}\ln(}{d(}$

input
```
int(cos(d*x+c)^2/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3/(a^2+b^2)^2*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^2*(((1/2*a^3+1/2*a*b^2
)*tan(d*x+c)+1/2*a^2*b+1/2*b^3)/(1+tan(d*x+c)^2)-1/2*ln(1+tan(d*x+c)^2)*b^
3+1/2*(a^3+3*a*b^2)*arctan(tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.28

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{b^3\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+\left(a^3+3\,ab^2\right)dx+\left(a^2b+b^3\right)\cos\left(dx\right.}{2\left(a^4+2\,a^2b^2+b^4\right)d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(b^3*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2) + (a^3 + 3*a*b^2)*d*x + (a^2*b + b^3)*cos(d*x + c)^2 + (a^3 + a*b^2
)*cos(d*x + c)*sin(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cos^2\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*tan(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**2/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.52

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{2\,b^3\log(b\tan(dx+c)+a)}{a^4+2\,a^2b^2+b^4}-\frac{b^3\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^3+3\,ab^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{a\tan(dx+c)+b}{\left(a^2+b^2\right)\tan(dx+c)^2+a^2+b^2}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(2*b^3*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - b^3*log(tan(d
*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 3*a*b^2)*(d*x + c)/(a^4 +
2*a^2*b^2 + b^4) + (a*tan(d*x + c) + b)/((a^2 + b^2)*tan(d*x + c)^2 + a^2
+ b^2))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.81

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{b^4\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^4bd+2\,a^2b^3d+b^5d} - \frac{b^3\log\left(\tan\left(dx+c\right)^2+1\right)}{2\left(a^4d+2\,a^2b^2d+b^4d\right)} + \frac{(a^3+3\,ab^2)(dx+c)}{2\left(a^4d+2\,a^2b^2d+b^4d\right)} + \frac{a^2b+b^3+(a^3+ab^2)\tan\left(dx+c\right)}{2\left(a^2+b^2\right)^2\left(\tan\left(dx+c\right)^2+1\right)d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
b^4*log(abs(b*tan(d*x + c) + a))/(a^4*b*d + 2*a^2*b^3*d + b^5*d) - 1/2*b^3
*log(tan(d*x + c)^2 + 1)/(a^4*d + 2*a^2*b^2*d + b^4*d) + 1/2*(a^3 + 3*a*b^
2)*(d*x + c)/(a^4*d + 2*a^2*b^2*d + b^4*d) + 1/2*(a^2*b + b^3 + (a^3 + a*b
^2)*tan(d*x + c))/((a^2 + b^2)^2*(tan(d*x + c)^2 + 1)*d)
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\cos\left(c+d\,x\right)^2\left(\frac{b}{2\left(a^2+b^2\right)}+\frac{a\tan(c+d\,x)}{2\left(a^2+b^2\right)}\right)}{d} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(2\,b+a\,1\mathrm{i}\right)}{4\,d\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a+b\,2\mathrm{i}\right)}{4\,d\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)} + \frac{b^3\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,{\left(a^2+b^2\right)}^2}$$

input
```
int(cos(c + d*x)^2/(a + b*tan(c + d*x)),x)
```

output
```
(cos(c + d*x)^2*(b/(2*(a^2 + b^2)) + (a*tan(c + d*x))/(2*(a^2 + b^2))))/d
- (log(tan(c + d*x) + 1i)*(a*1i + 2*b))/(4*d*(a*b*2i - a^2 + b^2)) - (log(
tan(c + d*x) - 1i)*(a + b*2i))/(4*d*(2*a*b - a^2*1i + b^2*1i)) + (b^3*log(
a + b*tan(c + d*x)))/(d*(a^2 + b^2)^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.87

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\cos(dx+c)\sin(dx+c)\,a^3+\cos(dx+c)\sin(dx+c)\,a\,b^2-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)b^3+2\log\left(\tan\left(\frac{dx}{2}\right.\right.}{2d}$$

input
```
int(cos(d*x+c)^2/(a+b*tan(d*x+c)),x)
```

output
```
(cos(c + d*x)*sin(c + d*x)*a**3 + cos(c + d*x)*sin(c + d*x)*a*b**2 - 2*log
(tan((c + d*x)/2)**2 + 1)*b**3 + 2*log(tan((c + d*x)/2)**2*a - 2*tan((c +
d*x)/2)*b - a)*b**3 - sin(c + d*x)**2*a**2*b - sin(c + d*x)**2*b**3 + a**3
*c + a**3*d*x + 2*a**2*b + 3*a*b**2*c + 3*a*b**2*d*x + 2*b**3)/(2*d*(a**4
+ 2*a**2*b**2 + b**4))
```

3.556 $\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{a(3a^4+10a^2b^2+15b^4)\,x}{8\,(a^2+b^2)^3} + \frac{b^5\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\,(a^2+b^2)\,d} + \frac{\cos^2(c+dx)\,(4b^3+a(3a^2+7b^2)\tan(c+dx))}{8\,(a^2+b^2)^2\,d}$$

output
```
1/8*a*(3*a^4+10*a^2*b^2+15*b^4)*x/(a^2+b^2)^3+b^5*ln(a*cos(d*x+c)+b*sin(d*
x+c))/(a^2+b^2)^3/d+1/4*cos(d*x+c)^4*(b+a*tan(d*x+c))/(a^2+b^2)/d+1/8*cos(
d*x+c)^2*(4*b^3+a*(3*a^2+7*b^2)*tan(d*x+c))/(a^2+b^2)^2/d
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.48

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{-\left(\left(8b^6+\sqrt{-b^2}(3a^5+10a^3b^2+15ab^4)\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)\right)+16b^6\log(a+b\tan(c+dx))-(8}{}$$

input
```
Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

output
```
(-((8*b^6 + Sqrt[-b^2]*(3*a^5 + 10*a^3*b^2 + 15*a*b^4))*Log[Sqrt[-b^2] - b
*Tan[c + d*x]]) + 16*b^6*Log[a + b*Tan[c + d*x]] - (8*b^6 - Sqrt[-b^2]*(3*
a^5 + 10*a^3*b^2 + 15*a*b^4))*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 4*b*(a^2
+ b^2)^2*Cos[c + d*x]^4*(b + a*Tan[c + d*x]) + 2*(a^2 + b^2)*Cos[c + d*x]^
2*(4*b^4 + a*b*(3*a^2 + 7*b^2)*Tan[c + d*x]))/(16*b*(a^2 + b^2)^3*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.61, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3987, 27, 496, 25, 686, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(c+dx)^4(a+b\tan(c+dx))}dx$$
$$\Big\downarrow \text{3987}$$
$$\frac{\int \frac{b^6}{(a+b\tan(c+dx))(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{bd}$$
$$\Big\downarrow \text{27}$$

$$\frac{b^5\int \frac{1}{(a+b\tan(c+dx))(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{496}$$

$$\frac{b^5\left(\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}-\frac{\int -\frac{3a^2+3b\tan(c+dx)a+4b^2}{(a+b\tan(c+dx))\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2(a^2+b^2)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^5\left(\frac{\int \frac{3a^2+3b\tan(c+dx)a+4b^2}{(a+b\tan(c+dx))\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}\right)}{d}$$

$$\Big\downarrow \text{686}$$

$$\frac{b^5\left(\frac{\frac{ab\left(3a^2+7b^2\right)\tan(c+dx)+4b^4}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{\int -\frac{3a^4+7b^2a^2+b\left(3a^2+7b^2\right)\tan(c+dx)a+8b^4}{(a+b\tan(c+dx))\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2\left(a^2+b^2\right)}}{4b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^5\left(\frac{\frac{\int \frac{3a^4+7b^2a^2+b\left(3a^2+7b^2\right)\tan(c+dx)a+8b^4}{(a+b\tan(c+dx))\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2\left(a^2+b^2\right)}+\frac{ab\left(3a^2+7b^2\right)\tan(c+dx)+4b^4}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)}}{4b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}\right)}{d}$$

$$\Big\downarrow \text{657}$$

$$\frac{b^5\left(\frac{\frac{\int \left(\frac{8b^4}{\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{3a^5+10b^2a^3+15b^4a-8b^5\tan(c+dx)}{\left(a^2+b^2\right)\left(\tan^2(c+dx)b^2+b^2\right)}\right)d(b\tan(c+dx))}{2b^2\left(a^2+b^2\right)}+\frac{ab\left(3a^2+7b^2\right)\tan(c+dx)+4b^4}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)}}{4b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+d\right.}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^5\left(\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}+\frac{\frac{ab\left(3a^2+7b^2\right)\tan(c+dx)+4b^4}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)}+\frac{-\frac{4b^4\log\left(b^2\tan^2(c+dx)+b^2\right)}{a^2+b^2}+\frac{8b^4\log(a+b\tan(c+dx))}{a^2+b^2}+\frac{a\left(3a^4+10a^2b^2+15\cdots\right.}{b\left(\cdots\right.}}{2b^2\left(a^2+b^2\right)}}{4b^2(a^2+b^2)}\right.}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

output
```
(b^5*((b^2 + a*b*Tan[c + d*x])/(4*b^2*(a^2 + b^2)*(b^2 + b^2*Tan[c + d*x]^
2)^2) + (((a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*ArcTan[Tan[c + d*x]])/(b*(a^2 +
 b^2)) + (8*b^4*Log[a + b*Tan[c + d*x]])/(a^2 + b^2) - (4*b^4*Log[b^2 + b^
2*Tan[c + d*x]^2])/(a^2 + b^2))/(2*b^2*(a^2 + b^2)) + (4*b^4 + a*b*(3*a^2
+ 7*b^2)*Tan[c + d*x])/(2*b^2*(a^2 + b^2)*(b^2 + b^2*Tan[c + d*x]^2)))/(4*
b^2*(a^2 + b^2))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 8.10 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{\frac{b^5\ln(a+b\tan(dx+c))}{(a^2+b^2)^3}+\frac{\frac{\left(\frac{3}{8}a^5+\frac{5}{4}a^3b^2+\frac{7}{8}ab^4\right)\tan(dx+c)^3+\left(\frac{1}{2}a^2b^3+\frac{1}{2}b^5\right)\tan(dx+c)^2+\left(\frac{7}{4}a^3b^2+\frac{9}{8}ab^4+\frac{5}{8}a^5\right)\tan(dx+c)+\frac{a^4b}{4}\cdots}{(1+\tan(dx+c)^2)^2}}{(a^2+b^2)^3}\cdots}{d}$
default	$\frac{\frac{b^5\ln(a+b\tan(dx+c))}{(a^2+b^2)^3}+\frac{\frac{\left(\frac{3}{8}a^5+\frac{5}{4}a^3b^2+\frac{7}{8}ab^4\right)\tan(dx+c)^3+\left(\frac{1}{2}a^2b^3+\frac{1}{2}b^5\right)\tan(dx+c)^2+\left(\frac{7}{4}a^3b^2+\frac{9}{8}ab^4+\frac{5}{8}a^5\right)\tan(dx+c)+\frac{a^4b}{4}\cdots}{(1+\tan(dx+c)^2)^2}}{(a^2+b^2)^3}\cdots}{d}$
risch	$\frac{9xab}{8ia^3-24iab^2+24a^2b-8b^3}+\frac{3ixa^2}{8ia^3-24iab^2+24a^2b-8b^3}-\frac{8ixb^2}{8ia^3-24iab^2+24a^2b-8b^3}-\frac{3e^{2i(dx+c)}b}{16(-2iab+a^2-b^2)d}-\cdots$

input
```
int(cos(d*x+c)^4/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^5/(a^2+b^2)^3*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^3*(((3/8*a^5+5/4*a^3*b
^2+7/8*a*b^4)*tan(d*x+c)^3+(1/2*a^2*b^3+1/2*b^5)*tan(d*x+c)^2+(7/4*a^3*b^2
+9/8*a*b^4+5/8*a^5)*tan(d*x+c)+1/4*a^4*b+a^2*b^3+3/4*b^5)/(1+tan(d*x+c)^2)
^2-1/2*b^5*ln(1+tan(d*x+c)^2)+1/8*(3*a^5+10*a^3*b^2+15*a*b^4)*arctan(tan(d
*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.37

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{4\,b^5\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/8*(4*b^5*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^
2 + b^2) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + (3*a^5 + 10*a^3*b^
2 + 15*a*b^4)*d*x + 4*(a^2*b^3 + b^5)*cos(d*x + c)^2 + (2*(a^5 + 2*a^3*b^2
 + a*b^4)*cos(d*x + c)^3 + (3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cos(d*x + c))*si
n(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [F]

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cos^4\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

input
```
integrate(cos(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**4/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.78

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{8\,b^5\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{4\,b^5\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(3\,a^5+10\,a^3b^2+15\,ab^4)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{4\,b^3\tan(dx+c)^2+(3\,a^3+7\,ab^2)\tan(dx+c)^3+2\,a^2b}{(a^4+2\,a^2b^2+b^4)\tan(dx+c)^4+a^4+2\,a^2b^2+b^4+}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/8*(8*b^5*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 4
*b^5*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3*a^5
+ 10*a^3*b^2 + 15*a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (
4*b^3*tan(d*x + c)^2 + (3*a^3 + 7*a*b^2)*tan(d*x + c)^3 + 2*a^2*b + 6*b^3
+ (5*a^3 + 9*a*b^2)*tan(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)^4
+ a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)^2))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.82

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{b^6\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^6bd+3\,a^4b^3d+3\,a^2b^5d+b^7d}$$
$$-\frac{b^5\log\left(\tan\left(dx+c\right)^2+1\right)}{2\left(a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d\right)}+\frac{(3\,a^5+10\,a^3b^2+15\,ab^4)(dx+c)}{8\left(a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d\right)}$$
$$+\frac{2\,a^4b+8\,a^2b^3+6\,b^5+(3\,a^5+10\,a^3b^2+7\,ab^4)\tan\left(dx+c\right)^3+4\left(a^2b^3+b^5\right)\tan\left(dx+c\right)^2+(5\,a^5+14}{8\left(a^2+b^2\right)^3\left(\tan\left(dx+c\right)^2+1\right)^2d}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
b^6*log(abs(b*tan(d*x + c) + a))/(a^6*b*d + 3*a^4*b^3*d + 3*a^2*b^5*d + b^
7*d) - 1/2*b^5*log(tan(d*x + c)^2 + 1)/(a^6*d + 3*a^4*b^2*d + 3*a^2*b^4*d
+ b^6*d) + 1/8*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*(d*x + c)/(a^6*d + 3*a^4*b^
2*d + 3*a^2*b^4*d + b^6*d) + 1/8*(2*a^4*b + 8*a^2*b^3 + 6*b^5 + (3*a^5 + 1
0*a^3*b^2 + 7*a*b^4)*tan(d*x + c)^3 + 4*(a^2*b^3 + b^5)*tan(d*x + c)^2 + (
5*a^5 + 14*a^3*b^2 + 9*a*b^4)*tan(d*x + c))/((a^2 + b^2)^3*(tan(d*x + c)^2
 + 1)^2*d)
```

Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.09

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{a^2\,b+3\,b^3}{4\,(a^4+2\,a^2\,b^2+b^4)}+\frac{b^3\,\tan(c+d\,x)^2}{2\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+d\,x)^3\,(3\,a^3+7\,a\,b^2)}{8\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+d\,x)\,(5\,a^3+9\,a\,b^2)}{8\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-a^2\,3\mathrm{i}+9\,a\,b+b^2\,8\mathrm{i}\right)}{16\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)} + \frac{b^5\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,{\left(a^2+b^2\right)}^3} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-3\,a^2+a\,b\,9\mathrm{i}+8\,b^2\right)}{16\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}$$

input
```
int(cos(c + d*x)^4/(a + b*tan(c + d*x)),x)
```

output
```
((a^2*b + 3*b^3)/(4*(a^4 + b^4 + 2*a^2*b^2)) + (b^3*tan(c + d*x)^2)/(2*(a^
4 + b^4 + 2*a^2*b^2)) + (tan(c + d*x)^3*(7*a*b^2 + 3*a^3))/(8*(a^4 + b^4 +
 2*a^2*b^2)) + (tan(c + d*x)*(9*a*b^2 + 5*a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)
))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1)) - (log(tan(c + d*x) - 1i)*(
9*a*b - a^2*3i + b^2*8i))/(16*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) + (b^
5*log(a + b*tan(c + d*x)))/(d*(a^2 + b^2)^3) - (log(tan(c + d*x) + 1i)*(a*
b*9i - 3*a^2 + 8*b^2))/(16*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.32

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{-2\cos(dx+c)\sin(dx+c)^3 a^5 - 4\cos(dx+c)\sin(dx+c)^3 a^3b^2 - 2\cos(dx+c)\sin(dx+c)^3 a\,b^4 + 5\,\mathrm{c}}{}$$

input
```
int(cos(d*x+c)^4/(a+b*tan(d*x+c)),x)
```

output
```
( - 2*cos(c + d*x)*sin(c + d*x)**3*a**5 - 4*cos(c + d*x)*sin(c + d*x)**3*a
**3*b**2 - 2*cos(c + d*x)*sin(c + d*x)**3*a*b**4 + 5*cos(c + d*x)*sin(c +
d*x)*a**5 + 14*cos(c + d*x)*sin(c + d*x)*a**3*b**2 + 9*cos(c + d*x)*sin(c
+ d*x)*a*b**4 - 8*log(tan((c + d*x)/2)**2 + 1)*b**5 + 8*log(tan((c + d*x)/
2)**2*a - 2*tan((c + d*x)/2)*b - a)*b**5 + 2*sin(c + d*x)**4*a**4*b + 4*si
n(c + d*x)**4*a**2*b**3 + 2*sin(c + d*x)**4*b**5 - 4*sin(c + d*x)**2*a**4*
b - 12*sin(c + d*x)**2*a**2*b**3 - 8*sin(c + d*x)**2*b**5 + 3*a**5*c + 3*a
**5*d*x + 4*a**4*b + 10*a**3*b**2*c + 10*a**3*b**2*d*x + 12*a**2*b**3 + 15
*a*b**4*c + 15*a*b**4*d*x + 8*b**5)/(8*d*(a**6 + 3*a**4*b**2 + 3*a**2*b**4
 + b**6))
```

3.557 $\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 140

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx = &-\frac{a(2a^2+3b^2)\operatorname{arctanh}(\sin(c+dx))}{2b^4d}\\ &-\frac{(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^4d}\\ &+\frac{(a^2+b^2)\sec(c+dx)}{b^3d}+\frac{\sec^3(c+dx)}{3bd}\\ &-\frac{a\sec(c+dx)\tan(c+dx)}{2b^2d}\end{aligned}$$

output
```
-1/2*a*(2*a^2+3*b^2)*arctanh(sin(d*x+c))/b^4/d-(a^2+b^2)^(3/2)*arctanh(cos
(d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)^(1/2))/b^4/d+(a^2+b^2)*sec(d*x+c)/b^3/d
+1/3*sec(d*x+c)^3/b/d-1/2*a*sec(d*x+c)*tan(d*x+c)/b^2/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 321 vs. $2(140) = 280$.

Time = 1.58 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.29

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{48(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sec^3(c+dx)\left(12a^2b+20b^3+12b(a^2+b^2)\cos(2(c+dx))+6\right.}{}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

output
```
(48*(a^2 + b^2)^(3/2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] +
 Sec[c + d*x]^3*(12*a^2*b + 20*b^3 + 12*b*(a^2 + b^2)*Cos[2*(c + d*x)] + 6
*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*a*b^2*C
os[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*a*(2*a^2 + 3*
b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]) - 6*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]] - 9*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] - 6*a*b^2*Sin[2*(c + d*x)]))/(24*b^4*d)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3989, 3042, 3967, 3042, 3989, 3042, 3967, 3042, 3988, 219, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sec(c+dx)^5}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3989}$$

$$\frac{(a^2+b^2)\int\frac{\sec^3(c+dx)}{a+b\tan(c+dx)}dx}{b^2}-\frac{\int\sec^3(c+dx)(a-b\tan(c+dx))dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int\frac{\sec(c+dx)^3}{a+b\tan(c+dx)}dx}{b^2}-\frac{\int\sec(c+dx)^3(a-b\tan(c+dx))dx}{b^2}$$

$$\Big\downarrow \text{3967}$$

$$\frac{(a^2+b^2)\int\frac{\sec(c+dx)^3}{a+b\tan(c+dx)}dx}{b^2}-\frac{a\int\sec^3(c+dx)dx-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int\frac{\sec(c+dx)^3}{a+b\tan(c+dx)}dx}{b^2}-\frac{a\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \text{3989}$$

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\int\frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{b^2}-\frac{\int\sec(c+dx)(a-b\tan(c+dx))dx}{b^2}\right)}{b^2}-\frac{a\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\int\frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{b^2}-\frac{\int\sec(c+dx)(a-b\tan(c+dx))dx}{b^2}\right)}{b^2}-\frac{a\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \text{3967}$$

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\int\frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{b^2}-\frac{a\int\sec(c+dx)dx-\frac{b\sec(c+dx)}{d}}{b^2}\right)}{b^2}-\frac{a\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2+b^2\right)\left(\frac{\left(a^2+b^2\right)\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\sec(c+dx)}{d}}{b^2}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \; 3988$$

$$\frac{\left(a^2+b^2\right)\left(-\frac{\left(a^2+b^2\right)\int \frac{1}{a^2+b^2-\cos^2(c+dx)(b-a\tan(c+dx))^2}d(\cos(c+dx)(b-a\tan(c+dx)))}{b^2d}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\sec(c+dx)}{d}}{b^2}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(a^2+b^2\right)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\sec(c+dx)}{d}}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \; 4255$$

$$\frac{\left(a^2+b^2\right)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\sec(c+dx)}{d}}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d}\right)}{b^2}-\frac{a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\left(a^2+b^2\right)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\sec(c+dx)}{d}}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d}\right)}{b^2}-\frac{a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \; 4257$$

$$\frac{(a^2+b^2)\left(-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{\frac{a\text{arctanh}(\sin(c+dx))}{d}-\frac{b\sec(c+dx)}{d}}{b^2}\right)}{b^2}-\frac{a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

output
```
((a^2 + b^2)*(-((Sqrt[a^2 + b^2]*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]
))/Sqrt[a^2 + b^2]])/(b^2*d)) - ((a*ArcTanh[Sin[c + d*x]])/d - (b*Sec[c +
d*x])/d)/b^2))/b^2 - (-1/3*(b*Sec[c + d*x]^3)/d + a*(ArcTanh[Sin[c + d*x]]
/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/b^2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3988
```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Simp[-f^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f
*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 3989
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_
Symbol] :> Simp[-(b^2)^(-1)   Int[Sec[e + f*x]^(m - 2)*(a - b*Tan[e + f*x])
, x], x] + Simp[(a^2 + b^2)/b^2   Int[Sec[e + f*x]^(m - 2)/(a + b*Tan[e + f
*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0] && IGtQ[(m - 1)
/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(130) = 260$.

Time = 15.61 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.92

method	result
derivativedivides	$\frac{\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{-a+b}{2b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{-2a^{2}+ab-3b^{2}}{2b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\left(2a^{2}+3b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b^{4}}-\frac{2\left(-a^{4}-2b^{2}a^{2}-b^{4}\right)}{}}{}$
default	$\frac{\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{-a+b}{2b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{-2a^{2}+ab-3b^{2}}{2b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\left(2a^{2}+3b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b^{4}}-\frac{2\left(-a^{4}-2b^{2}a^{2}-b^{4}\right)}{}}{}$
risch	$\frac{e^{i(dx+c)}\left(3iab\,e^{4i(dx+c)}+6a^{2}e^{4i(dx+c)}+6b^{2}e^{4i(dx+c)}+12a^{2}e^{2i(dx+c)}+20b^{2}e^{2i(dx+c)}-3iab+6a^{2}+6b^{2}\right)}{3d\,b^{3}\left(e^{2i(dx+c)}+1\right)^{3}}+\frac{a^{3}\ln\left(e^{i(dx\right.}\right.}{d\,b^{4}}$

input
```
int(sec(d*x+c)^5/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3/b/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-a+b)/b^2/(tan(1/2*d*x+1/2*c)+1)^
2-1/2*(-2*a^2+a*b-3*b^2)/b^3/(tan(1/2*d*x+1/2*c)+1)-1/2*a*(2*a^2+3*b^2)/b^
4*ln(tan(1/2*d*x+1/2*c)+1)-2/b^4*(-a^4-2*a^2*b^2-b^4)/(a^2+b^2)^(1/2)*arct
anh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-1/3/b/(tan(1/2*d*x+1
/2*c)-1)^3-1/2*(a+b)/b^2/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(2*a^2+a*b+3*b^2)/b^
3/(tan(1/2*d*x+1/2*c)-1)+1/2*a*(2*a^2+3*b^2)/b^4*ln(tan(1/2*d*x+1/2*c)-1))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.85

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{6\left(a^2+b^2\right)^{\frac{3}{2}}\cos\left(dx+c\right)^3\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)}{}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(6*(a^2 + b^2)^(3/2)*cos(d*x + c)^3*log(-(2*a*b*cos(d*x + c)*sin(d*x
+ c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos
(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 + b^2)) - 3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x
+ c) + 1) + 3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 6*
a*b^2*cos(d*x + c)*sin(d*x + c) + 4*b^3 + 12*(a^2*b + b^3)*cos(d*x + c)^2)
/(b^4*d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sec^5\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*tan(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*tan(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(132) = 264$.

Time = 0.13 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.58

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{2\left(6a^2+8b^2-\frac{3ab\sin(dx+c)}{\cos(dx+c)+1}+\frac{3ab\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{12\left(a^2+b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\left(a^2+2b^2\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)}{b^3-\frac{3b^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3b^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{b^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{3\left(2a^3+3ab^2\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^4}+\frac{3(}{}}{6d}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/6*(2*(6*a^2 + 8*b^2 - 3*a*b*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a*b*sin(
d*x + c)^5/(cos(d*x + c) + 1)^5 - 12*(a^2 + b^2)*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 6*(a^2 + 2*b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/(b^3 - 3
*b^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*b^3*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 - b^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - 3*(2*a^3 + 3*a*b^2
)*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^4 + 3*(2*a^3 + 3*a*b^2)*log(s
in(d*x + c)/(cos(d*x + c) + 1) - 1)/b^4 - 6*(a^4 + 2*a^2*b^2 + b^4)*log((b
 - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c
)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(132) = 264$.

Time = 0.27 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.99

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx =$$
$$-\frac{\frac{3\left(2a^3+3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^4}-\frac{3\left(2a^3+3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^4}+\frac{6\left(a^4+2a^2b^2+b^4\right)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{}\right.}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{}\right.}\right.}{\sqrt{a^2+b^2}b^4}}{}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(3*(2*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*a^
3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 6*(a^4 + 2*a^2*b^2 +
 b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*
tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 2
*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*tan(1/2*d*x + 1/2*c)^4 + 12*b^2*tan
(1/2*d*x + 1/2*c)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x +
 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*a^2 + 8*b^2)/((tan(1/2*d*x + 1/
2*c)^2 - 1)^3*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 724, normalized size of antiderivative = 5.17

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^5*(a + b*tan(c + d*x))),x)
```

output
```
(b^3*(cos(c + d*x) + cos(2*c + 2*d*x)/2 + cos(3*c + 3*d*x)/3 + 5/6) - b^2*
((a*sin(2*c + 2*d*x))/4 + (3*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)
)*cos(3*c + 3*d*x))/4 + (9*a*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2
 + (d*x)/2)))/4) + b*((3*a^2*cos(c + d*x))/4 + a^2/2 + (a^2*cos(2*c + 2*d*
x))/2 + (a^2*cos(3*c + 3*d*x))/4) + (atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 +
 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*
b^4 + 3*a^4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2)
 + a*b^4*cos(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c
/2 + (d*x)/2) + 4*a^2*b^3*sin(c/2 + (d*x)/2)))*cos(3*c + 3*d*x)*((a^2 + b^
2)^3)^(1/2))/2 - (3*a^3*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d
*x)/2)))/2 - (a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3
*d*x))/2 + (3*cos(c + d*x)*atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^
2*b^4 + 3*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*c
os(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x)
/2) + 4*a^2*b^3*sin(c/2 + (d*x)/2)))*((a^2 + b^2)^3)^(1/2))/2)/(b^4*d*((3*
cos(c + d*x))/4 + cos(3*c + 3*d*x)/4))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 549, normalized size of antiderivative = 3.92

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^5/(a+b*tan(d*x+c)),x)
```

output
```
( - 12*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**
2))*cos(c + d*x)*sin(c + d*x)**2*a**2*i - 12*sqrt(a**2 + b**2)*atan((tan((
c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*b**
2*i + 12*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b
**2))*cos(c + d*x)*a**2*i + 12*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*
i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*b**2*i + 6*cos(c + d*x)*log(tan((
c + d*x)/2) - 1)*sin(c + d*x)**2*a**3 + 9*cos(c + d*x)*log(tan((c + d*x)/2
) - 1)*sin(c + d*x)**2*a*b**2 - 6*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a
**3 - 9*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a*b**2 - 6*cos(c + d*x)*log
(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**3 - 9*cos(c + d*x)*log(tan((c +
d*x)/2) + 1)*sin(c + d*x)**2*a*b**2 + 6*cos(c + d*x)*log(tan((c + d*x)/2)
+ 1)*a**3 + 9*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a*b**2 + 2*cos(c + d*
x)*sin(c + d*x)**2*a**2*b + 3*cos(c + d*x)*sin(c + d*x)*a*b**2 - 2*cos(c +
 d*x)*a**2*b + 6*sin(c + d*x)**2*a**2*b + 6*sin(c + d*x)**2*b**3 - 6*a**2*
b - 8*b**3)/(6*cos(c + d*x)*b**4*d*(sin(c + d*x)**2 - 1))
```

3.558 $\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{a\operatorname{arctanh}(\sin(c+dx))}{b^2d} - \frac{\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d} + \frac{\sec(c+dx)}{bd}$$

output
```
-a*arctanh(sin(d*x+c))/b^2/d-(a^2+b^2)^(1/2)*arctanh(cos(d*x+c)*(b-a*tan(d
*x+c))/(a^2+b^2)^(1/2))/b^2/d+sec(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.38

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right) + a\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.\right.}{b^2d}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

output
```
(2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + a*
(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]]) + b*Sec[c + d*x])/(b^2*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3989, 3042, 3967, 3042, 3988, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^3}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3989}$$

$$\frac{(a^2+b^2)\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{b^2}-\frac{\int \sec(c+dx)(a-b\tan(c+dx))dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{b^2}-\frac{\int \sec(c+dx)(a-b\tan(c+dx))dx}{b^2}$$

$$\Big\downarrow \text{3967}$$

$$\frac{(a^2+b^2)\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{b^2}-\frac{a\int \sec(c+dx)dx-\frac{b\sec(c+dx)}{d}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\sec(c+dx)}{d}}{b^2}$$

$$\Big\downarrow \text{3988}$$

$$-\frac{\left(a^2+b^2\right)\int \frac{1}{a^2+b^2-\cos^2(c+dx)(b-a\tan(c+dx))^2}d(\cos(c+dx)(b-a\tan(c+dx)))}{b^2d}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\sec(c+dx)}{d}}{b^2}$$

$$\Big\downarrow \textbf{219}$$

$$-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\sec(c+dx)}{d}}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d}$$

$$\Big\downarrow \textbf{4257}$$

$$-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{\frac{a\text{arctanh}(\sin(c+dx))}{d}-\frac{b\sec(c+dx)}{d}}{b^2}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

output
```
-((Sqrt[a^2 + b^2]*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 +
b^2]])/(b^2*d)) - ((a*ArcTanh[Sin[c + d*x]])/d - (b*Sec[c + d*x])/d)/b^2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3988
```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Simp[-f^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f
*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 3989
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_
Symbol] :> Simp[-(b^2)^(-1)   Int[Sec[e + f*x]^(m - 2)*(a - b*Tan[e + f*x])
, x], x] + Simp[(a^2 + b^2)/b^2   Int[Sec[e + f*x]^(m - 2)/(a + b*Tan[e + f
*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0] && IGtQ[(m - 1)
/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 3.85 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.63

method	result
derivativedivides	$\frac{\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}-\frac{2\left(-a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\sqrt{a^2+b^2}}\right.}{b^2\sqrt{a^2+b^2}}}{d}$
default	$\frac{\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}-\frac{2\left(-a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\sqrt{a^2+b^2}}\right.}{b^2\sqrt{a^2+b^2}}}{d}$
risch	$\frac{2\,\mathrm{e}^{i(dx+c)}}{db\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d\,b^2}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d\,b^2}+\frac{\sqrt{a^2+b^2}\,\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{d\,b^2}-\frac{\sqrt{a^2+b^2}\,\ln\left(\mathrm{e}^{i(\right.}{}$

input
```
int(sec(d*x+c)^3/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b/(tan(1/2*d*x+1/2*c)+1)-a/b^2*ln(tan(1/2*d*x+1/2*c)+1)-1/b/(tan(1/
2*d*x+1/2*c)-1)+a/b^2*ln(tan(1/2*d*x+1/2*c)-1)-2/b^2*(-a^2-b^2)/(a^2+b^2)^
(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(77) = 154$.

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.42

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{a\cos(dx+c)\log(\sin(dx+c)+1) - a\cos(dx+c)\log(-\sin(dx+c)+1) - \sqrt{a^2+b^2}\cos(dx+c)\,\mathrm{lo}}{2\,b^2d\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x +
 c) + 1) - sqrt(a^2 + b^2)*cos(d*x + c)*log(-(2*a*b*cos(d*x + c)*sin(d*x +
 c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(
d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)
*cos(d*x + c)^2 + b^2)) - 2*b)/(b^2*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*tan(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(77) = 154$.

Time $= 0.12$ (sec) , antiderivative size $= 163$, normalized size of antiderivative $= 2.06$

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^2} - \frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^2} + \frac{\sqrt{a^2+b^2}\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{b^2} - \frac{2}{b-\frac{b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-(a*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^2 - a*log(sin(d*x + c)/(cos
(d*x + c) + 1) - 1)/b^2 + sqrt(a^2 + b^2)*log((b - a*sin(d*x + c)/(cos(d*x
 + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqr
t(a^2 + b^2)))/b^2 - 2/(b - b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

Giac [A] (verification not implemented)

Time $= 0.21$ (sec) , antiderivative size $= 136$, normalized size of antiderivative $= 1.72$

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2} - \frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2} + \frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{b^2} + \frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)b}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-(a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - a*log(abs(tan(1/2*d*x + 1/2*c
) - 1))/b^2 + sqrt(a^2 + b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*s
qrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/b
^2 + 2/((tan(1/2*d*x + 1/2*c)^2 - 1)*b))/d
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.92

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{2\,\mathrm{atanh}\left(\frac{64\,a^2\,\sqrt{a^2+b^2}}{64\,a^2\,b+\frac{64\,a^4}{b}+128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+128\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{128\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2+b^2}}{64\,a^2+\frac{64\,a^4}{b^2}+\frac{128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}+128\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{\cdots}{64\,a^4+1\cdots}\right.}{b^2\,d}$$
$$-\frac{2\,a\,\mathrm{atanh}\left(\frac{64\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^2+\frac{64\,a^4}{b^2}}+\frac{64\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^4+64\,a^2\,b^2}\right)}{b^2\,d}-\frac{2}{b\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int(1/(cos(c + d*x)^3*(a + b*tan(c + d*x))),x)
```

output
```
(2*atanh((64*a^2*(a^2 + b^2)^(1/2))/(64*a^2*b + (64*a^4)/b + 128*a^3*tan(c
/2 + (d*x)/2) + 128*a*b^2*tan(c/2 + (d*x)/2)) + (128*a*tan(c/2 + (d*x)/2)*
(a^2 + b^2)^(1/2))/(64*a^2 + (64*a^4)/b^2 + (128*a^3*tan(c/2 + (d*x)/2))/b
 + 128*a*b*tan(c/2 + (d*x)/2)) + (64*a^3*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1
/2))/(64*a^4 + 64*a^2*b^2 + 128*a*b^3*tan(c/2 + (d*x)/2) + 128*a^3*b*tan(c
/2 + (d*x)/2)))*(a^2 + b^2)^(1/2))/(b^2*d) - (2*a*atanh((64*a^2*tan(c/2 +
(d*x)/2))/(64*a^2 + (64*a^4)/b^2) + (64*a^4*tan(c/2 + (d*x)/2))/(64*a^4 +
64*a^2*b^2)))/(b^2*d) - 2/(b*d*(tan(c/2 + (d*x)/2)^2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.43

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{-2\sqrt{a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)ai-bi}{\sqrt{a^2+b^2}}\right)\cos(dx+c)\,i+\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a-\cos(dx+c)\,\mathrm{lo}\cdots}{\cos(dx+c)\,b^2d}$$

input
```
int(sec(d*x+c)^3/(a+b*tan(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2
))*cos(c + d*x)*i + cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a - cos(c + d*x
)*log(tan((c + d*x)/2) + 1)*a - cos(c + d*x)*b + b)/(cos(c + d*x)*b**2*d)
```

3.559 $\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}$$

output
```
-arctanh(cos(d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x]),x]
```

output
```
(2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3988, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3988}$$

$$-\frac{\int \frac{1}{a^2+b^2-\cos^2(c+dx)(b-a\tan(c+dx))^2}d(\cos(c+dx)(b-a\tan(c+dx)))}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

input
```
Int[Sec[c + d*x]/(a + b*Tan[c + d*x]),x]
```

output
```
-(ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]]/(Sqrt[a^2 +
 b^2]*d))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3988
```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Simp[-f^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f
*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
default	$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
risch	$\frac{\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$	88

input
```
int(sec(d*x+c)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(44) = 88.

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.85

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)}{2\,\sqrt{a^2+b^2}d}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2
*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*c
os(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))/(sqrt(a^2 +
b^2)*d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(
d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}d}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1
/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{d\,\sqrt{a^2+b^2}}$$

input
```
int(1/(cos(c + d*x)*(a + b*tan(c + d*x))),x)
```

output
```
-(2*atanh((b - a*tan(c/2 + (d*x)/2))/(a^2 + b^2)^(1/2)))/(d*(a^2 + b^2)^(1
/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{2\sqrt{a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)ai-bi}{\sqrt{a^2+b^2}}\right)i}{d\left(a^2+b^2\right)}$$

input
```
int(sec(d*x+c)/(a+b*tan(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2
))*i)/(d*(a**2 + b**2))
```

3.560 $\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{b^2\operatorname{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} + \frac{b\cos(c+dx)}{(a^2+b^2)\,d} + \frac{a\sin(c+dx)}{(a^2+b^2)\,d}$$

output
```
-b^2*arctanh(cos(d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/
d+b*cos(d*x+c)/(a^2+b^2)/d+a*sin(d*x+c)/(a^2+b^2)/d
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2b^2\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sqrt{a^2+b^2}(b\cos(c+dx)+a\sin(c+dx))}{(a^2+b^2)^{3/2}\,d}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x]),x]
```

output
```
(2*b^2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2
]*(b*Cos[c + d*x] + a*Sin[c + d*x]))/((a^2 + b^2)^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3990, 3042, 3967, 3042, 3117, 3988, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{3990}$$

$$\frac{\int \cos(c+dx)(a-b\tan(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{\int \frac{a-b\tan(c+dx)}{\sec(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3967}$$

$$\frac{a\int \cos(c+dx)dx+\frac{b\cos(c+dx)}{d}}{a^2+b^2}+\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+\frac{b\cos(c+dx)}{d}}{a^2+b^2}+\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)}{d}}{a^2+b^2}$$

$$\Big\downarrow \textbf{3988}$$

$$\frac{\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)}{d}}{a^2+b^2}-\frac{b^2\int\frac{1}{a^2+b^2-\cos^2(c+dx)(b-a\tan(c+dx))^2}d(\cos(c+dx)(b-a\tan(c+dx)))}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)}{d}}{a^2+b^2}-\frac{b^2\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

input
```
Int[Cos[c + d*x]/(a + b*Tan[c + d*x]),x]
```

output
```
-((b^2*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]])/((a^2
 + b^2)^(3/2)*d)) + ((b*Cos[c + d*x])/d + (a*Sin[c + d*x])/d)/(a^2 + b^2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3988
```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Simp[-f^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f
*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 3990
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_
Symbol] :> Simp[1/(a^2 + b^2)   Int[Sec[e + f*x]^m*(a - b*Tan[e + f*x]), x]
, x] + Simp[b^2/(a^2 + b^2)   Int[Sec[e + f*x]^(m + 2)/(a + b*Tan[e + f*x])
, x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[(m - 1)/2,
0]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

method	result	si
derivativedivides	$\frac{-\frac{2\left(-a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{\left(a^2+b^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{2b^2\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$	9
default	$\frac{-\frac{2\left(-a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{\left(a^2+b^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{2b^2\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$	9
risch	$-\frac{i e^{i(dx+c)}}{2(-ib+a)d}+\frac{i e^{-i(dx+c)}}{2(ib+a)d}+\frac{b^2\ln\left(e^{i(dx+c)}+\frac{ia^3+iab^2-a^2b-b^3}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d}-\frac{b^2\ln\left(e^{i(dx+c)}-\frac{ia^3+iab^2-a^2b-b^3}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d}$	1

input
```
int(cos(d*x+c)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/(a^2+b^2)*(-a*tan(1/2*d*x+1/2*c)-b)/(1+tan(1/2*d*x+1/2*c)^2)+2*b^2
/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(88) = 176$.

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.08

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\sqrt{a^2+b^2}b^2\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)+2\,(a^2b+b^3}{2\,(a^4+2\,a^2b^2+b^4)d}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(sqrt(a^2 + b^2)*b^2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^
2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*si
n(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2)) + 2*(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((
a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.58

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{b^2\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(b+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2+b^2+\frac{(a^2+b^2)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-(b^2*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a
*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2
*(b + a*sin(d*x + c)/(cos(d*x + c) + 1))/(a^2 + b^2 + (a^2 + b^2)*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.31

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{b^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\,\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)}{(a^2+b^2)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}}{d}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-(b^2*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*
tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a*
tan(1/2*d*x + 1/2*c) + b)/((a^2 + b^2)*(tan(1/2*d*x + 1/2*c)^2 + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.22

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{2\,b}{a^2+b^2}+\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2+b^2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{a^2\,b+b^3-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+b^2\right)}{{\left(a^2+b^2\right)}^{3/2}}\right)}{d\,{\left(a^2+b^2\right)}^{3/2}}$$

input
```
int(cos(c + d*x)/(a + b*tan(c + d*x)),x)
```

output
```
((2*b)/(a^2 + b^2) + (2*a*tan(c/2 + (d*x)/2))/(a^2 + b^2))/(d*(tan(c/2 + (
d*x)/2)^2 + 1)) - (2*b^2*atanh((a^2*b + b^3 - a*tan(c/2 + (d*x)/2)*(a^2 +
b^2))/(a^2 + b^2)^(3/2)))/(d*(a^2 + b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.31

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{-2\sqrt{a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)ai-bi}{\sqrt{a^2+b^2}}\right)b^2i+\cos\left(dx+c\right)a^2b+\cos\left(dx+c\right)b^3+\sin\left(dx+c\right)a^3+\sin\left(dx+c\right.}{d\left(a^4+2a^2b^2+b^4\right)}$$

input
```
int(cos(d*x+c)/(a+b*tan(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2
))*b**2*i + cos(c + d*x)*a**2*b + cos(c + d*x)*b**3 + sin(c + d*x)*a**3 +
sin(c + d*x)*a*b**2 - a**2*b - b**3)/(d*(a**4 + 2*a**2*b**2 + b**4))
```

3.561 $\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 165

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{b^4\operatorname{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}\,d} + \frac{b^3\cos(c+dx)}{(a^2+b^2)^2\,d} + \frac{b\cos^3(c+dx)}{3\,(a^2+b^2)\,d} + \frac{ab^2\sin(c+dx)}{(a^2+b^2)^2\,d} + \frac{a\sin(c+dx)}{(a^2+b^2)\,d} - \frac{a\sin^3(c+dx)}{3\,(a^2+b^2)\,d}$$

output
```
-b^4*arctanh(cos(d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)/
d+b^3*cos(d*x+c)/(a^2+b^2)^2/d+1/3*b*cos(d*x+c)^3/(a^2+b^2)/d+a*b^2*sin(d*
x+c)/(a^2+b^2)^2/d+a*sin(d*x+c)/(a^2+b^2)/d-1/3*a*sin(d*x+c)^3/(a^2+b^2)/d
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{24b^4\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sqrt{a^2+b^2}(3b(a^2+5b^2)\cos(c+dx)+b(a^2+b^2)\cos(3(c+dx))+2a(5a}{12\left(a^2+b^2\right)^{5/2}d}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

output
```
(24*b^4*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^
2]*(3*b*(a^2 + 5*b^2)*Cos[c + d*x] + b*(a^2 + b^2)*Cos[3*(c + d*x)] + 2*a*
(5*a^2 + 11*b^2 + (a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*(a^2 +
b^2)^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.92, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3990, 3042, 3967, 3042, 3113, 2009, 3990, 3042, 3967, 3042, 3117, 3988, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{3990}$$

$$\frac{\int \cos^3(c+dx)(a-b\tan(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b^2\int \frac{1}{\sec(c+dx)(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{\int \frac{a-b\tan(c+dx)}{\sec(c+dx)^3}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3967}$$

$$\frac{a\int \cos^3(c+dx)dx+\frac{b\cos^3(c+dx)}{3d}}{a^2+b^2}+\frac{b^2\int \frac{1}{\sec(c+dx)(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{b\cos^3(c+dx)}{3d}}{a^2+b^2}+\frac{b^2\int \frac{1}{\sec(c+dx)(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3113}$$

$$\frac{\frac{b\cos^3(c+dx)}{3d}-\frac{a\int (1-\sin^2(c+dx))d(-\sin(c+dx))}{d}}{a^2+b^2}+\frac{b^2\int \frac{1}{\sec(c+dx)(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^2\int \frac{1}{\sec(c+dx)(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{\frac{b\cos^3(c+dx)}{3d}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2+b^2}$$

$$\Big\downarrow \text{ 3990}$$

$$\frac{b^2\left(\frac{\int \cos(c+dx)(a-b\tan(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{\frac{b\cos^3(c+dx)}{3d}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b^2\left(\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{\int \frac{a-b\tan(c+dx)}{\sec(c+dx)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{\frac{b\cos^3(c+dx)}{3d}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2+b^2}$$

$$\Big\downarrow \text{ 3967}$$

$$\frac{b^2\left(\frac{a\int \cos(c+dx)dx+\frac{b\cos(c+dx)}{d}}{a^2+b^2}+\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{\frac{b\cos^3(c+dx)}{3d}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b^2\left(\frac{a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+\frac{b\cos(c+dx)}{d}}{a^2+b^2}+\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{\frac{b\cos^3(c+dx)}{3d}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2+b^2}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{b^2\left(\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)}{d}}{a^2+b^2}\right)}{a^2+b^2}+\frac{\frac{b\cos^3(c+dx)}{3d}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2+b^2}$$

$$\Big\downarrow \text{ 3988}$$

$$\frac{b^2\left(\frac{\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)}{d}}{a^2+b^2}-\frac{b^2\int \frac{1}{a^2+b^2-\cos^2(c+dx)(b-a\tan(c+dx))^2}d(\cos(c+dx)(b-a\tan(c+dx)))}{d(a^2+b^2)}\right)}{a^2+b^2}+\frac{\frac{b\cos^3(c+dx)}{3d}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2+b^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{b^2\left(\frac{\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)}{d}}{a^2+b^2}-\frac{b^2\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}\right)}{a^2+b^2}+\frac{\frac{b\cos^3(c+dx)}{3d}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2+b^2}$$

input
```
Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

output
```
(b^2*(-((b^2*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]])
/((a^2 + b^2)^(3/2)*d)) + ((b*Cos[c + d*x])/d + (a*Sin[c + d*x])/d)/(a^2 +
 b^2)))/(a^2 + b^2) + ((b*Cos[c + d*x]^3)/(3*d) - (a*(-Sin[c + d*x] + Sin[
c + d*x]^3/3))/d)/(a^2 + b^2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3988
```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Simp[-f^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f
*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 3990
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_
Symbol] :> Simp[1/(a^2 + b^2)   Int[Sec[e + f*x]^m*(a - b*Tan[e + f*x]), x]
, x] + Simp[b^2/(a^2 + b^2)   Int[Sec[e + f*x]^(m + 2)/(a + b*Tan[e + f*x])
, x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[(m - 1)/2,
0]
```

Maple [A] (verified)

Time = 4.01 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{\frac{2b^4 \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^4+2b^2a^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(\left(-a^3-2a\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-a^2b-2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(-\frac{2}{3}a^3-\frac{8}{3}a\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{\left(a^4+2b^2a^2+b^4\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$
default	$\frac{\frac{2b^4 \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^4+2b^2a^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(\left(-a^3-2a\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-a^2b-2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(-\frac{2}{3}a^3-\frac{8}{3}a\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{\left(a^4+2b^2a^2+b^4\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$
risch	$-\frac{5\,\mathrm{e}^{i(dx+c)}b}{8(-2iab+a^2-b^2)d}-\frac{3i\mathrm{e}^{i(dx+c)}a}{8(-2iab+a^2-b^2)d}-\frac{5\,\mathrm{e}^{-i(dx+c)}b}{8(ib+a)^2d}+\frac{3i\mathrm{e}^{-i(dx+c)}a}{8(ib+a)^2d}+\frac{b^4\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^5+2ia^3b^2+ia\,b^4}{(a^2+}\right.}{(a^2+b^2)^{\frac{5}{2}}\,d}$

input
```
int(cos(d*x+c)^3/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*
x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-2/(a^4+2*a^2*b^2+b^4)*((-a^3-2*a*b^2)*tan(1
/2*d*x+1/2*c)^5+(-a^2*b-2*b^3)*tan(1/2*d*x+1/2*c)^4+(-2/3*a^3-8/3*a*b^2)*t
an(1/2*d*x+1/2*c)^3-2*b^3*tan(1/2*d*x+1/2*c)^2+(-a^3-2*a*b^2)*tan(1/2*d*x+
1/2*c)-1/3*a^2*b-4/3*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.59

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{3\sqrt{a^2+b^2}b^4\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)+2\left(a^4b+2\right.}{6\,(a}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*sqrt(a^2 + b^2)*b^4*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*
sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)
^2 + b^2)) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 + 6*(a^2*b^3 + b^5
)*cos(d*x + c) + 2*(2*a^5 + 7*a^3*b^2 + 5*a*b^4 + (a^5 + 2*a^3*b^2 + a*b^4
)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [F]

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cos^3\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

input
```
integrate(cos(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**3/(a + b*tan(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(159) = 318$.

Time = 0.14 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.30

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{3\,b^4\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(a^2b+4\,b^3+\frac{6\,b^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\left(a^3+2\,ab^2\right)\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\left(a^3+4\,ab^2\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\left(a^2b+2\,b^3\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\cdots\right.}{a^4+2\,a^2b^2+b^4+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\cdots}}{3\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/3*(3*b^4*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/
(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b
^2 + b^4)*sqrt(a^2 + b^2)) - 2*(a^2*b + 4*b^3 + 6*b^3*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 3*(a^3 + 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) + 2*(a
^3 + 4*a*b^2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*(a^2*b + 2*b^3)*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4 + 3*(a^3 + 2*a*b^2)*sin(d*x + c)^5/(cos(d*
x + c) + 1)^5)/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + (a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6))/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.73

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx =$$

$$-\frac{\frac{3\,b^4\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+6\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{(\ldots}}{3\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/3*(3*b^4*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/ab
s(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 +
 b^4)*sqrt(a^2 + b^2)) - 2*(3*a^3*tan(1/2*d*x + 1/2*c)^5 + 6*a*b^2*tan(1/2
*d*x + 1/2*c)^5 + 3*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2
*c)^4 + 2*a^3*tan(1/2*d*x + 1/2*c)^3 + 8*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*
b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) + 6*a*b^2*tan(1/2*
d*x + 1/2*c) + a^2*b + 4*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*
c)^2 + 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.07

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{\frac{2a^2b}{3}+\frac{8b^3}{3}}{a^4+2a^2b^2+b^4}+\frac{4b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^4+2a^2b^2+b^4}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(2a^3+4ab^2)}{a^4+2a^2b^2+b^4}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{4a^3}{3}+\frac{16ab^2}{3}\right)}{a^4+2a^2b^2+b^4}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^3+2ab^2)}{a^4+2a^2b^2+b^4}+\frac{2}{}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)} - \frac{2b^4\,\mathrm{atanh}\left(\frac{a^4b+b^5+2a^2b^3-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^4+2a^2b^2+b^4)}{(a^2+b^2)^{5/2}}\right)}{d\,(a^2+b^2)^{5/2}}$$

input
```
int(cos(c + d*x)^3/(a + b*tan(c + d*x)),x)
```

output
```
(((2*a^2*b)/3 + (8*b^3)/3)/(a^4 + b^4 + 2*a^2*b^2) + (4*b^3*tan(c/2 + (d*x
)/2)^2)/(a^4 + b^4 + 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^5*(4*a*b^2 + 2*a^3))
/(a^4 + b^4 + 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^3*((16*a*b^2)/3 + (4*a^3)/3
))/(a^4 + b^4 + 2*a^2*b^2) + (2*tan(c/2 + (d*x)/2)*(2*a*b^2 + a^3))/(a^4 +
 b^4 + 2*a^2*b^2) + (2*b*tan(c/2 + (d*x)/2)^4*(a^2 + 2*b^2))/(a^4 + b^4 +
2*a^2*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2
+ (d*x)/2)^6 + 1)) - (2*b^4*atanh((a^4*b + b^5 + 2*a^2*b^3 - a*tan(c/2 + (
d*x)/2)*(a^4 + b^4 + 2*a^2*b^2))/(a^2 + b^2)^(5/2)))/(d*(a^2 + b^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.61

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{-6\sqrt{a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)ai-bi}{\sqrt{a^2+b^2}}\right)b^4i-\cos(dx+c)\sin(dx+c)^2a^4b-2\cos(dx+c)\sin(dx+c)^2a^2b^3}{}$$

input
```
int(cos(d*x+c)^3/(a+b*tan(d*x+c)),x)
```

output
```
( - 6*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2
))*b**4*i - cos(c + d*x)*sin(c + d*x)**2*a**4*b - 2*cos(c + d*x)*sin(c + d
*x)**2*a**2*b**3 - cos(c + d*x)*sin(c + d*x)**2*b**5 + cos(c + d*x)*a**4*b
 + 5*cos(c + d*x)*a**2*b**3 + 4*cos(c + d*x)*b**5 - sin(c + d*x)**3*a**5 -
 2*sin(c + d*x)**3*a**3*b**2 - sin(c + d*x)**3*a*b**4 + 3*sin(c + d*x)*a**
5 + 9*sin(c + d*x)*a**3*b**2 + 6*sin(c + d*x)*a*b**4 - a**4*b - a**2*b**3)
/(3*d*(a**6 + 3*a**4*b**2 + 3*a**2*b**4 + b**6))
```

3.562 $\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 178

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{6a(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^7d} + \frac{(5a^4+9a^2b^2+3b^4)\tan(c+dx)}{b^6d} - \frac{a(2a^2+3b^2)\tan^2(c+dx)}{b^5d} + \frac{(a^2+b^2)\tan^3(c+dx)}{b^4d} - \frac{a\tan^4(c+dx)}{2b^3d} + \frac{\tan^5(c+dx)}{5b^2d} - \frac{(a^2+b^2)^3}{b^7d(a+b\tan(c+dx))}$$

output

```
-6*a*(a^2+b^2)^2*ln(a+b*tan(d*x+c))/b^7/d+(5*a^4+9*a^2*b^2+3*b^4)*tan(d*x+
c)/b^6/d-a*(2*a^2+3*b^2)*tan(d*x+c)^2/b^5/d+(a^2+b^2)*tan(d*x+c)^3/b^4/d-1
/2*a*tan(d*x+c)^4/b^3/d+1/5*tan(d*x+c)^5/b^2/d-(a^2+b^2)^3/b^7/d/(a+b*tan(
d*x+c))
```

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.29

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{2b^6\sec^6(c+dx)+b^4\sec^4(c+dx)\left(a^2+4b^2-3ab\tan(c+dx)\right)-2\Big(8(a^2+b^2)^3+30a^2(a^2+b^2)^2\log(a+}{}$$

input
```
Integrate[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^2,x]
```

output
```
(2*b^6*Sec[c + d*x]^6 + b^4*Sec[c + d*x]^4*(a^2 + 4*b^2 - 3*a*b*Tan[c + d*
x]) - 2*(8*(a^2 + b^2)^3 + 30*a^2*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]] +
2*a*b*(-11*a^4 - 18*a^2*b^2 - 4*b^4 + 15*(a^2 + b^2)^2*Log[a + b*Tan[c + d
*x]])*Tan[c + d*x] - b^2*(15*a^4 + 29*a^2*b^2 + 8*b^4)*Tan[c + d*x]^2 + a*
b^3*(5*a^2 + 7*b^2)*Tan[c + d*x]^3 - 2*a^2*b^4*Tan[c + d*x]^4))/(10*b^7*d*
(a + b*Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^8}{(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3987}$$
$$\frac{\int \frac{\left(\tan^2(c+dx)b^2+b^2\right)^3}{b^6(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(\tan^2(c+dx)b^2+b^2)^3}{(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{b^7d}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(5\left(\frac{3(3a^2+b^2)b^2}{5a^4}+1\right)a^4-2b^3\tan^3(c+dx)a-2b(2a^2+3b^2)\tan(c+dx)a-\frac{6(a^2+b^2)^2a}{a+b\tan(c+dx)}+b^4\tan^4(c+dx)+3\right.}{b^7d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-ab^2(2a^2+3b^2)\tan^2(c+dx)-\frac{(a^2+b^2)^3}{a+b\tan(c+dx)}-6a(a^2+b^2)^2\log(a+b\tan(c+dx))+b^3(a^2+b^2)\tan^3(c+dx)-}{b^7d}$$

input
```
Int[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^2,x]
```

output
```
(-6*a*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]] + b*(5*a^4 + 9*a^2*b^2 + 3*b^4
)*Tan[c + d*x] - a*b^2*(2*a^2 + 3*b^2)*Tan[c + d*x]^2 + b^3*(a^2 + b^2)*Ta
n[c + d*x]^3 - (a*b^4*Tan[c + d*x]^4)/2 + (b^5*Tan[c + d*x]^5)/5 - (a^2 +
b^2)^3/(a + b*Tan[c + d*x]))/(b^7*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 254.47 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{\frac{\tan(dx+c)^5 b^4}{5}-\frac{a\,b^3\tan(dx+c)^4}{2}+a^2b^2\tan(dx+c)^3+b^4\tan(dx+c)^3-2a^3b\tan(dx+c)^2-3a\,b^3\tan(dx+c)^2+5a^4\tan(dx+c)+9b^2a^2\,\mathrm{t}}{b^6}}{d}$
default	$\frac{\frac{\frac{\tan(dx+c)^5 b^4}{5}-\frac{a\,b^3\tan(dx+c)^4}{2}+a^2b^2\tan(dx+c)^3+b^4\tan(dx+c)^3-2a^3b\tan(dx+c)^2-3a\,b^3\tan(dx+c)^2+5a^4\tan(dx+c)+9b^2a^2\,\mathrm{t}}{b^6}}{d}$
risch	$-\frac{4i(-15ia^5+8b^5+25a^2b^3+15a^4b-25ia^3b^2-8ia\,b^4+32b^5\mathrm{e}^{2i(dx+c)}-15ia^5\mathrm{e}^{10i(dx+c)}+60a^4b\,\mathrm{e}^{2i(dx+c)}+15a^4b\,\mathrm{e}^{8i(dx+}}{}$

input
```
int(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^6*(1/5*tan(d*x+c)^5*b^4-1/2*a*b^3*tan(d*x+c)^4+a^2*b^2*tan(d*x+c)
^3+b^4*tan(d*x+c)^3-2*a^3*b*tan(d*x+c)^2-3*a*b^3*tan(d*x+c)^2+5*a^4*tan(d*
x+c)+9*b^2*a^2*tan(d*x+c)+3*b^4*tan(d*x+c))-1/b^7*(a^6+3*a^4*b^2+3*a^2*b^4
+b^6)/(a+b*tan(d*x+c))-6*a/b^7*(a^4+2*a^2*b^2+b^4)*ln(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(174) = 348$.

Time = 0.12 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.17

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{4\,(15\,a^4b^2+25\,a^2b^4+8\,b^6)\cos{(dx+c)}^6-2\,b^6-2\,(15\,a^4b^2+25\,a^2b^4+8\,b^6)\cos{(dx+c)}^4-(5\,a^2b^4+$$

input
```
integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/10*(4*(15*a^4*b^2 + 25*a^2*b^4 + 8*b^6)*cos(d*x + c)^6 - 2*b^6 - 2*(15*
a^4*b^2 + 25*a^2*b^4 + 8*b^6)*cos(d*x + c)^4 - (5*a^2*b^4 + 4*b^6)*cos(d*x
 + c)^2 + 30*((a^6 + 2*a^4*b^2 + a^2*b^4)*cos(d*x + c)^6 + (a^5*b + 2*a^3*
b^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x +
 c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 30*((a^6 + 2*a^4*b^2 + a^2*b^4)*
cos(d*x + c)^6 + (a^5*b + 2*a^3*b^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c))*
log(cos(d*x + c)^2) + (3*a*b^5*cos(d*x + c) - 4*(15*a^5*b + 25*a^3*b^3 + 8
*a*b^5)*cos(d*x + c)^5 + 2*(5*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^3)*sin(d*x +
 c))/(a*b^7*d*cos(d*x + c)^6 + b^8*d*cos(d*x + c)^5*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^8{(c+dx)}}{(a+b\tan{(c+dx)})^2}\,dx$$

input
```
integrate(sec(d*x+c)**8/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**8/(a + b*tan(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{10\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)}{b^8\tan(dx+c)+ab^7} - \frac{2\,b^4\tan(dx+c)^5-5\,ab^3\tan(dx+c)^4+10\,(a^2b^2+b^4)\tan(dx+c)^3-10\,(2\,a^3b+3\,ab^3)\tan(dx+c)^2+10\,(5\,a^4+9\,a}{b^6}}{10\,d}$$

input
```
integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/10*(10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/(b^8*tan(d*x + c) + a*b^7) -
 (2*b^4*tan(d*x + c)^5 - 5*a*b^3*tan(d*x + c)^4 + 10*(a^2*b^2 + b^4)*tan(d
*x + c)^3 - 10*(2*a^3*b + 3*a*b^3)*tan(d*x + c)^2 + 10*(5*a^4 + 9*a^2*b^2
+ 3*b^4)*tan(d*x + c))/b^6 + 60*(a^5 + 2*a^3*b^2 + a*b^4)*log(b*tan(d*x +
c) + a)/b^7)/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.36

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{6\,(a^5+2\,a^3b^2+ab^4)\log(|b\tan(dx+c)+a|)}{b^7d} - \frac{a^6+3\,a^4b^2+3\,a^2b^4+b^6}{(b\tan(dx+c)+a)b^7d} + \frac{2\,b^8d^4\tan(dx+c)^5-5\,ab^7d^4\tan(dx+c)^4+10\,a^2b^6d^4\tan(dx+c)^3+10\,b^8d^4\tan(dx+c)^3-20\,a^3b^5}{}$$

input
```
integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-6*(a^5 + 2*a^3*b^2 + a*b^4)*log(abs(b*tan(d*x + c) + a))/(b^7*d) - (a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)/((b*tan(d*x + c) + a)*b^7*d) + 1/10*(2*b^8*d
^4*tan(d*x + c)^5 - 5*a*b^7*d^4*tan(d*x + c)^4 + 10*a^2*b^6*d^4*tan(d*x +
c)^3 + 10*b^8*d^4*tan(d*x + c)^3 - 20*a^3*b^5*d^4*tan(d*x + c)^2 - 30*a*b^
7*d^4*tan(d*x + c)^2 + 50*a^4*b^4*d^4*tan(d*x + c) + 90*a^2*b^6*d^4*tan(d*
x + c) + 30*b^8*d^4*tan(d*x + c))/(b^10*d^5)
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.45

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\tan(c+d\,x)^2\left(\frac{a^3}{b^5}-\frac{a\left(\frac{3}{b^2}+\frac{3\,a^2}{b^4}\right)}{b}\right)}{d} + \frac{\tan(c+d\,x)^5}{5\,b^2\,d} + \frac{\tan(c+d\,x)^3\left(\frac{1}{b^2}+\frac{a^2}{b^4}\right)}{d} - \frac{\tan(c+d\,x)\left(\frac{a^2\left(\frac{3}{b^2}+\frac{3\,a^2}{b^4}\right)}{b^2}-\frac{3}{b^2}+\frac{2\,a\left(\frac{2\,a^3}{b^5}-\frac{2\,a\left(\frac{3}{b^2}+\frac{3\,a^2}{b^4}\right)}{b}\right)}{b}\right)}{d} - \frac{a\tan(c+d\,x)^4}{2\,b^3\,d} - \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(6\,a^5+12\,a^3\,b^2+6\,a\,b^4\right)}{b^7\,d} - \frac{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}{b\,d\,\left(\tan\left(c+d\,x\right)\,b^7+a\,b^6\right)}$$

input
```
int(1/(cos(c + d*x)^8*(a + b*tan(c + d*x))^2),x)
```

output
```
(tan(c + d*x)^2*(a^3/b^5 - (a*(3/b^2 + (3*a^2)/b^4))/b))/d + tan(c + d*x)^
5/(5*b^2*d) + (tan(c + d*x)^3*(1/b^2 + a^2/b^4))/d - (tan(c + d*x)*((a^2*(
3/b^2 + (3*a^2)/b^4))/b^2 - 3/b^2 + (2*a*((2*a^3)/b^5 - (2*a*(3/b^2 + (3*a
^2)/b^4))/b))/b))/d - (a*tan(c + d*x)^4)/(2*b^3*d) - (log(a + b*tan(c + d*
x))*(6*a*b^4 + 6*a^5 + 12*a^3*b^2))/(b^7*d) - (a^6 + b^6 + 3*a^2*b^4 + 3*a
^4*b^2)/(b*d*(a*b^6 + b^7*tan(c + d*x)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7571, normalized size of antiderivative = 42.53

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input

```
int(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x)
```

output

```
(60*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**7*tan(c + d*x)*a*
*5*b**2 + 120*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**7*tan(c
 + d*x)*a**3*b**4 + 60*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)
**7*tan(c + d*x)*a*b**6 + 60*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c
+ d*x)**7*a**6*b + 120*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)
**7*a**4*b**3 + 60*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**7*
a**2*b**5 - 180*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**5*tan
(c + d*x)*a**5*b**2 - 360*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d
*x)**5*tan(c + d*x)*a**3*b**4 - 180*cos(c + d*x)*log(tan((c + d*x)/2) - 1)
*sin(c + d*x)**5*tan(c + d*x)*a*b**6 - 180*cos(c + d*x)*log(tan((c + d*x)/
2) - 1)*sin(c + d*x)**5*a**6*b - 360*cos(c + d*x)*log(tan((c + d*x)/2) - 1
)*sin(c + d*x)**5*a**4*b**3 - 180*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*s
in(c + d*x)**5*a**2*b**5 + 180*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(
c + d*x)**3*tan(c + d*x)*a**5*b**2 + 360*cos(c + d*x)*log(tan((c + d*x)/2)
 - 1)*sin(c + d*x)**3*tan(c + d*x)*a**3*b**4 + 180*cos(c + d*x)*log(tan((c
 + d*x)/2) - 1)*sin(c + d*x)**3*tan(c + d*x)*a*b**6 + 180*cos(c + d*x)*log
(tan((c + d*x)/2) - 1)*sin(c + d*x)**3*a**6*b + 360*cos(c + d*x)*log(tan((
c + d*x)/2) - 1)*sin(c + d*x)**3*a**4*b**3 + 180*cos(c + d*x)*log(tan((c +
 d*x)/2) - 1)*sin(c + d*x)**3*a**2*b**5 - 60*cos(c + d*x)*log(tan((c + d*x
)/2) - 1)*sin(c + d*x)*tan(c + d*x)*a**5*b**2 - 120*cos(c + d*x)*log(ta...
```

3.563 $\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{4a(a^2+b^2)\log(a+b\tan(c+dx))}{b^5d} + \frac{(3a^2+2b^2)\tan(c+dx)}{b^4d} - \frac{a\tan^2(c+dx)}{b^3d} + \frac{\tan^3(c+dx)}{3b^2d} - \frac{(a^2+b^2)^2}{b^5d(a+b\tan(c+dx))}$$

output
```
-4*a*(a^2+b^2)*ln(a+b*tan(d*x+c))/b^5/d+(3*a^2+2*b^2)*tan(d*x+c)/b^4/d-a*t
an(d*x+c)^2/b^3/d+1/3*tan(d*x+c)^3/b^2/d-(a^2+b^2)^2/b^5/d/(a+b*tan(d*x+c)
)
```

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{4b(2a^2+b^2)\tan(c+dx) - 2ab^2\tan^2(c+dx) + \frac{b^4\sec^4(c+dx)-4(a^2+b^2)(a^2+b^2+3a^2\log(a+b\tan(c+dx))+3ab\log(a+b\tan(}{a+b\tan(c+dx)}}{3b^5d}$$

input
```
Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

output
```
(4*b*(2*a^2 + b^2)*Tan[c + d*x] - 2*a*b^2*Tan[c + d*x]^2 + (b^4*Sec[c + d*
x]^4 - 4*(a^2 + b^2)*(a^2 + b^2 + 3*a^2*Log[a + b*Tan[c + d*x]] + 3*a*b*Lo
g[a + b*Tan[c + d*x]]*Tan[c + d*x]))/(a + b*Tan[c + d*x]))/(3*b^5*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^6}{(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3987}$$
$$\frac{\int \frac{(\tan^2(c+dx)b^2+b^2)^2}{b^4(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{bd}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{(\tan^2(c+dx)b^2+b^2)^2}{(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{b^5d}$$
$$\Big\downarrow \text{476}$$
$$\frac{\int \left(3\left(\frac{2b^2}{3a^2}+1\right)a^2-2b\tan(c+dx)a-\frac{4(a^2+b^2)a}{a+b\tan(c+dx)}+b^2\tan^2(c+dx)+\frac{(a^2+b^2)^2}{(a+b\tan(c+dx))^2}\right)d(b\tan(c+dx))}{b^5d}$$
$$\Big\downarrow \text{2009}$$

$$\frac{b(3a^2+2b^2)\tan(c+dx)-\frac{(a^2+b^2)^2}{a+b\tan(c+dx)}-4a(a^2+b^2)\log(a+b\tan(c+dx))-ab^2\tan^2(c+dx)+\frac{1}{3}b^3\tan^3(c+}{b^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

output
```
(-4*a*(a^2 + b^2)*Log[a + b*Tan[c + d*x]] + b*(3*a^2 + 2*b^2)*Tan[c + d*x]
 - a*b^2*Tan[c + d*x]^2 + (b^3*Tan[c + d*x]^3)/3 - (a^2 + b^2)^2/(a + b*Ta
n[c + d*x]))/(b^5*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 58.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{\frac{\frac{\tan(dx+c)^3 b^2}{3}-\tan(dx+c)^2 ab+3\tan(dx+c)a^2+2\tan(dx+c)b^2}{b^4}-\frac{4a\left(a^2+b^2\right)\ln(a+b\tan(dx+c))}{b^5}-\frac{a^4+2b^2a^2+b^4}{b^5(a+b\tan(dx+c))}}{d}$
default	$\frac{\frac{\frac{\tan(dx+c)^3 b^2}{3}-\tan(dx+c)^2 ab+3\tan(dx+c)a^2+2\tan(dx+c)b^2}{b^4}-\frac{4a\left(a^2+b^2\right)\ln(a+b\tan(dx+c))}{b^5}-\frac{a^4+2b^2a^2+b^4}{b^5(a+b\tan(dx+c))}}{d}$
risch	$-\frac{8i(2b^3-3ia^3-2ia\,b^2+3a^2b-3ia\,b^2e^{6i(dx+c)}-6ia\,b^2e^{4i(dx+c)}+4b^3e^{2i(dx+c)}-5ia\,b^2e^{2i(dx+c)}+3a^2b\,e^{4i(dx+c)}+6a^2b}{3\left(e^{2i(dx+c)}+1\right)^3\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)b^4d}$

input
```
int(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^4*(1/3*tan(d*x+c)^3*b^2-tan(d*x+c)^2*a*b+3*tan(d*x+c)*a^2+2*tan(d
*x+c)*b^2)-4*a/b^5*(a^2+b^2)*ln(a+b*tan(d*x+c))-1/b^5*(a^4+2*a^2*b^2+b^4)/
(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(114) = 228.

Time = 0.11 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.42

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{4\left(3\,a^2b^2+2\,b^4\right)\cos\left(dx+c\right)^4-b^4-2\left(3\,a^2b^2+2\,b^4\right)\cos\left(dx+c\right)^2+6\left(\left(a^4+a^2b^2\right)\cos\left(dx+c\right)^4+(a^3}{}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/3*(4*(3*a^2*b^2 + 2*b^4)*cos(d*x + c)^4 - b^4 - 2*(3*a^2*b^2 + 2*b^4)*c
os(d*x + c)^2 + 6*((a^4 + a^2*b^2)*cos(d*x + c)^4 + (a^3*b + a*b^3)*cos(d*
x + c)^3*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*c
os(d*x + c)^2 + b^2) - 6*((a^4 + a^2*b^2)*cos(d*x + c)^4 + (a^3*b + a*b^3)
*cos(d*x + c)^3*sin(d*x + c))*log(cos(d*x + c)^2) + 2*(a*b^3*cos(d*x + c)
- 2*(3*a^3*b + 2*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(a*b^5*d*cos(d*x + c
)^4 + b^6*d*cos(d*x + c)^3*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**6/(a + b*tan(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{3\,(a^4+2\,a^2b^2+b^4)}{b^6\tan(dx+c)+ab^5} - \frac{b^2\tan(dx+c)^3-3\,ab\tan(dx+c)^2+3\,(3\,a^2+2\,b^2)\tan(dx+c)}{b^4} + \frac{12\,(a^3+ab^2)\log(b\tan(dx+c)+a)}{b^5}}{3\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/3*(3*(a^4 + 2*a^2*b^2 + b^4)/(b^6*tan(d*x + c) + a*b^5) - (b^2*tan(d*x
+ c)^3 - 3*a*b*tan(d*x + c)^2 + 3*(3*a^2 + 2*b^2)*tan(d*x + c))/b^4 + 12*(
a^3 + a*b^2)*log(b*tan(d*x + c) + a)/b^5)/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.18

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= -\frac{4\,(a^3+ab^2)\log\left(|b\tan\left(dx+c\right)+a|\right)}{b^5d} - \frac{a^4+2\,a^2b^2+b^4}{(b\tan\left(dx+c\right)+a)b^5d}$$
$$+ \frac{b^4d^2\tan\left(dx+c\right)^3-3\,ab^3d^2\tan\left(dx+c\right)^2+9\,a^2b^2d^2\tan\left(dx+c\right)+6\,b^4d^2\tan\left(dx+c\right)}{3\,b^6d^3}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-4*(a^3 + a*b^2)*log(abs(b*tan(d*x + c) + a))/(b^5*d) - (a^4 + 2*a^2*b^2 +
 b^4)/((b*tan(d*x + c) + a)*b^5*d) + 1/3*(b^4*d^2*tan(d*x + c)^3 - 3*a*b^3
*d^2*tan(d*x + c)^2 + 9*a^2*b^2*d^2*tan(d*x + c) + 6*b^4*d^2*tan(d*x + c))
/(b^6*d^3)
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\tan(c+d\,x)^3}{3\,b^2\,d} + \frac{\tan(c+d\,x)\,\left(\frac{2}{b^2}+\frac{3\,a^2}{b^4}\right)}{d} - \frac{a\tan(c+d\,x)^2}{b^3\,d}$$
$$- \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(4\,a^3+4\,a\,b^2\right)}{b^5\,d}$$
$$- \frac{a^4+2\,a^2\,b^2+b^4}{b\,d\,\left(\tan\left(c+d\,x\right)\,b^5+a\,b^4\right)}$$

input
```
int(1/(cos(c + d*x)^6*(a + b*tan(c + d*x))^2),x)
```

output
```
tan(c + d*x)^3/(3*b^2*d) + (tan(c + d*x)*(2/b^2 + (3*a^2)/b^4))/d - (a*tan
(c + d*x)^2)/(b^3*d) - (log(a + b*tan(c + d*x))*(4*a*b^2 + 4*a^3))/(b^5*d)
 - (a^4 + b^4 + 2*a^2*b^2)/(b*d*(a*b^4 + b^5*tan(c + d*x)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 3952, normalized size of antiderivative = 34.07

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x)
```

output
```
(12*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**5*tan(c + d*x)*a*
*3*b**2 + 12*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**5*tan(c
+ d*x)*a*b**4 + 12*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**5*
a**4*b + 12*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**5*a**2*b*
*3 - 24*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**3*tan(c + d*x
)*a**3*b**2 - 24*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**3*ta
n(c + d*x)*a*b**4 - 24*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)
**3*a**4*b - 24*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**3*a**
2*b**3 + 12*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*tan(c + d*
x)*a**3*b**2 + 12*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*tan(
c + d*x)*a*b**4 + 12*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*a
**4*b + 12*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*a**2*b**3 +
 12*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**5*tan(c + d*x)*a*
*3*b**2 + 12*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**5*tan(c
+ d*x)*a*b**4 + 12*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**5*
a**4*b + 12*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**5*a**2*b*
*3 - 24*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**3*tan(c + d*x
)*a**3*b**2 - 24*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**3*ta
n(c + d*x)*a*b**4 - 24*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)
**3*a**4*b - 24*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**3*...
```

3.564 $\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2} \, dx = -\frac{2a\log(a+b\tan(c+dx))}{b^3d} + \frac{\tan(c+dx)}{b^2d} - \frac{a^2+b^2}{b^3d(a+b\tan(c+dx))}$$

output
```
-2*a*ln(a+b*tan(d*x+c))/b^3/d+tan(d*x+c)/b^2/d-(a^2+b^2)/b^3/d/(a+b*tan(d*
x+c))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2} \, dx = \frac{-2a\log(a+b\tan(c+dx)) + b\tan(c+dx) - \frac{a^2+b^2}{a+b\tan(c+dx)}}{b^3d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

output
```
(-2*a*Log[a + b*Tan[c + d*x]] + b*Tan[c + d*x] - (a^2 + b^2)/(a + b*Tan[c
+ d*x]))/(b^3*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^4}{(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3987}$$
$$\frac{\int \frac{\tan^2(c+dx)b^2+b^2}{b^2(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{bd}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{\tan^2(c+dx)b^2+b^2}{(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{b^3d}$$
$$\Big\downarrow \text{476}$$
$$\frac{\int \left(-\frac{2a}{a+b\tan(c+dx)}+\frac{a^2+b^2}{(a+b\tan(c+dx))^2}+1\right)d(b\tan(c+dx))}{b^3d}$$
$$\Big\downarrow \text{2009}$$
$$\frac{-\frac{a^2+b^2}{a+b\tan(c+dx)}-2a\log(a+b\tan(c+dx))+b\tan(c+dx)}{b^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

output
```
(-2*a*Log[a + b*Tan[c + d*x]] + b*Tan[c + d*x] - (a^2 + b^2)/(a + b*Tan[c
+ d*x]))/(b^3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 15.57 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\frac{\tan(dx+c)}{b^2}-\frac{a^2+b^2}{b^3(a+b\tan(dx+c))}-\frac{2a\ln(a+b\tan(dx+c))}{b^3}}{d}$	57
default	$\frac{\frac{\tan(dx+c)}{b^2}-\frac{a^2+b^2}{b^3(a+b\tan(dx+c))}-\frac{2a\ln(a+b\tan(dx+c))}{b^3}}{d}$	57
risch	$-\frac{4i\left(-ia\,e^{2i(dx+c)}+b-ia\right)}{\left(e^{2i(dx+c)}+1\right)\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)b^2d}+\frac{2a\ln\left(e^{2i(dx+c)}+1\right)}{b^3d}-\frac{2a\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{b^3d}$	136

input
```
int(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^2*tan(d*x+c)-1/b^3*(a^2+b^2)/(a+b*tan(d*x+c))-2/b^3*a*ln(a+b*tan(
d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(61) = 122$.

Time = 0.10 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.92

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{2\,b^2\cos\left(dx+c\right)^2 - 2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right) - b^2 + \left(a^2\cos\left(dx+c\right)^2 + ab\cos\left(dx+c\right)\sin\left(dx+c\right.}{ab^3d\,\mathrm{co}}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(2*b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) - b^2 + (a^2*cos(
d*x + c)^2 + a*b*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x
 + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*cos(d*x + c)^2 + a*b*cos(
d*x + c)*sin(d*x + c))*log(cos(d*x + c)^2))/(a*b^3*d*cos(d*x + c)^2 + b^4*
d*cos(d*x + c)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^4\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*tan(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{a^2+b^2}{b^4\tan(dx+c)+ab^3} + \frac{2\,a\log(b\tan(dx+c)+a)}{b^3} - \frac{\tan(dx+c)}{b^2}}{d}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-((a^2 + b^2)/(b^4*tan(d*x + c) + a*b^3) + 2*a*log(b*tan(d*x + c) + a)/b^3
 - tan(d*x + c)/b^2)/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{2\,a\log\left(|b\tan\left(dx+c\right)+a|\right)}{b^3d} + \frac{\tan\left(dx+c\right)}{b^2d} - \frac{a^2+b^2}{(b\tan\left(dx+c\right)+a)b^3d}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-2*a*log(abs(b*tan(d*x + c) + a))/(b^3*d) + tan(d*x + c)/(b^2*d) - (a^2 +
b^2)/((b*tan(d*x + c) + a)*b^3*d)
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\tan(c+d\,x)}{b^2\,d} - \frac{a^2+b^2}{b\,d\,\left(\tan\left(c+d\,x\right)\,b^3+a\,b^2\right)} - \frac{2\,a\,\ln\left(a+b\tan(c+d\,x)\right)}{b^3\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + b*tan(c + d*x))^2),x)
```

output
```
tan(c + d*x)/(b^2*d) - (a^2 + b^2)/(b*d*(a*b^2 + b^3*tan(c + d*x))) - (2*a
*log(a + b*tan(c + d*x)))/(b^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1477, normalized size of antiderivative = 24.21

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x)
```

output
```
(2*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**3*tan(c + d*x)*a*b
**2 + 2*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**3*a**2*b - 2*
cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*tan(c + d*x)*a*b**2 -
2*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*a**2*b + 2*cos(c + d
*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**3*tan(c + d*x)*a*b**2 + 2*cos(
c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**3*a**2*b - 2*cos(c + d*x)
*log(tan((c + d*x)/2) + 1)*sin(c + d*x)*tan(c + d*x)*a*b**2 - 2*cos(c + d*
x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)*a**2*b - 2*cos(c + d*x)*log(tan(
(c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)**3*tan(c + d*x)
*a*b**2 - 2*cos(c + d*x)*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b
- a)*sin(c + d*x)**3*a**2*b + 2*cos(c + d*x)*log(tan((c + d*x)/2)**2*a - 2
*tan((c + d*x)/2)*b - a)*sin(c + d*x)*tan(c + d*x)*a*b**2 + 2*cos(c + d*x)
*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)*a**2*b
 - cos(c + d*x)*sec(c + d*x)**4*sin(c + d*x)**3*b**3 + cos(c + d*x)*sec(c
+ d*x)**4*sin(c + d*x)*b**3 - 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*
tan(c + d*x)*a**2*b - 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**3 + 4
*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*tan(c + d*x)*a**2*b + 4*log(tan
((c + d*x)/2) - 1)*sin(c + d*x)**2*a**3 - 2*log(tan((c + d*x)/2) - 1)*tan(
c + d*x)*a**2*b - 2*log(tan((c + d*x)/2) - 1)*a**3 - 2*log(tan((c + d*x)/2
) + 1)*sin(c + d*x)**4*tan(c + d*x)*a**2*b - 2*log(tan((c + d*x)/2) + 1...
```

3.565 $\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{1}{bd(a+b\tan(c+dx))}$$

output
```
-1/b/d/(a+b*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

output
```
Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3987, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^2}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{1}{(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{17}$$

$$-\frac{1}{bd(a+b\tan(c+dx))}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

output
```
-(1/(b*d*(a + b*Tan[c + d*x])))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\frac{1}{bd(a+b\tan(dx+c))}$	21
default	$-\frac{1}{bd(a+b\tan(dx+c))}$	21
risch	$\frac{2i}{d(-ib+a)\left(-ib\,\mathrm{e}^{2i(dx+c)}+a\,\mathrm{e}^{2i(dx+c)}+ib+a\right)}$	47

input
```
int(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b/d/(a+b*tan(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(20) = 40$.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.85

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{b\cos(dx+c)-a\sin(dx+c)}{(a^3+ab^2)d\cos(dx+c)+(a^2b+b^3)d\sin(dx+c)}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(b*cos(d*x + c) - a*sin(d*x + c))/((a^3 + a*b^2)*d*cos(d*x + c) + (a^2*b
+ b^3)*d*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^2{(c+dx)}}{(a+b\tan{(c+dx)})^2}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*tan(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{1}{(b\tan{(dx+c)}+a)bd}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/((b*tan(d*x + c) + a)*b*d)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{1}{(b\tan{(dx+c)}+a)bd}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/((b*tan(d*x + c) + a)*b*d)
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx=-\frac{1}{b\,d\,\left(a+b\tan\left(c+d\,x\right)\right)}$$

input
```
int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x))^2),x)
```

output
```
-1/(b*d*(a + b*tan(c + d*x)))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 169, normalized size of antiderivative = 8.45

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{-\cos\left(dx+c\right)\sec\left(dx+c\right)^2\sin\left(dx+c\right)b+\sec\left(dx+c\right)^2\sin\left(dx+c\right)^2a-\sec\left(dx+c\right)^2a+\sin\left(dx\right.}{bd\left(\cos\left(dx+c\right)\sin\left(dx+c\right)\tan\left(dx+c\right)b^2+\cos\left(dx+c\right)\sin\left(dx+c\right)ab-\sin\left(dx+c\right)^2\tan\left(dx+c\right)ab\right.}$$

input
```
int(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x)
```

output
```
( - cos(c + d*x)*sec(c + d*x)**2*sin(c + d*x)*b + sec(c + d*x)**2*sin(c +
d*x)**2*a - sec(c + d*x)**2*a + sin(c + d*x)**2*tan(c + d*x)*b + sin(c + d
*x)**2*a)/(b*d*(cos(c + d*x)*sin(c + d*x)*tan(c + d*x)*b**2 + cos(c + d*x)
*sin(c + d*x)*a*b - sin(c + d*x)**2*tan(c + d*x)*a*b - sin(c + d*x)**2*a**
2 + tan(c + d*x)*a*b + a**2))
```

3.566 $\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 152

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{(a^4+6a^2b^2-3b^4)\,x}{2\,(a^2+b^2)^3} \\ &+ \frac{4ab^3\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d} \\ &+ \frac{b(a^2-3b^2)}{2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))} \\ &+ \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\,(a^2+b^2)\,d(a+b\tan(c+dx))}\end{aligned}$$

output
```
1/2*(a^4+6*a^2*b^2-3*b^4)*x/(a^2+b^2)^3+4*a*b^3*ln(a*cos(d*x+c)+b*sin(d*x+
c))/(a^2+b^2)^3/d+1/2*b*(a^2-3*b^2)/(a^2+b^2)^2/d/(a+b*tan(d*x+c))+1/2*cos
(d*x+c)^2*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 2.74 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.00

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{-\frac{ab\left(\left(-a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)-2\sqrt{-b^2}\log(a+b\tan(c+dx))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)\right)}{\sqrt{-b^2}(a^2+b^2)}+\frac{\cos^2(c+dx)(b+a\tan(}{a+b\tan(c+dx)}}{2(}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

output
```
(-((a*b*((-a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*Sqrt[-b^2]
*Log[a + b*Tan[c + d*x]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x
]]))/(Sqrt[-b^2]*(a^2 + b^2))) + (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(a
+ b*Tan[c + d*x]) + (b*(a^2 - 3*b^2)*((2*a + (-a^2 + b^2)/Sqrt[-b^2])*Log[
Sqrt[-b^2] - b*Tan[c + d*x]] - 4*a*Log[a + b*Tan[c + d*x]] + (2*a + (a^2 -
 b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (2*(a^2 + b^2))/(a +
b*Tan[c + d*x])))/(2*(a^2 + b^2)^2))/(2*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.38, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3987, 27, 496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(c+dx)^2(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{b^4}{(a+b\tan(c+dx))^2(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \; 27$$

$$\frac{b^3\int \frac{1}{(a+b\tan(c+dx))^2(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 496$$

$$\frac{b^3\left(\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)(a+b\tan(c+dx))}-\frac{\int -\frac{a^2+2b\tan(c+dx)a+3b^2}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2(a^2+b^2)}\right)}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{b^3\left(\frac{\int \frac{a^2+2b\tan(c+dx)a+3b^2}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)(a+b\tan(c+dx))}\right)}{d}$$

$$\Big\downarrow \; 657$$

$$\frac{b^3\left(\frac{\int \left(\frac{8ab^2}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))}+\frac{a^4+6b^2a^2-8b^3\tan(c+dx)a-3b^4}{\left(a^2+b^2\right)^2\left(\tan^2(c+dx)b^2+b^2\right)}+\frac{3b^2-a^2}{\left(a^2+b^2\right)(a+b\tan(c+dx))^2}\right)d(b\tan(c+dx))}{2b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2}\right.}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{b^3\left(\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)(a+b\tan(c+dx))}+\frac{\frac{a^2-3b^2}{\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{4ab^2\log\left(b^2\tan^2(c+dx)+b^2\right)}{\left(a^2+b^2\right)^2}+\frac{8ab^2\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2}+\frac{(a^4+6a}{}}{2b^2(a^2+b^2)}\right.}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

output
```
(b^3*((b^2 + a*b*Tan[c + d*x])/(2*b^2*(a^2 + b^2)*(a + b*Tan[c + d*x])*(b^
2 + b^2*Tan[c + d*x]^2)) + (((a^4 + 6*a^2*b^2 - 3*b^4)*ArcTan[Tan[c + d*x]
])/(b*(a^2 + b^2)^2) + (8*a*b^2*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^2 - (
4*a*b^2*Log[b^2 + b^2*Tan[c + d*x]^2])/(a^2 + b^2)^2 + (a^2 - 3*b^2)/((a^2
 + b^2)*(a + b*Tan[c + d*x])))/(2*b^2*(a^2 + b^2))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{-\frac{b^3}{\left(a^2+b^2\right)^2(a+b\tan(dx+c))}+\frac{4b^3a\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{\frac{\left(\frac{a^4}{2}-\frac{b^4}{2}\right)\tan(dx+c)+a^3b+a\,b^3}{1+\tan(dx+c)^2}-2a\,b^3\ln\left(1+\tan(dx+c)^2\right)+\frac{\left(a^4+6b^2a\right.}{}}{\left(a^2+b^2\right)^3}}{d}$
default	$\frac{-\frac{b^3}{\left(a^2+b^2\right)^2(a+b\tan(dx+c))}+\frac{4b^3a\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{\frac{\left(\frac{a^4}{2}-\frac{b^4}{2}\right)\tan(dx+c)+a^3b+a\,b^3}{1+\tan(dx+c)^2}-2a\,b^3\ln\left(1+\tan(dx+c)^2\right)+\frac{\left(a^4+6b^2a\right.}{}}{\left(a^2+b^2\right)^3}}{d}$
risch	$\frac{3ixb}{6ia^2b-2ib^3-2a^3+6a\,b^2}-\frac{xa}{6ia^2b-2ib^3-2a^3+6a\,b^2}-\frac{i\mathrm{e}^{2i(dx+c)}}{8(-2iab+a^2-b^2)d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8(2iab+a^2-b^2)d}-\frac{8ia\,b^3x}{a^6+3a^4b^2+3a^2}$

input
```
int(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b^3/(a^2+b^2)^2/(a+b*tan(d*x+c))+4*b^3/(a^2+b^2)^3*a*ln(a+b*tan(d*x+
c))+1/(a^2+b^2)^3*(((1/2*a^4-1/2*b^4)*tan(d*x+c)+a^3*b+a*b^3)/(1+tan(d*x+c
)^2)-2*a*b^3*ln(1+tan(d*x+c)^2)+1/2*(a^4+6*a^2*b^2-3*b^4)*arctan(tan(d*x+c
))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.84

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{(a^4b+2\,a^2b^3+b^5)\cos(dx+c)^3-(a^2b^3+3\,b^5-(a^5+6\,a^3b^2-3\,ab^4)dx)\cos(dx+c)+4\,(a^2b^3\cos(dx}{2\,((a^7+}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*((a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 - (a^2*b^3 + 3*b^5 - (a^5 +
6*a^3*b^2 - 3*a*b^4)*d*x)*cos(d*x + c) + 4*(a^2*b^3*cos(d*x + c) + a*b^4*s
in(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c
)^2 + b^2) - (a^3*b^2 - a*b^4 - (a^4*b + 6*a^2*b^3 - 3*b^5)*d*x - (a^5 + 2
*a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 + 3*a^5*b^2 + 3*a^3*
b^4 + a*b^6)*d*cos(d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d*sin(
d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.86

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{8\,ab^3\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{4\,ab^3\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(a^4+6\,a^2b^2-3\,b^4)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{2\,a^2b-2\,b^3+(a^2b-3\,b^3)\,\tan}{a^5+2\,a^3b^2+ab^4+(a^4b+2\,a^2b^3+b^5)\tan(dx+c)^3+(a^5}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*(8*a*b^3*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
 4*a*b^3*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (a^
4 + 6*a^2*b^2 - 3*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (2*
a^2*b - 2*b^3 + (a^2*b - 3*b^3)*tan(d*x + c)^2 + (a^3 + a*b^2)*tan(d*x + c
))/(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)^3 + (
a^5 + 2*a^3*b^2 + a*b^4)*tan(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*
x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.77

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{4\,ab^4\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^6bd+3\,a^4b^3d+3\,a^2b^5d+b^7d} - \frac{2\,ab^3\log\left(\tan\left(dx+c\right)^2+1\right)}{a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d} + \frac{(a^4+6\,a^2b^2-3\,b^4)(dx+c)}{2\,(a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d)} + \frac{a^2b\tan\left(dx+c\right)^2-3\,b^3\tan\left(dx+c\right)^2+a^3\tan\left(dx+c\right)+ab^2\tan\left(dx+c\right)+2\,a^2b-2\,b^3}{2\,(a^4d+2\,a^2b^2d+b^4d)\left(b\tan\left(dx+c\right)^3+a\tan\left(dx+c\right)^2+b\tan\left(dx+c\right)+a\right)}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
4*a*b^4*log(abs(b*tan(d*x + c) + a))/(a^6*b*d + 3*a^4*b^3*d + 3*a^2*b^5*d
+ b^7*d) - 2*a*b^3*log(tan(d*x + c)^2 + 1)/(a^6*d + 3*a^4*b^2*d + 3*a^2*b^
4*d + b^6*d) + 1/2*(a^4 + 6*a^2*b^2 - 3*b^4)*(d*x + c)/(a^6*d + 3*a^4*b^2*
d + 3*a^2*b^4*d + b^6*d) + 1/2*(a^2*b*tan(d*x + c)^2 - 3*b^3*tan(d*x + c)^
2 + a^3*tan(d*x + c) + a*b^2*tan(d*x + c) + 2*a^2*b - 2*b^3)/((a^4*d + 2*a
^2*b^2*d + b^4*d)*(b*tan(d*x + c)^3 + a*tan(d*x + c)^2 + b*tan(d*x + c) +
a))
```

Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.62

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{a^2\,b-b^3}{(a^2+b^2)^2}+\frac{\tan(c+d\,x)^2\,(a^2\,b-3\,b^3)}{2\,(a^4+2\,a^2\,b^2+b^4)}+\frac{a\tan(c+d\,x)}{2\,(a^2+b^2)}}{d\,\left(b\tan\left(c+d\,x\right)^3+a\tan\left(c+d\,x\right)^2+b\tan\left(c+d\,x\right)+a\right)} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-3\,b+a\,1\mathrm{i}\right)}{4\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(a-b\,3\mathrm{i}\right)}{4\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)} + \frac{4\,a\,b^3\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,(a^2+b^2)^3}$$

input
```
int(cos(c + d*x)^2/(a + b*tan(c + d*x))^2,x)
```

output
```
((a^2*b - b^3)/(a^2 + b^2)^2 + (tan(c + d*x)^2*(a^2*b - 3*b^3))/(2*(a^4 +
b^4 + 2*a^2*b^2)) + (a*tan(c + d*x))/(2*(a^2 + b^2)))/(d*(a + b*tan(c + d*
x) + a*tan(c + d*x)^2 + b*tan(c + d*x)^3)) + (log(tan(c + d*x) - 1i)*(a*1i
 - 3*b))/(4*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) + (log(tan(c + d*x) + 1
i)*(a - b*3i))/(4*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) + (4*a*b^3*log(a
+ b*tan(c + d*x)))/(d*(a^2 + b^2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 480, normalized size of antiderivative = 3.16

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{-8\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)a^2b^4+8\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b-a\right)}{}$$

input
```
int(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x)
```

output
```
( - 8*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*a**2*b**4 + 8*cos(c + d*x)
*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*a**2*b**4 - cos(c +
 d*x)*sin(c + d*x)**2*a**4*b**2 - 2*cos(c + d*x)*sin(c + d*x)**2*a**2*b**4
 - cos(c + d*x)*sin(c + d*x)**2*b**6 - cos(c + d*x)*a**6 + cos(c + d*x)*a*
*5*b*d*x + 6*cos(c + d*x)*a**3*b**3*d*x - cos(c + d*x)*a**2*b**4 - 3*cos(c
 + d*x)*a*b**5*d*x - 2*cos(c + d*x)*b**6 - 8*log(tan((c + d*x)/2)**2 + 1)*
sin(c + d*x)*a*b**5 + 8*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b -
 a)*sin(c + d*x)*a*b**5 - sin(c + d*x)**3*a**5*b - 2*sin(c + d*x)**3*a**3*
b**3 - sin(c + d*x)**3*a*b**5 + sin(c + d*x)*a**4*b**2*d*x + 6*sin(c + d*x
)*a**2*b**4*d*x - 3*sin(c + d*x)*b**6*d*x)/(2*b*d*(cos(c + d*x)*a**7 + 3*c
os(c + d*x)*a**5*b**2 + 3*cos(c + d*x)*a**3*b**4 + cos(c + d*x)*a*b**6 + s
in(c + d*x)*a**6*b + 3*sin(c + d*x)*a**4*b**3 + 3*sin(c + d*x)*a**2*b**5 +
 sin(c + d*x)*b**7))
```

3.567 $\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 235

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{3(a^6+5a^4b^2+15a^2b^4-5b^6)\,x}{8\,(a^2+b^2)^4} \\ &+ \frac{6ab^5\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d} \\ &+ \frac{3b(a^2-b^2)\,(a^2+5b^2)}{8\,(a^2+b^2)^3\,d(a+b\tan(c+dx))} \\ &+ \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\,(a^2+b^2)\,d(a+b\tan(c+dx))} \\ &- \frac{\cos^2(c+dx)\,(b(a^2-5b^2)-3a(a^2+3b^2)\tan(c+dx))}{8\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}\end{aligned}$$

output
```
3/8*(a^6+5*a^4*b^2+15*a^2*b^4-5*b^6)*x/(a^2+b^2)^4+6*a*b^5*ln(a*cos(d*x+c)
+b*sin(d*x+c))/(a^2+b^2)^4/d+3/8*b*(a^2-b^2)*(a^2+5*b^2)/(a^2+b^2)^3/d/(a+
b*tan(d*x+c))+1/4*cos(d*x+c)^4*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c
))-1/8*cos(d*x+c)^2*(b*(a^2-5*b^2)-3*a*(a^2+3*b^2)*tan(d*x+c))/(a^2+b^2)^2
/d/(a+b*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 2.21 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.77

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{4b\cos^4(c+dx)(b+a\tan(c+dx)) + \frac{2b\cos^2(c+dx)\left(-a^2b+5b^3+3a\left(a^2+3b^2\right)\tan(c+dx)\right)}{a^2+b^2} - \frac{\sqrt{-b^2}\left(6a\left(a^2+b^2\right)\left(a^2+3b^2\right)\left(\left(a-\sqrt{}\right.\right.\right.}{}}{}$$

input
```
Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

output
```
(4*b*Cos[c + d*x]^4*(b + a*Tan[c + d*x]) + (2*b*Cos[c + d*x]^2*(-(a^2*b) +
 5*b^3 + 3*a*(a^2 + 3*b^2)*Tan[c + d*x]))/(a^2 + b^2) - (Sqrt[-b^2]*(6*a*(
a^2 + b^2)*(a^2 + 3*b^2)*((a - Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]
] + 2*Sqrt[-b^2]*Log[a + b*Tan[c + d*x]] - (a + Sqrt[-b^2])*Log[Sqrt[-b^2]
 + b*Tan[c + d*x]])*(a + b*Tan[c + d*x]) + 3*(a^4 + 4*a^2*b^2 - 5*b^4)*(2*
Sqrt[-b^2]*(a^2 + b^2) + (-a^2 + b^2 + 2*a*Sqrt[-b^2])*Log[Sqrt[-b^2] - b*
Tan[c + d*x]]*(a + b*Tan[c + d*x]) - 4*a*Sqrt[-b^2]*Log[a + b*Tan[c + d*x]
]*(a + b*Tan[c + d*x]) + (a^2 - b^2 + 2*a*Sqrt[-b^2])*Log[Sqrt[-b^2] + b*T
an[c + d*x]]*(a + b*Tan[c + d*x]))))/(a^2 + b^2)^3)/(16*b*(a^2 + b^2)*d*(a
 + b*Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.37, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3987, 27, 496, 25, 686, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sec(c+dx)^4(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{b^6}{(a+b\tan(c+dx))^2(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^5\int \frac{1}{(a+b\tan(c+dx))^2(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{496}$$

$$\frac{b^5\left(\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2(a+b\tan(c+dx))}-\frac{\int -\frac{3a^2+4b\tan(c+dx)a+5b^2}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2(a^2+b^2)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^5\left(\frac{\int \frac{3a^2+4b\tan(c+dx)a+5b^2}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2(a+b\tan(c+dx))}\right)}{d}$$

$$\Big\downarrow \text{686}$$

$$\frac{b^5\left(\frac{-\frac{\int -\frac{3\left(a^4+2b^2a^2+2b\left(a^2+3b^2\right)\tan(c+dx)a+5b^4\right)}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2\left(a^2+b^2\right)}-\frac{b^2\left(a^2-5b^2\right)-3ab\left(a^2+3b^2\right)\tan(c+dx)}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)(a+b\tan(c+dx))}}{4b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2(a+b\tan(c+dx))}\right)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^5\left(\frac{\frac{3\int \frac{a^4+2b^2a^2+2b\left(a^2+3b^2\right)\tan(c+dx)a+5b^4}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2\left(a^2+b^2\right)}-\frac{b^2\left(a^2-5b^2\right)-3ab\left(a^2+3b^2\right)\tan(c+dx)}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)(a+b\tan(c+dx))}}{4b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2(a+b\tan(c+dx))}\right)}{d}$$

$$\Big\downarrow \text{657}$$

$$\frac{b^5\left(\frac{\frac{3\int\left(\frac{16ab^4}{(a^2+b^2)^2(a+b\tan(c+dx))}+\frac{a^6+5b^2a^4+15b^4a^2-16b^5\tan(c+dx)a-5b^6}{(a^2+b^2)^2(\tan^2(c+dx)b^2+b^2)}+\frac{-a^4-4b^2a^2+5b^4}{(a^2+b^2)(a+b\tan(c+dx))^2}\right)d(b\tan(c+dx))}{2b^2(a^2+b^2)}-\frac{b^2(a^2-5b^2)-3ab(a^2-}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)}}{4b^2(a^2+b^2)}\right.}{d}$$

$\downarrow$ 2009

$$\frac{b^5\left(\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2(a+b\tan(c+dx))}+\frac{\frac{3\left(-\frac{8ab^4\log(b^2\tan^2(c+dx)+b^2)}{(a^2+b^2)^2}+\frac{16ab^4\log(a+b\tan(c+dx))}{(a^2+b^2)^2}+\frac{a^4+4a^2b^2-5b^4}{(a^2+b^2)(a+b\tan(c+dx))}+\right.}{2b^2(a^2+b^2)}}{4b^2}\right.}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

output
```
(b^5*((b^2 + a*b*Tan[c + d*x])/(4*b^2*(a^2 + b^2)*(a + b*Tan[c + d*x])*(b^
2 + b^2*Tan[c + d*x]^2)^2) + (-1/2*(b^2*(a^2 - 5*b^2) - 3*a*b*(a^2 + 3*b^2
)*Tan[c + d*x])/(b^2*(a^2 + b^2)*(a + b*Tan[c + d*x])*(b^2 + b^2*Tan[c + d
*x]^2)) + (3*(((a^6 + 5*a^4*b^2 + 15*a^2*b^4 - 5*b^6)*ArcTan[Tan[c + d*x]]
)/(b*(a^2 + b^2)^2) + (16*a*b^4*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^2 - (
8*a*b^4*Log[b^2 + b^2*Tan[c + d*x]^2])/(a^2 + b^2)^2 + (a^4 + 4*a^2*b^2 -
5*b^4)/((a^2 + b^2)*(a + b*Tan[c + d*x]))))/(2*b^2*(a^2 + b^2)))/(4*b^2*(a
^2 + b^2))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 16.88 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\frac{\left(\frac{3}{8}a^6+\frac{15}{8}a^4b^2+\frac{5}{8}a^2b^4-\frac{7}{8}b^6\right)\tan(dx+c)^3+\left(2a^3b^3+2a\,b^5\right)\tan(dx+c)^2+\left(\frac{17}{8}a^4b^2+\frac{3}{8}a^2b^4-\frac{9}{8}b^6+\frac{5}{8}a^6\right)\tan(dx+c)+\frac{a^5b}{2}+3a^3b^3}{\left(1+\tan(dx+c)^2\right)^2}}{\left(a^2+b^2\right)^4}$
default	$\frac{\frac{\left(\frac{3}{8}a^6+\frac{15}{8}a^4b^2+\frac{5}{8}a^2b^4-\frac{7}{8}b^6\right)\tan(dx+c)^3+\left(2a^3b^3+2a\,b^5\right)\tan(dx+c)^2+\left(\frac{17}{8}a^4b^2+\frac{3}{8}a^2b^4-\frac{9}{8}b^6+\frac{5}{8}a^6\right)\tan(dx+c)+\frac{a^5b}{2}+3a^3b^3}{\left(1+\tan(dx+c)^2\right)^2}}{\left(a^2+b^2\right)^4}$
risch	$\frac{12ixab}{32ia^3b-32ia\,b^3-8a^4+48b^2a^2-8b^4}-\frac{3x\,a^2}{32ia^3b-32ia\,b^3-8a^4+48b^2a^2-8b^4}+\frac{15x\,b^2}{32ia^3b-32ia\,b^3-8a^4+48b^2a^2-8b^4}-$

input
```
int(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^4*(((3/8*a^6+15/8*a^4*b^2+5/8*a^2*b^4-7/8*b^6)*tan(d*x+c)
^3+(2*a^3*b^3+2*a*b^5)*tan(d*x+c)^2+(17/8*a^4*b^2+3/8*a^2*b^4-9/8*b^6+5/8*
a^6)*tan(d*x+c)+1/2*a^5*b+3*a^3*b^3+5/2*a*b^5)/(1+tan(d*x+c)^2)^2-3*a*b^5*
ln(1+tan(d*x+c)^2)+3/8*(a^6+5*a^4*b^2+15*a^2*b^4-5*b^6)*arctan(tan(d*x+c))
)-b^5/(a^2+b^2)^3/(a+b*tan(d*x+c))+6*b^5/(a^2+b^2)^4*a*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.80

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{4\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7\right)\cos\left(dx+c\right)^5-2\left(a^6b-3\,a^4b^3-9\,a^2b^5-5\,b^7\right)\cos\left(dx+c\right)^3+\left(3\,a^6b+8\,a^4\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/16*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^5 - 2*(a^6*b -
3*a^4*b^3 - 9*a^2*b^5 - 5*b^7)*cos(d*x + c)^3 + (3*a^6*b + 8*a^4*b^3 - 9*a
^2*b^5 - 30*b^7 + 6*(a^7 + 5*a^5*b^2 + 15*a^3*b^4 - 5*a*b^6)*d*x)*cos(d*x
+ c) + 48*(a^2*b^5*cos(d*x + c) + a*b^6*sin(d*x + c))*log(2*a*b*cos(d*x +
c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (3*a^5*b^2 + 22*a^3*
b^4 + 3*a*b^6 - 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^4 - 6
*(a^6*b + 5*a^4*b^3 + 15*a^2*b^5 - 5*b^7)*d*x - 6*(a^7 + 5*a^5*b^2 + 7*a^3
*b^4 + 3*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^9 + 4*a^7*b^2 + 6*a^5*b^
4 + 4*a^3*b^6 + a*b^8)*d*cos(d*x + c) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4
*a^2*b^7 + b^9)*d*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(228) = 456$.

Time = 0.13 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.14

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{48\,ab^5\log(b\tan(dx+c)+a)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{24\,ab^5\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{3\,(a^6+5\,a^4b^2+15\,a^2b^4-5\,b^6)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{4\,\cdots}{a^7+3\,a^5b^2+3\,a^3b^4+ab^6+(a^6b+\cdots}}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/8*(48*a*b^5*log(b*tan(d*x + c) + a)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2
*b^6 + b^8) - 24*a*b^5*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^
4 + 4*a^2*b^6 + b^8) + 3*(a^6 + 5*a^4*b^2 + 15*a^2*b^4 - 5*b^6)*(d*x + c)/
(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + (4*a^4*b + 20*a^2*b^3 -
8*b^5 + 3*(a^4*b + 4*a^2*b^3 - 5*b^5)*tan(d*x + c)^4 + 3*(a^5 + 4*a^3*b^2
+ 3*a*b^4)*tan(d*x + c)^3 + (5*a^4*b + 28*a^2*b^3 - 25*b^5)*tan(d*x + c)^2
 + (5*a^5 + 16*a^3*b^2 + 11*a*b^4)*tan(d*x + c))/(a^7 + 3*a^5*b^2 + 3*a^3*
b^4 + a*b^6 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)^5 + (a^7
+ 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c)^4 + 2*(a^6*b + 3*a^4*b^3 + 3
*a^2*b^5 + b^7)*tan(d*x + c)^3 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*t
an(d*x + c)^2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.70

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{6\,ab^6\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^8bd+4\,a^6b^3d+6\,a^4b^5d+4\,a^2b^7d+b^9d} - \frac{3\,ab^5\log\left(\tan\left(dx+c\right)^2+1\right)}{a^8d+4\,a^6b^2d+6\,a^4b^4d+4\,a^2b^6d+b^8d} + \frac{3\,(a^6+5\,a^4b^2+15\,a^2b^4-5\,b^6)(dx+c)}{8\,(a^8d+4\,a^6b^2d+6\,a^4b^4d+4\,a^2b^6d+b^8d)} + \frac{4\,a^6b+24\,a^4b^3+12\,a^2b^5-8\,b^7+3\,(a^6b+5\,a^4b^3-a^2b^5-5\,b^7)\tan\left(dx+c\right)^4+3\,(a^7+5\,a^5b^2+7\,a^3b^4}{8\,(a^2+b^2)^4(b\tan}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
6*a*b^6*log(abs(b*tan(d*x + c) + a))/(a^8*b*d + 4*a^6*b^3*d + 6*a^4*b^5*d
+ 4*a^2*b^7*d + b^9*d) - 3*a*b^5*log(tan(d*x + c)^2 + 1)/(a^8*d + 4*a^6*b^
2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b^8*d) + 3/8*(a^6 + 5*a^4*b^2 + 15*a^2*b
^4 - 5*b^6)*(d*x + c)/(a^8*d + 4*a^6*b^2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b
^8*d) + 1/8*(4*a^6*b + 24*a^4*b^3 + 12*a^2*b^5 - 8*b^7 + 3*(a^6*b + 5*a^4*
b^3 - a^2*b^5 - 5*b^7)*tan(d*x + c)^4 + 3*(a^7 + 5*a^5*b^2 + 7*a^3*b^4 + 3
*a*b^6)*tan(d*x + c)^3 + (5*a^6*b + 33*a^4*b^3 + 3*a^2*b^5 - 25*b^7)*tan(d
*x + c)^2 + (5*a^7 + 21*a^5*b^2 + 27*a^3*b^4 + 11*a*b^6)*tan(d*x + c))/((a
^2 + b^2)^4*(b*tan(d*x + c) + a)*(tan(d*x + c)^2 + 1)^2*d)
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.97

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{3\tan(c+dx)^4\,(a^4\,b+4\,a^2\,b^3-5\,b^5)}{8\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}+\frac{a^4\,b+5\,a^2\,b^3-2\,b^5}{2\,(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+dx)\,(5\,a^3+11\,a\,b^2)}{8\,(a^4+2\,a^2\,b^2+b^4)}+\frac{3\tan(c+dx)^3\,(a^3+3\,a\,b^2)}{8\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+dx)}{8\,(a^2+}}{d\,\left(b\tan\left(c+d\,x\right)^5+a\tan\left(c+d\,x\right)^4+2\,b\tan\left(c+d\,x\right)^3+2\,a\tan\left(c+d\,x\right)^2+b\tan\left(c+d\,x\right.\right.}$$
$$+\frac{3\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-a^2+a\,b\,4\mathrm{i}+5\,b^2\right)}{16\,d\,\left(a^4\,1\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6\mathrm{i}-4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$
$$+\frac{3\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a^2+a\,b\,4\mathrm{i}-5\,b^2\right)}{16\,d\,\left(a^4\,1\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6\mathrm{i}+4\,a\,b^3+b^4\,1\mathrm{i}\right)}+\frac{6\,a\,b^5\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,\left(a^2+b^2\right)^4}$$

input
```
int(cos(c + d*x)^4/(a + b*tan(c + d*x))^2,x)
```

output
```
((3*tan(c + d*x)^4*(a^4*b - 5*b^5 + 4*a^2*b^3))/(8*(a^6 + b^6 + 3*a^2*b^4
+ 3*a^4*b^2)) + (a^4*b - 2*b^5 + 5*a^2*b^3)/(2*(a^2 + b^2)*(a^4 + b^4 + 2*
a^2*b^2)) + (tan(c + d*x)*(11*a*b^2 + 5*a^3))/(8*(a^4 + b^4 + 2*a^2*b^2))
+ (3*tan(c + d*x)^3*(3*a*b^2 + a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c
+ d*x)^2*(5*a^4*b - 25*b^5 + 28*a^2*b^3))/(8*(a^2 + b^2)*(a^4 + b^4 + 2*a^
2*b^2)))/(d*(a + b*tan(c + d*x) + 2*a*tan(c + d*x)^2 + a*tan(c + d*x)^4 +
2*b*tan(c + d*x)^3 + b*tan(c + d*x)^5)) + (3*log(tan(c + d*x) + 1i)*(a*b*4
i - a^2 + 5*b^2))/(16*d*(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i)
) + (3*log(tan(c + d*x) - 1i)*(a*b*4i + a^2 - 5*b^2))/(16*d*(4*a*b^3 - 4*a
^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)) + (6*a*b^5*log(a + b*tan(c + d*x)))/
(d*(a^2 + b^2)^4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 752, normalized size of antiderivative = 3.20

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x)
```

output

```
( - 48*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*a**2*b**6 + 48*cos(c + d*
x)*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*a**2*b**6 + 2*cos
(c + d*x)*sin(c + d*x)**4*a**6*b**2 + 6*cos(c + d*x)*sin(c + d*x)**4*a**4*
b**4 + 6*cos(c + d*x)*sin(c + d*x)**4*a**2*b**6 + 2*cos(c + d*x)*sin(c + d
*x)**4*b**8 - 3*cos(c + d*x)*sin(c + d*x)**2*a**6*b**2 - 15*cos(c + d*x)*s
in(c + d*x)**2*a**4*b**4 - 21*cos(c + d*x)*sin(c + d*x)**2*a**2*b**6 - 9*c
os(c + d*x)*sin(c + d*x)**2*b**8 - 5*cos(c + d*x)*a**8 + 3*cos(c + d*x)*a*
*7*b*d*x - 17*cos(c + d*x)*a**6*b**2 + 15*cos(c + d*x)*a**5*b**3*d*x - 3*c
os(c + d*x)*a**4*b**4 + 45*cos(c + d*x)*a**3*b**5*d*x + cos(c + d*x)*a**2*
b**6 - 15*cos(c + d*x)*a*b**7*d*x - 8*cos(c + d*x)*b**8 - 48*log(tan((c +
d*x)/2)**2 + 1)*sin(c + d*x)*a*b**7 + 48*log(tan((c + d*x)/2)**2*a - 2*tan
((c + d*x)/2)*b - a)*sin(c + d*x)*a*b**7 + 2*sin(c + d*x)**5*a**7*b + 6*si
n(c + d*x)**5*a**5*b**3 + 6*sin(c + d*x)**5*a**3*b**5 + 2*sin(c + d*x)**5*
a*b**7 - 7*sin(c + d*x)**3*a**7*b - 27*sin(c + d*x)**3*a**5*b**3 - 33*sin(
c + d*x)**3*a**3*b**5 - 13*sin(c + d*x)**3*a*b**7 + 3*sin(c + d*x)*a**6*b*
*2*d*x + 15*sin(c + d*x)*a**4*b**4*d*x + 45*sin(c + d*x)*a**2*b**6*d*x - 1
5*sin(c + d*x)*b**8*d*x)/(8*b*d*(cos(c + d*x)*a**9 + 4*cos(c + d*x)*a**7*b
**2 + 6*cos(c + d*x)*a**5*b**4 + 4*cos(c + d*x)*a**3*b**6 + cos(c + d*x)*a
*b**8 + sin(c + d*x)*a**8*b + 4*sin(c + d*x)*a**6*b**3 + 6*sin(c + d*x)*a*
*4*b**5 + 4*sin(c + d*x)*a**2*b**7 + sin(c + d*x)*b**9))
```

3.568 $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 255

$$\begin{aligned}\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= -\frac{5a(a^2+b^2)\sec(c+dx)}{b^5d}-\frac{5a\sec^3(c+dx)}{3b^3d}\\&\quad+\frac{5(8a^4+12a^2b^2+3b^4)\operatorname{arcsinh}(\tan(c+dx))\sec(c+dx)}{8b^6d\sqrt{\sec^2(c+dx)}}\\&\quad+\frac{5a(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{b^6d\sqrt{\sec^2(c+dx)}}\\&\quad+\frac{5(4a^2+3b^2)\sec(c+dx)\tan(c+dx)}{8b^4d}\\&\quad+\frac{5\sec^3(c+dx)\tan(c+dx)}{4b^2d}-\frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))}\end{aligned}$$

output
```
-5*a*(a^2+b^2)*sec(d*x+c)/b^5/d-5/3*a*sec(d*x+c)^3/b^3/d+5/8*(8*a^4+12*a^2
*b^2+3*b^4)*arcsinh(tan(d*x+c))*sec(d*x+c)/b^6/d/(sec(d*x+c)^2)^(1/2)+5*a*
(a^2+b^2)^(3/2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)^(1
/2))*sec(d*x+c)/b^6/d/(sec(d*x+c)^2)^(1/2)+5/8*(4*a^2+3*b^2)*sec(d*x+c)*ta
n(d*x+c)/b^4/d+5/4*sec(d*x+c)^3*tan(d*x+c)/b^2/d-sec(d*x+c)^5/b/d/(a+b*tan
(d*x+c))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.21 (sec) , antiderivative size = 747, normalized size of antiderivative = 2.93

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])*(-48*b*(a^2 + b^2)^2 - 1
6*a*b*(12*a^2 + 13*b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x]) - 480*a*(a - I*b
)*(a + I*b)*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b
^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x]) - 30*(8*a^4 + 12*a^2*b^2 + 3*b^4)*L
og[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])
+ 30*(8*a^4 + 12*a^2*b^2 + 3*b^4)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
*(a*Cos[c + d*x] + b*Sin[c + d*x]) + (3*b^4*(a*Cos[c + d*x] + b*Sin[c + d*
x]))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4 + (b^2*(36*a^2 - 8*a*b + 21*b
^2)*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
])^2 - (16*a*b^3*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(Cos[
(c + d*x)/2] - Sin[(c + d*x)/2])^3 - (16*a*b*(12*a^2 + 13*b^2)*Sin[(c + d*
x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]) - (3*b^4*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^4 + (16*a*b^3*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*
x]))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - (b^2*(36*a^2 + 8*a*b + 21*b
^2)*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^2 + (16*a*b*(12*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[
c + d*x]))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(48*b^6*d*(a + b*Tan[c
+ d*x])^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.87, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3992, 492, 591, 25, 682, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{\sec(c+dx)^7}{(a+b\tan(c+dx))^2}dx$$

$\Big\downarrow$ 3992

$$\frac{\sec(c+dx)\int \frac{\left(\tan^2(c+dx)+1\right)^{5/2}}{(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$\Big\downarrow$ 492

$$\frac{\sec(c+dx)\left(\frac{5\int \frac{b\tan(c+dx)\left(\tan^2(c+dx)+1\right)^{3/2}}{a+b\tan(c+dx)}d(b\tan(c+dx))}{b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{5/2}}{a+b\tan(c+dx)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$\Big\downarrow$ 591

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{1}{4}\int -\frac{\left(a-\left(\frac{4a^2}{b^2}+3\right)b\tan(c+dx)\right)\sqrt{\tan^2(c+dx)+1}}{a+b\tan(c+dx)}d(b\tan(c+dx))-\frac{1}{12}\left(\tan^2(c+dx)+1\right)^{3/2}(4a-3b\tan(c+dx))\right)}{b^2}-\frac{(\tan^2(c+d}{a+b\tan}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$\Big\downarrow$ 25

$$\frac{\sec(c+dx)\left(\frac{5\left(-\frac{1}{4}\int \frac{\left(a-\left(\frac{4a^2}{b^2}+3\right)b\tan(c+dx)\right)\sqrt{\tan^2(c+dx)+1}}{a+b\tan(c+dx)}d(b\tan(c+dx))-\frac{1}{12}\left(\tan^2(c+dx)+1\right)^{3/2}(4a-3b\tan(c+dx))\right)}{b^2}-\frac{(\tan^2(c+d}{a+b\tan}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$\Big\downarrow$ 682

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{1}{4}\left(-\frac{1}{2}b^2\int \frac{ab^2\left(4a^2+5b^2\right)-b\left(8a^4+12b^2a^2+3b^4\right)\tan(c+dx)}{b^6(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-\frac{\sqrt{\tan^2(c+dx)+1}\left(8a\left(a^2+b^2\right)-b\left(4a^2+3b^2\right)\tan(c+dx)\right)}{2b^2}\right.\right.}{b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$\Big\downarrow$ 27

$$\sec(c+dx)\left(\frac{5\left(\frac{1}{4}\left(-\frac{\int \frac{ab^2\left(4a^2+5b^2\right)-b\left(8a^4+12b^2a^2+3b^4\right)\tan(c+dx)}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2b^4}-\frac{\sqrt{\tan^2(c+dx)+1}\left(8a\left(a^2+b^2\right)-b\left(4a^2+3b^2\right)\tan(c+dx)\right)}{2b^2}\right)-\frac{}{1}\right.}{b^2}\right.$$
$$\overline{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 719$$

$$\sec(c+dx)\left(\frac{5\left(\frac{1}{4}\left(-\frac{8a\left(a^2+b^2\right)^2\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-\left(8a^4+12a^2b^2+3b^4\right)\int \frac{1}{\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2b^4}-\frac{\sqrt{\tan^2(}}{}\right.\right.}{b^2}\right.$$
$$\overline{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 222$$

$$\sec(c+dx)\left(\frac{5\left(\frac{1}{4}\left(-\frac{8a\left(a^2+b^2\right)^2\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-b\left(8a^4+12a^2b^2+3b^4\right)\text{arcsinh}(\tan(c+dx))}{2b^4}-\frac{\sqrt{\tan^2(c+dx)+1}\left(8\right.}{}\right.\right.}{b^2}\right.$$
$$\overline{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 488$$

$$\sec(c+dx)\left(\frac{5\left(\frac{1}{4}\left(-\frac{-8a\left(a^2+b^2\right)^2\int \frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}-b\left(8a^4+12a^2b^2+3b^4\right)\text{arcsinh}(\tan(c+dx))}{2b^4}-\frac{\sqrt{\tan^2(c+dx)+1}\left(8a\left(a^2\right.\right.}{}\right.\right.}{b^2}\right.$$
$$\overline{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 219$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{1}{4}\left(-\frac{\sqrt{\tan^2(c+dx)+1}\left(8a\left(a^2+b^2\right)-b\left(4a^2+3b^2\right)\tan(c+dx)\right)}{2b^2}-\frac{-8ab\left(a^2+b^2\right)^{3/2}\text{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)-b\left(8a^4+12a^2b^2+3b^4\right)\text{ar}}{2b^4}\right.\right.}{b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*(-((1 + Tan[c + d*x]^2)^(5/2)/(a + b*Tan[c + d*x])) + (5*(-1
/12*((4*a - 3*b*Tan[c + d*x])*(1 + Tan[c + d*x]^2)^(3/2)) + (-1/2*(-(b*(8*
a^4 + 12*a^2*b^2 + 3*b^4)*ArcSinh[Tan[c + d*x]]) - 8*a*b*(a^2 + b^2)^(3/2)
*ArcTanh[(b^2*Tan[c + d*x])/Sqrt[a^2 + b^2]])/b^4 - ((8*a*(a^2 + b^2) - b*
(4*a^2 + 3*b^2)*Tan[c + d*x])*Sqrt[1 + Tan[c + d*x]^2])/(2*b^2))/4))/b^2))
/(b*d*Sqrt[Sec[c + d*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 591
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 2*p + 1)*x)/
(d^2*(n + 2*p + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 2*p + 1)*(n +
2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*Simp[a*c*d*n + (b*c^2*(2*p
 + 1) + a*d^2*(n + 2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &&
GtQ[p, 0] && LeQ[-1, n, 0] &&  !ILtQ[n + 2*p, 0]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 150.12 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.78

method	result
derivativedivides	$\frac{-\frac{1}{4b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{4a-3b}{6b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{12a^2-8ab+11b^2}{8b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{\left(40a^4+60b^2a^2+15b^4\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8b^6}-\frac{32a^3}{8b^5}}{}$
default	$\frac{-\frac{1}{4b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{4a-3b}{6b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{12a^2-8ab+11b^2}{8b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{\left(40a^4+60b^2a^2+15b^4\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8b^6}-\frac{32a^3}{8b^5}}{}$
risch	$-\frac{120ia^3b\,e^{3i(dx+c)}+60ia^3b\,e^{i(dx+c)}+75ia\,b^3e^{i(dx+c)}-190ia\,b^3e^{7i(dx+c)}+190ia\,b^3e^{3i(dx+c)}-120ia^3b\,e^{7i(dx+c)}-60i}{}$

input
```
int(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4/b^2/(tan(1/2*d*x+1/2*c)+1)^4-1/6*(4*a-3*b)/b^3/(tan(1/2*d*x+1/2*
c)+1)^3-1/8*(12*a^2-8*a*b+11*b^2)/b^4/(tan(1/2*d*x+1/2*c)+1)^2+1/8/b^6*(40
*a^4+60*a^2*b^2+15*b^4)*ln(tan(1/2*d*x+1/2*c)+1)-1/8*(32*a^3-12*a^2*b+40*a
*b^2-9*b^3)/b^5/(tan(1/2*d*x+1/2*c)+1)+1/4/b^2/(tan(1/2*d*x+1/2*c)-1)^4-1/
6*(-4*a-3*b)/b^3/(tan(1/2*d*x+1/2*c)-1)^3-1/8*(-12*a^2-8*a*b-11*b^2)/b^4/(
tan(1/2*d*x+1/2*c)-1)^2+1/8/b^6*(-40*a^4-60*a^2*b^2-15*b^4)*ln(tan(1/2*d*x
+1/2*c)-1)-1/8*(-32*a^3-12*a^2*b-40*a*b^2-9*b^3)/b^5/(tan(1/2*d*x+1/2*c)-1
)+2/b^6*(((a^4+2*a^2*b^2+b^4)*b^2/a*tan(1/2*d*x+1/2*c)+b*(a^4+2*a^2*b^2+b^
4))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)-5*a*(a^4+2*a^2*b^2+b
^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2))))
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.85

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{12\,b^5 - 30\,(8\,a^4b + 12\,a^2b^3 + 3\,b^5)\cos(dx+c)^4 + 10\,(4\,a^2b^3 + 3\,b^5)\cos(dx+c)^2 + 120\,((a^4 + a^2b^2)\cos(\cdots}{}$$

input
```
integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/48*(12*b^5 - 30*(8*a^4*b + 12*a^2*b^3 + 3*b^5)*cos(d*x + c)^4 + 10*(4*a^
2*b^3 + 3*b^5)*cos(d*x + c)^2 + 120*((a^4 + a^2*b^2)*cos(d*x + c)^5 + (a^3
*b + a*b^3)*cos(d*x + c)^4*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*
x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^
2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x +
c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 15*((8*a^5 + 12*a^3*b^2 + 3*a*b^
4)*cos(d*x + c)^5 + (8*a^4*b + 12*a^2*b^3 + 3*b^5)*cos(d*x + c)^4*sin(d*x
+ c))*log(sin(d*x + c) + 1) - 15*((8*a^5 + 12*a^3*b^2 + 3*a*b^4)*cos(d*x +
 c)^5 + (8*a^4*b + 12*a^2*b^3 + 3*b^5)*cos(d*x + c)^4*sin(d*x + c))*log(-s
in(d*x + c) + 1) - 10*(2*a*b^4*cos(d*x + c) + 3*(4*a^3*b^2 + 5*a*b^4)*cos(
d*x + c)^3)*sin(d*x + c))/(a*b^6*d*cos(d*x + c)^5 + b^7*d*cos(d*x + c)^4*s
in(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)**7/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**7/(a + b*tan(c + d*x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(239) = 478$.

Time = 0.14 (sec) , antiderivative size = 827, normalized size of antiderivative = 3.24

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/24*(2*(120*a^5 + 160*a^3*b^2 + 24*a*b^4 + (180*a^4*b + 245*a^2*b^3 + 24
*b^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*(48*a^5 + 68*a^3*b^2 + 15*a*b^4
)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(300*a^4*b + 385*a^2*b^3 + 48*b^
5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*(72*a^5 + 100*a^3*b^2 + 15*a*b
^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 48*(15*a^4*b + 20*a^2*b^3 + 3*b^
5)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 30*(16*a^5 + 20*a^3*b^2 + 3*a*b^4
)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*(60*a^4*b + 85*a^2*b^3 + 16*b^5)
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 30*(4*a^5 + 4*a^3*b^2 - a*b^4)*sin(
d*x + c)^8/(cos(d*x + c) + 1)^8 + 3*(20*a^4*b + 25*a^2*b^3 + 8*b^5)*sin(d*
x + c)^9/(cos(d*x + c) + 1)^9)/(a^2*b^5 + 2*a*b^6*sin(d*x + c)/(cos(d*x +
c) + 1) - 5*a^2*b^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 8*a*b^6*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 10*a^2*b^5*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 + 12*a*b^6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 10*a^2*b^5*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6 - 8*a*b^6*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 5
*a^2*b^5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 2*a*b^6*sin(d*x + c)^9/(cos
(d*x + c) + 1)^9 - a^2*b^5*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 120*(a
^4 + 2*a^2*b^2 + b^4)*a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(
a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sq
rt(a^2 + b^2)*b^6) - 15*(8*a^4 + 12*a^2*b^2 + 3*b^4)*log(sin(d*x + c)/(cos
(d*x + c) + 1) + 1)/b^6 + 15*(8*a^4 + 12*a^2*b^2 + 3*b^4)*log(sin(d*x +...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(239) = 478$.

Time = 0.30 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.08

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(15*(8*a^4 + 12*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b
^6 - 15*(8*a^4 + 12*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^
6 + 120*(a^5 + 2*a^3*b^2 + a*b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b -
 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)
))/(sqrt(a^2 + b^2)*b^6) + 48*(a^4*b*tan(1/2*d*x + 1/2*c) + 2*a^2*b^3*tan(
1/2*d*x + 1/2*c) + b^5*tan(1/2*d*x + 1/2*c) + a^5 + 2*a^3*b^2 + a*b^4)/((a
*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)*a*b^5) + 2*(36*a^2
*b*tan(1/2*d*x + 1/2*c)^7 + 27*b^3*tan(1/2*d*x + 1/2*c)^7 + 96*a^3*tan(1/2
*d*x + 1/2*c)^6 + 144*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 36*a^2*b*tan(1/2*d*x
+ 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^5 - 288*a^3*tan(1/2*d*x + 1/2*c)^4
 - 336*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*
b^3*tan(1/2*d*x + 1/2*c)^3 + 288*a^3*tan(1/2*d*x + 1/2*c)^2 + 304*a*b^2*ta
n(1/2*d*x + 1/2*c)^2 + 36*a^2*b*tan(1/2*d*x + 1/2*c) + 27*b^3*tan(1/2*d*x
+ 1/2*c) - 96*a^3 - 112*a*b^2)/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*b^5))/d
```

Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 2654, normalized size of antiderivative = 10.41

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^7*(a + b*tan(c + d*x))^2),x)
```

output
```
-((9*a*b^5)/64 + (15*a^5*b)/8 + (b^6*sin(c + d*x))/8 + (115*a^3*b^3)/48 +
(3*b^6*sin(3*c + 3*d*x))/16 + (b^6*sin(5*c + 5*d*x))/16 + (a^6*cos(c + d*x
)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*25i)/4 + (5*a*b^5*cos(2
*c + 2*d*x))/8 + (5*a^5*b*cos(2*c + 2*d*x))/2 + (5*a*b^5*cos(3*c + 3*d*x))
/16 + (25*a^5*b*cos(3*c + 3*d*x))/16 + (15*a*b^5*cos(4*c + 4*d*x))/64 + (5
*a^5*b*cos(4*c + 4*d*x))/8 + (a*b^5*cos(5*c + 5*d*x))/16 + (5*a^5*b*cos(5*
c + 5*d*x))/16 + (25*a^3*b^3*cos(c + d*x))/6 + (5*a^2*b^4*sin(c + d*x))/6
+ (5*a^4*b^2*sin(c + d*x))/8 + (a^6*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 +
 (d*x)/2))*cos(3*c + 3*d*x)*25i)/8 + (a^6*atan((sin(c/2 + (d*x)/2)*1i)/cos
(c/2 + (d*x)/2))*cos(5*c + 5*d*x)*5i)/8 + (10*a^3*b^3*cos(2*c + 2*d*x))/3
+ (25*a^3*b^3*cos(3*c + 3*d*x))/12 + (15*a^3*b^3*cos(4*c + 4*d*x))/16 + (5
*a^3*b^3*cos(5*c + 5*d*x))/12 + (95*a^2*b^4*sin(2*c + 2*d*x))/96 + (5*a^4*
b^2*sin(2*c + 2*d*x))/8 + (5*a^2*b^4*sin(3*c + 3*d*x))/4 + (15*a^4*b^2*sin
(3*c + 3*d*x))/16 + (25*a^2*b^4*sin(4*c + 4*d*x))/64 + (5*a^4*b^2*sin(4*c
+ 4*d*x))/16 + (5*a^2*b^4*sin(5*c + 5*d*x))/12 + (5*a^4*b^2*sin(5*c + 5*d*
x))/16 + (5*a*b^5*cos(c + d*x))/8 + (25*a^5*b*cos(c + d*x))/8 + (a*b^5*ata
n((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*45i)/64 + (
a^5*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*15
i)/8 + (a*b^5*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(5*c + 5
*d*x)*15i)/64 + (a^5*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6643, normalized size of antiderivative = 26.05

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x)
```

output
```
(240*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2)
)*cos(c + d*x)*sin(c + d*x)**6*tan(c + d*x)*a**4*b*i + 240*sqrt(a**2 + b**
2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c
 + d*x)**6*tan(c + d*x)*a**2*b**3*i + 240*sqrt(a**2 + b**2)*atan((tan((c +
 d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**6*a**5*i
 + 240*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**
2))*cos(c + d*x)*sin(c + d*x)**6*a**3*b**2*i - 720*sqrt(a**2 + b**2)*atan(
(tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)*
*4*tan(c + d*x)*a**4*b*i - 720*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*
i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**4*tan(c + d*x)*a**2
*b**3*i - 720*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**
2 + b**2))*cos(c + d*x)*sin(c + d*x)**4*a**5*i - 720*sqrt(a**2 + b**2)*ata
n((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x
)**4*a**3*b**2*i + 720*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)
/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*tan(c + d*x)*a**4*b*i + 7
20*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*
cos(c + d*x)*sin(c + d*x)**2*tan(c + d*x)*a**2*b**3*i + 720*sqrt(a**2 + b*
*2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(
c + d*x)**2*a**5*i + 720*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*
i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*a**3*b**2*i - 240*sq...
```

3.569 $\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 184

$$
\begin{aligned}
\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx = &-\frac{3a\sec(c+dx)}{b^3d} \\
&+\frac{3(2a^2+b^2)\operatorname{arcsinh}(\tan(c+dx))\sec(c+dx)}{2b^4d\sqrt{\sec^2(c+dx)}} \\
&+\frac{3a\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{b^4d\sqrt{\sec^2(c+dx)}} \\
&+\frac{3\sec(c+dx)\tan(c+dx)}{2b^2d}-\frac{\sec^3(c+dx)}{bd(a+b\tan(c+dx))}
\end{aligned}
$$

output
```
-3*a*sec(d*x+c)/b^3/d+3/2*(2*a^2+b^2)*arcsinh(tan(d*x+c))*sec(d*x+c)/b^4/d
/(sec(d*x+c)^2)^(1/2)+3*a*(a^2+b^2)^(1/2)*arctanh((b-a*tan(d*x+c))/(a^2+b^
2)^(1/2)/(sec(d*x+c)^2)^(1/2))*sec(d*x+c)/b^4/d/(sec(d*x+c)^2)^(1/2)+3/2*s
ec(d*x+c)*tan(d*x+c)/b^2/d-sec(d*x+c)^3/b/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.41 (sec) , antiderivative size = 709, normalized size of antiderivative = 3.85

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= -\frac{(a-ib)(a+ib)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))}{b^3d(a+b\tan(c+dx))^2}$$
$$-\frac{2a\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{b^3d(a+b\tan(c+dx))^2}$$
$$-\frac{6a\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\sqrt{a^2+b^2}\left(-b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2\cos\left(\frac{1}{2}(c+dx)\right)+b^2\cos\left(\frac{1}{2}(c+dx)\right)}\right)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{b^4d(a+b\tan(c+dx))^2}$$
$$-\frac{3(2a^2+b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{2b^4d(a+b\tan(c+dx))^2}$$
$$+\frac{3(2a^2+b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{2b^4d(a+b\tan(c+dx))^2}$$
$$+\frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{4b^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a+b\tan(c+dx))^2}$$
$$-\frac{2a\sec^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{b^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2}$$
$$-\frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{4b^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a+b\tan(c+dx))^2}$$
$$+\frac{2a\sec^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{b^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^2,x]
```

output
```
-(((a - I*b)*(a + I*b)*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(
b^3*d*(a + b*Tan[c + d*x])^2)) - (2*a*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*S
in[c + d*x])^2)/(b^3*d*(a + b*Tan[c + d*x])^2) - (6*a*Sqrt[a^2 + b^2]*ArcT
anh[(Sqrt[a^2 + b^2]*(-(b*Cos[(c + d*x)/2]) + a*Sin[(c + d*x)/2]))/(a^2*Co
s[(c + d*x)/2] + b^2*Cos[(c + d*x)/2])]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b
*Sin[c + d*x])^2)/(b^4*d*(a + b*Tan[c + d*x])^2) - (3*(2*a^2 + b^2)*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(2*b^4*d*(a + b*Tan[c + d*x])^2) + (3*(2*a^2 + b^2)*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^2)/(2*b^4*d*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2)/(4*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(
a + b*Tan[c + d*x])^2) - (2*a*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d
*x] + b*Sin[c + d*x])^2)/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a +
 b*Tan[c + d*x])^2) - (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
/(4*b^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a + b*Tan[c + d*x])^2)
+ (2*a*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2
)/(b^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3992, 492, 591, 25, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{492}$$

$$\frac{\sec(c+dx)\left(\frac{3\int \frac{b\tan(c+dx)\sqrt{\tan^2(c+dx)+1}}{a+b\tan(c+dx)}d(b\tan(c+dx))}{b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{a+b\tan(c+dx)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$\Big\downarrow$ 591

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{1}{2}\int -\frac{a-\left(\frac{2a^2}{b^2}+1\right)b\tan(c+dx)}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-\frac{1}{2}\sqrt{\tan^2(c+dx)+1}(2a-b\tan(c+dx))\right)}{b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{a+b\tan(c+dx)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$\Big\downarrow$ 25

$$\frac{\sec(c+dx)\left(\frac{3\left(-\frac{1}{2}\int \frac{a-\left(\frac{2a^2}{b^2}+1\right)b\tan(c+dx)}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-\frac{1}{2}\sqrt{\tan^2(c+dx)+1}(2a-b\tan(c+dx))\right)}{b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{a+b\tan(c+dx)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$\Big\downarrow$ 719

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{1}{2}\left(\left(\frac{2a^2}{b^2}+1\right)\int \frac{1}{\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-2a\left(\frac{a^2}{b^2}+1\right)\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))\right)-\frac{1}{2}\sqrt{\tan^2(c+\right.}{b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$\Big\downarrow$ 222

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{1}{2}\left(b\left(\frac{2a^2}{b^2}+1\right)\text{arcsinh}(\tan(c+dx))-2a\left(\frac{a^2}{b^2}+1\right)\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))\right)-\frac{1}{2}\sqrt{\tan^2(c+dx)+1}(2a\right.}{b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$\Big\downarrow$ 488

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{1}{2}\left(2a\left(\frac{a^2}{b^2}+1\right)\int\frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}+b\left(\frac{2a^2}{b^2}+1\right)\text{arcsinh}(\tan(c+dx))\right)-\frac{1}{2}\sqrt{\tan^2(c+dx)+1}(2a-b\tan(c}{b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{1}{2}\left(b\left(\frac{2a^2}{b^2}+1\right)\text{arcsinh}(\tan(c+dx))+\frac{2ab\left(\frac{a^2}{b^2}+1\right)\text{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)-\frac{1}{2}\sqrt{\tan^2(c+dx)+1}(2a-b\tan(c+dx))\right)}{b^2}-\right.}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*(-((1 + Tan[c + d*x]^2)^(3/2)/(a + b*Tan[c + d*x])) + (3*(((
1 + (2*a^2)/b^2)*b*ArcSinh[Tan[c + d*x]] + (2*a*(1 + a^2/b^2)*b*ArcTanh[(b
^2*Tan[c + d*x])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2])/2 - ((2*a - b*Tan[c +
d*x])*Sqrt[1 + Tan[c + d*x]^2])/2))/b^2))/(b*d*Sqrt[Sec[c + d*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 591
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 2*p + 1)*x)/
(d^2*(n + 2*p + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 2*p + 1)*(n +
2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*Simp[a*c*d*n + (b*c^2*(2*p
 + 1) + a*d^2*(n + 2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &&
GtQ[p, 0] && LeQ[-1, n, 0] &&  !ILtQ[n + 2*p, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 30.71 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{\frac{\frac{2\left(\frac{(a^2+b^2)b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b(a^2+b^2)\right)}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-6a\sqrt{a^2+b^2}\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^4}+\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-4a-b}{2b^3\left(\tan\left(\frac{dx}{2}+\cdots\right)\right)}}{d}$
default	$\frac{\frac{\frac{2\left(\frac{(a^2+b^2)b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b(a^2+b^2)\right)}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-6a\sqrt{a^2+b^2}\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^4}+\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-4a-b}{2b^3\left(\tan\left(\frac{dx}{2}+\cdots\right)\right)}}{d}$
risch	$-\frac{-3iab\,e^{5i(dx+c)}+6a^2e^{5i(dx+c)}+3b^2e^{5i(dx+c)}+12a^2e^{3i(dx+c)}+2b^2e^{3i(dx+c)}+3iab\,e^{i(dx+c)}+6a^2e^{i(dx+c)}+3b^2e^{i(dx+\cdots)}}{\left(e^{2i(dx+c)}+1\right)^2\left(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a\right)d\,b^3}$

input
```
int(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^4*(((a^2+b^2)*b^2/a*tan(1/2*d*x+1/2*c)+b*(a^2+b^2))/(a*tan(1/2*d*
x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)-3*a*(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*
tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))+1/2/b^2/(tan(1/2*d*x+1/2*c)-1)^2
-1/2*(-4*a-b)/b^3/(tan(1/2*d*x+1/2*c)-1)+1/2/b^4*(-6*a^2-3*b^2)*ln(tan(1/2
*d*x+1/2*c)-1)-1/2/b^2/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(4*a-b)/b^3/(tan(1/2*d
*x+1/2*c)+1)+1/2/b^4*(6*a^2+3*b^2)*ln(tan(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(172) = 344$.

Time = 0.14 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.93

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{6\,ab^2\cos(dx+c)\sin(dx+c)-2\,b^3+6\,(2\,a^2b+b^3)\cos(dx+c)^2-6\,\left(a^2\cos(dx+c)^3+ab\cos(dx+\cdots\right)}{\cdots}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/4*(6*a*b^2*cos(d*x + c)*sin(d*x + c) - 2*b^3 + 6*(2*a^2*b + b^3)*cos(d*
x + c)^2 - 6*(a^2*cos(d*x + c)^3 + a*b*cos(d*x + c)^2*sin(d*x + c))*sqrt(a
^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^
2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*
a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 3*((2
*a^3 + a*b^2)*cos(d*x + c)^3 + (2*a^2*b + b^3)*cos(d*x + c)^2*sin(d*x + c)
)*log(sin(d*x + c) + 1) + 3*((2*a^3 + a*b^2)*cos(d*x + c)^3 + (2*a^2*b + b
^3)*cos(d*x + c)^2*sin(d*x + c))*log(-sin(d*x + c) + 1))/(a*b^4*d*cos(d*x
+ c)^3 + b^5*d*cos(d*x + c)^2*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^5{(c+dx)}}{(a+b\tan{(c+dx)})^2}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*tan(c + d*x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(172) = 344$.

Time = 0.14 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.56

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{\frac{2\left(6a^3+2ab^2+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(9a^2b+2b^3)\sin(dx+c)}{\cos(dx+c)+1}-\frac{6(2a^3+ab^2)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4(3a^2b+b^3)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{(3a^2b+2b^3)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2b^3+\frac{2ab^4\sin(dx+c)}{\cos(dx+c)+1}-\frac{3a^2b^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4ab^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a^2b^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2ab^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2b^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}} - \frac{6\sqrt{}}{}}{2d}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(6*a^3 + 2*a*b^2 + 6*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (9*
a^2*b + 2*b^3)*sin(d*x + c)/(cos(d*x + c) + 1) - 6*(2*a^3 + a*b^2)*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 - 4*(3*a^2*b + b^3)*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + (3*a^2*b + 2*b^3)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2*b
^3 + 2*a*b^4*sin(d*x + c)/(cos(d*x + c) + 1) - 3*a^2*b^3*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 4*a*b^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^2*b
^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2*a*b^4*sin(d*x + c)^5/(cos(d*x +
 c) + 1)^5 - a^2*b^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - 6*sqrt(a^2 + b
^2)*a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a
*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/b^4 - 3*(2*a^2 + b^2)
*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^4 + 3*(2*a^2 + b^2)*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/b^4)/d
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.52

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{\frac{3\left(2a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^4} - \frac{3\left(2a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^4} + \frac{6\left(a^3+ab^2\right)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4} + 2(}{2d}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(3*(2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*a^2 + b
^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 6*(a^3 + a*b^2)*log(abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c
) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 2*(b*tan(1/2*d*x + 1
/2*c)^3 + 4*a*tan(1/2*d*x + 1/2*c)^2 + b*tan(1/2*d*x + 1/2*c) - 4*a)/((tan
(1/2*d*x + 1/2*c)^2 - 1)^2*b^3) + 4*(a^2*b*tan(1/2*d*x + 1/2*c) + b^3*tan(
1/2*d*x + 1/2*c) + a^3 + a*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d
*x + 1/2*c) - a)*a*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 585, normalized size of antiderivative = 3.18

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^5*(a + b*tan(c + d*x))^2),x)
```

output
```
(atanh((648*a^3*tan(c/2 + (d*x)/2))/(216*a*b^2 + 648*a^3 + (432*a^5)/b^2)
+ (432*a^5*tan(c/2 + (d*x)/2))/(216*a*b^4 + 432*a^5 + 648*a^3*b^2) + (216*
a*tan(c/2 + (d*x)/2))/(216*a + (648*a^3)/b^2 + (432*a^5)/b^4))*(6*a^2 + 3*
b^2))/(b^4*d) - ((2*(3*a^2 + b^2))/b^3 + (6*a^2*tan(c/2 + (d*x)/2)^4)/b^3
- (6*tan(c/2 + (d*x)/2)^2*(2*a^2 + b^2))/b^3 + (tan(c/2 + (d*x)/2)*(9*a^2
+ 2*b^2))/(a*b^2) - (4*tan(c/2 + (d*x)/2)^3*(3*a^2 + b^2))/(a*b^2) + (tan(
c/2 + (d*x)/2)^5*(3*a^2 + 2*b^2))/(a*b^2))/(d*(a + 2*b*tan(c/2 + (d*x)/2)
- 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 - a*tan(c/2 + (d*x)/
2)^6 - 4*b*tan(c/2 + (d*x)/2)^3 + 2*b*tan(c/2 + (d*x)/2)^5)) - (6*a*atanh(
(432*a^3*(a^2 + b^2)^(1/2))/(432*a^3*b + (432*a^5)/b + 864*a^4*tan(c/2 + (
d*x)/2) + 864*a^2*b^2*tan(c/2 + (d*x)/2)) + (864*a^2*tan(c/2 + (d*x)/2)*(a
^2 + b^2)^(1/2))/(432*a^3 + (432*a^5)/b^2 + 864*a^2*b*tan(c/2 + (d*x)/2) +
 (864*a^4*tan(c/2 + (d*x)/2))/b) + (432*a^4*tan(c/2 + (d*x)/2)*(a^2 + b^2)
^(1/2))/(432*a^5 + 432*a^3*b^2 + 864*a^4*b*tan(c/2 + (d*x)/2) + 864*a^2*b^
3*tan(c/2 + (d*x)/2)))*(a^2 + b^2)^(1/2))/(b^4*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 2966, normalized size of antiderivative = 16.12

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x)
```

output

```
(12*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))
*cos(c + d*x)*sin(c + d*x)**4*tan(c + d*x)*a**2*b*i + 12*sqrt(a**2 + b**2)
*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c +
 d*x)**4*a**3*i - 24*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/s
qrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*tan(c + d*x)*a**2*b*i - 24*
sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos
(c + d*x)*sin(c + d*x)**2*a**3*i + 12*sqrt(a**2 + b**2)*atan((tan((c + d*x
)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*tan(c + d*x)*a**2*b*i + 12
*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*co
s(c + d*x)*a**3*i + 12*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)
/sqrt(a**2 + b**2))*sin(c + d*x)**5*tan(c + d*x)*a*b**2*i + 12*sqrt(a**2 +
 b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*sin(c + d*x)**
5*a**2*b*i - 24*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a
**2 + b**2))*sin(c + d*x)**3*tan(c + d*x)*a*b**2*i - 24*sqrt(a**2 + b**2)*
atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*sin(c + d*x)**3*a**2*
b*i + 12*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b
**2))*sin(c + d*x)*tan(c + d*x)*a*b**2*i + 12*sqrt(a**2 + b**2)*atan((tan(
(c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*sin(c + d*x)*a**2*b*i - 6*cos(c
 + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*tan(c + d*x)*a**3*b - 3*
cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*tan(c + d*x)*a*b...
```

3.570 $\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\text{arctanh}(\sin(c+dx))}{b^2 d} + \frac{a\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d} - \frac{\sec(c+dx)}{bd(a+b\tan(c+dx))}$$

output
```
arctanh(sin(d*x+c))/b^2/d+a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^
(1/2))/b^2/(a^2+b^2)^(1/2)/d-sec(d*x+c)/b/d/(a+b*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.32

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{2a\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}\right.\right.}{b^2 d}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

output
```
-(((2*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2
] + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] + (b*Sec[c + d*x])/(a + b*Tan[c + d*x]))/(b^2*d))
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3992, 492, 605, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^3}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{\sqrt{\tan^2(c+dx)+1}}{(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{492}$$

$$\frac{\sec(c+dx)\left(\frac{\int \frac{b\tan(c+dx)}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{b^2}-\frac{\sqrt{\tan^2(c+dx)+1}}{a+b\tan(c+dx)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{605}$$

$$\frac{\sec(c+dx)\left(\frac{\int \frac{1}{\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-a\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{b^2}-\frac{\sqrt{\tan^2(c+dx)+1}}{a+b\tan(c+dx)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{222}$$

$$\frac{\sec(c+dx)\left(\frac{b\text{arcsinh}(\tan(c+dx))-a\int\frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{b^2}-\frac{\sqrt{\tan^2(c+dx)+1}}{a+b\tan(c+dx)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{488}$$

$$\frac{\sec(c+dx)\left(\frac{a\int\frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}+b\text{arcsinh}(\tan(c+dx))}{b^2}-\frac{\sqrt{\tan^2(c+dx)+1}}{a+b\tan(c+dx)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\text{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+b\text{arcsinh}(\tan(c+dx))}{b^2}-\frac{\sqrt{\tan^2(c+dx)+1}}{a+b\tan(c+dx)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*((b*ArcSinh[Tan[c + d*x]] + (a*b*ArcTanh[(b^2*Tan[c + d*x])/
Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2])/b^2 - Sqrt[1 + Tan[c + d*x]^2]/(a + b*T
an[c + d*x])))/(b*d*Sqrt[Sec[c + d*x]^2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 605
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[1/d   Int[x^(m - 1)*(a + b*x^2)^p, x], x] - Simp[c/d   Int[x^(m - 1
)*((a + b*x^2)^p/(c + d*x)), x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m,
 0] && LtQ[-1, p, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 7.90 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{\frac{\frac{2\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b\right)}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}}{d}$
default	$\frac{\frac{\frac{2\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b\right)}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}}{d}$
risch	$-\frac{2\,\mathrm{e}^{i(dx+c)}}{db\left(-ib\,\mathrm{e}^{2i(dx+c)}+a\,\mathrm{e}^{2i(dx+c)}+ib+a\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d\,b^2}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d\,b^2}-\frac{\ln(\mathrm{e}^{i(dx+c)}-}{d\,b^2}$

input
```
int(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^2*((b^2/a*tan(1/2*d*x+1/2*c)+b)/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1
/2*d*x+1/2*c)-a)-a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b
)/(a^2+b^2)^(1/2)))+1/b^2*ln(tan(1/2*d*x+1/2*c)+1)-1/b^2*ln(tan(1/2*d*x+1/
2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(87) = 174$.

Time = 0.13 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.22

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{2\,a^2b+2\,b^3-\left(a^2\cos\left(dx+c\right)+ab\sin\left(dx+c\right)\right)\sqrt{a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-}\right.}{}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*b + 2*b^3 - (a^2*cos(d*x + c) + a*b*sin(d*x + c))*sqrt(a^2 + b
^2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*
a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*co
s(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - ((a^3 + a*b
^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))*log(sin(d*x + c) + 1) + ((a
^3 + a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))*log(-sin(d*x + c) +
 1))/((a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^2*b^3 + b^5)*d*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^3\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*tan(c + d*x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(87) = 174$.

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.33

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{2\left(a+\frac{b\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2b+\frac{2ab^2\sin(dx+c)}{\cos(dx+c)+1}-\frac{a^2b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}} - \frac{a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^2} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^2} + \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^2}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(2*(a + b*sin(d*x + c)/(cos(d*x + c) + 1))/(a^2*b + 2*a*b^2*sin(d*x + c)/
(cos(d*x + c) + 1) - a^2*b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) - a*log((b
 - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c
)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) - log(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)/b^2 + log(sin(d*x + c)/(cos(d*x + c) + 1)
- 1)/b^2)/d
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.82

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^2} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2} + \frac{2\,\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
(a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + log(
abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^
2 + 2*(b*tan(1/2*d*x + 1/2*c) + a)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/
2*d*x + 1/2*c) - a)*a*b))/d
```

Mupad [B] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 383, normalized size of antiderivative = 4.21

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{b^2\,\sin\left(c+d\,x\right) - \frac{2\,\left(a^2\,\cos(c+d\,x)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sqrt{a^2+b^2}+a^3\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+1\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+2\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}+2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}}\right)\right.}{\sqrt{a^2+b^2}}}{a\,b^2}$$

input
```
int(1/(cos(c + d*x)^3*(a + b*tan(c + d*x))^2),x)
```

output
```
-(b^2*sin(c + d*x) - (2*(a^3*atan((a^2*sin(c/2 + (d*x)/2)*1i + b^2*sin(c/2
 + (d*x)/2)*2i + a*b*cos(c/2 + (d*x)/2)*1i)/(a*cos(c/2 + (d*x)/2)*(a^2 + b
^2)^(1/2) + 2*b*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)))*cos(c + d*x)*1i + a
^2*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*(a^2 + b^2)^(
1/2)))/(a^2 + b^2)^(1/2) + (2*b*((a*(a^2 + b^2)^(1/2))/2 + (a*cos(c + d*x)
*(a^2 + b^2)^(1/2))/2 - a^2*atan((a^2*sin(c/2 + (d*x)/2)*1i + b^2*sin(c/2
+ (d*x)/2)*2i + a*b*cos(c/2 + (d*x)/2)*1i)/(a*cos(c/2 + (d*x)/2)*(a^2 + b^
2)^(1/2) + 2*b*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)))*sin(c + d*x)*1i - a*
sin(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*(a^2 + b^2)^(1/2
)))/(a^2 + b^2)^(1/2))/(a*b^2*d*(a*cos(c + d*x) + b*sin(c + d*x)))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1912, normalized size of antiderivative = 21.01

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x)
```

output
```
(2*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*
cos(c + d*x)*sin(c + d*x)**2*tan(c + d*x)*a**2*b*i + 2*sqrt(a**2 + b**2)*a
tan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d
*x)**2*a**3*i - 2*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt
(a**2 + b**2))*cos(c + d*x)*tan(c + d*x)*a**2*b*i - 2*sqrt(a**2 + b**2)*at
an((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*a**3*i + 2
*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*si
n(c + d*x)**3*tan(c + d*x)*a*b**2*i + 2*sqrt(a**2 + b**2)*atan((tan((c + d
*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*sin(c + d*x)**3*a**2*b*i - 2*sqrt(a**
2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*sin(c + d*x
)*tan(c + d*x)*a*b**2*i - 2*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i -
 b*i)/sqrt(a**2 + b**2))*sin(c + d*x)*a**2*b*i - cos(c + d*x)*log(tan((c +
 d*x)/2) - 1)*sin(c + d*x)**2*tan(c + d*x)*a**3*b - cos(c + d*x)*log(tan((
c + d*x)/2) - 1)*sin(c + d*x)**2*tan(c + d*x)*a*b**3 - cos(c + d*x)*log(ta
n((c + d*x)/2) - 1)*sin(c + d*x)**2*a**4 - cos(c + d*x)*log(tan((c + d*x)/
2) - 1)*sin(c + d*x)**2*a**2*b**2 + cos(c + d*x)*log(tan((c + d*x)/2) - 1)
*tan(c + d*x)*a**3*b + cos(c + d*x)*log(tan((c + d*x)/2) - 1)*tan(c + d*x)
*a*b**3 + cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a**4 + cos(c + d*x)*log(t
an((c + d*x)/2) - 1)*a**2*b**2 + cos(c + d*x)*log(tan((c + d*x)/2) + 1)*si
n(c + d*x)**2*tan(c + d*x)*a**3*b + cos(c + d*x)*log(tan((c + d*x)/2) +...
```

3.571 $\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{b\sec(c+dx)}{(a^2+b^2)\,d(a+b\tan(c+dx))}$$

output
```
-a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d-
b*sec(d*x+c)/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{2a\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}} - \frac{b\sec(c+dx)}{(a^2+b^2)(a+b\tan(c+dx))}}{d}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

output
```
((2*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2
) - (b*Sec[c + d*x])/((a^2 + b^2)*(a + b*Tan[c + d*x])))/d
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3992, 491, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{1}{(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{491}$$

$$\frac{\sec(c+dx)\left(\frac{a\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}-\frac{b^2\sqrt{\tan^2(c+dx)+1}}{(a^2+b^2)(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{488}$$

$$\frac{\sec(c+dx)\left(-\frac{a\int \frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}}{a^2+b^2}-\frac{b^2\sqrt{\tan^2(c+dx)+1}}{(a^2+b^2)(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sec(c+dx)\left(-\frac{ab\operatorname{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{b^2\sqrt{\tan^2(c+dx)+1}}{(a^2+b^2)(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*(-((a*b*ArcTanh[(b^2*Tan[c + d*x])/Sqrt[a^2 + b^2]])/(a^2 +
b^2)^(3/2)) - (b^2*Sqrt[1 + Tan[c + d*x]^2])/((a^2 + b^2)*(a + b*Tan[c + d
*x]))))/(b*d*Sqrt[Sec[c + d*x]^2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 491
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b*(c/(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(a^2+b^2\right)}-\frac{b}{a^2+b^2}\right)}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$
default	$\frac{-\frac{2\left(-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(a^2+b^2\right)}-\frac{b}{a^2+b^2}\right)}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$
risch	$-\frac{2ib\,\mathrm{e}^{i(dx+c)}}{(-ia+b)d(ia+b)\left(b\,\mathrm{e}^{2i(dx+c)}+ia\,\mathrm{e}^{2i(dx+c)}-b+ia\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^3+ia\,b^2-a^2b-b^3}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^3\cdots}{}\right)}{\left(a^2+b^2\cdots\right)}$

input
```
int(sec(d*x+c)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-b^2/a/(a^2+b^2)*tan(1/2*d*x+1/2*c)-b/(a^2+b^2))/(a*tan(1/2*d*x+1
/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)+2*a/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan
(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(78) = 156$.

Time = 0.09 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.62

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{2\,a^2b+2\,b^3-\left(a^2\cos\left(dx+c\right)+ab\sin\left(dx+c\right)\right)\sqrt{a^2+b^2}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\cdots}{2\,ab\cos(dx+c)\sin(dx+c)+\cdots}\right.}{2\left(\left(a^5+2\,a^3b^2+ab^4\right)d\cos\left(dx+c\right)+\left(a^4b+2\,a^2b^3+b^5\right)d\sin\left(dx+c\cdots\right.\right.}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*b + 2*b^3 - (a^2*cos(d*x + c) + a*b*sin(d*x + c))*sqrt(a^2 + b
^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2
*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*c
os(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)))/((a^5 + 2*a
^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)/(a + b*tan(c + d*x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(78) = 156.

Time = 0.12 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.22

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{2\left(ab+\frac{b^2\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+a^2b^2+\frac{2\left(a^3b+ab^3\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{\left(a^4+a^2b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*s
in(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(
a*b + b^2*sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + a^2*b^2 + 2*(a^3*b + a*b
^3)*sin(d*x + c)/(cos(d*x + c) + 1) - (a^4 + a^2*b^2)*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2))/d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.68

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+ab\right)}{(a^3+ab^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}}{d}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-(a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*ta
n(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b^2*
tan(1/2*d*x + 1/2*c) + a*b)/((a^3 + a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b
*tan(1/2*d*x + 1/2*c) - a)))/d
```

Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.66

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{2\,b}{a^2+b^2}+\frac{2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,(a^2+b^2)}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)} + \frac{a\,\mathrm{atan}\left(\frac{a^2\,b\,1\mathrm{i}+b^3\,1\mathrm{i}-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2+b^2)\,1\mathrm{i}}{(a^2+b^2)^{3/2}}\right)\,2\mathrm{i}}{d\,(a^2+b^2)^{3/2}}$$

input
```
int(1/(cos(c + d*x)*(a + b*tan(c + d*x))^2),x)
```

output
```
(a*atan((a^2*b*1i + b^3*1i - a*tan(c/2 + (d*x)/2)*(a^2 + b^2)*1i)/(a^2 + b
^2)^(3/2))*2i)/(d*(a^2 + b^2)^(3/2)) - ((2*b)/(a^2 + b^2) + (2*b^2*tan(c/2
 + (d*x)/2))/(a*(a^2 + b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 +
 (d*x)/2)^2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.27

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{-2\sqrt{a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)ai-bi}{\sqrt{a^2+b^2}}\right)\cos(dx+c)\,a^2i-2\sqrt{a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)ai-bi}{\sqrt{a^2+b^2}}\right)\sin(dx+c)\,abi-}{d\left(\cos(dx+c)\,a^5+2\cos(dx+c)\,a^3b^2+\cos(dx+c)\,a\,b^4+\sin(dx+c)\,a^4b+2\sin(dx+c)\,a^2b^3+\sin(d\right.}$$

input
```
int(sec(d*x+c)/(a+b*tan(d*x+c))^2,x)
```

output
```
( - 2*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2
))*cos(c + d*x)*a**2*i - 2*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i -
b*i)/sqrt(a**2 + b**2))*sin(c + d*x)*a*b*i - a**2*b - b**3)/(d*(cos(c + d*
x)*a**5 + 2*cos(c + d*x)*a**3*b**2 + cos(c + d*x)*a*b**4 + sin(c + d*x)*a*
*4*b + 2*sin(c + d*x)*a**2*b**3 + sin(c + d*x)*b**5))
```

3.572 $\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 157

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{3ab^2\text{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{(a^2+b^2)^{5/2}\,d\sqrt{\sec^2(c+dx)}} + \frac{b(a^2-2b^2)\sec(c+dx)}{(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))}$$

output
```
-3*a*b^2*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)^(1/2))*se
c(d*x+c)/(a^2+b^2)^(5/2)/d/(sec(d*x+c)^2)^(1/2)+b*(a^2-2*b^2)*sec(d*x+c)/(
a^2+b^2)^2/d/(a+b*tan(d*x+c))+cos(d*x+c)*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b
*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.97

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\sec(c+dx)\left(12ab^2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)(a\cos(c+dx)+b\sin(c+dx))+(a^2+b^2)\left(3b(a^2\right.\right.}{2\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*(12*a*b^2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/
Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x]) + (a^2 + b^2)*(3*b*(a^2
 - b^2) + b*(a^2 + b^2)*Cos[2*(c + d*x)] + a*(a^2 + b^2)*Sin[2*(c + d*x)])
))/(2*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3992, 496, 25, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{1}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)+1\right)^{3/2}}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{496}$$

$$\frac{\sec(c+dx)\left(\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}-\frac{b^2\int -\frac{2b^2+a\tan(c+dx)b}{b^2(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sec(c+dx)\left(\frac{b^2\int \frac{2b^2+a\tan(c+dx)b}{b^2(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}+\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sec(c+dx)\left(\frac{\int \frac{2b^2+a\tan(c+dx)b}{(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}+\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \ 679$$

$$\frac{\sec(c+dx)\left(\frac{\frac{3ab^2\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}+\frac{b^2\left(a^2-2b^2\right)\sqrt{\tan^2(c+dx)+1}}{\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}+\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \ 488$$

$$\frac{\sec(c+dx)\left(\frac{\frac{b^2\left(a^2-2b^2\right)\sqrt{\tan^2(c+dx)+1}}{\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{3ab^2\int \frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}}{a^2+b^2}}{a^2+b^2}+\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \ 219$$

$$\frac{\sec(c+dx)\left(\frac{\frac{b^2(a^2-2b^2)\sqrt{\tan^2(c+dx)+1}}{(a^2+b^2)(a+b\tan(c+dx))}-\frac{3ab^3\operatorname{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}}{a^2+b^2}+\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*((b^2 + a*b*Tan[c + d*x])/((a^2 + b^2)*(a + b*Tan[c + d*x])*
Sqrt[1 + Tan[c + d*x]^2]) + ((-3*a*b^3*ArcTanh[(b^2*Tan[c + d*x])/Sqrt[a^2
 + b^2]])/(a^2 + b^2)^(3/2) + (b^2*(a^2 - 2*b^2)*Sqrt[1 + Tan[c + d*x]^2])
/((a^2 + b^2)*(a + b*Tan[c + d*x])))/(a^2 + b^2)))/(b*d*Sqrt[Sec[c + d*x]^
2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{-\frac{2b^2\left(\frac{-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{3a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2}-\frac{2\left(\left(-a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2ab\right)}{\left(a^4+2b^2a^2+b^4\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}{d}$
default	$\frac{-\frac{2b^2\left(\frac{-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{3a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2}-\frac{2\left(\left(-a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2ab\right)}{\left(a^4+2b^2a^2+b^4\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}{d}$
risch	$-\frac{ie^{i(dx+c)}}{2(-2iab+a^2-b^2)d}+\frac{ie^{-i(dx+c)}}{2(2iab+a^2-b^2)d}-\frac{2ib^3e^{i(dx+c)}}{(-ia+b)^2d(ia+b)^2\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)}+\frac{3b^2a\ln\left(e^{i(dx}\right.}{}$

input
```
int(cos(d*x+c)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b^2/(a^2+b^2)^2*((-b^2/a*tan(1/2*d*x+1/2*c)-b)/(a*tan(1/2*d*x+1/2*
c)^2-2*b*tan(1/2*d*x+1/2*c)-a)-3*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/
2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))-2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tan(
1/2*d*x+1/2*c)-2*a*b)/(1+tan(1/2*d*x+1/2*c)^2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.92

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$=\frac{2\,a^4b-2\,a^2b^3-4\,b^5+2\,(a^4b+2\,a^2b^3+b^5)\cos{(dx+c)}^2+2\,(a^5+2\,a^3b^2+ab^4)\cos{(dx+c)}\sin{(dx+c)}}{2\,((a^7+3\,a^5b^2+3\,a^3b^4+ab^6)d}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(2*a^4*b - 2*a^2*b^3 - 4*b^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c
)^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*(a^2*b^2*c
os(d*x + c) + a*b^3*sin(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)
*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^
2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (
a^2 - b^2)*cos(d*x + c)^2 + b^2))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*
d*cos(d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\cos\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(cos(c + d*x)/(a + b*tan(c + d*x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(151) = 302$.

Time = 0.13 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.22

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{3\,ab^2\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(2\,a^3b-ab^3-\frac{3\,ab^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\left(a^4+3\,a^2b^2-b^4\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{\left(a^4-a^2b^2+b^4\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right.}{a^6+2\,a^4b^2+a^2b^4+\frac{2\left(a^5b+2\,a^3b^3+ab^5\right)\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\left(a^5b+2\,a^3b^3+ab^5\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{\left(a^6+2\,a^4b^2+a^2b\right.}{(\cos(dx+c}}}{d}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(3*a*b^2*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b
 - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2
 + b^4)*sqrt(a^2 + b^2)) - 2*(2*a^3*b - a*b^3 - 3*a*b^3*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + (a^4 + 3*a^2*b^2 - b^4)*sin(d*x + c)/(cos(d*x + c) + 1
) - (a^4 - a^2*b^2 + b^4)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^6 + 2*a^
4*b^2 + a^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(d*x + c)/(cos(d*x + c)
 + 1) + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
- (a^6 + 2*a^4*b^2 + a^2*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4))/d
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.82

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{3\,ab^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}{(a^5+2\,a^3b^2+ab^4)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,b\tan\left(\frac{1}{2}\right.\right.}}{d}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-(3*a*b^2*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(
2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b
^4)*sqrt(a^2 + b^2)) - 2*(a^4*tan(1/2*d*x + 1/2*c)^3 - a^2*b^2*tan(1/2*d*x
 + 1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^3*tan(1/2*d*x + 1/2*c)^2
- a^4*tan(1/2*d*x + 1/2*c) - 3*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*
d*x + 1/2*c) - 2*a^3*b + a*b^3)/((a^5 + 2*a^3*b^2 + a*b^4)*(a*tan(1/2*d*x
+ 1/2*c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)))/
d
```

Mupad [B] (verification not implemented)

Time = 3.09 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.82

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{4\,a^2\,b-2\,b^3}{a^4+2\,a^2\,b^2+b^4}-\frac{6\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^4+2\,a^2\,b^2+b^4}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^4+3\,a^2\,b^2-b^4)}{a\,(a^4+2\,a^2\,b^2+b^4)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(2\,a^4-2\,a^2\,b^2+2\,b^4)}{a\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}$$
$$-\frac{6\,a\,b^2\,\mathrm{atanh}\left(\frac{a^4\,b+b^5+2\,a^2\,b^3-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^4+2\,a^2\,b^2+b^4)}{(a^2+b^2)^{5/2}}\right)}{d\,(a^2+b^2)^{5/2}}$$

input
```
int(cos(c + d*x)/(a + b*tan(c + d*x))^2,x)
```

output
```
((4*a^2*b - 2*b^3)/(a^4 + b^4 + 2*a^2*b^2) - (6*b^3*tan(c/2 + (d*x)/2)^2)/
(a^4 + b^4 + 2*a^2*b^2) + (2*tan(c/2 + (d*x)/2)*(a^4 - b^4 + 3*a^2*b^2))/(
a*(a^4 + b^4 + 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^3*(2*a^4 + 2*b^4 - 2*a^2*
b^2))/(a*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(
c/2 + (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^3)) - (6*a*b^2*atanh((a^4*b + b^
5 + 2*a^2*b^3 - a*tan(c/2 + (d*x)/2)*(a^4 + b^4 + 2*a^2*b^2))/(a^2 + b^2)^
(5/2)))/(d*(a^2 + b^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.57

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{-6\sqrt{a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)ai-bi}{\sqrt{a^2+b^2}}\right)\cos(dx+c)\,a^2b^3i-6\sqrt{a^2+b^2}\,atan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)ai-bi}{\sqrt{a^2+b^2}}\right)\sin(dx+c)\,a\,b^4}{}$$

input
```
int(cos(d*x+c)/(a+b*tan(d*x+c))^2,x)
```

output
```
( - 6*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2
))*cos(c + d*x)*a**2*b**3*i - 6*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a
*i - b*i)/sqrt(a**2 + b**2))*sin(c + d*x)*a*b**4*i + cos(c + d*x)*sin(c +
d*x)*a**5*b + 2*cos(c + d*x)*sin(c + d*x)*a**3*b**3 + cos(c + d*x)*sin(c +
 d*x)*a*b**5 + cos(c + d*x)*a**6 + 2*cos(c + d*x)*a**4*b**2 + cos(c + d*x)
*a**2*b**4 - sin(c + d*x)**2*a**4*b**2 - 2*sin(c + d*x)**2*a**2*b**4 - sin
(c + d*x)**2*b**6 + sin(c + d*x)*a**5*b + 2*sin(c + d*x)*a**3*b**3 + sin(c
 + d*x)*a*b**5 + 2*a**4*b**2 + a**2*b**4 - b**6)/(b*d*(cos(c + d*x)*a**7 +
 3*cos(c + d*x)*a**5*b**2 + 3*cos(c + d*x)*a**3*b**4 + cos(c + d*x)*a*b**6
 + sin(c + d*x)*a**6*b + 3*sin(c + d*x)*a**4*b**3 + 3*sin(c + d*x)*a**2*b*
*5 + sin(c + d*x)*b**7))
```

3.573 $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 241

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{5ab^4\operatorname{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{(a^2+b^2)^{7/2}\,d\sqrt{\sec^2(c+dx)}} + \frac{b(2a^4+9a^2b^2-8b^4)\sec(c+dx)}{3(a^2+b^2)^3\,d(a+b\tan(c+dx))} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{\cos(c+dx)\left(b(a^2-4b^2)-a(2a^2+7b^2)\tan(c+dx)\right)}{3(a^2+b^2)^2\,d(a+b\tan(c+dx))}$$

output
```
-5*a*b^4*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)^(1/2))*se
c(d*x+c)/(a^2+b^2)^(7/2)/d/(sec(d*x+c)^2)^(1/2)+1/3*b*(2*a^4+9*a^2*b^2-8*b
^4)*sec(d*x+c)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))+1/3*cos(d*x+c)^3*(b+a*tan(d*
x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))-1/3*cos(d*x+c)*(b*(a^2-4*b^2)-a*(2*a^2+
7*b^2)*tan(d*x+c))/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.03

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\sec(c+dx)\left(240ab^4\sqrt{a^2+b^2}\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)(a\cos(c+dx)+b\sin(c+dx))+(a^2+b^2)\left(15a\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*(240*a*b^4*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])
/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x]) + (a^2 + b^2)*(15*a^4*
b + 90*a^2*b^3 - 45*b^5 + 20*b^3*(a^2 + b^2)*Cos[2*(c + d*x)] + b*(a^2 + b
^2)^2*Cos[4*(c + d*x)] + 10*a^5*Sin[2*(c + d*x)] + 40*a^3*b^2*Sin[2*(c + d
*x)] + 30*a*b^4*Sin[2*(c + d*x)] + a^5*Sin[4*(c + d*x)] + 2*a^3*b^2*Sin[4*
(c + d*x)] + a*b^4*Sin[4*(c + d*x)])))/(24*(a^2 + b^2)^4*d*(a + b*Tan[c +
d*x]))
```

Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3992, 496, 25, 27, 686, 25, 25, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sec(c+dx)^3(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{ 3992}$$

$$\frac{\sec(c+dx)\int \frac{1}{(a+b\tan(c+dx))^2(\tan^2(c+dx)+1)^{5/2}}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{ 496}$$

$$\frac{\sec(c+dx)\left(\frac{ab\tan(c+dx)+b^2}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}(a+b\tan(c+dx))}-\frac{b^2\int -\frac{2\left(\frac{a^2}{b^2}+2\right)b^2+3a\tan(c+dx)b}{b^2(a+b\tan(c+dx))^2\left(\tan^2(c+dx)+1\right)^{3/2}}d(b\tan(c+dx))}{3(a^2+b^2)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sec(c+dx)\left(\frac{b^2\int \frac{2\left(a^2+2b^2\right)+3ab\tan(c+dx)}{b^2(a+b\tan(c+dx))^2\left(\tan^2(c+dx)+1\right)^{3/2}}d(b\tan(c+dx))}{3(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sec(c+dx)\left(\frac{\int \frac{2\left(a^2+2b^2\right)+3ab\tan(c+dx)}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)+1\right)^{3/2}}d(b\tan(c+dx))}{3(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{ 686}$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+7b^2\right)\tan(c+dx)+b^4\left(4-\frac{a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}-\frac{b^4\int -\frac{2\left(4-\frac{a^2}{b^2}\right)b^4+a\left(2a^2+7b^2\right)\tan(c+dx)b}{b^4(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}}{3(a^2+b^2)}+\frac{ab\tan(c+da}{3(a^2+b^2)(\tan^2(c+dx)+1)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sec(c+dx)\left(\frac{\frac{b^4\int -\frac{2b^2\left(a^2-4b^2\right)-ab\left(2a^2+7b^2\right)\tan(c+dx)}{b^4(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}+\frac{ab\left(2a^2+7b^2\right)\tan(c+dx)+b^4\left(4-\frac{a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}}{3(a^2+b^2)}+\frac{ab\tan(c+d}{3(a^2+b^2)(\tan^2(c+dx)+1)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+7b^2\right)\tan(c+dx)+b^4\left(4-\frac{a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}-\frac{b^4\int \frac{2b^2\left(a^2-4b^2\right)-ab\left(2a^2+7b^2\right)\tan(c+dx)}{b^4(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}}{3(a^2+b^2)}+\frac{ab\tan(c+dx}{3(a^2+b^2)(\tan^2(c+dx)+1)^3}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+7b^2\right)\tan(c+dx)+b^4\left(4-\frac{a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}-\frac{\int \frac{2b^2\left(a^2-4b^2\right)-ab\left(2a^2+7b^2\right)\tan(c+dx)}{(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}}{3(a^2+b^2)}+\frac{ab\tan(c+dx)+}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 679$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+7b^2\right)\tan(c+dx)+b^4\left(4-\frac{a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}-\frac{-\frac{15ab^4\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}-\frac{b^2\left(2a^4+9a^2b^2-8b^4\right)\sqrt{\tan^2(c+dx}}{\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}}{3(a^2+b^2)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 488$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab(2a^2+7b^2)\tan(c+dx)+b^4\left(4-\frac{a^2}{b^2}\right)}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}-\frac{\frac{15ab^4\int\frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}}{a^2+b^2}-\frac{b^2(2a^4+9a^2b^2-8b^4)\sqrt{\tan^2(c+dx)+1}}{(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}}{3(a^2+b^2)}+\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 219$$

$$\frac{\sec(c+dx)\left(\frac{ab\tan(c+dx)+b^2}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}(a+b\tan(c+dx))}+\frac{\frac{ab(2a^2+7b^2)\tan(c+dx)+b^4\left(4-\frac{a^2}{b^2}\right)}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}-\frac{\frac{15ab^5\text{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{b^2}{}}{a^2+}}{3(a^2+b^2)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*((b^2 + a*b*Tan[c + d*x])/(3*(a^2 + b^2)*(a + b*Tan[c + d*x]
)*(1 + Tan[c + d*x]^2)^(3/2)) + (((4 - a^2/b^2)*b^4 + a*b*(2*a^2 + 7*b^2)*
Tan[c + d*x])/((a^2 + b^2)*(a + b*Tan[c + d*x])*Sqrt[1 + Tan[c + d*x]^2])
- ((15*a*b^5*ArcTanh[(b^2*Tan[c + d*x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2
) - (b^2*(2*a^4 + 9*a^2*b^2 - 8*b^4)*Sqrt[1 + Tan[c + d*x]^2])/((a^2 + b^2
)*(a + b*Tan[c + d*x])))/(a^2 + b^2))/(3*(a^2 + b^2))))/(b*d*Sqrt[Sec[c +
d*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 8.43 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{-\frac{2b^4\left(\frac{-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{5a\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^4+2b^2a^2+b^4\right)\left(a^2+b^2\right)}-\frac{2\left(\left(-a^4-3b^2a^2+2b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-2a^3\right.\right.}{}}{}$
default	$\frac{-\frac{2b^4\left(\frac{-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{5a\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^4+2b^2a^2+b^4\right)\left(a^2+b^2\right)}-\frac{2\left(\left(-a^4-3b^2a^2+2b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-2a^3\right.\right.}{}}{}$
risch	$-\frac{i e^{3i(dx+c)}}{24(-2iab+a^2-b^2)d}-\frac{7\,e^{i(dx+c)}b}{8(-3ia^2b+ib^3+a^3-3a\,b^2)d}-\frac{3i e^{i(dx+c)}a}{8(-3ia^2b+ib^3+a^3-3a\,b^2)d}-\frac{7\,e^{-i(dx+c)}b}{8(ib+a)^3d}+\frac{3ie^{-i(dx}}{8(ib+a)}$

input
```
int(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-b^2/a*tan(1/2*d*x+1/2*c)-b)/(
a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)-5*a/(a^2+b^2)^(1/2)*arcta
nh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))-2/(a^2+b^2)/(a^4+2*a
^2*b^2+b^4)*((-a^4-3*a^2*b^2+2*b^4)*tan(1/2*d*x+1/2*c)^5+(-2*a^3*b-6*a*b^3
)*tan(1/2*d*x+1/2*c)^4+(-2/3*a^4-6*b^2*a^2+8/3*b^4)*tan(1/2*d*x+1/2*c)^3-8
*a*b^3*tan(1/2*d*x+1/2*c)^2+(-a^4-3*a^2*b^2+2*b^4)*tan(1/2*d*x+1/2*c)-2/3*
a^3*b-14/3*a*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^3)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.73

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{4\,a^6b + 22\,a^4b^3 + 2\,a^2b^5 - 16\,b^7 + 2\,(a^6b + 3\,a^4b^3 + 3\,a^2b^5 + b^7)\cos\left(dx+c\right)^4 - 2\,(a^6b - 2\,a^4b^3 - 7\,a^2b^5 -}{}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(4*a^6*b + 22*a^4*b^3 + 2*a^2*b^5 - 16*b^7 + 2*(a^6*b + 3*a^4*b^3 + 3*
a^2*b^5 + b^7)*cos(d*x + c)^4 - 2*(a^6*b - 2*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*
cos(d*x + c)^2 + 15*(a^2*b^4*cos(d*x + c) + a*b^5*sin(d*x + c))*sqrt(a^2 +
 b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 -
 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b
*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*((a^7
+ 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (2*a^7 + 11*a^5*b^2 + 16
*a^3*b^4 + 7*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^9 + 4*a^7*b^2 + 6*a^5*
b^4 + 4*a^3*b^6 + a*b^8)*d*cos(d*x + c) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 +
 4*a^2*b^7 + b^9)*d*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+b*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(229) = 458$.

Time = 0.17 (sec) , antiderivative size = 772, normalized size of antiderivative = 3.20

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/3*(15*a*b^4*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2
))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(2*a^5*b + 14*a^3*b^3 - 3*a*
b^5 - 15*a*b^5*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + (3*a^6 + 13*a^4*b^2 +
 22*a^2*b^4 - 3*b^6)*sin(d*x + c)/(cos(d*x + c) + 1) + (4*a^5*b + 28*a^3*b
^3 - 21*a*b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (a^6 - 9*a^4*b^2 - 46
*a^2*b^4 + 9*b^6)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*(2*a^5*b + 6*a^3
*b^3 - 5*a*b^5)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^6 + 3*a^4*b^2 + 3
8*a^2*b^4 - 9*b^6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*(a^6 + 3*a^4*b^
2 - 2*a^2*b^4 + b^6)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(a^8 + 3*a^6*b^2
 + 3*a^4*b^4 + a^2*b^6 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*sin(d*x
 + c)/(cos(d*x + c) + 1) + 2*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 6*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 +
a*b^7)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2*(a^8 + 3*a^6*b^2 + 3*a^4*b^
4 + a^2*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 2*(a^7*b + 3*a^5*b^3 +
3*a^3*b^5 + a*b^7)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - (a^8 + 3*a^6*b^2
+ 3*a^4*b^4 + a^2*b^6)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8))/d
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.82

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{\frac{15\,ab^4\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{6\,(b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+ab^5)}{(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)} - \frac{2\left(3\,a^4\tan\left(\frac{1}{2}\right.\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/3*(15*a*b^4*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))
/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 6*(b^6*tan(1/2*d*x + 1/2*c) + a*b^
5)/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*
tan(1/2*d*x + 1/2*c) - a)) - 2*(3*a^4*tan(1/2*d*x + 1/2*c)^5 + 9*a^2*b^2*t
an(1/2*d*x + 1/2*c)^5 - 6*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*a^3*b*tan(1/2*d*x
 + 1/2*c)^4 + 18*a*b^3*tan(1/2*d*x + 1/2*c)^4 + 2*a^4*tan(1/2*d*x + 1/2*c)
^3 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 8*b^4*tan(1/2*d*x + 1/2*c)^3 + 24
*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^4*tan(1/2*d*x + 1/2*c) + 9*a^2*b^2*tan
(1/2*d*x + 1/2*c) - 6*b^4*tan(1/2*d*x + 1/2*c) + 2*a^3*b + 14*a*b^3)/((a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 674, normalized size of antiderivative = 2.80

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{\frac{2\,(2\,a^4\,b+14\,a^2\,b^3-3\,b^5)}{3\,(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)} - \frac{10\,b^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6} + \frac{10\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(2\,a^4+6\,a^2\,b^2-5\,b^4)}{3\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)} + \frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(4\,a^4\,b+28\,a^2\,b^3-21\,b^5)}{3\,(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-2\,a\tan\left(\frac{c}{2}\right.\right.}$$
$$- \frac{10\,a\,b^4\,\mathrm{atanh}\left(\frac{2\,a^6\,b+2\,b^7+6\,a^2\,b^5+6\,a^4\,b^3-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}{2\,(a^2+b^2)^{7/2}}\right)}{d\,(a^2+b^2)^{7/2}}$$

input
```
int(cos(c + d*x)^3/(a + b*tan(c + d*x))^2,x)
```

output
```
((2*(2*a^4*b - 3*b^5 + 14*a^2*b^3))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)
) - (10*b^5*tan(c/2 + (d*x)/2)^6)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (1
0*b*tan(c/2 + (d*x)/2)^4*(2*a^4 - 5*b^4 + 6*a^2*b^2))/(3*(a^6 + b^6 + 3*a^
2*b^4 + 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^2*(4*a^4*b - 21*b^5 + 28*a^2*b
^3))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (2*tan(c/2 + (d*x)/2)^7*(a^
6 + b^6 - 2*a^2*b^4 + 3*a^4*b^2))/(a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
+ (2*tan(c/2 + (d*x)/2)*(3*a^6 - 3*b^6 + 22*a^2*b^4 + 13*a^4*b^2))/(3*a*(a
^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + (2*tan(c/2 + (d*x)/2)^5*(a^6 - 9*b^6
+ 38*a^2*b^4 + 3*a^4*b^2))/(3*a*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (2*
tan(c/2 + (d*x)/2)^3*(a^6 + 9*b^6 - 46*a^2*b^4 - 9*a^4*b^2))/(3*a*(a^2 + b
^2)*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 2*a*tan(c/2
 + (d*x)/2)^2 - 2*a*tan(c/2 + (d*x)/2)^6 - a*tan(c/2 + (d*x)/2)^8 + 6*b*ta
n(c/2 + (d*x)/2)^3 + 6*b*tan(c/2 + (d*x)/2)^5 + 2*b*tan(c/2 + (d*x)/2)^7))
 - (10*a*b^4*atanh((2*a^6*b + 2*b^7 + 6*a^2*b^5 + 6*a^4*b^3 - 2*a*tan(c/2
+ (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))/(2*(a^2 + b^2)^(7/2))))/(d
*(a^2 + b^2)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.71

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{-3\cos(dx+c)\sin(dx+c)^3 a^3b^5 + 3\cos(dx+c)\sin(dx+c)a^7b + 14\cos(dx+c)\sin(dx+c)a^5b^3 + 19}{}$$

input
```
int(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x)
```

output
```
( - 30*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**
2))*cos(c + d*x)*a**2*b**5*i - 30*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)
*a*i - b*i)/sqrt(a**2 + b**2))*sin(c + d*x)*a*b**6*i - cos(c + d*x)*sin(c
+ d*x)**3*a**7*b - 3*cos(c + d*x)*sin(c + d*x)**3*a**5*b**3 - 3*cos(c + d*
x)*sin(c + d*x)**3*a**3*b**5 - cos(c + d*x)*sin(c + d*x)**3*a*b**7 + 3*cos
(c + d*x)*sin(c + d*x)*a**7*b + 14*cos(c + d*x)*sin(c + d*x)*a**5*b**3 + 1
9*cos(c + d*x)*sin(c + d*x)*a**3*b**5 + 8*cos(c + d*x)*sin(c + d*x)*a*b**7
 + 3*cos(c + d*x)*a**8 + 14*cos(c + d*x)*a**6*b**2 + 19*cos(c + d*x)*a**4*
b**4 + 8*cos(c + d*x)*a**2*b**6 + sin(c + d*x)**4*a**6*b**2 + 3*sin(c + d*
x)**4*a**4*b**4 + 3*sin(c + d*x)**4*a**2*b**6 + sin(c + d*x)**4*b**8 - sin
(c + d*x)**2*a**6*b**2 - 8*sin(c + d*x)**2*a**4*b**4 - 13*sin(c + d*x)**2*
a**2*b**6 - 6*sin(c + d*x)**2*b**8 + 3*sin(c + d*x)*a**7*b + 14*sin(c + d*
x)*a**5*b**3 + 19*sin(c + d*x)*a**3*b**5 + 8*sin(c + d*x)*a*b**7 + 2*a**6*
b**2 + 16*a**4*b**4 + 11*a**2*b**6 - 3*b**8)/(3*b*d*(cos(c + d*x)*a**9 + 4
*cos(c + d*x)*a**7*b**2 + 6*cos(c + d*x)*a**5*b**4 + 4*cos(c + d*x)*a**3*b
**6 + cos(c + d*x)*a*b**8 + sin(c + d*x)*a**8*b + 4*sin(c + d*x)*a**6*b**3
 + 6*sin(c + d*x)*a**4*b**5 + 4*sin(c + d*x)*a**2*b**7 + sin(c + d*x)*b**9
))
```

3.574 $\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 185

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{3(a^2+b^2)(5a^2+b^2)\log(a+b\tan(c+dx))}{b^7d} - \frac{a(10a^2+9b^2)\tan(c+dx)}{b^6d} + \frac{3(2a^2+b^2)\tan^2(c+dx)}{2b^5d} - \frac{a\tan^3(c+dx)}{b^4d} + \frac{\tan^4(c+dx)}{4b^3d} - \frac{(a^2+b^2)^3}{2b^7d(a+b\tan(c+dx))^2} + \frac{6a(a^2+b^2)^2}{b^7d(a+b\tan(c+dx))}$$

output
```
3*(a^2+b^2)*(5*a^2+b^2)*ln(a+b*tan(d*x+c))/b^7/d-a*(10*a^2+9*b^2)*tan(d*x+
c)/b^6/d+3/2*(2*a^2+b^2)*tan(d*x+c)^2/b^5/d-a*tan(d*x+c)^3/b^4/d+1/4*tan(d
*x+c)^4/b^3/d-1/2*(a^2+b^2)^3/b^7/d/(a+b*tan(d*x+c))^2+6*a*(a^2+b^2)^2/b^7
/d/(a+b*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.47

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{2(a^2+b^2)\left(19a^4+16a^2b^2-3b^4+6a^2(5a^2+b^2)\log(a+b\tan(c+dx))\right)+b^6\sec^6(c+dx)+4ab(4a^4+1}{}$$

input
```
Integrate[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^3,x]
```

output
```
(2*(a^2 + b^2)*(19*a^4 + 16*a^2*b^2 - 3*b^4 + 6*a^2*(5*a^2 + b^2)*Log[a +
b*Tan[c + d*x]]) + b^6*Sec[c + d*x]^6 + 4*a*b*(4*a^4 + 17*a^2*b^2 + 11*b^4
 + 6*(5*a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Tan[c + d*x]])*Tan[c + d*x] + 4*b
^2*(-13*a^4 - 10*a^2*b^2 + 3*(5*a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Tan[c + d
*x]])*Tan[c + d*x]^2 - 20*a*b^3*(a^2 + b^2)*Tan[c + d*x]^3 + 4*a^2*b^4*Tan
[c + d*x]^4 + b^4*Sec[c + d*x]^4*(a^2 + 3*b^2 - 2*a*b*Tan[c + d*x]))/(4*b^
7*d*(a + b*Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^8}{(a+b\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3987}$$
$$\frac{\int \frac{\left(\tan^2(c+dx)b^2+b^2\right)^3}{b^6(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(\tan^2(c+dx)b^2+b^2)^3}{(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{b^7 d}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(-10\left(\frac{9b^2}{10a^2}+1\right)a^3-3b^2\tan^2(c+dx)a-\frac{6(a^2+b^2)^2a}{(a+b\tan(c+dx))^2}+b^3\tan^3(c+dx)+3b(2a^2+b^2)\tan(c+dx)+\frac{3(5a}{a+}\right.}{b^7 d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3}{2}b^2(2a^2+b^2)\tan^2(c+dx)-ab(10a^2+9b^2)\tan(c+dx)+\frac{6a(a^2+b^2)^2}{a+b\tan(c+dx)}-\frac{(a^2+b^2)^3}{2(a+b\tan(c+dx))^2}+3(a^2+b^2)(5a^2+}{b^7 d}$$

input
```
Int[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^3,x]
```

output
```
(3*(a^2 + b^2)*(5*a^2 + b^2)*Log[a + b*Tan[c + d*x]] - a*b*(10*a^2 + 9*b^2
)*Tan[c + d*x] + (3*b^2*(2*a^2 + b^2)*Tan[c + d*x]^2)/2 - a*b^3*Tan[c + d*
x]^3 + (b^4*Tan[c + d*x]^4)/4 - (a^2 + b^2)^3/(2*(a + b*Tan[c + d*x])^2) +
 (6*a*(a^2 + b^2)^2)/(a + b*Tan[c + d*x]))/(b^7*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.05

$$\frac{\frac{\frac{\tan(dx+c)^4 b^3}{4}-\tan(dx+c)^3 a\,b^2+3\tan(dx+c)^2 a^2 b+\frac{3\tan(dx+c)^2 b^3}{2}-10a^3\tan(dx+c)-9\tan(dx+c)a\,b^2}{b^6}+\frac{(15a^4+18b^2a^2+3b^4)\ln(a+b\tan(dx+c))}{b^7}}{d}$$

input
```
int(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x)
```

output
```
1/d*(1/b^6*(1/4*tan(d*x+c)^4*b^3-tan(d*x+c)^3*a*b^2+3*tan(d*x+c)^2*a^2*b+3
/2*tan(d*x+c)^2*b^3-10*a^3*tan(d*x+c)-9*tan(d*x+c)*a*b^2)+(15*a^4+18*a^2*b
^2+3*b^4)/b^7*ln(a+b*tan(d*x+c))-1/2/b^7*(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a+
b*tan(d*x+c))^2+6*a/b^7*(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(179) = 358.

Time = 0.14 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.57

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{8\left(15\,a^4b^2+13\,a^2b^4\right)\cos\left(dx+c\right)^6+b^6-2\left(45\,a^4b^2+44\,a^2b^4+3\,b^6\right)\cos\left(dx+c\right)^4+\left(5\,a^2b^4+3\,b^6\right)\cos(\ldots}{\ldots}$$

input
```
integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(8*(15*a^4*b^2 + 13*a^2*b^4)*cos(d*x + c)^6 + b^6 - 2*(45*a^4*b^2 + 44
*a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + (5*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 + 6*
((5*a^6 + a^4*b^2 - 5*a^2*b^4 - b^6)*cos(d*x + c)^6 + 2*(5*a^5*b + 6*a^3*b
^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c) + (5*a^4*b^2 + 6*a^2*b^4 + b^6)*co
s(d*x + c)^4)*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x +
c)^2 + b^2) - 6*((5*a^6 + a^4*b^2 - 5*a^2*b^4 - b^6)*cos(d*x + c)^6 + 2*(5
*a^5*b + 6*a^3*b^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c) + (5*a^4*b^2 + 6*a
^2*b^4 + b^6)*cos(d*x + c)^4)*log(cos(d*x + c)^2) - 2*(a*b^5*cos(d*x + c)
+ 2*(15*a^5*b - 2*a^3*b^3 - 13*a*b^5)*cos(d*x + c)^5 + 10*(a^3*b^3 + a*b^5
)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^8*d*cos(d*x + c)^5*sin(d*x + c) + b
^9*d*cos(d*x + c)^4 + (a^2*b^7 - b^9)*d*cos(d*x + c)^6)
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^8{(c+dx)}}{(a+b\tan{(c+dx)})^3}\,dx$$

input
```
integrate(sec(d*x+c)**8/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**8/(a + b*tan(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{2\left(11a^6+21a^4b^2+9a^2b^4-b^6+12\left(a^5b+2a^3b^3+ab^5\right)\tan(dx+c)\right)}{b^9\tan(dx+c)^2+2ab^8\tan(dx+c)+a^2b^7} + \frac{b^3\tan(dx+c)^4-4ab^2\tan(dx+c)^3+6\left(2a^2b+b^3\right)\tan(dx+c)^2-4\left(10a^3+9\right.}{b^6}}{4d}$$

input
```
integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/4*(2*(11*a^6 + 21*a^4*b^2 + 9*a^2*b^4 - b^6 + 12*(a^5*b + 2*a^3*b^3 + a*
b^5)*tan(d*x + c))/(b^9*tan(d*x + c)^2 + 2*a*b^8*tan(d*x + c) + a^2*b^7) +
 (b^3*tan(d*x + c)^4 - 4*a*b^2*tan(d*x + c)^3 + 6*(2*a^2*b + b^3)*tan(d*x
+ c)^2 - 4*(10*a^3 + 9*a*b^2)*tan(d*x + c))/b^6 + 12*(5*a^4 + 6*a^2*b^2 +
b^4)*log(b*tan(d*x + c) + a)/b^7)/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.19

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{3\,(5\,a^4+6\,a^2b^2+b^4)\log\left(|b\tan\left(dx+c\right)+a|\right)}{b^7d} + \frac{11\,a^6+21\,a^4b^2+9\,a^2b^4-b^6+12\,(a^5b+2\,a^3b^3+ab^5)\tan\left(dx+c\right)}{2\,(b\tan\left(dx+c\right)+a)^2b^7d} + \frac{b^9d^3\tan\left(dx+c\right)^4-4\,ab^8d^3\tan\left(dx+c\right)^3+12\,a^2b^7d^3\tan\left(dx+c\right)^2+6\,b^9d^3\tan\left(dx+c\right)^2-40\,a^3b^6d^3}{4\,b^{12}d^4}$$

input
```
integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
3*(5*a^4 + 6*a^2*b^2 + b^4)*log(abs(b*tan(d*x + c) + a))/(b^7*d) + 1/2*(11
*a^6 + 21*a^4*b^2 + 9*a^2*b^4 - b^6 + 12*(a^5*b + 2*a^3*b^3 + a*b^5)*tan(d
*x + c))/((b*tan(d*x + c) + a)^2*b^7*d) + 1/4*(b^9*d^3*tan(d*x + c)^4 - 4*
a*b^8*d^3*tan(d*x + c)^3 + 12*a^2*b^7*d^3*tan(d*x + c)^2 + 6*b^9*d^3*tan(d
*x + c)^2 - 40*a^3*b^6*d^3*tan(d*x + c) - 36*a*b^8*d^3*tan(d*x + c))/(b^12
*d^4)
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.26

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{11\,a^6+21\,a^4\,b^2+9\,a^2\,b^4-b^6}{2\,b}+\tan(c+d\,x)\,\left(6\,a^5+12\,a^3\,b^2+6\,a\,b^4\right)}{d\,\left(a^2\,b^6+2\,a\,b^7\,\tan\left(c+d\,x\right)+b^8\,\tan\left(c+d\,x\right)^2\right)} + \frac{\tan(c+d\,x)^2\,\left(\frac{3}{2\,b^3}+\frac{3\,a^2}{b^5}\right)}{d}+\frac{\tan(c+d\,x)^4}{4\,b^3\,d} + \frac{\tan(c+d\,x)\,\left(\frac{8\,a^3}{b^6}-\frac{3\,a\,\left(\frac{3}{b^3}+\frac{6\,a^2}{b^5}\right)}{b}\right)}{d}-\frac{a\tan(c+d\,x)^3}{b^4\,d} + \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(15\,a^4+18\,a^2\,b^2+3\,b^4\right)}{b^7\,d}$$

input
```
int(1/(cos(c + d*x)^8*(a + b*tan(c + d*x))^3),x)
```

output
```
((11*a^6 - b^6 + 9*a^2*b^4 + 21*a^4*b^2)/(2*b) + tan(c + d*x)*(6*a*b^4 + 6
*a^5 + 12*a^3*b^2))/(d*(a^2*b^6 + b^8*tan(c + d*x)^2 + 2*a*b^7*tan(c + d*x
))) + (tan(c + d*x)^2*(3/(2*b^3) + (3*a^2)/b^5))/d + tan(c + d*x)^4/(4*b^3
*d) + (tan(c + d*x)*((8*a^3)/b^6 - (3*a*(3/b^3 + (6*a^2)/b^5))/b))/d - (a*
tan(c + d*x)^3)/(b^4*d) + (log(a + b*tan(c + d*x))*(15*a^4 + 3*b^4 + 18*a^
2*b^2))/(b^7*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13621, normalized size of antiderivative = 73.63

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x)
```

output
```
( - 120*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**7*tan(c + d*x
)**2*a**5*b**3 - 144*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**
7*tan(c + d*x)**2*a**3*b**5 - 24*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*si
n(c + d*x)**7*tan(c + d*x)**2*a*b**7 - 240*cos(c + d*x)*log(tan((c + d*x)/
2) - 1)*sin(c + d*x)**7*tan(c + d*x)*a**6*b**2 - 288*cos(c + d*x)*log(tan(
(c + d*x)/2) - 1)*sin(c + d*x)**7*tan(c + d*x)*a**4*b**4 - 48*cos(c + d*x)
*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**7*tan(c + d*x)*a**2*b**6 - 120*co
s(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**7*a**7*b - 144*cos(c +
d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**7*a**5*b**3 - 24*cos(c + d*x)
*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**7*a**3*b**5 + 360*cos(c + d*x)*lo
g(tan((c + d*x)/2) - 1)*sin(c + d*x)**5*tan(c + d*x)**2*a**5*b**3 + 432*co
s(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**5*tan(c + d*x)**2*a**3*
b**5 + 72*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**5*tan(c + d
*x)**2*a*b**7 + 720*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**5
*tan(c + d*x)*a**6*b**2 + 864*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c
 + d*x)**5*tan(c + d*x)*a**4*b**4 + 144*cos(c + d*x)*log(tan((c + d*x)/2)
- 1)*sin(c + d*x)**5*tan(c + d*x)*a**2*b**6 + 360*cos(c + d*x)*log(tan((c
+ d*x)/2) - 1)*sin(c + d*x)**5*a**7*b + 432*cos(c + d*x)*log(tan((c + d*x)
/2) - 1)*sin(c + d*x)**5*a**5*b**3 + 72*cos(c + d*x)*log(tan((c + d*x)/2)
- 1)*sin(c + d*x)**5*a**3*b**5 - 360*cos(c + d*x)*log(tan((c + d*x)/2) ...
```

3.575 $\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 121

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{2(3a^2+b^2)\log(a+b\tan(c+dx))}{b^5d} - \frac{3a\tan(c+dx)}{b^4d} + \frac{\tan^2(c+dx)}{2b^3d} - \frac{(a^2+b^2)^2}{2b^5d(a+b\tan(c+dx))^2} + \frac{4a(a^2+b^2)}{b^5d(a+b\tan(c+dx))}$$

output
```
2*(3*a^2+b^2)*ln(a+b*tan(d*x+c))/b^5/d-3*a*tan(d*x+c)/b^4/d+1/2*tan(d*x+c)
^2/b^3/d-1/2*(a^2+b^2)^2/b^5/d/(a+b*tan(d*x+c))^2+4*a*(a^2+b^2)/b^5/d/(a+b
*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 2.42 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.16

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{b^4\sec^4(c+dx)}{2(a+b\tan(c+dx))^2} - 2a\left(-2a\log(a+b\tan(c+dx)) + b\tan(c+dx) - \frac{a^2+b^2}{a+b\tan(c+dx)}\right) + 2(a^2+b^2)\left(\log(a+b\right.}{b^5d}$$

input
```
Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

output
```
((b^4*Sec[c + d*x]^4)/(2*(a + b*Tan[c + d*x])^2) - 2*a*(-2*a*Log[a + b*Tan
[c + d*x]] + b*Tan[c + d*x] - (a^2 + b^2)/(a + b*Tan[c + d*x])) + 2*(a^2 +
 b^2)*(Log[a + b*Tan[c + d*x]] + (3*a^2 - b^2 + 4*a*b*Tan[c + d*x])/(2*(a
+ b*Tan[c + d*x])^2)))/(b^5*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^6}{(a+b\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3987}$$
$$\frac{\int \frac{(\tan^2(c+dx)b^2+b^2)^2}{b^4(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{bd}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{(\tan^2(c+dx)b^2+b^2)^2}{(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{b^5d}$$
$$\Big\downarrow \text{476}$$
$$\frac{\int \left(\frac{(a^2+b^2)^2}{(a+b\tan(c+dx))^3}-\frac{4a(a^2+b^2)}{(a+b\tan(c+dx))^2}-3a+b\tan(c+dx)+\frac{2(3a^2+b^2)}{a+b\tan(c+dx)}\right)d(b\tan(c+dx))}{b^5d}$$
$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(a^2+b^2)^2}{2(a+b\tan(c+dx))^2}+\frac{4a(a^2+b^2)}{a+b\tan(c+dx)}+2(3a^2+b^2)\log(a+b\tan(c+dx))-3ab\tan(c+dx)+\frac{1}{2}b^2\tan^2(c+dx)}{b^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

output
```
(2*(3*a^2 + b^2)*Log[a + b*Tan[c + d*x]] - 3*a*b*Tan[c + d*x] + (b^2*Tan[c
 + d*x]^2)/2 - (a^2 + b^2)^2/(2*(a + b*Tan[c + d*x])^2) + (4*a*(a^2 + b^2)
)/(a + b*Tan[c + d*x]))/(b^5*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 164.80 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{-\frac{b\tan(dx+c)^2}{2}+3a\tan(dx+c)}{b^4}+\frac{(6a^2+2b^2)\ln(a+b\tan(dx+c))}{b^5}-\frac{a^4+2b^2a^2+b^4}{2b^5(a+b\tan(dx+c))^2}+\frac{4a(a^2+b^2)}{b^5(a+b\tan(dx+c))}}{d}$
default	$\frac{-\frac{-\frac{b\tan(dx+c)^2}{2}+3a\tan(dx+c)}{b^4}+\frac{(6a^2+2b^2)\ln(a+b\tan(dx+c))}{b^5}-\frac{a^4+2b^2a^2+b^4}{2b^5(a+b\tan(dx+c))^2}+\frac{4a(a^2+b^2)}{b^5(a+b\tan(dx+c))}}{d}$
risch	$\frac{12ia^3-12iab^2-24a^2b+4b^3e^{6i(dx+c)}-4iab^2e^{2i(dx+c)}+12a^2be^{6i(dx+c)}+12ia^3e^{6i(dx+c)}+36ia^3e^{4i(dx+c)}+4b^3e^{2i(dx+c}}{(e^{2i(dx+c)}+1)^2(be^{2i(dx+c)}+iae^{2i(dx+c)}-b+ia)^2}$

input
```
int(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^4*(-1/2*b*tan(d*x+c)^2+3*a*tan(d*x+c))+(6*a^2+2*b^2)/b^5*ln(a+b*
tan(d*x+c))-1/2/b^5*(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c))^2+4*a/b^5*(a^2+b^
2)/(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(117) = 234.

Time = 0.12 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.93

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{24\,a^2b^2\cos(dx+c)^4+b^4-2\,(9\,a^2b^2+b^4)\cos(dx+c)^2+2\,((3\,a^4-2\,a^2b^2-b^4)\cos(dx+c)^4+2\,(3\,a^3b}{}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(24*a^2*b^2*cos(d*x + c)^4 + b^4 - 2*(9*a^2*b^2 + b^4)*cos(d*x + c)^2
+ 2*((3*a^4 - 2*a^2*b^2 - b^4)*cos(d*x + c)^4 + 2*(3*a^3*b + a*b^3)*cos(d*
x + c)^3*sin(d*x + c) + (3*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(2*a*b*cos(d*
x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*((3*a^4 - 2*a^
2*b^2 - b^4)*cos(d*x + c)^4 + 2*(3*a^3*b + a*b^3)*cos(d*x + c)^3*sin(d*x +
 c) + (3*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(cos(d*x + c)^2) - 4*(a*b^3*cos
(d*x + c) + 3*(a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^6*d*cos
(d*x + c)^3*sin(d*x + c) + b^7*d*cos(d*x + c)^2 + (a^2*b^5 - b^7)*d*cos(d*
x + c)^4)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx=\int \frac{\sec^6{(c+dx)}}{(a+b\tan{(c+dx)})^3}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**6/(a + b*tan(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.06

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx=\frac{\frac{7a^4+6a^2b^2-b^4+8(a^3b+ab^3)\tan(dx+c)}{b^7\tan(dx+c)^2+2ab^6\tan(dx+c)+a^2b^5}+\frac{b\tan(dx+c)^2-6a\tan(dx+c)}{b^4}+\frac{4(3a^2+b^2)\log(b\tan(dx+c)+a)}{b^5}}{2d}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*((7*a^4 + 6*a^2*b^2 - b^4 + 8*(a^3*b + a*b^3)*tan(d*x + c))/(b^7*tan(d
*x + c)^2 + 2*a*b^6*tan(d*x + c) + a^2*b^5) + (b*tan(d*x + c)^2 - 6*a*tan(
d*x + c))/b^4 + 4*(3*a^2 + b^2)*log(b*tan(d*x + c) + a)/b^5)/d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{2\,(3\,a^2+b^2)\log\left(|b\tan\left(dx+c\right)+a|\right)}{b^5d} + \frac{b^3d\tan\left(dx+c\right)^2-6\,ab^2d\tan\left(dx+c\right)}{2\,b^6d^2} + \frac{7\,a^4+6\,a^2b^2-b^4+8\,(a^3b+ab^3)\tan\left(dx+c\right)}{2\,(b\tan\left(dx+c\right)+a)^2b^5d}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
2*(3*a^2 + b^2)*log(abs(b*tan(d*x + c) + a))/(b^5*d) + 1/2*(b^3*d*tan(d*x
+ c)^2 - 6*a*b^2*d*tan(d*x + c))/(b^6*d^2) + 1/2*(7*a^4 + 6*a^2*b^2 - b^4
+ 8*(a^3*b + a*b^3)*tan(d*x + c))/((b*tan(d*x + c) + a)^2*b^5*d)
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.18

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{7\,a^4+6\,a^2\,b^2-b^4}{2\,b}+\tan(c+d\,x)\,\left(4\,a^3+4\,a\,b^2\right)}{d\,\left(a^2\,b^4+2\,a\,b^5\tan\left(c+d\,x\right)+b^6\tan\left(c+d\,x\right)^2\right)} + \frac{\tan(c+d\,x)^2}{2\,b^3\,d} - \frac{3\,a\tan(c+d\,x)}{b^4\,d} + \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(6\,a^2+2\,b^2\right)}{b^5\,d}$$

input
```
int(1/(cos(c + d*x)^6*(a + b*tan(c + d*x))^3),x)
```

output
```
((7*a^4 - b^4 + 6*a^2*b^2)/(2*b) + tan(c + d*x)*(4*a*b^2 + 4*a^3))/(d*(a^2
*b^4 + b^6*tan(c + d*x)^2 + 2*a*b^5*tan(c + d*x))) + tan(c + d*x)^2/(2*b^3
*d) - (3*a*tan(c + d*x))/(b^4*d) + (log(a + b*tan(c + d*x))*(6*a^2 + 2*b^2
))/(b^5*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 7160, normalized size of antiderivative = 59.17

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x)
```

output
```
( - 24*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**5*tan(c + d*x)
**2*a**3*b**3 - 8*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**5*t
an(c + d*x)**2*a*b**5 - 48*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c +
d*x)**5*tan(c + d*x)*a**4*b**2 - 16*cos(c + d*x)*log(tan((c + d*x)/2) - 1)
*sin(c + d*x)**5*tan(c + d*x)*a**2*b**4 - 24*cos(c + d*x)*log(tan((c + d*x
)/2) - 1)*sin(c + d*x)**5*a**5*b - 8*cos(c + d*x)*log(tan((c + d*x)/2) - 1
)*sin(c + d*x)**5*a**3*b**3 + 48*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*si
n(c + d*x)**3*tan(c + d*x)**2*a**3*b**3 + 16*cos(c + d*x)*log(tan((c + d*x
)/2) - 1)*sin(c + d*x)**3*tan(c + d*x)**2*a*b**5 + 96*cos(c + d*x)*log(tan
((c + d*x)/2) - 1)*sin(c + d*x)**3*tan(c + d*x)*a**4*b**2 + 32*cos(c + d*x
)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**3*tan(c + d*x)*a**2*b**4 + 48*co
s(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**3*a**5*b + 16*cos(c + d
*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**3*a**3*b**3 - 24*cos(c + d*x)*
log(tan((c + d*x)/2) - 1)*sin(c + d*x)*tan(c + d*x)**2*a**3*b**3 - 8*cos(c
 + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*tan(c + d*x)**2*a*b**5 - 48
*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*tan(c + d*x)*a**4*b**
2 - 16*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*tan(c + d*x)*a*
*2*b**4 - 24*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*a**5*b -
8*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*a**3*b**3 - 24*cos(c
 + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**5*tan(c + d*x)**2*a**3*...
```

3.576 $\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\log(a+b\tan(c+dx))}{b^3d} - \frac{a^2+b^2}{2b^3d(a+b\tan(c+dx))^2} + \frac{2a}{b^3d(a+b\tan(c+dx))}$$

output
```
ln(a+b*tan(d*x+c))/b^3/d-1/2*(a^2+b^2)/b^3/d/(a+b*tan(d*x+c))^2+2*a/b^3/d/
(a+b*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\log(a+b\tan(c+dx)) - \frac{a^2+b^2}{2(a+b\tan(c+dx))^2} + \frac{2a}{a+b\tan(c+dx)}}{b^3d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Log[a + b*Tan[c + d*x]] - (a^2 + b^2)/(2*(a + b*Tan[c + d*x])^2) + (2*a)/
(a + b*Tan[c + d*x]))/(b^3*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^4}{(a+b\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3987}$$
$$\frac{\int \frac{\tan^2(c+dx)b^2+b^2}{b^2(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{bd}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{\tan^2(c+dx)b^2+b^2}{(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{b^3d}$$
$$\Big\downarrow \text{476}$$
$$\frac{\int \left(-\frac{2a}{(a+b\tan(c+dx))^2}+\frac{1}{a+b\tan(c+dx)}+\frac{a^2+b^2}{(a+b\tan(c+dx))^3}\right)d(b\tan(c+dx))}{b^3d}$$
$$\Big\downarrow \text{2009}$$
$$\frac{-\frac{a^2+b^2}{2(a+b\tan(c+dx))^2}+\frac{2a}{a+b\tan(c+dx)}+\log(a+b\tan(c+dx))}{b^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Log[a + b*Tan[c + d*x]] - (a^2 + b^2)/(2*(a + b*Tan[c + d*x])^2) + (2*a)/
(a + b*Tan[c + d*x]))/(b^3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 36.72 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{-\frac{a^2+b^2}{2b^3(a+b\tan(dx+c))^2}+\frac{2a}{b^3(a+b\tan(dx+c))}+\frac{\ln(a+b\tan(dx+c))}{b^3}}{d}$	63
default	$\frac{-\frac{a^2+b^2}{2b^3(a+b\tan(dx+c))^2}+\frac{2a}{b^3(a+b\tan(dx+c))}+\frac{\ln(a+b\tan(dx+c))}{b^3}}{d}$	63
risch	$\frac{-2a^2 e^{2i(dx+c)}+2b^2 e^{2i(dx+c)}+4iab\,e^{2i(dx+c)}-2a^2-2iab}{b^2(ia+b)\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^2 d}+\frac{\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{b^3 d}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{b^3 d}$	160

input
```
int(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*(a^2+b^2)/b^3/(a+b*tan(d*x+c))^2+2/b^3*a/(a+b*tan(d*x+c))+1/b^3*
ln(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(67) = 134$.

Time = 0.11 (sec) , antiderivative size = 284, normalized size of antiderivative = 4.12

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{4\,a^2b^2\cos(dx+c)^2 - 3\,a^2b^2 - b^4 - 2\,(a^3b - ab^3)\cos(dx+c)\sin(dx+c) + \left(a^2b^2 + b^4 + (a^4 - b^4)\cos(d\right.}{}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(4*a^2*b^2*cos(d*x + c)^2 - 3*a^2*b^2 - b^4 - 2*(a^3*b - a*b^3)*cos(d*
x + c)*sin(d*x + c) + (a^2*b^2 + b^4 + (a^4 - b^4)*cos(d*x + c)^2 + 2*(a^3
*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c)
 + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*b^2 + b^4 + (a^4 - b^4)*cos(d*
x + c)^2 + 2*(a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(cos(d*x + c)^2
))/((a^4*b^3 - b^7)*d*cos(d*x + c)^2 + 2*(a^3*b^4 + a*b^6)*d*cos(d*x + c)*
sin(d*x + c) + (a^2*b^5 + b^7)*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*tan(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{4\,ab\tan(dx+c)+3\,a^2-b^2}{b^5\tan(dx+c)^2+2\,ab^4\tan(dx+c)+a^2b^3}+\frac{2\,\log(b\tan(dx+c)+a)}{b^3}}{2\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*((4*a*b*tan(d*x + c) + 3*a^2 - b^2)/(b^5*tan(d*x + c)^2 + 2*a*b^4*tan(
d*x + c) + a^2*b^3) + 2*log(b*tan(d*x + c) + a)/b^3)/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\log\left(|b\tan\left(dx+c\right)+a|\right)}{b^3d}+\frac{4\,a\tan\left(dx+c\right)+\frac{3\,a^2-b^2}{b}}{2\left(b\tan\left(dx+c\right)+a\right)^2b^2d}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
log(abs(b*tan(d*x + c) + a))/(b^3*d) + 1/2*(4*a*tan(d*x + c) + (3*a^2 - b^
2)/b)/((b*tan(d*x + c) + a)^2*b^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{3\,a^2-b^2}{2\,b^3}+\frac{2\,a\tan(c+d\,x)}{b^2}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)}+\frac{\ln\left(a+b\tan(c+d\,x)\right)}{b^3\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + b*tan(c + d*x))^3),x)
```

output
```
((3*a^2 - b^2)/(2*b^3) + (2*a*tan(c + d*x))/b^2)/(d*(a^2 + b^2*tan(c + d*x
)^2 + 2*a*b*tan(c + d*x))) + log(a + b*tan(c + d*x))/(b^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3133, normalized size of antiderivative = 45.41

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x)
```

output
```
( - 4*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**3*tan(c + d*x)*
*2*a*b**3 - 8*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**3*tan(c
 + d*x)*a**2*b**2 - 4*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*
*3*a**3*b + 4*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*tan(c +
d*x)**2*a*b**3 + 8*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*tan
(c + d*x)*a**2*b**2 + 4*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x
)*a**3*b - 4*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**3*tan(c
+ d*x)**2*a*b**3 - 8*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**
3*tan(c + d*x)*a**2*b**2 - 4*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c
+ d*x)**3*a**3*b + 4*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)*t
an(c + d*x)**2*a*b**3 + 8*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d
*x)*tan(c + d*x)*a**2*b**2 + 4*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(
c + d*x)*a**3*b + 4*cos(c + d*x)*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*
x)/2)*b - a)*sin(c + d*x)**3*tan(c + d*x)**2*a*b**3 + 8*cos(c + d*x)*log(t
an((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)**3*tan(c + d
*x)*a**2*b**2 + 4*cos(c + d*x)*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)
/2)*b - a)*sin(c + d*x)**3*a**3*b - 4*cos(c + d*x)*log(tan((c + d*x)/2)**2
*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)*tan(c + d*x)**2*a*b**3 - 8*cos
(c + d*x)*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*
x)*tan(c + d*x)*a**2*b**2 - 4*cos(c + d*x)*log(tan((c + d*x)/2)**2*a - ...
```

3.577 $\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{1}{2bd(a+b\tan(c+dx))^2}$$

output
```
-1/2/b/d/(a+b*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{1}{2bd(a+b\tan(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

output
```
-1/2*1/(b*d*(a + b*Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3987, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^2}{(a+b\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3987}$$
$$\frac{\int \frac{1}{(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{bd}$$
$$\Big\downarrow \text{17}$$
$$-\frac{1}{2bd(a+b\tan(c+dx))^2}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

output
```
-1/2*1/(b*d*(a + b*Tan[c + d*x])^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 7.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{1}{2bd(a+b\tan(dx+c))^2}$	21
default	$-\frac{1}{2bd(a+b\tan(dx+c))^2}$	21
risch	$\frac{2ia\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}+2ia}{\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^2 d(ia+b)^2}$	77

input
```
int(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/d/(a+b*tan(d*x+c))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(20) = 40.

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 6.45

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{4\,a^2b\cos\left(dx+c\right)^2 - a^2b + b^3 - 2\,(a^3-ab^2)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,\left((a^6+a^4b^2-a^2b^4-b^6)d\cos\left(dx+c\right)^2 + 2\,(a^5b+2\,a^3b^3+ab^5)d\cos\left(dx+c\right)\sin\left(dx+c\right) + (a^4b^2+\right.}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2*(4*a^2*b*cos(d*x + c)^2 - a^2*b + b^3 - 2*(a^3 - a*b^2)*cos(d*x + c)*
sin(d*x + c))/((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b
 + 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 +
 b^6)*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^2{(c+dx)}}{(a+b\tan{(c+dx)})^3}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*tan(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{1}{2\,(b\tan{(dx+c)}+a)^2bd}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2/((b*tan(d*x + c) + a)^2*b*d)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{1}{2\,(b\tan{(dx+c)}+a)^2bd}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2/((b*tan(d*x + c) + a)^2*b*d)
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{1}{d\,\left(2\,a^2\,b+4\,a\,b^2\,\tan{(c+d\,x)}+2\,b^3\,\tan{(c+d\,x)}^2\right)}$$

input
```
int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x))^3),x)
```

output
```
-1/(d*(2*a^2*b + 2*b^3*tan(c + d*x)^2 + 4*a*b^2*tan(c + d*x)))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\sin{(dx+c)}^2-1}{2bd\,\left(2\cos{(dx+c)}\sin{(dx+c)}\,ab-\sin{(dx+c)}^2\,a^2+\sin{(dx+c)}^2\,b^2+a^2\right)}$$

input
```
int(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x)
```

output
```
(sin(c + d*x)**2 - 1)/(2*b*d*(2*cos(c + d*x)*sin(c + d*x)*a*b - sin(c + d*
x)**2*a**2 + sin(c + d*x)**2*b**2 + a**2))
```

3.578 $\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 202

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{a(a^4+10a^2b^2-15b^4)\,x}{2\,(a^2+b^2)^4} + \frac{2b^3(5a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d} + \frac{b(a^2-2b^2)}{2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{ab(a^2-11b^2)}{2\,(a^2+b^2)^3\,d(a+b\tan(c+dx))}$$

output
```
1/2*a*(a^4+10*a^2*b^2-15*b^4)*x/(a^2+b^2)^4+2*b^3*(5*a^2-b^2)*ln(a*cos(d*x
+c)+b*sin(d*x+c))/(a^2+b^2)^4/d+1/2*b*(a^2-2*b^2)/(a^2+b^2)^2/d/(a+b*tan(d
*x+c))^2+1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+
1/2*a*b*(a^2-11*b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 458 vs. $2(202) = 404$.

Time = 6.20 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.27

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{b^3\left(\frac{\cos^2(c+dx)(b^2+ab\tan(c+dx))}{2b^4(a^2+b^2)(a+b\tan(c+dx))^2} - \frac{(2a^2-4b^2)\left(-\frac{\left(3a^2-b^2-\frac{a^3-3ab^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3}+\frac{\left(3a^2-b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3}-\frac{\left(3a^2-b^2+\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b^3*((Cos[c + d*x]^2*(b^2 + a*b*Tan[c + d*x]))/(2*b^4*(a^2 + b^2)*(a + b*
Tan[c + d*x])^2) - ((2*a^2 - 4*b^2)*(-1/2*((3*a^2 - b^2 - (a^3 - 3*a*b^2)/
Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(a^2 + b^2)^3 + ((3*a^2 - b^
2)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^3 - ((3*a^2 - b^2 + (a^3 - 3*a*b^2
)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^3) - 1/(2*(
a^2 + b^2)*(a + b*Tan[c + d*x])^2) - (2*a)/((a^2 + b^2)^2*(a + b*Tan[c + d
*x]))) - 3*a*(-1/2*((2*a - (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[
c + d*x]])/(a^2 + b^2)^2 + (2*a*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^2 - (
(2*a + (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 +
 b^2)^2) - 1/((a^2 + b^2)*(a + b*Tan[c + d*x]))))/(2*b^2*(a^2 + b^2))))/d
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3987, 27, 496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{b^4}{(a+b\tan(c+dx))^3(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^3\int \frac{1}{(a+b\tan(c+dx))^3(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{496}$$

$$\frac{b^3\left(\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)(a+b\tan(c+dx))^2}-\frac{\int -\frac{a^2+3b\tan(c+dx)a+4b^2}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2(a^2+b^2)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^3\left(\frac{\int \frac{a^2+3b\tan(c+dx)a+4b^2}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)(a+b\tan(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \text{657}$$

$$\frac{b^3\left(\frac{\int\left(-\frac{a\left(a^2-11b^2\right)}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}+\frac{4\left(5a^2b^2-b^4\right)}{\left(a^2+b^2\right)^3(a+b\tan(c+dx))}+\frac{a\left(a^4+10b^2a^2-15b^4\right)-4b^3\left(5a^2-b^2\right)\tan(c+dx)}{\left(a^2+b^2\right)^3\left(\tan^2(c+dx)b^2+b^2\right)}-\frac{2\left(a^2-2b^2\right)}{\left(a^2+b^2\right)(a+b\tan(c+dx))^3}\right)d(b\tan}{2b^2(a^2+b^2)}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^3\left(\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)(a+b\tan(c+dx))^2}+\frac{\frac{a\left(a^2-11b^2\right)}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))}+\frac{a^2-2b^2}{\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{2b^2\left(5a^2-b^2\right)\log\left(b^2\tan^2(c+dx)+\right.}{\left(a^2+b^2\right)^3}}{2b^2(a^2+b^2}\right.}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b^3*((b^2 + a*b*Tan[c + d*x])/(2*b^2*(a^2 + b^2)*(a + b*Tan[c + d*x])^2*(
b^2 + b^2*Tan[c + d*x]^2)) + ((a*(a^4 + 10*a^2*b^2 - 15*b^4)*ArcTan[Tan[c
+ d*x]])/(b*(a^2 + b^2)^3) + (4*b^2*(5*a^2 - b^2)*Log[a + b*Tan[c + d*x]])
/(a^2 + b^2)^3 - (2*b^2*(5*a^2 - b^2)*Log[b^2 + b^2*Tan[c + d*x]^2])/(a^2
+ b^2)^3 + (a^2 - 2*b^2)/((a^2 + b^2)*(a + b*Tan[c + d*x])^2) + (a*(a^2 -
11*b^2))/((a^2 + b^2)^2*(a + b*Tan[c + d*x]))))/(2*b^2*(a^2 + b^2))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 10.33 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{-\frac{b^3}{2\left(a^2+b^2\right)^2(a+b\tan(dx+c))^2}-\frac{4b^3a}{\left(a^2+b^2\right)^3(a+b\tan(dx+c))}+\frac{2b^3\left(5a^2-b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^4}+\frac{\left(\frac{1}{2}a^5-a^3b^2-\frac{3}{2}a\,b^4\right)\tan(dx+c}{1+\tan(dx+c}}{d}$
default	$\frac{-\frac{b^3}{2\left(a^2+b^2\right)^2(a+b\tan(dx+c))^2}-\frac{4b^3a}{\left(a^2+b^2\right)^3(a+b\tan(dx+c))}+\frac{2b^3\left(5a^2-b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^4}+\frac{\left(\frac{1}{2}a^5-a^3b^2-\frac{3}{2}a\,b^4\right)\tan(dx+c}{1+\tan(dx+c}}{d}$
risch	$\frac{4ixb}{8ia^3b-8ia\,b^3-2a^4+12b^2a^2-2b^4}-\frac{xa}{8ia^3b-8ia\,b^3-2a^4+12b^2a^2-2b^4}-\frac{ie^{2i(dx+c)}}{8(-3ia^2b+ib^3+a^3-3a\,b^2)d}+\frac{ie^{-2}}{8(3ia^2b-ib}$

input
```
int(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*b^3/(a^2+b^2)^2/(a+b*tan(d*x+c))^2-4*b^3/(a^2+b^2)^3*a/(a+b*tan(
d*x+c))+2*b^3*(5*a^2-b^2)/(a^2+b^2)^4*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^4*(((
1/2*a^5-a^3*b^2-3/2*a*b^4)*tan(d*x+c)+3/2*a^4*b+a^2*b^3-1/2*b^5)/(1+tan(d*
x+c)^2)+1/4*(-20*a^2*b^3+4*b^5)*ln(1+tan(d*x+c)^2)+1/2*(a^5+10*a^3*b^2-15*
a*b^4)*arctan(tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. 2(194) = 388.

Time = 0.12 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.49

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{3\,a^4b^3-16\,a^2b^5+b^7-2\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7\right)\cos\left(dx+c\right)^4-2\left(a^5b^2+10\,a^3b^4-15\,ab^6\right)dx-(}{}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/4*(3*a^4*b^3 - 16*a^2*b^5 + b^7 - 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^
7)*cos(d*x + c)^4 - 2*(a^5*b^2 + 10*a^3*b^4 - 15*a*b^6)*d*x - (a^6*b - a^4
*b^3 - 45*a^2*b^5 - 3*b^7 + 2*(a^7 + 9*a^5*b^2 - 25*a^3*b^4 + 15*a*b^6)*d*
x)*cos(d*x + c)^2 - 4*(5*a^2*b^5 - b^7 + (5*a^4*b^3 - 6*a^2*b^5 + b^7)*cos
(d*x + c)^2 + 2*(5*a^3*b^4 - a*b^6)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*c
os(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*((a^7 + 3
*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 2*(a^5*b^2 - 3*a^3*b^4 + 6*
a*b^6 - (a^6*b + 10*a^4*b^3 - 15*a^2*b^5)*d*x)*cos(d*x + c))*sin(d*x + c))
/((a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*d*cos(d*x
+ c)^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*d*cos(d*x +
 c)*sin(d*x + c) + (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(cos(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(194) = 388$.

Time = 0.16 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.27

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{(a^5+10\,a^3b^2-15\,ab^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{4\,(5\,a^2b^3-b^5)\log(b\tan(dx+c)+a)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{2\,(5\,a^2b^3-b^5)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{}{a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6+(a}}{}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*((a^5 + 10*a^3*b^2 - 15*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8) + 4*(5*a^2*b^3 - b^5)*log(b*tan(d*x + c) + a)/(a^8 + 4*
a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 2*(5*a^2*b^3 - b^5)*log(tan(d*x +
 c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + (3*a^4*b - 10
*a^2*b^3 - b^5 + (a^3*b^2 - 11*a*b^4)*tan(d*x + c)^3 + 2*(a^4*b - 6*a^2*b^
3 - b^5)*tan(d*x + c)^2 + (a^5 + 3*a^3*b^2 - 10*a*b^4)*tan(d*x + c))/(a^8
+ 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8
)*tan(d*x + c)^4 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c)^
3 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*tan(d*x + c)^2 + 2*(a^
7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.83

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{(a^5+10\,a^3b^2-15\,ab^4)(dx+c)}{2\,(a^8d+4\,a^6b^2d+6\,a^4b^4d+4\,a^2b^6d+b^8d)} - \frac{(5\,a^2b^3-b^5)\log\left(\tan\left(dx+c\right)^2+1\right)}{a^8d+4\,a^6b^2d+6\,a^4b^4d+4\,a^2b^6d+b^8d} + \frac{2\,(5\,a^2b^4-b^6)\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^8bd+4\,a^6b^3d+6\,a^4b^5d+4\,a^2b^7d+b^9d} + \frac{3\,a^6b-7\,a^4b^3-11\,a^2b^5-b^7+(a^5b^2-10\,a^3b^4-11\,ab^6)\tan\left(dx+c\right)^3+2\,(a^6b-5\,a^4b^3-7\,a^2b^5-b^7)}{2\,(a^2+b^2)^4(b\tan\left(dx+c\right)+a)^2\left(\tan\left(dx+c\right)^2\right.}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(a^5 + 10*a^3*b^2 - 15*a*b^4)*(d*x + c)/(a^8*d + 4*a^6*b^2*d + 6*a^4*b
^4*d + 4*a^2*b^6*d + b^8*d) - (5*a^2*b^3 - b^5)*log(tan(d*x + c)^2 + 1)/(a
^8*d + 4*a^6*b^2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b^8*d) + 2*(5*a^2*b^4 - b
^6)*log(abs(b*tan(d*x + c) + a))/(a^8*b*d + 4*a^6*b^3*d + 6*a^4*b^5*d + 4*
a^2*b^7*d + b^9*d) + 1/2*(3*a^6*b - 7*a^4*b^3 - 11*a^2*b^5 - b^7 + (a^5*b^
2 - 10*a^3*b^4 - 11*a*b^6)*tan(d*x + c)^3 + 2*(a^6*b - 5*a^4*b^3 - 7*a^2*b
^5 - b^7)*tan(d*x + c)^2 + (a^7 + 4*a^5*b^2 - 7*a^3*b^4 - 10*a*b^6)*tan(d*
x + c))/((a^2 + b^2)^4*(b*tan(d*x + c) + a)^2*(tan(d*x + c)^2 + 1)*d)
```

Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.07

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{10\,b^3}{{\left(a^2+b^2\right)}^3}-\frac{12\,b^5}{{\left(a^2+b^2\right)}^4}\right)}{d} - \frac{\frac{-3\,a^4\,b+10\,a^2\,b^3+b^5}{2\,\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{{\tan(c+d\,x)}^3\,\left(11\,a\,b^4-a^3\,b^2\right)}{2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{{\tan(c+d\,x)}^2\,\left(-a^4\,b+6\,a^2\,b^3+b^5\right)}{\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{a\tan(c+d\,x)\,\left(a^4+3\,a^2\,b^2-10\,b^4\right)}{2\,\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left({\tan\left(c+d\,x\right)}^2\,\left(a^2+b^2\right)+a^2+b^2\,{\tan\left(c+d\,x\right)}^4+2\,a\,b\tan\left(c+d\,x\right)+2\,a\,b\,{\tan\left(c+d\,x\right)}^3\right)} + \frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(b+\frac{a\,1{}\mathrm{i}}{4}\right)}{d\,\left(a^4-a^3\,b\,4{}\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4{}\mathrm{i}+b^4\right)} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a+b\,4{}\mathrm{i}\right)}{4\,d\,\left(a^4\,1{}\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6{}\mathrm{i}+4\,a\,b^3+b^4\,1{}\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^2/(a + b*tan(c + d*x))^3,x)
```

output
```
(log(a + b*tan(c + d*x))*((10*b^3)/(a^2 + b^2)^3 - (12*b^5)/(a^2 + b^2)^4)
)/d - ((b^5 - 3*a^4*b + 10*a^2*b^3)/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)
) + (tan(c + d*x)^3*(11*a*b^4 - a^3*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)) + (tan(c + d*x)^2*(b^5 - a^4*b + 6*a^2*b^3))/((a^2 + b^2)*(a^4 + b
^4 + 2*a^2*b^2)) - (a*tan(c + d*x)*(a^4 - 10*b^4 + 3*a^2*b^2))/(2*(a^2 + b
^2)*(a^4 + b^4 + 2*a^2*b^2)))/(d*(tan(c + d*x)^2*(a^2 + b^2) + a^2 + b^2*t
an(c + d*x)^4 + 2*a*b*tan(c + d*x) + 2*a*b*tan(c + d*x)^3)) + (log(tan(c +
 d*x) + 1i)*((a*1i)/4 + b))/(d*(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^
2)) + (log(tan(c + d*x) - 1i)*(a + b*4i))/(4*d*(4*a*b^3 - 4*a^3*b + a^4*1i
 + b^4*1i - a^2*b^2*6i))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1303, normalized size of antiderivative = 6.45

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x)
```

output
```
( - 80*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*a**3*b**5 +
16*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*a*b**7 + 80*cos(
c + d*x)*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x
)*a**3*b**5 - 16*cos(c + d*x)*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/
2)*b - a)*sin(c + d*x)*a*b**7 - 2*cos(c + d*x)*sin(c + d*x)**3*a**7*b - 6*
cos(c + d*x)*sin(c + d*x)**3*a**5*b**3 - 6*cos(c + d*x)*sin(c + d*x)**3*a*
*3*b**5 - 2*cos(c + d*x)*sin(c + d*x)**3*a*b**7 + 4*cos(c + d*x)*sin(c + d
*x)*a**6*b**2*c + 4*cos(c + d*x)*sin(c + d*x)*a**6*b**2*d*x + 40*cos(c + d
*x)*sin(c + d*x)*a**4*b**4*c + 40*cos(c + d*x)*sin(c + d*x)*a**4*b**4*d*x
- 60*cos(c + d*x)*sin(c + d*x)*a**2*b**6*c - 60*cos(c + d*x)*sin(c + d*x)*
a**2*b**6*d*x + 40*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a**4*b**4
- 48*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a**2*b**6 + 8*log(tan((c
 + d*x)/2)**2 + 1)*sin(c + d*x)**2*b**8 - 40*log(tan((c + d*x)/2)**2 + 1)*
a**4*b**4 + 8*log(tan((c + d*x)/2)**2 + 1)*a**2*b**6 - 40*log(tan((c + d*x
)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)**2*a**4*b**4 + 48*log(t
an((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)**2*a**2*b**6
 - 8*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)**2
*b**8 + 40*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*a**4*b**4
 - 8*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*a**2*b**6 + 2*s
in(c + d*x)**4*a**6*b**2 + 6*sin(c + d*x)**4*a**4*b**4 + 6*sin(c + d*x)...
```

3.579 $\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 295

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{3a(a^6+7a^4b^2+35a^2b^4-35b^6)\,x}{8\,(a^2+b^2)^5} \\ &\quad+\frac{3b^5(7a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^5\,d} \\ &\quad+\frac{3b(a^4+5a^2b^2-4b^4)}{8\,(a^2+b^2)^3\,d(a+b\tan(c+dx))^2} \\ &\quad+\frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} \\ &\quad+\frac{3ab(a^4+6a^2b^2-27b^4)}{8\,(a^2+b^2)^4\,d(a+b\tan(c+dx))} \\ &\quad-\frac{\cos^2(c+dx)\,(2b(a^2-3b^2)-a(3a^2+11b^2)\tan(c+dx))}{8\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2}\end{aligned}$$

output
```
3/8*a*(a^6+7*a^4*b^2+35*a^2*b^4-35*b^6)*x/(a^2+b^2)^5+3*b^5*(7*a^2-b^2)*ln
(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^5/d+3/8*b*(a^4+5*a^2*b^2-4*b^4)/(a^2
+b^2)^3/d/(a+b*tan(d*x+c))^2+1/4*cos(d*x+c)^4*(b+a*tan(d*x+c))/(a^2+b^2)/d
/(a+b*tan(d*x+c))^2+3/8*a*b*(a^4+6*a^2*b^2-27*b^4)/(a^2+b^2)^4/d/(a+b*tan(
d*x+c))-1/8*cos(d*x+c)^2*(2*b*(a^2-3*b^2)-a*(3*a^2+11*b^2)*tan(d*x+c))/(a^
2+b^2)^2/d/(a+b*tan(d*x+c))^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 596 vs. $2(295) = 590$.

Time = 6.16 (sec) , antiderivative size = 596, normalized size of antiderivative = 2.02

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{b^5\left(\frac{\cos^4(c+dx)(b^2+ab\tan(c+dx))}{4b^6(a^2+b^2)(a+b\tan(c+dx))^2} - \frac{\frac{\cos^2(c+dx)\left(5a^2b^2-3b^2\left(a^2+2b^2\right)+b\left(-5ab^2-3a\left(a^2+2b^2\right)\right)\tan(c+dx)\right)}{2b^4\left(a^2+b^2\right)(a+b\tan(c+dx))^2} - \frac{\left(-3a^2\left(3a^2+11b^2\right)+3\left(a^4+a^2b^2+8b^4\right.\right.}{}}{}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b^5*((Cos[c + d*x]^4*(b^2 + a*b*Tan[c + d*x]))/(4*b^6*(a^2 + b^2)*(a + b*
Tan[c + d*x])^2) - ((Cos[c + d*x]^2*(5*a^2*b^2 - 3*b^2*(a^2 + 2*b^2) + b*(
-5*a*b^2 - 3*a*(a^2 + 2*b^2))*Tan[c + d*x]))/(2*b^4*(a^2 + b^2)*(a + b*Tan
[c + d*x])^2) - ((-3*a^2*(3*a^2 + 11*b^2) + 3*(a^4 + a^2*b^2 + 8*b^4))*(-1
/2*((3*a^2 - b^2 - (a^3 - 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c +
d*x]])/(a^2 + b^2)^3 + ((3*a^2 - b^2)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)
^3 - ((3*a^2 - b^2 + (a^3 - 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c
+ d*x]])/(2*(a^2 + b^2)^3) - 1/(2*(a^2 + b^2)*(a + b*Tan[c + d*x])^2) - (2
*a)/((a^2 + b^2)^2*(a + b*Tan[c + d*x]))) + 3*a*(3*a^2 + 11*b^2)*(-1/2*((2
*a - (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(a^2 + b^2)
^2 + (2*a*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^2 - ((2*a + (a^2 - b^2)/Sqr
t[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^2) - 1/((a^2 + b
^2)*(a + b*Tan[c + d*x]))))/(2*b^2*(a^2 + b^2)))/(4*b^2*(a^2 + b^2))))/d
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.31, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3987, 27, 496, 25, 686, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^4(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{b^6}{(a+b\tan(c+dx))^3(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^5\int \frac{1}{(a+b\tan(c+dx))^3(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{496}$$

$$\frac{b^5\left(\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2(a+b\tan(c+dx))^2}-\frac{\int -\frac{3\left(a^2+2b^2\right)+5ab\tan(c+dx)}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2(a^2+b^2)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^5\left(\frac{\int \frac{3\left(a^2+2b^2\right)+5ab\tan(c+dx)}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2(a+b\tan(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \text{686}$$

$$\frac{b^5\left(\frac{-\frac{\int -\frac{3\left(a^4+b^2a^2+b\left(3a^2+11b^2\right)\tan(c+dx)a+8b^4\right)}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2\left(a^2+b^2\right)}-\frac{2b^2\left(a^2-3b^2\right)-ab\left(3a^2+11b^2\right)\tan(c+dx)}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)(a+b\tan(c+dx))^2}}{4b^2(a^2+b^2)}+\frac{ab\tan(c-}{4b^2(a^2+b^2)(b^2\tan^2(c+dx}\right.}{d}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{b^5\left(\frac{\frac{3\int \frac{a^4+b^2a^2+b\left(3a^2+11b^2\right)\tan(c+dx)a+8b^4}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2\left(a^2+b^2\right)}-\frac{2b^2\left(a^2-3b^2\right)-ab\left(3a^2+11b^2\right)\tan(c+dx)}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)(a+b\tan(c+dx))^2}}{4b^2(a^2+b^2)}+\frac{ab\tan(c+dx)-}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2}\right.}{d}$$

$$\Big\downarrow \textbf{657}$$

$$\frac{b^5\left(\frac{\frac{3\int\left(-\frac{a\left(a^4+6b^2a^2-27b^4\right)}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}+\frac{8\left(7a^2b^4-b^6\right)}{\left(a^2+b^2\right)^3(a+b\tan(c+dx))}+\frac{a\left(a^6+7b^2a^4+35b^4a^2-35b^6\right)-8b^5\left(7a^2-b^2\right)\tan(c+dx)}{\left(a^2+b^2\right)^3\left(\tan^2(c+dx)b^2+b^2\right)}-\frac{2\left(a^4+5b^2a^2-4b^4\right)}{\left(a^2+b^2\right)(a+b\tan(c+dx))^3}\right)}{2b^2\left(a^2+b^2\right)}}{4b^2(a^2+b^2)}\right.}{d}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{b^5\left(\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2(a+b\tan(c+dx))^2}+\frac{3\left(-\frac{4b^4\left(7a^2-b^2\right)\log\left(b^2\tan^2(c+dx)+b^2\right)}{\left(a^2+b^2\right)^3}+\frac{8b^4\left(7a^2-b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3}+\frac{a\left(a^4+6\right.}{\left(a^2+b^2\right)^2(}\right.}{2b^2}\right.}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b^5*((b^2 + a*b*Tan[c + d*x])/(4*b^2*(a^2 + b^2)*(a + b*Tan[c + d*x])^2*(
b^2 + b^2*Tan[c + d*x]^2)^2) + (-1/2*(2*b^2*(a^2 - 3*b^2) - a*b*(3*a^2 + 1
1*b^2)*Tan[c + d*x])/(b^2*(a^2 + b^2)*(a + b*Tan[c + d*x])^2*(b^2 + b^2*Ta
n[c + d*x]^2)) + (3*((a*(a^6 + 7*a^4*b^2 + 35*a^2*b^4 - 35*b^6)*ArcTan[Tan
[c + d*x]])/(b*(a^2 + b^2)^3) + (8*b^4*(7*a^2 - b^2)*Log[a + b*Tan[c + d*x
]])/(a^2 + b^2)^3 - (4*b^4*(7*a^2 - b^2)*Log[b^2 + b^2*Tan[c + d*x]^2])/(a
^2 + b^2)^3 + (a^4 + 5*a^2*b^2 - 4*b^4)/((a^2 + b^2)*(a + b*Tan[c + d*x])^
2) + (a*(a^4 + 6*a^2*b^2 - 27*b^4))/((a^2 + b^2)^2*(a + b*Tan[c + d*x]))))
/(2*b^2*(a^2 + b^2)))/(4*b^2*(a^2 + b^2))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 41.04 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{b^5}{2(a^2+b^2)^3(a+b\tan(dx+c))^2}-\frac{6b^5a}{(a^2+b^2)^4(a+b\tan(dx+c))}+\frac{3b^5(7a^2-b^2)\ln(a+b\tan(dx+c))}{(a^2+b^2)^5}+\frac{\left(\frac{3}{8}a^7+\frac{21}{8}a^5b^2-\frac{15}{8}a^3b^4-\frac{33}{8}a$
default	$-\frac{b^5}{2(a^2+b^2)^3(a+b\tan(dx+c))^2}-\frac{6b^5a}{(a^2+b^2)^4(a+b\tan(dx+c))}+\frac{3b^5(7a^2-b^2)\ln(a+b\tan(dx+c))}{(a^2+b^2)^5}+\frac{\left(\frac{3}{8}a^7+\frac{21}{8}a^5b^2-\frac{15}{8}a^3b^4-\frac{33}{8}a$
risch	$\frac{15xab}{8ia^5-80ia^3b^2+40ia\,b^4+40a^4b-80a^2b^3+8b^5}-\frac{42ib^5a^2c}{d(a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10})}-\frac{ie^{2i(dx-}}{8(-4ia^3b+4ia\,b^3+}$

input
```
int(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*b^5/(a^2+b^2)^3/(a+b*tan(d*x+c))^2-6*b^5/(a^2+b^2)^4*a/(a+b*tan(
d*x+c))+3*b^5*(7*a^2-b^2)/(a^2+b^2)^5*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^5*(((
3/8*a^7+21/8*a^5*b^2-15/8*a^3*b^4-33/8*a*b^6)*tan(d*x+c)^3+(5*a^4*b^3+4*a^
2*b^5-b^7)*tan(d*x+c)^2+(19/8*a^5*b^2-39/8*a*b^6+5/8*a^7-25/8*a^3*b^4)*tan
(d*x+c)+3/4*a^6*b+25/4*a^4*b^3+17/4*b^5*a^2-5/4*b^7)/(1+tan(d*x+c)^2)^2+3/
16*(-56*a^2*b^5+8*b^7)*ln(1+tan(d*x+c)^2)+3/8*(a^7+7*a^5*b^2+35*a^3*b^4-35
*a*b^6)*arctan(tan(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(284) = 568$.

Time = 0.18 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.27

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3} \, dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/32*(9*a^6*b^3 + 95*a^4*b^5 - 141*a^2*b^7 - 3*b^9 - 8*(a^8*b + 4*a^6*b^3
 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^6 + 8*(a^8*b - 6*a^4*b^5 - 8*
a^2*b^7 - 3*b^9)*cos(d*x + c)^4 - 12*(a^7*b^2 + 7*a^5*b^4 + 35*a^3*b^6 - 3
5*a*b^8)*d*x - (15*a^8*b + 82*a^6*b^3 + 68*a^4*b^5 - 498*a^2*b^7 - 51*b^9
+ 12*(a^9 + 6*a^7*b^2 + 28*a^5*b^4 - 70*a^3*b^6 + 35*a*b^8)*d*x)*cos(d*x +
 c)^2 - 48*(7*a^2*b^7 - b^9 + (7*a^4*b^5 - 8*a^2*b^7 + b^9)*cos(d*x + c)^2
 + 2*(7*a^3*b^6 - a*b^8)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)
*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*(4*(a^9 + 4*a^7*b^2
+ 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cos(d*x + c)^5 + 2*(3*a^9 + 20*a^7*b^2 +
42*a^5*b^4 + 36*a^3*b^6 + 11*a*b^8)*cos(d*x + c)^3 - (3*a^7*b^2 + 53*a^5*b
^4 - 15*a^3*b^6 + 159*a*b^8 - 12*(a^8*b + 7*a^6*b^3 + 35*a^4*b^5 - 35*a^2*
b^7)*d*x)*cos(d*x + c))*sin(d*x + c))/((a^12 + 4*a^10*b^2 + 5*a^8*b^4 - 5*
a^4*b^8 - 4*a^2*b^10 - b^12)*d*cos(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10
*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 + a*b^11)*d*cos(d*x + c)*sin(d*x + c) +
(a^10*b^2 + 5*a^8*b^4 + 10*a^6*b^6 + 10*a^4*b^8 + 5*a^2*b^10 + b^12)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(cos(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(284) = 568$.

Time = 0.15 (sec) , antiderivative size = 738, normalized size of antiderivative = 2.50

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/8*(3*(a^7 + 7*a^5*b^2 + 35*a^3*b^4 - 35*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b
^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 24*(7*a^2*b^5 - b^7)*lo
g(b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*
b^8 + b^10) - 12*(7*a^2*b^5 - b^7)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b
^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + (6*a^6*b + 44*a^4*b^3 -
 62*a^2*b^5 - 4*b^7 + 3*(a^5*b^2 + 6*a^3*b^4 - 27*a*b^6)*tan(d*x + c)^5 +
6*(a^6*b + 6*a^4*b^3 - 13*a^2*b^5 - 2*b^7)*tan(d*x + c)^4 + (3*a^7 + 23*a^
5*b^2 + 61*a^3*b^4 - 151*a*b^6)*tan(d*x + c)^3 + 2*(5*a^6*b + 37*a^4*b^3 -
 73*a^2*b^5 - 9*b^7)*tan(d*x + c)^2 + (5*a^7 + 26*a^5*b^2 + 49*a^3*b^4 - 6
8*a*b^6)*tan(d*x + c))/(a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8
 + (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*tan(d*x + c)^6 + 2
*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)^5 + (a^1
0 + 6*a^8*b^2 + 14*a^6*b^4 + 16*a^4*b^6 + 9*a^2*b^8 + 2*b^10)*tan(d*x + c)
^4 + 4*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)^3
+ (2*a^10 + 9*a^8*b^2 + 16*a^6*b^4 + 14*a^4*b^6 + 6*a^2*b^8 + b^10)*tan(d*
x + c)^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x +
 c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(284) = 568$.

Time = 0.24 (sec) , antiderivative size = 613, normalized size of antiderivative = 2.08

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{3\,(a^7+7\,a^5b^2+35\,a^3b^4-35\,ab^6)(dx+c)}{8\,(a^{10}d+5\,a^8b^2d+10\,a^6b^4d+10\,a^4b^6d+5\,a^2b^8d+b^{10}d)} - \frac{3\,(7\,a^2b^5-b^7)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,(a^{10}d+5\,a^8b^2d+10\,a^6b^4d+10\,a^4b^6d+5\,a^2b^8d+b^{10}d)} + \frac{3\,(7\,a^2b^6-b^8)\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^{10}bd+5\,a^8b^3d+10\,a^6b^5d+10\,a^4b^7d+5\,a^2b^9d+b^{11}d} + \frac{3\,a^5b^2\tan\left(dx+c\right)^5+18\,a^3b^4\tan\left(dx+c\right)^5-81\,ab^6\tan\left(dx+c\right)^5+6\,a^6b\tan\left(dx+c\right)^4+36\,a^4b^3\tan\left(\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
3/8*(a^7 + 7*a^5*b^2 + 35*a^3*b^4 - 35*a*b^6)*(d*x + c)/(a^10*d + 5*a^8*b^
2*d + 10*a^6*b^4*d + 10*a^4*b^6*d + 5*a^2*b^8*d + b^10*d) - 3/2*(7*a^2*b^5
 - b^7)*log(tan(d*x + c)^2 + 1)/(a^10*d + 5*a^8*b^2*d + 10*a^6*b^4*d + 10*
a^4*b^6*d + 5*a^2*b^8*d + b^10*d) + 3*(7*a^2*b^6 - b^8)*log(abs(b*tan(d*x
+ c) + a))/(a^10*b*d + 5*a^8*b^3*d + 10*a^6*b^5*d + 10*a^4*b^7*d + 5*a^2*b
^9*d + b^11*d) + 1/8*(3*a^5*b^2*tan(d*x + c)^5 + 18*a^3*b^4*tan(d*x + c)^5
 - 81*a*b^6*tan(d*x + c)^5 + 6*a^6*b*tan(d*x + c)^4 + 36*a^4*b^3*tan(d*x +
 c)^4 - 78*a^2*b^5*tan(d*x + c)^4 - 12*b^7*tan(d*x + c)^4 + 3*a^7*tan(d*x
+ c)^3 + 23*a^5*b^2*tan(d*x + c)^3 + 61*a^3*b^4*tan(d*x + c)^3 - 151*a*b^6
*tan(d*x + c)^3 + 10*a^6*b*tan(d*x + c)^2 + 74*a^4*b^3*tan(d*x + c)^2 - 14
6*a^2*b^5*tan(d*x + c)^2 - 18*b^7*tan(d*x + c)^2 + 5*a^7*tan(d*x + c) + 26
*a^5*b^2*tan(d*x + c) + 49*a^3*b^4*tan(d*x + c) - 68*a*b^6*tan(d*x + c) +
6*a^6*b + 44*a^4*b^3 - 62*a^2*b^5 - 4*b^7)/((a^8*d + 4*a^6*b^2*d + 6*a^4*b
^4*d + 4*a^2*b^6*d + b^8*d)*(b*tan(d*x + c)^3 + a*tan(d*x + c)^2 + b*tan(d
*x + c) + a)^2)
```

Mupad [B] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 715, normalized size of antiderivative = 2.42

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{3a^6b+22a^4b^3-31a^2b^5-2b^7}{4(a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8)} + \frac{\tan(c+dx)\,(5a^7+26a^5b^2+49a^3b^4-68ab^6)}{8(a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8)} + \frac{3\tan(c+dx)^5\,(a^5b^2+6a^3b^4-27ab^6)}{8(a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8)} + \frac{\tan(c+dx)}{8(a^8}}{d\,\left(\tan(c+dx)^2\,(2a^2+b^2)+\tan(c+dx)^4\,(a^2+2b^2)+a^2+b^2\tan(c+\right.}$$
$$+ \frac{\ln(a+b\tan(c+dx))\left(\frac{21b^5}{(a^2+b^2)^4}-\frac{24b^7}{(a^2+b^2)^5}\right)}{d}$$
$$+ \frac{3\ln(\tan(c+dx)-\mathrm{i})\,(-a^2\,1\mathrm{i}+5ab+b^2\,8\mathrm{i})}{16d\,(a^5+a^4b\,5\mathrm{i}-10a^3b^2-a^2b^3\,10\mathrm{i}+5ab^4+b^5\,1\mathrm{i})}$$
$$+ \frac{3\ln(\tan(c+dx)+1\mathrm{i})\,(a^2\,1\mathrm{i}+5ab-b^2\,8\mathrm{i})}{16d\,(a^5-a^4b\,5\mathrm{i}-10a^3b^2+a^2b^3\,10\mathrm{i}+5ab^4-b^5\,1\mathrm{i})}$$

input
```
int(cos(c + d*x)^4/(a + b*tan(c + d*x))^3,x)
```

output
```
((3*a^6*b - 2*b^7 - 31*a^2*b^5 + 22*a^4*b^3)/(4*(a^8 + b^8 + 4*a^2*b^6 + 6
*a^4*b^4 + 4*a^6*b^2)) + (tan(c + d*x)*(5*a^7 - 68*a*b^6 + 49*a^3*b^4 + 26
*a^5*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (3*tan(c
+ d*x)^5*(6*a^3*b^4 - 27*a*b^6 + a^5*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a
^4*b^4 + 4*a^6*b^2)) + (tan(c + d*x)^3*(3*a^7 - 151*a*b^6 + 61*a^3*b^4 + 2
3*a^5*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (3*tan(c
 + d*x)^4*(a^6*b - 2*b^7 - 13*a^2*b^5 + 6*a^4*b^3))/(4*(a^8 + b^8 + 4*a^2*
b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (tan(c + d*x)^2*(5*a^6*b - 9*b^7 - 73*a^2*
b^5 + 37*a^4*b^3))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)))/(d
*(tan(c + d*x)^2*(2*a^2 + b^2) + tan(c + d*x)^4*(a^2 + 2*b^2) + a^2 + b^2*
tan(c + d*x)^6 + 2*a*b*tan(c + d*x) + 4*a*b*tan(c + d*x)^3 + 2*a*b*tan(c +
 d*x)^5)) + (log(a + b*tan(c + d*x))*((21*b^5)/(a^2 + b^2)^4 - (24*b^7)/(a
^2 + b^2)^5))/d + (3*log(tan(c + d*x) - 1i)*(5*a*b - a^2*1i + b^2*8i))/(16
*d*(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)) + (3*lo
g(tan(c + d*x) + 1i)*(5*a*b + a^2*1i - b^2*8i))/(16*d*(5*a*b^4 - a^4*b*5i
+ a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 1475, normalized size of antiderivative = 5.00

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x)
```

output
```
( - 672*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*a**3*b**7 +
 96*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*a*b**9 + 672*co
s(c + d*x)*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d
*x)*a**3*b**7 - 96*cos(c + d*x)*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x
)/2)*b - a)*sin(c + d*x)*a*b**9 + 4*cos(c + d*x)*sin(c + d*x)**5*a**9*b +
16*cos(c + d*x)*sin(c + d*x)**5*a**7*b**3 + 24*cos(c + d*x)*sin(c + d*x)**
5*a**5*b**5 + 16*cos(c + d*x)*sin(c + d*x)**5*a**3*b**7 + 4*cos(c + d*x)*s
in(c + d*x)**5*a*b**9 - 14*cos(c + d*x)*sin(c + d*x)**3*a**9*b - 72*cos(c
+ d*x)*sin(c + d*x)**3*a**7*b**3 - 132*cos(c + d*x)*sin(c + d*x)**3*a**5*b
**5 - 104*cos(c + d*x)*sin(c + d*x)**3*a**3*b**7 - 30*cos(c + d*x)*sin(c +
 d*x)**3*a*b**9 + 12*cos(c + d*x)*sin(c + d*x)*a**8*b**2*d*x + 84*cos(c +
d*x)*sin(c + d*x)*a**6*b**4*d*x + 420*cos(c + d*x)*sin(c + d*x)*a**4*b**6*
d*x - 420*cos(c + d*x)*sin(c + d*x)*a**2*b**8*d*x + 336*log(tan((c + d*x)/
2)**2 + 1)*sin(c + d*x)**2*a**4*b**6 - 384*log(tan((c + d*x)/2)**2 + 1)*si
n(c + d*x)**2*a**2*b**8 + 48*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*
b**10 - 336*log(tan((c + d*x)/2)**2 + 1)*a**4*b**6 + 48*log(tan((c + d*x)/
2)**2 + 1)*a**2*b**8 - 336*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*
b - a)*sin(c + d*x)**2*a**4*b**6 + 384*log(tan((c + d*x)/2)**2*a - 2*tan((
c + d*x)/2)*b - a)*sin(c + d*x)**2*a**2*b**8 - 48*log(tan((c + d*x)/2)**2*
a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)**2*b**10 + 336*log(tan((c + ...
```

3.580 $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 257

$$\begin{aligned}
\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{5(4a^2+b^2)\sec(c+dx)}{2b^5d}+\frac{5\sec^3(c+dx)}{6b^3d} \\
&\quad -\frac{5a(4a^2+3b^2)\operatorname{arcsinh}(\tan(c+dx))\sec(c+dx)}{2b^6d\sqrt{\sec^2(c+dx)}} \\
&\quad -\frac{5\sqrt{a^2+b^2}(4a^2+b^2)\operatorname{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{2b^6d\sqrt{\sec^2(c+dx)}} \\
&\quad -\frac{5a\sec(c+dx)\tan(c+dx)}{b^4d}-\frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2}+\frac{5a\sec^3(c+dx)}{2b^3d(a+b\tan(c+dx))}
\end{aligned}$$

output
```
5/2*(4*a^2+b^2)*sec(d*x+c)/b^5/d+5/6*sec(d*x+c)^3/b^3/d-5/2*a*(4*a^2+3*b^2
)*arcsinh(tan(d*x+c))*sec(d*x+c)/b^6/d/(sec(d*x+c)^2)^(1/2)-5/2*(a^2+b^2)^
(1/2)*(4*a^2+b^2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)^
(1/2))*sec(d*x+c)/b^6/d/(sec(d*x+c)^2)^(1/2)-5*a*sec(d*x+c)*tan(d*x+c)/b^4
/d-1/2*sec(d*x+c)^5/b/d/(a+b*tan(d*x+c))^2+5/2*a*sec(d*x+c)^3/b^3/d/(a+b*t
an(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 688, normalized size of antiderivative = 2.68

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{6b^2(a^2+b^2)^2\sin(c+dx)}{a}+\frac{6(a-ib)(a+ib)b(8a^2-b^2)(a\cos(c+dx)+b\sin(c+dx))}{a}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*((6*b^2*(a^2 + b^2)^2*Si
n[c + d*x])/a + (6*(a - I*b)*(a + I*b)*b*(8*a^2 - b^2)*(a*Cos[c + d*x] + b
*Sin[c + d*x]))/a + 2*b*(36*a^2 + 13*b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x]
)^2 + 60*Sqrt[a^2 + b^2]*(4*a^2 + b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/S
qrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + 30*a*(4*a^2 + 3*b^2)
*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] + b*Sin[c + d*x]
)^2 - 30*a*(4*a^2 + 3*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*Cos
[c + d*x] + b*Sin[c + d*x])^2 + (b^2*(-9*a + b)*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^3*Sin[(c + d*x)/
2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])^3 + (2*b*(36*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b^3*Sin[(c + d*x)/2]
*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^3 + (b^2*(9*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^2 - (2*b*(36*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(12*b
^6*d*(a + b*Tan[c + d*x])^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.87, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3992, 492, 590, 25, 27, 682, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^7}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{\left(\tan^2(c+dx)+1\right)^{5/2}}{(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{492}$$

$$\frac{\sec(c+dx)\left(\frac{5\int \frac{b\tan(c+dx)\left(\tan^2(c+dx)+1\right)^{3/2}}{(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{2b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{5/2}}{2(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{590}$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{\left(\tan^2(c+dx)+1\right)^{3/2}(4a+b\tan(c+dx))}{3(a+b\tan(c+dx))}-\int -\frac{\left(b^2-4ab\tan(c+dx)\right)\sqrt{\tan^2(c+dx)+1}}{b^2(a+b\tan(c+dx))}d(b\tan(c+dx))\right)}{2b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{5/2}}{2(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\int \frac{\left(b^2-4ab\tan(c+dx)\right)\sqrt{\tan^2(c+dx)+1}}{b^2(a+b\tan(c+dx))}d(b\tan(c+dx))+\frac{\left(\tan^2(c+dx)+1\right)^{3/2}(4a+b\tan(c+dx))}{3(a+b\tan(c+dx))}\right)}{2b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{5/2}}{2(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{\int \frac{\left(b^2-4ab\tan(c+dx)\right)\sqrt{\tan^2(c+dx)+1}}{a+b\tan(c+dx)}d(b\tan(c+dx))}{b^2}+\frac{\left(\tan^2(c+dx)+1\right)^{3/2}(4a+b\tan(c+dx))}{3(a+b\tan(c+dx))}\right)}{2b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{5/2}}{2(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 682$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{\frac{1}{2}b^2\int \frac{2\left(\left(\frac{2a^2}{b^2}+1\right)b^4-ab\left(4a^2+3b^2\right)\tan(c+dx)\right)}{b^4(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))+\sqrt{\tan^2(c+dx)+1}\left(b^2\left(\frac{4a^2}{b^2}+1\right)-2ab\tan(c+dx)\right)}{b^2}+\frac{\left(\tan^2(c+dx)+1\right)^{3/\ldots}}{3(a+b\tan\ldots}\ldots\right)}{2b^2}\ldots\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{\frac{\int \frac{b^2\left(2a^2+b^2\right)-ab\left(4a^2+3b^2\right)\tan(c+dx)}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{b^2}+\sqrt{\tan^2(c+dx)+1}\left(b^2\left(\frac{4a^2}{b^2}+1\right)-2ab\tan(c+dx)\right)}{b^2}+\frac{\left(\tan^2(c+dx)+1\right)^{3/2}(4a+b\ldots}{3(a+b\tan(c+dx)\ldots}\ldots\right)}{2b^2}\ldots\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 719$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{\frac{\left(a^2+b^2\right)\left(4a^2+b^2\right)\int\frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-a\left(4a^2+3b^2\right)\int\frac{1}{\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{b^2}}{b^2}+\sqrt{\tan^2(c+dx)+\cdots}\right.}{2b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 222$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{\frac{\left(a^2+b^2\right)\left(4a^2+b^2\right)\int\frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-ab\left(4a^2+3b^2\right)\text{arcsinh}(\tan(c+dx))}{b^2}}{b^2}+\sqrt{\tan^2(c+dx)+1}\left(b^2\left(\frac{4a^2}{b^2}\cdots\right.\right.\right.}{2b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 488$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{\frac{-\left(a^2+b^2\right)\left(4a^2+b^2\right)\int\frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}-ab\left(4a^2+3b^2\right)\text{arcsinh}(\tan(c+dx))}{b^2}}{b^2}+\sqrt{\tan^2(c+dx)+1}\left(b^2\left(\frac{4a^2}{b^2}+1\right)\cdots\right.\right.}{2b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 219$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{\frac{-ab(4a^2+3b^2)\operatorname{arcsinh}(\tan(c+dx))-b\sqrt{a^2+b^2}(4a^2+b^2)\operatorname{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2}+\sqrt{\tan^2(c+dx)+1}\left(b^2\left(\frac{4a^2}{b^2}+1\right)-2ab\tan(c+dx\right.}{b^2}\right.}{2b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]*(-1/2*(1 + Tan[c + d*x]^2)^(5/2)/(a + b*Tan[c + d*x])^2 + (5
*(((4*a + b*Tan[c + d*x])*(1 + Tan[c + d*x]^2)^(3/2))/(3*(a + b*Tan[c + d*
x])) + ((-(a*b*(4*a^2 + 3*b^2)*ArcSinh[Tan[c + d*x]]) - b*Sqrt[a^2 + b^2]*
(4*a^2 + b^2)*ArcTanh[(b^2*Tan[c + d*x])/Sqrt[a^2 + b^2]])/b^2 + ((1 + (4*
a^2)/b^2)*b^2 - 2*a*b*Tan[c + d*x])*Sqrt[1 + Tan[c + d*x]^2])/b^2))/(2*b^2
)))/(b*d*Sqrt[Sec[c + d*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 590
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 1)*x)/(d^2*(
n + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 1)*(n + 2*p + 2)))   Int[(
c + d*x)^(n + 1)*(a + b*x^2)^(p - 1)*(a*d*(n + 1) + b*c*(2*p + 1)*x), x], x
] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n, -1] &&  !ILtQ[n + 2*p +
1, 0]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 291.07 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.73

method	result
derivativedivides	$-\frac{2\left(\frac{\frac{b^2\left(7a^4+5b^2a^2-2b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{b\left(8a^6-9a^4b^2-15a^2b^4+2b^6\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a^2}-\frac{b^2\left(25a^4+23b^2a^2-2b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-4}{\left(a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}\right.}{b^6}$
default	$-\frac{2\left(\frac{\frac{b^2\left(7a^4+5b^2a^2-2b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{b\left(8a^6-9a^4b^2-15a^2b^4+2b^6\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a^2}-\frac{b^2\left(25a^4+23b^2a^2-2b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-4}{\left(a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}\right.}{b^6}$
risch	$60a^4e^{9i(dx+c)}-15b^4e^{9i(dx+c)}+240a^4e^{7i(dx+c)}-20b^4e^{7i(dx+c)}+360a^4e^{5i(dx+c)}+22b^4e^{5i(dx+c)}+15a^2b^2e^{9i(dx+c)}+14$

input
```
int(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/b^6*((1/2*b^2*(7*a^4+5*a^2*b^2-2*b^4)/a*tan(1/2*d*x+1/2*c)^3+1/2*b
*(8*a^6-9*a^4*b^2-15*a^2*b^4+2*b^6)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*b^2*(25*a
^4+23*a^2*b^2-2*b^4)/a*tan(1/2*d*x+1/2*c)-4*a^4*b-7/2*a^2*b^3+1/2*b^5)/(a*
tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)^2-5/2*(4*a^4+5*a^2*b^2+b^4)
/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
)+1/3/b^3/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-3*a+b)/b^4/(tan(1/2*d*x+1/2*c)+1)
^2-1/2*(-12*a^2+3*a*b-5*b^2)/b^5/(tan(1/2*d*x+1/2*c)+1)-5/2*a*(4*a^2+3*b^2
)/b^6*ln(tan(1/2*d*x+1/2*c)+1)-1/3/b^3/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(3*a+b
)/b^4/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(12*a^2+3*a*b+5*b^2)/b^5/(tan(1/2*d*x+1
/2*c)-1)+5/2*a*(4*a^2+3*b^2)/b^6*ln(tan(1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. 2(237) = 474.

Time = 0.18 (sec) , antiderivative size = 564, normalized size of antiderivative = 2.19

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{4\,b^5+30\,(4\,a^4b+a^2b^3-b^5)\cos{(dx+c)}^4+20\,(2\,a^2b^3+b^5)\cos{(dx+c)}^2+15\,((4\,a^4-3\,a^2b^2-b^4)\cos{(}}{}$$

input
```
integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(4*b^5 + 30*(4*a^4*b + a^2*b^3 - b^5)*cos(d*x + c)^4 + 20*(2*a^2*b^3
+ b^5)*cos(d*x + c)^2 + 15*((4*a^4 - 3*a^2*b^2 - b^4)*cos(d*x + c)^5 + 2*(
4*a^3*b + a*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^2*b^2 + b^4)*cos(d*x +
 c)^3)*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)
*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(
d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
 b^2)) - 15*((4*a^5 - a^3*b^2 - 3*a*b^4)*cos(d*x + c)^5 + 2*(4*a^4*b + 3*a
^2*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^3
)*log(sin(d*x + c) + 1) + 15*((4*a^5 - a^3*b^2 - 3*a*b^4)*cos(d*x + c)^5 +
 2*(4*a^4*b + 3*a^2*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^3*b^2 + 3*a*b^
4)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 10*(a*b^4*cos(d*x + c) - 6*(3*
a^3*b^2 + 2*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^7*d*cos(d*x + c)^4
*sin(d*x + c) + b^8*d*cos(d*x + c)^3 + (a^2*b^6 - b^8)*d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^7{(c+dx)}}{(a+b\tan{(c+dx)})^3}\,dx$$

input
```
integrate(sec(d*x+c)**7/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**7/(a + b*tan(c + d*x))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(237) = 474$.

Time = 0.14 (sec) , antiderivative size = 902, normalized size of antiderivative = 3.51

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/6*(2*(60*a^6 + 35*a^4*b^2 - 3*a^2*b^4 + (210*a^5*b + 125*a^3*b^3 - 6*a*b
^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*(120*a^6 - 10*a^4*b^2 - 55*a^2*b^4
 + 3*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(330*a^5*b + 205*a^3*b^3
 - 12*a*b^5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*(180*a^6 - 95*a^4*b^2
 - 120*a^2*b^4 + 9*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 12*(60*a^5*b
 + 35*a^3*b^3 - 3*a*b^5)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 6*(40*a^6 -
 30*a^4*b^2 - 35*a^2*b^4 + 3*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*
(50*a^5*b + 25*a^3*b^3 - 4*a*b^5)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3*
(20*a^6 - 15*a^4*b^2 - 15*a^2*b^4 + 2*b^6)*sin(d*x + c)^8/(cos(d*x + c) +
1)^8 + 3*(10*a^5*b + 5*a^3*b^3 - 2*a*b^5)*sin(d*x + c)^9/(cos(d*x + c) + 1
)^9)/(a^4*b^5 + 4*a^3*b^6*sin(d*x + c)/(cos(d*x + c) + 1) - 16*a^3*b^6*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 24*a^3*b^6*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 - 16*a^3*b^6*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 4*a^3*b^6*sin(d
*x + c)^9/(cos(d*x + c) + 1)^9 - a^4*b^5*sin(d*x + c)^10/(cos(d*x + c) + 1
)^10 - (5*a^4*b^5 - 4*a^2*b^7)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*(5*
a^4*b^5 - 6*a^2*b^7)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2*(5*a^4*b^5 -
6*a^2*b^7)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + (5*a^4*b^5 - 4*a^2*b^7)*s
in(d*x + c)^8/(cos(d*x + c) + 1)^8) - 15*(4*a^3 + 3*a*b^2)*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/b^6 + 15*(4*a^3 + 3*a*b^2)*log(sin(d*x + c)/(cos
(d*x + c) + 1) - 1)/b^6 - 15*(4*a^4 + 5*a^2*b^2 + b^4)*log((b - a*sin(d...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(237) = 474$.

Time = 0.40 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.98

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/6*(15*(4*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^6 - 15*(4*
a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^6 + 15*(4*a^4 + 5*a^2*
b^2 + b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs
(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6
) + 2*(9*a*b*tan(1/2*d*x + 1/2*c)^5 + 36*a^2*tan(1/2*d*x + 1/2*c)^4 + 18*b
^2*tan(1/2*d*x + 1/2*c)^4 - 72*a^2*tan(1/2*d*x + 1/2*c)^2 - 24*b^2*tan(1/2
*d*x + 1/2*c)^2 - 9*a*b*tan(1/2*d*x + 1/2*c) + 36*a^2 + 14*b^2)/((tan(1/2*
d*x + 1/2*c)^2 - 1)^3*b^5) + 6*(7*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 5*a^3*b^3
*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 8*a^6*tan(1/2*d
*x + 1/2*c)^2 - 9*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*b^4*tan(1/2*d*x
+ 1/2*c)^2 + 2*b^6*tan(1/2*d*x + 1/2*c)^2 - 25*a^5*b*tan(1/2*d*x + 1/2*c)
- 23*a^3*b^3*tan(1/2*d*x + 1/2*c) + 2*a*b^5*tan(1/2*d*x + 1/2*c) - 8*a^6 -
 7*a^4*b^2 + a^2*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c
) - a)^2*a^2*b^5))/d
```

Mupad [B] (verification not implemented)

Time = 4.03 (sec) , antiderivative size = 1203, normalized size of antiderivative = 4.68

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^7*(a + b*tan(c + d*x))^3),x)
```

output
```
((60*a^4 - 3*b^4 + 35*a^2*b^2)/(3*b^5) + (tan(c/2 + (d*x)/2)*(210*a^4 - 6*
b^4 + 125*a^2*b^2))/(3*a*b^4) + (tan(c/2 + (d*x)/2)^8*(20*a^6 + 2*b^6 - 15
*a^2*b^4 - 15*a^4*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^6*(40*a^6 + 3*b^
6 - 35*a^2*b^4 - 30*a^4*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^2*(120*a^6
 + 3*b^6 - 55*a^2*b^4 - 10*a^4*b^2))/(3*a^2*b^5) + (2*tan(c/2 + (d*x)/2)^4
*(180*a^6 + 9*b^6 - 120*a^2*b^4 - 95*a^4*b^2))/(3*a^2*b^5) + (tan(c/2 + (d
*x)/2)^9*(10*a^4 - 2*b^4 + 5*a^2*b^2))/(a*b^4) - (2*tan(c/2 + (d*x)/2)^7*(
50*a^4 - 4*b^4 + 25*a^2*b^2))/(a*b^4) + (4*tan(c/2 + (d*x)/2)^5*(60*a^4 -
3*b^4 + 35*a^2*b^2))/(a*b^4) - (2*tan(c/2 + (d*x)/2)^3*(330*a^4 - 12*b^4 +
 205*a^2*b^2))/(3*a*b^4))/(d*(tan(c/2 + (d*x)/2)^8*(5*a^2 - 4*b^2) - tan(c
/2 + (d*x)/2)^2*(5*a^2 - 4*b^2) + tan(c/2 + (d*x)/2)^4*(10*a^2 - 12*b^2) -
 tan(c/2 + (d*x)/2)^6*(10*a^2 - 12*b^2) - a^2*tan(c/2 + (d*x)/2)^10 + a^2
- 16*a*b*tan(c/2 + (d*x)/2)^3 + 24*a*b*tan(c/2 + (d*x)/2)^5 - 16*a*b*tan(c
/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2)^9 + 4*a*b*tan(c/2 + (d*x)/2)))
- (atanh((3000*a^2*tan(c/2 + (d*x)/2))/(3000*a^2 + (7000*a^4)/b^2 + (4000*
a^6)/b^4) + (7000*a^4*tan(c/2 + (d*x)/2))/(7000*a^4 + 3000*a^2*b^2 + (4000
*a^6)/b^2) + (4000*a^6*tan(c/2 + (d*x)/2))/(4000*a^6 + 3000*a^2*b^4 + 7000
*a^4*b^2))*(15*a*b^2 + 20*a^3))/(b^6*d) + (5*atanh((1000*a^2*(a^2 + b^2)^(
1/2))/(1000*a^2*b + (5000*a^4)/b + (4000*a^6)/b^3 + 10000*a^3*tan(c/2 + (d
*x)/2) + 2000*a*b^2*tan(c/2 + (d*x)/2) + (8000*a^5*tan(c/2 + (d*x)/2))/...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10737, normalized size of antiderivative = 41.78

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x)
```

output
```
( - 240*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b*
*2))*cos(c + d*x)*sin(c + d*x)**6*tan(c + d*x)**2*a**4*b**2*i + 180*sqrt(a
**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d
*x)*sin(c + d*x)**6*tan(c + d*x)**2*a**2*b**4*i + 60*sqrt(a**2 + b**2)*ata
n((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x
)**6*tan(c + d*x)**2*b**6*i - 480*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)
*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**6*tan(c + d*x)*a
**5*b*i + 360*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**
2 + b**2))*cos(c + d*x)*sin(c + d*x)**6*tan(c + d*x)*a**3*b**3*i + 120*sqr
t(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c
+ d*x)*sin(c + d*x)**6*tan(c + d*x)*a*b**5*i - 240*sqrt(a**2 + b**2)*atan(
(tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)*
*6*a**6*i + 180*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a
**2 + b**2))*cos(c + d*x)*sin(c + d*x)**6*a**4*b**2*i + 60*sqrt(a**2 + b**
2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c
 + d*x)**6*a**2*b**4*i + 720*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i
- b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**4*tan(c + d*x)**2*a**
4*b**2*i - 300*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a*
*2 + b**2))*cos(c + d*x)*sin(c + d*x)**4*tan(c + d*x)**2*a**2*b**4*i - 120
*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))...
```

3.581 $\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{3a\text{arctanh}(\sin(c+dx))}{b^4d} - \frac{3(2a^2+b^2)\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^4\sqrt{a^2+b^2}d} - \frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{3\sec(c+dx)(2a+b\tan(c+dx))}{2b^3d(a+b\tan(c+dx))}$$

output
```
-3*a*arctanh(sin(d*x+c))/b^4/d-3/2*(2*a^2+b^2)*arctanh((b*cos(d*x+c)-a*sin
(d*x+c))/(a^2+b^2)^(1/2))/b^4/(a^2+b^2)^(1/2)/d-1/2*sec(d*x+c)^3/b/d/(a+b*
tan(d*x+c))^2+3/2*sec(d*x+c)*(2*a+b*tan(d*x+c))/b^3/d/(a+b*tan(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 396 vs. $2(148) = 296$.

Time = 2.22 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.68

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{b^2(a^2+b^2)\sin(c+dx)}{a}+\frac{(2a-b)b(2a+b)(a\cos(c+dx)+b\sin(c+dx))}{a}+2b(a\,c\right.}{}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*((b^2*(a^2 + b^2)*Sin[c
+ d*x])/a + ((2*a - b)*b*(2*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x]))/a +
2*b*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + (6*(2*a^2 + b^2)*ArcTanh[(-b + a
*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/S
qrt[a^2 + b^2] + 6*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d
*x] + b*Sin[c + d*x])^2 - 6*a*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*
Cos[c + d*x] + b*Sin[c + d*x])^2 + (2*b*Sin[(c + d*x)/2]*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b*Sin[(c +
d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])))/(2*b^4*d*(a + b*Tan[c + d*x])^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3992, 492, 590, 25, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{492}$$

$$\frac{\sec(c+dx)\left(\frac{3\int \frac{b\tan(c+dx)\sqrt{\tan^2(c+dx)+1}}{(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{2b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{2(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{590}$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{\sqrt{\tan^2(c+dx)+1}(2a+b\tan(c+dx))}{a+b\tan(c+dx)}-\int -\frac{b^2-2ab\tan(c+dx)}{b^2(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))\right)}{2b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{2(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\int \frac{b^2-2ab\tan(c+dx)}{b^2(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))+\frac{\sqrt{\tan^2(c+dx)+1}(2a+b\tan(c+dx))}{a+b\tan(c+dx)}\right)}{2b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{2(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{\int \frac{b^2-2ab\tan(c+dx)}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{b^2}+\frac{\sqrt{\tan^2(c+dx)+1}(2a+b\tan(c+dx))}{a+b\tan(c+dx)}\right)}{2b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{2(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{719}$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{\left(2a^2+b^2\right)\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-2a\int \frac{1}{\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{b^2}+\frac{\sqrt{\tan^2(c+dx)+1}(2a+b\tan(c+dx))}{a+b\tan(c+dx)}\right.}{2b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 222$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{\left(2a^2+b^2\right)\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-2ab\,\text{arcsinh}(\tan(c+dx))}{b^2}+\frac{\sqrt{\tan^2(c+dx)+1}(2a+b\tan(c+dx))}{a+b\tan(c+dx)}\right)}{2b^2}-\frac{(\mathrm{ta}}{2(\mathrm{c}}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 488$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{-\left(2a^2+b^2\right)\int \frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}-2ab\,\text{arcsinh}(\tan(c+dx))}{b^2}+\frac{\sqrt{\tan^2(c+dx)+1}(2a+b\tan(c+dx))}{a+b\tan(c+dx)}\right)}{2b^2}-\frac{(\tan^2(c}{2(a+b\,\mathrm{t}}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 219$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{-\frac{b\left(2a^2+b^2\right)\text{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-2ab\,\text{arcsinh}(\tan(c+dx))}{b^2}+\frac{\sqrt{\tan^2(c+dx)+1}(2a+b\tan(c+dx))}{a+b\tan(c+dx)}\right)}{2b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{2(a+b\tan(c+dx))^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]*(-1/2*(1 + Tan[c + d*x]^2)^(3/2)/(a + b*Tan[c + d*x])^2 + (3
*((-2*a*b*ArcSinh[Tan[c + d*x]] - (b*(2*a^2 + b^2)*ArcTanh[(b^2*Tan[c + d*
x])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2])/b^2 + ((2*a + b*Tan[c + d*x])*Sqrt[
1 + Tan[c + d*x]^2])/(a + b*Tan[c + d*x])))/(2*b^2)))/(b*d*Sqrt[Sec[c + d*
x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 590
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 1)*x)/(d^2*(
n + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 1)*(n + 2*p + 2)))   Int[(
c + d*x)^(n + 1)*(a + b*x^2)^(p - 1)*(a*d*(n + 1) + b*c*(2*p + 1)*x), x], x
] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n, -1] &&  !ILtQ[n + 2*p +
1, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 93.88 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{\frac{1}{b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^4}-\frac{1}{b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^4}-\frac{2\left(\frac{\frac{b^2\left(3a^2-2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}}{}\right.}{}}{d}$
default	$\frac{\frac{1}{b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^4}-\frac{1}{b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^4}-\frac{2\left(\frac{\frac{b^2\left(3a^2-2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}}{}\right.}{}}{d}$
risch	$\frac{-9iab\,\mathrm{e}^{5i(dx+c)}+6a^2\mathrm{e}^{5i(dx+c)}-3b^2\mathrm{e}^{5i(dx+c)}+12a^2\mathrm{e}^{3i(dx+c)}+2b^2\mathrm{e}^{3i(dx+c)}+9iab\,\mathrm{e}^{i(dx+c)}+6a^2\mathrm{e}^{i(dx+c)}-3b^2\mathrm{e}^{i(dx+c)}}{\left(\mathrm{e}^{2i(dx+c)}+1\right)\left(-ib\,\mathrm{e}^{2i(dx+c)}+a\,\mathrm{e}^{2i(dx+c)}+ib+a\right)^2b^3d}$

input
```
int(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^3/(tan(1/2*d*x+1/2*c)+1)-3*a/b^4*ln(tan(1/2*d*x+1/2*c)+1)-1/b^3/(
tan(1/2*d*x+1/2*c)-1)+3*a/b^4*ln(tan(1/2*d*x+1/2*c)-1)-2/b^4*((1/2*b^2*(3*
a^2-2*b^2)/a*tan(1/2*d*x+1/2*c)^3+1/2*b*(4*a^4-9*a^2*b^2+2*b^4)/a^2*tan(1/
2*d*x+1/2*c)^2-1/2*b^2*(13*a^2-2*b^2)/a*tan(1/2*d*x+1/2*c)-2*a^2*b+1/2*b^3
)/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)^2-3/2*(2*a^2+b^2)/(a^2
+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(138) = 276$.

Time = 0.14 (sec) , antiderivative size = 513, normalized size of antiderivative = 3.47

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{4\,a^2b^3+4\,b^5+6\,(2\,a^4b+a^2b^3-b^5)\cos(dx+c)^2+18\,(a^3b^2+ab^4)\cos(dx+c)\sin(dx+c)+3\,((2\,a^4-}{}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(4*a^2*b^3 + 4*b^5 + 6*(2*a^4*b + a^2*b^3 - b^5)*cos(d*x + c)^2 + 18*(
a^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*((2*a^4 - a^2*b^2 - b^4)*co
s(d*x + c)^3 + 2*(2*a^3*b + a*b^3)*cos(d*x + c)^2*sin(d*x + c) + (2*a^2*b^
2 + b^4)*cos(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x +
c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d
*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*
cos(d*x + c)^2 + b^2)) - 6*((a^5 - a*b^4)*cos(d*x + c)^3 + 2*(a^4*b + a^2*
b^3)*cos(d*x + c)^2*sin(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c))*log(sin
(d*x + c) + 1) + 6*((a^5 - a*b^4)*cos(d*x + c)^3 + 2*(a^4*b + a^2*b^3)*cos
(d*x + c)^2*sin(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x +
c) + 1))/((a^4*b^4 - b^8)*d*cos(d*x + c)^3 + 2*(a^3*b^5 + a*b^7)*d*cos(d*x
 + c)^2*sin(d*x + c) + (a^2*b^6 + b^8)*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*tan(c + d*x))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(138) = 276$.

Time = 0.14 (sec) , antiderivative size = 518, normalized size of antiderivative = 3.50

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{2\left(6a^4-a^2b^2+\frac{(21a^3b-2ab^3)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2(6a^4-9a^2b^2+b^4)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4(6a^3b-ab^3)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{(6a^4-9a^2b^2+2b^4)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(3a^3b-2ab^3)\cdots}{(\cos(dx+\cdots}\cdots}{a^4b^3+\frac{4a^3b^4\sin(dx+c)}{\cos(dx+c)+1}-\frac{8a^3b^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{4a^3b^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^4b^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{(3a^4b^3-4a^2b^5)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{(3a^4b^3-4a^2b^5)\sin(dx+c)\cdots}{(\cos(dx+c)+1)^4}\cdots}\cdots}{2d}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(2*(6*a^4 - a^2*b^2 + (21*a^3*b - 2*a*b^3)*sin(d*x + c)/(cos(d*x + c)
+ 1) - 2*(6*a^4 - 9*a^2*b^2 + b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4
*(6*a^3*b - a*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + (6*a^4 - 9*a^2*b^
2 + 2*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (3*a^3*b - 2*a*b^3)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5)/(a^4*b^3 + 4*a^3*b^4*sin(d*x + c)/(cos(d*x
 + c) + 1) - 8*a^3*b^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4*a^3*b^4*sin
(d*x + c)^5/(cos(d*x + c) + 1)^5 - a^4*b^3*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 - (3*a^4*b^3 - 4*a^2*b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (3*a^
4*b^3 - 4*a^2*b^5)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 6*a*log(sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)/b^4 + 6*a*log(sin(d*x + c)/(cos(d*x + c) + 1)
 - 1)/b^4 - 3*(2*a^2 + b^2)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + s
qrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*b^4))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(138) = 276$.

Time = 0.38 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.12

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4}-\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4}+\frac{3\,(2\,a^2+b^2)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^4}+\frac{4}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}}{}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*(6*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 6*a*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/b^4 + 3*(2*a^2 + b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2
*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 +
b^2)))/(sqrt(a^2 + b^2)*b^4) + 4/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^3) + 2*(3
*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*tan
(1/2*d*x + 1/2*c)^2 - 9*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 2*b^4*tan(1/2*d*x
 + 1/2*c)^2 - 13*a^3*b*tan(1/2*d*x + 1/2*c) + 2*a*b^3*tan(1/2*d*x + 1/2*c)
 - 4*a^4 + a^2*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c)
- a)^2*a^2*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 2.87 (sec) , antiderivative size = 1311, normalized size of antiderivative = 8.86

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^5*(a + b*tan(c + d*x))^3),x)
```

output
```
((6*a^2 - b^2)/b^3 - (2*tan(c/2 + (d*x)/2)^2*(6*a^4 + b^4 - 9*a^2*b^2))/(a
^2*b^3) + (tan(c/2 + (d*x)/2)*(21*a^2 - 2*b^2))/(a*b^2) + (tan(c/2 + (d*x)
/2)^4*(6*a^4 + 2*b^4 - 9*a^2*b^2))/(a^2*b^3) - (4*tan(c/2 + (d*x)/2)^3*(6*
a^2 - b^2))/(a*b^2) + (tan(c/2 + (d*x)/2)^5*(3*a^2 - 2*b^2))/(a*b^2))/(d*(
tan(c/2 + (d*x)/2)^4*(3*a^2 - 4*b^2) - tan(c/2 + (d*x)/2)^2*(3*a^2 - 4*b^2
) - a^2*tan(c/2 + (d*x)/2)^6 + a^2 - 8*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*ta
n(c/2 + (d*x)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2))) - (6*a*atanh(tan(c/2 + (d*
x)/2)))/(b^4*d) + (atan((((2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b^5 +
 (8*tan(c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5 + 72*a^5*b^3))/b^9 - (3*(2*a
^2 + b^2)*(a^2 + b^2)^(1/2)*((8*tan(c/2 + (d*x)/2)*(12*a*b^10 + 24*a^3*b^8
))/b^9 - 48*a^2 + (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan(
c/2 + (d*x)/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(2*(b^6 + a^2*b^4))))/(2*(b
^6 + a^2*b^4)))*3i)/(2*(b^6 + a^2*b^4)) + ((2*a^2 + b^2)*(a^2 + b^2)^(1/2)
*((288*a^4)/b^5 + (8*tan(c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5 + 72*a^5*b^
3))/b^9 - (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(48*a^2 - (8*tan(c/2 + (d*x)/
2)*(12*a*b^10 + 24*a^3*b^8))/b^9 + (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(32*
a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(2*(b^6 +
a^2*b^4))))/(2*(b^6 + a^2*b^4)))*3i)/(2*(b^6 + a^2*b^4)))/((16*(54*a^4 + 2
7*a^2*b^2))/b^8 - (16*tan(c/2 + (d*x)/2)*(216*a^5 + 108*a^3*b^2))/b^9 - (3
*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b^5 + (8*tan(c/2 + (d*x)/2)...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 7694, normalized size of antiderivative = 51.99

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x)
```

output

```
( - 24*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**
2))*cos(c + d*x)*sin(c + d*x)**4*tan(c + d*x)**2*a**4*b**2*i + 12*sqrt(a**
2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x
)*sin(c + d*x)**4*tan(c + d*x)**2*a**2*b**4*i + 12*sqrt(a**2 + b**2)*atan(
(tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)*
*4*tan(c + d*x)**2*b**6*i - 48*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*
i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**4*tan(c + d*x)*a**5
*b*i + 24*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 +
b**2))*cos(c + d*x)*sin(c + d*x)**4*tan(c + d*x)*a**3*b**3*i + 24*sqrt(a**
2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x
)*sin(c + d*x)**4*tan(c + d*x)*a*b**5*i - 24*sqrt(a**2 + b**2)*atan((tan((
c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**4*a**
6*i + 12*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b
**2))*cos(c + d*x)*sin(c + d*x)**4*a**4*b**2*i + 12*sqrt(a**2 + b**2)*atan
((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)
**4*a**2*b**4*i + 48*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/s
qrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*tan(c + d*x)**2*a**4*b**2*i
 - 12*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2
))*cos(c + d*x)*sin(c + d*x)**2*tan(c + d*x)**2*b**6*i + 96*sqrt(a**2 + b*
*2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*s...
```

3.582 $\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d} - \frac{\sec(c+dx)(b-a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}$$

output
```
-1/2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/
d-1/2*sec(d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.39

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\left(a^2+b^2\right)\left(-b\cos(c+dx)+a\sin(c+dx)\right)+2\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)(a\cos(c+dx)+b\sin(}{2(a-ib)^2(a+ib)^2d(a\cos(c+dx)+b\sin(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

output
```
((a^2 + b^2)*(-(b*Cos[c + d*x]) + a*Sin[c + d*x]) + 2*Sqrt[a^2 + b^2]*ArcT
anh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^2)/(2*(a - I*b)^2*(a + I*b)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2
)
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3992, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^3}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{\sqrt{\tan^2(c+dx)+1}}{(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{486}$$

$$\frac{\sec(c+dx)\left(\frac{\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2(a^2+b^2)}-\frac{\sqrt{\tan^2(c+dx)+1}(b^2-ab\tan(c+dx))}{2(a^2+b^2)(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{488}$$

$$\frac{\sec(c+dx)\left(-\frac{\int \frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}}{2(a^2+b^2)}-\frac{\sqrt{\tan^2(c+dx)+1}(b^2-ab\tan(c+dx))}{2(a^2+b^2)(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\downarrow \text{219}$$

$$\frac{\sec(c+dx)\left(-\frac{b\text{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}-\frac{\sqrt{\tan^2(c+dx)+1}(b^2-ab\tan(c+dx))}{2(a^2+b^2)(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]*(-1/2*(b*ArcTanh[(b^2*Tan[c + d*x])/Sqrt[a^2 + b^2]])/(a^2 +
 b^2)^(3/2) - ((b^2 - a*b*Tan[c + d*x])*Sqrt[1 + Tan[c + d*x]^2])/(2*(a^2
+ b^2)*(a + b*Tan[c + d*x])^2)))/(b*d*Sqrt[Sec[c + d*x]^2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(87) = 174$.

Time = 16.75 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.01

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a(a^2+b^2)}-\frac{b(a^2-2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2(a^2+b^2)a^2}-\frac{(a^2-2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^2+b^2)a}+\frac{b}{2a^2+2b^2}\right)}{\left(a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}{2\sqrt{a^2+b^2}}\right.}{(a^2+b^2)^{\frac{3}{2}}}}{d}$
default	$\frac{-\frac{2\left(-\frac{(a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a(a^2+b^2)}-\frac{b(a^2-2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2(a^2+b^2)a^2}-\frac{(a^2-2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^2+b^2)a}+\frac{b}{2a^2+2b^2}\right)}{\left(a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}{2\sqrt{a^2+b^2}}\right.}{(a^2+b^2)^{\frac{3}{2}}}}{d}$
risch	$\frac{e^{i(dx+c)}\left(ia\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}-ia+b\right)}{\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^2(-ia+b)d(ia+b)}+\frac{\ln\left(e^{i(dx+c)}+\frac{ia^3+ia\,b^2-a^2b-b^3}{(a^2+b^2)^{\frac{3}{2}}}\right)}{2(a^2+b^2)^{\frac{3}{2}}d}-\frac{\ln\left(e^{i(dx+c)}-\frac{ia^3+ia\,b^2}{(a^2+}\right.}{2(a^2+b^2)^{\frac{3}{2}}d}$

input
```
int(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(a^2+2*b^2)/a/(a^2+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*b*(a^2-2*b^
2)/(a^2+b^2)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*(a^2-2*b^2)/(a^2+b^2)/a*tan(1/2*
d*x+1/2*c)+1/2*b/(a^2+b^2))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)
-a)^2+1/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)
^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(88) = 176.

Time = 0.10 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.09

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)\sqrt{a^2+b^2}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\,c}{2\,ab\cos(dx+c)}\right.}{4\left(\left(a^6+a^4b^2-a^2b^4-b^6\right)d\cos\left(dx+c\right)^2+2\left(a^5b+2\,a^3b^3+\right.\right.}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*
sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*
x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c
)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))
- 2*(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^6 + a^4
*b^2 - a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d*c
os(d*x + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^6)*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^3\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*tan(c + d*x))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(88) = 176$.

Time = 0.13 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.43

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{\frac{2\left(a^2b-\frac{(a^3-2\,ab^2)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(a^2b-2\,b^3)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(a^3+2\,ab^2)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^6+a^4b^2+\frac{4\,(a^5b+a^3b^3)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\,(a^6-a^4b^2-2\,a^2b^4)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4\,(a^5b+a^3b^3)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{(a^6+a^4b^2)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}}\right.}{(a^2+b^2)^{\frac{3}{2}}}}{2\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(2*(a^2*b - (a^3 - 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) - (a^2*b
 - 2*b^3)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (a^3 + 2*a*b^2)*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3)/(a^6 + a^4*b^2 + 4*(a^5*b + a^3*b^3)*sin(d*x + c
)/(cos(d*x + c) + 1) - 2*(a^6 - a^4*b^2 - 2*a^2*b^4)*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 - 4*(a^5*b + a^3*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
 (a^6 + a^4*b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + log((b - a*sin(d*x
 + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x +
 c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(88) = 176$.

Time = 0.35 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.33

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\right.}{(a^4+a^2b^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right.}}{2\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*(log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*
tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^
3*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^2*b*tan(1/2*
d*x + 1/2*c)^2 - 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*x + 1/2*c) -
 2*a*b^2*tan(1/2*d*x + 1/2*c) - a^2*b)/((a^4 + a^2*b^2)*(a*tan(1/2*d*x + 1
/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

Mupad [B] (verification not implemented)

Time = 3.03 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.74

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-2\,b^2\right)}{a\,\left(a^2+b^2\right)}-\frac{b}{a^2+b^2}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(a^2+2\,b^2\right)}{a\,\left(a^2+b^2\right)}+\frac{b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^2-2\,b^2\right)}{a^2\,\left(a^2+b^2\right)}}{d\,\left(a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2-4\,b^2\right)+a^2-4\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$
$$+\frac{\mathrm{atanh}\left(\frac{\left(2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{2\,a^2\,b+2\,b^3}{a^2+b^2}\right)\,\left(\frac{a^2}{2}+\frac{b^2}{2}\right)}{{\left(a^2+b^2\right)}^{3/2}}\right)}{d\,{\left(a^2+b^2\right)}^{3/2}}$$

input
```
int(1/(cos(c + d*x)^3*(a + b*tan(c + d*x))^3),x)
```

output
```
((tan(c/2 + (d*x)/2)*(a^2 - 2*b^2))/(a*(a^2 + b^2)) - b/(a^2 + b^2) + (tan
(c/2 + (d*x)/2)^3*(a^2 + 2*b^2))/(a*(a^2 + b^2)) + (b*tan(c/2 + (d*x)/2)^2
*(a^2 - 2*b^2))/(a^2*(a^2 + b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2
+ (d*x)/2)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*ta
n(c/2 + (d*x)/2))) + atanh(((2*a*tan(c/2 + (d*x)/2) - (2*a^2*b + 2*b^3)/(a
^2 + b^2))*(a^2/2 + b^2/2))/(a^2 + b^2)^(3/2))/(d*(a^2 + b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2842, normalized size of antiderivative = 29.92

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x)
```

output
```
( - 4*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2
))*cos(c + d*x)*sin(c + d*x)**2*tan(c + d*x)**2*a**2*b**3*i + 4*sqrt(a**2
+ b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*
sin(c + d*x)**2*tan(c + d*x)**2*b**5*i - 8*sqrt(a**2 + b**2)*atan((tan((c
+ d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*tan(c
 + d*x)*a**3*b**2*i + 8*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i
)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*tan(c + d*x)*a*b**4*i -
4*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*c
os(c + d*x)*sin(c + d*x)**2*a**4*b*i + 4*sqrt(a**2 + b**2)*atan((tan((c +
d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*a**2*b*
*3*i + 4*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b
**2))*cos(c + d*x)*tan(c + d*x)**2*a**2*b**3*i + 8*sqrt(a**2 + b**2)*atan(
(tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*tan(c + d*x)*
a**3*b**2*i + 4*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a
**2 + b**2))*cos(c + d*x)*a**4*b*i - 8*sqrt(a**2 + b**2)*atan((tan((c + d*
x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*sin(c + d*x)**3*tan(c + d*x)**2*a*b**4
*i - 16*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b*
*2))*sin(c + d*x)**3*tan(c + d*x)*a**2*b**3*i - 8*sqrt(a**2 + b**2)*atan((
tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*sin(c + d*x)**3*a**3*b**2*i
 + 8*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b*...
```

3.583 $\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 155

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{(2a^2-b^2)\operatorname{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{2\left(a^2+b^2\right)^{5/2}d\sqrt{\sec^2(c+dx)}} - \frac{b\sec(c+dx)}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{3ab\sec(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))}$$

output
```
-1/2*(2*a^2-b^2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)^(
1/2))*sec(d*x+c)/(a^2+b^2)^(5/2)/d/(sec(d*x+c)^2)^(1/2)-1/2*b*sec(d*x+c)/(
a^2+b^2)/d/(a+b*tan(d*x+c))^2-3/2*a*b*sec(d*x+c)/(a^2+b^2)^2/d/(a+b*tan(d*
x+c))
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{2(2a^2-b^2)\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{b\sec(c+dx)\left(4a^2+b^2+3ab\tan(c+dx)\right)}{(a^2+b^2)^2(a+b\tan(c+dx))^2}}{2d}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

output
```
((2*(2*a^2 - b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2
 + b^2)^(5/2) - (b*Sec[c + d*x]*(4*a^2 + b^2 + 3*a*b*Tan[c + d*x]))/((a^2
+ b^2)^2*(a + b*Tan[c + d*x])^2))/(2*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 3992, 498, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{1}{(a+b\tan(c+dx))^3\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{498}$$

$$\frac{\sec(c+dx)\left(-\frac{\int -\frac{2a-b\tan(c+dx)}{(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2(a^2+b^2)}-\frac{b^2\sqrt{\tan^2(c+dx)+1}}{2(a^2+b^2)(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sec(c+dx)\left(\frac{\int \frac{2a-b\tan(c+dx)}{(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2(a^2+b^2)}-\frac{b^2\sqrt{\tan^2(c+dx)+1}}{2(a^2+b^2)(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 679$$

$$\frac{\sec(c+dx)\left(\frac{\frac{(2a^2-b^2)\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}-\frac{3ab^2\sqrt{\tan^2(c+dx)+1}}{(a^2+b^2)(a+b\tan(c+dx))}}{2(a^2+b^2)}-\frac{b^2\sqrt{\tan^2(c+dx)+1}}{2(a^2+b^2)(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 488$$

$$\frac{\sec(c+dx)\left(\frac{-\frac{(2a^2-b^2)\int \frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}}{a^2+b^2}-\frac{3ab^2\sqrt{\tan^2(c+dx)+1}}{(a^2+b^2)(a+b\tan(c+dx))}}{2(a^2+b^2)}-\frac{b^2\sqrt{\tan^2(c+dx)+1}}{2(a^2+b^2)(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 219$$

$$\frac{\sec(c+dx)\left(\frac{-\frac{b(2a^2-b^2)\text{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{3ab^2\sqrt{\tan^2(c+dx)+1}}{(a^2+b^2)(a+b\tan(c+dx))}}{2(a^2+b^2)}-\frac{b^2\sqrt{\tan^2(c+dx)+1}}{2(a^2+b^2)(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]*(-1/2*(b^2*Sqrt[1 + Tan[c + d*x]^2])/((a^2 + b^2)*(a + b*Tan
[c + d*x])^2) + (-((b*(2*a^2 - b^2)*ArcTanh[(b^2*Tan[c + d*x])/Sqrt[a^2 +
b^2]])/(a^2 + b^2)^(3/2)) - (3*a*b^2*Sqrt[1 + Tan[c + d*x]^2])/((a^2 + b^2
)*(a + b*Tan[c + d*x])))/(2*(a^2 + b^2))))/(b*d*Sqrt[Sec[c + d*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.81

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b^{2}\left(5a^{2}+2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2a\left(a^{4}+2b^{2}a^{2}+b^{4}\right)}-\frac{b\left(4a^{4}-7b^{2}a^{2}-2b^{4}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2\left(a^{4}+2b^{2}a^{2}+b^{4}\right)a^{2}}+\frac{b^{2}\left(11a^{2}+2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^{4}+2b^{2}a^{2}+b^{4}\right)a}+\frac{b\left(4a^{2}+b^{2}\right)}{2a^{4}+4b^{2}a^{2}+2b^{4}}\right)}{\left(a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^{2}}+\frac{(2a}{}}{d}$
default	$\frac{-\frac{2\left(-\frac{b^{2}\left(5a^{2}+2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2a\left(a^{4}+2b^{2}a^{2}+b^{4}\right)}-\frac{b\left(4a^{4}-7b^{2}a^{2}-2b^{4}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2\left(a^{4}+2b^{2}a^{2}+b^{4}\right)a^{2}}+\frac{b^{2}\left(11a^{2}+2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^{4}+2b^{2}a^{2}+b^{4}\right)a}+\frac{b\left(4a^{2}+b^{2}\right)}{2a^{4}+4b^{2}a^{2}+2b^{4}}\right)}{\left(a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^{2}}+\frac{(2a}{}}{d}$
risch	$-\frac{b\,e^{i(dx+c)}\left(-3iab\,e^{2i(dx+c)}+4a^{2}e^{2i(dx+c)}+b^{2}e^{2i(dx+c)}+3iab+4a^{2}+b^{2}\right)}{(ib+a)^{2}\left(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a\right)^{2}d(-ib+a)^{2}}+\frac{\ln\left(e^{i(dx+c)}+\frac{ia^{5}+2ia^{3}b^{2}+ia\,b^{4}-a^{4}b-2}{\left(a^{2}+b^{2}\right)^{\frac{5}{2}}}\right.}{(a^{2}+b^{2})^{\frac{5}{2}}d}$

input
```
int(sec(d*x+c)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*b^2*(5*a^2+2*b^2)/a/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^3
-1/2*b*(4*a^4-7*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)/a^2*tan(1/2*d*x+1/2*c)^
2+1/2*b^2*(11*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4)/a*tan(1/2*d*x+1/2*c)+1/2*b*(4
*a^2+b^2)/(a^4+2*a^2*b^2+b^4))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2
*c)-a)^2+(2*a^2-b^2)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*
tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(143) = 286$.

Time = 0.10 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.27

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\left(2\,a^2b^2-b^4+\left(2\,a^4-3\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2+2\left(2\,a^3b-ab^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right)\right)\sqrt{a^2+b^2}\,\mathrm{lo}}{4\left(\left(a^8+2\,a^6b^2-2\,a^2b^6-b^8\right)d\cos\left(dx+c\right)^2+2\right.}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/4*((2*a^2*b^2 - b^4 + (2*a^4 - 3*a^2*b^2 + b^4)*cos(d*x + c)^2 + 2*(2*a
^3*b - a*b^3)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*
x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^
2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x +
c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*co
s(d*x + c) + 6*(a^3*b^2 + a*b^4)*sin(d*x + c))/((a^8 + 2*a^6*b^2 - 2*a^2*b
^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*c
os(d*x + c)*sin(d*x + c) + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)/(a + b*tan(c + d*x))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(143) = 286$.

Time = 0.13 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.66

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{(2a^2-b^2)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(4a^4b+a^2b^3+\frac{(11a^3b^2+2ab^4)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(4a^4b-7a^2b^3-2b^5)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\cdots\right.}{a^8+2a^6b^2+a^4b^4+\frac{4(a^7b+2a^5b^3+a^3b^5)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2(a^8-3a^4b^4-2a^2b^6)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4(a^7b+2\cdots}{\cdots}}}{2d}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*((2*a^2 - b^2)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2
+ b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 +
 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(4*a^4*b + a^2*b^3 + (11*a^3*b^2 +
2*a*b^4)*sin(d*x + c)/(cos(d*x + c) + 1) - (4*a^4*b - 7*a^2*b^3 - 2*b^5)*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 - (5*a^3*b^2 + 2*a*b^4)*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3)/(a^8 + 2*a^6*b^2 + a^4*b^4 + 4*(a^7*b + 2*a^5*b^3 +
a^3*b^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*(a^8 - 3*a^4*b^4 - 2*a^2*b^6)
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*(a^7*b + 2*a^5*b^3 + a^3*b^5)*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + (a^8 + 2*a^6*b^2 + a^4*b^4)*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(143) = 286$.

Time = 0.32 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.89

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{(2a^2-b^2)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(5a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-7a^2b^3\tan\cdots\right.}{(a^6+2a^4b^2+a^2b^4)\left(a\tan\cdots\right.}}{2d}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*((2*a^2 - b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 +
b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a
^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*a
*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 7*a^2*b^3*t
an(1/2*d*x + 1/2*c)^2 - 2*b^5*tan(1/2*d*x + 1/2*c)^2 - 11*a^3*b^2*tan(1/2*
d*x + 1/2*c) - 2*a*b^4*tan(1/2*d*x + 1/2*c) - 4*a^4*b - a^2*b^3)/((a^6 + 2
*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) -
 a)^2))/d
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.86

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\ln\left(\left(a^2+b^2\right)^{5/2}-a^4\,b-b^5-2\,a^2\,b^3+a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\,b^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,a^3\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2-\right.}{d\,\left(a^2+b^2\right)^{5/2}}$$
$$-\frac{\ln\left(\left(a^2+b^2\right)^{5/2}+a^4\,b+b^5+2\,a^2\,b^3-a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,b^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,a^3\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a\right.}{2\,d\,\left(a^2+b^2\right)^{5/2}}$$
$$-\frac{\frac{4\,a^2\,b+b^3}{a^4+2\,a^2\,b^2+b^4}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^2-2\,b^2\right)\,\left(4\,a^2\,b+b^3\right)}{a^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(11\,a^2\,b+2\,b^3\right)}{a\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(5\,a^2\,b+2\,b^3\right)}{a\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2-4\,b^2\right)+a^2-4\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

input
```
int(1/(cos(c + d*x)*(a + b*tan(c + d*x))^3),x)
```

output
```
(log((a^2 + b^2)^(5/2) - a^4*b - b^5 - 2*a^2*b^3 + a^5*tan(c/2 + (d*x)/2)
+ a*b^4*tan(c/2 + (d*x)/2) + 2*a^3*b^2*tan(c/2 + (d*x)/2))*(a^2 - b^2/2))/
(d*(a^2 + b^2)^(5/2)) - (log((a^2 + b^2)^(5/2) + a^4*b + b^5 + 2*a^2*b^3 -
 a^5*tan(c/2 + (d*x)/2) - a*b^4*tan(c/2 + (d*x)/2) - 2*a^3*b^2*tan(c/2 + (
d*x)/2))*(2*a^2 - b^2))/(2*d*(a^2 + b^2)^(5/2)) - ((4*a^2*b + b^3)/(a^4 +
b^4 + 2*a^2*b^2) - (tan(c/2 + (d*x)/2)^2*(a^2 - 2*b^2)*(4*a^2*b + b^3))/(a
^2*(a^4 + b^4 + 2*a^2*b^2)) + (b*tan(c/2 + (d*x)/2)*(11*a^2*b + 2*b^3))/(a
*(a^4 + b^4 + 2*a^2*b^2)) - (b*tan(c/2 + (d*x)/2)^3*(5*a^2*b + 2*b^3))/(a*
(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2
)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2 + (
d*x)/2)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 5579, normalized size of antiderivative = 35.99

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)/(a+b*tan(d*x+c))^3,x)
```

output
```
( - 24*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**
2))*cos(c + d*x)*sin(c + d*x)**2*tan(c + d*x)**2*a**6*b**3*i + 36*sqrt(a**
2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x
)*sin(c + d*x)**2*tan(c + d*x)**2*a**4*b**5*i - 12*sqrt(a**2 + b**2)*atan(
(tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)*
*2*tan(c + d*x)**2*a**2*b**7*i - 48*sqrt(a**2 + b**2)*atan((tan((c + d*x)/
2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*tan(c + d*x)
*a**7*b**2*i + 72*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt
(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*tan(c + d*x)*a**5*b**4*i - 24*
sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos
(c + d*x)*sin(c + d*x)**2*tan(c + d*x)*a**3*b**6*i - 24*sqrt(a**2 + b**2)*
atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c +
d*x)**2*a**8*b*i + 36*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/
sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)**2*a**6*b**3*i - 12*sqrt(a**2
 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)
*sin(c + d*x)**2*a**4*b**5*i + 24*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)
*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*tan(c + d*x)**2*a**6*b**3*i -
12*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*
cos(c + d*x)*tan(c + d*x)**2*a**4*b**5*i + 48*sqrt(a**2 + b**2)*atan((tan(
(c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*tan(c + d*x)*a*...
```

3.584 $\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 221

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx = &-\frac{3b^2(4a^2-b^2)\operatorname{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{2\left(a^2+b^2\right)^{7/2}d\sqrt{\sec^2(c+dx)}}\\&+\frac{b(2a^2-3b^2)\sec(c+dx)}{2\left(a^2+b^2\right)^2d(a+b\tan(c+dx))^2}\\&+\frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))^2}\\&+\frac{ab(2a^2-13b^2)\sec(c+dx)}{2\left(a^2+b^2\right)^3d(a+b\tan(c+dx))}\end{aligned}$$

output
```
-3/2*b^2*(4*a^2-b^2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^
2)^(1/2))*sec(d*x+c)/(a^2+b^2)^(7/2)/d/(sec(d*x+c)^2)^(1/2)+1/2*b*(2*a^2-3
*b^2)*sec(d*x+c)/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2+cos(d*x+c)*(b+a*tan(d*x+
c))/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/2*a*b*(2*a^2-13*b^2)*sec(d*x+c)/(a^2+
b^2)^3/d/(a+b*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.83

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{-\frac{12b^2(-4a^2+b^2)\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} + \frac{\sec^2(c+dx)\left(b(11a^4-22a^2b^2-3b^4)\cos(c+dx)+b(a^2+b^2)^2\cos(3(c+dx))+2a\left(a^4+4a^2\right.\right.}{(a^2+b^2)^3(a+b\tan(c+dx))^2}}{4d}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

output
```
((-12*b^2*(-4*a^2 + b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]
])/(a^2 + b^2)^(7/2) + (Sec[c + d*x]^2*(b*(11*a^4 - 22*a^2*b^2 - 3*b^4)*Co
s[c + d*x] + b*(a^2 + b^2)^2*Cos[3*(c + d*x)] + 2*a*(a^4 + 4*a^2*b^2 - 12*
b^4 + (a^2 + b^2)^2*Cos[2*(c + d*x)])*Sin[c + d*x]))/((a^2 + b^2)^3*(a + b
*Tan[c + d*x])^2))/(4*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 3992, 496, 25, 27, 688, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(c+dx)(a+b\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{3992}$$
$$\frac{\sec(c+dx)\int \frac{1}{(a+b\tan(c+dx))^3(\tan^2(c+dx)+1)^{3/2}}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 496$$

$$\frac{\sec(c+dx)\left(\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{b^2\int -\frac{3b^2+2a\tan(c+dx)b}{b^2(a+b\tan(c+dx))^3\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sec(c+dx)\left(\frac{b^2\int \frac{3b^2+2a\tan(c+dx)b}{b^2(a+b\tan(c+dx))^3\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}+\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sec(c+dx)\left(\frac{\int \frac{3b^2+2a\tan(c+dx)b}{(a+b\tan(c+dx))^3\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}+\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 688$$

$$\frac{\sec(c+dx)\left(\frac{\frac{\left(2a^2-3b^2\right)\sqrt{\tan^2(c+dx)+1}}{2\left(\frac{a^2}{b^2}+1\right)(a+b\tan(c+dx))^2}-\frac{b^2\int -\frac{10a-\left(3-\frac{2a^2}{b^2}\right)b\tan(c+dx)}{(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2\left(a^2+b^2\right)}}{a^2+b^2}+\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sec(c+dx)\left(\frac{\frac{b^2\int \frac{10a-\left(3-\frac{2a^2}{b^2}\right)b\tan(c+dx)}{(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2\left(a^2+b^2\right)}+\frac{\left(2a^2-3b^2\right)\sqrt{\tan^2(c+dx)+1}}{2\left(\frac{a^2}{b^2}+1\right)(a+b\tan(c+dx))^2}}{a^2+b^2}+\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 679$$

$$\frac{\sec(c+dx)\left(\frac{\frac{b^2\left(\frac{3\left(4a^2-b^2\right)\int\frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}+\frac{a\left(2a^2-13b^2\right)\sqrt{\tan^2(c+dx)+1}}{\left(a^2+b^2\right)(a+b\tan(c+dx))}\right)}{2\left(a^2+b^2\right)}+\frac{\left(2a^2-3b^2\right)\sqrt{\tan^2(c+dx)+1}}{2\left(\frac{a^2}{b^2}+1\right)(a+b\tan(c+dx))^2}}{a^2+b^2}+\frac{}{(a}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$\Big\downarrow$ 488

$$\frac{\sec(c+dx)\left(\frac{\frac{b^2\left(\frac{a\left(2a^2-13b^2\right)\sqrt{\tan^2(c+dx)+1}}{\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{3\left(4a^2-b^2\right)\int\frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}}{a^2+b^2}\right)}{2\left(a^2+b^2\right)}+\frac{\left(2a^2-3b^2\right)\sqrt{\tan^2(c+dx)+1}}{2\left(\frac{a^2}{b^2}+1\right)(a+b\tan(c+dx))^2}}{a^2+b^2}+\frac{}{(a^2+b^2)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$\Big\downarrow$ 219

$$\frac{\sec(c+dx)\left(\frac{\frac{b^2\left(\frac{a\left(2a^2-13b^2\right)\sqrt{\tan^2(c+dx)+1}}{\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{3b\left(4a^2-b^2\right)\text{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}\right)}{2\left(a^2+b^2\right)}+\frac{\left(2a^2-3b^2\right)\sqrt{\tan^2(c+dx)+1}}{2\left(\frac{a^2}{b^2}+1\right)(a+b\tan(c+dx))^2}}{a^2+b^2}+\frac{ab\tan(c}{(a^2+b^2)\sqrt{\tan^2(c+dx}}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]*((b^2 + a*b*Tan[c + d*x])/((a^2 + b^2)*(a + b*Tan[c + d*x])^
2*Sqrt[1 + Tan[c + d*x]^2]) + (((2*a^2 - 3*b^2)*Sqrt[1 + Tan[c + d*x]^2])/
(2*(1 + a^2/b^2)*(a + b*Tan[c + d*x])^2) + (b^2*((-3*b*(4*a^2 - b^2)*ArcTa
nh[(b^2*Tan[c + d*x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + (a*(2*a^2 - 13
*b^2)*Sqrt[1 + Tan[c + d*x]^2])/((a^2 + b^2)*(a + b*Tan[c + d*x]))))/(2*(a
^2 + b^2)))/(a^2 + b^2)))/(b*d*Sqrt[Sec[c + d*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 4.96 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{-\frac{2\left(\left(-a^{3}+3a\,b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3a^{2}b+b^{3}\right)}{\left(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}-\frac{2b^{2}\left(\frac{-\frac{b^{2}\left(9a^{2}+2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2a}-\frac{b\left(8a^{4}-15b^{2}a^{2}-2b^{4}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2a^{2}}+\frac{b^{2}(2}{}}{\left(a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^{2}}\right.}{(a^{2}}}{d}$
default	$\frac{-\frac{2\left(\left(-a^{3}+3a\,b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3a^{2}b+b^{3}\right)}{\left(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}-\frac{2b^{2}\left(\frac{-\frac{b^{2}\left(9a^{2}+2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2a}-\frac{b\left(8a^{4}-15b^{2}a^{2}-2b^{4}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2a^{2}}+\frac{b^{2}(2}{}}{\left(a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^{2}}\right.}{(a^{2}}}{d}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}}{2(-3ia^{2}b+ib^{3}+a^{3}-3a\,b^{2})d}+\frac{i\mathrm{e}^{-i(dx+c)}}{2(3ia^{2}b-ib^{3}+a^{3}-3a\,b^{2})d}+\frac{b^{3}\mathrm{e}^{i(dx+c)}\left(-7iab\,\mathrm{e}^{2i(dx+c)}+8a^{2}\mathrm{e}^{2i(dx+c)}+b^{2}\mathrm{e}^{2i(dx-}\right.}{(-ia+b)^{3}\left(b\,\mathrm{e}^{2i(dx+c)}+ia\,\mathrm{e}^{2i(dx+c)}-b+ia\right)}$

input
```
int(cos(d*x+c)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*((-a^3+3*a*b^2)*tan(1/2*d*x+1/2*c)-3
*a^2*b+b^3)/(1+tan(1/2*d*x+1/2*c)^2)-2*b^2/(a^2+b^2)^3*((-1/2*b^2*(9*a^2+2
*b^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(8*a^4-15*a^2*b^2-2*b^4)/a^2*tan(1/2*d*
x+1/2*c)^2+1/2*b^2*(23*a^2+2*b^2)/a*tan(1/2*d*x+1/2*c)+4*a^2*b+1/2*b^3)/(a
*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)^2-3/2*(4*a^2-b^2)/(a^2+b^2
)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(209) = 418$.

Time = 0.12 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.17

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{4\left(a^{6}b+3\,a^{4}b^{3}+3\,a^{2}b^{5}+b^{7}\right)\cos\left(dx+c\right)^{3}-3\left(4\,a^{2}b^{4}-b^{6}+\left(4\,a^{4}b^{2}-5\,a^{2}b^{4}+b^{6}\right)\cos\left(dx+c\right)^{2}+2\left(4\,a\right.\right.}{4\left(\left(a^{10}+\right.\right.}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^3 - 3*(4*a^2*b^4
 - b^6 + (4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(4*a^3*b^3 - a*b
^5)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin
(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(
b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2
- b^2)*cos(d*x + c)^2 + b^2)) + 2*(4*a^6*b - 10*a^4*b^3 - 17*a^2*b^5 - 3*b
^7)*cos(d*x + c) + 2*(2*a^5*b^2 - 11*a^3*b^4 - 13*a*b^6 + 2*(a^7 + 3*a^5*b
^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^10 + 3*a^8*b^2 +
 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*d*cos(d*x + c)^2 + 2*(a^9*b + 4
*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) + (a
^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\cos{(c+dx)}}{(a+b\tan{(c+dx)})^3}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(cos(c + d*x)/(a + b*tan(c + d*x))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(209) = 418$.

Time $= 0.14$ (sec) , antiderivative size $= 658$, normalized size of antiderivative $= 2.98$

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(3*(4*a^2*b^2 - b^4)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqr
t(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(6*a^6*b - 10*a^4
*b^3 - a^2*b^5 + (2*a^7 + 18*a^5*b^2 - 31*a^3*b^4 - 2*a*b^6)*sin(d*x + c)/
(cos(d*x + c) + 1) - 2*(2*a^6*b - 2*a^4*b^3 + 12*a^2*b^5 + b^7)*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 - 2*(2*a^7 + 2*a^5*b^2 + 15*a^3*b^4)*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 - (2*a^6*b - 30*a^4*b^3 + 15*a^2*b^5 + 2*b^7)*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 + (2*a^7 - 6*a^5*b^2 + 9*a^3*b^4 + 2*a*b^
6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^10 + 3*a^8*b^2 + 3*a^6*b^4 + a^
4*b^6 + 4*(a^9*b + 3*a^7*b^3 + 3*a^5*b^5 + a^3*b^7)*sin(d*x + c)/(cos(d*x
+ c) + 1) - (a^10 - a^8*b^2 - 9*a^6*b^4 - 11*a^4*b^6 - 4*a^2*b^8)*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 - (a^10 - a^8*b^2 - 9*a^6*b^4 - 11*a^4*b^6 - 4
*a^2*b^8)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*(a^9*b + 3*a^7*b^3 + 3*a
^5*b^5 + a^3*b^7)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + (a^10 + 3*a^8*b^2
+ 3*a^6*b^4 + a^4*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6))/d
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.81

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{3\,(4\,a^2b^2-b^4)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{4\,(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^2b-b^3)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)} - \frac{2\left(9\,a^3b^4\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*(3*(4*a^2*b^2 - b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(
a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 4*(a^3*tan(1/2*d*x + 1/
2*c) - 3*a*b^2*tan(1/2*d*x + 1/2*c) + 3*a^2*b - b^3)/((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*(9*a^3*b^4*tan(1/2*d*x +
 1/2*c)^3 + 2*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 8*a^4*b^3*tan(1/2*d*x + 1/2*c
)^2 - 15*a^2*b^5*tan(1/2*d*x + 1/2*c)^2 - 2*b^7*tan(1/2*d*x + 1/2*c)^2 - 2
3*a^3*b^4*tan(1/2*d*x + 1/2*c) - 2*a*b^6*tan(1/2*d*x + 1/2*c) - 8*a^4*b^3
- a^2*b^5)/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*(a*tan(1/2*d*x + 1/2*c
)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

Mupad [B] (verification not implemented)

Time = 4.51 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.76

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{-6a^4b+10a^2b^3+b^5}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(2a^5+2a^3b^2+15ab^4)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(2a^6b-30a^4b^3+15a^2b^5+2b^7)}{a^2(a^6+3a^4b^2+3a^2b^4+b^6)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2}{a(a^6+3}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+a^2-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(a^2-4b^2)-\tan\left(\frac{c}{2}+\frac{d}{2}\right.\right.} - \frac{\operatorname{atan}\left(\frac{-1i\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^7+a^6b\,1i-3i\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^5b^2+a^4b^3\,3i-3i\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^3b^4+a^2b^5\,3i-1i\tan\left(\frac{c}{2}+\frac{dx}{2}\right)ab^6+b^7\,1i}{(a^2+b^2)^{7/2}}\right)(3b^4-}{d\,(a^2+b^2)^{7/2}}$$

input
```
int(cos(c + d*x)/(a + b*tan(c + d*x))^3,x)
```

output
```
- ((b^5 - 6*a^4*b + 10*a^2*b^3)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (2*t
an(c/2 + (d*x)/2)^3*(15*a*b^4 + 2*a^5 + 2*a^3*b^2))/(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2) + (tan(c/2 + (d*x)/2)^4*(2*a^6*b + 2*b^7 + 15*a^2*b^5 - 30*a
^4*b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)*(
2*a^6 - 2*b^6 - 31*a^2*b^4 + 18*a^4*b^2))/(a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)) - (tan(c/2 + (d*x)/2)^5*(2*a^6 + 2*b^6 + 9*a^2*b^4 - 6*a^4*b^2))/(
a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^2*(2*a^6*b
+ b^7 + 12*a^2*b^5 - 2*a^4*b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
)/(d*(a^2*tan(c/2 + (d*x)/2)^6 + a^2 - tan(c/2 + (d*x)/2)^2*(a^2 - 4*b^2)
- tan(c/2 + (d*x)/2)^4*(a^2 - 4*b^2) - 4*a*b*tan(c/2 + (d*x)/2)^5 + 4*a*b*
tan(c/2 + (d*x)/2))) - (atan((a^6*b*1i + b^7*1i + a^2*b^5*3i + a^4*b^3*3i
- a^7*tan(c/2 + (d*x)/2)*1i - a*b^6*tan(c/2 + (d*x)/2)*1i - a^3*b^4*tan(c/
2 + (d*x)/2)*3i - a^5*b^2*tan(c/2 + (d*x)/2)*3i)/(a^2 + b^2)^(7/2))*(3*b^4
 - 12*a^2*b^2)*1i)/(d*(a^2 + b^2)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1025, normalized size of antiderivative = 4.64

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(d*x+c)/(a+b*tan(d*x+c))^3,x)
```

output
```
( - 96*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**
2))*cos(c + d*x)*sin(c + d*x)*a**3*b**4*i + 24*sqrt(a**2 + b**2)*atan((tan
((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)*a*b*
*6*i + 48*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 +
b**2))*sin(c + d*x)**2*a**4*b**3*i - 60*sqrt(a**2 + b**2)*atan((tan((c + d
*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*sin(c + d*x)**2*a**2*b**5*i + 12*sqrt
(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*sin(c +
 d*x)**2*b**7*i - 48*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/s
qrt(a**2 + b**2))*a**4*b**3*i + 12*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2
)*a*i - b*i)/sqrt(a**2 + b**2))*a**2*b**5*i - 4*cos(c + d*x)*sin(c + d*x)*
*2*a**6*b**2 - 12*cos(c + d*x)*sin(c + d*x)**2*a**4*b**4 - 12*cos(c + d*x)
*sin(c + d*x)**2*a**2*b**6 - 4*cos(c + d*x)*sin(c + d*x)**2*b**8 + 4*cos(c
 + d*x)*sin(c + d*x)*a**7*b + 16*cos(c + d*x)*sin(c + d*x)*a**5*b**3 - 10*
cos(c + d*x)*sin(c + d*x)*a**3*b**5 - 22*cos(c + d*x)*sin(c + d*x)*a*b**7
+ 12*cos(c + d*x)*a**6*b**2 - 8*cos(c + d*x)*a**4*b**4 - 22*cos(c + d*x)*a
**2*b**6 - 2*cos(c + d*x)*b**8 - 4*sin(c + d*x)**3*a**7*b - 12*sin(c + d*x
)**3*a**5*b**3 - 12*sin(c + d*x)**3*a**3*b**5 - 4*sin(c + d*x)**3*a*b**7 -
 2*sin(c + d*x)**2*a**8 - 6*sin(c + d*x)**2*a**6*b**2 + 13*sin(c + d*x)**2
*a**4*b**4 + 6*sin(c + d*x)**2*a**2*b**6 - 11*sin(c + d*x)**2*b**8 + 4*sin
(c + d*x)*a**7*b + 16*sin(c + d*x)*a**5*b**3 - 10*sin(c + d*x)*a**3*b**...
```

3.585 $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 310

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{5b^4(6a^2-b^2)\,\mathrm{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{2\,(a^2+b^2)^{9/2}\,d\sqrt{\sec^2(c+dx)}} + \frac{b(4a^4+24a^2b^2-15b^4)\sec(c+dx)}{6\,(a^2+b^2)^3\,d(a+b\tan(c+dx))^2} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{ab(4a^4+28a^2b^2-81b^4)\sec(c+dx)}{6\,(a^2+b^2)^4\,d(a+b\tan(c+dx))} - \frac{\cos(c+dx)\,(b(2a^2-5b^2)-a(2a^2+9b^2)\tan(c+dx))}{3\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2}$$

output
```
-5/2*b^4*(6*a^2-b^2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^
2)^(1/2))*sec(d*x+c)/(a^2+b^2)^(9/2)/d/(sec(d*x+c)^2)^(1/2)+1/6*b*(4*a^4+2
4*a^2*b^2-15*b^4)*sec(d*x+c)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))^2+1/3*cos(d*x+
c)^3*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/6*a*b*(4*a^4+28*a^2
*b^2-81*b^4)*sec(d*x+c)/(a^2+b^2)^4/d/(a+b*tan(d*x+c))-1/3*cos(d*x+c)*(b*(
2*a^2-5*b^2)-a*(2*a^2+9*b^2)*tan(d*x+c))/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.20

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{9b(a^4+14a^2b^2-3b^4)(a\cos(c+dx)+b\sin(c+dx))^2}{(a^2+b^2)^4}+\frac{6b^6\tan(c+dx)}{a(a^2+b^2)^3}+\frac{9a(a^4+\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])*((9*b*(a^4 + 14*a^2*b^2
- 3*b^4)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a^2 + b^2)^4 + (6*b^6*Tan[c
 + d*x])/(a*(a^2 + b^2)^3) + (9*a*(a^4 + 6*a^2*b^2 - 11*b^4)*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2*Tan[c + d*x])/(a^2 + b^2)^4 - (6*b^5*(12*a^2 + b^2)
*(a + b*Tan[c + d*x]))/(a*(a^2 + b^2)^4) - (60*b^4*(-6*a^2 + b^2)*ArcTanh[
(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*Cos[c + d*x]*(a + b*Tan[c + d*x
])^2)/(a^2 + b^2)^(9/2) - (b*(-3*a^2 + b^2)*Cos[c + d*x]*Cos[3*(c + d*x)]*
(a + b*Tan[c + d*x])^2)/(a^2 + b^2)^3 + (a*(a^2 - 3*b^2)*Cos[c + d*x]*Sin[
3*(c + d*x)]*(a + b*Tan[c + d*x])^2)/(a^2 + b^2)^3))/(12*d*(a + b*Tan[c +
d*x])^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 3992, 496, 25, 27, 686, 25, 25, 27, 688, 25, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \; 3992$$

$$\frac{\sec(c+dx)\int \frac{1}{(a+b\tan(c+dx))^3(\tan^2(c+dx)+1)^{5/2}}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 496$$

$$\frac{\sec(c+dx)\left(\frac{ab\tan(c+dx)+b^2}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}(a+b\tan(c+dx))^2}-\frac{b^2\int -\frac{\left(\frac{2a^2}{b^2}+5\right)b^2+4a\tan(c+dx)b}{b^2(a+b\tan(c+dx))^3\left(\tan^2(c+dx)+1\right)^{3/2}}d(b\tan(c+dx))}{3(a^2+b^2)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sec(c+dx)\left(\frac{b^2\int \frac{2a^2+4b\tan(c+dx)a+5b^2}{b^2(a+b\tan(c+dx))^3\left(\tan^2(c+dx)+1\right)^{3/2}}d(b\tan(c+dx))}{3(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sec(c+dx)\left(\frac{\int \frac{2a^2+4b\tan(c+dx)a+5b^2}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)+1\right)^{3/2}}d(b\tan(c+dx))}{3(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 686$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{b^4\int -\frac{3\left(5-\frac{2a^2}{b^2}\right)b^4+2a\left(2a^2+9b^2\right)\tan(c+dx)b}{b^4(a+b\tan(c+dx))^3\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}}{3(a^2+b^2)}+\frac{ab\tan(c}{3(a^2+b^2)(\tan^2(c+dx)+}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sec(c+dx)\left(\frac{\frac{b^4\int -\frac{3b^2\left(2a^2-5b^2\right)-2ab\left(2a^2+9b^2\right)\tan(c+dx)}{b^4(a+b\tan(c+dx))^3\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}+\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}}{3(a^2+b^2)}+\frac{ab\tan(}{3(a^2+b^2)(\tan^2(c+dx)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{b^4\int \frac{3b^2\left(2a^2-5b^2\right)-2ab\left(2a^2+9b^2\right)\tan(c+dx)}{b^4(a+b\tan(c+dx))^3\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}}{3(a^2+b^2)}+\frac{ab\tan(c-}{3(a^2+b^2)(\tan^2(c+dx)+}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{\int \frac{3b^2\left(2a^2-5b^2\right)-2ab\left(2a^2+9b^2\right)\tan(c+dx)}{(a+b\tan(c+dx))^3\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}}{3(a^2+b^2)}+\frac{ab\tan(c+d}{3(a^2+b^2)(\tan^2(c+dx)+1)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{688}$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{-\frac{b^2\int -\frac{2ab^2\left(2a^2-33b^2\right)-b\left(4a^4+24b^2a^2-15b^4\right)\tan(c+dx)}{b^2(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2\left(a^2+b^2\right)}-\frac{b^2\left(4a^4+24a^2b^2\right.}{2\left(a^2+b^2\right.}}{a^2+b^2}}{3(a^2+b^2)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{\frac{b^2\int\frac{2ab^2\left(2a^2-33b^2\right)-b\left(4a^4+24b^2a^2-15b^4\right)\tan(c+dx)}{b^2(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2\left(a^2+b^2\right)}-\frac{b^2\left(4a^4+24a^2b^2-1\right.}{2\left(a^2+b^2\right)(a}}{a^2+b^2}}{3(a^2+b^2)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$\downarrow$ 27

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{\frac{\int\frac{2ab^2\left(2a^2-33b^2\right)-b\left(4a^4+24b^2a^2-15b^4\right)\tan(c+dx)}{(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2\left(a^2+b^2\right)}-\frac{b^2\left(4a^4+24a^2b^2-15b^4\right.}{2\left(a^2+b^2\right)(a+b}}{a^2+b^2}}{3(a^2+b^2)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$\downarrow$ 679

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{\frac{-\frac{15b^4\left(6a^2-b^2\right)\int\frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}-\frac{ab^2\left(4a^4+28a^2b^2-81b^4\right.}{\left(a^2+b^2\right)(a+b\tan}}{2\left(a^2+b^2\right)}}{a^2+b^2}}{3(a^2+b^2)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$\downarrow$ 488

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{\frac{\frac{15b^4\left(6a^2-b^2\right)\int\frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}}{a^2+b^2}-\frac{ab^2\left(4a^4+28a^2b^2-81b^4\right)\sqrt{\tan^2(}}{\left(a^2+b^2\right)(a+b\tan(c+dx)}}{2\left(a^2+b^2\right)}}{a^2+b^2}}{3(a^2+b^2)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$\downarrow$ 219

$$\frac{\sec(c+dx)\left(\frac{ab\tan(c+dx)+b^2}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}(a+b\tan(c+dx))^2}+\frac{\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{\frac{15b^5\left(6a^2-b^2\right)\text{arctanh}\left(\frac{b^2\tan(}{\sqrt{a^2-}}\right.}{\left(a^2+b^2\right)^{3/2}}}{}}{}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

input

```
Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

output

```
(Sec[c + d*x]*((b^2 + a*b*Tan[c + d*x])/(3*(a^2 + b^2)*(a + b*Tan[c + d*x]
)^2*(1 + Tan[c + d*x]^2)^(3/2)) + (((5 - (2*a^2)/b^2)*b^4 + a*b*(2*a^2 + 9
*b^2)*Tan[c + d*x])/((a^2 + b^2)*(a + b*Tan[c + d*x])^2*Sqrt[1 + Tan[c + d
*x]^2]) - (-1/2*(b^2*(4*a^4 + 24*a^2*b^2 - 15*b^4)*Sqrt[1 + Tan[c + d*x]^2
])/((a^2 + b^2)*(a + b*Tan[c + d*x])^2) + ((15*b^5*(6*a^2 - b^2)*ArcTanh[(
b^2*Tan[c + d*x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (a*b^2*(4*a^4 + 28
*a^2*b^2 - 81*b^4)*Sqrt[1 + Tan[c + d*x]^2])/((a^2 + b^2)*(a + b*Tan[c + d
*x])))/(2*(a^2 + b^2)))/(a^2 + b^2))/(3*(a^2 + b^2))))/(b*d*Sqrt[Sec[c + d
*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 24.19 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.47

method	result
derivativedivides	$-\frac{2\left(\left(-a^5-4a^3b^2+9a\,b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-3a^4b-12a^2b^3+3b^5\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(-\frac{2}{3}a^5-\frac{32}{3}a^3b^2+14a\,b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-20a\right.\right.}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\left(a^2+b^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$
default	$-\frac{2\left(\left(-a^5-4a^3b^2+9a\,b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-3a^4b-12a^2b^3+3b^5\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(-\frac{2}{3}a^5-\frac{32}{3}a^3b^2+14a\,b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-20a\right.\right.}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\left(a^2+b^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$
risch	$-\frac{ie^{3i(dx+c)}}{24(-3ia^2b+ib^3+a^3-3a\,b^2)d}-\frac{9\,e^{i(dx+c)}b}{8(-4ia^3b+4ia\,b^3+a^4-6b^2a^2+b^4)d}-\frac{3ie^{i(dx+c)}a}{8(-4ia^3b+4ia\,b^3+a^4-6b^2a^2+b^4)d}-\frac{}{8(}$

input
```
int(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)*((-a^5-4*a^3*b^2+9*a*b^4)*
tan(1/2*d*x+1/2*c)^5+(-3*a^4*b-12*a^2*b^3+3*b^5)*tan(1/2*d*x+1/2*c)^4+(-2/
3*a^5-32/3*a^3*b^2+14*a*b^4)*tan(1/2*d*x+1/2*c)^3+(-20*a^2*b^3+4*b^5)*tan(
1/2*d*x+1/2*c)^2+(-a^5-4*a^3*b^2+9*a*b^4)*tan(1/2*d*x+1/2*c)-a^4*b-32/3*a^
2*b^3+7/3*b^5)/(1+tan(1/2*d*x+1/2*c)^2)^3-2*b^4/(a^2+b^2)/(a^6+3*a^4*b^2+3
*a^2*b^4+b^6)*((-1/2*b^2*(13*a^2+2*b^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(12*a
^4-23*a^2*b^2-2*b^4)/a^2*tan(1/2*d*x+1/2*c)^2+1/2*b^2*(35*a^2+2*b^2)/a*tan
(1/2*d*x+1/2*c)+6*a^2*b+1/2*b^3)/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1
/2*c)-a)^2-5/2*(6*a^2-b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/
2*c)-2*b)/(a^2+b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(294) = 588$.

Time = 0.14 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.00

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{4\left(a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9\right)\cos\left(dx+c\right)^5-4\left(2\,a^8b+a^6b^3-9\,a^4b^5-13\,a^2b^7-5\,b^9\right)\cos\left(dx+\right.}{}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^5 -
 4*(2*a^8*b + a^6*b^3 - 9*a^4*b^5 - 13*a^2*b^7 - 5*b^9)*cos(d*x + c)^3 - 1
5*(6*a^2*b^6 - b^8 + (6*a^4*b^4 - 7*a^2*b^6 + b^8)*cos(d*x + c)^2 + 2*(6*a
^3*b^5 - a*b^7)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(
d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(
a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x
+ c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*(8*a^8*b + 64*a^6*b^3 - 16*a
^4*b^5 - 87*a^2*b^7 - 15*b^9)*cos(d*x + c) + 2*(4*a^7*b^2 + 32*a^5*b^4 - 5
3*a^3*b^6 - 81*a*b^8 + 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)
*cos(d*x + c)^4 + 2*(2*a^9 + 15*a^7*b^2 + 33*a^5*b^4 + 29*a^3*b^6 + 9*a*b^
8)*cos(d*x + c)^2)*sin(d*x + c))/((a^12 + 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b
^8 - 4*a^2*b^10 - b^12)*d*cos(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10*a^7*
b^5 + 10*a^5*b^7 + 5*a^3*b^9 + a*b^11)*d*cos(d*x + c)*sin(d*x + c) + (a^10
*b^2 + 5*a^8*b^4 + 10*a^6*b^6 + 10*a^4*b^8 + 5*a^2*b^10 + b^12)*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+b*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. $2(294) = 588$.

Time $= 0.20$ (sec) , antiderivative size $= 1229$, normalized size of antiderivative $= 3.96$

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/6*(15*(6*a^2*b^4 - b^6)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sq
rt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/
((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*sqrt(a^2 + b^2)) - 2*(6*a
^8*b + 64*a^6*b^3 - 50*a^4*b^5 - 3*a^2*b^7 + (6*a^9 + 48*a^7*b^2 + 202*a^5
*b^4 - 161*a^3*b^6 - 6*a*b^8)*sin(d*x + c)/(cos(d*x + c) + 1) + 2*(6*a^8*b
 + 56*a^6*b^3 - 14*a^4*b^5 - 67*a^2*b^7 - 3*b^9)*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 4*(2*a^9 - 4*a^7*b^2 - 86*a^5*b^4 + 133*a^3*b^6 + 3*a*b^8)*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*(8*a^8*b + 28*a^6*b^3 + 188*a^4*b^5
- 156*a^2*b^7 - 9*b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2*(2*a^9 + 4*
a^7*b^2 + 62*a^5*b^4 - 255*a^3*b^6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
2*(14*a^8*b + 56*a^6*b^3 - 246*a^4*b^5 + 141*a^2*b^7 + 9*b^9)*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 - 4*(2*a^9 + 8*a^7*b^2 + 42*a^5*b^4 + 33*a^3*b^6 -
 3*a*b^8)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*(2*a^8*b + 8*a^6*b^3 - 7
8*a^4*b^5 + 23*a^2*b^7 + 2*b^9)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 3*(2
*a^9 + 8*a^7*b^2 - 18*a^5*b^4 + 13*a^3*b^6 + 2*a*b^8)*sin(d*x + c)^9/(cos(
d*x + c) + 1)^9)/(a^12 + 4*a^10*b^2 + 6*a^8*b^4 + 4*a^6*b^6 + a^4*b^8 + 4*
(a^11*b + 4*a^9*b^3 + 6*a^7*b^5 + 4*a^5*b^7 + a^3*b^9)*sin(d*x + c)/(cos(d
*x + c) + 1) + (a^12 + 8*a^10*b^2 + 22*a^8*b^4 + 28*a^6*b^6 + 17*a^4*b^8 +
 4*a^2*b^10)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 8*(a^11*b + 4*a^9*b^3 +
 6*a^7*b^5 + 4*a^5*b^7 + a^3*b^9)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 -...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(294) = 588$.

Time = 0.41 (sec) , antiderivative size = 640, normalized size of antiderivative = 2.06

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/6*(15*(6*a^2*b^4 - b^6)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt
(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*sqrt(a^2 + b^2)) - 6*(13*a^3*
b^6*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^8*tan(1/2*d*x + 1/2*c)^3 + 12*a^4*b^5*t
an(1/2*d*x + 1/2*c)^2 - 23*a^2*b^7*tan(1/2*d*x + 1/2*c)^2 - 2*b^9*tan(1/2*
d*x + 1/2*c)^2 - 35*a^3*b^6*tan(1/2*d*x + 1/2*c) - 2*a*b^8*tan(1/2*d*x + 1
/2*c) - 12*a^4*b^5 - a^2*b^7)/((a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 +
 a^2*b^8)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2) - 4
*(3*a^5*tan(1/2*d*x + 1/2*c)^5 + 12*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*a*
b^4*tan(1/2*d*x + 1/2*c)^5 + 9*a^4*b*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b^3*t
an(1/2*d*x + 1/2*c)^4 - 9*b^5*tan(1/2*d*x + 1/2*c)^4 + 2*a^5*tan(1/2*d*x +
 1/2*c)^3 + 32*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 42*a*b^4*tan(1/2*d*x + 1/2
*c)^3 + 60*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 12*b^5*tan(1/2*d*x + 1/2*c)^2
+ 3*a^5*tan(1/2*d*x + 1/2*c) + 12*a^3*b^2*tan(1/2*d*x + 1/2*c) - 27*a*b^4*
tan(1/2*d*x + 1/2*c) + 3*a^4*b + 32*a^2*b^3 - 7*b^5)/((a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8)*(tan(1/2*d*x + 1/2*c)^2 + 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 6.24 (sec) , antiderivative size = 1128, normalized size of antiderivative = 3.64

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^3/(a + b*tan(c + d*x))^3,x)
```

output
```
((2*tan(c/2 + (d*x)/2)^5*(2*a^7 - 255*a*b^6 + 62*a^3*b^4 + 4*a^5*b^2))/(3*
(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (6*a^6*b - 3*b^7 - 50*a
^2*b^5 + 64*a^4*b^3)/(3*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) +
 (2*tan(c/2 + (d*x)/2)^2*(6*a^6*b - 3*b^7 - 64*a^2*b^5 + 50*a^4*b^3))/(3*a
^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)^9*(2*a^8 + 2
*b^8 + 13*a^2*b^6 - 18*a^4*b^4 + 8*a^6*b^2))/(a*(a^8 + b^8 + 4*a^2*b^6 + 6
*a^4*b^4 + 4*a^6*b^2)) - (4*tan(c/2 + (d*x)/2)^7*(2*a^6 - 3*b^6 + 36*a^2*b
^4 + 6*a^4*b^2))/(3*a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c/2 + (d
*x)/2)^8*(2*a^8*b + 2*b^9 + 23*a^2*b^7 - 78*a^4*b^5 + 8*a^6*b^3))/(a^2*(a^
8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (2*tan(c/2 + (d*x)/2)^4*(8
*a^8*b - 9*b^9 - 156*a^2*b^7 + 188*a^4*b^5 + 28*a^6*b^3))/(3*a^2*(a^2 + b^
2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2)^6*(14*a^8*
b + 9*b^9 + 141*a^2*b^7 - 246*a^4*b^5 + 56*a^6*b^3))/(3*a^2*(a^2 + b^2)*(a
^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)*(6*a^8 - 6*b^8 -
161*a^2*b^6 + 202*a^4*b^4 + 48*a^6*b^2))/(3*a*(a^2 + b^2)*(a^6 + b^6 + 3*a
^2*b^4 + 3*a^4*b^2)) - (4*tan(c/2 + (d*x)/2)^3*(2*a^8 + 3*b^8 + 133*a^2*b^
6 - 86*a^4*b^4 - 4*a^6*b^2))/(3*a*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a
^4*b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^6*(2*a^2 - 12
*b^2) - tan(c/2 + (d*x)/2)^4*(2*a^2 - 12*b^2) + a^2 + tan(c/2 + (d*x)/2)^2
*(a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^8*(a^2 + 4*b^2) + 8*a*b*tan(c/2 + (...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1324, normalized size of antiderivative = 4.27

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x)
```

output
```
( - 720*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b*
*2))*cos(c + d*x)*sin(c + d*x)*a**3*b**6*i + 120*sqrt(a**2 + b**2)*atan((t
an((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*cos(c + d*x)*sin(c + d*x)*a*
b**8*i + 360*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2
 + b**2))*sin(c + d*x)**2*a**4*b**5*i - 420*sqrt(a**2 + b**2)*atan((tan((c
 + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*sin(c + d*x)**2*a**2*b**7*i + 60*
sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*sin
(c + d*x)**2*b**9*i - 360*sqrt(a**2 + b**2)*atan((tan((c + d*x)/2)*a*i - b
*i)/sqrt(a**2 + b**2))*a**4*b**5*i + 60*sqrt(a**2 + b**2)*atan((tan((c + d
*x)/2)*a*i - b*i)/sqrt(a**2 + b**2))*a**2*b**7*i + 4*cos(c + d*x)*sin(c +
d*x)**4*a**8*b**2 + 16*cos(c + d*x)*sin(c + d*x)**4*a**6*b**4 + 24*cos(c +
 d*x)*sin(c + d*x)**4*a**4*b**6 + 16*cos(c + d*x)*sin(c + d*x)**4*a**2*b**
8 + 4*cos(c + d*x)*sin(c + d*x)**4*b**10 - 28*cos(c + d*x)*sin(c + d*x)**2
*a**6*b**4 - 84*cos(c + d*x)*sin(c + d*x)**2*a**4*b**6 - 84*cos(c + d*x)*s
in(c + d*x)**2*a**2*b**8 - 28*cos(c + d*x)*sin(c + d*x)**2*b**10 + 12*cos(
c + d*x)*sin(c + d*x)*a**9*b + 84*cos(c + d*x)*sin(c + d*x)*a**7*b**3 + 22
0*cos(c + d*x)*sin(c + d*x)*a**5*b**5 + 26*cos(c + d*x)*sin(c + d*x)*a**3*
b**7 - 122*cos(c + d*x)*sin(c + d*x)*a*b**9 + 12*cos(c + d*x)*a**8*b**2 +
140*cos(c + d*x)*a**6*b**4 + 28*cos(c + d*x)*a**4*b**6 - 106*cos(c + d*x)*
a**2*b**8 - 6*cos(c + d*x)*b**10 + 4*sin(c + d*x)**5*a**9*b + 16*sin(c ...
```

3.586 $\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx = -\frac{6ad^4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2b(d\sec(e+fx))^{7/2}}{7f}+\frac{6ad^3\sqrt{d\sec(e+fx)}\sin(e+fx)}{5f}+\frac{2ad(d\sec(e+fx))^{5/2}\sin(e+fx)}{5f}$$

output
```
-6/5*a*d^4*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(d*sec
(f*x+e))^(1/2)+2/7*b*(d*sec(f*x+e))^(7/2)/f+6/5*a*d^3*(d*sec(f*x+e))^(1/2)
*sin(f*x+e)/f+2/5*a*d*(d*sec(f*x+e))^(5/2)*sin(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.57

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx = \frac{(d\sec(e+fx))^{7/2}\left(40b-168a\cos^{\frac{7}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)+70a\sin(2(e+fx))+\right.}{140f}$$

input
```
Integrate[(d*Sec[e + f*x])^(7/2)*(a + b*Tan[e + f*x]),x]
```

output
```
((d*Sec[e + f*x])^(7/2)*(40*b - 168*a*Cos[e + f*x]^(7/2)*EllipticE[(e + f*
x)/2, 2] + 70*a*Sin[2*(e + f*x)] + 21*a*Sin[4*(e + f*x)]))/(140*f)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3967, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int (d\sec(e+fx))^{7/2}dx+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{5}d^2\int (d\sec(e+fx))^{3/2}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}d^2\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{5}d^2\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-d^2\int\frac{1}{\sqrt{d\sec(e+fx)}}dx\right)+\frac{2d\sin(e+fx)(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}d^2\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-d^2\int\frac{1}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx\right)+\frac{2d\sin(e+fx)(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{3}{5}d^2\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-\frac{d^2\int\sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\right)+\frac{2d\sin(e+fx)(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}d^2\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-\frac{d^2\int\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\right)+\frac{2d\sin(e+fx)(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{3}{5}d^2\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-\frac{2d^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\right)+\frac{2d\sin(e+fx)(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

input
```
Int[(d*Sec[e + f*x])^(7/2)*(a + b*Tan[e + f*x]),x]
```

output
```
(2*b*(d*Sec[e + f*x])^(7/2))/(7*f) + a*((2*d*(d*Sec[e + f*x])^(5/2)*Sin[e
+ f*x])/(5*f) + (3*d^2*((-2*d^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e +
 f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*d*Sqrt[d*Sec[e + f*x]]*Sin[e + f*x])/f))
/5)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 45.29 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.86

method	result
parts	$\frac{2a\sqrt{d\sec(fx+e)}\,d^3\left(3\sin(fx+e)+\tan(fx+e)+\sec(fx+e)\tan(fx+e)+i\left(3\cos(fx+e)^2+6\cos(fx+e)+3\right)\operatorname{EllipticF}(i(\csc(fx+e)-\cdots}{\cdots}$
default	$\frac{2\sqrt{d\sec(fx+e)}\left(21i\left(-\cos(fx+e)^2-2\cos(fx+e)-1\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,a\operatorname{EllipticE}(i(\csc(fx+e)-\cot(fx+e)),i)+21i\cdots}{\cdots}$

input
```
int((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/5*a/f*(d*sec(f*x+e))^(1/2)*d^3/(1+cos(f*x+e))*(3*sin(f*x+e)+tan(f*x+e)+s
ec(f*x+e)*tan(f*x+e)+I*(3*cos(f*x+e)^2+6*cos(f*x+e)+3)*EllipticF(I*(csc(f*
x+e)-cot(f*x+e)),I)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)+I*(-3*cos(f*x+e)^2-6*cos(f*x+e)-3)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)
),I)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2))+2/7*b*(d*
sec(f*x+e))^(7/2)/f
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.20

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx = \frac{-21i\sqrt{2}ad^{\frac{7}{2}}\cos(fx+e)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+\cdots}{\cdots}$$

input
```
integrate((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
1/35*(-21*I*sqrt(2)*a*d^(7/2)*cos(f*x + e)^3*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 21*I*sqrt(2)*a*d^(7
/2)*cos(f*x + e)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f
*x + e) - I*sin(f*x + e))) + 2*(5*b*d^3 + 7*(3*a*d^3*cos(f*x + e)^3 + a*d^
3*cos(f*x + e))*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(7/2)*(a+b*tan(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{7}{2}}(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(7/2)*(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx=\int (d\sec(fx+e))^{\frac{7}{2}}(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(7/2)*(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx=\int \left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}(a+b\tan(e+f\,x))\,dx$$

input
```
int((d/cos(e + f*x))^(7/2)*(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(7/2)*(a + b*tan(e + f*x)), x)
```

Reduce [F]

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx=\frac{\sqrt{d}\,d^3\left(2\sqrt{\sec(fx+e)}\,\sec(fx+e)^3 b+7\left(\int \sqrt{\sec(fx+e)}\,\sec(fx+e)^3 dx\right) af\right)}{7f}$$

input
```
int((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x)
```

output
```
(sqrt(d)*d**3*(2*sqrt(sec(e + f*x))*sec(e + f*x)**3*b + 7*int(sqrt(sec(e +
 f*x))*sec(e + f*x)**3,x)*a*f))/(7*f)
```

3.587 $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 92

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \frac{2ad^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3f} + \frac{2b(d\sec(e+fx))^{5/2}}{5f} + \frac{2ad(d\sec(e+fx))^{3/2}\sin(e+fx)}{3f}$$

output
```
2/3*a*d^2*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(d*sec(f
*x+e))^(1/2)/f+2/5*b*(d*sec(f*x+e))^(5/2)/f+2/3*a*d*(d*sec(f*x+e))^(3/2)*s
in(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.63

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \frac{(d\sec(e+fx))^{5/2}\left(6b+10a\cos^{\frac{5}{2}}(e+fx)\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+5a\sin(2(e+fx)\right.}{15f}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x]),x]
```

output
```
((d*Sec[e + f*x])^(5/2)*(6*b + 10*a*Cos[e + f*x]^(5/2)*EllipticF[(e + f*x)
/2, 2] + 5*a*Sin[2*(e + f*x)]))/(15*f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3967, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int (d\sec(e+fx))^{5/2}dx+\frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{3}d^2\int \sqrt{d\sec(e+fx)}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}d^2\int \sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{1}{3}d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2d^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3f}+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

input
```
Int[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x]),x]
```

output
```
(2*b*(d*Sec[e + f*x])^(5/2))/(5*f) + a*((2*d^2*Sqrt[Cos[e + f*x]]*Elliptic
F[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*f) + (2*d*(d*Sec[e + f*x])^(3/2
)*Sin[e + f*x])/(3*f))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 37.91 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.12

method	result
default	$\frac{\left(\frac{2a\tan(fx+e)}{3}+\frac{2\sec(fx+e)^2 b}{5}+\frac{2i(1+\cos(fx+e))\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,a\,\mathrm{EllipticF}(i(-\csc(fx+e)+\cot(fx+e)),i)}{3}\right)d^2\sqrt{d\sec(fx+}}{f}$
parts	$\frac{a\left(-\frac{2i(1+\cos(fx+e))\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)}{3}+\frac{2\tan(fx+e)}{3}\right)d^2\sqrt{d\sec(fx+e)}}{f}+\frac{2b(d\sec(}{5}$

input
```
int((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/f*(2/3*a*tan(f*x+e)+2/5*sec(f*x+e)^2*b+2/3*I*(1+cos(f*x+e))*(1/(1+cos(f*
x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*a*EllipticF(I*(-csc(f*x+e)+
cot(f*x+e)),I))*d^2*(d*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.34

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \frac{-5i\sqrt{2}ad^{\frac{5}{2}}\cos(fx+e)^2 \,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+5}{}$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*a*d^(5/2)*cos(f*x + e)^2*weierstrassPInverse(-4, 0, cos
(f*x + e) + I*sin(f*x + e)) + 5*I*sqrt(2)*a*d^(5/2)*cos(f*x + e)^2*weierst
rassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(5*a*d^2*cos(f*x +
e)*sin(f*x + e) + 3*b*d^2)*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \int (d\sec(e+fx))^{\frac{5}{2}}\,(a+b\tan(e+fx))\,dx$$

input
```
integrate((d*sec(f*x+e))**(5/2)*(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx=\int (d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx=\int (d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx=\int \left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}(a+b\tan(e+f\,x))\,dx$$

input
```
int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x)), x)
```

Reduce [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \frac{\sqrt{d}\,d^2\left(2\sqrt{\sec(fx+e)}\,\sec(fx+e)^2 b+5\left(\int \sqrt{\sec(fx+e)}\,\sec(fx+e)^2\,dx\right)af\right)}{5f}$$

input
```
int((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x)
```

output
```
(sqrt(d)*d**2*(2*sqrt(sec(e + f*x))*sec(e + f*x)**2*b + 5*int(sqrt(sec(e +
 f*x))*sec(e + f*x)**2,x)*a*f))/(5*f)
```

3.588 $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = -\frac{2ad^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2b(d\sec(e+fx))^{3/2}}{3f} + \frac{2ad\sqrt{d\sec(e+fx)}\sin(e+fx)}{f}$$

output
```
-2*a*d^2*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(d*sec(f
*x+e))^(1/2)+2/3*b*(d*sec(f*x+e))^(3/2)/f+2*a*d*(d*sec(f*x+e))^(1/2)*sin(f
*x+e)/f
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = \frac{(d\sec(e+fx))^{3/2}\left(2b-6a\cos^{\frac{3}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)+3a\sin(2(e+fx))\right)}{3f}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x]),x]
```

output
```
((d*Sec[e + f*x])^(3/2)*(2*b - 6*a*Cos[e + f*x]^(3/2)*EllipticE[(e + f*x)/
2, 2] + 3*a*Sin[2*(e + f*x)]))/(3*f)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3967, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int (d\sec(e+fx))^{3/2}dx+\frac{2b(d\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b(d\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-d^2\int \frac{1}{\sqrt{d\sec(e+fx)}}dx\right)+\frac{2b(d\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-d^2\int \frac{1}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b(d\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-\frac{d^2\int \sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\right)+\frac{2b(d\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-\frac{d^2\int\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\right)+\frac{2b(d\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-\frac{2d^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\right)+\frac{2b(d\sec(e+fx))^{3/2}}{3f}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x]),x]
```

output
```
(2*b*(d*Sec[e + f*x])^(3/2))/(3*f) + a*((-2*d^2*EllipticE[(e + f*x)/2, 2])
/(f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*d*Sqrt[d*Sec[e + f*x]]*S
in[e + f*x])/f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 12.40 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.26

method	result
default	$\frac{2d\sqrt{d\sec(fx+e)}\left(i\left(3\cos(fx+e)^2+6\cos(fx+e)+3\right)\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\ \mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{1+\cos(fx+e)}}\,a+i\left(-3\right.\right.}{3f(1+}$
parts	$\frac{2a\left(i\left(\cos(fx+e)^2+2\cos(fx+e)+1\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\ \mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)+i\left(-\cos(fx+e)^2-2\cos\right.\right.}{f(1+\cos(fx+e))}$

input
```
int((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/3/f*d*(d*sec(f*x+e))^(1/2)/(1+cos(f*x+e))*(I*(3*cos(f*x+e)^2+6*cos(f*x+e
)+3)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e))
,I)*(1/(1+cos(f*x+e)))^(1/2)*a+I*(-3*cos(f*x+e)^2-6*cos(f*x+e)-3)*(cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(1+cos
(f*x+e)))^(1/2)*a+3*a*sin(f*x+e)+b*(1+sec(f*x+e)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = \frac{-3i\sqrt{2}ad^{\frac{3}{2}}\cos(fx+e)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e}{}$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
1/3*(-3*I*sqrt(2)*a*d^(3/2)*cos(f*x + e)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2)*a*d^(3/2)*c
os(f*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e)
 - I*sin(f*x + e))) + 2*(3*a*d*cos(f*x + e)*sin(f*x + e) + b*d)*sqrt(d/cos
(f*x + e)))/(f*cos(f*x + e))
```

Sympy [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = \int (d\sec{(e+fx)})^{\frac{3}{2}}\,(a+b\tan{(e+fx)})\,dx$$

input
```
integrate((d*sec(f*x+e))**(3/2)*(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = \int (d\sec{(fx+e)})^{\frac{3}{2}}\,(b\tan{(fx+e)}+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}(a+b\tan(e+f\,x))\,dx$$

input
```
int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x)), x)
```

Reduce [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = \frac{\sqrt{d}\,d\left(2\sqrt{\sec(fx+e)}\,\sec(fx+e)\,b+3\left(\int\sqrt{\sec(fx+e)}\,\sec(fx+e)\,dx\right)af\right)}{3f}$$

input
```
int((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x)
```

output
```
(sqrt(d)*d*(2*sqrt(sec(e + f*x))*sec(e + f*x)*b + 3*int(sqrt(sec(e + f*x))
*sec(e + f*x),x)*a*f))/(3*f)
```

3.589 $\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 58

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{2b\sqrt{d\sec(e+fx)}}{f}+\frac{2a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{f}$$

output
```
2*b*(d*sec(f*x+e))^(1/2)/f+2*a*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/
2*e,2^(1/2))*(d*sec(f*x+e))^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.72

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{2\left(b+a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\right)\sqrt{d\sec(e+fx)}}{f}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]),x]
```

output
```
(2*(b + a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])*Sqrt[d*Sec[e + f*x
]])/f
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))dx$$
$$\Big\downarrow \ 3967$$
$$a\int \sqrt{d\sec(e+fx)}dx+\frac{2b\sqrt{d\sec(e+fx)}}{f}$$
$$\Big\downarrow \ 3042$$
$$a\int \sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}dx+\frac{2b\sqrt{d\sec(e+fx)}}{f}$$
$$\Big\downarrow \ 4258$$
$$a\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx+\frac{2b\sqrt{d\sec(e+fx)}}{f}$$
$$\Big\downarrow \ 3042$$
$$a\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2b\sqrt{d\sec(e+fx)}}{f}$$
$$\Big\downarrow \ 3120$$
$$\frac{2a\sqrt{\cos(e+fx)}\operatorname{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{f}+\frac{2b\sqrt{d\sec(e+fx)}}{f}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]),x]
```

output
```
(2*b*Sqrt[d*Sec[e + f*x]])/f + (2*a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)
/2, 2]*Sqrt[d*Sec[e + f*x]])/f
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.43

method	result
default	$\frac{\left(2i(1+\cos(fx+e))\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,a\,\text{EllipticF}(i(-\csc(fx+e)+\cot(fx+e)),i)+2b\right)\sqrt{d\sec(fx+e)}}{f}$
parts	$-\frac{2ia(1+\cos(fx+e))\sqrt{d\sec(fx+e)}\,\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\text{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)}{f}+\frac{2b\sqrt{d\sec(fx+e)}}{f}$

input
```
int((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/f*(2*I*a*(1+cos(f*x+e))*EllipticF(I*(-csc(f*x+e)+cot(f*x+e)),I)*(1/(1+co
s(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+2*b)*(d*sec(f*x+e))^(1/
2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.28

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx$$
$$= \frac{-i\sqrt{2}a\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+i\sqrt{2}a\sqrt{d}\text{weierstrassPInverse}(-4,}{f}$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x
+ e)) + I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*si
n(f*x + e)) + 2*b*sqrt(d/cos(f*x + e)))/f
```

Sympy [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \int \sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)*(a+b*tan(f*x+e)),x)
```

output
```
Integral(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \int \sqrt{d\sec(fx+e)}(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \int \sqrt{d\sec(fx+e)}(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{2\left(b+a\sqrt{\cos(e+fx)}\,\mathrm{F}\left(\frac{e}{2}+\frac{fx}{2}\middle|2\right)\right)\sqrt{\frac{d}{\cos(e+fx)}}}{f}$$

input
```
int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x)),x)
```

output
```
(2*(b + a*cos(e + f*x)^(1/2)*ellipticF(e/2 + (f*x)/2, 2))*(d/cos(e + f*x))
^(1/2))/f
```

Reduce [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx$$
$$=\frac{\sqrt{d}\left(2\sqrt{\sec\left(fx+e\right)}\,b+\left(\int\sqrt{\sec\left(fx+e\right)}dx\right)af\right)}{f}$$

input
```
int((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x)
```

output
```
(sqrt(d)*(2*sqrt(sec(e + f*x))*b + int(sqrt(sec(e + f*x)),x)*a*f))/f
```

3.590 $\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 58

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx=-\frac{2b}{f\sqrt{d\sec(e+fx)}}+\frac{2aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}$$

output
```
-2*b/f/(d*sec(f*x+e))^(1/2)+2*a*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/co
s(f*x+e)^(1/2)/(d*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx=\frac{-2b\sqrt{\cos(e+fx)}+2aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[(a + b*Tan[e + f*x])/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(-2*b*Sqrt[Cos[e + f*x]] + 2*a*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e +
f*x]]*Sqrt[d*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3967, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \frac{1}{\sqrt{d\sec(e+fx)}}dx-\frac{2b}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2b}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\int \sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2aE\left(\left.\frac{1}{2}(e+fx)\right|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b}{f\sqrt{d\sec(e+fx)}}$$

input
```
Int[(a + b*Tan[e + f*x])/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(-2*b)/(f*Sqrt[d*Sec[e + f*x]]) + (2*a*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[
Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.91 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.33

method	result
parts	$\frac{2a\left(i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)(-\cos(fx+e)-2-\sec(fx+e))+i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(f}{1+\cos(}}\right.}{f(1+\cos(fx+e))\sqrt{d\sec(fx+e)}}$
risch	$-\frac{i(-ib+a)\sqrt{2}}{f\sqrt{\frac{d\,\mathrm{e}^{i(fx+e)}}{\mathrm{e}^{2i(fx+e)}+1}}}-\frac{ia\left(-\frac{2\left(d\,\mathrm{e}^{2i(fx+e)}+d\right)}{d\sqrt{\mathrm{e}^{i(fx+e)}\left(d\,\mathrm{e}^{2i(fx+e)}+d\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(fx+e)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(fx+e)}-i\right)}\,\sqrt{i\mathrm{e}^{i(fx+e)}}\,\left(-2i\,\mathrm{EllipticE}\left(\sqrt{-i\left(\mathrm{e}^{i(}\right.}\right.\right.}{\sqrt{d\,\mathrm{e}^{3i(fx+e)}+d\,\mathrm{e}^{i(fx+}}}}{f\sqrt{\frac{d\,\mathrm{e}^{i(fx+e)}}{\mathrm{e}^{2i(fx+e)}+1}}\left(\mathrm{e}^{2i(fx+e)}\right.}$
default	$\frac{i\,\mathrm{EllipticF}(i(-\csc(fx+e)+\cot(fx+e)),i)\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}\,\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,a(4\cos(fx+e)+8+4\sec(fx+e))+i\,\mathrm{Elli}}{}$

input
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a/f/(1+cos(f*x+e))/(d*sec(f*x+e))^(1/2)*(I*(1/(1+cos(f*x+e)))^(1/2)*(cos
(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(-cos
(f*x+e)-2-sec(f*x+e))+I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(2+cos(f*x+e)+sec(f*x+e))+
sin(f*x+e))-2*b/f/(d*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.53

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx$$
$$= \frac{i\sqrt{2}a\sqrt{d}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))-i\sqrt{2}a\sqrt{d}\mathrm{we}}{}$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*a*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(f*x + e) + I*sin(f*x + e))) - I*sqrt(2)*a*sqrt(d)*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - 2*b*sqrt(d/co
s(f*x + e))*cos(f*x + e))/(d*f)
```

Sympy [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(1/2),x)
```

output
```
Integral((a + b*tan(e + f*x))/sqrt(d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{b\tan(fx+e)+a}{\sqrt{d\sec(fx+e)}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)/sqrt(d*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{b\tan(fx+e)+a}{\sqrt{d\sec(fx+e)}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)/sqrt(d*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{a+b\tan(e+f\,x)}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(1/2),x)
```

output
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx = \frac{\sqrt{d}\left(\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)}dx\right)a+\left(\int \frac{\sqrt{\sec(fx+e)}\,\tan(fx+e)}{\sec(fx+e)}dx\right)b\right)}{d}$$

input
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*(int(sqrt(sec(e + f*x))/sec(e + f*x),x)*a + int((sqrt(sec(e + f*x
))*tan(e + f*x))/sec(e + f*x),x)*b))/d
```

3.591 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = -\frac{2b}{3f(d\sec(e+fx))^{3/2}} + \frac{2a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3d^2f} + \frac{2a\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}$$

output
```
-2/3*b/f/(d*sec(f*x+e))^(3/2)+2/3*a*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f
*x+1/2*e,2^(1/2))*(d*sec(f*x+e))^(1/2)/d^2/f+2/3*a*sin(f*x+e)/d/f/(d*sec(f
*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{d\sec(e+fx)}\left(b+b\cos(2(e+fx))-2a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)-a\sin(2(e+fx))\right)}{3d^2f}$$

input
```
Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(3/2),x]
```

output
```
-1/3*(Sqrt[d*Sec[e + f*x]]*(b + b*Cos[2*(e + f*x)] - 2*a*Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2] - a*Sin[2*(e + f*x)]))/(d^2*f)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3967, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \frac{1}{(d\sec(e+fx))^{3/2}}dx-\frac{2b}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx-\frac{2b}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{\int \sqrt{d\sec(e+fx)}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)-\frac{2b}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\int \sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)-\frac{2b}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)-\frac{2b}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)-\frac{2b}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)-\frac{2b}{3f(d\sec(e+fx))^{3/2}}$$

input
```
Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(-2*b)/(3*f*(d*Sec[e + f*x])^(3/2)) + a*((2*Sqrt[Cos[e + f*x]]*EllipticF[(
e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*d^2*f) + (2*Sin[e + f*x])/(3*d*f*S
qrt[d*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.36 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{-\frac{2ia\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\operatorname{EllipticF}(i(-\csc(fx+e)+\cot(fx+e)),i)(-1-\sec(fx+e))}{3}+\frac{2a\sin(fx+e)}{3}-\frac{2b\cos(fx+e)}{3}}{f\sqrt{d\sec(fx+e)}\,d}$	103
parts	$\frac{a\left(-\frac{2i\operatorname{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,(1+\sec(fx+e))}{3}+\frac{2\sin(fx+e)}{3}\right)}{f\sqrt{d\sec(fx+e)}\,d}-\frac{2b}{3f(d\sec(fx+e))^{\frac{3}{2}}}$	108

input
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-2/3*I*a*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*E
llipticF(I*(-csc(f*x+e)+cot(f*x+e)),I)*(-1-sec(f*x+e))+2/3*a*sin(f*x+e)-2/
3*b*cos(f*x+e))/(d*sec(f*x+e))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = \frac{-i\sqrt{2}a\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\sin{(fx+e)})+i\sqrt{2}a\sqrt{d}$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(
f*x + e)) + I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) -
I*sin(f*x + e)) - 2*(b*cos(f*x + e)^2 - a*cos(f*x + e)*sin(f*x + e))*sqrt(
d/cos(f*x + e)))/(d^2*f)
```

Sympy [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{a+b\tan{(e+fx)}}{(d\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{b\tan{(fx+e)}+a}{(d\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx=\int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx=\int \frac{a+b\tan(e+f\,x)}{{\left(\frac{d}{\cos(e+f\,x)}\right)}^{3/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(3/2),x)
```

output
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{d}\,\left(\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^{2}}dx\right)a+\left(\int \frac{\sqrt{\sec(fx+e)}\,\tan(fx+e)}{\sec(fx+e)^{2}}dx\right)b\right)}{d^{2}}$$

input
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*(int(sqrt(sec(e + f*x))/sec(e + f*x)**2,x)*a + int((sqrt(sec(e + 
f*x))*tan(e + f*x))/sec(e + f*x)**2,x)*b))/d**2
```

3.592 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx = -\frac{2b}{5f(d\sec(e+fx))^{5/2}} + \frac{6aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2a\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}$$

output
```
-2/5*b/f/(d*sec(f*x+e))^(5/2)+6/5*a*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/
d^2/f/cos(f*x+e)^(1/2)/(d*sec(f*x+e))^(1/2)+2/5*a*sin(f*x+e)/d/f/(d*sec(f*
x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.79

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx = \frac{2\sqrt{d\sec(e+fx)}\left(3a\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)+\cos^2(e+fx)(-b\cos(e+\right.}{5d^3f}$$

input
```
Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/2),x]
```

output
```
(2*Sqrt[d*Sec[e + f*x]]*(3*a*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2]
+ Cos[e + f*x]^2*(-(b*Cos[e + f*x]) + a*Sin[e + f*x])))/(5*d^3*f)
```

Rubi [A] (verified)

Time = 0.45 (sec), antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3967, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \frac{1}{(d\sec(e+fx))^{5/2}}dx-\frac{2b}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx-\frac{2b}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{3\int \frac{1}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2b}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3\int \frac{1}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{5d^2}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2b}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{3\int\sqrt{\cos(e+fx)}dx}{5d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2b}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3\int\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{5d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2b}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{6E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2b}{5f(d\sec(e+fx))^{5/2}}$$

input
```
Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/2),x]
```

output
```
(-2*b)/(5*f*(d*Sec[e + f*x])^(5/2)) + a*((6*EllipticE[(e + f*x)/2, 2])/(5*
d^2*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*Sin[e + f*x])/(5*d*f*(
d*Sec[e + f*x])^(3/2)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 19.63 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.22

method	result
parts	$\frac{2a\left(\sin(fx+e)\left(\cos(fx+e)^2+\cos(fx+e)+3\right)-3i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,(2+\cos(fx+e)+\sec(fx+e))\,\mathrm{EllipticF}(i(\csc(fx+e)-\right.}{5f(1+\cos(fx+e))\sqrt{d\sec}}$
default	$\frac{-\frac{6i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,(2+\cos(fx+e)+\sec(fx+e))a\,\mathrm{EllipticE}(i(-\csc(fx+e)+\cot(fx+e)),i)}{5}+\frac{6i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,(2-}{}}{f(1+\cos(fx+e))}$

input
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*a/f/(1+cos(f*x+e))/(d*sec(f*x+e))^(1/2)/d^2*(sin(f*x+e)*(cos(f*x+e)^2+
cos(f*x+e)+3)-3*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)*(2+cos(f*x+e)+sec(f*x+e))*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)+3*I*(1
/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(csc(
f*x+e)-cot(f*x+e)),I)*(2+cos(f*x+e)+sec(f*x+e)))-2/5*b/f/(d*sec(f*x+e))^(5
/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx = \frac{3i\sqrt{2}a\sqrt{d}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(3*I*sqrt(2)*a*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(f*x + e) + I*sin(f*x + e))) - 3*I*sqrt(2)*a*sqrt(d)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - 2*(b*
cos(f*x + e)^3 - a*cos(f*x + e)^2*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^3
*f)
```

Sympy [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(5/2),x)
```

output
```
Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{a+b\tan(e+f\,x)}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(5/2),x)
```

output
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{a + b\tan(e + fx)}{(d\sec(e + fx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^3}dx\right)a + \left(\int \frac{\sqrt{\sec(fx+e)}\,\tan(fx+e)}{\sec(fx+e)^3}dx\right)b\right)}{d^3}$$

input
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(d)*(int(sqrt(sec(e + f*x))/sec(e + f*x)**3,x)*a + int((sqrt(sec(e +
f*x))*tan(e + f*x))/sec(e + f*x)**3,x)*b))/d**3
```

3.593 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = -\frac{2b}{7f(d\sec(e+fx))^{7/2}} + \frac{10a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{21d^4f} + \frac{2a\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}} + \frac{10a\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}}$$

output
```
-2/7*b/f/(d*sec(f*x+e))^(7/2)+10/21*a*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2
*f*x+1/2*e,2^(1/2))*(d*sec(f*x+e))^(1/2)/d^4/f+2/7*a*sin(f*x+e)/d/f/(d*sec
(f*x+e))^(5/2)+10/21*a*sin(f*x+e)/d^3/f/(d*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{d\sec(e+fx)}\left(-9b-12b\cos(2(e+fx))-3b\cos(4(e+fx))+40a\sqrt{\cos(e+\right.}{84d^4f}$$

input
```
Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(7/2),x]
```

output
```
(Sqrt[d*Sec[e + f*x]]*(-9*b - 12*b*Cos[2*(e + f*x)] - 3*b*Cos[4*(e + f*x)]
 + 40*a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + 26*a*Sin[2*(e + f*x
)] + 3*a*Sin[4*(e + f*x)]))/(84*d^4*f)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3967, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \frac{1}{(d\sec(e+fx))^{7/2}}dx-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{5\int \frac{1}{(d\sec(e+fx))^{3/2}}dx}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{5\left(\frac{\int\sqrt{d\sec(e+fx)}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5\left(\frac{\int\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{5\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{5\left(\frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

input
```
Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(7/2),x]
```

output
```
(-2*b)/(7*f*(d*Sec[e + f*x])^(7/2)) + a*((2*Sin[e + f*x])/(7*d*f*(d*Sec[e
+ f*x])^(5/2)) + (5*((2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[
d*Sec[e + f*x]])/(3*d^2*f) + (2*Sin[e + f*x])/(3*d*f*Sqrt[d*Sec[e + f*x]])
))/(7*d^2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 21.79 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

method	result
default	$\frac{\frac{2i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,a\,\mathrm{EllipticF}(i(-\csc(fx+e)+\cot(fx+e)),i)(5+5\sec(fx+e))}{21}+\frac{2\sin(fx+e)\left(3\cos(fx+e)^2+5\right)a}{21}-\frac{2\cos(fx+e)^3 b}{7}}{f\sqrt{d\sec(fx+e)}\,d^3}$
parts	$-\frac{2a\left(\sin(fx+e)\left(-3\cos(fx+e)^2-5\right)+i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)(5+5\sec(fx+e))\right)}{21f\sqrt{d\sec(fx+e)}\,d^3}-$

input
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(2/21*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*a*E
llipticF(I*(-csc(f*x+e)+cot(f*x+e)),I)*(5+5*sec(f*x+e))+2/21*sin(f*x+e)*(3
*cos(f*x+e)^2+5)*a-2/7*cos(f*x+e)^3*b)/(d*sec(f*x+e))^(1/2)/d^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.96

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx=\frac{-5i\sqrt{2}a\sqrt{d}\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+5i\sqrt{2}a\sqrt{}}{}$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*s
in(f*x + e)) + 5*I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x +
e) - I*sin(f*x + e)) - 2*(3*b*cos(f*x + e)^4 - (3*a*cos(f*x + e)^3 + 5*a*c
os(f*x + e))*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^4*f)
```

Sympy [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{a+b\tan{(e+fx)}}{(d\sec{(e+fx)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(7/2),x)
```

output
```
Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(7/2), x)
```

Maxima [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{b\tan{(fx+e)}+a}{(d\sec{(fx+e)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(7/2), x)
```

Giac [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{b\tan{(fx+e)}+a}{(d\sec{(fx+e)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{a+b\tan(e+f\,x)}{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(7/2),x)
```

output
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{d}\left(\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^4}dx\right)a+\left(\int \frac{\sqrt{\sec(fx+e)}\,\tan(fx+e)}{\sec(fx+e)^4}dx\right)b\right)}{d^4}$$

input
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x)
```

output
```
(sqrt(d)*(int(sqrt(sec(e + f*x))/sec(e + f*x)**4,x)*a + int((sqrt(sec(e +
f*x))*tan(e + f*x))/sec(e + f*x)**4,x)*b))/d**4
```

3.594 $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 143

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx = \frac{2(7a^2-2b^2)\,d^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{21f} + \frac{18ab(d\sec(e+fx))^{5/2}}{35f} + \frac{2(7a^2-2b^2)\,d(d\sec(e+fx))^{3/2}\sin(e+fx)}{21f} + \frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

output
```
2/21*(7*a^2-2*b^2)*d^2*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2^(1
/2))*(d*sec(f*x+e))^(1/2)/f+18/35*a*b*(d*sec(f*x+e))^(5/2)/f+2/21*(7*a^2-2
*b^2)*d*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/f+2/7*b*(d*sec(f*x+e))^(5/2)*(a+b*
tan(f*x+e))/f
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx = \frac{2d^2\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\left(5(7a^2-2b^2)\cos^{\frac{5}{2}}(e+fx)\,\text{EllipticF}\left(\frac{1}{2}(e+\right.\right.}{105f(a\cos(e+fx)+b\sin(}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2,x]
```

output
```
(2*d^2*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2*(5*(7*a^2 - 2*b^2)*Cos[
e + f*x]^(5/2)*EllipticF[(e + f*x)/2, 2] + (5*(7*a^2 - 2*b^2)*Sin[2*(e + f
*x)])/2 + 3*b*(14*a + 5*b*Tan[e + f*x])))/(105*f*(a*Cos[e + f*x] + b*Sin[e
 + f*x])^2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2dx$$

$$\Big\downarrow \text{ 3993}$$

$$\frac{2}{7}\int \frac{1}{2}(d\sec(e+fx))^{5/2}\left(7a^2+9b\tan(e+fx)a-2b^2\right)dx+\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\int (d\sec(e+fx))^{5/2}\left(7a^2+9b\tan(e+fx)a-2b^2\right)dx+\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\int (d\sec(e+fx))^{5/2}\left(7a^2+9b\tan(e+fx)a-2b^2\right)dx+\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

$$\Big\downarrow \; 3967$$

$$\frac{1}{7}\left(\left(7a^2-2b^2\right)\int (d\sec(e+fx))^{5/2}dx+\frac{18ab(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\left(7a^2-2b^2\right)\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{18ab(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

$$\Big\downarrow \; 4255$$

$$\frac{1}{7}\left(\left(7a^2-2b^2\right)\left(\frac{1}{3}d^2\int \sqrt{d\sec(e+fx)}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{18ab(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\left(7a^2-2b^2\right)\left(\frac{1}{3}d^2\int \sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{18ab(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

$$\Big\downarrow \; 4258$$

$$\frac{1}{7}\left(\left(7a^2-2b^2\right)\left(\frac{1}{3}d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)+\right.$$
$$\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\left(7a^2-2b^2\right)\left(\frac{1}{3}d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right.\right.$$
$$\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{7}\left(\left(7a^2-2b^2\right)\left(\frac{2d^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3f}+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)\right.$$
$$\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

input
```
Int[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2,x]
```

output
```
((18*a*b*(d*Sec[e + f*x])^(5/2))/(5*f) + (7*a^2 - 2*b^2)*((2*d^2*Sqrt[Cos[
e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*f) + (2*d*(d*
Sec[e + f*x])^(3/2)*Sin[e + f*x])/(3*f)))/7 + (2*b*(d*Sec[e + f*x])^(5/2)*
(a + b*Tan[e + f*x]))/(7*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 165.45 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.43

method	result
default	$\left(\frac{4\sec(fx+e)^2 ab}{5}+\frac{2b^2\left(-10\tan(fx+e)+15\tan(fx+e)\sec(fx+e)^2\right)}{105}+\frac{2\tan(fx+e)a^2}{3}-\frac{2i(1+\cos(fx+e))\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\sqrt{\frac{1}{1+\cos(fx+e)}}\ \text{Ellipti}}{3}\right.$
parts	$\frac{a^2\left(-\frac{2i(1+\cos(fx+e))\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\ \text{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)}{3}+\frac{2\tan(fx+e)}{3}\right)d^2\sqrt{d\sec(fx+e)}}{f}+\frac{b^2\left(-\frac{4\,\mathrm{t}}{}\right.}{}$

input
```
int((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(4/5*sec(f*x+e)^2*a*b+2/105*b^2*(-10*tan(f*x+e)+15*tan(f*x+e)*sec(f*x+
e)^2)+2/3*tan(f*x+e)*a^2-2/3*I*(1+cos(f*x+e))*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*a^2+
4/21*I*(1+cos(f*x+e))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))
^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*b^2)*(d*sec(f*x+e))^(1/2)*d^
2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.20

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx = \frac{-5i\sqrt{2}(7a^2-2b^2)d^{\frac{5}{2}}\cos(fx+e)^3\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(}{}$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/105*(-5*I*sqrt(2)*(7*a^2 - 2*b^2)*d^(5/2)*cos(f*x + e)^3*weierstrassPInv
erse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 5*I*sqrt(2)*(7*a^2 - 2*b^2)*d
^(5/2)*cos(f*x + e)^3*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x
+ e)) + 2*(42*a*b*d^2*cos(f*x + e) + 5*((7*a^2 - 2*b^2)*d^2*cos(f*x + e)^2
 + 3*b^2*d^2)*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx = \int (d\sec{(e+fx)})^{\frac{5}{2}}\,(a+b\tan{(e+fx)})^2\,dx$$

input
```
integrate((d*sec(f*x+e))**(5/2)*(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx = \int (d\sec{(fx+e)})^{\frac{5}{2}}\,(b\tan{(fx+e)}+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx = \int (d\sec{(fx+e)})^{\frac{5}{2}}\,(b\tan{(fx+e)}+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}(a+b\tan(e+f\,x))^2\,dx$$

input
```
int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^2, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx = \frac{\sqrt{d}\,d^2\left(4\sqrt{\sec(fx+e)}\,\sec(fx+e)^2\,ab+5\left(\int\sqrt{\sec(fx+e)}\,\sec(fx+e)^2\tan(fx\right.\right.}{5f}$$

input
```
int((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x)
```

output
```
(sqrt(d)*d**2*(4*sqrt(sec(e + f*x))*sec(e + f*x)**2*a*b + 5*int(sqrt(sec(e
 + f*x))*sec(e + f*x)**2*tan(e + f*x)**2,x)*b**2*f + 5*int(sqrt(sec(e + f*
x))*sec(e + f*x)**2,x)*a**2*f))/(5*f)
```

3.595 $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 143

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx = -\frac{2(5a^2-2b^2)\,d^2E\left(\frac{1}{2}(e+fx)\middle|\,2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{14ab(d\sec(e+fx))^{3/2}}{15f} + \frac{2(5a^2-2b^2)\,d\sqrt{d\sec(e+fx)}\sin(e+fx)}{5f} + \frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

output

```
-2/5*(5*a^2-2*b^2)*d^2*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^
(1/2)/(d*sec(f*x+e))^(1/2)+14/15*a*b*(d*sec(f*x+e))^(3/2)/f+2/5*(5*a^2-2*b
^2)*d*(d*sec(f*x+e))^(1/2)*sin(f*x+e)/f+2/5*b*(d*sec(f*x+e))^(3/2)*(a+b*ta
n(f*x+e))/f
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx =$$
$$-\frac{2d^2(a+b\tan(e+fx))^2\left(3(5a^2-2b^2)\cos^{\frac{3}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)+\left(-\frac{15a^2}{2}+3b^2\right)\sin(2(e+fx))-\right.}{15f\sqrt{d\sec(e+fx)}(a\cos(e+fx)+b\sin(e+fx))^2}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2,x]
```

output
```
(-2*d^2*(a + b*Tan[e + f*x])^2*(3*(5*a^2 - 2*b^2)*Cos[e + f*x]^(3/2)*Ellip
ticE[(e + f*x)/2, 2] + ((-15*a^2)/2 + 3*b^2)*Sin[2*(e + f*x)] - b*(10*a +
3*b*Tan[e + f*x])))/(15*f*Sqrt[d*Sec[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e +
 f*x])^2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2dx$$
$$\Big\downarrow \text{3993}$$
$$\frac{2}{5}\int \frac{1}{2}(d\sec(e+fx))^{3/2}\left(5a^2+7b\tan(e+fx)a-2b^2\right)dx+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int (d\sec(e+fx))^{3/2}\left(5a^2+7b\tan(e+fx)a-2b^2\right)dx+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int (d\sec(e+fx))^{3/2}\left(5a^2+7b\tan(e+fx)a-2b^2\right)dx+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{5}\left(\left(5a^2-2b^2\right)\int (d\sec(e+fx))^{3/2}dx+\frac{14ab(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\left(5a^2-2b^2\right)\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{14ab(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{5}\left(\left(5a^2-2b^2\right)\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-d^2\int \frac{1}{\sqrt{d\sec(e+fx)}}dx\right)+\frac{14ab(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\left(5a^2-2b^2\right)\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-d^2\int \frac{1}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx\right)+\frac{14ab(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{5}\left(\left(5a^2-2b^2\right)\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-\frac{d^2\int\sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\right)+\frac{14ab(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\left(5a^2-2b^2\right)\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-\frac{d^2\int\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\right)+\frac{14ab(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{5}\left(\left(5a^2-2b^2\right)\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-\frac{2d^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\right)+\frac{14ab(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2,x]
```

output
```
((14*a*b*(d*Sec[e + f*x])^(3/2))/(3*f) + (5*a^2 - 2*b^2)*((-2*d^2*Elliptic
E[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*d*Sqrt
[d*Sec[e + f*x]]*Sin[e + f*x])/f))/5 + (2*b*(d*Sec[e + f*x])^(3/2)*(a + b*
Tan[e + f*x]))/(5*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 23.29 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.74

method	result
default	$\frac{2\sqrt{d\sec(fx+e)}\left(15i\left(-\cos(fx+e)^2-2\cos(fx+e)-1\right)\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\sqrt{\frac{1}{1+\cos(fx+e)}}\,a^2\,\mathrm{EllipticE}(i(\csc(fx+e)-\cot(fx+e)),i)+6i(}{}$
parts	$\frac{2a^2\left(i\left(\cos(fx+e)^2+2\cos(fx+e)+1\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)+i\left(-\cos(fx+e)^2-2\cos\right.}{f(1+\cos(fx+e))}$

input
```
int((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/15/f*(d*sec(f*x+e))^(1/2)/(1+cos(f*x+e))*(15*I*(-cos(f*x+e)^2-2*cos(f*x+
e)-1)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*a^2*Ellip
ticE(I*(csc(f*x+e)-cot(f*x+e)),I)+6*I*(cos(f*x+e)^2+2*cos(f*x+e)+1)*(cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*b^2*EllipticE(I*(csc(
f*x+e)-cot(f*x+e)),I)+15*I*(cos(f*x+e)^2+2*cos(f*x+e)+1)*(cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*a^2*EllipticF(I*(csc(f*x+e)-cot(
f*x+e)),I)+6*I*(-cos(f*x+e)^2-2*cos(f*x+e)-1)*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*(1/(1+cos(f*x+e)))^(1/2)*b^2*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)+
15*a^2*sin(f*x+e)+10*b*a*(1+sec(f*x+e))+3*b^2*(-2*sin(f*x+e)+tan(f*x+e)+se
c(f*x+e)*tan(f*x+e)))*d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.20

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx = \frac{-3i\sqrt{2}(5a^2-2b^2)d^{\frac{3}{2}}\cos(fx+e)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\ldots$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/15*(-3*I*sqrt(2)*(5*a^2 - 2*b^2)*d^(3/2)*cos(f*x + e)^2*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sq
rt(2)*(5*a^2 - 2*b^2)*d^(3/2)*cos(f*x + e)^2*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*(10*a*b*d*cos(f*x
 + e) + 3*((5*a^2 - 2*b^2)*d*cos(f*x + e)^2 + b^2*d)*sin(f*x + e))*sqrt(d/
cos(f*x + e)))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx = \int (d\sec{(e+fx)})^{\frac{3}{2}}\,(a+b\tan{(e+fx)})^2\,dx$$

input
```
integrate((d*sec(f*x+e))**(3/2)*(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}\,(b\tan(fx+e)+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}\,(b\tan(fx+e)+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx=\int \left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}(a+b\tan(e+f\,x))^2\,dx$$

input
```
int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^2, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx=\frac{\sqrt{d}\,d\Big(4\sqrt{\sec(fx+e)}\,\sec(fx+e)\,ab+3\Big(\int\sqrt{\sec(fx+e)}\,\sec(fx+e)\tan(fx+e}{3f}$$

input
```
int((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x)
```

output
```
(sqrt(d)*d*(4*sqrt(sec(e + f*x))*sec(e + f*x)*a*b + 3*int(sqrt(sec(e + f*x
))*sec(e + f*x)*tan(e + f*x)**2,x)*b**2*f + 3*int(sqrt(sec(e + f*x))*sec(e
 + f*x),x)*a**2*f))/(3*f)
```

3.596 $\int \sqrt{d \sec(e+fx)}(a+b\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \sqrt{d \sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \frac{10ab\sqrt{d\sec(e+fx)}}{3f} + \frac{2(3a^2-2b^2)\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3f} + \frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

output
```
10/3*a*b*(d*sec(f*x+e))^(1/2)/f+2/3*(3*a^2-2*b^2)*cos(f*x+e)^(1/2)*Inverse
JacobiAM(1/2*f*x+1/2*e,2^(1/2))*(d*sec(f*x+e))^(1/2)/f+2/3*b*(d*sec(f*x+e)
)^(1/2)*(a+b*tan(f*x+e))/f
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$$
$$=\frac{2\sec^2(e+fx)\sqrt{d\sec(e+fx)}\left((3a^2-2b^2)\cos^{\frac{5}{2}}(e+fx)\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+b\cos(e+fx)(6a\cos\right.}{3f}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2,x]
```

output
```
(2*Sec[e + f*x]^2*Sqrt[d*Sec[e + f*x]]*((3*a^2 - 2*b^2)*Cos[e + f*x]^(5/2)
*EllipticF[(e + f*x)/2, 2] + b*Cos[e + f*x]*(6*a*Cos[e + f*x] + b*Sin[e +
f*x])))/(3*f)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2dx$$

$$\Big\downarrow \text{3993}$$

$$\frac{2}{3}\int \frac{1}{2}\sqrt{d\sec(e+fx)}\left(3a^2+5b\tan(e+fx)a-2b^2\right)dx+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \sqrt{d\sec(e+fx)}\left(3a^2+5b\tan(e+fx)a-2b^2\right)dx+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \sqrt{d\sec(e+fx)}\left(3a^2+5b\tan(e+fx)a-2b^2\right)dx+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{3}\left(\left(3a^2-2b^2\right)\int \sqrt{d\sec(e+fx)}dx+\frac{10ab\sqrt{d\sec(e+fx)}}{f}\right)+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\left(3a^2-2b^2\right)\int \sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}dx+\frac{10ab\sqrt{d\sec(e+fx)}}{f}\right)+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}\left(\left(3a^2-2b^2\right)\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx+\frac{10ab\sqrt{d\sec(e+fx)}}{f}\right)+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\left(3a^2-2b^2\right)\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{10ab\sqrt{d\sec(e+fx)}}{f}\right)+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{3}\left(\frac{2\left(3a^2-2b^2\right)\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{f}+\frac{10ab\sqrt{d\sec(e+fx)}}{f}\right)+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2,x]
```

output
```
((10*a*b*Sqrt[d*Sec[e + f*x]])/f + (2*(3*a^2 - 2*b^2)*Sqrt[Cos[e + f*x]]*E
llipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/f)/3 + (2*b*Sqrt[d*Sec[e +
f*x]]*(a + b*Tan[e + f*x]))/(3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 15.45 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.58

method	result
default	$\frac{\left(-2i(1+\cos(fx+e))\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\sqrt{\frac{1}{1+\cos(fx+e)}}\ \mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)a^2+\frac{4i(1+\cos(fx+e))\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\sqrt{\frac{1}{1+c}}}{}\right.}{f}$
parts	$-\frac{2ia^2(1+\cos(fx+e))\sqrt{d\sec(fx+e)}\,\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\ \mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)}{f}+\frac{b^2\left(\frac{2\tan(fx+e)}{3}+\frac{4i}{}\right.}{}$

input
```
int((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-2*I*(1+cos(f*x+e))*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*a^2*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)+4/3*I*(1+cos(f*x+e))*
(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*b^2*EllipticF(I
*(csc(f*x+e)-cot(f*x+e)),I)+4*a*b+2/3*tan(f*x+e)*b^2)*(d*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.29

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$$
$$=\frac{\sqrt{2}(-3i\,a^2+2i\,b^2)\sqrt{d}\cos(fx+e)\ \mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e))+\sqrt{2}(3i\,a^2}{}$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-3*I*a^2 + 2*I*b^2)*sqrt(d)*cos(f*x + e)*weierstrassPInverse
(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + sqrt(2)*(3*I*a^2 - 2*I*b^2)*sqrt(
d)*cos(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))
+ 2*(6*a*b*cos(f*x + e) + b^2*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f
*x + e))
```

Sympy [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \int \sqrt{d\sec{(e+fx)}}(a+b\tan{(e+fx)})^2\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)*(a+b*tan(f*x+e))**2,x)
```

output
```
Integral(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \int \sqrt{d\sec{(fx+e)}}(b\tan{(fx+e)}+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \int \sqrt{d\sec(fx+e)}(b\tan(fx+e)+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \int \sqrt{\frac{d}{\cos(e+f\,x)}}\,(a+b\tan(e+f\,x))^2\,dx$$

input
```
int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^2, x)
```

Reduce [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \frac{\sqrt{d}\left(4\sqrt{\sec(fx+e)}\,ab+\left(\int \sqrt{\sec(fx+e)}dx\right)a^2f+\left(\int \sqrt{\sec(fx+e)}\,\tan(fx+e)^2\,dx\right)b^2f\right)}{f}$$

input
```
int((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x)
```

output
```
(sqrt(d)*(4*sqrt(sec(e + f*x))*a*b + int(sqrt(sec(e + f*x)),x)*a**2*f + in
t(sqrt(sec(e + f*x))*tan(e + f*x)**2,x)*b**2*f))/f
```

3.597 $\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx = -\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2(a^2-2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

output
```
-6*a*b/f/(d*sec(f*x+e))^(1/2)+2*(a^2-2*b^2)*EllipticE(sin(1/2*f*x+1/2*e),2
^(1/2))/f/cos(f*x+e)^(1/2)/(d*sec(f*x+e))^(1/2)+2*b*(a+b*tan(f*x+e))/f/(d*
sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx = \frac{\frac{2(a^2-2b^2)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}+2b(-2a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[(a + b*Tan[e + f*x])^2/Sqrt[d*Sec[e + f*x]],x]
```

output
```
((2*(a^2 - 2*b^2)*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] + 2*b*(-2*
a + b*Tan[e + f*x]))/(f*Sqrt[d*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3993}$$

$$2\int \frac{a^2+3b\tan(e+fx)a-2b^2}{2\sqrt{d\sec(e+fx)}}dx+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{a^2+3b\tan(e+fx)a-2b^2}{\sqrt{d\sec(e+fx)}}dx+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+3b\tan(e+fx)a-2b^2}{\sqrt{d\sec(e+fx)}}dx+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3967}$$

$$\left(a^2-2b^2\right)\int \frac{1}{\sqrt{d\sec(e+fx)}}dx-\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\left(a^2-2b^2\right)\int \frac{1}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{(a^2-2b^2)\int \sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2-2b^2)\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2(a^2-2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

input
```
Int[(a + b*Tan[e + f*x])^2/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(-6*a*b)/(f*Sqrt[d*Sec[e + f*x]]) + (2*(a^2 - 2*b^2)*EllipticE[(e + f*x)/2
, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*b*(a + b*Tan[e + f*
x]))/(f*Sqrt[d*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 14.79 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.97

method	result
parts	$\frac{2a^2\left(i\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\ \text{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)(-\cos(fx+e)-2-\sec(fx+e))+i\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(}{1+\cos}}\right.}{f(1+\cos(fx+e))\sqrt{d\sec(fx+e)}}$
default	$\frac{\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}+4\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}-2\cos(fx+e)+2}{1+\cos(fx+e)}\right)ab-\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}+2\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}}{1+\cos(fx+e)}\right.}{}$

input
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^2/f/(1+cos(f*x+e))/(d*sec(f*x+e))^(1/2)*(I*(1/(1+cos(f*x+e)))^(1/2)*(c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(-c
os(f*x+e)-2-sec(f*x+e))+I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(2+cos(f*x+e)+sec(f*x+e)
)+sin(f*x+e))-2*b^2/f/(1+cos(f*x+e))/(d*sec(f*x+e))^(1/2)*(sin(f*x+e)-tan(
f*x+e)-2*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(2+c
os(f*x+e)+sec(f*x+e))*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)+2*I*(1/(1+cos
(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(csc(f*x+e)-
cot(f*x+e)),I)*(2+cos(f*x+e)+sec(f*x+e)))-4*a*b/f/(d*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.28

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx$$
$$= \frac{\sqrt{2}(i\,a^2-2i\,b^2)\sqrt{d}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\,\sin{(fx+e)}))+\sqrt{}}{}$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*(I*a^2 - 2*I*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(2)*(-I*a^2 + 2*I*b^2)*sq
rt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*s
in(f*x + e))) - 2*(2*a*b*cos(f*x + e) - b^2*sin(f*x + e))*sqrt(d/cos(f*x +
 e)))/(d*f)
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(a+b\tan{(e+fx)})^2}{\sqrt{d\sec{(e+fx)}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(1/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**2/sqrt(d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^2}{\sqrt{d\sec{(fx+e)}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^2/sqrt(d*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^2}{\sqrt{d\sec{(fx+e)}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^2/sqrt(d*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(a+b\tan(e+f\,x))^2}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(1/2),x)
```

output
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx = \frac{\sqrt{d}\left(\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)}dx\right)a^2+\left(\int \frac{\sqrt{\sec(fx+e)}\,\tan(fx+e)^2}{\sec(fx+e)}dx\right)b^2+2\left(\int \frac{\sqrt{\sec(fx+e)}\,\tan(fx+e)}{\sec(fx+e)}dx\right)ab\right)}{d}$$

input
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*(int(sqrt(sec(e + f*x))/sec(e + f*x),x)*a**2 + int((sqrt(sec(e +
f*x))*tan(e + f*x)**2)/sec(e + f*x),x)*b**2 + 2*int((sqrt(sec(e + f*x))*ta
n(e + f*x))/sec(e + f*x),x)*a*b))/d
```

3.598 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2ab}{3f(d\sec(e+fx))^{3/2}} + \frac{2(a^2+2b^2)\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3d^2f} + \frac{2(a^2+2b^2)\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}} - \frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

output
```
2/3*a*b/f/(d*sec(f*x+e))^(3/2)+2/3*(a^2+2*b^2)*cos(f*x+e)^(1/2)*InverseJac
obiAM(1/2*f*x+1/2*e,2^(1/2))*(d*sec(f*x+e))^(1/2)/d^2/f+2/3*(a^2+2*b^2)*si
n(f*x+e)/d/f/(d*sec(f*x+e))^(1/2)-2*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(3
/2)
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.73

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sec^2(e+fx)\left(-2ab-2ab\cos(2(e+fx))+2(a^2+2b^2)\sqrt{\cos(e+fx)}\,\text{Ellipti}\right.}{3f(d\sec(e+fx))^{3/}}$$

input
```
Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(Sec[e + f*x]^2*(-2*a*b - 2*a*b*Cos[2*(e + f*x)] + 2*(a^2 + 2*b^2)*Sqrt[Co
s[e + f*x]]*EllipticF[(e + f*x)/2, 2] + a^2*Sin[2*(e + f*x)] - b^2*Sin[2*(
e + f*x)]))/(3*f*(d*Sec[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \; 3993$$

$$-2\int -\frac{a^2-b\tan(e+fx)a+2b^2}{2(d\sec(e+fx))^{3/2}}dx-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\int \frac{a^2-b\tan(e+fx)a+2b^2}{(d\sec(e+fx))^{3/2}}dx-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\int \frac{a^2-b\tan(e+fx)a+2b^2}{(d\sec(e+fx))^{3/2}}dx-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3967}$$

$$\left(a^2+2b^2\right)\int \frac{1}{(d\sec(e+fx))^{3/2}}dx+\frac{2ab}{3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\left(a^2+2b^2\right)\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{2ab}{3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 4256}$$

$$\left(a^2+2b^2\right)\left(\frac{\int \sqrt{d\sec(e+fx)}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)+\frac{2ab}{3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\left(a^2+2b^2\right)\left(\frac{\int \sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)+\frac{2ab}{3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$\left(a^2+2b^2\right)\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)+\frac{2ab}{3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\left(a^2+2b^2\right)\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)+\frac{2ab}{3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3120}$$

$$(a^2+2b^2)\left(\frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)+\frac{2ab}{3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

input
```
Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(2*a*b)/(3*f*(d*Sec[e + f*x])^(3/2)) + (a^2 + 2*b^2)*((2*Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*d^2*f) + (2*Sin[e + f
*x])/(3*d*f*Sqrt[d*Sec[e + f*x]])) - (2*b*(a + b*Tan[e + f*x]))/(f*(d*Sec[
e + f*x])^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 15.51 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.35

method	result
default	$\frac{-\frac{2i\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\sqrt{\frac{1}{1+\cos(fx+e)}}\,a^2 \operatorname{EllipticF}(i(-\csc(fx+e)+\cot(fx+e)),i)(-1-\sec(fx+e))}{3}-\frac{2i\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\sqrt{\frac{1}{1+\cos(fx+e)}}\,b^2 \operatorname{EllipticF}}{}}{f\sqrt{d\sec(fx+e)}\,d}$
parts	$\frac{a^2\left(-\frac{2i \operatorname{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,(1+\sec(fx+e))}{3}+\frac{2\sin(fx+e)}{3}\right)}{f\sqrt{d\sec(fx+e)}\,d}+\frac{b^2\left(-\frac{2i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{}}{}\right.}{}$

input
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-2/3*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*a^2
*EllipticF(I*(-csc(f*x+e)+cot(f*x+e)),I)*(-1-sec(f*x+e))-2/3*I*(cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*b^2*EllipticF(I*(-csc(f*x+
e)+cot(f*x+e)),I)*(-2-2*sec(f*x+e))+2/3*a^2*sin(f*x+e)-4/3*cos(f*x+e)*a*b-
2/3*sin(f*x+e)*b^2)/(d*sec(f*x+e))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}(-i\,a^2-2i\,b^2)\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e)}{}$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-I*a^2 - 2*I*b^2)*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x
 + e) + I*sin(f*x + e)) + sqrt(2)*(I*a^2 + 2*I*b^2)*sqrt(d)*weierstrassPIn
verse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*(2*a*b*cos(f*x + e)^2 - (a
^2 - b^2)*cos(f*x + e)*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^2*f)
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(3/2),x)
```

output
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^2}dx\right)a^2+\left(\int \frac{\sqrt{\sec(fx+e)}\,\tan(fx+e)^2}{\sec(fx+e)^2}dx\right)b^2+2\left(\int \frac{\sqrt{\sec(fx+e)}\,\tan}{\sec(fx+e}\right.\right.}{d^2}$$

input
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*(int(sqrt(sec(e + f*x))/sec(e + f*x)**2,x)*a**2 + int((sqrt(sec(e
 + f*x))*tan(e + f*x)**2)/sec(e + f*x)**2,x)*b**2 + 2*int((sqrt(sec(e + f*
x))*tan(e + f*x))/sec(e + f*x)**2,x)*a*b))/d**2
```

3.599 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 145

$$\begin{aligned}\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx &= -\frac{2ab}{15f(d\sec(e+fx))^{5/2}}\\ &\quad+\frac{2(3a^2+2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\\ &\quad+\frac{2(3a^2+2b^2)\sin(e+fx)}{15df(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}\end{aligned}$$

output

```
-2/15*a*b/f/(d*sec(f*x+e))^(5/2)+2/5*(3*a^2+2*b^2)*EllipticE(sin(1/2*f*x+1
/2*e),2^(1/2))/d^2/f/cos(f*x+e)^(1/2)/(d*sec(f*x+e))^(1/2)+2/15*(3*a^2+2*b
^2)*sin(f*x+e)/d/f/(d*sec(f*x+e))^(3/2)-2/3*b*(a+b*tan(f*x+e))/f/(d*sec(f*
x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = \frac{\left(6a^2+4b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)+2\cos^{\frac{3}{2}}(e+fx)\left(-2ab\cos(e+fx)+\left(a^2-b^2\right)\right.}{5f\cos^{\frac{5}{2}}(e+fx)(d\sec(e+fx))^{5/2}}$$

input
```
Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/2),x]
```

output
```
((6*a^2 + 4*b^2)*EllipticE[(e + f*x)/2, 2] + 2*Cos[e + f*x]^(3/2)*(-2*a*b*
Cos[e + f*x] + (a^2 - b^2)*Sin[e + f*x]))/(5*f*Cos[e + f*x]^(5/2)*(d*Sec[e
 + f*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3993}$$

$$-\frac{2}{3}\int -\frac{3a^2+b\tan(e+fx)a+2b^2}{2(d\sec(e+fx))^{5/2}}dx-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3a^2+b\tan(e+fx)a+2b^2}{(d\sec(e+fx))^{5/2}}dx-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3a^2+b\tan(e+fx)a+2b^2}{(d\sec(e+fx))^{5/2}}dx-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{3}\left(\left(3a^2+2b^2\right)\int \frac{1}{(d\sec(e+fx))^{5/2}}dx-\frac{2ab}{5f(d\sec(e+fx))^{5/2}}\right)-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\left(3a^2+2b^2\right)\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx-\frac{2ab}{5f(d\sec(e+fx))^{5/2}}\right)-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{1}{3}\left(\left(3a^2+2b^2\right)\left(\frac{3\int \frac{1}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2ab}{5f(d\sec(e+fx))^{5/2}}\right)-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\left(3a^2+2b^2\right)\left(\frac{3\int \frac{1}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{5d^2}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2ab}{5f(d\sec(e+fx))^{5/2}}\right)-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}\left(\left(3a^2+2b^2\right)\left(\frac{3\int \sqrt{\cos(e+fx)}dx}{5d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2ab}{5f(d\sec(e+fx))^{5/2}}\right)-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\left(3a^2+2b^2\right)\left(\frac{3\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{5d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2ab}{5f(d\sec(e+fx))^{5/2}}\right)-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{3}\left((3a^2+2b^2)\left(\frac{6E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2ab}{5f(d\sec(e+fx))^{5/2}}\right)-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

input
```
Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/2),x]
```

output
```
((-2*a*b)/(5*f*(d*Sec[e + f*x])^(5/2)) + (3*a^2 + 2*b^2)*((6*EllipticE[(e
+ f*x)/2, 2])/(5*d^2*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*Sin[e
 + f*x])/(5*d*f*(d*Sec[e + f*x])^(3/2))))/3 - (2*b*(a + b*Tan[e + f*x]))/(
3*f*(d*Sec[e + f*x])^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 25.42 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.74

method	result
default	$-\frac{\frac{6i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,(2+\cos(fx+e)+\sec(fx+e))a^2\,\mathrm{EllipticE}(i(-\csc(fx+e)+\cot(fx+e)),i)}{5}-\frac{4i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,(2}{}}{}$
parts	$\frac{2a^2\left(\sin(fx+e)\left(\cos(fx+e)^2+\cos(fx+e)+3\right)-3i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,(2+\cos(fx+e)+\sec(fx+e))\,\mathrm{EllipticF}(i(\csc(fx+e)\right.}{5f(1+\cos(fx+e))\sqrt{d\,\mathrm{se}}}$

input
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d^2/f/(1+cos(f*x+e))/(d*sec(f*x+e))^(1/2)*(-3*I*(1/(1+cos(f*x+e)))^(1/
2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(2+cos(f*x+e)+sec(f*x+e))*a^2*Ellipti
cE(I*(-csc(f*x+e)+cot(f*x+e)),I)-2*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*(2+cos(f*x+e)+sec(f*x+e))*b^2*EllipticE(I*(-csc(f*x+
e)+cot(f*x+e)),I)+3*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*(2+cos(f*x+e)+sec(f*x+e))*a^2*EllipticF(I*(-csc(f*x+e)+cot(f*x+e)),
I)+2*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(2+cos(f
*x+e)+sec(f*x+e))*b^2*EllipticF(I*(-csc(f*x+e)+cot(f*x+e)),I)+sin(f*x+e)*(
cos(f*x+e)^2+cos(f*x+e)+3)*a^2+b*a*(-2*cos(f*x+e)^3-2*cos(f*x+e)^2)+sin(f*
x+e)*(-cos(f*x+e)^2-cos(f*x+e)+2)*b^2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}(3i\,a^2+2i\,b^2)\sqrt{d}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos{(f}$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(sqrt(2)*(3*I*a^2 + 2*I*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(2)*(-3*I*a^2 - 2*I
*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x +
e) - I*sin(f*x + e))) - 2*(2*a*b*cos(f*x + e)^3 - (a^2 - b^2)*cos(f*x + e)
^2*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^3*f)
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(a+b\tan{(e+fx)})^2}{(d\sec{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(5/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(5/2),x)
```

output
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^3}dx\right)a^2 + \left(\int \frac{\sqrt{\sec(fx+e)}\tan(fx+e)^2}{\sec(fx+e)^3}dx\right)b^2 + 2\left(\int \frac{\sqrt{\sec(fx+e)}\tan(fx+e)}{\sec(fx+e)^3}dx\right)ab\right)}{d^3}$$

input
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(d)*(int(sqrt(sec(e + f*x))/sec(e + f*x)**3,x)*a**2 + int((sqrt(sec(e
 + f*x))*tan(e + f*x)**2)/sec(e + f*x)**3,x)*b**2 + 2*int((sqrt(sec(e + f*
x))*tan(e + f*x))/sec(e + f*x)**3,x)*a*b))/d**3
```

3.600 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 184

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = -\frac{6ab}{35f(d\sec(e+fx))^{7/2}} + \frac{2(5a^2+2b^2)\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{21d^4f} + \frac{2(5a^2+2b^2)\sin(e+fx)}{35df(d\sec(e+fx))^{5/2}} + \frac{2(5a^2+2b^2)\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}} - \frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

output

```
-6/35*a*b/f/(d*sec(f*x+e))^(7/2)+2/21*(5*a^2+2*b^2)*cos(f*x+e)^(1/2)*Inver
seJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(d*sec(f*x+e))^(1/2)/d^4/f+2/35*(5*a^2+2
*b^2)*sin(f*x+e)/d/f/(d*sec(f*x+e))^(5/2)+2/21*(5*a^2+2*b^2)*sin(f*x+e)/d^
3/f/(d*sec(f*x+e))^(1/2)-2/5*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(7/2)
```

Mathematica [A] (verified)

Time = 2.85 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \frac{-18ab\cos(e+fx)-6ab\cos(3(e+fx))+\frac{4(5a^2+2b^2)\operatorname{EllipticF}\left(\frac{1}{2}(e+fx),2\right)}{\sqrt{\cos(e+fx)}}+23a^2}{42d^3f\sqrt{d\sec}}$$

input
```
Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(7/2),x]
```

output
```
(-18*a*b*Cos[e + f*x] - 6*a*b*Cos[3*(e + f*x)] + (4*(5*a^2 + 2*b^2)*Ellipt
icF[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] + 23*a^2*Sin[e + f*x] + 5*b^2*Sin[
e + f*x] + 3*a^2*Sin[3*(e + f*x)] - 3*b^2*Sin[3*(e + f*x)])/(42*d^3*f*Sqrt
[d*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3993}$$

$$-\frac{2}{5}\int -\frac{5a^2+3b\tan(e+fx)a+2b^2}{2(d\sec(e+fx))^{7/2}}dx-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{5a^2+3b\tan(e+fx)a+2b^2}{(d\sec(e+fx))^{7/2}}dx-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{5a^2+3b\tan(e+fx)a+2b^2}{(d\sec(e+fx))^{7/2}}dx-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{5}\left(\left(5a^2+2b^2\right)\int \frac{1}{(d\sec(e+fx))^{7/2}}dx-\frac{6ab}{7f(d\sec(e+fx))^{7/2}}\right)-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\left(5a^2+2b^2\right)\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx-\frac{6ab}{7f(d\sec(e+fx))^{7/2}}\right)-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{1}{5}\left(\left(5a^2+2b^2\right)\left(\frac{5\int \frac{1}{(d\sec(e+fx))^{3/2}}dx}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{6ab}{7f(d\sec(e+fx))^{7/2}}\right)-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\left(5a^2+2b^2\right)\left(\frac{5\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{6ab}{7f(d\sec(e+fx))^{7/2}}\right)-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{1}{5}\left(\left(5a^2+2b^2\right)\left(\frac{5\left(\frac{\int \sqrt{d\sec(e+fx)}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{6ab}{7f(d\sec(e+fx))^{7/2}}\right)-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left((5a^2+2b^2)\left(\frac{5\left(\frac{\int\sqrt{d\csc(e+fx+\frac{\pi}{2})}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{6ab}{7f(d\sec(e+fx))^{7/2}}\right)+$$

$$\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$\Big\downarrow$ 4258

$$\frac{1}{5}\left((5a^2+2b^2)\left(\frac{5\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{}{7f(d\sec}\right.$$

$$\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left((5a^2+2b^2)\left(\frac{5\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{}{7f(}\right.$$

$$\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$\Big\downarrow$ 3120

$$\frac{1}{5}\left((5a^2+2b^2)\left(\frac{5\left(\frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{}{7f}\right.$$

$$\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

input
```
Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(7/2),x]
```

output
```
((-6*a*b)/(7*f*(d*Sec[e + f*x])^(7/2)) + (5*a^2 + 2*b^2)*((2*Sin[e + f*x])
/(7*d*f*(d*Sec[e + f*x])^(5/2)) + (5*((2*Sqrt[Cos[e + f*x]]*EllipticF[(e +
 f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*d^2*f) + (2*Sin[e + f*x])/(3*d*f*Sqrt
[d*Sec[e + f*x]])))/(7*d^2)))/5 - (2*b*(a + b*Tan[e + f*x]))/(5*f*(d*Sec[e
 + f*x])^(7/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 26.70 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.15

method	result
default	$-\frac{2\left(i\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\sqrt{\frac{1}{1+\cos(fx+e)}}\,a^2\,\mathrm{EllipticF}(i(-\csc(fx+e)+\cot(fx+e)),i)(-5-5\sec(fx+e))+i\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\sqrt{\frac{1}{1+\cos(fx+e}}\right.}{}$
parts	$-\frac{2a^2\left(\sin(fx+e)\left(-3\cos(fx+e)^2-5\right)+i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)(5+5\sec(fx+e))\right)}{21f\sqrt{d\sec(fx+e)}\,d^3}$

input
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21/f/(d*sec(f*x+e))^(1/2)/d^3*(I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(
1+cos(f*x+e)))^(1/2)*a^2*EllipticF(I*(-csc(f*x+e)+cot(f*x+e)),I)*(-5-5*sec
(f*x+e))+I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*b^2*
EllipticF(I*(-csc(f*x+e)+cot(f*x+e)),I)*(-2-2*sec(f*x+e))+sin(f*x+e)*(-3*c
os(f*x+e)^2-5)*a^2+6*cos(f*x+e)^3*a*b+sin(f*x+e)*(3*cos(f*x+e)^2-2)*b^2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{2}(-5i\,a^2-2i\,b^2)\sqrt{d}\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e}{}$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

output
```
1/21*(sqrt(2)*(-5*I*a^2 - 2*I*b^2)*sqrt(d)*weierstrassPInverse(-4, 0, cos(
f*x + e) + I*sin(f*x + e)) + sqrt(2)*(5*I*a^2 + 2*I*b^2)*sqrt(d)*weierstra
ssPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*(6*a*b*cos(f*x + e)^4
 - (3*(a^2 - b^2)*cos(f*x + e)^3 + (5*a^2 + 2*b^2)*cos(f*x + e))*sin(f*x +
 e))*sqrt(d/cos(f*x + e)))/(d^4*f)
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(a+b\tan{(e+fx)})^2}{(d\sec{(e+fx)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(7/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(7/2), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^2}{(d\sec{(fx+e)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(7/2), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(7/2),x)
```

output
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{d}\left(\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^4}dx\right)a^2+\left(\int \frac{\sqrt{\sec(fx+e)}\,\tan(fx+e)^2}{\sec(fx+e)^4}dx\right)b^2+2\left(\int \frac{\sqrt{\sec(fx+e)}\,\tan}{\sec(fx+e)}\right.\right.}{d^4}$$

input
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x)
```

output
```
(sqrt(d)*(int(sqrt(sec(e + f*x))/sec(e + f*x)**4,x)*a**2 + int((sqrt(sec(e
 + f*x))*tan(e + f*x)**2)/sec(e + f*x)**4,x)*b**2 + 2*int((sqrt(sec(e + f*
x))*tan(e + f*x))/sec(e + f*x)**4,x)*a*b))/d**4
```

3.601 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 184

$$\begin{aligned}\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx &= -\frac{10ab}{63f(d\sec(e+fx))^{9/2}}\\ &+\frac{2(7a^2+2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|\,2\right)}{15d^4f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(7a^2+2b^2)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}}\\ &+\frac{2(7a^2+2b^2)\sin(e+fx)}{45d^3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}\end{aligned}$$

output
```
-10/63*a*b/f/(d*sec(f*x+e))^(9/2)+2/15*(7*a^2+2*b^2)*EllipticE(sin(1/2*f*x
+1/2*e),2^(1/2))/d^4/f/cos(f*x+e)^(1/2)/(d*sec(f*x+e))^(1/2)+2/63*(7*a^2+2
*b^2)*sin(f*x+e)/d/f/(d*sec(f*x+e))^(7/2)+2/45*(7*a^2+2*b^2)*sin(f*x+e)/d^
3/f/(d*sec(f*x+e))^(3/2)-2/7*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(9/2)
```

Mathematica [A] (verified)

Time = 3.52 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.68

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = \frac{\frac{48(7a^2+2b^2)E\left(\frac{1}{2}(e+fx)|2\right)}{\sqrt{\cos(e+fx)}} + 4\cos(e+fx)\left(-30ab\cos(e+fx) - 10ab\cos(3(e+f\right.}{360d^4f\sqrt{d\sec(e+}}$$

input
```
Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(9/2),x]
```

output
```
((48*(7*a^2 + 2*b^2)*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] + 4*Cos
[e + f*x]*(-30*a*b*Cos[e + f*x] - 10*a*b*Cos[3*(e + f*x)] + 2*(19*a^2 - b^
2 + 5*(a^2 - b^2)*Cos[2*(e + f*x)])*Sin[e + f*x]))/(360*d^4*f*Sqrt[d*Sec[e
 + f*x]])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{ 3993}$$

$$-\frac{2}{7}\int -\frac{7a^2+5b\tan(e+fx)a+2b^2}{2(d\sec(e+fx))^{9/2}}dx - \frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{7}\int \frac{7a^2+5b\tan(e+fx)a+2b^2}{(d\sec(e+fx))^{9/2}}dx - \frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{7a^2+5b\tan(e+fx)a+2b^2}{(d\sec(e+fx))^{9/2}}dx-\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\int \frac{1}{(d\sec(e+fx))^{9/2}}dx-\frac{10ab}{9f(d\sec(e+fx))^{9/2}}\right)-\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{9/2}}dx-\frac{10ab}{9f(d\sec(e+fx))^{9/2}}\right)-\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{7\int \frac{1}{(d\sec(e+fx))^{5/2}}dx}{9d^2}+\frac{2\sin(e+fx)}{9df(d\sec(e+fx))^{7/2}}\right)-\frac{10ab}{9f(d\sec(e+fx))^{9/2}}\right)-\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{7\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9d^2}+\frac{2\sin(e+fx)}{9df(d\sec(e+fx))^{7/2}}\right)-\frac{10ab}{9f(d\sec(e+fx))^{9/2}}\right)-\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)}{9d^2}+\frac{2\sin(e+fx)}{9df(d\sec(e+fx))^{7/2}}\right)-\frac{10ab}{9f(d\sec(e+fx))^{9/2}}\right)-\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{5d^2}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)}{9d^2}+\frac{2\sin(e+fx)}{9df(d\sec(e+fx))^{7/2}}\right)-\frac{10ab}{9f(d\sec(e+fx))^{9/2}}\right.$$

$$\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \; 4258$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{7\left(\frac{3\int \sqrt{\cos(e+fx)}dx}{5d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)}{9d^2}+\frac{2\sin(e+fx)}{9df(d\sec(e+fx))^{7/2}}\right)-\frac{10ab}{9f(d\sec(e+fx)}\right.$$

$$\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{7\left(\frac{3\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{5d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)}{9d^2}+\frac{2\sin(e+fx)}{9df(d\sec(e+fx))^{7/2}}\right)-\frac{10ab}{9f(d\sec(e+fx}\right.$$

$$\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \; 3119$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{7\left(\frac{6E\left(\frac{1}{2}(e+fx)|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)}{9d^2}+\frac{2\sin(e+fx)}{9df(d\sec(e+fx))^{7/2}}\right)-\frac{10ab}{9f(d\sec(e+fx}\right.$$

$$\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

input
```
Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(9/2),x]
```

output
```
((-10*a*b)/(9*f*(d*Sec[e + f*x])^(9/2)) + (7*a^2 + 2*b^2)*((2*Sin[e + f*x]
)/(9*d*f*(d*Sec[e + f*x])^(7/2)) + (7*((6*EllipticE[(e + f*x)/2, 2])/(5*d^
2*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*Sin[e + f*x])/(5*d*f*(d*
Sec[e + f*x])^(3/2))))/(9*d^2)))/7 - (2*b*(a + b*Tan[e + f*x]))/(7*f*(d*Se
c[e + f*x])^(9/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 39.14 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.40

method	result
default	$\frac{14i\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,(2+\cos(fx+e)+\sec(fx+e))a^2\,\mathrm{EllipticE}(i(\csc(fx+e)-\cot(fx+e)),i)}{15}+\frac{4i\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,(2+c}{}$
parts	$\frac{2a^2\left(\sin(fx+e)\left(5\cos(fx+e)^4+5\cos(fx+e)^3+7\cos(fx+e)^2+7\cos(fx+e)+21\right)+21i\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\mathrm{EllipticE}(i(cs}{45f(1}$

input
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/f/(1+cos(f*x+e))/(d*sec(f*x+e))^(1/2)/d^4*(21*I*(1/(1+cos(f*x+e)))^(1
/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(2+cos(f*x+e)+sec(f*x+e))*a^2*Ellipt
icE(I*(csc(f*x+e)-cot(f*x+e)),I)+6*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*(2+cos(f*x+e)+sec(f*x+e))*b^2*EllipticE(I*(csc(f*x+e
)-cot(f*x+e)),I)-21*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*(2+cos(f*x+e)+sec(f*x+e))*a^2*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I
)-6*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(2+cos(f*
x+e)+sec(f*x+e))*b^2*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)+sin(f*x+e)*(5*
cos(f*x+e)^4+5*cos(f*x+e)^3+7*cos(f*x+e)^2+7*cos(f*x+e)+21)*a^2+b*a*(-10*c
os(f*x+e)^5-10*cos(f*x+e)^4)+sin(f*x+e)*(-5*cos(f*x+e)^4-5*cos(f*x+e)^3+2*
cos(f*x+e)^2+2*cos(f*x+e)+6)*b^2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx =$$
$$-\frac{3\sqrt{2}(-7i\,a^2-2i\,b^2)\sqrt{d}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e)))}{}$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

output
```
-1/45*(3*sqrt(2)*(-7*I*a^2 - 2*I*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt(2)*(7*I*a^2
 + 2*I*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
f*x + e) - I*sin(f*x + e))) + 2*(10*a*b*cos(f*x + e)^5 - (5*(a^2 - b^2)*co
s(f*x + e)^4 + (7*a^2 + 2*b^2)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(d/cos(f*
x + e)))/(d^5*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(9/2), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(9/2),x)
```

output
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = \frac{\sqrt{d}\left(\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^5}dx\right)a^2 + \left(\int \frac{\sqrt{\sec(fx+e)}\,\tan(fx+e)^2}{\sec(fx+e)^5}dx\right)b^2 + 2\left(\int \frac{\sqrt{\sec(fx+e)}\,\tan}{\sec(fx+e)}\right.\right.}{d^5}$$

input
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x)
```

output
```
(sqrt(d)*(int(sqrt(sec(e + f*x))/sec(e + f*x)**5,x)*a**2 + int((sqrt(sec(e
 + f*x))*tan(e + f*x)**2)/sec(e + f*x)**5,x)*b**2 + 2*int((sqrt(sec(e + f*
x))*tan(e + f*x))/sec(e + f*x)**5,x)*a*b))/d**5
```

3.602 $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 215

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx = \frac{2b(3a^2-b^2)\,d^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}}{5f} + \frac{2b^3d^2\sec^4(e+fx)\sqrt{d\sec(e+fx)}}{9f} + \frac{2a(7a^2-6b^2)\,d^2\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{21f\sqrt[4]{\sec^2(e+fx)}} + \frac{2a(7a^2-6b^2)\,d^2\sqrt{d\sec(e+fx)}\tan(e+fx)}{21f} + \frac{6ab^2d^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}\tan(e+fx)}{7f}$$

output
```
2/5*b*(3*a^2-b^2)*d^2*sec(f*x+e)^2*(d*sec(f*x+e))^(1/2)/f+2/9*b^3*d^2*sec(
f*x+e)^4*(d*sec(f*x+e))^(1/2)/f+2/21*a*(7*a^2-6*b^2)*d^2*InverseJacobiAM(1
/2*arctan(tan(f*x+e)),2^(1/2))*(d*sec(f*x+e))^(1/2)/f/(sec(f*x+e)^2)^(1/4)
+2/21*a*(7*a^2-6*b^2)*d^2*(d*sec(f*x+e))^(1/2)*tan(f*x+e)/f+6/7*a*b^2*d^2*
sec(f*x+e)^2*(d*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 2.36 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.73

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx =$$
$$-\frac{2d(d\sec(e+fx))^{3/2}\left(63b(-3a^2+b^2)\cos^2(e+fx)-15a(7a^2-6b^2)\cos^{\frac{9}{2}}(e+fx)\,\text{EllipticF}\left(\frac{1}{2}(e+fx),\right.\right.}{315f(a\cos(e+fx)}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3,x]
```

output
```
(-2*d*(d*Sec[e + f*x])^(3/2)*(63*b*(-3*a^2 + b^2)*Cos[e + f*x]^2 - 15*a*(7
*a^2 - 6*b^2)*Cos[e + f*x]^(9/2)*EllipticF[(e + f*x)/2, 2] - 15*a*(7*a^2 -
 6*b^2)*Cos[e + f*x]^3*Sin[e + f*x] - (5*b^2*(14*b + 27*a*Sin[2*(e + f*x)]
))/2)*(a + b*Tan[e + f*x])^3)/(315*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3994, 497, 27, 25, 676, 211, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\int (a+b\tan(e+fx))^3\sqrt[4]{\tan^2(e+fx)+1}d(b\tan(e+fx))}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{497}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2}{9}b^2\int -\frac{(a+b\tan(e+fx))\left(\left(4-\frac{9a^2}{b^2}\right)b^2-13ab\tan(e+fx)\right)\sqrt[4]{\tan^2(e+fx)+1}}{2b^2}d(b\tan(e+fx))+\frac{2}{9}b^2(\mathrm{ta}}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\downarrow$ 27

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2}{9}b^2\left(\tan^2(e+fx)+1\right)^{5/4}(a+b\tan(e+fx))^2-\frac{1}{9}\int -\left((a+b\tan(e+fx))\left(9a^2+13b\tan(e\right.\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\downarrow$ 25

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{1}{9}\int(a+b\tan(e+fx))\left(9a^2+13b\tan(e+fx)a-4b^2\right)\sqrt[4]{\tan^2(e+fx)+1}d(b\tan(e+fx))-\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\downarrow$ 676

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{1}{9}\left(\frac{9}{7}a\left(7a^2-6b^2\right)\int\sqrt[4]{\tan^2(e+fx)+1}d(b\tan(e+fx))+\frac{4}{5}b^2\left(11a^2-2b^2\right)\left(\tan^2(e+fx)+\right.\right.\right.}{bf\sqrt[4]{\sec^2(e+}}$$

$\downarrow$ 211

$$d^2\sqrt{d\sec(e+fx)}\left(\frac{1}{9}\left(\frac{9}{7}a\left(7a^2-6b^2\right)\left(\frac{1}{3}\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))+\frac{2}{3}b\sqrt[4]{\tan^2(e+fx)+1}\tan(e+fx\right.\right.\right.$$

$\downarrow$ 229

$$d^2\sqrt{d\sec(e+fx)}\left(\frac{1}{9}\left(\frac{9}{7}a\left(7a^2-6b^2\right)\left(\frac{2}{3}b\,\mathrm{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)+\frac{2}{3}b\tan(e+fx)\sqrt[4]{\tan^2(e+fx)}\right.\right.\right.$$

input
```
Int[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3,x]
```

output
```
(d^2*Sqrt[d*Sec[e + f*x]]*((2*b^2*(a + b*Tan[e + f*x])^2*(1 + Tan[e + f*x]
^2)^(5/4))/9 + ((4*b^2*(11*a^2 - 2*b^2)*(1 + Tan[e + f*x]^2)^(5/4))/5 + (2
6*a*b^3*Tan[e + f*x]*(1 + Tan[e + f*x]^2)^(5/4))/7 + (9*a*(7*a^2 - 6*b^2)*
((2*b*EllipticF[ArcTan[Tan[e + f*x]]/2, 2])/3 + (2*b*Tan[e + f*x]*(1 + Tan
[e + f*x]^2)^(1/4))/3))/7)/9))/(b*f*(Sec[e + f*x]^2)^(1/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 922.67 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.08

method	result
default	$\frac{d^2\left(\left(\frac{2\sec(fx+e)^4}{9}-\frac{2\sec(fx+e)^2}{5}\right)b^3+\frac{2\tan(fx+e)a^3}{3}+\frac{6\sec(fx+e)^2a^2b}{5}+\left(\frac{6\tan(fx+e)\sec(fx+e)^2}{7}-\frac{4\tan(fx+e)}{7}\right)b^2a-\frac{2i(1+\cos(fx+e)}{}\right.}{}$
parts	$\frac{a^3\left(-\frac{2i(1+\cos(fx+e))\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)}{3}+\frac{2\tan(fx+e)}{3}\right)d^2\sqrt{d\sec(fx+e)}}{f}+\frac{2b^3\left(\frac{(ds}{}\right.}{}$

input
```
int((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
d^2/f*((2/9*sec(f*x+e)^4-2/5*sec(f*x+e)^2)*b^3+2/3*tan(f*x+e)*a^3+6/5*sec(
f*x+e)^2*a^2*b+(6/7*tan(f*x+e)*sec(f*x+e)^2-4/7*tan(f*x+e))*b^2*a-2/3*I*(1
+cos(f*x+e))*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*El
lipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*a^3+4/7*I*(1+cos(f*x+e))*(1/(1+cos(f*
x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(csc(f*x+e)-cot
(f*x+e)),I)*b^2*a)*(d*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.94

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx = \frac{-15i\sqrt{2}(7a^3-6ab^2)d^{\frac{5}{2}}\cos(fx+e)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/315*(-15*I*sqrt(2)*(7*a^3 - 6*a*b^2)*d^(5/2)*cos(f*x + e)^4*weierstrassP
Inverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 15*I*sqrt(2)*(7*a^3 - 6*a*
b^2)*d^(5/2)*cos(f*x + e)^4*weierstrassPInverse(-4, 0, cos(f*x + e) - I*si
n(f*x + e)) + 2*(35*b^3*d^2 + 63*(3*a^2*b - b^3)*d^2*cos(f*x + e)^2 + 15*(
9*a*b^2*d^2*cos(f*x + e) + (7*a^3 - 6*a*b^2)*d^2*cos(f*x + e)^3)*sin(f*x +
 e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^4)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/2)*(a+b*tan(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e)+a)^3\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}(a+b\tan(e+f\,x))^3\,dx$$

input
```
int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^3,x)
```

output
```
int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^3, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx = \frac{\sqrt{d}\,d^2\left(10\sqrt{\sec(fx+e)}\,\sec(fx+e)^2\tan(fx+e)^2b^3+54\sqrt{\sec(fx+e)}\,\sec(fx+\right.}{}$$

input

```
int((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x)
```

output

```
(sqrt(d)*d**2*(10*sqrt(sec(e + f*x))*sec(e + f*x)**2*tan(e + f*x)**2*b**3
+ 54*sqrt(sec(e + f*x))*sec(e + f*x)**2*a**2*b - 8*sqrt(sec(e + f*x))*sec(
e + f*x)**2*b**3 + 135*int(sqrt(sec(e + f*x))*sec(e + f*x)**2*tan(e + f*x)
**2,x)*a*b**2*f + 45*int(sqrt(sec(e + f*x))*sec(e + f*x)**2,x)*a**3*f))/(4
5*f)
```

3.603 $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 190

$$\begin{aligned}\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx &= \frac{2b(3a^2-b^2)\,(d\sec(e+fx))^{3/2}}{3f}\\ &+\frac{2b^3\sec^2(e+fx)(d\sec(e+fx))^{3/2}}{7f}\\ &-\frac{2a(5a^2-6b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{5f\sec^2(e+fx)^{3/4}}\\ &+\frac{2a(5a^2-6b^2)\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{5f}\\ &+\frac{6ab^2(d\sec(e+fx))^{3/2}\tan(e+fx)}{5f}\end{aligned}$$

output
```
2/3*b*(3*a^2-b^2)*(d*sec(f*x+e))^(3/2)/f+2/7*b^3*sec(f*x+e)^2*(d*sec(f*x+e
))^(3/2)/f-2/5*a*(5*a^2-6*b^2)*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/
2))*(d*sec(f*x+e))^(3/2)/f/(sec(f*x+e)^2)^(3/4)+2/5*a*(5*a^2-6*b^2)*cos(f*
x+e)*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/f+6/5*a*b^2*(d*sec(f*x+e))^(3/2)*tan(
f*x+e)/f
```

Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.82

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx =$$
$$-\frac{d\sqrt{d\sec(e+fx)}\left(70b(-3a^2+b^2)\cos^2(e+fx)+42a(5a^2-6b^2)\cos^{\frac{7}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)-42a(5a\right.}{105f(a\cos(e+fx)+b\sin}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3,x]
```

output
```
-1/105*(d*Sqrt[d*Sec[e + f*x]]*(70*b*(-3*a^2 + b^2)*Cos[e + f*x]^2 + 42*a*
(5*a^2 - 6*b^2)*Cos[e + f*x]^(7/2)*EllipticE[(e + f*x)/2, 2] - 42*a*(5*a^2
 - 6*b^2)*Cos[e + f*x]^3*Sin[e + f*x] - 3*b^2*(10*b + 21*a*Sin[2*(e + f*x)
]))*(a + b*Tan[e + f*x])^3)/(f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3994, 497, 27, 25, 676, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx$$
$$\Big\downarrow \ 3042$$
$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3dx$$
$$\Big\downarrow \ 3994$$
$$\frac{(d\sec(e+fx))^{3/2}\int \frac{(a+b\tan(e+fx))^3}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{bf\sec^2(e+fx)^{3/4}}$$
$$\Big\downarrow \ 497$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2}{7}b^2\int -\frac{(a+b\tan(e+fx))\left(\left(4-\frac{7a^2}{b^2}\right)b^2-11ab\tan(e+fx)\right)}{2b^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+\frac{2}{7}b^2\left(\tan^2(e+fx)+1\right)^{3/4}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \; 27$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2}{7}b^2\left(\tan^2(e+fx)+1\right)^{3/4}(a+b\tan(e+fx))^2-\frac{1}{7}\int -\frac{(a+b\tan(e+fx))(7a^2+11b\tan(e+fx)a-4b^2)}{\sqrt[4]{\tan^2(e+fx)+1}}d\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \; 25$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{1}{7}\int \frac{(a+b\tan(e+fx))(7a^2+11b\tan(e+fx)a-4b^2)}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+\frac{2}{7}b^2\left(\tan^2(e+fx)+1\right)^{3/4}(a+b\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \; 676$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{1}{7}\left(\frac{7}{5}a(5a^2-6b^2)\int \frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+\frac{4}{3}b^2(9a^2-2b^2)\left(\tan^2(e+fx)+\right.\right.\right.}{bf\sec^2(e+fx}$$

$$\Big\downarrow \; 225$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{1}{7}\left(\frac{7}{5}a(5a^2-6b^2)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int \frac{1}{(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))\right)+\frac{4}{3}b^2(9a^2\right.\right.}{bf}$$

$$\Big\downarrow \; 212$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{1}{7}\left(\frac{7}{5}a(5a^2-6b^2)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)+\frac{4}{3}b^2(9a^2-2\right.\right.}{bf\,\mathrm{se}}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3,x]
```

output
```
((d*Sec[e + f*x])^(3/2)*((2*b^2*(a + b*Tan[e + f*x])^2*(1 + Tan[e + f*x]^2
)^(3/4))/7 + ((4*b^2*(9*a^2 - 2*b^2)*(1 + Tan[e + f*x]^2)^(3/4))/3 + (22*a
*b^3*Tan[e + f*x]*(1 + Tan[e + f*x]^2)^(3/4))/5 + (7*a*(5*a^2 - 6*b^2)*(-2
*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*b*Tan[e + f*x])/(1 + Tan[e +
f*x]^2)^(1/4)))/5)/7))/(b*f*(Sec[e + f*x]^2)^(3/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 26.26 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.30

method	result
default	$-\frac{2d\sqrt{d\sec(fx+e)}\left(105i\left(\cos(fx+e)^2+2\cos(fx+e)+1\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,a^3\,\mathrm{EllipticE}(i(\csc(fx+e)-\cot(fx+e)),i)+}{}$
parts	$\frac{2a^3\left(i\left(\cos(fx+e)^2+2\cos(fx+e)+1\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)+i\left(-\cos(fx+e)^2-2\cos\right.}{f(1+\cos(fx+e))}$

input
```
int((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/105*d/f*(d*sec(f*x+e))^(1/2)/(1+cos(f*x+e))*(105*I*(cos(f*x+e)^2+2*cos(
f*x+e)+1)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*a^3*E
llipticE(I*(csc(f*x+e)-cot(f*x+e)),I)+126*I*(-cos(f*x+e)^2-2*cos(f*x+e)-1)
*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*a*b^2*Elliptic
E(I*(csc(f*x+e)-cot(f*x+e)),I)+105*I*(-cos(f*x+e)^2-2*cos(f*x+e)-1)*(1/(1+
cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*a^3*EllipticF(I*(csc(
f*x+e)-cot(f*x+e)),I)+126*I*(cos(f*x+e)^2+2*cos(f*x+e)+1)*(1/(1+cos(f*x+e)
))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*a*b^2*EllipticF(I*(csc(f*x+e)-c
ot(f*x+e)),I)-105*a^3*sin(f*x+e)+105*b*a^2*(-1-sec(f*x+e))+63*a*b^2*(2*sin
(f*x+e)-tan(f*x+e)-sec(f*x+e)*tan(f*x+e))+5*b^3*(-3*sec(f*x+e)^3-3*sec(f*x
+e)^2+7*sec(f*x+e)+7))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.05

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx = \frac{-21i\sqrt{2}(5a^3-6ab^2)d^{\frac{3}{2}}\cos(fx+e)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-\ldots}{\ldots}$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/105*(-21*I*sqrt(2)*(5*a^3 - 6*a*b^2)*d^(3/2)*cos(f*x + e)^3*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 21
*I*sqrt(2)*(5*a^3 - 6*a*b^2)*d^(3/2)*cos(f*x + e)^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*(15*b^3*d
+ 35*(3*a^2*b - b^3)*d*cos(f*x + e)^2 + 21*(3*a*b^2*d*cos(f*x + e) + (5*a^
3 - 6*a*b^2)*d*cos(f*x + e)^3)*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(
f*x + e)^3)
```

Sympy [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx = \int (d\sec(e+fx))^{\frac{3}{2}}(a+b\tan(e+fx))^3\,dx$$

input
```
integrate((d*sec(f*x+e))**(3/2)*(a+b*tan(f*x+e))**3,x)
```

output
```
Integral((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e)+a)^3\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}(a+b\tan(e+f\,x))^3\,dx$$

input
```
int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^3,x)
```

output
```
int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^3, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx = \frac{\sqrt{d}\,d\left(6\sqrt{\sec(fx+e)}\sec(fx+e)\tan(fx+e)^2 b^3 + 42\sqrt{\sec(fx+e)}\sec(fx+e)\,a\right.}{}$$

input
```
int((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x)
```

output
```
(sqrt(d)*d*(6*sqrt(sec(e + f*x))*sec(e + f*x)*tan(e + f*x)**2*b**3 + 42*sq
rt(sec(e + f*x))*sec(e + f*x)*a**2*b - 8*sqrt(sec(e + f*x))*sec(e + f*x)*b
**3 + 63*int(sqrt(sec(e + f*x))*sec(e + f*x)*tan(e + f*x)**2,x)*a*b**2*f +
 21*int(sqrt(sec(e + f*x))*sec(e + f*x),x)*a**3*f))/(21*f)
```

3.604 $\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 139

$$\begin{aligned}\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx &= \frac{2b(3a^2-b^2)\sqrt{d\sec(e+fx)}}{f} + \frac{2b^3\sec^2(e+fx)\sqrt{d\sec(e+fx)}}{5f} \\ &\quad + \frac{2a(a^2-2b^2)\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{f\sqrt[4]{\sec^2(e+fx)}} \\ &\quad + \frac{2ab^2\sqrt{d\sec(e+fx)}\tan(e+fx)}{f}\end{aligned}$$

output
```
2*b*(3*a^2-b^2)*(d*sec(f*x+e))^(1/2)/f+2/5*b^3*sec(f*x+e)^2*(d*sec(f*x+e))
^(1/2)/f+2*a*(a^2-2*b^2)*InverseJacobiAM(1/2*arctan(tan(f*x+e)),2^(1/2))*(
d*sec(f*x+e))^(1/2)/f/(sec(f*x+e)^2)^(1/4)+2*a*b^2*(d*sec(f*x+e))^(1/2)*ta
n(f*x+e)/f
```

Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.95

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx = -\frac{2\sqrt{d\sec(e+fx)}\left(5b(-3a^2+b^2)\cos^3(e+fx)-5a(a^2-2b^2)\cos^{\frac{7}{2}}(e+fx)\operatorname{EllipticF}\left(\frac{1}{2}(e+fx),2\right)-\right.}{5f(a\cos(e+fx)+b\sin(e+fx))^3}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3,x]
```

output
```
(-2*Sqrt[d*Sec[e + f*x]]*(5*b*(-3*a^2 + b^2)*Cos[e + f*x]^3 - 5*a*(a^2 - 2
*b^2)*Cos[e + f*x]^(7/2)*EllipticF[(e + f*x)/2, 2] - (b^2*Cos[e + f*x]*(2*
b + 5*a*Sin[2*(e + f*x)]))/2)*(a + b*Tan[e + f*x])^3)/(5*f*(a*Cos[e + f*x]
 + b*Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3994, 497, 27, 25, 676, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt{d\sec(e+fx)}\int \frac{(a+b\tan(e+fx))^3}{(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{497}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{2}{5}b^2\int -\frac{(a+b\tan(e+fx))\left(\left(4-\frac{5a^2}{b^2}\right)b^2-9ab\tan(e+fx)\right)}{2b^2(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))+\frac{2}{5}b^2\sqrt[4]{\tan^2(e+fx)+1}(a+b}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{2}{5}b^2\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2-\frac{1}{5}\int -\frac{(a+b\tan(e+fx))(5a^2+9b\tan(e+fx)a-4b^2)}{(\tan^2(e+fx)+1)^{3/4}}d(b\tan(\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{1}{5}\int \frac{(a+b\tan(e+fx))(5a^2+9b\tan(e+fx)a-4b^2)}{(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))+\frac{2}{5}b^2\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 676$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{1}{5}\left(5a(a^2-2b^2)\int \frac{1}{(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))+4b^2(7a^2-2b^2)\sqrt[4]{\tan^2(e+fx)+1}+6ab\right.\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 229$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{1}{5}\left(10ab(a^2-2b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)+4b^2(7a^2-2b^2)\sqrt[4]{\tan^2(e+fx)+1}+\right.\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3,x]
```

output
```
(Sqrt[d*Sec[e + f*x]]*((2*b^2*(a + b*Tan[e + f*x])^2*(1 + Tan[e + f*x]^2)^
(1/4))/5 + (10*a*b*(a^2 - 2*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2] + 4*
b^2*(7*a^2 - 2*b^2)*(1 + Tan[e + f*x]^2)^(1/4) + 6*a*b^3*Tan[e + f*x]*(1 +
 Tan[e + f*x]^2)^(1/4))/5))/(b*f*(Sec[e + f*x]^2)^(1/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 23.49 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.32

method	result
default	$\frac{\left(4i(1+\cos(fx+e))\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)b^2a+2a\,b^2\tan(fx+e)-2i(1+\cos(fx+e))\sqrt{}\right.}{f}$
parts	$-\frac{2ia^3(1+\cos(fx+e))\sqrt{d\sec(fx+e)}\,\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)}{f}-\frac{b^3\sqrt{d\sec(fx+e)}\left(\right.}{}$

input
```
int((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(4*I*(1+cos(f*x+e))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)
))^(1/2)*b^2*a*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)+2*a*b^2*tan(f*x+e)-2
*I*(1+cos(f*x+e))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/
2)*a^3*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)+6*a^2*b+(2/5*sec(f*x+e)^2-2)
*b^3)*(d*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.18

$$\int\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx =$$
$$-\frac{5\sqrt{2}(i\,a^3-2i\,ab^2)\sqrt{d}\cos{(fx+e)}^2\,\mathrm{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\,\sin{(fx+e)})+5\sqrt{2}(}{}$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
-1/5*(5*sqrt(2)*(I*a^3 - 2*I*a*b^2)*sqrt(d)*cos(f*x + e)^2*weierstrassPInv
erse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 5*sqrt(2)*(-I*a^3 + 2*I*a*b^2
)*sqrt(d)*cos(f*x + e)^2*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f
*x + e)) - 2*(5*a*b^2*cos(f*x + e)*sin(f*x + e) + b^3 + 5*(3*a^2*b - b^3)*
cos(f*x + e)^2)*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx = \int \sqrt{d\sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)^3\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)*(a+b*tan(f*x+e))**3,x)
```

output
```
Integral(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**3, x)
```

Maxima [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx = \int \sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)^3\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^3, x)
```

Giac [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx = \int \sqrt{d\sec(fx+e)}(b\tan(fx+e)+a)^3\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx = \int \sqrt{\frac{d}{\cos(e+f\,x)}}\,(a+b\tan(e+f\,x))^3\,dx$$

input
```
int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^3,x)
```

output
```
int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^3, x)
```

Reduce [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx$$
$$= \frac{\sqrt{d}\left(2\sqrt{\sec(fx+e)}\tan(fx+e)^2b^3+30\sqrt{\sec(fx+e)}\,a^2b-8\sqrt{\sec(fx+e)}\,b^3+5\left(\int\sqrt{\sec(fx+e)}\right.\right.}{5f}$$

input
```
int((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x)
```

output
```
(sqrt(d)*(2*sqrt(sec(e + f*x))*tan(e + f*x)**2*b**3 + 30*sqrt(sec(e + f*x)
)*a**2*b - 8*sqrt(sec(e + f*x))*b**3 + 5*int(sqrt(sec(e + f*x)),x)*a**3*f
+ 15*int(sqrt(sec(e + f*x))*tan(e + f*x)**2,x)*a*b**2*f))/(5*f)
```

3.605 $\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 178

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = -\frac{2b(3a^2-b^2)}{f\sqrt{d\sec(e+fx)}}+\frac{2b^3\sec^2(e+fx)}{3f\sqrt{d\sec(e+fx)}} + \frac{2a(a^2-6b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{f\sqrt{d\sec(e+fx)}} - \frac{2a(a^2-6b^2)\tan(e+fx)}{f\sqrt{d\sec(e+fx)}}+\frac{2a(a^2-3b^2)\tan(e+fx)}{f\sqrt{d\sec(e+fx)}}$$

output
```
-2*b*(3*a^2-b^2)/f/(d*sec(f*x+e))^(1/2)+2/3*b^3*sec(f*x+e)^2/f/(d*sec(f*x+
e))^(1/2)+2*a*(a^2-6*b^2)*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(
sec(f*x+e)^2)^(1/4)/f/(d*sec(f*x+e))^(1/2)-2*a*(a^2-6*b^2)*tan(f*x+e)/f/(d
*sec(f*x+e))^(1/2)+2*a*(a^2-3*b^2)*tan(f*x+e)/f/(d*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.85 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.73

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx$$
$$= \frac{d\Big(6a(a^2-6b^2)\cos^{\frac{3}{2}}(e+fx)E\big(\frac{1}{2}(e+fx)\big|2\big)+b(-9a^2+5b^2+(-9a^2+3b^2)\cos(2(e+fx))+9ab\sin(2}{3f(d\sec(e+fx))^{3/2}(a\cos(e+fx)+b\sin(e+fx))^3}$$

input
```
Integrate[(a + b*Tan[e + f*x])^3/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(d*(6*a*(a^2 - 6*b^2)*Cos[e + f*x]^(3/2)*EllipticE[(e + f*x)/2, 2] + b*(-9
*a^2 + 5*b^2 + (-9*a^2 + 3*b^2)*Cos[2*(e + f*x)] + 9*a*b*Sin[2*(e + f*x)])
)*(a + b*Tan[e + f*x])^3)/(3*f*(d*Sec[e + f*x])^(3/2)*(a*Cos[e + f*x] + b*
Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3994, 495, 27, 25, 676, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3994}$$
$$\frac{\sqrt[4]{\sec^2(e+fx)}\int \frac{(a+b\tan(e+fx))^3}{(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{495}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(2b^2\int\frac{(a+b\tan(e+fx))\left(\left(4-\frac{a^2}{b^2}\right)b^2-5ab\tan(e+fx)\right)}{2b^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2(b^2-ab\tan(e+fx))}{\sqrt[4]{\tan^2(e+fx)+1}}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\int-\frac{(a+b\tan(e+fx))(a^2+5b\tan(e+fx)a-4b^2)}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2(b^2-ab\tan(e+fx))}{\sqrt[4]{\tan^2(e+fx)+1}}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(-\int\frac{(a+b\tan(e+fx))(a^2+5b\tan(e+fx)a-4b^2)}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-\frac{2(b^2-ab\tan(e+fx))(a+b\tan(e+fx))^2}{\sqrt[4]{\tan^2(e+fx)+1}}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{676}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(-a(a^2-6b^2)\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-\frac{4}{3}b^2(3a^2-2b^2)\left(\tan^2(e+fx)+1\right)^{3/4}-\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{225}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(-a(a^2-6b^2)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int\frac{1}{(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))\right)-\frac{4}{3}b^2(3a^2-2b^2)\right.(}{bf\sqrt{d\sec(e+f}}$$

$$\Big\downarrow \text{212}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(-a(a^2-6b^2)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-\frac{4}{3}b^2(3a^2-2b^2)\right.(\tan}{bf\sqrt{d\sec(e+fx}}$$

input
```
Int[(a + b*Tan[e + f*x])^3/Sqrt[d*Sec[e + f*x]],x]
```

output
```
((Sec[e + f*x]^2)^(1/4)*((-2*(a + b*Tan[e + f*x])^2*(b^2 - a*b*Tan[e + f*x
]))/(1 + Tan[e + f*x]^2)^(1/4) - (4*b^2*(3*a^2 - 2*b^2)*(1 + Tan[e + f*x]^
2)^(3/4))/3 - 2*a*b^3*Tan[e + f*x]*(1 + Tan[e + f*x]^2)^(3/4) - a*(a^2 - 6
*b^2)*(-2*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*b*Tan[e + f*x])/(1 +
 Tan[e + f*x]^2)^(1/4))))/(b*f*Sqrt[d*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 21.91 (sec) , antiderivative size = 627, normalized size of antiderivative = 3.52

method	result
parts	$\frac{2a^3\left(i\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\text{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)(-\cos(fx+e)-2-\sec(fx+e))+i\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(}{1+\cos}}\right.}{f(1+\cos(fx+e))\sqrt{d\sec(fx+e)}}$
default	Expression too large to display

input
```
int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^3/f/(1+cos(f*x+e))/(d*sec(f*x+e))^(1/2)*(I*(1/(1+cos(f*x+e)))^(1/2)*(c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(-c
os(f*x+e)-2-sec(f*x+e))+I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(2+cos(f*x+e)+sec(f*x+e)
)+sin(f*x+e))-1/6*b^3/f/(1+cos(f*x+e))/(d*sec(f*x+e))^(1/2)/(-cos(f*x+e)/(
1+cos(f*x+e))^2)^(1/2)*(3*ln((2*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^
(1/2)+2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(1+cos(f*x+e)))
-3*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2*(-cos(f*x+e)/
(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(1+cos(f*x+e)))+(-cos(f*x+e)/(1+cos(
f*x+e))^2)^(1/2)*(-12*cos(f*x+e)-12-4*sec(f*x+e)-4*sec(f*x+e)^2))-6*a*b^2/
f/(1+cos(f*x+e))/(d*sec(f*x+e))^(1/2)*(sin(f*x+e)-tan(f*x+e)-2*I*(1/(1+cos
(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(2+cos(f*x+e)+sec(f*x+e)
)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)+2*I*(1/(1+cos(f*x+e)))^(1/2)*(cos
(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(2+co
s(f*x+e)+sec(f*x+e)))-6*a^2*b/(d*sec(f*x+e))^(1/2)/f
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx =$$

$$-\frac{3\sqrt{2}(-i\,a^3+6i\,ab^2)\sqrt{d}\cos(fx+e)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(3*sqrt(2)*(-I*a^3 + 6*I*a*b^2)*sqrt(d)*cos(f*x + e)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt
(2)*(I*a^3 - 6*I*a*b^2)*sqrt(d)*cos(f*x + e)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - 2*(9*a*b^2*cos(f*x
+ e)*sin(f*x + e) + b^3 - 3*(3*a^2*b - b^3)*cos(f*x + e)^2)*sqrt(d/cos(f*x
 + e)))/(d*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(a+b\tan{(e+fx)})^3}{\sqrt{d\sec{(e+fx)}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(1/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**3/sqrt(d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^3}{\sqrt{d\sec{(fx+e)}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^3/sqrt(d*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^3}{\sqrt{d\sec{(fx+e)}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^3/sqrt(d*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(a+b\tan(e+f\,x))^3}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(1/2),x)
```

output
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = \frac{\sqrt{d}\left(\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)}dx\right)a^3+\left(\int \frac{\sqrt{\sec(fx+e)}\,\tan(fx+e)^3}{\sec(fx+e)}dx\right)b^3+3\left(\int \frac{\sqrt{\sec(fx+e)}\,\tan(fx+e)^2}{\sec(fx+e)}dx\right)a\,b^2+3\left(\int \frac{\sqrt{\sec(}{}\right.\right.}{d}$$

input
```
int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*(int(sqrt(sec(e + f*x))/sec(e + f*x),x)*a**3 + int((sqrt(sec(e +
f*x))*tan(e + f*x)**3)/sec(e + f*x),x)*b**3 + 3*int((sqrt(sec(e + f*x))*ta
n(e + f*x)**2)/sec(e + f*x),x)*a*b**2 + 3*int((sqrt(sec(e + f*x))*tan(e +
f*x))/sec(e + f*x),x)*a**2*b))/d
```

3.606 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 149

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = -\frac{2b(3a^2-b^2)}{3f(d\sec(e+fx))^{3/2}}+\frac{2b^3\sec^2(e+fx)}{f(d\sec(e+fx))^{3/2}} + \frac{2a(a^2+6b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3f(d\sec(e+fx))^{3/2}} + \frac{2a(a^2-3b^2)\tan(e+fx)}{3f(d\sec(e+fx))^{3/2}}$$

output
```
-2/3*b*(3*a^2-b^2)/f/(d*sec(f*x+e))^(3/2)+2*b^3*sec(f*x+e)^2/f/(d*sec(f*x+
e))^(3/2)+2/3*a*(a^2+6*b^2)*InverseJacobiAM(1/2*arctan(tan(f*x+e)),2^(1/2)
)*(sec(f*x+e)^2)^(3/4)/f/(d*sec(f*x+e))^(3/2)+2/3*a*(a^2-3*b^2)*tan(f*x+e)
/f/(d*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.79

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sec^2(e+fx)\left(-3a^2b+7b^3+(-3a^2b+b^3)\cos(2(e+fx))+2a(a^2+6b^2)\sqrt{c\ldots}\right.}{3f(d\sec(\ldots}$$

input
```
Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(Sec[e + f*x]^2*(-3*a^2*b + 7*b^3 + (-3*a^2*b + b^3)*Cos[2*(e + f*x)] + 2*
a*(a^2 + 6*b^2)*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + a^3*Sin[2*(
e + f*x)] - 3*a*b^2*Sin[2*(e + f*x)]))/(3*f*(d*Sec[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3994, 495, 27, 676, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sec^2(e+fx)^{3/4}\int \frac{(a+b\tan(e+fx))^3}{(\tan^2(e+fx)+1)^{7/4}}d(b\tan(e+fx))}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{495}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{2}{3}b^2\int\frac{(a+b\tan(e+fx))\left(\left(\frac{a^2}{b^2}+4\right)b^2-3ab\tan(e+fx)\right)}{2b^2(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2(b^2-ab\tan(e+fx))}{3(\tan^2(e+fx)+1)^{3/4}}\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{1}{3}\int\frac{(a+b\tan(e+fx))(a^2-3b\tan(e+fx)a+4b^2)}{(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2(b^2-ab\tan(e+fx))}{3(\tan^2(e+fx)+1)^{3/4}}\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 676$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{1}{3}\left(a(a^2+6b^2)\int\frac{1}{(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))-4b^2(a^2-2b^2)\sqrt[4]{\tan^2(e+fx)+1}-2ab^3\right.\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 229$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{1}{3}\left(2ab(a^2+6b^2)\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-4b^2(a^2-2b^2)\sqrt[4]{\tan^2(e+fx)+1}-2a\right.\right.}{bf(d\sec(e+fx))^{3/2}}$$

input
```
Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(3/2),x]
```

output
```
((Sec[e + f*x]^2)^(3/4)*((-2*(a + b*Tan[e + f*x])^2*(b^2 - a*b*Tan[e + f*x
]))/(3*(1 + Tan[e + f*x]^2)^(3/4)) + (2*a*b*(a^2 + 6*b^2)*EllipticF[ArcTan
[Tan[e + f*x]]/2, 2] - 4*b^2*(a^2 - 2*b^2)*(1 + Tan[e + f*x]^2)^(1/4) - 2*
a*b^3*Tan[e + f*x]*(1 + Tan[e + f*x]^2)^(1/4))/3))/(b*f*(d*Sec[e + f*x])^(
3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 21.58 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.41

method	result
default	$\frac{\left(\frac{2\cos(fx+e)}{3}+2\sec(fx+e)\right)b^3+\frac{2a^3\sin(fx+e)}{3}+\frac{2i\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\sqrt{\frac{1}{1+\cos(fx+e)}}\,a\,b^2\,\mathrm{EllipticF}(i(-\csc(fx+e)+\cot(fx+e)),i)(6+6\sec(fx+e}{3}}{df\sqrt{d\sec(fx}}$
parts	$\frac{a^3\left(-\frac{2i\,\mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}(1+\sec(fx+e))}{3}+\frac{2\sin(fx+e)}{3}\right)}{f\sqrt{d\sec(fx+e)}\,d}+\frac{b^3\left(-\frac{\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{}{(}}}{}\right)}{}\right.}{}$

input
```
int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/f*((2/3*cos(f*x+e)+2*sec(f*x+e))*b^3+2/3*a^3*sin(f*x+e)+2/3*I*(cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*a*b^2*EllipticF(I*(-csc
(f*x+e)+cot(f*x+e)),I)*(6+6*sec(f*x+e))-2*cos(f*x+e)*a^2*b-2*a*b^2*sin(f*x
+e)+2/3*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*a^3*E
llipticF(I*(-csc(f*x+e)+cot(f*x+e)),I)*(1+sec(f*x+e)))/(d*sec(f*x+e))^(1/2
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}(-i\,a^3-6i\,ab^2)\sqrt{d}\mathrm{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\,\sin{(fx+e}}{}$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-I*a^3 - 6*I*a*b^2)*sqrt(d)*weierstrassPInverse(-4, 0, cos(f
*x + e) + I*sin(f*x + e)) + sqrt(2)*(I*a^3 + 6*I*a*b^2)*sqrt(d)*weierstras
sPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(3*b^3 - (3*a^2*b - b^
3)*cos(f*x + e)^2 + (a^3 - 3*a*b^2)*cos(f*x + e)*sin(f*x + e))*sqrt(d/cos(
f*x + e)))/(d^2*f)
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(a+b\tan{(e+fx)})^3}{(d\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**3/(d*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^3}{(d\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^3}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(3/2),x)
```

output
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^2}dx\right)a^3+\left(\int \frac{\sqrt{\sec(fx+e)}\,\tan(fx+e)^3}{\sec(fx+e)^2}dx\right)b^3+3\left(\int \frac{\sqrt{\sec(fx+e)}\,\tan}{\sec(fx+e}\right.\right.}{d^2}$$

input
```
int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*(int(sqrt(sec(e + f*x))/sec(e + f*x)**2,x)*a**3 + int((sqrt(sec(e
 + f*x))*tan(e + f*x)**3)/sec(e + f*x)**2,x)*b**3 + 3*int((sqrt(sec(e + f*
x))*tan(e + f*x)**2)/sec(e + f*x)**2,x)*a*b**2 + 3*int((sqrt(sec(e + f*x))
*tan(e + f*x))/sec(e + f*x)**2,x)*a**2*b))/d**2
```

3.607 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 167

$$\begin{aligned}\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx &= -\frac{2b^3}{d^2f\sqrt{d\sec(e+fx)}}-\frac{2b(3a^2-b^2)\cos^2(e+fx)}{5d^2f\sqrt{d\sec(e+fx)}}\\ &+\frac{6a(a^2+2b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}}\\ &+\frac{2a(a^2-3b^2)\cos(e+fx)\sin(e+fx)}{5d^2f\sqrt{d\sec(e+fx)}}\end{aligned}$$

output
```
-2*b^3/d^2/f/(d*sec(f*x+e))^(1/2)-2/5*b*(3*a^2-b^2)*cos(f*x+e)^2/d^2/f/(d*
sec(f*x+e))^(1/2)+6/5*a*(a^2+2*b^2)*EllipticE(sin(1/2*arctan(tan(f*x+e))),
2^(1/2))*(sec(f*x+e)^2)^(1/4)/d^2/f/(d*sec(f*x+e))^(1/2)+2/5*a*(a^2-3*b^2)
*cos(f*x+e)*sin(f*x+e)/d^2/f/(d*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.68 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{d\sec(e+fx)}\left(-b(9a^2+17b^2)\cos(e+fx)-3a^2b\cos(3(e+fx))+b^3\cos(\right.}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(5/2),x]
```

output
```
(Sqrt[d*Sec[e + f*x]]*(-(b*(9*a^2 + 17*b^2)*Cos[e + f*x]) - 3*a^2*b*Cos[3*
(e + f*x)] + b^3*Cos[3*(e + f*x)] + 12*a*(a^2 + 2*b^2)*Sqrt[Cos[e + f*x]]*
EllipticE[(e + f*x)/2, 2] + a^3*Sin[e + f*x] - 3*a*b^2*Sin[e + f*x] + a^3*
Sin[3*(e + f*x)] - 3*a*b^2*Sin[3*(e + f*x)]))/(10*d^3*f)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3994, 495, 27, 675, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \; 3994$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\int \frac{(a+b\tan(e+fx))^3}{(\tan^2(e+fx)+1)^{9/4}}d(b\tan(e+fx))}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \; 495$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2}{5}b^2\int\frac{(a+b\tan(e+fx))\left(\left(\frac{3a^2}{b^2}+4\right)b^2-ab\tan(e+fx)\right)}{2b^2(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2(b^2-ab\tan(e+fx))}{5(\tan^2(e+fx)+1)^{5/4}}\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{1}{5}\int\frac{(a+b\tan(e+fx))(3a^2-b\tan(e+fx)a+4b^2)}{(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2(b^2-ab\tan(e+fx))}{5(\tan^2(e+fx)+1)^{5/4}}\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \textbf{675}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{1}{5}\left(-3a(a^2+2b^2)\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-\frac{4b^2(a^2+2b^2)}{\sqrt[4]{\tan^2(e+fx)+1}}+\frac{2ab(3a^2+5b^2)}{\sqrt[4]{\tan^2(e+}}\right.\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \textbf{225}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{1}{5}\left(-3a(a^2+2b^2)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int\frac{1}{(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))\right)-\frac{4b^2(a^2+}{\sqrt[4]{\tan^2(e+}}\right.\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \textbf{212}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{1}{5}\left(-3a(a^2+2b^2)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-\frac{4b^2(a^2+2b^2)}{\sqrt[4]{\tan^2(e+fx}}\right.\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

input
```
Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(5/2),x]
```

output
```
((Sec[e + f*x]^2)^(1/4)*((-2*(a + b*Tan[e + f*x])^2*(b^2 - a*b*Tan[e + f*x
]))/(5*(1 + Tan[e + f*x]^2)^(5/4)) + ((-4*b^2*(a^2 + 2*b^2))/(1 + Tan[e +
f*x]^2)^(1/4) + (2*a*b*(3*a^2 + 5*b^2)*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^
(1/4) - 3*a*(a^2 + 2*b^2)*(-2*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*
b*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^(1/4)))/5))/(b*d^2*f*Sqrt[d*Sec[e + f
*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 27.08 (sec) , antiderivative size = 652, normalized size of antiderivative = 3.90

method	result
parts	$\frac{2a^3\left(\sin(fx+e)\left(\cos(fx+e)^2+\cos(fx+e)+3\right)-3i\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,(2+\cos(fx+e)+\sec(fx+e))\,\mathrm{EllipticF}(i(\csc(fx+e)}{5f(1+\cos(fx+e))\sqrt{d\,\mathrm{se}}}$
default	Expression too large to display

input
```
int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*a^3/f/(1+cos(f*x+e))/(d*sec(f*x+e))^(1/2)/d^2*(sin(f*x+e)*(cos(f*x+e)^
2+cos(f*x+e)+3)-3*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*(2+cos(f*x+e)+sec(f*x+e))*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)+3*I*
(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(cs
c(f*x+e)-cot(f*x+e)),I)*(2+cos(f*x+e)+sec(f*x+e)))+1/10*b^3/f/d^2/(d*sec(f
*x+e))^(1/2)*((-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*ln((2*cos(f*x+e)*(-cos(
f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(
f*x+e)+1)/(1+cos(f*x+e)))*(-5-5*sec(f*x+e))+(-cos(f*x+e)/(1+cos(f*x+e))^2)
^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2*(-cos(f*x
+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(1+cos(f*x+e)))*(5+5*sec(f*x+e))
+4*cos(f*x+e)^2-20)+6/5*a*b^2/f/(1+cos(f*x+e))/(d*sec(f*x+e))^(1/2)/d^2*(s
in(f*x+e)*(-cos(f*x+e)^2-cos(f*x+e)+2)-2*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)*(2+cos(f*x+e)+sec(f*x+e))*EllipticF(I*(csc(f*x
+e)-cot(f*x+e)),I)+2*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(2+cos(f*x+e)+sec(f*x+e)))-
6/5*a^2*b/f/(d*sec(f*x+e))^(5/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.98

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{3\sqrt{2}(-i\,a^3-2i\,ab^2)\sqrt{d}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e)))}{}$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-1/5*(3*sqrt(2)*(-I*a^3 - 2*I*a*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt(2)*(I*a^3 +
2*I*a*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f
*x + e) - I*sin(f*x + e))) + 2*(5*b^3*cos(f*x + e) + (3*a^2*b - b^3)*cos(f
*x + e)^3 - (a^3 - 3*a*b^2)*cos(f*x + e)^2*sin(f*x + e))*sqrt(d/cos(f*x +
e)))/(d^3*f)
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(a+b\tan{(e+fx)})^3}{(d\sec{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(5/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**3/(d*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^3}{(d\sec{(fx+e)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^3}{(d\sec{(fx+e)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^3}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(5/2),x)
```

output
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^3}dx\right)a^3+\left(\int \frac{\sqrt{\sec(fx+e)}\tan(fx+e)^3}{\sec(fx+e)^3}dx\right)b^3+3\left(\int \frac{\sqrt{\sec(fx+e)}\tan}{\sec(fx+e}\right.\right.}{d^3}$$

input
```
int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(d)*(int(sqrt(sec(e + f*x))/sec(e + f*x)**3,x)*a**3 + int((sqrt(sec(e
 + f*x))*tan(e + f*x)**3)/sec(e + f*x)**3,x)*b**3 + 3*int((sqrt(sec(e + f*
x))*tan(e + f*x)**2)/sec(e + f*x)**3,x)*a*b**2 + 3*int((sqrt(sec(e + f*x))
*tan(e + f*x))/sec(e + f*x)**3,x)*a**2*b))/d**3
```

3.608 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 211

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = -\frac{2b^3}{3d^2f(d\sec(e+fx))^{3/2}}-\frac{2b(3a^2-b^2)\cos^2(e+fx)}{7d^2f(d\sec(e+fx))^{3/2}}+\frac{2a(5a^2+6b^2)\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{21d^2f(d\sec(e+fx))^{3/2}}+\frac{2a(a^2-3b^2)\cos(e+fx)\sin(e+fx)}{7d^2f(d\sec(e+fx))^{3/2}}+\frac{2a(5a^2+6b^2)\tan(e+fx)}{21d^2f(d\sec(e+fx))^{3/2}}$$

output
```
-2/3*b^3/d^2/f/(d*sec(f*x+e))^(3/2)-2/7*b*(3*a^2-b^2)*cos(f*x+e)^2/d^2/f/(
d*sec(f*x+e))^(3/2)+2/21*a*(5*a^2+6*b^2)*InverseJacobiAM(1/2*arctan(tan(f*
x+e)),2^(1/2))*(sec(f*x+e)^2)^(3/4)/d^2/f/(d*sec(f*x+e))^(3/2)+2/7*a*(a^2-
3*b^2)*cos(f*x+e)*sin(f*x+e)/d^2/f/(d*sec(f*x+e))^(3/2)+2/21*a*(5*a^2+6*b^
2)*tan(f*x+e)/d^2/f/(d*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 3.66 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.71

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\left(4(5a^3+6ab^2)\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+\sqrt{\text{co}\right.}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(7/2),x]
```

output
```
(Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]*(4*(5*a^3 + 6*a*b^2)*EllipticF[(e
 + f*x)/2, 2] + Sqrt[Cos[e + f*x]]*(-(b*(27*a^2 + 19*b^2)*Cos[e + f*x]) +
(-9*a^2*b + 3*b^3)*Cos[3*(e + f*x)] + 2*a*(13*a^2 + 3*b^2 + 3*(a^2 - 3*b^2
)*Cos[2*(e + f*x)])*Sin[e + f*x])))/(42*d^4*f)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3994, 495, 27, 675, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \; 3994$$

$$\frac{\sec^2(e+fx)^{3/4}\int \frac{(a+b\tan(e+fx))^3}{(\tan^2(e+fx)+1)^{11/4}}d(b\tan(e+fx))}{bd^2f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 495$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{2}{7}b^2\int\frac{(a+b\tan(e+fx))\left(\left(\frac{5a^2}{b^2}+4\right)b^2+a\tan(e+fx)b\right)}{2b^2(\tan^2(e+fx)+1)^{7/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2(b^2-ab\tan(e+fx))}{7(\tan^2(e+fx)+1)^{7/4}}\right)}{bd^2f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow 27$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{1}{7}\int\frac{(a+b\tan(e+fx))(5a^2+b\tan(e+fx)a+4b^2)}{(\tan^2(e+fx)+1)^{7/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2(b^2-ab\tan(e+fx))}{7(\tan^2(e+fx)+1)^{7/4}}\right)}{bd^2f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow 675$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{1}{7}\left(\frac{1}{3}a(5a^2+6b^2)\int\frac{1}{(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))-\frac{4b^2(3a^2+2b^2)}{3(\tan^2(e+fx)+1)^{3/4}}+\frac{2ab(5a^2+3b^2)\tan(e+fx)}{3(\tan^2(e+fx)+1)^{3/4}}\right)\ldots\right)}{bd^2f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow 229$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{1}{7}\left(\frac{2}{3}ab(5a^2+6b^2)\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\frac{4b^2(3a^2+2b^2)}{3(\tan^2(e+fx)+1)^{3/4}}+\frac{2ab(5a^2+3b^2)\tan(e+fx)}{3(\tan^2(e+fx)+1)^{3/4}}\right)\ldots\right)}{bd^2f(d\sec(e+fx))^{3/2}}$$

input
```
Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(7/2),x]
```

output
```
((Sec[e + f*x]^2)^(3/4)*((-2*(a + b*Tan[e + f*x])^2*(b^2 - a*b*Tan[e + f*x
]))/(7*(1 + Tan[e + f*x]^2)^(7/4)) + ((2*a*b*(5*a^2 + 6*b^2)*EllipticF[Arc
Tan[Tan[e + f*x]]/2, 2])/3 - (4*b^2*(3*a^2 + 2*b^2))/(3*(1 + Tan[e + f*x]^
2)^(3/4)) + (2*a*b*(5*a^2 + 3*b^2)*Tan[e + f*x])/(3*(1 + Tan[e + f*x]^2)^(
3/4)))/7))/(b*d^2*f*(d*Sec[e + f*x])^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 28.53 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.14

method	result
default	$\frac{2i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,a^3\,\text{EllipticF}(i(-\csc(fx+e)+\cot(fx+e)),i)(5+5\sec(fx+e))}{21}+\frac{2i\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\sqrt{\frac{1}{1+\cos(fx+e)}}\,a\,b^2\,\text{EllipticF}(}{21}$
parts	$-\frac{2a^3\left(\sin(fx+e)\left(-3\cos(fx+e)^2-5\right)+i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\text{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)(5+5\sec(fx+e))\right)}{21f\sqrt{d\sec(fx+e)}\,d^3}$

input
```
int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^3/f*(2/21*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*a^3*EllipticF(I*(-csc(f*x+e)+cot(f*x+e)),I)*(5+5*sec(f*x+e))+2/21*I*(1/(1
+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*a*b^2*EllipticF(I*(-
csc(f*x+e)+cot(f*x+e)),I)*(6+6*sec(f*x+e))+2/21*sin(f*x+e)*(3*cos(f*x+e)^2
+5)*a^3-6/7*cos(f*x+e)^3*a^2*b+2/21*sin(f*x+e)*(-9*cos(f*x+e)^2+6)*a*b^2+2
/21*b^3*(3*cos(f*x+e)^3-7*cos(f*x+e)))/(d*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{2}(-5i\,a^3-6i\,ab^2)\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\,\sin{(fx+}}{}$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

output
```
1/21*(sqrt(2)*(-5*I*a^3 - 6*I*a*b^2)*sqrt(d)*weierstrassPInverse(-4, 0, co
s(f*x + e) + I*sin(f*x + e)) + sqrt(2)*(5*I*a^3 + 6*I*a*b^2)*sqrt(d)*weier
strassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*(7*b^3*cos(f*x +
e)^2 + 3*(3*a^2*b - b^3)*cos(f*x + e)^4 - (3*(a^3 - 3*a*b^2)*cos(f*x + e)^
3 + (5*a^3 + 6*a*b^2)*cos(f*x + e))*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d
^4*f)
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(a+b\tan{(e+fx)})^3}{(d\sec{(e+fx)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(7/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**3/(d*sec(e + f*x))**(7/2), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^3}{(d\sec{(fx+e)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(7/2), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^3}{(d\sec{(fx+e)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx=\int \frac{(a+b\tan(e+f\,x))^3}{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(7/2),x)
```

output
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx=\frac{\sqrt{d}\left(\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^4}dx\right)a^3+\left(\int \frac{\sqrt{\sec(fx+e)}\tan(fx+e)^3}{\sec(fx+e)^4}dx\right)b^3+3\left(\int \frac{\sqrt{\sec(fx+e)}\tan}{\sec(fx+e}\right.\right.}{d^4}$$

input
```
int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x)
```

output
```
(sqrt(d)*(int(sqrt(sec(e + f*x))/sec(e + f*x)**4,x)*a**3 + int((sqrt(sec(e
 + f*x))*tan(e + f*x)**3)/sec(e + f*x)**4,x)*b**3 + 3*int((sqrt(sec(e + f*
x))*tan(e + f*x)**2)/sec(e + f*x)**4,x)*a*b**2 + 3*int((sqrt(sec(e + f*x))
*tan(e + f*x))/sec(e + f*x)**4,x)*a**2*b))/d**4
```

3.609 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 227

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx = -\frac{2b^3\cos^2(e+fx)}{5d^4f\sqrt{d\sec(e+fx)}} - \frac{2b(3a^2-b^2)\cos^4(e+fx)}{9d^4f\sqrt{d\sec(e+fx)}} + \frac{2a(7a^2+6b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}} + \frac{2a(7a^2+6b^2)\cos(e+fx)\sin(e+fx)}{45d^4f\sqrt{d\sec(e+fx)}} + \frac{2a(a^2-3b^2)\cos^3(e+fx)\sin(e+fx)}{9d^4f\sqrt{d\sec(e+fx)}}$$

output
```
-2/5*b^3*cos(f*x+e)^2/d^4/f/(d*sec(f*x+e))^(1/2)-2/9*b*(3*a^2-b^2)*cos(f*x
+e)^4/d^4/f/(d*sec(f*x+e))^(1/2)+2/15*a*(7*a^2+6*b^2)*EllipticE(sin(1/2*ar
ctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(1/4)/d^4/f/(d*sec(f*x+e))^(1/2)
+2/45*a*(7*a^2+6*b^2)*cos(f*x+e)*sin(f*x+e)/d^4/f/(d*sec(f*x+e))^(1/2)+2/9
*a*(a^2-3*b^2)*cos(f*x+e)^3*sin(f*x+e)/d^4/f/(d*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 5.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.64

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx = \frac{\frac{48a(7a^2+6b^2)E\left(\frac{1}{2}(e+fx)|2\right)}{\sqrt{\cos(e+fx)}} + 4\cos(e+fx)\left(-3b(15a^2+7b^2)\cos(e+fx)+5b(-\right.}{360d^4f}$$

input
```
Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(9/2),x]
```

output
```
((48*a*(7*a^2 + 6*b^2)*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] + 4*C
os[e + f*x]*(-3*b*(15*a^2 + 7*b^2)*Cos[e + f*x] + 5*b*(-3*a^2 + b^2)*Cos[3
*(e + f*x)] + 2*a*(19*a^2 - 3*b^2 + 5*(a^2 - 3*b^2)*Cos[2*(e + f*x)])*Sin[
e + f*x]))/(360*d^4*f*Sqrt[d*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3994, 495, 27, 675, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\int \frac{(a+b\tan(e+fx))^3}{(\tan^2(e+fx)+1)^{13/4}}d(b\tan(e+fx))}{bd^4f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{495}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2}{9}b^2\int\frac{(a+b\tan(e+fx))\left(\left(\frac{7a^2}{b^2}+4\right)b^2+3a\tan(e+fx)b\right)}{2b^2(\tan^2(e+fx)+1)^{9/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2(b^2-ab\tan(e+fx))}{9(\tan^2(e+fx)+1)^{9/4}}\right)}{bd^4f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{1}{9}\int\frac{(a+b\tan(e+fx))(7a^2+3b\tan(e+fx)a+4b^2)}{(\tan^2(e+fx)+1)^{9/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2(b^2-ab\tan(e+fx))}{9(\tan^2(e+fx)+1)^{9/4}}\right)}{bd^4f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \textbf{675}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{1}{9}\left(\frac{3}{5}a(7a^2+6b^2)\int\frac{1}{(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))-\frac{4b^2(5a^2+2b^2)}{5(\tan^2(e+fx)+1)^{5/4}}+\frac{2ab(7a^2+b^2)\tan(e+fx)}{5(\tan^2(e+fx)+1)^{5/4}}\right)\right.}{bd^4f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \textbf{212}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{1}{9}\left(\frac{6}{5}ab(7a^2+6b^2)E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)-\frac{4b^2(5a^2+2b^2)}{5(\tan^2(e+fx)+1)^{5/4}}+\frac{2ab(7a^2+b^2)\tan(e+fx)}{5(\tan^2(e+fx)+1)^{5/4}}\right)-\right.}{bd^4f\sqrt{d\sec(e+fx)}}$$

input
```
Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(9/2),x]
```

output
```
((Sec[e + f*x]^2)^(1/4)*((-2*(a + b*Tan[e + f*x])^2*(b^2 - a*b*Tan[e + f*x
]))/(9*(1 + Tan[e + f*x]^2)^(9/4)) + ((6*a*b*(7*a^2 + 6*b^2)*EllipticE[Arc
Tan[Tan[e + f*x]]/2, 2])/5 - (4*b^2*(5*a^2 + 2*b^2))/(5*(1 + Tan[e + f*x]^
2)^(5/4)) + (2*a*b*(7*a^2 + b^2)*Tan[e + f*x])/(5*(1 + Tan[e + f*x]^2)^(5/
4)))/9))/(b*d^4*f*Sqrt[d*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 39.86 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.17

method	result
default	$\frac{14i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,(2+\cos(fx+e)+\sec(fx+e))a^3\,\mathrm{EllipticE}(i(\csc(fx+e)-\cot(fx+e)),i)}{15}+\frac{4i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,(2+c}{}$
parts	$\frac{2a^3\left(\sin(fx+e)\left(5\cos(fx+e)^4+5\cos(fx+e)^3+7\cos(fx+e)^2+7\cos(fx+e)+21\right)+21i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\mathrm{EllipticE}(i(\mathrm{cs}}{45f(1}$

input
```
int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/d^4/f*(21*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*(2+cos(f*x+e)+sec(f*x+e))*a^3*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)+18*
I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(2+cos(f*x+e)
+sec(f*x+e))*a*b^2*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-21*I*(1/(1+cos(f
*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(2+cos(f*x+e)+sec(f*x+e))*
a^3*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)-18*I*(1/(1+cos(f*x+e)))^(1/2)*(
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(2+cos(f*x+e)+sec(f*x+e))*a*b^2*EllipticF
(I*(csc(f*x+e)-cot(f*x+e)),I)+sin(f*x+e)*(5*cos(f*x+e)^4+5*cos(f*x+e)^3+7*
cos(f*x+e)^2+7*cos(f*x+e)+21)*a^3+b*a^2*(-15*cos(f*x+e)^5-15*cos(f*x+e)^4)
+3*sin(f*x+e)*(-5*cos(f*x+e)^4-5*cos(f*x+e)^3+2*cos(f*x+e)^2+2*cos(f*x+e)+
6)*a*b^2+b^3*(5*cos(f*x+e)^5+5*cos(f*x+e)^4-9*cos(f*x+e)^3-9*cos(f*x+e)^2)
)/(1+cos(f*x+e))/(d*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx =$$

$$-\frac{3\sqrt{2}(-7i\,a^3-6i\,ab^2)\sqrt{d}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e))}{}$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

output
```
-1/45*(3*sqrt(2)*(-7*I*a^3 - 6*I*a*b^2)*sqrt(d)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt(2)*(7*I*a
^3 + 6*I*a*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(f*x + e) - I*sin(f*x + e))) + 2*(9*b^3*cos(f*x + e)^3 + 5*(3*a^2*b - b
^3)*cos(f*x + e)^5 - (5*(a^3 - 3*a*b^2)*cos(f*x + e)^4 + (7*a^3 + 6*a*b^2)
*cos(f*x + e)^2)*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^5*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\tan(e + fx))^3}{(d\sec(e + fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a + b\tan(e + fx))^3}{(d\sec(e + fx))^{9/2}}\,dx = \int \frac{(b\tan(fx + e) + a)^3}{(d\sec(fx + e))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(9/2), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx=\int \frac{(b\tan(fx+e)+a)^3}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx=\int \frac{(a+b\tan(e+f\,x))^3}{\left(\frac{d}{\cos(e+f\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(9/2),x)
```

output
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx=\frac{\sqrt{d}\left(\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^5}dx\right)a^3+\left(\int \frac{\sqrt{\sec(fx+e)}\tan(fx+e)^3}{\sec(fx+e)^5}dx\right)b^3+3\left(\int \frac{\sqrt{\sec(fx+e)}\tan}{\sec(fx+e}\right.\right.}{d^5}$$

input
```
int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x)
```

output
```
(sqrt(d)*(int(sqrt(sec(e + f*x))/sec(e + f*x)**5,x)*a**3 + int((sqrt(sec(e
 + f*x))*tan(e + f*x)**3)/sec(e + f*x)**5,x)*b**3 + 3*int((sqrt(sec(e + f*
x))*tan(e + f*x)**2)/sec(e + f*x)**5,x)*a*b**2 + 3*int((sqrt(sec(e + f*x))
*tan(e + f*x))/sec(e + f*x)**5,x)*a**2*b))/d**5
```

3.610 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 267

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = -\frac{2b^3\cos^2(e+fx)}{7d^4f(d\sec(e+fx))^{3/2}} - \frac{2b(3a^2-b^2)\cos^4(e+fx)}{11d^4f(d\sec(e+fx))^{3/2}} + \frac{10a(3a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{77d^4f(d\sec(e+fx))^{3/2}} + \frac{6a(3a^2+2b^2)\cos(e+fx)\sin(e+fx)}{77d^4f(d\sec(e+fx))^{3/2}} + \frac{2a(a^2-3b^2)\cos^3(e+fx)\sin(e+fx)}{11d^4f(d\sec(e+fx))^{3/2}} + \frac{10a(3a^2+2b^2)\tan(e+fx)}{77d^4f(d\sec(e+fx))^{3/2}}$$

output
```
-2/7*b^3*cos(f*x+e)^2/d^4/f/(d*sec(f*x+e))^(3/2)-2/11*b*(3*a^2-b^2)*cos(f*
x+e)^4/d^4/f/(d*sec(f*x+e))^(3/2)+10/77*a*(3*a^2+2*b^2)*InverseJacobiAM(1/
2*arctan(tan(f*x+e)),2^(1/2))*(sec(f*x+e)^2)^(3/4)/d^4/f/(d*sec(f*x+e))^(3
/2)+6/77*a*(3*a^2+2*b^2)*cos(f*x+e)*sin(f*x+e)/d^4/f/(d*sec(f*x+e))^(3/2)+
2/11*a*(a^2-3*b^2)*cos(f*x+e)^3*sin(f*x+e)/d^4/f/(d*sec(f*x+e))^(3/2)+10/7
7*a*(3*a^2+2*b^2)*tan(f*x+e)/d^4/f/(d*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 7.57 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx =$$
$$-\frac{\cos^{\frac{7}{2}}(e+fx)\sqrt{d\sec(e+fx)}\left(-80(3a^3+2ab^2)\operatorname{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+\sqrt{\cos(e+fx)}((210a^2b+62b^3}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(11/2),x]
```

output
```
-1/616*(Cos[e + f*x]^(7/2)*Sqrt[d*Sec[e + f*x]]*(-80*(3*a^3 + 2*a*b^2)*Ell
ipticF[(e + f*x)/2, 2] + Sqrt[Cos[e + f*x]]*((210*a^2*b + 62*b^3)*Cos[e +
f*x] + 3*(35*a^2*b + 3*b^3)*Cos[3*(e + f*x)] + 21*a^2*b*Cos[5*(e + f*x)] -
 7*b^3*Cos[5*(e + f*x)] - 290*a^3*Sin[e + f*x] - 142*a*b^2*Sin[e + f*x] -
57*a^3*Sin[3*(e + f*x)] + 39*a*b^2*Sin[3*(e + f*x)] - 7*a^3*Sin[5*(e + f*x
)] + 21*a*b^2*Sin[5*(e + f*x)]))*(a + b*Tan[e + f*x])^3)/(d^6*f*(a*Cos[e +
 f*x] + b*Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 3994, 495, 27, 675, 215, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}dx$$
$$\Big\downarrow \; 3994$$

$$\frac{\sec^2(e+fx)^{3/4}\int\frac{(a+b\tan(e+fx))^3}{(\tan^2(e+fx)+1)^{15/4}}d(b\tan(e+fx))}{bd^4f(d\sec(e+fx))^{3/2}}$$

$\downarrow$ 495

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{2}{11}b^2\int\frac{(a+b\tan(e+fx))\left(\left(\frac{9a^2}{b^2}+4\right)b^2+5a\tan(e+fx)b\right)}{2b^2(\tan^2(e+fx)+1)^{11/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2(b^2-ab\tan(e+fx)}{11(\tan^2(e+fx)+1)^{11/4}}\right.}{bd^4f(d\sec(e+fx))^{3/2}}$$

$\downarrow$ 27

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{1}{11}\int\frac{(a+b\tan(e+fx))(9a^2+5b\tan(e+fx)a+4b^2)}{(\tan^2(e+fx)+1)^{11/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2(b^2-ab\tan(e+fx))}{11(\tan^2(e+fx)+1)^{11/4}}\right)}{bd^4f(d\sec(e+fx))^{3/2}}$$

$\downarrow$ 675

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{1}{11}\left(\frac{15}{7}a(3a^2+2b^2)\int\frac{1}{(\tan^2(e+fx)+1)^{7/4}}d(b\tan(e+fx))-\frac{4b^2(7a^2+2b^2)}{7(\tan^2(e+fx)+1)^{7/4}}+\frac{2ab(9a^2-b^2)\tan(e+f}{7(\tan^2(e+fx)+1)^{7/}}\right.\right.}{bd^4f(d\sec(e+fx))^{3/2}}$$

$\downarrow$ 215

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{1}{11}\left(\frac{15}{7}a(3a^2+2b^2)\left(\frac{1}{3}\int\frac{1}{(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))+\frac{2b\tan(e+fx)}{3(\tan^2(e+fx)+1)^{3/4}}\right)-\frac{4b^2(7a^2+2b}{7(\tan^2(e+fx)+}\right.\right.}{bd^4f(d\sec(e+fx))^{3/2}}$$

$\downarrow$ 229

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{1}{11}\left(\frac{15}{7}a(3a^2+2b^2)\left(\frac{2}{3}b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)+\frac{2b\tan(e+fx)}{3(\tan^2(e+fx)+1)^{3/4}}\right)-\frac{4b^2(7a^2}{7(\tan^2(e+f}\right.\right.}{bd^4f(d\sec(e+fx))^{3/2}}$$

input
```
Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(11/2),x]
```

output
```
((Sec[e + f*x]^2)^(3/4)*((-2*(a + b*Tan[e + f*x])^2*(b^2 - a*b*Tan[e + f*x
]))/(11*(1 + Tan[e + f*x]^2)^(11/4)) + ((-4*b^2*(7*a^2 + 2*b^2))/(7*(1 + T
an[e + f*x]^2)^(7/4)) + (2*a*b*(9*a^2 - b^2)*Tan[e + f*x])/(7*(1 + Tan[e +
 f*x]^2)^(7/4)) + (15*a*(3*a^2 + 2*b^2)*((2*b*EllipticF[ArcTan[Tan[e + f*x
]]/2, 2])/3 + (2*b*Tan[e + f*x])/(3*(1 + Tan[e + f*x]^2)^(3/4))))/7)/11))/
(b*d^4*f*(d*Sec[e + f*x])^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 47.36 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.99

method	result
default	$\frac{2i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,a^3\,\mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)(-15-15\sec(fx+e))}{77}+\frac{2i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,a\,b^2\,\mathrm{Elliptic}$
parts	$\frac{a^3\left(-\frac{2\sin(fx+e)\left(-7\cos(fx+e)^4-9\cos(fx+e)^2-15\right)}{77}-\frac{30i\,\mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,(1+\sec(fx+e)}{77}\right.}{f\sqrt{d\sec(fx+e)}\,d^5}$

input
```
int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^5/f*(2/77*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*a^3*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(-15-15*sec(f*x+e))+2/77*I*(1/
(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*a*b^2*EllipticF(I*
(csc(f*x+e)-cot(f*x+e)),I)*(-10-10*sec(f*x+e))+2/77*sin(f*x+e)*(7*cos(f*x+
e)^4+9*cos(f*x+e)^2+15)*a^3-6/11*cos(f*x+e)^5*a^2*b+2/77*sin(f*x+e)*(-21*c
os(f*x+e)^4+6*cos(f*x+e)^2+10)*a*b^2+2/77*b^3*(7*cos(f*x+e)^5-11*cos(f*x+e
)^3))/(d*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.77

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx =$$
$$-\frac{5\sqrt{2}(3i\,a^3+2i\,ab^2)\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\,\sin{(fx+e)})+5\sqrt{2}(-3i\,a^3-2i\,ab^2)}{}$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x, algorithm="fricas")
```

output
```
-1/77*(5*sqrt(2)*(3*I*a^3 + 2*I*a*b^2)*sqrt(d)*weierstrassPInverse(-4, 0,
cos(f*x + e) + I*sin(f*x + e)) + 5*sqrt(2)*(-3*I*a^3 - 2*I*a*b^2)*sqrt(d)*
weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(11*b^3*cos(
f*x + e)^4 + 7*(3*a^2*b - b^3)*cos(f*x + e)^6 - (7*(a^3 - 3*a*b^2)*cos(f*x
 + e)^5 + 3*(3*a^3 + 2*a*b^2)*cos(f*x + e)^3 + 5*(3*a^3 + 2*a*b^2)*cos(f*x
 + e))*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^6*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{(d\sec(fx+e))^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(11/2), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{(d\sec(fx+e))^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^3}{\left(\frac{d}{\cos(e+f\,x)}\right)^{11/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(11/2),x)
```

output
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(11/2), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \frac{\sqrt{d}\left(\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^6}dx\right)a^3+\left(\int \frac{\sqrt{\sec(fx+e)}\,\tan(fx+e)^3}{\sec(fx+e)^6}dx\right)b^3+3\left(\int \frac{\sqrt{\sec(fx+e)}\,\tan}{\sec(fx+e}\right.\right.}{d^6}$$

input
```
int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x)
```

output
```
(sqrt(d)*(int(sqrt(sec(e + f*x))/sec(e + f*x)**6,x)*a**3 + int((sqrt(sec(e
 + f*x))*tan(e + f*x)**3)/sec(e + f*x)**6,x)*b**3 + 3*int((sqrt(sec(e + f*
x))*tan(e + f*x)**2)/sec(e + f*x)**6,x)*a*b**2 + 3*int((sqrt(sec(e + f*x))
*tan(e + f*x))/sec(e + f*x)**6,x)*a**2*b))/d**6
```

3.611 $\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 456

$$\begin{aligned}
\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx &= \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf} \\
&+ \frac{(a^2+b^2)^{3/4}\,d^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{b^{5/2}f\sec^2(e+fx)^{3/4}} \\
&- \frac{(a^2+b^2)^{3/4}\,d^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{b^{5/2}f\sec^2(e+fx)^{3/4}} \\
&+ \frac{2ad^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}} \\
&- \frac{2ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2f} \\
&- \frac{a\sqrt{a^2+b^2}d^2\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2}}{b^3f\sec^2(e+fx)^{3/4}} \\
&+ \frac{a\sqrt{a^2+b^2}d^2\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e}}{b^3f\sec^2(e+fx)^{3/4}}
\end{aligned}$$

output
```
2/3*d^2*(d*sec(f*x+e))^(3/2)/b/f+(a^2+b^2)^(3/4)*d^2*arctan(b^(1/2)*(sec(f
*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/b^(5/2)/f/(sec(f*x+e)
^2)^(3/4)-(a^2+b^2)^(3/4)*d^2*arctanh(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^
2)^(1/4))*(d*sec(f*x+e))^(3/2)/b^(5/2)/f/(sec(f*x+e)^2)^(3/4)+2*a*d^2*Elli
pticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(3/2)/b^2/f/(sec
(f*x+e)^2)^(3/4)-2*a*d^2*cos(f*x+e)*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/b^2/f-
a*(a^2+b^2)^(1/2)*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b
^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b^3/f/(sec(f*x+e)^
2)^(3/4)+a*(a^2+b^2)^(1/2)*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),
b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b^3/f/(sec
(f*x+e)^2)^(3/4)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7284 vs. $2(456) = 912$.

Time = 84.17 (sec) , antiderivative size = 7284, normalized size of antiderivative = 15.97

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.71, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 3994, 493, 27, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{ 3994}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\int \frac{\left(\tan^2(e+fx)+1\right)^{3/4}}{a+b\tan(e+fx)}d(b\tan(e+fx))}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{ 493}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\int \frac{b^2-ab\tan(e+fx)}{b^2(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+\frac{2}{3}\left(\tan^2(e+fx)+1\right)^{3/4}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{\int \frac{b^2-ab\tan(e+fx)}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{b^2}+\frac{2}{3}\left(\tan^2(e+fx)+1\right)^{3/4}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{ 719}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{(a^2+b^2)\int \frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-a\int \frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{b^2}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{ 225}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{(a^2+b^2)\int \frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int \frac{1}{\left(\tan^2(e+fx)\right.}\right.}{b^2}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow 212$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{(a^2+b^2)\int \frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan\right.\right.}{b^2}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow 504$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{(a^2+b^2)\left(a\int \frac{1}{\sqrt[4]{\tan^2(e+fx)+1}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}\left(a^2-b^2\tan\right.}\right.}{b^2}\right.}{bf\sec^2(e+fx)}$$

$$\Big\downarrow 310$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\int\right.}{}\right.}{bf}$$

$$\Big\downarrow 353$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\frac{1}{2}\int\right.}{}\right.}{b}$$

$\Big\downarrow$ 73

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\int\right.}{}\right.$$

$bf\,$se

$\Big\downarrow$ 827

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\right.}{}\right.$$

$\Big\downarrow$ 218

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\right.}{}\right.$$

$\Big\downarrow$ 221

$$\frac{d^2 (d\sec(e+fx))^{3/2} \left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b} - 2b^2 \left(\cdots \right.\right.}{\cdots} \right.}{bf \cdots}$$

$$\Big\downarrow \; 993$$

$$\frac{d^2 (d\sec(e+fx))^{3/2} \left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int \frac{1}{\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\tan^2(e+fx)+1} - \cdots \right.}{b} \right.}{\cdots} \right.}{\cdots}$$

$$\Big\downarrow \; 1537$$

$$\frac{d^2 (d\sec(e+fx))^{3/2} \left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int \frac{1}{\sqrt{1-b^2\tan^2(e+fx)}\sqrt{b^2\tan^2(e+fx)+1}\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)}d\sqrt[4]{\tan^2(\cdots} \right.}{\cdots} \right.}{\cdots} \right.}{\cdots}$$

$$\Big\downarrow \; 412$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{b\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\tan^2(e+fx)+1}\right),-1\right)}{2\sqrt{a^2+b^2}}-\frac{b\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\ldots\right.}{}\right.}{b}\right.}{}\right.}{}$$

input
```
Int[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x]),x]
```

output
```
(d^2*(d*Sec[e + f*x])^(3/2)*((2*(1 + Tan[e + f*x]^2)^(3/4))/3 + ((a^2 + b^
2)*(-2*b^2*(-1/2*ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(b^(3/2)
*(a^2 + b^2)^(1/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2
*b^(3/2)*(a^2 + b^2)^(1/4))) + (2*a*Cot[e + f*x]*(-1/2*(b*EllipticPi[-(b/S
qrt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1])/Sqrt[a^2 + b^2]
+ (b*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1]
)/(2*Sqrt[a^2 + b^2]))*Sqrt[-Tan[e + f*x]^2])/b) - a*(-2*b*EllipticE[ArcTa
n[Tan[e + f*x]]/2, 2] + (2*b*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^(1/4)))/b^
2))/(b*f*(Sec[e + f*x]^2)^(3/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7820 vs. $2(395) = 790$.

Time = 31.31 (sec) , antiderivative size = 7821, normalized size of antiderivative = 17.15

method	result	size
default	Expression too large to display	7821

input
```
int((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(7/2)/(a+b*tan(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}{a+b\tan(e+f\,x)}\,dx$$

input
```
int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx = \sqrt{d}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sec(fx+e)^3}{\tan(fx+e)\,b+a}dx\right)d^3$$

input
```
int((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x)
```

output
```
sqrt(d)*int((sqrt(sec(e + f*x))*sec(e + f*x)**3)/(tan(e + f*x)*b + a),x)*d
**3
```

3.612 $\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 396

$$\begin{aligned}\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx &= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf}\\
&-\frac{\sqrt[4]{a^2+b^2}\,d^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}}\\
&-\frac{\sqrt[4]{a^2+b^2}\,d^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}}\\
&-\frac{2ad^2\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}}\\
&+\frac{ad^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}}\\
&+\frac{ad^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}}\end{aligned}$$

output
```
2*d^2*(d*sec(f*x+e))^(1/2)/b/f-(a^2+b^2)^(1/4)*d^2*arctan(b^(1/2)*(sec(f*x
+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(1/2)/b^(3/2)/f/(sec(f*x+e)^2
)^(1/4)-(a^2+b^2)^(1/4)*d^2*arctanh(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)
^(1/4))*(d*sec(f*x+e))^(1/2)/b^(3/2)/f/(sec(f*x+e)^2)^(1/4)-2*a*d^2*Invers
eJacobiAM(1/2*arctan(tan(f*x+e)),2^(1/2))*(d*sec(f*x+e))^(1/2)/b^2/f/(sec(
f*x+e)^2)^(1/4)+a*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b
^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^2/f/(sec(f*x+e)^
2)^(1/4)+a*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2
),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^2/f/(sec(f*x+e)^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 24.34 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.73

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \frac{d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\left(-\sqrt{b}\sqrt[4]{a^2+b^2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\tan(\right.}{}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x]),x]
```

output
```
(d^2*Cot[e + f*x]*Sqrt[d*Sec[e + f*x]]*(-(Sqrt[b]*(a^2 + b^2)^(1/4)*ArcTan
[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Tan[e + f*x]) - Sqrt[
b]*(a^2 + b^2)^(1/4)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^
(1/4)]*Tan[e + f*x] + 2*b*(Sec[e + f*x]^2)^(1/4)*Tan[e + f*x] - a*Hypergeo
metric2F1[1/2, 3/4, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x]^2 + a*EllipticPi[-(
b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]
^2] + a*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*
Sqrt[-Tan[e + f*x]^2]))/(b^2*f*(Sec[e + f*x]^2)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.76, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3042, 3994, 493, 27, 719, 229, 504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\int \frac{\sqrt[4]{\tan^2(e+fx)+1}}{a+b\tan(e+fx)}d(b\tan(e+fx))}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{493}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\int \frac{b^2-ab\tan(e+fx)}{b^2(a+b\tan(e+fx))(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))+2\sqrt[4]{\tan^2(e+fx)+1}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\int \frac{b^2-ab\tan(e+fx)}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{b^2}+2\sqrt[4]{\tan^2(e+fx)+1}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{719}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-a\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{b^2}+2\sqrt[4]{\tan^2(e\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\downarrow$ 229

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\int\frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{b^2}+2\sqrt[4]{\tan^2(e}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\downarrow$ 504

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\left(a\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int\frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(\right.}{b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\downarrow$ 312

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\left(\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{-\frac{\tan(e+fx)}{b}}\left(\frac{\tan(e+fx)}{b}+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d\left(b^2\tan^2(e+fx)\right)}{2b}-\int\frac{}{\left(\tan^2(e\right.}\right.}{b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\downarrow$ 118

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\left(-\int\frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int-\frac{}{\sqrt{1-b^4\tan^4(e+fx)}}}{}\right.}{b^2}\right.}{bf\sqrt[4]{\sec^2(e+f}}$$

$\downarrow$ 25

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\int \frac{}{\left(\tan^2(\cdots\right.}\right.}{b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}\cdots}$$

$\downarrow$ 353

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\frac{1}{2}\int \frac{}{\left(\tan\cdots\right.}\right.}{b^2}\right.}{bf\sqrt[4]{\sec^2(e+f\cdots}}$$

$\downarrow$ 73

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\int \frac{}{-\mathrm{t}\cdots}\right.}{b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}\cdots}$$

$\downarrow$ 756

$$\frac{d^2\sqrt{d\sec(e+fx)}\left((a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\int \cdots\right.\right.\right.}{\cdots}$$

$\downarrow$ 218

$$\frac{d^2\sqrt{d\sec(e+fx)}\left((a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\int \cdots\right.\right.\right.}{bf\sqrt[4]{\mathrm{s}\cdots}}$$

$\downarrow$ 221

$$\frac{d^2\sqrt{d\sec(e+fx)}\left((a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\mathrm{arc}\cdots}{\cdots}\right.\right.\right.}{bf\sqrt[4]{\sec^2(e+\cdots}}$$

$\downarrow$ 925

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int \frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{2\left(a^2+b^2\right)}-\frac{b^2\int \cdots}{\cdots}\cdots\right.}{b}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$\downarrow$ 1537

$$d^2\sqrt{d\sec(e+fx)}\left((a^2+b^2)-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-\sqrt[4]{\frac{\tan(e+fx)}{b}}+1}\sqrt{\sqrt[4]{\frac{\tan(e+fx)}{b}}\cdots}}}{2\left(a^2+b^2\right)}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 412

$$d^2\sqrt{d\sec(e+fx)} \Bigg((a^2+b^2) \Bigg(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx) \Bigg(-\frac{b^2\,\mathrm{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\frac{\tan(e+fx)}{b}+1}\right),-1\right)}{2(a^2+b^2)} - \frac{b^2\,\mathrm{EllipticPi}\Big(}{}}{b}$$

input
```
Int[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x]),x]
```

output
```
(d^2*Sqrt[d*Sec[e + f*x]]*(2*(1 + Tan[e + f*x]^2)^(1/4) + (-2*a*b*Elliptic
F[ArcTan[Tan[e + f*x]]/2, 2] + (a^2 + b^2)*(-2*b^2*(ArcTan[(b^(3/2)*Tan[e
+ f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4)) + ArcTanh[(b^(3/2
)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4))) - (2*a*C
ot[e + f*x]*(-1/2*(b^2*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan[e
+ f*x]/b)^(1/4)], -1])/(a^2 + b^2) - (b^2*EllipticPi[b/Sqrt[a^2 + b^2], Ar
cSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(2*(a^2 + b^2)))*Sqrt[-Tan[e + f*x]
^2])/b))/b^2))/(b*f*(Sec[e + f*x]^2)^(1/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3130 vs. $2(342) = 684$.

Time = 28.03 (sec) , antiderivative size = 3131, normalized size of antiderivative = 7.91

method	result	size
default	Expression too large to display	3131

input
```
int((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*sec(f*x+e)^4*(-csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^3*d^2*(d*sec(f*x+e)
)^(1/2)/(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^2*(1+cos(f*x+e))^4*(-4*I*cos(f*x
+e)/(1+cos(f*x+e))^2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1
/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/
2)*(a^2+b^2)^(1/2)*EllipticPi(I*(csc(f*x+e)-cot(f*x+e)),-1/(b+(a^2+b^2)^(1
/2))^2*a^2,I)*a^3+4*I*cos(f*x+e)/(1+cos(f*x+e))^2*(1/(1+cos(f*x+e)))^(1/2)
*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/
2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2
)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(a^2+b^2)^(1/2)*EllipticPi(I*(csc(f*x+e)-c
ot(f*x+e)),-1/(-b+(a^2+b^2)^(1/2))^2*a^2,I)*a^3+4*I*cos(f*x+e)/(1+cos(f*x+
e))^2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF
(I*(csc(f*x+e)-cot(f*x+e)),I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^
2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2
-2*a^2*b-2*b^3)/a^4)^(1/2)*a^3*b-(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(2
*(2*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2*(-cos(f*x+e)/(1+cos(
f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(1+cos(f*x+e)))*(b*((a^2+b^2)^(1/2)*a^2+2*(
a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a
^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*a^4+(-cos(f*x+e)/(1+cos(f*x+e)
)^2)^(3/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)...
```

Fricas [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(f*x + e))*d^2*sec(f*x + e)^2/(b*tan(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(e+fx))^{\frac{5}{2}}}{a+b\tan(e+fx)}\,dx$$

input
```
integrate((d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**(5/2)/(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{a+b\tan\left(e+f\,x\right)}\,dx$$

input
```
int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \sqrt{d}\left(\int \frac{\sqrt{\sec\left(fx+e\right)}\,\sec\left(fx+e\right)^{2}}{\tan\left(fx+e\right)b+a}dx\right)d^{2}$$

input
```
int((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x)
```

output
```
sqrt(d)*int((sqrt(sec(e + f*x))*sec(e + f*x)**2)/(tan(e + f*x)*b + a),x)*d
**2
```

3.613 $\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 334

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{\sqrt{b}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{\sqrt{b}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}} - \frac{a\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}}{b\sqrt{a^2+b^2}f\sec^2(e+fx)^{3/4}} + \frac{a\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}}{b\sqrt{a^2+b^2}f\sec^2(e+fx)^{3/4}}$$

output

```
arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/
b^(1/2)/(a^2+b^2)^(1/4)/f/(sec(f*x+e)^2)^(3/4)-arctanh(b^(1/2)*(sec(f*x+e)
^2)^(1/4)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/b^(1/2)/(a^2+b^2)^(1/4)/f/
(sec(f*x+e)^2)^(3/4)-a*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+
b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b/(a^2+b^2)^(1/2)
/f/(sec(f*x+e)^2)^(3/4)+a*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^
2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b/(a^2+b^2)^(1/
2)/f/(sec(f*x+e)^2)^(3/4)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 2.04 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.83

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx =$$

$$-\frac{12d^2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},\frac{1}{4},\frac{3}{2},\frac{a-ib}{a+b\tan(e+fx)}\right.}{bf\sqrt{d\sec(e+fx)}\left((a+ib)\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},\frac{5}{4},\frac{5}{2},\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+(a-ib)\,\text{AppellF1}\left(\frac{3}{2},\frac{5}{4},\frac{1}{4},\right.\right.}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x]),x]
```

output
```
(-12*d^2*AppellF1[1/2, 1/4, 1/4, 3/2, (a - I*b)/(a + b*Tan[e + f*x]), (a +
 I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x]))/(b*f*Sqrt[d*Sec[e + f*x]
]*((a + I*b)*AppellF1[3/2, 1/4, 5/4, 5/2, (a - I*b)/(a + b*Tan[e + f*x]),
(a + I*b)/(a + b*Tan[e + f*x])] + (a - I*b)*AppellF1[3/2, 5/4, 1/4, 5/2, (
a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 6*AppellF
1[1/2, 1/4, 1/4, 3/2, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan
[e + f*x])]*(a + b*Tan[e + f*x])))
```

Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.74, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3994, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{(d\sec(e+fx))^{3/2}\int \frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{504}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(a\int \frac{1}{\sqrt[4]{\tan^2(e+fx)+1}(a^2-b^2\tan^2(e+fx))}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}(a^2-b^2\tan^2(e\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{310}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\int \frac{}{\sqrt[4]{\tan^2}}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{353}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\frac{1}{2}\int \frac{}{\sqrt[4]{\tan}}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\int \frac{\sqrt[4]{t}}{-\tan}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{827}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\frac{\int \frac{}{\sqrt{a}}}{}\right.\right.}{bf\sec^2(e+fx)^{3/4}}$$

$\downarrow$ 218

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\frac{\int\frac{}{\sqrt{a}}}{}\right.\right.}{bf\sec^2(e+fx)^{3/4}}$$

$\downarrow$ 221

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\frac{\text{arct}}{}\right.\right.}{bf\sec^2(e+fx)^{3/4}}$$

$\downarrow$ 993

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int\frac{1}{\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\tan^2(e+fx)+1}-\frac{1}{2}b\int\frac{}{(\tan}\right.}{b}\right.}{bf\sec^2(e-}$$

$\downarrow$ 1537

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int\frac{1}{\sqrt{1-b^2\tan^2(e+fx)}\sqrt{b^2\tan^2(e+fx)+1}\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)}d\sqrt[4]{\tan^2(e+fx}\right.}{b}\right.}{}$$

$\downarrow$ 412

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{b\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\tan^2(e+fx)+1}\right),-1\right)}{2\sqrt{a^2+b^2}}-\frac{b\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\text{ar}\right.}{}\right.}{b}\right.}{bf\sec^2(e+fx)^{3/4}}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x]),x]
```

output
```
((d*Sec[e + f*x])^(3/2)*(-2*b^2*(-1/2*ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 +
 b^2)^(1/4)]/(b^(3/2)*(a^2 + b^2)^(1/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/
(a^2 + b^2)^(1/4)]/(2*b^(3/2)*(a^2 + b^2)^(1/4))) + (2*a*Cot[e + f*x]*(-1/
2*(b*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]^2)^(1/4)],
-1])/Sqrt[a^2 + b^2] + (b*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e
+ f*x]^2)^(1/4)], -1])/(2*Sqrt[a^2 + b^2]))*Sqrt[-Tan[e + f*x]^2])/b))/(b*
f*(Sec[e + f*x]^2)^(3/4))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3617 vs. $2(282) = 564$.

Time = 7.69 (sec) , antiderivative size = 3618, normalized size of antiderivative = 10.83

method	result	size
default	Expression too large to display	3618

input
```
int((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2*d/f*(d*sec(f*x+e))^(1/2)*(1+cos(f*x+e))*(4*I*b*a^3*(1/(1+cos(f*x+e)))^
(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticPi(I*(-csc(f*x+e)+cot(f*x+
e)),-1/(-b+(a^2+b^2)^(1/2))^2*a^2,I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(
1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1
/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(a^2+b^2)^(1/2)+4*I*(-b*((a^2+b^2)^(1/2)
*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(a^2+b^2)^(1/2)*(b*((
a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(1/(1+c
os(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-csc(f*x+
e)+cot(f*x+e)),I)*a*b^3-4*I*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2
-2*a^2*b-2*b^3)/a^4)^(1/2)*(a^2+b^2)^(3/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+
b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-csc(f*x+e)+cot(f*x+e)),I)*a*b+4*I*
b*a^3*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticP
i(I*(-csc(f*x+e)+cot(f*x+e)),-1/(b+(a^2+b^2)^(1/2))^2*a^2,I)*(b*((a^2+b^2)
^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^
(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(a^2+b^2)^(1/2)+
(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(
-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(a^2+b^2)^(3/2)*(b*((a^2+b^2)^(1/2)*a^
2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-...
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   catd
ef: division by zero
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec{(e+fx)})^{\frac{3}{2}}}{a+b\tan{(e+fx)}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**(3/2)/(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec{(fx+e)})^{\frac{3}{2}}}{b\tan{(fx+e)}+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec{(fx+e)})^{\frac{3}{2}}}{b\tan{(fx+e)}+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{a+b\tan\left(e+f\,x\right)}\,dx$$

input
```
int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \sqrt{d}\left(\int \frac{\sqrt{\sec{(fx+e)}}\,\sec{(fx+e)}}{\tan{(fx+e)}\,b+a}dx\right)d$$

input
```
int((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x)
```

output
```
sqrt(d)*int((sqrt(sec(e + f*x))*sec(e + f*x))/(tan(e + f*x)*b + a),x)*d
```

3.614 $\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 324

$$\begin{aligned}\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = &-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)^{3/4}f\sqrt[4]{\sec^2(e+fx)}}\\ &-\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)^{3/4}f\sqrt[4]{\sec^2(e+fx)}}\\ &+\frac{a\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}\\ &+\frac{a\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}\end{aligned}$$

output
```
-b^(1/2)*arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(d*sec(f*x+e
))^(1/2)/(a^2+b^2)^(3/4)/f/(sec(f*x+e)^2)^(1/4)-b^(1/2)*arctanh(b^(1/2)*(s
ec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^(3/4)/f
/(sec(f*x+e)^2)^(1/4)+a*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2
+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)/f/(sec
(f*x+e)^2)^(1/4)+a*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^
(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)/f/(sec(f*x+e
)^2)^(1/4)
```

Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$$
$$= \frac{\sqrt{d\sec(e+fx)}\left(-\sqrt{b}\sqrt[4]{a^2+b^2}\left(\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\right)+a\cot\right.}{}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x]),x]
```

output
```
(Sqrt[d*Sec[e + f*x]]*(-(Sqrt[b]*(a^2 + b^2)^(1/4)*(ArcTan[(Sqrt[b]*(Sec[e
 + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)] + ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(
1/4))/(a^2 + b^2)^(1/4)])) + a*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]
), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]^2] + a*Cot[e + f
*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt
[-Tan[e + f*x]^2]))/((a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.77, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 3994, 504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}dx$$
$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt{d\sec(e+fx)}\int \frac{1}{(a+b\tan(e+fx))(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{504}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(a\int \frac{1}{(\tan^2(e+fx)+1)^{3/4}(a^2-b^2\tan^2(e+fx))}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{(\tan^2(e+fx)+1)^{3/4}(a^2-b^2\tan^2(e+fx))}d(b\ta\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{312}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{-\frac{\tan(e+fx)}{b}}\left(\frac{\tan(e+fx)}{b}+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b^2\tan^2(e+fx))}{2b}-\int \frac{}{(\tan^2(e+fx)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{118}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(-\int \frac{b\tan(e+fx)}{(\tan^2(e+fx)+1)^{3/4}(a^2-b^2\tan^2(e+fx))}d(b\tan(e+fx))-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int -\frac{}{\sqrt{1-b^4\tan^4(e+}}}{}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\int \frac{}{(\tan^2(e+fx)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{353}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\frac{1}{2}\int \frac{}{\left(\frac{\tan(e+fx)}{b}\right.}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\downarrow$ 73

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\int\frac{}{-\tan^4(e}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\downarrow$ 756

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\int\frac{}{\sqrt{a^2+b}}}{}\right.\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\downarrow$ 218

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\int\frac{}{\sqrt{a^2+b}}}{}\right.\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\downarrow$ 221

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\arctan\left(\right.}{2\sqrt{b}}\right.\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\downarrow$ 925

$$\frac{\sqrt{d\sec(e+fx)}\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{2\left(a^2+b^2\right)}-\frac{b^2\int\frac{\cdots}{\left(\frac{\tan^2(e+f\cdots}{\sqrt{a^2+b\cdots}}\right.}\cdots}{\cdots}\right.\cdots}{b}\cdots\right.}{bf\sqrt[4]{\sec^2(e+\cdots}}$$

$$\Big\downarrow \text{1537}$$

$$\sqrt{d\sec(e+fx)}\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}\sqrt{\sqrt[4]{\frac{\tan(e+fx)}{b}+1}+1}}d\cdots}{2\left(a^2+b^2\right)}\cdots\right.}{\cdots}\cdots\right.$$

$$\Big\downarrow \text{412}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\frac{\tan(e+fx)}{b}+1}\right),-1\right)}{2\left(a^2+b^2\right)}-\frac{b^2\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}}\cdots\right.}{\cdots}\right.\cdots}{b}\cdots\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x]),x]
```

output
```
(Sqrt[d*Sec[e + f*x]]*(-2*b^2*(ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(
1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2
+ b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4))) - (2*a*Cot[e + f*x]*(-1/2*(b^
2*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1]
)/(a^2 + b^2) - (b^2*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e + f*x
]/b)^(1/4)], -1])/(2*(a^2 + b^2)))*Sqrt[-Tan[e + f*x]^2])/b))/(b*f*(Sec[e
+ f*x]^2)^(1/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3036 vs. $2(276) = 552$.

Time = 7.28 (sec) , antiderivative size = 3037, normalized size of antiderivative = 9.37

method	result	size
default	Expression too large to display	3037

input
```
int((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*(4*I*b*a^3*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*EllipticPi(I*(csc(f*x+e)-cot(f*x+e)),-1/(b+(a^2+b^2)^(1/2))^2*a^2,I)*(b*
((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*(
(a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(a^2+b
^2)^(1/2)-4*I*b*a^3*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*EllipticPi(I*(csc(f*x+e)-cot(f*x+e)),-1/(-b+(a^2+b^2)^(1/2))^2*a^2,I)
*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(
-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(a
^2+b^2)^(1/2)-4*I*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*
b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b
^3)/a^4)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*a^5-4*I*(b*((a^2+b^2)^(1/2)*a^2+2*(
a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a
^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*(cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*a^3*b^
2-(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)
^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^
(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)+2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(1+
cos(f*x+e)))*a^4*b+(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(b*((a^2+b^2)^(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d \sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \int \frac{\sqrt{d\sec{(e+fx)}}}{a+b\tan{(e+fx)}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e)),x)
```

output
```
Integral(sqrt(d*sec(e + f*x))/(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \int \frac{\sqrt{d\sec{(fx+e)}}}{b\tan{(fx+e)}+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \int \frac{\sqrt{d\sec{(fx+e)}}}{b\tan{(fx+e)}+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx=\int \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}}{a+b\tan\left(e+f\,x\right)}\,dx$$

input
```
int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx=\sqrt{d}\left(\int \frac{\sqrt{\sec\left(fx+e\right)}}{\tan\left(fx+e\right)b+a}dx\right)$$

input
```
int((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x)
```

output
```
sqrt(d)*int(sqrt(sec(e + f*x))/(tan(e + f*x)*b + a),x)
```

3.615 $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 451

$$\begin{aligned}
&\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx \\
&= \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{5/4} f\sqrt{d\sec(e+fx)}} \\
&\quad - \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{5/4} f\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{2aE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2) f\sqrt{d\sec(e+fx)}} - \frac{2a\tan(e+fx)}{(a^2+b^2) f\sqrt{d\sec(e+fx)}} \\
&\quad - \frac{ab\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)^{3/2} f\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{ab\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)^{3/2} f\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{2(b+a\tan(e+fx))}{(a^2+b^2) f\sqrt{d\sec(e+fx)}}
\end{aligned}$$

output
```
b^(3/2)*arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2
)^(1/4)/(a^2+b^2)^(5/4)/f/(d*sec(f*x+e))^(1/2)-b^(3/2)*arctanh(b^(1/2)*(se
c(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(5/4)/f/
(d*sec(f*x+e))^(1/2)+2*a*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(s
ec(f*x+e)^2)^(1/4)/(a^2+b^2)/f/(d*sec(f*x+e))^(1/2)-2*a*tan(f*x+e)/(a^2+b^
2)/f/(d*sec(f*x+e))^(1/2)-a*b*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-
b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^
(3/2)/f/(d*sec(f*x+e))^(1/2)+a*b*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4
),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2
)^(3/2)/f/(d*sec(f*x+e))^(1/2)+2*(b+a*tan(f*x+e))/(a^2+b^2)/f/(d*sec(f*x+e
))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 2.54 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx =$$

$$-\frac{28d\,\text{AppellF1}\left(\frac{5}{2},\frac{5}{4},\frac{5}{4},\frac{7}{2},\frac{a-ib}{a+b\tan(e+fx)}\right.}{5bf(d\sec(e+fx))^{3/2}\left(5(a+ib)\,\text{AppellF1}\left(\frac{7}{2},\frac{5}{4},\frac{9}{4},\frac{9}{2},\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+5(a-ib)\,\text{AppellF1}\right.}$$

input
```
Integrate[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])),x]
```

output
```
(-28*d*AppellF1[5/2, 5/4, 5/4, 7/2, (a - I*b)/(a + b*Tan[e + f*x]), (a + I
*b)/(a + b*Tan[e + f*x])]*(a*Cos[e + f*x] + b*Sin[e + f*x]))/(5*b*f*(d*Sec
[e + f*x])^(3/2)*(5*(a + I*b)*AppellF1[7/2, 5/4, 9/4, 9/2, (a - I*b)/(a +
b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 5*(a - I*b)*AppellF1[7/
2, 9/4, 5/4, 9/2, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e +
 f*x])] + 14*AppellF1[5/2, 5/4, 5/4, 7/2, (a - I*b)/(a + b*Tan[e + f*x]),
(a + I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x])))
```

Rubi [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.76, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3042, 3994, 496, 27, 25, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}dx$$
$$\Big\downarrow \text{3994}$$
$$\frac{\sqrt[4]{\sec^2(e+fx)}\int \frac{1}{(a+b\tan(e+fx))(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))}{bf\sqrt{d\sec(e+fx)}}$$
$$\Big\downarrow \text{496}$$
$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{2b^2\int -\frac{\left(1-\frac{a^2}{b^2}\right)b^2-ab\tan(e+fx)}{2b^2(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}\right)}{bf\sqrt{d\sec(e+fx)}}$$
$$\Big\downarrow \text{27}$$
$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\int -\frac{a^2+b\tan(e+fx)a-b^2}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}+\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}\right)}{bf\sqrt{d\sec(e+fx)}}$$
$$\Big\downarrow \text{25}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{\int\frac{a^2+b\tan(e+fx)a-b^2}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 719}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-b^2\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+f}}}{a^2+b^2}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 225}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int\frac{1}{(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))\right)-b^2\int\frac{}{(a+b\tan}}{a^2+b^2}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 212}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\int\frac{}{(a+b\tan(e+fx)}}{a^2+b^2}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 504}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\left(a\int\frac{}{\sqrt[4]{\tan^2(}}\right.}{}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow 310$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\left(\frac{2a\sqrt{-\tan^2(e+f}}{}\right.}{}\right.}{bf\sqrt{d}}$$

$$\Big\downarrow 353$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\left(\frac{2a\sqrt{-\tan^2(e+f}}{}\right.}{}\right.}{bf\sqrt{}}$$

$$\Big\downarrow 73$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\left(\frac{2a\sqrt{-\tan^2(e+f}}{}\right.}{}\right.}{bf\sqrt{d\,\mathrm{se}}}$$

$$\Big\downarrow 827$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\left(\frac{2a\sqrt{-\tan^2(e+f}}{}\right.}{}\right.}{}$$

$\downarrow$ 218

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\left(\frac{2a\sqrt{-\tan^2(e+f}}{}\right.}{}\right.}{}$$

$\downarrow$ 221

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\left(\frac{2a\sqrt{-\tan^2(e+f}}{}\right.}{}\right.}{bf\sqrt{d}}$$

$\downarrow$ 993

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-b^2\left(\frac{2a\sqrt{-\tan^2(e+f}}{}\right.}{}\right.$$

$$\downarrow \text{1537}$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-b^2\left(\frac{2a\sqrt{-\tan^2(e+f}}{}\right.}{}\right.$$

$$\downarrow \text{412}$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-b^2\left(\frac{2a\sqrt{-\tan^2(e+f}}{}\right.}{}\right.$$

input
```
Int[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])),x]
```

output
```
((Sec[e + f*x]^2)^(1/4)*((2*(b^2 + a*b*Tan[e + f*x]))/((a^2 + b^2)*(1 + Ta
n[e + f*x]^2)^(1/4)) - (-(b^2*(-2*b^2*(-1/2*ArcTan[(b^(3/2)*Tan[e + f*x])/
(a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 + b^2)^(1/4)) + ArcTanh[(b^(3/2)*Tan[e +
f*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2)*(a^2 + b^2)^(1/4))) + (2*a*Cot[e + f*x
]*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]^2)^(1
/4)], -1])/Sqrt[a^2 + b^2] + (b*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 +
Tan[e + f*x]^2)^(1/4)], -1])/(2*Sqrt[a^2 + b^2]))*Sqrt[-Tan[e + f*x]^2])/b
)) + a*(-2*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*b*Tan[e + f*x])/(1
+ Tan[e + f*x]^2)^(1/4)))/(a^2 + b^2)))/(b*f*Sqrt[d*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4861 vs. $2(392) = 784$.

Time = 9.16 (sec) , antiderivative size = 4862, normalized size of antiderivative = 10.78

method	result	size
default	Expression too large to display	4862

input
```
int(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(1+cos(f*x+e))/(d*sec(f*x+e))^(1/2)/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(
1/2)*(4*(a^2+b^2)^(3/2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(-b*((a^2+b^2
)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(b*((a^2+b^2)^
(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*a^3*sin(f*x+e)-(
a^2+b^2)^(1/2)*ln(2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*
b-2*b^3)/a^4)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+
2*b^3)/a^4)^(1/2)*a^2*b^3+(a^2+b^2)^(3/2)*ln(2)*(-b*((a^2+b^2)^(1/2)*a^2+2
*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(
a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*a^2*b-4*(a^2+b^2)^(1/2)*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)
*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b
^2+2*a^2*b+2*b^3)/a^4)^(1/2)*a^3*b^2*sin(f*x+e)-2*ln(2*(2*cos(f*x+e)*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos
(f*x+e)+1)/(1+cos(f*x+e)))*(a^2+b^2)^(3/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2
+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b
^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*a^2*b+2*ln(2*(2*cos(f*x+e)*(-cos(f
*x+e)/(1+cos(f*x+e))^2)^(1/2)+2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f
*x+e)+1)/(1+cos(f*x+e)))*(a^2+b^2)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b
^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2
)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*a^2*b^3+2*ln((2*cos(f*x+e)*(-cos(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e)),x)
```

output
```
Integral(1/(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\sqrt{d\sec(fx+e)}(b\tan(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\sqrt{d\sec\left(fx+e\right)}(b\tan\left(fx+e\right)+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\sqrt{\frac{d}{\cos(e+f\,x)}}\,\left(a+b\tan\left(e+f\,x\right)\right)}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))),x)
```

output
```
int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)\tan(fx+e)b+\sec(fx+e)a}dx\right)}{d}$$

input
```
int(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x)
```

output
```
(sqrt(d)*int(sqrt(sec(e + f*x))/(sec(e + f*x)*tan(e + f*x)*b + sec(e + f*x
)*a),x))/d
```

3.616 $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 422

$$\begin{aligned}
&\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx = \\
&-\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{(a^2+b^2)^{7/4}f(d\sec(e+fx))^{3/2}} \\
&-\frac{b^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{(a^2+b^2)^{7/4}f(d\sec(e+fx))^{3/2}} \\
&+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3(a^2+b^2)f(d\sec(e+fx))^{3/2}} \\
&+\frac{ab^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)^2f(d\sec(e+fx))^{3/2}} \\
&+\frac{ab^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)^2f(d\sec(e+fx))^{3/2}} \\
&+\frac{2(b+a\tan(e+fx))}{3(a^2+b^2)f(d\sec(e+fx))^{3/2}}
\end{aligned}$$

output
```
-b^(5/2)*arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(sec(f*x+e)^
2)^(3/4)/(a^2+b^2)^(7/4)/f/(d*sec(f*x+e))^(3/2)-b^(5/2)*arctanh(b^(1/2)*(s
ec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^(7/4)/f
/(d*sec(f*x+e))^(3/2)+2/3*a*InverseJacobiAM(1/2*arctan(tan(f*x+e)),2^(1/2)
)*(sec(f*x+e)^2)^(3/4)/(a^2+b^2)/f/(d*sec(f*x+e))^(3/2)+a*b^2*cot(f*x+e)*E
llipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(3/4)*
(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(3/2)+a*b^2*cot(f*x+e)*
EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(3/4)*
(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(3/2)+2/3*(b+a*tan(f*x+
e))/(a^2+b^2)/f/(d*sec(f*x+e))^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.27 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.99

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx = \frac{a^2b\sec^2(e+fx)+b^3\sec^2(e+fx)+a^2b\cos(2(e+fx))\sec^2}{}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])),x]
```

output
```
(a^2*b*Sec[e + f*x]^2 + b^3*Sec[e + f*x]^2 + a^2*b*Cos[2*(e + f*x)]*Sec[e
+ f*x]^2 + b^3*Cos[2*(e + f*x)]*Sec[e + f*x]^2 - 3*b^(5/2)*(a^2 + b^2)^(1/
4)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x
]^2)^(3/4) - 3*b^(5/2)*(a^2 + b^2)^(1/4)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)
^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(3/4) + 2*a^3*Tan[e + f*x] + 2
*a*b^2*Tan[e + f*x] + a*(a^2 + b^2)*Hypergeometric2F1[1/2, 3/4, 3/2, -Tan[
e + f*x]^2]*(Sec[e + f*x]^2)^(3/4)*Tan[e + f*x] + 3*a*b^2*Cot[e + f*x]*Ell
ipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e +
 f*x]^2)^(3/4)*Sqrt[-Tan[e + f*x]^2] + 3*a*b^2*Cot[e + f*x]*EllipticPi[b/S
qrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/4)
*Sqrt[-Tan[e + f*x]^2])/(3*(a^2 + b^2)^2*f*(d*Sec[e + f*x])^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.77, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3042, 3994, 496, 27, 719, 229, 504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sec^2(e+fx)^{3/4}\int \frac{1}{(a+b\tan(e+fx))(\tan^2(e+fx)+1)^{7/4}}d(b\tan(e+fx))}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{496}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{2(ab\tan(e+fx)+b^2)}{3(a^2+b^2)(\tan^2(e+fx)+1)^{3/4}}-\frac{2b^2\int -\frac{\left(\frac{a^2}{b^2}+3\right)b^2+a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{3(a^2+b^2)}\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\int \frac{a^2+b\tan(e+fx)a+3b^2}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{3(a^2+b^2)}+\frac{2(ab\tan(e+fx)+b^2)}{3(a^2+b^2)(\tan^2(e+fx)+1)^{3/4}}\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{719}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{3b^2\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))+a\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{3(a^2+b^2)}+\frac{2(ab\tan(e+fx)+}{3(a^2+b^2)(\tan^2(e+fa}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 229$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{3b^2\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))+2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{3(a^2+b^2)}+\frac{2(ab\tan(e+fx)+\cdots}{3(a^2+b^2)(\tan^2(e+fx\cdots}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 504$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{3b^2\left(a\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))\right)}{3(a^2+b^2)}\cdots\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 312$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{3b^2\left(\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{-\frac{\tan(e+fx)}{b}}\left(\frac{\tan(e+fx)}{b}+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d\left(b^2\tan^2(e+fx)\right)}{2b}-\int \frac{b\cdots}{\left(\tan^2(e+fx)+1\right)\cdots}\right.}{3(a^2+b^2)}\cdots\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 118$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{3b^2\left(-\int \frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int -\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan\cdots\right.}}{b}\cdots\right.}{3(a^2+b^2)}\cdots\right.}{bf(d\sec(e+fx))^{3\cdots}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{3b^2\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\int\frac{b}{\left(\tan^2(e+fx)+1\right.}\right.}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 353$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{3b^2\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\frac{1}{2}\int\frac{}{\left(\frac{\tan(e+fx)}{b}\right.}\right.}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3}}$$

$$\Big\downarrow \ 73$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{3b^2\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\int\frac{}{-\tan^4(e+f}\right.}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 756$$

$$\sec^2(e+fx)^{3/4}\left(3b^2\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\int\frac{}{\sqrt{a^2+b^2-}}}{}\right.\right.\right.$$

$\downarrow$ 218

$$\sec^2(e+fx)^{3/4}\left(\frac{3b^2\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\int\frac{}{\sqrt{a^2+b^2-}}}{}\right.\right.}{3(a^2+b^2)}\right.$$

$$bf(d\sec($$

$\downarrow$ 221

$$\sec^2(e+fx)^{3/4}\left(\frac{3b^2\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\arctan\left(\frac{b^{3/}}{\sqrt[4]{}}\right.}{2\sqrt{b}\left(a^2\right.}\right.\right.}{3(a^2+b^2)}\right.$$

$$bf(d\sec(e+fx))$$

$\downarrow$ 925

$$\text{sec}^2(e+fx)^{3/4} \left(3b^2 \left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int \frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{2\left(a^2+b^2\right)} - \frac{b^2\int \frac{\tan^2(e+}{\left(\frac{\tan^2(e+}{\sqrt{a^2+}}\right.}}{}\right.}{b}\right.\right.}{}$$

$$\downarrow 1537$$

$$\frac{\sec^2(e+fx)^{3/4}}{\left(\frac{3b^2\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-\sqrt[4]{\frac{\tan(e+fx)}{b}}+1}\sqrt{\sqrt[4]{\frac{\tan(e+fx)}{b}}+1+1}}d}{2(a^2+b^2)}\right.}{}\right.}{}\right.}$$

$\downarrow$ 412

$$\frac{\sec^2(e+fx)^{3/4}\left(3b^2\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\frac{\tan(e+fx)}{b}+1}\right),-1\right)}{2\left(a^2+b^2\right)}-\frac{b^2\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}}\right.}{}\right.}{b}\right.\right.}{}$$

input
```
Int[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])),x]
```

output
```
((Sec[e + f*x]^2)^(3/4)*((2*(b^2 + a*b*Tan[e + f*x]))/(3*(a^2 + b^2)*(1 +
Tan[e + f*x]^2)^(3/4)) + (2*a*b*EllipticF[ArcTan[Tan[e + f*x]]/2, 2] + 3*b
^2*(-2*b^2*(ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a
^2 + b^2)^(3/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sq
rt[b]*(a^2 + b^2)^(3/4))) - (2*a*Cot[e + f*x]*(-1/2*(b^2*EllipticPi[-(b/Sq
rt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(a^2 + b^2) - (b^
2*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(
2*(a^2 + b^2)))*Sqrt[-Tan[e + f*x]^2])/b))/(3*(a^2 + b^2))))/(b*f*(d*Sec[e
 + f*x])^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3758 vs. $2(364) = 728$.

Time = 11.08 (sec) , antiderivative size = 3759, normalized size of antiderivative = 8.91

method	result	size
default	Expression too large to display	3759

input
```
int(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-1/6/f/(1+cos(f*x+e))/(d*sec(f*x+e))^(1/2)/(-cos(f*x+e)/(1+cos(f*x+e))^2)^
(1/2)/d*(3*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^
4)^(1/2)*arctanh(1/2*((a^2+b^2)^(1/2)*cos(f*x+e)*b-a^2*cos(f*x+e)-b^2*cos(
f*x+e)-b*(a^2+b^2)^(1/2)+b^2)/(1+cos(f*x+e))/(-cos(f*x+e)/(1+cos(f*x+e))^2
)^(1/2)/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)
^(1/2)/a^2)*(a^2+b^2)^(3/2)*b^4-3*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2
)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*arctanh(1/2*((a^2+b^2)^(1/2)*cos(f*x+e)*b-
a^2*cos(f*x+e)-b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2)+b^2)/(1+cos(f*x+e))/(-cos(
f*x+e)/(1+cos(f*x+e))^2)^(1/2)/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*
b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*(a^2+b^2)^(1/2)*b^6-3*(b*((a^2+b^2)^(1/
2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*arctanh(1/2*((a^2+b
^2)^(1/2)*cos(f*x+e)*b-a^2*cos(f*x+e)-b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2)+b^2
)/(1+cos(f*x+e))/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)/(-b*((a^2+b^2)^(1/2)
*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*a^4*b^3-3*(b*((a
^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*arctanh(
1/2*((a^2+b^2)^(1/2)*cos(f*x+e)*b-a^2*cos(f*x+e)-b^2*cos(f*x+e)-b*(a^2+b^2
)^(1/2)+b^2)/(1+cos(f*x+e))/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)/(-b*((a^2
+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*a^2*b
^5+3*arctanh(1/2*((a^2+b^2)^(1/2)*cos(f*x+e)*b+a^2*cos(f*x+e)+b^2*cos(f*x+
e)-b*(a^2+b^2)^(1/2)-b^2)/(1+cos(f*x+e))/(-cos(f*x+e)/(1+cos(f*x+e))^2)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx = \int \frac{1}{(d\sec{(e+fx)})^{\frac{3}{2}}\,(a+b\tan{(e+fx)})}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e)),x)
```

output
```
Integral(1/((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx = \int \frac{1}{(d\sec{(fx+e)})^{\frac{3}{2}}\,(b\tan{(fx+e)}+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)), x)
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx = \int \frac{1}{(d\sec{(fx+e)})^{\frac{3}{2}}\,(b\tan{(fx+e)}+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}\,(a+b\tan\left(e+f\,x\right))}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))),x)
```

output
```
int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))), x)
```

Reduce [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^2\tan(fx+e)b+\sec(fx+e)^2a}dx\right)}{d^2}$$

input
```
int(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x)
```

output
```
(sqrt(d)*int(sqrt(sec(e + f*x))/(sec(e + f*x)**2*tan(e + f*x)*b + sec(e +
f*x)**2*a),x))/d**2
```

3.617 $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 593

$$\begin{aligned}
\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx &= \frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{9/4}\,d^2f\sqrt{d\sec(e+fx)}} \\
&- \frac{b^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{9/4}\,d^2f\sqrt{d\sec(e+fx)}} \\
&+ \frac{2ab^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^2\,d^2f\sqrt{d\sec(e+fx)}} \\
&+ \frac{6aE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\,(a^2+b^2)\,d^2f\sqrt{d\sec(e+fx)}} - \frac{2ab^2\tan(e+fx)}{(a^2+b^2)^2\,d^2f\sqrt{d\sec(e+fx)}} \\
&- \frac{ab^3\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)^{5/2}\,d^2f\sqrt{d\sec(e+fx)}} \\
&+ \frac{ab^3\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)^{5/2}\,d^2f\sqrt{d\sec(e+fx)}} \\
&+ \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\,(a^2+b^2)\,d^2f\sqrt{d\sec(e+fx)}} + \frac{2(b^3+ab^2\tan(e+fx))}{(a^2+b^2)^2\,d^2f\sqrt{d\sec(e+fx)}}
\end{aligned}$$

output
```
b^(7/2)*arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2
)^(1/4)/(a^2+b^2)^(9/4)/d^2/f/(d*sec(f*x+e))^(1/2)-b^(7/2)*arctanh(b^(1/2)
*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(9/4
)/d^2/f/(d*sec(f*x+e))^(1/2)+2*a*b^2*EllipticE(sin(1/2*arctan(tan(f*x+e)))
,2^(1/2))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^2/d^2/f/(d*sec(f*x+e))^(1/2)+6/5*
a*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(1/4)/(a^2
+b^2)/d^2/f/(d*sec(f*x+e))^(1/2)-2*a*b^2*tan(f*x+e)/(a^2+b^2)^2/d^2/f/(d*s
ec(f*x+e))^(1/2)-a*b^3*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+
b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(5/2)/d
^2/f/(d*sec(f*x+e))^(1/2)+a*b^3*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4)
,b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)
^(5/2)/d^2/f/(d*sec(f*x+e))^(1/2)+2/5*cos(f*x+e)^2*(b+a*tan(f*x+e))/(a^2+b
^2)/d^2/f/(d*sec(f*x+e))^(1/2)+2*(b^3+a*b^2*tan(f*x+e))/(a^2+b^2)^2/d^2/f/
(d*sec(f*x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 33.06 (sec) , antiderivative size = 17838, normalized size of antiderivative = 30.08

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 425, normalized size of antiderivative = 0.72, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3994, 496, 27, 686, 27, 25, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}dx$$

$$\Big\downarrow \text{ 3994}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\int \frac{1}{(a+b\tan(e+fx))(\tan^2(e+fx)+1)^{9/4}}d(b\tan(e+fx))}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 496}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{5(a^2+b^2)(\tan^2(e+fx)+1)^{5/4}}-\frac{2b^2\int -\frac{\left(\frac{3a^2}{b^2}+5\right)b^2+3a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))}{5(a^2+b^2)}\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\int \frac{3a^2+3b\tan(e+fx)a+5b^2}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))}{5(a^2+b^2)}+\frac{2(ab\tan(e+fx)+b^2)}{5(a^2+b^2)(\tan^2(e+fx)+1)^{5/4}}\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 686}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{2b^4\int -\frac{\left(-\frac{3a^4}{b^4}-\frac{8a^2}{b^2}+5\right)b^4-ab\left(3a^2+8b^2\right)\tan(e+fx)}{2b^4(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}}{5(a^2+b^2)}+\frac{2(ab\tan(e+\cdots}{5(a^2+b^2)(\tan^2(e\cdots}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{\int -\frac{3a^4+8b^2a^2+b\left(3a^2+8b^2\right)\tan(e+fx)a-5b^4}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}+\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}}{5(a^2+b^2)}+\frac{2(ab\tan(e+fx)+b}{5(a^2+b^2)(\tan^2(e+fx)-}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{\int \frac{3a^4+8b^2a^2+b\left(3a^2+8b^2\right)\tan(e+fx)a-5b^4}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}}{5(a^2+b^2)}+\frac{2(ab\tan(e+fx)+b^2)}{5(a^2+b^2)(\tan^2(e+fx)+}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{719}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-5b^4\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(}}}{a^2+b^2}}{5(a^2+b^2)}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{225}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))\right)-5b^4\int -}{a^2+b^2}}{5(a^2+b^2)}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$\Big\downarrow$ 212

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\int}{a^2+b^2}}{5(a^2+b^2)}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$\Big\downarrow$ 504

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(a\int\right.}{}}{5(a^2+b^2}\right.}{bd^2f\sqrt{}}$$

$\Big\downarrow$ 310

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(\frac{2a\sqrt{-\tan^2}\cdots}{\cdots}\right.\cdots}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 353$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(\frac{2a\sqrt{-\tan^2}\cdots}{\cdots}\right.\cdots}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 73$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(\frac{2a\sqrt{-\tan^2}\cdots}{\cdots}\right.\cdots}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \text{827}$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(\frac{2a\sqrt{-\tan^2}}{}\right.}{}}{}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(\frac{2a\sqrt{-\tan^2}}{}\right.}{}}{}\right.$$

$$\Big\downarrow \text{221}$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(\frac{2a\sqrt{-\tan^2}}{}\right.}{}}{}\right.$$

$$\Big\downarrow \ 993$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(\frac{2a\sqrt{-\tan^2}}{}\right.}{}}{}\right.$$

$$\Big\downarrow \ 1537$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(\frac{2a\sqrt{-\tan^2}}{}\right.}{}}{}\right.$$

$$\Big\downarrow \text{412}$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(\frac{2a\sqrt{-\tan^2}}{}\right.}{}}{}\right.$$

input
```
Int[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])),x]
```

output
```
((Sec[e + f*x]^2)^(1/4)*((2*(b^2 + a*b*Tan[e + f*x]))/(5*(a^2 + b^2)*(1 +
Tan[e + f*x]^2)^(5/4)) + ((2*(5*b^4 + a*b*(3*a^2 + 8*b^2)*Tan[e + f*x]))/(
(a^2 + b^2)*(1 + Tan[e + f*x]^2)^(1/4)) - (-5*b^4*(-2*b^2*(-1/2*ArcTan[(b^
(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 + b^2)^(1/4)) + ArcTa
nh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2)*(a^2 + b^2)^(1/4))
) + (2*a*Cot[e + f*x]*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1
+ Tan[e + f*x]^2)^(1/4)], -1])/Sqrt[a^2 + b^2] + (b*EllipticPi[b/Sqrt[a^2
+ b^2], ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1])/(2*Sqrt[a^2 + b^2]))*Sqrt
[-Tan[e + f*x]^2])/b) + a*(3*a^2 + 8*b^2)*(-2*b*EllipticE[ArcTan[Tan[e + f
*x]]/2, 2] + (2*b*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^(1/4)))/(a^2 + b^2))/
(5*(a^2 + b^2))))/(b*d^2*f*Sqrt[d*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6386 vs. $2(525) = 1050$.

Time = 16.63 (sec) , antiderivative size = 6387, normalized size of antiderivative = 10.77

method	result	size
default	Expression too large to display	6387

input
```
int(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \int \frac{1}{(d\sec{(e+fx)})^{\frac{5}{2}}\,(a+b\tan{(e+fx)})}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e)),x)
```

output
```
Integral(1/((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}(a+b\tan(e+f\,x))}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))),x)
```

output
```
int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))), x)
```

Reduce [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^3\tan(fx+e)b+\sec(fx+e)^3a}dx\right)}{d^3}$$

input
```
int(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x)
```

output
```
(sqrt(d)*int(sqrt(sec(e + f*x))/(sec(e + f*x)**3*tan(e + f*x)*b + sec(e +
f*x)**3*a),x))/d**3
```

3.618 $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 480

$$
\begin{aligned}
\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = & -\frac{3ad^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2b^{5/2}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}} \\
& + \frac{3ad^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2b^{5/2}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}} \\
& - \frac{3d^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}} \\
& + \frac{3d^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2f} \\
& + \frac{3a^2d^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}}{2b^3\sqrt{a^2+b^2}f\sec^2(e+fx)^{3/4}} \\
& - \frac{3a^2d^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}}{2b^3\sqrt{a^2+b^2}f\sec^2(e+fx)^{3/4}} \\
& - \frac{d^2(d\sec(e+fx))^{3/2}}{bf(a+b\tan(e+fx))}
\end{aligned}
$$

output
```
-3/2*a*d^2*arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(d*sec(f*x
+e))^(3/2)/b^(5/2)/(a^2+b^2)^(1/4)/f/(sec(f*x+e)^2)^(3/4)+3/2*a*d^2*arctan
h(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/b^(5/
2)/(a^2+b^2)^(1/4)/f/(sec(f*x+e)^2)^(3/4)-3*d^2*EllipticE(sin(1/2*arctan(t
an(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(3/2)/b^2/f/(sec(f*x+e)^2)^(3/4)+3*d^2
*cos(f*x+e)*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/b^2/f+3/2*a^2*d^2*cot(f*x+e)*E
llipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*
(-tan(f*x+e)^2)^(1/2)/b^3/(a^2+b^2)^(1/2)/f/(sec(f*x+e)^2)^(3/4)-3/2*a^2*d
^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(
f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b^3/(a^2+b^2)^(1/2)/f/(sec(f*x+e)^2)^(
3/4)-d^2*(d*sec(f*x+e))^(3/2)/b/f/(a+b*tan(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.99 (sec) , antiderivative size = 1129, normalized size of antiderivative = 2.35

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^2,x]
```

output
```
(Cos[e + f*x]*(d*Sec[e + f*x])^(7/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(
(3*Cos[e + f*x])/(a*b) + (3*Sin[e + f*x])/b^2 - 1/(b*(a*Cos[e + f*x] + b*S
in[e + f*x]))))/(f*(a + b*Tan[e + f*x])^2) + (3*(d*Sec[e + f*x])^(7/2)*(a*
Cos[e + f*x] + b*Sin[e + f*x])^2*(-((a*EllipticE[ArcSin[Tan[(e + f*x)/2]],
 -1]*Sqrt[1 + Tan[(e + f*x)/2]^2])/Sqrt[1 - Tan[(e + f*x)/2]^2]) + (2*a*El
lipticF[ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 + Tan[(e + f*x)/2]^2])/Sqrt[1
 - Tan[(e + f*x)/2]^2] + (-2*Sqrt[2]*a*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[
Sqrt[((1 + I)*(1 + Tan[(e + f*x)/2]))/(I + Tan[(e + f*x)/2])]/Sqrt[2]], 2]
*Sqrt[-((1 + I*Tan[(e + f*x)/2])/(I + Tan[(e + f*x)/2]))] + Sqrt[2]*a^2*Sq
rt[a^2 + b^2]*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b +
Sqrt[a^2 + b^2]), ArcSin[Sqrt[((1 + I)*(1 + Tan[(e + f*x)/2]))/(I + Tan[(e
 + f*x)/2])]/Sqrt[2]], 2]*Sqrt[-((1 + I*Tan[(e + f*x)/2])/(I + Tan[(e + f*
x)/2]))] + a^2*(a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b
+ Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[((1 + I)*(1 +
Tan[(e + f*x)/2]))/(I + Tan[(e + f*x)/2])]/Sqrt[2]], 2]*Sqrt[-((2 + (2*I)*
Tan[(e + f*x)/2])/(I + Tan[(e + f*x)/2]))] - a^3*EllipticPi[((1 + I)*(a -
I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[((1 + I)*
(1 + Tan[(e + f*x)/2]))/(I + Tan[(e + f*x)/2])]/Sqrt[2]], 2]*Sqrt[-((2 + (
2*I)*Tan[(e + f*x)/2])/(I + Tan[(e + f*x)/2]))] - I*a^2*b*EllipticPi[((1 +
 I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sq...
```

Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.68, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 3994, 492, 605, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}dx$$
$$\Big\downarrow \text{3994}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\int\frac{\left(\tan^2(e+fx)+1\right)^{3/4}}{(a+b\tan(e+fx))^2}d(b\tan(e+fx))}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \ 492$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\int\frac{b\tan(e+fx)}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2b^2}-\frac{\left(\tan^2(e+fx)+1\right)^{3/4}}{a+b\tan(e+fx)}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \ 605$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-a\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))\right)}{2b^2}-\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \ 225$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(-a\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))+\frac{2b}{\sqrt[4]{\tan^2}}\right.}{2b^2}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \ 212$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(-a\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)+\frac{2b\tan(e+}{\sqrt[4]{\tan^2(e+}}\right.}{2b^2}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \ 504$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(-a\left(a\int \frac{1}{\sqrt[4]{\tan^2(e+fx)+1}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}\left(a^2-b^2\tan^2(\cdots\right.}\cdots\right.\right.}{2b^2}\cdots\right.}{bf\sec^2(e+fx)^{3/4}\cdots}$$

$$\Big\downarrow 310$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\int \frac{\cdots}{\sqrt[4]{\mathrm{t}\cdots}}\cdots\right.\right.}{\cdots}\cdots\right.}{bf\sec^2(\cdots}$$

$$\Big\downarrow 353$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\frac{1}{2}\int \frac{\cdots}{\sqrt[4]{\cdots}}\cdots\right.\right.}{\cdots}\cdots\right.}{bf\sec^2\cdots}$$

$$\Big\downarrow 73$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\int \cdots\right.\right.}{2b\cdots}\cdots\right.}{bf\sec^2(e\cdots}$$

$\downarrow$ 827

$$d^2(d\sec(e+fx))^{3/2}\left(3\left(-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\int\right.\right.\right.\right.$$

$\downarrow$ 218

$$d^2(d\sec(e+fx))^{3/2}\left(3\left(-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\int\right.\right.\right.\right.$$

$\downarrow$ 221

$$d^2(d\sec(e+fx))^{3/2}\left(3\left(-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(a\right.\right.\right.\right.$$

$bf\sec^2($

$\downarrow$ 993

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int\frac{1}{\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\tan^2(e+fx)+1}-\frac{1}{2}b\cdots\right.}{b}\cdots\right.\right.}{\cdots}\cdots\right.$$

$\Big\downarrow$ 1537

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int\frac{1}{\sqrt{1-b^2\tan^2(e+fx)}\sqrt{b^2\tan^2(e+fx)+1}\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)}d\sqrt[4]{\tan^2(e\cdots\right.}{\cdots}\cdots\right.\right.}{\cdots}\cdots\right.$$

$\Big\downarrow$ 412

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{b\,\mathrm{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\tan^2(e+fx)+1}\right),-1\right)}{2\sqrt{a^2+b^2}}-\frac{b\,\mathrm{EllipticPi}\left(-\frac{\cdots}{\sqrt{a\cdots}}\cdots\right)}{\cdots}\cdots\right.}{b}\cdots\right.\right.}{\cdots}\cdots\right.$$

input
```
Int[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^2,x]
```

output
```
(d^2*(d*Sec[e + f*x])^(3/2)*(-((1 + Tan[e + f*x]^2)^(3/4)/(a + b*Tan[e + f
*x])) + (3*(-2*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*b*Tan[e + f*x])
/(1 + Tan[e + f*x]^2)^(1/4) - a*(-2*b^2*(-1/2*ArcTan[(b^(3/2)*Tan[e + f*x]
)/(a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 + b^2)^(1/4)) + ArcTanh[(b^(3/2)*Tan[e
+ f*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2)*(a^2 + b^2)^(1/4))) + (2*a*Cot[e + f
*x]*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]^2)^
(1/4)], -1])/Sqrt[a^2 + b^2] + (b*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1
+ Tan[e + f*x]^2)^(1/4)], -1])/(2*Sqrt[a^2 + b^2]))*Sqrt[-Tan[e + f*x]^2])
/b)))/(2*b^2)))/(b*f*(Sec[e + f*x]^2)^(3/4))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 605
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[1/d   Int[x^(m - 1)*(a + b*x^2)^p, x], x] - Simp[c/d   Int[x^(m - 1
)*((a + b*x^2)^p/(c + d*x)), x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m,
 0] && LtQ[-1, p, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 12831 vs. $2(413) = 826$.

Time = 244.87 (sec) , antiderivative size = 12832, normalized size of antiderivative = 26.73

method	result	size
default	Expression too large to display	12832

input
```
int((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(f*x + e))*d^3*sec(f*x + e)^3/(b^2*tan(f*x + e)^2 + 2*a
*b*tan(f*x + e) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(7/2)/(a+b*tan(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(d \sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d \sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}{(a+b\tan(e+f\,x))^2}\,dx$$

input
```
int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(d \sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \frac{\sqrt{d}\,d^3\left(-10\sqrt{\sec(fx+e)}\,\sec(fx+e)^3\tan(fx+e)+35\left(\int \frac{\sqrt{\sec(fx+e)}\,\sec(fx\cdots}{\tan(fx+e)^2b^2+2\,\mathrm{ta}\cdots}\right.\right.}{}$$

input
```
int((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x)
```

output

```
(sqrt(d)*d**3*( - 10*sqrt(sec(e + f*x))*sec(e + f*x)**3*tan(e + f*x) + 35*
int((sqrt(sec(e + f*x))*sec(e + f*x)**3*tan(e + f*x)**3)/(tan(e + f*x)**2*
b**2 + 2*tan(e + f*x)*a*b + a**2),x)*tan(e + f*x)*b**2*f + 35*int((sqrt(se
c(e + f*x))*sec(e + f*x)**3*tan(e + f*x)**3)/(tan(e + f*x)**2*b**2 + 2*tan
(e + f*x)*a*b + a**2),x)*a*b*f + 45*int((sqrt(sec(e + f*x))*sec(e + f*x)**
3*tan(e + f*x)**2)/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*t
an(e + f*x)*a*b*f + 45*int((sqrt(sec(e + f*x))*sec(e + f*x)**3*tan(e + f*x
)**2)/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*a**2*f + 28*in
t((sqrt(sec(e + f*x))*sec(e + f*x)**3)/(tan(e + f*x)**2*b**2 + 2*tan(e + f
*x)*a*b + a**2),x)*tan(e + f*x)*a*b*f + 28*int((sqrt(sec(e + f*x))*sec(e +
 f*x)**3)/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*a**2*f))/(
18*a*f*(tan(e + f*x)*b + a))
```

3.619 $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 440

$$\begin{aligned}\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx &= \frac{ad^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2b^{3/2}\left(a^2+b^2\right)^{3/4}f\sqrt[4]{\sec^2(e+fx)}}\\
&+\frac{ad^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2b^{3/2}\left(a^2+b^2\right)^{3/4}f\sqrt[4]{\sec^2(e+fx)}}\\
&+\frac{d^2\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}}\\
&-\frac{a^2d^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{2b^2\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}}\\
&-\frac{a^2d^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{2b^2\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}}\\
&-\frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))}\end{aligned}$$

output
```
1/2*a*d^2*arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(d*sec(f*x+
e))^(1/2)/b^(3/2)/(a^2+b^2)^(3/4)/f/(sec(f*x+e)^2)^(1/4)+1/2*a*d^2*arctanh
(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(1/2)/b^(3/2
)/(a^2+b^2)^(3/4)/f/(sec(f*x+e)^2)^(1/4)+d^2*InverseJacobiAM(1/2*arctan(ta
n(f*x+e)),2^(1/2))*(d*sec(f*x+e))^(1/2)/b^2/f/(sec(f*x+e)^2)^(1/4)-1/2*a^2
*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*s
ec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^2/(a^2+b^2)/f/(sec(f*x+e)^2)^(1/4
)-1/2*a^2*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2)
,I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^2/(a^2+b^2)/f/(sec(f*x+e)
^2)^(1/4)-d^2*(d*sec(f*x+e))^(1/2)/b/f/(a+b*tan(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.20 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.71

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \frac{(d\sec(e+fx))^{5/2}(a\cos(e+fx)+b\sin(e+fx))^2\left(-\frac{2b\cos(e+fx)}{a\cos(e+fx)+b\sin(e+fx)}+\right.}{}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^2,x]
```

output
```
((d*Sec[e + f*x])^(5/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*((-2*b*Cos[e +
 f*x])/(a*Cos[e + f*x] + b*Sin[e + f*x]) + (Hypergeometric2F1[1/2, 3/4, 3/
2, -Tan[e + f*x]^2]*Tan[e + f*x] + (a*(a*EllipticPi[-(b/Sqrt[a^2 + b^2]),
ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Tan[e + f*x] + a*EllipticPi[b/Sqrt[a^2
 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Tan[e + f*x] + Sqrt[b]*(a^2 +
 b^2)^(1/4)*(ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)] +
ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)])*Sqrt[-Tan[e +
 f*x]^2]))/((a^2 + b^2)*Sqrt[-Tan[e + f*x]^2]))/(Sec[e + f*x]^2)^(1/4)))/(
2*b^2*f*(a + b*Tan[e + f*x])^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.70, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3042, 3994, 492, 605, 229, 504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\int \frac{\sqrt[4]{\tan^2(e+fx)+1}}{(a+b\tan(e+fx))^2}d(b\tan(e+fx))}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{492}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\int \frac{b\tan(e+fx)}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2b^2}-\frac{\sqrt[4]{\tan^2(e+fx)+1}}{a+b\tan(e+fx)}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{605}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-a\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2b^2}-\frac{\sqrt[4]{\tan^2(e+fx)+}}{a+b\tan(e+fx)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{229}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2b^2}-\frac{\sqrt[4]{\tan^2(e+fx)+}}{a+b\tan(e+fx)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{504}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(a\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int\frac{b}{\left(\tan^2(e+fx)+1\right.}\right.}{2b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{312}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{-\frac{\tan(e+fx)}{b}}\left(\frac{\tan(e+fx)}{b}+1\right)^{3/4}\left(a^2-b^2\tan^2(e+f\right.}}{2b}\right.}{2b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{118}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(-\int\frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\frac{2a\sqrt{-\tan^2(e+fx)}}{}\right.}{2b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\cdots}}{b}\cdots\right.}{2b^2}\cdots\right.}{bf\sqrt[4]{\sec^2(e+fx)}\cdots}$$

$$\Big\downarrow \; 353$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\cdots}}{b}\cdots\right.}{2b^2}\cdots\right.}{bf\sqrt[4]{\sec^2(e+fx)}\cdots}$$

$$\Big\downarrow \; 73$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\cdots}}{b}\cdots\right.}{2b^2}\cdots\right.}{bf\sqrt[4]{\sec^2(e+fx)}\cdots}$$

$$\Big\downarrow \; 756$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\cdots}}{b}\cdots\right.}{\cdots}\right.}{\cdots}$$

$$\Big\downarrow \; 218$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\cdots}}{b}\cdots\right.}{2b^2\cdots}\right.}{bf\sqrt[4]{\sec^2(\cdots}}$$

$$\Big\downarrow \; 221$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\cdots}}{b}\cdots\right.}{2b^2\cdots}\right.}{bf\sqrt[4]{\sec^2(e+fx)}\cdots}$$

$$\Big\downarrow \; 925$$

$$d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\ }}{2\left(a^2+b^2\right)}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{1537}$$

$$d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-\sqrt[4]{\frac{\tan(e+f}{b}}}}}{}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \; 412$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\mathrm{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\,\mathrm{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\frac{\tan(e+}{b}}\right.\right.}{2\left(a^2+b^2\right)}\right.}{}\right.}{}\right.}{}$$

input
```
Int[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^2,x]
```

output
```
(d^2*Sqrt[d*Sec[e + f*x]]*(-((1 + Tan[e + f*x]^2)^(1/4)/(a + b*Tan[e + f*x
])) + (2*b*EllipticF[ArcTan[Tan[e + f*x]]/2, 2] - a*(-2*b^2*(ArcTan[(b^(3/
2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4)) + ArcTan
h[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4)))
 - (2*a*Cot[e + f*x]*(-1/2*(b^2*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1
 + Tan[e + f*x]/b)^(1/4)], -1])/(a^2 + b^2) - (b^2*EllipticPi[b/Sqrt[a^2 +
 b^2], ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(2*(a^2 + b^2)))*Sqrt[-Tan
[e + f*x]^2])/b))/(2*b^2)))/(b*f*(Sec[e + f*x]^2)^(1/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 605
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[1/d   Int[x^(m - 1)*(a + b*x^2)^p, x], x] - Simp[c/d   Int[x^(m - 1
)*((a + b*x^2)^p/(c + d*x)), x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m,
 0] && LtQ[-1, p, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2819 vs. $2(378) = 756$.

Time = 124.40 (sec) , antiderivative size = 2820, normalized size of antiderivative = 6.41

method	result	size
default	Expression too large to display	2820

input
```
int((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*d^2/f*(cos(f*x+e)*(1+cos(f*x+e))*(a^2+b^2)^(1/2)*arctanh(1/2*((a^2+b^2
)^(1/2)*cos(f*x+e)*b-a^2*cos(f*x+e)-b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2)+b^2)/
(1+cos(f*x+e))/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)/(-b*((a^2+b^2)^(1/2)*a
^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*(-cos(f*x+e)/(1+co
s(f*x+e))^2)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2
*b^3)/a^4)^(1/2)*a+(1+cos(f*x+e))*sin(f*x+e)*(a^2+b^2)^(1/2)*arctanh(1/2*(
(a^2+b^2)^(1/2)*cos(f*x+e)*b-a^2*cos(f*x+e)-b^2*cos(f*x+e)-b*(a^2+b^2)^(1/
2)+b^2)/(1+cos(f*x+e))/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)/(-b*((a^2+b^2)
^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*(-cos(f*x+
e)/(1+cos(f*x+e))^2)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2
*a^2*b+2*b^3)/a^4)^(1/2)*b+cos(f*x+e)*(-cos(f*x+e)-1)*b*arctanh(1/2*((a^2+
b^2)^(1/2)*cos(f*x+e)*b-a^2*cos(f*x+e)-b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2)+b^
2)/(1+cos(f*x+e))/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)/(-b*((a^2+b^2)^(1/2
)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*(-cos(f*x+e)/(1
+cos(f*x+e))^2)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*
b+2*b^3)/a^4)^(1/2)*a+(-cos(f*x+e)-1)*sin(f*x+e)*arctanh(1/2*((a^2+b^2)^(1
/2)*cos(f*x+e)*b-a^2*cos(f*x+e)-b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2)+b^2)/(1+c
os(f*x+e))/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)/(-b*((a^2+b^2)^(1/2)*a^2+2
*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*...
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   catd
ef: division by zero
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(e+fx))^{\frac{5}{2}}}{(a+b\tan(e+fx))^2}\,dx$$

input
```
integrate((d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**(5/2)/(a + b*tan(e + f*x))**2, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{\left(a+b\tan\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \frac{\sqrt{d}\,d^2\left(-6\sqrt{\sec\left(fx+e\right)}\,\sec\left(fx+e\right)^2\tan\left(fx+e\right)+15\left(\int \frac{\sqrt{\sec(fx+e)}\,\sec(fx+}{\tan(fx+e)^2b^2+2\tan}\right.\right.}{}$$

input
```
int((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x)
```

output
```
(sqrt(d)*d**2*( - 6*sqrt(sec(e + f*x))*sec(e + f*x)**2*tan(e + f*x) + 15*i
nt((sqrt(sec(e + f*x))*sec(e + f*x)**2*tan(e + f*x)**3)/(tan(e + f*x)**2*b
**2 + 2*tan(e + f*x)*a*b + a**2),x)*tan(e + f*x)*b**2*f + 15*int((sqrt(sec
(e + f*x))*sec(e + f*x)**2*tan(e + f*x)**3)/(tan(e + f*x)**2*b**2 + 2*tan(
e + f*x)*a*b + a**2),x)*a*b*f + 21*int((sqrt(sec(e + f*x))*sec(e + f*x)**2
*tan(e + f*x)**2)/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*ta
n(e + f*x)*a*b*f + 21*int((sqrt(sec(e + f*x))*sec(e + f*x)**2*tan(e + f*x)
**2)/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*a**2*f + 20*int
((sqrt(sec(e + f*x))*sec(e + f*x)**2)/(tan(e + f*x)**2*b**2 + 2*tan(e + f*
x)*a*b + a**2),x)*tan(e + f*x)*a*b*f + 20*int((sqrt(sec(e + f*x))*sec(e +
f*x)**2)/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*a**2*f))/(1
4*a*f*(tan(e + f*x)*b + a))
```

3.620 $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 477

$$
\begin{aligned}
\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx &= \frac{a\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2\sqrt{b}\left(a^2+b^2\right)^{5/4}f\sec^2(e+fx)^{3/4}} \\
&- \frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2\sqrt{b}\left(a^2+b^2\right)^{5/4}f\sec^2(e+fx)^{3/4}} \\
&- \frac{E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{(a^2+b^2)\,f\sec^2(e+fx)^{3/4}} \\
&+ \frac{\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{(a^2+b^2)\,f} \\
&- \frac{a^2\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}}{2b\left(a^2+b^2\right)^{3/2}f\sec^2(e+fx)^{3/4}} \\
&+ \frac{a^2\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}}{2b\left(a^2+b^2\right)^{3/2}f\sec^2(e+fx)^{3/4}} \\
&- \frac{b(d\sec(e+fx))^{3/2}}{(a^2+b^2)\,f(a+b\tan(e+fx))}
\end{aligned}
$$

output
```
1/2*a*arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^
(3/2)/b^(1/2)/(a^2+b^2)^(5/4)/f/(sec(f*x+e)^2)^(3/4)-1/2*a*arctanh(b^(1/2)
*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/b^(1/2)/(a^2+b
^2)^(5/4)/f/(sec(f*x+e)^2)^(3/4)-EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(
1/2))*(d*sec(f*x+e))^(3/2)/(a^2+b^2)/f/(sec(f*x+e)^2)^(3/4)+cos(f*x+e)*(d*
sec(f*x+e))^(3/2)*sin(f*x+e)/(a^2+b^2)/f-1/2*a^2*cot(f*x+e)*EllipticPi((se
c(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^
2)^(1/2)/b/(a^2+b^2)^(3/2)/f/(sec(f*x+e)^2)^(3/4)+1/2*a^2*cot(f*x+e)*Ellip
ticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan
(f*x+e)^2)^(1/2)/b/(a^2+b^2)^(3/2)/f/(sec(f*x+e)^2)^(3/4)-b*(d*sec(f*x+e))
^(3/2)/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 29.43 (sec) , antiderivative size = 6560, normalized size of antiderivative = 13.75

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^2,x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.72, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 3994, 498, 27, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{(d\sec(e+fx))^{3/2}\int \frac{1}{(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{498}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(-\frac{\int -\frac{2a+b\tan(e+fx)}{2(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}-\frac{b^2\left(\tan^2(e+fx)+1\right)^{3/4}}{(a^2+b^2)(a+b\tan(e+fx))}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{\int \frac{2a+b\tan(e+fx)}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2(a^2+b^2)}-\frac{b^2\left(\tan^2(e+fx)+1\right)^{3/4}}{(a^2+b^2)(a+b\tan(e+fx))}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{719}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\int \frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+\int \frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2(a^2+b^2)}-\frac{b^2(\tan}{(a^2+b^2)}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{225}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\int \frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))+\frac{2b\tan(e+f}{\sqrt[4]{\tan^2(e+f}}}{2(a^2+b^2)}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{212}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)+\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+\cdots}}}{2(a^2+b^2)}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{504}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\left(a\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int\frac{b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}\left(a^2-b^2\tan^2(e+fx)\right)}d\cdots\right.}{2(a^2+b^2)}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{310}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\int\frac{\cdots}{\sqrt[4]{\tan^2(e\cdots}}\right.}{2(a^2+b^2\cdots}\right.}{bf\sec^2(e\cdots}$$

$$\Big\downarrow \text{353}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\frac{1}{2}\int\frac{\cdots}{\sqrt[4]{\tan(e\cdots}}\right.}{2(a^2+\cdots}\right.}{bf\sec^2(e\cdots}$$

$$\Big\downarrow \text{73}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\int\frac{\sqrt[4]{\tan(\cdots}}{-\tan^4(\cdots}\cdots}{2(a^2+b^2)\cdots}\cdots}{bf\sec^2(e+\cdots}$$

$\downarrow$ 827

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\frac{\int\frac{\cdots}{\sqrt{a^2+b^2}\cdots}}{\cdots}\cdots}{\cdots}\cdots}{\cdots}$$

$\downarrow$ 218

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\frac{\int\frac{\cdots}{\sqrt{a^2+b^2}\cdots}}{\cdots}\cdots}{\cdots}\cdots}{b\cdots}$$

$\downarrow$ 221

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\frac{\text{arctanh}}{2b^{3/}}\right.\right.}{2(a^2+b^2)}\right.}{bf\sec^2(e}$$

$$\downarrow 993$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int\frac{1}{\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\tan^2(e+fx)+1}-\frac{1}{2}b\int\frac{}{\left(\tan^2(\right.}\right.}{b}\right.}{}\right.}{}$$

$$\downarrow 1537$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int\frac{1}{\sqrt{1-b^2\tan^2(e+fx)}\sqrt{b^2\tan^2(e+fx)+1}\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)}d\sqrt[4]{\tan^2(e+fx)}\right.}{b}\right.}{}\right.}{}$$

$$\downarrow 412$$

$$(d\sec(e+fx))^{3/2}\left(\frac{a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{b\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\tan^2(e+fx)+1}\right),-1\right)}{2\sqrt{a^2+b^2}}-\frac{b\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\text{arc}\right.}{}\right.}{b}\right.}{}\right.$$

input
```
Int[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^2,x]
```

output
```
((d*Sec[e + f*x])^(3/2)*(-((b^2*(1 + Tan[e + f*x]^2)^(3/4))/((a^2 + b^2)*(
a + b*Tan[e + f*x]))) + (-2*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*b*
Tan[e + f*x])/(1 + Tan[e + f*x]^2)^(1/4) + a*(-2*b^2*(-1/2*ArcTan[(b^(3/2)
*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 + b^2)^(1/4)) + ArcTanh[(b
^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2)*(a^2 + b^2)^(1/4))) + (
2*a*Cot[e + f*x]*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan
[e + f*x]^2)^(1/4)], -1])/Sqrt[a^2 + b^2] + (b*EllipticPi[b/Sqrt[a^2 + b^2
], ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1])/(2*Sqrt[a^2 + b^2]))*Sqrt[-Tan
[e + f*x]^2])/b))/(2*(a^2 + b^2))))/(b*f*(Sec[e + f*x]^2)^(3/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10258 vs. $2(410) = 820$.

Time = 7.00 (sec) , antiderivative size = 10259, normalized size of antiderivative = 21.51

method	result	size
default	Expression too large to display	10259

input
```
int((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec{(e+fx)})^{\frac{3}{2}}}{(a+b\tan{(e+fx)})^2}\,dx$$

input
```
integrate((d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**(3/2)/(a + b*tan(e + f*x))**2, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{(a+b\tan(e+f\,x))^2}\,dx$$

input
```
int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \frac{\sqrt{d}\,d\left(-2\sqrt{\sec(fx+e)}\,\sec(fx+e)\tan(fx+e)+3\left(\int \frac{\sqrt{\sec(fx+e)}\,\sec(fx+e)\,\mathrm{ta}}{\tan(fx+e)^2b^2+2\tan(fx+}\right.\right.}{}$$

input
```
int((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x)
```

output
```
(sqrt(d)*d*( - 2*sqrt(sec(e + f*x))*sec(e + f*x)*tan(e + f*x) + 3*int((sqr
t(sec(e + f*x))*sec(e + f*x)*tan(e + f*x)**3)/(tan(e + f*x)**2*b**2 + 2*ta
n(e + f*x)*a*b + a**2),x)*tan(e + f*x)*b**2*f + 3*int((sqrt(sec(e + f*x))*
sec(e + f*x)*tan(e + f*x)**3)/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b +
 a**2),x)*a*b*f + 5*int((sqrt(sec(e + f*x))*sec(e + f*x)*tan(e + f*x)**2)/
(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*tan(e + f*x)*a*b*f +
 5*int((sqrt(sec(e + f*x))*sec(e + f*x)*tan(e + f*x)**2)/(tan(e + f*x)**2*
b**2 + 2*tan(e + f*x)*a*b + a**2),x)*a**2*f + 12*int((sqrt(sec(e + f*x))*s
ec(e + f*x))/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*tan(e +
 f*x)*a*b*f + 12*int((sqrt(sec(e + f*x))*sec(e + f*x))/(tan(e + f*x)**2*b*
*2 + 2*tan(e + f*x)*a*b + a**2),x)*a**2*f))/(10*a*f*(tan(e + f*x)*b + a))
```

3.621 $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 430

$$\begin{aligned}
\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx &= -\frac{3a\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2(a^2+b^2)^{7/4}f\sqrt[4]{\sec^2(e+fx)}} \\
&\quad - \frac{3a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2(a^2+b^2)^{7/4}f\sqrt[4]{\sec^2(e+fx)}} \\
&\quad - \frac{\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}} \\
&\quad + \frac{3a^2\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{2(a^2+b^2)^2f\sqrt[4]{\sec^2(e+fx)}} \\
&\quad + \frac{3a^2\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{2(a^2+b^2)^2f\sqrt[4]{\sec^2(e+fx)}} \\
&\quad - \frac{b\sqrt{d\sec(e+fx)}}{(a^2+b^2)f(a+b\tan(e+fx))}
\end{aligned}$$

output
```
-3/2*a*b^(1/2)*arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(d*sec
(f*x+e))^(1/2)/(a^2+b^2)^(7/4)/f/(sec(f*x+e)^2)^(1/4)-3/2*a*b^(1/2)*arctan
h(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(1/2)/(a^2+
b^2)^(7/4)/f/(sec(f*x+e)^2)^(1/4)-InverseJacobiAM(1/2*arctan(tan(f*x+e)),2
^(1/2))*(d*sec(f*x+e))^(1/2)/(a^2+b^2)/f/(sec(f*x+e)^2)^(1/4)+3/2*a^2*cot(
f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e)
)^(1/2)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^2/f/(sec(f*x+e)^2)^(1/4)+3/2*a^2*c
ot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+
e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^2/f/(sec(f*x+e)^2)^(1/4)-b*(d*se
c(f*x+e))^(1/2)/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.62 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$$
$$= \frac{\sec^2(e+fx)\sqrt{d\sec(e+fx)}(a\cos(e+fx)+b\sin(e+fx))^2\left(-\frac{b}{a(a-ib)(a+ib)}+\frac{b^2\sin(e+fx)}{a(a-ib)(a+ib)(a\cos(e+fx)+b\sin(}\right.}{f(a+b\tan(e+fx))^2}$$
$$+\frac{\sqrt{d\sec(e+fx)}\sec^2(e+fx)^{3/4}(a\cos(e+fx)+b\sin(e+fx))^2\left(-\left((a^2+b^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.\right.}{}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^2,x]
```

output
```
(Sec[e + f*x]^2*Sqrt[d*Sec[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(
-(b/(a*(a - I*b)*(a + I*b))) + (b^2*Sin[e + f*x])/(a*(a - I*b)*(a + I*b)*(
a*Cos[e + f*x] + b*Sin[e + f*x]))))/(f*(a + b*Tan[e + f*x])^2) + (Sqrt[d*S
ec[e + f*x]]*(Sec[e + f*x]^2)^(3/4)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(-
((a^2 + b^2)*Hypergeometric2F1[1/2, 3/4, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x
]) + 3*a*(-(Sqrt[b]*(a^2 + b^2)^(1/4)*(ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1
/4))/(a^2 + b^2)^(1/4)] + ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 +
b^2)^(1/4)])) + a*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Se
c[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]^2] + a*Cot[e + f*x]*EllipticP
i[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x
]^2])))/(2*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x])^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.75, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3994, 498, 27, 719, 229, 504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt{d\sec(e+fx)}\int \frac{1}{(a+b\tan(e+fx))^2(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{498}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(-\frac{\int -\frac{2a-b\tan(e+fx)}{2(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{a^2+b^2}-\frac{b^2\sqrt[4]{\tan^2(e+fx)+1}}{(a^2+b^2)(a+b\tan(e+fx))}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\int \frac{2a-b\tan(e+fx)}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2(a^2+b^2)}-\frac{b^2\sqrt[4]{\tan^2(e+fx)+1}}{(a^2+b^2)(a+b\tan(e+fx))}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 719$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2(a^2+b^2)}-\frac{b^2\sqrt[4]{\tan^2(e+fx)+}}{(a^2+b^2)(a+b\tan(e+f}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 229$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{2(a^2+b^2)}-\frac{b^2\sqrt[4]{\tan^2(e+fx)+}}{(a^2+b^2)(a+b\tan(e+f}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 504$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\left(a\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))\right)}{2(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 312$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\left(\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{-\frac{\tan(e+fx)}{b}}\left(\frac{\tan(e+fx)}{b}+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d\left(b^2\tan^2(e+fx)\right)}{2b}-\int \frac{b\tan}{\left(\tan^2(e+fx)+1\right)^{3}}\right.}{2(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 118$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\left(-\int \frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int -\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4\right.}}{b}\right.}{2(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\int \frac{b\,t}{\left(\tan^2(e+fx)+1\right)}\right.}{2(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\Big\downarrow$ 353

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\frac{1}{2}\int \frac{}{\left(\frac{\tan(e+fx)}{b}+\right.}\right.}{2(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\Big\downarrow$ 73

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\int\frac{1}{-\tan^4(e+fx}\right.}{2(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 756$$

$$\sqrt{d\sec(e+fx)}\left(\frac{3a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\int\frac{}{\sqrt{a^2+b^2-b}}}{}\right.\right.}{}\right.$$

$$\Big\downarrow \; 218$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\int\frac{}{\sqrt{a^2+b^2-b}}}{}\right.\right.}{2(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(}}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\arctan\left(\frac{b^{3/2}}{\sqrt[4]{\cdots}}\right)}{2\sqrt{b}\left(a^2\cdots\right)}\cdots\right.\right)}{2(a^2+b^2)}\cdots\right)}{bf\sqrt[4]{\sec^2(e+fx)}\cdots}$$

$$\Big\downarrow \; 925$$

$$\frac{\sqrt{d\sec(e+fx)}\left(3a\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{2\left(a^2+b^2\right)}-\frac{b^2\int\cdots}{\left(\frac{\tan^2(e+fx)}{\sqrt{a^2+b^2}}\cdots\right)}\cdots\right)}{b}\cdots\right)\cdots\right)}{\cdots}$$

$$\Big\downarrow \; 1537$$

$$\sqrt{d\sec(e+fx)}\left(3a\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-\sqrt[4]{\frac{\tan(e+fx)}{b}}+1}\sqrt{\sqrt[4]{\frac{\tan(e+fx)}{b}}+1+1}}d\sqrt[4]{}}{2\left(a^2+b^2\right)}\right.}{}\right.\right.$$

$\Big\downarrow$ 412

$$\sqrt{d\sec(e+fx)}\left(\frac{\left(3a\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\frac{\tan(e+fx)}{b}+1}\right),-1\right)}{2(a^2+b^2)}-\frac{b^2\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}}\right.}{}\right.}{b}\right.\right.}{}\right.$$

input
```
Int[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^2,x]
```

output
```
(Sqrt[d*Sec[e + f*x]]*(-((b^2*(1 + Tan[e + f*x]^2)^(1/4))/((a^2 + b^2)*(a
+ b*Tan[e + f*x]))) + (-2*b*EllipticF[ArcTan[Tan[e + f*x]]/2, 2] + 3*a*(-2
*b^2*(ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b
^2)^(3/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*
(a^2 + b^2)^(3/4))) - (2*a*Cot[e + f*x]*(-1/2*(b^2*EllipticPi[-(b/Sqrt[a^2
 + b^2]), ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(a^2 + b^2) - (b^2*Elli
pticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(2*(a^2
 + b^2)))*Sqrt[-Tan[e + f*x]^2])/b))/(2*(a^2 + b^2))))/(b*f*(Sec[e + f*x]^
2)^(1/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7437 vs. $2(368) = 736$.

Time = 10.56 (sec) , antiderivative size = 7438, normalized size of antiderivative = 17.30

method	result	size
default	Expression too large to display	7438

input
```
int((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral(sqrt(d*sec(e + f*x))/(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\sqrt{d\sec(fx+e)}}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)}}{\left(b\tan\left(fx+e\right)+a\right)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}}{\left(a+b\tan\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$$
$$= \frac{\sqrt{d}\left(2\sqrt{\sec\left(fx+e\right)}\,\tan\left(fx+e\right)+4\left(\int \frac{\sqrt{\sec(fx+e)}}{\tan(fx+e)^2b^2+2\tan(fx+e)ab+a^2}dx\right)\tan\left(fx+e\right)abf+4\left(\int \frac{}{\tan(fx+e)}\right.\right.}{}$$

input
```
int((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x)
```

output
```
(sqrt(d)*(2*sqrt(sec(e + f*x))*tan(e + f*x) + 4*int(sqrt(sec(e + f*x))/(ta
n(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*tan(e + f*x)*a*b*f + 4*
int(sqrt(sec(e + f*x))/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),
x)*a**2*f - int((sqrt(sec(e + f*x))*tan(e + f*x)**3)/(tan(e + f*x)**2*b**2
 + 2*tan(e + f*x)*a*b + a**2),x)*tan(e + f*x)*b**2*f - int((sqrt(sec(e + f
*x))*tan(e + f*x)**3)/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),x
)*a*b*f - 3*int((sqrt(sec(e + f*x))*tan(e + f*x)**2)/(tan(e + f*x)**2*b**2
 + 2*tan(e + f*x)*a*b + a**2),x)*tan(e + f*x)*a*b*f - 3*int((sqrt(sec(e +
f*x))*tan(e + f*x)**2)/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),
x)*a**2*f))/(6*a*f*(tan(e + f*x)*b + a))
```

3.622 $\int \frac{1}{\sqrt{d \sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 555

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$$
$$= \frac{5ab^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{9/4}f\sqrt{d\sec(e+fx)}}$$
$$-\frac{5ab^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{9/4}f\sqrt{d\sec(e+fx)}}$$
$$+\frac{(2a^2-3b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{\left(a^2+b^2\right)^2f\sqrt{d\sec(e+fx)}}$$
$$-\frac{(2a^2-3b^2)\tan(e+fx)}{\left(a^2+b^2\right)^2f\sqrt{d\sec(e+fx)}}$$
$$-\frac{5a^2b\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^{5/2}f\sqrt{d\sec(e+fx)}}$$
$$+\frac{5a^2b\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^{5/2}f\sqrt{d\sec(e+fx)}}$$
$$+\frac{b(2a^2-3b^2)\sec^2(e+fx)}{\left(a^2+b^2\right)^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$+\frac{2(b+a\tan(e+fx))}{\left(a^2+b^2\right)f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$

output
```
5/2*a*b^(3/2)*arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(sec(f*
x+e)^2)^(1/4)/(a^2+b^2)^(9/4)/f/(d*sec(f*x+e))^(1/2)-5/2*a*b^(3/2)*arctanh
(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(1/4)/(a^2+b
^2)^(9/4)/f/(d*sec(f*x+e))^(1/2)+(2*a^2-3*b^2)*EllipticE(sin(1/2*arctan(ta
n(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/2
)-(2*a^2-3*b^2)*tan(f*x+e)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/2)-5/2*a^2*b*co
t(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)
^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(5/2)/f/(d*sec(f*x+e))^(1/2)+5/2
*a^2*b*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(se
c(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(5/2)/f/(d*sec(f*x+e))^(
1/2)+b*(2*a^2-3*b^2)*sec(f*x+e)^2/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/2)/(a+b*
tan(f*x+e))+2*(b+a*tan(f*x+e))/(a^2+b^2)/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f
*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 29.96 (sec) , antiderivative size = 8379, normalized size of antiderivative = 15.10

$$\int \frac{1}{\sqrt{d \sec(e+fx)}(a+b\tan(e+fx))^2} \, dx = \text{Result too large to show}$$

input
```
Integrate[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 431, normalized size of antiderivative = 0.78, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3994, 496, 27, 25, 688, 27, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \; 3994$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\int \frac{1}{(a+b\tan(e+fx))^2(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \; 496$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{2b^2\int -\frac{\left(3-\frac{a^2}{b^2}\right)b^2+a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\int -\frac{a^2-b\tan(e+fx)a-3b^2}{(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}+\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{\int \frac{a^2-b\tan(e+fx)a-3b^2}{(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \; 688$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{-\frac{b^2\int\frac{2a\left(4-\frac{a^2}{b^2}\right)+\left(3-\frac{2a^2}{b^2}\right)b\tan(e+fx)}{2(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}-\frac{b^2(\cdots}{\cdots}}{a^2+b^2}\cdots\right.}{bf\sqrt{d\sec(e+fx)}\cdots}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{-\frac{b^2\int\frac{2a\left(4-\frac{a^2}{b^2}\right)+\left(3-\frac{2a^2}{b^2}\right)b\tan(e+fx)}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}-\frac{b^2(\cdots}{\cdots}}{a^2+b^2}\cdots\right.}{bf\sqrt{d\sec(e+fx)}\cdots}$$

$$\Big\downarrow 719$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(\left(3-\frac{2a^2}{b^2}\right)\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+5a\int\frac{\cdots}{(a+\cdots}\cdots\right.}{2\left(a^2+b^2\right)\cdots}}{\cdots}\cdots\right.}{bf\sqrt{d\sec(e+fx)}\cdots}$$

$$\Big\downarrow 225$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(\left(3-\frac{2a^2}{b^2}\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{5/4}}d\cdots\right.\right.}{\cdots}}{\cdots}\cdots\right.}{bf\sqrt{d\sec(e+fx)}\cdots}$$

$$\Big\downarrow \ 212$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+(\right.}{2(}}{}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \ 504$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(a\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}(a^2-b^2\tan^2(e+fx))}d(b\tan(e-\right.\right.}{}}{}\right.$$

$$\Big\downarrow \ 310$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(\right.}}{b}\right.\right.}{}}{}\right.$$

$$\Big\downarrow \ 353$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(\cdots\right.}\cdots}{b}\cdots\right.\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 73

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(\cdots\right.}\cdots}{b}\cdots\right.\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 827

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(\cdots\right.}\cdots}{b}\cdots\right.\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 218

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e\right.}}{b}\right.\right.}{}}{}\right.$$

$$\Big\downarrow \text{221}$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e\right.}}{b}\right.\right.}{}}{}\right.$$

$$\Big\downarrow \text{993}$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int\frac{1}{\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)}\right.}{}\right.\right.}{}}{}\right.$$

$$\Big\downarrow \text{1537}$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int\frac{\cdots}{\sqrt{1-b^2\tan^2(e+fx)}\sqrt{b^2\tan\cdots}}\cdots\right.}{\cdots}\cdots\right.\right.}{\cdots}}{\cdots}\cdots\right.$$

$$\Big\downarrow \; 412$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{b\,\mathrm{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\cdots\right.\right.}{2\sqrt{\cdots}}\cdots\right.}{\cdots}\cdots\right.\right.}{\cdots}}{\cdots}\cdots\right.$$

input

```
Int[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2),x]
```

output
```
((Sec[e + f*x]^2)^(1/4)*((2*(b^2 + a*b*Tan[e + f*x]))/((a^2 + b^2)*(a + b*
Tan[e + f*x])*(1 + Tan[e + f*x]^2)^(1/4)) - (-((b^2*(2*a^2 - 3*b^2)*(1 + T
an[e + f*x]^2)^(3/4))/((a^2 + b^2)*(a + b*Tan[e + f*x]))) - (b^2*(5*a*(-2*
b^2*(-1/2*ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 +
 b^2)^(1/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2
)*(a^2 + b^2)^(1/4))) + (2*a*Cot[e + f*x]*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2
 + b^2]), ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1])/Sqrt[a^2 + b^2] + (b*El
lipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1])/(2*Sq
rt[a^2 + b^2]))*Sqrt[-Tan[e + f*x]^2])/b) + (3 - (2*a^2)/b^2)*(-2*b*Ellipt
icE[ArcTan[Tan[e + f*x]]/2, 2] + (2*b*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^(
1/4))))/(2*(a^2 + b^2)))/(a^2 + b^2)))/(b*f*Sqrt[d*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 15132 vs. $2(486) = 972$.

Time = 12.45 (sec) , antiderivative size = 15133, normalized size of antiderivative = 27.27

method	result	size
default	Expression too large to display	15133

input
```
int(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\sqrt{d\sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral(1/(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\sqrt{d\sec\left(fx+e\right)}(b\tan\left(fx+e\right)+a)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\sqrt{d\sec\left(fx+e\right)}(b\tan\left(fx+e\right)+a)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\sqrt{\frac{d}{\cos(e+f\,x)}}\,(a+b\tan(e+f\,x))^2}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^2),x)
```

output
```
int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)\tan(fx+e)^2b^2+2\sec(fx+e)\tan(fx+e)ab+\sec(fx+e)a^2}dx\right)}{d}$$

input
```
int(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x)
```

output
```
(sqrt(d)*int(sqrt(sec(e + f*x))/(sec(e + f*x)*tan(e + f*x)**2*b**2 + 2*sec
(e + f*x)*tan(e + f*x)*a*b + sec(e + f*x)*a**2),x))/d
```

3.623 $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 520

$$
\begin{aligned}
\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx =
&-\frac{7ab^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{2\left(a^2+b^2\right)^{11/4}f(d\sec(e+fx))^{3/2}}\\
&-\frac{7ab^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{2\left(a^2+b^2\right)^{11/4}f(d\sec(e+fx))^{3/2}}\\
&+\frac{(2a^2-5b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}}\\
&+\frac{7a^2b^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^3 f(d\sec(e+fx))^{3/2}}\\
&+\frac{7a^2b^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^3 f(d\sec(e+fx))^{3/2}}\\
&+\frac{b(2a^2-5b^2)\sec^2(e+fx)}{3\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\\
&+\frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}
\end{aligned}
$$

output
```
-7/2*a*b^(5/2)*arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(sec(f
*x+e)^2)^(3/4)/(a^2+b^2)^(11/4)/f/(d*sec(f*x+e))^(3/2)-7/2*a*b^(5/2)*arcta
nh(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(3/4)/(a^2
+b^2)^(11/4)/f/(d*sec(f*x+e))^(3/2)+1/3*(2*a^2-5*b^2)*InverseJacobiAM(1/2*
arctan(tan(f*x+e)),2^(1/2))*(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^2/f/(d*sec(f*x+
e))^(3/2)+7/2*a^2*b^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b
^2)^(1/2),I)*(sec(f*x+e)^2)^(3/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^3/f/(d*s
ec(f*x+e))^(3/2)+7/2*a^2*b^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/
(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(3/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^3/
f/(d*sec(f*x+e))^(3/2)+1/3*b*(2*a^2-5*b^2)*sec(f*x+e)^2/(a^2+b^2)^2/f/(d*s
ec(f*x+e))^(3/2)/(a+b*tan(f*x+e))+2/3*(b+a*tan(f*x+e))/(a^2+b^2)/f/(d*sec(
f*x+e))^(3/2)/(a+b*tan(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.86 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.68

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \frac{\frac{2(a^2+b^2)\sec^2(e+fx)\left(3a^2b-4b^3+b(a^2+b^2)\cos(2(e+fx))+a(a^2+b^2)\sin(2(e+f}{a+b\tan(e+fx)}}{}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2),x]
```

output
```
((2*(a^2 + b^2)*Sec[e + f*x]^2*(3*a^2*b - 4*b^3 + b*(a^2 + b^2)*Cos[2*(e +
 f*x)] + a*(a^2 + b^2)*Sin[2*(e + f*x)]))/(a + b*Tan[e + f*x]) + (Sec[e +
f*x]^2)^(3/4)*((2*a^4 - 3*a^2*b^2 - 5*b^4)*Hypergeometric2F1[1/2, 3/4, 3/2
, -Tan[e + f*x]^2]*Tan[e + f*x] + 21*a*b^2*(-(Sqrt[b]*(a^2 + b^2)^(1/4)*(A
rcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)] + ArcTanh[(Sqrt[
b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)])) + a*Cot[e + f*x]*EllipticP
i[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e +
f*x]^2] + a*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x
]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]^2])))/(6*(a^2 + b^2)^3*f*(d*Sec[e + f*
x])^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.85 (sec) , antiderivative size = 413, normalized size of antiderivative = 0.79, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3994, 496, 27, 688, 27, 719, 229, 504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sec^2(e+fx)^{3/4}\int \frac{1}{(a+b\tan(e+fx))^2(\tan^2(e+fx)+1)^{7/4}}d(b\tan(e+fx))}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{496}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{2(ab\tan(e+fx)+b^2)}{3(a^2+b^2)(\tan^2(e+fx)+1)^{3/4}(a+b\tan(e+fx))}-\frac{2b^2\int -\frac{\left(\frac{a^2}{b^2}+5\right)b^2+3a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))^2\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{3(a^2+b^2)}\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\int \frac{a^2+3b\tan(e+fx)a+5b^2}{(a+b\tan(e+fx))^2\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{3(a^2+b^2)}+\frac{2(ab\tan(e+fx)+b^2)}{3(a^2+b^2)(\tan^2(e+fx)+1)^{3/4}(a+b\tan(e+fx))}\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{688}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(2a^2-5b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{b^2\int -\frac{2a\left(\frac{a^2}{b^2}+8\right)-\left(5-\frac{2a^2}{b^2}\right)b\tan(e+fx)}{2(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{a^2+b^2}}{3(a^2+b^2)}+\frac{2(ab\tan}{3(a^2+b^2)(\tan^2(e+fx}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\int\frac{2a\left(\frac{a^2}{b^2}+8\right)-\left(5-\frac{2a^2}{b^2}\right)b\tan(e+fx)}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}+\frac{b^2\left(2a^2-5b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}}{3(a^2+b^2)}+\frac{2(ab\tan(e}{3(a^2+b^2)(\tan^2(e+fx)+}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 719$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(21a\int\frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-\left(5-\frac{2a^2}{b^2}\right)\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))\right)}{2\left(a^2+b^2\right)}+\frac{b^2\left(2a^2-5b^2\right)\sqrt[4]{}}{\left(a^2+b^2\right.}}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 229$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(21a\int\frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-2b\left(5-\frac{2a^2}{b^2}\right)\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\right)}{2\left(a^2+b^2\right)}+\frac{b^2\left(2a^2-5b^2\right)\sqrt[4]{}}{\left(a^2+b^2\right)(}}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 504$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(21a\left(a\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))\right)}{2\left(a^2+b^2\right)}}{3(a^2+b^2)}\right.}{bf(d\sec(e+}$$

$$\Big\downarrow \ 312$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(21a\left(\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{-\frac{\tan(e+fx)}{b}}\left(\frac{\tan(e+fx)}{b}+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d\left(b^2\tan^2(e+fx)\right)}{2b}-\int \frac{}{\left(\tan^2(e+fx)\right.}\right.\right.}{2\left(a^2+b^2\right)}}{3(}\right.}{}$$

$$\Big\downarrow \ 118$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(21a\left(-\int \frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int -\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan\right.}}{b}\right.\right.}{2\left(a^2+b^2\right)}}{}\right.}{}$$

$$\Big\downarrow \ 25$$

$$\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(21a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\int\frac{\cdots}{\left(\tan^2(e+fx\cdots\right.}\cdots\right.\right.}{2\left(a^2+b^2\right)\cdots}\cdots\right.$$

$$\Big\downarrow 353$$

$$\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(21a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\frac{1}{2}\int\frac{\cdots}{\left(\frac{\tan(e+f\cdots}{b}\cdots\right.}\cdots\right.\right.}{2\left(a^2+b^2\right)\cdots}\cdots\right.$$

$$\Big\downarrow 73$$

$$\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(21a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\int\frac{\cdots}{-\tan^4(e\cdots}\cdots\right.\right.}{2\left(a^2+b^2\right)\cdots}\cdots\right.$$

$\downarrow$ 756

$$\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(21a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\int\frac{}{\sqrt{a^2+b}}}{}\right.\right.\right.}{}\right.$$

$\downarrow$ 218

$$\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(21a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\int\frac{}{\sqrt{a^2+b}}}{}\right.\right.\right.}{2\left(a^2+\right.}\right.$$

$\downarrow$ 221

$$\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(21a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\arctan\left(\cdots\right.}{2\sqrt{b}\cdots}\right.\right.\right.}{2\left(a^2+b^2\right)\cdots}\right.$$

$$\Big\downarrow \; 925$$

$$\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(21a\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int \frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{2\left(a^2+b^2\right)}-\frac{b^2\int \frac{\tan^2(\cdots}{\left(\cdots\sqrt{\cdots}\right.}\cdots}{\cdots}\right.}{b}\right.\right.}{\cdots}\right.$$

$$\Big\downarrow \; 1537$$

$$\sec^2(e+fx)^{3/4} \left(\frac{b^2 \left(21a \left(- \frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx) \left(- \frac{b^2 \int \frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-\sqrt[4]{\frac{\tan(e+fx)}{b}}+1}\sqrt{\sqrt[4]{\frac{\tan(e+fx)}{b}}+1-}}}{2(a^2+b^2)} \right.}{} \right.\right.}{} \right.$$

$\downarrow$ 412

$$\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(21a\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\frac{\tan(e+fx)}{b}+1}\right),-1\right)}{2(a^2+b^2)}-\frac{b^2\,\text{EllipticPi}\left(\frac{}{\sqrt{a^2}}\right.}{}\right.}{b}\right.\right.}{}\right.$$

input
```
Int[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2),x]
```

output
```
((Sec[e + f*x]^2)^(3/4)*((2*(b^2 + a*b*Tan[e + f*x]))/(3*(a^2 + b^2)*(a +
b*Tan[e + f*x])*(1 + Tan[e + f*x]^2)^(3/4)) + ((b^2*(2*a^2 - 5*b^2)*(1 + T
an[e + f*x]^2)^(1/4))/((a^2 + b^2)*(a + b*Tan[e + f*x])) + (b^2*(-2*(5 - (
2*a^2)/b^2)*b*EllipticF[ArcTan[Tan[e + f*x]]/2, 2] + 21*a*(-2*b^2*(ArcTan[
(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4)) +
ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(
3/4))) - (2*a*Cot[e + f*x]*(-1/2*(b^2*EllipticPi[-(b/Sqrt[a^2 + b^2]), Arc
Sin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(a^2 + b^2) - (b^2*EllipticPi[b/Sqrt
[a^2 + b^2], ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(2*(a^2 + b^2)))*Sqr
t[-Tan[e + f*x]^2])/b)))/(2*(a^2 + b^2)))/(3*(a^2 + b^2))))/(b*f*(d*Sec[e
+ f*x])^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8276 vs. $2(450) = 900$.

Time = 14.77 (sec) , antiderivative size = 8277, normalized size of antiderivative = 15.92

method	result	size
default	Expression too large to display	8277

input
```
int(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec{(e+fx)})^{\frac{3}{2}}\,(a+b\tan{(e+fx)})^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral(1/((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2), x)
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}(a+b\tan(e+f\,x))^2}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^2),x)
```

output
```
int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^2\tan(fx+e)^2b^2+2\sec(fx+e)^2\tan(fx+e)ab+\sec(fx+e)^2a^2}\right.}{d^2}$$

input
```
int(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x)
```

output
```
(sqrt(d)*int(sqrt(sec(e + f*x))/(sec(e + f*x)**2*tan(e + f*x)**2*b**2 + 2*
sec(e + f*x)**2*tan(e + f*x)*a*b + sec(e + f*x)**2*a**2),x))/d**2
```

3.624 $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 691

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \frac{9ab^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{13/4}d^2f\sqrt{d\sec(e+fx)}} - \frac{9ab^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{13/4}d^2f\sqrt{d\sec(e+fx)}} + \frac{9a^2b^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} + \frac{3(2a^2-7b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}} - \frac{9a^2b^2\tan(e+fx)}{\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} - \frac{9a^2b^3\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^{7/2}d^2f\sqrt{d\sec(e+fx)}} + \frac{9a^2b^3\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^{7/2}d^2f\sqrt{d\sec(e+fx)}} + \frac{b(2a^2-7b^2)}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{9a(b^3+ab^2\tan(e+fx))}{\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}}$$

output
```
9/2*a*b^(7/2)*arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(sec(f*
x+e)^2)^(1/4)/(a^2+b^2)^(13/4)/d^2/f/(d*sec(f*x+e))^(1/2)-9/2*a*b^(7/2)*ar
ctanh(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(1/4)/(
a^2+b^2)^(13/4)/d^2/f/(d*sec(f*x+e))^(1/2)+9*a^2*b^2*EllipticE(sin(1/2*arc
tan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^3/d^2/f/(d*sec(f*
x+e))^(1/2)+3/5*(2*a^2-7*b^2)*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2
))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^2/d^2/f/(d*sec(f*x+e))^(1/2)-9*a^2*b^2*t
an(f*x+e)/(a^2+b^2)^3/d^2/f/(d*sec(f*x+e))^(1/2)-9/2*a^2*b^3*cot(f*x+e)*El
lipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(
-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(7/2)/d^2/f/(d*sec(f*x+e))^(1/2)+9/2*a^2*b^
3*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f*x
+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(7/2)/d^2/f/(d*sec(f*x+e))^(1
/2)+1/5*b*(2*a^2-7*b^2)/(a^2+b^2)^2/d^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*
x+e))+2/5*cos(f*x+e)^2*(b+a*tan(f*x+e))/(a^2+b^2)/d^2/f/(d*sec(f*x+e))^(1/
2)/(a+b*tan(f*x+e))+9*a*(b^3+a*b^2*tan(f*x+e))/(a^2+b^2)^3/d^2/f/(d*sec(f*
x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 28.18 (sec) , antiderivative size = 9161, normalized size of antiderivative = 13.26

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 538, normalized size of antiderivative = 0.78, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.840, Rules used = {3042, 3994, 496, 27, 686, 27, 688, 27, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\int \frac{1}{(a+b\tan(e+fx))^2(\tan^2(e+fx)+1)^{9/4}}d(b\tan(e+fx))}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{496}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{5(a^2+b^2)(\tan^2(e+fx)+1)^{5/4}(a+b\tan(e+fx))}-\frac{2b^2\int -\frac{\left(\frac{3a^2}{b^2}+7\right)b^2+5a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))^2\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))}{5(a^2+b^2)}\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\int \frac{3a^2+5b\tan(e+fx)a+7b^2}{(a+b\tan(e+fx))^2\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))}{5(a^2+b^2)}+\frac{2\left(ab\tan(e+fx)+b^2\right)}{5(a^2+b^2)(\tan^2(e+fx)+1)^{5/4}(a+b\tan(e+fx))}\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{686}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{2b^4\int\frac{3\left(a^4+6b^2a^2-b\left(a^2+4b^2\right)\tan(e+fx)a-7b^4\right)}{2b^4(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}}{5(a^2+b^2)}+\frac{}{5(}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\int\frac{a^4+6b^2a^2-b\left(a^2+4b^2\right)\tan(e+fx)a-7b^4}{(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}}{5(a^2+b^2)}+\frac{}{5(a^2+b}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$\Big\downarrow$ 688

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(-\frac{b^2\int-\frac{2a\left(\frac{a^4}{b^2}+5a^2-11b^2\right)b^2+\left(2a^4+10b^2a^2-7b^4\right)\tan(e+fx)b}{2b^2(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e-}{a^2+b^2}\right.}{a^2+b^2}}{5(a^2+b^2)}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\int\frac{2a\left(a^4+5b^2a^2-11b^4\right)+b\left(2a^4+10b^2a^2-7b^4\right)\tan(e+fx)}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}-\right.}{a^2+b^2}}{5(a^2+b^2)}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$\downarrow$ 719

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-15ab^4}{2\left(a^2+b^2\right)}\right.}{}}{5(a^2+b^2)}\right.}{bd^2f\sqrt{}}$$

$\downarrow$ 225

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{5/4}}\right.}{}\right.}{}}{5(a^2}\right.}{}$$

$\Big\downarrow$ 212

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+\right.\right.}{2(}\right.}{}}{5(a^2+b^2}\right.}{b}$$

$\Big\downarrow$ 504

$$\frac{\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+\right.\right.}{}\right.}{}}{\sqrt[4]{\sec^2(e+fx)}}$$

$\downarrow$ 310

$$\frac{\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+\right.\right.}{}\right.}{}}{\sqrt[4]{\sec^2(e+fx)}}$$

$\downarrow$ 353

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+\right.\right.}{}\right.}{}\right.$$

$$\Big\downarrow \textbf{73}$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+\right.\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{827}$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+\right.\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 221

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$\big\downarrow$ 993

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+\right.\right.}{}\right.}{}\right.}{}$$

$$\downarrow 1537$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+\right.\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{412}$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{5(a^2+b^2)(\tan^2(e+fx)+1)^{5/4}(a+b\tan(e+fx))}+\frac{\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right.}{}\right.}{}}{}\right.$$

input
```
Int[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2),x]
```

output
```
((Sec[e + f*x]^2)^(1/4)*((2*(b^2 + a*b*Tan[e + f*x]))/(5*(a^2 + b^2)*(a +
b*Tan[e + f*x])*(1 + Tan[e + f*x]^2)^(5/4)) + ((2*((7 - (2*a^2)/b^2)*b^4 +
 3*a*b*(a^2 + 4*b^2)*Tan[e + f*x]))/((a^2 + b^2)*(a + b*Tan[e + f*x])*(1 +
 Tan[e + f*x]^2)^(1/4)) - (3*(-((b^2*(2*a^4 + 10*a^2*b^2 - 7*b^4)*(1 + Tan
[e + f*x]^2)^(3/4))/((a^2 + b^2)*(a + b*Tan[e + f*x]))) + (-15*a*b^4*(-2*b
^2*(-1/2*ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 +
b^2)^(1/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2)
*(a^2 + b^2)^(1/4))) + (2*a*Cot[e + f*x]*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2
+ b^2]), ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1])/Sqrt[a^2 + b^2] + (b*Ell
ipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1])/(2*Sqr
t[a^2 + b^2]))*Sqrt[-Tan[e + f*x]^2])/b) + (2*a^4 + 10*a^2*b^2 - 7*b^4)*(-
2*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*b*Tan[e + f*x])/(1 + Tan[e +
 f*x]^2)^(1/4)))/(2*(a^2 + b^2))))/(a^2 + b^2))/(5*(a^2 + b^2))))/(b*d^2*f
*Sqrt[d*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 17615 vs. 2(611) = 1222.

Time = 18.59 (sec) , antiderivative size = 17616, normalized size of antiderivative = 25.49

method	result	size
default	Expression too large to display	17616

input
```
int(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(e+fx))^{\frac{5}{2}}\,(a+b\tan(e+fx))^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral(1/((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}\,(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^2), x)
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}\,(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}\,(a+b\tan\left(e+f\,x\right))^2}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^2),x)
```

output
```
int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^3\tan(fx+e)^2b^2+2\sec(fx+e)^3\tan(fx+e)ab+\sec(fx+e)^3a^2}\right.}{d^3}$$

input
```
int(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x)
```

output
```
(sqrt(d)*int(sqrt(sec(e + f*x))/(sec(e + f*x)**3*tan(e + f*x)**2*b**2 + 2*
sec(e + f*x)**3*tan(e + f*x)*a*b + sec(e + f*x)**3*a**2),x))/d**3
```

3.625 $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 583

$$
\begin{aligned}
\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx &= \frac{3(a^2+2b^2)\,d^2\arctan\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8b^{5/2}\left(a^2+b^2\right)^{5/4}f\sec^2(e+fx)^{3/4}} \\
&- \frac{3(a^2+2b^2)\,d^2\text{arctanh}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8b^{5/2}\left(a^2+b^2\right)^{5/4}f\sec^2(e+fx)^{3/4}} \\
&+ \frac{3ad^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4b^2\left(a^2+b^2\right)f\sec^2(e+fx)^{3/4}} \\
&- \frac{3ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4b^2\left(a^2+b^2\right)f} \\
&- \frac{3a(a^2+2b^2)\,d^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\text{ta}}}{8b^3\left(a^2+b^2\right)^{3/2}f\sec^2(e+fx)^{3/4}} \\
&+ \frac{3a(a^2+2b^2)\,d^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2}}{8b^3\left(a^2+b^2\right)^{3/2}f\sec^2(e+fx)^{3/4}} \\
&- \frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2} + \frac{3ad^2(d\sec(e+fx))^{3/2}}{4b\left(a^2+b^2\right)f(a+b\tan(e+fx))}
\end{aligned}
$$

output
```
3/8*(a^2+2*b^2)*d^2*arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(
d*sec(f*x+e))^(3/2)/b^(5/2)/(a^2+b^2)^(5/4)/f/(sec(f*x+e)^2)^(3/4)-3/8*(a^
2+2*b^2)*d^2*arctanh(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(d*sec(
f*x+e))^(3/2)/b^(5/2)/(a^2+b^2)^(5/4)/f/(sec(f*x+e)^2)^(3/4)+3/4*a*d^2*Ell
ipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(3/2)/b^2/(a^2+
b^2)/f/(sec(f*x+e)^2)^(3/4)-3/4*a*d^2*cos(f*x+e)*(d*sec(f*x+e))^(3/2)*sin(
f*x+e)/b^2/(a^2+b^2)/f-3/8*a*(a^2+2*b^2)*d^2*cot(f*x+e)*EllipticPi((sec(f*
x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(
1/2)/b^3/(a^2+b^2)^(3/2)/f/(sec(f*x+e)^2)^(3/4)+3/8*a*(a^2+2*b^2)*d^2*cot(
f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))
^(3/2)*(-tan(f*x+e)^2)^(1/2)/b^3/(a^2+b^2)^(3/2)/f/(sec(f*x+e)^2)^(3/4)-1/
2*d^2*(d*sec(f*x+e))^(3/2)/b/f/(a+b*tan(f*x+e))^2+3/4*a*d^2*(d*sec(f*x+e))
^(3/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 29.85 (sec) , antiderivative size = 7817, normalized size of antiderivative = 13.41

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^3,x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.68, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3994, 492, 594, 27, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{ 3994}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\int \frac{\left(\tan^2(e+fx)+1\right)^{3/4}}{(a+b\tan(e+fx))^3}d(b\tan(e+fx))}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{ 492}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\int \frac{b\tan(e+fx)}{(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{4b^2}-\frac{\left(\tan^2(e+fx)+1\right)^{3/4}}{2(a+b\tan(e+fx))^2}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{ 594}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{ab^2\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{b^2\int -\frac{2b^2-ab\tan(e+fx)}{2b^2(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}\right)}{4b^2}-\frac{\left(\tan^2(e+fx)+1\right)^{3/4}}{2(a+b\tan(e+fx))^2}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{\int \frac{2b^2-ab\tan(e+fx)}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}+\frac{ab^2\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}\right)}{4b^2}-\frac{\left(\tan^2(e+fx)+1\right)^{3/4}}{2(a+b\tan(e+fx))^2}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{719}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{\left(a^2+2b^2\right)\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-a\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}+\right.}{4b^2}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{225}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{\left(a^2+2b^2\right)\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int\frac{1}{\left(\tan^2(e+fx)\right.}\right.}{2\left(a^2+b^2\right)}\right.}{4b^2}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{212}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{\left(a^2+2b^2\right)\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\right.\right.}{2\left(a^2+b^2\right)}\right.}{4b^2}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$\downarrow$ 504

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{\left(a^2+2b^2\right)\left(a\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int\frac{b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}\left(a^2-b^2\tan\right.}\right.}{2\left(a^2+b^2\right)}}{4b}\right.}{bf\sec^2(e+}$$

$\downarrow$ 310

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{\left(a^2+2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\int\right.}{}\right.}{}\right.$$

$\downarrow$ 353

$$\frac{3\left(\frac{\left(a^2+2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\frac{1}{2}\cdots\right.}{\cdots}\right.}{d^2(d\sec(e+fx))^{3/2}\left(\cdots\right.}$$

$\downarrow$ 73

$$\frac{3\left(\frac{\left(a^2+2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2\cdots\right.}{2(\cdots}\right.}{d^2(d\sec(e+fx))^{3/2}\left(\cdots\right.}$$

$\downarrow$ 827

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{\left(a^2+2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2\right.}{}\right.}{}\right.$$

$\downarrow$ 218

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{\left(a^2+2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2\right.}{}\right.}{}\right.$$

$\downarrow$ 221

$$\frac{\left(\frac{3\left(\frac{\left(a^2+2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2}{}}{}\right.}{}}{d^2(d\sec(e+fx))^{3/2}}$$

$\downarrow$ 993

$$\frac{\left(\frac{3\left(\frac{\left(a^2+2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int \frac{1}{\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\tan^2(e+fx)+}\right.}{b}}{}}{}\right.}{}}{d^2(d\sec(e+fx))^{3/2}}$$

$\downarrow$ 1537

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{\left(a^2+2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int\frac{1}{\sqrt{1-b^2\tan^2(e+fx)}\sqrt{b^2\tan^2(e+fx)+1}\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)}d\sqrt[4]{\tan\right.}{}\right.}{}\right.}{}\right.$$

$\downarrow$ 412

$$\frac{d^2 (d \sec(e+fx))^{3/2} \left(\frac{3 \left(\frac{(a^2+2b^2) \left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx) \left(\frac{b \operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}}, \arcsin\left(\sqrt[4]{\tan^2(e+fx)+1}\right), -1\right)}{2\sqrt{a^2+b^2}} - \frac{b \operatorname{EllipticPi}}{\cdots} \cdots \right.}{b} \cdots \right.}{\cdots} \right.}{\cdots} \right.}{\cdots}$$

input

```
Int[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^3,x]
```

output
```
(d^2*(d*Sec[e + f*x])^(3/2)*(-1/2*(1 + Tan[e + f*x]^2)^(3/4)/(a + b*Tan[e
+ f*x])^2 + (3*((a*b^2*(1 + Tan[e + f*x]^2)^(3/4))/((a^2 + b^2)*(a + b*Tan
[e + f*x])) + ((a^2 + 2*b^2)*(-2*b^2*(-1/2*ArcTan[(b^(3/2)*Tan[e + f*x])/(
a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 + b^2)^(1/4)) + ArcTanh[(b^(3/2)*Tan[e + f
*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2)*(a^2 + b^2)^(1/4))) + (2*a*Cot[e + f*x]
*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]^2)^(1/
4)], -1])/Sqrt[a^2 + b^2] + (b*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + T
an[e + f*x]^2)^(1/4)], -1])/(2*Sqrt[a^2 + b^2]))*Sqrt[-Tan[e + f*x]^2])/b)
 - a*(-2*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*b*Tan[e + f*x])/(1 +
Tan[e + f*x]^2)^(1/4)))/(2*(a^2 + b^2))))/(4*b^2)))/(b*f*(Sec[e + f*x]^2)^
(3/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31691 vs. $2(506) = 1012$.

Time = 1424.86 (sec) , antiderivative size = 31692, normalized size of antiderivative = 54.36

method	result	size
default	Expression too large to display	31692

input
```
int((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(7/2)/(a+b*tan(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{(b\tan(fx+e)+a)^3}\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}{(a+b\tan(e+f\,x))^3}\,dx$$

input
```
int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x))^3,x)
```

output
```
int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x))^3, x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx = \sqrt{d}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sec(fx+e)^3}{\tan(fx+e)^3\,b^3+3\tan(fx+e)^2\,a\,b^2+3\tan(fx+e)\,a^2b+a^3}dx\right)$$

input
```
int((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x)
```

output
```
sqrt(d)*int((sqrt(sec(e + f*x))*sec(e + f*x)**3)/(tan(e + f*x)**3*b**3 + 3
*tan(e + f*x)**2*a*b**2 + 3*tan(e + f*x)*a**2*b + a**3),x)*d**3
```

3.626 $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 532

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \frac{(a^2-2b^2)\,d^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8b^{3/2}\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}}$$

$$+\frac{(a^2-2b^2)\,d^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8b^{3/2}\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}}$$

$$+\frac{ad^2\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{4b^2\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}}$$

$$-\frac{a(a^2-2b^2)\,d^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(}}{8b^2\left(a^2+b^2\right)^2f\sqrt[4]{\sec^2(e+fx)}}$$

$$-\frac{a(a^2-2b^2)\,d^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e}}{8b^2\left(a^2+b^2\right)^2f\sqrt[4]{\sec^2(e+fx)}}$$

$$-\frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2}+\frac{ad^2\sqrt{d\sec(e+fx)}}{4b\left(a^2+b^2\right)f(a+b\tan(e+fx))}$$

output
```
1/8*(a^2-2*b^2)*d^2*arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(
d*sec(f*x+e))^(1/2)/b^(3/2)/(a^2+b^2)^(7/4)/f/(sec(f*x+e)^2)^(1/4)+1/8*(a^
2-2*b^2)*d^2*arctanh(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(d*sec(
f*x+e))^(1/2)/b^(3/2)/(a^2+b^2)^(7/4)/f/(sec(f*x+e)^2)^(1/4)+1/4*a*d^2*Inv
erseJacobiAM(1/2*arctan(tan(f*x+e)),2^(1/2))*(d*sec(f*x+e))^(1/2)/b^2/(a^2
+b^2)/f/(sec(f*x+e)^2)^(1/4)-1/8*a*(a^2-2*b^2)*d^2*cot(f*x+e)*EllipticPi((
sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e
)^2)^(1/2)/b^2/(a^2+b^2)^2/f/(sec(f*x+e)^2)^(1/4)-1/8*a*(a^2-2*b^2)*d^2*co
t(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e
))^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^2/(a^2+b^2)^2/f/(sec(f*x+e)^2)^(1/4)-1/2*
d^2*(d*sec(f*x+e))^(1/2)/b/f/(a+b*tan(f*x+e))^2+1/4*a*d^2*(d*sec(f*x+e))^(
1/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.91 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.66

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \frac{d(d\sec(e+fx))^{3/2}(a\cos(e+fx)+b\sin(e+fx))^3\left(-\frac{2b(a^2+b^2)\sec^2(e+fx)(a^2+2}{(a+b\tan(e+f}\right.}{}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^3,x]
```

output
```
(d*(d*Sec[e + f*x])^(3/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*((-2*b*(a^2
+ b^2)*Sec[e + f*x]^2*(a^2 + 2*b^2 - a*b*Tan[e + f*x]))/(a + b*Tan[e + f*x
])^2 + (Sec[e + f*x]^2)^(3/4)*(a*(a^2 + b^2)*Hypergeometric2F1[1/2, 3/4, 3
/2, -Tan[e + f*x]^2]*Tan[e + f*x] + ((a^2 - 2*b^2)*(a*EllipticPi[-(b/Sqrt[
a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Tan[e + f*x] + a*Elliptic
Pi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Tan[e + f*x] + S
qrt[b]*(a^2 + b^2)^(1/4)*(ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b
^2)^(1/4)] + ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)])*
Sqrt[-Tan[e + f*x]^2]))/Sqrt[-Tan[e + f*x]^2])))/(8*b^2*(a^2 + b^2)^2*f*(a
 + b*Tan[e + f*x])^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.70, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.760, Rules used = {3042, 3994, 492, 594, 27, 719, 229, 504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\int \frac{\sqrt[4]{\tan^2(e+fx)+1}}{(a+b\tan(e+fx))^3}d(b\tan(e+fx))}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{492}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\int \frac{b\tan(e+fx)}{(a+b\tan(e+fx))^2\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{4b^2}-\frac{\sqrt[4]{\tan^2(e+fx)+1}}{2(a+b\tan(e+fx))^2}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{594}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{ab^2\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{b^2\int -\frac{2b^2+a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{a^2+b^2}}{4b^2}-\frac{\sqrt[4]{\tan^2(e+fx)+1}}{2(a+b\tan(e+fx))^2}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{\int \frac{2b^2+a\tan(e+fx)b}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}+\frac{ab^2\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}}{4b^2}-\frac{\sqrt[4]{\tan^2(e+fx)+1}}{2(a+b\tan(e+fx))^2}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 719$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{a\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-\left(a^2-2b^2\right)\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}+\frac{ab^2\sqrt[4]{\tan^2(e+\cdots}}{\left(a^2+b^2\right)(a+b\tan(\cdots}}{4b^2}\cdots\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 229$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}+\frac{ab^2\sqrt[4]{\tan^2(e+f\cdots}}{\left(a^2+b^2\right)(a+b\tan(e\cdots}}{4b^2}\cdots\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 504$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(a\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int \frac{\cdots}{\left(\tan^2(e+fx)+\cdots\right.}\cdots\right.}{2\left(a^2+b^2\right)}}{4b^2}\cdots\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 312$$

$$d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{-\frac{\tan(e+fx)}{b}}\left(\frac{\tan(e+fx)}{b}+1\right)^{3/4}\left(a^2-b^2\tan^2\right.}}{2b}\right.}{2\left(a^2+b^2\right)}}{4b^2}\right.$$

$$bf\sqrt[4]{\sec^2}$$

$\downarrow$ 118

$$d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(-\int\frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\frac{2a\sqrt{-\tan^2(e+f}}{}\right.}{2\left(a^2+b^2\right)}}{4b}\right.$$

$$bf\sqrt[4]{\mathrm{se}}$$

$\downarrow$ 25

$$d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}}{b}\right.}{2\left(a^2+b^2\right)}}{4b^2}\right.$$

$$bf\sqrt[4]{\sec}$$

$\downarrow$ 353

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d}{b}\right.}{2\left(a^2+b^2\right)}}{4}\right.}{bf\sqrt[4]{s}}$$

$\downarrow$ 73

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d}{b}\right.}{2\left(a^2+b^2\right)}}{4b^2}\right.}{bf\sqrt[4]{\sec}}$$

$\downarrow$ 756

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d}{b}\right.}{}\right.}{}$$

$\downarrow$ 218

$$d^2\sqrt{d\sec(e+fx)}\left(\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d}{b}\right.}{2\left(a^2+b^2\right.}\right.$$

$\downarrow$ 221

$$d^2\sqrt{d\sec(e+fx)}\left(\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d}{b}\right.}{2\left(a^2+b^2\right)}\right.$$

$bf\sqrt[4]{}$

$\downarrow$ 925

$$d^2\sqrt{d\sec(e+fx)}\left(\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-b^4\tan^4(e+fx}}}{2\left(a^2+b^2\right)}\right.}{}\right.}{}\right.$$

$\downarrow$ 1537

$$d^2\sqrt{d\sec(e+fx)}\left(\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-\sqrt[4]{\frac{\tan(e}{b}}}}}{}\right)}{}\right)}{}\right)$$

$\downarrow$ 412

$$\frac{d^2\sqrt{d\sec(e+fx)}}{\left(\frac{2ab\,\mathrm{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\,\mathrm{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\frac{\tan(}{}}\right.\right.}{2\left(a^2+b^2\right)}\right.}{}\right.}{}\right.}$$

input
```
Int[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^3,x]
```

output
```
(d^2*Sqrt[d*Sec[e + f*x]]*(-1/2*(1 + Tan[e + f*x]^2)^(1/4)/(a + b*Tan[e +
f*x])^2 + ((a*b^2*(1 + Tan[e + f*x]^2)^(1/4))/((a^2 + b^2)*(a + b*Tan[e +
f*x])) + (2*a*b*EllipticF[ArcTan[Tan[e + f*x]]/2, 2] - (a^2 - 2*b^2)*(-2*b
^2*(ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2
)^(3/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a
^2 + b^2)^(3/4))) - (2*a*Cot[e + f*x]*(-1/2*(b^2*EllipticPi[-(b/Sqrt[a^2 +
 b^2]), ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(a^2 + b^2) - (b^2*Ellipt
icPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(2*(a^2 +
 b^2)))*Sqrt[-Tan[e + f*x]^2])/b))/(2*(a^2 + b^2)))/(4*b^2)))/(b*f*(Sec[e
+ f*x]^2)^(1/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 17717 vs. $2(462) = 924$.

Time = 1442.46 (sec) , antiderivative size = 17718, normalized size of antiderivative = 33.30

method	result	size
default	Expression too large to display	17718

input
```
int((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{(d\sec{(e+fx)})^{\frac{5}{2}}}{(a+b\tan{(e+fx)})^3}\,dx$$

input
```
integrate((d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**3,x)
```

output
```
Integral((d*sec(e + f*x))**(5/2)/(a + b*tan(e + f*x))**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e)+a)^3}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{(a+b\tan(e+f\,x))^3}\,dx$$

input
```
int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x))^3,x)
```

output
```
int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x))^3, x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \sqrt{d}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sec(fx+e)^2}{\tan(fx+e)^3\,b^3+3\tan(fx+e)^2\,a\,b^2+3\tan(fx+e)\,a^2b+a^3}dx\right)$$

input
```
int((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x)
```

output
```
sqrt(d)*int((sqrt(sec(e + f*x))*sec(e + f*x)**2)/(tan(e + f*x)**3*b**3 + 3
*tan(e + f*x)**2*a*b**2 + 3*tan(e + f*x)*a**2*b + a**3),x)*d**2
```

3.627 $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 566

$$
\begin{aligned}
\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx &= \frac{(3a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8\sqrt{b}\,(a^2+b^2)^{9/4}\,f\sec^2(e+fx)^{3/4}} \\
&- \frac{(3a^2-2b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8\sqrt{b}\,(a^2+b^2)^{9/4}\,f\sec^2(e+fx)^{3/4}} \\
&- \frac{5aE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|\,2\right)(d\sec(e+fx))^{3/2}}{4\,(a^2+b^2)^2\,f\sec^2(e+fx)^{3/4}} \\
&+ \frac{5a\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4\,(a^2+b^2)^2\,f} \\
&- \frac{a(3a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2\cdots}}{8b\,(a^2+b^2)^{5/2}\,f\sec^2(e+fx)^{3/4}} \\
&+ \frac{a(3a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e\cdots}}{8b\,(a^2+b^2)^{5/2}\,f\sec^2(e+fx)^{3/4}} \\
&- \frac{b(d\sec(e+fx))^{3/2}}{2\,(a^2+b^2)\,f(a+b\tan(e+fx))^2} - \frac{5ab(d\sec(e+fx))^{3/2}}{4\,(a^2+b^2)^2\,f(a+b\tan(e+fx))}
\end{aligned}
$$

output
```
1/8*(3*a^2-2*b^2)*arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(d*
sec(f*x+e))^(3/2)/b^(1/2)/(a^2+b^2)^(9/4)/f/(sec(f*x+e)^2)^(3/4)-1/8*(3*a^
2-2*b^2)*arctanh(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(d*sec(f*x+
e))^(3/2)/b^(1/2)/(a^2+b^2)^(9/4)/f/(sec(f*x+e)^2)^(3/4)-5/4*a*EllipticE(s
in(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(3/2)/(a^2+b^2)^2/f/(se
c(f*x+e)^2)^(3/4)+5/4*a*cos(f*x+e)*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/(a^2+b^
2)^2/f-1/8*a*(3*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(
a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b/(a^2+b^2)^(
5/2)/f/(sec(f*x+e)^2)^(3/4)+1/8*a*(3*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec
(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)
^(1/2)/b/(a^2+b^2)^(5/2)/f/(sec(f*x+e)^2)^(3/4)-1/2*b*(d*sec(f*x+e))^(3/2)
/(a^2+b^2)/f/(a+b*tan(f*x+e))^2-5/4*a*b*(d*sec(f*x+e))^(3/2)/(a^2+b^2)^2/f
/(a+b*tan(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 30.15 (sec) , antiderivative size = 7905, normalized size of antiderivative = 13.97

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^3,x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 413, normalized size of antiderivative = 0.73, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3994, 498, 27, 688, 27, 25, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}dx$$

$\Big\downarrow$ 3042

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}dx$$

$\Big\downarrow$ 3994

$$\frac{(d\sec(e+fx))^{3/2}\int \frac{1}{(a+b\tan(e+fx))^3\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{bf\sec^2(e+fx)^{3/4}}$$

$\Big\downarrow$ 498

$$\frac{(d\sec(e+fx))^{3/2}\left(-\frac{\int -\frac{4a-b\tan(e+fx)}{2(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2(a^2+b^2)}-\frac{b^2(\tan^2(e+fx)+1)^{3/4}}{2(a^2+b^2)(a+b\tan(e+fx))^2}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$\Big\downarrow$ 27

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{\int \frac{4a-b\tan(e+fx)}{(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{4(a^2+b^2)}-\frac{b^2(\tan^2(e+fx)+1)^{3/4}}{2(a^2+b^2)(a+b\tan(e+fx))^2}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$\Big\downarrow$ 688

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{-\frac{b^2\int \frac{\left(2-\frac{8a^2}{b^2}\right)b^2-5ab\tan(e+fx)}{2b^2(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}-\frac{5ab^2\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}}{4(a^2+b^2)}-\frac{b^2(\tan^2(e+fx)+1)^{3/}}{2(a^2+b^2)(a+b\tan(e+f}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$\Big\downarrow$ 27

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{-\frac{\int -\frac{2\left(4a^2-b^2\right)+5ab\tan(e+fx)}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}-\frac{5ab^2\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}}{4(a^2+b^2)}-\frac{b^2(\tan^2(e+fx)+1)^{3/4}}{2(a^2+b^2)(a+b\tan(e+fx))^2}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{\frac{\int \frac{2\left(4a^2-b^2\right)+5ab\tan(e+fx)}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}-\frac{5ab^2\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}}{4(a^2+b^2)}-\frac{b^2(\tan^2(e+fx)+1)^{3/4}}{2(a^2+b^2)(a+b\tan(e+fx))^2}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \textcolor{blue}{719}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{\frac{\left(3a^2-2b^2\right)\int \frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+5a\int \frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}-\frac{5ab}{(a^2}\right.}{4(a^2+b^2)}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \textcolor{blue}{225}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{\frac{\left(3a^2-2b^2\right)\int \frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+5a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int \frac{1}{\left(\tan^2(e+fx)+1\right)}\right.}{2\left(a^2+b^2\right)}}{4(a^2+b^2)}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \textcolor{blue}{212}$$

$$(d\sec(e+fx))^{3/2}\left(\frac{\frac{\left(3a^2-2b^2\right)\int \frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+5a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(\right.\right.}{2\left(a^2+b^2\right)}}{4(a^2+b^2)}\right.$$

$$\overline{\qquad bf\sec^2(e+fx)^{3/4}}$$

$\downarrow$ 504

$$(d\sec(e+fx))^{3/2}\left(\frac{\frac{\left(3a^2-2b^2\right)\left(a\int \frac{1}{\sqrt[4]{\tan^2(e+fx)+1}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}\left(a^2-b^2\tan^2(e+\right.}\right.}{2\left(a^2+b^2\right)}}{4(a^2+b^2)}\right.$$

$$\overline{\qquad bf\sec^2(e-}$$

$\downarrow$ 310

$$(d\sec(e+fx))^{3/2}\left(\frac{\frac{\left(3a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\int \frac{}{\sqrt[4]{}}\right.}{2\left(a^2\right.}}{}\right.$$

$\downarrow$ 353

$$(d\sec(e+fx))^{3/2}\left(\frac{\left(3a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\frac{1}{2}\int\right.}{2\left(\right.}\right.$$

$$\downarrow 73$$

$$(d\sec(e+fx))^{3/2}\left(\frac{\left(3a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\int\right.}{2\left(a^2+\right.}\right.$$

$$\downarrow 827$$

$$(d\sec(e+fx))^{3/2}\left(\left(3a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\int\right.\right.\right.$$

$$\downarrow 218$$

$$(d\sec(e+fx))^{3/2}\left(\frac{\left(3a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\int\right.\right.}{}\right.$$

$$\Big\downarrow \; 221$$

$$(d\sec(e+fx))^{3/2}\left(\frac{\left(3a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(a\right.\right.}{2\left(a^2\right.}\right.$$

$$\Big\downarrow \; 993$$

$$(d\sec(e+fx))^{3/2}\left(\frac{\left(3a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int\frac{1}{\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\tan^2(e+fx)+1}-\frac{1}{2}\right.}{b}\right.}{}\right.$$

$$\Big\downarrow \; 1537$$

$$(d\sec(e+fx))^{3/2}\left(\frac{\left(3a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int\frac{1}{\sqrt{1-b^2\tan^2(e+fx)}\sqrt{b^2\tan^2(e+fx)+1}\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)}d\sqrt[4]{\tan^2(e\cdots}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 412$$

$$(d\sec(e+fx))^{3/2}\left(\frac{\left(3a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{b\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\tan^2(e+fx)+1}\right),-1\right)}{2\sqrt{a^2+b^2}}-\frac{b\,\text{EllipticPi}\left(-\frac{\cdots}{\sqrt{\cdots}}\right.}{\cdots}\right.}{b}\cdots\right.}{\cdots}}{\cdots}\right.$$

input
```
Int[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^3,x]
```

output
```
((d*Sec[e + f*x])^(3/2)*(-1/2*(b^2*(1 + Tan[e + f*x]^2)^(3/4))/((a^2 + b^2
)*(a + b*Tan[e + f*x])^2) + ((-5*a*b^2*(1 + Tan[e + f*x]^2)^(3/4))/((a^2 +
 b^2)*(a + b*Tan[e + f*x])) + ((3*a^2 - 2*b^2)*(-2*b^2*(-1/2*ArcTan[(b^(3/
2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 + b^2)^(1/4)) + ArcTanh[
(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2)*(a^2 + b^2)^(1/4))) +
 (2*a*Cot[e + f*x]*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + T
an[e + f*x]^2)^(1/4)], -1])/Sqrt[a^2 + b^2] + (b*EllipticPi[b/Sqrt[a^2 + b
^2], ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1])/(2*Sqrt[a^2 + b^2]))*Sqrt[-T
an[e + f*x]^2])/b) + 5*a*(-2*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*b
*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^(1/4)))/(2*(a^2 + b^2)))/(4*(a^2 + b^2
))))/(b*f*(Sec[e + f*x]^2)^(3/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 25456 vs. $2(489) = 978$.

Time = 12.89 (sec) , antiderivative size = 25457, normalized size of antiderivative = 44.98

method	result	size
default	Expression too large to display	25457

input
```
int((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx=\int \frac{(d\sec{(e+fx)})^{\frac{3}{2}}}{(a+b\tan{(e+fx)})^3}\,dx$$

input
```
integrate((d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**3,x)
```

output
```
Integral((d*sec(e + f*x))**(3/2)/(a + b*tan(e + f*x))**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx=\text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx=\int \frac{(d\sec{(fx+e)})^{\frac{3}{2}}}{(b\tan{(fx+e)}+a)^3}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{(a+b\tan\left(e+f\,x\right))^3}\,dx$$

input
```
int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x))^3,x)
```

output
```
int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x))^3, x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = \sqrt{d}\left(\int \frac{\sqrt{\sec\left(fx+e\right)}\,\sec\left(fx+e\right)}{\tan\left(fx+e\right)^3 b^3+3\tan\left(fx+e\right)^2 a\,b^2+3\tan\left(fx+e\right)a^2b+a^3}dx\right)$$

input
```
int((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x)
```

output
```
sqrt(d)*int((sqrt(sec(e + f*x))*sec(e + f*x))/(tan(e + f*x)**3*b**3 + 3*ta
n(e + f*x)**2*a*b**2 + 3*tan(e + f*x)*a**2*b + a**3),x)*d
```

3.628 $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 515

$$\begin{aligned}
&\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx \\
&= -\frac{3\sqrt{b}(5a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8(a^2+b^2)^{11/4}f\sqrt[4]{\sec^2(e+fx)}} \\
&\quad -\frac{3\sqrt{b}(5a^2-2b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8(a^2+b^2)^{11/4}f\sqrt[4]{\sec^2(e+fx)}} \\
&\quad -\frac{7a\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{4(a^2+b^2)^2f\sqrt[4]{\sec^2(e+fx)}} \\
&\quad +\frac{3a(5a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2}}{8(a^2+b^2)^3f\sqrt[4]{\sec^2(e+fx)}} \\
&\quad +\frac{3a(5a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(}}{8(a^2+b^2)^3f\sqrt[4]{\sec^2(e+fx)}} \\
&\quad -\frac{b\sqrt{d\sec(e+fx)}}{2(a^2+b^2)f(a+b\tan(e+fx))^2}-\frac{7ab\sqrt{d\sec(e+fx)}}{4(a^2+b^2)^2f(a+b\tan(e+fx))}
\end{aligned}$$

output
```
-3/8*b^(1/2)*(5*a^2-2*b^2)*arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(
1/4))*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^(11/4)/f/(sec(f*x+e)^2)^(1/4)-3/8*b^(
1/2)*(5*a^2-2*b^2)*arctanh(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(
d*sec(f*x+e))^(1/2)/(a^2+b^2)^(11/4)/f/(sec(f*x+e)^2)^(1/4)-7/4*a*InverseJ
acobiAM(1/2*arctan(tan(f*x+e)),2^(1/2))*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^2/f
/(sec(f*x+e)^2)^(1/4)+3/8*a*(5*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e
)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2
)/(a^2+b^2)^3/f/(sec(f*x+e)^2)^(1/4)+3/8*a*(5*a^2-2*b^2)*cot(f*x+e)*Ellipt
icPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(
f*x+e)^2)^(1/2)/(a^2+b^2)^3/f/(sec(f*x+e)^2)^(1/4)-1/2*b*(d*sec(f*x+e))^(1
/2)/(a^2+b^2)/f/(a+b*tan(f*x+e))^2-7/4*a*b*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^
2/f/(a+b*tan(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.32 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$$

$$= \frac{(d\sec(e+fx))^{3/2}(a\cos(e+fx)+b\sin(e+fx))^3\left(-\frac{2b\sec^2(e+fx)(9a^2+2b^2+7ab\tan(e+fx))}{(a-ib)^2(a+ib)^2(a+b\tan(e+fx))^2}+\frac{\sec^2(e+fx)^{3/4}\left(-7a\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^3,x]
```

output
```
((d*Sec[e + f*x])^(3/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*((-2*b*Sec[e +
 f*x]^2*(9*a^2 + 2*b^2 + 7*a*b*Tan[e + f*x]))/((a - I*b)^2*(a + I*b)^2*(a
+ b*Tan[e + f*x])^2) + ((Sec[e + f*x]^2)^(3/4)*(-7*a*(a^2 + b^2)*Hypergeom
etric2F1[1/2, 3/4, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] + 3*(5*a^2 - 2*b^2)*
(-(Sqrt[b]*(a^2 + b^2)^(1/4)*(ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2
 + b^2)^(1/4)] + ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4
)])) + a*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x
]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]^2] + a*Cot[e + f*x]*EllipticPi[b/Sqrt[
a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]^2]))/(
a^2 + b^2)^3))/(8*d*f*(a + b*Tan[e + f*x])^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.76, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3994, 498, 27, 688, 27, 25, 719, 229, 504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt{d\sec(e+fx)}\int \frac{1}{(a+b\tan(e+fx))^3(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{498}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(-\frac{\int -\frac{4a-3b\tan(e+fx)}{2(a+b\tan(e+fx))^2\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2(a^2+b^2)}-\frac{b^2\sqrt[4]{\tan^2(e+fx)+1}}{2(a^2+b^2)(a+b\tan(e+fx))^2}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\int\frac{4a-3b\tan(e+fx)}{(a+b\tan(e+fx))^2\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{4(a^2+b^2)}-\frac{b^2\sqrt[4]{\tan^2(e+fx)+1}}{2(a^2+b^2)(a+b\tan(e+fx))^2}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\Big\downarrow$ 688

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{-\frac{b^2\int\frac{\left(6-\frac{8a^2}{b^2}\right)b^2+7a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{a^2+b^2}-\frac{7ab^2\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}}{4(a^2+b^2)}-\frac{b^2\sqrt[4]{\tan^2(e+fx)+1}}{2(a^2+b^2)(a+b\tan(e+fx))^2}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{-\frac{\int-\frac{2\left(4a^2-3b^2\right)-7ab\tan(e+fx)}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}-\frac{7ab^2\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}}{4(a^2+b^2)}-\frac{b^2\sqrt[4]{\tan^2(e+fx)+1}}{2(a^2+b^2)(a+b\tan(e+fx))^2}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\frac{\int\frac{2\left(4a^2-3b^2\right)-7ab\tan(e+fx)}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}-\frac{7ab^2\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}}{4(a^2+b^2)}-\frac{b^2\sqrt[4]{\tan^2(e+fx)+1}}{2(a^2+b^2)(a+b\tan(e+fx))^2}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\Big\downarrow$ 719

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\int\frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-7a\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}-\frac{7ab^2\sqrt[4]{\tan^2(e+}}{\left(a^2+b^2\right)(a+b\tan}}{4(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\Big\downarrow$ 229

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\int\frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-14ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{2\left(a^2+b^2\right)}-\frac{7ab^2\sqrt[4]{\tan^2(e+f}}{\left(a^2+b^2\right)(a+b\tan(e}}{4(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\Big\downarrow$ 504

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\left(a\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int\frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+f\right.}{2\left(a^2+b^2\right)}}{4(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$\Big\downarrow$ 312

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\left(\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{-\frac{\tan(e+fx)}{b}}\left(\frac{\tan(e+fx)}{b}+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d\left(b^2\tan^2(e+fx)\right)}{2b}-\int\frac{}{\left(\tan^2(e+\right.}\right.}{2\left(a^2+b^2\right)}}{4(a^2+b^2}\right.}{bf\sqrt[4]{se}}$$

$\Big\downarrow$ 118

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\left(-\int\frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int-\frac{}{\sqrt{1-b^4\tan^4(e+fx)}\left(-\right.}}{b}\right.}{2\left(a^2+b^2\right)}}{4(a^2+\right.}{bf\sqrt[4]{}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\int\frac{}{\left(\tan^2(e\right.}\right.}{2\left(a^2+b^2\right)}}{4(a^2+b^2}\right.}{bf\sqrt[4]{\mathrm{se}}}$$

$$\Big\downarrow \; 353$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\frac{1}{2}\int\frac{}{\left(\tan\right.}\right.}{2\left(a^2+b^2\right)}}{4(a^2-}\right.}{bf\sqrt[4]{}}$$

$$\Big\downarrow \; 73$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\int\frac{\cdots}{-\tan\cdots}\cdots\right)}{2\left(a^2+b^2\right)}\cdots}{4(a^2+b^2\cdots}\cdots\right)}{bf\sqrt[4]{\sec\cdots}}$$

$$\Big\downarrow \; 756$$

$$\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\int\frac{\cdots}{\sqrt{\cdots}}}{\cdots}\cdots\right)\right)}{\cdots}}{\cdots}\cdots\right)$$

$$\Big\downarrow \; 218$$

$$\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\int\frac{\cdots}{\sqrt{\cdots}}}{\cdots}\cdots\right)\right)}{2\left(a^2+b^2\right)\cdots}}{\cdots}\cdots\right)$$

$\downarrow$ 221

$$\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\mathrm{arcta}\ldots}{\ldots}\right.\right.}{2\left(a^2+b^2\right)}}{4(a^2\ldots}\right.$$

$$bf\sqrt[4]{\ldots}$$

$\downarrow$ 925

$$\sqrt{d\sec(e+fx)}\left(\frac{3\left(5a^2-2b^2\right)\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{2\left(a^2+b^2\right)}-\frac{b^2\int\ldots}{\ldots}\right.}{b}\right.}{\ldots}\right.$$

$\downarrow$ 1537

$$\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\left(2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int \frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-\sqrt[4]{\frac{\tan(e+fx)}{b}}+1}\sqrt{\sqrt[4]{\frac{\tan(e+fx)}{b}}+}}}{2\left(a^2+b^2\right)}\right.\right.}{}-\frac{}{}}{}\right.$$

$\downarrow$ 412

$$\sqrt{d\sec(e+fx)}\left(\frac{3\left(5a^2-2b^2\right)\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\frac{\tan(e+fx)}{b}+1}\right),-1\right)}{2\left(a^2+b^2\right)}-\frac{b^2\operatorname{EllipticPi}\left(\right.}{}\right.}{b}\right.}{}\right.$$

input

```
Int[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^3,x]
```

output

```
(Sqrt[d*Sec[e + f*x]]*(-1/2*(b^2*(1 + Tan[e + f*x]^2)^(1/4))/((a^2 + b^2)*
(a + b*Tan[e + f*x])^2) + ((-7*a*b^2*(1 + Tan[e + f*x]^2)^(1/4))/((a^2 + b
^2)*(a + b*Tan[e + f*x])) + (-14*a*b*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]
+ 3*(5*a^2 - 2*b^2)*(-2*b^2*(ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/
4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2 +
b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4))) - (2*a*Cot[e + f*x]*(-1/2*(b^2*
EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/
(a^2 + b^2) - (b^2*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e + f*x]/
b)^(1/4)], -1])/(2*(a^2 + b^2)))*Sqrt[-Tan[e + f*x]^2])/b))/(2*(a^2 + b^2)
))/(4*(a^2 + b^2))))/(b*f*(Sec[e + f*x]^2)^(1/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 756

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 925

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1537

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31117 vs. $2(445) = 890$.

Time = 14.84 (sec) , antiderivative size = 31118, normalized size of antiderivative = 60.42

method	result	size
default	Expression too large to display	31118

input
```
int((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{\sqrt{d\sec{(e+fx)}}}{(a+b\tan{(e+fx)})^3}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**3,x)
```

output
```
Integral(sqrt(d*sec(e + f*x))/(a + b*tan(e + f*x))**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{\sqrt{d\sec(fx+e)}}{(b\tan(fx+e)+a)^3}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}}{(a+b\tan(e+f\,x))^3}\,dx$$

input
```
int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x))^3,x)
```

output
```
int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x))^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$$
$$=\sqrt{d}\left(\int \frac{\sqrt{\sec(fx+e)}}{\tan(fx+e)^3 b^3+3\tan(fx+e)^2 a\,b^2+3\tan(fx+e)\,a^2b+a^3}dx\right)$$

input
```
int((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x)
```

output
```
sqrt(d)*int(sqrt(sec(e + f*x))/(tan(e + f*x)**3*b**3 + 3*tan(e + f*x)**2*a
*b**2 + 3*tan(e + f*x)*a**2*b + a**3),x)
```

3.629 $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 664

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$$
$$= \frac{5b^{3/2}(7a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{8\,(a^2+b^2)^{13/4}\,f\sqrt{d\sec(e+fx)}}$$
$$- \frac{5b^{3/2}(7a^2-2b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{8\,(a^2+b^2)^{13/4}\,f\sqrt{d\sec(e+fx)}}$$
$$+ \frac{a(8a^2-37b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|\,2\right)\sqrt[4]{\sec^2(e+fx)}}{4\,(a^2+b^2)^3\,f\sqrt{d\sec(e+fx)}}$$
$$- \frac{a(8a^2-37b^2)\tan(e+fx)}{4\,(a^2+b^2)^3\,f\sqrt{d\sec(e+fx)}}$$
$$- \frac{5ab(7a^2-2b^2)\cot(e+fx)\,\mathrm{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan}}{8\,(a^2+b^2)^{7/2}\,f\sqrt{d\sec(e+fx)}}$$
$$+ \frac{5ab(7a^2-2b^2)\cot(e+fx)\,\mathrm{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2}}{8\,(a^2+b^2)^{7/2}\,f\sqrt{d\sec(e+fx)}}$$
$$+ \frac{b(4a^2-5b^2)\sec^2(e+fx)}{2\,(a^2+b^2)^2\,f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$+ \frac{2(b+a\tan(e+fx))}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$+ \frac{ab(8a^2-37b^2)\sec^2(e+fx)}{4\,(a^2+b^2)^3\,f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$

output
```
5/8*b^(3/2)*(7*a^2-2*b^2)*arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1
/4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(13/4)/f/(d*sec(f*x+e))^(1/2)-5/8*b^(3
/2)*(7*a^2-2*b^2)*arctanh(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(s
ec(f*x+e)^2)^(1/4)/(a^2+b^2)^(13/4)/f/(d*sec(f*x+e))^(1/2)+1/4*a*(8*a^2-37
*b^2)*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(1/4)/
(a^2+b^2)^3/f/(d*sec(f*x+e))^(1/2)-1/4*a*(8*a^2-37*b^2)*tan(f*x+e)/(a^2+b^
2)^3/f/(d*sec(f*x+e))^(1/2)-5/8*a*b*(7*a^2-2*b^2)*cot(f*x+e)*EllipticPi((s
ec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)
^2)^(1/2)/(a^2+b^2)^(7/2)/f/(d*sec(f*x+e))^(1/2)+5/8*a*b*(7*a^2-2*b^2)*cot
(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2
)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(7/2)/f/(d*sec(f*x+e))^(1/2)+1/2*b
*(4*a^2-5*b^2)*sec(f*x+e)^2/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*
x+e))^2+2*(b+a*tan(f*x+e))/(a^2+b^2)/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e
))^2+1/4*a*b*(8*a^2-37*b^2)*sec(f*x+e)^2/(a^2+b^2)^3/f/(d*sec(f*x+e))^(1/2
)/(a+b*tan(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 31.79 (sec) , antiderivative size = 14652, normalized size of antiderivative = 22.07

$$\int \frac{1}{\sqrt{d \sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.77, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.880, Rules used = {3042, 3994, 496, 27, 25, 688, 27, 688, 27, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\int \frac{1}{(a+b\tan(e+fx))^3(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{496}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{2b^2\int -\frac{\left(5-\frac{a^2}{b^2}\right)b^2+3a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))^3\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+f}{a^2+b^2}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\int -\frac{a^2-3b\tan(e+fx)a-5b^2}{(a+b\tan(e+fx))^3\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}+\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{\int\frac{a^2-3b\tan(e+fx)a-5b^2}{(a+b\tan(e+fx))^3\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \; 688$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\int\frac{4a\left(8-\frac{a^2}{b^2}\right)-\left(5-\frac{4a^2}{b^2}\right)b\tan(e+fx)}{2(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}-\cdots}{a^2+b^2}\cdots\right.}{bf\sqrt{d\sec(e+fx)}\cdots}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\int\frac{4a\left(8-\frac{a^2}{b^2}\right)-\left(5-\frac{4a^2}{b^2}\right)b\tan(e+fx)}{(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{4\left(a^2+b^2\right)}-\cdots}{a^2+b^2}\cdots\right.}{bf\sqrt{d\sec(e+fx)}\cdots}$$

$$\Big\downarrow \; 688$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{b^2\int\frac{\left(\frac{8a^4}{b^4}-\frac{72a^2}{b^2}+10\right)b^4+a\cdots}{2b^4(a+b\tan(e+fx))\sqrt[4]{\cdots}}\cdots}{\cdots}\right.}{4\left(a^2+b^2\right)}\cdots}{\cdots}\cdots\right.}{bf\sqrt{d\sec(e+fx)}\cdots}$$

$\downarrow$ 27

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{\int\frac{2\left(4a^4-36b^2a^2+5b^4\right)+ab\left(\right.}{(a+b\tan(e+fx))\sqrt[4]{\tan}}}{2b}\right)}{4\left(a^2+b^2\right)}}{a}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$\downarrow$ 719

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\int\frac{}{\sqrt[4]{\tan^2(e}}}{}\right)}{}}{}\right)}{bf\sqrt{d\sec(e}}$$

$\downarrow$ 225

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan\cdots}{\sqrt[4]{\tan^2(e\cdots}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{bf\cdots}$$

$\downarrow$ 212

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan\cdots}{\sqrt[4]{\tan^2(e\cdots}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{bf\sqrt{\cdots}}$$

$\downarrow$ 504

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan}{\sqrt[4]{\tan^2(e}}\right.}{}\right.}{}}{}\right.$$

$\downarrow$ 310

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan}{\sqrt[4]{\tan^2(e}}\right.}{}\right.}{}}{}\right.$$

$\downarrow$ 353

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan\cdots}{\sqrt[4]{\tan^2(e\cdots}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 73

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan\cdots}{\sqrt[4]{\tan^2(e\cdots}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 827

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan\cdots}{\sqrt[4]{\tan^2(e\cdots}}\right.}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 218

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan\cdots}{\sqrt[4]{\tan^2(e\cdots}}\right.}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 221

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a(8a^2-37b^2)(\tan^2(e+fx)+1)^{3/4}}{(a^2+b^2)(a+b\tan(e+fx))}-\frac{a(8a^2-37b^2)\left(\frac{2b\tan}{\sqrt[4]{\tan^2(e}}\right.}{}\right.}{}}{}\right.$$

$\downarrow$ 993

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a(8a^2-37b^2)(\tan^2(e+fx)+1)^{3/4}}{(a^2+b^2)(a+b\tan(e+fx))}-\frac{a(8a^2-37b^2)\left(\frac{2b\tan}{\sqrt[4]{\tan^2(e}}\right.}{}\right.}{}}{}\right.$$

$\downarrow$ 1537

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{-b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan\cdots}{\sqrt[4]{\tan^2(e\cdots}}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\downarrow 412$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a(8a^2-37b^2)(\tan^2(e+fx)+1)^{3/4}}{(a^2+b^2)(a+b\tan(e+fx))}-\frac{a(8a^2-37b^2)\left(\frac{2b\tan}{\sqrt[4]{\tan^2(e}}\right.}{}\right.}{}}{}\right.$$

input
```
Int[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3),x]
```

output
```
((Sec[e + f*x]^2)^(1/4)*((2*(b^2 + a*b*Tan[e + f*x]))/((a^2 + b^2)*(a + b*
Tan[e + f*x])^2*(1 + Tan[e + f*x]^2)^(1/4)) - (-1/2*(b^2*(4*a^2 - 5*b^2)*(
1 + Tan[e + f*x]^2)^(3/4))/((a^2 + b^2)*(a + b*Tan[e + f*x])^2) - (b^2*((a
*(8*a^2 - 37*b^2)*(1 + Tan[e + f*x]^2)^(3/4))/((a^2 + b^2)*(a + b*Tan[e +
f*x])) - (-5*b^2*(7*a^2 - 2*b^2)*(-2*b^2*(-1/2*ArcTan[(b^(3/2)*Tan[e + f*x
])/(a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 + b^2)^(1/4)) + ArcTanh[(b^(3/2)*Tan[e
 + f*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2)*(a^2 + b^2)^(1/4))) + (2*a*Cot[e +
f*x]*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]^2)
^(1/4)], -1])/Sqrt[a^2 + b^2] + (b*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1
 + Tan[e + f*x]^2)^(1/4)], -1])/(2*Sqrt[a^2 + b^2]))*Sqrt[-Tan[e + f*x]^2]
)/b) + a*(8*a^2 - 37*b^2)*(-2*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*
b*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^(1/4)))/(2*b^2*(a^2 + b^2))))/(4*(a^2
 + b^2)))/(a^2 + b^2)))/(b*f*Sqrt[d*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 33495 vs. 2(585) = 1170.

Time = 16.92 (sec) , antiderivative size = 33496, normalized size of antiderivative = 50.45

method	result	size
default	Expression too large to display	33496

input
```
int(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{\sqrt{d\sec{(e+fx)}}\,(a+b\tan{(e+fx)})^3}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**3,x)
```

output
```
Integral(1/(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{\sqrt{d\sec{(fx+e)}}(b\tan{(fx+e)}+a)^3}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{\sqrt{d\sec{(fx+e)}}(b\tan{(fx+e)}+a)^3}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{\sqrt{\frac{d}{\cos(e+f\,x)}}\,(a+b\tan(e+f\,x))^3}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^3),x)
```

output
```
int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)\tan(fx+e)^3b^3+3\sec(fx+e)\tan(fx+e)^2a\,b^2+3\sec(fx+e)\tan(fx+e)a^2b+\sec(fx+e)a^3}dx\right)}{d}$$

input
```
int(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x)
```

output
```
(sqrt(d)*int(sqrt(sec(e + f*x))/(sec(e + f*x)*tan(e + f*x)**3*b**3 + 3*sec
(e + f*x)*tan(e + f*x)**2*a*b**2 + 3*sec(e + f*x)*tan(e + f*x)*a**2*b + se
c(e + f*x)*a**3),x))/d
```

3.630 $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 620

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx =$$

$$-\frac{7b^{5/2}(9a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{8\left(a^2+b^2\right)^{15/4}f(d\sec(e+fx))^{3/2}}$$
$$-\frac{7b^{5/2}(9a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{8\left(a^2+b^2\right)^{15/4}f(d\sec(e+fx))^{3/2}}$$
$$+\frac{a(8a^2-69b^2)\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{12\left(a^2+b^2\right)^3f(d\sec(e+fx))^{3/2}}$$
$$+\frac{7ab^2(9a^2-2b^2)\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2}}{8\left(a^2+b^2\right)^4f(d\sec(e+fx))^{3/2}}$$
$$+\frac{7ab^2(9a^2-2b^2)\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e}}{8\left(a^2+b^2\right)^4f(d\sec(e+fx))^{3/2}}$$
$$+\frac{b(4a^2-7b^2)\sec^2(e+fx)}{6\left(a^2+b^2\right)^2f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}$$
$$+\frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}$$
$$+\frac{ab(8a^2-69b^2)\sec^2(e+fx)}{12\left(a^2+b^2\right)^3f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}$$

output
```
-7/8*b^(5/2)*(9*a^2-2*b^2)*arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(
1/4))*(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^(15/4)/f/(d*sec(f*x+e))^(3/2)-7/8*b^(
5/2)*(9*a^2-2*b^2)*arctanh(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/4))*(
sec(f*x+e)^2)^(3/4)/(a^2+b^2)^(15/4)/f/(d*sec(f*x+e))^(3/2)+1/12*a*(8*a^2-
69*b^2)*InverseJacobiAM(1/2*arctan(tan(f*x+e)),2^(1/2))*(sec(f*x+e)^2)^(3/
4)/(a^2+b^2)^3/f/(d*sec(f*x+e))^(3/2)+7/8*a*b^2*(9*a^2-2*b^2)*cot(f*x+e)*E
llipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(3/4)*
(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^4/f/(d*sec(f*x+e))^(3/2)+7/8*a*b^2*(9*a^2-
2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(se
c(f*x+e)^2)^(3/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^4/f/(d*sec(f*x+e))^(3/2)
+1/6*b*(4*a^2-7*b^2)*sec(f*x+e)^2/(a^2+b^2)^2/f/(d*sec(f*x+e))^(3/2)/(a+b*
tan(f*x+e))^2+2/3*(b+a*tan(f*x+e))/(a^2+b^2)/f/(d*sec(f*x+e))^(3/2)/(a+b*t
an(f*x+e))^2+1/12*a*b*(8*a^2-69*b^2)*sec(f*x+e)^2/(a^2+b^2)^3/f/(d*sec(f*x
+e))^(3/2)/(a+b*tan(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.44 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.66

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \frac{\frac{2(a^2+b^2)\sec^3(e+fx)\left(b(22a^4-63a^2b^2-8b^4)\cos(e+fx)+2b(a^2+b^2)^2\cos(3(e+f}{(a+b\tan(e+fx)}}{}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3),x]
```

output
```
((2*(a^2 + b^2)*Sec[e + f*x]^3*(b*(22*a^4 - 63*a^2*b^2 - 8*b^4)*Cos[e + f*
x] + 2*b*(a^2 + b^2)^2*Cos[3*(e + f*x)] + a*(4*a^4 + 16*a^2*b^2 - 65*b^4 +
 4*(a^2 + b^2)^2*Cos[2*(e + f*x)])*Sin[e + f*x]))/(a + b*Tan[e + f*x])^2 +
 (Sec[e + f*x]^2)^(3/4)*(a*(8*a^4 - 61*a^2*b^2 - 69*b^4)*Hypergeometric2F1
[1/2, 3/4, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] + (21*b^2*(-9*a^2 + 2*b^2)*(
a*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Tan
[e + f*x] + a*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)]
, -1]*Tan[e + f*x] + Sqrt[b]*(a^2 + b^2)^(1/4)*(ArcTan[(Sqrt[b]*(Sec[e + f
*x]^2)^(1/4))/(a^2 + b^2)^(1/4)] + ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4)
)/(a^2 + b^2)^(1/4)])*Sqrt[-Tan[e + f*x]^2]))/Sqrt[-Tan[e + f*x]^2]))/(24*
(a^2 + b^2)^4*f*(d*Sec[e + f*x])^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 495, normalized size of antiderivative = 0.80, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 3994, 496, 27, 688, 27, 688, 27, 25, 719, 229, 504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sec^2(e+fx)^{3/4}\int \frac{1}{(a+b\tan(e+fx))^3(\tan^2(e+fx)+1)^{7/4}}d(b\tan(e+fx))}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{496}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{2(ab\tan(e+fx)+b^2)}{3(a^2+b^2)(\tan^2(e+fx)+1)^{3/4}(a+b\tan(e+fx))^2}-\frac{2b^2\int -\frac{\left(\frac{a^2}{b^2}+7\right)b^2+5a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))^3\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{3(a^2+b^2)}\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\int \frac{a^2+5b\tan(e+fx)a+7b^2}{(a+b\tan(e+fx))^3\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{3(a^2+b^2)}+\frac{2(ab\tan(e+fx)+b^2)}{3(a^2+b^2)(\tan^2(e+fx)+1)^{3/4}(a+b\tan(e+fx))^2}\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 688$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{\left(4a^2-7b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}{2\left(\frac{a^2}{b^2}+1\right)(a+b\tan(e+fx))^2}-\frac{b^2\int -\frac{4a\left(\frac{a^2}{b^2}+12\right)-3\left(7-\frac{4a^2}{b^2}\right)b\tan(e+fx)}{2(a+b\tan(e+fx))^2\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}}{3(a^2+b^2)}+\frac{2(ab\tan}{3(a^2+b^2)(\tan^2(e+fx)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\int \frac{4a\left(\frac{a^2}{b^2}+12\right)-3\left(7-\frac{4a^2}{b^2}\right)b\tan(e+fx)}{(a+b\tan(e+fx))^2\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{4\left(a^2+b^2\right)}+\frac{\left(4a^2-7b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}{2\left(\frac{a^2}{b^2}+1\right)(a+b\tan(e+fx))^2}}{3(a^2+b^2)}+\frac{2(ab\tan(e-}{3(a^2+b^2)(\tan^2(e+fx)+}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 688$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(\frac{a\left(8a^2-69b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{b^2\int\frac{\left(-\frac{8a^4}{b^4}-\frac{120a^2}{b^2}+42\right)b^4-ab\left(8a^2-69b^2\right)\tan(e+fx)}{2b^4(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{a^2+b^2}\right)}{4\left(a^2+b^2\right)}+\frac{\left(4a^2-7\right.}{2}}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$\downarrow$ 27

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(\frac{a\left(8a^2-69b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{\int-\frac{2\left(4a^4+60b^2a^2-21b^4\right)+ab\left(8a^2-69b^2\right)\tan(e+fx)}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2b^2\left(a^2+b^2\right)}\right)}{4\left(a^2+b^2\right)}+\frac{\left(4a^2-7b^2\right)}{2\left(\frac{a^2}{b^2}\right.}}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$\downarrow$ 25

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(\frac{\int\frac{2\left(4a^4+60b^2a^2-21b^4\right)+ab\left(8a^2-69b^2\right)\tan(e+fx)}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2b^2\left(a^2+b^2\right)}+\frac{a\left(8a^2-69b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}\right)}{4\left(a^2+b^2\right)}+\frac{\left(4a^2-7b^2\right)}{2\left(\frac{a^2}{b^2}+\right.}}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$\downarrow$ 719

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\int\frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))+a\left(8a^2-69b^2\right)\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2b^2\left(a^2+b^2\right)}+\frac{a}{}\right.}{4\left(a^2+b^2\right)}}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx}$$

$\downarrow$ 229

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\int\frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))+2ab\left(8a^2-69b^2\right)\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{2b^2\left(a^2+b^2\right)}+\frac{a\left(8\right.}{}\right.}{4\left(a^2+b^2\right)}}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx}$$

$\downarrow$ 504

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\left(a\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int\frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan\right.}{2b^2\left(a^2+b^2\right)}\right.}{4\left(a^2+b^2\right)}\right.}{}$$

$\downarrow$ 312

$$\frac{\sec^2(e+fx)^{3/4}}{\left(\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\left(\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{-\frac{\tan(e+fx)}{b}}\left(\frac{\tan(e+fx)}{b}+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d\left(b^2\tan^2(e+fx)\right)}{2b}-\int \cdots\right)}{2b^2\left(a^2+b^2\right)}\cdots\right)}{\cdots}\cdots\right)}$$

$\downarrow$ 118

$$\frac{\sec^2(e+fx)^{3/4}}{\left(\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\left(-\int\frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int-\frac{\cdots}{\sqrt{1-b^4\tan^4(e\cdots}}}{\cdots}\right)}{2b^2\left(a^2+b^2\right)}\cdots\right)}{\cdots}\cdots\right)}$$

$\downarrow$ 25

$$\frac{\sec^2(e+fx)^{3/4}}{\left(\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\int\right.}{2b^2\left(a^2+b^2\right)}}{}\right.}$$

$\downarrow$ 353

$$\frac{\sec^2(e+fx)^{3/4}}{\left(\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\frac{1}{2}\right.}{2b^2\left(a^2+b^2\right)}\right.}{}\right.}$$

$\downarrow$ 73

$$\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b\right.}{2b^2\left(a^2+b^2\right)}\right.}{}\right.}{}$$

$$\downarrow 756$$

$$\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{218}$$

$$\frac{\sec^2(e+fx)^{3/4}\Bigg(\dfrac{b^2\Bigg(\dfrac{21b^2\left(9a^2-2b^2\right)\Bigg(\dfrac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b}{}}{}}{}$$

$\downarrow$ 221

$$\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b\right.}{2b^2\left(a^2+b^2\right.}}{}}{}\right.$$

$\downarrow$ 925

$$\frac{\sec^2(e+fx)^{3/4}}{\left(\frac{b^2 \left(21b^2\left(9a^2-2b^2\right) - \frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int \frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{2\left(a^2+b^2\right)} - \cdots \right.}{b} \cdots \right.}{\cdots} \cdots \right.}$$

$\downarrow$ 1537

$$\sec^2(e+fx)^{3/4} \left(\frac{b^2 \left(21b^2\left(9a^2-2b^2\right) - \frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx) \left(-\frac{b^2 \int \frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-\sqrt[4]{\frac{\tan(e+fx)}{b}}+1}\sqrt{\sqrt[4]{\frac{\tan(e+}{b}}}}}{2\left(a^2+b^2\right)} \right.}{\left(\right.} \right.}{} \right.$$

$$\Big\downarrow \text{412}$$

$$\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(21b^2\left(9a^2-2b^2\right)-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\frac{\tan(e+fx)}{b}+1}\right),-1\right)}{2\left(a^2+b^2\right)}-\frac{b^2\,\text{Ell}}{}\right.}{b}\right.}{}\right.$$

input
```
Int[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3),x]
```

output
```
((Sec[e + f*x]^2)^(3/4)*((2*(b^2 + a*b*Tan[e + f*x]))/(3*(a^2 + b^2)*(a +
b*Tan[e + f*x])^2*(1 + Tan[e + f*x]^2)^(3/4)) + (((4*a^2 - 7*b^2)*(1 + Tan
[e + f*x]^2)^(1/4))/(2*(1 + a^2/b^2)*(a + b*Tan[e + f*x])^2) + (b^2*((a*(8
*a^2 - 69*b^2)*(1 + Tan[e + f*x]^2)^(1/4))/((a^2 + b^2)*(a + b*Tan[e + f*x
])) + (2*a*b*(8*a^2 - 69*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2] + 21*b^
2*(9*a^2 - 2*b^2)*(-2*b^2*(ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)
]/(2*Sqrt[b]*(a^2 + b^2)^(3/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^
2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4))) - (2*a*Cot[e + f*x]*(-1/2*(b^2*El
lipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(a
^2 + b^2) - (b^2*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e + f*x]/b)
^(1/4)], -1])/(2*(a^2 + b^2)))*Sqrt[-Tan[e + f*x]^2])/b))/(2*b^2*(a^2 + b^
2))))/(4*(a^2 + b^2)))/(3*(a^2 + b^2))))/(b*f*(d*Sec[e + f*x])^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 32826 vs. $2(546) = 1092$.

Time = 20.39 (sec) , antiderivative size = 32827, normalized size of antiderivative = 52.95

method	result	size
default	Expression too large to display	32827

input
```
int(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{(d\sec{(e+fx)})^{\frac{3}{2}}\,(a+b\tan{(e+fx)})^3}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**3,x)
```

output
```
Integral(1/((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))**3), x)
```

Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{(d\sec{(fx+e)})^{\frac{3}{2}}\,(b\tan{(fx+e)}+a)^3}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^3), x)
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{(d\sec{(fx+e)})^{\frac{3}{2}}\,(b\tan{(fx+e)}+a)^3}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx=\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}\,(a+b\tan\left(e+f\,x\right))^3}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^3),x)
```

output
```
int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^3), x)
```

Reduce [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx=\frac{\sqrt{d}\,\Big(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^2\tan(fx+e)^3b^3+3\sec(fx+e)^2\tan(fx+e)^2a\,b^2+3\sec(fx+e)}}{d^2}$$

input
```
int(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x)
```

output
```
(sqrt(d)*int(sqrt(sec(e + f*x))/(sec(e + f*x)**2*tan(e + f*x)**3*b**3 + 3*
sec(e + f*x)**2*tan(e + f*x)**2*a*b**2 + 3*sec(e + f*x)**2*tan(e + f*x)*a*
*2*b + sec(e + f*x)**2*a**3),x))/d**2
```

3.631 $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 820

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx = \frac{9b^{7/2}(11a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+f}{8\,(a^2+b^2)^{17/4}\,d^2f\sqrt{d\sec(e+fx)}}$$
$$-\frac{9b^{7/2}(11a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{8\,(a^2+b^2)^{17/4}\,d^2f\sqrt{d\sec(e+fx)}}$$
$$+\frac{9ab^2(11a^2-2b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{4\,(a^2+b^2)^4\,d^2f\sqrt{d\sec(e+fx)}}$$
$$+\frac{3a(8a^2-109b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{20\,(a^2+b^2)^3\,d^2f\sqrt{d\sec(e+fx)}}$$
$$-\frac{9ab^2(11a^2-2b^2)\tan(e+fx)}{4\,(a^2+b^2)^4\,d^2f\sqrt{d\sec(e+fx)}}$$
$$-\frac{9ab^3(11a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan}}{8\,(a^2+b^2)^{9/2}\,d^2f\sqrt{d\sec(e+fx)}}$$
$$+\frac{9ab^3(11a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(}}{8\,(a^2+b^2)^{9/2}\,d^2f\sqrt{d\sec(e+fx)}}$$
$$+\frac{b(4a^2-9b^2)}{10\,(a^2+b^2)^2\,d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$+\frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\,(a^2+b^2)\,d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$+\frac{ab(8a^2-109b^2)}{20\,(a^2+b^2)^3\,d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$+\frac{9(11a^2-2b^2)\,(b^3+ab^2\tan(e+fx))}{4\,(a^2+b^2)^4\,d^2f\sqrt{d\sec(e+fx)}}$$

output
```
9/8*b^(7/2)*(11*a^2-2*b^2)*arctan(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(
1/4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(17/4)/d^2/f/(d*sec(f*x+e))^(1/2)-9/8
*b^(7/2)*(11*a^2-2*b^2)*arctanh(b^(1/2)*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(1/
4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(17/4)/d^2/f/(d*sec(f*x+e))^(1/2)+9/4*a
*b^2*(11*a^2-2*b^2)*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*
x+e)^2)^(1/4)/(a^2+b^2)^4/d^2/f/(d*sec(f*x+e))^(1/2)+3/20*a*(8*a^2-109*b^2
)*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(1/4)/(a^2
+b^2)^3/d^2/f/(d*sec(f*x+e))^(1/2)-9/4*a*b^2*(11*a^2-2*b^2)*tan(f*x+e)/(a^
2+b^2)^4/d^2/f/(d*sec(f*x+e))^(1/2)-9/8*a*b^3*(11*a^2-2*b^2)*cot(f*x+e)*El
lipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(
-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(9/2)/d^2/f/(d*sec(f*x+e))^(1/2)+9/8*a*b^3*
(11*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2
),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(9/2)/d^2/f/(d*s
ec(f*x+e))^(1/2)+1/10*b*(4*a^2-9*b^2)/(a^2+b^2)^2/d^2/f/(d*sec(f*x+e))^(1/
2)/(a+b*tan(f*x+e))^2+2/5*cos(f*x+e)^2*(b+a*tan(f*x+e))/(a^2+b^2)/d^2/f/(d
*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2+1/20*a*b*(8*a^2-109*b^2)/(a^2+b^2)^3
/d^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))+9/4*(11*a^2-2*b^2)*(b^3+a*b^2
*tan(f*x+e))/(a^2+b^2)^4/d^2/f/(d*sec(f*x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 29.14 (sec) , antiderivative size = 15481, normalized size of antiderivative = 18.88

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 1.16 (sec) , antiderivative size = 626, normalized size of antiderivative = 0.76, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 3994, 496, 27, 686, 27, 25, 688, 27, 688, 27, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\int \frac{1}{(a+b\tan(e+fx))^3(\tan^2(e+fx)+1)^{9/4}}d(b\tan(e+fx))}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{496}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{5(a^2+b^2)(\tan^2(e+fx)+1)^{5/4}(a+b\tan(e+fx))^2}-\frac{2b^2\int -\frac{3\left(\frac{a^2}{b^2}+3\right)b^2+7a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))^3\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))}{5(a^2+b^2)}\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\int \frac{3\left(a^2+3b^2\right)+7ab\tan(e+fx)}{(a+b\tan(e+fx))^3\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))}{5(a^2+b^2)}+\frac{2\left(ab\tan(e+fx)+b^2\right)}{5(a^2+b^2)(\tan^2(e+fx)+1)^{5/4}(a+b\tan(e+fx))^2}\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{686}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{2b^4\int -\frac{3\left(\left(-\frac{a^4}{b^4}-\frac{12a^2}{b^2}+15\right)b^4+a\left(3a^2+16b^2\right)\tan(e+fx)b\right)}{2b^4(a+b\tan(e+fx))^3\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}}{5(a^2+b^2)}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{3\int -\frac{a^4+12b^2a^2-b\left(3a^2+16b^2\right)\tan(e+fx)a-15b^4}{(a+b\tan(e+fx))^3\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}+\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}}{5(a^2+b^2)}+\frac{\ }{5(a^2\cdots}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{3\int \frac{a^4+12b^2a^2-b\left(3a^2+16b^2\right)\tan(e+fx)a-15b^4}{(a+b\tan(e+fx))^3\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}}{5(a^2+b^2)}+\frac{\ }{5(a^2+\cdots}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \; 688$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(-\frac{b^2\int-\frac{4ab^2\left(\frac{a^4}{b^2}+9a^2-31b^2\right)-b\left(4a^4+28b^2a^2-15b^4\right)\tan(e+fx)}{2b^2(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan}{2\left(a^2+b^2\right)}\right.}{a^2+b^2}}{5(a^2+b^2)}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{\int\frac{4a\left(a^4+9b^2a^2-31b^4\right)-b\left(4a^4+28b^2a^2-15b^4\right)\tan(e+fx)}{(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{4\left(a^2+b^2\right)}\right.}{a^2+b^2}}{5(a^2+b^2)}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{688}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{3\left(\frac{-\frac{b^2\int -\frac{\left(\frac{8a^6}{b^2}+64a^4-304b^2a^2+30b^4\right)b^2+a\left(8a^4+64b^2a^2-139b^4\right)\tan(e+\cdots}{2b^2(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}\cdots}{a^2+b^2}\cdots}{4\left(a^2+\cdots\right.}\cdots\right)}{5(a^2+b^2)\cdots}\right)}{bd^2\cdots}$$

$$\Big\downarrow\ 27$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{3\left(\frac{\frac{\int \frac{2\left(4a^6+32b^2a^4-152b^4a^2+15b^6\right)+ab\left(8a^4+64b^2a^2-139b^4\right)\tan(e+fx)}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}\,d\cdots}{2\left(a^2+b^2\right)}\cdots}{4\left(a^2+b^2\right)\cdots}\cdots\right)}{5(a^2+b^2)\cdots}\right)}{bd^2f\cdots}$$

$$\Big\downarrow\ 719$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-15}{2(}\right.}{}\right.$$

$\downarrow$ 225

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int\frac{1}{\left(\tan^2(e+fx)+\right.}\right.}{}\right.}{}\right.$$

$\downarrow$ 212

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\mathrm{ta}\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 504

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\mathrm{ta}\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 310

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\mathrm{ta}\right.\right.}{}\right.}{}\right.}{}$$

$\downarrow$ 353

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(ta\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 73

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\mathrm{ta}\right.\right.}{}\right.}{}\right.}{}$$

$\downarrow$ 827

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\mathrm{ta}\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$\downarrow$ 218

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(ta\right.\right.}{}\right.}{}\right.}{}$$

$$\downarrow 221$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\mathrm{ta}\right.\right.}{}\right.}{}\right.}{}$$

$\downarrow$ 993

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\text{ta}\right.\right.}{}\right.}{}}{}\right.$$

$\downarrow$ 1537

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\mathrm{ta}\right.\right.}{}\right.}{}\right.$$

↓ 412

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{5(a^2+b^2)(\tan^2(e+fx)+1)^{5/4}(a+b\tan(e+fx))^2}+\frac{\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2\right.}{}\right.}{}}{}\right.$$

input
```
Int[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3),x]
```

output
```
((Sec[e + f*x]^2)^(1/4)*((2*(b^2 + a*b*Tan[e + f*x]))/(5*(a^2 + b^2)*(a +
b*Tan[e + f*x])^2*(1 + Tan[e + f*x]^2)^(5/4)) + ((2*((9 - (4*a^2)/b^2)*b^4
 + a*b*(3*a^2 + 16*b^2)*Tan[e + f*x]))/((a^2 + b^2)*(a + b*Tan[e + f*x])^2
*(1 + Tan[e + f*x]^2)^(1/4)) - (3*(-1/2*(b^2*(4*a^4 + 28*a^2*b^2 - 15*b^4)
*(1 + Tan[e + f*x]^2)^(3/4))/((a^2 + b^2)*(a + b*Tan[e + f*x])^2) + (-((a*
b^2*(8*a^4 + 64*a^2*b^2 - 139*b^4)*(1 + Tan[e + f*x]^2)^(3/4))/((a^2 + b^2
)*(a + b*Tan[e + f*x]))) + (-15*b^4*(11*a^2 - 2*b^2)*(-2*b^2*(-1/2*ArcTan[
(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 + b^2)^(1/4)) + Ar
cTanh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2)*(a^2 + b^2)^(1/
4))) + (2*a*Cot[e + f*x]*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[
(1 + Tan[e + f*x]^2)^(1/4)], -1])/Sqrt[a^2 + b^2] + (b*EllipticPi[b/Sqrt[a
^2 + b^2], ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1])/(2*Sqrt[a^2 + b^2]))*S
qrt[-Tan[e + f*x]^2])/b) + a*(8*a^4 + 64*a^2*b^2 - 139*b^4)*(-2*b*Elliptic
E[ArcTan[Tan[e + f*x]]/2, 2] + (2*b*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^(1/
4)))/(2*(a^2 + b^2)))/(4*(a^2 + b^2))))/(a^2 + b^2))/(5*(a^2 + b^2))))/(b*
d^2*f*Sqrt[d*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36193 vs. $2(730) = 1460$.

Time = 27.50 (sec) , antiderivative size = 36194, normalized size of antiderivative = 44.14

method	result	size
default	Expression too large to display	36194

input
```
int(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{(d\sec(e+fx))^{\frac{5}{2}}\,(a+b\tan(e+fx))^3}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**3,x)
```

output
```
Integral(1/((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x))**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx=\int \frac{1}{(d\sec\left(fx+e\right))^{\frac{5}{2}}\left(b\tan\left(fx+e\right)+a\right)^3}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx=\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}\left(a+b\tan\left(e+f\,x\right)\right)^3}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^3),x)
```

output
```
int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^3), x)
```

Reduce [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx=\frac{\sqrt{d}\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^3\tan(fx+e)^3b^3+3\sec(fx+e)^3\tan(fx+e)^2a\,b^2+3\sec(fx+e)}\right.}{d^3}$$

input
```
int(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x)
```

output
```
(sqrt(d)*int(sqrt(sec(e + f*x))/(sec(e + f*x)**3*tan(e + f*x)**3*b**3 + 3*
sec(e + f*x)**3*tan(e + f*x)**2*a*b**2 + 3*sec(e + f*x)**3*tan(e + f*x)*a*
*2*b + sec(e + f*x)**3*a**3),x))/d**3
```

3.632 $\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx = \frac{3b(d\sec(e+fx))^{5/3}}{5f} + \frac{3ad\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sin(e+fx)}{2f\sqrt{\sin^2(e+fx)}}$$

output
```
3/5*b*(d*sec(f*x+e))^(5/3)/f+3/2*a*d*hypergeom([-1/3, 1/2],[2/3],cos(f*x+e
)^2)*(d*sec(f*x+e))^(2/3)*sin(f*x+e)/f/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx = \frac{3d(d\sec(e+fx))^{2/3}\left(b\sec(e+fx)+a\csc(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\sec^2\right.\right.}{5f}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x]),x]
```

output
```
(3*d*(d*Sec[e + f*x])^(2/3)*(b*Sec[e + f*x] + a*Csc[e + f*x]*Hypergeometri
c2F1[1/2, 5/6, 11/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]))/(5*f)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int (d\sec(e+fx))^{5/3}dx+\frac{3b(d\sec(e+fx))^{5/3}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/3}dx+\frac{3b(d\sec(e+fx))^{5/3}}{5f}$$

$$\Big\downarrow \text{4259}$$

$$a\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\int \frac{1}{\left(\frac{\cos(e+fx)}{d}\right)^{5/3}}dx+\frac{3b(d\sec(e+fx))^{5/3}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\int \frac{1}{\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{5/3}}dx+\frac{3b(d\sec(e+fx))^{5/3}}{5f}$$

$$\Big\downarrow \text{3122}$$

$$\frac{3ad\sin(e+fx)(d\sec(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}}+\frac{3b(d\sec(e+fx))^{5/3}}{5f}$$

input
```
Int[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x]),x]
```

output
```
(3*b*(d*Sec[e + f*x])^(5/3))/(5*f) + (3*a*d*Hypergeometric2F1[-1/3, 1/2, 2
/3, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(2/3)*Sin[e + f*x])/(2*f*Sqrt[Sin[e +
 f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int (d\sec (fx+e))^{\frac{5}{3}} (a+b\tan (fx+e))\, dx$$

input
```
int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\, dx = \int (d\sec (fx+e))^{\frac{5}{3}} (b\tan (fx+e)+a)\, dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
integral((b*d*sec(f*x + e)*tan(f*x + e) + a*d*sec(f*x + e))*(d*sec(f*x + e
))^(2/3), x)
```

Sympy [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\, dx = \int (d\sec (e+fx))^{\frac{5}{3}} (a+b\tan (e+fx))\, dx$$

input
```
integrate((d*sec(f*x+e))**(5/3)*(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**(5/3)*(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{5}{3}}(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{5}{3}}(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}(a+b\tan(e+f\,x))\,dx$$

input
```
int((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x)), x)
```

Reduce [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx = \frac{d^{\frac{5}{3}}\left(3\sec\left(fx+e\right)^{\frac{5}{3}}b+5\left(\int \sec\left(fx+e\right)^{\frac{5}{3}}dx\right)af\right)}{5f}$$

input
```
int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x)
```

output
```
(d**(2/3)*d*(3*sec(e + f*x)**(2/3)*sec(e + f*x)*b + 5*int(sec(e + f*x)**(2
/3)*sec(e + f*x),x)*a*f))/(5*f)
```

3.633 $\int \sqrt[3]{d \sec(e+fx)}(a+b\tan(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \sqrt[3]{d \sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{3b\sqrt[3]{d\sec(e+fx)}}{f} - \frac{3ad\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(e+fx)\right)\sin(e+fx)}{2f(d\sec(e+fx))^{2/3}\sqrt{\sin^2(e+fx)}}$$

output
```
3*b*(d*sec(f*x+e))^(1/3)/f-3/2*a*d*hypergeom([1/3, 1/2],[4/3],cos(f*x+e)^2
)*sin(f*x+e)/f/(d*sec(f*x+e))^(2/3)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.78

$$\int \sqrt[3]{d \sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{3\sqrt[3]{d\sec(e+fx)}\left(b+a\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{f}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x]),x]
```

output
```
(3*(d*Sec[e + f*x])^(1/3)*(b + a*Cot[e + f*x]*Hypergeometric2F1[1/6, 1/2,
7/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]))/f
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sqrt[3]{d\sec(e+fx)}dx+\frac{3b\sqrt[3]{d\sec(e+fx)}}{f}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt[3]{d\csc\left(e+fx+\frac{\pi}{2}\right)}dx+\frac{3b\sqrt[3]{d\sec(e+fx)}}{f}$$

$$\Big\downarrow \text{4259}$$

$$a\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\int \frac{1}{\sqrt[3]{\frac{\cos(e+fx)}{d}}}dx+\frac{3b\sqrt[3]{d\sec(e+fx)}}{f}$$

$$\Big\downarrow \text{3042}$$

$$a\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\int \frac{1}{\sqrt[3]{\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}}}dx+\frac{3b\sqrt[3]{d\sec(e+fx)}}{f}$$

$$\Big\downarrow \text{3122}$$

$$\frac{3b\sqrt[3]{d\sec(e+fx)}}{f}-\frac{3ad\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{2/3}}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x]),x]
```

output
```
(3*b*(d*Sec[e + f*x])^(1/3))/f - (3*a*d*Hypergeometric2F1[1/3, 1/2, 4/3, C
os[e + f*x]^2]*Sin[e + f*x])/(2*f*(d*Sec[e + f*x])^(2/3)*Sqrt[Sin[e + f*x]
^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{1}{3}} (a+b\tan(fx+e))\, dx$$

input
```
int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x)
```

Fricas [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\, dx = \int (d\sec(fx+e))^{\frac{1}{3}} (b\tan(fx+e)+a)\, dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a), x)
```

Sympy [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\, dx = \int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\, dx$$

input
```
integrate((d*sec(f*x+e))**(1/3)*(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}(a+b\tan(e+f\,x))\,dx$$

input
```
int((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x)), x)
```

Reduce [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{d^{\frac{1}{3}}\left(3\sec\left(fx+e\right)^{\frac{1}{3}}b+\left(\int\sec\left(fx+e\right)^{\frac{1}{3}}dx\right)af\right)}{f}$$

input
```
int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x)
```

output
```
(d**(1/3)*(3*sec(e + f*x)**(1/3)*b + int(sec(e + f*x)**(1/3),x)*a*f))/f
```

3.634 $\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3b}{f\sqrt[3]{d\sec(e+fx)}} - \frac{3ad\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(e+fx)\right)\sin(e+fx)}{4f(d\sec(e+fx))^{4/3}\sqrt{\sin^2(e+fx)}}$$

output
```
-3*b/f/(d*sec(f*x+e))^(1/3)-3/4*a*d*hypergeom([1/2, 2/3],[5/3],cos(f*x+e)^
2)*sin(f*x+e)/f/(d*sec(f*x+e))^(4/3)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.78

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3\left(b+a\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{f\sqrt[3]{d\sec(e+fx)}}$$

input
```
Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
```

output
```
(-3*(b + a*Cot[e + f*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[e + f*x]^2]*
Sqrt[-Tan[e + f*x]^2]))/(f*(d*Sec[e + f*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \frac{1}{\sqrt[3]{d\sec(e+fx)}}dx-\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt[3]{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{4259}$$

$$a\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\int \sqrt[3]{\frac{\cos(e+fx)}{d}}dx-\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\int \sqrt[3]{\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}}dx-\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3ad\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(e+fx)\right)}{4f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{4/3}}-\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}$$

input
```
Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
```

output
```
(-3*b)/(f*(d*Sec[e + f*x])^(1/3)) - (3*a*d*Hypergeometric2F1[1/2, 2/3, 5/3
, Cos[e + f*x]^2]*Sin[e + f*x])/(4*f*(d*Sec[e + f*x])^(4/3)*Sqrt[Sin[e + f
*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{a+b\tan \left(fx+e\right)}{\left(d\sec \left(fx+e\right)\right)^{\frac{1}{3}}}dx$$

input
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

output
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx=\int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(2/3)*(b*tan(f*x + e) + a)/(d*sec(f*x + e)), x)
```

Sympy [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx=\int \frac{a+b\tan\left(e+fx\right)}{\sqrt[3]{d\sec\left(e+fx\right)}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(1/3),x)
```

output
```
Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx=\int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{a+b\tan(e+f\,x)}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(1/3),x)
```

output
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\left(\int \frac{\tan(fx+e)}{\sec(fx+e)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{1}{\sec(fx+e)^{\frac{1}{3}}}dx\right)a}{d^{\frac{1}{3}}}$$

input
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

output
```
(int(tan(e + f*x)/sec(e + f*x)**(1/3),x)*b + int(1/sec(e + f*x)**(1/3),x)*
a)/d**(1/3)
```

3.635 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{3b}{5f(d\sec(e+fx))^{5/3}} - \frac{3ad\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(e+fx)\right)\sin(e+fx)}{8f(d\sec(e+fx))^{8/3}\sqrt{\sin^2(e+fx)}}$$

output
```
-3/5*b/f/(d*sec(f*x+e))^(5/3)-3/8*a*d*hypergeom([1/2, 4/3],[7/3],cos(f*x+e
)^2)*sin(f*x+e)/f/(d*sec(f*x+e))^(8/3)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{3\left(b+a\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{5f(d\sec(e+fx))^{5/3}}$$

input
```
Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

output
```
(-3*(b + a*Cot[e + f*x]*Hypergeometric2F1[-5/6, 1/2, 1/6, Sec[e + f*x]^2]*
Sqrt[-Tan[e + f*x]^2]))/(5*f*(d*Sec[e + f*x])^(5/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}dx$$
$$\Big\downarrow \text{3967}$$
$$a\int \frac{1}{(d\sec(e+fx))^{5/3}}dx-\frac{3b}{5f(d\sec(e+fx))^{5/3}}$$
$$\Big\downarrow \text{3042}$$
$$a\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/3}}dx-\frac{3b}{5f(d\sec(e+fx))^{5/3}}$$
$$\Big\downarrow \text{4259}$$
$$a\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\int \left(\frac{\cos(e+fx)}{d}\right)^{5/3}dx-\frac{3b}{5f(d\sec(e+fx))^{5/3}}$$
$$\Big\downarrow \text{3042}$$
$$a\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\int \left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{5/3}dx-\frac{3b}{5f(d\sec(e+fx))^{5/3}}$$
$$\Big\downarrow \text{3122}$$
$$-\frac{3ad\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{8/3}}-\frac{3b}{5f(d\sec(e+fx))^{5/3}}$$

input
```
Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

output
```
(-3*b)/(5*f*(d*Sec[e + f*x])^(5/3)) - (3*a*d*Hypergeometric2F1[1/2, 4/3, 7
/3, Cos[e + f*x]^2]*Sin[e + f*x])/(8*f*(d*Sec[e + f*x])^(8/3)*Sqrt[Sin[e +
 f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{a+b\tan\left(fx+e\right)}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}dx$$

input
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

output
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{b\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)/(d^2*sec(f*x + e)^2),
 x)
```

Sympy [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(5/3),x)
```

output
```
Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(5/3), x)
```

Maxima [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{b\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

Giac [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{a+b\tan(e+f\,x)}{{\left(\frac{d}{\cos(e+f\,x)}\right)}^{5/3}}\,dx$$

input
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(5/3),x)
```

output
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(5/3), x)
```

Reduce [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx=\frac{\left(\int \frac{\tan(fx+e)}{\sec(fx+e)^{\frac{5}{3}}}dx\right)b+\left(\int \frac{1}{\sec(fx+e)^{\frac{5}{3}}}dx\right)a}{d^{\frac{5}{3}}}$$

input
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

output
```
(int(tan(e + f*x)/(sec(e + f*x)**(2/3)*sec(e + f*x)),x)*b + int(1/(sec(e +
 f*x)**(2/3)*sec(e + f*x)),x)*a)/(d**(2/3)*d)
```

3.636 $\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx = \frac{33ab(d\sec(e+fx))^{5/3}}{40f} + \frac{3(8a^2-3b^2)\,d\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sin(e+fx)}{16f\sqrt{\sin^2(e+fx)}} + \frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

output
```
33/40*a*b*(d*sec(f*x+e))^(5/3)/f+3/16*(8*a^2-3*b^2)*d*hypergeom([-1/3, 1/2
],[2/3],cos(f*x+e)^2)*(d*sec(f*x+e))^(2/3)*sin(f*x+e)/f/(sin(f*x+e)^2)^(1/
2)+3/8*b*(d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))/f
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx = \frac{3(d\sec(e+fx))^{5/3}\left(b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{6},\frac{11}{6},\sec^2(e+fx)\right)\tan(e+fx)+a\right.}{5f\sqrt{-\tan}}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2,x]
```

output
```
(3*(d*Sec[e + f*x])^(5/3)*(b^2*Hypergeometric2F1[-1/2, 5/6, 11/6, Sec[e +
f*x]^2]*Tan[e + f*x] + a*(-(a*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[e + f*
x]^2]*Tan[e + f*x]) + 2*b*Sqrt[-Tan[e + f*x]^2])))/(5*f*Sqrt[-Tan[e + f*x]
^2])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2dx$$

$$\Big\downarrow \text{3993}$$

$$\frac{3}{8}\int \frac{1}{3}(d\sec(e+fx))^{5/3}\left(8a^2+11b\tan(e+fx)a-3b^2\right)dx+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int (d\sec(e+fx))^{5/3}\left(8a^2+11b\tan(e+fx)a-3b^2\right)dx+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int (d\sec(e+fx))^{5/3}\left(8a^2+11b\tan(e+fx)a-3b^2\right)dx+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{8}\left(\left(8a^2-3b^2\right)\int (d\sec(e+fx))^{5/3}dx+\frac{33ab(d\sec(e+fx))^{5/3}}{5f}\right)+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\left(8a^2-3b^2\right)\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/3}dx+\frac{33ab(d\sec(e+fx))^{5/3}}{5f}\right)+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

$$\Big\downarrow \text{4259}$$

$$\frac{1}{8}\left(\left(8a^2-3b^2\right)\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\int \frac{1}{\left(\frac{\cos(e+fx)}{d}\right)^{5/3}}dx+\frac{33ab(d\sec(e+fx))^{5/3}}{5f}\right)+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\left(8a^2-3b^2\right)\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\int \frac{1}{\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{5/3}}dx+\frac{33ab(d\sec(e+fx))^{5/3}}{5f}\right)+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

$$\Big\downarrow \text{3122}$$

$$\frac{1}{8}\left(\frac{3d\left(8a^2-3b^2\right)\sin(e+fx)(d\sec(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}}+\frac{33ab(d\sec(e+f}{5f}\right.$$
$$\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

input
```
Int[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2,x]
```

output
```
((33*a*b*(d*Sec[e + f*x])^(5/3))/(5*f) + (3*(8*a^2 - 3*b^2)*d*Hypergeometr
ic2F1[-1/3, 1/2, 2/3, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(2/3)*Sin[e + f*x])
/(2*f*Sqrt[Sin[e + f*x]^2]))/8 + (3*b*(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e
+ f*x]))/(8*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{5}{3}} (a+b\tan(fx+e))^2 \, dx$$

input
```
int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2 \, dx = \int (d\sec(fx+e))^{\frac{5}{3}} (b\tan(fx+e)+a)^2 \, dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((b^2*d*sec(f*x + e)*tan(f*x + e)^2 + 2*a*b*d*sec(f*x + e)*tan(f*x
 + e) + a^2*d*sec(f*x + e))*(d*sec(f*x + e))^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2 \, dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/3)*(a+b*tan(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{5}{3}}(b\tan(fx+e)+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{5}{3}}(b\tan(fx+e)+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}(a+b\tan(e+f\,x))^2\,dx$$

input
```
int((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))^2, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx = \frac{d^{\frac{5}{3}}\left(6\sec\left(fx+e\right)^{\frac{5}{3}}ab+5\left(\int\sec\left(fx+e\right)^{\frac{5}{3}}\tan\left(fx+e\right)^{2}dx\right)b^{2}f+5\left(\int\sec\left(fx+e\right)\right.\right.}{5f}$$

input
```
int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x)
```

output
```
(d**(2/3)*d*(6*sec(e + f*x)**(2/3)*sec(e + f*x)*a*b + 5*int(sec(e + f*x)**
(2/3)*sec(e + f*x)*tan(e + f*x)**2,x)*b**2*f + 5*int(sec(e + f*x)**(2/3)*s
ec(e + f*x),x)*a**2*f))/(5*f)
```

3.637 $\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\begin{aligned}\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx &= \frac{21ab\sqrt[3]{d\sec(e+fx)}}{4f} \\ &\quad - \frac{3(4a^2-3b^2)\,d\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(e+fx)\right)\sin(e+fx)}{8f(d\sec(e+fx))^{2/3}\sqrt{\sin^2(e+fx)}} \\ &\quad + \frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}\end{aligned}$$

output
```
21/4*a*b*(d*sec(f*x+e))^(1/3)/f-3/8*(4*a^2-3*b^2)*d*hypergeom([1/3, 1/2],[
4/3],cos(f*x+e)^2)*sin(f*x+e)/f/(d*sec(f*x+e))^(2/3)/(sin(f*x+e)^2)^(1/2)+
3/4*b*(d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))/f
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.89

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \frac{3\sqrt[3]{d\sec(e+fx)}\left(\frac{b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6},\frac{7}{6},\sec^2(e+fx)\right)\tan(e+fx)}{\sqrt{-\tan^2(e+fx)}}+a\left(2b+a\cot(e+fx)\,\text{Hypergeometric2F}\right.\right.}{f}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2,x]
```

output
```
(3*(d*Sec[e + f*x])^(1/3)*((b^2*Hypergeometric2F1[-1/2, 1/6, 7/6, Sec[e +
f*x]^2]*Tan[e + f*x])/Sqrt[-Tan[e + f*x]^2] + a*(2*b + a*Cot[e + f*x]*Hype
rgeometric2F1[1/6, 1/2, 7/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2])))/f
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2dx$$

$$\Big\downarrow \text{3993}$$

$$\frac{3}{4}\int \frac{1}{3}\sqrt[3]{d\sec(e+fx)}\left(4a^2+7b\tan(e+fx)a-3b^2\right)dx+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \sqrt[3]{d\sec(e+fx)}\left(4a^2+7b\tan(e+fx)a-3b^2\right)dx+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\int \sqrt[3]{d\sec(e+fx)}\left(4a^2+7b\tan(e+fx)a-3b^2\right)dx+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

$$\Big\downarrow \; 3967$$

$$\frac{1}{4}\left(\left(4a^2-3b^2\right)\int \sqrt[3]{d\sec(e+fx)}dx+\frac{21ab\sqrt[3]{d\sec(e+fx)}}{f}\right)+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\left(4a^2-3b^2\right)\int \sqrt[3]{d\csc\left(e+fx+\frac{\pi}{2}\right)}dx+\frac{21ab\sqrt[3]{d\sec(e+fx)}}{f}\right)+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

$$\Big\downarrow \; 4259$$

$$\frac{1}{4}\left(\left(4a^2-3b^2\right)\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\int \frac{1}{\sqrt[3]{\frac{\cos(e+fx)}{d}}}dx+\frac{21ab\sqrt[3]{d\sec(e+fx)}}{f}\right)+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\left(4a^2-3b^2\right)\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\int \frac{1}{\sqrt[3]{\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}}}dx+\frac{21ab\sqrt[3]{d\sec(e+fx)}}{f}\right)+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

$$\Big\downarrow \; 3122$$

$$\frac{1}{4}\left(\frac{21ab\sqrt[3]{d\sec(e+fx)}}{f}-\frac{3d(4a^2-3b^2)\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{2/3}}\right)+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2,x]
```

output
```
((21*a*b*(d*Sec[e + f*x])^(1/3))/f - (3*(4*a^2 - 3*b^2)*d*Hypergeometric2F
1[1/3, 1/2, 4/3, Cos[e + f*x]^2]*Sin[e + f*x])/(2*f*(d*Sec[e + f*x])^(2/3)
*Sqrt[Sin[e + f*x]^2]))/4 + (3*b*(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x
]))/(4*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{1}{3}} (a+b\tan(fx+e))^2 \, dx$$

input
```
int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2 \, dx = \int (d\sec(fx+e))^{\frac{1}{3}} (b\tan(fx+e)+a)^2 \, dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^
(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \int \sqrt[3]{d\sec{(e+fx)}}(a+b\tan{(e+fx)})^2\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/3)*(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \int (d\sec{(fx+e)})^{\frac{1}{3}}\,(b\tan{(fx+e)}+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \int (d\sec{(fx+e)})^{\frac{1}{3}}\,(b\tan{(fx+e)}+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{1/3}\left(a+b\tan(e+f\,x)\right)^2\,dx$$

input
```
int((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))^2, x)
```

Reduce [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$$
$$= \frac{d^{\frac{1}{3}}\left(6\sec\left(fx+e\right)^{\frac{1}{3}}ab+\left(\int \sec\left(fx+e\right)^{\frac{1}{3}}dx\right)a^2f+\left(\int \sec\left(fx+e\right)^{\frac{1}{3}}\tan\left(fx+e\right)^2dx\right)b^2f\right)}{f}$$

input
```
int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x)
```

output
```
(d**(1/3)*(6*sec(e + f*x)**(1/3)*a*b + int(sec(e + f*x)**(1/3),x)*a**2*f +
 int(sec(e + f*x)**(1/3)*tan(e + f*x)**2,x)*b**2*f))/f
```

3.638 $\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{15ab}{2f\sqrt[3]{d\sec(e+fx)}} - \frac{3(2a^2-3b^2)\,d\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(e+fx)\right)\sin(e+fx)}{8f(d\sec(e+fx))^{4/3}\sqrt{\sin^2(e+fx)}} + \frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

output
```
-15/2*a*b/f/(d*sec(f*x+e))^(1/3)-3/8*(2*a^2-3*b^2)*d*hypergeom([1/2, 2/3],
[5/3],cos(f*x+e)^2)*sin(f*x+e)/f/(d*sec(f*x+e))^(4/3)/(sin(f*x+e)^2)^(1/2)
+3/2*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(1/3)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx =$$
$$-\frac{3\left(b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},\sec^2(e+fx)\right)\tan(e+fx)+a\left(-a\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\right.\right.\right.}{f\sqrt[3]{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

input
```
Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

output
```
(-3*(b^2*Hypergeometric2F1[-1/2, -1/6, 5/6, Sec[e + f*x]^2]*Tan[e + f*x] +
 a*(-(a*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[e + f*x]^2]*Tan[e + f*x]) +
2*b*Sqrt[-Tan[e + f*x]^2])))/(f*(d*Sec[e + f*x])^(1/3)*Sqrt[-Tan[e + f*x]^
2])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3993}$$
$$\frac{3}{2}\int \frac{2a^2+5b\tan(e+fx)a-3b^2}{3\sqrt[3]{d\sec(e+fx)}}dx+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$
$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{2a^2+5b\tan(e+fx)a-3b^2}{\sqrt[3]{d\sec(e+fx)}}dx+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{2a^2+5b\tan(e+fx)a-3b^2}{\sqrt[3]{d\sec(e+fx)}}dx+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{2}\left(\left(2a^2-3b^2\right)\int \frac{1}{\sqrt[3]{d\sec(e+fx)}}dx-\frac{15ab}{f\sqrt[3]{d\sec(e+fx)}}\right)+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\left(2a^2-3b^2\right)\int \frac{1}{\sqrt[3]{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{15ab}{f\sqrt[3]{d\sec(e+fx)}}\right)+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{4259}$$

$$\frac{1}{2}\left(\left(2a^2-3b^2\right)\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\int \sqrt[3]{\frac{\cos(e+fx)}{d}}dx-\frac{15ab}{f\sqrt[3]{d\sec(e+fx)}}\right)+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\left(2a^2-3b^2\right)\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\int \sqrt[3]{\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}}dx-\frac{15ab}{f\sqrt[3]{d\sec(e+fx)}}\right)+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{1}{2}\left(-\frac{3d\left(2a^2-3b^2\right)\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(e+fx)\right)}{4f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{4/3}}-\frac{15ab}{f\sqrt[3]{d\sec(e+fx)}}\right)+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

input

```
Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

output
```
((-15*a*b)/(f*(d*Sec[e + f*x])^(1/3)) - (3*(2*a^2 - 3*b^2)*d*Hypergeometri
c2F1[1/2, 2/3, 5/3, Cos[e + f*x]^2]*Sin[e + f*x])/(4*f*(d*Sec[e + f*x])^(4
/3)*Sqrt[Sin[e + f*x]^2]))/2 + (3*b*(a + b*Tan[e + f*x]))/(2*f*(d*Sec[e +
f*x])^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{(a+b\tan(fx+e))^2}{(d\sec(fx+e))^{\frac{1}{3}}}dx$$

input
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

output
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^
(2/3)/(d*sec(f*x + e)), x)
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(1/3),x)
```

output
```
Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(1/3),x)
```

output
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\left(\int \frac{\tan(fx+e)^2}{\sec(fx+e)^{\frac{1}{3}}}dx\right)b^2+2\left(\int \frac{\tan(fx+e)}{\sec(fx+e)^{\frac{1}{3}}}dx\right)ab+\left(\int \frac{1}{\sec(fx+e)^{\frac{1}{3}}}dx\right)a^2}{d^{\frac{1}{3}}}$$

input
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

output
```
(int(tan(e + f*x)**2/sec(e + f*x)**(1/3),x)*b**2 + 2*int(tan(e + f*x)/sec(
e + f*x)**(1/3),x)*a*b + int(1/sec(e + f*x)**(1/3),x)*a**2)/d**(1/3)
```

3.639 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$
\begin{aligned}
\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx &= \frac{3ab}{10f(d\sec(e+fx))^{5/3}} \\
&\quad - \frac{3(2a^2+3b^2)\,d\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(e+fx)\right)\sin(e+fx)}{16f(d\sec(e+fx))^{8/3}\sqrt{\sin^2(e+fx)}} \\
&\quad - \frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}
\end{aligned}
$$

output
```
3/10*a*b/f/(d*sec(f*x+e))^(5/3)-3/16*(2*a^2+3*b^2)*d*hypergeom([1/2, 4/3],
[7/3],cos(f*x+e)^2)*sin(f*x+e)/f/(d*sec(f*x+e))^(8/3)/(sin(f*x+e)^2)^(1/2)
-3/2*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(5/3)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$
\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{3\Big(b^2\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{6},\sec^2(e+fx)\right)\sin(e+fx)+a\Big(-a\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\text{s}\right.}{5df(d\sec(e+fx))^{2/3}\sqrt{-\tan^2(e+fx)}}
$$

input
```
Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
```

output
```
(-3*(b^2*Hypergeometric2F1[-5/6, -1/2, 1/6, Sec[e + f*x]^2]*Sin[e + f*x] +
 a*(-(a*Hypergeometric2F1[-5/6, 1/2, 1/6, Sec[e + f*x]^2]*Sin[e + f*x]) +
2*b*Cos[e + f*x]*Sqrt[-Tan[e + f*x]^2])))/(5*d*f*(d*Sec[e + f*x])^(2/3)*Sq
rt[-Tan[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3993}$$

$$-\frac{3}{2}\int -\frac{2a^2-b\tan(e+fx)a+3b^2}{3(d\sec(e+fx))^{5/3}}dx-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{2a^2-b\tan(e+fx)a+3b^2}{(d\sec(e+fx))^{5/3}}dx-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{2a^2-b\tan(e+fx)a+3b^2}{(d\sec(e+fx))^{5/3}}dx-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{2}\left(\left(2a^2+3b^2\right)\int \frac{1}{(d\sec(e+fx))^{5/3}}dx+\frac{3ab}{5f(d\sec(e+fx))^{5/3}}\right)-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\left(2a^2+3b^2\right)\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/3}}dx+\frac{3ab}{5f(d\sec(e+fx))^{5/3}}\right)-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{4259}$$

$$\frac{1}{2}\left(\left(2a^2+3b^2\right)\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\int \left(\frac{\cos(e+fx)}{d}\right)^{5/3}dx+\frac{3ab}{5f(d\sec(e+fx))^{5/3}}\right)-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\left(2a^2+3b^2\right)\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\int \left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{5/3}dx+\frac{3ab}{5f(d\sec(e+fx))^{5/3}}\right)-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{1}{2}\left(\frac{3ab}{5f(d\sec(e+fx))^{5/3}}-\frac{3d\left(2a^2+3b^2\right)\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{8/3}}\right)-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

input
```
Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
```

output
```
((3*a*b)/(5*f*(d*Sec[e + f*x])^(5/3)) - (3*(2*a^2 + 3*b^2)*d*Hypergeometri
c2F1[1/2, 4/3, 7/3, Cos[e + f*x]^2]*Sin[e + f*x])/(8*f*(d*Sec[e + f*x])^(8
/3)*Sqrt[Sin[e + f*x]^2]))/2 - (3*b*(a + b*Tan[e + f*x]))/(2*f*(d*Sec[e +
f*x])^(5/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{(a+b\tan(fx+e))^2}{(d\sec(fx+e))^{\frac{5}{3}}}dx$$

input
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

output
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
```

output
```
integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^
(1/3)/(d^2*sec(f*x + e)^2), x)
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(5/3),x)
```

output
```
Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(5/3), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}\,dx$$

input
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(5/3),x)
```

output
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(5/3), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \frac{\left(\int \frac{\tan(fx+e)^2}{\sec(fx+e)^{\frac{5}{3}}}dx\right)b^2 + 2\left(\int \frac{\tan(fx+e)}{\sec(fx+e)^{\frac{5}{3}}}dx\right)ab + \left(\int \frac{1}{\sec(fx+e)^{\frac{5}{3}}}dx\right)a^2}{d^{\frac{5}{3}}}$$

input

```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

output

```
(int(tan(e + f*x)**2/(sec(e + f*x)**(2/3)*sec(e + f*x)),x)*b**2 + 2*int(ta
n(e + f*x)/(sec(e + f*x)**(2/3)*sec(e + f*x)),x)*a*b + int(1/(sec(e + f*x)
**(2/3)*sec(e + f*x)),x)*a**2)/(d**(2/3)*d)
```

3.640 $\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 552

$$
\begin{aligned}
\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx &= -\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{2b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}} \\
&+\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{2b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}} \\
&-\frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}} \\
&+\frac{\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{4b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}} \\
&-\frac{\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{4b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}} \\
&+\frac{\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{af\sec^2(e+fx)^{5/6}}
\end{aligned}
$$

output
```
1/2*3^(1/2)*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)*3^(1/2)/(
a^2+b^2)^(1/6))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(1/6)/f/(sec(f*x+e)
^2)^(5/6)+1/2*3^(1/2)*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)*
3^(1/2)/(a^2+b^2)^(1/6))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(1/6)/f/(s
ec(f*x+e)^2)^(5/6)-arctanh(b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(
d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(1/6)/f/(sec(f*x+e)^2)^(5/6)+1/4*ln(
(a^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(
f*x+e)^2)^(1/3))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(1/6)/f/(sec(f*x+e
)^2)^(5/6)-1/4*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(
1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^
(1/6)/f/(sec(f*x+e)^2)^(5/6)+AppellF1(1/2,1,1/6,3/2,b^2*tan(f*x+e)^2/a^2,-
tan(f*x+e)^2)*(d*sec(f*x+e))^(5/3)*tan(f*x+e)/a/f/(sec(f*x+e)^2)^(5/6)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 14.60 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.50

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx =$$

$$-\frac{24d^2\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{6},\frac{1}{6},\frac{4}{3},\frac{a-ib}{a+b\tan(e+fx)}\right.}{bf\sqrt[3]{d\sec(e+fx)}\left((a+ib)\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{6},\frac{7}{6},\frac{7}{3},\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+(a-ib)\,\text{AppellF1}\left(\frac{4}{3},\frac{7}{6},\frac{1}{6}\right.\right.}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x]),x]
```

output
```
(-24*d^2*AppellF1[1/3, 1/6, 1/6, 4/3, (a - I*b)/(a + b*Tan[e + f*x]), (a +
 I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x]))/(b*f*(d*Sec[e + f*x])^(1
/3)*((a + I*b)*AppellF1[4/3, 1/6, 7/6, 7/3, (a - I*b)/(a + b*Tan[e + f*x])
, (a + I*b)/(a + b*Tan[e + f*x])] + (a - I*b)*AppellF1[4/3, 7/6, 1/6, 7/3,
 (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 8*Appel
lF1[1/3, 1/6, 1/6, 4/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*T
an[e + f*x])]*(a + b*Tan[e + f*x])))
```

Rubi [A] (warning: unable to verify)

Time = 0.74 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.69, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3994, 504, 333, 353, 73, 825, 27, 221, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \ 3994$$

$$\frac{(d\sec(e+fx))^{5/3}\int \frac{1}{(a+b\tan(e+fx))\sqrt[6]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{bf\sec^2(e+fx)^{5/6}}$$

$$\Big\downarrow \ 504$$

$$\frac{(d\sec(e+fx))^{5/3}\left(a\int \frac{1}{\sqrt[6]{\tan^2(e+fx)+1}(a^2-b^2\tan^2(e+fx))}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{\sqrt[6]{\tan^2(e+fx)+1}(a^2-b^2\tan^2(e+fx))}\right.}{bf\sec^2(e+fx)^{5/6}}$$

$$\Big\downarrow \ 333$$

$$\frac{(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\int \frac{b\tan(e+fx)}{\sqrt[6]{\tan^2(e+fx)+1}(a^2-b^2\tan^2(e+fx))}d\right.}{bf\sec^2(e+fx)^{5/6}}$$

$$\Big\downarrow \ 353$$

$$\frac{(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\frac{1}{2}\int \frac{1}{\sqrt[6]{\frac{\tan(e+fx)}{b}+1}(a^2-b^2\tan^2(e+fx))}\right.}{bf\sec^2(e+fx)^{5/6}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\int\frac{b^4\tan^4(e+fx)}{-\tan^6(e+fx)b^8+b^2+a^2}d\sqrt[6]{\frac{\tan(e+f}{b}}\right.}{bf\sec^2(e+fx)^{5/6}}$$

$$\Big\downarrow \text{825}$$

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{8/3}\tan^2(e+fx)}d\sqrt[6]{\tan(e}}{3b^{4/3}}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{8/3}\tan^2(e+fx)}d\sqrt[6]{\tan(e}}{3b^{4/3}}\right.\right.$$

$$\Big\downarrow \text{221}$$

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(-\frac{\int\frac{\tan(e+fx)b^{4/3}+\sqrt[6]{a^2+}}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)}}{6b^{4/3}\sqrt[6]{}}\right.\right.$$

$$\Big\downarrow \text{1142}$$

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(-\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2-}}}{}\right.\right.$$

$\downarrow$ 25

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(-\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2-}}}{}\right.\right.$$

$\downarrow$ 27

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(-\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2-}}}{}\right.\right.$$

$\downarrow$ 1082

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(-\frac{\frac{3\int\frac{1}{\frac{2b^{4/3}\tan(e+fx)}{\sqrt[6]{a^2+b^2}}-4}d\left(1-\frac{2b^{4/3}\tan(e+f}{\sqrt[6]{a^2+b^2}}\right.}{\sqrt[3]{b}}}{}\right.\right.$$

$\downarrow$ 217

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(-\frac{-\frac{1}{2}\int\frac{\sqrt[6]{a^2+b^2}-2b^{4/3}\,\text{ta}}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\,\tan(e+}}{}\right.\right.$$

$\downarrow$ 1103

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(-\frac{\frac{\log\left(\sqrt[3]{a^2+b^2}-b^{4/3}\sqrt[6]{a^2+b^2}\,\tan(e\right.}{2\sqrt[3]{b}}}{6}\right.\right.$$

input

```
Int[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x]),x]
```

output

```
((d*Sec[e + f*x])^(5/3)*(-3*b^2*(ArcTanh[(b^(4/3)*Tan[e + f*x])/(a^2 + b^2
)^(1/6)]/(3*b^(5/3)*(a^2 + b^2)^(1/6)) - (-((Sqrt[3]*ArcTan[(1 - (2*b^(4/3
)*Tan[e + f*x])/(a^2 + b^2)^(1/6))/Sqrt[3]])/b^(1/3)) + Log[(a^2 + b^2)^(1
/3) - b^(4/3)*(a^2 + b^2)^(1/6)*Tan[e + f*x] + b^(8/3)*Tan[e + f*x]^2]/(2*
b^(1/3)))/(6*b^(4/3)*(a^2 + b^2)^(1/6)) - ((Sqrt[3]*ArcTan[(1 + (2*b^(4/3)
*Tan[e + f*x])/(a^2 + b^2)^(1/6))/Sqrt[3]])/b^(1/3) - Log[(a^2 + b^2)^(1/3
) + b^(4/3)*(a^2 + b^2)^(1/6)*Tan[e + f*x] + b^(8/3)*Tan[e + f*x]^2]/(2*b^
(1/3)))/(6*b^(4/3)*(a^2 + b^2)^(1/6))) + (b*AppellF1[1/2, 1, 1/6, 3/2, (b^
2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a))/(b*f*(Sec[e + f*
x]^2)^(5/6))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{a+b\tan(fx+e)}dx$$

input
```
int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(e+fx))^{\frac{5}{3}}}{a+b\tan(e+fx)}\,dx$$

input
```
integrate((d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**(5/3)/(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/3)/(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/3)/(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}{a+b\tan(e+f\,x)}\,dx$$

input
```
int((d/cos(e + f*x))^(5/3)/(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(5/3)/(a + b*tan(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = d^{\frac{5}{3}}\left(\int \frac{\sec(fx+e)^{\frac{5}{3}}}{\tan(fx+e)\,b+a}dx\right)$$

input
```
int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

output
```
d**(2/3)*int((sec(e + f*x)**(2/3)*sec(e + f*x))/(tan(e + f*x)*b + a),x)*d
```

3.641 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 552

$$
\begin{aligned}
\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx &= \frac{\sqrt{3}b^{2/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{2\left(a^2+b^2\right)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} \\
&- \frac{\sqrt{3}b^{2/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{2\left(a^2+b^2\right)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} \\
&- \frac{b^{2/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{\left(a^2+b^2\right)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} \\
&+ \frac{b^{2/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{4\left(a^2+b^2\right)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} \\
&- \frac{b^{2/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{4\left(a^2+b^2\right)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} \\
&+ \frac{\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{af\sqrt[6]{\sec^2(e+fx)}}
\end{aligned}
$$

output
```
-1/2*3^(1/2)*b^(2/3)*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)*
3^(1/2)/(a^2+b^2)^(1/6))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(5/6)/f/(sec(f*x+e
)^2)^(1/6)-1/2*3^(1/2)*b^(2/3)*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^
2)^(1/6)*3^(1/2)/(a^2+b^2)^(1/6))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(5/6)/f/(
sec(f*x+e)^2)^(1/6)-b^(2/3)*arctanh(b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)
^(1/6))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(5/6)/f/(sec(f*x+e)^2)^(1/6)+1/4*b^
(2/3)*ln((a^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2
/3)*(sec(f*x+e)^2)^(1/3))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(5/6)/f/(sec(f*x+
e)^2)^(1/6)-1/4*b^(2/3)*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*
x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)
^(5/6)/f/(sec(f*x+e)^2)^(1/6)+AppellF1(1/2,1,5/6,3/2,b^2*tan(f*x+e)^2/a^2,
-tan(f*x+e)^2)*(d*sec(f*x+e))^(1/3)*tan(f*x+e)/a/f/(sec(f*x+e)^2)^(1/6)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.66 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx =$$

$$-\frac{48d^2\,\text{AppellF1}\left(\frac{5}{3},\frac{5}{6},\frac{5}{6},\frac{8}{3},\frac{\cdots}{a+b\cdots}\right.}{5bf(d\sec(e+fx))^{5/3}\left(5(a+ib)\,\text{AppellF1}\left(\frac{8}{3},\frac{5}{6},\frac{11}{6},\frac{11}{3},\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+5(a-ib)\,\text{AppellF}\cdots\right.}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x]),x]
```

output
```
(-48*d^2*AppellF1[5/3, 5/6, 5/6, 8/3, (a - I*b)/(a + b*Tan[e + f*x]), (a +
 I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x]))/(5*b*f*(d*Sec[e + f*x])^
(5/3)*(5*(a + I*b)*AppellF1[8/3, 5/6, 11/6, 11/3, (a - I*b)/(a + b*Tan[e +
 f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 5*(a - I*b)*AppellF1[8/3, 11/6,
5/6, 11/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]
 + 16*AppellF1[5/3, 5/6, 5/6, 8/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*
b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x])))
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.67, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3994, 504, 333, 353, 73, 754, 27, 221, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int \frac{1}{(a+b\tan(e+fx))(\tan^2(e+fx)+1)^{5/6}}d(b\tan(e+fx))}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{504}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\left(a\int \frac{1}{(\tan^2(e+fx)+1)^{5/6}(a^2-b^2\tan^2(e+fx))}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{(\tan^2(e+fx)+1)^{5/6}(a^2-b^2\tan^2(e+fx))}d(b\tan(e+fx))\right)}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{333}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\int \frac{b\tan(e+fx)}{(\tan^2(e+fx)+1)^{5/6}(a^2-b^2\tan^2(e+fx))}d(b\tan(e+fx))\right)}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{353}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\frac{1}{2}\int \frac{1}{\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}(a^2-b^2\tan^2(e+fx))}d(b^2\tan^2(e+fx))\right)}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\int\frac{1}{-\tan^6(e+fx)b^8+b^2+a^2}d\sqrt[6]{\frac{\tan(e+fx)}{b}}\right.}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{754}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{8/3}\tan^2(e+fx)}d\sqrt[6]{\frac{\tan(e+}{b}}}{3(a^2+b^2)^{2/3}}\right.\right.}{}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{8/3}\tan^2(e+fx)}d\sqrt[6]{\frac{\tan(e+}{b}}}{3(a^2+b^2)^{2/3}}\right.\right.}{}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\int\frac{2\sqrt[6]{a^2+b^2}-b^{4/3}\tan(e+fx)}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+}}{6(a^2+b^2)^5}\right.\right.}{}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{1}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}}}{}\right.\right.}{}$$

$\big\downarrow$ 25

$$\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{1}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\ldots}\ldots}{\ldots}\right.\right.$$

$\big\downarrow$ 27

$$\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{1}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\ldots}\ldots}{\ldots}\right.\right.$$

$\big\downarrow$ 1082

$$\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\frac{1}{2}\int\frac{\sqrt[6]{a^2+b^2}-2b^{4/3}\tan(e+fx\ldots}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}\ldots}\ldots}{\ldots}\right.\right.$$

$\big\downarrow$ 217

$$\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\frac{1}{2}\int\frac{\sqrt[6]{a^2+b^2}-2b^{4/3}\tan(e+fx\ldots}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}\ldots}\ldots}{\ldots}\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2b^{4/3}\tan(e+fx)}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a^2}\right.}{}}{6(a}\right.\right.$$

input
```
Int[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x]),x]
```

output
```
((d*Sec[e + f*x])^(1/3)*(-3*b^2*(ArcTanh[(b^(4/3)*Tan[e + f*x])/(a^2 + b^2
)^(1/6)]/(3*b^(1/3)*(a^2 + b^2)^(5/6)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(4/3
)*Tan[e + f*x])/(a^2 + b^2)^(1/6))/Sqrt[3]])/b^(1/3)) - Log[(a^2 + b^2)^(1
/3) - b^(4/3)*(a^2 + b^2)^(1/6)*Tan[e + f*x] + b^(8/3)*Tan[e + f*x]^2]/(2*
b^(1/3)))/(6*(a^2 + b^2)^(5/6)) + ((Sqrt[3]*ArcTan[(1 + (2*b^(4/3)*Tan[e +
 f*x])/(a^2 + b^2)^(1/6))/Sqrt[3]])/b^(1/3) + Log[(a^2 + b^2)^(1/3) + b^(4
/3)*(a^2 + b^2)^(1/6)*Tan[e + f*x] + b^(8/3)*Tan[e + f*x]^2]/(2*b^(1/3)))/
(6*(a^2 + b^2)^(5/6))) + (b*AppellF1[1/2, 1, 5/6, 3/2, (b^2*Tan[e + f*x]^2
)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a))/(b*f*(Sec[e + f*x]^2)^(1/6))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{a+b\tan(fx+e)}dx$$

input
```
int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**(1/3)/(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx=\int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}{a+b\tan(e+f\,x)}\,dx$$

input
```
int((d/cos(e + f*x))^(1/3)/(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(1/3)/(a + b*tan(e + f*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx=d^{\frac{1}{3}}\left(\int \frac{\sec(fx+e)^{\frac{1}{3}}}{\tan(fx+e)\,b+a}dx\right)$$

input
```
int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

output
```
d**(1/3)*int(sec(e + f*x)**(1/3)/(tan(e + f*x)*b + a),x)
```

3.642 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 579

$$
\begin{aligned}
\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx &= \frac{3b}{(a^2+b^2)\,f\,\sqrt[3]{d\sec(e+fx)}} \\
&- \frac{\sqrt{3}b^{4/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{7/6}f\sqrt[3]{d\sec(e+fx)}} \\
&+ \frac{\sqrt{3}b^{4/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{7/6}f\sqrt[3]{d\sec(e+fx)}} \\
&- \frac{b^{4/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{\left(a^2+b^2\right)^{7/6}f\sqrt[3]{d\sec(e+fx)}} \\
&+ \frac{b^{4/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[6]{\sec^2(e+fx)}}{4\left(a^2+b^2\right)^{7/6}f\sqrt[3]{d\sec(e+fx)}} \\
&- \frac{b^{4/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[6]{\sec^2(e+fx)}}{4\left(a^2+b^2\right)^{7/6}f\sqrt[3]{d\sec(e+fx)}} \\
&+ \frac{\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{af\sqrt[3]{d\sec(e+fx)}}
\end{aligned}
$$

output
```
3*b/(a^2+b^2)/f/(d*sec(f*x+e))^(1/3)+1/2*3^(1/2)*b^(4/3)*arctan(-1/3*3^(1/
2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)*3^(1/2)/(a^2+b^2)^(1/6))*(sec(f*x+e)^2
)^(1/6)/(a^2+b^2)^(7/6)/f/(d*sec(f*x+e))^(1/3)+1/2*3^(1/2)*b^(4/3)*arctan(
1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)*3^(1/2)/(a^2+b^2)^(1/6))*(sec
(f*x+e)^2)^(1/6)/(a^2+b^2)^(7/6)/f/(d*sec(f*x+e))^(1/3)-b^(4/3)*arctanh(b^
(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)
^(7/6)/f/(d*sec(f*x+e))^(1/3)+1/4*b^(4/3)*ln((a^2+b^2)^(1/3)-b^(1/3)*(a^2+
b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^
2)^(1/6)/(a^2+b^2)^(7/6)/f/(d*sec(f*x+e))^(1/3)-1/4*b^(4/3)*ln((a^2+b^2)^(
1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(
1/3))*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(7/6)/f/(d*sec(f*x+e))^(1/3)+AppellF1
(1/2,1,7/6,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(sec(f*x+e)^2)^(1/6)*ta
n(f*x+e)/a/f/(d*sec(f*x+e))^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 48.79 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx =$$

$$-\frac{60d\,\mathrm{AppellF1}\left(\frac{7}{3},\frac{7}{6},\frac{7}{6},\frac{10}{3},\frac{a-ib}{a+b\tan(e+fx)}\ldots\right.}{7bf(d\sec(e+fx))^{4/3}\left(7(a+ib)\,\mathrm{AppellF1}\left(\frac{10}{3},\frac{7}{6},\frac{13}{6},\frac{13}{3},\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+7(a-ib)\,\mathrm{Appell}\ldots\right.}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])),x]
```

output
```
(-60*d*AppellF1[7/3, 7/6, 7/6, 10/3, (a - I*b)/(a + b*Tan[e + f*x]), (a +
I*b)/(a + b*Tan[e + f*x])]*(a*Cos[e + f*x] + b*Sin[e + f*x]))/(7*b*f*(d*Se
c[e + f*x])^(4/3)*(7*(a + I*b)*AppellF1[10/3, 7/6, 13/6, 13/3, (a - I*b)/(
a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 7*(a - I*b)*AppellF
1[10/3, 13/6, 7/6, 13/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*
Tan[e + f*x])] + 20*AppellF1[7/3, 7/6, 7/6, 10/3, (a - I*b)/(a + b*Tan[e +
 f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x])))
```

Rubi [A] (warning: unable to verify)

Time = 0.78 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.73, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3042, 3994, 504, 333, 353, 61, 73, 825, 27, 221, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\int \frac{1}{(a+b\tan(e+fx))(\tan^2(e+fx)+1)^{7/6}}d(b\tan(e+fx))}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{504}$$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\left(a\int \frac{1}{(\tan^2(e+fx)+1)^{7/6}(a^2-b^2\tan^2(e+fx))}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{(\tan^2(e+fx)+1)^{7/6}(a^2-b^2\tan^2(e+fx))}d(b\tan\right.}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{333}$$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\int \frac{b\tan(e+fx)}{(\tan^2(e+fx)+1)^{7/6}(a^2-b^2\tan^2(e+fx))}d(b\tan(e\right.}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{353}$$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\frac{1}{2}\int \frac{1}{\left(\frac{\tan(e+fx)}{b}+1\right)^{7/6}(a^2-b^2\tan^2(e+fx))}d(b^2\tan\right.}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}-\frac{b^2\int\frac{1}{\sqrt[6]{\frac{\tan(e+fx)}{b}+1}\left(a^2-b^2\tan^2(e+fx)\right)}d(b^2\tan^2(e+fx))}{a^2+b^2}\right)+\frac{b\tan(\cdots}{\cdots}\right.}{bf\sqrt[3]{d\sec(e+fx)}}$$

$\downarrow$ 73

$$\frac{\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}-\frac{6b^4\int\frac{b^4\tan^4(e+fx)}{-\tan^6(e+fx)b^8+b^2+a^2}d\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}{a^2+b^2}\right)+\frac{b\tan(e+fx)\,\mathrm{Appel}\cdots}{\cdots}\right.}{bf\sqrt[3]{d\sec(e+fx)}}$$

$\downarrow$ 825

$$\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}-\frac{6b^4\left(\frac{\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{8/3}\tan^2(e+fx)}d\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}{3b^{4/3}}+\frac{\int-\frac{\cdots}{2\left(\tan^2(e+\cdots\right.}}{\cdots}\right.}{\cdots}\right.\right.$$

$\downarrow$ 27

$$\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}-\frac{6b^4\left(\frac{\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{8/3}\tan^2(e+fx)}d\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}{3b^{4/3}}-\frac{\int\frac{\cdots}{\tan^2(e+fx)b^{8}\cdots}}{\cdots}\right.}{\cdots}\right.\right.$$

$$\downarrow 221$$

$$\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}}+1}-\frac{6b^4\left(-\frac{\int\frac{\tan(e+fx)b^{4/3}+\sqrt[6]{a^2+b^2}}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}d\sqrt[6]{\frac{\tan(e}{b}}}{6b^{4/3}\sqrt[6]{a^2+b^2}}\right.}{}\right.\right.$$

$$\downarrow 1142$$

$$\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}}+1}-\frac{6b^4\left(-\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{1}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b}}}{}\right.}{}\right.\right.$$

$$\downarrow 25$$

$$\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}}+1}-\frac{6b^4\left(-\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{1}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b}}}{}\right.}{}\right.\right.$$

$\downarrow$ 27

$$\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}}+1}-\frac{6b^4\left(-\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{1}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b}}}{}\right.}{}\right.\right.$$

$\downarrow$ 1082

$$\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}-\frac{6b^4\left(-\frac{\frac{3\int\frac{1}{\frac{2b^{4/3}\tan(e+fx)}{\sqrt[6]{a^2+b^2}}-4}d\left(1-\frac{2b^{4/3}\tan(e+fx)}{\sqrt[6]{a^2+b^2}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[6]{a^2-}}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a}}}{6b^{4/3}\sqrt[6]{a^2+}}\right.}{}\right.\right.$$

$\downarrow$ 217

$$\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}-\frac{6b^4\left(-\frac{-\frac{1}{2}\int\frac{\sqrt[6]{a^2+b^2}-2b^{4/3}\tan(e+fx)}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}d\sqrt[6]{\tan}}{6b^{4/3}\sqrt[6]{a^2+b^2}}\right.}{}\right.\right.$$

$\downarrow$ 1103

$$\sqrt[6]{\sec^2(e+fx)}\left(\frac{b\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}+\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}-\frac{6b^4\left(-\frac{\log\left(\right.}{}\right.}{}\right.\right.$$

input
```
Int[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])),x]
```

output
```
((Sec[e + f*x]^2)^(1/6)*((b*AppellF1[1/2, 1, 7/6, 3/2, (b^2*Tan[e + f*x]^2
)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a + ((-6*b^4*(ArcTanh[(b^(4/3)*Tan[e
 + f*x])/(a^2 + b^2)^(1/6)]/(3*b^(5/3)*(a^2 + b^2)^(1/6)) - (-((Sqrt[3]*Ar
cTan[(1 - (2*b^(4/3)*Tan[e + f*x])/(a^2 + b^2)^(1/6))/Sqrt[3]])/b^(1/3)) +
 Log[(a^2 + b^2)^(1/3) - b^(4/3)*(a^2 + b^2)^(1/6)*Tan[e + f*x] + b^(8/3)*
Tan[e + f*x]^2]/(2*b^(1/3)))/(6*b^(4/3)*(a^2 + b^2)^(1/6)) - ((Sqrt[3]*Arc
Tan[(1 + (2*b^(4/3)*Tan[e + f*x])/(a^2 + b^2)^(1/6))/Sqrt[3]])/b^(1/3) - L
og[(a^2 + b^2)^(1/3) + b^(4/3)*(a^2 + b^2)^(1/6)*Tan[e + f*x] + b^(8/3)*Ta
n[e + f*x]^2]/(2*b^(1/3)))/(6*b^(4/3)*(a^2 + b^2)^(1/6))))/(a^2 + b^2) + (
6*b^2)/((a^2 + b^2)*(1 + Tan[e + f*x]/b)^(1/6)))/2))/(b*f*(d*Sec[e + f*x])
^(1/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [F]

$$\int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}(a+b\tan(fx+e))}dx$$

input
```
int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

output
```
int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx=\text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx=\int \frac{1}{\sqrt[3]{d\sec(e+fx)}\,(a+b\tan(e+fx))}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e)),x)
```

output
```
Integral(1/((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx=\int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}(b\tan(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx=\int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}(b\tan(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx=\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}(a+b\tan(e+f\,x))}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))),x)
```

output
```
int(1/((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \frac{\int \frac{1}{\sec(fx+e)^{\frac{1}{3}}\tan(fx+e)b+\sec(fx+e)^{\frac{1}{3}}a}dx}{d^{\frac{1}{3}}}$$

input
```
int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

output
```
int(1/(sec(e + f*x)**(1/3)*tan(e + f*x)*b + sec(e + f*x)**(1/3)*a),x)/d**(
1/3)
```

3.643 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 581

$$\begin{aligned}\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx &= \frac{3b}{5(a^2+b^2)f(d\sec(e+fx))^{5/3}} \\
&+ \frac{\sqrt{3}b^{8/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{2(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}} \\
&- \frac{\sqrt{3}b^{8/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{2(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}} \\
&- \frac{b^{8/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}} \\
&+ \frac{b^{8/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{4(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}} \\
&- \frac{b^{8/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{4(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}} \\
&+ \frac{\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{af(d\sec(e+fx))^{5/3}}\end{aligned}$$

output
```
3/5*b/(a^2+b^2)/f/(d*sec(f*x+e))^(5/3)-1/2*3^(1/2)*b^(8/3)*arctan(-1/3*3^(
1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)*3^(1/2)/(a^2+b^2)^(1/6))*(sec(f*x+e)
^2)^(5/6)/(a^2+b^2)^(11/6)/f/(d*sec(f*x+e))^(5/3)-1/2*3^(1/2)*b^(8/3)*arct
an(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)*3^(1/2)/(a^2+b^2)^(1/6))*(
sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(11/6)/f/(d*sec(f*x+e))^(5/3)-b^(8/3)*arctan
h(b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(sec(f*x+e)^2)^(5/6)/(a^2+
b^2)^(11/6)/f/(d*sec(f*x+e))^(5/3)+1/4*b^(8/3)*ln((a^2+b^2)^(1/3)-b^(1/3)*
(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*
x+e)^2)^(5/6)/(a^2+b^2)^(11/6)/f/(d*sec(f*x+e))^(5/3)-1/4*b^(8/3)*ln((a^2+
b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e
)^2)^(1/3))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(11/6)/f/(d*sec(f*x+e))^(5/3)+A
ppellF1(1/2,1,11/6,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(sec(f*x+e)^2)^
(5/6)*tan(f*x+e)/a/f/(d*sec(f*x+e))^(5/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 6862 vs. $2(581) = 1162$.

Time = 103.42 (sec) , antiderivative size = 6862, normalized size of antiderivative = 11.81

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 0.80 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.71, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 3994, 504, 333, 353, 61, 73, 754, 27, 221, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}dx$$

$$\Big\downarrow \text{ 3994}$$

$$\frac{\sec^2(e+fx)^{5/6}\int \frac{1}{(a+b\tan(e+fx))(\tan^2(e+fx)+1)^{11/6}}d(b\tan(e+fx))}{bf(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{ 504}$$

$$\frac{\sec^2(e+fx)^{5/6}\left(a\int \frac{1}{(\tan^2(e+fx)+1)^{11/6}(a^2-b^2\tan^2(e+fx))}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{(\tan^2(e+fx)+1)^{11/6}(a^2-b^2\tan^2(e+fx))}d(b\right.}{bf(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{ 333}$$

$$\frac{\sec^2(e+fx)^{5/6}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\int \frac{b\tan(e+fx)}{(\tan^2(e+fx)+1)^{11/6}(a^2-b^2\tan^2(e+fx))}d(b\tan\right.}{bf(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{ 353}$$

$$\frac{\sec^2(e+fx)^{5/6}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\frac{1}{2}\int \frac{1}{\left(\frac{\tan(e+fx)}{b}+1\right)^{11/6}(a^2-b^2\tan^2(e+fx))}d(b^2\right.}{bf(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{b^2\int \frac{1}{\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}\left(a^2-b^2\tan^2(e+fx)\right)}d(b^2\tan^2(e+fx))}{a^2+b^2}\right)+\frac{b\tan(e+fx)\operatorname{Ap}}{}\right.}{bf(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\int\frac{1}{-\tan^6(e+fx)b^8+b^2+a^2}d\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}{a^2+b^2}\right)+\frac{b\tan(e+fx)\operatorname{AppellF1}}{}\right.\right.}{bf(d\sec(e+fx))^{5/3}}$$

$\downarrow$ 754

$$\frac{\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\left(\frac{\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{8/3}\tan^2(e+fx)}d\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}{3\left(a^2+b^2\right)^{2/3}}+\frac{\int}{2\left(\tan^2(e+fx)b^{8/}\right.}\right.}{}\right.\right.}{}$$

$\downarrow$ 27

$$\frac{\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\left(\frac{\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{8/3}\tan^2(e+fx)}d\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}{3\left(a^2+b^2\right)^{2/3}}+\frac{\int\frac{2}{\tan^2(e+fx)b^{8/3}-}}{}\right.}{}\right.\right.}{}$$

$\downarrow$ 221

$$\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\left(\frac{\int \frac{2\sqrt[6]{a^2+b^2}-b^{4/3}\tan(e+fx)}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}d\sqrt[6]{\frac{\tan(e+fx)}{b}}\cdots}{6\left(a^2+b^2\right)^{5/6}}\cdots\right.}{\cdots}\cdots\right.\right.$$

$$\Big\downarrow \; 1142$$

$$\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\left(\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int \frac{1}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}d\sqrt[6]{\cdots}}{\cdots}\cdots\right.}{\cdots}\cdots\right.\right.$$

$$\Big\downarrow \; 25$$

$$\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\left(\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{1}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}d\sqrt[6]{\cdots}}{\cdots}\right.}{\cdots}\right.\right.$$

$\downarrow$ 27

$$\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\left(\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{1}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}d\sqrt[6]{\cdots}}{\cdots}\right.}{\cdots}\right.\right.$$

$\downarrow$ 1082

$$\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\left(\frac{\frac{1}{2}\int\frac{\sqrt[6]{a^2+b^2}-2b^{4/3}\tan(e+fx)}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}d\sqrt[6]{\frac{\tan(e+\ldots}{b}}}{6\left(a^2+b^2\right)^{5/6}}\right.}{\ldots}\right.\right.$$

$$\Big\downarrow \; 217$$

$$\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\left(\frac{\frac{1}{2}\int\frac{\sqrt[6]{a^2+b^2}-2b^{4/3}\tan(e+fx)}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}d\sqrt[6]{\frac{\tan(e+\ldots}{b}}}{6\left(a^2+b^2\right)^{5/6}}\right.}{\ldots}\right.\right.$$

$$\Big\downarrow \; 1103$$

$$\sec^2(e+fx)^{5/6}\left(\frac{b\tan(e+fx)\,\mathrm{AppellF1}\left(\frac{1}{2},1,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}+\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\left(-\frac{\sqrt{3}\,\mathrm{arcta}}{}\right.}{}\right.\right.$$

input
```
Int[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])),x]
```

output
```
((Sec[e + f*x]^2)^(5/6)*((b*AppellF1[1/2, 1, 11/6, 3/2, (b^2*Tan[e + f*x]^
2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a + ((-6*b^4*(ArcTanh[(b^(4/3)*Tan[
e + f*x])/(a^2 + b^2)^(1/6)]/(3*b^(1/3)*(a^2 + b^2)^(5/6)) + (-((Sqrt[3]*A
rcTan[(1 - (2*b^(4/3)*Tan[e + f*x])/(a^2 + b^2)^(1/6))/Sqrt[3]])/b^(1/3))
- Log[(a^2 + b^2)^(1/3) - b^(4/3)*(a^2 + b^2)^(1/6)*Tan[e + f*x] + b^(8/3)
*Tan[e + f*x]^2]/(2*b^(1/3)))/(6*(a^2 + b^2)^(5/6)) + ((Sqrt[3]*ArcTan[(1
+ (2*b^(4/3)*Tan[e + f*x])/(a^2 + b^2)^(1/6))/Sqrt[3]])/b^(1/3) + Log[(a^2
 + b^2)^(1/3) + b^(4/3)*(a^2 + b^2)^(1/6)*Tan[e + f*x] + b^(8/3)*Tan[e + f
*x]^2]/(2*b^(1/3)))/(6*(a^2 + b^2)^(5/6))))/(a^2 + b^2) + (6*b^2)/(5*(a^2
+ b^2)*(1 + Tan[e + f*x]/b)^(5/6)))/2))/(b*f*(d*Sec[e + f*x])^(5/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [F]

$$\int \frac{1}{\left(d\sec \left(fx+e\right)\right)^{\frac{5}{3}} \left(a+b\tan \left(fx+e\right)\right)}dx$$

input
```
int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

output
```
int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx=\text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx=\int \frac{1}{\left(d\sec\left(e+fx\right)\right)^{\frac{5}{3}}\left(a+b\tan\left(e+fx\right)\right)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e)),x)
```

output
```
Integral(1/((d*sec(e + f*x))**(5/3)*(a + b*tan(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx=\int \frac{1}{(d\sec(fx+e))^{\frac{5}{3}}\,(b\tan(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)), x)
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx=\int \frac{1}{(d\sec(fx+e))^{\frac{5}{3}}\,(b\tan(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx=\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}\,(a+b\tan(e+f\,x))}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))),x)
```

output
```
int(1/((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))), x)
```

Reduce [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx = \frac{\int \frac{1}{\sec(fx+e)^{\frac{5}{3}}\tan(fx+e)b+\sec(fx+e)^{\frac{5}{3}}a}dx}{d^{\frac{5}{3}}}$$

input
```
int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

output
```
int(1/(sec(e + f*x)**(2/3)*sec(e + f*x)*tan(e + f*x)*b + sec(e + f*x)**(2/
3)*sec(e + f*x)*a),x)/(d**(2/3)*d)
```

3.644 $\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 687

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx = -\frac{a\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{2\sqrt{3}b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx)^{5/6}}$$
$$+\frac{a\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{2\sqrt{3}b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx)^{5/6}}$$
$$-\frac{a\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{3b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx)^{5/6}}$$
$$+\frac{a\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{12b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx)^{5/6}}$$
$$-\frac{a\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{12b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx)^{5/6}}$$
$$+\frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{a^2f\sec^2(e+fx)^{5/6}}$$
$$+\frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{1}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan^3(e+fx)}{3a^4f\sec^2(e+fx)^{5/6}}$$
$$-\frac{ab(d\sec(e+fx))^{5/3}}{(a^2+b^2)\,f\,(a^2-b^2\tan^2(e+fx))}$$

output
```
1/6*a*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)*3^(1/2)/(a^2+b^
2)^(1/6))*(d*sec(f*x+e))^(5/3)*3^(1/2)/b^(2/3)/(a^2+b^2)^(7/6)/f/(sec(f*x+
e)^2)^(5/6)+1/6*a*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)*3^(1
/2)/(a^2+b^2)^(1/6))*(d*sec(f*x+e))^(5/3)*3^(1/2)/b^(2/3)/(a^2+b^2)^(7/6)/
f/(sec(f*x+e)^2)^(5/6)-1/3*a*arctanh(b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2
)^(1/6))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(7/6)/f/(sec(f*x+e)^2)^(5/
6)+1/12*a*ln((a^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+
b^(2/3)*(sec(f*x+e)^2)^(1/3))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(7/6)
/f/(sec(f*x+e)^2)^(5/6)-1/12*a*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*
(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(d*sec(f*x+e))^(5/3)/b^
(2/3)/(a^2+b^2)^(7/6)/f/(sec(f*x+e)^2)^(5/6)+AppellF1(1/2,2,1/6,3/2,b^2*ta
n(f*x+e)^2/a^2,-tan(f*x+e)^2)*(d*sec(f*x+e))^(5/3)*tan(f*x+e)/a^2/f/(sec(f
*x+e)^2)^(5/6)+1/3*b^2*AppellF1(3/2,2,1/6,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*
x+e)^2)*(d*sec(f*x+e))^(5/3)*tan(f*x+e)^3/a^4/f/(sec(f*x+e)^2)^(5/6)-a*b*(
d*sec(f*x+e))^(5/3)/(a^2+b^2)/f/(a^2-b^2*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 118.43 (sec) , antiderivative size = 8003, normalized size of antiderivative = 11.65

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x])^2,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 545, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3994, 505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3994}$$

$$\frac{(d\sec(e+fx))^{5/3}\int \frac{1}{(a+b\tan(e+fx))^2\sqrt[6]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{bf\sec^2(e+fx)^{5/6}}$$

$$\Big\downarrow \text{ 505}$$

$$\frac{(d\sec(e+fx))^{5/3}\int \left(\frac{a^2}{\sqrt[6]{\tan^2(e+fx)+1}(a^2-b^2\tan^2(e+fx))^2}-\frac{2b\tan(e+fx)a}{\sqrt[6]{\tan^2(e+fx)+1}(a^2-b^2\tan^2(e+fx))^2}+\frac{\ldots}{\sqrt[6]{\tan^2(e\ldots}}\right.}{bf\sec^2(e+fx)^{5/6}}$$

$$\Big\downarrow \text{ 2009}$$

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2}-\frac{a\sqrt[3]{b}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\tan^2(e+fx)+1}\ldots}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right.}{2\sqrt{3}(a^2+b^2)^{7/6}}\ldots\right.$$

input
```
Int[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x])^2,x]
```

output
```
((d*Sec[e + f*x])^(5/3)*(-1/2*(a*b^(1/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(1
+ Tan[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))])/(Sqrt[3]*(a^2 + b^2
)^(7/6)) + (a*b^(1/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(1 + Tan[e + f*x]^2)^(
1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))])/(2*Sqrt[3]*(a^2 + b^2)^(7/6)) - (a*b^(
1/3)*ArcTanh[(b^(1/3)*(1 + Tan[e + f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)])/(3*(
a^2 + b^2)^(7/6)) + (a*b^(1/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3)*(a^2 + b^2)
^(1/6)*(1 + Tan[e + f*x]^2)^(1/6) + b^(2/3)*(1 + Tan[e + f*x]^2)^(1/3)])/(
12*(a^2 + b^2)^(7/6)) - (a*b^(1/3)*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 +
b^2)^(1/6)*(1 + Tan[e + f*x]^2)^(1/6) + b^(2/3)*(1 + Tan[e + f*x]^2)^(1/3)
])/(12*(a^2 + b^2)^(7/6)) + (b*AppellF1[1/2, 2, 1/6, 3/2, (b^2*Tan[e + f*x
]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a^2 + (b^3*AppellF1[3/2, 2, 1/6,
5/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x]^3)/(3*a^4) -
(a*b^2*(1 + Tan[e + f*x]^2)^(5/6))/((a^2 + b^2)*(a^2 - b^2*Tan[e + f*x]^2)
)))/(b*f*(Sec[e + f*x]^2)^(5/6))
```

Defintions of rubi rules used

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{(a+b\tan(fx+e))^{2}}dx$$

input
```
int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(e+fx))^{\frac{5}{3}}}{(a+b\tan(e+fx))^{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**(5/3)/(a + b*tan(e + f*x))**2, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/3)/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}{(a+b\tan(e+f\,x))^2}\,dx$$

input
```
int((d/cos(e + f*x))^(5/3)/(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(5/3)/(a + b*tan(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx = \frac{d^{\frac{5}{3}}\left(-3\sec\left(fx+e\right)^{\frac{5}{3}}\tan\left(fx+e\right)+5\left(\int\frac{\sec(fx+e)^{\frac{5}{3}}\tan(fx+e)^{3}}{\tan(fx+e)^{2}b^{2}+2\tan(fx+e)ab+a^{2}}dx\right)\tan\right.}{}$$

input
```
int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

output
```
(d**(2/3)*d*( - 3*sec(e + f*x)**(2/3)*sec(e + f*x)*tan(e + f*x) + 5*int((s
ec(e + f*x)**(2/3)*sec(e + f*x)*tan(e + f*x)**3)/(tan(e + f*x)**2*b**2 + 2
*tan(e + f*x)*a*b + a**2),x)*tan(e + f*x)*b**2*f + 5*int((sec(e + f*x)**(2
/3)*sec(e + f*x)*tan(e + f*x)**3)/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a
*b + a**2),x)*a*b*f + 8*int((sec(e + f*x)**(2/3)*sec(e + f*x)*tan(e + f*x)
**2)/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*tan(e + f*x)*a*
b*f + 8*int((sec(e + f*x)**(2/3)*sec(e + f*x)*tan(e + f*x)**2)/(tan(e + f*
x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*a**2*f + 15*int((sec(e + f*x)**
(2/3)*sec(e + f*x))/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*
tan(e + f*x)*a*b*f + 15*int((sec(e + f*x)**(2/3)*sec(e + f*x))/(tan(e + f*
x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*a**2*f))/(12*a*f*(tan(e + f*x)*
b + a))
```

3.645 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 687

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \frac{5ab^{2/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{2\sqrt{3}\left(a^2+b^2\right)^{11/6}f\sqrt[6]{\sec^2(e+fx)}}$$
$$-\frac{5ab^{2/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{2\sqrt{3}\left(a^2+b^2\right)^{11/6}f\sqrt[6]{\sec^2(e+fx)}}$$
$$-\frac{5ab^{2/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{3\left(a^2+b^2\right)^{11/6}f\sqrt[6]{\sec^2(e+fx)}}$$
$$+\frac{5ab^{2/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{12\left(a^2+b^2\right)^{11/6}f\sqrt[6]{\sec^2(e+fx)}}$$
$$-\frac{5ab^{2/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{12\left(a^2+b^2\right)^{11/6}f\sqrt[6]{\sec^2(e+fx)}}$$
$$+\frac{\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{a^2f\sqrt[6]{\sec^2(e+fx)}}$$
$$+\frac{b^2\operatorname{AppellF1}\left(\frac{3}{2},2,\frac{5}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^3(e+fx)}{3a^4f\sqrt[6]{\sec^2(e+fx)}}$$
$$-\frac{ab\sqrt[3]{d\sec(e+fx)}}{(a^2+b^2)f\left(a^2-b^2\tan^2(e+fx)\right)}$$

output
```
-5/6*a*b^(2/3)*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)*3^(1/2
)/(a^2+b^2)^(1/6))*(d*sec(f*x+e))^(1/3)*3^(1/2)/(a^2+b^2)^(11/6)/f/(sec(f*
x+e)^2)^(1/6)-5/6*a*b^(2/3)*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^
(1/6)*3^(1/2)/(a^2+b^2)^(1/6))*(d*sec(f*x+e))^(1/3)*3^(1/2)/(a^2+b^2)^(11/
6)/f/(sec(f*x+e)^2)^(1/6)-5/3*a*b^(2/3)*arctanh(b^(1/3)*(sec(f*x+e)^2)^(1/
6)/(a^2+b^2)^(1/6))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(11/6)/f/(sec(f*x+e)^2)
^(1/6)+5/12*a*b^(2/3)*ln((a^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+
e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(
11/6)/f/(sec(f*x+e)^2)^(1/6)-5/12*a*b^(2/3)*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^
2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(d*sec(f*x
+e))^(1/3)/(a^2+b^2)^(11/6)/f/(sec(f*x+e)^2)^(1/6)+AppellF1(1/2,2,5/6,3/2,
b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(d*sec(f*x+e))^(1/3)*tan(f*x+e)/a^2/f/
(sec(f*x+e)^2)^(1/6)+1/3*b^2*AppellF1(3/2,2,5/6,5/2,b^2*tan(f*x+e)^2/a^2,-
tan(f*x+e)^2)*(d*sec(f*x+e))^(1/3)*tan(f*x+e)^3/a^4/f/(sec(f*x+e)^2)^(1/6)
-a*b*(d*sec(f*x+e))^(1/3)/(a^2+b^2)/f/(a^2-b^2*tan(f*x+e)^2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4560 vs. $2(687) = 1374$.

Time = 70.80 (sec) , antiderivative size = 4560, normalized size of antiderivative = 6.64

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x])^2,x]
```

output
```
((d*Sec[e + f*x])^(1/3)*((-2*b^2*AppellF1[7/6, 1/2, 1, 13/6, Sec[e + f*x]^
2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]
^(10/3))/(21*(a^2 + b^2)^2*Sqrt[1 - Sec[e + f*x]^2]) - (7*(3*a^2 - 2*b^2)*
AppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2
)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]^(4/3))/(3*(-1 + Sec[e + f*x]^2)*(
7*(a^2 + b^2)*AppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]
^2)/(a^2 + b^2)] + 3*(2*b^2*AppellF1[7/6, 1/2, 2, 13/6, Sec[e + f*x]^2, (b
^2*Sec[e + f*x]^2)/(a^2 + b^2)] + (a^2 + b^2)*AppellF1[7/6, 3/2, 1, 13/6,
Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)])*Sec[e + f*x]^2)*(-a^2 +
 b^2*(-1 + Sec[e + f*x]^2))) + (b^(2/3)*((-10*(-1)^(5/6)*a*ArcTan[Sqrt[3]
- (2*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6)])/(a^2 + b^2
)^(11/6) + (10*(-1)^(5/6)*a*ArcTan[Sqrt[3] + (2*(-1)^(1/6)*b^(1/3)*Sec[e +
 f*x]^(1/3))/(a^2 + b^2)^(1/6)])/(a^2 + b^2)^(11/6) + (20*(-1)^(5/6)*a*Arc
Tan[((-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6)])/(a^2 + b^2
)^(11/6) - (5*(-1)^(5/6)*Sqrt[3]*a*Log[(a^2 + b^2)^(1/3) - (-1)^(1/6)*Sqrt
[3]*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^(1/3) + (-1)^(1/3)*b^(2/3)*Sec[
e + f*x]^(2/3)])/(a^2 + b^2)^(11/6) + (5*(-1)^(5/6)*Sqrt[3]*a*Log[(a^2 + b
^2)^(1/3) + (-1)^(1/6)*Sqrt[3]*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^(1/3
) + (-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(2/3)])/(a^2 + b^2)^(11/6) - (12*a*b^(
1/3)*Sec[e + f*x]^(1/3))/((a^2 + b^2)*(a^2 + b^2 - b^2*Sec[e + f*x]^2))...
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 545, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3994, 505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int \frac{1}{(a+b\tan(e+fx))^2(\tan^2(e+fx)+1)^{5/6}}d(b\tan(e+fx))}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{505}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int \left(\frac{a^2}{(\tan^2(e+fx)+1)^{5/6}(a^2-b^2\tan^2(e+fx))^2}-\frac{2b\tan(e+fx)a}{(\tan^2(e+fx)+1)^{5/6}(a^2-b^2\tan^2(e+fx))^2}+\frac{b^2\tan^2(e+f}{(\tan^2(e+fx)+1)^{5/6}(b^2\tan}\right.}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2}+\frac{5ab^{5/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\tan^2(e+fx)+1}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)}{2\sqrt{3}(a^2+b^2)^{11/6}}\right.}{}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x])^2,x]
```

output
```
((d*Sec[e + f*x])^(1/3)*((5*a*b^(5/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(1 + T
an[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))])/(2*Sqrt[3]*(a^2 + b^2)
^(11/6)) - (5*a*b^(5/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(1 + Tan[e + f*x]^2)
^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))])/(2*Sqrt[3]*(a^2 + b^2)^(11/6)) - (5*
a*b^(5/3)*ArcTanh[(b^(1/3)*(1 + Tan[e + f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)])
/(3*(a^2 + b^2)^(11/6)) + (5*a*b^(5/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3)*(a^
2 + b^2)^(1/6)*(1 + Tan[e + f*x]^2)^(1/6) + b^(2/3)*(1 + Tan[e + f*x]^2)^(
1/3)])/(12*(a^2 + b^2)^(11/6)) - (5*a*b^(5/3)*Log[(a^2 + b^2)^(1/3) + b^(1
/3)*(a^2 + b^2)^(1/6)*(1 + Tan[e + f*x]^2)^(1/6) + b^(2/3)*(1 + Tan[e + f*
x]^2)^(1/3)])/(12*(a^2 + b^2)^(11/6)) + (b*AppellF1[1/2, 2, 5/6, 3/2, (b^2
*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a^2 + (b^3*AppellF1[3
/2, 2, 5/6, 5/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x]^3
)/(3*a^4) - (a*b^2*(1 + Tan[e + f*x]^2)^(1/6))/((a^2 + b^2)*(a^2 - b^2*Tan
[e + f*x]^2))))/(b*f*(Sec[e + f*x]^2)^(1/6))
```

Defintions of rubi rules used

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{(a+b\tan(fx+e))^2}dx$$

input
```
int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\sqrt[3]{d\sec{(e+fx)}}}{(a+b\tan{(e+fx)})^2}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**(1/3)/(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}{(a+b\tan(e+f\,x))^2}\,dx$$

input
```
int((d/cos(e + f*x))^(1/3)/(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(1/3)/(a + b*tan(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$$
$$= \frac{d^{\frac{1}{3}}\left(3\sec(fx+e)^{\frac{1}{3}}\tan(fx+e)+3\left(\int \frac{\sec(fx+e)^{\frac{1}{3}}}{\tan(fx+e)^2b^2+2\tan(fx+e)ab+a^2}dx\right)\tan(fx+e)abf+3\left(\int \frac{}{\tan(fx+e)^2}\right.\right.}{}$$

input
```
int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

output
```
(d**(1/3)*(3*sec(e + f*x)**(1/3)*tan(e + f*x) + 3*int(sec(e + f*x)**(1/3)/
(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*tan(e + f*x)*a*b*f +
 3*int(sec(e + f*x)**(1/3)/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a*
*2),x)*a**2*f - int((sec(e + f*x)**(1/3)*tan(e + f*x)**3)/(tan(e + f*x)**2
*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*tan(e + f*x)*b**2*f - int((sec(e + f
*x)**(1/3)*tan(e + f*x)**3)/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a
**2),x)*a*b*f - 4*int((sec(e + f*x)**(1/3)*tan(e + f*x)**2)/(tan(e + f*x)*
*2*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*tan(e + f*x)*a*b*f - 4*int((sec(e
+ f*x)**(1/3)*tan(e + f*x)**2)/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b
+ a**2),x)*a**2*f))/(6*a*f*(tan(e + f*x)*b + a))
```

3.646 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 715

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \frac{7ab}{(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}} - \frac{7ab^{4/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{2\sqrt{3}\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} + \frac{7ab^{4/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{2\sqrt{3}\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} - \frac{7ab^{4/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{3\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} + \frac{7ab^{4/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[6]{\sec^2(e+fx)}}{12\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} - \frac{7ab^{4/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[6]{\sec^2(e+fx)}}{12\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} + \frac{\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{a^2 f\sqrt[3]{d\sec(e+fx)}} + \frac{b^2\operatorname{AppellF1}\left(\frac{3}{2},2,\frac{7}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan^3(e+fx)}{3a^4 f\sqrt[3]{d\sec(e+fx)}} - \frac{ab}{(a^2+b^2)\,f\sqrt[3]{d\sec(e+fx)}\,(a^2-b^2\tan^2(e+fx))}$$

output
```
7*a*b/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/3)+7/6*a*b^(4/3)*arctan(-1/3*3^(1/2)
+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)*3^(1/2)/(a^2+b^2)^(1/6))*(sec(f*x+e)^2)^
(1/6)*3^(1/2)/(a^2+b^2)^(13/6)/f/(d*sec(f*x+e))^(1/3)+7/6*a*b^(4/3)*arctan
(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)*3^(1/2)/(a^2+b^2)^(1/6))*(se
c(f*x+e)^2)^(1/6)*3^(1/2)/(a^2+b^2)^(13/6)/f/(d*sec(f*x+e))^(1/3)-7/3*a*b^
(4/3)*arctanh(b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(sec(f*x+e)^2)
^(1/6)/(a^2+b^2)^(13/6)/f/(d*sec(f*x+e))^(1/3)+7/12*a*b^(4/3)*ln((a^2+b^2)
^(1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)
^(1/3))*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(13/6)/f/(d*sec(f*x+e))^(1/3)-7/12*
a*b^(4/3)*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+
b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(13/6)/f/(d*s
ec(f*x+e))^(1/3)+AppellF1(1/2,2,7/6,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2
)*(sec(f*x+e)^2)^(1/6)*tan(f*x+e)/a^2/f/(d*sec(f*x+e))^(1/3)+1/3*b^2*Appel
lF1(3/2,2,7/6,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(sec(f*x+e)^2)^(1/6)
*tan(f*x+e)^3/a^4/f/(d*sec(f*x+e))^(1/3)-a*b/(a^2+b^2)/f/(d*sec(f*x+e))^(1
/3)/(a^2-b^2*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 128.70 (sec) , antiderivative size = 18832, normalized size of antiderivative = 26.34

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 574, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3994, 505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\int \frac{1}{(a+b\tan(e+fx))^2(\tan^2(e+fx)+1)^{7/6}}d(b\tan(e+fx))}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{505}$$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\int \left(\frac{a^2}{(\tan^2(e+fx)+1)^{7/6}(a^2-b^2\tan^2(e+fx))^2}-\frac{2b\tan(e+fx)a}{(\tan^2(e+fx)+1)^{7/6}(a^2-b^2\tan^2(e+fx))^2}+\frac{b^2\tan^2(e+fx}{(\tan^2(e+fx)+1)^{7/6}(b^2\tan}\right.}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2}-\frac{7ab^{7/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\tan^2(e+fx)+1}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)}{2\sqrt{3}(a^2+b^2)^{13/6}}\right.}{}$$

input
```
Int[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2),x]
```

output
```
((Sec[e + f*x]^2)^(1/6)*((-7*a*b^(7/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(1 +
Tan[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))])/(2*Sqrt[3]*(a^2 + b^2
)^(13/6)) + (7*a*b^(7/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(1 + Tan[e + f*x]^2
)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))])/(2*Sqrt[3]*(a^2 + b^2)^(13/6)) - (7
*a*b^(7/3)*ArcTanh[(b^(1/3)*(1 + Tan[e + f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]
)/(3*(a^2 + b^2)^(13/6)) + (7*a*b^(7/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3)*(a
^2 + b^2)^(1/6)*(1 + Tan[e + f*x]^2)^(1/6) + b^(2/3)*(1 + Tan[e + f*x]^2)^
(1/3)])/(12*(a^2 + b^2)^(13/6)) - (7*a*b^(7/3)*Log[(a^2 + b^2)^(1/3) + b^(
1/3)*(a^2 + b^2)^(1/6)*(1 + Tan[e + f*x]^2)^(1/6) + b^(2/3)*(1 + Tan[e + f
*x]^2)^(1/3)])/(12*(a^2 + b^2)^(13/6)) + (b*AppellF1[1/2, 2, 7/6, 3/2, (b^
2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a^2 + (b^3*AppellF1[
3/2, 2, 7/6, 5/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x]^
3)/(3*a^4) + (7*a*b^2)/((a^2 + b^2)^2*(1 + Tan[e + f*x]^2)^(1/6)) - (a*b^2
)/((a^2 + b^2)*(1 + Tan[e + f*x]^2)^(1/6)*(a^2 - b^2*Tan[e + f*x]^2))))/(b
*f*(d*Sec[e + f*x])^(1/3))
```

Defintions of rubi rules used

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [F]

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(a+b\tan\left(fx+e\right)\right)^{2}}dx$$

input
```
int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

output
```
int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\sqrt[3]{d\sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)^{2}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral(1/((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}(a+b\tan(e+f\,x))^2}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))^2),x)
```

output
```
int(1/((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$$
$$= \frac{\int \frac{1}{\sec(fx+e)^{\frac{1}{3}}\tan(fx+e)^2b^2+2\sec(fx+e)^{\frac{1}{3}}\tan(fx+e)ab+\sec(fx+e)^{\frac{1}{3}}a^2}dx}{d^{\frac{1}{3}}}$$

input
```
int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

output
```
int(1/(sec(e + f*x)**(1/3)*tan(e + f*x)**2*b**2 + 2*sec(e + f*x)**(1/3)*ta
n(e + f*x)*a*b + sec(e + f*x)**(1/3)*a**2),x)/d**(1/3)
```

3.647 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 717

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx = \frac{11ab}{5\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{5/3}}$$
$$+\frac{11ab^{8/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{2\sqrt{3}\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}}$$
$$-\frac{11ab^{8/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{2\sqrt{3}\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}}$$
$$-\frac{11ab^{8/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{3\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}}$$
$$+\frac{11ab^{8/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{12\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}}$$
$$-\frac{11ab^{8/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{12\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}}$$
$$+\frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{a^2f(d\sec(e+fx))^{5/3}}$$
$$+\frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{11}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan^3(e+fx)}{3a^4f(d\sec(e+fx))^{5/3}}$$
$$-\frac{ab}{\left(a^2+b^2\right)f(d\sec(e+fx))^{5/3}\left(a^2-b^2\tan^2(e+fx)\right)}$$

output
```
11/5*a*b/(a^2+b^2)^2/f/(d*sec(f*x+e))^(5/3)-11/6*a*b^(8/3)*arctan(-1/3*3^(
1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)*3^(1/2)/(a^2+b^2)^(1/6))*(sec(f*x+e)
^2)^(5/6)*3^(1/2)/(a^2+b^2)^(17/6)/f/(d*sec(f*x+e))^(5/3)-11/6*a*b^(8/3)*a
rctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)*3^(1/2)/(a^2+b^2)^(1/6)
)*(sec(f*x+e)^2)^(5/6)*3^(1/2)/(a^2+b^2)^(17/6)/f/(d*sec(f*x+e))^(5/3)-11/
3*a*b^(8/3)*arctanh(b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(sec(f*x
+e)^2)^(5/6)/(a^2+b^2)^(17/6)/f/(d*sec(f*x+e))^(5/3)+11/12*a*b^(8/3)*ln((a
^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*
x+e)^2)^(1/3))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(17/6)/f/(d*sec(f*x+e))^(5/3
)-11/12*a*b^(8/3)*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2
)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(17/6
)/f/(d*sec(f*x+e))^(5/3)+AppellF1(1/2,2,11/6,3/2,b^2*tan(f*x+e)^2/a^2,-tan
(f*x+e)^2)*(sec(f*x+e)^2)^(5/6)*tan(f*x+e)/a^2/f/(d*sec(f*x+e))^(5/3)+1/3*
b^2*AppellF1(3/2,2,11/6,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(sec(f*x+e
)^2)^(5/6)*tan(f*x+e)^3/a^4/f/(d*sec(f*x+e))^(5/3)-a*b/(a^2+b^2)/f/(d*sec(
f*x+e))^(5/3)/(a^2-b^2*tan(f*x+e)^2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 10396 vs. $2(717) = 1434$.

Time = 73.69 (sec) , antiderivative size = 10396, normalized size of antiderivative = 14.50

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3994, 505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sec^2(e+fx)^{5/6}\int \frac{1}{(a+b\tan(e+fx))^2(\tan^2(e+fx)+1)^{11/6}}d(b\tan(e+fx))}{bf(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{505}$$

$$\frac{\sec^2(e+fx)^{5/6}\int \left(\frac{a^2}{(\tan^2(e+fx)+1)^{11/6}(a^2-b^2\tan^2(e+fx))^2}-\frac{2b\tan(e+fx)a}{(\tan^2(e+fx)+1)^{11/6}(a^2-b^2\tan^2(e+fx))^2}+\frac{b^2\tan^2(e\text{-}}{(\tan^2(e+fx)+1)^{11/6}(b^2}\right.}{bf(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{2009}$$

$$\sec^2(e+fx)^{5/6}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2}+\frac{11ab^{11/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\tan^2(e+fx)+}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right.}{2\sqrt{3}(a^2+b^2)^{17/6}}\right.$$

input
```
Int[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2),x]
```

output
```
((Sec[e + f*x]^2)^(5/6)*((11*a*b^(11/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(1 +
 Tan[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))])/(2*Sqrt[3]*(a^2 + b^
2)^(17/6)) - (11*a*b^(11/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(1 + Tan[e + f*x
]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))])/(2*Sqrt[3]*(a^2 + b^2)^(17/6)) -
 (11*a*b^(11/3)*ArcTanh[(b^(1/3)*(1 + Tan[e + f*x]^2)^(1/6))/(a^2 + b^2)^(
1/6)])/(3*(a^2 + b^2)^(17/6)) + (11*a*b^(11/3)*Log[(a^2 + b^2)^(1/3) - b^(
1/3)*(a^2 + b^2)^(1/6)*(1 + Tan[e + f*x]^2)^(1/6) + b^(2/3)*(1 + Tan[e + f
*x]^2)^(1/3)])/(12*(a^2 + b^2)^(17/6)) - (11*a*b^(11/3)*Log[(a^2 + b^2)^(1
/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(1 + Tan[e + f*x]^2)^(1/6) + b^(2/3)*(1 +
Tan[e + f*x]^2)^(1/3)])/(12*(a^2 + b^2)^(17/6)) + (b*AppellF1[1/2, 2, 11/6
, 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a^2 + (b^3
*AppellF1[3/2, 2, 11/6, 5/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Ta
n[e + f*x]^3)/(3*a^4) + (11*a*b^2)/(5*(a^2 + b^2)^2*(1 + Tan[e + f*x]^2)^(
5/6)) - (a*b^2)/((a^2 + b^2)*(1 + Tan[e + f*x]^2)^(5/6)*(a^2 - b^2*Tan[e +
 f*x]^2))))/(b*f*(d*Sec[e + f*x])^(5/3))
```

Defintions of rubi rules used

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [F]

$$\int \frac{1}{\left(d \sec \left(f x+e\right)\right)^{\frac{5}{3}} \left(a+b \tan \left(f x+e\right)\right)^{2}}d x$$

input
```
int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

output
```
int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx=\int \frac{1}{\left(d \sec \left(e+f x\right)\right)^{\frac{5}{3}} \left(a+b \tan \left(e+f x\right)\right)^{2}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral(1/((d*sec(e + f*x))**(5/3)*(a + b*tan(e + f*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{3}}(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2), x)
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{3}}(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}(a+b\tan(e+f\,x))^2}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))^2),x)
```

output
```
int(1/((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx = \frac{\int \frac{1}{\sec(fx+e)^{\frac{5}{3}}\tan(fx+e)^2 b^2+2\sec(fx+e)^{\frac{5}{3}}\tan(fx+e)ab+\sec(fx+e)^{\frac{5}{3}}a^2}dx}{d^{\frac{5}{3}}}$$

input
```
int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

output
```
int(1/(sec(e + f*x)**(2/3)*sec(e + f*x)*tan(e + f*x)**2*b**2 + 2*sec(e + f
*x)**(2/3)*sec(e + f*x)*tan(e + f*x)*a*b + sec(e + f*x)**(2/3)*sec(e + f*x
)*a**2),x)/(d**(2/3)*d)
```

3.648 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 169

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx = \frac{b(3a^2-b^2)\,(d\sec(e+fx))^m}{fm} + \frac{b^3\sec^2(e+fx)(d\sec(e+fx))^m}{f(2+m)} + \frac{3ab^2(d\sec(e+fx))^m\tan(e+fx)}{f(1+m)} - \frac{a(3b^2-a^2(1+m))\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-\frac{m}{2},\frac{3}{2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-}}{f(1+m)}$$

output
```
b*(3*a^2-b^2)*(d*sec(f*x+e))^m/f/m+b^3*sec(f*x+e)^2*(d*sec(f*x+e))^m/f/(2+
m)+3*a*b^2*(d*sec(f*x+e))^m*tan(f*x+e)/f/(1+m)-a*(3*b^2-a^2*(1+m))*hyperge
om([1/2, 1-1/2*m],[3/2],-tan(f*x+e)^2)*(d*sec(f*x+e))^m*tan(f*x+e)/f/(1+m)
/((sec(f*x+e)^2)^(1/2*m))
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.94

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx$$
$$= \frac{(d\sec(e+fx))^m \left(3ab^2(2+m)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)\tan(e+fx) - a^3(2+m$$
$$fm($$

input
```
Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

output
```
((d*Sec[e + f*x])^m*(3*a*b^2*(2 + m)*Hypergeometric2F1[-1/2, m/2, (2 + m)/
2, Sec[e + f*x]^2]*Tan[e + f*x] - a^3*(2 + m)*Hypergeometric2F1[1/2, m/2,
(2 + m)/2, Sec[e + f*x]^2]*Tan[e + f*x] + b*((3*a^2 - b^2)*(2 + m) + b^2*m
*Sec[e + f*x]^2)*Sqrt[-Tan[e + f*x]^2]))/(f*m*(2 + m)*Sqrt[-Tan[e + f*x]^2
])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3994, 497, 25, 27, 676, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^3(d\sec(e+fx))^m\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a+b\tan(e+fx))^3(d\sec(e+fx))^m dx$$
$$\Big\downarrow \text{3994}$$
$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m \int (a+b\tan(e+fx))^3\left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}d(b\tan(e+fx))}{bf}$$

$$\Big\downarrow \ 497$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(\frac{b^2\int -\frac{(a+b\tan(e+fx))\left(b^2\left(2-\frac{a^2(m+2)}{b^2}\right)-ab(m+4)\tan(e+fx)\right)\left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}}{b^2}d(b\tan(e+fx))}{m+2}\right.}{bf}$$

$$\Big\downarrow \ 25$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(\frac{b^2\left(\tan^2(e+fx)+1\right)^{m/2}(a+b\tan(e+fx))^2}{m+2}-\frac{b^2\int\frac{(a+b\tan(e+fx))\left(-\left((m+2)a^2\right)-b(m+4)\tan(e+fx\right.}{b^2}}{m+2}\right.}{bf}$$

$$\Big\downarrow \ 27$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(\frac{b^2\left(\tan^2(e+fx)+1\right)^{m/2}(a+b\tan(e+fx))^2}{m+2}-\frac{\int(a+b\tan(e+fx))(-((m+2)a^2)-b(m+4)\tan(e+}{m+}\right.}{bf}$$

$$\Big\downarrow \ 676$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(\frac{b^2\left(\tan^2(e+fx)+1\right)^{m/2}(a+b\tan(e+fx))^2}{m+2}-\frac{\frac{a(m+2)\left(3b^2-a^2(m+1)\right)\int\left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}d(b\tan}{m+1}}{}\right.}{bf}$$

$$\Big\downarrow \ 237$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(\frac{b^2\left(\tan^2(e+fx)+1\right)^{m/2}(a+b\tan(e+fx))^2}{m+2}-\frac{\frac{ab(m+2)\left(3b^2-a^2(m+1)\right)\tan(e+fx)\,\text{Hypergeometric2F}}{m+1}}{}\right.}{bf}$$

input
```
Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

output
```
((d*Sec[e + f*x])^m*((b^2*(a + b*Tan[e + f*x])^2*(1 + Tan[e + f*x]^2)^(m/2
))/(2 + m) - ((a*b*(2 + m)*(3*b^2 - a^2*(1 + m))*Hypergeometric2F1[1/2, (2
 - m)/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(1 + m) + (2*b^2*(b^2 - a^2*(
3 + m))*(1 + Tan[e + f*x]^2)^(m/2))/m - (a*b^3*(4 + m)*Tan[e + f*x]*(1 + T
an[e + f*x]^2)^(m/2))/(1 + m))/(2 + m)))/(b*f*(Sec[e + f*x]^2)^(m/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [F]

$$\int (d\sec (fx+e))^m (a+b\tan (fx+e))^3 dx$$

input
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

output
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3 dx = \int (b\tan (fx+e)+a)^3(d\sec (fx+e))^m dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x +
e) + a^3)*(d*sec(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3 dx = \int (d\sec (e+fx))^m (a+b\tan (e+fx))^3 dx$$

input
```
integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e))**3,x)
```

output
```
Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x))**3, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx = \int (b\tan(fx+e)+a)^3(d\sec(fx+e))^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^3*(d*sec(f*x + e))^m, x)
```

Giac [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx = \int (b\tan(fx+e)+a)^3(d\sec(fx+e))^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^3*(d*sec(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^m (a+b\tan(e+f\,x))^3\,dx$$

input
```
int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^3,x)
```

output
```
int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^3, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx$$
$$= \frac{d^m\left(\sec\left(fx+e\right)^m\tan\left(fx+e\right)^2b^3m+3\sec\left(fx+e\right)^m a^2bm+6\sec\left(fx+e\right)^m a^2b-2\sec\left(fx+e\right)^m b^3+\right.}{}$$

input
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

output
```
(d**m*(sec(e + f*x)**m*tan(e + f*x)**2*b**3*m + 3*sec(e + f*x)**m*a**2*b*m
 + 6*sec(e + f*x)**m*a**2*b - 2*sec(e + f*x)**m*b**3 + int(sec(e + f*x)**m
,x)*a**3*f*m**2 + 2*int(sec(e + f*x)**m,x)*a**3*f*m + 3*int(sec(e + f*x)**
m*tan(e + f*x)**2,x)*a*b**2*f*m**2 + 6*int(sec(e + f*x)**m*tan(e + f*x)**2
,x)*a*b**2*f*m))/(f*m*(m + 2))
```

3.649 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx = \frac{ab(2+m)(d\sec(e+fx))^m}{fm(1+m)} + \frac{d(b^2-a^2(1+m))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^{-1+m}\sin(e+fx)}{f(1-m)(1+m)\sqrt{\sin^2(e+fx)}} + \frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))}{f(1+m)}$$

output
```
a*b*(2+m)*(d*sec(f*x+e))^m/f/m/(1+m)+d*(b^2-a^2*(1+m))*hypergeom([1/2, 1/2
-1/2*m],[3/2-1/2*m],cos(f*x+e)^2)*(d*sec(f*x+e))^(-1+m)*sin(f*x+e)/f/(1-m)
/(1+m)/(sin(f*x+e)^2)^(1/2)+b*(d*sec(f*x+e))^m*(a+b*tan(f*x+e))/f/(1+m)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.81

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx = \frac{(d\sec(e+fx))^m\Big(b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)\tan(e+fx)+a\Big(-a\,\text{Hypergeomet}}{fm\sqrt{-\tan^2(e+fx)}}$$

input
```
Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```

output
```
((d*Sec[e + f*x])^m*(b^2*Hypergeometric2F1[-1/2, m/2, (2 + m)/2, Sec[e + f
*x]^2]*Tan[e + f*x] + a*(-(a*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[e
+ f*x]^2]*Tan[e + f*x]) + 2*b*Sqrt[-Tan[e + f*x]^2])))/(f*m*Sqrt[-Tan[e +
f*x]^2])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3993, 25, 3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^2(d\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))^2(d\sec(e+fx))^m dx$$

$$\Big\downarrow \text{3993}$$

$$\frac{\int -(d\sec(e+fx))^m\left(-\left((m+1)a^2\right)-b(m+2)\tan(e+fx)a+b^2\right)dx}{m+1}+\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^m}{f(m+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^m}{f(m+1)}-\frac{\int (d\sec(e+fx))^m\left(-\left((m+1)a^2\right)-b(m+2)\tan(e+fx)a+b^2\right)dx}{m+1}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^m}{f(m+1)}-\frac{\int (d\sec(e+fx))^m\left(-\left((m+1)a^2\right)-b(m+2)\tan(e+fx)a+b^2\right)dx}{m+1}$$

$$\Big\downarrow \text{ 3967}$$

$$\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^m}{f(m+1)}-\frac{\left(b^2-a^2(m+1)\right)\int (d\sec(e+fx))^m dx-\frac{ab(m+2)(d\sec(e+fx))^m}{fm}}{m+1}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^m}{f(m+1)}-\frac{\left(b^2-a^2(m+1)\right)\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx-\frac{ab(m+2)(d\sec(e+fx))^m}{fm}}{m+1}$$

$$\Big\downarrow \text{ 4259}$$

$$\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^m}{f(m+1)}-\frac{\left(b^2-a^2(m+1)\right)\left(\frac{\cos(e+fx)}{d}\right)^m (d\sec(e+fx))^m \int \left(\frac{\cos(e+fx)}{d}\right)^{-m} dx-\frac{ab(m+2)(d\sec(e+fx))^m}{fm}}{m+1}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^m}{f(m+1)}-\frac{\left(b^2-a^2(m+1)\right)\left(\frac{\cos(e+fx)}{d}\right)^m (d\sec(e+fx))^m \int \left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-m} dx-\frac{ab(m+2)(d\sec(e+fx))^m}{fm}}{m+1}$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^m}{f(m+1)}-\frac{-\frac{d\left(b^2-a^2(m+1)\right)\sin(e+fx)(d\sec(e+fx))^{m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)}{f(1-m)\sqrt{\sin^2(e+fx)}}-\frac{ab(m+2)(d\sec(e+fx))^m}{fm}}{m+1}$$

input

```
Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```

output

```
-((-((a*b*(2 + m)*(d*Sec[e + f*x])^m)/(f*m)) - (d*(b^2 - a^2*(1 + m))*Hype
rgeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^
(-1 + m)*Sin[e + f*x])/(f*(1 - m)*Sqrt[Sin[e + f*x]^2]))/(1 + m)) + (b*(d*
Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(1 + m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int (d\sec(fx+e))^m (a+b\tan(fx+e))^2\,dx$$

input
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx = \int (b\tan(fx+e)+a)^2(d\sec(fx+e))^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^
m, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx = \int (d\sec(e+fx))^m (a+b\tan(e+fx))^2\,dx$$

input
```
integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx = \int (b\tan(fx+e)+a)^2(d\sec(fx+e))^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^2*(d*sec(f*x + e))^m, x)
```

Giac [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx = \int (b\tan(fx+e)+a)^2(d\sec(fx+e))^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^2*(d*sec(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^m (a+b\tan(e+f\,x))^2\,dx$$

input
```
int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^2, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx$$
$$=\frac{d^m\left(2\sec(fx+e)^m ab+\left(\int \sec(fx+e)^m\,dx\right)a^2fm+\left(\int \sec(fx+e)^m\tan(fx+e)^2\,dx\right)b^2fm\right)}{fm}$$

input
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

output
```
(d**m*(2*sec(e + f*x)**m*a*b + int(sec(e + f*x)**m,x)*a**2*f*m + int(sec(e
 + f*x)**m*tan(e + f*x)**2,x)*b**2*f*m))/(f*m)
```

3.650 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx = \frac{b(d\sec(e+fx))^m}{fm} - \frac{ad\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^{-1+m}\sin(e+fx)}{f(1-m)\sqrt{\sin^2(e+fx)}}$$

output
```
b*(d*sec(f*x+e))^m/f/m-a*d*hypergeom([1/2, 1/2-1/2*m],[3/2-1/2*m],cos(f*x+
e)^2)*(d*sec(f*x+e))^(-1+m)*sin(f*x+e)/f/(1-m)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.70

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx = \frac{(d\sec(e+fx))^m\left(b+a\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{fm}$$

input
```
Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]),x]
```

output
```
((d*Sec[e + f*x])^m*(b + a*Cot[e + f*x]*Hypergeometric2F1[1/2, m/2, (2 + m
)/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]))/(f*m)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))(d\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))(d\sec(e+fx))^m dx$$

$$\Big\downarrow \text{3967}$$

$$a\int (d\sec(e+fx))^m dx+\frac{b(d\sec(e+fx))^m}{fm}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx+\frac{b(d\sec(e+fx))^m}{fm}$$

$$\Big\downarrow \text{4259}$$

$$a\left(\frac{\cos(e+fx)}{d}\right)^m (d\sec(e+fx))^m\int \left(\frac{\cos(e+fx)}{d}\right)^{-m}dx+\frac{b(d\sec(e+fx))^m}{fm}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\cos(e+fx)}{d}\right)^m (d\sec(e+fx))^m\int \left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-m}dx+\frac{b(d\sec(e+fx))^m}{fm}$$

$$\Big\downarrow \text{3122}$$

$$\frac{b(d\sec(e+fx))^m}{fm}-\frac{ad\sin(e+fx)(d\sec(e+fx))^{m-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)}{f(1-m)\sqrt{\sin^2(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]),x]
```

output
```
(b*(d*Sec[e + f*x])^m)/(f*m) - (a*d*Hypergeometric2F1[1/2, (1 - m)/2, (3 -
 m)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + m)*Sin[e + f*x])/(f*(1 - m)*
Sqrt[Sin[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int (d\sec(fx+e))^m (a+b\tan(fx+e))\,dx$$

input
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx = \int (b\tan(fx+e)+a)(d\sec(fx+e))^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e) + a)*(d*sec(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx = \int (d\sec(e+fx))^m\,(a+b\tan(e+fx))\,dx$$

input
```
integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx = \int (b\tan(fx+e)+a)(d\sec(fx+e))^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)*(d*sec(f*x + e))^m, x)
```

Giac [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx=\int (b\tan(fx+e)+a)(d\sec(fx+e))^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)*(d*sec(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx=\int \left(\frac{d}{\cos(e+f\,x)}\right)^m(a+b\tan(e+f\,x))\,dx$$

input
```
int((d/cos(e + f*x))^m*(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^m*(a + b*tan(e + f*x)), x)
```

Reduce [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx =\frac{d^m\left(\sec(fx+e)^m b+\left(\int \sec(fx+e)^m\,dx\right)afm\right)}{fm}$$

input
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

output
```
(d**m*(sec(e + f*x)**m*b + int(sec(e + f*x)**m,x)*a*f*m))/(f*m)
```

3.651 $\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 141

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{2+m}{2},\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)(d\sec(e+fx))^m}{(a^2+b^2)\,fm} + \frac{\text{AppellF1}\left(\frac{1}{2},1,1-\frac{m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\tan(e+fx)}{af}$$

output
```
-b*hypergeom([1, 1/2*m],[1+1/2*m],b^2*sec(f*x+e)^2/(a^2+b^2))*(d*sec(f*x+e
))^m/(a^2+b^2)/f/m+AppellF1(1/2,1,1-1/2*m,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*
x+e)^2)*(d*sec(f*x+e))^m*tan(f*x+e)/a/f/((sec(f*x+e)^2)^(1/2*m))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.80 (sec) , antiderivative size = 1158, normalized size of antiderivative = 8.21

$$\int \frac{(d \sec(e+fx))^m}{a+b \tan(e+fx)} \, dx = \text{Too large to display}$$

input
```
Integrate[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

output
```
((d*Sec[e + f*x])^m*(b - b*(Sec[e + f*x]^2)^(m/2) + a*m*Hypergeometric2F1[
1/2, 1 - m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] + (b*AppellF1[-m, -1/2*m,
 -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f
*x])]*(Sec[e + f*x]^2)^(m/2))/(((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x
]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))))/(f*(a + b
*Tan[e + f*x])*(a*m*Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e + f*x]^2]*
Sec[e + f*x]^2 - b*m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x] + (b*m*AppellF1[-
m, -1/2*m, -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b
*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x])/(((b*(-I + Tan[e + f*
x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*
x]))^(m/2)) + (b*(Sec[e + f*x]^2)^(m/2)*(-1/2*((a - I*b)*b*m^2*AppellF1[1
- m, 1 - m/2, -1/2*m, 2 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a
+ b*Tan[e + f*x])]*Sec[e + f*x]^2)/((1 - m)*(a + b*Tan[e + f*x])^2) - ((a
+ I*b)*b*m^2*AppellF1[1 - m, -1/2*m, 1 - m/2, 2 - m, (a - I*b)/(a + b*Tan[
e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*Sec[e + f*x]^2)/(2*(1 - m)*(a +
 b*Tan[e + f*x])^2)))/(((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2
)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)) - (b*m*AppellF1[-m,
 -1/2*m, -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*T
an[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e
 + f*x]))^(-1 - m/2)*(-((b^2*Sec[e + f*x]^2*(-I + Tan[e + f*x]))/(a + b...
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3994, 504, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\int \frac{\left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}}{a+b\tan(e+fx)}d(b\tan(e+fx))}{bf}$$

$$\Big\downarrow \text{504}$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(a\int \frac{\left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}}{a^2-b^2\tan^2(e+fx)}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)\left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}}{a^2-b^2\tan^2(e+fx)}d(b\tan(e+fx))\right)}{bf}$$

$$\Big\downarrow \text{333}$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{2-m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\int \frac{b\tan(e+fx)\left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}}{a^2-b^2\tan^2(e+fx)}d(b\tan(e+fx))\right)}{bf}$$

$$\Big\downarrow \text{353}$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{2-m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\frac{1}{2}\int \frac{\left(\frac{\tan(e+fx)}{b}+1\right)^{\frac{m-2}{2}}}{a^2-b^2\tan^2(e+fx)}\right.$$

$$\Big\downarrow \text{78}$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{2-m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\frac{b^2\left(\frac{\tan(e+fx)}{b}+1\right)^{m/2}\text{Hy}}{}\right.}{bf}$$

input
```
Int[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

output
```
((d*Sec[e + f*x])^m*((b*AppellF1[1/2, 1, (2 - m)/2, 3/2, (b^2*Tan[e + f*x]
^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a - (b^2*Hypergeometric2F1[1, m/2,
 (2 + m)/2, (b^2 + b^2*Tan[e + f*x]^2)/(a^2 + b^2)]*(1 + Tan[e + f*x]/b)^(
m/2))/((a^2 + b^2)*m)))/(b*f*(Sec[e + f*x]^2)^(m/2))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [F]

$$\int \frac{(d\sec (fx+e))^m}{a+b\tan (fx+e)}dx$$

input
```
int((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx=\int \frac{(d\sec (fx+e))^m}{b\tan (fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec{(e+fx)})^m}{a+b\tan{(e+fx)}}\,dx$$

input
```
integrate((d*sec(f*x+e))**m/(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**m/(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec{(fx+e)})^m}{b\tan{(fx+e)}+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec{(fx+e)})^m}{b\tan{(fx+e)}+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^m}{a+b\tan\left(e+f\,x\right)}\,dx$$

input
```
int((d/cos(e + f*x))^m/(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^m/(a + b*tan(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = d^m\left(\int \frac{\sec\left(fx+e\right)^m}{\tan\left(fx+e\right)b+a}dx\right)$$

input
```
int((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

output
```
d**m*int(sec(e + f*x)**m/(tan(e + f*x)*b + a),x)
```

3.652 $\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 227

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = -\frac{2ab\,\text{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{2+m}{2},\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)(d\sec(e+fx))^m}{(a^2+b^2)^2 fm} + \frac{\text{AppellF1}\left(\frac{1}{2},2,1-\frac{m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\tan(e+fx)}{a^2 f} + \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,1-\frac{m}{2},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\tan^3(e+}{3a^4 f}$$

output
```
-2*a*b*hypergeom([2, 1/2*m],[1+1/2*m],b^2*sec(f*x+e)^2/(a^2+b^2))*(d*sec(f
*x+e))^m/(a^2+b^2)^2/f/m+AppellF1(1/2,2,1-1/2*m,3/2,b^2*tan(f*x+e)^2/a^2,-
tan(f*x+e)^2)*(d*sec(f*x+e))^m*tan(f*x+e)/a^2/f/((sec(f*x+e)^2)^(1/2*m))+1
/3*b^2*AppellF1(3/2,2,1-1/2*m,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(d*s
ec(f*x+e))^m*tan(f*x+e)^3/a^4/f/((sec(f*x+e)^2)^(1/2*m))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.08 (sec) , antiderivative size = 2453, normalized size of antiderivative = 10.81

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```

output
```
((d*Sec[e + f*x])^m*((-2*a*b*(-1 + (Sec[e + f*x]^2)^(m/2)))/m + (a^2 - b^2
)*Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] + (2*
a*b*AppellF1[-m, -1/2*m, -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a
 + I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2))/(m*((b*(-I + Tan[e +
 f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e +
 f*x]))^(m/2)) + (b*(a^2 + b^2)*AppellF1[1 - m, -1/2*m, -1/2*m, 2 - m, (a
- I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]
^2)^(m/2))/((-1 + m)*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*
((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(a + b*Tan[e + f*x])))
)/(f*(a + b*Tan[e + f*x])^2*((a^2 - b^2)*Hypergeometric2F1[1/2, 1 - m/2, 3
/2, -Tan[e + f*x]^2]*Sec[e + f*x]^2 - 2*a*b*(Sec[e + f*x]^2)^(m/2)*Tan[e +
 f*x] + (2*a*b*AppellF1[-m, -1/2*m, -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e
+ f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*
x])/(((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e +
 f*x]))/(a + b*Tan[e + f*x]))^(m/2)) - (b^2*(a^2 + b^2)*AppellF1[1 - m, -1
/2*m, -1/2*m, 2 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[
e + f*x])]*(Sec[e + f*x]^2)^(1 + m/2))/((-1 + m)*((b*(-I + Tan[e + f*x]))/
(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^
(m/2)*(a + b*Tan[e + f*x])^2) + (b*(a^2 + b^2)*m*AppellF1[1 - m, -1/2*m, -
1/2*m, 2 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + ...
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3994, 505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\int \frac{(\tan^2(e+fx)+1)^{\frac{m-2}{2}}}{(a+b\tan(e+fx))^2}d(b\tan(e+fx))}{bf}$$

$$\Big\downarrow \text{505}$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\int \left(\frac{a^2(\tan^2(e+fx)+1)^{\frac{m-2}{2}}}{(a^2-b^2\tan^2(e+fx))^2}-\frac{2ab\tan(e+fx)(\tan^2(e+fx)+1)^{\frac{m-2}{2}}}{(a^2-b^2\tan^2(e+fx))^2}+\frac{b^2\tan^2(e+fx)(\tan^2(e+\cdots}{(b^2\tan^2(e+fx)-\cdots}\cdots}{bf}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{2-m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2}-\frac{2ab^2(\tan^2(e+fx)+1)^{m/2}\cdots}{\cdots}\cdots\right.}{bf}$$

input
```
Int[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```

output
```
((d*Sec[e + f*x])^m*((b*AppellF1[1/2, 2, (2 - m)/2, 3/2, (b^2*Tan[e + f*x]
^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a^2 + (b^3*AppellF1[3/2, 2, (2 - m
)/2, 5/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x]^3)/(3*a^
4) - (2*a*b^2*Hypergeometric2F1[2, m/2, (2 + m)/2, (b^2 + b^2*Tan[e + f*x]
^2)/(a^2 + b^2)]*(1 + Tan[e + f*x]^2)^(m/2))/((a^2 + b^2)^2*m)))/(b*f*(Sec
[e + f*x]^2)^(m/2))
```

Defintions of rubi rules used

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Maple [F]

$$\int \frac{(d\sec (fx+e))^m}{(a+b\tan (fx+e))^2}dx$$

input
```
int((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^m}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^m/(b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2
), x)
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$$

input
```
integrate((d*sec(f*x+e))**m/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**m/(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^m}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx=\int \frac{(d\sec(fx+e))^m}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^m}{(a+b\tan(e+f\,x))^2}\,dx$$

input
```
int((d/cos(e + f*x))^m/(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^m/(a + b*tan(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$$
$$=\frac{d^m\left(-\sec(fx+e)^m+\left(\int \frac{\sec(fx+e)^m\tan(fx+e)^2}{\tan(fx+e)^2b^2+2\tan(fx+e)ab+a^2}dx\right)\tan(fx+e)\,b^2fm-\left(\int \frac{\sec(fx+e)^m\tan(fx+e)^2}{\tan(fx+e)^2b^2+2\tan(fx+e)ab}\right.}{}$$

input
```
int((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
```

output

```
(d**m*( - sec(e + f*x)**m + int((sec(e + f*x)**m*tan(e + f*x)**2)/(tan(e +
 f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*tan(e + f*x)*b**2*f*m - int(
(sec(e + f*x)**m*tan(e + f*x)**2)/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a
*b + a**2),x)*tan(e + f*x)*b**2*f + int((sec(e + f*x)**m*tan(e + f*x)**2)/
(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*a*b*f*m - int((sec(e
 + f*x)**m*tan(e + f*x)**2)/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a
**2),x)*a*b*f + int((sec(e + f*x)**m*tan(e + f*x))/(tan(e + f*x)**2*b**2 +
 2*tan(e + f*x)*a*b + a**2),x)*tan(e + f*x)*a*b*f*m + int((sec(e + f*x)**m
*tan(e + f*x))/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2),x)*a**2*
f*m))/(b*f*(tan(e + f*x)*b + a))
```

3.653 $\int (d\sec(e+fx))^m (a+b\tan(e+fx))^n \, dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\int (d\sec(e+fx))^m (a+b\tan(e+fx))^n \, dx = \frac{b \operatorname{AppellF1}\left(1+n, 1-\frac{m}{2}, 1-\frac{m}{2}, 2+n, \frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}, \frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right) (d\sec(e+fx))^m (a+b\tan(e+fx))^{1+n}}{(a^2+b^2) f (1+n)}$$

output
```
b*AppellF1(1+n,1-1/2*m,1-1/2*m,2+n,(a+b*tan(f*x+e))/(a-(-b^2)^(1/2)),(a+b*
tan(f*x+e))/(a+(-b^2)^(1/2)))*(d*sec(f*x+e))^m*(a+b*tan(f*x+e))^(1+n)/(a^2
+b^2)/f/(1+n)/((1-(a+b*tan(f*x+e))/(a-(-b^2)^(1/2)))^(1/2*m))/((1-(a+b*tan
(f*x+e))/(a+(-b^2)^(1/2)))^(1/2*m))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.06 (sec) , antiderivative size = 699, normalized size of antiderivative = 3.84

$$\int (d\sec(e+fx))^m (a+b\tan(e+fx))^n \, dx = \frac{}{f\left(2b \operatorname{AppellF1}\left(1+n, 1-\frac{m}{2}, 1-\frac{m}{2}, 2+n, \frac{a+b\tan(e+fx)}{a-ib}, \frac{a+b\tan(e+fx)}{a+ib}\right)\sec^2(e+fx) + 2n \operatorname{AppellF1}\left(1 - \right.\right.}$$

input
```
Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

output
```
(2*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b)
, (a + b*Tan[e + f*x])/(a + I*b)]*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(1 + n))/(f*(2*b*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f
*x])/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2 + 2*n*Appel
lF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b
*Tan[e + f*x])/(a + I*b)]*(b - a*Tan[e + f*x]) - (b*(-2 + m)*(1 + n)*((a -
 I*b)*AppellF1[2 + n, 1 - m/2, 2 - m/2, 3 + n, (a + b*Tan[e + f*x])/(a - I
*b), (a + b*Tan[e + f*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 2 - m/2,
1 - m/2, 3 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[e + f*x])/(a +
I*b)])*Sec[e + f*x]^2*(a + b*Tan[e + f*x]))/((a - I*b)*(a + I*b)*(2 + n))
+ 2*(m + n)*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/
(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Tan[e + f*x]*(a + b*Tan[e + f*x
]) - (m*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a -
 I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2*(a + b*Tan[e + f*x])
)/(-I + Tan[e + f*x]) - (m*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b
*Tan[e + f*x])/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2*(
a + b*Tan[e + f*x]))/(I + Tan[e + f*x])))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3995, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n dx$$

$$\Big\downarrow \text{3995}$$

$$\frac{b(d\sec(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{-m/2}\left(1-\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)^{-m/2}\int(a+b\tan(e+fx))^n\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{\frac{m-}{2}}}{f\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{150}$$

$$\frac{b(d\sec(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{-m/2}\left(1-\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)^{-m/2}(a+b\tan(e+fx))^{n+1}\operatorname{AppellF1}\left(n+1,\frac{2-n}{2}\right.}{f(n+1)\left(a^2+b^2\right)}$$

input
```
Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

output
```
(b*AppellF1[1 + n, (2 - m)/2, (2 - m)/2, 2 + n, (a + b*Tan[e + f*x])/(a -
Sqrt[-b^2]), (a + b*Tan[e + f*x])/(a + Sqrt[-b^2])]*(d*Sec[e + f*x])^m*(a
+ b*Tan[e + f*x])^(1 + n))/((a^2 + b^2)*f*(1 + n)*(1 - (a + b*Tan[e + f*x]
)/(a - Sqrt[-b^2]))^(m/2)*(1 - (a + b*Tan[e + f*x])/(a + Sqrt[-b^2]))^(m/2
))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3995
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*(a^2 + b^2)^(IntPart[m/2]
- 1)*((d*Sec[e + f*x])^(2*FracPart[m/2])/(f*b^(2*IntPart[m/2] - 1)*(1 - (a
+ b*Tan[e + f*x])/(a - Rt[-b^2, 2]))^FracPart[m/2]*(1 - (a + b*Tan[e + f*x]
)/(a + Rt[-b^2, 2]))^FracPart[m/2]))   Subst[Int[x^n*(1 - x/(a - Rt[-b^2, 2
]))^(m/2 - 1)*(1 - x/(a + Rt[-b^2, 2]))^(m/2 - 1), x], x, a + b*Tan[e + f*x
]], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 + b^2, 0] &&  !Integer
Q[m] &&  !IntegerQ[n]
```

Maple [F]

$$\int (d\sec (fx+e))^m (a+b\tan (fx+e))^n \, dx$$

input
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

output
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n \, dx = \int (d\sec (fx+e))^m (b\tan (fx+e)+a)^n \, dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx = \int (d\sec{(e+fx)})^m\,(a+b\tan{(e+fx)})^n\,dx$$

input
```
integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e))**n,x)
```

output
```
Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x))**n, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx = \int (d\sec{(fx+e)})^m\,(b\tan{(fx+e)}+a)^n\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Giac [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx = \int (d\sec{(fx+e)})^m\,(b\tan{(fx+e)}+a)^n\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^m (a+b\tan(e+f\,x))^n\,dx$$

input
```
int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^n,x)
```

output
```
int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^n, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx = d^m\left(\int \sec(fx+e)^m(\tan(fx+e)\,b+a)^n\,dx\right)$$

input
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

output
```
d**m*int(sec(e + f*x)**m*(tan(e + f*x)*b + a)**n,x)
```

3.654 $\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 161

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx=\frac{(a^2+b^2)^2\,(a+b\tan(c+dx))^{1+n}}{b^5d(1+n)}-\frac{4a(a^2+b^2)\,(a+b\tan(c+dx))^{2+n}}{b^5d(2+n)}+\frac{2(3a^2+b^2)\,(a+b\tan(c+dx))^{3+n}}{b^5d(3+n)}-\frac{4a(a+b\tan(c+dx))^{4+n}}{b^5d(4+n)}+\frac{(a+b\tan(c+dx))^{5+n}}{b^5d(5+n)}$$

output
```
(a^2+b^2)^2*(a+b*tan(d*x+c))^(1+n)/b^5/d/(1+n)-4*a*(a^2+b^2)*(a+b*tan(d*x+
c))^(2+n)/b^5/d/(2+n)+2*(3*a^2+b^2)*(a+b*tan(d*x+c))^(3+n)/b^5/d/(3+n)-4*a
*(a+b*tan(d*x+c))^(4+n)/b^5/d/(4+n)+(a+b*tan(d*x+c))^(5+n)/b^5/d/(5+n)
```

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$= \frac{(a+b\tan(c+dx))^{1+n}\left(b^4\sec^4(c+dx)+4(a^2+b^2)\left(\frac{a^2+b^2}{1+n}-\frac{2a(a+b\tan(c+dx))}{2+n}+\frac{(a+b\tan(c+dx))^2}{3+n}\right)-4a(a+\right.}{b^5d(5+n)}$$

input
```
Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^n,x]
```

output
```
((a + b*Tan[c + d*x])^(1 + n)*(b^4*Sec[c + d*x]^4 + 4*(a^2 + b^2)*((a^2 +
b^2)/(1 + n) - (2*a*(a + b*Tan[c + d*x]))/(2 + n) + (a + b*Tan[c + d*x])^2
/(3 + n)) - 4*a*(a + b*Tan[c + d*x])*((a^2 + b^2)/(2 + n) - (2*a*(a + b*Ta
n[c + d*x]))/(3 + n) + (a + b*Tan[c + d*x])^2/(4 + n))))/(b^5*d*(5 + n))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+b\tan(c+dx))^n dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{(a+b\tan(c+dx))^n\left(\tan^2(c+dx)b^2+b^2\right)^2}{b^4}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (a+b\tan(c+dx))^n \left(\tan^2(c+dx)b^2+b^2\right)^2 d(b\tan(c+dx))}{b^5 d}$$

$$\Big\downarrow \color{blue}{476}$$

$$\frac{\int \left(\left(a^2+b^2\right)^2 (a+b\tan(c+dx))^n - 4a\left(a^2+b^2\right)(a+b\tan(c+dx))^{n+1} + 2\left(3a^2+b^2\right)(a+b\tan(c+dx))^{n+2} - \right.}{b^5 d}$$

$$\Big\downarrow \color{blue}{2009}$$

$$\frac{\frac{\left(a^2+b^2\right)^2(a+b\tan(c+dx))^{n+1}}{n+1} - \frac{4a\left(a^2+b^2\right)(a+b\tan(c+dx))^{n+2}}{n+2} + \frac{2\left(3a^2+b^2\right)(a+b\tan(c+dx))^{n+3}}{n+3} - \frac{4a(a+b\tan(c+dx))^{n+4}}{n+4} + \frac{(a+b\tan}{}}{b^5 d}$$

input
```
Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^n,x]
```

output
```
(((a^2 + b^2)^2*(a + b*Tan[c + d*x])^(1 + n))/(1 + n) - (4*a*(a^2 + b^2)*(
a + b*Tan[c + d*x])^(2 + n))/(2 + n) + (2*(3*a^2 + b^2)*(a + b*Tan[c + d*x
])^(3 + n))/(3 + n) - (4*a*(a + b*Tan[c + d*x])^(4 + n))/(4 + n) + (a + b*
Tan[c + d*x])^(5 + n)/(5 + n))/(b^5*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(161) = 322$.

Time = 0.33 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.87

$$\frac{\tan(dx+c)^5 \, e^{n\ln(a+b\tan(dx+c))}}{d(5+n)}+\frac{a(b^4n^4+14b^4n^3+4a^2b^2n^2+71b^4n^2+36a^2b^2n+154b^4n+24a^4+80b^2a^2}{b^5d(n^5+15n^4+85n^3+225n^2+274n+120)}$$

input
```
int(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x)
```

output
```
1/d/(5+n)*tan(d*x+c)^5*exp(n*ln(a+b*tan(d*x+c)))+a*(b^4*n^4+14*b^4*n^3+4*a
^2*b^2*n^2+71*b^4*n^2+36*a^2*b^2*n+154*b^4*n+24*a^4+80*a^2*b^2+120*b^4)/b^
5/d/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)*exp(n*ln(a+b*tan(d*x+c)))+a*n/b/
d/(n^2+9*n+20)*tan(d*x+c)^4*exp(n*ln(a+b*tan(d*x+c)))-2*(-b^2*n^2+2*a^2*n-
9*b^2*n-20*b^2)/b^2/d/(n^3+12*n^2+47*n+60)*tan(d*x+c)^3*exp(n*ln(a+b*tan(d
*x+c)))-(-b^4*n^4+4*a^2*b^2*n^3-14*b^4*n^3+36*a^2*b^2*n^2-71*b^4*n^2+24*a^
4*n+80*a^2*b^2*n-154*b^4*n-120*b^4)/b^4/(n^5+15*n^4+85*n^3+225*n^2+274*n+1
20)/d*tan(d*x+c)*exp(n*ln(a+b*tan(d*x+c)))+2*(b^2*n^2+9*b^2*n+6*a^2+20*b^2
)*a/b^3/d*n/(n^4+14*n^3+71*n^2+154*n+120)*tan(d*x+c)^2*exp(n*ln(a+b*tan(d*
x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(161) = 322$.

Time = 0.13 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.61

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n \, dx$$
$$= \frac{(8(3a^5+10a^3b^2+15ab^4-(a^3b^2-3ab^4)n^2+3(a^3b^2+5ab^4)n)\cos(dx+c)^5+4(2ab^4n^3+3(a^3b^2+}{}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
(8*(3*a^5 + 10*a^3*b^2 + 15*a*b^4 - (a^3*b^2 - 3*a*b^4)*n^2 + 3*(a^3*b^2 +
 5*a*b^4)*n)*cos(d*x + c)^5 + 4*(2*a*b^4*n^3 + 3*(a^3*b^2 + 3*a*b^4)*n^2 +
 (3*a^3*b^2 + 7*a*b^4)*n)*cos(d*x + c)^3 + (a*b^4*n^4 + 6*a*b^4*n^3 + 11*a
*b^4*n^2 + 6*a*b^4*n)*cos(d*x + c) + (b^5*n^4 + 10*b^5*n^3 + 35*b^5*n^2 +
50*b^5*n + 24*b^5 + 8*(8*b^5 - (3*a^2*b^3 - b^5)*n^2 - 3*(a^4*b + 3*a^2*b^
3 - 2*b^5)*n)*cos(d*x + c)^4 + 4*(8*b^5 - (a^2*b^3 - b^5)*n^3 - (3*a^2*b^3
 - 7*b^5)*n^2 - 2*(a^2*b^3 - 7*b^5)*n)*cos(d*x + c)^2)*sin(d*x + c))*((a*c
os(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^n/((b^5*d*n^5 + 15*b^5*d*n^4 +
 85*b^5*d*n^3 + 225*b^5*d*n^2 + 274*b^5*d*n + 120*b^5*d)*cos(d*x + c)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**6*(a+b*tan(d*x+c))**n,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.78

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$= \frac{\frac{(b\tan(dx+c)+a)^{n+1}}{b(n+1)} + \frac{2\left(\left(n^{2}+3\,n+2\right)b^{3}\tan(dx+c)^{3}+\left(n^{2}+n\right)ab^{2}\tan(dx+c)^{2}-2\,a^{2}bn\tan(dx+c)+2\,a^{3}\right)(b\tan(dx+c)+a)^{n}}{(n^{3}+6\,n^{2}+11\,n+6)b^{3}} + \frac{\left(\left(n^{4}+10\,n^{3}\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
((b*tan(d*x + c) + a)^(n + 1)/(b*(n + 1)) + 2*((n^2 + 3*n + 2)*b^3*tan(d*x
 + c)^3 + (n^2 + n)*a*b^2*tan(d*x + c)^2 - 2*a^2*b*n*tan(d*x + c) + 2*a^3)
*(b*tan(d*x + c) + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^4 + 10*n^3 +
35*n^2 + 50*n + 24)*b^5*tan(d*x + c)^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^
4*tan(d*x + c)^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*tan(d*x + c)^3 + 12*(n^2
+ n)*a^3*b^2*tan(d*x + c)^2 - 24*a^4*b*n*tan(d*x + c) + 24*a^5)*(b*tan(d*x
 + c) + a)^n/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5))/d
```

Giac [F]

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n \sec(dx+c)^6\,dx$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sec(d*x + c)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx = \int \frac{(a+b\tan(c+d\,x))^n}{\cos(c+d\,x)^6}\,dx$$

input
```
int((a + b*tan(c + d*x))^n/cos(c + d*x)^6,x)
```

output
```
int((a + b*tan(c + d*x))^n/cos(c + d*x)^6, x)
```

Reduce [F]

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx = \int (\tan(dx+c)\,b+a)^n \sec(dx+c)^6\,dx$$

input
```
int(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*b + a)**n*sec(c + d*x)**6,x)
```

3.655 $\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(a^2+b^2)\,(a+b\tan(c+dx))^{1+n}}{b^3d(1+n)} - \frac{2a(a+b\tan(c+dx))^{2+n}}{b^3d(2+n)} + \frac{(a+b\tan(c+dx))^{3+n}}{b^3d(3+n)}$$

output
```
(a^2+b^2)*(a+b*tan(d*x+c))^(1+n)/b^3/d/(1+n)-2*a*(a+b*tan(d*x+c))^(2+n)/b^
3/d/(2+n)+(a+b*tan(d*x+c))^(3+n)/b^3/d/(3+n)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(a+b\tan(c+dx))^{1+n}\left(\frac{a^2+b^2}{1+n}-\frac{2a(a+b\tan(c+dx))}{2+n}+\frac{(a+b\tan(c+dx))^2}{3+n}\right)}{b^3d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

output
```
((a + b*Tan[c + d*x])^(1 + n)*((a^2 + b^2)/(1 + n) - (2*a*(a + b*Tan[c + d
*x]))/(2 + n) + (a + b*Tan[c + d*x])^2/(3 + n)))/(b^3*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4(a+b\tan(c+dx))^ndx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{(a+b\tan(c+dx))^n(\tan^2(c+dx)b^2+b^2)}{b^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (a+b\tan(c+dx))^n\left(\tan^2(c+dx)b^2+b^2\right)d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(\left(a^2+b^2\right)(a+b\tan(c+dx))^n-2a(a+b\tan(c+dx))^{n+1}+(a+b\tan(c+dx))^{n+2}\right)d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a^2+b^2)(a+b\tan(c+dx))^{n+1}}{n+1}-\frac{2a(a+b\tan(c+dx))^{n+2}}{n+2}+\frac{(a+b\tan(c+dx))^{n+3}}{n+3}}{b^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

output
```
(((a^2 + b^2)*(a + b*Tan[c + d*x])^(1 + n))/(1 + n) - (2*a*(a + b*Tan[c +
d*x])^(2 + n))/(2 + n) + (a + b*Tan[c + d*x])^(3 + n)/(3 + n))/(b^3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(88) = 176$.

Time = 181.34 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.32

method	result
derivativedivides	$\frac{\tan(dx+c)^3 \mathrm{e}^{n \ln(a+b\tan(dx+c))}}{d(3+n)}+\frac{a(b^2n^2+5b^2n+2a^2+6b^2)\mathrm{e}^{n \ln(a+b\tan(dx+c))}}{b^3 d(n^3+6n^2+11n+6)}+\frac{an\tan(dx+c)^2 \mathrm{e}^{n \ln(a+b\tan(dx+}}{bd(n^2+5n+6)}$
default	$\frac{\tan(dx+c)^3 \mathrm{e}^{n \ln(a+b\tan(dx+c))}}{d(3+n)}+\frac{a(b^2n^2+5b^2n+2a^2+6b^2)\mathrm{e}^{n \ln(a+b\tan(dx+c))}}{b^3 d(n^3+6n^2+11n+6)}+\frac{an\tan(dx+c)^2 \mathrm{e}^{n \ln(a+b\tan(dx+}}{bd(n^2+5n+6)}$

input
```
int(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
1/d/(3+n)*tan(d*x+c)^3*exp(n*ln(a+b*tan(d*x+c)))+a*(b^2*n^2+5*b^2*n+2*a^2+
6*b^2)/b^3/d/(n^3+6*n^2+11*n+6)*exp(n*ln(a+b*tan(d*x+c)))+a*n/b/d/(n^2+5*n
+6)*tan(d*x+c)^2*exp(n*ln(a+b*tan(d*x+c)))-(-b^2*n^2+2*a^2*n-5*b^2*n-6*b^2
)/b^2/(n^3+6*n^2+11*n+6)/d*tan(d*x+c)*exp(n*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.00

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\left(2\left(2\,ab^2n+a^3+3\,ab^2\right)\cos\left(dx+c\right)^3+\left(ab^2n^2+ab^2n\right)\cos\left(dx+c\right)+\left(b^3n^2+3\,b^3n+2\,b^3+2\left(2\,b^3-(a\right.\right.}{\left(b^3dn^3+6\,b^3dn^2+11\,b^3dn+6\,b^3d\right)\cos\left(dx+\right.}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
(2*(2*a*b^2*n + a^3 + 3*a*b^2)*cos(d*x + c)^3 + (a*b^2*n^2 + a*b^2*n)*cos(
d*x + c) + (b^3*n^2 + 3*b^3*n + 2*b^3 + 2*(2*b^3 - (a^2*b - b^3)*n)*cos(d*
x + c)^2)*sin(d*x + c))*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^n
/((b^3*d*n^3 + 6*b^3*d*n^2 + 11*b^3*d*n + 6*b^3*d)*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^n\sec^4\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + dx))**n*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$=\frac{\frac{(b\tan(dx+c)+a)^{n+1}}{b(n+1)}+\frac{\left((n^2+3n+2)b^3\tan(dx+c)^3+(n^2+n)ab^2\tan(dx+c)^2-2a^2bn\tan(dx+c)+2a^3\right)(b\tan(dx+c)+a)^n}{(n^3+6n^2+11n+6)b^3}}{d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
((b*tan(d*x + c) + a)^(n + 1)/(b*(n + 1)) + ((n^2 + 3*n + 2)*b^3*tan(d*x +
 c)^3 + (n^2 + n)*a*b^2*tan(d*x + c)^2 - 2*a^2*b*n*tan(d*x + c) + 2*a^3)*(
b*tan(d*x + c) + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3))/d
```

Giac [F]

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan(dx+c)+a)^n\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx=\int \frac{(a+b\tan(c+d\,x))^n}{\cos(c+d\,x)^4}\,dx$$

input
```
int((a + b*tan(c + d*x))^n/cos(c + d*x)^4,x)
```

output
```
int((a + b*tan(c + d*x))^n/cos(c + d*x)^4, x)
```

Reduce [F]

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int (\tan(dx+c)\,b+a)^n \sec(dx+c)^4\,dx$$

input
```
int(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*b + a)**n*sec(c + d*x)**4,x)
```

3.656 $\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(a+b\tan(c+dx))^{1+n}}{bd(1+n)}$$

output
```
(a+b*tan(d*x+c))^(1+n)/b/d/(1+n)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(a+b\tan(c+dx))^{1+n}}{bd(1+n)}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

output
```
(a + b*Tan[c + d*x])^(1 + n)/(b*d*(1 + n))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3987, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(c+dx)^2(a+b\tan(c+dx))^n dx$$

$$\Big\downarrow \text{ 3987}$$

$$\frac{\int (a+b\tan(c+dx))^n d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(a+b\tan(c+dx))^{n+1}}{bd(n+1)}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

output
```
(a + b*Tan[c + d*x])^(1 + n)/(b*d*(1 + n))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 15.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{(a+b\tan(dx+c))^{1+n}}{bd(1+n)}$	27
default	$\frac{(a+b\tan(dx+c))^{1+n}}{bd(1+n)}$	27

input
```
int(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
(a+b*tan(d*x+c))^(1+n)/b/d/(1+n)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(26) = 52.

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(a\cos(dx+c)+b\sin(dx+c))\left(\frac{a\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)}\right)^n}{(bdn+bd)\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
(a*cos(d*x + c) + b*sin(d*x + c))*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d
*x + c))^n/((b*d*n + b*d)*cos(d*x + c))
```

Sympy [F]

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (a+b\tan(c+dx))^n \sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(b\tan(dx+c)+a)^{n+1}}{bd(n+1)}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
(b*tan(d*x + c) + a)^(n + 1)/(b*d*(n + 1))
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(b\tan(dx+c)+a)^{n+1}}{bd(n+1)}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
(b*tan(d*x + c) + a)^(n + 1)/(b*d*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx = \begin{cases} \frac{\ln(a+b\tan(c+dx))}{b\,d} & \text{if } n=-1 \\ \frac{(a+b\tan(c+dx))^{n+1}}{b\,d\,(n+1)} & \text{if } n\neq -1 \end{cases}$$

input
```
int((a + b*tan(c + d*x))^n/cos(c + d*x)^2,x)
```

output
```
piecewise(n == -1, log(a + b*tan(c + d*x))/(b*d), n ~= -1, (a + b*tan(c +
d*x))^(n + 1)/(b*d*(n + 1)))
```

Reduce [F]

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (\tan(dx+c)\,b+a)^n \sec(dx+c)^2\,dx$$

input
```
int(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*b + a)**n*sec(c + d*x)**2,x)
```

3.657 $\int (a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 12, antiderivative size = 167

$$\int (a+b\tan(c+dx))^n\,dx = \frac{b\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+n}}{2\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)d(1+n)} - \frac{b\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+n}}{2\sqrt{-b^2}\left(a+\sqrt{-b^2}\right)d(1+n)}$$

output
```
1/2*b*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)))*(a+b*tan
(d*x+c))^(1+n)/(-b^2)^(1/2)/(a-(-b^2)^(1/2))/d/(1+n)-1/2*b*hypergeom([1, 1
+n],[2+n],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))*(a+b*tan(d*x+c))^(1+n)/(-b^2)
^(1/2)/(a+(-b^2)^(1/2))/d/(1+n)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.71

$$\int (a+b\tan(c+dx))^n\,dx$$
$$=\frac{\left((a+ib)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-ib}\right)-(a-ib)\,\text{Hypergeometric2F1}\left(1,1+n,2\right.\right.}{2(a+ib)(ia+b)d(1+n)}$$

input
```
Integrate[(a + b*Tan[c + d*x])^n,x]
```

output
```
(((a + I*b)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - I
*b)] - (a - I*b)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(
a + I*b)])*(a + b*Tan[c + d*x])^(1 + n))/(2*(a + I*b)*(I*a + b)*d*(1 + n))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3966, 485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(c+dx))^n\,dx$$
$$\Big\downarrow \; 3042$$
$$\int (a+b\tan(c+dx))^n dx$$
$$\Big\downarrow \; 3966$$
$$\frac{b\int \frac{(a+b\tan(c+dx))^n}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{d}$$
$$\Big\downarrow \; 485$$

$$\frac{b\int \left(\frac{\sqrt{-b^2}(a+b\tan(c+dx))^n}{2b^2\left(\sqrt{-b^2}-b\tan(c+dx)\right)}+\frac{\sqrt{-b^2}(a+b\tan(c+dx))^n}{2b^2\left(b\tan(c+dx)+\sqrt{-b^2}\right)}\right)d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\left(\frac{(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{2\sqrt{-b^2}(n+1)\left(a-\sqrt{-b^2}\right)}-\frac{(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c-}{a+\sqrt{-b}}\right.}{2\sqrt{-b^2}(n+1)\left(a+\sqrt{-b^2}\right)}\right.}{d}$$

input
```
Int[(a + b*Tan[c + d*x])^n,x]
```

output
```
(b*((Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^
2])]*(a + b*Tan[c + d*x])^(1 + n))/(2*Sqrt[-b^2]*(a - Sqrt[-b^2])*(1 + n))
 - (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2
])]*(a + b*Tan[c + d*x])^(1 + n))/(2*Sqrt[-b^2]*(a + Sqrt[-b^2])*(1 + n)))
)/d
```

Defintions of rubi rules used

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3966
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Su
bst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && NeQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (a+b\tan(dx+c))^n \, dx$$

input
```
int((a+b*tan(d*x+c))^n,x)
```

output
```
int((a+b*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (a+b\tan(c+dx))^n \, dx = \int (b\tan(dx+c)+a)^n \, dx$$

input
```
integrate((a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n, x)
```

Sympy [F]

$$\int (a+b\tan(c+dx))^n \, dx = \int (a+b\tan(c+dx))^n \, dx$$

input
```
integrate((a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n, x)
```

Maxima [F]

$$\int (a + b\tan(c + dx))^n \, dx = \int (b\tan(dx + c) + a)^n \, dx$$

input
```
integrate((a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (a + b\tan(c + dx))^n \, dx = \int (b\tan(dx + c) + a)^n \, dx$$

input
```
integrate((a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\tan(c + dx))^n \, dx = \int (a + b\tan(c + d\,x))^n \, dx$$

input
```
int((a + b*tan(c + d*x))^n,x)
```

output
```
int((a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int (a+b\tan(c+dx))^n\,dx = \int (\tan(dx+c)\,b+a)^n\,dx$$

input
```
int((a+b*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*b + a)**n,x)
```

3.658 $\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 272

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx = -\frac{\left(\sqrt{-b^2}\left(1+\frac{a^2}{b^2}-n\right)-an\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+n}}{4\left(1+\frac{a^2}{b^2}\right)b\left(a-\sqrt{-b^2}\right)d(1+n)} + \frac{b\left(\sqrt{-b^2}\left(1+\frac{a^2}{b^2}-n\right)+an\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+}}{4\left(a^2+b^2\right)\left(a+\sqrt{-b^2}\right)d(1+n)} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2\left(a^2+b^2\right)d}$$

output

```
-1/4*((-b^2)^(1/2)*(1+a^2/b^2-n)-a*n)*hypergeom([1, 1+n],[2+n],(a+b*tan(d*
x+c))/(a-(-b^2)^(1/2)))*(a+b*tan(d*x+c))^(1+n)/(1+a^2/b^2)/b/(a-(-b^2)^(1/
2))/d/(1+n)+1/4*b*((-b^2)^(1/2)*(1+a^2/b^2-n)+a*n)*hypergeom([1, 1+n],[2+n
],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))*(a+b*tan(d*x+c))^(1+n)/(a^2+b^2)/(a+(
-b^2)^(1/2))/d/(1+n)+1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))*(a+b*tan(d*x+c))^(1
+n)/(a^2+b^2)/d
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.83

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$= \frac{(a+b\tan(c+dx))^{1+n}\left(-\frac{\left(\sqrt{-b^2}(a^2-b^2(-1+n))-ab^2n\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{\left(a-\sqrt{-b^2}\right)(1+n)}+\frac{\left(a^2\sqrt{-b^2}+(-b^2)^{3}\right.}{}\right.}{4b\,(a^2+b^2)\,d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

output
```
((a + b*Tan[c + d*x])^(1 + n)*(-(((Sqrt[-b^2]*(a^2 - b^2*(-1 + n)) - a*b^2
*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2
])])/((a - Sqrt[-b^2])*(1 + n))) + ((a^2*Sqrt[-b^2] + (-b^2)^(3/2)*(-1 + n
) + a*b^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a +
Sqrt[-b^2])])/((a + Sqrt[-b^2])*(1 + n)) + 2*b*Cos[c + d*x]^2*(b + a*Tan[c
 + d*x])))/(4*b*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3987, 27, 496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^n}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{b^4(a+b\tan(c+dx))^n}{(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \; 27$$

$$\frac{b^3\int \frac{(a+b\tan(c+dx))^n}{(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 496$$

$$\frac{b^3\left(\frac{(ab\tan(c+dx)+b^2)(a+b\tan(c+dx))^{n+1}}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}-\frac{\int -\frac{(a+b\tan(c+dx))^n\left(a^2-bn\tan(c+dx)a+b^2(1-n)\right)}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2(a^2+b^2)}\right)}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{b^3\left(\frac{\int \frac{(a+b\tan(c+dx))^n\left(a^2-bn\tan(c+dx)a+b^2(1-n)\right)}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2(a^2+b^2)}+\frac{(ab\tan(c+dx)+b^2)(a+b\tan(c+dx))^{n+1}}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \; 657$$

$$\frac{b^3\left(\frac{\int \left(\frac{\left(anb^2+\sqrt{-b^2}\left(a^2+b^2(1-n)\right)\right)(a+b\tan(c+dx))^n}{2b^2\left(\sqrt{-b^2}-b\tan(c+dx)\right)}+\frac{\left(\sqrt{-b^2}\left(a^2+b^2(1-n)\right)-ab^2n\right)(a+b\tan(c+dx))^n}{2b^2\left(b\tan(c+dx)+\sqrt{-b^2}\right)}\right)d(b\tan(c+dx))}{2b^2(a^2+b^2)}+\frac{(ab\tan(c+dx)+b^2)(a+\cdots}{2b^2(a^2+b^2)(b^2\tan\cdots}\right.}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{b^3\left(\frac{\frac{\left(\sqrt{-b^2}\left(a^2+b^2(1-n)\right)+ab^2n\right)(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{2b^2(n+1)\left(a+\sqrt{-b^2}\right)}-\frac{\left(\sqrt{-b^2}\left(a^2+b^2(1-n)\right)-ab^2n\right)(a+b\tan(c+dx))^{n\cdots}}{2b^2(n+1)\cdots}}{2b^2(a^2+b^2)}\right.}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

output
```
(b^3*(((a + b*Tan[c + d*x])^(1 + n)*(b^2 + a*b*Tan[c + d*x]))/(2*b^2*(a^2
+ b^2)*(b^2 + b^2*Tan[c + d*x]^2)) + (-1/2*((Sqrt[-b^2]*(a^2 + b^2*(1 - n)
) - a*b^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a -
Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(b^2*(a - Sqrt[-b^2])*(1 + n))
+ ((Sqrt[-b^2]*(a^2 + b^2*(1 - n)) + a*b^2*n)*Hypergeometric2F1[1, 1 + n,
2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n)
)/(2*b^2*(a + Sqrt[-b^2])*(1 + n)))/(2*b^2*(a^2 + b^2))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [F]

$$\int \cos (dx+c)^2 (a+b\tan (dx+c))^n dx$$

input
```
int(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n dx = \int (b\tan (dx+c)+a)^n \cos (dx+c)^2 \, dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

Sympy [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n dx = \int (a+b\tan (c+dx))^n \cos^2 (c+dx) \, dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*cos(c + d*x)**2, x)
```

Maxima [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int \cos(c+d\,x)^2\,(a+b\tan(c+d\,x))^n\,dx$$

input
```
int(cos(c + d*x)^2*(a + b*tan(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)^2*(a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (\tan(dx+c)\,b+a)^n \cos(dx+c)^2\,dx$$

input
```
int(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*b + a)**n*cos(c + d*x)**2,x)
```

3.659 $\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 432

$$
\begin{aligned}
&\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx \\
&= \frac{\left(a\left(5+\frac{3a^2}{b^2}-2n\right)n-\frac{\sqrt{-b^2}(3a^4+a^2b^2(6-2n-n^2)+b^4(3-4n+n^2))}{b^4}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c}{a-\sqrt{-}}\right.}{16\left(1+\frac{a^2}{b^2}\right)^2b\left(a-\sqrt{-b^2}\right)d(1+n)} \\
&+\frac{\left(a\left(5+\frac{3a^2}{b^2}-2n\right)n+\frac{\sqrt{-b^2}(3a^4+a^2b^2(6-2n-n^2)+b^4(3-4n+n^2))}{b^4}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan}{a+}\right.}{16\left(1+\frac{a^2}{b^2}\right)^2b\left(a+\sqrt{-b^2}\right)d(1+n)} \\
&+\frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4(a^2+b^2)d} \\
&+\frac{b\cos^2(c+dx)(a+b\tan(c+dx))^{1+n}\left(b^2(3-n)+a^2(1+n)+ab\left(5+\frac{3a^2}{b^2}-2n\right)\tan(c+dx)\right)}{8(a^2+b^2)^2d}
\end{aligned}
$$

output
```
1/16*(a*(5+3*a^2/b^2-2*n)*n-(-b^2)^(1/2)*(3*a^4+a^2*b^2*(-n^2-2*n+6)+b^4*(
n^2-4*n+3))/b^4)*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)
))*(a+b*tan(d*x+c))^(1+n)/(1+a^2/b^2)^2/b/(a-(-b^2)^(1/2))/d/(1+n)+1/16*(a
*(5+3*a^2/b^2-2*n)*n+(-b^2)^(1/2)*(3*a^4+a^2*b^2*(-n^2-2*n+6)+b^4*(n^2-4*n
+3))/b^4)*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))*(a+b
*tan(d*x+c))^(1+n)/(1+a^2/b^2)^2/b/(a+(-b^2)^(1/2))/d/(1+n)+1/4*cos(d*x+c)
^4*(b+a*tan(d*x+c))*(a+b*tan(d*x+c))^(1+n)/(a^2+b^2)/d+1/8*b*cos(d*x+c)^2*
(a+b*tan(d*x+c))^(1+n)*(b^2*(3-n)+a^2*(1+n)+a*b*(5+3*a^2/b^2-2*n)*tan(d*x+
c))/(a^2+b^2)^2/d
```

Mathematica [A] (verified)

Time = 2.92 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.83

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$= \frac{(a+b\tan(c+dx))^{1+n}\left(\frac{\left(ab^2\left(3a^2+b^2(5-2n)\right)n+\sqrt{-b^2}\left(-3a^4-b^4\left(3-4n+n^2\right)+a^2b^2\left(-6+2n+n^2\right)\right)\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(}{a-\sqrt{}}\right.}{a-\sqrt{-b^2}}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

output
```
((a + b*Tan[c + d*x])^(1 + n)*((((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n + Sqrt[-
b^2]*(-3*a^4 - b^4*(3 - 4*n + n^2) + a^2*b^2*(-6 + 2*n + n^2)))*Hypergeome
tric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])])/(a - Sqrt
[-b^2]) + ((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n + Sqrt[-b^2]*(3*a^4 + b^4*(3 -
 4*n + n^2) - a^2*b^2*(-6 + 2*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n
, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])])/(a + Sqrt[-b^2]))/((a^2 + b^2)*(
1 + n)) + 4*b*Cos[c + d*x]^4*(b + a*Tan[c + d*x]) - (2*b*Cos[c + d*x]^2*(b
^3*(-3 + n) - a^2*b*(1 + n) - a*(3*a^2 + b^2*(5 - 2*n))*Tan[c + d*x]))/(a^
2 + b^2)))/(16*b*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3987, 27, 496, 25, 686, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^n}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{b^6(a+b\tan(c+dx))^n}{(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^5\int \frac{(a+b\tan(c+dx))^n}{(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{496}$$

$$\frac{b^5\left(\frac{(ab\tan(c+dx)+b^2)(a+b\tan(c+dx))^{n+1}}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}-\frac{\int -\frac{(a+b\tan(c+dx))^n\left(3a^2+b(2-n)\tan(c+dx)a+b^2(3-n)\right)}{\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2(a^2+b^2)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^5\left(\frac{\int \frac{(a+b\tan(c+dx))^n\left(3a^2+b(2-n)\tan(c+dx)a+b^2(3-n)\right)}{\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2(a^2+b^2)}+\frac{(ab\tan(c+dx)+b^2)(a+b\tan(c+dx))^{n+1}}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}\right)}{d}$$

$$\Big\downarrow \text{686}$$

$$b^5\left(\frac{\frac{(a+b\tan(c+dx))^{n+1}\left(ab\left(3a^2+b^2(5-2n)\right)\tan(c+dx)+b^2\left(a^2(n+1)+b^2(3-n)\right)\right)}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{\int -\frac{(a+b\tan(c+dx))^n\left(3a^4+b^2\left(-n^2-2n+6\right)a^2-b\left(3a^2+b^2(5-2n)\right)n\tan}{\tan^2(c+dx)b^2+b^2}}{2b^2\left(a^2+b^2\right)}}{4b^2(a^2+b^2)}\right.$$
$$\overline{\qquad\qquad d \qquad\qquad}$$

$\big\downarrow$ 25

$$b^5\left(\frac{\frac{\int\frac{(a+b\tan(c+dx))^n\left(3a^4+b^2\left(-n^2-2n+6\right)a^2-b\left(3a^2+b^2(5-2n)\right)n\tan(c+dx)a+b^4\left(n^2-4n+3\right)\right)}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2\left(a^2+b^2\right)}+\frac{\left(ab\left(3a^2+b^2(5-2n)\right)\tan(c+dx)+b^2\left(a\right.\right.}{2b^2\left(a^2+b^2\right)\left(b^2\right.}}{4b^2(a^2+b^2)}\right.$$
$$\overline{\qquad\qquad d \qquad\qquad}$$

$\big\downarrow$ 657

$$b^5\left(\frac{\frac{\int\left(\frac{\left(a\left(3a^2+b^2(5-2n)\right)nb^2+\sqrt{-b^2}\left(3a^4+b^2\left(-n^2-2n+6\right)a^2+b^4\left(n^2-4n+3\right)\right)\right)(a+b\tan(c+dx))^n}{2b^2\left(\sqrt{-b^2}-b\tan(c+dx)\right)}+\frac{\left(\sqrt{-b^2}\left(3a^4+b^2\left(-n^2-2n+6\right)a^2+b^4\left(n^2-4n+3\right)\right)-\right.}{2b^2\left(b\tan(c+dx)+\right.}\right.}{2b^2\left(a^2+b^2\right)}}{4b^2(a^2+b^2)}\right.$$

$\big\downarrow$ 2009

$$b^5\left(\frac{(ab\tan(c+dx)+b^2)(a+b\tan(c+dx))^{n+1}}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}+\frac{\frac{\left(ab\left(3a^2+b^2(5-2n)\right)\tan(c+dx)+b^2\left(a^2(n+1)+b^2(3-n)\right)\right)(a+b\tan(c+dx))^{n+1}}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)}+\frac{\left(ab^2n\left(3a^2+b^2(5-2n\right.\right.}{}}{}\right.$$

input
```
Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

output
```
(b^5*(((a + b*Tan[c + d*x])^(1 + n)*(b^2 + a*b*Tan[c + d*x]))/(4*b^2*(a^2
+ b^2)*(b^2 + b^2*Tan[c + d*x]^2)^2) + (((a + b*Tan[c + d*x])^(1 + n)*(b^2
*(b^2*(3 - n) + a^2*(1 + n)) + a*b*(3*a^2 + b^2*(5 - 2*n))*Tan[c + d*x]))/
(2*b^2*(a^2 + b^2)*(b^2 + b^2*Tan[c + d*x]^2)) + (((a*b^2*(3*a^2 + b^2*(5
- 2*n))*n - Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2*n - n^2) + b^4*(3 - 4*n + n
^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b
^2])]*(a + b*Tan[c + d*x])^(1 + n))/(2*b^2*(a - Sqrt[-b^2])*(1 + n)) + ((a
*b^2*(3*a^2 + b^2*(5 - 2*n))*n + Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2*n - n^
2) + b^4*(3 - 4*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c
 + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(2*b^2*(a + Sqrt[
-b^2])*(1 + n)))/(2*b^2*(a^2 + b^2)))/(4*b^2*(a^2 + b^2))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [F]

$$\int \cos(dx+c)^4 (a+b\tan(dx+c))^n \, dx$$

input
```
int(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan(dx+c)+a)^n\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan(dx+c)+a)^n\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

Giac [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int \cos(c+d\,x)^4\,(a+b\tan(c+d\,x))^n\,dx$$

input
```
int(cos(c + d*x)^4*(a + b*tan(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)^4*(a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int (\tan(dx+c)\,b+a)^n\cos(dx+c)^4\,dx$$

input
```
int(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*b + a)**n*cos(c + d*x)**4,x)
```

3.660 $\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\operatorname{AppellF1}\left(1+n,-\frac{1}{2},-\frac{1}{2},2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\sec(c+dx)(a+b\tan(c+dx))^{1+n}}{bd(1+n)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}}$$

output
```
AppellF1(1+n,-1/2,-1/2,2+n,(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)),(a+b*tan(d*x+
c))/(a+(-b^2)^(1/2)))*sec(d*x+c)*(a+b*tan(d*x+c))^(1+n)/b/d/(1+n)/(1-(a+b*
tan(d*x+c))/(a-(-b^2)^(1/2)))^(1/2)/(1-(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))^
(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.55 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.92

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{2(a-ib)(a+ib)(2+n)\operatorname{AppellF1}\left(1-\right.}{bd(1+n)\left(2\left(a^2+b^2\right)(2+n)\operatorname{AppellF1}\left(1+n,-\frac{1}{2},-\frac{1}{2},2+n,\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)-\left((a-ib)\operatorname{Ap}\right.\right.}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

output
```
(2*(a - I*b)*(a + I*b)*(2 + n)*AppellF1[1 + n, -1/2, -1/2, 2 + n, (a + b*T
an[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Sec[c + d*x]*(a +
b*Tan[c + d*x])^(1 + n))/(b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 +
n, -1/2, -1/2, 2 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])
/(a + I*b)] - ((a - I*b)*AppellF1[2 + n, -1/2, 1/2, 3 + n, (a + b*Tan[c +
d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 +
n, 1/2, -1/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/
(a + I*b)])*(a + b*Tan[c + d*x])))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3992, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3(a+b\tan(c+dx))^ndx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int (a+b\tan(c+dx))^n\sqrt{\tan^2(c+dx)+1}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{514}$$

$$\frac{\sqrt{\tan^2(c+dx)+1}\sec(c+dx)\int (a+b\tan(c+dx))^n\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}d(a+b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sqrt{\tan^2(c+dx)+1}\sec(c+dx)(a+b\tan(c+dx))^{n+1}\operatorname{AppellF1}\left(n+1,-\frac{1}{2},-\frac{1}{2},n+2,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{bd(n+1)\sqrt{\sec^2(c+dx)}\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

output
```
(AppellF1[1 + n, -1/2, -1/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]),
 (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Sec[c + d*x]*(a + b*Tan[c + d*x])^
(1 + n)*Sqrt[1 + Tan[c + d*x]^2])/(b*d*(1 + n)*Sqrt[Sec[c + d*x]^2]*Sqrt[1
 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*Sqrt[1 - (a + b*Tan[c + d*x])/(a
 + Sqrt[-b^2])])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \sec (dx+c)^3 (a+b\tan (dx+c))^n\,dx$$

input
```
int(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

output
```
int(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan (dx+c)+a)^n \sec (dx+c)^3 \,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (a+b\tan (c+dx))^n \sec^3 (c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*sec(c + d*x)**3, x)
```

Maxima [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

Giac [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int \frac{(a+b\tan(c+d\,x))^n}{\cos(c+d\,x)^3}\,dx$$

input
```
int((a + b*tan(c + d*x))^n/cos(c + d*x)^3,x)
```

output
```
int((a + b*tan(c + d*x))^n/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (\tan(dx+c)\,b+a)^n \sec(dx+c)^3\,dx$$

input
```
int(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*b + a)**n*sec(c + d*x)**3,x)
```

3.661 $\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 159

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$= \frac{\operatorname{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos(c+dx)(a+b\tan(c+dx))^{1+n}\sqrt{1-\frac{a+b\tan(c+}{a-\sqrt{-b}}}}{bd(1+n)}$$

output
```
AppellF1(1+n,1/2,1/2,2+n,(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)),(a+b*tan(d*x+c)
)/(a+(-b^2)^(1/2)))*cos(d*x+c)*(a+b*tan(d*x+c))^(1+n)*(1-(a+b*tan(d*x+c))/
(a-(-b^2)^(1/2)))^(1/2)*(1-(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))^(1/2)/b/d/(1
+n)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.18 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.14

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$= \frac{2(a^2+b^2)^2(2+n)\operatorname{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,\right.}{(a-ib)(a+ib)bd(1+n)\left(2(a^2+b^2)(2+n)\operatorname{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)+\right.}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

output
```
(2*(a^2 + b^2)^2*(2 + n)*AppellF1[1 + n, 1/2, 1/2, 2 + n, (a + b*Tan[c + d
*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Cos[c + d*x]^3*(-I + Tan[c
 + d*x])*(I + Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*(a +
I*b)*b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n, 1/2, 1/2, 2 + n, (
a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + ((a - I*b
)*AppellF1[2 + n, 1/2, 3/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*
Tan[c + d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 3/2, 1/2, 3 + n, (a +
 b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)])*(a + b*Tan[c
+ d*x])))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3992, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(c+dx)(a+b\tan(c+dx))^ndx$$
$$\Big\downarrow \text{3992}$$
$$\frac{\sec(c+dx)\int \frac{(a+b\tan(c+dx))^n}{\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$
$$\Big\downarrow \text{514}$$
$$\frac{\sec(c+dx)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}\int \frac{(a+b\tan(c+dx))^n}{\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}}d(a+b\tan(c+dx))}{bd\sqrt{\tan^2(c+dx)+1}\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sec(c+dx)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}(a+b\tan(c+dx))^{n+1}\operatorname{AppellF1}\left(n+1,\frac{1}{2},\frac{1}{2},n+2,\frac{a+b\tan(c+dx}{a-\sqrt{-b^2}}\right.}{bd(n+1)\sqrt{\tan^2(c+dx)+1}\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

output
```
(AppellF1[1 + n, 1/2, 1/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]), (
a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Sec[c + d*x]*(a + b*Tan[c + d*x])^(1
 + n)*Sqrt[1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*Sqrt[1 - (a + b*Tan[
c + d*x])/(a + Sqrt[-b^2])])/(b*d*(1 + n)*Sqrt[Sec[c + d*x]^2]*Sqrt[1 + Ta
n[c + d*x]^2])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \sec(dx+c)\,(a+b\tan(dx+c))^n\,dx$$

input
```
int(sec(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

output
```
int(sec(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

Sympy [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx = \int (a+b\tan(c+dx))^n\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n \sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n \sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx = \int \frac{(a+b\tan(c+d\,x))^n}{\cos(c+d\,x)}\,dx$$

input
```
int((a + b*tan(c + d*x))^n/cos(c + d*x),x)
```

output
```
int((a + b*tan(c + d*x))^n/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx = \int \left(\tan\left(dx+c\right)b+a\right)^n \sec\left(dx+c\right)dx$$

input
```
int(sec(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*b + a)**n*sec(c + d*x),x)
```

3.662 $\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 161

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$= \frac{\text{AppellF1}\left(1+n,\frac{3}{2},\frac{3}{2},2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos^3(c+dx)(a+b\tan(c+dx))^{1+n}\left(1-\frac{a+b\tan(c}{a-\sqrt{-}}\right.}{bd(1+n)}$$

output
```
AppellF1(1+n,3/2,3/2,2+n,(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)),(a+b*tan(d*x+c)
)/(a+(-b^2)^(1/2)))*cos(d*x+c)^3*(a+b*tan(d*x+c))^(1+n)*(1-(a+b*tan(d*x+c)
)/(a-(-b^2)^(1/2)))^(3/2)*(1-(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))^(3/2)/b/d/
(1+n)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.08 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.12

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$= \frac{2(a^2+b^2)^2(2+n)\,\text{AppellF1}\left(1+n,\frac{3}{2},\frac{3}{2},2+n,\right.}{(a-ib)(a+ib)bd(1+n)\left(2(a^2+b^2)(2+n)\,\text{AppellF1}\left(1+n,\frac{3}{2},\frac{3}{2},2+n,\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)+3\right.}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

output
```
(2*(a^2 + b^2)^2*(2 + n)*AppellF1[1 + n, 3/2, 3/2, 2 + n, (a + b*Tan[c + d
*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Cos[c + d*x]^5*(-I + Tan[c
 + d*x])*(I + Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*(a +
I*b)*b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n, 3/2, 3/2, 2 + n, (
a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + 3*((a - I
*b)*AppellF1[2 + n, 3/2, 5/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a +
b*Tan[c + d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 5/2, 3/2, 3 + n, (a
 + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)])*(a + b*Tan[
c + d*x])))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3992, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^n}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{(a+b\tan(c+dx))^n}{(\tan^2(c+dx)+1)^{3/2}}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{514}$$

$$\frac{\sec(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{3/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{3/2}\int \frac{(a+b\tan(c+dx))^n}{\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{3/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{3/2}}d(a+b\tan(c+dx))}{bd\left(\tan^2(c+dx)+1\right)^{3/2}\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sec(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{3/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{3/2}(a+b\tan(c+dx))^{n+1}\text{AppellF1}\left(n+1,\frac{3}{2},\frac{3}{2},n+2,\frac{a+b}{a}\right.}{bd(n+1)\left(\tan^2(c+dx)+1\right)^{3/2}\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

output
```
(AppellF1[1 + n, 3/2, 3/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]), (
a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Sec[c + d*x]*(a + b*Tan[c + d*x])^(1
 + n)*(1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]))^(3/2)*(1 - (a + b*Tan[c
+ d*x])/(a + Sqrt[-b^2]))^(3/2))/(b*d*(1 + n)*Sqrt[Sec[c + d*x]^2]*(1 + Ta
n[c + d*x]^2)^(3/2))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \cos (dx + c) (a + b \tan (dx + c))^n \, dx$$

input
```
int(cos(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos(c + dx)(a + b\tan(c + dx))^n \, dx = \int (b \tan (dx + c) + a)^n \cos (dx + c) \, dx$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

Sympy [F]

$$\int \cos(c + dx)(a + b\tan(c + dx))^n \, dx = \int (a + b \tan (c + dx))^n \cos (c + dx) \, dx$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*cos(c + d*x), x)
```

Maxima [F]

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx = \int \cos(c+d\,x)\,\left(a+b\tan(c+d\,x)\right)^n\,dx$$

input
```
int(cos(c + d*x)*(a + b*tan(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)*(a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx = \int (\tan(dx+c)\,b+a)^n \cos(dx+c)\,dx$$

input
```
int(cos(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*b + a)**n*cos(c + d*x),x)
```

3.663 $\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 161

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\text{AppellF1}\left(1+n,\frac{5}{2},\frac{5}{2},2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos^5(c+dx)(a+b\tan(c+dx))^{1+n}\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right.}{bd(1+n)}$$

output
```
AppellF1(1+n,5/2,5/2,2+n,(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)),(a+b*tan(d*x+c)
)/(a+(-b^2)^(1/2)))*cos(d*x+c)^5*(a+b*tan(d*x+c))^(1+n)*(1-(a+b*tan(d*x+c)
)/(a-(-b^2)^(1/2)))^(5/2)*(1-(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))^(5/2)/b/d/
(1+n)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.07 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.12

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{2(a^2+b^2)^2(2+n)\,\text{AppellF1}\left(1+n,\frac{5}{2},\frac{5}{2},2+n,\right.}{(a-ib)(a+ib)bd(1+n)\left(2\left(a^2+b^2\right)(2+n)\,\text{AppellF1}\left(1+n,\frac{5}{2},\frac{5}{2},2+n,\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)+\right.}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

output
```
(2*(a^2 + b^2)^2*(2 + n)*AppellF1[1 + n, 5/2, 5/2, 2 + n, (a + b*Tan[c + d
*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Cos[c + d*x]^7*(-I + Tan[c
 + d*x])*(I + Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*(a +
I*b)*b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n, 5/2, 5/2, 2 + n, (
a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + 5*((a - I
*b)*AppellF1[2 + n, 5/2, 7/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a +
b*Tan[c + d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 7/2, 5/2, 3 + n, (a
 + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)])*(a + b*Tan[
c + d*x])))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3992, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^n}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{(a+b\tan(c+dx))^n}{(\tan^2(c+dx)+1)^{5/2}}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{514}$$

$$\frac{\sec(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{5/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{5/2}\int \frac{(a+b\tan(c+dx))^n}{\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{5/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{5/2}}d(a+b\tan(c+\ldots}{bd\left(\tan^2(c+dx)+1\right)^{5/2}\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 150$$

$$\frac{\sec(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{5/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{5/2}(a+b\tan(c+dx))^{n+1}\operatorname{AppellF1}\left(n+1,\frac{5}{2},\frac{5}{2},n+2,\frac{a+b}{c}\right.}{bd(n+1)\left(\tan^2(c+dx)+1\right)^{5/2}\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

output
```
(AppellF1[1 + n, 5/2, 5/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]), (
a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Sec[c + d*x]*(a + b*Tan[c + d*x])^(1
 + n)*(1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]))^(5/2)*(1 - (a + b*Tan[c
+ d*x])/(a + Sqrt[-b^2]))^(5/2))/(b*d*(1 + n)*Sqrt[Sec[c + d*x]^2]*(1 + Ta
n[c + d*x]^2)^(5/2))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \cos (dx+c)^3 (a+b\tan (dx+c))^n \, dx$$

input
```
int(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n \, dx = \int (b\tan (dx+c)+a)^n \cos (dx+c)^3 \, dx$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n \, dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

Giac [F]

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int \cos(c+d\,x)^3\,(a+b\tan(c+d\,x))^n\,dx$$

input
```
int(cos(c + d*x)^3*(a + b*tan(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)^3*(a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (\tan(dx+c)\,b+a)^n \cos(dx+c)^3\,dx$$

input
```
int(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

output
```
int((tan(c + d*x)*b + a)**n*cos(c + d*x)**3,x)
```

3.664 $\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 124

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx=-\frac{2ia(e\cos(c+dx))^{7/2}}{7d}+\frac{10a(e\cos(c+dx))^{7/2}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\cos^{\frac{7}{2}}(c+dx)}+\frac{2a(e\cos(c+dx))^{7/2}\tan(c+dx)}{7d}+\frac{10a(e\cos(c+dx))^{7/2}\sec^2(c+dx)\tan(c+dx)}{21d}$$

output
```
-2/7*I*a*(e*cos(d*x+c))^(7/2)/d+10/21*a*(e*cos(d*x+c))^(7/2)*InverseJacobi
AM(1/2*d*x+1/2*c,2^(1/2))/d/cos(d*x+c)^(7/2)+2/7*a*(e*cos(d*x+c))^(7/2)*ta
n(d*x+c)/d+10/21*a*(e*cos(d*x+c))^(7/2)*sec(d*x+c)^2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx=\frac{ae^3\sqrt{e\cos(c+dx)}(\cos(dx)-i\sin(dx))\left(10\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(c+dx)-i\right.}{}$$

input
```
Integrate[(e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*e^3*Sqrt[e*Cos[c + d*x]]*(Cos[d*x] - I*Sin[d*x])*(10*EllipticF[(c + d*x
)/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]) + Sqrt[Cos[c + d*x]]*(-8*I + (2*I)
*Cos[2*(c + d*x)] + 5*Sin[2*(c + d*x)]))*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x
]))/(21*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.27, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 3998, 3042, 3967, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))(e\cos(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))(e\cos(c+dx))^{7/2}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\int \frac{i\tan(c+dx)a+a}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\int \frac{i\tan(c+dx)a+a}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3967}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(a\int \frac{1}{(e\sec(c+dx))^{7/2}}dx-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(a\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \; 4256$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(a\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(a\left(\frac{5\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \; 4256$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(a\left(\frac{5\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(a\left(\frac{5\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \; 4258$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(a\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(a\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \textcolor{blue}{3120}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(a\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{}{7d(e\sec}\right.$$

input
```
Int[(e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*((((-2*I)/7)*a)/(d*(e*Sec[c
+ d*x])^(7/2)) + a*((2*Sin[c + d*x])/(7*d*e*(e*Sec[c + d*x])^(5/2)) + (5*(
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d
*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(106) = 212$.

Time = 11.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.85

method	result
parts	$-\frac{2a\sqrt{e\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, e^{4}\left(48\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+5\sqrt{\frac{1}{2}-}\right.}{21\sqrt{-e\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{e\left(2\cos\right.}}$
default	$-\frac{2a\,e^{4}\left(48i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+48\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-96i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-72\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+72i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+56\right.}{21}$

input
```
int((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/21*a*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^4*(48*
cos(1/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*
cos(1/2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(
-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)
/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d-2/7*I*a*(e*cos(d*x+c))^(7/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \frac{\left(-40i\sqrt{\frac{1}{2}}ae^{\frac{7}{2}}e^{(i\,dx+i\,c)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{\frac{1}{2}}\left(-3i\,ae^{3}e^{(4i\,dx+4i\,c)}\right.\right.}{42\,d}$$

input
```
integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/42*(-40*I*sqrt(1/2)*a*e^(7/2)*e^(I*d*x + I*c)*weierstrassPInverse(-4, 0,
 e^(I*d*x + I*c)) + sqrt(1/2)*(-3*I*a*e^3*e^(4*I*d*x + 4*I*c) - 16*I*a*e^3
*e^(2*I*d*x + 2*I*c) + 7*I*a*e^3)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*
I*d*x - 1/2*I*c))*e^(-I*d*x - I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(7/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \int (e\cos(dx+c))^{\frac{7}{2}}(i\,a\tan(dx+c)+a)\,dx$$

input
```
integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*cos(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a), x)
```

Giac [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \int (e\,\cos(c+d\,x))^{7/2}\,(a+a\tan(c+d\,x)\,1\mathrm{i})\,dx$$

input
```
int((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \sqrt{e}\,a\,e^3\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\tan(dx+c)\,dx\right)i + \int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)$$

input
```
int((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
sqrt(e)*a*e**3*(int(sqrt(cos(c + d*x))*cos(c + d*x)**3*tan(c + d*x),x)*i +
 int(sqrt(cos(c + d*x))*cos(c + d*x)**3,x))
```

3.665 $\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 90

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx=-\frac{2ia(e\cos(c+dx))^{5/2}}{5d}+\frac{6a(e\cos(c+dx))^{5/2}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a(e\cos(c+dx))^{5/2}\tan(c+dx)}{5d}$$

output
```
-2/5*I*a*(e*cos(d*x+c))^(5/2)/d+6/5*a*(e*cos(d*x+c))^(5/2)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(5/2)+2/5*a*(e*cos(d*x+c))^(5/2)*tan(d
*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.76 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.78

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx=\frac{e^2\sqrt{e\cos(c+dx)}(\cos(dx)-i\sin(dx))\left(\left(9\cos(c-dx-\arctan(\tan(c)))(\csc(c)-i\operatorname{s}\right.\right.}{}$$

input
```
Integrate[(e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(e^2*Sqrt[e*Cos[c + d*x]]*(Cos[d*x] - I*Sin[d*x])*((9*Cos[c - d*x - ArcTan
[Tan[c]]]*(Csc[c] - I*Sec[c]) + 3*Cos[c + d*x + ArcTan[Tan[c]]]*(Csc[c] -
I*Sec[c]) - Csc[c]*Sqrt[Sec[c]^2]*(Cos[d*x] + I*Sin[d*x])*(6*Cos[c] + 3*Co
s[c + 2*d*x] + 3*Cos[3*c + 2*d*x] + (2*I)*Sin[c] - (4*I)*Sin[c + 2*d*x] -
(2*I)*Sin[3*c + 2*d*x]))*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2] + (6*I)*Hyperge
ometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + Arc
Tan[Tan[c]]]*(I + Tan[c]))*(a + I*a*Tan[c + d*x]))/(10*d*Sqrt[Sec[c]^2]*Sq
rt[Sin[d*x + ArcTan[Tan[c]]]^2])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3998, 3042, 3967, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))(e\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))(e\cos(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\int \frac{i\tan(c+dx)a+a}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\int \frac{i\tan(c+dx)a+a}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3967}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(a\int \frac{1}{(e\sec(c+dx))^{5/2}}dx-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(a\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 4256$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(a\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(a\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 4258$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(a\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(a\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3119$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}\right)$$

input
```
Int[(e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2)*((((-2*I)/5)*a)/(d*(e*Sec[c
+ d*x])^(5/2)) + a*((6*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*
x]]*Sqrt[e*Sec[c + d*x]]) + (2*Sin[c + d*x])/(5*d*e*(e*Sec[c + d*x])^(3/2)
)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(77) = 154$.

Time = 9.01 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.28

method	result
default	$\frac{2a\,e^3\left(8i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^7+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-12i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^5-8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+6i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^3+2\sin\left(\frac{dx}{2}+\right.\right.}{5\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 e}}$
parts	$-\frac{2a\sqrt{e\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,e^3\left(-8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\right.\right.}{5\sqrt{-e\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{e\left(2\cos\left(\right.\right.}}$
risch	$-\frac{i\left(\mathrm{e}^{2i(dx+c)}+7\right)\sqrt{2}\,e^2 a\sqrt{e\left(\mathrm{e}^{2i(dx+c)}+1\right)\mathrm{e}^{-i(dx+c)}}}{10d}-\frac{3i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}}{}\right.}{}$

input
```
int((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/5/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^3*(8*I*sin(
1/2*d*x+1/2*c)^7+8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*I*sin(1/2*d*
x+1/2*c)^5-8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+6*I*sin(1/2*d*x+1/2*c
)^3+2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-
I*sin(1/2*d*x+1/2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \frac{12i\sqrt{\frac{1}{2}}ae^{\frac{5}{2}}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{\frac{1}{2}}\left(-i\,ae\right.}{5\,d}$$

input
```
integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/5*(12*I*sqrt(1/2)*a*e^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, e^(I*d*x + I*c))) + sqrt(1/2)*(-I*a*e^2*e^(2*I*d*x + 2*I*c) + 5*I*a*
e^2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(5/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \int (e\cos(dx+c))^{\frac{5}{2}}(i\,a\tan(dx+c)+a)\,dx$$

input
```
integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*cos(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a), x)
```

Giac [F]

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \int (e\cos(dx+c))^{\frac{5}{2}}(i\,a\tan(dx+c)+a)\,dx$$

input
```
integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*cos(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \int (e\,\cos\left(c+d\,x\right))^{5/2}\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)\,dx$$

input
```
int((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \sqrt{e}\,a\,e^{2}\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{2}\tan\left(dx+c\right)dx\right)i+\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{2}dx\right)$$

input
```
int((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
sqrt(e)*a*e**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**2*tan(c + d*x),x)*i +
 int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x))
```

3.666 $\int (e\cos (c+dx))^{3/2}(a+ia\tan (c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 90

$$\int (e\cos (c+dx))^{3/2}(a+ia\tan (c+dx))\,dx=-\frac{2ia(e\cos (c+dx))^{3/2}}{3d}+\frac{2a(e\cos (c+dx))^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\cos ^{\frac{3}{2}}(c+dx)}+\frac{2a(e\cos (c+dx))^{3/2}\tan (c+dx)}{3d}$$

output
```
-2/3*I*a*(e*cos(d*x+c))^(3/2)/d+2/3*a*(e*cos(d*x+c))^(3/2)*InverseJacobiAM
(1/2*d*x+1/2*c,2^(1/2))/d/cos(d*x+c)^(3/2)+2/3*a*(e*cos(d*x+c))^(3/2)*tan(
d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int (e\cos (c+dx))^{3/2}(a+ia\tan (c+dx))\,dx=\frac{2ae\sqrt{\cos (c+dx)}\sqrt{e\cos (c+dx)}\left(\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(i\cos (c)+\sin (c))+\sqrt{\cos}\right.}{3d}$$

input
```
Integrate[(e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(2*a*e*Sqrt[Cos[c + d*x]]*Sqrt[e*Cos[c + d*x]]*(EllipticF[(c + d*x)/2, 2]*
(I*Cos[c] + Sin[c]) + Sqrt[Cos[c + d*x]]*(Cos[d*x] + I*Sin[d*x]))*(Cos[d*x
] - I*Sin[d*x])*(-I + Tan[c + d*x]))/(3*d)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3998, 3042, 3967, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))(e\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))(e\cos(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\int \frac{i\tan(c+dx)a+a}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\int \frac{i\tan(c+dx)a+a}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3967}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(a\int \frac{1}{(e\sec(c+dx))^{3/2}}dx-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(a\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(a\left(\frac{\int\sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(a\left(\frac{\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(a\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(a\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(a\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3}}\right.$$

input
```
Int[(e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(e*Cos[c + d*x])^(3/2)*(e*Sec[c + d*x])^(3/2)*((((-2*I)/3)*a)/(d*(e*Sec[c
+ d*x])^(3/2)) + a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e
*Sec[c + d*x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]]))
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(76) = 152$.

Time = 4.54 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.87

method	result
default	$-\frac{2a\,e^2\left(4i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^5+4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^3-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}\right.}\right.}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2e+e}\,d}$
parts	$-\frac{2a\sqrt{e\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,e^2\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin}\right.}{3\sqrt{-e\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{e\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}\,d}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}\sqrt{2}\,ea\sqrt{e\left(\mathrm{e}^{2i(dx+c)}+1\right)\mathrm{e}^{-i(dx+c)}}}{3d}+\frac{2\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\ \mathrm{EllipticF}\left(\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right.}{3d\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx+c)}}\,\left(\mathrm{e}^{2i}\right.}$

input
```
int((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^2*(4*I*sin
(1/2*d*x+1/2*c)^5+4*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*I*sin(1/2*d*
x+1/2*c)^3-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+I*sin(1/2*d*x+1/2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = -\frac{2\left(i\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\,aee^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+2i\sqrt{\frac{1}{2}}ae^{\frac{3}{2}}\,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{3\,d}$$

input
```
integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2/3*(I*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*a*e*e^(1/2*I*d*x + 1/2*I
*c) + 2*I*sqrt(1/2)*a*e^(3/2)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))
/d
```

Sympy [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(3/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \int (e\cos(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)\,dx$$

input
```
integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*cos(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a), x)
```

Giac [F]

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \int (e\cos(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)\,dx$$

input
```
integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*cos(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \int (e\cos(c+dx))^{3/2}\,(a+a\tan(c+dx)\,1\mathrm{i})\,dx$$

input
```
int((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \sqrt{e}\,ae\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\tan(dx+c)\,dx\right)i + \int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)$$

input
```
int((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
sqrt(e)*a*e*(int(sqrt(cos(c + d*x))*cos(c + d*x)*tan(c + d*x),x)*i + int(s
qrt(cos(c + d*x))*cos(c + d*x),x))
```

3.667 $\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx = -\frac{2ia\sqrt{e\cos(c+dx)}}{d} + \frac{2a\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

output
```
-2*I*a*(e*cos(d*x+c))^(1/2)/d+2*a*(e*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.26 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.20

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx$$
$$= \frac{a\cos(c)\sqrt{e\cos(c+dx)}\sin(c)(\cos(dx)-i\sin(dx))\left({}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)(-i\csc(\right.}{}$$

input
```
Integrate[Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*Cos[c]*Sqrt[e*Cos[c + d*x]]*Sin[c]*(Cos[d*x] - I*Sin[d*x])*(Hypergeomet
ricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*((-I)*Csc[c] - Se
c[c])*Sec[c]*Sin[d*x + ArcTan[Tan[c]]] + Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]
*(2*Cos[d*x + ArcTan[Tan[c]]]*Csc[c]*(I*Csc[c] + Sec[c]) + Sec[c]*((-2*I)*
Cos[c + d*x]*Csc[c]^2*Sqrt[Sec[c]^2] + (I*Csc[c] + Sec[c])*Sin[d*x + ArcTa
n[Tan[c]]]))*(-I + Tan[c + d*x]))/(d*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan
[Tan[c]]]^2])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.42, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3998, 3042, 3967, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))\sqrt{e\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))\sqrt{e\cos(c+dx)}dx$$

$$\Big\downarrow \text{3998}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{i\tan(c+dx)a+a}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{i\tan(c+dx)a+a}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3967}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(a\int \frac{1}{\sqrt{e\sec(c+dx)}}dx-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(a\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{ 4258}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{a\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{ 3119}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)$$

input
```
Int[Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

output
```
Sqrt[e*Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]*(((-2*I)*a)/(d*Sqrt[e*Sec[c + d*
x]]) + (2*a*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
+ d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 3.77 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.80

method	result
default	$\frac{2ae\left(2i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}-i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}e+e}\,d}$
parts	$\frac{2a\sqrt{e\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,e\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-e\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{e\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}\,d}-\frac{2ia\sqrt{e\cos(dx+}}{d}$
risch	$-\frac{2i\sqrt{2}\,a\sqrt{e\left(\mathrm{e}^{2i(dx+c)}+1\right)\mathrm{e}^{-i(dx+c)}}}{d}-\frac{i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\left(-2i\,\mathrm{E}\right.}{\sqrt{e\,\mathrm{e}^{3i}}}\right.}{}$

input
```
int((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e*(2*I*sin(1/2*
d*x+1/2*c)^3+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-I*sin(1/2*d*x+1/2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.45

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx$$
$$= \frac{4i\,\sqrt{\frac{1}{2}}a\sqrt{e}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{d}$$

input
```
integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
4*I*sqrt(1/2)*a*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
e^(I*d*x + I*c)))/d
```

Sympy [F]

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx = ia\left(\int \left(-i\sqrt{e\cos\left(c+dx\right)}\right)\,dx + \int \sqrt{e\cos\left(c+dx\right)}\tan\left(c+dx\right)dx\right)$$

input
```
integrate((e*cos(d*x+c))**(1/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*sqrt(e*cos(c + d*x)), x) + Integral(sqrt(e*cos(c + d*x))*
tan(c + d*x), x))
```

Maxima [F]

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx=\int \sqrt{e\cos\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)\,dx$$

input
```
integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*cos(d*x + c))*(I*a*tan(d*x + c) + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx=\text{Timed out}$$

input
```
integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx=\int \sqrt{e\,\cos\left(c+d\,x\right)}\,\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)\,dx$$

input
```
int((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx = \sqrt{e}\,a\left(\int \sqrt{\cos(dx+c)}dx + \left(\int \sqrt{\cos(dx+c)}\,\tan(dx+c)\,dx\right)i\right)$$

input
```
int((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
sqrt(e)*a*(int(sqrt(cos(c + d*x)),x) + int(sqrt(cos(c + d*x))*tan(c + d*x)
,x)*i)
```

3.668 $\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx=\frac{2ia}{d\sqrt{e\cos(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{e\cos(c+dx)}}$$

output
```
2*I*a/d/(e*cos(d*x+c))^(1/2)+2*a*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+
1/2*c,2^(1/2))/d/(e*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.38 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.38

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx=$$

$$-\frac{\sqrt{2}a\sqrt{e\cos(c+dx)}(-i+\cot(c))\left(\sqrt{2}\sqrt{\csc^2(c)}+i\cos(c+dx)\sqrt{1+\cos(2dx-2\arctan(\cot(c)))}\csc\right.}{}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

output
```
-((Sqrt[2]*a*Sqrt[e*Cos[c + d*x]]*(-I + Cot[c])*(Sqrt[2]*Sqrt[Csc[c]^2] +
I*Cos[c + d*x]*Sqrt[1 + Cos[2*d*x - 2*ArcTan[Cot[c]]]]*Csc[c]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot
[c]]])*Sin[c]*(Cos[d*x] - I*Sin[d*x])*(-I + Tan[c + d*x]))/(d*e*Sqrt[Csc[c
]^2]))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.42, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3998, 3042, 3967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3967}$$

$$\frac{a\int \sqrt{e\sec(c+dx)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}}{d}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

input
```
Int[(a + I*a*Tan[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

output
```
(((2*I)*a*Sqrt[e*Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d)/(Sqrt[e*Cos[c + d*x]]*Sqrt[e*Sec[c +
d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

method	result	size
parts	$\frac{2ia}{d\sqrt{e\cos(dx+c)}}+\frac{2a\sqrt{\cos(dx+c)}\,\mathrm{InverseJacobiAM}\left(\frac{dx}{2}+\frac{c}{2},\sqrt{2}\right)}{d\sqrt{e\cos(dx+c)}}$	55
default	$\frac{2\left(-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}e+e}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)d}$	94

input
```
int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I*a/d/(e*cos(d*x+c))^(1/2)+2*a*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+
1/2*c,2^(1/2))/d/(e*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int\frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx=$$
$$-\frac{4\left(-i\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}ae^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+\sqrt{\frac{1}{2}}\left(i\,ae^{(2i\,dx+2i\,c)}+i\,a\right)\sqrt{e}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+}\right.\right.}{dee^{(2i\,dx+2i\,c)}+de}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-4*(-I*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*a*e^(1/2*I*d*x + 1/2*I*c)
 + sqrt(1/2)*(I*a*e^(2*I*d*x + 2*I*c) + I*a)*sqrt(e)*weierstrassPInverse(-
4, 0, e^(I*d*x + I*c)))/(d*e*e^(2*I*d*x + 2*I*c) + d*e)
```

Sympy [F]

$$\int \frac{a + ia\tan(c + dx)}{\sqrt{e\cos(c + dx)}}\,dx = ia\left(\int \left(-\frac{i}{\sqrt{e\cos{(c + dx)}}}\right)dx + \int \frac{\tan{(c + dx)}}{\sqrt{e\cos{(c + dx)}}}\,dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(1/2),x)
```

output
```
I*a*(Integral(-I/sqrt(e*cos(c + d*x)), x) + Integral(tan(c + d*x)/sqrt(e*c
os(c + d*x)), x))
```

Maxima [F]

$$\int \frac{a + ia\tan(c + dx)}{\sqrt{e\cos(c + dx)}}\,dx = \int \frac{i\,a\tan{(dx + c)} + a}{\sqrt{e\cos{(dx + c)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{-4,[1]%%%},0]:[1,0,%%%{1,[1]%%%}]%%},[2,1]%%%}+%%%
{%%%{8,[2
```

Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx = \frac{2\,a\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{e\,\cos\left(c+d\,x\right)}} + \frac{a\,\cos\left(c+d\,x\right)\,\sqrt{e\,\cos\left(c+d\,x\right)}\,4{}\mathrm{i}}{d\,e\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(1/2),x)
```

output
```
(2*a*cos(c + d*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(d*(e*cos(c + d*x))^(
1/2)) + (a*cos(c + d*x)*(e*cos(c + d*x))^(1/2)*4i)/(d*e*(cos(2*c + 2*d*x)
+ 1))
```

Reduce [F]

$$\int \frac{a + ia\tan(c + dx)}{\sqrt{e\cos(c + dx)}}\,dx = \frac{\sqrt{e}\,a\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx + \left(\int \frac{\sqrt{\cos(dx+c)}\,\tan(dx+c)}{\cos(dx+c)}dx\right)i\right)}{e}$$

input
```
int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*a*(int(sqrt(cos(c + d*x))/cos(c + d*x),x) + int((sqrt(cos(c + d*x
))*tan(c + d*x))/cos(c + d*x),x)*i))/e
```

3.669 $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx=\frac{2ia}{3d(e\cos(c+dx))^{3/2}}-\frac{2a\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d(e\cos(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}$$

output
```
2/3*I*a/d/(e*cos(d*x+c))^(3/2)-2*a*cos(d*x+c)^(3/2)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d/(e*cos(d*x+c))^(3/2)+2*a*sin(d*x+c)/d/e/(e*cos(d*x+c))^(
1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.02 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.13

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx=\frac{(\cos(dx)-i\sin(dx))\left(\frac{6\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\arctan(\tan(c)))}{\sqrt{\sin^2(dx+\arctan(\tan(c)))}}\right.}{}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

output
```
((Cos[d*x] - I*Sin[d*x])*((6*Cos[c + d*x]*HypergeometricPFQ[{-1/2, -1/4},
{3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*(1 - I*Tan[c
]))/Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2] + (Csc[c] - I*Sec[c])*(-3*Cos[c + d*
x]*(3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]]) + (4*
(I + 3*Cos[d*x]*Cos[c + d*x]*Csc[c])*Tan[c])/Sqrt[Sec[c]^2]))*(a + I*a*Tan
[c + d*x]))/(6*d*e*Sqrt[e*Cos[c + d*x]]*Sqrt[Sec[c]^2])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3998, 3042, 3967, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3967}$$

$$\frac{a\int (e\sec(c+dx))^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 4255$$

$$\frac{a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 4258$$

$$\frac{a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3119$$

$$\frac{a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

output
```
((((2*I)/3)*a*(e*Sec[c + d*x])^(3/2))/d + a*((-2*e^2*EllipticE[(c + d*x)/2
, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*
x]]*Sin[c + d*x])/d))/((e*Cos[c + d*x])^(3/2)*(e*Sec[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(80) = 160$.

Time = 2.88 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.40

method	result
default	$\frac{2\left(12\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-6\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-6\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right.}{3\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\right.}}$
parts	$-\frac{2a\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}e+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}e}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\right.}}{e\sqrt{-e\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{e\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}\,d}$

input
```
int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e*(12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-6*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)-I*sin(1/2*d*x+1/2*c))*a/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.62

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{4\left(\sqrt{\frac{1}{2}}\left(3i\,ae^{(4i\,dx+4i\,c)}+i\,ae^{(2i\,dx+2i\,c)}\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+3\sqrt{\frac{1}{2}}\left(i\,ae^{(4i\,dx+4i\,c)}+2i\,ae^{(2i\,dx+2i\,c}\right.\right.}{3\left(de^{2}e^{(4i\,dx+4i\,c)}+2\,de^{2}e^{(2i\,dx+2i\,c)}+\right.}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-4/3*(sqrt(1/2)*(3*I*a*e^(4*I*d*x + 4*I*c) + I*a*e^(2*I*d*x + 2*I*c))*sqrt
(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c) + 3*sqrt(1/2)*(I*a*e^
(4*I*d*x + 4*I*c) + 2*I*a*e^(2*I*d*x + 2*I*c) + I*a)*sqrt(e)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(d*e^2*e^(4*I*d*x
+ 4*I*c) + 2*d*e^2*e^(2*I*d*x + 2*I*c) + d*e^2)
```

Sympy [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx = ia\left(\int\left(-\frac{i}{(e\cos(c+dx))^{\frac{3}{2}}}\right)dx+\int\frac{\tan(c+dx)}{(e\cos(c+dx))^{\frac{3}{2}}}\,dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(3/2),x)
```

output
```
I*a*(Integral(-I/(e*cos(c + d*x))**(3/2), x) + Integral(tan(c + d*x)/(e*co
s(c + d*x))**(3/2), x))
```

Maxima [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx = \int \frac{i\,a\tan(dx+c)+a}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx=\int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx=\int \frac{a+a\tan(c+d\,x)\ 1\mathrm{i}}{\left(e\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{e}\,a\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^{2}}dx+\left(\int \frac{\sqrt{\cos(dx+c)}\,\tan(dx+c)}{\cos(dx+c)^{2}}dx\right)i\right)}{e^{2}}$$

input
```
int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*a*(int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x) + int((sqrt(cos(c +
d*x))*tan(c + d*x))/cos(c + d*x)**2,x)*i))/e**2
```

3.670 $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2ia}{5d(e\cos(c+dx))^{5/2}} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d(e\cos(c+dx))^{5/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{3d(e\cos(c+dx))^{5/2}}$$

output
```
2/5*I*a/d/(e*cos(d*x+c))^(5/2)+2/3*a*cos(d*x+c)^(5/2)*InverseJacobiAM(1/2*
d*x+1/2*c,2^(1/2))/d/(e*cos(d*x+c))^(5/2)+2/3*a*cos(d*x+c)*sin(d*x+c)/d/(e
*cos(d*x+c))^(5/2)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.59

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx = \frac{a\left(6i+10\cos^{\frac{5}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))\right)}{15d(e\cos(c+dx))^{5/2}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

output
```
(a*(6*I + 10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 5*Sin[2*(c + d
*x)]))/(15*d*(e*Cos[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3998, 3042, 3967, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int (e\sec(c+dx))^{5/2}(i\tan(c+dx)a+a)dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e\sec(c+dx))^{5/2}(i\tan(c+dx)a+a)dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3967}$$

$$\frac{a\int (e\sec(c+dx))^{5/2}dx+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{a\left(\frac{1}{3}e^2\int \sqrt{e\sec(c+dx)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{1}{3}e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{a\left(\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

output
```
((((2*I)/5)*a*(e*Sec[c + d*x])^(5/2))/d + a*((2*e^2*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d) + (2*e*(e*Sec[c + d*x])
^(3/2)*Sin[c + d*x])/(3*d)))/((e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2
))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(82) = 164$.

Time = 2.94 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.71

method	result
parts	$-\frac{2a\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(}{}}}\right.}{3e^2\sqrt{-e\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin}$
default	$-\frac{2\left(20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+20\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-20\sqrt{\frac{1}{2}-\frac{\cos}{}}\right.}{15\left(4\sin\right.}$

input
```
int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*a*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/e^2*(e*(2*cos(1/2*d*x
+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1
/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(e*(
2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d+2/5*I*a/d/(e*cos(d*x+c))^(5/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(81) = 162$.

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.88

$$\int \frac{a + ia\tan(c + dx)}{(e\cos(c + dx))^{5/2}}\,dx =$$

$$-\frac{4\left(\sqrt{\frac{1}{2}}\left(5i\,ae^{(5i\,dx+5i\,c)} - 12i\,ae^{(3i\,dx+3i\,c)} - 5i\,ae^{(i\,dx+i\,c)}\right)\sqrt{ee^{(2i\,dx+2i\,c)} + e}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)} + 5\sqrt{\frac{1}{2}}\left(i\,ae^{(6i\,dx+6i}\right.\right.}{15\left(de^3e^{(6i\,dx+6i\,c)} + 3\,de^3e^{(4i\,dx+4i\,c)} + 3\,d\right.}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-4/15*(sqrt(1/2)*(5*I*a*e^(5*I*d*x + 5*I*c) - 12*I*a*e^(3*I*d*x + 3*I*c) -
 5*I*a*e^(I*d*x + I*c))*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/
2*I*c) + 5*sqrt(1/2)*(I*a*e^(6*I*d*x + 6*I*c) + 3*I*a*e^(4*I*d*x + 4*I*c)
+ 3*I*a*e^(2*I*d*x + 2*I*c) + I*a)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I
*d*x + I*c)))/(d*e^3*e^(6*I*d*x + 6*I*c) + 3*d*e^3*e^(4*I*d*x + 4*I*c) + 3
*d*e^3*e^(2*I*d*x + 2*I*c) + d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + ia \tan(c + dx)}{(e \cos(c + dx))^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a + ia \tan(c + dx)}{(e \cos(c + dx))^{5/2}} \, dx = \int \frac{i\,a \tan(dx + c) + a}{(e \cos(dx + c))^{\frac{5}{2}}} \, dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{a + ia \tan(c + dx)}{(e \cos(c + dx))^{5/2}} \, dx = \int \frac{i\,a \tan(dx + c) + a}{(e \cos(dx + c))^{\frac{5}{2}}} \, dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx=\int \frac{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(5/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{e}\,a\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx+\left(\int \frac{\sqrt{\cos(dx+c)}\,\tan(dx+c)}{\cos(dx+c)^3}dx\right)i\right)}{e^3}$$

input
```
int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*a*(int(sqrt(cos(c + d*x))/cos(c + d*x)**3,x) + int((sqrt(cos(c +
d*x))*tan(c + d*x))/cos(c + d*x)**3,x)*i))/e**3
```

3.671 $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 130

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2ia}{7d(e\cos(c+dx))^{7/2}} - \frac{6a\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d(e\cos(c+dx))^{7/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} + \frac{6a\cos^3(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}}$$

output
```
2/7*I*a/d/(e*cos(d*x+c))^(7/2)-6/5*a*cos(d*x+c)^(7/2)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d/(e*cos(d*x+c))^(7/2)+2/5*a*cos(d*x+c)*sin(d*x+c)/d/(e*
cos(d*x+c))^(7/2)+6/5*a*cos(d*x+c)^3*sin(d*x+c)/d/(e*cos(d*x+c))^(7/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.97 (sec) , antiderivative size = 666, normalized size of antiderivative = 5.12

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx = \frac{\cos^5(c+dx)\left(\csc(c)\sec(c)\left(\frac{6\cos(c)}{5}-\frac{6}{5}i\sin(c)\right)+\sec^4(c+dx)\left(\frac{2}{7}i\cos(c)+\frac{2\sin}{7}\right.\right.}{}$$

$$-\frac{3i\cos^{\frac{9}{2}}(c+dx)\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\arctan(\tan(c)))\tan(c)}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx+\arctan(\tan(c)))\sqrt{1+\tan^2(c)}}\sqrt{1+\tan^2(c)}}\right.}{5d(e\cos(c+dx))^{7/2}(\cos(dx)+i\sin}$$

$$+\frac{3\cos^{\frac{9}{2}}(c+dx)\cot(c)\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\arctan(\tan(c)))\tan(c)}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx+\arctan(\tan(c)))\sqrt{1+\tan^2(c)}}\sqrt{1+\tan}}\right.}{5d(e\cos(c+dx))^{7/2}(\cos(dx)+i}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```

output
```
(Cos[c + d*x]^5*(Csc[c]*Sec[c]*((6*Cos[c])/5 - ((6*I)/5)*Sin[c]) + Sec[c +
 d*x]^4*(((2*I)/7)*Cos[c] + (2*Sin[c])/7) + Sec[c]*Sec[c + d*x]^3*((2*Cos[
c])/5 - ((2*I)/5)*Sin[c])*Sin[d*x] + Sec[c]*Sec[c + d*x]*((6*Cos[c])/5 - (
(6*I)/5)*Sin[c])*Sin[d*x] + Sec[c + d*x]^2*((2*Cos[c])/5 - ((2*I)/5)*Sin[c
])*Tan[c])*(a + I*a*Tan[c + d*x]))/(d*(e*Cos[c + d*x])^(7/2)*(Cos[d*x] + I
*Sin[d*x])) - (((3*I)/5)*Cos[c + d*x]^(9/2)*((HypergeometricPFQ[{-1/2, -1/
4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/
(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2
]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[
Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]])*(a + I*a*Tan[c + d*x
]))/(d*(e*Cos[c + d*x])^(7/2)*(Cos[d*x] + I*Sin[d*x])) + (3*Cos[c + d*x]^(
9/2)*Cot[c]*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[
c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c
]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]
*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2]])*(a + I*a*Tan[c + d*x]))/(5*d*(e*Cos[c + d*x])^(7/...
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 3998, 3042, 3967, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int (e\sec(c+dx))^{7/2}(i\tan(c+dx)a+a)dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e\sec(c+dx))^{7/2}(i\tan(c+dx)a+a)dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3967}$$

$$\frac{a\int (e\sec(c+dx))^{7/2}dx+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{a\left(\frac{3}{5}e^2\int (e\sec(c+dx))^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{3}{5}e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```

output
```
((((2*I)/7)*a*(e*Sec[c + d*x])^(7/2))/d + a*((2*e*(e*Sec[c + d*x])^(5/2)*S
in[c + d*x])/(5*d) + (3*e^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Co
s[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x]
)/d))/5))/((e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(113) = 226$.

Time = 3.50 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.96

method	result
parts	$-\frac{2a\sqrt{e\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(24\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-12\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin}\right.}{}$
default	$\frac{2\left(336\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-168\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-504\sin\left(\frac{dx}{2}\right.\right.}{}$

input
```
int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*a*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^4/sin(1
/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*
d*x+1/2*c)^2-1)*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2))*(-2*sin(1/2*d*x+1/2*c)
^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d+
2/7*I*a/d/(e*cos(d*x+c))^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.71

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx =$$
$$-\frac{4\left(\sqrt{\frac{1}{2}}\left(21i\,ae^{(8i\,dx+8i\,c)}+77i\,ae^{(6i\,dx+6i\,c)}+23i\,ae^{(4i\,dx+4i\,c)}+7i\,ae^{(2i\,dx+2i\,c)}\right)\sqrt{ee^{(2i\,dx+2i\,c)}+ee}^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right.}{35\left(de^4e^{(8i\,dx+8i\,c)}\right.}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-4/35*(sqrt(1/2)*(21*I*a*e^(8*I*d*x + 8*I*c) + 77*I*a*e^(6*I*d*x + 6*I*c)
+ 23*I*a*e^(4*I*d*x + 4*I*c) + 7*I*a*e^(2*I*d*x + 2*I*c))*sqrt(e*e^(2*I*d*
x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c) + 21*sqrt(1/2)*(I*a*e^(8*I*d*x +
8*I*c) + 4*I*a*e^(6*I*d*x + 6*I*c) + 6*I*a*e^(4*I*d*x + 4*I*c) + 4*I*a*e^(
2*I*d*x + 2*I*c) + I*a)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, e^(I*d*x + I*c))))/(d*e^4*e^(8*I*d*x + 8*I*c) + 4*d*e^4*e^(6*I*d*x
 + 6*I*c) + 6*d*e^4*e^(4*I*d*x + 4*I*c) + 4*d*e^4*e^(2*I*d*x + 2*I*c) + d*
e^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + ia \tan(c + dx)}{(e \cos(c + dx))^{7/2}} \, dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a + ia \tan(c + dx)}{(e \cos(c + dx))^{7/2}} \, dx = \int \frac{i \, a \tan\left(dx + c\right) + a}{\left(e \cos\left(dx + c\right)\right)^{\frac{7}{2}}} \, dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{a + ia \tan(c + dx)}{(e \cos(c + dx))^{7/2}} \, dx = \int \frac{i \, a \tan\left(dx + c\right) + a}{\left(e \cos\left(dx + c\right)\right)^{\frac{7}{2}}} \, dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + ia\tan(c + dx)}{(e\cos(c + dx))^{7/2}}\,dx = \int \frac{a + a\tan(c + d\,x)\ 1\mathrm{i}}{(e\ \cos(c + d\,x))^{7/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(7/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{a + ia\tan(c + dx)}{(e\cos(c + dx))^{7/2}}\,dx = \frac{\sqrt{e}\,a\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx + \left(\int \frac{\sqrt{\cos(dx+c)}\ \tan(dx+c)}{\cos(dx+c)^4}dx\right)i\right)}{e^4}$$

input
```
int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x)
```

output
```
(sqrt(e)*a*(int(sqrt(cos(c + d*x))/cos(c + d*x)**4,x) + int((sqrt(cos(c +
d*x))*tan(c + d*x))/cos(c + d*x)**4,x)*i))/e**4
```

3.672 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 190

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2(e\cos(c+dx))^{7/2}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7a^2d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+dx)}{15a^2d} + \frac{6(e\cos(c+dx))^{7/2}\tan(c+dx)}{35a^2d} + \frac{2(e\cos(c+dx))^{7/2}\sec^2(c+dx)\tan(c+dx)}{7a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{7/2}}{15d\,(a^2+ia^2\tan(c+dx))}$$

output

```
2/7*(e*cos(d*x+c))^(7/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^2/d/cos(
d*x+c)^(7/2)+2/15*cos(d*x+c)*(e*cos(d*x+c))^(7/2)*sin(d*x+c)/a^2/d+6/35*(e
*cos(d*x+c))^(7/2)*tan(d*x+c)/a^2/d+2/7*(e*cos(d*x+c))^(7/2)*sec(d*x+c)^2*
tan(d*x+c)/a^2/d+4/15*I*cos(d*x+c)^2*(e*cos(d*x+c))^(7/2)/d/(a^2+I*a^2*tan
(d*x+c))
```

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.82

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{e^3\sqrt{e\cos(c+dx)}\left(-240\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(2(c+dx))+i\sin(2(c\right.}{}$$

input
```
Integrate[(e*Cos[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(e^3*Sqrt[e*Cos[c + d*x]]*(-240*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)
] + I*Sin[2*(c + d*x)]) + Sqrt[Cos[c + d*x]]*((-296*I)*Cos[c + d*x] + (68*
I)*Cos[3*(c + d*x)] + (4*I)*Cos[5*(c + d*x)] + 134*Sin[c + d*x] - 117*Sin[
3*(c + d*x)] - 11*Sin[5*(c + d*x)])))/(840*a^2*d*Cos[c + d*x]^(5/2)*(-I +
Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3998, 3042, 3981, 3042, 4256, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\int \frac{1}{(e\sec(c+dx))^{7/2}(i\tan(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\int \frac{1}{(e\sec(c+dx))^{7/2}(i\tan(c+dx)a+a)^2}dx$$

$$\Big\downarrow \ \color{blue}{3981}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{11e^2\int \frac{1}{(e\sec(c+dx))^{11/2}}dx}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}\right)$$

$$\Big\downarrow \ \color{blue}{3042}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{11e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{11/2}}dx}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}\right)$$

$$\Big\downarrow \ \color{blue}{4256}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{11e^2\left(\frac{9\int \frac{1}{(e\sec(c+dx))^{7/2}}dx}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}\right)$$

$$\Big\downarrow \ \color{blue}{3042}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{11e^2\left(\frac{9\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}\right)$$

$$\Big\downarrow \ \color{blue}{4256}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{11e^2\left(\frac{9\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(

$$(e\cos(c+dx))^{7/2}(e\sec(c+ dx))^{7/2}\left(\frac{11e^2\left(\frac{9\left(\frac{5\int\frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\right.$$

$$\Big\downarrow \; 4256$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+ dx))^{7/2}\left(\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{\int\sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{}{15d\left(a^2+ia^2\tan(c\right.}$$

$$\Big\downarrow \; 3042$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{}{15d\left(a^2+ia^2\tan\right.}\right.$$

$$\Big\downarrow \; 4258$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{}{15}\right.$$

$$\Big\downarrow \; 3042$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\right.$$

$$\Big\downarrow \; 3120$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}\right.$$

input
```
Int[(e*Cos[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*((11*e^2*((2*Sin[c + d*x])/(
11*d*e*(e*Sec[c + d*x])^(9/2)) + (9*((2*Sin[c + d*x])/(7*d*e*(e*Sec[c + d*
x])^(5/2)) + (5*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Se
c[c + d*x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(
7*e^2)))/(11*e^2)))/(15*a^2) + (((4*I)/15)*e^2)/(d*(e*Sec[c + d*x])^(11/2)
*(a^2 + I*a^2*Tan[c + d*x])))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(167) = 334.

Time = 14.99 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.04

method	result
default	$-\frac{2e^4\left(-14i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)+3584\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{16}+25088i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-12544\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+6272i\sin\left(\right.\right.}{}$

input
```
int((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/105/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(-14
*I*sin(1/2*d*x+1/2*c)+3584*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^16+25088*
I*sin(1/2*d*x+1/2*c)^11-12544*sin(1/2*d*x+1/2*c)^14*cos(1/2*d*x+1/2*c)+627
2*I*sin(1/2*d*x+1/2*c)^7+19264*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)+14
336*I*sin(1/2*d*x+1/2*c)^15-16800*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)
+224*I*sin(1/2*d*x+1/2*c)^3+9104*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-2
5088*I*sin(1/2*d*x+1/2*c)^13-3128*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-
3584*I*sin(1/2*d*x+1/2*c)^17+700*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-1
5680*I*sin(1/2*d*x+1/2*c)^9-90*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-1568*I*sin(1/2*d*x+1/2*c)^5)/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.79

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(-960i\sqrt{\frac{1}{2}}e^{\frac{7}{2}}e^{(7i\,dx+7i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{\frac{1}{2}}\left(-15i\,e^{3}\right.\right.}{}$$

input
```
integrate((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/1680*(-960*I*sqrt(1/2)*e^(7/2)*e^(7*I*d*x + 7*I*c)*weierstrassPInverse(-
4, 0, e^(I*d*x + I*c)) + sqrt(1/2)*(-15*I*e^3*e^(10*I*d*x + 10*I*c) - 185*
I*e^3*e^(8*I*d*x + 8*I*c) + 430*I*e^3*e^(6*I*d*x + 6*I*c) + 162*I*e^3*e^(4
*I*d*x + 4*I*c) + 49*I*e^3*e^(2*I*d*x + 2*I*c) + 7*I*e^3)*sqrt(e*e^(2*I*d*
x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c))*e^(-7*I*d*x - 7*I*c)/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*cos(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\,\cos(c+d\,x))^{7/2}}{(a+a\tan(c+d\,x)\;1\mathrm{i})^2}\,dx$$

input
```
int((e*cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e*cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^3}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)e^3}{a^2}$$

input
```
int((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - sqrt(e)*int((sqrt(cos(c + d*x))*cos(c + d*x)**3)/(tan(c + d*x)**2 - 2*
tan(c + d*x)*i - 1),x)*e**3)/a**2
```

3.673 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 154

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{42(e\cos(c+dx))^{5/2}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\cos(c+dx)(e\cos(c+dx))^{5/2}\sin(c+dx)}{13a^2d} + \frac{14(e\cos(c+dx))^{5/2}\tan(c+dx)}{65a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{5/2}}{13d\,(a^2+ia^2\tan(c+dx))}$$

output
```
42/65*(e*cos(d*x+c))^(5/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos
(d*x+c)^(5/2)+2/13*cos(d*x+c)*(e*cos(d*x+c))^(5/2)*sin(d*x+c)/a^2/d+14/65*
(e*cos(d*x+c))^(5/2)*tan(d*x+c)/a^2/d+4/13*I*cos(d*x+c)^2*(e*cos(d*x+c))^(
5/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.99 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.01

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{(e\cos(c+dx))^{5/2}\sec^5(c+dx)(\cos(dx)+i\sin(dx))^2\left(-21\cos(c)\,{}_2F_1\left(-\frac{1}{2},-\right.\right.}{}$$

input
```
Integrate[(e*Cos[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5*(Cos[d*x] + I*Sin[d*x])^2*(-21*Cos[
c]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin
[d*x + ArcTan[Tan[c]]] - (42*I)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos
[d*x + ArcTan[Tan[c]]]^2]*Sin[c]*Sin[d*x + ArcTan[Tan[c]]] + (21*I)*(3*Cos
[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Sqrt[Sin[d*x +
 ArcTan[Tan[c]]]^2] + (21*(3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x +
 ArcTan[Tan[c]]])*Cot[c]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])/2 - (Cos[c + d
*x]*Csc[c]*Sqrt[Sec[c]^2]*(Cos[2*d*x] - I*Sin[2*d*x])*(178*Cos[c + 2*d*x]
+ 158*Cos[3*c + 2*d*x] - 9*Cos[3*c + 4*d*x] + 9*Cos[5*c + 4*d*x] - (88*I)*
Sin[c] + (208*I)*Sin[c + 2*d*x] + (128*I)*Sin[3*c + 2*d*x] - (4*I)*Sin[3*c
 + 4*d*x] + (4*I)*Sin[5*c + 4*d*x])*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])/8 +
 21*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Si
n[c]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c] - (21*(3*Cos[c - d*x - ArcTan[Tan[c]
]] + Cos[c + d*x + ArcTan[Tan[c]]])*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]*Tan[
c])/2))/(65*d*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]*(a + I*a*Ta
n[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3998, 3042, 3981, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \ 3998$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\int \frac{1}{(e\sec(c+dx))^{5/2}(i\tan(c+dx)a+a)^2}dx$$

$$\Big\downarrow \ 3042$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\int \frac{1}{(e\sec(c+dx))^{5/2}(i\tan(c+dx)a+a)^2}dx$$

$$\Big\downarrow \ 3981$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{9e^2\int \frac{1}{(e\sec(c+dx))^{9/2}}dx}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{9e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}}dx}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \ 4256$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{9e^2\left(\frac{7\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{9e^2\left(\frac{7\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \ 4256$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{9e^2\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+d}\right.$$

$$\Big\downarrow \ 3042$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{9e^2\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c}\right.$$

$$\Big\downarrow \ 4258$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{9e^2\left(\frac{7\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec}\right.$$

$$\Big\downarrow \ 3042$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{9e^2\left(\frac{7\left(\frac{3\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\se$$

$\Big\downarrow$ 3119

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{9e^2\left(\frac{7\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\se$$

input
```
Int[(e*Cos[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2)*((9*e^2*((2*Sin[c + d*x])/(9
*d*e*(e*Sec[c + d*x])^(7/2)) + (7*((6*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*
Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*Sin[c + d*x])/(5*d*e*(e*Sec[
c + d*x])^(3/2))))/(9*e^2)))/(13*a^2) + (((4*I)/13)*e^2)/(d*(e*Sec[c + d*x
])^(9/2)*(a^2 + I*a^2*Tan[c + d*x])))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(136) = 272.

Time = 12.76 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.28

method	result
default	$\frac{2e^3\left(4480i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}+1280\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-140i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-3840\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+10i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$

input
```
int((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/65/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(4480*
I*sin(1/2*d*x+1/2*c)^13+1280*sin(1/2*d*x+1/2*c)^14*cos(1/2*d*x+1/2*c)-140*
I*sin(1/2*d*x+1/2*c)^3-3840*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)+10*I*
sin(1/2*d*x+1/2*c)+4960*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-6720*I*si
n(1/2*d*x+1/2*c)^11-3520*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-2800*I*si
n(1/2*d*x+1/2*c)^7+1496*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-1280*I*sin
(1/2*d*x+1/2*c)^15-376*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5600*I*sin(
1/2*d*x+1/2*c)^9+44*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+21*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)+840*I*sin(1/2*d*x+1/2*c)^5)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.91

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(672i\sqrt{\frac{1}{2}}e^{\frac{5}{2}}e^{(6i\,dx+6i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,c}\right.\right.\right.}{}$$

input
```
integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/520*(672*I*sqrt(1/2)*e^(5/2)*e^(6*I*d*x + 6*I*c)*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(1/2)*(-13*I*e^2*e^(8*I
*d*x + 8*I*c) + 386*I*e^2*e^(6*I*d*x + 6*I*c) + 88*I*e^2*e^(4*I*d*x + 4*I*
c) + 30*I*e^2*e^(2*I*d*x + 2*I*c) + 5*I*e^2)*sqrt(e*e^(2*I*d*x + 2*I*c) +
e)*e^(-1/2*I*d*x - 1/2*I*c))*e^(-6*I*d*x - 6*I*c)/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*cos(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\,\cos(c+d\,x))^{5/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}\,dx$$

input
```
int((e*cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e*cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)^2}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)e^2}{a^2}$$

input
```
int((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - sqrt(e)*int((sqrt(cos(c + d*x))*cos(c + d*x)**2)/(tan(c + d*x)**2 - 2*
tan(c + d*x)*i - 1),x)*e**2)/a**2
```

3.674 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 154

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{10(e\cos(c+dx))^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{33a^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\cos(c+dx)(e\cos(c+dx))^{3/2}\sin(c+dx)}{11a^2d} + \frac{10(e\cos(c+dx))^{3/2}\tan(c+dx)}{33a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{3/2}}{11d\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
10/33*(e*cos(d*x+c))^(3/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^2/d/co
s(d*x+c)^(3/2)+2/11*cos(d*x+c)*(e*cos(d*x+c))^(3/2)*sin(d*x+c)/a^2/d+10/33
*(e*cos(d*x+c))^(3/2)*tan(d*x+c)/a^2/d+4/11*I*cos(d*x+c)^2*(e*cos(d*x+c))^
(3/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.85

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{(e\cos(c+dx))^{3/2}\left(-20\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(2(c+dx))+i\sin(2(c-$$

$$66a^2d\,\text{co}$$

input
```
Integrate[(e*Cos[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((e*Cos[c + d*x])^(3/2)*(-20*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] +
 I*Sin[2*(c + d*x)]) + Sqrt[Cos[c + d*x]]*((-28*I)*Cos[c + d*x] + (4*I)*Co
s[3*(c + d*x)] + 13*Sin[c + d*x] - 7*Sin[3*(c + d*x)])))/(66*a^2*d*Cos[c +
 d*x]^(7/2)*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3998, 3042, 3981, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\int \frac{1}{(e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\int \frac{1}{(e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3981}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{7e^2\int\frac{1}{(e\sec(c+dx))^{7/2}}dx}{11a^2}+\frac{4ie^2}{11d\,(a^2+ia^2\tan(c+dx))\,(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{7e^2\int\frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{11a^2}+\frac{4ie^2}{11d\,(a^2+ia^2\tan(c+dx))\,(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{7e^2\left(\frac{5\int\frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\,(a^2+ia^2\tan(c+dx))\,(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{7e^2\left(\frac{5\int\frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\,(a^2+ia^2\tan(c+dx))\,(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{7e^2\left(\frac{5\left(\frac{\int\sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\,(a^2+ia^2\tan(c+dx))\,(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{7e^2\left(\frac{5\left(\frac{\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+d\right.}\right.$$

$\downarrow$ 4258

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{7e^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+d\right.}\right.$$

$\downarrow$ 3042

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{7e^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{}{11d\left(a^2+ia^2\tan(c-\right.}\right.$$

$\downarrow$ 3120

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{7e^2\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{}{11d\left(a^2+ia^2\tan(c\right.}\right.$$

input
```
Int[(e*Cos[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(e*Cos[c + d*x])^(3/2)*(e*Sec[c + d*x])^(3/2)*((7*e^2*((2*Sin[c + d*x])/(7
*d*e*(e*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*
Sec[c + d*x]])))/(7*e^2)))/(11*a^2) + (((4*I)/11)*e^2)/(d*(e*Sec[c + d*x])
^(7/2)*(a^2 + I*a^2*Tan[c + d*x])))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(135) = 270$.

Time = 10.14 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.05

method	result
default	$-\frac{2e^2\left(-384i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}+384\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1152i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-960\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-1440i\sin\left(\frac{d}{}\right.\right.}{}$

input
```
int((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/33/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*(-384
*I*sin(1/2*d*x+1/2*c)^13+384*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)+1152
*I*sin(1/2*d*x+1/2*c)^11-960*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-1440
*I*sin(1/2*d*x+1/2*c)^9+1008*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+960*I
*sin(1/2*d*x+1/2*c)^7-552*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-360*I*si
n(1/2*d*x+1/2*c)^5+176*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+72*I*sin(1/
2*d*x+1/2*c)^3-28*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-6*I*sin(1/2*d*x+1/2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.75

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(-80i\sqrt{\frac{1}{2}}e^{\frac{3}{2}}e^{(5i\,dx+5i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{\frac{1}{2}}\left(-11i\,ee^{(\right.\right.}}{}$$

input
```
integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/132*(-80*I*sqrt(1/2)*e^(3/2)*e^(5*I*d*x + 5*I*c)*weierstrassPInverse(-4,
 0, e^(I*d*x + I*c)) + sqrt(1/2)*(-11*I*e*e^(6*I*d*x + 6*I*c) + 41*I*e*e^(
4*I*d*x + 4*I*c) + 15*I*e*e^(2*I*d*x + 2*I*c) + 3*I*e)*sqrt(e*e^(2*I*d*x +
 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c))*e^(-5*I*d*x - 5*I*c)/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*cos(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\,\cos(c+d\,x))^{3/2}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^2}\,dx$$

input
```
int((e*cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e*cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\cos(dx+c)}\,\cos(dx+c)}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)e}{a^2}$$

input
```
int((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - sqrt(e)*int((sqrt(cos(c + d*x))*cos(c + d*x))/(tan(c + d*x)**2 - 2*tan
(c + d*x)*i - 1),x)*e)/a**2
```

3.675 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 120

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{\cos(c+dx)}} + \frac{2i\sqrt{e\cos(c+dx)}}{9d(a+ia\tan(c+dx))^2} + \frac{2i\sqrt{e\cos(c+dx)}}{9d\,(a^2+ia^2\tan(c+dx))}$$

output
```
2/3*(e*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d
*x+c)^(1/2)+2/9*I*(e*cos(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^2+2/9*I*(e*cos
(d*x+c))^(1/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.34 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\sqrt{e\cos(c+dx)}\sec^3(c+dx)(\cos(dx)+i\sin(dx))^2\left(6\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sec(c)(\cdots\right.}{\cdots}$$

input
```
Integrate[Sqrt[e*Cos[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^3*(Cos[d*x] + I*Sin[d*x])^2*(6*Hypergeo
metricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sec[c]*(Cos[2*
c] + I*Sin[2*c])*Sin[d*x + ArcTan[Tan[c]]] + Sqrt[Sin[d*x + ArcTan[Tan[c]]
]^2]*(Cos[c + d*x]*Csc[c]*Sqrt[Sec[c]^2]*(Cos[2*d*x] - I*Sin[2*d*x])*(7*Co
s[c + 2*d*x] + 5*Cos[3*c + 2*d*x] - (4*I)*(Sin[c] - 2*Sin[c + 2*d*x] - Sin
[3*c + 2*d*x])) - 3*Cos[c + d*x + ArcTan[Tan[c]]]*(I + Cot[c])^2*Tan[c] +
9*Cos[c - d*x - ArcTan[Tan[c]]]*(-2*I - Cot[c] + Tan[c]))))/(18*a^2*d*Sqrt
[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3998, 3042, 3981, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3998}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{5e^2\int\frac{1}{(e\sec(c+dx))^{5/2}}dx}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{5e^2\int\frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}\right)$$

$\Big\downarrow$ 4256

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{5e^2\left(\frac{3\int\frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+$$

$\Big\downarrow$ 3042

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{5e^2\left(\frac{3\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec$$

$\Big\downarrow$ 4258

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{5e^2\left(\frac{3\int\sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e$$

$\Big\downarrow$ 3042

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{5e^2\left(\frac{3\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)($$

$\Big\downarrow$ 3119

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{5e^2\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(\right.$$

input
```
Int[Sqrt[e*Cos[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
Sqrt[e*Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]*((5*e^2*((6*EllipticE[(c + d*x)/
2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*Sin[c + d*x]
)/(5*d*e*(e*Sec[c + d*x])^(3/2))))/(9*a^2) + (((4*I)/9)*e^2)/(d*(e*Sec[c +
 d*x])^(5/2)*(a^2 + I*a^2*Tan[c + d*x])))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(104) = 208$.

Time = 7.92 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.31

method	result
default	$\frac{2e\left(-64i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}+64\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+160i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-160i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+1\right.}{}$

input
```
int((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/9/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e*(-64*I*si
n(1/2*d*x+1/2*c)^11+64*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+160*I*sin(
1/2*d*x+1/2*c)^9-128*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-160*I*sin(1/2
*d*x+1/2*c)^7+104*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+80*I*sin(1/2*d*x
+1/2*c)^5-40*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-20*I*sin(1/2*d*x+1/2*
c)^3+6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
+2*I*sin(1/2*d*x+1/2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(15i\,e^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+24i\,\sqrt{\frac{1}{2}}\sqrt{e}e^{(4i\,dx+4i\,c)}\mathrm{weierstrassZ}\right.}{18\,a^2d}$$

input
```
integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/18*(sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(15*I*e^(4*I*d*x + 4*I*c)
+ 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(-1/2*I*d*x - 1/2*I*c) + 24*I*sqrt(1/2)*s
qrt(e)*e^(4*I*d*x + 4*I*c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, e^(I*d*x + I*c))))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sqrt{e\cos{(c+dx)}}}{\tan^2{(c+dx)}-2i\tan{(c+dx)}-1}\,dx}{a^2}$$

input
```
integrate((e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sqrt(e*cos(c + d*x))/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x
)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\sqrt{e\cos(dx+c)}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*cos(d*x + c))/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\sqrt{e\,\cos(c+d\,x)}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}\,dx$$

input
```
int((e*cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e*cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sqrt{e}\left(\int \frac{\sqrt{\cos(dx+c)}}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)}{a^2}$$

input
```
int((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - sqrt(e)*int(sqrt(cos(c + d*x))/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1
),x))/a**2
```

3.676 $\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 120

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7a^2d\sqrt{e\cos(c+dx)}} + \frac{2i}{7d\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2} + \frac{2i}{7d\sqrt{e\cos(c+dx)}\,(a^2+ia^2\tan(c+dx))}$$

output
```
2/7*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^2/d/(e*cos(d
*x+c))^(1/2)+2/7*I/d/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2+2/7*I/d/(e*
cos(d*x+c))^(1/2)/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.32

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \frac{\left(-i\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sqrt{\cos(c+dx)}\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)+4i\sin^3\left(\frac{1}{2}(\right.\right.\right.}{7a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{e\cos(c+dx)}(-i+}$$

input
```
Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(((-I)*Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Sqrt[Cos[c + d*x]]*(3*Cos[(c
+ d*x)/2] + Cos[(3*(c + d*x))/2] + (4*I)*Sin[(c + d*x)/2]^3) + 2*EllipticF
[(c + d*x)/2, 2]*((-I)*Cos[(3*(c + d*x))/2] + Sin[(3*(c + d*x))/2])))/(7*a
^2*d*Cos[c + d*x]^(3/2)*Sqrt[e*Cos[c + d*x]]*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3998, 3042, 3981, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2\sqrt{e\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^2\sqrt{e\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^2}dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^2}dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3981}$$

$$\frac{\frac{3e^2\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{\frac{3e^2\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3e^2\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{3e^2\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
((3*e^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*
x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*a^2) +
 (((4*I)/7)*e^2)/(d*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^2*Tan[c + d*x])))/(S
qrt[e*Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(103) = 206$.

Time = 6.01 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.99

method	result
default	$-\frac{2\left(-32i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+64i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-48\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-48i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+28\sin\right.}{7a^{2}\sin}$

input
```
int(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/7/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(-32*I*sin
(1/2*d*x+1/2*c)^9+32*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+64*I*sin(1/2*
d*x+1/2*c)^7-48*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-48*I*sin(1/2*d*x+1
/2*c)^5+28*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+16*I*sin(1/2*d*x+1/2*c)
^3-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*I
*sin(1/2*d*x+1/2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx$$
$$=\frac{\left(\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}-4i\,\sqrt{\frac{1}{2}}\sqrt{e}e^{(3i\,dx+3i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(\right.}\right.}{7\,a^{2}de}$$

input
```
integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

output
```
1/7*(sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(3*I*e^(2*I*d*x + 2*I*c) +
I)*e^(-1/2*I*d*x - 1/2*I*c) - 4*I*sqrt(1/2)*sqrt(e)*e^(3*I*d*x + 3*I*c)*we
ierstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-3*I*d*x - 3*I*c)/(a^2*d*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx$$
$$= -\frac{\int \frac{1}{\sqrt{e\cos{(c+dx)}}\tan^2{(c+dx)}-2i\sqrt{e\cos{(c+dx)}}\tan{(c+dx)}-\sqrt{e\cos{(c+dx)}}}\,dx}{a^2}$$

input
```
integrate(1/(e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(1/(sqrt(e*cos(c + d*x))*tan(c + d*x)**2 - 2*I*sqrt(e*cos(c + d*x
))*tan(c + d*x) - sqrt(e*cos(c + d*x))), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\cos(dx+c)}(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*cos(d*x + c))*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\,\cos(c+d\,x)}\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{\sqrt{\cos(dx+c)}\,\tan(dx+c)^2-2\sqrt{\cos(dx+c)}\,\tan(dx+c)i-\sqrt{\cos(dx+c)}}dx}{\sqrt{e}\,a^2}$$

input
```
int(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(1/(sqrt(cos(c + d*x))*tan(c + d*x)**2 - 2*sqrt(cos(c + d*x))*tan(c
 + d*x)*i - sqrt(cos(c + d*x))),x))/(sqrt(e)*a**2)
```

3.677 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \frac{2\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{3/2}} + \frac{4i\cos^2(c+dx)}{5d(e\cos(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
2/5*cos(d*x+c)^(3/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/(e*cos(d*
x+c))^(3/2)+4/5*I*cos(d*x+c)^2/d/(e*cos(d*x+c))^(3/2)/(a^2+I*a^2*tan(d*x+c
))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \frac{2\cos^2(c+dx)\left(1+2^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{1}{2}(1\right.\right.}{5a^2d(e\cos(c}$$

input
```
Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(2*Cos[c + d*x]^2*(1 + 2^(3/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (1 + I*Ta
n[c + d*x])/2]*(1 - I*Tan[c + d*x])^(1/4)*(1 + I*Tan[c + d*x]) - I*Tan[c +
 d*x]))/(5*a^2*d*(e*Cos[c + d*x])^(3/2)*(-I + Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3998, 3042, 3981, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int \frac{(e\sec(c+dx))^{3/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(e\sec(c+dx))^{3/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3981}$$

$$\frac{\frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5a^2}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a^2}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{e^2\int\sqrt{\cos(c+dx)}dx}{5a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

input
```
Int[1/((e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
((2*e^2*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (((4*I)/5)*e^2)/(d*Sqrt[e*Sec[c + d*x]]*(a^2 + I*a^2*Tan[c +
d*x])))/((e*Cos[c + d*x])^(3/2)*(e*Sec[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(82) = 164$.

Time = 4.66 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.24

method	result
default	$\frac{-\frac{32i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{5}+\frac{32\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}+\frac{48i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{32\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}-\frac{24i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{5}+\frac{8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos(}{5}}{e\,a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}e+e}\,d}$

input
```
int(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/5/e/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(-16*I*si
n(1/2*d*x+1/2*c)^7+16*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+24*I*sin(1/2
*d*x+1/2*c)^5-16*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-12*I*sin(1/2*d*x+
1/2*c)^3+4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)+2*I*sin(1/2*d*x+1/2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.02

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(-2i\,e^{(2i\,dx+2i\,c)}-i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}-2i\,\sqrt{\frac{1}{2}}\sqrt{e}e^{(2i\,dx+2i\,c)}\text{weierstrassZeta}\left(-4,0,\text{we}\right.\right.}{5\,a^2de^2}$$

input
```
integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

output
```
-2/5*(sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(-2*I*e^(2*I*d*x + 2*I*c)
- I)*e^(-1/2*I*d*x - 1/2*I*c) - 2*I*sqrt(1/2)*sqrt(e)*e^(2*I*d*x + 2*I*c)*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))*e^(-2
*I*d*x - 2*I*c)/(a^2*d*e^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\cos(dx+c))^{\frac{3}{2}}(ia\tan(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*cos(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\cos(c+dx))^{3/2}\,(a+a\tan(c+dx)\,1i)^2}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

Reduce [F]

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{\sqrt{\cos(dx+c)}\,\cos(dx+c)\tan(dx+c)^2-2\sqrt{\cos(dx+c)}\,\cos(dx+c)\tan(dx+c)i-\sqrt{\cos(dx+c)}\,\cos(dx+c)}dx}{\sqrt{e}\,a^2e}$$

input

```
int(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x)
```

output

```
( - int(1/(sqrt(cos(c + d*x))*cos(c + d*x)*tan(c + d*x)**2 - 2*sqrt(cos(c
+ d*x))*cos(c + d*x)*tan(c + d*x)*i - sqrt(cos(c + d*x))*cos(c + d*x)),x))
/(sqrt(e)*a**2*e)
```

3.678 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{2\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d(e\cos(c+dx))^{5/2}} + \frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{5/2}\,(a^2+ia^2\tan(c+dx))}$$

output
```
-2/3*cos(d*x+c)^(5/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^2/d/(e*cos(
d*x+c))^(5/2)+4/3*I*cos(d*x+c)^2/d/(e*cos(d*x+c))^(5/2)/(a^2+I*a^2*tan(d*x
+c))
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.26

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \frac{2\sqrt{\cos(c+dx)}(\cos(dx)+i\sin(dx))^2\left(\text{EllipticF}\left(\frac{1}{2}(c+dx\right.\right.}{3a^2d(e\cos(c}$$

input
```
Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2*(EllipticF[(c + d*x)/2, 2]
*(Cos[2*c] + I*Sin[2*c]) + 2*Sqrt[Cos[c + d*x]]*((-I)*Cos[c - d*x] + Sin[c
 - d*x])))/(3*a^2*d*(e*Cos[c + d*x])^(5/2)*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3998, 3042, 3981, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int \frac{(e\sec(c+dx))^{5/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(e\sec(c+dx))^{5/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3981}$$

$$\frac{-\frac{e^2\int \sqrt{e\sec(c+dx)}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{-\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{-\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^2d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

input
```
Int[1/((e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
((-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]]
)/(3*a^2*d) + (((4*I)/3)*e^2*Sqrt[e*Sec[c + d*x]])/(d*(a^2 + I*a^2*Tan[c +
 d*x])))/((e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(81) = 162$.

Time = 3.46 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.86

method	result
default	$-\frac{2\left(-8i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^5+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^3-4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\right.}\right.}{3e^2a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2e+e}\,d}$

input
```
int(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3/e^2/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(-8*I*
sin(1/2*d*x+1/2*c)^5+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+8*I*sin(1/2
*d*x+1/2*c)^3-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-2*I*sin(1/2*d*x+1/2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.86

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx =$$
$$-\frac{4\left(-i\sqrt{\frac{1}{2}}\sqrt{e}e^{(i\,dx+i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)-i\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right)e^{(-i\,dx-i}}{3\,a^2de^3}$$

input
```
integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

output
```
-4/3*(-I*sqrt(1/2)*sqrt(e)*e^(I*d*x + I*c)*weierstrassPInverse(-4, 0, e^(I
*d*x + I*c)) - I*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x -
 1/2*I*c))*e^(-I*d*x - I*c)/(a^2*d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\cos(dx+c))^{\frac{5}{2}}\,(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*cos(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\,\cos(c+d\,x))^{5/2}\,(a+a\tan(c+d\,x)\;1\mathrm{i})^2}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

Reduce [F]

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\tan(dx+c)^2-2\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\tan(dx+c)i-\sqrt{\cos(dx+c)}\,\cos(dx+c)^2}dx}{\sqrt{e}\,a^2e^2}$$

input
```
int(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(1/(sqrt(cos(c + d*x))*cos(c + d*x)**2*tan(c + d*x)**2 - 2*sqrt(cos
(c + d*x))*cos(c + d*x)**2*tan(c + d*x)*i - sqrt(cos(c + d*x))*cos(c + d*x
)**2),x))/(sqrt(e)*a**2*e**2)
```

3.679 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 122

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \frac{6\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(e\cos(c+dx))^{7/2}} - \frac{6\cos^3(c+dx)\sin(c+dx)}{a^2d(e\cos(c+dx))^{7/2}} + \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{7/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
6*cos(d*x+c)^(7/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/(e*cos(d*x+
c))^(7/2)-6*cos(d*x+c)^3*sin(d*x+c)/a^2/d/(e*cos(d*x+c))^(7/2)+4*I*cos(d*x
+c)^2/d/(e*cos(d*x+c))^(7/2)/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.97 (sec) , antiderivative size = 1106, normalized size of antiderivative = 9.07

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2*((-2*I)*Cos[c - d*x]*Sqrt[Co
s[c + d*x]] + 2*Sqrt[Cos[c + d*x]]*Sin[c - d*x]))/(d*(e*Cos[c + d*x])^(7/2
)*(a + I*a*Tan[c + d*x])^2) + (3*Cos[c]*Cos[c + d*x]^(3/2)*(Cos[d*x] + I*S
in[d*x])^2*(-((Cos[d*x - ArcTan[Cot[c]]]*Cot[c]*HypergeometricPFQ[{-1/2, -
1/4}, {3/4}, Sin[d*x - ArcTan[Cot[c]]]^2])/(Sqrt[1 + Cot[c]^2]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan
[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])) + ((Cos[d*x - ArcTan[Cot
[c]]]*Cot[c])/Sqrt[1 + Cot[c]^2] + (2*Sqrt[1 + Cot[c]^2]*Sin[c]^2*Sin[d*x
- ArcTan[Cot[c]]])/(Cos[c]^2 + Sin[c]^2))/Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]
*Sin[d*x - ArcTan[Cot[c]]])]))/(d*(e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c +
d*x])^2) + ((3*I)*Cos[c + d*x]^(3/2)*Sin[c]*(Cos[d*x] + I*Sin[d*x])^2*(-((
Cos[d*x - ArcTan[Cot[c]]]*Cot[c]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Si
n[d*x - ArcTan[Cot[c]]]^2])/(Sqrt[1 + Cot[c]^2]*Sqrt[1 - Sin[d*x - ArcTan[
Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqr
t[1 + Sin[d*x - ArcTan[Cot[c]]]])) + ((Cos[d*x - ArcTan[Cot[c]]]*Cot[c])/S
qrt[1 + Cot[c]^2] + (2*Sqrt[1 + Cot[c]^2]*Sin[c]^2*Sin[d*x - ArcTan[Cot[c]
]])/(Cos[c]^2 + Sin[c]^2))/Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]))/(d*(e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2) - ((3*
I)*Cos[c]*Cos[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])^2*((HypergeometricPFQ
[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan...
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3998, 3042, 3981, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{7/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{7/2}}dx$$
$$\Big\downarrow \text{3998}$$

$$\frac{\int \frac{(e\sec(c+dx))^{7/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{(e\sec(c+dx))^{7/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3981}$$

$$\frac{-\frac{3e^2\int(e\sec(c+dx))^{3/2}dx}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{3e^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

input
```
Int[1/((e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
((-3*e^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*
Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/a^2 + ((4*I)*
e^2*(e*Sec[c + d*x])^(3/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))))/((e*Cos[c + d*
x])^(7/2)*(e*Sec[c + d*x])^(7/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && !IntegerQ[m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.11

method	result
default	$-\frac{2\left(4i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-3\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}-2i\sin\left(\frac{dx}{2}\right.}{e^{3}a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}e+e}\,d}$

input
```
int(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/e^3/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(4*I*sin
(1/2*d*x+1/2*c)^3+2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)-2*I*sin(1/2*d*x+1/2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{4\left(\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(-3i\,e^{(2i\,dx+2i\,c)}-2i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+3\sqrt{\frac{1}{2}}\sqrt{e}\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)\mathrm{weierstrassZeta}\left(\right.\right.}{a^{2}de^{4}e^{(2i\,dx+2i\,c)}+a^{2}de^{4}}$$

input
```
integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

output
```
-4*(sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(-3*I*e^(2*I*d*x + 2*I*c) -
2*I)*e^(-1/2*I*d*x - 1/2*I*c) + 3*sqrt(1/2)*sqrt(e)*(-I*e^(2*I*d*x + 2*I*c
) - I)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))
)/(a^2*d*e^4*e^(2*I*d*x + 2*I*c) + a^2*d*e^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cos(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx=\int \frac{1}{(e\cos(dx+c))^{\frac{7}{2}}(ia\tan(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*cos(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx=\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\tan(c+dx)\,1\mathrm{i})^2}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

Reduce [F]

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx=-\frac{\int \frac{1}{\sqrt{\cos(dx+c)}\cos(dx+c)^3\tan(dx+c)^2-2\sqrt{\cos(dx+c)}\cos(dx+c)^3\tan(dx+c)i-\sqrt{\cos(dx+c)}\cos(dx+c)^3}dx}{\sqrt{e}\,a^2e^3}$$

input
```
int(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(1/(sqrt(cos(c + d*x))*cos(c + d*x)**3*tan(c + d*x)**2 - 2*sqrt(cos
(c + d*x))*cos(c + d*x)**3*tan(c + d*x)*i - sqrt(cos(c + d*x))*cos(c + d*x
)**3),x))/(sqrt(e)*a**2*e**3)
```

3.680 $\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 126

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \frac{10\cos^{\frac{9}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d(e\cos(c+dx))^{9/2}} + \frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(e\cos(c+dx))^{9/2}} - \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{9/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
10/3*cos(d*x+c)^(9/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/a^2/d/(e*cos(
d*x+c))^(9/2)+10/3*cos(d*x+c)^3*sin(d*x+c)/a^2/d/(e*cos(d*x+c))^(9/2)-4*I*
cos(d*x+c)^2/d/(e*cos(d*x+c))^(9/2)/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.53

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \frac{2\left(-6i\cos(c+dx)+5\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right.\right.}{3a^2de^3(e\cos(c+dx))^{3/2}}$$

input
```
Integrate[1/((e*Cos[c + d*x])^(9/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(2*((-6*I)*Cos[c + d*x] + 5*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] -
 Sin[c + d*x]))/(3*a^2*d*e^3*(e*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 3998, 3042, 3981, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int \frac{(e\sec(c+dx))^{9/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(e\sec(c+dx))^{9/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3981}$$

$$\frac{\frac{5e^2\int (e\sec(c+dx))^{5/2}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{5e^2\left(\frac{1}{3}e^2\int\sqrt{e\sec(c+dx)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5e^2\left(\frac{1}{3}e^2\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{5e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{5e^2\left(\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

input
```
Int[1/((e*Cos[c + d*x])^(9/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
((5*e^2*((2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c
+ d*x]])/(3*d) + (2*e*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)))/a^2 - (
(4*I)*e^2*(e*Sec[c + d*x])^(5/2))/(d*(a^2 + I*a^2*Tan[c + d*x])))/((e*Cos[
c + d*x])^(9/2)*(e*Sec[c + d*x])^(9/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.65

method	result
default	$\frac{-\frac{20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{3}-8i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\frac{4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{3}+\frac{10\sqrt{\frac{1}{2}-\cdots}}{\cdots}}{\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}e+e\,e^{4}\cdots}}$

input
```
int(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*
c)^2*e+e)^(1/2)/e^4*(-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12
*I*sin(1/2*d*x+1/2*c)^3+2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+6*I*sin(1/2*d*x+1/2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.14

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx =$$
$$-\frac{4\left(\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(5i\,e^{(3i\,dx+3i\,c)}+7i\,e^{(i\,dx+i\,c)}\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+5\sqrt{\frac{1}{2}}\sqrt{e}\left(i\,e^{(4i\,dx+4i\,c)}+2i\,e^{(2i\,dx+2i\,c)}\cdots\right.\right.}{3\left(a^2de^5e^{(4i\,dx+4i\,c)}+2\,a^2de^5e^{(2i\,dx+2i\,c)}+a^2de^5\right)}$$

input
```
integrate(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

output
```
-4/3*(sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(5*I*e^(3*I*d*x + 3*I*c) +
 7*I*e^(I*d*x + I*c))*e^(-1/2*I*d*x - 1/2*I*c) + 5*sqrt(1/2)*sqrt(e)*(I*e^
(4*I*d*x + 4*I*c) + 2*I*e^(2*I*d*x + 2*I*c) + I)*weierstrassPInverse(-4, 0
, e^(I*d*x + I*c)))/(a^2*d*e^5*e^(4*I*d*x + 4*I*c) + 2*a^2*d*e^5*e^(2*I*d*
x + 2*I*c) + a^2*d*e^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cos(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\cos(dx+c))^{\frac{9}{2}}(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*cos(d*x + c))^(9/2)*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\,\cos\left(c+d\,x\right))^{9/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(9/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e*cos(c + d*x))^(9/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

Reduce [F]

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{\sqrt{\cos(dx+c)}\,\cos(dx+c)^4\tan(dx+c)^2-2\sqrt{\cos(dx+c)}\,\cos(dx+c)^4\tan(dx+c)i-\sqrt{\cos(dx+c)}\,\cos(dx+c)^4}dx}{\sqrt{e}\,a^2e^4}$$

input
```
int(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(1/(sqrt(cos(c + d*x))*cos(c + d*x)**4*tan(c + d*x)**2 - 2*sqrt(cos
(c + d*x))*cos(c + d*x)**4*tan(c + d*x)*i - sqrt(cos(c + d*x))*cos(c + d*x
)**4),x))/(sqrt(e)*a**2*e**4)
```

3.681 $\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 164

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{14\cos^{\frac{11}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{11/2}}+\frac{14\cos^3(c+dx)\sin(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}} + \frac{14\cos^5(c+dx)\sin(c+dx)}{5a^2d(e\cos(c+dx))^{11/2}}-\frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
-14/5*cos(d*x+c)^(11/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/(e*cos
(d*x+c))^(11/2)+14/15*cos(d*x+c)^3*sin(d*x+c)/a^2/d/(e*cos(d*x+c))^(11/2)+
14/5*cos(d*x+c)^5*sin(d*x+c)/a^2/d/(e*cos(d*x+c))^(11/2)-4/3*I*cos(d*x+c)^
2/d/(e*cos(d*x+c))^(11/2)/(a^2+I*a^2*tan(d*x+c))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.59 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.52

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\cos^3(c+dx)(\cos(dx)+i\sin(dx))^2\left(7\cos(c)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\right.\right.}{}$$

input
```
Integrate[1/((e*Cos[c + d*x])^(11/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(Cos[c + d*x]^3*(Cos[d*x] + I*Sin[d*x])^2*(7*Cos[c]*HypergeometricPFQ[{-1/
2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]] -
(7*(3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Cot[c
]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])/2 + (Csc[c]*Sqrt[Sec[c]^2]*Sec[c + d*
x]^2*(Cos[2*c] + I*Sin[2*c])*(36*Cos[d*x] + 27*Cos[2*c + d*x] + 21*Cos[2*c
 + 3*d*x] + (20*I)*Sin[d*x] - (20*I)*Sin[2*c + d*x])*Sqrt[Sin[d*x + ArcTan
[Tan[c]]]^2])/6 + (7*I)*(2*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x
+ ArcTan[Tan[c]]]^2]*Sin[c]*Sin[d*x + ArcTan[Tan[c]]] - (3*Cos[c - d*x - A
rcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Sqrt[Sin[d*x + ArcTan[Tan[
c]]]^2]) + (7*(-2*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[
Tan[c]]]^2]*Sin[c]*Sin[d*x + ArcTan[Tan[c]]] + (3*Cos[c - d*x - ArcTan[Tan
[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])*
Tan[c])/2))/(5*d*(e*Cos[c + d*x])^(11/2)*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + Arc
Tan[Tan[c]]]^2]*(a + I*a*Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3998, 3042, 3981, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{11/2}}\,dx$$

$$\Big\downarrow \ \textbf{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{11/2}}dx$$

$$\Big\downarrow \ \textbf{3998}$$

$$\frac{\int \frac{(e\sec(c+dx))^{11/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \ \textbf{3042}$$

$$\frac{\int \frac{(e\sec(c+dx))^{11/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \ \textbf{3981}$$

$$\frac{\frac{7e^2\int (e\sec(c+dx))^{7/2}dx}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \ \textbf{3042}$$

$$\frac{\frac{7e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \ \textbf{4255}$$

$$\frac{\frac{7e^2\left(\frac{3}{5}e^2\int (e\sec(c+dx))^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \ \textbf{3042}$$

$$\frac{\frac{7e^2\left(\frac{3}{5}e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \ \textbf{4255}$$

$$\frac{\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \ \textbf{3042}$$

$$\frac{\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \textbf{4258}$$

$$\frac{\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \textbf{3119}$$

$$\frac{\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

input

```
Int[1/((e*Cos[c + d*x])^(11/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output

```
((7*e^2*((2*e*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d) + (3*e^2*((-2*e^2
*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) +
(2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/5))/(3*a^2) - (((4*I)/3)*e^2*(
e*Sec[c + d*x])^(7/2))/(d*(a^2 + I*a^2*Tan[c + d*x])))/((e*Cos[c + d*x])^(
11/2)*(e*Sec[c + d*x])^(11/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(146) = 292.

Time = 5.31 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.96

method	result
default	$\frac{\frac{112\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}-\frac{56\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{5}-\frac{112\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\right.}{5}}{\left(4\,\mathrm{s}\right.}$

input
```
int(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/15/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/a^2/sin(1/2*d*x+1/2
*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^5*(168*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)-84*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-168*sin(1
/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+84*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*
x+1/2*c)^2+20*I*sin(1/2*d*x+1/2*c)^3+36*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)-21*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-10*I*sin(1/2*d*x+1/2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.14

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{4\left(\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(21i\,e^{(6i\,dx+6i\,c)}+56i\,e^{(4i\,dx+4i\,c)}+47i\,e^{(2i\,dx+2i\,c)}\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+21\sqrt{\frac{1}{2}}\sqrt{e}\left(i\,e^{(6i\,dx}\right.\right.}{15\left(a^{2}de^{6}e^{(6i\,dx+6i\,c)}+3\,a^{2}de^{6}e^{(4i\,dx}\right.}$$

input
```
integrate(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="frica
s")
```

output
```
-4/15*(sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(21*I*e^(6*I*d*x + 6*I*c)
 + 56*I*e^(4*I*d*x + 4*I*c) + 47*I*e^(2*I*d*x + 2*I*c))*e^(-1/2*I*d*x - 1/
2*I*c) + 21*sqrt(1/2)*sqrt(e)*(I*e^(6*I*d*x + 6*I*c) + 3*I*e^(4*I*d*x + 4*
I*c) + 3*I*e^(2*I*d*x + 2*I*c) + I)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, e^(I*d*x + I*c))))/(a^2*d*e^6*e^(6*I*d*x + 6*I*c) + 3*a^2*d*e^
6*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^6*e^(2*I*d*x + 2*I*c) + a^2*d*e^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cos(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\cos(dx+c))^{\frac{11}{2}}(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac"
)
```

output
```
integrate(1/((e*cos(d*x + c))^(11/2)*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\cos(c+d\,x))^{11/2}\,(a+a\tan(c+d\,x)\,1\mathrm{i})^2}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(11/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e*cos(c + d*x))^(11/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

Reduce [F]

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{\sqrt{\cos(dx+c)}\cos(dx+c)^5\tan(dx+c)^2-2\sqrt{\cos(dx+c)}\cos(dx+c)^5\tan(dx+c)i-\sqrt{\cos(dx+c)}\cos(dx+c)^5}dx}{\sqrt{e}\,a^2e^5}$$

input
```
int(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - int(1/(sqrt(cos(c + d*x))*cos(c + d*x)**5*tan(c + d*x)**2 - 2*sqrt(cos
(c + d*x))*cos(c + d*x)**5*tan(c + d*x)*i - sqrt(cos(c + d*x))*cos(c + d*x
)**5),x))/(sqrt(e)*a**2*e**5)
```

3.682 $\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 179

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{12ia(e\cos(c+dx))^{7/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} + \frac{32ia(e\cos(c+dx))^{7/2}\sec^4(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}{7d} - \frac{16i(e\cos(c+dx))^{7/2}\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d}$$

output
```
12/35*I*a*(e*cos(d*x+c))^(7/2)*sec(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)+32/
35*I*a*(e*cos(d*x+c))^(7/2)*sec(d*x+c)^4/d/(a+I*a*tan(d*x+c))^(1/2)-2/7*I*
(e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2)/d-16/35*I*(e*cos(d*x+c))^(7/
2)*sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.45

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{e^3\sqrt{e\cos(c+dx)}(35i\cos(c+dx)+i\cos(3(c+dx))+70\sin(c+dx)+}{70d}$$

input
```
Integrate[(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(e^3*Sqrt[e*Cos[c + d*x]]*((35*I)*Cos[c + d*x] + I*Cos[3*(c + d*x)] + 70*S
in[c + d*x] + 6*Sin[3*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(70*d)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3998, 3042, 3978, 3042, 3983, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{6a\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{6a\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3983}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{6a\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{6a\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3978}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{6a\left(\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{6a\left(\frac{4\left(\frac{2a\int\frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right.$$

$$\Big\downarrow \; 3969$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{6a\left(\frac{4\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)$$

input
```
Int[(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*((((-2*I)/7)*Sqrt[a + I*a*Ta
n[c + d*x]])/(d*(e*Sec[c + d*x])^(7/2)) + (6*a*(((2*I)/5)/(d*(e*Sec[c + d*
x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (4*((((4*I)/3)*a*Sqrt[e*Sec[c + d*
x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((2*I)/3)*Sqrt[a + I*a*Tan[c +
d*x]])/(d*(e*Sec[c + d*x])^(3/2))))/(5*a)))/(7*e^2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

Maple [A] (verified)

Time = 10.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.44

method	result	size
default	$-\frac{2e^{3}\sqrt{e\cos(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,\left(-6\sin(dx+c)\cos(dx+c)^{2}-i\cos(dx+c)^{3}-16\sin(dx+c)-8i\cos(dx+c)\right)}{35d}$	79

input
```
int((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35/d*e^3*(e*cos(d*x+c))^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*(-6*sin(d*x+c)
*cos(d*x+c)^2-I*cos(d*x+c)^3-16*sin(d*x+c)-8*I*cos(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.56

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\left(-5i\,e^{3}e^{(6i\,dx+6i\,c)}-35i\,e^{3}e^{(4i\,dx+4i\,c)}+105i\,e^{3}e^{(2i\,dx+2i\,c)}+7i\,e^{3}\right)}{140\,d}$$

input
```
integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/140*sqrt(2)*sqrt(1/2)*(-5*I*e^3*e^(6*I*d*x + 6*I*c) - 35*I*e^3*e^(4*I*d*
x + 4*I*c) + 105*I*e^3*e^(2*I*d*x + 2*I*c) + 7*I*e^3)*sqrt(e*e^(2*I*d*x +
2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-5/2*I*d*x - 5/2*I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(7/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.13

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\left(7i\,e^3\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-5i\,e^3\cos\left(\frac{7}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx\right.\right.\right.\right.}{}$$

input
```
integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
1/140*(7*I*e^3*cos(5/2*d*x + 5/2*c) - 5*I*e^3*cos(7/5*arctan2(sin(5/2*d*x
+ 5/2*c), cos(5/2*d*x + 5/2*c))) - 35*I*e^3*cos(3/5*arctan2(sin(5/2*d*x +
5/2*c), cos(5/2*d*x + 5/2*c))) + 105*I*e^3*cos(1/5*arctan2(sin(5/2*d*x + 5
/2*c), cos(5/2*d*x + 5/2*c))) + 7*e^3*sin(5/2*d*x + 5/2*c) + 5*e^3*sin(7/5
*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 35*e^3*sin(3/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 105*e^3*sin(1/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqrt(a)*sqrt(e)/d
```

Giac [F(-2)]

Exception generated.

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.54

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{e^3\sqrt{e\,\cos(c+d\,x)}\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\left(\sin(c+d\,x)+\frac{3\,\sin(}{}\right.}{d}$$

input
```
int((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(e^3*(e*cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
1))/(cos(2*c + 2*d*x) + 1))^(1/2)*((cos(c + d*x)*1i)/2 + sin(c + d*x) + (c
os(3*c + 3*d*x)*1i)/70 + (3*sin(3*c + 3*d*x))/35))/d
```

Reduce [F]

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx=\sqrt{e}\,\sqrt{a}\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)e^3$$

input
```
int((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
sqrt(e)*sqrt(a)*int(sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x))*cos(c + d*
x)**3,x)*e**3
```

3.683 $\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 132

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{8ia(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{5d} - \frac{16i(e\cos(c+dx))^{5/2}\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d}$$

output
```
8/15*I*a*(e*cos(d*x+c))^(5/2)*sec(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)-2/5*
I*(e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2)/d-16/15*I*(e*cos(d*x+c))^(
5/2)*sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.48

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{ie^2\sqrt{e\cos(c+dx)}(-15+\cos(2(c+dx))-4i\sin(2(c+dx)))\sqrt{a+ia\,\mathrm{t}}}{15d}$$

input
```
Integrate[(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I/15)*e^2*Sqrt[e*Cos[c + d*x]]*(-15 + Cos[2*(c + d*x)] - (4*I)*Sin[2*(c
+ d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3998, 3042, 3978, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3983}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3969}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{4a\left(\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)$$

input
```
Int[(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2)*((((-2*I)/5)*Sqrt[a + I*a*Ta
n[c + d*x]])/(d*(e*Sec[c + d*x])^(5/2)) + (4*a*(((2*I)/3)/(d*Sqrt[e*Sec[c
+ d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)*Sqrt[a + I*a*Tan[c + d*x]
])/(a*d*Sqrt[e*Sec[c + d*x]])))/(5*e^2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

Maple [A] (verified)

Time = 9.75 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{2\left(i\cos(dx+c)^{2}+4\cos(dx+c)\sin(dx+c)-8i\right)e^{2}\sqrt{e\cos(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}}{15d}$	62
risch	$-\frac{ie^{2}\sqrt{2}\,\sqrt{e\cos(dx+c)}\,\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,(30-2\cos(2dx+2c)+8i\sin(2dx+2c))}{30d}$	74

input
```
int((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*(I*cos(d*x+c)^2+4*cos(d*x+c)*sin(d*x+c)-8*I)*e^2*(e*cos(d*x+c))^(1/
2)*(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.65

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\left(-3i\,e^{2}e^{(4i\,dx+4i\,c)}-30i\,e^{2}e^{(2i\,dx+2i\,c)}+5i\,e^{2}\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\cdots}}{30\,d}$$

input
```
integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/30*sqrt(2)*sqrt(1/2)*(-3*I*e^2*e^(4*I*d*x + 4*I*c) - 30*I*e^2*e^(2*I*d*x
 + 2*I*c) + 5*I*e^2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*e^(-3/2*I*d*x - 3/2*I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.12

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\left(5i\,e^2\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\,e^2\cos\left(\frac{5}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx\right.\right.\right.\right.}{}$$

input
```
integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
1/30*(5*I*e^2*cos(3/2*d*x + 3/2*c) - 3*I*e^2*cos(5/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) - 30*I*e^2*cos(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 5*e^2*sin(3/2*d*x + 3/2*c) + 3*e^2*sin(5/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 30*e^2*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)*sqrt(e)/d
```

Giac [F(-2)]

Exception generated.

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.64

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{e^2\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}+\right.}{15\,d}$$

input
```
int((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(e^2*(e*cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*1i + 4*sin(2*c + 2*d*x
) - 15i))/(15*d)
```

Reduce [F]

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx=\sqrt{e}\,\sqrt{a}\left(\int\sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)e^2$$

input
```
int((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
sqrt(e)*sqrt(a)*int(sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x))*cos(c + d*
x)**2,x)*e**2
```

3.684 $\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 85

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx=\frac{4iae\sqrt{e\cos(c+dx)}\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}-\frac{2i(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{3d}$$

output
```
4/3*I*a*e*(e*cos(d*x+c))^(1/2)*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-2/3*I
*(e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx=\frac{2e\sqrt{e\cos(c+dx)}(i\cos(c+dx)+2\sin(c+dx))\sqrt{a+ia\tan(c+dx)}}{3d}$$

input
```
Integrate[(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*e*Sqrt[e*Cos[c + d*x]]*(I*Cos[c + d*x] + 2*Sin[c + d*x])*Sqrt[a + I*a*T
an[c + d*x]])/(3*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3998, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3969}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)$$

input
```
Int[(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(e*Cos[c + d*x])^(3/2)*(e*Sec[c + d*x])^(3/2)*((((4*I)/3)*a*Sqrt[e*Sec[c +
 d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((2*I)/3)*Sqrt[a + I*a*Tan[c
 + d*x]])/(d*(e*Sec[c + d*x])^(3/2)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

Maple [A] (verified)

Time = 9.60 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{2e\sqrt{e\cos(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,(i\cos(dx+c)+2\sin(dx+c))}{3d}$	50
risch	$-\frac{ie\sqrt{2}\,\sqrt{e\cos(dx+c)}\,\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,(-2\cos(dx+c)+4i\sin(dx+c))}{3d}$	65

input
```
int((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*e*(e*cos(d*x+c))^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*(I*cos(d*x+c)+2*si
n(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.80

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(-i\,ee^{(2i\,dx+2i\,c)}+3i\,e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\right.}}{3\,d}$$

input
```
integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/3*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(-I*e*e^(2*I*d*x + 2
*I*c) + 3*I*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/
d
```

Sympy [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\left(-i\,e\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3i\,e\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+e\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,e\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}\sqrt{e}}{3\,d}$$

input
```
integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
1/3*(-I*e*cos(3/2*d*x + 3/2*c) + 3*I*e*cos(1/2*d*x + 1/2*c) + e*sin(3/2*d*
x + 3/2*c) + 3*e*sin(1/2*d*x + 1/2*c))*sqrt(a)*sqrt(e)/d
```

Giac [F(-2)]

Exception generated.

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2\,e\,\sqrt{e\,\left(2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 2\mathrm{i}+2\,\sin\left(c+d\,x\right)-\mathrm{i}\right)\,\sqrt{}}{3\,d}$$

input
```
int((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(2*e*(e*(2*cos(c/2 + (d*x)/2)^2 - 1))^(1/2)*(2*sin(c + d*x) + cos(c/2 + (d
*x)/2)^2*2i - 1i)*((a*(sin(2*c + 2*d*x)*1i + 2*cos(c + d*x)^2))/(2*cos(c +
 d*x)^2))^(1/2))/(3*d)
```

Reduce [F]

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \sqrt{e}\,\sqrt{a}\,\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)e$$

input
```
int((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
sqrt(e)*sqrt(a)*int(sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x))*cos(c + d*
x),x)*e
```

3.685 $\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx=-\frac{2i\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

output
```
-2*I*(e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx=-\frac{2i\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

input
```
Integrate[Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 3998, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}dx$$

$$\Big\downarrow \ 3998$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \ 3969$$

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{d}$$

input
```
Int[Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

Maple [A] (verified)

Time = 11.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

method	result	size
orering	$-\frac{2i\sqrt{e\cos(dx+c)}\,\sqrt{a+ia\tan(dx+c)}}{d}$	31
default	$-\frac{2i\sqrt{e\cos(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}}{d}$	32
risch	$-\frac{2i\sqrt{2}\,\sqrt{e\cos(dx+c)}\,\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}{d}$	46

input
```
int((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I*(e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.44

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{d}$$

input
```
integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
-2*I*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/d
```

Sympy [F]

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{e\cos{(c+dx)}}\sqrt{ia\left(\tan{(c+dx)}-i\right)}\,dx$$

input
```
integrate((e*cos(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(e*cos(c + d*x))*sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(28) = 56.

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.11

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i\sqrt{a}\sqrt{e}\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}{d\sqrt{-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

input
```
integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
-2*I*sqrt(a)*sqrt(e)*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 1)/(d*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - 1))
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

input
```
int((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx$$
$$= \frac{\sqrt{e}\,\sqrt{a}\,i\left(-2\sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cos(dx+c)}-\left(\int\frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{\cos(dx+c)}dx\right)d+\left(\int\sqrt{\tan(d\right.\right.}{d}$$

input
```
int((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*sqrt(a)*i*( - 2*sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x)) - int
((sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x))*sin(c + d*x))/cos(c + d*x),x
)*d + int(sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x))*tan(c + d*x),x)*d))/
d
```

3.686 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 230

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = \frac{i\sqrt{2}\sqrt{a}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}} - \frac{i\sqrt{2}\sqrt{a}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}} + \frac{i\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}(1+\cos(c+dx)+i\sin(c+dx))}\right)}{d\sqrt{e}}$$

output
```
I*2^(1/2)*a^(1/2)*arctan(1-2^(1/2)*(e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))
^(1/2)/a^(1/2)/e^(1/2))/d/e^(1/2)-I*2^(1/2)*a^(1/2)*arctan(1+2^(1/2)*(e*co
s(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/d/e^(1/2)+I*2^(1
/2)*a^(1/2)*arctanh(2^(1/2)*(e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/
a^(1/2)/e^(1/2)/(1+cos(d*x+c)+I*sin(d*x+c)))/d/e^(1/2)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = \frac{ie^{-\frac{3}{2}idx}\left(-e^{-2ic}\right)^{3/4}\left(1+e^{2i(c+dx)}\right)\left(\arctan\left(\frac{e^{\frac{idx}{2}}}{\sqrt[4]{-e^{-2ic}}}\right)-\operatorname{arctanh}\left(\frac{e^{\frac{idx}{2}}}{\sqrt[4]{-e^{-2ic}}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\cos(c+dx)}}$$

input
```
Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Cos[c + d*x]],x]
```

output
```
(I*(-E^((-2*I)*c))^(3/4)*(1 + E^((2*I)*(c + d*x)))*(ArcTan[E^((I/2)*d*x)/(
-E^((-2*I)*c))^(1/4)] - ArcTanh[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)])*Sqrt
[a + I*a*Tan[c + d*x]])/(d*E^(((3*I)/2)*d*x)*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.43, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3996, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}dx$$

$$\Big\downarrow \ 3996$$

$$-\frac{4ia\int \frac{e\cos(c+dx)(i\tan(c+dx)a+a)}{a^2e^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2e^2}d\left(\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\right)}{d}$$

$$\Big\downarrow \ 826$$

$$-\frac{4ia\left(\frac{1}{2}\int \frac{ae+\cos(c+dx)(i\tan(c+dx)a+a)e}{a^2e^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2e^2}d\left(\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\right)-\frac{1}{2}\int \frac{ae-e\cos(c+dx)(i\tan(c+dx}{a^2e^2+\cos^2(c+dx)(i\tan(c+dx)}\right.}{d}$$

$\downarrow$ 1476

$$-\frac{4ia\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{ae+\cos(c+dx)(i\tan(c+dx)a+a)e-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\sqrt{e}}d\left(\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\right.\right.\right.}{}$$

$\downarrow$ 1082

$$-\frac{4ia\left(\frac{1}{2}\left(\frac{\int \frac{1}{-e\cos(c+dx)(i\tan(c+dx)a+a)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-e\cos(c+dx)(i\tan(c+dx)a+a)-1}d\left(\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{i}}{\sqrt{a}\sqrt{}}\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}\right.\right.}{d}$$

$\downarrow$ 217

$$-\frac{4ia\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}\right)-\frac{1}{2}\int \frac{ae-e\cos(c+dx)(i\tan(}{a^2e^2+\cos^2(c+dx)(i\tan(}\right.}{d}$$

$\downarrow$ 1479

$$-\frac{4ia\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{a}\sqrt{e}-2\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}}{ae+\cos(c+dx)(i\tan(c+dx)a+a)e-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\sqrt{e}}d\left(\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}+\frac{\int -\frac{\sqrt{}}{ae+\cos(c+dx)(}}{}\right.\right.}{}$$

$\downarrow$ 25

$$-\frac{4ia\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{a}\sqrt{e}-2\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}}{ae+\cos(c+dx)(i\tan(c+dx)a+a)e-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\sqrt{e}}d\left(\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{\sqrt{2}}{ae+\cos(c+dx)(i}}{}\right.\right.}{}$$

$\downarrow$ 27

$$-\frac{4ia\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{a}\sqrt{e}-2\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}}{ae+\cos(c+dx)(i\tan(c+dx)a+a)e-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\sqrt{e}}d\left(\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{}{ae+\cos(c+dx)(i}}{}\right.\right.}{}$$

$\downarrow$ 1103

$$-\frac{4ia\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(e\cos(c+dx)(a+ia\tan\right.}{}\right.}{}$$

input
```
Int[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Cos[c + d*x]],x]
```

output
```
((-4*I)*a*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c
+ d*x]])/(Sqrt[a]*Sqrt[e])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[
2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e])]/(Sq
rt[2]*Sqrt[a]*Sqrt[e]))/2 + (Log[a*e - Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[e*Cos[
c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]] + e*Cos[c + d*x]*(a + I*a*Tan[c + d*x
])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]) - Log[a*e + Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[e
*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]] + e*Cos[c + d*x]*(a + I*a*Tan[c
+ d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3996
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-4*(b/f)   Subst[Int[x^2/(a^2*d^2 + x^4), x], x,
 Sqrt[d*Cos[e + f*x]]*Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, d, e, f
}, x] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 11.55 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.88

method	result
default	$\frac{\cos(dx+c)\sqrt{a(1+i\tan(dx+c))}\left(i\,\mathrm{arctanh}\left(\frac{-\cot(dx+c)+\csc(dx+c)-1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+i\,\mathrm{arctanh}\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+\mathrm{arctanh}\left(\frac{-\cot(d}{2\sqrt{}}\right.\right.}{d(-\sin(dx+c)+i\cos(dx+c)+i)\sqrt{e\cos(dx+c)}\sqrt{\frac{1}{\cos(dx+c)+1}}}$

input

```
int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/d*cos(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*(I*arctanh(1/2*(-cot(d*x+c)+csc(
d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))+I*arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)
*(-cot(d*x+c)+csc(d*x+c)+1))+arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(1/(co
s(d*x+c)+1))^(1/2))-arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(
d*x+c)+1)))/(-sin(d*x+c)+I*cos(d*x+c)+I)/(e*cos(d*x+c))^(1/2)/(1/(cos(d*x+
c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.36

$$
\begin{aligned}
&\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx \\
&= -\frac{1}{2}\sqrt{\frac{4i\,a}{d^2e}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right. \\
&\qquad \left. +\frac{1}{2}i\,de\sqrt{\frac{4i\,a}{d^2e}}\right) \\
&\quad +\frac{1}{2}\sqrt{\frac{4i\,a}{d^2e}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right. \\
&\qquad \left. -\frac{1}{2}i\,de\sqrt{\frac{4i\,a}{d^2e}}\right) \\
&\quad -\frac{1}{2}\sqrt{-\frac{4i\,a}{d^2e}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right. \\
&\qquad \left. +\frac{1}{2}i\,de\sqrt{-\frac{4i\,a}{d^2e}}\right) \\
&\quad +\frac{1}{2}\sqrt{-\frac{4i\,a}{d^2e}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right. \\
&\qquad \left. -\frac{1}{2}i\,de\sqrt{-\frac{4i\,a}{d^2e}}\right)
\end{aligned}
$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
-1/2*sqrt(4*I*a/(d^2*e))*log(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c)
+ e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c) + 1/2*I*d*
e*sqrt(4*I*a/(d^2*e))) + 1/2*sqrt(4*I*a/(d^2*e))*log(sqrt(2)*sqrt(1/2)*sqr
t(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d
*x - 1/2*I*c) - 1/2*I*d*e*sqrt(4*I*a/(d^2*e))) - 1/2*sqrt(-4*I*a/(d^2*e))*
log(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c) + 1/2*I*d*e*sqrt(-4*I*a/(d^2*e))) +
 1/2*sqrt(-4*I*a/(d^2*e))*log(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c)
 + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c) - 1/2*I*d
*e*sqrt(-4*I*a/(d^2*e)))
```

Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = \int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\sqrt{e\cos\left(c+dx\right)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))/sqrt(e*cos(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1400 vs. $2(168) = 336$.

Time = 0.37 (sec) , antiderivative size = 1400, normalized size of antiderivative = 6.09

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
1/4*(-2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqr
t(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, sqr
t(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2
*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) - 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1) - 2*sqrt(2)*arctan2(sqrt(2)*sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))), sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 1) + 2*sqrt(2)*arctan2(-sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))), -sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + 1) + I*sqrt(2)*log(2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + ia\tan(c + dx)}}{\sqrt{e\cos(c + dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = \int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{\sqrt{e\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)}{e}$$

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*sqrt(a)*int((sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x)))/cos(c +
 d*x),x))/e
```

3.687 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 470

$$
\begin{aligned}
\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx &= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} \\
&\quad - \frac{ia^{3/2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&\quad + \frac{ia^{3/2}e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&\quad - \frac{ia^{3/2}e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}\left(\sqrt{a}+\cos(c+dx)\left(\sqrt{a}-i\sqrt{a}\tan(c+dx)\right)\right)}\right)\sec(c+dx)}{\sqrt{2}d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
\end{aligned}
$$

output
```
I*a/d/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*a^(3/2)*e^(3/2)*
arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(
1/2))*sec(d*x+c)*2^(1/2)/d/(e*cos(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2)/(a-I*
a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+1/2*I*a^(3/2)*e^(3/2)*arctan(
1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*s
ec(d*x+c)*2^(1/2)/d/(e*cos(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2)/(a-I*a*tan(d
*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*a^(3/2)*e^(3/2)*arctanh(2^(1/2
)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)/(a^(1/2)+cos(d*x+c
)*(a^(1/2)-I*a^(1/2)*tan(d*x+c))))*sec(d*x+c)*2^(1/2)/d/(e*cos(d*x+c))^(3/
2)/(e*sec(d*x+c))^(3/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 3.20 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx = \frac{ie^{-\frac{1}{2}i(c+dx)}\cos^2(c+dx)\left(2\sqrt{2}\cos\left(\frac{1}{2}(c+dx)\right)+2\arctan\left(1-\sqrt{2}e^{\frac{1}{2}i(c+dx)}\right)c\right.}{}$$

input
```
Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(3/2),x]
```

output
```
(I*Cos[c + d*x]^2*(2*Sqrt[2]*Cos[(c + d*x)/2] + 2*ArcTan[1 - Sqrt[2]*E^((I
/2)*(c + d*x))]*Cos[c + d*x] - 2*ArcTan[1 + Sqrt[2]*E^((I/2)*(c + d*x))]*C
os[c + d*x] + Cos[c + d*x]*Log[1 - Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c +
 d*x))] - Cos[c + d*x]*Log[1 + Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x
))] - (2*I)*Sqrt[2]*Sin[(c + d*x)/2])*(Cos[c + d*x] + I*Sin[c + d*x])*Sqrt
[a + I*a*Tan[c + d*x]])/(Sqrt[2]*d*E^((I/2)*(c + d*x))*(1 + E^((2*I)*(c +
d*x)))*(e*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.93, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3998, 3042, 3979, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3998}$$

$$\frac{\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3979}$$

$$\frac{\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3980}$$

$$\frac{\frac{ae\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{ae\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3976}$$

$$\frac{\frac{2ia^2e^3\sec(c+dx)\int \frac{\cos(c+dx)(a-ia\tan(c+dx))}{e\left(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2\right)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{826}$$

$$\frac{\frac{2ia^2e^3\sec(c+dx)\left(\frac{\int \frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+d\cdots}{d\sqrt{a+ia\tan\cdots}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c\cdots}{e}}\cdots}{2e}}{2e}\cdots\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\cdots}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3\cdots}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\cdots\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\cdots}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\cdots}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\,\mathrm{t}\cdots}{e}\right.}\cdots}{2\sqrt{2}\sqrt{a}\sqrt{e}}\cdots\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+\cdots}}}{(e\cos(c+dx))^{3/2}(e\cdots}$$

$$\Big\downarrow 25$$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx}}$$

$$(e\cos(c+dx))^{3/2}(e\,\mathrm{se}$$

$$\Big\downarrow 27$$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx}{e}}}{2\sqrt{2}\sqrt{a}e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c$$

$$\Big\downarrow 1103$$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+d\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}$$

input
```
Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(3/2),x]
```

output
```
((I*a*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + ((2*I)*a^2*
e^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*S
qrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqr
t[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*
Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a -
I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*
x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a -
 I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d
*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[a - I*a*Ta
n[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]))/((e*Cos[c + d*x])^(3/2)*(e*Sec[c
+ d*x])^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

Maple [A] (verified)

Time = 11.57 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.57

method	result
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\left(i\cos(dx+c)\operatorname{arctanh}\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+i\cos(dx+c)\operatorname{arctanh}\left(\frac{-\cot(dx+c)+\csc(dx+c)-1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+2i\sqrt{\frac{}{\mathrm{co}}}\right.}{2d(-\sin(dx+c)+i}$

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(a*(1+I*tan(d*x+c)))^(1/2)*(I*cos(d*x+c)*arctanh(1/2/(1/(cos(d*x+c)+
1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+I*cos(d*x+c)*arctanh(1/2*(-cot(d*x+c
)+csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))+2*I*(1/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)+cos(d*x+c)*arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d
*x+c)+1))-cos(d*x+c)*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(1/(cos(d*x+c)
+1))^(1/2))-2*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2))/(-sin(d*x+c)+I*cos(
d*x+c)+I)/(1/(cos(d*x+c)+1))^(1/2)/e/(e*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx = \frac{4i\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-\left(de^{2}e^{(2i\,dx+2i\,c)}+de^{2}\right)\sqrt{}}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
1/2*(4*I*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - (d*e^2*e^(2*I*d*x + 2*I*c) +
d*e^2)*sqrt(I*a/(d^2*e^3))*log(d*e^2*sqrt(I*a/(d^2*e^3)) + sqrt(2)*sqrt(1/
2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1
/2*I*d*x - 1/2*I*c)) + (d*e^2*e^(2*I*d*x + 2*I*c) + d*e^2)*sqrt(I*a/(d^2*e
^3))*log(-d*e^2*sqrt(I*a/(d^2*e^3)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x
+ 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c))
+ (d*e^2*e^(2*I*d*x + 2*I*c) + d*e^2)*sqrt(-I*a/(d^2*e^3))*log(d*e^2*sqrt(
-I*a/(d^2*e^3)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) - (d*e^2*e^(2*I*d*x
+ 2*I*c) + d*e^2)*sqrt(-I*a/(d^2*e^3))*log(-d*e^2*sqrt(-I*a/(d^2*e^3)) + s
qrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)))/(d*e^2*e^(2*I*d*x + 2*I*c) + d*e^2)
```

Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\left(e\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))/(e*cos(c + d*x))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1836 vs. $2(360) = 720$.

Time = 0.30 (sec) , antiderivative size = 1836, normalized size of antiderivative = 3.91

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output
```
-8*(2*(sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*ar
ctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(
2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2
)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*cos(2*d*
x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*cos(2*d*x + 2*c) + I
*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 1) - 2*(-I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*s
in(2*d*x + 2*c) - I*sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 2*(I*sqrt(2)*cos(2*
d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + I*sqrt(2))*arctan2(-sqrt(2)*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{e}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)}{e^2}$$

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(e)*sqrt(a)*int((sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x)))/cos(c +
 d*x)**2,x))/e**2
```

3.688 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 404

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx = \frac{3i\sqrt{a}e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{3i\sqrt{a}e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} + \frac{3i\sqrt{a}e^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}\left(\sqrt{a}+\cos(c+dx)\left(\sqrt{a}+i\sqrt{a}\tan(c+dx)\right)\right)}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} + \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d(e\cos(c+dx))^{5/2}}$$

output
```
3/8*I*a^(1/2)*e^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^
(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2)/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^
(5/2)-3/8*I*a^(1/2)*e^(5/2)*arctan(1+2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1
/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2)/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*
x+c))^(5/2)+3/8*I*a^(1/2)*e^(5/2)*arctanh(2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c
))^(1/2)/(e*sec(d*x+c))^(1/2)/(a^(1/2)+cos(d*x+c)*(a^(1/2)+I*a^(1/2)*tan(d
*x+c))))*2^(1/2)/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)+1/2*I*a/d/(e*
cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2)-3/4*I*cos(d*x+c)^2*(a+I*a*tan(d
*x+c))^(1/2)/d/(e*cos(d*x+c))^(5/2)
```

Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\left(\frac{3ie^{-\frac{1}{2}i(2c+5dx)}\left(-e^{-2ic}\right)^{3/4}\left(1+e^{2i(c+dx)}\right)^2\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(\arctan\left(\cdots\right.\right.}{4\sqrt{2}}\cdots\right.}{\cdots}$$

input
```
Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((((3*I)/4)*(-E^((-2*I)*c))^(3/4)*(1 + E^((2*I)*(c + d
*x)))^2*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(ArcTan[E^((I/2)*d
*x)/(-E^((-2*I)*c))^(1/4)] - ArcTanh[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)])
)/(Sqrt[2]*E^((I/2)*(2*c + 5*d*x))) - (3*I)*Cos[c + d*x]^(3/2) + 2*Sqrt[Co
s[c + d*x]]*(I*Cos[c + d*x] + Sin[c + d*x]))*Sqrt[a + I*a*Tan[c + d*x]])/(
4*d*(e*Cos[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.10, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3998, 3042, 3979, 3042, 3982, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \ 3998$$

$$\frac{\int (e\sec(c+dx))^{5/2}\sqrt{i\tan(c+dx)a+a}dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int (e\sec(c+dx))^{5/2}\sqrt{i\tan(c+dx)a+a}dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3979$$

$$\frac{\frac{3}{4}a\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{5/2}}{2d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{3}{4}a\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{5/2}}{2d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3982$$

$$\frac{\frac{3}{4}a\left(\frac{e^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}\right)+\frac{ia(e\sec(c+dx))^{5/2}}{2d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{3}{4}a\left(\frac{e^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}\right)+\frac{ia(e\sec(c+dx))^{5/2}}{2d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3976$$

$$\frac{\frac{3}{4}a\left(-\frac{2ie^4\int \frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e\left(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2\right)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}\right)+\frac{ia(e\sec(c+dx))^{5/2}}{2d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \ 826$$

$$\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\int \frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\frac{ie^2\sqrt{a+ia\tan(c+}}{a}\right.$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}$$

$$\Big\downarrow \ 1476$$

$$\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}}{2e}}{2e}\right.}{d}\right.$$

$$(e\cos(c+dx))^{5/2}$$

$$\Big\downarrow \ 1082$$

$$\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\right.}{d}\right.$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+$$

$$\Big\downarrow \ 217$$

$$\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\right.$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}$$

$\Big\downarrow$ 1479

$$\frac{\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)\cdots}{e}\cdots\right)}\cdots}{2\sqrt{2}\sqrt{a}\sqrt{e}}\cdots\right)}{d}\cdots\right)}{(e\cos\cdots}$$

$\Big\downarrow$ 25

$$\frac{\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+\cdots}{e}\cdots\right)}\cdots}{2\sqrt{2}\sqrt{a}\sqrt{e}}\cdots\right)}{d}\cdots\right)}{(e\cos(\cdots}$$

$\Big\downarrow$ 27

$$\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d}{2\sqrt{2}\sqrt{a}e}\right)}{d}\right.$$

$$\big\downarrow \; 1103$$

$$\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right)}{d}\right.$$

$$(e\cos(c+dx))^{5/2}(e\,s$$

input
```
Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(5/2),x]
```

output
```
(((I/2)*a*(e*Sec[c + d*x])^(5/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*a*((
(-2*I)*e^4*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sq
rt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt
[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(S
qrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sq
rt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan
[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*S
qrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Ta
n[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/d - (I*e^2*Sqrt[e*Sec[c
+ d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)))/4)/((e*Cos[c + d*x])^(5/2)*(e*
Sec[c + d*x])^(5/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

Maple [A] (verified)

Time = 11.60 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.72

method	result
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\left(3i\cos(dx+c)\operatorname{arctanh}\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)-3\cos(dx+c)\operatorname{arctanh}\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+3i\cos\right.}{8}$

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*(a*(1+I*tan(d*x+c)))^(1/2)/(-sin(d*x+c)+I*cos(d*x+c)+I)/(e*cos(d*x+c
))^(1/2)/(1/(cos(d*x+c)+1))^(1/2)/e^2*(3*I*cos(d*x+c)*arctanh(1/2/(1/(cos(
d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))-3*cos(d*x+c)*arctanh(1/2/(1/(
cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+3*I*cos(d*x+c)*arctanh(1/
2*(-cot(d*x+c)+csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))+3*cos(d*x+c)*arctan
h(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))+2*I*(1/(cos(d*x
+c)+1))^(1/2)*(3*sin(d*x+c)+2*tan(d*x+c))+2*(1/(cos(d*x+c)+1))^(1/2)*(1+3*
cos(d*x+c)-2*sec(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
1/2*(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*(-3*I*e^(4*I*d*x + 4*I*c) + I*e^(2*I*d*x + 2*I*c))*e^(-1/2*
I*d*x - 1/2*I*c) - (d*e^3*e^(4*I*d*x + 4*I*c) + 2*d*e^3*e^(2*I*d*x + 2*I*c
) + d*e^3)*sqrt(9/16*I*a/(d^2*e^5))*log(4/3*I*d*e^3*sqrt(9/16*I*a/(d^2*e^5
)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) + (d*e^3*e^(4*I*d*x + 4*I*c) + 2*
d*e^3*e^(2*I*d*x + 2*I*c) + d*e^3)*sqrt(9/16*I*a/(d^2*e^5))*log(-4/3*I*d*e
^3*sqrt(9/16*I*a/(d^2*e^5)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c)
 + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) - (d*e^3
*e^(4*I*d*x + 4*I*c) + 2*d*e^3*e^(2*I*d*x + 2*I*c) + d*e^3)*sqrt(-9/16*I*a
/(d^2*e^5))*log(4/3*I*d*e^3*sqrt(-9/16*I*a/(d^2*e^5)) + sqrt(2)*sqrt(1/2)*
sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*
I*d*x - 1/2*I*c)) + (d*e^3*e^(4*I*d*x + 4*I*c) + 2*d*e^3*e^(2*I*d*x + 2*I*
c) + d*e^3)*sqrt(-9/16*I*a/(d^2*e^5))*log(-4/3*I*d*e^3*sqrt(-9/16*I*a/(d^2
*e^5)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)))/(d*e^3*e^(4*I*d*x + 4*I*c) +
 2*d*e^3*e^(2*I*d*x + 2*I*c) + d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + ia\tan(c + dx)}}{(e\cos(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2249 vs. $2(300) = 600$.

Time = 0.47 (sec) , antiderivative size = 2249, normalized size of antiderivative = 5.57

$$\int \frac{\sqrt{a + ia\tan(c + dx)}}{(e\cos(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
-32*(6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*
sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2)*cos(4*d*x
+ 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqr
t(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*
d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + s
qrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
+ 6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin
(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(-I*sqrt(2)*cos(4*d*x
 + 4*c) - 2*I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt
(2)*sin(2*d*x + 2*c) - I*sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(I*sqrt(2...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + ia\tan(c + dx)}}{(e\cos(c + dx))^{5/2}}\,dx = \int \frac{\sqrt{a + a\tan\left(c + d\,x\right)\,1\mathrm{i}}}{\left(e\,\cos\left(c + d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(5/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a + ia\tan(c + dx)}}{(e\cos(c + dx))^{5/2}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)}{e^3}$$

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(e)*sqrt(a)*int((sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x)))/cos(c +
 d*x)**3,x))/e**3
```

3.689 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 571

$$\begin{aligned}
\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx &= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{5ia^{3/2}e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{5ia^{3/2}e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{5ia^{3/2}e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}\left(\sqrt{a}+\cos(c+dx)\left(\sqrt{a}-i\sqrt{a}\tan(c+dx)\right)\right)}\right)\sec(c+dx)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{5i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d(e\cos(c+dx))^{7/2}}
\end{aligned}$$

output
```
1/3*I*a/d/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2)+5/8*I*a*cos(d*x+c)
^2/d/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2)-5/16*I*a^(3/2)*e^(7/2)*
arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(
1/2))*sec(d*x+c)*2^(1/2)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)/(a-I*
a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+5/16*I*a^(3/2)*e^(7/2)*arctan
(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*
sec(d*x+c)*2^(1/2)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)/(a-I*a*tan(
d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-5/16*I*a^(3/2)*e^(7/2)*arctanh(2^(1
/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)/(a^(1/2)+cos(d*x
+c)*(a^(1/2)-I*a^(1/2)*tan(d*x+c))))*sec(d*x+c)*2^(1/2)/d/(e*cos(d*x+c))^(
7/2)/(e*sec(d*x+c))^(7/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2
)-5/12*I*cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d/(e*cos(d*x+c))^(7/2)
```

Mathematica [A] (verified)

Time = 2.75 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\left(-\frac{40}{3}i\cos^{\frac{3}{2}}(c+dx)+\frac{5}{8}ie^{-\frac{7}{2}i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3}\sqrt{e^{-i(c+dx)}\left(1\right.}\right.}{}$$

input
```
Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(7/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(((-40*I)/3)*Cos[c + d*x]^(3/2) + (((5*I)/8)*(1 + E^((
2*I)*(c + d*x)))^3*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(2*ArcT
an[1 - Sqrt[2]*E^((I/2)*(c + d*x))] - 2*ArcTan[1 + Sqrt[2]*E^((I/2)*(c + d
*x))] + Log[1 - Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))] - Log[1 + S
qrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))]))/E^(((7*I)/2)*(c + d*x)) +
(32*Sqrt[Cos[c + d*x]]*(I*Cos[c + d*x] + Sin[c + d*x]))/3 + 20*Cos[c + d*x
]^(5/2)*(I*Cos[c + d*x] + Sin[c + d*x]))*Sqrt[a + I*a*Tan[c + d*x]])/(32*d
*(e*Cos[c + d*x])^(7/2))
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 538, normalized size of antiderivative = 0.94, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3998, 3042, 3979, 3042, 3982, 3042, 3979, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int (e\sec(c+dx))^{7/2}\sqrt{i\tan(c+dx)a+a}dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e\sec(c+dx))^{7/2}\sqrt{i\tan(c+dx)a+a}dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3979}$$

$$\frac{\frac{5}{6}a\int \frac{(e\sec(c+dx))^{7/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{7/2}}{3d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{6}a\int \frac{(e\sec(c+dx))^{7/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{7/2}}{3d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3982}$$

$$\frac{\frac{5}{6}a\left(\frac{3e^2\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}\right)+\frac{ia(e\sec(c+dx))^{7/2}}{3d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{6}a\left(\frac{3e^2\int(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}\right)+\frac{ia(e\sec(c+dx))^{7/2}}{3d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3979}$$

$$\frac{\frac{5}{6}a\left(\frac{3e^2\left(\frac{1}{2}a\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}\right)+\frac{ia(e\sec(c+dx))^{7/2}}{3d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{6}a\left(\frac{3e^2\left(\frac{1}{2}a\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}\right)+\frac{ia(e\sec(c+dx))^{7/2}}{3d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3980}$$

$$\frac{\frac{5}{6}a\left(\frac{3e^2\left(\frac{ae\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}\right)+\frac{ia(e\sec(c+dx))^{7/2}}{3d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{6}a\left(\frac{3e^2\left(\frac{ae\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}\right)+\frac{ia(e\sec(c+dx))^{7/2}}{3d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3976}$$

$$\frac{\frac{5}{6}a\left(\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\int\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e\left(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2\right)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}\right)+\frac{ia(e\sec(c+dx))^{7/2}}{3d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{826}$$

$$\frac{\frac{5}{6}a\left(\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\int \frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec \cdots}{d\sqrt{a+\cdots}}\cdots\right)}{4a}\cdots\right)}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{5}{6}a\left(\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(\cdots}{\cdots}}\cdots}{2e}}{2e}\cdots\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\cdots\right)}{4a}\cdots\right)}{(e\,c\cdots}$$

$$\Big\downarrow \; 1082$$

$$\frac{5}{6}a\left(\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int\frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int\frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c}{\sqrt{a}\sqrt{e\sec(c+d}}\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{4a}\right.}{(e\cos(c+dx}$$

$$\Big\downarrow \; 217$$

$$\frac{5}{6}a\left(\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan}}{\sqrt{e\sec(}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{4a}\right.}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/}}$$

$$\Big\downarrow \; 1479$$

$$\frac{5}{6}a\left(\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c}{\sqrt{e\sec(c+dx}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+}{}\right.}}{2\sqrt{2}\sqrt{c}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+}}\right.}{}\right.$$

$\big\downarrow$ 25

$$\frac{5}{6}a\left(\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+d}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)}{}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia}}\right.}{}\right.$$

$\downarrow$ 27

$$\frac{5}{6}a\left(\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-i}{e}}}{2\sqrt{2}\sqrt{a}e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c}}\right.}{4a}\right.$$

$$\Big\downarrow \; 1103$$

$$\frac{5}{6}a\left(\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{4a}\right.}{(e\cos}$$

input
```
Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(7/2),x]
```

output
```
(((I/3)*a*(e*Sec[c + d*x])^(7/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (5*a*((
(-1/2*I)*e^2*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d) + (3
*e^2*((I*a*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + ((2*I)
*a^2*e^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt
[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[2
]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqr
t[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt
[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c
 + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqr
t[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[
c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[a - I
*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])))/(4*a)))/6)/((e*Cos[c + d*x]
)^(7/2)*(e*Sec[c + d*x])^(7/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

Maple [A] (verified)

Time = 11.81 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.55

method	result
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\left(15i\cos(dx+c)\operatorname{arctanh}\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+15\cos(dx+c)\operatorname{arctanh}\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+15\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d*(a*(1+I*tan(d*x+c)))^(1/2)/(-sin(d*x+c)+I*cos(d*x+c)+I)/(e*cos(d*x+
c))^(1/2)/(1/(cos(d*x+c)+1))^(1/2)/e^3*(15*I*cos(d*x+c)*arctanh(1/2/(1/(co
s(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+15*cos(d*x+c)*arctanh(1/2/(
1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+15*I*cos(d*x+c)*arctan
h(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))-15*cos(d*x+c)*a
rctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))+2*I*(1/(co
s(d*x+c)+1))^(1/2)*(15*sin(d*x+c)+10*tan(d*x+c)+8*sec(d*x+c)*tan(d*x+c))-2
*(1/(cos(d*x+c)+1))^(1/2)*(5+15*cos(d*x+c)-2*sec(d*x+c)+8*sec(d*x+c)^2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="fric
as")
```

output
```
1/12*(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*(-5*I*e^(5*I*d*x + 5*I*c) + 42*I*e^(3*I*d*x + 3*I*c) + 15*
I*e^(I*d*x + I*c))*e^(-1/2*I*d*x - 1/2*I*c) - 6*(d*e^4*e^(6*I*d*x + 6*I*c)
 + 3*d*e^4*e^(4*I*d*x + 4*I*c) + 3*d*e^4*e^(2*I*d*x + 2*I*c) + d*e^4)*sqrt
(25/64*I*a/(d^2*e^7))*log(8/5*d*e^4*sqrt(25/64*I*a/(d^2*e^7)) + sqrt(2)*sq
rt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
e^(-1/2*I*d*x - 1/2*I*c)) + 6*(d*e^4*e^(6*I*d*x + 6*I*c) + 3*d*e^4*e^(4*I*
d*x + 4*I*c) + 3*d*e^4*e^(2*I*d*x + 2*I*c) + d*e^4)*sqrt(25/64*I*a/(d^2*e^
7))*log(-8/5*d*e^4*sqrt(25/64*I*a/(d^2*e^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^
(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1
/2*I*c)) + 6*(d*e^4*e^(6*I*d*x + 6*I*c) + 3*d*e^4*e^(4*I*d*x + 4*I*c) + 3*
d*e^4*e^(2*I*d*x + 2*I*c) + d*e^4)*sqrt(-25/64*I*a/(d^2*e^7))*log(8/5*d*e^
4*sqrt(-25/64*I*a/(d^2*e^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c
) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) - 6*(d*
e^4*e^(6*I*d*x + 6*I*c) + 3*d*e^4*e^(4*I*d*x + 4*I*c) + 3*d*e^4*e^(2*I*d*x
 + 2*I*c) + d*e^4)*sqrt(-25/64*I*a/(d^2*e^7))*log(-8/5*d*e^4*sqrt(-25/64*I
*a/(d^2*e^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)))/(d*e^4*e^(6*I*d*x + 6
*I*c) + 3*d*e^4*e^(4*I*d*x + 4*I*c) + 3*d*e^4*e^(2*I*d*x + 2*I*c) + d*e^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + ia\tan(c + dx)}}{(e\cos(c + dx))^{7/2}}\, dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2661 vs. $2(433) = 866$.

Time = 0.46 (sec) , antiderivative size = 2661, normalized size of antiderivative = 4.66

$$\int \frac{\sqrt{a + ia\tan(c + dx)}}{(e\cos(c + dx))^{7/2}}\, dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="maxi
ma")
```

output
```
-192*(30*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2
)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(6*d*x + 6*c) + 3*I*sqrt(2)*sin(4*d*x +
4*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 30*(sqrt(2)*cos(6*d*x + 6*c) + 3*
sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(6*d*
x + 6*c) + 3*I*sqrt(2)*sin(4*d*x + 4*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + s
qrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
 + 30*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*c
os(2*d*x + 2*c) + I*sqrt(2)*sin(6*d*x + 6*c) + 3*I*sqrt(2)*sin(4*d*x + 4*c
) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 30*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqr
t(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(6*d*x +
 6*c) + 3*I*sqrt(2)*sin(4*d*x + 4*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt
(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) -
30*(-I*sqrt(2)*cos(6*d*x + 6*c) - 3*I*sqrt(2)*cos(4*d*x + 4*c) - 3*I*sqrt(
2)*cos(2*d*x + 2*c) + sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + ...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + ia\tan(c + dx)}}{(e\cos(c + dx))^{7/2}}\, dx = \int \frac{\sqrt{a + a\tan\left(c + d\,x\right)\,1{}\mathrm{i}}}{{\left(e\,\cos\left(c + d\,x\right)\right)}^{7/2}}\, dx$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(7/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a + ia\tan(c + dx)}}{(e\cos(c + dx))^{7/2}}\, dx = \frac{\sqrt{e}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)^4} dx\right)}{e^4}$$

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x)
```

output
```
(sqrt(e)*sqrt(a)*int((sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x)))/cos(c +
 d*x)**4,x))/e**4
```

3.690 $\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 175

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}} + \frac{16i(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{12i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{35ad} - \frac{32i(e\cos(c+dx))^{5/2}\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35ad}$$

output
```
2/7*I*(e*cos(d*x+c))^(5/2)/d/(a+I*a*tan(d*x+c))^(1/2)+16/35*I*(e*cos(d*x+c
))^(5/2)*sec(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)-12/35*I*(e*cos(d*x+c))^(5
/2)*(a+I*a*tan(d*x+c))^(1/2)/a/d-32/35*I*(e*cos(d*x+c))^(5/2)*sec(d*x+c)^2
*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.46

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{ie^3(35\cos(c+dx)+\cos(3(c+dx))+70i\sin(c+dx)+6i\sin(3(c+dx)))}{70d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(e*Cos[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-1/70*I)*e^3*(35*Cos[c + d*x] + Cos[3*(c + d*x)] + (70*I)*Sin[c + d*x] +
 (6*I)*Sin[3*(c + d*x)]))/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3998, 3042, 3983, 3042, 3978, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3983}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{6\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{6\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3978}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{6\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{6\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3983}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{6\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{6\left(\frac{4a\left(\frac{2\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec($$

$$\Big\downarrow \textbf{3969}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{6\left(\frac{4a\left(\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec($$

input
```
Int[(e*Cos[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2)*(((2*I)/7)/(d*(e*Sec[c + d*x
])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (6*((((-2*I)/5)*Sqrt[a + I*a*Tan[c
+ d*x]])/(d*(e*Sec[c + d*x])^(5/2)) + (4*a*(((2*I)/3)/(d*Sqrt[e*Sec[c + d*
x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(
a*d*Sqrt[e*Sec[c + d*x]])))/(5*e^2)))/(7*a))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

Maple [A] (verified)

Time = 9.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.40

method	result	size
default	$-\frac{2e^{2}\sqrt{e\cos(dx+c)}\left(-6\cos(dx+c)\sin(dx+c)+i\cos(dx+c)^{2}-16\tan(dx+c)+8i\right)}{35d\sqrt{a(1+i\tan(dx+c))}}$	70

input
```
int((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35/d*e^2*(e*cos(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)*(-6*cos(d*x+c)
*sin(d*x+c)+I*cos(d*x+c)^2-16*tan(d*x+c)+8*I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.59

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\left(-7i\,e^{2}e^{(6i\,dx+6i\,c)}-105i\,e^{2}e^{(4i\,dx+4i\,c)}+35i\,e^{2}e^{(2i\,dx+2i\,c)}+5i\,e^{2}\right)\sqrt{ee^{(2}}}{140\,ad}$$

input
```
integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/140*sqrt(2)*sqrt(1/2)*(-7*I*e^2*e^(6*I*d*x + 6*I*c) - 105*I*e^2*e^(4*I*d
*x + 4*I*c) + 35*I*e^2*e^(2*I*d*x + 2*I*c) + 5*I*e^2)*sqrt(e*e^(2*I*d*x +
2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-7/2*I*d*x - 7/2*I*c)/(a*
d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.15

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(5i\,e^2\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)-7i\,e^2\cos\left(\frac{5}{7}\,\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right.\right.\right.\right.}{}$$

input
```
integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
1/140*(5*I*e^2*cos(7/2*d*x + 7/2*c) - 7*I*e^2*cos(5/7*arctan2(sin(7/2*d*x
+ 7/2*c), cos(7/2*d*x + 7/2*c))) + 35*I*e^2*cos(3/7*arctan2(sin(7/2*d*x +
7/2*c), cos(7/2*d*x + 7/2*c))) - 105*I*e^2*cos(1/7*arctan2(sin(7/2*d*x + 7
/2*c), cos(7/2*d*x + 7/2*c))) + 5*e^2*sin(7/2*d*x + 7/2*c) + 7*e^2*sin(5/7
*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 35*e^2*sin(3/7*arc
tan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 105*e^2*sin(1/7*arctan
2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(e)/(sqrt(a)*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.63

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{e^2\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,28{}\mathrm{i}+\cos\right.}{140\,a\,d}$$

input
```
int((e*cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(e^2*(e*cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*28i + cos(4*c + 4*d*x)
*5i + 42*sin(2*c + 2*d*x) + 5*sin(4*c + 4*d*x) - 105i))/(140*a*d)
```

Reduce [F]

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\,e^2\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)i+\int \frac{\sqrt{\tan(dx+c)i+1}\,\cdots}{\mathrm{t}\cdots}\cdots\right)}{a}$$

input
```
int((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*sqrt(a)*e**2*( - int((sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x))
*cos(c + d*x)**2*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*i + int((sqrt(tan(
c + d*x)*i + 1)*sqrt(cos(c + d*x))*cos(c + d*x)**2)/(tan(c + d*x)**2 + 1),
x)))/a
```

3.691 $\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 126

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i(e\cos(c+dx))^{3/2}}{5d\sqrt{a+ia\tan(c+dx)}} + \frac{16i(e\cos(c+dx))^{3/2}\sec^2(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} - \frac{8i(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{15ad}$$

output
```
2/5*I*(e*cos(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)+16/15*I*(e*cos(d*x+c
))^(3/2)*sec(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)-8/15*I*(e*cos(d*x+c))^(3/
2)*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.50

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{ie^2(-15+\cos(2(c+dx))+4i\sin(2(c+dx)))}{15d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(e*Cos[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-1/15*I)*e^2*(-15 + Cos[2*(c + d*x)] + (4*I)*Sin[2*(c + d*x)]))/(d*Sqrt[
e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3998, 3042, 3983, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3983}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3978}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{4\left(\frac{2a\int\frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{4\left(\frac{2a\int\frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3969}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{4\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)$$

input
```
Int[(e*Cos[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(e*Cos[c + d*x])^(3/2)*(e*Sec[c + d*x])^(3/2)*(((2*I)/5)/(d*(e*Sec[c + d*x
])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (4*((((4*I)/3)*a*Sqrt[e*Sec[c + d*x
]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((2*I)/3)*Sqrt[a + I*a*Tan[c + d
*x]])/(d*(e*Sec[c + d*x])^(3/2))))/(5*a))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

Maple [A] (verified)

Time = 8.99 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{2e\sqrt{e\cos\left(dx+c\right)}\,\left(i\left(-\cos\left(dx+c\right)+8\sec\left(dx+c\right)\right)+4\sin\left(dx+c\right)\right)}{15d\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}}$	61

input
```
int((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*e*(e*cos(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)*(I*(-cos(d*x+c)+8
*sec(d*x+c))+4*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.66

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\left(-5i\,ee^{(4i\,dx+4i\,c)}+30i\,ee^{(2i\,dx+2i\,c)}+3i\,e\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}}}}{30\,ad}$$

input

```
integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output

```
1/30*sqrt(2)*sqrt(1/2)*(-5*I*e*e^(4*I*d*x + 4*I*c) + 30*I*e*e^(2*I*d*x + 2
*I*c) + 3*I*e)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*e^(-5/2*I*d*x - 5/2*I*c)/(a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input

```
integrate((e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output

```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.08

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(3i\,e\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-5i\,e\cos\left(\frac{3}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right.\right.}{}$$

input

```
integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
1/30*(3*I*e*cos(5/2*d*x + 5/2*c) - 5*I*e*cos(3/5*arctan2(sin(5/2*d*x + 5/2
*c), cos(5/2*d*x + 5/2*c))) + 30*I*e*cos(1/5*arctan2(sin(5/2*d*x + 5/2*c),
 cos(5/2*d*x + 5/2*c))) + 3*e*sin(5/2*d*x + 5/2*c) + 5*e*sin(3/5*arctan2(s
in(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 30*e*sin(1/5*arctan2(sin(5/2
*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqrt(e)/(sqrt(a)*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.79

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{e\sqrt{e\cos(c+dx)}\sqrt{\frac{a(\cos(2c+2dx)+1+\sin(2c+2dx)\,1i)}{\cos(2c+2dx)+1}}\,(35\sin(c+dx)+3\sin(3}{30\,a\,d}$$

input
```
int((e*cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(e*(e*cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1)
)/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*25i + 35*sin(c + d*x) + cos(
3*c + 3*d*x)*3i + 3*sin(3*c + 3*d*x)))/(30*a*d)
```

Reduce [F]

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\,e\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cos(dx+c)}\,\cos(dx+c)\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)i+\int \frac{\sqrt{\tan(dx+c)i+1}}{\tan(}\right.}{a}$$

input
```
int((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*sqrt(a)*e*( - int((sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x))*co
s(c + d*x)*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*i + int((sqrt(tan(c + d*
x)*i + 1)*sqrt(cos(c + d*x))*cos(c + d*x))/(tan(c + d*x)**2 + 1),x)))/a
```

3.692 $\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 80

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i\sqrt{e\cos(c+dx)}}{3d\sqrt{a+ia\tan(c+dx)}} - \frac{4i\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{3ad}$$

output
```
2/3*I*(e*cos(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^(1/2)-4/3*I*(e*cos(d*x+c))
^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2\sqrt{e\cos(c+dx)}(-i+2\tan(c+dx))}{3d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sqrt[e*Cos[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*Sqrt[e*Cos[c + d*x]]*(-I + 2*Tan[c + d*x]))/(3*d*Sqrt[a + I*a*Tan[c + d
*x]])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3998, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3998}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3983}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3969}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\right)$$

input
```
Int[Sqrt[e*Cos[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
Sqrt[e*Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]*(((2*I)/3)/(d*Sqrt[e*Sec[c + d*x
]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(a
*d*Sqrt[e*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

Maple [A] (verified)

Time = 9.75 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

method	result	size
default	$-\frac{2\sqrt{e\cos(dx+c)}\,(i-2\tan(dx+c))}{3d\sqrt{a(1+i\tan(dx+c))}}$	42
risch	$-\frac{i\sqrt{2}\,\sqrt{e\cos(dx+c)}\,(3\,\mathrm{e}^{2i(dx+c)}-1)}{3(\mathrm{e}^{2i(dx+c)}+1)\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	72

input
```
int((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d*(e*cos(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)*(I-2*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{3\,ad}$$

input
```
integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/3*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*(-3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3/2*I*d*x - 3/2*I*c)/(a*d
)
```

Sympy [F]

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sqrt{e\cos\left(c+dx\right)}}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

input
```
integrate((e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(e*cos(c + d*x))/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{e}\left(i\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\cos\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)+\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\sin\right.}{3\sqrt{a}d}$$

input
```
integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
1/3*sqrt(e)*(I*cos(3/2*d*x + 3/2*c) - 3*I*cos(1/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/(sqrt(a)*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)-3\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}{3\,a\,d}$$

input
```
int((e*cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
((e*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) - 3i)*((a*
(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2
))/(3*a*d)
```

Reduce [F]

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\sqrt{e}\,\sqrt{a}\,i\left(-2\sqrt{\tan\left(dx+c\right)i+1}\,\sqrt{\cos\left(dx+c\right)}-\left(\int\frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{\cos(dx+c)\tan(dx+c)^2+\cos(dx+c)}dx\right)\tan\left(dx+c\right)^2 d\,\cdots}{}$$

input
```
int((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*sqrt(a)*i*( - 2*sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x)) - int
((sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x))*sin(c + d*x))/(cos(c + d*x)*
tan(c + d*x)**2 + cos(c + d*x)),x)*tan(c + d*x)**2*d - int((sqrt(tan(c + d
*x)*i + 1)*sqrt(cos(c + d*x))*sin(c + d*x))/(cos(c + d*x)*tan(c + d*x)**2
+ cos(c + d*x)),x)*d - 4*int((sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x))*
tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*tan(c + d*x)**2*d - 4*int((sqrt(tan
(c + d*x)*i + 1)*sqrt(cos(c + d*x))*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)
*d))/(a*d*(tan(c + d*x)**2 + 1))
```

3.693 $\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

output
```
2*I/d/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[1/(Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(2*I)/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 3998, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}dx$$
$$\Big\downarrow \text{3998}$$
$$\frac{\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$
$$\Big\downarrow \text{3969}$$
$$\frac{2i}{d\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}$$

input
```
Int[1/(Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(2*I)/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

Maple [A] (verified)

Time = 14.65 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

method	result	size
orering	$\frac{2i}{d\sqrt{e\cos(dx+c)}\,\sqrt{a+ia\tan(dx+c)}}$	31
default	$\frac{2i}{d\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\cos(dx+c)}}$	32
risch	$\frac{i\sqrt{2}}{\sqrt{e\cos(dx+c)}\,\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	46

input
```
int(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
2*I/d/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(28) = 56.

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i\,\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{ade}$$

input
```
integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output
```
2*I*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a*d*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\cos{(c+dx)}}\sqrt{ia\left(\tan{(c+dx)}-i\right)}}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*cos(c + d*x))*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i\sqrt{-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}{\sqrt{a}d\sqrt{e}\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

input
```
integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
2*I*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)/(sqrt(a)*d*sqrt(e)*sqrt
(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - 1))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
int(1/((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cos(dx+c)}\,\tan(dx+c)}{\cos(dx+c)\tan(dx+c)^2+\cos(dx+c)}dx\right)i+\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cos(dx+c)}}{\cos(dx+c)\tan(dx+c)^2+\cos(dx+c)}dx\right)}{ae}$$

input
```
int(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x))*tan(
c + d*x))/(cos(c + d*x)*tan(c + d*x)**2 + cos(c + d*x)),x)*i + int((sqrt(t
an(c + d*x)*i + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)*tan(c + d*x)**2 + cos
(c + d*x)),x)))/(a*e)
```

3.694 $\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 350

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i\sqrt{2}\sqrt{a}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)\sec(c+dx)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{i\sqrt{2}\sqrt{a}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)\sec(c+dx)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{i\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}(1+\cos(c+dx)-i\sin(c+dx))}\right)\sec(c+dx)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

output
```
-I*2^(1/2)*a^(1/2)*arctan(1-2^(1/2)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c)
)^(1/2)/a^(1/2)/e^(1/2))*sec(d*x+c)/d/e^(3/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+
I*a*tan(d*x+c))^(1/2)+I*2^(1/2)*a^(1/2)*arctan(1+2^(1/2)*(e*cos(d*x+c))^(1
/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*sec(d*x+c)/d/e^(3/2)/(a-I*a*
tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-I*2^(1/2)*a^(1/2)*arctanh(2^(1/
2)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/e^(1/2)/(1+cos(d*
x+c)-I*sin(d*x+c)))*sec(d*x+c)/d/e^(3/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*t
an(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.60

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{ie^{\frac{1}{2}i(c+dx)}\left(2\arctan\left(1-\sqrt{2}e^{\frac{1}{2}i(c+dx)}\right)-2\arctan\left(1+\sqrt{2}e\right.\right.}{\sqrt{2}de\sqrt{\frac{ae^2}{1+e}}}$$

input
```
Integrate[1/((e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(I*E^((I/2)*(c + d*x))*(2*ArcTan[1 - Sqrt[2]*E^((I/2)*(c + d*x))] - 2*ArcT
an[1 + Sqrt[2]*E^((I/2)*(c + d*x))] + Log[1 - Sqrt[2]*E^((I/2)*(c + d*x))
+ E^(I*(c + d*x))] - Log[1 + Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))
]))/(Sqrt[2]*d*e*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*S
qrt[(e*(1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x))])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3997, 3042, 3996, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3997}$$

$$\frac{\sec(c+dx)\int \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}dx}{e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec(c+dx)\int \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}dx}{e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3996}$$

$$\frac{4ia\sec(c+dx)\int \frac{e\cos(c+dx)(a-ia\tan(c+dx))}{a^2e^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2e^2}d\left(\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{826}$$

$$\frac{4ia\sec(c+dx)\left(\frac{1}{2}\int \frac{ae+\cos(c+dx)(a-ia\tan(c+dx))e}{a^2e^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2e^2}d\left(\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)-\frac{1}{2}\int \frac{ae-e\cos(c+dx}{a^2e^2+\cos^2(c+dx}\right.}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{1476}$$

$$4ia\sec(c+dx)\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{ae+\cos(c+dx)(a-ia\tan(c+dx))e-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\sqrt{e}}d\left(\sqrt{e\cos(c+dx)}\sqrt{a-ia\,t\right.\right.\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{4ia\sec(c+dx)\left(\frac{1}{2}\left(\frac{\int \frac{1}{-e\cos(c+dx)(a-ia\tan(c+dx))-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-e\cos(c+dx)(a-ia\tan(c+dx))-1}d\left(\frac{\sqrt{2}\sqrt{}}{}\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}\right.\right.}{de\sqrt{a-ia\tan(c+dx)}\sqrt{}}$$

$$\Big\downarrow \text{217}$$

$$\frac{4ia\sec(c+dx)\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}\right)-\frac{1}{2}\int \frac{ae-e\cos}{a^2e^2+\cos^2(}\right.}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{1479}$$

$$4ia\sec(c+dx)\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{a}\sqrt{e}-2\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{ae+\cos(c+dx)(a-ia\tan(c+dx))e-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\sqrt{e}}d\left(\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}+\frac{\int -}{}\right.\right.$$

$$\Big\downarrow \text{25}$$

$$4ia\sec(c+dx)\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{a}\sqrt{e}-2\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{ae+\cos(c+dx)(a-ia\tan(c+dx))e-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\sqrt{e}}d\left(\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}} - \frac{\int \frac{}{a}\right.\right.$$

$$\Big\downarrow \; 27$$

$$4ia\sec(c+dx)\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{a}\sqrt{e}-2\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{ae+\cos(c+dx)(a-ia\tan(c+dx))e-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\sqrt{e}}d\left(\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}} - \frac{\int \frac{}{a}\right.\right.$$

$$\Big\downarrow \; 1103$$

$$\frac{4ia\sec(c+dx)\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}\right) + \frac{1}{2}\left(\frac{\log\left(e\cos(c+\right.}{}\right.\right.}{de\sqrt{a-ia\tan}}$$

input
```
Int[1/((e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
((4*I)*a*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a - I*a*Tan[c +
 d*x]])/(Sqrt[a]*Sqrt[e])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[2
]*Sqrt[e*Cos[c + d*x]]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e])]/(Sqr
t[2]*Sqrt[a]*Sqrt[e]))/2 + (Log[a*e - Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[e*Cos[c
 + d*x]]*Sqrt[a - I*a*Tan[c + d*x]] + e*Cos[c + d*x]*(a - I*a*Tan[c + d*x]
)]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]) - Log[a*e + Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[e*
Cos[c + d*x]]*Sqrt[a - I*a*Tan[c + d*x]] + e*Cos[c + d*x]*(a - I*a*Tan[c +
 d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/2)*Sec[c + d*x])/(d*e*Sqrt[a - I*a*Ta
n[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3996
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-4*(b/f)   Subst[Int[x^2/(a^2*d^2 + x^4), x], x,
 Sqrt[d*Cos[e + f*x]]*Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, d, e, f
}, x] && EqQ[a^2 + b^2, 0]
```

rule 3997
```
Int[1/((cos[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)*Sqrt[(a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]]), x_Symbol] :> Simp[1/(d*Cos[e + f*x]*Sqrt[a - b*Tan[e + f*x]]
*Sqrt[a + b*Tan[e + f*x]])   Int[Sqrt[a - b*Tan[e + f*x]]/Sqrt[d*Cos[e + f*
x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Maple [A] (warning: unable to verify)

Time = 14.54 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.66

method	result
default	$\frac{\sec(dx+c)\left(i\,\mathrm{arctanh}\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)-i\,\mathrm{arctanh}\left(\frac{\cot(dx+c)-\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+\mathrm{arctanh}\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+\mathrm{ar}}{4d\sqrt{a(1+i\tan(dx+c))}\,\sqrt{\frac{1}{\cos(dx+c)+}}}$

input
```
int(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/4/d*sec(d*x+c)*(I*arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(
d*x+c)+1))-I*arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1
))+arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+arctan
h(1/2/(1/(cos(d*x+c)+1))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)))*(csc(d*x+c)^2*(
1-cos(d*x+c))^2-1)*(cot(d*x+c)-csc(d*x+c)+I)/(a*(1+I*tan(d*x+c)))^(1/2)*(c
os(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)/e/(e*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.97

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{1}{2}\sqrt{\frac{4i}{ad^2e^3}}\log\left(\frac{1}{2}ade^2\sqrt{\frac{4i}{ad^2e^3}} + \sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right) + \frac{1}{2}\sqrt{\frac{4i}{ad^2e^3}}\log\left(-\frac{1}{2}ade^2\sqrt{\frac{4i}{ad^2e^3}} + \sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right) + \frac{1}{2}\sqrt{-\frac{4i}{ad^2e^3}}\log\left(\frac{1}{2}ade^2\sqrt{-\frac{4i}{ad^2e^3}} + \sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right) - \frac{1}{2}\sqrt{-\frac{4i}{ad^2e^3}}\log\left(-\frac{1}{2}ade^2\sqrt{-\frac{4i}{ad^2e^3}} + \sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right)$$

input
```
integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output
```
-1/2*sqrt(4*I/(a*d^2*e^3))*log(1/2*a*d*e^2*sqrt(4*I/(a*d^2*e^3)) + sqrt(2)
*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*e^(-1/2*I*d*x - 1/2*I*c)) + 1/2*sqrt(4*I/(a*d^2*e^3))*log(-1/2*a*d*e^2*
sqrt(4*I/(a*d^2*e^3)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) + 1/2*sqrt(-4*
I/(a*d^2*e^3))*log(1/2*a*d*e^2*sqrt(-4*I/(a*d^2*e^3)) + sqrt(2)*sqrt(1/2)*
sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*
I*d*x - 1/2*I*c)) - 1/2*sqrt(-4*I/(a*d^2*e^3))*log(-1/2*a*d*e^2*sqrt(-4*I/
(a*d^2*e^3)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c))
```

Sympy [F]

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\cos(c+dx))^{\frac{3}{2}}\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/((e*cos(c + d*x))**(3/2)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(264) = 528$.

Time = 0.25 (sec) , antiderivative size = 714, normalized size of antiderivative = 2.04

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
-1/4*(2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/
2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1
, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2
*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2
(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - 2*
sqrt(2)*arctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqrt(2)*cos(1
/2*d*x + 1/2*c) + cos(d*x + c) + 1) + 2*sqrt(2)*arctan2(-sqrt(2)*sin(1/2*d
*x + 1/2*c) + sin(d*x + c), -sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) +
 1) + I*sqrt(2)*log(2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + 2*(sqrt(
2)*cos(1/2*d*x + 1/2*c) + 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2*d*x
 + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 1) - I*sqrt(2)*log(-2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x +
1/2*c) - 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1)*cos(d*x + c) + cos(d*x + c)^
2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*
sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
int(1/((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{\sqrt{e}\,\sqrt{a}\,\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cos(dx+c)}\,\tan(dx+c)}{\cos(dx+c)^2\tan(dx+c)^2+\cos(dx+c)^2}dx\right)i+\int \frac{\sqrt{}}{\cos(d}\right.}{a\,e^2}$$

input
```
int(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x))*tan(
c + d*x))/(cos(c + d*x)**2*tan(c + d*x)**2 + cos(c + d*x)**2),x)*i + int((
sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**2*tan(c + d*x)
**2 + cos(c + d*x)**2),x)))/(a*e**2)
```

3.695 $\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 360

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{ie^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{ie^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} + \frac{ie^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}(\sqrt{a}+\cos(c+dx)(\sqrt{a}+i\sqrt{a}\tan(c+dx)))}\right)}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}}$$

output
```
1/2*I*e^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e
*sec(d*x+c))^(1/2))*2^(1/2)/a^(1/2)/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^
(5/2)-1/2*I*e^(5/2)*arctan(1+2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1
/2)/(e*sec(d*x+c))^(1/2))*2^(1/2)/a^(1/2)/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*
x+c))^(5/2)+1/2*I*e^(5/2)*arctanh(2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)
/(e*sec(d*x+c))^(1/2)/(a^(1/2)+cos(d*x+c)*(a^(1/2)+I*a^(1/2)*tan(d*x+c))))
*2^(1/2)/a^(1/2)/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)-I*cos(d*x+c)^
2*(a+I*a*tan(d*x+c))^(1/2)/a/d/(e*cos(d*x+c))^(5/2)
```

Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.69

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{ie^{ic-\frac{idx}{2}}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-2e^{\frac{3idx}{2}}+\left(-e^{-2ic}\right)^{3/4}\left(1+e^{2i(c+dx)}\right)\mathrm{a}\right.}{d\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\sqrt{\cos(c+dx)}}$$

input
```
Integrate[1/((e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(I*E^(I*c - (I/2)*d*x)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-2
*E^(((3*I)/2)*d*x) + (-E^((-2*I)*c))^(3/4)*(1 + E^((2*I)*(c + d*x)))*ArcTa
n[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)] - (-E^((-2*I)*c))^(3/4)*(1 + E^((2*
I)*(c + d*x)))*ArcTanh[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)]))/(d*Sqrt[(1 +
 E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*Sqrt[Cos[c + d*x]]*(e*Cos[c + d*x])
^(5/2)*Sec[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.11, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 3998, 3042, 3982, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3982}$$

$$\frac{\frac{e^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{e^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3976}$$

$$\frac{-\frac{2ie^4\int \frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e\left(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2\right)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{826}$$

$$\frac{-\frac{2ie^4\left(\frac{\int \frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{-\frac{2ie^4\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}}{2e}\right)}{d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{-\frac{2ie^4\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos}{a^2+\cos}}{}\right)}{d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$\Big\downarrow$ 217

$$\frac{-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\frac{ie^2\sqrt{\cdots}}{\cdots}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$\Big\downarrow$ 1479

$$\frac{-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right)}\cdots}{2\sqrt{2}\sqrt{a}\sqrt{e}}\cdots\right)}{d}\cdots}{(e\cos(c+dx))^{5/2}\cdots}$$

$\Big\downarrow$ 25

$$\frac{-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right)}d\frac{\sqrt{\cdots}}{\cdots}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\cdots\right)}{d}\cdots}{(e\cos(c+dx))^{5/2}(e\cdots}$$

$\Big\downarrow$ 27

$$\frac{-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(\cdots}}{\sqrt{e\sec\cdots}}}{2\sqrt{2}\sqrt{a}e}\cdots\right)}{d}\cdots}{(e\cos(c+dx))^{5/2}(e\sec(\cdots}$$

$\Big\downarrow$ 1103

$$-\frac{\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\mathrm{lo}}{}}{2e}\right.}{d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/}}$$

input
```
Int[1/((e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(((-2*I)*e^4*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(
Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sq
rt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/
(Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*
Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*T
an[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]
*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*
Tan[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/d - (I*e^2*Sqrt[e*Sec[
c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d))/((e*Cos[c + d*x])^(5/2)*(e*Se
c[c + d*x])^(5/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

Maple [A] (verified)

Time = 14.76 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.83

method	result
default	$\frac{4\sqrt{\frac{1}{\cos(dx+c)+1}}\,(\sin(dx+c)+\tan(dx+c))-4i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}+(\cos(dx+c)-\sin(dx+c)+1)\operatorname{arctanh}\left(\frac{-\cot(dx+c)+\csc(}{2\sqrt{\frac{1}{\cos(dx+c}}}\right.}{}$

input
```
int(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/4/d/(cos(d*x+c)+1)/(e*cos(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)/(1/(c
os(d*x+c)+1))^(1/2)/e^2*(4*(1/(cos(d*x+c)+1))^(1/2)*(sin(d*x+c)+tan(d*x+c)
)-4*I*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)+(cos(d*x+c)-sin(d*x+c)+1)*ar
ctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+(cos(d*x+c)
+sin(d*x+c)+1)*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(
1/2))+I*(-cos(d*x+c)+sin(d*x+c)-1)*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/
(1/(cos(d*x+c)+1))^(1/2))+I*(cos(d*x+c)+sin(d*x+c)+1)*arctanh(1/2/(1/(cos(
d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.39

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{-4i\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)} - (ad$$

input
```
integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output
```
1/2*(-4*I*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) - (a*d*e^3*e^(2*I*d*x + 2*I*c)
 + a*d*e^3)*sqrt(I/(a*d^2*e^5))*log(I*a*d*e^3*sqrt(I/(a*d^2*e^5)) + sqrt(2
)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*e^(-1/2*I*d*x - 1/2*I*c)) + (a*d*e^3*e^(2*I*d*x + 2*I*c) + a*d*e^3)*sq
rt(I/(a*d^2*e^5))*log(-I*a*d*e^3*sqrt(I/(a*d^2*e^5)) + sqrt(2)*sqrt(1/2)*s
qrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I
*d*x - 1/2*I*c)) - (a*d*e^3*e^(2*I*d*x + 2*I*c) + a*d*e^3)*sqrt(-I/(a*d^2*
e^5))*log(I*a*d*e^3*sqrt(-I/(a*d^2*e^5)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I
*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I
*c)) + (a*d*e^3*e^(2*I*d*x + 2*I*c) + a*d*e^3)*sqrt(-I/(a*d^2*e^5))*log(-I
*a*d*e^3*sqrt(-I/(a*d^2*e^5)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*
c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)))/(a*d*
e^3*e^(2*I*d*x + 2*I*c) + a*d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2147 vs. $2(274) = 548$.

Time = 0.42 (sec) , antiderivative size = 2147, normalized size of antiderivative = 5.96

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
-8*(2*(sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + sqrt(2))*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1) + 2*(sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + sqrt(2))*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(sqrt(2)*cos(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*arctan2(sqrt(2)*cos(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, sqrt(2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(sqrt(2)*cos(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin(4
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*arctan2
(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1,
 -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1
) + 2*(-I*sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) - I*sqrt(2))*arctan2(sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{5/2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
int(1/((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cos(dx+c)}\,\tan(dx+c)}{\cos(dx+c)^3\tan(dx+c)^2+\cos(dx+c)^3}dx\right)i+\int \frac{\sqrt{}}{\cos(d}\right.}{a\,e^3}$$

input
```
int(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x))*tan(
c + d*x))/(cos(c + d*x)**3*tan(c + d*x)**2 + cos(c + d*x)**3),x)*i + int((
sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**3*tan(c + d*x)
**2 + cos(c + d*x)**3),x)))/(a*e**3)
```

3.696 $\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 534

$$
\begin{aligned}
\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{3i\sqrt{a}e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{4\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{3i\sqrt{a}e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{4\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{3i\sqrt{a}e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}\left(\sqrt{a}+\cos(c+dx)\left(\sqrt{a}-i\sqrt{a}\tan(c+dx)\right)\right)}\right)\sec(c+dx)}{4\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad(e\cos(c+dx))^{7/2}}
\end{aligned}
$$

output
```
3/4*I*cos(d*x+c)^2/d/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2)-3/8*I*a
^(1/2)*e^(7/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(
e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+
c))^(7/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+3/8*I*a^(1/2)*
e^(7/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d
*x+c))^(1/2))*sec(d*x+c)*2^(1/2)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/
2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-3/8*I*a^(1/2)*e^(7/2)
*arctanh(2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)/(a^
(1/2)+cos(d*x+c)*(a^(1/2)-I*a^(1/2)*tan(d*x+c))))*sec(d*x+c)*2^(1/2)/d/(e*
cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan
(d*x+c))^(1/2)-1/2*I*cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/a/d/(e*cos(d*x+
c))^(7/2)
```

Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.46

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}ie^{\frac{1}{2}i(c+dx)}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{5/2}\left(2\,\mathrm{ar}\right.\right.}{}$$

input
```
Integrate[1/((e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(((3*I)/4)*E^((I/2)*(c + d*x))*((1 + E^((2*I)*(c + d*x
)))/E^(I*(c + d*x)))^(5/2)*(2*ArcTan[1 - Sqrt[2]*E^((I/2)*(c + d*x))] - 2*
ArcTan[1 + Sqrt[2]*E^((I/2)*(c + d*x))] + Log[1 - Sqrt[2]*E^((I/2)*(c + d*
x)) + E^(I*(c + d*x))] - Log[1 + Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d
*x))]) + 4*Sqrt[Cos[c + d*x]]*(I*Cos[c + d*x] + 2*Sin[c + d*x])))/(16*d*(e
*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 493, normalized size of antiderivative = 0.92, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3998, 3042, 3982, 3042, 3979, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int \frac{(e\sec(c+dx))^{7/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(e\sec(c+dx))^{7/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3982}$$

$$\frac{\frac{3e^2\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3e^2\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3979}$$

$$\frac{\frac{3e^2\left(\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3e^2\left(\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \; 3980$$

$$\frac{\frac{3e^2\left(\frac{ae\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3e^2\left(\frac{ae\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \; 3976$$

$$\frac{\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\int\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e\left(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2\right)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \; 826$$

$$\frac{\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\int\frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx}{d\sqrt{a+ia\tan(}}\right.}{4a}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\ta}{e}}}{2e}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{4a}}{(e\cos(c+dx))^{7/}}$$

$\Big\downarrow$ 1082

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int -\frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int -\frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+\cdots\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\cdots\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\cdots\right)}{4a}\cdots}{(e\cos(c+dx))^{7/2}(e\sec(c\cdots}$$

$\Big\downarrow$ 217

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+d\cdots}}{\sqrt{e\sec(c+dx)}}\cdots}{2e}\cdots\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\cdots\right)}{4a}\cdots}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$\Big\downarrow$ 1479

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-\cdots}{\cdots}\right)}\cdots}{2\sqrt{2}\sqrt{a}\sqrt{e}}\cdots\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(\cdots}}\cdots\right)}{4a}\cdots}{(e\,\cdots}$$

$\Big\downarrow$ 25

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\,\mathrm{t}}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+}}\right.}{4a}}{(e\cos}$$

$$\Big\downarrow \; 27$$

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c}{e}}}{2\sqrt{2}\sqrt{a}e}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{4a}}{(e\cos(c+}$$

$$\Big\downarrow \; 1103$$

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{4a}}{(e\cos(c+dx))^{7/2}(}$$

input
```
Int[1/((e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(((-1/2*I)*e^2*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d) +
(3*e^2*((I*a*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + ((2*
I)*a^2*e^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sq
rt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt
[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(S
qrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sq
rt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan
[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*S
qrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Ta
n[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[a -
 I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])))/(4*a))/((e*Cos[c + d*x])^
(7/2)*(e*Sec[c + d*x])^(7/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

Maple [A] (verified)

Time = 15.12 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.60

method	result
default	$\frac{i(3\sin(dx+c)-3\cos(dx+c)-3)\,\mathrm{arctanh}\left(\frac{-\cot(dx+c)+\csc(dx+c)+1}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+(3\sin(dx+c)+3\cos(dx+c)+3)\,\mathrm{arctanh}\left(\frac{-\cot(dx+c)+\csc(dx-}{2\sqrt{\frac{1}{\cos(dx+c)+1}}}\right.}{}$

input
```
int(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/16/d/(cos(d*x+c)+1)/(e*cos(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)/(1/(
cos(d*x+c)+1))^(1/2)/e^3*(I*(3*sin(d*x+c)-3*cos(d*x+c)-3)*arctanh(1/2/(1/(
cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+(3*sin(d*x+c)+3*cos(d*x+c
)+3)*arctanh(1/2/(1/(cos(d*x+c)+1))^(1/2)*(-cot(d*x+c)+csc(d*x+c)+1))+I*(3
*sin(d*x+c)+3*cos(d*x+c)+3)*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)/(1/(cos
(d*x+c)+1))^(1/2))+(-3*sin(d*x+c)+3*cos(d*x+c)+3)*arctanh(1/2*(-cot(d*x+c)
+csc(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2))+(1/(cos(d*x+c)+1))^(1/2)*(8*tan(d
*x+c)+8*sec(d*x+c)*tan(d*x+c))+I*(1/(cos(d*x+c)+1))^(1/2)*(4+4*sec(d*x+c))
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.13

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output
```
1/2*(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*(-I*e^(3*I*d*x + 3*I*c) + 3*I*e^(I*d*x + I*c))*e^(-1/2*I*d*
x - 1/2*I*c) - (a*d*e^4*e^(4*I*d*x + 4*I*c) + 2*a*d*e^4*e^(2*I*d*x + 2*I*c
) + a*d*e^4)*sqrt(9/16*I/(a*d^2*e^7))*log(4/3*a*d*e^4*sqrt(9/16*I/(a*d^2*e
^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) + (a*d*e^4*e^(4*I*d*x + 4*I*c)
+ 2*a*d*e^4*e^(2*I*d*x + 2*I*c) + a*d*e^4)*sqrt(9/16*I/(a*d^2*e^7))*log(-4
/3*a*d*e^4*sqrt(9/16*I/(a*d^2*e^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x
+ 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c))
+ (a*d*e^4*e^(4*I*d*x + 4*I*c) + 2*a*d*e^4*e^(2*I*d*x + 2*I*c) + a*d*e^4)*
sqrt(-9/16*I/(a*d^2*e^7))*log(4/3*a*d*e^4*sqrt(-9/16*I/(a*d^2*e^7)) + sqrt
(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*e^(-1/2*I*d*x - 1/2*I*c)) - (a*d*e^4*e^(4*I*d*x + 4*I*c) + 2*a*d*e^4
*e^(2*I*d*x + 2*I*c) + a*d*e^4)*sqrt(-9/16*I/(a*d^2*e^7))*log(-4/3*a*d*e^4
*sqrt(-9/16*I/(a*d^2*e^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c)
+ e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)))/(a*d*e^4
*e^(4*I*d*x + 4*I*c) + 2*a*d*e^4*e^(2*I*d*x + 2*I*c) + a*d*e^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cos(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2254 vs. $2(406) = 812$.

Time = 0.34 (sec) , antiderivative size = 2254, normalized size of antiderivative = 4.22

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
-32*(6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*
sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2)*cos(4*d*x
+ 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqr
t(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*
d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + s
qrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
+ 6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin
(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 6*(-I*sqrt(2)*cos(4*d*x
 + 4*c) - 2*I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt
(2)*sin(2*d*x + 2*c) - I*sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 6*(I*sqrt(2...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\,\cos\left(c+d\,x\right))^{7/2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
int(1/((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{e}\,\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cos(dx+c)}\,\tan(dx+c)}{\cos(dx+c)^4\tan(dx+c)^2+\cos(dx+c)^4}dx\right)i+\int \frac{\sqrt{}}{\cos(d}\right.}{a\,e^4}$$

input
```
int(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(e)*sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x))*tan(
c + d*x))/(cos(c + d*x)**4*tan(c + d*x)**2 + cos(c + d*x)**4),x)*i + int((
sqrt(tan(c + d*x)*i + 1)*sqrt(cos(c + d*x)))/(cos(c + d*x)**4*tan(c + d*x)
**2 + cos(c + d*x)**4),x)))/(a*e**4)
```

3.697 $\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 26, antiderivative size = 109

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-\frac{m}{2}+n}a(e\cos(c+dx))^m \operatorname{Hypergeometric2F1}\left(-\frac{m}{2},\frac{1}{2}(2+m-2n),1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+}{dm}$$

output
```
-I*2^(-1/2*m+n)*a*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 1+1/2*m-n],[1-1/2*m]
,1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1+1/2*m-n)*(a+I*a*tan(d*x+c))^(-1
+n)/d/m
```

Mathematica [A] (verified)

Time = 13.50 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.85

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \frac{i2^{-m+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^{-m+n}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\cos^{-m}(c+dx)(e\cos(c+dx)}{}$$

input
```
Integrate[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*2^(-m + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^n*
(1 + E^((2*I)*(c + d*x)))^(-m + n)*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*
x)))^m*(e*Cos[c + d*x])^m*Hypergeometric2F1[-m + n, -1/2*m + n, 1 - m/2 +
n, -E^((2*I)*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(m - 2*n)*Cos[c + d*
x]^m*Sec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3998, 3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\cos(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\cos(c+dx))^mdx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int (e\sec(c+dx))^{-m}(i\tan(c+dx)a+a)^ndx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int (e\sec(c+dx))^{-m}(i\tan(c+dx)a+a)^ndx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{n-\frac{m}{2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{n-\frac{m}{2}}\,dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int(a-ia\tan(c+dx))^{-\frac{m}{2}-1}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a2^{-\frac{m}{2}+n-1}(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^n(e\cos(c+dx))^m(1+i\tan(c+dx))^{\frac{1}{2}(m-2n)}\int\left(\frac{1}{2}i\tan(c+\right.}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{i2^{n-\frac{m}{2}}(a+ia\tan(c+dx))^n(e\cos(c+dx))^m(1+i\tan(c+dx))^{\frac{1}{2}(m-2n)}\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{1}{2}(m-2n+\right.}{dm}$$

input
```
Int[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(-1/2*m + n)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (2 + m -
 2*n)/2, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^((m - 2*n)/
2)*(a + I*a*Tan[c + d*x])^n)/(d*m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\cos (dx+c))^m (a+ia\tan (dx+c))^n\,dx$$

input
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx=\int (e\cos (dx+c))^m (i\,a\tan (dx+c)+a)^n\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(I*d*m*x
+ I*c*m + m*log(a*e) - m*log(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c)
+ 1))), x)
```

Sympy [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int (e\cos(c+dx))^m\,(ia(\tan(c+dx)-i))^n\,dx$$

input
```
integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*cos(c + d*x))**m*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int (e\cos(dx+c))^m\,(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((e*cos(d*x + c))^m*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int (e\cos(dx+c))^m\,(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*cos(d*x + c))^m*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int (e\,\cos\left(c+d\,x\right))^m\,\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)^n\,dx$$

input
```
int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx = e^m\left(\int \left(\tan\left(dx+c\right)ai+a\right)^n\cos\left(dx+c\right)^m dx\right)$$

input
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

output
```
e**m*int((tan(c + d*x)*a*i + a)**n*cos(c + d*x)**m,x)
```

3.698 $\int (e\cos(c+dx))^m (a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int (e\cos(c+dx))^m (a+ia\tan(c+dx))^2\,dx = -\frac{i 2^{2-\frac{m}{2}} a^2 (e\cos(c+dx))^m \,\text{Hypergeometric2F1}\left(\frac{1}{2}(-2+m),-\frac{m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\,\text{ta}}{dm}$$

output
```
-I*2^(2-1/2*m)*a^2*(e*cos(d*x+c))^m*hypergeom([-1/2*m, -1+1/2*m],[1-1/2*m]
,1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m)/d/m
```

Mathematica [A] (verified)

Time = 8.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int (e\cos(c+dx))^m (a+ia\tan(c+dx))^2\,dx = -\frac{i 2^{2-\frac{m}{2}} a^2 (e\cos(c+dx))^m \,\text{Hypergeometric2F1}\left(\frac{1}{2}(-2+m),-\frac{m}{2},1-\frac{m}{2},-\frac{1}{2}i(i+\tan(c+dx))\right)(1+i\,\text{t}}{dm}$$

input
```
Integrate[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*2^(2 - m/2)*a^2*(e*Cos[c + d*x])^m*Hypergeometric2F1[(-2 + m)/2, -1/
2*m, 1 - m/2, (-1/2*I)*(I + Tan[c + d*x])]*(1 + I*Tan[c + d*x])^(m/2))/(d*
m)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3998, 3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^2(e\cos(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^2(e\cos(c+dx))^m dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int (e\sec(c+dx))^{-m}(i\tan(c+dx)a+a)^2dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int (e\sec(c+dx))^{-m}(i\tan(c+dx)a+a)^2dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{2-\frac{m}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{2-\frac{m}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int(a-ia\tan(c+dx))^{-\frac{m}{2}-1}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^3 2^{1-\frac{m}{2}}(1+i\tan(c+dx))^{m/2}(a-ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{1-\frac{m}{2}}(a-ia\tan(c}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{ia^2 2^{2-\frac{m}{2}}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{m-2}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

input
```
Int[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*2^(2 - m/2)*a^2*(e*Cos[c + d*x])^m*Hypergeometric2F1[(-2 + m)/2, -1/
2*m, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(d*m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\cos (dx+c))^{m} (a+ia\tan (dx+c))^{2} dx$$

input
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

output
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

Fricas [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \int (i\,a\tan (dx+c)+a)^{2}(e\cos (dx+c))^{m}\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(4*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*a^2*e^(4*I
*d*x + 4*I*c)/(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx$$
$$= -a^2\left(\int \left(-(e\cos{(c+dx)})^m\right)\,dx + \int (e\cos{(c+dx)})^m\tan^2{(c+dx)}\,dx + \int \left(-2i(e\cos{(c+dx)})^m\tan{(c+dx)}\right)\,dx\right)$$

input
```
integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(-(e*cos(c + d*x))**m, x) + Integral((e*cos(c + d*x))**m*ta
n(c + d*x)**2, x) + Integral(-2*I*(e*cos(c + d*x))**m*tan(c + d*x), x))
```

Maxima [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \int (i\,a\tan{(dx+c)}+a)^2(e\cos{(dx+c)})^m\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2*(e*cos(d*x + c))^m, x)
```

Giac [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \int (i\,a\tan(dx+c)+a)^2(e\cos(dx+c))^m\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2*(e*cos(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \int (e\,\cos(c+d\,x))^m\,(a+a\tan(c+d\,x)\,1\mathrm{i})^2\,dx$$

input
```
int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx = e^m a^2\left(\int \cos(dx+c)^m\,dx - \left(\int \cos(dx+c)^m\tan(dx+c)^2\,dx\right) + 2\left(\int \cos(dx+c)^m\tan(dx+c)\,dx\right)i\right)$$

input
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

output
```
e**m*a**2*(int(cos(c + d*x)**m,x) - int(cos(c + d*x)**m*tan(c + d*x)**2,x)
 + 2*int(cos(c + d*x)**m*tan(c + d*x),x)*i)
```

3.699 $\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = -\frac{i2^{1-\frac{m}{2}}a(e\cos(c+dx))^m \,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx)}{dm}$$

output
```
-I*2^(1-1/2*m)*a*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 1/2*m],[1-1/2*m],1/2-
1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m)/d/m
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = -\frac{a(e\cos(c+dx))^m\left(i(1+m)+m\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)\sqrt{\sin^2(}\right.}{dm(1+m)}$$

input
```
Integrate[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

output
```
-((a*(e*Cos[c + d*x])^m*(I*(1 + m) + m*Cot[c + d*x]*Hypergeometric2F1[1/2,
 (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))/(d*m*(1 + m)
))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3998, 3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))(e\cos(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))(e\cos(c+dx))^m dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int (e\sec(c+dx))^{-m}(i\tan(c+dx)a+a)dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int (e\sec(c+dx))^{-m}(i\tan(c+dx)a+a)dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{1-\frac{m}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{1-\frac{m}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int(a-ia\tan(c+dx))^{-\frac{m}{2}-1}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \textit{80}$$

$$\frac{a^2 2^{-m/2}(1+i\tan(c+dx))^{m/2}(a-ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{-m/2}(a-ia\tan(c}{d}$$

$$\Big\downarrow \textit{79}$$

$$-\frac{ia2^{1-\frac{m}{2}}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m\,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

input
```
Int[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*2^(1 - m/2)*a*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, m/2, 1 -
m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(d*m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\cos (dx+c))^{m} (a+ia\tan (dx+c))\, dx$$

input
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

output
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

Fricas [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = \int (i\,a\tan (dx+c)+a)(e\cos (dx+c))^{m}\, dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
integral(2*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*a*e^(2*I*d
*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = ia\left(\int\left(-i(e\cos{(c+dx)})^m\right)\,dx + \int (e\cos{(c+dx)})^m\tan{(c+dx)}\,dx\right)$$

input
```
integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*(e*cos(c + d*x))**m, x) + Integral((e*cos(c + d*x))**m*ta
n(c + d*x), x))
```

Maxima [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = \int (i\,a\tan{(dx+c)}+a)(e\cos{(dx+c)})^m\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)*(e*cos(d*x + c))^m, x)
```

Giac [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = \int (i\,a\tan(dx+c)+a)(e\cos(dx+c))^m\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)*(e*cos(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = \int (e\,\cos(c+d\,x))^m\,(a+a\tan(c+d\,x)\;1\mathrm{i})\,dx$$

input
```
int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = e^m a\left(\int \cos(dx+c)^m\,dx + \left(\int \cos(dx+c)^m\tan(dx+c)\,dx\right)i\right)$$

input
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

output
```
e**m*a*(int(cos(c + d*x)**m,x) + int(cos(c + d*x)**m*tan(c + d*x),x)*i)
```

3.700 $\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = -\frac{i2^{-1-\frac{m}{2}}(e\cos(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{4+m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+d$$
$$adm$$

output
```
-I*2^(-1-1/2*m)*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 2+1/2*m],[1-1/2*m],1/2
-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m)/a/d/m
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 433 vs. 2(86) = 172.

Time = 4.82 (sec) , antiderivative size = 433, normalized size of antiderivative = 5.03

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = -\frac{2^{-m/2}\cos(c+dx)(e\cos(c+dx))^m\left(1-2\cos^2(c+dx)+i\sin(2(c+dx))\right)^{m/2}\left(2^{m/2}(2+m)\,\text{Hypergeor}}{ad(1+m)(2+m)\left(-i\sin(c+dx)\left(\left(1-2\cos^2(c+dx)+i\sin(2(c+dx))\right)^{m/2}((\cos(c)+i\,\text{si}}$$

input
```
Integrate[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

output
```
-((Cos[c + d*x]*(e*Cos[c + d*x])^m*(1 - 2*Cos[c + d*x]^2 + I*Sin[2*(c + d*
x)])^(m/2)*(2^(m/2)*(2 + m)*Hypergeometric2F1[1 + m, (2 + m)/2, 2 + m, 2*C
os[c + d*x]*(Cos[c + d*x] - I*Sin[c + d*x])]*((Cos[c] - I*Sin[c])*Sin[c]*(
I + Tan[d*x]))^(m/2) - 2*(1 + m)*Hypergeometric2F1[-1 - m/2, m/2, -1/2*m,
(1 + I*Tan[c + d*x])/2]*((Cos[c] + I*Sin[c])*Sin[c]*(-I + Tan[d*x]))^(m/2)
*(1 - I*Tan[c + d*x])^(m/2)))/(2^(m/2)*a*d*(1 + m)*(2 + m)*((-I)*Sin[c + d
*x]*((1 - 2*Cos[c + d*x]^2 + I*Sin[2*(c + d*x)])^(m/2)*((Cos[c] + I*Sin[c]
)*Sin[c]*(-I + Tan[d*x]))^(m/2) - ((Cos[c] - I*Sin[c])*Sin[c]*(I + Tan[d*x
]))^(m/2)) + Cos[c + d*x]*((1 - 2*Cos[c + d*x]^2 + I*Sin[2*(c + d*x)])^(m/
2)*((Cos[c] + I*Sin[c])*Sin[c]*(-I + Tan[d*x]))^(m/2) + ((Cos[c] - I*Sin[c
])*Sin[c]*(I + Tan[d*x]))^(m/2))*(-I + Tan[c + d*x])))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3998, 3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \text{3998}$$
$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int \frac{(e\sec(c+dx))^{-m}}{i\tan(c+dx)a+a}dx$$
$$\Big\downarrow \text{3042}$$
$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int \frac{(e\sec(c+dx))^{-m}}{i\tan(c+dx)a+a}dx$$
$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int(a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{-\frac{m}{2}-1}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int(a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{-\frac{m}{2}-1}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int(a-ia\tan(c+dx))^{-\frac{m}{2}-1}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{-\frac{m}{2}-2}(1+i\tan(c+dx))^{m/2}(a-ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{-\frac{m}{2}-2}(a-ia\tan(c}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{i2^{-\frac{m}{2}-1}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m\,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m+4}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{adm}$$

input
```
Int[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*2^(-1 - m/2)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (4 + m)/2,
 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(a*d*m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(e\cos(dx+c))^m}{a+ia\tan(dx+c)}dx$$

input
```
int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

output
```
int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

Fricas [F]

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\cos(dx+c))^m}{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
integral(1/2*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*(e^(2*I*
d*x + 2*I*c) + 1)*e^(-2*I*d*x - 2*I*c)/a, x)
```

Sympy [F]

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{(e\cos(c+dx))^m}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate((e*cos(d*x+c))**m/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral((e*cos(c + d*x))**m/(tan(c + d*x) - I), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\cos(dx+c))^m}{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*cos(d*x + c))^m/(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\,\cos(c+d\,x))^m}{a+a\tan(c+d\,x)\,1\mathrm{i}}\,dx$$

input
```
int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \frac{e^m\left(\int \frac{\cos(dx+c)^m}{\tan(dx+c)i+1}dx\right)}{a}$$

input
```
int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

output
```
(e**m*int(cos(c + d*x)**m/(tan(c + d*x)*i + 1),x))/a
```

3.701 $\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = -\frac{i2^{-2-\frac{m}{2}}(e\cos(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{6+m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+d}{a^2dm}$$

output
```
-I*2^(-2-1/2*m)*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 3+1/2*m],[1-1/2*m],1/2
-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m)/a^2/d/m
```

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.16

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = -\frac{i2^{-m/2}(e\cos(c+dx))^m \text{Hypergeometric2F1}\left(-2-\frac{m}{2},\frac{2+m}{2},-1-\frac{m}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(1-i\tan(}{a^2d(4+m)(-i+\tan(c+dx))^2}$$

input
```
Integrate[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-2 - m/2, (2 + m)/2, -1 - m/2,
(1 + I*Tan[c + d*x])/2]*(1 - I*Tan[c + d*x])^(m/2))/(2^(m/2)*a^2*d*(4 + m)
*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3998, 3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3998}$$
$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int \frac{(e\sec(c+dx))^{-m}}{(i\tan(c+dx)a+a)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int \frac{(e\sec(c+dx))^{-m}}{(i\tan(c+dx)a+a)^2}dx$$
$$\Big\downarrow \text{3986}$$
$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{-\frac{m}{2}-2}dx$$
$$\Big\downarrow \text{3042}$$
$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{-\frac{m}{2}-2}dx$$
$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int(a-ia\tan(c+dx))^{-\frac{m}{2}-1}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{ 80}$$

$$\frac{2^{-\frac{m}{2}-3}(1+i\tan(c+dx))^{m/2}(a-ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{-\frac{m}{2}-3}(a-ia\tan(c}{ad}$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{i2^{-\frac{m}{2}-2}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m\,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m+6}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{a^2dm}$$

input
```
Int[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*2^(-2 - m/2)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (6 + m)/2,
 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(a^2*d*m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(e\cos(dx+c))^m}{(a+ia\tan(dx+c))^2}dx$$

input
```
int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

output
```
int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

Fricas [F]

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\cos(dx+c))^m}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(1/4*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*(e^(4*I*
d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*e^(-4*I*d*x - 4*I*c)/a^2, x)
```

Sympy [F]

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{(e\cos(c+dx))^m}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate((e*cos(d*x+c))**m/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral((e*cos(c + d*x))**m/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)
/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\cos(dx+c))^m}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*cos(d*x + c))^m/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\cos(c+d\,x))^m}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}\,dx$$

input
```
int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = -\frac{e^m\left(\int \frac{\cos(dx+c)^m}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)}{a^2}$$

input
```
int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - e**m*int(cos(c + d*x)**m/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x))/
a**2
```

3.702 $\int (e\cos(c+dx))^m \sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 105

$$\int (e\cos(c+dx))^m \sqrt{a+ia\tan(c+dx)}\,dx = -\frac{i2^{\frac{1}{2}-\frac{m}{2}}a(e\cos(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{1+m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+d$$
$$dm\sqrt{a+ia\tan(c+dx)}$$

output
```
-I*2^(1/2-1/2*m)*a*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 1/2+1/2*m],[1-1/2*m
],1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2+1/2*m)/d/m/(a+I*a*tan(d*x+c)
)^(1/2)
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.16

$$\int (e\cos(c+dx))^m \sqrt{a+ia\tan(c+dx)}\,dx = \frac{i2^{-m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}-m}\left(ee^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m \text{Hypergeometric2F1}\left(\frac{1}{2}-m,\frac{1-m}{2},\frac{3-m}{2},-e^{2i(c+dx)}\right)\sqrt{}}{d(-1+m)}$$

input
```
Integrate[(e*Cos[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(I*(1 + E^((2*I)*(c + d*x)))^(1/2 - m)*((e*(1 + E^((2*I)*(c + d*x))))/E^(I
*(c + d*x)))^m*Hypergeometric2F1[1/2 - m, (1 - m)/2, (3 - m)/2, -E^((2*I)*
(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])/(2^m*d*(-1 + m))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3998, 3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^m dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int (e\sec(c+dx))^{-m}\sqrt{i\tan(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int (e\sec(c+dx))^{-m}\sqrt{i\tan(c+dx)a+a}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{\frac{1-m}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{\frac{1-m}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int(a-ia\tan(c+dx))^{-\frac{m}{2}-1}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2 2^{-\frac{m}{2}-\frac{1}{2}}(1+i\tan(c+dx))^{\frac{m+1}{2}}(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{\frac{1}{2}(-m-1)+\frac{m}{2}}(e\cos(c+dx))^m\int\left(\frac{1}{2}i\,\mathrm{ta}\right.}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{ia2^{\frac{1}{2}-\frac{m}{2}}(1+i\tan(c+dx))^{\frac{m+1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(-m-1)+\frac{m}{2}}(e\cos(c+dx))^m\,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m+1}{2}\right.}{dm}$$

input
```
Int[(e*Cos[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*2^(1/2 - m/2)*a*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (1 + m)
/2, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^((1 + m)/2)*(a +
 I*a*Tan[c + d*x])^((-1 - m)/2 + m/2))/(d*m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (e\cos (dx+c))^m \sqrt{a+ia\tan (dx+c)}dx$$

input
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int (e\cos(c+dx))^m \sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan (dx+c)+a}(e\cos (dx+c))^m\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(2)*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c), x)
```

Sympy [F]

$$\int (e\cos(c+dx))^m \sqrt{a+ia\tan(c+dx)}\,dx = \int (e\cos{(c+dx)})^m \sqrt{ia(\tan{(c+dx)}-i)}\,dx$$

input
```
integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((e*cos(c + d*x))**m*sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [F]

$$\int (e\cos(c+dx))^m \sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan{(dx+c)}+a}(e\cos{(dx+c)})^m\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*(e*cos(d*x + c))^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (e\cos(c+dx))^m \sqrt{a+ia\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \int (e\,\cos\left(c+d\,x\right))^m\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int (e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \frac{e^m\sqrt{a}\,i\left(-2\sqrt{\tan\left(dx+c\right)i+1}\,\cos\left(dx+c\right)^m-2\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\cos(dx+c)^m\sin(dx+c)}{\cos(dx+c)}dx\right)dm+\left(\int \sqrt{\tan}\right.\right.}{d}$$

input
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(e**m*sqrt(a)*i*( - 2*sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**m - 2*int((sq
rt(tan(c + d*x)*i + 1)*cos(c + d*x)**m*sin(c + d*x))/cos(c + d*x),x)*d*m +
 int(sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**m*tan(c + d*x),x)*d))/d
```

3.703 $\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 105

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i2^{-\frac{1}{2}-\frac{m}{2}}a(e\cos(c+dx))^m \,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{3+m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+}{dm(a+ia\tan(c+dx))^{3/2}}$$

output
```
-I*2^(-1/2-1/2*m)*a*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 3/2+1/2*m],[1-1/2*
m],1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(3/2+1/2*m)/d/m/(a+I*a*tan(d*x+c
))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.67 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.36

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i4^{-m}\left(1+e^{2i(c+dx)}\right)\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\left(ee^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\cos^{-m}(c+dx)\,\text{Hypergeometr}}{d(1+m)\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(e*Cos[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(I*(1 + E^((2*I)*(c + d*x)))*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^m
*((e*(1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x)))^m*Hypergeometric2F1[1, (2
 + m)/2, (1 - m)/2, -E^((2*I)*(c + d*x))])/(4^m*d*(1 + m)*Cos[c + d*x]^m*S
qrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3998, 3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int \frac{(e\sec(c+dx))^{-m}}{\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int \frac{(e\sec(c+dx))^{-m}}{\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{\frac{1}{2}(-m-1)}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{\frac{1}{2}(-m-1)}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int(a-ia\tan(c+dx))^{-\frac{m}{2}-1}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow 80$$

$$\frac{a2^{-\frac{m}{2}-\frac{3}{2}}(1+i\tan(c+dx))^{\frac{m+1}{2}}(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{\frac{1}{2}(-m-1)+\frac{m}{2}}(e\cos(c+dx))^m\int\left(\frac{1}{2}i\tan\right.}{d}$$

$$\Big\downarrow 79$$

$$-\frac{i2^{-\frac{m}{2}-\frac{1}{2}}(1+i\tan(c+dx))^{\frac{m+1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(-m-1)+\frac{m}{2}}(e\cos(c+dx))^m\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m+3}{2}\right.}{dm}$$

input
```
Int[(e*Cos[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*2^(-1/2 - m/2)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (3 + m)/
2, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^((1 + m)/2)*(a +
I*a*Tan[c + d*x])^((-1 - m)/2 + m/2))/(d*m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(e\cos(dx+c))^m}{\sqrt{a+ia\tan(dx+c)}}dx$$

input
```
int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\cos(dx+c))^m}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(1/2*sqrt(2)*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(-I*d*x - I*
c)/a, x)
```

Sympy [F]

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\cos(c+dx))^m}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate((e*cos(d*x+c))**m/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((e*cos(c + d*x))**m/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [F]

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\cos(dx+c))^m}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*cos(d*x + c))^m/sqrt(I*a*tan(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\,\cos(c+d\,x))^m}{\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}\,dx$$

input
```
int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{2e^m\sqrt{a}\,i\left(-\sqrt{\tan(dx+c)\,i+1}\,\cos(dx+c)^m - \left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\cos(dx+c)^m\sin(dx+c)}{\cos(dx+c)\tan(dx+c)^2+\cos(dx+c)}dx\right)\tan(dx+c)^2\,dm}{}$$

input
```
int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*e**m*sqrt(a)*i*( - sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**m - int((sqrt
(tan(c + d*x)*i + 1)*cos(c + d*x)**m*sin(c + d*x))/(cos(c + d*x)*tan(c + d
*x)**2 + cos(c + d*x)),x)*tan(c + d*x)**2*d*m - int((sqrt(tan(c + d*x)*i +
 1)*cos(c + d*x)**m*sin(c + d*x))/(cos(c + d*x)*tan(c + d*x)**2 + cos(c +
d*x)),x)*d*m - 2*int((sqrt(tan(c + d*x)*i + 1)*cos(c + d*x)**m*tan(c + d*x
))/(tan(c + d*x)**2 + 1),x)*tan(c + d*x)**2*d - 2*int((sqrt(tan(c + d*x)*i
 + 1)*cos(c + d*x)**m*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*d))/(a*d*(tan
(c + d*x)**2 + 1))
```

3.704 $\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 178

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx = -\frac{b(3a^2-b^2)(d\cos(e+fx))^m}{fm} + \frac{b^3(d\cos(e+fx))^m\sec^2(e+fx)}{f(2-m)} + \frac{3ab^2(d\cos(e+fx))^m\tan(e+fx)}{f(1-m)} - \frac{a(3b^2-a^2(1-m))(d\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{3}{2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}}{f(1-m)}$$

output

```
-b*(3*a^2-b^2)*(d*cos(f*x+e))^m/f/m+b^3*(d*cos(f*x+e))^m*sec(f*x+e)^2/f/(2
-m)+3*a*b^2*(d*cos(f*x+e))^m*tan(f*x+e)/f/(1-m)-a*(3*b^2-a^2*(1-m))*(d*cos
(f*x+e))^m*hypergeom([1/2, 1+1/2*m],[3/2],-tan(f*x+e)^2)*(sec(f*x+e)^2)^(1
/2*m)*tan(f*x+e)/f/(1-m)
```

Mathematica [A] (verified)

Time = 4.49 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.19

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx$$
$$=\frac{\cos(e+fx)(d\cos(e+fx))^m\left(-\frac{b^3}{-2+m}+\frac{b(-3a^2+b^2)\cos^2(e+fx)}{m}-\frac{a(a^2-3b^2)\cos^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2}\right.}{(1+m)\sqrt{\sin^2(e+fx)}}\right.}{f(a\cos(e+fx)+b\sin}$$

input
```
Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

output
```
(Cos[e + f*x]*(d*Cos[e + f*x])^m*(-(b^3/(-2 + m)) + (b*(-3*a^2 + b^2)*Cos[
e + f*x]^2)/m - (a*(a^2 - 3*b^2)*Cos[e + f*x]^3*Hypergeometric2F1[1/2, (1
+ m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/((1 + m)*Sqrt[Sin[e + f*x
]^2]) - (3*a*b^2*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Cos[e + f*x
]^2]*Sin[2*(e + f*x)])/(2*(-1 + m)*Sqrt[Sin[e + f*x]^2]))*(a + b*Tan[e + f
*x])^3)/(f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3998, 3042, 3994, 497, 25, 27, 676, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^3(d\cos(e+fx))^m\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int (a+b\tan(e+fx))^3(d\cos(e+fx))^m dx$$
$$\Big\downarrow \text{ 3998}$$
$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx))^3dx$$

$$\Big\downarrow \; 3042$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx))^3dx$$

$$\Big\downarrow \; 3994$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\int (a+b\tan(e+fx))^3\left(\tan^2(e+fx)+1\right)^{-\frac{m}{2}-1}d(b\tan(e+fx))}{bf}$$

$$\Big\downarrow \; 497$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\Bigg(\frac{b^2\int -\frac{(a+b\tan(e+fx))\left(b^2\left(2-\frac{a^2(2-m)}{b^2}\right)-ab(4-m)\tan(e+fx)\right)\left(\tan^2(e+fx)+1\right)^{-\frac{m}{2}-1}}{b^2}d(b\tan(e+fx)}{2-m}}{bf}$$

$$\Big\downarrow \; 25$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\Bigg(\frac{b^2\left(\tan^2(e+fx)+1\right)^{-m/2}(a+b\tan(e+fx))^2}{2-m}-\frac{b^2\int\frac{(a+b\tan(e+fx))\left(-\left((2-m)a^2\right)-b(4-m)\tan(e+fx}{b^2}}{2-m}}{bf}$$

$$\Big\downarrow \; 27$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\Bigg(\frac{b^2\left(\tan^2(e+fx)+1\right)^{-m/2}(a+b\tan(e+fx))^2}{2-m}-\frac{\int (a+b\tan(e+fx))\left(-\left((2-m)a^2\right)-b(4-m)\tan(e+}{2-}}{bf}$$

$$\Big\downarrow \; 676$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\Bigg(\frac{b^2\left(\tan^2(e+fx)+1\right)^{-m/2}(a+b\tan(e+fx))^2}{2-m}-\frac{a(2-m)\left(3b^2-a^2(1-m)\right)\int\left(\tan^2(e+fx)+1\right)^{-\frac{m}{2}-1}d(b\,t}{1-m}}{bf}$$

$$\Big\downarrow \; 237$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\Bigg(\frac{b^2\left(\tan^2(e+fx)+1\right)^{-m/2}(a+b\tan(e+fx))^2}{2-m}-\frac{ab(2-m)\left(3b^2-a^2(1-m)\right)\tan(e+fx)\,\text{Hypergeometric2F}}{1-m}}{bf}$$

input
```
Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

output
```
((d*Cos[e + f*x])^m*(Sec[e + f*x]^2)^(m/2)*((b^2*(a + b*Tan[e + f*x])^2)/(
(2 - m)*(1 + Tan[e + f*x]^2)^(m/2)) - ((a*b*(3*b^2 - a^2*(1 - m))*(2 - m)*
Hypergeometric2F1[1/2, (2 + m)/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(1 -
 m) - (2*b^2*(b^2 - a^2*(3 - m)))/(m*(1 + Tan[e + f*x]^2)^(m/2)) - (a*b^3*
(4 - m)*Tan[e + f*x])/((1 - m)*(1 + Tan[e + f*x]^2)^(m/2)))/(2 - m)))/(b*f
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

Maple [F]

$$\int (d\cos(fx+e))^m (a+b\tan(fx+e))^3\,dx$$

input
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

output
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

Fricas [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx = \int (b\tan(fx+e)+a)^3(d\cos(fx+e))^m\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x +
e) + a^3)*(d*cos(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx = \int (d\cos{(e+fx)})^m\,(a+b\tan{(e+fx)})^3\,dx$$

input
```
integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e))**3,x)
```

output
```
Integral((d*cos(e + f*x))**m*(a + b*tan(e + f*x))**3, x)
```

Maxima [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx = \int (b\tan{(fx+e)}+a)^3(d\cos{(fx+e)})^m\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^3*(d*cos(f*x + e))^m, x)
```

Giac [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx = \int (b\tan{(fx+e)}+a)^3(d\cos{(fx+e)})^m\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^3*(d*cos(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx = \int (d\,\cos\left(e+f\,x\right))^m\,\left(a+b\tan(e+f\,x)\right)^3\,dx$$

input
```
int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^3,x)
```

output
```
int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^3, x)
```

Reduce [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx = d^m\left(\left(\int \cos\left(fx+e\right)^m dx\right)a^3 + \left(\int \cos\left(fx+e\right)^m \tan\left(fx+e\right)^3 dx\right)b^3 + 3\left(\int \cos\left(fx+e\right)^m \tan\left(fx+e\right)^2 dx\right)a\,b^2 + 3\left(\int \cos\left(fx+e\right)^m \tan\left(fx+e\right) dx\right)a^2b\right)$$

input
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

output
```
d**m*(int(cos(e + f*x)**m,x)*a**3 + int(cos(e + f*x)**m*tan(e + f*x)**3,x)
*b**3 + 3*int(cos(e + f*x)**m*tan(e + f*x)**2,x)*a*b**2 + 3*int(cos(e + f*
x)**m*tan(e + f*x),x)*a**2*b)
```

3.705 $\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = -\frac{ab(2-m)(d\cos(e+fx))^m}{f(1-m)m} + \frac{(b^2-a^2(1-m))\cos(e+fx)(d\cos(e+fx))^m \,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sin(e-}{f(1-m)(1+m)\sqrt{\sin^2(e+fx)}} + \frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))}{f(1-m)}$$

output
```
-a*b*(2-m)*(d*cos(f*x+e))^m/f/(1-m)/m+(b^2-a^2*(1-m))*cos(f*x+e)*(d*cos(f*
x+e))^m*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/f/
(1-m)/(1+m)/(sin(f*x+e)^2)^(1/2)+b*(d*cos(f*x+e))^m*(a+b*tan(f*x+e))/f/(1-
m)
```

Mathematica [A] (verified)

Time = 3.48 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.90

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx$$
$$= \frac{(d\cos(e+fx))^m \left(2ab\left(-1+\sec^2(e+fx)^{m/2}\right)+b^2m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{3}{2},-\tan^2(e+fx)\right)\sec^2}{}$$

input
```
Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```

output
```
((d*Cos[e + f*x])^m*(2*a*b*(-1 + (Sec[e + f*x]^2)^(m/2)) + b^2*m*Hypergeom
etric2F1[1/2, m/2, 3/2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*
x] + (a^2 - b^2)*m*Hypergeometric2F1[1/2, (2 + m)/2, 3/2, -Tan[e + f*x]^2]
*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]))/(f*m)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 3998, 3042, 3993, 25, 3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^2(d\cos(e+fx))^m\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a+b\tan(e+fx))^2(d\cos(e+fx))^mdx$$
$$\Big\downarrow \text{3998}$$
$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx))^2dx$$
$$\Big\downarrow \text{3042}$$
$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx))^2dx$$

$$\Big\downarrow \ 3993$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(\frac{\int -(d\sec(e+fx))^{-m}\left(-((1-m)a^2)-b(2-m)\tan(e+fx)a+b^2\right)dx}{1-m}+\frac{b(a+b\tan(e+fx))(d\sec(e$$

$$\Big\downarrow \ 25$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^{-m}}{f(1-m)}-\frac{\int(d\sec(e+fx))^{-m}\left(-((1-m)a^2)-b(2-m)\tan(e+fx)a\right.}{1-m}\right.$$

$$\Big\downarrow \ 3042$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^{-m}}{f(1-m)}-\frac{\int(d\sec(e+fx))^{-m}\left(-((1-m)a^2)-b(2-m)\tan(e+fx)a\right.}{1-m}\right.$$

$$\Big\downarrow \ 3967$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^{-m}}{f(1-m)}-\frac{(b^2-a^2(1-m))\int(d\sec(e+fx))^{-m}dx+\frac{ab(2-m)(d\sec(e+fx))}{fm}}{1-m}\right.$$

$$\Big\downarrow \ 3042$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^{-m}}{f(1-m)}-\frac{(b^2-a^2(1-m))\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-m}dx+\frac{ab(2-m)(d\sec}{fm}}{1-m}\right.$$

$$\Big\downarrow \ 4259$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^{-m}}{f(1-m)}-\frac{(b^2-a^2(1-m))\left(\frac{\cos(e+fx)}{d}\right)^{-m}(d\sec(e+fx))^{-m}\int\left(\frac{\cos(e+f}{d}\right.}{1-m}\right.$$

$$\Big\downarrow \ 3042$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^{-m}}{f(1-m)}-\frac{(b^2-a^2(1-m))\left(\frac{\cos(e+fx)}{d}\right)^{-m}(d\sec(e+fx))^{-m}\int\left(\frac{\sin(e+f}{d}\right.}{1-m}\right.$$

$$\Big\downarrow \ 3122$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^{-m}}{f(1-m)}-\frac{\frac{ab(2-m)(d\sec(e+fx))^{-m}}{fm}-\frac{d(b^2-a^2(1-m))\sin(e+fx)(d\sec(e+fx))^{-m-1}}{f(m+1)\sqrt{\cdots}}}{1-m}\right.$$

input
```
Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```

output
```
(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m*(-(((a*b*(2 - m))/(f*m*(d*Sec[e + f*
x])^m) - (d*(b^2 - a^2*(1 - m))*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/
2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 - m)*Sin[e + f*x])/(f*(1 + m)*Sqrt
[Sin[e + f*x]^2]))/(1 - m)) + (b*(a + b*Tan[e + f*x]))/(f*(1 - m)*(d*Sec[e
 + f*x])^m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int (d\cos (fx+e))^m (a+b\tan (fx+e))^2\,dx$$

input
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

output
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = \int (b\tan (fx+e)+a)^2(d\cos (fx+e))^m\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*cos(f*x + e))^
m, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = \int (d\cos{(e+fx)})^m\,(a+b\tan{(e+fx)})^2\,dx$$

input
```
integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*cos(e + f*x))**m*(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = \int (b\tan{(fx+e)}+a)^2(d\cos{(fx+e)})^m\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^2*(d*cos(f*x + e))^m, x)
```

Giac [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = \int (b\tan{(fx+e)}+a)^2(d\cos{(fx+e)})^m\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^2*(d*cos(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = \int (d\,\cos\left(e+f\,x\right))^m\,\left(a+b\tan(e+f\,x)\right)^2\,dx$$

input
```
int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^2,x)
```

output
```
int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^2, x)
```

Reduce [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = d^m\left(\left(\int \cos\left(fx+e\right)^m dx\right)a^2 + \left(\int \cos\left(fx+e\right)^m \tan\left(fx+e\right)^2 dx\right)b^2 + 2\left(\int \cos\left(fx+e\right)^m \tan\left(fx+e\right) dx\right)ab\right)$$

input
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

output
```
d**m*(int(cos(e + f*x)**m,x)*a**2 + int(cos(e + f*x)**m*tan(e + f*x)**2,x)
*b**2 + 2*int(cos(e + f*x)**m*tan(e + f*x),x)*a*b)
```

3.706 $\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = -\frac{b(d\cos(e+fx))^m}{fm} - \frac{a(d\cos(e+fx))^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sin(e+fx)}{df(1+m)\sqrt{\sin^2(e+fx)}}$$

output
```
-b*(d*cos(f*x+e))^m/f/m-a*(d*cos(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m],
[3/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/d/f/(1+m)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.83

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = -\frac{(d\cos(e+fx))^m\left(b+bm+am\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sqrt{\sin^2(e\right.}{fm(1+m)}$$

input
```
Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]),x]
```

output
```
-(((d*Cos[e + f*x])^m*(b + b*m + a*m*Cot[e + f*x]*Hypergeometric2F1[1/2, (
1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2]))/(f*m*(1 + m)))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3998, 3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))(d\cos(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))(d\cos(e+fx))^m dx$$

$$\Big\downarrow \text{3998}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx))dx$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx))dx$$

$$\Big\downarrow \text{3967}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(a\int (d\sec(e+fx))^{-m}dx-\frac{b(d\sec(e+fx))^{-m}}{fm}\right)$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(a\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-m}dx-\frac{b(d\sec(e+fx))^{-m}}{fm}\right)$$

$$\Big\downarrow \text{4259}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(a\left(\frac{\cos(e+fx)}{d}\right)^{-m}(d\sec(e+fx))^{-m}\int\left(\frac{\cos(e+fx)}{d}\right)^m dx-\frac{b(d\sec(e+fx))^{-m}}{fm}\right)$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(a\left(\frac{\cos(e+fx)}{d}\right)^{-m}(d\sec(e+fx))^{-m}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^m dx-\frac{b(d\sec(e+fx))^{-m}}{fm}\right)$$

$$\Big\downarrow \text{3122}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(-\frac{ad\sin(e+fx)(d\sec(e+fx))^{-m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx)\right)}{f(m+1)\sqrt{\sin^2(e+fx)}}-\frac{b(d\sec(e+fx))^{-m}}{fm}\right)$$

input
```
Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]),x]
```

output
```
(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m*(-(b/(f*m*(d*Sec[e + f*x])^m)) - (a*
d*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(d*Sec[e +
f*x])^(-1 - m)*Sin[e + f*x])/(f*(1 + m)*Sqrt[Sin[e + f*x]^2]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int (d\cos(fx+e))^m (a+b\tan(fx+e))\,dx$$

input
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

output
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

Fricas [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx=\int (b\tan(fx+e)+a)(d\cos(fx+e))^m\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e) + a)*(d*cos(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = \int (d\cos{(e+fx)})^m\,(a+b\tan{(e+fx)})\,\,dx$$

input
```
integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*cos(e + f*x))**m*(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = \int (b\tan{(fx+e)}+a)(d\cos{(fx+e)})^m\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)*(d*cos(f*x + e))^m, x)
```

Giac [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = \int (b\tan{(fx+e)}+a)(d\cos{(fx+e)})^m\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)*(d*cos(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx=\int (d\,\cos\left(e+f\,x\right))^m\,\left(a+b\tan(e+f\,x)\right)\,dx$$

input
```
int((d*cos(e + f*x))^m*(a + b*tan(e + f*x)),x)
```

output
```
int((d*cos(e + f*x))^m*(a + b*tan(e + f*x)), x)
```

Reduce [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx=d^m\left(\left(\int \cos\left(fx+e\right)^m dx\right)a+\left(\int \cos\left(fx+e\right)^m \tan\left(fx+e\right)dx\right)b\right)$$

input
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

output
```
d**m*(int(cos(e + f*x)**m,x)*a + int(cos(e + f*x)**m*tan(e + f*x),x)*b)
```

3.707 $\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx = \frac{b(d\cos(e+fx))^m \operatorname{Hypergeometric2F1}\left(1,-\frac{m}{2},1-\frac{m}{2},\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{(a^2+b^2)\,fm} + \frac{\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{2+m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\tan(e+fx)}{af}$$

output
```
b*(d*cos(f*x+e))^m*hypergeom([1, -1/2*m],[1-1/2*m],b^2*sec(f*x+e)^2/(a^2+b
^2))/(a^2+b^2)/f/m+AppellF1(1/2,1,1+1/2*m,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*
x+e)^2)*(d*cos(f*x+e))^m*(sec(f*x+e)^2)^(1/2*m)*tan(f*x+e)/a/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.40 (sec) , antiderivative size = 869, normalized size of antiderivative = 6.21

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx$$

$$= \frac{}{fm\left(2a^3\cos^2(e+fx)+2a^3m\cos^2(e+fx)-2ab^2\sin^2(e+fx)-2ab^2m\sin^2(e+fx)+a^2b\sin(2(e+f\right.}$$

input
```
Integrate[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

output
```
(2*(1 + m)*Cos[e + f*x]*(d*Cos[e + f*x])^m*(a*Cos[e + f*x] + b*Sin[e + f*x
])*(b - b*(Sec[e + f*x]^2)^(m/2) + a*m*Hypergeometric2F1[1/2, 1 + m/2, 3/2
, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x] - b*AppellF1[m, m/2
, m/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x
])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e +
f*x]))/(a + b*Tan[e + f*x]))^(m/2)))/(f*m*(2*a^3*Cos[e + f*x]^2 + 2*a^3*m*
Cos[e + f*x]^2 - 2*a*b^2*Sin[e + f*x]^2 - 2*a*b^2*m*Sin[e + f*x]^2 + a^2*b
*Sin[2*(e + f*x)] + a^2*b*m*Sin[2*(e + f*x)] - 2*b^3*Sin[e + f*x]^2*Tan[e
+ f*x] - 2*b^3*m*Sin[e + f*x]^2*Tan[e + f*x] + (a - I*b)*b^2*m*AppellF1[1
+ m, 1 + m/2, m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b
*Tan[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(
I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2) + a*b^2*m*AppellF1[1 + m, m
/2, 1 + m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e
 + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Ta
n[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2) + I*b^3*m*AppellF1[1 + m, m/2, 1
+ m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x
])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e +
f*x]))/(a + b*Tan[e + f*x]))^(m/2) + 2*b^2*(1 + m)*AppellF1[m, m/2, m/2, 1
 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*((b*
(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x])...
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 3998, 3042, 3994, 504, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\cos (e+fx))^m}{a+b\tan (e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos (e+fx))^m}{a+b\tan (e+fx)}dx$$

$$\Big\downarrow \text{3998}$$

$$(d\cos (e+fx))^m (d\sec (e+fx))^m \int \frac{(d\sec (e+fx))^{-m}}{a+b\tan (e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$(d\cos (e+fx))^m (d\sec (e+fx))^m \int \frac{(d\sec (e+fx))^{-m}}{a+b\tan (e+fx)}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sec ^2(e+fx)^{m/2} (d\cos (e+fx))^m \int \frac{\left(\tan ^2(e+fx)+1\right)^{-\frac{m}{2}-1}}{a+b\tan (e+fx)}d(b\tan (e+fx))}{bf}$$

$$\Big\downarrow \text{504}$$

$$\frac{\sec ^2(e+fx)^{m/2} (d\cos (e+fx))^m \left(a\int \frac{\left(\tan ^2(e+fx)+1\right)^{-\frac{m}{2}-1}}{a^2-b^2\tan ^2(e+fx)}d(b\tan (e+fx))-\int \frac{b\tan (e+fx)\left(\tan ^2(e+fx)+1\right)^{-\frac{m}{2}-1}}{a^2-b^2\tan ^2(e+fx)}d(b\tan$$

$$\Big\downarrow \text{333}$$

$$\frac{\sec ^2(e+fx)^{m/2} (d\cos (e+fx))^m \left(\frac{b\tan (e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{m+2}{2},\frac{3}{2},\frac{b^2\tan ^2(e+fx)}{a^2},-\tan ^2(e+fx)\right)}{a}-\int \frac{b\tan (e+fx)\left(\tan ^2(e+fx)\right.}{a^2-b^2\tan ^2(e+f}\right.}{bf}$$

$$\Big\downarrow \text{353}$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\left(\frac{b\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1,\frac{m+2}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\frac{1}{2}\int\frac{\left(\frac{\tan(e+fx)}{b}+1\right)^{-\frac{m}{2}-1}}{a^2-b^2\tan^2(e+fx)}\right.}{bf}$$

$$\Big\downarrow \text{78}$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\left(\frac{b\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1,\frac{m+2}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}+\frac{b^2\left(\frac{\tan(e+fx)}{b}+1\right)^{-m/2}\text{Hy}}{}\right.}{bf}$$

input
```
Int[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

output
```
((d*Cos[e + f*x])^m*(Sec[e + f*x]^2)^(m/2)*((b*AppellF1[1/2, 1, (2 + m)/2,
 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a + (b^2*Hy
pergeometric2F1[1, -1/2*m, 1 - m/2, (b^2 + b^2*Tan[e + f*x]^2)/(a^2 + b^2)
])/((a^2 + b^2)*m*(1 + Tan[e + f*x]/b)^(m/2))))/(b*f)
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(d\cos(fx+e))^m}{a+b\tan(fx+e)}dx$$

input
```
int((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

output
```
int((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx=\int \frac{(d\cos(fx+e))^m}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*cos(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx=\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx$$

input
```
integrate((d*cos(f*x+e))**m/(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*cos(e + f*x))**m/(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx=\int \frac{(d\cos(fx+e))^m}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\cos(fx+e))^m}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\cos(e+f\,x))^m}{a+b\tan(e+f\,x)}\,dx$$

input
```
int((d*cos(e + f*x))^m/(a + b*tan(e + f*x)),x)
```

output
```
int((d*cos(e + f*x))^m/(a + b*tan(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx = d^m\left(\int \frac{\cos(fx+e)^m}{\tan(fx+e)\,b+a}dx\right)$$

input
```
int((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

output
```
d**m*int(cos(e + f*x)**m/(tan(e + f*x)*b + a),x)
```

3.708 $\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 227

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \frac{2ab(d\cos(e+fx))^m \operatorname{Hypergeometric2F1}\left(2,-\frac{m}{2},1-\frac{m}{2},\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{(a^2+b^2)^2 fm} + \frac{\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{2+m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\cos(e+fx))^m \sec^2(e+fx)^{m/2}\tan(e+fx)}{a^2 f} + \frac{b^2\operatorname{AppellF1}\left(\frac{3}{2},2,\frac{2+m}{2},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\cos(e+fx))^m \sec^2(e+fx)^{m/2}\tan^3(e+fx)}{3a^4 f}$$

output
```
2*a*b*(d*cos(f*x+e))^m*hypergeom([2, -1/2*m],[1-1/2*m],b^2*sec(f*x+e)^2/(a
^2+b^2))/(a^2+b^2)^2/f/m+AppellF1(1/2,2,1+1/2*m,3/2,b^2*tan(f*x+e)^2/a^2,-
tan(f*x+e)^2)*(d*cos(f*x+e))^m*(sec(f*x+e)^2)^(1/2*m)*tan(f*x+e)/a^2/f+1/3
*b^2*AppellF1(3/2,2,1+1/2*m,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(d*cos
(f*x+e))^m*(sec(f*x+e)^2)^(1/2*m)*tan(f*x+e)^3/a^4/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 15.68 (sec) , antiderivative size = 2502, normalized size of antiderivative = 11.02

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```

output
```
((d*Cos[e + f*x])^m*((2*a*b*(-1 + (Sec[e + f*x]^2)^(-1/2*m)))/m + a^2*Hype
rgeometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] - b^2*Hyper
geometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] - (2*a*b*App
ellF1[m, m/2, m/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b
*Tan[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(
I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))/(m*(Sec[e + f*x]^2)^(m/2))
 - (b*(a^2 + b^2)*AppellF1[1 + m, m/2, m/2, 2 + m, (a - I*b)/(a + b*Tan[e
+ f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*T
an[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))/(
(1 + m)*(Sec[e + f*x]^2)^(m/2)*(a + b*Tan[e + f*x]))))/(f*(a + b*Tan[e + f
*x])^2*(a^2*Hypergeometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Sec[e +
f*x]^2 - b^2*Hypergeometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Sec[e +
 f*x]^2 - (2*a*b*Tan[e + f*x])/(Sec[e + f*x]^2)^(m/2) + (2*a*b*AppellF1[m,
 m/2, m/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e +
 f*x])]*Tan[e + f*x]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*
((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))/(Sec[e + f*x]^2)^(m/2
) + (b^2*(a^2 + b^2)*AppellF1[1 + m, m/2, m/2, 2 + m, (a - I*b)/(a + b*Tan
[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(1 - m/2)*((b
*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/
(a + b*Tan[e + f*x]))^(m/2))/((1 + m)*(a + b*Tan[e + f*x])^2) + (b*(a^2...
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3998, 3042, 3994, 505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3998}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int \frac{(d\sec(e+fx))^{-m}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int \frac{(d\sec(e+fx))^{-m}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\int \frac{(\tan^2(e+fx)+1)^{-\frac{m}{2}-1}}{(a+b\tan(e+fx))^2}d(b\tan(e+fx))}{bf}$$

$$\Big\downarrow \text{505}$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\int \left(\frac{a^2(\tan^2(e+fx)+1)^{-\frac{m}{2}-1}}{(a^2-b^2\tan^2(e+fx))^2}-\frac{2ab\tan(e+fx)(\tan^2(e+fx)+1)^{-\frac{m}{2}-1}}{(a^2-b^2\tan^2(e+fx))^2}+\frac{b^2\tan^2(e+fx)(\tan^2(e}{(b^2\tan^2(e+f}\right.}{bf}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{m+2}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2}+\frac{2ab^2(\tan^2(e+fx)+1)^{-m/2}}{}\right.}{bf}$$

input
```
Int[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```

output
```
((d*Cos[e + f*x])^m*(Sec[e + f*x]^2)^(m/2)*((b*AppellF1[1/2, 2, (2 + m)/2,
 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a^2 + (b^3*
AppellF1[3/2, 2, (2 + m)/2, 5/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2
]*Tan[e + f*x]^3)/(3*a^4) + (2*a*b^2*Hypergeometric2F1[2, -1/2*m, 1 - m/2,
 (b^2 + b^2*Tan[e + f*x]^2)/(a^2 + b^2)])/((a^2 + b^2)^2*m*(1 + Tan[e + f*
x]^2)^(m/2))))/(b*f)
```

Defintions of rubi rules used

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(d\cos(fx+e))^m}{(a+b\tan(fx+e))^2}dx$$

input
```
int((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
```

output
```
int((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\cos(fx+e))^m}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((d*cos(f*x + e))^m/(b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2
), x)
```

Sympy [F]

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$$

input
```
integrate((d*cos(f*x+e))**m/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*cos(e + f*x))**m/(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\cos(fx+e))^m}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\cos(fx+e))^m}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\,\cos(e+f\,x))^m}{(a+b\tan(e+f\,x))^2}\,dx$$

input
```
int((d*cos(e + f*x))^m/(a + b*tan(e + f*x))^2,x)
```

output
```
int((d*cos(e + f*x))^m/(a + b*tan(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = d^m\left(\int \frac{\cos(fx+e)^m}{\tan(fx+e)^2 b^2 + 2\tan(fx+e)\,ab + a^2}dx\right)$$

input
```
int((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
```

output
```
d**m*int(cos(e + f*x)**m/(tan(e + f*x)**2*b**2 + 2*tan(e + f*x)*a*b + a**2
),x)
```

3.709 $\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx$$
$$=\frac{b\,\mathrm{AppellF1}\left(1+n,\frac{2+m}{2},\frac{2+m}{2},2+n,\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}},\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)(d\cos(e+fx))^m(a+b\tan(e+fx))^{1+n}\,(}{(a^2+b^2)\,f(1+n)}$$

output
```
b*AppellF1(1+n,1+1/2*m,1+1/2*m,2+n,(a+b*tan(f*x+e))/(a-(-b^2)^(1/2)),(a+b*
tan(f*x+e))/(a+(-b^2)^(1/2)))*(d*cos(f*x+e))^m*(a+b*tan(f*x+e))^(1+n)*(1-(
a+b*tan(f*x+e))/(a-(-b^2)^(1/2)))^(1/2*m)*(1-(a+b*tan(f*x+e))/(a+(-b^2)^(1
/2)))^(1/2*m)/(a^2+b^2)/f/(1+n)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.04 (sec) , antiderivative size = 698, normalized size of antiderivative = 3.84

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx$$
$$=\frac{}{f\left(2b\,\mathrm{AppellF1}\left(1+n,1+\frac{m}{2},1+\frac{m}{2},2+n,\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)\sec^2(e+fx)+2n\,\mathrm{AppellF1}\left(1+\right.\right.}$$

input
```
Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

output
```
(2*AppellF1[1 + n, 1 + m/2, 1 + m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b)
, (a + b*Tan[e + f*x])/(a + I*b)]*(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^
(1 + n))/(f*(2*b*AppellF1[1 + n, 1 + m/2, 1 + m/2, 2 + n, (a + b*Tan[e + f
*x])/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2 + 2*n*Appel
lF1[1 + n, 1 + m/2, 1 + m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b
*Tan[e + f*x])/(a + I*b)]*(b - a*Tan[e + f*x]) + (b*(2 + m)*(1 + n)*((a -
I*b)*AppellF1[2 + n, 1 + m/2, 2 + m/2, 3 + n, (a + b*Tan[e + f*x])/(a - I*
b), (a + b*Tan[e + f*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 2 + m/2, 1
 + m/2, 3 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[e + f*x])/(a + I
*b)])*Sec[e + f*x]^2*(a + b*Tan[e + f*x]))/((a - I*b)*(a + I*b)*(2 + n)) +
 2*(-m + n)*AppellF1[1 + n, 1 + m/2, 1 + m/2, 2 + n, (a + b*Tan[e + f*x])/
(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Tan[e + f*x]*(a + b*Tan[e + f*x
]) + (m*AppellF1[1 + n, 1 + m/2, 1 + m/2, 2 + n, (a + b*Tan[e + f*x])/(a -
 I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2*(a + b*Tan[e + f*x])
)/(-I + Tan[e + f*x]) + (m*AppellF1[1 + n, 1 + m/2, 1 + m/2, 2 + n, (a + b
*Tan[e + f*x])/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2*(
a + b*Tan[e + f*x]))/(I + Tan[e + f*x])))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3998, 3042, 3995, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n dx$$

$$\Big\downarrow \text{3998}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx))^n dx$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int(d\sec(e+fx))^{-m}(a+b\tan(e+fx))^ndx$$

$$\Big\downarrow \text{3995}$$

$$\frac{b(d\cos(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{m/2}\left(1-\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)^{m/2}\int(a+b\tan(e+fx))^n\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{-\frac{m}{2}-1}}{f\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{150}$$

$$\frac{b(d\cos(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{m/2}\left(1-\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)^{m/2}(a+b\tan(e+fx))^{n+1}\operatorname{AppellF1}\left(n+1,\frac{m+2}{2},\right.}{f(n+1)\left(a^2+b^2\right)}$$

input
```
Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

output
```
(b*AppellF1[1 + n, (2 + m)/2, (2 + m)/2, 2 + n, (a + b*Tan[e + f*x])/(a -
Sqrt[-b^2]), (a + b*Tan[e + f*x])/(a + Sqrt[-b^2])]*(d*Cos[e + f*x])^m*(a
+ b*Tan[e + f*x])^(1 + n)*(1 - (a + b*Tan[e + f*x])/(a - Sqrt[-b^2]))^(m/2
)*(1 - (a + b*Tan[e + f*x])/(a + Sqrt[-b^2]))^(m/2))/((a^2 + b^2)*f*(1 + n
))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3995
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*(a^2 + b^2)^(IntPart[m/2]
- 1)*((d*Sec[e + f*x])^(2*FracPart[m/2])/(f*b^(2*IntPart[m/2] - 1)*(1 - (a
+ b*Tan[e + f*x])/(a - Rt[-b^2, 2]))^FracPart[m/2]*(1 - (a + b*Tan[e + f*x]
)/(a + Rt[-b^2, 2]))^FracPart[m/2]))   Subst[Int[x^n*(1 - x/(a - Rt[-b^2, 2
]))^(m/2 - 1)*(1 - x/(a + Rt[-b^2, 2]))^(m/2 - 1), x], x, a + b*Tan[e + f*x
]], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 + b^2, 0] &&  !Integer
Q[m] &&  !IntegerQ[n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

Maple [F]

$$\int (d\cos(fx+e))^m (a+b\tan(fx+e))^n \, dx$$

input
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

output
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n \, dx = \int (d\cos(fx+e))^m (b\tan(fx+e)+a)^n \, dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*cos(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx = \int (d\cos(e+fx))^m\,(a+b\tan(e+fx))^n\,dx$$

input
```
integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e))**n,x)
```

output
```
Integral((d*cos(e + f*x))**m*(a + b*tan(e + f*x))**n, x)
```

Maxima [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx = \int (d\cos(fx+e))^m\,(b\tan(fx+e)+a)^n\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*cos(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Giac [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx = \int (d\cos(fx+e))^m\,(b\tan(fx+e)+a)^n\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*cos(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx = \int (d\,\cos\left(e+f\,x\right))^m\,\left(a+b\tan(e+f\,x)\right)^n\,dx$$

input
```
int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^n,x)
```

output
```
int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^n, x)
```

Reduce [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx = d^m\left(\int (\tan\left(fx+e\right)b+a)^n\cos\left(fx+e\right)^m dx\right)$$

input
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

output
```
d**m*int((tan(e + f*x)*b + a)**n*cos(e + f*x)**m,x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                   ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```python
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
       return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
       if expn.parent() is SR:
          return expn.operator() is None
       if expn.parent() in (ZZ, QQ, AA, QQbar):
          return expn in expn.parent() # Should always return True
       if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
          return expn in expn.parent().base_ring() or expn in expn.parent().gens()

       return False

    except AttributeError as error:
       print("Exception,AttributeError in  is_atom")
       print ("cought exception" , type(error).__name__ )
       return False


def expnType(expn):

    if debug:
       print (">>>>>Enter expnType, expn=", expn)
       print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
       return 1
    elif type(expn)==list:   #isinstance(expn,list):
       return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
       if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
          return 1
       else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
       if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file